UNEMPLOYMENT (FEARS) AND DEFLATIONARY SPIRALS

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Abstract

The interaction of incomplete markets and sticky nominal wages is shown to magnify business cycles even though these two features—in isolation—dampen them. During recessions, fears of unemployment stir up precautionary sentiments that induce agents to save more. The additional savings may be used as investments in both a productive asset (equity) and an unproductive nominal liquid asset. The desire to hold the nominal liquid asset puts deflationary pressure on the economy which, provided that nominal wages aresticky, increases labor costs, and reduces firm profits. Lower profits repress the desire to save in equity, which increases (the fear of) unemployment, and so on. This mechanism causes the model to behave differently from its complete markets version and is quantitatively important even if monetary policy counteracts the desire to hold more of the liquid asset by lowering the interest rate. The deflationary pressure yields a mean-reverting reduction in the price level, which implies an increase in expected inflation and a decrease in the expected real interest rate even if the policy rate does not adjust. Thus, our mechanism is different from the typical zero lower bound argument. Due to the deflationary spiral, our model also behaves differently from its incomplete market version without aggregate uncertainty, especially in terms of the impact of unemployment insurance on average employment levels. (JEL: E12, E24, E32, E41, J64, J65)

The editor in charge of this paper was Dirk Krueger.

Acknowledgments: We would like to thank Christian Bayer, Chris Carroll, V.V. Chari, Zeno Enders, Marc Giannoni, Greg Kaplan, Dirk Krueger, Per Krusell, Alisdair McKay, Kurt Mitman, Michael Reiter, Rana Sajedi, Victor Rios-Rull, Richard Rogerson, Petr Sedlacek, Jiri Slacalek, and three anonymous referees for useful comments. We thank Felix Koenig for excellent research assistance. Den Haan and Riegler gratefully acknowledge support through the grant “Working Towards a Stable and Sustainable Growth Path,” funded by the Economic and Social Research Council (ESRC). Rendahl and Riegler gratefully acknowledge financial support from the ADEMU project, “A Dynamic Economic and Monetary Union,” funded by the European Union’s Horizon 2020 Program under grant agreement No 649396. Rendahl also gratefully acknowledges financial support from the Institute for New Economic Thinking (INET). Rendahl is a Research Affiliate at CEPR.

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1. Introduction

The empirical literature documents that workers suffer substantial losses in both earnings and consumption levels during unemployment. For instance, Kolsrud et al. (2015) use Swedish data to document that consumption expenditures drop on average by 26.3% during the first year of an unemployment spell. Krueger, Mitman, and Perri (2017) document that during the great recession US households reduced consumption expenditures by more than the slowdown in their disposable income alone would indicate. A plausible explanation for this finding is increased uncertainty about future job prospects. The observed inability to insure against unemployment spells has motivated several researchers to develop business cycle models with a focus on incomplete markets. The hope (and expectation) has been that such models would not only generate more realistic behavior for individual variables, but also be able to generate volatile and prolonged business cycles without relying on large and persistent exogenous shocks. Although, in existing models, individual consumption is indeed much more volatile than aggregate consumption, aggregate variables are often not substantially more volatile than their counterparts in the corresponding complete markets (or representative-agent) version. Krusell, Mukoyama, and Sahin (2010), for instance, find that imperfect risk sharing does not help in generating more volatile business cycles. McKay and Reis (2016) document that a decrease in unemployment benefits—which exacerbates market incompleteness—actually decreases the volatility of aggregate consumption. The reason is that a decrease in unemployment benefits increases precautionary savings, investment, and the capital stock, and ultimately makes the economy as a whole better equipped to smooth consumption.

We develop a model in which the inability to insure against unemployment risk generates business cycles that are much more volatile than the corresponding complete markets version. Moreover, although the only aggregate exogenous shock has a small standard deviation, the outcome of key exercises, such as changes in unemployment benefits, depends crucially on whether there is aggregate uncertainty. This result is obtained by combining incomplete asset markets with incomplete adjustments of the nominal wage rate to changes in the price level. Markets are incomplete because agents can invest in only two assets, both with payoffs that depend on aggregate outcomes. Those are a productive asset (equity) and an unproductive asset (money). Our mechanism operates both when the unproductive asset earns interest and when it does not. In the version considered in this paper, it does earn interest. The key assumption is that nominal wages are expressed in units of this (unproductive) asset and are somewhat rigid. In addition, the impact of shocks is prolonged by Diamond–Mortensen–Pissarides search frictions in the labor market.

1. Appendix A provides a more detailed discussion of the empirical literature investigating the behavior of individual consumption during unemployment spells.
2. We discuss the empirical motivation for these assumptions in Section 2 and Appendix A.
Before explaining why the combination of incomplete markets and sticky nominal wages amplifies business cycles, we first explain why these features by themselves dampen business cycles in our model in which aggregate fluctuations are caused by productivity shocks. First, consider a model in which there are complete markets, but nominal wages do not respond one-for-one to price level changes. A negative productivity shock induces agents to reduce their demand for money, because the present is worse than the future, and agents would like to smooth consumption. Moreover, at lower activity levels, less money is needed for transactions purposes. The decline in money demand puts upward pressure on the price level. Provided that nominal wages are sticky, the resulting downward pressure on real wages mitigates the reduction in profits caused by the direct negative effect of a decline in productivity. The result is a muted aggregate downturn, because a smaller reduction in profits implies a smaller decline in employment. Next, consider a model in which nominal wages are flexible, but workers cannot fully insure themselves against unemployment risk. Forward-looking agents understand that a persistent negative productivity shock increases the risk of being unemployed in the near future. If workers are not fully insured against this risk, the desire to save increases for precautionary reasons. However, increased savings leads to an increase in demand for all assets, including productive assets such as firm ownership. This counteracting effect alleviates the initial reduction in demand for productive assets that was induced by the direct negative effect of a reduced productivity level, and therefore dampens the increase in unemployment. In either case, sticky nominal wages or incomplete markets lead—in isolation—to a muted business cycle.

Why does the combination of incomplete markets and sticky nominal wages lead to the opposite results? As before, the increased probability of being unemployed in the near future increases agents’ desire to save more in all assets. However, the increased desire to hold money puts downward pressure on the price level, which in turn increases real labor costs and reduces profits. This latter effect counters any positive effect that increased precautionary savings might have on the demand for productive investments. Once started, this channel will reinforce itself. That is, if precautionary savings lead—through downward pressure on prices—to increased unemployment, then this will in turn lead to a further increase in precautionary savings, and so on. When does this process come to an end? At some point, the expanding number of workers searching for a new job reduces the expected cost of hiring, which makes it attractive to resume job-creating investments.

In addition to endogenizing unemployment, the presence of search frictions in the labor market adds further dynamics to this propagation mechanism. First, the value of a firm—i.e., the price of equity—is forward-looking. As a consequence, a prolonged increase in real labor costs leads to a sharp reduction in economic activity already in the present, with an associated higher risk of unemployment. Second, with low job-finding rates unemployment becomes a slow moving variable. Thus, the increase in unemployment is more persistent than the reduction in productivity itself.
Our mechanism is quite different from the one emphasized in the zero lower bound (ZLB) literature. A key feature in this literature is a deflationary pressure that manifests itself in a reduction in expected inflation—or even deflation—combined with the inability to reduce the nominal interest rate to values below zero. This leads to an increase in the real interest rate, which in turn leads to a deterioration of the economy and further deflationary pressure. In our framework, however, the deflationary pressure means a reduction in the price level that actually goes together with an increase in expected inflation and a substantial decrease in the real interest rate.

We use our framework to study the advantages of alternative unemployment insurance (UI) policies. Specifically, we document that the effects of changes in unemployment benefits on the behavior of aggregate variables differ from the effects in other models. In the existing literature, increased unemployment benefits bring forth adverse aggregate consequences that eclipse the gains of reduced income volatility. In particular, with lower fluctuations in individual income the precautionary motive weakens and aggregate investment falls. The result is a decline in average employment and output, with adverse effects on welfare. This channel is important in our model as well. However, in the version of our model with aggregate uncertainty, there are two quantitatively important factors that push average employment in the opposite direction and can overturn the negative effect associated with the reduction in precautionary savings. The first is that the demand for the productive asset can increase, because an increase in the level of unemployment benefits stabilizes asset prices as well as business cycles. The second is that the nonlinearity in the matching process is such that increases in employment during expansions are smaller than reductions during recessions. Consequently, a reduction in volatility can lead to an increase in average employment.

An important aspect of our model is that precautionary savings can be used for investments in both the productive asset (firm ownership) and the unproductive asset (money). This complicates the analysis, because the numerical procedure requires a simultaneous solution for a portfolio choice problem for each agent and for equilibrium prices. Our numerical analysis ensures that the market for firm ownership (equity) is in equilibrium and all agents owning equity discount future equity returns with the correct, that is, their own individual-specific, intertemporal marginal rate of substitution (MRS). In contrast, a typical set of assumptions in the macroeconomics literature on the impact of precautionary savings is that workers jointly own the productive asset at equal shares, that these shares cannot be sold, and that discounting of the returns of this

4. See, for example, Young (2004) and Krusell et al. (2010).
5. In our environment, we avoid the unresolved theoretical corporate finance issue on evaluating alternative earnings streams in the presence of heterogeneous ownership, because we only have to price an earnings stream that is not affected by firm decisions and ownership shares are traded in a competitive market. Our challenge is not a theoretical but a computational one.
asset occurs with some average MRS or an MRS based on aggregate consumption.\textsuperscript{6,7} One exception is Krusell et al. (2010) who—like us—allow trade in the productive asset and discount agents’ returns on this asset with the correct MRS.\textsuperscript{8}

In Section 2, we provide empirical motivation for the key assumptions underlying our model: sticky nominal wages and workers’ inability to insure against unemployment risk. In Section 3, we describe the model. In Section 4, we discuss the calibration of our model. In Sections 5 and 6, we describe the behavior of individual and aggregate variables, respectively. In Section 7, we document that the model is capable of describing key characteristics of Eurozone economic aggregates during the great recessions and why the mechanism of our model is quite different from that emphasized in the ZLB literature. In Section 8, we discuss how business cycle behavior is affected by alternative UI policies.

2. Empirical Motivation

Figure 1 displays the behavior of gross domestic product (GDP), the unemployment rate, and stock prices for the Eurozone during the great recession.\textsuperscript{9} What are the factors behind the observed large and persistent drops in real activity and stock prices? The mechanism proposed in this paper is motivated by the behavior of prices, nominal wages, and unit labor costs in the Eurozone during this economic downturn. The first observation is that the growth in the price level slowed considerably during the crisis relative to the trend. This is documented in the top panel of Figure 2, which plots the GDP deflator for the Eurozone alongside its precrisis trend. To investigate whether nominal wages followed the slowdown in inflation, the second panel of Figure 2 displays nominal hourly earnings together with its precrisis trend. We find that nominal wages continued to grow at precrisis rates or above, despite a substantial reduction in inflation rates. This means that real wages \textit{increased} relative to trend.\textsuperscript{10}

\begin{thebibliography}{10}
\bibitem{7} An alternative simplifying assumption is that the only agents who are allowed to invest in the productive asset are agents that are not affected by idiosyncratic risk (of any kind). Examples are Rudanko (2009), Bils, Chang, and Kim (2011), Challe et al. (2017), and Challe and Ragot (2016). Bayer et al. (2014) analyze a more challenging problem than ours, in which firms are engaged in intertemporal decision making. However, in contrast to our model, these firms are assumed to be risk neutral, consume their own profits, and discount the future at a constant geometric rate.
\bibitem{8} The procedure in Krusell et al. (2010) is only exact if the aggregate shock can take on as many realizations as there are assets and no agents are at the short-selling constraint. Our procedure does not require such restrictions, which is important, because the fraction of agents at the constraint is nontrivial in our model.
\bibitem{9} Details on data sources are given in Appendix B.
\bibitem{10} Similarly, Daly, Hobijn, and Lucking (2012) and Daly and Hobijn (2014) document that real wages increased during the recent recession in the United States.
\end{thebibliography}
The observed increases in real wages are not necessarily due to a combination of low inflation and downward nominal wage rigidity. It is possible that solid real wage growth reflects an increase in labor productivity, for example, because workers that are laid off are less productive than those that are not. To shed light on this possibility, we compare the nominal unit labor cost with the price level.\footnote{The nominal unit labor cost is defined as the cost of producing one unit of output, i.e., the nominal wage rate divided by labor productivity. The price index used as comparison is the price index used in defining labor productivity.} The results are shown...
FIGURE 2. Key Eurozone variables. The precrisis trend is defined as the time path the variable would have followed if its growth rate before and beyond the fourth quarter of 2007 had been equal to the average growth rate over the five years preceding 2007. Shaded areas mark CEPR recessions. Data sources are given in Appendix B.

in the bottom panel of Figure 2. The panel shows that nominal unit labor costs have grown faster than prices since the onset of the crisis, whereas the opposite was true before the crisis. This indicates that real labor costs increased during the crisis even if one corrects for productivity.  

12 The observation that real unit labor costs are not constant over the business cycle is interesting in itself. If the real wage rate is equal to the marginal product of labor and the marginal product is proportional to average labor productivity—properties that hold in several business cycle models—then real unit labor costs would be constant.
These observations are consistent with the hypothesis that the combination of deflationary pressure and nominal wage stickiness increased labor costs.\footnote{Throughout this paper, we will use the term deflationary pressure broadly. In particular, we also use it—as is the case here—to indicate a slowdown in inflation relative to trend.} Labor hoarding combined with stickiness of real wages would also lead to an increase in real unit labor costs.\footnote{This could not explain the observed increase in real wages, unless it was accompanied by a composition effect.} As documented in Appendix A, however, there is convincing evidence that nominal wages do not fully respond to changes in prices. Thus, it seems reasonable to assume that at least part of the observed increase in real wage costs is due to the combination of deflationary pressure and sticky nominal wages.

The pattern displayed in Figure 2 is not universally true in all economic downturns. In fact, when we repeat the exercise for the United States, we find that real wages increased relative to the precrisis trend, like they did in the Eurozone, but that real unit labor costs did not.

3. Model

The economy consists of a unit mass of households, a large mass of potential firms, and one government. The mass of active firms is denoted $q_t$, and all firms are identical. Households are ex-ante homogeneous but differ ex-post in terms of their employment status (employed or unemployed) and their asset holdings.

\textit{Notation.} Upper (lower) case variables denote nominal (real) variables. Variables with subscript $i$ are household specific. Variables without a subscript $i$ are either aggregate variables or variables that are identical across agents, such as prices.

3.1. Households

Each household consists of one worker who is either employed, $e_{i,t} = 1$, or unemployed, $e_{i,t} = 0$. The period-$t$ budget constraint of household $i$ is given by

$$
P_t c_{i,t} + J_t (q_{i,t+1} - (1 - \delta) q_{i,t}) + L_{i,t+1}
= (1 - \tau_t) W_t e_{i,t} + \mu (1 - \tau_t) W_t (1 - e_{i,t}) + D_t q_{i,t} + R_{t-1} L_{i,t},
$$

(1)

where $c_{i,t}$ denotes consumption of household $i$, $P_t$ the price of the consumption good, $L_{i,t+1}$ the amount of the liquid asset bought in period $t$, $R_t$ the gross nominal interest rate on this asset, $W_t$ the nominal wage rate, $\tau_t$ the tax rate on nominal labor income, and $\mu$ the replacement rate. The variable $q_{i,t}$ denotes the amount of equity held at the beginning of period $t$. One unit of equity pays out nominal dividends $D_t$. Firms are identical except that a fraction $\delta$ of all firms go out of business each period. We assume...
that households hold a diversified portfolio of equity, which means that each investor’s portfolio also depreciates at rate $\delta$. When the term $q_{i,t+1} - (1 - \delta)q_{i,t}$ is positive, the worker is buying equity, and vice versa. The nominal value of this transaction is equal to $J_t(q_{i,t+1} - (1 - \delta)q_{i,t})$, where $J_t$ denotes the nominal price of equity ex dividend.

Households are not allowed to take short positions in equity, that is $q_{i,t} \leq 0$. (2)

The household maximizes the objective function

$$
\mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^{t} \left( \frac{c_{i,t}^{1-\gamma} - 1}{1 - \gamma} + \chi \frac{L_{i,t+1} - (1 - \delta)q_{i,t}}{1 - \zeta} \right) \right],
$$

subject to constraints (1), (2), and with $L_{i,0}$ and $q_{i,0}$ given.

The first-order conditions are given as

$$
c_{i,t}^{1-\gamma} \equiv \beta \mathbb{E}_{t} \left[ c_{i,t+1}^{1-\gamma} \frac{P_{t}}{P_{t+1}} \right] + \chi \left( \frac{L_{i,t+1} - (1 - \delta)q_{i,t}}{P_{t}} \right)^{-\zeta},
$$

$$
c_{i,t}^{-\gamma} \geq \beta \mathbb{E}_{t} \left[ c_{i,t+1}^{-\gamma} \left( \frac{D_{t+1} + (1 - \delta)J_{t+1}}{P_{t}} \right) \frac{P_{t}}{P_{t+1}} \right],
$$

$$
0 = q_{i,t+1} \left( c_{i,t}^{1-\gamma} - \beta \mathbb{E}_{t} \left[ c_{i,t+1}^{1-\gamma} \left( \frac{D_{t+1} + (1 - \delta)J_{t+1}}{P_{t}} \right) \frac{P_{t}}{P_{t+1}} \right] \right).
$$

Equation (3) represents the Euler equation with respect to the liquid asset; equation (4) the Euler equation with respect to equity; and equation (5) captures the complementary slackness condition associated with the short-selling constraint in equation (2). 15

Characteristics of the Liquid Asset. The utility function captures the idea that the liquid asset facilitates transactions within the period or more generally provides other benefits than just earning a rate of return. Thus, we could think of the liquid asset as money. We want to allow for a broader interpretation and assume that the liquid asset does earn interest. 16 Important for our mechanism is that an increase in

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15. The utility specification implies that agents will always invest a strictly positive amount in the liquid asset. Short positions in the liquid asset would become possible if the argument of the utility function is equal to $(L_{i,t+1} + \Phi)/P_{t}$ with $\Phi > 0$ instead of $L_{i,t+1}/P_{t}$. At higher values of $\Phi$, agents can take larger short positions in the liquid asset and are, thus, better insured against unemployment risk. Increases in $\chi$—while keeping $\Phi$ equal to zero—have similar implications, because higher values of $\chi$ imply higher average levels of financial assets.

16. Our mechanism would go through if the liquid asset does not earn any interest, that is, when $R_{i} = 1$ in each period. In fact, we reduce the quantitative importance of our mechanism by introducing realistic cyclical behavior for $R_{i}$.
uncertainty about future consumption levels could lead to a precautionary increase in the demand for the liquid asset. Equation (3) shows that this framework allows for this insurance role of the liquid asset in a flexible way. Increased uncertainty about future consumption levels would increase $E_t[c_{t,t+1}^{-\gamma} P_t / P_{t+1}]$, which would put upward pressure on $L_{t,t+1}/P_t$. The parameter $\xi$ controls the strength of this effect. Another salient feature of this setup is that the investment portfolio of the less wealthy will be skewed towards the liquid asset. The utility aspect is also helpful in solving for the households’ portfolio problems. In this economy, agents can only invest in two assets and there is no financial intermediation. Thus, we think of the liquid asset representing a broader category than just money and this broader category could, for example, also include short-term government bonds.

The following characteristics of the liquid asset are important. First, as discussed above, agents hold the liquid asset not only for transactions motives, but also to insure themselves against unemployment risk. Second, it serves as the unit of account. In particular, wages are expressed in units of this asset. This means that real wages are affected if nominal wages do not respond one-for-one to changes in the price level. Third, equilibrium in the market for liquid assets implies that investors as a whole cannot shift into and out of this safer but unproductive asset unless the supply adjusts. The desire to do so when uncertainty about the future increases does play a key role in our model. It is important for our story that there is not an additional agent in the economy who is always willing to absorb risk and thereby channel any increase in aggregate demand for the safe liquid assets into productive but risky investments without asking a premium in return.

3.2. Active Firms

An active firm produces $z_t$ units of the output good in each period, where $z_t$ is an exogenous stochastic variable that is identical across firms. The value of $z_t$ follows a first-order Markov process with a low (recession) and a high (expansion) value.

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17. Monetary frameworks such as shopping time models and transaction costs models incorporate a specific transactions technology, but these can be expressed as models in which the amount of liquidity enters the utility function directly. See Feenstra (1986) and Den Haan (1990) for details. A cash-in-advance model is a special case of these frameworks but is a lot more restrictive in terms of its implications for velocity defined here as the ratio of aggregate consumption over real money holdings. It is essential for our mechanism that the demand for liquidity is not largely pinned down by the current consumption level but can respond to changes in uncertainty about the future through the impact of the latter on the expected MRS.

18. The least wealthy will only hold the liquid assets. They would like to go short in equity but are prevented from doing so because of the short-sale constraint.

19. The transactions component “anchors” the portfolio and avoids large swings in the portfolio decision.

20. Telyukova (2013) documents that households hold more liquid assets than they need for buying goods.

21. Although the model is solvable for richer processes, this simple specification for $z_t$ helps in keeping the computational burden manageable.
There is one worker attached to each active firm. Thus, the mass of active firms, $q_t$, is equal to the economy-wide employment rate. The nominal wage rate, $W_t$, is the only cost to the firm. Consequently, nominal firm profits, $D_t$, are given by

$$D_t = P_t z_t - W_t. \tag{6}$$

**Wage Setting.** The matching friction creates a surplus and we have to take a stand on how this surplus is divided between the worker and the owner of the firm. One possibility is Nash bargaining. Another popular approach is to assume that the relationship is only severed when the surplus is negative and wages remain constant until the situation is such that either wage adjustment is required to prevent the worker or the firm owner from severing the relationship. There are several other possibilities and the empirical literature provides only limited guidance. We want wage setting to be consistent with the following two properties. First, as discussed in Appendix A, there is ample evidence that nominal wages do not fully adjust to changes in the price level. Second and consistent with the matching literature, any wage setting rule should not lead to inefficient breakups, that is, to severance of the relationship while the surplus is positive. Incorporating (nominal) wage stickiness is problematic for the usual Nash bargaining setup, because it assumes that wages are renegotiated every period. Nash bargaining is also computationally challenging in our environment, because individual asset holdings would affect the worker’s bargaining position and thus the wage.\textsuperscript{22} We also do not want to assume that wages are constant through time because there clearly is some adjustment of nominal wages to inflation and real wages to economic activity. Instead we use the following flexible rule for the nominal wage

$$W_t = \omega_0 \left( \frac{z_t}{\bar{z}} \right)^{\omega_z} \bar{z} \left( \frac{P_t}{P^*_t} \right)^{\omega_P} \bar{P}_t, \tag{7}$$

where $\bar{z}$ is the average productivity level, $P_t$ is the price level, and $\bar{P}_t = (1 + \pi)^t$ is the trend price level. The parameter $\omega_P$ controls how responsive wages are to changes in the price level. If $\omega_P$ is equal to 1, for instance, then nominal wages adjust one-for-one to changes in $P_t$. In contrast, nominal wages are entirely unresponsive to changes in $P_t$ if $\omega_P$ is equal to zero. The coefficient $\omega_z$ indicates the sensitivity of the wage rate to changes in productivity and—together with $\omega_P$—controls how wages vary with business cycle conditions. The coefficient $\omega_0$ indicates the fraction of output that goes to the worker when $z_t$ and $P_t$ take on their average values, and pins down the steady-state value of firm profits in real terms.

The specified wage is necessarily in the worker’s bargaining set, because parameters are chosen such that the wage always exceeds unemployment benefits; there is neither home production nor any disutility from working; and the probability of remaining employed exceeds the probability of finding a job. The parameters are chosen such

\textsuperscript{22} If firms pay different wages, then we would have to price heterogeneous firms. As discussed below, pricing a homogeneous firm when its owners are heterogeneous is already quite challenging.
that the wage rate is also in the firm owner’s bargaining set, that is, it is never so high that the firm would rather fire the worker than remain in the relationship.

**Wages of New and Existing Relationships.** What matters in labor market matching models is the flexibility of wages of newly hired workers. Haefke, Sonntag, and van Rens (2013) argue that wages of new hires respond almost one-to-one to changes in labor productivity. Gertler, Huckfeldt, and Trigari (2016), however, argue that this result reflects changes in the composition of new hires and that—after correcting for such composition effects—the wages of new hires are not more cyclical than wages of existing employees. More importantly, however, what matters for our paper is whether nominal wages respond to changes in the price level, and this question is not addressed in either paper. Druant et al. (2009) find that many firms do not adjust wages to inflation. We would think that their results apply to new as well as old matches, because their survey evidence focuses on the firms’ main occupational groups not on the wages of individual workers.

### 3.3. Firm Creation and Equity Market

Agents that would like to increase their equity position in firm ownership, that is, agents for whom \( q_{i,t+1} - (1 - \delta)q_{i,t} > 0 \), can do so by buying equity at the price \( J_t \) from agents that would like to sell equity, that is, from agents for whom \( q_{i,t+1} - (1 - \delta)q_{i,t} < 0 \). Alternatively, agents who would like to obtain additional equity can also acquire new firms by creating them.

How many new firms are created by investing \( v_{i,t} \) real units depends on the number of unemployed workers, \( u_t \), and the aggregate amount invested, \( v_t \). In particular, the aggregate number of new firms created is equal to

\[
h_t = q_{i,t+1} - (1 - \delta)q_t = \psi v_t^\eta u_t^{1-\eta}
\]

and an individual investment of \( v_{i,t} \) results in \( (h_t/v_t) v_{i,t} \) new firms with certainty. There is no risk in this transaction and new firms are identical to existing firms. Consequently, the cost of creating one new firm, \( v_t/h_t \), has to be equal to the real market price, \( J_t/P_t \). Setting \( v_t/h_t \) equal to \( J_t/P_t \) and using equation (8) gives

\[
v_t = \left( \psi \frac{J_t}{P_t} \right)^{1/(1-\eta)} u_t.
\]

Thus, investment in new firms is increasing in \( J_t/P_t \) and increasing in the mass of workers looking for a job, \( u_t \).

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23. An alternative interpretation of the matching friction is that an investment of one unit results in the creation of one firm with probability \((h_t/v_t)\). Our approach implicitly assumes that this matching risk is diversified.
Equilibrium in the equity market requires that the supply of equity is equal to the demand for equity. That is,

\[ h_t + \int_{i \in A_-} ((1 - \delta)q_i - q(e_i, q_i, L_i; s_t))dF_t(e_i, q_i, L_i) \]

\[ = \int_{i \in A_+} q(e_i, q_i, L_i; s_t) - (1 - \delta)q_i) dF_t(e_i, q_i, L_i), \]  

(10)

with

\[ \mathcal{A}_- = \{ i : q(e_i, q_i, L_i; s_t) - (1 - \delta)q_i \leq 0 \}, \]

\[ \mathcal{A}_+ = \{ i : q(e_i, q_i, L_i; s_t) - (1 - \delta)q_i \geq 0 \}, \]

where \( F_t(e_i, q_i, L_t) \) denotes the cross-sectional cumulative distribution function in period \( t \) of the three individual state variables: the employment state, \( e_i \), liquid asset holdings, \( L_i \), and equity holdings, \( q_i \). The variable \( s_t \) denotes the set of aggregate state variables and its elements are discussed in Section 3.6.

Combining the last three equations gives

\[ \psi^{1/(1-\eta)} \left( \frac{F_t}{F_t} \right)^{\eta/(1-\eta)} u_t \]

\[ = \int_{i \in A} (q(e_i, q_i, L_i; s_t) - (1 - \delta)q_i) dF_t(e_i, q_i, L_i), \]  

(11)

where \( A \) is the set of all households, that is, \( A = \{ A_+ \cup A_- \} \).

Our representation of the “matching market” looks somewhat different than usual. As documented in Appendix C, however, it is equivalent to the standard search-and-matching setup. Our way of “telling the story” has two advantages. First, the only investors are households. That is, we do not have entrepreneurs who fulfill a crucial arbitrage role in the standard setup but attach no value to their existence or activities pursued. Second, all agents in our economy have access to the same two assets, firm ownership and money. In contrast, households and entrepreneurs have different investment opportunities in the standard setup. 24

3.4. Government Budget Constraint

The overall government budget constraint is given by

\[ \tau_t q_t W_t + L_{t+1} - R_{t-1} L_t = (1 - q_t) \mu (1 - \tau_t) W_t. \]  

(12)

24. There is one other minor difference. In our formulation, there is no parameter for the cost of posting a vacancy, and there is no variable representing the number of vacancies. Our version only contains the product, that is, the total amount invested in creating new firms. In the standard setup, the vacancy posting cost parameter is not identified unless we have data on vacancies. The reason is that different combinations of this parameter and the scalings coefficient of the matching function can generate the exact same model outcomes as long as the numbers of vacancies are not taken into consideration.
An increase in $q_t$, that is, an increase in employment, means that there is an increase in the tax base and a reduction in the number of unemployed. Both lead to a reduction in the tax rate. UI is the main component affecting the tax rate. The other component consists of net revenues of the central bank. The central bank supplies the liquid asset and sets the interest rate. Its net revenues are equal to $L_{t+1} - R_{t-1}L_t$.

3.5. Monetary Policy and Market for Liquid Assets

In our environment, business cycles are amplified by an interaction of deflationary pressure due to precautionary savings and sticky nominal wages. The obvious monetary policy to fight against this mechanism consists of reducing price level variations. Therefore, we consider a monetary policy that allows for doing so. In particular, we assume that the interest rate rule is set by the central bank according to

$$R_t = v_0 (1 + \pi) \left( \frac{P_t}{\bar{P}_t} \right)^{\nu_P},$$

where $\bar{P}_t$ is the trend price level. The parameter $v_0$ determines the average nominal interest rate and $\nu_P$ the intensity with which the central banks pursue price level stability. The central bank can reduce price and business cycle volatility by increasing $\nu_P$. Because nominal wage rigidity is defined relative to the trend price level and this is the only nominal rigidity, it makes sense for the central bank to also target the price level relative to the long-run trend.

Equilibrium in the market for liquid assets requires that the aggregate demand by households equals the supply by the central bank. That is,

$$L_{t+1} = \int_{l \in A} L(e_i, q_i, L_i; s_i) dF_i(e_i, q_i, L_i).$$

The supply of the liquid asset grows at rate $\pi$. That is, monetary policy mainly affects the economy by changing the interest rate not by changing the supply of the liquid asset. This is consistent with the behavior of monetary aggregates in the Eurozone. In Section 7, we consider the case when $R_t$ is constrained to be nonnegative.

25. During recessions expected inflation increases because aggregate productivity is a mean-reverting process. Consequently, interest rates would rise during recessions if the central bank would target expected inflation. Interest rates decrease during recessions according to our choice of the monetary policy rule.

26. Consequently, the trend price level grows at rate $\pi$ and the steady-state inflation rate equals $\pi$.

27. Specifically, the amount the government has to repay each period scaled by trend supply, $R_{-1}L_i(1 + \pi)^{-t}$, is fixed and equal to $L_t$. This assumption ensures that $R_{-1}$ is not a state variable, which would be the case if, for example, $L(1 + \pi)^{-t}$ is instead held constant. This is an obvious computational benefit. Our approach does imply that there are some changes in the supply of the liquid asset. Relative to the alternative of a fixed growth rate for $L$, these changes weaken our mechanism because supply $L_{t+1}$ increases when $R_t$ is low, that is, when there is deflationary pressure.

28. See Section 7 and Appendix A for a detailed discussion.
That is, the only place in the paper where the level of steady-state inflation, $\pi$, matters. In the remainder of the paper, we will therefore refer to nominal variables as the value scaled by the trend price level.

### 3.6. Equilibrium and Model Solution

In equilibrium, the following conditions hold: (i) asset demands are determined by the households’ optimality conditions, (ii) the cost of creating a new firm equals the market price of an existing firm, (iii) the demand for equity from households that want to buy equity equals the creation of new firms plus the supply of equity from households that want to sell equity, (iv) the demand for the liquid assets from households equals the supply by the central bank, (v) the overall government’s budget constraint is satisfied, and (vi) the interest rate is set according to the central bank’s interest rate rule.

The state variables for agent $i$ are individual asset holdings, employment status, and the aggregate state variables. The latter consists of the aggregate productivity level, $z_t$, and the cross-sectional joint distribution of employment status and asset holdings, $F_t$. We use an algorithm similar to the one used in Krusell and Smith (1998) to solve for the laws of motion of aggregate variables. The numerical procedure is discussed in Appendix F.2.

### 3.7. Discounting Firm Profits Taking into Account Idiosyncratic Risk

With incomplete markets and heterogeneous firm ownership, the question arises how to discount future firm profits. There are two separate issues.

A long outstanding and unresolved debate in corporate finance deals with firm decision making when owners are heterogeneous and markets are incomplete. That is, how should one compare alternative hypothetical future cash flows when firm owners have different marginal rates of substitutions? We avoid this issue in our model even though firms are owned by heterogeneous households. It obviously is not an issue for active firms, because they do not take any decisions. It is also not an issue for creating a new firm, because this is a static decision that does not involve any risk. Consequently, arbitrage ensures that the cost of creating one firm, $v_t/h_t$, equals its market value, $J_t/P_t$.

Firm profits do show up in households’ Euler equations. Because agents have different marginal rates of substitution, they will discount future firm profits differently. In contrast to the first issue, this does not lead to any theoretical challenges. As indicated in equations (3), (4), and (5), theory stipulates that future firm profits should be discounted with the household’s own, individual-specific, MRS, just as future

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29. Without constraints on the nominal interest rate, the model is identical to one in which the steady-state level of inflation is equal to zero after rescaling all nominal variables with the trend price level and adjusting the nominal interest rate with the steady-state inflation rate. See Appendix D for details.

30. See, for example, Grossman and Hart (1979).
returns on the liquid asset are discounted with the household’s own MRS. The fact that there is co-ownership does not make equity holdings different from any other asset. Although this does not raise any theoretical challenges, it does raise computational challenges, because all agents have their own individual-specific MRS. First, one has to solve a portfolio problem. This is nontrivial in environments like ours in which (idiosyncratic) uncertainty plays a key role in portfolio composition. Second, this has to be done in general equilibrium. That is, we have to construct an algorithm that finds the MRS and the demands for both assets for all individual households and equilibrium prices that solve the simultaneous system of equations consisting of Euler equations and equilibrium conditions. At this equilibrium solution, all agents holding equity discount future dividends with the correct, that is, their own individual-specific MRS.

In the literature on precautionary savings and idiosyncratic risk, one can find two approaches to avoid this computational challenge. The first approach assumes some form of communal ownership of the productive asset, with fixed ownership shares that can never be sold no matter how keen an agent would be to do so. Investment decisions in the productive asset are then reduced to one aggregate investment decision, that is, it is determined by only one “Euler equation” using an MRS based either on aggregate consumption; on an average of the MRS of all agents; or on risk neutral geometric discounting.31 The second approach assumes that there exist two distinct types of agents: One faces idiosyncratic risk but cannot invest in the productive asset, whereas the other can invest in the productive asset but is not affected by any type of idiosyncratic risk. Because there is no ex-post heterogeneity within the group of the latter type, their analysis lends itself to a representative agent, which then dictates the aggregate investment decisions in the economy.32

Both approaches simplify the analysis considerably, but both limit our understanding of the effect of idiosyncratic risk on business cycles through precautionary savings. The reason is the following. Productive assets, such as firms in our model, generate a flow of profits and a reduction in real interest rates (or discount rates more generally) put upward pressure on the value of such productive assets, which would stimulate investment in this asset. When an increase in precautionary savings lowers discount rates, this would stimulate investment in the productive asset, not depress it. Increased investment in the productive asset typically leads to an increased demand for labor, which in turn would reduce unemployment and idiosyncratic uncertainty. This effect is very direct in labor market matching models in which investment in the productive asset is equivalent to job creation.33

32. Examples are Rudanko (2009), Bils et al. (2011), McKay and Reis (2016), Challe et al. (2017), and Challe and Ragot (2016).
33. But an increase in capital investment would also lead to an increase in the demand for labor if it increases the marginal product of labor.
We avoid the unresolved corporate finance issue by having an environment in which firm profits follow directly from the law of motion for productivity and the wage rule. However, we do not make simplifying assumptions in terms of how heterogeneous agents value future firm profits. That is all agents evaluate future firm earnings with the correct, that is, their \textit{individual-specific} MRS.\footnote{Krusell et al. (2010) also describe a procedure in which agents discount firm profits with their own individual-specific MRS. However, their procedure requires that the number of assets is equal to the number of realizations of the aggregate shock and that borrowing and short-sale constraints are not binding. Firm profits can then be discounted with the prices of the two corresponding contingent claims. Our procedure allows investors to be constrained and the number of realizations of the aggregate shock can exceed the number of assets.} Consequently, we can analyze the question whether an increase in precautionary savings induced by an increase in idiosyncratic uncertainty could lead to an increased demand for equity as well as an increase in the demand for the liquid asset. As discussed below, key for the answer to this question is the difference in risk characteristics of the two assets.

4. Calibration

We start with a discussion of model parameters for which we can directly calibrate the appropriate values. Next we discuss the remaining parameters for which the values are chosen indirectly to ensure that the model aligns well with a set of empirical observations. Finally, we discuss parameters for the representative-agent model. The calibration targets are constructed using Eurozone data and the model period is one quarter.\footnote{We use data from 1980 to 2012, whenever possible. Details are given in Appendix B.}

4.1. Directly Calibrated Parameters

UI regimes vary a lot across Eurozone countries. Esser et al. (2013) report that net replacement rates for insured workers vary from 20\% in Malta to just above 90\% in Portugal. Most countries have net replacement rates between 50\% and 70\% with an average duration of around one year. Coverage ratios vary from about 50\% in Italy to 100\% in Finland, Ireland, and Greece. Net replacement rates for workers that are not covered by an UI scheme are much lower. In most countries, replacement rates of workers only receiving unemployment/social assistance are less than 40\%. In the model, the replacement rate, $\mu$, is set equal to 50\% and—for computational convenience— is assumed to last for the entire duration of the unemployment spell. A replacement rate of 50\% is possibly a bit less than the average observed, but this is compensated for by the longer duration of unemployment benefits in the model and the universal coverage.

The curvature parameter in the utility component for liquidity services, $\zeta$, plays an important role, because it directly affects the impact of changes in future job security
on the demand for the liquid asset. With more curvature, the demand for the liquid asset is less sensitive to changes in the expected MRS and increased concerns about future job prospects would generate less deflationary pressure. We follow Lucas (2000) and target a demand elasticity equal to $-0.5$. The resulting value of $\zeta$ is equal to 2.\textsuperscript{36}

The other key parameter in money demand functions is the elasticity with respect to a transactions volume measure. Our transactions variable is consumption and the elasticity of demand for the liquid asset with respect to consumption is equal to $\gamma/\zeta$, where $\gamma$ is the coefficient of relative risk aversion. By setting $\gamma = 2$, this elasticity takes on the standard unit value and we are conservative in terms of agents’ risk aversion.\textsuperscript{37,38}

A key aspect of the mechanism emphasized in this paper is that nominal wages do not fully adjust to changes in the price level, that is, we need $\omega_p < 1$. Direct estimates of $\omega_p$ are not available. We want to make a conservative choice, because this is such a key parameter. Our benchmark value for $\omega_p$ is equal to 0.7, which means that a 1% increase in the price level leads to an 0.7% increase in nominal wages. This seems conservative given that Druant et al. (2009) report that only 6% of European firms adjust wages (of their main occupational groups) more than once a year and only 50% do so once a year.\textsuperscript{39}

The two values for $z_t$ are 0.978 and 1.023 and the probability of switching is equal to 0.025. With these values, we match the key characteristics of the typical process for exogenous productivity.\textsuperscript{40} Although we rely on regular magnitudes for changes in $z_t$, a sustained drop in $z_t$ generates a drop in aggregate output that resembles the observed drop in Eurozone output during the great recession. That is, the deflationary mechanism magnifies shocks considerably.

Based on the empirical estimates in Petrongolo and Pissarides (2001), the elasticity of the job finding rate with respect to tightness, $\eta$, is set equal to 0.5. In our model, the

\textsuperscript{36} The demand elasticity of $L_{i,t+1}$ with respect to the expected MRS is equal to the demand elasticity with respect to the interest rate of an asset with a risk-free nominal payoff and no liquidity benefits, $\bar{R}_t$. The Euler equation of this (hypothetical) asset is given by

$$
1 = \beta \bar{R}_t E_t \left[ \left( \frac{c_{t+1}}{c_{t,t}} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \right].
$$

Using $R_t/ \bar{R}_t \approx 1 + R_t - \bar{R}_t$ and equation (3), we get

$$
\ln \left( \frac{L_{i,t+1}}{P_t} \right) \approx -\zeta^{-1} \ln \left( \bar{R}_t - R_t \right) + \zeta^{-1} \left( \ln \chi + \gamma \ln c_{t,t} \right).
$$

\textsuperscript{37} For example, Lucas (2000) imposes a unit elasticity.

\textsuperscript{38} This elasticity is also equal to 1 in standard cash-in-advance models. Our framework is more flexible than a cash-in-advance model, because we can vary $\zeta$ to ensure the right sensitivity of demand for the liquid asset to the expected MRS.

\textsuperscript{39} Moreover, even if firms adjust wages for inflation they typically do so using backward looking measures of inflation, which reduces the responsiveness to changes in inflationary pressure.

\textsuperscript{40} That is, $\mathbb{E}[\ln z_t] = 0$, $\mathbb{E}_t[\ln z_{t+1}] = 0.95 \ln z_t$, and $\mathbb{E}_t[\ln z_{t+1} - \mathbb{E}_t[\ln z_{t+1}]]^2 = 0.0072$. 


TABLE 1. Summary of directly calibrated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>Replacement rate</td>
<td>0.5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.98</td>
</tr>
<tr>
<td>$I$</td>
<td>Steady-state supply of liquid assets</td>
<td>0.215</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Steady-state inflation rate</td>
<td>0.0022</td>
</tr>
<tr>
<td>$\nu_0$</td>
<td>Steady-state gross real interest rate</td>
<td>1.005</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Coefficient of risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Curvature liquidity utility component</td>
<td>2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of matching function</td>
<td>0.5</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>Nominal wage rigidity</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Notes: This table lists the directly calibrated parameter values of the model. The motivation is described in the main text. The process for productivity, $z$, is such that $z \sim \{0.9778, 1.0227\}$, and $Pr(z_{t+1} = z_t) = 0.975$. One period in the model corresponds to one quarter. The remaining parameters can be found in Table 2.

presence of idiosyncratic risk lowers average real rates of return. To ensure that real rates of return are not unrealistically small, we set the discount factor, $\beta$, equal to 0.98, which is below its usual value of 0.99. 41 The average inflation rate in the Eurozone over the 1999–2007 period is equal to 0.87% on an annual basis (using the GDP deflator). The steady-state value of $\pi$ is set to match this. The value of $\nu_0$ is chosen such that the steady-state real return on the liquid asset is equal to 50 basis points on a quarterly basis. 42 Finally, the scaling coefficient affecting the trend value of the liquid asset, $I$, is chosen such that the average value of the price level relative to its trend value, $P_t / \bar{P}_t$, is equal to 1. Table 1 provides a summary of the directly calibrated parameters.

4.2. Calibration Targets and Indirectly Chosen Parameters

For the quantitative importance of our mechanism to be credible, we want to ensure that our choice of parameter values does not exaggerate the importance of unemployment risk. Given the emphasis on deflationary pressure and labor costs, we want our model to have reasonable predictions for these variables as well.

Regarding the importance of idiosyncratic risk, we want our model to be consistent with four empirical properties. The unemployment rate should not increase to unrealistically high levels during recessions. If that would be the case, then we would exaggerate the magnitude of changes in idiosyncratic risk over the business cycle. Consequently, it is important that our model generates a reasonable amount of volatility in the cyclical employment rate relative to the volatility of the cyclical output component. For the same reason, we want the average unemployment rate and the

41. At this relatively high 8% annual discount rate, the average real rate of return on equity is 3.06% on an annual basis in the economy with aggregate uncertainty.

42. The steady-state annual nominal interest rate is equal to 2.90%. The ECB deposit rate for overnight deposits fluctuated between 1% and 3.75% in the period from 1999 to 2007. The slightly higher level of the real interest rate in our model helps agents insure against idiosyncratic risk and weakens our mechanism.
### Table 2: Summary of indirectly calibrated parameters.

<table>
<thead>
<tr>
<th>Calibration target</th>
<th>Data (range)</th>
<th>Model Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment duration</td>
<td>[3.54, 3.71]</td>
<td>3.64</td>
<td>$\omega_0$</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>10.7%</td>
<td>10.7%</td>
<td>$\omega_z$</td>
</tr>
<tr>
<td>Wealth to income ratio</td>
<td>[0.213, 0.312]</td>
<td>0.243</td>
<td>$\chi$</td>
</tr>
<tr>
<td>Volatility of employment to output</td>
<td>[0.39, 0.67]</td>
<td>0.51</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Deflationary pressure</td>
<td>[−3.25%, −1.26%]</td>
<td>−2.61%</td>
<td>$\nu_p$</td>
</tr>
<tr>
<td>Real unit labor cost increase</td>
<td>[3.74%, 4.38%]</td>
<td>4.26%</td>
<td>$\psi$</td>
</tr>
</tbody>
</table>

Notes: This table lists the empirical targets and their corresponding model moments. The six calibrated parameters are jointly determined and do not correspond one-by-one to a specific target. Their listed order is therefore irrelevant. The calculations and targets are described in the main text.

The average length of an unemployment spell to be close to their empirical counterparts. Finally, we ensure that agents are not unrealistically poor. 43

The last two targets are related to the behavior of prices and labor costs. As discussed in Section 2, the Eurozone economy was characterized by deflationary pressure as well as increases in real labor costs during the great recession. Consequently, we want the model to generate a nontrivial decline in the price level and an increase in real unit labor costs relative to their trend values when economic activity deteriorates.

There are six parameters that are not yet determined and these are used to ensure that the model is consistent with aforementioned targets. Those parameters are the job destruction rate, $\delta$, the parameter characterizing the efficiency in the matching market, $\psi$, the parameter that controls the share of firm revenues going to workers, $\omega_0$, the slope coefficients in the wage setting rule characterizing the sensitivity to changes in $z_t$, $\omega_z$, the scale parameter that characterizes the liquidity benefits of money, $\chi$, and the parameter in the monetary policy rule that affects the sensitivity of the interest rate to inflationary pressure, $\nu_p$.

Table 2 reports the chosen parameter values as well as the calibration targets and the model counterparts. In the remainder of this section, we present a detailed discussion of the empirical targets and the choice of parameter values. 44

**Targets for Individual Wealth Levels.** There is not a lot of empirical guidance on what the empirical counterparts are for wealth levels at the start of unemployment spells. Therefore, we have constructed some evidence ourselves using data from the Eurosystem Household Finance Consumer Survey (HFCS). 45

The HFCS provides household- and individual-level data collected from more than 62,000 households in a harmonized way for 15 Eurozone countries in 2010. In our

43. In contrast, agents in the model of Ravn and Sterk (2017) have zero wealth and cannot smooth consumption at all. McKay, Nakamura, and Steinsson (2016) and Auclert (2017) impose a zero aggregate wealth to income ratio.

44. In Appendix B, we provide additional details on the data used to construct the empirical targets.

45. See Appendix B.3 for details on how we constructed the statistics reported here.
TABLE 3. Median financial wealth to annual labor income ratios.

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
<th>Expansion</th>
<th>Recession</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross wealth to income ratio</td>
<td>0.312</td>
<td>0.342</td>
<td>0.243</td>
<td></td>
</tr>
<tr>
<td>Net wealth to income ratio</td>
<td>0.213</td>
<td>0.342</td>
<td>0.243</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table shows the ratio of the indicated financial wealth variable to annual labor income. Details are given in Appendix B.3.

model, workers only hold two financial assets, the liquid asset and equity. In reality individuals also hold other assets, the most important one being real estate. We exclude this wealth component as it is likely to be too illiquid to be a useful source of insurance against the decline in income experienced during a typical unemployment spell. Similarly, we ignore mortgage debt and other collateralized debt when constructing our measure of net financial wealth under the assumption that collateralized debt is not under pressure to be redeemed during unemployment spells. The financial assets we include are bank deposits, both sight accounts and savings accounts, and an extended set of financial assets that we could in principle use for consumption smoothing because they can be relatively easily sold.

The HFCS only reports income before taxes. Workers pay taxes in our model, but these are very low, because taxes are only used to finance unemployment benefits and interest payments. To make sure that the observed data are comparable to the model data, we use after tax income for both and we use a tax rate of 29.5% for the HFCS income data.46

Table 3 provides information on the median value for the financial wealth to income ratio.47 The table indicates that the median agent’s level of financial assets is sufficient to cover the income loss during an unemployment spell lasting 2.5 quarters.48 As indicated in Table 2, this is at least one quarter less than the average unemployment spell for European workers. Using net financial assets, the median worker only has enough funds to cover an unemployment spell lasting 1.7 quarters. Thus, the data

46. This rate is an average of the average income tax rate for a set of 18 Eurozone countries using 2015 data from the OECD.

47. We focus on households with either one or two individuals employed and nobody unemployed. In our model, all workers have the same labor income. That is, of course, not true in the data. If we look at the median value for the wealth to income ratio for the whole sample, then we would find that both some rich and some poor agents have wealth to income ratios above the median. The rich for obvious reasons and the poor because some households in the data have quite low labor income, but still some bank deposits. To make sure that this is not an issue, we divided the data into ten income deciles according to their labor income and calculated the median for each decile. The average across deciles is equal to 0.216 for net financial assets and 0.306 for financial assets. These numbers are very similar to the ones reported here. Appendix B.3 provides additional details on the data used.

48. Using a net replacement rate of 50%.
indicate that the median household does not have a lot of financial wealth to insure against unemployment spells.\textsuperscript{49}

The HFCS data are for 2010. Because the data are from a recession year, it would make most sense to focus on the model outcomes for a recession as well. In our model, agents cannot borrow, so net financial assets are equal to total financial assets. Whether we should compare the model outcome with gross financial assets or financial assets net of liabilities depends on whether we think that these liabilities could be easily rolled over during unemployment spells. Kolsrud et al. (2015) report that the amount of consumption that is financed out of an increase in debt actually decreases following a displacement. Our strategy is to choose parameter values such that the average level of the ratio of financial assets over income for employed workers is in between the empirical counterparts using net and gross financial assets.

\textit{Other Calibration Targets Related to Idiosyncratic Risk.} Observed volatility of cyclical employment relative to observed volatility of cyclical output varies between 0.39 and 0.67 for the set of Eurozone countries for which quarterly data for a sufficiently long period are available.\textsuperscript{50} The observed average Eurozone unemployment rate is equal to 10.7\%.

The last target related to idiosyncratic risk is the average duration of unemployment spells. Most data sources divide the unemployed workers into different groups, for example, those that are unemployed for less than one month or more than one year. Direct information is provided in Pellizzari (2006) and Tatsiramos (2009) who use individual data from different waves of the European Community Household Panel. Pellizzari (2006) reports estimates for average unemployment duration for European workers ranging from 3.54 to 3.71 quarters.\textsuperscript{51}

\textit{Calibration Targets for Deflationary Pressure and Labor Costs Increase.} Figure 2 documents that the observed Eurozone price level dropped considerably relative to its trend value during the great recession. In the beginning of 2010, the gap is 1.26\%. At the end of the sample, the gap has increased to 3.25\%. Over the same period, real unit labor costs increased from a level that was 3.74\% above its trend value to a level that was 4.38\% above its trend level. We choose parameters such that the price drop and the change in real unit labor costs are in the specified ranges during a sustained recession.\textsuperscript{52}

\textsuperscript{49} Using US data, Gruber (2001) finds that the median agent holds enough gross financial assets to cover 73\% of the average net-income loss during an unemployment spell. In terms of net financial assets, the median agents does not even have enough to cover 10\% of the average net-income loss.

\textsuperscript{50} See Appendix B for details.

\textsuperscript{51} Tatsiramos (2009) reports estimates for a few individual Eurozone countries ranging from 2.83 to 4.80 quarters.

\textsuperscript{52} What really matters are the changes relative to the drop in GDP, but our model does generate a drop in GDP that is comparable to the one observed in the great recession for the Eurozone area.
Values of Calibrated Parameters and Model Outcomes. It should be emphasized already at this point that each parameter of the model is not directly related to one moment of the data, but rather to several. As such, our calibration strategy involves choosing an array of parameters such that the model aligns well with an array of moments. The mapping between parameter values and model properties is complex, and we need a costly computer algorithm to figure out what the model predictions are for a given set of parameter values.\(^53\)

Key parameters affecting the volatility of employment are \(\omega_0\) and \(\omega_z\). Decreasing \(\omega_z\) and/or increasing \(\omega_0\) increase employment volatility by making profits more volatile. If we choose a high value of \(\omega_z\), then we would need a higher value of \(\omega_0\) to get enough employment volatility. But a high value of \(\omega_0\) means a low profit share and a low value of equity holdings, a key household wealth component. For our results to be credible, it is important that agents are not too poor, which prevents us from setting \(\omega_0\) too high, which in turn prevents us from setting \(\omega_z\) too high. The calibrated values for \(\omega_0\) and \(\omega_z\) are equal to 0.967 and 0.25, respectively.

The parameter \(\omega_0\) and the parameter characterizing the liquidity benefits of the liquid asset, \(\chi\), are important for the wealth of individual agents. At the chosen parameter values, the model generates a value for the median wealth level that is in the target ranges for this statistic. Specifically, the median agent in our model is not unrealistically poor. Agents in our model are a bit richer than their empirical counterparts when we consider net financial assets. The two financial wealth ratios are 0.24 (model) and 0.22 (data). When we consider gross financial assets, then the opposite is true and the two statistics are 0.24 (model) and 0.31 (data).

Our calibration strategy is conservative by focusing on the median agent, because there are quite a few workers in the data that have a lot less assets than the median agent, which is much less true in the model. For example, the observed fifth and tenth percentile values for gross financial assets over labor income are equal to 0 and 0.012, respectively, whereas, they are equal to 0.076 and 0.117 in the model.

The average unemployment rate in the model matches its empirical counterpart exactly. The model equivalent for the average duration is equal to 3.64 quarters, which is in the middle of the target range.

The model predicts a sharp drop in the price level followed by a quick partial recovery if the economy starts out in an expansion and then faces the low productivity outcome for a sustained period. After 10 quarters, the price level is 3.1% below its starting value and after 20 quarters it has basically stabilized at a level that is 2.6% below its starting value. Although the sharp initial drop generated by the model is not observed in the data, we are conservative in terms of the longer term deflationary pressure that we consider. Recall that the observed variation in the price level was a drop equal to 3.25% that is a significantly larger drop than the one predicted by the model. Several parameter values are important for the behavior of prices in the

53. Given the complexity of the numerical algorithm, we cannot simply use a minimization routine that will find the best parameter values given a set of desired model properties.
model. For prices to respond negatively to a negative productivity shock, we need the parameters related to precautionary saving and demand for the liquid asset to be such that our mechanism is sufficiently strong. For the price drop to be sufficiently pronounced, it is also important that monetary policy does not fully undo changes in prices. This is a reasonable property to impose on the model, because the European Central Bank (ECB) was not capable of avoiding some deflationary pressure in the Eurozone. Our choice of \( \nu_p \) ensures that monetary policy reduces the drop in the price level during a prolonged recession by 51%. Thus, monetary policy has a nontrivial impact on model outcomes.

Parameter Values in the Representative-Agent Model. We will compare the results of our model with those generated by the corresponding representative-agent economy. Parameter values in the representative-agent model are identical to those in the heterogeneous-agent model, except for \( \beta \). We choose the value of \( \beta \) for the representative-agent model such that average employment equals its respective value in the heterogeneous-agent model.\(^{54}\)

5. Agents’ Consumption, Investment, and Portfolio Decisions

In Section 5.1, we describe key aspects of the behavior of individual consumption, and in particular its behavior during an unemployment spell. In Section 5.2, we focus on the individuals’ investment portfolio decisions. Appendix E provides more detailed information on both topics.

5.1. Postdisplacement Consumption

Consumption during the first year of an unemployment spell is on average 25.2% below predisplacement levels. Although not an explicit calibration target, we would like our model to have a reasonable outcome in this dimension. Unfortunately, we are not aware of any studies documenting representative information on the magnitude of consumption drops during unemployment spells for the Eurozone. However, Kolsrud et al. (2015) study consumption behavior of a particular group of Swedish unemployment workers for whom individual consumption dropped by on average 26.3% after one year of unemployment. Using household consumption levels, this drop is equal to 15.8%.\(^{55}\) A drawback of this study is that it focuses on a group of workers with a relatively high replacement rate of 72%, which exceeds the average level for the

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54. The resulting value of \( \beta \) in the representative-agent model is equal to 0.99. Without this adjustment, the agent in the representative-agent economy would have a more shortsighted investment horizon due to a lower intertemporal MRS and average employment would be lower.

55. This is the average consumption level during the first year of an unemployment spell relative to the average consumption level during the first two years before the start of the unemployment spell for workers that were employed in all eight quarters before the unemployment spell.
Eurozone and Sweden as well as the 50% replacement rate in the model. Moreover, it focuses on the precrisis period. We could expect consumption drops to be larger during recessions. Furthermore, everything else being equal, higher replacement rates should facilitate consumption smoothing. However, higher replacement rates may reduce precautionary savings that hampers consumption smoothing. Despite the caveats in making a comparison, we think that the Swedish study provides some support for our model’s prediction regarding the order of magnitude for postdisplacement consumption drops.

There are several reasons why the fall in consumption is of such a nontrivial magnitude. One reason is, of course, that unemployment benefits are only half as big as labor income. But a key factor affecting the magnitude of the drop is the average level of wealth at the beginning of an unemployment spell. We calibrated our parameters such that median holdings of financial assets relative to income are in line with their empirical counterparts. However, in the data and in the model, workers are not well insured against unemployment. In our model, the median agent’s asset holdings are equal to 58.2% of the average net-income loss during unemployment spells.\(^\text{56}\)

The question arises why infinitely lived agents do not build a wealth buffer that insulates them better against this consumption volatility, as is the case in the model of Krusell and Smith (1998).\(^\text{57}\) By choosing a low value for the scaling coefficient affecting the utility of liquid assets, \(\chi\), we directly limit the magnitude of one of the two wealth components. The other wealth component is the value of equity holdings, \(\text{J}_{t,q_{i,t}}\). Elevated uncertainty about future individual consumption increases the expected value of an agent’s MRS, which would increase the price of equity, \(\text{J}_t\). As a consequence, the number of new firms as well as the total number of shares outstanding would therefore rise. However, there are several reasons why this component of wealth is not very large in our model. First, equity returns are very volatile in our model making it an unattractive asset to insure against income losses. Second, the equity price, \(\text{J}_t\), cannot increase by too much, because the presence of a liquid asset with a positive transactions benefit puts a lower bound on the average real return on equity. Moreover, the nonlinearity of the matching function dampens the impact of an increase in equity prices on the creation of new firms. Lastly, the equity price depends positively on the average share of output going to firm owners, \(1 - \omega_0\). To generate sufficient volatility in employment, we chose a relatively high value for \(\omega_0\), which reduces the value of \(\text{J}_t\).

For all these reasons, agents in our model do not build up large buffers of real money balances or equity to insure themselves against the large declines in consumption upon and during unemployment.

Another aspect affecting consumption during unemployment is the ability to borrow. In our model, agents cannot go short in any asset, and they would presumably

\(^{56}\) Whereas the calibration focused on wealth levels during recessions, this is the unconditional average and, thus, includes both recessions and expansions.

\(^{57}\) For the model of Krusell and Smith (1998), solved in Den Haan and Rendahl (2010) using a 15% unemployment replacement rate, the average postdisplacement consumption level has dropped by only 5% after one year of being unemployed.
hold less financial assets if they had the option to borrow. Kolsrud et al. (2015) report, however, that the amount of consumption that is financed out of an increase in debt actually decreases following a displacement.

State Dependence of Consumption Drop. Figure 3 presents a scatter plot of the reduction in consumption (y-axis) and beginning-of-period cash on hand (x-axis), where both are measured in the period when the agent becomes unemployed. There are two distinct patterns, one for expansions and one for recessions. Figure 3 documents that the drop in consumption is, on average, much more severe if the unemployment spell initiates in a recession. The figure also underscores the nontrivial role played by the agents’ wealth levels. In particular, during recessions, the decline in consumption varies from 20.0% for the richest agent to 35.9% for the poorest. This

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58. Cash on hand is equal to the sum of nonasset income (here unemployment benefits), money balances, dividends, and the value of equity holdings.

59. The level of employment is also important for the observed decline in consumption, which explains the scatter of observations. In particular, the fall in the level of consumption is smaller at the beginning of an expansion and larger at the beginning of a recession. The reason is that expected investment returns are higher (lower) at the beginning of the expansion (recession), which would put upward (downward) pressure on consumption when the income effect dominates the substitution effect.
range increases during an expansion: The richest agent faces a modest drop of 9.46%, whereas the poorest agent can expect to see consumption fall by basically the same amount as in a recession.

There are several reasons why consumption falls by more during recessions. First, job finding rates are lower in recessions than in expansions. As a consequence, agents anticipate longer unemployment spells and will, for a given amount of cash on hand, therefore reduce consumption more sharply. A second factor that affects agents’ reduced consumption is the amount of cash on hand they hold. Because the price of equity declines in recessions, so does the agents’ cash on hand. Indeed, the average value of cash on hand held by a newly unemployed agent in a recession is 23.8% below the corresponding number in an expansion.

In reality, a typical worker may not face such a large decline in the value of their equity position when the economy enters a recession. After all, quite a few workers do not own equity at all. We think, however, that the cyclicality of postdisplacement consumption behavior that is driven by the cyclicality of equity prices captures real world phenomena. First, although not all workers hold equity, many hold assets such as housing that also have volatile and cyclical prices. Second, unemployed workers may receive loans, and or handouts, from financial intermediaries, affluent family members, or friends whose ability and willingness may be affected by the value of their assets.

5.2. Investment Decisions

In our heterogeneous-agent model, the demand for the liquid asset increases during recessions, whereas it decreases in the representative-agent version. In this section, we shed light on this difference.

Cash on Hand and Demand for the Liquid Asset. A key result of this paper is that the interaction between sticky nominal wages and the inability to insure against unemployment risk deepens recessions. An integral part of the mechanism underlying this result is the upward pressure on demand for the liquid asset that emerges during recessions when job prospects worsen. Consumption smoothing motives reduce demand for all assets, including the liquid asset, during recessions. With incomplete markets, however, there are two additional mechanisms. These lead to an increase in the aggregate demand for the liquid asset and are strong enough to more than offset the reduction induced by the standard consumption smoothing motive. First, as documented in Figure 4, for given cash-on-hand levels, all agents demand more of the liquid asset during recessions. This is consistent with the observed shift towards more liquid assets during the great recession. Countercyclical demand for the liquid asset stands in sharp contrast with the representative-agent version of our economy, in which aggregate demand for the liquid asset unambiguously decreases during recessions.60

Second, for equal cash-on-hand levels unemployed agents demand more of the liquid

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60. Cf. Bayer et al. (2014).
FIGURE 4. Demand for liquid asset (real). This figure displays the amount invested in the liquid asset as a function of beginning-of-period cash on hand for workers of the indicated employment status and for both outcomes of aggregate productivity.

asset than employed agents, and there are more unemployed agents in the economy during recessions.

To see that this is a remarkable result, consider the economy without aggregate uncertainty. The price level, wages, and the equity price are all constant in this economy. The first-order condition for equity, equation (4), then implies that the expected MRS does not depend on the employment status for any of the agents that are not at the short-sale constraint.\footnote{In this case, equation (4) can be rearranged as

\[
J = \beta (D + (1 - \delta)J) \mathbb{E}_t \left[ \left( \frac{e_{i,t+1}}{e_{i,t}} \right)^{-\gamma} \right],
\]

which implies that the individual MRS, \( \beta \mathbb{E}_t \left[ \left( e_{i,t+1}/e_{i,t} \right)^{-\gamma} \right] \), is pinned down by aggregate prices only, and is therefore not affected by employment status.} The first-order condition for the liquid asset, equation (3), then specifies that the real value of the liquid asset and consumption must move in the same direction. Because unemployed agents consume less than employed agents with
the same level of cash on hand, they should hold less of the liquid asset. Although this simple reasoning abstracts from the presence of the short-sale constraint, the implication is that absent aggregate uncertainty unemployed agents would hold less of the liquid asset than employed agents with the same amount of cash on hand. In contrast, as indicated in Figure 4, the opposite is true in the economy with aggregate uncertainty.

Liquid Asset Holdings During Unemployment Spells. The analysis above focused on demand for the liquid asset taking cash-on-hand levels as given. Here, we discuss the demand for liquid assets during an unemployment spell taking into account the fall in cash on hand. Consumers cushion the drop in consumption following displacement by selling assets. Although the total amount of financial assets, and the amount invested in equity, sharply decrease, the amount held in the liquid asset actually increases during the first two quarters of an unemployment spell. The loss of labor income means that workers’ cash-on-hand levels drop when they become unemployed. This reduces the demand for the liquid assets. However, and as discussed above, the unemployed actually hold more liquid assets for a given level of cash on hand. The last effect dominates in the beginning of an unemployment spell.

6. Economic Aggregates Over the Business Cycle

In the previous section, we showed that the inability of agents to insure against unemployment risk means that workers face a sharp drop in consumption when they become unemployed. We also discussed how imperfect insurance affects demand for the safer liquid asset in ways that are not present in an economy with complete markets. In this section, we discuss what this implies for aggregate activity. In particular, we document and explain why the interactions between sticky nominal wages, gloomy outlooks regarding future employment prospects, and the inability to insure against unemployment risk can deepen recessions. We compare the business cycle properties of the economy with imperfect risk sharing to those of an economy with full risk sharing. We first do this when nominal wages are sticky, that is, when $\omega_p < 1$. Subsequently, we discuss the same comparison when nominal wages are not sticky, i.e., $\omega_p = 1$.

62. A similar reasoning can be used to make clear that it is also remarkable that employed as well as unemployed agents hold more liquid asset when aggregate economic conditions deteriorate. Envisage a partial equilibrium version of our model with aggregate uncertainty in which all prices are constant, and there is no short-sale constraint. Markets are still incomplete because the agents cannot insure fully against unemployment risk. As discussed in the text, consumption and demand for the liquid asset move in the same direction under these conditions. But the reduction in the job finding rate lowers consumption. Thus, in real terms the demand for the liquid asset must decrease as well.

63. See Appendix E.
FIGURE 5. Impulse responses with sticky nominal wages. These graphs illustrate the difference between the expected time path when the economy is in an expansion in period 0 and the expected time path conditional on being in a recession in period 1. $\omega_p = 0.7$, that is, nominal wages increases with 0.7% when prices increase with 1%.

6.1. The Role of Imperfect Insurance When Nominal Wages are Sticky

Figure 5 shows the impulse response functions (IRFs) of key aggregate variables to a negative productivity shock for our benchmark economy and for the corresponding representative-agent economy. The responses for output and employment document that the economy with incomplete risk sharing faces a much deeper recession than the economy with complete risk sharing. In particular, in response to a 4.3% drop...
in productivity, expected employment falls by 3.19% in the heterogeneous-agent economy and by only 1.81% in the representative-agent economy.\textsuperscript{64}

The key aspect in understanding this large difference is the behavior of the price level. In the representative-agent economy, the reduction in real activity decreases the demand for the liquid asset and \textit{increases} the price level. In our benchmark calibration wages are sticky ($\omega_p = 0.7$), and a 1% increase in the price level leads to a 0.7% increase in nominal wages and therefore a 0.3% \textit{decrease} in real wages. Figure 5 documents that the real wage drops by 2.0% in the representative-agent economy, which is driven by the direct effect of the reduction in $z_t$, because $\omega_z > 0$, and the indirect effect through $P_t$, because $\omega_p < 1$. Thus, the direct effect of the reduction of productivity, $z_t$, on profits is counteracted, because nominal wages do not fully respond to the increase in the price level. That is, our starting point is an economy in which the sluggish response of nominal wages to changes in prices actually \textit{dampens} the economic downturn.

In contrast, the price level falls in the heterogeneous-agent economy. This fall is caused by an \textit{increase} in the aggregate demand for the safer asset. As documented in Figure 5, the heterogeneous-agent model predicts a much smaller decrease in the real wage rate. In fact, initially the real wage rate \textit{increases} because the indirect effect through deflationary pressure is so strong that it dominates the direct effect of the reduction in $z_t$. Overall, real wages are procyclical. The correlation between real wages and output is equal to 0.24. After applying the Hodrick-Prescott filter the correlation drops to $-0.24$.\textsuperscript{65} Similarly, real unit labor costs increase by more when markets are incomplete because deflationary pressure induced by increased idiosyncratic risk increases real wage costs when $\omega_p < 1$.\textsuperscript{66} Real unit labor costs are countercyclical, also after HP-filtering.\textsuperscript{67}

Whereas sticky nominal wages reduce the depth of recessions in the representative-agent economy, they worsen recessions in the heterogeneous-agent economy. This is a quantitatively important effect, because a reduction in the price level (for any reason) starts a self-reinforcing process that deepens recessions. In particular, the reduction

\textsuperscript{64.} In our benchmark calibration, productivity takes on only two values. The IRFs are calculated as follows. The starting point is period $s$, when productivity takes on its “expansion” value and employment is equal to its limiting value, that is, its value when the economy has been in an expansion for a long time. We then calculate the following two time paths for each variable. The “no-shock” time path is the expected time path from this point onward. The “shock” time path is the expected time path when the productivity switches to the low value in period $s + 1$. The IRF is the difference between these two time paths.

\textsuperscript{65.} Small changes in the value of $\omega_z$ can change these numbers substantially without affecting the main results. For example, using a calibration for which $\omega_z = 0.3$ and all calibration targets are met, the correlation coefficients are equal to 0.74 and 0.04 for the filtered and unfiltered series, respectively. For this calibration, the short-lived initial increase in the real wage rate becomes less important and the subsequent sustained decline becomes more important. The latter (procyclical) component is less important for filtered series because it is largely picked up by the trend term.

\textsuperscript{66.} Real unit labor costs increase because $\omega_z < 1$ and because $\omega_p < 1$.

\textsuperscript{67.} Real unit labor costs are calculated as total wages over total output. If we include hiring costs, then real unit labor costs are slightly less countercyclical. Although hiring workers is cheaper when there are more unemployed workers these costs are too small relative to the wage bill to have a big impact.
in the price level puts upward pressure on real wages, which reduces profits. The fall in profits reduces investment in new jobs, which in turn reduces employment. Because this reduction in employment is persistent, employment prospects worsen. With elevated risk there is a further increase in the demand for the liquid asset, which in turn puts additional upward pressure on the price level, and so on. The impulse responses show that this mechanism is powerful enough to completely overturn the dampening effect that sticky nominal wages have in an economy with complete risk sharing.

Our mechanism has the flavor of self-fulfilling beliefs, that is, agents worry about future employment prospects that induces them to save more that leads to deflationary pressure, which in turn leads to higher real wage costs, which leads to higher unemployment that indeed justifies agents’ concerns about future employment. Although this is a powerful mechanism, there is a counterforce. In particular, a higher unemployment rate increases the number of jobs that can be created at a given level of investment. For the results reported here, this counterforce is strong enough to ensure stability. For some parameter values, the fluctuations could very well become so large that no nonexplosive solution exists. 68

6.2. Role of Imperfect Insurance When Nominal Wages are not Sticky

In this section, we discuss business cycle properties when changes in the price level leave real wages unaffected, that is, $\omega_p = 1$. Real wages are then necessarily procyclical, because $\omega_z > 0$. Figure 6 plots the IRFs for the heterogeneous-agent economy and the IRFs for the corresponding representative-agent economy. There are several similarities with our benchmark results, but also one essential difference. We start with the similarities.

A negative productivity shock still has a direct negative effect on profits, which leads to a reduced demand for equity (firm ownership), which in turn means that fewer jobs are created. Also, increased concerns about employment prospects still induce agents in the heterogeneous-agents economy to increase their demand for the liquid asset, which again is strong enough to push the price level down, whereas it increases in the representative-agent economy. 69

68. In particular, changes in parameter values that substantially enhance the deflationary mechanism make it computationally challenging, or even impossible, to find an accurate solution. This does not prove that a stationary solution does not exist, but it would be consistent with this hypothesis. If such a solution would exist, however, it is likely to have complex nonlinear features. This result is in contrast to standard perturbation methods that impose that aggregate shocks of any size will not destabilize the economy as long as arbitrarily small shocks do not. For example, the technique developed in Reiter (2009) to solve models with heterogeneous agents relies on a perturbation solution for changes in the aggregate shock, which implies that the solution is imposed to be stable—for shocks of any size—as long as the Blanchard–Kahn conditions are satisfied, that is, when the solution is stable for small shocks. Our experience suggests that this may impose stability where there is none.

69. In the representative-agent economy, there is a small decrease in the price level in the first period. The reason is that the largest drops in consumption and output occur with quite a delay, for consumption in the sixth quarter. Consequently, the demand for money increases for consumption smoothing motives at the outset of the recession.
There is also a striking difference. In the economy with flexible nominal wages, recessions are less severe when agents cannot insure themselves against unemployment risk. The reason is the following. Increased uncertainty, alongside with an expected reduction in individual consumption, increase the expected value of the MRS. This affects the first-order condition of the liquid asset as well as the first-order condition of the productive investment, because each agent’s future revenues of both assets are (correctly) discounted by the agent’s own individual MRS. Because wages are flexible,
the associated rise in the price level bears no consequence on the return on equity, and
the chain of events underlying the deflationary spiral breaks down.

Thus, if the rise in precautionary savings is partially used as productive investments,
then this would *dampen* the reduction in the demand for equity induced by the direct
negative effect of the productivity shock on profits. The IRFs document that this is
indeed the case when nominal wages respond one-for-one to changes in the price
level. Whereas the biggest drop in employment is 3.44ppt in the representative-agent
economy, it is equal to 2.66ppt in the heterogeneous-agent economy. These results
make clear that a researcher would bias the model predictions if this dampening aspect
of precautionary savings is not allowed to operate, for example, because there is
communal firm ownership.\textsuperscript{70}

In our benchmark economy, we allow this channel to operate, but the effect
is dominated by the *interaction* between sticky nominal wages and uninsurable
unemployment risk. Increased uncertainty may increase the demand for equity, but
it will also increase the demand for the liquid asset. The latter depresses the price
level, which, provided that nominal wages are somewhat sticky, increases real wages.
The rise in real wages reduces profits, which in turn lowers the demand for equity.
This channel dominates any positive effect that precautionary savings may have on the
demand for equity.

\textit{Temperance}. In all cases considered, we find that recessions are less severe in the
heterogeneous-agent economy than in the representative-agent economy if nominal
wages respond one-for-one to change in the price level. That is, this dampening effect
is very robust. During the 90s, several papers argued that an increase in idiosyncratic
risk could lead to a *reduction* in the demand for a risky asset when investors can save
through both a risky and a risk-free asset even though it would increase total savings.
This effect is referred to as *temperance*.\textsuperscript{71} In the presence of temperance, increased
concerns about future employment prospects could start a reinforcing mechanism even
when wages are fully flexible. However, we find that temperance is not important in
our environment.

The first reason is that it is a partial equilibrium result. In general, equilibrium
prices adjust. This is important. Suppose that the economy *as a whole* can increase
savings through the risky investment, but not through the risk-free investment. Then,
the relative price of the risk-free asset would increase making the riskier asset more
attractive. This plays a role in our economy, because the only way the economy as a
whole can do something now to have more goods in the future is by investing more
in the productive asset, that is, in the risky asset. There are several other features,
typically present in macroeconomic models, that make temperance less likely. One is
that the temperance result relies on idiosyncratic risk to be sufficiently independent
of investment risk. In macroeconomic models, that is not the case. The amount of

\textsuperscript{70} See footnote 6 for a list of papers following this approach.

\textsuperscript{71} See Kimball (1990), Kimball (1992), Gollier and Pratt (1996), and Elmendorf and Kimball (2000).
idiosyncratic risk depends on the level of the wage rate. But the level of the wage rate is often correlated with the return of the risky asset, because both are affected by the same shocks. Another feature that works against the temperance result is the short-sale constraint on equity, which directly prevents a reduction in the demand for equity, at least for some agents. In our model, diminishing returns on the transactions aspect of money also work against temperance. This makes increased investment in the risk-free asset less attractive relative to a framework in which the return remains fixed.

It may be the case that temperance can be generated in models with different utility functions, for example, if the utility function is such that the price of risk increases during recessions. We leave this for future research.

7. Prolonged Recessions

In the previous section, we discussed the expected behavior of macroeconomic aggregates when productivity switches from its high to its low value. When presenting this behavior, we took into account the probability that productivity would return to the high value as indicated by our driving process. Here, we discuss the results when productivity switches to the low value and stays low for a sustained period.

Figure 7 reports the results. The black solid lines report the responses for our benchmark calibration and monetary policy rule. The grey lines report the results when the nominal interest rate is held constant. The responses for output and employment simply become more persistent when the responses are conditioned on there being no recovery. Most of the decline in aggregate activity has taken place within 10 quarters. Relative to the observed drops discussed in Section 2, this is somewhat too slow for output and somewhat too fast for employment. Even though we use a typical TFP process and the drop in productivity is moderate, the model generates a drop in output relative to trend that is just slightly below its empirical counterpart; the limiting value of the drop is 9.2% in the model, whereas observed GDP is 10.2% below its trend value at the end of 2013. The model also does a good job in matching the increase in the unemployment rate. From the first quarter of 2008 to the end of our sample, the Eurozone unemployment rate increased by 4.8ppt. In the model, the unemployment rate increases with 4.6ppt.

72. The consequences of idiosyncratic risk are reduced when the wage rate falls. In the extreme case when the wage rate is zero, there is no unemployment risk.

73. In the model considered here, they are both directly affected by $z_t$.

74. We considered models with different degrees of risk aversion, but this does not seem to matter for this particular issue.

75. The asset price panel is replaced with a panel that documents the response of the policy rate. During a sustained recession, asset prices drop sharply at the outset of the crisis. In contrast to the data, the generated time path does not display a partial recovery.
FIGURE 7. Role of monetary policy during a prolonged recession. These graphs illustrate the behavior of the indicated variables when the productivity $z_t$ switches to the low value and stays low. Under the baseline scenario, the interest rate adjusts.

The model generates a large initial drop in the price level (relative to its trend value) followed by a partial recovery. In contrast, the observed fall in the price level is more gradual. Nevertheless, we do not think that we exaggerate the extent of the deflationary pressure because the gap between trend and actual value reaches 3.25% at the end of the sample, whereas it only reaches 2.61% in the model.\footnote{The model would predict a more gradual drop in the price level if the economy faced a sequence of smaller negative reductions in $z_t$ as opposed to one large immediate decline.}

\footnote{76}
Real wage rates increase in the beginning of the recession, but then start to decline. After ten periods, they are 0.19% below the precrisis value. In contrast, average Eurozone wages have increased relative to their trend values throughout the economic slowdown. The response of real unit labor costs resembles its empirical counterpart more closely. Both display a sharp initial increase followed by a partial recovery.

To summarize, the model does a good job in capturing several qualitative and quantitative aspects of economic aggregates during the great recession. In the remainder of this section, we consider two alternative monetary policies. In the first case, monetary policy is constrained and interest rates cannot be lowered. In the second, monetary policy is substantially more aggressive than in the benchmark.

**Alternative I: No Interest Rate Reductions Possible.** For our benchmark calibration, the lowest value of the annualized nominal interest rate is equal to 0.85%. Thus, the nominal interest rate is always high enough to avoid the ZLB.

The importance of constraints on monetary policy is documented in Figure 7, which plots the responses when the nominal interest rate remains constant. When the interest rate cannot adjust, deflationary pressure is stronger and the recession is deeper. Moreover, the behavior of real wages is closer to its empirical counterpart. Specifically, the real wage response no longer turns negative and the initial response is substantially higher.

The result that the recession is deeper when interest rates do not decline raises the question whether the dynamics explored in this paper resemble, or even mimic, those emphasized in the ZLB literature. It does not. To understand why, it is important to notice that the dynamics underlying the ZLB literature hinges on the idea that a negative demand shock—which is similar to the precautionary amplification mechanism in this paper—gives rise to a **decline** in expected inflation and quite often even deflation. With nominal interest rates at zero (or at some constant, unchanged, level), this decline in expected inflation raises real interest rates, which further propagates the initial shock to demand, and the process reinforces itself. This mechanism contrasts markedly to the one explored in this paper, in which a negative effect on demand—through the precautionary savings in liquid assets—gives rise to a **rise** in expected inflation. The rise in expected inflation lowers the real interest rate, which therefore alleviates some of the adverse consequences of the initial shock. In fact, the real interest rate falls

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77. The average value is 2.89%

78. See, for example, Eggertsson and Woodford (2003), Christiano, Eichenbaum, and Rebelo (2011), and Eggertsson and Krugman (2012).

79. Gimeno and Ortega (2016) document that one-year ahead Eurozone inflation expectations drop sharply in 2008 and then rapidly increase up to levels above 3%. After 2011, these expectations gradually decline. Inflation expectations, thus, seem well anchored, which is not captured by our model (and not by standard ZLB models either). What matters for our model, however, is not expected inflation but realized deflationary pressure. In this dimension, our model is consistent with the data. If expected inflation would decrease or increase by less (perhaps because agents are not fully rational), then real interest rates would be even higher in our model and the downturn would be more severe.
to levels that are substantially below the nominal interest rate, whereas the opposite is true in the ZLB literature.

Interestingly, a more reactive interest rate policy, that is, a higher value of $\nu_p$, does not necessarily lead to a larger reduction in the real interest rate during recessions. The reason is the following. A more aggressive monetary policy response dampens the recession that reduces the increase in the demand for the liquid asset, which in turn reduces deflationary pressure and, thus, the increase in the expected inflation rate and the reduction in the real rate. Specifically, in the experiment discussed in this section, the quarterly real interest rate decreases with up to 2.99ppt when monetary policy responds according to equation (13) and $\nu_p$ is set to its calibrated value, and decreases with up to 3.60ppt when the nominal interest rate is kept fixed.

**Alternative II: More Aggressive Monetary Policy.** At the end of 2006, the ECB rate on the deposit facility was equal to 2.50%. In 2007, it was increased in steps to 3.00%. During 2008, its behavior was slightly erratic with six changes in different directions. On November 12 2008, the rate was lowered to 2.75% and this reduction was followed by a series of reductions until it was set equal to 0.25% on April 8 2009. Subsequently, it gradually increased until it was set equal to 0.75% on July 13 2011. After this, it was decreased until it reached negative territory on June 11 2014 when it was set equal to $-0.10\%$.

In our benchmark experiment, the interest rate also drops quite fast and with a substantial amount. Its starting position, at the end of a sustained expansion, is equal to 3.23%. It drops to 0.85% when the economy enters the recession. This is the lowest level because the interest rate follows the price level and the latter drops to its lowest level in the first period of the recession. This sharp 2.38ppt drop resembles the observed policy response following the collapse of Lehman brothers, except that the adjustment occurs in one quarter in our model whereas the ECB took several quarters to lower its policy rate. After the initial fast drop, the model’s interest rate gradually increases as deflationary pressure subsides. After one year, it is equal to 1.33ppt below its precrisis level and after two years this gap has been reduced to 0.85ppt.

To summarize, the model’s predicted total decline in the interest rate is somewhat smaller and definitely less persistent than the observed changes in the ECB policy rate. The strength of the interest rate response is controlled by the parameter, $\nu_p$, which is calibrated to ensure that the model captures the observed sustained deflationary pressure. Obviously, the ECB had to do much more than just fight deflationary pressure. Thus, it is not surprising that the ECB responded more aggressively than the model’s prediction.

Although the model’s policy response is consistent with the observed amount of deflationary pressure, it would be of interest to see whether the mechanism emphasized...
in this paper is still present if interest rates dropped by more and in particular would remain low for a sustained period. Specifically, we assume that the interest rate drops with 3.21ppt to 0% in the first period of the recession. Moreover, the interest rate is kept at 0% for ten periods as long as the economy stays in the recession regime. The interest rate will be set again according to the normal policy rule as described in equation (13) when the economy gets out of the recession and also after ten periods in case it stays in the recession regime during these ten periods. This policy change is unanticipated and agents assume that it will never happen again. Consequently, after ten periods agents’ policy rules and the laws of motion of aggregates are identical to those of our benchmark economy. We calculate the behavior of the economy during these first ten periods working backwards starting from the tenth period. 82

The IRFs for the case when the economy is in the recession regime for a sustained time period and the interest rate is set equal to zero during the first ten periods are added to Figure 7. They are indicated with dark dashed lines and those associated with our benchmark policy with a dark solid line. Relative to the benchmark policy, this more expansionary policy leads to a much smaller drop in prices and a substantial reduction in the increase in real unit labor costs. In contrast, the impact on employment is negligible. Why does the period of lower labor costs not lead to higher employment? Recall that the decision to invest in jobs depends on the net present value (NPV) of future profits. The presence of uncertainty lowers the discount rate that means that an increase in profits that affects a relatively short time period only has a limited impact on the NPV. The second reason is that this more aggressive monetary policy is also not able to avoid deflationary pressure. There is still a substantial drop in the price level. During the first ten periods of the recession, the average price level is 2.65% below its precrisis level. Recall that in the economy with complete markets in which the drop in employment is much smaller, there is an increase in the price level.

Monetary Aggregates. The central bank can diminish deflationary pressure either by lowering the interest paid on the liquid asset or by increasing its supply. In this paper, we have focused on the first; the supply of the liquid asset is assumed to increase at a fixed rate. During the great recession, the ECB adopted several liquidity injection programs. These were followed by large increases in bank reserves. However, these increases were temporary. 83 More importantly, these increases in bank reserves did not lead to increases in monetary aggregates. As documented in Appendix A, the M1, M2, and M3 aggregates grew at a slower rate following the outbreak of the crisis. We can expect that at least part of this slowdown is due to a reduction in the demand for money as aggregate activity and transactions decline. This mechanism is also present in our model. To generate the observed decline in monetary aggregates, we would have

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82. Although we will display the results for the case when the economy remains in a recession for a sustained time period, agents do take into account that the economy could switch back to the expansion regime and they understand that this would mean the end of the unusual monetary policy even if the switch occurs during the first ten periods.

83. In contrast, US bank reserves kept on growing strongly because the outset of the crisis and did not display such reversals. See Cukierman (2016) for a discussion.
had to reduce the supply of the liquid asset, which would have put more downward pressure on prices.

8. The Role of Unemployment Insurance for Business Cycles

In this section, we illustrate the effects of alternative UI policies. In our model, changes in such policies affect the economy quite differently compared to many other studies. Our results do not only differ from those of the standard labor search business cycle model with a representative agent, but also from those with heterogeneous agents, such as Krusell et al. (2010) and McKay and Reis (2016). As our objective is to highlight these differences as succinctly and transparently as possible, we will make two assumptions: First, the monetary policy rule is inactive and does not respond to changes in the price level. Second, the wage rule is left unaltered. Relaxing these two assumptions does not alter the qualitative result that changes in UI has different effects in our model compared to the literature. They do, however, impact the magnitude of the differences.

The experiment we consider is straightforward. Specifically, we solve the model without and with aggregate risk for a range of values of the replacement rate, $\mu$. Figure 8 illustrates the impact of changes in the replacement rate on employment levels. The value of the replacement rate, $\mu$, is provided on the $x$-axis, and the resulting employment rate on the $y$-axis.

First, consider the case without aggregate uncertainty. An increase in the replacement rate means that agents are better insured against idiosyncratic risk, which lowers the expected value of their MRS. The latter triggers a decrease in precautionary savings, which decreases investment and employment.

84. It ought to be noted, however, that though the nominal interest rate is assumed to be constant, the economy is still stabilized because movements in the price level affects real money balances and therefore the marginal utility of holding the liquid asset.

85. Although we make this assumption for simplicity, it is not unreasonable to assume that the wage-setting rule remains unchanged when UI changed because several empirical papers find that UI benefits do not have a significant effect on wages. Examples are Card, Chetty, and Weber (2007), Lalive (2007), van Ours and Vodopiec (2008), and Le Barbanchon (2016). However, not all papers reach this conclusion. Schmieder, Till von, and Binder (2016) find that more generous UI benefits have a significant negative effect on wages and Nekoei and Weber (2017) find that UI benefits have a positive effect on re-employment wages.

86. In Den Haan et al. (2015), we relax this assumption and allow for changes to the wage rule and show that the same mechanisms apply, and that the presence of aggregate uncertainty still leads to qualitatively different outcomes.

87. At low values of $\mu$, changes in the replacement rate have little effect on the employment level. The reason is that the presence of money puts a lower bound on the expected return on equity, and therefore an upper bound on the expected MRS. As a consequence, equity prices are bounded from above, which—through the free-entry condition—implies that employment is as well. In the model without aggregate uncertainty, the presence of money implies that the real return on firm ownership cannot be less than the real interest rate on money.
For the case with aggregate uncertainty, we report the unconditional, the average “expansion”, and the average “recession” employment levels. For values of $\mu$ greater than 0.56, all three employment levels decrease. As the replacement rate increases beyond about 0.6, the average expansion and the average recession employment level form a band around the no-aggregate-uncertainty employment level with a roughly constant width. Thus, for these values of $\mu$ the model with aggregate uncertainty and the model without aggregate uncertainty have similar predictions on the impact of changes in UI on average employment levels.

When $\mu$ is below 0.6, however, our deflationary mechanism is quantitatively important and the recession and expansion employment levels form a much wider band around the average. An increase in the replacement rate then leads to a reduction in aggregate volatility. Consider, for example, the case of a rise in $\mu$ from 0.45 to 0.55.

88. These results are consistent with the empirical results of Di Maggio and Kermani (2016), who show that economic activity is less responsive to shocks in counties with more generous unemployment benefits. Moreover, they provide support for the view that this result is due to a demand channel.
The increase in the replacement rate leads to a 52.9% decrease in the standard deviation of the employment rate. The reason for this decline is that improved insurance lowers the strength of the deflationary mechanism. Indeed, the standard deviation of individual consumption is reduced by 20.2%. As explained in Section 6, the large impact of the replacement rate on business cycle volatility is due to the self-reinforcing nature of our mechanism. The figure documents that this self-reinforcing process is much stronger at lower levels of $\mu$.

It is interesting to note the highly nonlinear effects of changes in the replacement rate on the employment rate levels during recessions. The average employment rate conditional on being in recessions is first increasing in $\mu$, reaches a peak around 0.54, and then continues to decline. It should be noted that the average employment rate in recessions is significantly larger than its benchmark value, corresponding to $\mu = 0.5$, even for values of $\mu$ as high as 0.7.

The figure also documents that the increase in $\mu$ not only decreases aggregate volatility, it can also increase the average employment rate. In particular, the increase of $\mu$ from 0.45 to 0.55 increases the average employment rate with 0.67ppt. In contrast, in the version of our model without aggregate uncertainty the same increase in the replacement rate leads to a decrease in average employment of 0.76ppt. Such comparative statics typically result in similar answers for economies with and without aggregate uncertainty, because aggregate uncertainty is relatively small. Volatility of the only aggregate exogenous random variable, productivity, is indeed modest in our model. Nevertheless, the economy with aggregate uncertainty responds to a change in the replacement rate quite differently than the economy without aggregate uncertainty.

In the remainder of this section, we explain why an increase in UI can lead to an increase in the average employment rate, when the value of the replacement rate is such that the deflationary mechanism is quantitatively important. The starting point is the 0.76ppt decrease in employment in the economy without aggregate uncertainty mentioned above. In the economy with aggregate uncertainty, however, there are two additional effects associated with an increase in the replacement rate that increase the demand for equity and, thus, job creation. These have to be added to the effect found in the economy without aggregate uncertainty. The first effect is that more insurance reduces the risk of holding equity, because an increase in the replacement rate not only reduces the volatility of real activity, but also leads to a substantial reduction in stock price volatility. In fact, if $\mu$ increases from 0.45 to 0.55 the standard deviation of the real equity price drops by 50.0%. This reduction in risk and increased demand for equity leads to more job creation and an increase in average employment of 0.80ppt.\textsuperscript{89} The second effect is related to the nonlinearity of the matching process; that is, increases

\textsuperscript{89} We calculate this as follows. If there is no aggregate uncertainty, then the increase in $\mu$ leads to a decrease in employment of 0.76ppt and a decrease in real equity value of 8.4%. If there is aggregate uncertainty, then the same change in $\mu$ leads to an increase in the average real equity value by 0.29%. The difference between the 0.29% increase and the 8.4% decrease is due to the reduction in volatility. Using the zero-profit condition, we calculate that this price increase corresponds to a 0.80ppt increase in employment.
in equity prices have a smaller effect on job creation than decreases.\textsuperscript{90} For the same change in the replacement rate as before, the decrease in the volatility of the real equity price increases average employment through this channel with 0.64ppt.\textsuperscript{91} We now have the ingredients to explain why the employment rate increases with 0.67ppt. If we add the 0.64ppt increase due to the nonlinearity of the matching function to the 0.80ppt due the lower volatility of the stock price and then subtract the 0.76ppt decrease due to the reduction in savings because of better individual insurance, we get an increase in average employment of 0.68ppt.\textsuperscript{92}

In this experiment, we abstract from several counteracting effects that may overturn the above results. For instance, Carlstrom and Fuerst (2006) and Johnston and Mas (2016) argue that the disincentive effects of UI on search behavior can be quite large. Taking such effects into account is, therefore, likely to be important to assess the total impact of changes in UI on average employment levels. The main point made in this section is, however, that the answer to this question can depend strongly on whether we consider a model with or a model without aggregate uncertainty. The reasons for this difference are likely to be present in other models that do take these counteracting effects into account.

\section{Concluding Comments}

The properties of our model depend crucially on whether the deflationary mechanism is sufficiently powerful. If it is not powerful enough, then the model properties are close to the outcomes of a representative-agent version of the model. In particular, the presence of nominal sticky wages would then dampen the effects of productivity shocks, and an increase in the replacement rate would decrease the average employment rate. If the deflationary mechanism is strong enough, however, then our model predicts the opposite. In so far as the conditions that affect the strength of the deflationary mechanism vary across time and place, we can also expect business cycle properties to

\textsuperscript{90} Ignoring transitions—which occur fast in this model—Equation (11) implies that

$$\psi^{1/(1-n)} \left( \frac{J}{P} \right)^{n/(1-n)} (1-q) \approx \delta q.$$  

Because $\eta = 1/2$, we get that

$$q \approx 1 - \frac{\delta}{\delta + \psi^2 J/P}.$$  

Thus, $q$ is a concave function of $J/P$. This nonlinearity is also present in a model with complete markets as discussed in Jung and Kuester (2011).

\textsuperscript{91} We calculate this as follows. When $\mu = 0.45$, the introduction of aggregate uncertainty leads to a reduction in employment of 1.75ppt of which 0.90ppt can be explained by the reduction in the average equity price. The remainder of 0.84ppt is, thus, due to the nonlinearity of the matching function. When $\mu = 0.55$, this nonlinearity effect is only 0.20ppt. Thus, when $\mu$ increases from 0.45 to 0.55 in the economy with aggregate uncertainty, there is a reduction of the impact of the nonlinearity on average employment of 0.84ppt–0.20ppt=0.64ppt.

\textsuperscript{92} The fact that the numbers of the three effects roughly add up to the total effect of 0.67ppt means that the interaction of the different effects either cancels out or is small.
vary across time and place. The same is true for the effects of changes in UI. Whether the deflationary mechanism is operative or not may depend on relatively small changes. For example, the mechanism is quantitatively very important when the replacement rate is around the benchmark value of 50%, but not when the replacement rate exceeds 60%. The message is that for this type of model in which the strength of a deflationary spiral depends strongly on parameter values, it may be difficult to predict business cycle behavior and the consequences of policy changes, because this may require reliable knowledge of model parameter values.

Appendix A: Further Empirical Motivation

In this appendix, we provide more empirical motivation for our model and the underlying assumptions. First, we review some evidence in support of our assumption that nominal wages do not respond one-for-one to price changes. Second, we discuss the inability of individuals to insure themselves against unemployment spells. Lastly, we discuss whether savings respond to an increase in idiosyncratic uncertainty.

Nominal Wage Stickiness and Inflation. There are many papers that document that nominal wages are sticky. However, what is important for our paper is the question of to which extent nominal wages adjust to aggregate shocks and, in particular, to changes in the aggregate price level. Druant et al. (2009) provide survey evidence for a sample of European firms with a focus on the wages of the firms’ main occupational groups; these would not change for reasons such as promotion. Another attractive feature of this study is that it explicitly investigates whether nominal wages adjust to inflation or not. In their survey, only 29.7% of Eurozone firms indicate that they have an internal policy of taking inflation into account when setting wages, and only half of these firms do so by using automatic indexation. Moreover, most firms that take inflation into account are backward looking. Both findings imply that real wages increase (or decrease by less) when inflation rates fall.

Papers that document nominal wage rigidity typically highlight the importance of downward nominal wage rigidity. Suppose there is downward, but no upward nominal wage rigidity. Does this imply that all nominal wages respond fully to changes in aggregate prices as long as aggregate prices increase? The answer is no. The reason is that firms are heterogeneous and a fraction of firms can still be constrained by the inability to adjust nominal wages downward. In fact, downward nominal wage rigidity is supported by the empirical finding that the distribution of firms’ nominal wage changes has a large mass-point at zero. The fraction of firms that is affected by this constraint would increase if the aggregate price level increases by less. In fact, Daly et al. (2012) document that the fraction of US workers with a constant nominal wage

93. See, for example, Dickens et al. (2007), Druant et al. (2009), Barattieri, Basu, and Gottschalk (2014), Daly et al. (2012), and Daly and Hobijn (2014).

94. See Barattieri et al. (2014), Dickens et al. (2007), Daly et al. (2012), and Daly and Hobijn (2014).
increased from 11.2% in 2007 to 16% in 2011, whereas the fraction of workers facing a reduction in nominal wages was roughly unchanged. This indicates that there is upward pressure on real wages when the inflation rate falls, even if it remains positive and nominal wages are only rigid downward.

Inability to Insure Against Unemployment Risk. An important feature of our model is that workers are poorly insured against unemployment risk. That is, that consumption decreases considerably following a displacement. Using Swedish data, Kolsrud et al. (2015) document that expenditures on consumption goods drop sharply during the first year of an unemployment spell, after which they settle down at 26.3% below the predisplacement level. This sharp fall is remarkable given that Sweden has a quite generous unemployment benefits program. As is discussed in Section 4, one reason is that the amount of assets workers hold at the start of an unemployment spell is low. Another reason is that average borrowing actually decreases during observed unemployment spells.

Using US data, Stephens Jr. (2004), Saporta-Eksten (2014), Aguiar and Hurst (2005), and Chodorow-Reich and Karabarbounis (2016) provide empirical support for substantial drops in consumption follow job loss, even when expenditures on durables are not included. Using Canadian survey data, Browning and Crossley (2001) find that workers that have been unemployed for six months report that their total consumption expenditures level during the last month is 14% below consumption in the month before unemployment.

Savings and Idiosyncratic Uncertainty. The idea that idiosyncratic uncertainty plays an important role in the savings decisions of individuals has a rich history in the economics literature. From a theoretical point of view, Kimball (1992) shows that idiosyncratic uncertainty increases savings when the third-order derivative of the utility function with respect to consumption is positive and/or the agent faces borrowing constraints. Moreover, idiosyncratic uncertainty regarding unemployment is more important in recessions that are characterized by a prolonged downturn and an increase

95. Similarly, at http://nadaesgratis.es/?p=39350, Marcel Jansen documents that from 2008 to 2013 there was a massive increase in the fraction of Spanish workers with no change in the nominal wage. There is some increase in the fraction of workers with a decrease in the nominal wage, but this increase is small relative to the increase in the spike of the histogram at constant nominal wages.

96. Using the four 1992–1996 waves of the Health and Retirement Study, Stephens Jr. (2004) finds that annual food consumption is 16% lower when a worker reports that he/she is no longer working for the employer of the previous wave either because of a layoff, business closure, or business relocation, that is, the worker was displaced between two waves. Similar results are found using the Panel Study of Income Dynamics (PSID). Using the 1999–2009 biannual waves of the PSID, Saporta-Eksten (2014) finds that job loss leads to a drop in total consumption of 17%. About half of this loss occurs before job loss and the other half around job loss. The drop before job loss suggests that either the worker anticipated the layoff or labor income was already under pressure. Moreover, this drop in consumption is very persistent and is only slightly less than 17% six years after displacement. Using data for food and services, Chodorow-Reich and Karabarbounis (2016) find that the consumption level of workers that are unemployed for a full year is 21% below the consumption level of employed workers. Using scanner data for food consumption, Aguiar and Hurst (2005) report a drop of 19%.
in the average duration of unemployment spells. Krueger, Cramer, and Cho (2014) document that during the recent recession the number of long-term unemployed increased in Canada, France, Italy, Sweden, the UK, and the US. The only case in which they found a decrease is Germany. The results are particularly striking for the US. During the recent recession, the amount of workers who were out of work for more than half a year relative to all unemployed workers reached a peak of 45%, whereas the highest peak observed in previous recessions was about 25%.

Several papers have provided empirical support for the hypothesis that increases in idiosyncratic uncertainty increases savings. Using 1992–98 data from the British Household Panel Survey, Benito (2006) finds that an individual whose level of idiosyncratic uncertainty would move from the bottom to the top of the cross-sectional distribution reduces consumption by 11%. An interesting aspect of this study is that the result holds both for a measure of idiosyncratic uncertainty based on an individuals’ own perceptions as well as on an econometric specification.97 Further empirical evidence for this relationship during the recent downturn can be found in Alan, Crossley, and Low (2012). They argue that the observed sharp rise in the savings ratio of the UK private sector is driven by increases in uncertainty, rather than other explanations such as tightening of credit standards. In line with the mechanism emphasized in this paper, Carroll (1992) argues that employment uncertainty is especially important because unemployment spells are the reason for the most drastic fluctuations in household income. In addition, Carroll (1992) provides empirical evidence to support the view that the fear of unemployment leads to an increased desire to save even when controlling for expected income growth.

**Monetary Aggregates.** Figure A.1 plots Eurozone monetary aggregates.

**Appendix B: Data Sources**

**B.1. Data Used for Figures 1, 2, and A.1**

- Eurozone GDP and its implicit price deflator are from the Federal Reserve Economic Data (FRED). Data are seasonally adjusted. Here, the Eurozone consists of the 18 countries that were members in 2014.
- Eurozone harmonized unemployment rate, total, all persons from the FRED. Data are seasonally adjusted.
- Eurozone total share price index for all shares from the FRED, originally from the OECD main economic indicators. Data are not seasonally adjusted.
- Eurozone private sector hourly earnings are from OECD.STATExtracts (MEI). The target series for hourly earnings correspond to seasonally adjusted average total earnings paid per employed person per hour, including overtime pay and regularly recurring cash supplements. Data are seasonally adjusted.

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97. Although the sign is correct, the results based on individuals’ own perceptions are not significant.
Figure A.1. Eurozone monetary aggregates. The precrisis trend is defined as the time path the variable would have followed if its growth rate before and beyond the fourth quarter of 2007 had been equal to the average growth rate over the five years preceding 2007. Shaded areas mark CEPR recessions. Data sources are given in Appendix B.

- Unit labor costs are from OECD.Stat. Data are for the total economy. Unit labor costs are calculated as the ratio of total labor costs to real output. Data are seasonally adjusted.
- Monetary aggregates are from the FRED. Data are not seasonally adjusted.

B.2. Data Used for Calibration

- Average unemployment rate: Average unemployment rate for the four large Eurozone economies, France, Germany, Italy, and Spain for 1980Q1–2012Q4. Data are from OECD.Stat.
Average unemployment duration: Average unemployment duration in Europe in the period from 1980Q1 to 2012Q4. Data are from OECD.Stat. This is annual data. The data series for Europe is used because no data for the Eurozone is available, nor data for the big Eurozone countries. Starting in 1992, separate data is given for Europe, the European Union with 21 countries, and the European Union with 28 countries, and the series are quite similar over this sample period.

Employment rate and GDP. Data are from OECD.Stat. Quarterly employment rate series for Eurozone countries are only available for relatively short samples. The longest series are available for France (from 1990Q1), for Germany (from 1991Q1), and for Italy (from 1995Q1). When we use samples up to 2012Q4, we get for the ratio of the standard deviation of HP-filtered employment over HP-filtered output 0.672 for France, 0.391 for Germany, and 0.505 for Italy.

HFCS data are discussed in the next subsection.

B.3. HFCS Data: Details and Additional Results

Excluded Households. To ensure that the data are used resemble the households of our model, we exclude types on agents that are not present in our model. In particular, we apply the following filters:

1. Always exclude the entire household if any household member is (according to PE0100a)
   (a) #2 (sick/maternity leave),
   (b) #5 retiree or early retiree,
   (c) #6 permanently disabled,
   (d) #7 compulsory military service, and
   (e) #9 other not working for pay.

2. Always exclude the entire household if nobody in the household is
   (a) either #1 (employed) or #3 (unemployed).

3. Exclude the entire household if there are more than two people either employed or unemployed.

4. Exclude the entire household if any in the household is aged 65 years or higher.

5. Exclude the entire household if nobody is aged 20 years or older.

6. Exclude individuals without reported labor force status.

Employed and Unemployed Households. In the main paper, we report results for two groups. The “employed” households are households in which nobody is unemployed and either 1 or 2 household members are working. The “unemployed” households are households in which nobody is working and either 1 or 2 household members are unemployed. We also looked at subgroups, such as, households with 1 household member working. The results for these subgroups are very similar to the ones for the broader group.
Weighting and Imputation. If an answer to a particular question is missing, then HFCS imputes the answer. An imputed answer consists of five different answers. We calculate statistics using all the data including imputed responses. We average the imputed observations to get one observation per individual or household.\footnote{An issue in the data is that occasionally flags indicate that there is no imputation, but there are still five different numbers given. We treat those cases like imputations.} HFCS provides weights with each observation. The weights are provided to make the sample representative. We calculate statistics using the provided weights.

Definitions of Variables. The variables used are defined as follows:

1. Labor income: Income for employed households is defined as the sum of gross cash employee income (PG0110) across household plus the sum of gross self-employment income (PG0210) across household members.

2. Unemployment benefits: Income for unemployed households is defined as the sum of regular social transfers across households. These include unemployment benefits (PG0510) and gross income from regular social transfers (HG0110). We do not include pension benefits, because we focus on working age population.

3. Liquid assets: The sum of sight accounts (HD110) plus savings accounts (HD1210).

4. Financial assets: The sum of sight accounts (HD110), savings accounts (HD1210), mutual funds (HD1320a-f), bonds (HD1420), publicly traded shares (HD1510), and managed accounts (HD1620).\footnote{ECB (2013) refer to this group of assets plus nonself-employment private business wealth as liquid assets. The latter asset is not very large. We exclude it, because it seems unlikely this type of asset could be easily sold. Note that in our framework “liquid” refers the ability of assets to facilitate transactions during the period.}

Appendix C: Equivalence With Standard Matching Framework

In the standard matching framework, new firms are created by “entrepreneurs” who post vacancies, \( \tilde{v}_t \), at a cost equal to \( \kappa \) per vacancy. The number of vacancies is pinned down by a free-entry condition. In the description of the model above, such additional agents are not introduced. Instead, creation of new firms is carried out by investors wanting to increase their equity holdings.

Although, the “story” we tell is somewhat different, our equations can be shown to be identical to those of the standard matching model. The free-entry condition in the standard matching model is given by

\[
\kappa = \frac{\tilde{h}_t}{\tilde{v}_t} \frac{J_t}{P_t},
\]

(C.1)
where
\[ \tilde{h}_t = \tilde{\psi} \tilde{v}_t^\eta u_t^{1-\eta}. \] (C.2)

Each vacancy leads to the creation of \( \tilde{h}_t / \tilde{v}_t \) new firms, which can be sold to households at price \( J_t \).

Equilibrium in the equity market requires that the net demand for equity by households is equal to the supply of new equity by entrepreneurs, that is
\[ \int_i \left( q(e_i, q_i, L_i; s_i) - (1 - \delta)q_i \right) dF_i(e_i, q_i, L_i) = \tilde{\psi} \tilde{v}_t^\eta u_t^{1-\eta}. \] (C.3)

Using equations (C.1) and (C.2), this equation can be rewritten as
\[ \int_i \left( q(e_i, q_i, L_i; s_i) - (1 - \delta)q_i \right) dF_i(e_i, q_i, L_i) = \tilde{\psi}^{1/(1-\eta)} \left( \frac{J_t}{\kappa} \right)^{\eta/(1-\eta)} u_t. \] (C.4)

This is equivalent to equation (11) if
\[ \tilde{\psi} = \psi \kappa^\eta. \] (C.5)

It only remains to establish that the number of new jobs created is the same in the two setups, that is,
\[ h_t = \tilde{h}_t \] (C.6)

or
\[ \psi v_t^\eta u_t^{1-\eta} = \tilde{\psi} \tilde{v}_t^\eta u_t^{1-\eta}. \] (C.7)

From equations (C.1) and (C.2), we get that
\[ \tilde{v}_t = \left( \frac{\tilde{\psi} J_t}{\kappa P_t} \right)^{1/(1-\eta)} u_t. \] (C.8)

Substituting this expression for \( \tilde{v}_t \) and the expression from equation (9) for \( v_t \) into equation (C.7) gives indeed that \( h_t = \tilde{h}_t \). Moreover, the total amount spent on creating new firms in our representation, \( v_t \), is equal to the number of vacancies times the posting cost in the traditional representation, \( \kappa \tilde{v}_t \).

The focus of this paper is on the effect of negative shocks on the savings and investment behavior of agents in the economy when markets are incomplete. We think that our way of telling the story behind the equations has the following two advantages. First, there is only one type of investor, namely, the household and there are no additional investors such as zombie entrepreneurs (poor souls who get no
positive benefits out of fulfilling a crucial role in the economy). Second, all agents have access to investment in the same two assets, namely equity and the liquid asset, whereas in the standard labor market model there are households and entrepreneurs and they have different investment opportunities.

Appendix D: Stationarity Inducing Transformation

In this section, we show that the model can be transformed into a system with only stationary variables. The trend price level grows at a constant rate \( \pi \). Thus, \( \bar{P}_t = (1 + \pi) \bar{P}_{t-1} \). According to equation (13), monetary policy is set according to

\[
R_t = v_0 (1 + \pi) \left( \frac{P_t}{\bar{P}_t} \right)^{\nu_p}.
\]

Define \( \tilde{R}_t \) as monetary policy net of trend inflation. That is,

\[
\tilde{R}_t = \frac{R_t}{1 + \pi} = v_0 \left( \frac{P_t}{\bar{P}_t} \right)^{\nu_p}.
\]

The consumers’ budget constraint is given by

\[
P_t c_{i,t} + J_t(q_{i,t+1} - (1 - \delta)q_{i,t}) + L_{i,t+1} = (1 - \tau_t)W_te_{i,t} + \mu(1 - \tau_t)W_t(1 - e_{i,t}) + D_tq_{i,t} + R_{t-1}L_{i,t}.
\]

Divide through by \( \bar{P}_t \), define \( \tilde{X}_t = X_t / \bar{P}_t \) for \( X_t \in \{P_t, J_t, W_t, D_t\} \), and \( \tilde{L}_{i,t} = L_{i,t} / \bar{P}_{t-1} \). Then the above equation is equivalent to

\[
\tilde{P}_t c_{i,t} + \tilde{J}_t(q_{i,t+1} - (1 - \delta)q_{i,t}) + \tilde{L}_{i,t+1} = (1 - \tau_t)\tilde{W}_te_{i,t} + \mu(1 - \tau_t)\tilde{W}_t(1 - e_{i,t}) + \tilde{D}_tq_{i,t} + \tilde{R}_t\tilde{L}_{i,t},
\]

with

\[
\tilde{W}_t = \omega_0 \left( \frac{\tilde{z}_t}{\bar{z}} \right)^{\omega_z} \bar{z} \left( \frac{\tilde{P}_t}{\bar{P}} \right)^{\omega_p},
\]

and

\[
\tilde{D}_t = \tilde{P}_tz_t - \tilde{W}_t.
\]

100. We could argue that entrepreneurs are part of the household, but with heterogeneous households the question arises which households they belong to.
Lastly, the government’s budget constraint net of trend inflation is given by

\[ \tau_t q_t \widehat{W}_t + \widehat{L}_{t+1} - \widehat{R}_{t-1} \widehat{L}_t = (1 - q_t) \mu (1 - \tau_t) \widehat{W}_t. \] (D.7)

Equations (D.2)–(D.7) show that the economy can be stationarized by rescaling by the trend price level.

**Appendix E: Consumption and Portfolio Decisions: Additional Results**

In this appendix, we provide some more detailed information regarding the agents’ consumption, investment, and portfolio decisions. Figure E.1 displays the average postdisplacement change in consumption. Relative to the results in Kolsrud et al. (2015), the model captures the drop in consumption during the first year following a displacement, but misses that consumption stops falling after the first year. Figure E.2, however, documents that most unemployment spells do not exceed one year.
FIGURE E.2. Distribution of the unemployed. The black bars measure the fraction of unemployed at various durations conditional on being in an expansion at the time of displacement. The grey bars provide the corresponding measure conditional on being in a recession.

Figure E.3 displays the complete cumulative distribution function of the value of assets at the beginning of an unemployment spell relative to the average net-income loss. As discussed in the main text, agents in the bottom of the wealth distribution are substantially richer than their real world counterparts, even if we focus on gross assets.

**Portfolio Composition.** Figure E.4 presents a scatter plot of the liquid asset’s share in the agents’ investment portfolios (y-axis) and the total amount invested (x-axis). The figure can be characterized reasonably well as follows. First, the fraction invested in money is higher at lower investment levels. Second, conditional on the amount invested, this fraction also increases when an agent becomes unemployed. Third, conditional on the amount invested and employment status, this fraction increases when the economy enters a recession. These three properties imply that the portfolio share invested in money increases during a recession. Without large enough increases in money portfolio shares, aggregate demand for money would decrease during recessions, like it does in the representative-agent model. This is because the total amount of funds carried over into the next period decreases during recessions, which in turn implies that the value of agents’ portfolios is lower.
Which forces explain the observed patterns? The first is that the transaction benefits of money are subject to diminishing returns. As a consequence, agents whose total demand for financial assets is high tend to invest a smaller fraction in money. This explains why the fraction invested in money is generally lower for asset-rich agents.

The second driving force is that money is less risky than equity. Therefore, agents whose total demand for financial assets is high relative to their nonasset income invest a larger fraction in money. For a given portfolio value, this explains why the fraction invested in money increases when a worker becomes unemployed, and why the fraction increases when the economy enters a recession.

Financial Assets During Unemployment Spells. Figures E.5 and E.6 document how the demand for equity and money holdings behave following a job displacement. The remarkable feature is that the demand for money actually increases during the first couple periods of an unemployment spell.

Unemployment Benefits and Unemployment Duration. As discussed above, it is not clear from empirical studies whether changes in unemployment benefits affect wages. There is much more empirical support for the hypothesis that more generous
benefits increase unemployment duration (see, for instance, Le Barbanchon (2016) for an overview). Several of these studies identify the effect of unemployment benefits on unemployment duration by considering changes in benefits that affect workers differently. These results may, thus, not be relevant for our general equilibrium experiment in which everybody is affected by the same increase in the replacement rate. If a large share of the unemployed search less intensely, then this provides improved opportunities of finding a job for those who actively search.\textsuperscript{101}

Appendix F: Solution Algorithm

F.1. Solution Algorithm for Representative-Agent Model

F.1.1. Algorithm. To solve the representative-agent models, we use a standard projection method, which solves for $q_{t+1}$ and $P_t$ on a grid and approximates the outcomes in-between gridpoints with piecewise linear interpolation.

F.1.2. Accuracy. Petrosky-Nadeau and Zhang (2017) consider a search and matching model with a representative agent. They show that it is not a trivial exercise to solve

\textsuperscript{101} Lalive, Landais, and Zweimüller (2015) argue that these externalities are quantitatively important.
FIGURE E.5. Postdisplacement equity holdings. The black line illustrates the average path for equity holdings of an individual that becomes unemployed in period 1, conditional on being in an expansion at the time of displacement. The grey line illustrates the equivalent path conditional being in a recession.

This model accurately, even though fluctuations are limited. Our representative-agent model is even simpler than the one considered in Petrosky-Nadeau and Zhang (2017). Nevertheless, we document here that both the linear and the log-linear perturbation solution are clearly not accurate. We also document that the projection solution is accurate.

To establish accuracy, we use the dynamic Euler-equation errors described in Den Haan (2010). The test compares simulated time series generated by the numerical solution for the policy rules with alternative time series. The alternative time paths are calculated using the exact equations of the model in each period; the approximation is not used, except when evaluating next period’s choices inside the expectations operator. This test is similar to the standard Euler equation test but reveals better whether (small) errors accumulate over time. If a numerical solution is accurate, then the two procedures generate very similar time paths.

Figure F.1 displays part of the generated time series and clearly documents that the linear perturbation solution has a substantial systematic error, whereas our projection solution does not. Table F.1 provides a more complete picture.
In Appendix F.2.1, we document how we solve the individual problem taking as given perceived laws of motion for prices and aggregate state variables. In Appendix F.2.2, we document how to generate time series for the variables of this economy, including the complete cross-sectional distribution, taking the individual policy rules as given. The simulation is needed to update the laws of motion for the aggregate variables and to characterize the properties of the model. We make a particularly strong effort in ensuring that markets clear exactly such that there is no “leakage” during the simulation. This is important since simulations play a key role in finding the numerical solution and in characterizing model properties.\footnote{If the equilibrium does not hold exactly, then the extent to which there is a disequilibrium is likely to accumulate over time, unless the inaccuracy would happen to be \textit{exactly} zero on average. Such accumulation is problematic, because long-time series are needed to obtain accurate representations of model properties.}
FIGURE F.1. Accuracy representative-agent solution. These graphs plot the time series for the employment rate generated with the indicated solution method and the exact solution according to the Euler equation when the approximation is only used to evaluate next period’s choices.

F.2.1. Solving for Individual Policy Functions. When solving for the individual policy functions, aggregate laws of motion as specified in Appendix F.2.2 are taken as given. Let \( \tilde{x}_i \) denote an individual cash on hand at the perceived prices. That is,

\[
\tilde{x}_i = e_i (1 - \tau) \frac{\tilde{W}}{\tilde{P}} + (1 - e_i) \mu (1 - \tau) \frac{\tilde{W}}{\tilde{P}} + q_i \left( \frac{\tilde{D}}{\tilde{P}} + (1 - \delta) \frac{\tilde{J}}{\tilde{P}} \right) + \frac{L_i R_{-1}}{\tilde{P}}. \tag{F.1}
\]
TABLE F.1. Accuracy comparison—Representative Agent Model.

<table>
<thead>
<tr>
<th></th>
<th>Projection</th>
<th>Linear perturbation</th>
<th>Log-linear perturbation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average error (%)</td>
<td>0.84e(−5)</td>
<td>1.01</td>
<td>0.43</td>
</tr>
<tr>
<td>Maximum error (%)</td>
<td>0.28e(−4)</td>
<td>1.76</td>
<td>0.74</td>
</tr>
<tr>
<td>Average unemployment rate (%)</td>
<td>11.5</td>
<td>10.7</td>
<td>10.7</td>
</tr>
<tr>
<td>Standard deviation of employment</td>
<td>2.91</td>
<td>2.63</td>
<td>2.87</td>
</tr>
</tbody>
</table>

Notes: These results are based on a sample of 100,000 observations.

Individual policy functions for equity, $q'_i = q(\tilde{x}_i, e_i, q, z)$, and the liquid asset, $L'_i = L(\tilde{x}_i, e_i, q, z)$, are obtained by iteration:

1. Using initial guesses for $q'_i$ and $L'_i$, a policy function for consumption can be calculated from the agent’s budget constraint

$$c(\tilde{x}_i, e_i, q, z) = \tilde{x}_i - \frac{q'_i \tilde{j} + L'_i}{\tilde{P}}.$$ (F.2)

2. Conditional on the realizations of the aggregate shock and the agent’s employment state, cash on hand and consumption in the next period can be calculated as

$$\tilde{x}'(e'_i, z') = e'_i (1 - \tau') \frac{\tilde{W}'}{\tilde{P}'} + (1 - e'_i) \mu (1 - \tau') \frac{\tilde{W}'}{\tilde{P}'} + q'_i \left( \frac{\tilde{j}'}{\tilde{P}'} + (1 - \delta) \frac{\tilde{j}'}{\tilde{P}'} \right) + \frac{L'_i \tilde{R}}{\tilde{P}'}.$$ (F.3)

and

$$c'(e'_i, z') = c(\tilde{x}'(e'_i, z'), e'_i, q', z').$$ (F.4)

3. Using the individual and aggregate transition probabilities, the expectations

$$\mathbb{E} \left[ c^{\gamma} \frac{\tilde{D}'}{\tilde{J}} + (1 - \delta) \frac{\tilde{j}'}{\tilde{P}} \right], \quad \text{and} \quad \mathbb{E} \left[ c^{\gamma} \frac{\tilde{P}}{\tilde{P}'} \right],$$ (F.5)

in the first-order conditions (3) and (4) can be calculated. Then, the first-order condition for equity holdings gives an updated guess for consumption of agents holding positive amounts of equity

$$c^{\text{new}}(\tilde{x}_i, e_i, q, z) = \left( \beta \mathbb{E} \left[ c^{\gamma} \frac{\tilde{D}'}{\tilde{J}} + (1 - \delta) \frac{\tilde{j}'}{\tilde{P}} \right] ^{-\frac{1}{\gamma}} \right).$$ (F.6)
The first-order condition for the liquid asset gives an updated policy function for it

$$L^{\text{new}}(\tilde{x}_i, e_i, q, z) = \tilde{P} \chi^{\frac{1}{\gamma}} \left( c^{\text{new}}(\tilde{x}_i, e_i, q, z)^{-\gamma} - \beta \mathbb{E} \left[ c^{t-\gamma} \frac{\tilde{P}}{P_t} \right] \right)^{-\frac{1}{\gamma}}.$$  \hfill (F.7)

The budget constraint in the current period gives the updated policy function for equity

$$q^{\text{new}}(\tilde{x}_i, e_i, q, z) = \max \left( 0, \frac{\tilde{x}_i \tilde{P} - c^{\text{new}}(\tilde{x}_i, e_i, q, z) \tilde{P} - L^{\text{new}}(\tilde{x}_i, e_i, q, z)}{J} \right).$$  \hfill (F.8)

For agents with a binding short-sale constraint, updated policy functions for consumption and liquid assets are instead calculated using only the first-order condition for liquid assets and the budget constraint

$$c^{\text{new,constraint}}(\tilde{x}_i, e_i, q, z) = \left( \beta \mathbb{E} \left[ c^{t-\gamma} \frac{\tilde{P}}{P_t} \right] + \chi \left( \frac{L^t_i}{\tilde{P}} \right)^{-\frac{1}{\gamma}} \right)^{-\frac{1}{\gamma}},$$  \hfill (F.9)

$$L^{\text{new,constraint}}(\tilde{x}_i, e_i, q, z) = \tilde{x}_i \tilde{P} - c^{\text{new,constraint}}(\tilde{x}_i, e_i, q, z) \tilde{P}.$$  \hfill (F.10)

4. A weighted average of the initial guesses and the new policy functions is used to update the initial guesses. The procedure is repeated from step (i) until the differences between initial and updated policy functions become sufficiently small.

F.2.2. Simulation and Solving for Laws of Motion of Key Aggregate Variables. The perceived laws of motion for the real stock price, $\tilde{J}/\tilde{P}$ and the price level, $\tilde{P}$, are given by the following two polynomials (using a total of 12 coefficients)

$$\ln \frac{\tilde{J}}{\tilde{P}} = a_0(z) + a_1(z) \ln q + a_2(z) (\ln q)^2.$$  \hfill (F.11)

$$\ln \tilde{P} = b_0(z) + b_1(z) \ln q + b_2(z) (\ln q)^2.$$  \hfill (F.12)

Note that $q$ is not only the level of employment, but also the number of firms, and the aggregate amount of equity shares held. We only use the first moment of the distribution of equity holdings, as in Krusell and Smith (1997), but we use a nonlinear function.\textsuperscript{103} To update the coefficients of this law of motion, we run a regression using simulated data. In this appendix, we describe how to simulate this economy taking the

\textsuperscript{103} Note that the first-moment of liquid asset holdings only depends on last period’s interest rate, because $R_{t-1} L_t$ is fixed and equal to $L$. 

Downloaded from https://academic.oup.com/jeea/advance-article-abstract/doi/10.1093/jeea/jvx040/4653491 by University of Cambridge user on 27 November 2017
policy rules of the individual agents as given. We start by describing the general idea and then turn to the particulars.

**General Idea of the Simulation Part of the Algorithm.** Policy functions are typically functions of the state variables, that is, functions of *predetermined* endogenous variables and *exogenous* random variables. These functions incorporate the effect that prices have on agents’ choices, but this formulation does not allow for prices to adjust if market clearing does not hold *exactly* when choices of the individuals are aggregated. If we used the true policy functions, then the equilibrium would hold exactly by definition. Unfortunately, this will not be true for numerical approximations, not even for very accurate ones. Because long simulations are needed, errors accumulate, driving supply and demand further apart, unless these errors happen to be exactly zero on average. Our simulation procedure is such that equilibrium does hold exactly. The cost of achieving this is that actual prices, \(J\) and \(P\), will be different from perceived prices, \(\tilde{J}\) and \(\tilde{P}\) and some of the actual individual choices will be different from those according to the original policy functions.\(^{104}\) These are errors too, but there is no reason that these will accumulate. In fact, we will document that perceived prices are close to actual prices in Appendix F.2.3.

**Preliminaries.** To simulate this economy, we need laws of motions for perceived prices, \(\tilde{J}(q, z)\) and \(\tilde{P}(q, z)\), as well as individual policy functions, \(q'_{i} = q(\tilde{x}_{i}, e_{i}, q, z)\) and \(L'_{i} = L(\tilde{x}_{i}, e_{i}, q, z)\). At the beginning of each period, we would also need the joint distribution of employment status, \(e_{i}\), and cash on hand, \(x_{i}\). This distribution is given by \(\psi(\tilde{x}_{i}, e_{i})\), where the tilde indicates that cash on hand is evaluated at perceived prices. The distribution is such that,

\[
\int_{\tilde{x}_{i}} \int_{e_{i}} \tilde{x}_{i} d\psi_{i} = zq + (1 - \delta)q \frac{\tilde{J}}{\tilde{P}} + \frac{\tilde{L}}{\tilde{P} \tilde{R}}, \tag{F.13}
\]

where the dependence of prices on the aggregate state variables has been suppressed. Below, we discuss how we construct a histogram for the cross-sectional distribution each period and show that this property is satisfied. We do not specify a joint distribution of equity and liquid asset holdings. As discussed below, we do know each agent’s level of beginning-of-period equity holdings, \(q_{i}\), and liquid asset holdings, \(L_{i}\).

A household’s cash-on-hand level is given by

\[
\tilde{x}_{i} = e_{i}(1 - \tau) \frac{\tilde{W}}{\tilde{P}} + (1 - e_{i})\mu(1 - \tau) \frac{\tilde{W}}{\tilde{P}} + q_{i} \left( \frac{\tilde{D}}{\tilde{P}} + (1 - \delta) \frac{\tilde{J}}{\tilde{P}} \right) + \frac{L_{i} R_{-1}}{\tilde{P}}. \tag{F.14}
\]

\(^{104}\). Throughout this appendix, perceived variables have a tilde and actual outcomes do not.
and the household can spend this on consumption and asset purchases, that is,
\[ \bar{x}_i = c_i + q_i' \frac{\tilde{J}}{\tilde{P}} + \frac{L_i'}{P}. \]  

(F.15)

The government has a balanced budget each period, that is,
\[ \tau = \mu \frac{1 - q}{q + \mu(1 - q)} + \frac{\tilde{R} - 1}{\tilde{R}} \frac{1}{q + \mu(1 - q)} \frac{\tilde{L}}{W}. \]

(F.16)

Even if the numerical solutions for \( q_i' \), \( L_i' \), \( \tilde{J} \), and \( \tilde{P} \) are very accurate, it is unlikely that equilibrium is exactly satisfied if we aggregate \( q_i' \) and \( L_i' \) across agents. To impose equilibrium exactly, we modify the numerical approximations for equity and liquid asset holdings such that they are no longer completely pinned down by exogenous random variables and predetermined variables, but instead depend directly—to at least some extent—on prices.\(^\text{105}\) In the remainder of this section, we explain how we do this and how we solve for equilibrium prices.

**Modification and Imposing Equilibrium.** To impose equilibrium we adjust \( q_i' \), \( L_i' \), \( \tilde{J} \), and \( \tilde{P} \). The equilibrium outcomes are denoted by \( \bar{q}_i, \bar{L}_i, \bar{J}, \) and \( \bar{P} \). The individual’s demand for assets is modified as follows:

\[ q_{i+1} = \frac{\tilde{J}}{J/P} q_i', \]  

(F.17)

\[ L_{i+1} = \frac{P}{\tilde{P}} L_i'. \]  

(F.18)

We will first discuss how equilibrium prices are determined and then discuss why this is a sensible modification. An important accuracy criterion is that this modification of the policy functions is small, that is, actual and perceived laws of motions are very similar.\(^\text{106}\)

We solve for the actual law of motion for employment, \( q_{+1} \), the number of new firms created, \( h \), the amount spent on creating new firms in real terms, \( v = hJ/P \), the market clearing asset price, \( J \), and the market clearing price level, \( P \), from the following equations\(^\text{107}\)

\[ q_{+1} = (1 - \delta)q + h. \]  

(F.19)

---

\(^\text{105}\) The policy functions \( q(x, \varepsilon, q, z) \) and \( L(x, \varepsilon, q, z) \) do depend on prices, but this dependence is captured by the aggregate state variables.

\(^\text{106}\) As explained above, it is important to do a modification like this to ensure that equilibrium holds exactly, even if the solution is very accurate and the modification small.

\(^\text{107}\) Recall that we define variables slightly different and \( v \) is not the number of vacancies, but the amount spent on creating new firms.
\[ h = \psi v^\eta (1 - q)^{1-\eta}, \quad (F.20) \]

\[ v = h J / P, \quad (F.21) \]

\[ h = \int \int_{\tilde{x}_i, e_i} (q_{i,+1}(\tilde{x}_i, e_i, q, z) - (1 - \delta)q_i) d\psi_i \]

\[ = \int \int_{\tilde{x}_i, e_i} \left( \frac{\tilde{J}}{J/P} q(\tilde{x}_i, e_i, q, z) - (1 - \delta)q_i \right) d\psi_i, \quad (F.22) \]

\[ \frac{L}{R} = \int \int_{\tilde{x}_i, e_i} L_{i,+1}(\tilde{x}_i, e_i, q, z) d\psi_i = \int \int_{\tilde{x}_i, e_i} \frac{P}{P} L(\tilde{x}_i, e_i, q, z) d\psi_i. \quad (F.23) \]

In particular, the distribution satisfies

\[ \int \int_{\tilde{x}_i, e_i} q_{i,+1} d\psi_i = q_{+1}. \quad (F.24) \]

**Logic Behind the Modification.** Recall that \( q(\tilde{x}_i, e_i, q, z) \) and \( L(\tilde{x}_i, e_i, q, z) \) are derived using perceived prices, \( \tilde{J}(q, z) \) and \( \tilde{P}(q, z) \). Now suppose that—in a particular period—aggregation of \( q(\tilde{x}_i, e_i, q, z) \) indicates that the demand for equity exceeds the supply for equity. This indicates that \( \tilde{J}(q, z) \) is too low in that period. By exactly imposing equilibrium, we increase the asset price and lower the demand for equity. Note that our modification is such that any possible misperception on prices does not affect the real amount each agent spends, but only the number of assets bought.

Throughout this section, the value of cash on hand that is used as the argument of the policy functions is constructed using perceived prices. In principle, the equilibrium prices that have been obtained could be used to update the definition of cash on hand and we could iterate on this until convergence. This would make the simulation more expensive. Moreover, our converged solutions are such that perceived and actual prices are close to each other, which means that this iterative procedure would not add much.

**Equilibrium in the Goods Market.** It remains to show that our modification is such that the goods market is in equilibrium as well. That is, Walras’ law is not wrecked by our modification.

From the budget constraint, we get that actual resources of agent \( i \) are equal to

\[ x_i = e_i (1 - \tau) \frac{W}{P} + (1 - e_i) \mu (1 - \tau) \frac{W}{P} + \left( \frac{D}{P} + (1 - \delta) \frac{J}{P} \right) q_i + \frac{L_j R_{-1}}{P} \quad (F.25) \]
and actual expenditures are equal to

\[ x_i = c_i + \frac{J_i}{P} q_{i,+1} + \frac{L_{i,+1}}{P}. \]  

(F.26)

The value of \( c_i \) adjusts to ensure this equation holds. Aggregation gives

\[ x = zq + \frac{J}{P} (1 - \delta) q + \frac{\bar{L}}{\bar{P} \bar{R}} \]  

(F.27)

and

\[ x = c + \frac{J}{P} \int \int q_{i,+1} d\psi_i + \int \int \frac{L_{i,+1}}{P} d\psi_i = c + \frac{J}{P} q_{+1} + \frac{\bar{L}}{\bar{P} \bar{R}}. \]  

(F.28)

Equation (F.27) uses the definition of dividends together with equation (F.16). Equation (F.28) follows from the construction of \( J \) and \( P \).

Because

\[ \frac{J}{P} q_{+1} - \frac{J}{P} (1 - \delta) q = v, \]  

(F.29)

we get

\[ zq = c + v, \]  

(F.30)

which means that we have goods market clearing in each and every time period.

**Implementation.** To simulate the economy, we use the “nonstochastic simulation method” developed in Young (2010). This procedure discretizes the cross-sectional distribution of agents’ characteristics using a histogram. This procedure would be computer intensive if we characterized the cross-sectional distribution of both equity and liquid asset holdings. Instead, we just characterize the cross-sectional distribution of cash-on-hand for the employed and unemployed. Let \( \psi(\tilde{x}_{i,-1}, e_{i,-1}) \) denote last period’s cross-sectional distribution of the cash-on-hand level and employment status. The objective is to calculate \( \psi(\tilde{x}_i, e_i) \).

1. As discussed above, given \( \psi(\tilde{x}_{i,-1}, e_{i,-1}) \) and the policy functions, we can calculate last period’s equilibrium outcome for the total number of firms (jobs) carried into the current period, \( q \); the job-finding rate, \( h_{-1}/(1 - q_{-1}) \); last period’s prices, \( J_{-1} \) and \( P_{-1} \); and for each individual the equilibrium asset holdings brought into the current period, \( q_i(\tilde{x}_{i,-1}, e_{i,-1}) \) and \( L_i(\tilde{x}_{i,-1}, e_{i,-1}) \).

2. Current employment, \( q \), together with the current technology shock, \( z \), allows us to calculate perceived prices \( \tilde{J} \) and \( \bar{P} \).

3. Using the perceived prices together with the asset holdings \( q_i \) and \( L_i \), we calculate perceived cash on hand conditional on last-period’s cash-on-hand level and both
the past and the present employment status. That is,

$$\tilde{x}(e_i, \tilde{x}_{i-1}, e_{i-1}) = e_i (1 - \tau) \frac{\tilde{W}}{\tilde{P}} + (1 - e_i) \mu (1 - \tau) \frac{\tilde{W}}{\tilde{P}} + q(\tilde{x}_{i-1}, e_{i-1}) \left( \frac{\tilde{D}}{\tilde{P}} + (1 - \delta) \frac{\tilde{j}}{\tilde{P}} \right) + \frac{L(\tilde{x}_{i-1}, e_{i-1})}{\tilde{P}} R_{i-1}. \quad (F.31)$$

4. Using last period’s distribution $\psi(\tilde{x}_{i-1}, e_{i-1})$ together with last-period’s transition probabilities, we can calculate the joint distribution of current perceived cash on hand, $\tilde{x}_i$, past employment status, and present employment status, $\tilde{\psi}(\tilde{x}_i, e_i, e_{i-1})$.

5. Next, we retrieve the current period’s distribution as

$$\psi(\tilde{x}_i, 1) = \tilde{\psi}(\tilde{x}_i, 1, 1) + \tilde{\psi}(\tilde{x}_i, 1, 0), \quad (F.32)$$

$$\psi(\tilde{x}_i, 0) = \tilde{\psi}(\tilde{x}_i, 0, 1) + \tilde{\psi}(\tilde{x}_i, 0, 0). \quad (F.33)$$

6. Even though we never explicitly calculate a multidimensional histogram, in each period we do have information on the joint cross-sectional distribution of cash on hand at perceived prices and asset holdings.

**Details.** Our wage-setting rule (7), contains $\tilde{P}$, an indicator for the average price level. For convenience, we use the average between the long-run expansion and the long-run recession value.\(^{108}\) Because it is a constant, it could be combined with the scaling factor, $\omega_0$. The properties of the algorithm are improved by including $\tilde{P}$. If a term like $\tilde{P}$ was not included, then average wages would change across iteration steps. Moreover, without such a term, then recalibrating $\omega_0$ would be more involved, for example, if we compare the case with and the case without aggregate uncertainty.

We use a simulation of 2,000 observations to estimate the coefficients of the laws of motion for aggregate variables. The first 150 observations are dropped to ensure the results are not affected by the specification of the initial state. The histogram that we use to track the cross-sectional distribution has 2,000 grid points. Statistics reported in the main text that are obtained by simulation are from a sample of 11,000 observations for aggregate variables and from a sample of 11,000,000 observations for idiosyncratic variables.

**F.2.3. Accuracy.** Conditional on perceived laws of motion for the price level and the employment rate, individual policy rules can be solved accurately using common numerical tools even though the presence of a portfolio problem makes the individual

\(^{108}\) This actually is a good approximation of the average price level.
optimization problem a bit more complex than the standard setup in heterogeneous-agent models. The key measure of accuracy is, therefore, whether the perceived laws of motion for the price level and the employment rate coincide with the corresponding laws of motion that are implied by the individual policy rules and market clearing. Figure F.2 shows that both perceived laws of motion track the implied market clearing outcome very closely.

References


