Fifty shades of greywater: an outcome perspective on wastewater system operations’ performance

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Abstract

Tightening of regulation on the quality of water and wastewater services often leads to the growth of capital expenditure in industry and higher bills for water customers. Applying an outcome-orientated approach to the economic regulation of the sector in England and Wales, shifts the focus away from how many assets a business has, towards whether a company is capable of delivering through its asset base, what ultimately matters to their customers. Although this approach sounds agreeable and almost common sense, its translation into metrics is a challenge with industry requiring a better understanding of the behavioural implications of a regulatory system built around outcomes. This paper contributes to the current debate on how to best address industry’s concerns, whilst meeting the regulator’s expectations. A way to reconcile performance measurement with a genuine outcome orientation by means of Input-Output Analysis (IOA) is outlined. IOA is identified as a suitable tool to manipulate the empirical data commonly available in industry for the purpose of tackling technical planning, environmental evaluation and costing problems for a multi-stage, multi-input, multi-output (and possibly multi-location) supply network. A simplified wastewater system underpinned by real-life industry data is used throughout the paper for illustrative purposes.

Keywords: wastewater system; supply network; performance; modelling; Input-Output Analysis (IOA)

Introduction

In Europe the tightening of regulation on the quality of water and wastewater services has driven growth of capital expenditures on additional treatment works which, in turn, has almost unavoidably resulted in higher bills for the final customers (ICIS Chemical Business, 2005; Wessex Water Services Plc, 2012). In England and Wales an independent economic regulator, Ofwat (henceforth: the regulator) sets binding price controls for each company that provides water and wastewater services over a multi-year period. This is known as a price review. Controls typically link the allowable revenues to what are deemed efficient levels of capital, as well as operational expenditures, adjusted by a reward and penalty mechanism. For the period 2015-20 the allowed expenditure across 18 wholesale water and wastewater businesses amounts to £40.4 billion (OFWAT, 2014b, pp. A3.4).
The current price review has sanctioned the adoption of an innovative, outcome-orientated approach to the economic regulation of the water and wastewater service business. The approach shifts the focus from how many assets a business has to whether it is capable of delivering through its asset base what ultimately matters to its customers (OFWAT, 2014c). Businesses set measures referred to as ‘performance commitments’ (PCs) to demonstrate how well a set of outcomes is delivered over the price control period. Outcome delivery incentives (ODIs) of both reputational and financial natures, reward the ability and penalise the inability of businesses to meet their commitments.

Although the idea of outcomes defined as ‘what ultimately matters to the customer’ sounds agreeable and almost common sense, its translation into metrics is a challenge. Industry recognise that the quantification and monitoring of outcome-based performance measures is not straightforward, and demands a better understanding of the behavioural implications of a regulatory system built around outcomes (ICS consulting, 2015).

This paper aims to speak to the current debate on how to best address industry’s concerns whilst meeting the regulator’s expectations. An approach to reconcile performance measurement with a genuine outcome orientation for use in the water and wastewater service industry is outlined. This is achieved through the use of a two-stage model. At the conceptual stage a blueprint of the delivery system of interest is created and the technological knowledge about the relevant operations involved is mapped. At the quantitative level empirical data commonly available in industry is identified and manipulated to evaluate how the delivery system has performed, or is expected to perform in achieving its ultimate purpose(s) – or outcome(s). Input-Output Analysis (IOA) is employed as a suitable tool to manipulate empirical data about a multi-stage, multi-input, multi-output (and possibly multi-location) supply network in a mathematically rigorous, yet practical way.

The remainder of this paper is as follows. Basic concepts such as input, output and outcomes are investigated. This is achieved using evidence from the servitization of business in other industries, chiefly defence aerospace as well as existing frameworks on performance metrics. An application of IOA is outlined to reconcile performance management with a genuine outcome orientation making reference to a simplified wastewater system underpinned by industry data. The paper closes with a discussion of practical recommendations for the water and wastewater businesses, current limitations, and suggestions for further research.

**Input, output, outcome orientation: evidence from the literature**

As mentioned in the introduction, the distinctive trait of the regulatory approach adopted in the water and wastewater sector in England and Wales is an orientation towards ‘outcomes’. Instead of setting targets in terms of ‘length of pipe built’, so to speak, the focus is on what customers ultimately want companies to deliver. Outcome orientation is not unfamiliar in other industrial settings. In defence aerospace the outcome orientation underpins the shift from a business centered on selling ‘asset and support’ to one aiming to deliver value ‘in-use’ by enabling customers to attain beneficial service outcomes through incentivized contractual mechanisms known as performance-based contracts—PBC (Baines and Lightfoot, 2013). The challenge in PBC is to identify and agree on contractually binding outcome metrics which capture what customers ultimately derive value from (Selviaridis and Wynstra, 2014).

Doost (1996) defines ‘outcome’ as some level of accomplishment of the enterprise. Therefore, the distinction between output and outcome depends on where the boundaries of the analysis are drawn. For example, a manufacturing department in a company may be given
credit for producing more units of a product than anticipated whilst cutting on departmental expenditures. However, such push on productivity is detrimental from an outcome perspective if the result is poor quality and excess inventory of unsold or returned items. By contrast, an input orientation focuses on how much has been spent over a period of time (e.g., calendar year) on categories such as labour, goods, services, and capital disregard what has been accomplished as a result of that spending. An input orientation characterises the public sector because in National Accounts it is a convention to equate the sector’s output to the inputs used up in producing such output (Anagboso and Spence, 2009).

The basic ‘input-transformation-output’ structure is at the core of generic operations models regardless of which specific industry they refer to (Waller, 2003). Transformation may occur in a multitude of steps and locations. For an organization it can be difficult to express quantitatively what the ultimate deliverable downstream from the outputs of each transformation is. For example, the defence sector’s outcome should ideally consist of rather vague ‘units of security’ or ‘units of peace and stability’ (Anagboso and Spence, 2009). How one conceptually frames the ultimate deliverable determines the metrics and models chosen.

Neely et al (2000) review several performance measurement frameworks only one of which explicitly differentiates between input, process, output and outcome measures. Later reviews tend to ignore this distinction despite a focus on the supply chain (e.g., Elrod et al., 2015). Applications of the ‘input-transformation-output-outcome’ structure include the performance evaluation of logistic processes (Stainer, 1997), healthcare systems (WHO, 2010), defence supply chains (Klapper et al., 1999), and service-based production processes (Yalley and Sekhon, 2014). Although the difference between inputs, output and outcomes is recognised, the firm is often addressed aggregately as a single transformation stage.

In the water and wastewater service sector similar concepts of input, output and outcome are used (OFWAT, 2011). However, the practical implementation of these concepts through metrics is shaped by the different viewpoints involved (Figure 1). Figure 1(a) depicts the ideal relationship between outcomes, PCs and ODIs emphasizing the centrality of outcomes. If the viewpoint of the regulator is taken (Figure 1(b)) the focus is placed on the allowable expenditure for each business during the price review period. The PCs of a company are taken into account insofar as they trigger adjustments in the allowable expenditure (OFWAT, 2014c). Hence, the relevant metrics are the explanatory variables included in the econometric models used by the regulator to estimate company-specific expenditures. Conversely, the perspective taken by industry (Figure 1(c)) starts from a situation where a company failing to meet its PCs is concerned with identifying possible causes of non-compliance, and favours the use of reliability engineering techniques (Tynemarch Systems, 2015).

Identifying and measuring only adverse outcomes such as PCs failures can be counterproductive, as demonstrated in the field of aviation safety (Hollnagel et al., 2013). More important is to investigate how the work within a delivery system is continually adjusted to succeed under varying conditions in everyday activities. A key idea in operations management is that performance is attained through the actions a business undertakes, and that performance measurement is ultimately the process of quantifying such actions (Neely et al., 2005). Activities are, by definition, purposeful because they are performed to contribute towards the realization of one or more outcomes (BS ISO/IEC, 2002). Hence, a performance measure which relates to an outcome should provide an assessment of the result that occurs from carrying out a set of activities compared to their intended purpose (Klapper et al., 1999).
Figure 1 – Perspectives on the outcome approach in the water and wastewater industry regulation: (a) ideal view; (b) regulator’s view; and (c) industry’s view
Making reference to the water and wastewater industry, Figure 2 contrasts a ‘cause-effect’ outlook centered on what caused failure to meet a PC with a ‘means-ends’ outlook centered on what enables the achievement of a PC. The former is triggered by one-off non-compliance and proceeds backwards with the purpose of eliminating its causes. The latter is forward-looking because it brings to the fore the purposefulness of everyday activities, and drives attention on the aspects of those activities that may increase the chances of achieving some outcome of interest. In the process industry, a means-ends outlook underpins successful approaches to integrate material, energy and cost flow analysis (Möller, 2010).

**Proposed approach**

In this section, the ‘input-transformation-output-outcome’ structure described earlier is used as the building block for a two-stage model of a system of operations enabling an integrated evaluation of technical, environmental and economic performance. The model encompasses:

- Visualization and conceptual modelling of a system of operations
- Mathematical manipulation of quantitative data about a system of operations.

The system of operations of interest here is a wastewater system, which is a “system providing the functions of collection, transport, treatment and discharge of wastewater” (BS, 2014). As most real-life systems of operations, a wastewater system consists of multiple interdependent transformation stages whereby each activity may be characterized by multiple inputs and multiple outputs (MIMO), including byproducts, that potentially contribute towards multiple and often conflicting outcomes. A technical overview of wastewater systems’ operations is beyond the scope of this work and can be found elsewhere (Mihelcic and Zimmerman, 2010; Spellman, 2003).

Each modelling stage is illustrated next, making reference to a hypothetical wastewater system underpinned by a real industrial case.
**Visualisation and conceptual modelling**

For modelling purposes it is often appropriate to start with a top-level conceptual representation of the subject matter. This allows specifying the boundaries and scope of the analysis as well as the perspective taken by the analyst before going into further detail by pursuing empirical data to add to the picture. For a wastewater system such a top-level view is shown in Figure 3. Following the IDEF0 conceptual modelling language a ‘single-box’ diagram called context diagram is used (National Institute of Standards and Technology, 1993). The context diagram is detailed later on through child diagrams to capture a snapshot of the wastewater system’s configuration with reference to a specific period of time. Both the context diagram and the child diagrams use the building blocks summarised in Table 1.

The outputs of a context diagram are the system’s outcomes because they represent final deliverables meeting a demand which is exogenous with respect to the system’s boundaries. The outcomes shown in Figure 3 answer the question ‘which exogenous demand are the wastewater system’s operations meant to meet?’ For wastewater systems the answer to this question is likely to change over time. For example, the generation of electricity from wastewater biomass can be viewed as either a mean of reducing the dependency of wastewater treatment works (WwTWs) on the national power grid, or as an opportunity to manage those plants as if they were power generation plants (Logan, 2005).

The wastewater system depicted aggregately in Figure 3 may correspond to the entire portfolio of a company’s WwTWs, an individual WwTW, or a network of WwTWs operating within the boundaries of a geographical area. Often it is deemed appropriate to define an area at the river catchment level for the purpose of assessing and managing more effectively the contribution of multiple WwTWs to the eutrophication of surface watercourses (Wessex Water Services Plc, 2012). These different levels are illustrated in Figure 4(a) for an industrial case (with sensible information omitted).
A context diagram can be used to categorise the data reported by businesses regarding their PCs. Using a company’s publicly available data (Wessex Water Services Plc, 2014) Table 2 reveals that the PCs for its wholesale wastewater line of business mostly focus on controls (e.g., regulatory constraints) and mechanisms (e.g., its asset-base). This demonstrates that distinguishing between genuine outcomes and regulatory constraints can be difficult. By contrast, details about the inputs and outputs of a wastewater system’s operations are required by other types of regulatory reporting (OFWAT, 2014a).

From a context diagram the analyst progressively develops a blueprint of the main functions that constitute the wastewater system of interest and the interrelationships between such functions as child diagrams. Figure 4(b) shows the first child diagram, called ‘A0’ diagram, derived from the context diagram in Figure 4(a). One way of looking at Figure 4(b) is to follow the influents and effluents (blue-coloured arrows) from wastewater collection through different stages of treatment (aggregately represented by box 1). By-products (red-coloured

### Table 1 – Building blocks of an IDEF0 conceptual model

<table>
<thead>
<tr>
<th>Building block</th>
<th>Pictorial representation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Functions</strong></td>
<td>Boxes</td>
<td>Purposeful transformations - neither a specific organisational unit, nor a piece of equipment. May correspond to atomic operations or aggregated processes, or the whole subject matter of interest depending on the level of granularity chosen.</td>
</tr>
<tr>
<td><strong>Inputs</strong></td>
<td>Arrows pointing towards a box</td>
<td>What is being acted upon to produce an effect by executing a function. Can be acquired from other economic delivery systems through market transactions, or provided freely by the natural environment.</td>
</tr>
<tr>
<td><strong>Outputs</strong></td>
<td>Arrows directed from a box to another</td>
<td>What is meant to be accomplished performing a transformation. Emissions into environmental media and other by-products of a transformation also qualify as outputs.</td>
</tr>
<tr>
<td><strong>Mechanisms</strong></td>
<td>Arrows pointing towards a box from below</td>
<td>The ‘operant’ resources employed to act upon the inputs for the transformation to take place. Physical assets and human resources typically fall into this category.</td>
</tr>
<tr>
<td><strong>Controls</strong></td>
<td>Arrows pointing towards a box from above</td>
<td>Specify under which circumstances a transformation is meant to take place for the intended results to be achieved. Examples include compliance with a specific regulation, the occurrence of physical conditions, or the availability of certain equipment.</td>
</tr>
</tbody>
</table>

### Table 2 Reclassification of PCs for a wastewater wholesale business line according to IDEF0

<table>
<thead>
<tr>
<th>Outcome category</th>
<th>Performance Commitment (PC)</th>
<th>Input</th>
<th>Output</th>
<th>Mechanism</th>
<th>Controls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improved bathing waters</td>
<td>Agreed schemes delivered</td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>Beaches passing EU standards</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EA’s rating</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rivers, lakes and estuaries protected</td>
<td>Monitored CSO’s</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td></td>
<td>River water quality improved</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Sewage flooding minimised</td>
<td>Internal flooding incidents</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Risk of flooding from public sewers due to hydraulic inadequacy</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td></td>
<td>North Bristol sewer scheme</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resilient services</td>
<td>Collapses and bursts on sewerage network</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reduced carbon footprint</td>
<td>Greenhouse gas emissions</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Proportion of energy self-generated</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

Notes: CSO = combine sewage overflows; EA = environmental agency
arrows) such as sludge flow from the functions generating them to the functions downstream (represented aggregately by boxes 2 and 3) which are in charge of treating the effluent using different technologies E.g. biologically, while pollutants (green-coloured arrows) are released directly into environmental media. The labels ‘A1’ and ‘A2’ underneath boxes 1 and 2 indicate that child diagrams exist for those functions, although not shown here. The input flow of by-products to the treatment functions equivalently expresses the output flow of ‘treatment service’ provided. In Figure 4(b) treatment services are represented as black arrows similarly to the goods and services purchased.
A ‘service’ perspective on wastewater treatment requires answering, starting from the top-level outcomes shown in the context diagram, the question ‘which means are needed to pursue this end?’ rather than ‘how much influent goes into this piece of equipment?’ Thanks to a ‘service’ perspective it is possible to represent functions such as box 3 in Figure 4(b), which are seldom recognised as part of wastewater operations modelling but are responsible for an asset base being in such a state that the physical flow is as depicted by the analyst.

The analyst’s subjective viewpoint shapes what can be seen in a conceptual model so outlined, and so does that of each individual taking part to the construction of the model. Hence, the usefulness of such a model resides in its ability to provide a baseline communication vehicle underpinned by a shared understanding of a system of operations.

Gathering and manipulation of quantitative data
The blueprint in Figure 4(b) provides a starting point to gather quantitative data that can be manipulated mathematically. For the purpose of this research data were gathered through a collaboration with industry for an individual WwTW, which therefore constitutes the relevant wastewater system. The data gathered were complemented by insights from the literature. Detail on the data gathering process, and the specific numerical values obtained are provided elsewhere (Settanni, 2015) and will not be disclosed in this paper for confidentiality.

Quantitative data for WwTW operations were obtained in the form of a mass balance. Mass balances are commonly used for process control in WwTWs (Puig et al., 2008). A recent trend in the process industry is to use mass balances to model networks of alternative processing technologies (also called ‘superstructure’) for plant synthesis by enterprise-wide optimization (Quaglia et al., 2012). This approach has been applied to WwTWs design (Bozkurt et al., 2015). By using mass balances only a subset of the conceptual model outlined earlier can be investigated quantitatively. Such subset is shown in Figure 5.

The first step is to visualize the data gathered from an existing mass balance. This is achieved by means of the Sankey diagram shown in Figure 6. A Sankey diagram is a quantitative data visualisation approach which follows the requirement of conservation of mass through a system of interdependent operations (Schmidt, 2008).
Figure 6 – Sankey diagram of physical flows through a WwTW from a “Means-ends” perspective (service provision). All flows in the legend are expressed in the units indicated per day. Details omitted for confidentiality. The diagram has been realised using e!Sankey (www.e-sankey.com)
### Table 3 – Tabular representation of the flows visualised in the Sankey diagram (only non-zero elements shown; formal notation replaces numerical values).

<table>
<thead>
<tr>
<th>Description</th>
<th>Flow index</th>
<th>Unit of measurement</th>
<th>Production process index</th>
<th>Final demand (net production)</th>
<th>Emissions (+) Purchases (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drainage service [Final demand]</td>
<td>a</td>
<td>[m³/day]</td>
<td>ZₐA ZₐC ZₐE</td>
<td></td>
<td>∑₊nₐ=AZₐₐn</td>
</tr>
<tr>
<td>Primary treatment services @ Site 1</td>
<td>b</td>
<td>[m³/day]</td>
<td>ZₐA ZₐE</td>
<td></td>
<td>∑₊nₐ=AZₐₐn</td>
</tr>
<tr>
<td>Secondary treatment services @ Site 1</td>
<td>c</td>
<td>[m³/day]</td>
<td>ZₐB ZₐC ZₐD</td>
<td></td>
<td>∑₊nₐ=AZₐₐn</td>
</tr>
<tr>
<td>Primary treatment services @ Site 2</td>
<td>d</td>
<td>[m³/day]</td>
<td>ZₐA ZₐB ZₐD ZₐE</td>
<td></td>
<td>∑₊nₐ=AZₐₐn</td>
</tr>
<tr>
<td>Secondary treatment services @ Site 2</td>
<td>e</td>
<td>[m³/day]</td>
<td>ZₐA ZₐD ZₐE</td>
<td></td>
<td>∑₊nₐ=AZₐₐn</td>
</tr>
<tr>
<td>Primary treatment services @ Site 3</td>
<td>f</td>
<td>[m³/day]</td>
<td>ZₐA ZₐD ZₐE</td>
<td></td>
<td>∑₊nₐ=AZₐₐn</td>
</tr>
<tr>
<td>Secondary treatment services @ Site 3</td>
<td>g</td>
<td>[m³/day]</td>
<td>ZₐA ZₐD ZₐE</td>
<td></td>
<td>∑₊nₐ=AZₐₐn</td>
</tr>
<tr>
<td>Tertiary treatment service</td>
<td>h</td>
<td>[m³/day]</td>
<td>ZₐC ZₐD ZₐE</td>
<td></td>
<td>∑₊nₐ=AZₐₐn</td>
</tr>
<tr>
<td>Raw sludge treatment service</td>
<td>i</td>
<td>[kg/day]</td>
<td>Z₁B Z₁D Z₁E</td>
<td></td>
<td>∑₊nₐ=AZ₁ₐn</td>
</tr>
<tr>
<td>Thickened sludge treatment service</td>
<td>j</td>
<td>[kg/day]</td>
<td>Z₁B Z₁D Z₁E</td>
<td></td>
<td>∑₊nₐ=AZ₁ₐn</td>
</tr>
<tr>
<td>Digested sludge treatment service</td>
<td>k</td>
<td>[kg/day]</td>
<td>Z₁B Z₁D Z₁E</td>
<td></td>
<td>∑₊nₐ=AZ₁ₐn</td>
</tr>
<tr>
<td>Digester gas treatment service</td>
<td>l</td>
<td>[m³/day]</td>
<td>Z₁B Z₁D Z₁E</td>
<td></td>
<td>∑₊nₐ=AZ₁ₐn</td>
</tr>
<tr>
<td>Final effluent</td>
<td>1</td>
<td>[m³/day]</td>
<td>w₁₁H</td>
<td></td>
<td>∑₊nₐ=W₁₁ₐn</td>
</tr>
<tr>
<td>Electricity</td>
<td>2</td>
<td>[kWh]</td>
<td>w₂₁L</td>
<td></td>
<td>∑₊nₐ=W₂₁ₐn</td>
</tr>
<tr>
<td>Solid waste disposal service</td>
<td>1</td>
<td>[kg/day]</td>
<td>v₁₄</td>
<td></td>
<td>∑₊nₐ=v₁₄ₐn</td>
</tr>
<tr>
<td>Polymer consumption</td>
<td>2</td>
<td>[kg/day]</td>
<td>v₁₄</td>
<td></td>
<td>∑₊nₐ=v₁₄ₐn</td>
</tr>
<tr>
<td>Ferric sulphate consumption</td>
<td>3</td>
<td>[L/day]</td>
<td>v₃₆</td>
<td></td>
<td>∑₊nₐ=v₃₆ₐn</td>
</tr>
<tr>
<td>Methanol consumption</td>
<td>4</td>
<td>[m³/day]</td>
<td>v₄₆</td>
<td></td>
<td>∑₊nₐ=v₄₆ₐn</td>
</tr>
<tr>
<td>Anionic polyelectrolyte consumption</td>
<td>5</td>
<td>[L/day]</td>
<td>v₅₆</td>
<td></td>
<td>∑₊nₐ=v₅₆ₐn</td>
</tr>
<tr>
<td>Electric energy consumption</td>
<td>6</td>
<td>[kWh/day]</td>
<td>v₆₄ v₆₅ v₆₆ v₆₇ v₆₈ v₆₉</td>
<td></td>
<td>∑₊nₐ=v₆₄ₐn</td>
</tr>
<tr>
<td>Thermal energy consumption</td>
<td>7</td>
<td>[MJ]</td>
<td>v₇₄ v₇₅ v₇₆ v₇₇ v₇₈</td>
<td></td>
<td>∑₊nₐ=v₇₄ₐn</td>
</tr>
<tr>
<td>Greenhouse gases emission</td>
<td>1</td>
<td>[kg CO₂eq/day]</td>
<td>g₁₄</td>
<td></td>
<td>∑₊nₐ=g₁₄ₐn</td>
</tr>
<tr>
<td>N emissions through effluent</td>
<td>2</td>
<td>[kg Ntot/day]</td>
<td>g₁₄</td>
<td></td>
<td>∑₊nₐ=g₁₄ₐn</td>
</tr>
<tr>
<td>BOD emissions through effluent</td>
<td>3</td>
<td>[kg BOD/day]</td>
<td>g₁₄</td>
<td></td>
<td>∑₊nₐ=g₁₄ₐn</td>
</tr>
<tr>
<td>Air volume</td>
<td>4</td>
<td>[Nm³/day]</td>
<td>g₄₅ g₄₆ g₄₇ g₄₈ g₄₉</td>
<td></td>
<td>∑₊nₐ=g₄₅ₐn</td>
</tr>
<tr>
<td>Biomass</td>
<td>5</td>
<td>[kg /day]</td>
<td>g₅₅ g₅₆ g₅₇ g₅₈ g₅₉</td>
<td></td>
<td>∑₊nₐ=g₅₅ₐn</td>
</tr>
</tbody>
</table>
A ‘service viewpoint’ on wastewater treatment was taken in constructing the diagram in Figure 6. A fictitious exogenous demand for service outcomes ‘pulls’ the system of operations—namely: to drain an area and receive the tankered wastes imported to site (a Licensed Treatment Center—LTC), and to treat raw sludge imported from other WwTWs. These services are displayed as outcomes on the rightmost side of the diagram. The demand of final deliverables propagates backward, triggering directly and indirectly a demand for intermediate outputs (goods and services) provided within the system, as well as inputs purchased exogenously or provided by the natural environment. The by-products generated while meeting such demand are recycled or treated within the boundaries of the WwTW, thus creating feedback loops (e.g., between functions ‘F’ and ‘G’). By-products which are disposed of are represented as an input of disposal service to the function generating them.

A special case in Figure 6 is electrical energy, which is artfully viewed as the by-product of providing a ‘digester gas treatment service’. This is the result of subjective modeling choices based on assumptions made by the analyst about which outcomes actually ‘pull’ the system. If a purely physical viewpoint was chosen instead of a service viewpoint, the final effluent released into a water body would be the final deliverable in Figure 6 whereas raw wastewater, tankered waste and imported sludge would be regarded as inputs.

A mathematical counterpart of the diagram in Figure 6 is a set of matrices and vectors the elements of which can be arranged in tabular form as shown in Table 3. The elements of the table with non-zero values have been colour-coded to facilitate the connection with Figure 6. Specific numerical values for the WwTW are omitted, hence the link between Table 3 and the diagram in Figure 6 will be expressed only through formal notation as described next.

Table 3 is equivalent to the following matrix equation:

$$[Z \ W \ V \ G] s = [y \ w \ v \ g]$$  \hspace{1cm} (1)

Matrix $Z = \begin{bmatrix} z_{aA} & z_{aB} & \cdots & z_{aL} \\ z_{bA} & z_{bB} & \cdots & z_{bL} \\ \vdots & \vdots & \ddots & \vdots \\ z_{lA} & z_{lB} & \cdots & z_{lL} \end{bmatrix}$ is called the ‘technology matrix’, and corresponds to the $12 \times 12$ upper partition of Table 3 defined by rows $a, \ldots, l$ and columns $A, \ldots, L$ is. It has the following characteristics:

- Material flows are reported row-wise, functions are reported column-wise.
- By convention outputs have positive sign, inputs have negative sign. The sign is not shown in the formal notation.
- At the intersection between the generic row $i$ and column $j$ ($i \neq j$) one reads the amount of $i$-th material flow employed by the $j$-th function as an input. For example, the element $z_{bA} < 0$ is the amount of flow $b$ into function $A$, and corresponds to the arc directed from function $B$ towards $A$ in the diagram (Figure 6).
- The main output of the $j$-th function is on the $j$-th row. Hence, the main outputs can be read along the main diagonal, where the column and row indexes are equal ($i = j$). For example $z_{BB} > 0$ is a measure of $B$’s main output and corresponds to the width of the base of the arc leaving $B$ in Figure 6, regardless its destination.
- No function produces another function’s output (there is no substitution).
The ‘final demand vector’ \( \mathbf{y} = \left[ \begin{array}{c} y_a = \sum_{n=a}^{N} x_{an} \\ y_b = \sum_{i}^{B} z_{bn} \\ y_1 = \sum_{n=a}^{N} x_{1n} \end{array} \right] \) is a 12 × 1 vector which corresponds to the column of Table 3 labelled ‘Final demand’. Its elements are greater than or equal to zero, and correspond to the demand of final deliverables that the system must meet.

The ‘by-product matrix’ \( \mathbf{W} = \left[ \begin{array}{ccc} w_{1A} & \cdots & w_{1L} \\ w_{2A} & \cdots & w_{2L} \end{array} \right] \) is the 2 × 12 partition of Table 3 defined by rows 13-14 and columns \( A, \ldots, L \). In the case considered here matrix \( \mathbf{W} \) has a particular configuration due to the ‘service viewpoint’ taken: it has non-zero elements that correspond to the treated effluent (\( w_{1H} > 0 \)) and the electricity generated (\( w_{2L} > 0 \)) only.

The ‘value added’ matrix \( \mathbf{V} = \left[ \begin{array}{ccc} v_{1A} & \cdots & v_{1L} \\ \vdots & \ddots & \vdots \\ v_{7A} & \cdots & v_{7L} \end{array} \right] \) is the 7 × 12 partition of Table 3 defined by rows 15-21 and columns \( A, \ldots, L \). Its elements, if non zero, have negative sign and denote exogenously purchased inputs. For example \( v_{1K} < 0 \) is the input of exogenously purchased services due to the generation of sludge cake if assumed to be disposed of.

The ‘environmental intervention matrix’ \( \mathbf{G} = \left[ \begin{array}{ccc} g_{1A} & \cdots & g_{1L} \\ \vdots & \ddots & \vdots \\ g_{5A} & \cdots & g_{5L} \end{array} \right] \) is the 5 × 12 partition of Table 3 defined by rows 22-26 and columns \( A, \ldots, L \). Its elements, if non-zero, record the total amount of environmental resources utilised (negative sign) and emissions generated (positive sign) by each function.

Finally, \( \mathbf{w}, \mathbf{v} \) and \( \mathbf{g} \) are vectors that correspond to the three partitions under the column heading ‘Emissions (+) Purchases (-)’ in Table 3; whereas \( \mathbf{s} \) is a 12 × 1 vector which specifies the ‘activity levels’ at which each operation \( A, \ldots, L \) within the system is required to perform in order to meet the final demand \( \mathbf{y} \) while sustaining themselves. Since the flows in Table 3 are already balanced, \( \mathbf{s} \) is in fact a unity vector (all its elements are equal to 1).

**Analysis**

The WwTW model described by equation (1) and visualised in Figure 6 allows a range of mathematical manipulations. These manipulations enable businesses in the water and wastewater sector to evaluate whether the actions they have undertaken, or will undertake, contribute toward the attainment or non-attainment of their PCs. The following manipulations are discussed next: quantifying resource requirements and environmental aspects; costing; evaluating productivity, effectiveness, efficiency and profitability. A spatial dimension can be added to the analysis but this is left to future research. Detailed numerical examples are provided elsewhere (Settanni, 2015).

**Joint evaluation of resources and environmental aspects**

The following part of equation (1): \( \mathbf{y} = \mathbf{Zs} \) expresses the fundamental physical balance which governs the net production within the system boundaries, and can be used for planning purposes. Scenarios can be created by changing the demand of some or all the final deliverables in \( \mathbf{y} \), and then by computing how this affects the demand of inputs and natural resources, as well as the by-products and emissions generated given the interdependencies among the system’s elements. This requires specifying a 12 × 1 vector \( \mathbf{y}_{\text{scenario}} \) obtained by changing the values in \( \mathbf{y} \) as desired (or by simulation), and calculating the unknown vector of activity levels \( \mathbf{s}_{\text{scenario}} \) corresponding to the envisaged scenario as follows:
\[ s_{\text{scenario}} = Z^{-1} y_{\text{scenario}} \]  

(2)

Where \( Z^{-1} \), if exists, is the inverse of matrix \( Z \) (the mathematical conditions for the existence of the inverse of a matrix are not discussed here). Knowing \( s_{\text{scenario}} \) it is possible to obtain:

- The total amount of electricity and final effluent generated as \( w^* = WZ^{-1} y_{\text{scenario}}^{*} \)
- The total amount of exogenously acquired goods and services as \( v^* = VZ^{-1} y_{\text{scenario}}^{*} \)
- The total amount of environmental resources utilised and emissions generated as \( g^* = GZ^{-1} y_{\text{scenario}}^{*} \)

The amount of environmental resources utilised and emissions generated \( g^* \) can be used to verify whether emissions into environmental media such as CO\(_2\), Nitrogen etc. are within limits if a certain level of plant activity is pursued. It can also be used as the starting point for environmental impact analysis, but this requires known characterisation factors for the elements in \( g^* \). The link between individual plants’ operations and broader sustainability analysis in a cradle-to-grave perspective is examined elsewhere (Heijungs et al., 2013).

Costing

Using a similar model, it is possible to jointly evaluate the unit monetary worth of each output within the wastewater system described so far. First, one must know values taken by the elements of the following vectors:

- \( p_v \): purchase prices of exogenously purchased inputs
- \( p_w \): charges for by-product disposal, or revenues if by-products are sold instead
- \( p_g \): value of tradable permits, environmental taxes etc. associated with emission in the atmosphere (such as CO\(_2\)) or into water bodies (such as Phosphorous).

These vectors have the same size as \( v, w, g \), respectively, and their entries, if different from zero, have a sign such that \( q = [p_v' \ p_w' \ p_g'] [W \ V \ G] \geq 0 \) (superscript ‘ denotes transposition). The unknown unit monetary worth of the output of each function \( A, \ldots, L \) corresponds to an entry of a \( 1 \times 12 \) vector \( p \). Each value must cover the known direct costs \( q \) and the unknown monetary worth of the outputs transferred-in from other functions:

\[ q = pZ \]  

(3)

Also in this case it is possible to formulate scenarios given by changing \( p_v, p_w, \) and \( p_g \):

\[ p_{\text{scenario}} = q_{\text{scenario}}^{*} Z^{-1} \]  

(4)

Equations (1-4) are the foundations of Input Output Analysis (IOA). IOA was originally developed in economics to investigate the techno-economic implications of alternative scenarios given a blueprint of the interrelationships among industries within an economic system (Leontief, 1986). Further refinements take into account interdependencies between production and the generation and treatment of waste (Nakamura and Kondo, 2009). The principles of IOA have also been applied for the evaluation of material and energy flows in manufacturing systems, be them individual plants (Xue, 2007) or supply chains (Albino et al., 2002), as well as to develop computational structures underpinning analytical sustainability evaluations in a life-cycle perspective (Heijungs et al., 2013).

Productivity

Productivity is fundamentally an input to output relationship measured as a prescribed output to the resources consumed. The productivity analysis of a multi-stage, multi-input, multi-
output production system suing IOA consists of determining the *technical coefficients* which form the ‘structural matrix’ of such a system (Leontief, 1986). The structural dependencies determined by the technology in use within the techno-economic system being investigated are exposed as ratios or coefficients of each input to the total output of which it becomes part.

Given the notation used above, it is necessary to disaggregate the technological matrix $Z$ into main inputs (off-diagonal elements) and main outputs (on-diagonal elements):

$$Z = \hat{x} + X = \begin{bmatrix} z_{aA} & 0 & \ldots & 0 \\ 0 & z_{bB} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & z_{lL} \end{bmatrix} + \begin{bmatrix} 0 & z_{aB} & \ldots & z_{aL} \\ z_{bA} & 0 & \ldots & z_{bL} \\ \vdots & \vdots & \ddots & \vdots \\ z_{lA} & z_{lB} & \ldots & 0 \end{bmatrix}$$

(5)

Where $\hat{x} \geq 0; X \leq 0$; superscript $\hat{}$ denotes vector diagonalisation. The structural matrix is:

$$A = -X\hat{x}^{-1}$$

(6)

Matrix $A$’s generic element $a_{ij}$ is a technical coefficient and expresses the quantity of the $i$-th function’s output that goes into the $j$-th function per unit of its total main output $j$. Additional technical coefficients can be evaluated in a similar way to measure the quantity of exogenously acquired goods, services, and environmental resources that goes into the $j$-th function per unit of its total output $j$, as well as the quantity of by-products and emissions into the environment generated by that function per unit of output. An example is given in Figure 7 for the functions $C, E, G$ which correspond to different biological treatment technologies within the WwTW considered here.

In industrial practice most PCs tend to be formulated and reported in absolute rather than relative terms. Examples include the self-generation of electricity from wastewater biomass, greenhouse gases emissions in the atmosphere and of nutrients in water bodies. Hence the technical coefficients approach can be used to improve the current formulation of PCs.

**Effectiveness**

Effectiveness is the ability of an organization to fulfil its objectives. It implies that the firm consists of multiple transformation stages whereby downstream from intermediate outputs is outcome, which reflects the ultimate achievement of the firm.

Assuming that the elements in the final demand vector $y$ are the ‘outcomes’ that a system of operations is supposed to pursue, IOA provides a pragmatic insight into the effectiveness of a multi-stage, MIMO delivery system. Using the structural matrix obtained earlier one can determine how much the output of each function would increase to match a variation in the final demand considering that it contributes to the final delivery both directly and indirectly by supplying many or most other functions. To achieve this, the planning problem in equation (2) is reformulated to include the structural matrix $A$ as follows:

$$x = (I - A)^{-1}y$$

(7)

Where the matrix $A^* = (I - A)^{-1}$ if it exists, is known as the Leontief inverse. The generic element $a_{ij}^*$ of matrix $A^*$ indicates by how much the output $x_i$ of the $i$-th process would increase if the quantity of the good or service $j$ absorbed by the final demand, $y_j$, had been increased by one unit. Such an increase would affect process $i$ directly if $i = j$ and indirectly when $i \neq j$ insofar as the $i$-th process provides inputs to some or all other processes which, in turn, directly or indirectly contribute to the final delivery (Leontief, 1986).
For illustrative purposes, Figure 8 shows the values taken by the Leontief’s inverse elements corresponding to the same functions considered earlier.

**Efficiency and Profitability**

Efficiency is a grade which measures the comparison between actual and standard output for a given array of inputs. For example, a grade of 50% means the firm should be able to double its output given the inputs used and hence it can be said to be output inefficient (Hackman, 2008). Efficiency is therefore an inherently comparative concept, which entails comparing and contrasting expected input-to-output ratios with actuals, or historical actuals over time. Most approaches would look at the firm in an aggregated way, assuming a single production stage. An example is Barbiroli (1996), who presents a detailed set of indicators to addresses manifold aspects of efficiency in production including waste generation, natural resource use,
recycling and emissions into the environment. Another example is water use efficiency which tends to be more emphasised in corporate reporting practices than water sources and destinations after use (Sodhi and Yatskovskaya, 2014). Efficiency evaluation in a multi-stage settings being possible, it is problematic and mostly limited to two stages (Agrell and Hatami-Marbini, 2013) and strictly sequential configurations (e.g., Troutt, 2001).

Computing efficiency requires shifting from the multi-stage representation used so far to a single-stage representation. This is achieved by focusing on vectors $y, w, v$, and $g$ obtained earlier, and corresponding to the totals in Table 3. Using historical values for these vectors one may determine if it is possible to achieve an output equal to or greater than the output observed in a reference time period by employing less than the amount of input observed at that time. The metric thus obtained, called ‘radial input efficiency’, requires setting up a mathematical programming problem which is described elsewhere (Hackman 2008).

Profitability analysis is based on historical, aggregated input-output data, too. Conceptually, it brings together the technical analysis performed in equations (1-2), and the monetary evaluation analysis performed in equations (3-4). Using the same vectors mentioned above, a series of indexes can be computed to figure out how well a firm performed between two time periods in both technical and monetary terms and to verify if a productivity gain has occurred. Details about how to calculate the indexes are given elsewhere (Hackman 2008).

In the case considered here realistic efficiency and profitability analysis could not be conducted. In the absence of historical data a simulation was carried out on the data gathered one-off to generate an artificial history. Existing procedures for efficiency and profitability analysis were then applied for illustrative purposes. Details about computations and results are provided elsewhere (Settanni, 2015).

Spatial analysis
As mentioned earlier, a wastewater system may involve a multitude of WwTWs operating over a geographical area, e.g. a catchment. Businesses often respond to regulatory requirements by ‘sweating’ existing assets while spreading the risk of compliance across multiple sites. Hence, it has practical relevance to extend the previous analysis to include multiple locations.

Extensions of IOA have been developed for regional analysis in macroeconomics (Leontief, 1986), and for the exploration of the effects of spatial variables on the economic and environmental performance of multi-locations supply chains (Yazan et al., 2011). However, the exploration of such extensions of IOA to scale up the approach presented above in order to deal simultaneously with more WwTWs within a region is left to future research.

Concluding remarks
This paper has considered some implications of the outcome-orientated approach promoted by the regulator of the water sector in England and Wales for the evaluation of the multi-faceted performance of wastewater systems operations. The use of concepts such as input, output and outcome for business performance evaluation were reviewed critically, including evidence from other sectors (defence, healthcare, etc.) and existing frameworks. An approach to reconcile performance measurement with a genuine outcome orientation for use in the water and wastewater service industry was outlined. To achieve this, a two-stage model covering both the conceptual stage, and the quantitative data collection and analysis stage
was applied to a simplified wastewater system underpinned by real company data. A range of performance evaluations allowed by the proposed model of the wastewater system of interest was illustrated, although details were not disclosed.

On a practical side, the research has highlighted the potential to systematise blueprinting of a wastewater system’s operation by managing existing mass balances of individual WwTWs while avoiding overly complicated mathematics. However, the focus on quantitative physical flows is at the same time a major limitation of this research because it neglects insights about what happens ‘behind the scenes’ to enable those flows to occur as depicted. For example, the system of equipment support activities which ensures asset availability was captured in the conceptual model, but not in the quantitative model. Future research should look beyond physical flows to capture service operations. It should also explore spatial analysis, which is becoming increasingly important as industry considers spreading the risk of meeting their performance commitments across multiple plants and locations.

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