THE STABILITY OF CIRCULATION IN BUBBLE COLUMNS

by

David John Söderberg

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A dissertation submitted for the degree of Doctor of Philosophy in the University of Cambridge

October 1980
In memory of my father

Lieut. Cdr. J.D. Söderberg (RN).
The work described in this dissertation was carried out in the Department of Chemical Engineering, University of Cambridge, between October 1977 and October 1980, and is the original and independent work of the author except where specifically acknowledged in the text. Neither the whole nor any part of this dissertation has been previously submitted at any place.

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Churchill College, Cambridge

David J. Söderberg

October, 1980
The Stability of Circulation in Bubble Columns - by D.J. Söderberg

SUMMARY

This dissertation is concerned with the steady-state circulation of a two column, "riser-downcomer", circulating system, and with the instabilities which may arise during its operation. The system itself consists of a long-limbed 'U' tube with a tank on top to allow the disengagement of air so that clear liquid is returned to the downcomer.

In the downcomer, air injected in the form of bubbles is carried downwards and around the U bend up into the riser. In the riser bubbles move upwards with the liquid flow and are disengaged in the large tank at the top of the apparatus. The downcomer, containing clear liquid above the injector and bubbly liquid below the injector, is heavier than the riser which contains only bubbly liquid. Thus circulation is maintained by the net density difference between the two limbs.

An apparatus about 10 m tall and with limbs of 0.24 m diameter has been built: experiments led to the following conclusions.

(i) The steady-state circulation of the system may be adequately described by a simple theory. The data are well correlated by using a bubble slip velocity of 0.5 m/s.

(ii) The system is subject to two different instabilities, namely:

(a) When a slug initially present at the downcomer injector succeeds in rising further up the downcomer against the downward flowing liquid. This instability occurs when the liquid velocity falls below 1.1 m/s.

(b) Unstable oscillations in the circulating liquid velocity. These oscillations appear as a perturbation velocity, \( v \), which is superimposed upon the steady-state circulation velocity, \( V_0 \). The form
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(b) Unstable oscillations in the circulating liquid velocity. These oscillations appear as a perturbation velocity, \( v \), which is superimposed upon the steady-state circulation velocity, \( V_0 \). The form
of the perturbation is \( v = D^*e^{(\beta+i\omega)t} \), where \( D^* \) is a constant and \( \beta \) and \( \omega \) relate to the oscillation damping and period respectively. The boundary of the instability is shown to be a contour of the form \( \beta = 0 \), since for \( \beta > 0 \) the perturbation is undamped.

Theoretical stability plots have been constructed for the two instability boundaries (a) and (b); good agreement is obtained with experimental results. These plots can be used as the basis for the design of large scale industrial units.
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NOMENCLATURE (*COMPLEX DIMENSIONS)

\[ a = \frac{P_A}{\rho g} \]
Equivalent head of one atmosphere, in metres of water (m)

\[ a_B \]
Reduced flow area at bend (m²)

\[ a_S \]
Bubble interfacial area per unit volume (m⁻¹)

\[ A \]
Tube cross-sectional area (m²)

\[ A_D, A_R \]
Constants defined in section 5.3 (DIMENSIONLESS)

\[ A_S \]
Area of sparger holes (m²)

\[ b \]
A specific depth in Appendix 1 (m)

\[ B_D, B_R \]
Constants defined in section 5.3 (DIMENSIONLESS)

\[ c \]
A specific depth in Appendix 1 (m)

\[ C_{BN} \]
Concentration of nitrogen in bubble (kmol/m³)

\[ C_{BO} \]
Concentration of oxygen in bubble (kmol/m³)

\[ C_{N}^*, C_{O}^* \]
Saturated concentration of nitrogen in liquid (kmol/m³)

\[ C_{O}^* \]
Saturated concentration of oxygen in liquid (kmol/m³)

\[ d \]
Bubble diameter (m)

\[ d_o \]
Orifice diameter (m)

\[ D \]
Column diameter (m)

\[ D^* \]
Pre-exponential constant defined in section 5.3 (m/s)

\[ E_M = \frac{g d^2 (p - \rho_G)}{\sigma} \]
Eötvös number (DIMENSIONLESS)

\[ \text{or } = \frac{gd^2 \rho}{\sigma} \]

\[ f \]
Defined by equation 5.20 (*)

\[ f_e \]
Equivalent friction factor (DIMENSIONLESS)

\[ f_{TP} \]
Two-phase friction factor (DIMENSIONLESS)

\[ f(\epsilon) \]
A function of voidage (DIMENSIONLESS)

\[ Fr = \frac{(V_s)^2}{gD} \]
Proude number for bubble rise velocities (DIMENSIONLESS)

\[ F_M \]
Constant introduced in equation A2.4 (DIMENSIONLESS)

\[ g \]
Acceleration due to gravity (m/s²)

\[ G_M \]
Constant introduced in equation A2.6 (s⁻¹)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>h</td>
<td>Depth (m)</td>
</tr>
<tr>
<td>I(S1), I(S2), I(S3)</td>
<td>Imaginary parts of series S1, S2, S3 (DIMENSIONLESS)</td>
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<tr>
<td>K</td>
<td>Resistance forces coefficient (kg/m)</td>
</tr>
<tr>
<td>K*</td>
<td>Flow distribution parameter of Brown et al. (1969) (DIMENSIONLESS)</td>
</tr>
<tr>
<td>K_LNI</td>
<td>Mass transfer coefficient for nitrogen (m/s)</td>
</tr>
<tr>
<td>K_LOX</td>
<td>Mass transfer coefficient for oxygen (m/s)</td>
</tr>
<tr>
<td>K_ORIFICE</td>
<td>Resistance forces coefficient due to orifice plate in single-phase flow (kg/m)</td>
</tr>
<tr>
<td>K_S</td>
<td>Sparger constant in section 4.4 (m^{7/2}kg^{-1/2})</td>
</tr>
<tr>
<td>K_SF</td>
<td>Single phase resistance forces coefficient (kg/m)</td>
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<tr>
<td>K_SYSTEM</td>
<td>Resistance forces coefficient due to system only in single-phase flow (kg/m)</td>
</tr>
<tr>
<td>L</td>
<td>Liquid depth in apparatus (m)</td>
</tr>
<tr>
<td>L_D</td>
<td>Height of downcomer injector above bottom of apparatus (m)</td>
</tr>
<tr>
<td>L_E</td>
<td>Total equivalent pipe length (m)</td>
</tr>
<tr>
<td>L_R</td>
<td>Height of riser injector above bottom of apparatus (m)</td>
</tr>
<tr>
<td>m = (d_0/D)^2</td>
<td>Area ratio of orifice (DIMENSIONLESS)</td>
</tr>
<tr>
<td>M = gu^3/\rho\sigma^3</td>
<td>Morton number (DIMENSIONLESS)</td>
</tr>
<tr>
<td>M_E</td>
<td>Equivalent mass of the system (kg)</td>
</tr>
<tr>
<td>n</td>
<td>Richardson and Zaki index (DIMENSIONLESS)</td>
</tr>
<tr>
<td>N</td>
<td>Number of velocity heads lost (DIMENSIONLESS)</td>
</tr>
<tr>
<td>N_BEST</td>
<td>Best fit value of N for a particular apparatus geometry (DIMENSIONLESS)</td>
</tr>
<tr>
<td>N_SYSTEM</td>
<td>Number of velocity heads lost for system only in single-phase flow (DIMENSIONLESS)</td>
</tr>
<tr>
<td>P</td>
<td>Constant in equation A4.5 (DIMENSIONLESS)</td>
</tr>
<tr>
<td>P_A</td>
<td>Atmospheric pressure (kg/ms^2)</td>
</tr>
<tr>
<td>P_BC</td>
<td>Hydrostatic pressure above riser injector (kg/ms^2)</td>
</tr>
<tr>
<td>P_h</td>
<td>Hydrostatic pressure at depth h (kg/ms^2)</td>
</tr>
<tr>
<td>P_o</td>
<td>Compressor air supply pressure (kg/ms^2)</td>
</tr>
<tr>
<td>P_s</td>
<td>Riser air injection pressure (kg/ms^2)</td>
</tr>
</tbody>
</table>
Hydrostatic pressure acting on bubble (kg/ms²)
Pressure drop due to resisting forces (kg/ms²)
Pressure drop per unit length for gas flowing alone (kg/m²s²)
Pressure drop per unit length for liquid flowing alone (kg/m²s²)
Two-phase pressure drop per unit length (kg/m²s²)
Downcomer air supply rate (average or injection value specified) (m³/s)
Riser air supply rate (average or injection value specified) (m³/s)
Total air supply rate (average or injection value specified) (m³/s)
Bubble radius (m)
Dimensionless radius of Wallis (1974) (DIMENSIONLESS)
Initial bubble radius (m)
Gas constant (kg m²/s² kmol K)
Radius of curvature of spherical cap bubble (m)
Reynolds number of rising bubble (DIMENSIONLESS)
Real parts of series S1,S2,S3 (DIMENSIONLESS)
Complex infinite series introduced in equations A4.7,A4.8,A4.9 respectively (DIMENSIONLESS)
Defined by equation 5.19 (*)
Time that a bubble has been undergoing mass transfer (s)
Period of oscillation of liquid velocity (s)
Absolute temperature in Kelvin (K)
Single bubble terminal rise velocity (m/s)
Superficial liquid velocity (m/s)
Average liquid velocity (m/s)
Superficial gas velocity (m/s)
Small perturbation velocity (m/s)
Air velocity through holes in riser sparger (m/s)
\[ V \quad \text{System circulating velocity (m/s)} \]
\[ dV \quad \text{Velocity perturbation in section 5.1 (m/s)} \]
\[ V^* \quad \text{Dimensionless speed of Wallis (1974) (DIMENSIONLESS)} \]
\[ V_B \quad \text{Liquid velocity at entry into bend (m/s)} \]
\[ V_B^* \quad \text{Dimensionless speed (DIMENSIONLESS)} \]
\[ V_B \quad \text{Liquid velocity at bottom of bend (m/s)} \]
\[ V_{BU} \quad \text{Volume of bubble (m}^3) \]
\[ V_C \quad \text{Measured central pitot velocity (m/s)} \]
\[ V_{CD} \quad \text{Characteristic velocity of Wallis (1962) (m/s)} \]
\[ V_D^* \quad \text{Dimensionless speed (DIMENSIONLESS)} \]
\[ V_f \quad \text{Absolute liquid velocity below downcomer injector (m/s)} \]
\[ V_i \quad \text{A chosen system velocity: see section 4.5 (m/s)} \]
\[ V_M = U_G + U_L \quad \text{Total superficial velocity of mixture (m/s)} \]
\[ V_R \quad \text{Absolute liquid velocity below the riser injector (m/s)} \]
\[ V_R^* \quad \text{Dimensionless speed (DIMENSIONLESS)} \]
\[ V_B \quad \text{Absolute liquid velocity above the riser injector (m/s)} \]
\[ V_s \quad \text{Bubble slip velocity (m/s)} \]
\[ V_{SI} \quad \text{Measured side pitot velocity (m/s)} \]
\[ V_o \quad \text{Steady state circulation velocity (m/s)} \]
\[ V_1 \quad \text{Lower (unstable) circulation velocity in section 4.3 (m/s)} \]
\[ V_2 \quad \text{Higher (stable) circulation velocity in section 4.3 (m/s)} \]
\[ We = \rho U_b^2 d/\sigma \quad \text{Weber number (DIMENSIONLESS)} \]
\[ x, y \quad \text{General depths as defined in Figure 19 and Figure 8 (m)} \]
\[ x_B \quad \text{Height of standing bubble in bend (see Figure 11) (m)} \]
\[ X \quad \text{Parameter used by Lockhart and Martinelli (1949) (DIMENSIONLESS)} \]
\[ z \quad \text{Mole fraction of oxygen in bubble (DIMENSIONLESS)} \]
\[ z_o \quad \text{Initial mole fraction of oxygen in bubble (DIMENSIONLESS)} \]
<table>
<thead>
<tr>
<th>Greek Letters</th>
<th>Description</th>
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<tr>
<td>( \alpha )</td>
<td>Exponent of perturbation velocity ( v = D e^{\alpha t} ) (s(^{-1}))</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Real part of ( \alpha ) (s(^{-1}))</td>
</tr>
<tr>
<td>( \delta = L - L_R )</td>
<td>Riser injection depth in section 4.4 (m)</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Voidage (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \varepsilon(x) )</td>
<td>Voidage as a function of position (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \varepsilon(x,t) )</td>
<td>Voidage as a function of position and time (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \bar{\varepsilon}, \varepsilon_{bc} )</td>
<td>Average voidage (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \varepsilon' )</td>
<td>Extra voidage above riser injector due to riser air (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \varepsilon'(y), \varepsilon'(t) )</td>
<td>( \varepsilon' ) as a function of position, and as a function of time (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \varepsilon_{1I}, \varepsilon'_{1I}(t) )</td>
<td>Voidages at downcomer injector (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \varepsilon'<em>{1I}, \varepsilon'</em>{1}(t) )</td>
<td>Extra voidages at riser injector due to riser air (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \varepsilon_{1}, \varepsilon_{1}(t) )</td>
<td>Voidages beneath downcomer injector (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \varepsilon_{2}, \varepsilon_{2}(t) )</td>
<td>Voidages beneath riser injector (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \varepsilon_{3} )</td>
<td>Voidage (total) above riser injector (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \varepsilon_{R} )</td>
<td>Voidage in riser in section 4.4 (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \varepsilon_{0} )</td>
<td>Voidage which would exist at atmospheric pressure (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \delta \varepsilon_{1} )</td>
<td>Change in voidage at downcomer injector (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \delta \varepsilon_{3} )</td>
<td>Change in voidage (total) at riser injector (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Surface tension (kg/s(^2))</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Liquid density (kg/m(^3))</td>
</tr>
<tr>
<td>( \rho_G )</td>
<td>Gas density (kg/m(^3))</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Liquid viscosity (kg/m.s)</td>
</tr>
<tr>
<td>( \mu_G )</td>
<td>Gas viscosity (kg/m.s)</td>
</tr>
<tr>
<td>( \eta = X/(1+X) )</td>
<td>Parameter introduced by Chisholm and Sutherland (1969-70) (DIMENSIONLESS)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>( \lambda_{cr} )</td>
<td>Critical wavelength for splitting by Taylor Instability (m)</td>
</tr>
<tr>
<td>( \phi_G )</td>
<td>Two-phase multiplier (DIMENSIONLESS)</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Imaginary part of ( \alpha ) (s(^{-1}))</td>
</tr>
</tbody>
</table>
A two-column, "riser-downcomer", circulating system is a concept which has emerged during the last ten years. An illustration of the type of system under consideration is shown in Figure 1(b). It consists of a long-limbed 'U'-tube with a tank on top both to complete the hydraulic circuit, and to disengage the air injected into the system. The objective is to induce circulation, up the riser and down the downcomer, by introducing air into one or both of the limbs.

In the downcomer, air injected in the form of bubbles is carried downwards. In the riser, bubbles move upwards with the liquid flow and are disengaged in the tank at the top. Circulation is maintained by the density difference between the two legs.

The system offers several advantages over traditional column and reactor designs. Besides a lack of moving parts, the ratio of the (power consumed by the mixing) to (mass transfer of oxygen achieved) is far lower than for a conventional system. Whereas in a sparged stirred tank power is consumed both in stirring and also in air compression, the "riser-downcomer" system only requires a compressed air supply. There is the possibility of use in large scale biological systems where high oxygen mass transfer rates are required: the hydrostatic pressure giving higher concentrations of dissolved oxygen than are possible in conventional shallow tanks.

In this dissertation the basic operating principles of such a system are outlined and the relevance of various two-phase flow phenomena are discussed. A simple steady-state circulation theory is given, which can be successfully applied in practice.

Instabilities in the system are discussed and are shown to be of two different kinds:
(i) The first instability involves the "breakthrough" of a slug from its position at the downcomer injector.

(ii) The second involves an undamped oscillatory velocity perturbation superimposed upon the steady-state circulation velocity.

Theoretical explanations are given for these instabilities, and a theoretically predicted plot of stable and unstable operating regions is compared with the experimental findings.
CHAPTER ONE: LITERATURE SURVEY

The following presents a general discussion of bubbly two-phase flow, to provide a background for the reader, of the factors affecting the present work. Specific literature concerning both the experimental apparatus and theories relating to its operation, will be discussed later in the relevant chapters.

1.1 Bubble Types and Flow Maps
1.1.1 Flow Regimes of Interest

This work is concerned with a circulating system operating mainly in the bubbly regime of two-phase air-water flow. It is also concerned with the appearance of slugs in the system, and so this regime is also of some interest. Within the bubbly flow regime three separate bubble types exist, namely spherical bubbles, ellipsoidal or spheroidal bubbles, and spherical cap bubbles. The three bubble types may be distinguished from slugs by the fact that they can be formed when air is injected into an infinite expanse of liquid in the form of a single bubble, whereas a slug almost fills the duct within which it is confined.

Since the bubbly and slug regimes are only two of several in two-phase flow, it is of great interest to be able to determine the particular regime that one encounters for given values of the system physical properties and phase flowrates. Attempts have been made in the past to obtain flow regime maps for horizontal flows, the classical one being due to Baker (1954). More recently flow regime maps for upward and downward gas/liquid flows have been presented by Golan and Stenning (1969-70) and Spedding and Nguyen (1980); these are more applicable to the present work.

Golan and Stenning (1969-70) found that there were considerable
differences between upward and downward gas/liquid flows and this led them to the conclusion that vertically upward flow maps should not be used for predicting behaviour in downward flow. Spedding and Nguyen (1980) also conclude that a general flow regime map which takes into account phase flow rates and pipe inclination, is not possible at this stage. They also point out that pipe diameter could be expected to have a significant effect on the accuracy of flow regime maps. Since flow maps all seem to have been based on data collected from small tubes, with diameters around 50 mm or less, their accurate application to larger systems may well be doubtful.

1.1.2 Bubble Shape Regions

Workers have indicated the position of boundaries between spherical, ellipsoidal, and spherical cap bubbles. Peebles and Garber (1953) give the boundaries of a region where ellipsoidal bubbles, with spiralling zig-zag paths, may be found. Habermann and Morton (1953) present their results as a plot of \( U_{b\infty} \) against Reynolds number, with observations that for liquids with a low Morton number, the transition from spherical to ellipsoidal occurs at \( \text{Re} \approx 250 \), and the transition to spherical cap at a Weber number of about twenty.

Grace (1973) gives boundaries between the three bubble types mentioned, on a plot of \( \log(\text{Re}) \) against \( \log(\text{Eo}) \), with the Morton number as a plotting parameter. He points out that it is not as easy to distinguish between the spherical and ellipsoidal regions as it is between the spherical cap and other regions: spherical cap bubbles are said to occur for \( (\text{Re}) > 1.2 \) and \( (\text{Eo}) > 40 \). He also points out that no ellipsoidal region occurs for values of the Morton number greater than ten, and that bubbles are spherical at low Reynolds numbers regardless of how large the Eötvös number is.
Coppus and Rietema (1980) dispute that spherical bubbles exist for large \( \text{Eo} \) and small \( \text{Re} \), they suggest that the condition for transition from the spherical to the ellipsoidal region is given by \( \text{Re} \frac{M^4}{\rho} > 0.5 \). However, Coppus and Rietema have made a mistake in not considering the solution of the Navier-Stokes equation for creeping flow around a fluid sphere, as was originally done by Hadamard (1911) and Rybczynski (1911). Clift et al. (1978a) conclude that the results of these earlier workers show that if Reynolds numbers are very low, bubbles and drops remain spherical no matter how small the surface tension forces. This conclusion was also drawn by Batchelor (1967).
1.2 Slip Velocities and the Influence of Other Bubbles

In order to predict certain characteristics of the circulating loop system, it is necessary to have an understanding of the relationship between bubble slip velocity and other system parameters such as voidage.

1.2.1 Single Bubble Rise Velocities

Correlations have been produced for the rise velocity of bubbles, \( U_{\text{bo}} \), in an infinite expanse of liquid, in the absence of impurities. In this work we are mainly concerned with bubbles larger than 3 mm in diameter, and so the only theoretical equations of interest concern the rise velocities of spherical caps and slugs which will be mentioned later. Correlations for the rise velocities of bubbles tend to cover the three shape regimes of interest namely spherical, ellipsoidal and spherical cap. Two of the earliest, already mentioned, are those of Habermann and Morton (1953), and of Peebles and Garber (1953), the latter giving equations for \( U_{\text{bo}} \) in four regions noted. We are mainly interested in single bubbles in the range of equivalent diameter from 3 to 8 mm, for which the rise velocity is nearly constant. More recent correlations include that of Grace (1973) and Coppus and Rietema (1980), already mentioned, together with that of Wallis (1974). The last named correlation is in terms of a dimensionless speed

\[ V^* = U_{\text{bo}} \left( \frac{\rho^2}{\mu g(\rho-\rho_G)} \right)^{1/3} \]

and a dimensionless radius

\[ r^* = r \left( \frac{\rho g(\rho-\rho_G)}{\mu^2} \right)^{1/3} \]

with Wallis (1974) commenting that bubble behaviour is greatly influenced by the "cleanliness" of the system; impurities can have a large effect on bubble behaviour and will be discussed later.

In the case of spherical cap bubbles, where the effects of surface tension and viscosity are negligible (and the equivalent diameter of
the bubble is less than about one quarter of the tube diameter), the rise velocity is accurately predicted by the equation of Davies and Taylor (1950)

\[ U_{b,oo} = \frac{2gR_c}{3} \]

where \( g \) is the acceleration due to gravity and \( R_c \) is the radius of curvature of the spherical cap. The effects of tube wall proximity on the rise velocity of spherical cap bubbles have been given by Wallis (1969a) on the basis of the work of Collins (1967), wall effects only becoming apparent when the bubble diameter is greater than about one eighth of the tube diameter. (This may well be the case in the work of Peebles and Garber (1953) on a 26 mm ID tube.) Whilst discussing spherical cap bubbles, Wegener and Parlange (1973) state that if wall effects become important, reduced rise speed and higher relative curvature are noticed: these effects are also apparent when high viscosity liquids are used.

In the case of slugs, using a crude theory, Davies and Taylor (1950) predicted that \( V_s = 0.33\sqrt{gD} \) for the case of a slug travelling up a circular tube filled with water and emptying at the bottom. Dumitrescu (1943) showed that \( V_s = 0.35\sqrt{gD} \) for slugs injected into a tube closed at the top. The smaller values obtained experimentally by Davies and Taylor (1950) were probably due to a scale effect. Nicklin et al. (1962) did experiments on systems where the liquid was not stationary. They showed that slugs rose relative to the liquid ahead of them at the rise velocity predicted by Dumitrescu, and that the slug rise velocity in upward flow was given by

\[ V_s = 1.2 U_L + 0.35\sqrt{gD} \quad \text{(for Re > 8000)} \]

This result was confirmed at high Reynolds numbers by Grace and
Clift (1979). A correlation for the rise velocity of both spherical caps and slugs has been given by White and Beardmore (1962) in terms of three dimensionless groups, the Froude number \( Fr = \frac{V^2_s}{gD} \), \( E_3 = \frac{gD^2 \rho}{\sigma} \) and \( M \). Slugs in downward liquid flow will be discussed in section 4.1.1.

1.2.2 Effects of other Bubbles

In the circulating system, large bubbles may be present in amongst swarms of smaller ones. This can be due to the method of gas injection, phase redistribution at bends, or coalescence in general. This phenomenon was observed in both the small and the large apparatus used in this work (see Figure 1(a) and (b)), and has been observed in bubble columns by other authors, notably Hills and Darton (1976), though also by Bridge et al. (1964), Gomezplata et al. (1972), and Lockett and Kirkpatrick (1975). Hills (1975) developed an expression for the theoretical trajectory followed by a small bubble when overtaken by a "two-dimensional" circular cap: "Caps" may be taken from now on as referring to bubbles which are intermediate between spherical cap bubbles and slugs. In a more recent paper, Hills and Darton (1976) showed that in all but the narrowest column considered, considerable enhancement of the rising velocity was found with large bubbles (or slugs) rising through clouds of smaller ones. They attributed this enhancement in rise velocity to small scale eddies in the liquid, produced by the bubble swarm, which distort the upper surface of the cap by changing the flow pattern around the bubble, upon which depends its rising velocity.

Relationships between \( V_s \), \( U_{\infty} \) and \( \epsilon \) which have been proposed by various authors are listed by Lockett and Kirkpatrick (1975), they are:
(1) \( V_s = U_{b_{\infty}} \) (Turner (1966)).

(2) \( V_s = \frac{U_{b_{\infty}}}{1-\varepsilon} \) (Davidson and Harrison (1966)).

(3) \( V_s = U_{b_{\infty}} (1-\varepsilon)^{n-1} \) (with various values of \( n \), derived from an equation due to Richardson and Zaki (1954)).

(4) \( V_s = \frac{U_{b_{\infty}}}{1-\varepsilon} (1-\varepsilon^{5/3}) \) (Marrucci (1965)).

Their conclusion is that the Richardson and Zaki equation, with \( n = 2.39 \) is best, if a correction factor of the form \( f(\varepsilon) = 1+2.55 \varepsilon^3 \) is added. Using riser air only in an apparatus similar to that used in this work, Hills (1976) found that his experimental results were well correlated by the equations

\[
\frac{U_G}{\varepsilon} = 0.24 + 1.35 V_M^{0.93} \quad \text{for} \quad U_L > 0.3 \text{ m/s}
\]

(where \( V_M \) is the total superficial velocity of the mixture), and

\[
V_s = 0.24 + 4.0 \varepsilon^{1.72} \quad \text{for} \quad U_L \leq 0.3 \text{ m/s}
\]

Finally, Kubota et al. (1978) suggest the use of an equation of the form

\[
\frac{V_s}{U_{b_{\infty}}} = 1 + 1.35 (\sqrt{gD/U_{b_{\infty}}})\varepsilon
\]

for use in such a circulating system, with \( U_{b_{\infty}} \) equal to 0.3 m/s.

It is interesting to note that the later authors' correlations show slip velocity increasing with voidage, whereas those correlations suggested by Lockett and Kirkpatrick (1975) (with the exception of that due to Davidson and Harrison (1966)), do not. This difference of opinion may well be due to the presence of larger bubbles or bubble clusters in the later works. In fact Lockett and Kirkpatrick (1975) state that the presence of these larger bubbles leads to a breakdown of their proposed relationship. Liquid circulation is another reason stated, but this will be dealt with in a later section.

The method proposed by Sriram and Mann (1977) can be used to
investigate liquid motion and bubble rise velocities by dynamic measurement of the gas disengagement from a bubble column. However, there remains a difficulty in separating the rise velocities from the liquid motion, since results depend upon the circulation model used to describe the system.

Once the relationship between \( V_s \), \( U_{b_{\infty}} \) and \( \varepsilon \) is known, a material balance may be performed on the system to provide a relationship first noted by Lapidus and Elgin (1957), namely:

\[
V_s = \left| \frac{U_G}{\varepsilon} - \frac{U_L}{(1-\varepsilon)} \right|
\]

The Lapidus and Elgin equation was rearranged by Wallis (1962), using a characteristic velocity \( V_{CD} \), into the form

\[
V_{CD} = \frac{V_s \varepsilon (1-\varepsilon)}{U_G (1-\varepsilon) - U_L \varepsilon}
\]

Thus knowing \( V_s = f(\varepsilon).U_{b_{\infty}} \), \( V_{CD} \) can be plotted as a function of \( \varepsilon \). This is useful in predicting operating conditions in a bubble column as a function of \( U_L \) and \( U_G \), as has clearly been shown by Whalley et al. (1972) amongst others.
1.3 Impurities, Coalescence and Breakup

1.3.1 Coalescence and Breakup

The relative rates of coalescence and breakup in a bubble column play an important part in determining the holdup in the system. In work on pure liquids Kirkpatrick and Lockett (1974), using an apparatus similar to that of Davidson and Kirk (1969), recognised two basic types of bubble coalescence in a swarm of 5 mm diameter bubbles, depending upon the approach velocity of the bubbles (i.e. how fast they are coming together). At low approach velocities they proposed that rapid coalescence would result, since film rupture could occur before the approaching bubbles were brought to rest. The observed lack of coalescence in bubble swarms was attributed to a high approach velocity, the bubbles bouncing apart before film rupture could occur.

Following the two dimensional theory of Hills (1975) on a circular "cap" bubble overtaking a smaller bubble, Otake et al. (1977) made observations on the interactions between two three dimensional "cap" type bubbles, specifically one bubble following in the wake of the other rising bubble. They concluded that there was a critical distance of about 3–4 bubble diameters at which the leading bubble began to exert a noticeable influence on the following one, coalescence or breakup occurring depending upon the overlap of the projected area of the leading on the following bubble.

It has been pointed out that the rate of bubble coalescence in a bubble column is not evenly spread up the column height. The experimental evidence of Mamucci and Nicodemo (1967) and Otake et al. (1977) suggests that coalescence appears to be most evident in the region near the gas distributor. Otake et al. (1977) also stated that bubble breakup took place more readily than bubble coalescence as \( U_G \) was increased, this is contrary to the conclusions of Mamucci and
Nicodemo (1967).

In the case of two slugs rising simultaneously through a vertical tube, Clift et al. (1974) found that the rear slug accelerates and eventually coalesces with the leading slug. Their results suggested that two slugs will always coalesce if a great enough distance is allowed, and they provided a model to predict the velocity of one slug relative to the other in such a system.

The breakup mechanism of larger bubbles has been commented upon by Clift and Grace (1972), Henriksen and Østergaard (1974), and Grace et al. (1978). Clift and Grace (1972) stated that bubbles formed by coalescence or by injection in a fluidised bed may be large enough to divide by the instability proposed by Taylor (1950). This conclusion was applied to large bubbles in liquids by Henriksen and Østergaard (1974), and confirmatory experimental data from several two-phase systems was summarised by Grace et al. (1978).

1.3.2 Impurities

Impurities are well known to affect the coalescence rates and bubble size in a bubble column, thus altering the holdup of gas in the system. Their effects have been investigated by several authors, amongst them Bridge et al. (1964), Marucci and Nicodemo (1967), Lee and Meyrick (1970), Anderson and Quinn (1970), and Whalley et al. (1972). Authors agree that the effect of a solute is to decrease bubble coalescence, leading to larger holdups in bubble columns than is observed with pure water, and up to a 50% decrease in holdup is quoted by Anderson and Quinn (1970), when changing from tap water to distilled water.

In the case of electrolytes a possible explanation for the lack of coalescence concerns the presence of electrolyte hindering film
drainage. Stretching of the film as it thins leads to a concentration gradient being set up, since as an element of film is stretched the same volume of liquid presents a larger interfacial area and the total deficiency of electrolyte in the surface layers increases. This concentration gradient sets up interfacial tension gradients sufficient to oppose any further stretching of the film, after which film drainage takes place by the slow process of viscous flow between two immobilised surfaces. Kirkpatrick and Lockett (1974) state that in concentrated electrolyte systems where coalescence is inhibited, the effect of approach velocity is unimportant.

In the case of highly polar surface active agents, another explanation for the lack of coalescence has been proposed by Marrucci and Nicodemo (1967). Due to the polar nature of the molecules, they concentrate and align themselves at the interface, leading to the formation of an electrical double layer and hence surface potential, which hinders coalescence. Anderson and Quinn (1970) propose that axial profiles of any impurities present may be set up during experimental runs, leading to hysteresis effects in repeated plots of $U_G$ against $\varepsilon$.

It is well known that the effect of impurities in water on spherical bubbles is to make them behave as if they were solid rather than fluid spheres, leading to lower bubble rise velocities. This effect is very marked for bubbles less than 0.5 mm in diameter where only trace impurities are needed to bring about this change. Clift et al. (1978b) discuss the effects of impurities on single spherical, ellipsoidal, and spherical cap bubbles, noting that surface active contaminants affect single bubble rise velocities most strongly in the ellipsoidal region. Spherical cap bubbles are not affected to any great extent by impurities, since surface tension forces cease to be
important in this region, this was noted experimentally by Habermann and Morton (1953).

Throughout this work Cambridge tapwater, which has a calcium content of about 300 ppm as quoted by the Cambridge Water Company (1980), was used.
1.4 **Radial Voidage Distribution and Circulation Cells**

In the following, the radial voidage distributions found and the possibility of circulation cells within bubble columns will be discussed. More detailed discussions of bubbles in downflow and the effect of bends are given in other sections.

1.4.1 **Radial Voidage Distribution**

An assumption inherent in most two-phase flow calculations and measurements is that homogeneous voidage exists. However, evidence has been presented to show that this is generally not the case, either for large or for small bubbles. Lockett and Kirkpatrick (1975) observed that large bubbles in bubble columns have "preferential tracks" up which they rise. Martin (1976) observed another type of radial variation with slugs in downflow, he noted that they oscillate back and forth across the tube keeping the slug nose in a velocity field always less than the maximum.

In the case of small bubbles, the form of the voidage distribution in cocurrent upward flow is the subject of some debate. Sato and Sekoguchi (1975) and Rouhani (1976), amongst others, claim to have observed radial peaking of voidage near the tube walls. They propose that in bubbly flow, some of the bubbles near the wall form centres of rotation for a layer of the surrounding liquid. These so-called "rolling vortices" would be formed in the turbulent boundary layer by the deceleration of fluid particles approaching the viscous sublayer. Thus the centripetal forces produced by the vortices on the bubbles would tend to collect the bubbles in a "bubble sublayer".

Other authors have found no evidence of radial peaking near the tube walls, and observe a central maximum in voidage. Hills (1979), using an apparatus similar to that used in this work, concludes that
radial voidage measurement gives no indication of voidage peaking near the walls in such a circulating system. The same conclusion can be reached from the results of Hoang and Davis (1980) who give voidage profiles observed 1.5 pipe diameters prior to entry into a bend (which is before the bend begins to influence the voidage profile). They used liquid velocities between 3.5 and 9 m/s and conclude that for the lower liquid velocities the voidage profile is centrally peaked; at higher velocities the profile is a flattened maximum over the central 30% of the tube diameter.

Brown et al. (1969) have produced a model which takes into account a radial variation of both liquid velocity and voidage. In their model, the assumption of parabolic type profiles for the liquid velocity and void fractions were made, and the result can be expressed in the form of a modification to the material balance equation of Lapidus and Elgin (1957) involving a single distribution parameter, $K^*$:

$$ V_s = \frac{U_G - U_L}{\varepsilon (K^* - \varepsilon)} $$

(1.3)

Stepanek (1970) noted that the profiles proposed by Brown et al. (1969) do not fulfill the condition of zero velocity and void-fraction (found by other workers) at the pipe wall. However, he was able to show that the modified equation involving $K^*$ was of a fully general nature and not dependent upon the type of void-fraction and velocity profiles assumed. Correlations for $K^*$ in terms of the Reynolds number of the flow have been proposed by Gomezplata et al. (1972) for both upflow and downflow. For downflow the correlation predicts that the voidage is at a minimum in the centre of the column, contrary to the results of Hoang and Davis (1980) who found a central peak in downflow voidage 9 pipe diameters downstream of a bend.
1.4.2 Circulation Cells

Radial voidage variation within a bubble column can provide the driving force for the setting up of a circulation cell within the column: the so-called "Gulf Stream" effect. This effect has been investigated for a column with a similar height and diameter, at low gas rates, by Freedman and Davidson (1969). They established the circulation rate within the column by means of a pressure balance, and found that it was of the same order as the bubble rise velocity. The work was extended by Whalley and Davidson (1974), this time using an energy balance to establish circulation rates. They predict that in wide shallow columns, where multiple circulation cells have been observed, the number of cells sets itself to give minimum liquid vorticity.

The presence of a circulation cell within a bubble column can have a dramatic effect upon the gas holdup in the system. Kirkpatrick and Lockett (1974) stated that in their column the recirculation became important as the voidage increased above 0.2. Hills (1974) observed large liquid velocities, of the order of 0.5 m/s, at gas rates above 0.04 m/s, in the centre of a column with no net liquid flow. Anderson and Quinn (1970) noted that when the column diameter was increased, the gas holdup decreased, due to the greater circulation.

The importance of a knowledge of circulation cells in modelling holdup and bubble size distributions in a bubble column becomes obvious from the work of Sriram and Mann (1977), since the presence of liquid circulation significantly alters the predicted bubble size spectrum. Describing a system very similar to that used in this work, namely an airlift tower fermenter, Orazem et al. (1979) discovered evidence of a circulation cell at the top of the downcomer whilst investigating incoming air entrainment into the downflow section: the effective disengagement tank used in this work should avoid this added complication.
1.5 Two-Phase Friction Correlations, and the Effect of Bends

1.5.1 Correlations for Two-Phase Frictional Pressure Drop and Holdup

In a circulating loop type system the stable operating velocity is determined by a balance between the density driving forces and the frictional resistance forces, thus a knowledge of the latter is vital to any theory for circulation developed.

Reasonable correlations of the two-phase frictional pressure drop have now been in use for about the last thirty years, the first of these for two component flow being due to Lockhart and Martinelli (1949). This was originally derived for horizontal flow, but has found wide use in vertical flow as well. In it, a two-phase multiplier \( \delta_G \) defined by

\[
\delta_G^2 = \frac{(dp/dz)_{TP}}{(dp/dz)_G}
\]

(where \((dp/dz)_{TP}\) is the two-phase pressure drop, and \((dp/dz)_G\) is the pressure drop for the gas phase flowing alone) is correlated with a parameter \( X \), where \( X \) is defined by

\[
X^2 = \frac{(dp/dz)_L}{(dp/dz)_G}
\]

(where \((dp/dz)_L\) is the pressure drop for the liquid phase flowing alone). Also the liquid fraction \((1-\varepsilon)\) is correlated with \( X \). The reason for not correlating \( \delta^2 \) against \( X \) or \( X^2 \) was originally because of a shortage of log-log graph paper! Baroczy (1966) extended this method, by recognising the existence of a mass flowrate influence and the influence of system pressure, which were not accounted for by the earlier correlation. His correlations involved the use of a mass flux and a "property index", here defined as \( \left(\frac{\mu}{\mu_G}\right)^{0.2} \frac{\rho_G}{\rho} \), where \( \mu \) and \( \mu_G \) are the liquid and gas dynamic viscosities. There was no real reason for the 0.2 factor originally except for a suitable graphical representation. A later correlation by Chisholm and
Sutherland (1969-70) followed Baroczy's use of a property index, here defined as \((\mu_G/\mu)^{0.125} (\rho/\rho_G)^{0.5}\). They also used a function of \(X\) (the Martinelli Parameter) of the form \(\eta = X/(1+X)\) for the abscissae of the Charts.

Papers have been written comparing the various correlations for pressure drop and holdup, and indicating those in best agreement with the available data at the time. Dukler et al. (1964) tested five correlations and found that Lockhart-Martinelli (1949) was in best agreement for correlation of pressure drop, whilst that due to Hughmark (1962), though poor, was in best agreement for holdup correlation. Gregory (1975) tested correlations for holdup with data for pipes inclined at a maximum of 10° to the horizontal. Again the conclusion reached was that the Hughmark (1962) correlation was in best agreement for bubble and slug flow. A more recent paper by Collier (1976) gives guidance for choosing correlations for two-phase pressure drop in certain geometrical configurations, e.g. enlargements, contractions, bends, tees etc.

As opposed to correlations, Davis (1974) has developed an analytical expression for the two-phase friction factor based on integration of the one-dimensional momentum equation for turbulent two-phase flow of a bubbly air-water mixture. This can be used to predict trends in the friction factor.

As an alternative to using two-phase friction factors, it has been proposed by certain authors to make use of a single-phase friction factor and a modified velocity to take account of the nature of the flow. These include Wallis (1969b), Owens (1961), and Gomezplata and Nichols (1967), all also assuming homogeneous flow. Single-phase friction factors have also been used by Kubota et al. (1978), and Hills (1976), to describe a circulating system similar to that used in
this work. In contrast, Hsu and Duduković (1980), also looking at such a circulating system, found that the data deviated widely from the predictions for single-phase flow and that they did not conform well to the Lockhart and Martinelli (1949) correlation either.

Experimentally the effect of liquid properties on frictional pressure drops in 25.4 mm diameter tubes has been investigated in upward and downward vertical flow of a liquid containing bubbles by Ohinowo and Charles (1974). They concluded that increasing \( \mu \) and \( \rho \) and decreasing \( \sigma \) leads to a rise in downflow pressure drop and a decrease in upflow pressure drop (though for high liquid flow rates the pressure drop in upflow increases). Data for large diameter tubes are notable for their absence.

### 1.5.2 The Effect of Bends and Obstructions

In the circulating loop system, because of the U bend at the bottom, we are very interested in any work on the effects of bends on two-phase flow phase distributions.

Hoang and Davis (1980) propose a criterion for phase separation in a bend, namely that where the turbulent kinetic energy exceeds the potential energy difference across the pipe radius it is expected that separation does not occur, and vice versa. Gardner and Neller (1969-70) describe the forces competing at a bend:

1. Centrifugal action - concentrating liquid at the outside of the bend.
2. Secondary flow - as described by Goldstein (1938a) where liquid from the outside of the bend is taken to the inside of the bend. This is generally a much smaller effect than centrifugal action in a circulating system like that used in this work.
3. Gravity - which makes the lightest phase rise to the highest point of the bend.
They conclude that "A bend or bend system seems to be an effective agglomerator of bubbles."

The effect on the flow downstream of a bend is mentioned by Golan and Stenning (1969-70), who had U bends both at the bottom, and inverted at the top of their apparatus. The effect of the upper bend, which had water at its inner radius at all flowrates, seemed to disappear after about ten diameters of downflow. Flow separation in the lower U bend was found to be more severe than in the upper U, though the effect of the lower U on phase distribution was observed to disappear after about four tube diameters. The authors ascribe this to the fact that gravity tends to cause a back flow of the liquid phase in upflow, which contributes to this rapid flow redistribution.

Gardner and Neller (1969-70) noted that an air pocket tended to form behind an obstruction placed in the flow: see section 2.2.4 on the air injectors in this work. Addition of air to the main stream tended to make this disappear.
2.1 Small Apparatus

A small apparatus, whose characteristics are shown in Figure 1(a) was built as an initial step towards understanding the operating features of a circulating "riser-downcomer" column system.

The apparatus consisted of a downcomer and riser leg of equal lengths, constructed from 50.8 mm ID perspex tubing. The riser and downcomer were connected at the bottom by a U bend, fabricated from four straight pieces of the same perspex tubing, see enlarged view of Figure 1(a). The flow pattern produced by a U section of this nature is much more difficult to quantify than that produced by a smooth U. However, since the small apparatus was used mainly for qualitative observations, and much more time was to be spent in the building of a larger apparatus, the U actually used was judged adequate for the purpose.

The downcomer and riser tubes were joined at the top to a gas disengagement tank, with a central dividing baffle. The ratio of the cross-sectional area of this tank to the combined cross-sectional areas of the tubes was 11:1. The tank produced adequate disengagement at all but the highest gas flow rates. A constant overall liquid height of 4.1 m, shown in Figure 1(a), was used for experiments on this apparatus. This constant level was maintained by the inclusion of an overflow pipe in conjunction with a refill pipe in the disengagement tank.

Air could be injected into the system at a known flowrate through the riser and downcomer injection points, which were situated on both tubes 0.25 m above the lowest point of the U bend. The air injectors were open-ended pieces of 6.35 mm ID copper tubing. The
air flowrate was measured by rotameters which had previously been calibrated at their line pressure using a "soap-film" flowmeter. The air supply for the apparatus was provided by the \(414 \text{kN/m}^2\) gauge compressed air line in the laboratory. The air flowing through the rotameters was maintained at \(103 \text{kN/m}^2\) downstream of a pressure reducing valve, before reduction to its injection value through diaphragm valves. In this apparatus, riser superficial gas velocities varied from 0.02 m/s to 0.07 m/s (based on atmospheric pressure).

Single-phase liquid velocity measurements could be made using a calibrated pitot tube at the top end of the downcomer leg. This pitot tube had previously been calibrated on an apparatus used for water flow measurements; it was also calibrated "in situ", by disconnecting the bottom U section and allowing a known amount of water from the mains supply to flow around the hydraulic circuit. The results of the two calibrating runs were in good agreement. The highest liquid velocity achieved in this apparatus was about 0.7 m/s at the highest riser air rate used.

The main purpose of the small apparatus was to observe qualitatively the flow in riser/downcomer columns, in order to gain an insight into the design of a much larger apparatus for quantitative work, see Figure 1(b). It soon became clear that a larger apparatus would avoid some of the undesirable aspects shown on the smaller scale, and that some of the flow phenomena present on the small scale would appear much more clearly on a large unit.

It is useful to list the main findings from the small apparatus, and its principal failings; a complete discussion of the observations will be deferred until the relevant sections of this dissertation.

(i) The small apparatus operated at liquid velocities below 0.7 m/s: however, since the ratio of tube wall area to liquid volume
was much less for the large apparatus, its operating velocity would be considerably higher.

(ii) With the greater height of the large apparatus, the injection of air much higher up the downcomer leg would be possible, whilst still maintaining liquid circulation. Also, due to the larger diameter tubes used, their "wall effects" on a given size of bubble would be far less than in the small apparatus.

(iii) In the absence of liquid circulation, whilst running on both riser and downcomer air, oscillations of a similar nature to those described by Garland and Davidson (1975), were observed on the small apparatus. These were expected to occur in the large apparatus under similar conditions.

(iv) The U section on the small apparatus produced a spiralling effect on the liquid flow; this effect could clearly be seen on any bubbles which were carried around the bend, but disappears when a smooth U is used.

(v) There was a tendency for slugs to form and fill the tube when downcomer air was injected. In a larger diameter apparatus there is more chance of these slugs being broken up before they can fill the tube.
2.2 Large Apparatus

2.2.1 The Columns and their Modular Construction

Following initial experiments on the small apparatus already described, a large apparatus, see Figure 1(b), was built to investigate the stable and unstable regions of operation of a two column "riser-downcomer" system.

In this apparatus the riser and downcomer were constructed from lengths of 254 mm external diameter, 6.35 mm thick, cast perspex tubing. The overall height of the apparatus was 10.2 m, and the perspex tubing constituted about 7.4 m of this total height. The overall liquid height was 9.85 m. Perspex was chosen for the tubes because of its transparency and ease of machining.

The riser and downcomer were fabricated in sections of overall length 1.23 m, see Figure 2(a), including perspex flanges cemented on at each end, which were 12.7 mm thick and 356 mm OD. A "step" was cut into each flange, from the internal diameter of the tube out to its external diameter of 254 mm and to a depth of 6.35 mm (half the flange thickness). The final assembly had a continuous smooth bore, maximising the structural strength of the section, and minimising the possibility of a leak developing at the flange/tube interface.

The spacing between the centres of the vertical riser and downcomer columns was 767 mm, see Figure 1(b): this distance was fixed by the dimensions of the U bend joining the columns.

Besides these 1.23 m sections, two special sections were constructed for the riser and downcomer injector units, see Figure 3(a) and Figure 3(b). These special sections consisted of two shorter modules of length 308 mm and a longer module twice this length. One of the shorter modules from each of these sections was modified to contain the design used for the riser or downcomer injection unit:
these designs will be discussed later. This modular construction gave maximum flexibility to injector positioning within the large apparatus.

Modules and sections were joined together using 10 mm bolts, and leak tight seals between the flanges were formed by the 'O' rings fitted, see Figure 2(a).

At this point it is worth mentioning four other large pieces of apparatus, used by other workers, similar in some respects to the system described. Shipley (1975) describes an apparatus which has two 457 mm diameter columns. However, this apparatus seems to have suffered from end effects, since it was only 5 m tall. Hills (1976) presents data on an apparatus about 11 m tall, with downcomer and riser bores of 149 mm. This apparatus had most of its tube length constructed from opaque PVC, five transparent sections being added to observe the flow. Martin (1976) mentions a system with a 140 mm bore clear plastic downcomer, using a 200 mm diameter intake pipe to supply water to the system. Finally, Smith (1979) has a large piece of apparatus with a 152 mm diameter riser and a 102 mm diameter downcomer, both manufactured from sections of QVF glass. The overall height of this apparatus is about 15 m.

The Cambridge apparatus here described, is unique in being capable of circulating with air supplied to the downcomer injector only.

2.2.2 Support and other Column Structural Considerations

Before discussing the column structural considerations, a property of perspex of which all designers should take note must be mentioned. If the surface tensile stress of a sample of perspex exceeds a critical value, the phenomenon known as crazing may occur, ultimately leading to sample penetration by cracks. In common with other properties, crazing is a time dependent phenomenon, and so it may occur at lower
stresses after longer periods of loading.

Perspex materials are more susceptible to stress crazing in the presence of certain organic chemicals or other vapours, especially those by which it is dissolved. For this reason special care was taken when using perspex cement to join the tubes and flanges, to ensure that they were never in a stressed state during cementing. Cast perspex tubing was chosen in preference to "rolled and seamed" tubing, since the residual stress level in the latter is far higher.

The first structural criterion that the tube must fulfil is that it is able to withstand the maximum hydraulic head generated within the system. A chart given by Ross (1974) shows that a 254 mm external diameter perspex tube, with a wall thickness of 6.35 mm, can withstand an internal pressure of about 400 kN/m². The maximum internal pressure, at the bottom of the apparatus is 100 kN/m²: hence this tube is suitable. Manufacturer's data on the 229 mm internal diameter QVF 90° bends, chosen for the bottom U, showed that these too were suitable for this internal pressure.

The apparatus was also designed to withstand a reduction in pressure to about 25 kN/m² absolute in the gas disengagement tank, for reasons which will be explained. This meant that the pressure at the bottom of the apparatus would be 125 kN/m² absolute. Thus a bubble travelling up one of the columns would experience a pressure ratio of five. Under normal operation, with the disengagement tank at atmospheric pressure, a column height of about 40 m is required to achieve this pressure ratio. Although in this work reduced pressure operation was never used, future work may well use reduced pressure to simulate the performance of a much larger column system, without all the physical hinderances of large scale operation.

Structural calculations were thus performed to ensure that the
tube walls would not buckle under the external pressure. These calculations involved the use of data on Young's modulus and Poisson's ratio, which were taken from an ICI (1975) publication. Timoshenko and Gere (1961) give a series of curves which can be used to calculate the permissible external pressure on a given piece of tube, without the occurrence of buckling. In order to use these curves, the mode of buckling (determined by the circumferential wave number), the tube geometry and structural properties, and the tube boundary conditions must be known. The recommendations of Calladine (1978) were used for this, i.e. a circumferential wave number of two, and the boundary conditions being the tube "held circular" at both ends. On this basis a tube section of length 1.23 m, with an external diameter of 254 mm, and a wall thickness of 6.35 mm should withstand an external to internal pressure difference of 140 kN/m². Since the maximum external to internal pressure difference expected is 100-25 = 75 kN/m², these tube sections should prove adequate for reduced pressure operation. Due to the additional hydrostatic head, the minimum pressure experienced by the QVF sections at the bottom of the apparatus would be 125 kN/m² absolute. Thus they would never be expected to run under reduced pressure. (The design of the head tank etc. for reduced pressure operation will be discussed later.)

It was now necessary to consider the best way to prevent lateral movement of the apparatus, under operating conditions, due to its large height to diameter ratio. In order to prevent this movement it was decided to use "collar" supports around each column, see Figure 2(b) and also Figure 1(b). Each "collar" consisted of two halves, which could be bolted together. Each half was made from a rectangular block of wood, with a semicircular hole cut out from the long side, of a radius about 5 mm larger than the riser and
downcomer tubes. The semicircular hole had a piece of 6 mm thick rubber sheet glued to its perimeter. Thus when the two column halves were bolted together around a column a tight fit was produced. The "collar" was then attached by "dexion" beams to a rigid "dexion" structure, which stood surrounding the columns. Plate 1 shows this structure and the columns inside it. (The collars are attached behind the horizontal beams crossing the support structure.) It was found that of the six collars made for each column, only three were needed during normal operation of the apparatus. The support structure itself was attached to the side of the laboratory, adding further rigidity to the apparatus. The horizontal "dexion" beams also allowed easy and safe access to the structure.

The U bend was supported underneath two sheets of aluminium (610 x 381 x 16 mm), which in turn were supported on top of and between two 'I' beams (178 x 102 x 6.35 mm), see Figure 4. Thus the aluminium sheets were acting both as flanges and as supports. The riser and downcomer columns were bolted down on top of these two aluminium sheets, the bolts passing down through the sheets to pull the U bend up from below. These aluminium sheets had a large central hole to continue the bore of the columns and of the U. Since the internal diameter of both the riser and downcomer was 241 mm, whilst that of the QVF U bend was 229 mm, it was necessary to taper the hole in the aluminium. The seal between the columns and the aluminium was provided by rubber gaskets, while a teflon gasket provided that between the aluminium and the U bend.

During normal operation of the apparatus, it was necessary to maintain the perspex columns under an axial compressive stress; a tensile stress might have cracked the tube. The apparatus itself contained about two tonnes of water, so a failure of this nature was
extremely undesirable. To maintain the axial compressive stress in the columns, they were jacked up on top of the aluminium sheets using the bolts provided, see Figure 4. Since the weight of the top tank was about 1\frac{1}{2} tonnes unladen, this was effectively an "immovable object" on top of the columns, and so this jacking up resulted in the necessary axial compressive stress. Sixteen 12 mm jacking bolts were provided, but in fact only the outer set of eight were found to be necessary under normal conditions.

2.2.3 The Diffusers, Head Tank, and U Bend

In the design of the large apparatus, a great deal of attention was paid to minimising hydraulic losses and hence maximising the circulating liquid velocity in the "riser-downcomer" system. For this reason it was decided to incorporate two identical diffuser tubes at the top of the riser and downcomer columns, see Figure 1(b) and Plate 2.

The effects of sudden enlargements and contractions upon liquid flow are described by Coulson and Richardson (1964): they state that to minimise the hydraulic losses for expansion from a circular pipe, a tapering section with an angle of 7° should be used. At the top of the riser, the bubbly liquid has to expand from the perspex column into the head tank. The diffuser at the top of the riser saved some of the kinetic energy of this fluid, by conversion into pressure energy. The diffuser at the top of the downcomer minimised entry losses between the head tank and the downcomer column.

The diffusers were made of mild steel, protected against corrosion with two coats of galvanising paint, inside and out; two coats of a plastic resin were added to the internal surfaces in contact with the flowing fluid. The truncated conical section of each diffuser was made from 3.2 mm thick, rolled and seamed steel sheet. Two steel flanges,
6.35 mm thick, were welded one onto each end of these sections. The flanges were used to attach the diffuser to the head tank at the large diameter end, and the perspex column at the other end.

Each diffuser was 1.22 m long, with internal end diameters of 241 mm (column end) and 541 mm (head tank end). The flange external diameters were 356 mm and 643 mm respectively. Attachment to the perspex columns was by twelve 10 mm diameter bolts; the diffusers were joined to the head tank by twenty four 10 mm diameter set screws at a PCD of 592 mm. The seals between the diffusers, the head tank, and the columns, were provided by greased rubber gaskets. Since the riser and downcomer were vertical, the spacing between the diffuser centres was the same as that between the column centres. Finally calculations based on the largest diameters of the diffusers, using the curves of Timoshenko and Gere (1961), showed that the diffusers were suitable for reduced pressure operation.

The operation of the "riser-downcomer" column system depends upon disengagement of the injected air at the top of the riser, allowing only liquid to flow down the downcomer. In order to achieve this, a disengagement tank as shown in Figure 1(b) and also in Plate 2, was constructed. This tank was in the form of a rectangular box, of internal size 1.78 m long, by 0.71 m wide, by 0.86 m high. The surface area ratio of the tank to that of the columns was thus 14:1, and this provided good gas disengagement for all but the highest air rate experiments.

A baffle was provided half way along the length of the head tank, to subdivide it into a riser section and a downcomer section. This baffle was the same width as the tank (0.71 m), and 0.46 m tall. It was made from 6 mm thick aluminium plate. During normal operation, the whole apparatus was filled with water to such a depth
that there was 0.19 m of water above the top of the baffle. Since average voidages in the columns were less than ten percent, the liquid level in the tank would not change by very much during an experimental "run" (due to the large tank:column area ratio). The baffle was provided to stop bubbles "short-circuiting" from the riser to the downcomer columns; it allowed bubbles to remain for long enough in the head tank for complete disengagement.

As the apparatus was to be built to withstand operation under reduced pressure, the disengagement tank was designed accordingly. The tank body was constructed from 12.7 mm thick mild steel, and in addition to this strengthening was added as follows. Internal reinforcements made from $102 \times 76.2 \times 12.7$ mm and $76.2 \times 76.2 \times 12.7$ mm angles were welded into the tank. These angles provided horizontal as well as vertical strengthening. The spacing between the vertical angles was about 0.23 m, and they reached from the bottom to the top of the tank; horizontal angles were welded around the tank perimeter, internally at the bottom of the tank, and externally at the top. A vertical angle "ring" was provided half way along the length of the tank. This provided extra strength, and had the baffle attached to it.

To compensate for the loss of strength caused by cutting holes in the bottom of the tank to accommodate the diffusers, strengthening rings were welded externally to the tank bottom surrounding the holes. These rings had internal and external diameters of 541 mm and 762 mm respectively, and a thickness of 12.7 mm. This increased tank bottom thickness also allowed tap holes to be drilled to accommodate the diffuser flange set screws, without breaching the internal surface of the tank.

A tank lid was built for use of the apparatus requiring reduced pressure conditions. The lid had the same area as the tank and was
9.5 mm thick. It had strengthening angles welded to its external surface, which corresponded to the pattern of the angles inside the tank. For the experimental conditions of this work the lid was never actually used. As with the diffusers, the tank and lid were given two coats of galvanising paint both inside and out, followed by two coats of a plastic resin on all the internal surfaces.

The weight of the tank construction, being one and a half tonnes empty, was sufficient to merit a sturdy support structure. Support was provided by four steel columns, made from I beams with welded on end plates: the I beams were $152 \times 152 \times 6.35$ mm. The bases of these columns were bolted onto a baseplate, the shape of a grillage, made from the same I beams, and with an overall length and breadth of 3 m and 1.93 m respectively. The function of this baseplate was to distribute the tank weight over the major beams under the laboratory floor. All of the support structure was painted in the same manner as the tank exterior.

Some noteworthy features of the U bend will now be mentioned, see Figure 4. The U was formed from two 90° QVF bends with internal diameters of 229 mm. The two 90° bends were not identical since one had a small side pipe, for draining, projecting from it. The two bends were held together by a standard coupling, the seal between the two pieces being provided by a teflon gasket. The drainage pipe was located half way around the perimeter on the outer radius of one 90° bend: it had an internal diameter of 25.4 mm, and using the valve attached to it the apparatus could be drained in under ten minutes.

The apparatus emptied into a channel in the floor of the laboratory. This channel led into a large underfloor tank, whose volume was an order of magnitude greater than that of the apparatus. Thus even if all of the water in the apparatus escaped due to a
rupture, it could easily be contained in this tank. The underfloor tank was pumped out to drain after every three or four experimental runs, so that it was always kept nearly empty.

2.2.4 The Air Injectors

The operation of a circulating "riser-downcomer" column system depends upon the effective introduction of air into the columns. Three criteria must be fulfilled by any air injection device.

(i) The first criterion is that the air injector must create only a small pressure drop, to maximise the circulation velocity.

(ii) In order to propose a simple model for liquid circulation, bubbles in the system must have a constant slip velocity. Habermann and Morton (1953), amongst others, showed that single bubbles with diameters between 3 and 8 mm had a terminal rise velocity of about 0.25 m/s in the absence of wall effects. Thus the constant slip velocity assumption has more meaning if the air injector produces bubbles mainly in this size range.

(iii) The third criterion to be fulfilled is that any injector must be capable of dispersing the bubbles formed into the flowing liquid within a short distance of the injection point. This is because of an homogeneous voidage assumption to be made in the circulation theory; this will be explained in a later section.

Qualitative experiments showed that a different design of injector was required in the riser column to that in the downcomer column. Thus the design of the injector for the riser column will be discussed first, and it is shown pictorially in Figures 3(a) and 5(a) and also in Plate 1. The shape chosen for the riser injector was that of a symmetrical aerofoil, and a full discussion of the properties of this shape is given by Goldstein (1938b). The dependence of drag on the thickness parameter of the aerofoil is discussed, the thickness
parameter being defined as the ratio of the maximum thickness to the chord length, see Figure 5(a). The total drag on an aerofoil is the sum of the form drag and the skin friction drag. As the thickness ratio is increased the form drag per unit length increases, whereas skin friction drag per unit length decreases. Thus there is an optimum thickness ratio which produces the minimum total drag: this is shown to occur at a thickness ratio of about 0.25, and so the riser injector design was based on this figure.

Goldstein (1938b) provides a chart showing the normal pressure distribution along the length of such symmetrical aerofoils. This chart shows that the positions along the length of the aerofoil where the aerofoil surface pressure is greater than the stream pressure are near the aerofoil nose, which is a stagnation point, and along the rear 20% of the aerofoil chord length. Based on these results, it was decided to inject air through the tail of the aerofoil at a dimensionless distance of about 0.85 along the chord length, measured from the front end. Any air injected here should be swept away from the surface of the aerofoil, and into the flowing liquid, since the surface pressure is higher than the stream pressure.

A symmetrical aerofoil was built with a length of 0.20 m and a width of 0.05 m. The aerofoil was made from eight perspex sections, each 24 mm thick. Specially moulded sections of aerofoil, made from car body filler, were added to each side of the aerofoil so that it fitted vertically into the 241 mm internal diameter riser column used, see Figure 1(b) and Figure 3(a). Air was supplied into the centre of the nose of the aerofoil by a 19 mm external diameter brass tube, which was 0.43 m long. A second brass tube of the same diameter was joined to the end of the first tube at right angles, and then left the riser through a hole in the special riser injection module.
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Air was supplied to holes in the tail of the aerofoil by a 'T' shaped manifold within the aerofoil structure. There were twelve holes on each side of the aerofoil tail, see Figure 5(a); these holes were 2 mm in diameter. All of the holes were equally spaced in the middle four perspex sections of the aerofoil, to avoid possible wall effects on the air distribution. During experiments, two different positions were used for the riser injector unit: these were with the injector holes at heights of 3.65 m and 5.18 m above the lowest part of the U bend.

The aerofoil was inverted and tried in the downcomer column, to check its suitability as a downcomer injector. However, it was found that low pressure pockets formed beneath the aerofoil at the tube walls: these rapidly filled with air and led to "downcomer slug breakthrough" (which will be discussed later) under normal operating conditions. The same problem of low pressure pockets beneath the injector, was found with all designs which involved obstructing the downward liquid flow in the downcomer. For this reason it was decided to use a downcomer injector which would not obstruct the liquid flow at all. This design is shown in Figure 3(b) and Plate 1.

The 308 mm long downcomer injection module was taken and fifty equally spaced holes, of diameter 2 mm, were drilled around the tube circumference, half way along its length. A perspex manifold box of dimensions 0.31 x 0.31 x 0.05 m was now constructed around this ring of holes. The manifold box itself was made from pieces of 10 mm thick perspex sheet. Air was supplied into this manifold box through a 19 mm external diameter plastic pipe, cemented into a hole in the side of the manifold.

The downcomer module was now tested in the large apparatus. It was found that although long thin "streaks" of air were formed
underneath each hole during circulation, see Figure 5(b), these did not join up to form "downcomer slugs" under most operating conditions. These "streaks" of air persisted down the side of the tube to about one tube diameter beneath the downcomer injector holes. Downstream of this position, the "streaks" broke up to form bubbles which appeared to be in the required size range. As with the riser injection module, the downcomer injection module was used in two different positions during experiments on the apparatus. These were with the injector holes at 2.25 m and 3.47 m above the lowest point of the U bend.

2.2.5 Pitot Tube and Rotameter Arrangements

In this section the installation of the devices used for measuring the air supply rate, and the single phase liquid velocity, in the large apparatus will be discussed. The calibration procedure adopted for the rotameters and pitot tubes used will be left for discussion later.

The simplest measure of the circulation rate is given by the single phase liquid velocity in the downcomer: this may be measured by a number of devices. The measurement device had to be chosen to give the minimum hydraulic loss. With this in mind, it was decided to use pitot tubes, because of their simplicity and very small effect on the liquid flow.

It was decided to use two pitot tubes situated at the same level in the downcomer column but at different radial positions, see Figure 6(a), to check the radial dependence of any perturbations in liquid velocity, and also to enable calculation of the liquid circulation rate from two different measurements. The first pitot tube was situated on the centre-line of the downcomer tube, so that it would experience the maximum liquid velocity (which should be about 1.22 times the mean velocity for fully developed turbulent flow).
The second pitot tube was situated at a dimensionless radius of 0.758 from the centre-line: this is where the liquid velocity should have its mean value on the basis of a one seventh power law liquid velocity profile.

The pitot tubes were placed in the 615 mm long module above the 308 mm long downcomer injection module, see Figure 1(b). The reason for placing the pitot tubes in this position was to allow the turbulent single phase liquid flow to develop, after entering the downcomer column. When the position of the downcomer injector was raised, the pitot tube section was moved upwards by the same amount. (The pitot tubes were then recalibrated, as will be mentioned later.) The pitot tubes themselves were made from 3.2 mm external diameter, stainless steel, hypodermic tubing. The vertical length of the dynamic pitot tubes was 0.14 m: a common static tube was used in conjunction with both dynamic tubes, this being flush with the tube wall and at the same height as the open ends of the dynamic tubes.

The exit holes, for the three pieces of hypodermic tubing leaving the downcomer column, were made watertight with epoxy resin. Epoxy resin was also used to cement small, three way, plastic taps onto the external ends of the exiting tubes: two of these taps can clearly be seen in Plate 1. These taps were used to bleed air from the system if any collected in the pitot tubes, and also to flush out any foreign bodies which might block the tubes.

The difference between the dynamic and static liquid heads was measured by two pressure transducers, and the measuring arrangement may be seen in Figure 6(a). The pressure difference was applied across a diaphragm inside the transducer, causing it to deflect. This deflection altered the capacitance of the system, and was used to give a measure of the magnitude of the pressure drop. The transducers were
manufactured by S.E. Labs. (Eng.) Ltd., and were of the type D5964.25WG: three way plastic taps were cemented to all the inlet ports of the transducers to aid in bleeding air from the system. One inlet port of each transducer was connected hydraulically to the common static pitot tube, whilst the other two inlet ports were connected to the two dynamic pitot tubes. The signals produced by the pressure difference across each transducer were fed into separate signal amplifiers before being displayed as two pen traces on a Tekman 220 series, two (heated) pen chart recorder. To cut out signal noise, i.e. frequencies shorter than one second, simple smoothing circuits were added to each amplifier output, as shown in Figure 6(a).

With regard to the measurement of the air supply rates, Figure 6(b) shows the layout of the air supply system for the large apparatus. This is also partly visible in Plate 1. As with the small apparatus, the air supply for the large apparatus was provided by the 414 kN/m² gauge compressed air line in the laboratory. However, in this case, the pressure maintained in the rotameters downstream of a pressure reducing valve was 172 kN/m² gauge. A bursting disc, rated at 206 kN/m² gauge, was incorporated into the rotameter line. The Norgren pressure reducing valve used ensured that the downstream pressure was maintained at its set value over a wide range of air flowrates.

For both the riser and downcomer injectors, a small or large flow rotameter could be selected. The small flow rotameter provided an air flowrate from 0.0006 to 0.0058 m³/s, at atmospheric pressure: the large flow rotameter from 0.0026 to 0.0290 m³/s. The maximum flowrate used in this work was about 0.008 m³/s, which corresponds to a column superficial gas velocity of 0.17 m/s (again at atmospheric pressure). Control of the air flowrates used was provided
by diaphragm valves, situated between the ball valves and the column injector units. Since the fully opened gate and ball valves provided little resistance to the air flow, the pressure drop from the rotameter line pressure to the injector pressure was nearly all across the diaphragm valves. Two extra air supplies, through the small rotameters, were also provided, but during this work they were never needed.
CHAPTER THREE: STEADY STATE CIRCULATION

3.1 Operating Procedure

The principles of operation of a circulating system of the type used in this work have been discussed elsewhere by Hines et al. (1975) and by Kubota et al. (1978). A simple steady state circulation theory to describe the system will be given later and compared with experimental data obtained from the large apparatus.

The basic method of operation is as follows; see Figure 1(b). To begin with air is injected into the riser leg through the injection point there. This air travels up the riser until it breaks surface in the gas disengagement tank. The riser is now full of a bubbly liquid whose bulk density is lower than that of the clear liquid in the downcomer leg. This density difference is the driving force which causes liquid to travel around the circuit until a steady-state operating velocity is reached in the system, where the density driving force is equal to the hydraulic losses in the circuit. Thus we now have a circulating system operating in what may be called an "air lift pump" mode.

It is now possible to begin to inject air gradually into the downcomer, and provided that this air is present as bubbles and that the circulation velocity is greater than the rise velocity of these bubbles, they will be carried downwards and around the U bend into the riser. (It is also possible for the air to be present as slugs provided in that case that the circulation velocity is greater than the rise velocity of these slugs.) Circulation in the system will be maintained provided that the density of the clear liquid part of the downcomer (above the air injection point) plus the bubbly section of the downcomer (below the air injection point) is still greater than
the bulk bubbly liquid density of the riser. If circulation has been satisfactorily maintained, it may now be possible to switch off the air supply to the riser and allow the system to circulate on "downcomer air only". Circulation is maintained by the net density difference between the downcomer and riser legs, provided that the liquid velocity is still greater than the bubble slip velocity.

3.1.1 Qualitative Observations

Whilst performing experiments on the large apparatus, several features of the two-phase flow in the circuit became obvious. Summaries of these features are presented here together with references to other parts of this dissertation where they are discussed more fully.

(1) The occurrence of "downcomer slugs" was found to give rise to one type of instability, see 4.1 and 4.3, and to be the end result of another different type of instability: see Chapter 5.

(2) Phase separation was noted in the U bend, with air at the inner radius of the bend for all flowrates: see 1.5.2. This also has a direct bearing on hydraulic losses in the circuit: see 3.4.4.

(3) During operation large bubbles were seen to rise rapidly up the riser column: see 1.2.2. This affects the overall bubble slip velocity in the system: see 3.4.1.

(4) The riser aerofoil, see 2.2.4, produced a "jetting" effect on the air injected there so that an homogeneous voidage distribution was not established for some distance downstream of the injection point. This "jetting" effect has important consequences when predicting the steady state circulation rate: see 3.5.

(5) The disengagement tank used, see 2.2.3, proved adequate at all but the highest experimental air rates used. At the highest rates some bubbles were seen to be re-entrained in the downward flow into
The downcomer: this leads to a small decrease in the density driving force.
3.2 Pitot Tube and Rotameter Calibration

The use of two pitot tubes to measure the single phase velocity in the downcomer has previously been mentioned. In order to use these pitot tubes to measure liquid velocities during experiments, it was necessary to calibrate them against a known measure of velocity. This was done by using an aluminium orifice plate, designed according to BS 1042 (1964), which had pressure tappings one tube diameter upstream and one half a tube diameter downstream of the orifice.

During calibration the orifice plate was positioned between two tube flanges just below the pitot tube module in the downcomer, as is shown in Figure 6(a). A spacing flange whose thickness was the same as the orifice plate, was fitted into the riser column at the same time. This presented no resistance to the flow, being flush with the inside of the riser column, and kept both the riser and downcomer lengths equal. The aluminium orifice plate was designed such that it could measure average liquid velocities between 0.1 and 3 m/s: this being the maximum expected velocity range of the apparatus. The diameter of the orifice, \( d_o \), was 0.187 m giving it an area ratio \( m = \left(\frac{d_o}{D}\right)^2 \) of 0.6014. The basic discharge coefficient was then found from the British Standard to be 0.607.

The procedure adopted when calibrating the pitot tubes was as follows. Firstly the pressure transducers themselves were calibrated by imposing a known head of water across them and noting their readings. (It was found that the pressure transducers had a fairly linear response with increasing pressure difference, until saturation of the amplifiers occurred corresponding to a head difference greater than 16 cm of water.) The pitot tube tappings were then connected across the two transducers as shown in Figure 6(a). Pressure differences between the orifice plate tappings were measured either
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using one of the transducers at lower pressure differences, or by connecting the tappings across a mercury manometer for higher pressure differences.

With the orifice plate in place, a known amount of riser air was injected which caused the system to begin to circulate. When a steady state velocity had been reached the head differences across the orifice plate, and between the static and the dynamic pitot tubes were measured. The British Standard enabled the conversion of the pressure drop across the orifice plate to an average liquid velocity. Thus a plot of pitot tube head difference against average liquid velocity could be prepared: see Figure 7(a). When the pitot tubes had been calibrated over a range of operating velocities, the orifice plate and spacer flange were removed from the system, to minimise the hydraulic losses in normal operation. By repeating the procedure at known riser air rates, with the orifice plate in place and the pitot tubes removed from the downcomer, it was shown that the presence of the pitot tubes had no effect on the orifice plate readings. Similarly, the orifice plate was far enough downstream of the pitot tubes so as not to disturb the flow that they were experiencing.

The calibration procedure was repeated when the downcomer injector section and pitot tube unit on top of it were moved further up the downcomer to investigate a different apparatus geometry. Figure 7(b) shows the calibration curves for the pitot tubes in the upper position. It has already been stated that in fully developed turbulent flow the centre-line velocity would be expected to be 1.22 times the mean liquid velocity, and that the side pitot tube was positioned where the mean velocity would be expected to occur on a 1/7th power law basis. Figures 7(a) and 7(b) clearly show that the centre-line velocities experienced by the centre pitot tube are less than expected and thus
the flow is still developing. Further, the velocity experienced by
the centre pitot tube in the upper position is less than it experienced
in the lower position, this too would be expected for a developing flow
profile. It can be seen, however, that the curves for the side pitot
tube lie closer to its predicted behaviour than do those of the centre
pitot tube. This is due to the shape of the developing flow profile
where the magnitude of the centre velocity changes much more than does
the magnitude of the velocity at the position of the side pitot tube.
Since the pitot tubes are 28 pipe diameters into the flow in their
lower position, and 23 in their upper position, the observation of
developing flow agrees with the comments of Goldstein (1938c) who
states that at least 40 pipe diameters are required for a turbulent
velocity profile to develop fully.

The four rotameters used to supply a known amount of air to the
system were calibrated as follows. The rotameter to be calibrated
was taken and supplied with air at a constant pressure of 172 kN/m²,
which was to be the supply pressure to the large apparatus. This
pressure was reduced across a flow controlling valve downstream of
the rotameter, before being exhausted to atmosphere across one of a
choice of several standard orifice plates. The pressure drop produced
across the standard orifice plate chosen was used with BS 1042 (1964)
to calculate the atmospheric flow rate of air through the rotameter,
and hence a calibration curve for each rotameter was built up.
3.3 A Simple Steady-State Circulation Theory

In this section a simple steady-state circulation theory is proposed. It is similar to theories put forward by Hines et al. (1975) and by Kubota et al. (1978). However, these authors did not provide any experimental data with which to compare their theories, whereas data from this work are used to test the model in section 3.5.

Certain assumptions are made in order to simplify the analysis, these are:

(i) That the voidage in the system is invariant with depth. The variation of voidage with depth is discussed in Appendix 1, and a full derivation making an allowance for the effect of hydrostatic head upon the system voidage is given in Appendix 3.

(ii) Following Turner (1966), it is assumed that the bubble slip velocity \( V_s \) is independent of the voidage: furthermore it is assumed that all the bubbles in the system have the same slip velocity.

(iii) The resistance forces in the circuit are assumed to have the form \( KV_o^2 \), where \( V_o \) is the steady-state circulation velocity. A full discussion of the resisting forces in the circulating system will be given in section 3.4.

(iv) That the system is circulating with air supplied to both limbs, at a rate \( Q_R \) to the riser and \( Q_D \) to the downcomer.

(v) Voidages are assumed small enough such that \( V_o \) does not change significantly around the circuit.

(vi) Mass transfer effects are assumed small, such that bubbles injected into the system do not change in size. This assumption is discussed in detail in Appendix 2.

(vii) The gas density is much less than the liquid density.

The system itself is shown in Figure 8(a). The voidage beneath the downcomer injector is constant at \( \varepsilon_1 \). In the riser, it is \( \varepsilon_2 \).
below the riser injector and $\varepsilon_3$ above it. The additional voidage due to the injection of air into the riser is given by $\varepsilon'$, and hence $\varepsilon_3$ is the sum of $\varepsilon_2$ and $\varepsilon'$.

Considering a mass balance at the downcomer injector gives

$$Q_D = A\varepsilon_1(V_o - V_s)$$  \hspace{1cm} (3.1)

Because of the effect of bubble slip velocity, this balance must be rewritten in the riser as

$$Q_D = A\varepsilon_2(V_o + V_s)$$  \hspace{1cm} (3.2)

Above the riser injector the total air flowrate is given by

$$Q_{TOT} = Q_R + Q_D = A(\varepsilon' + \varepsilon_2)(V_o + V_s) = A\varepsilon_3(V_o + V_s)$$  \hspace{1cm} (3.3)

The equation of motion of the system is given by $M_e(dV/dt) = \text{Density driving force minus Resisting forces}$. Since we are dealing with steady-state circulation, the acceleration term is zero.

The density driving force is given by

$$\rho gA(L - L_D) + \rho gA_D(1 - \varepsilon_1) - \rho gA_L(1 - \varepsilon_2) - \rho gA(L - L_R)(1 - \varepsilon_3)$$  \hspace{1cm} (3.4)

Thus substituting in for $\varepsilon_3$, the equation of motion becomes

$$KV_o^2 = \rho gA_L\varepsilon_2 + (L - L_R)(\varepsilon_2 + \varepsilon') - L_D\varepsilon_1$$  \hspace{1cm} (3.5)

The values for $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon'$ may be found from equations 3.1, 3.2, 3.3 giving

$$KV_o^2 = \rho g \left[ \frac{Q_D}{(V_o + V_s)} - \frac{Q_D}{(V_o - V_s)} + \frac{Q_R}{(V_o + V_s)} \right]$$  \hspace{1cm} (3.6)

This equation may be rearranged in terms of six dimensionless groups as is shown by equation A3.11 in Appendix 3.

In real terms the parameter $K$ is not particularly meaningful,
and so it may be replaced by \( N \), the number of velocity heads lost in the system, defined by

\[
KV_0^2 = \Delta P_f A = N \rho g \left( \frac{V_o^2}{2g} \right) \tag{3.7}
\]

Thus equation 3.6 becomes

\[
N = \frac{2g}{AV_0^2} \left\{ \frac{Q_D L}{(V_o + V_s)} - \frac{Q_D L_D}{(V_o - V_s)} + \frac{Q_R (L - L_R)}{(V_o + V_s)} \right\} \tag{3.8}
\]

Equation 3.8 shows that from a knowledge of \( Q_D, Q_R, L, L_D, L_R, V_s, N \) and \( A \), the circulation velocity in the circuit, \( V_o \), can be calculated.

### 3.3.1 Multiple Steady States

In order to investigate the predictions of the steady-state circulation theory the left and right hand sides of equation 3.6, the resisting and driving forces, were plotted against the steady-state liquid circulation velocity \( V_o \), for specified values of \( K, L, L_D, L_R, Q_D, Q_R \) and \( V_s \). Figure 9(i)-(iv) shows four such plots which investigate the effect of changing \( Q_D, Q_R, L_D \) and \( L_R \). The curve representing the resisting forces is the same for all of the plots, since it is independent of the injector position and air flowrates in this simple theory. It should be noted that in the presence of downcomer air, the driving force curves all asymptote downwards to minus infinity at \( V_o = 0.3 \). This is because the slip velocity is set at 0.3 m/s in these plots, and hence at a lower value of \( V_o \) bubbles would no longer be forced down the downcomer, and circulation in the manner already described would no longer be possible.

The main conclusion to be drawn from these plots, is that for fixed values of the other parameters there is the possibility of more than one operating point where the driving forces are equal to the resisting forces. Operating points can be stable, unstable or...
marginally stable, as shown by point B on Figure 9(i), point A on Figure 9(i) and point C on Figure 9(ii) respectively. At a stable operating point an increase in liquid velocity causes resisting forces to dominate over driving forces, whereas a decrease in the liquid velocity causes driving forces to dominate over resisting forces.

Multiple operating points have also been noted by Kubota et al. (1978), though their comments as to the stability or otherwise of the operating points are not clear.

From Figure 9, certain facts emerge about the effects of \( Q_D, Q_R, L_D \) and \( L_R \) upon the circulating system.

(i) The effect of \( L_D \) is such that for high enough values of \( L_D \), and low values of \( Q_R \), no operating points will exist at all; i.e. the resisting forces are always greater than the driving forces.

(ii) Increasing \( Q_R \) increases the stable operating velocity.

(iii) For the low values of \( L_D \) \( (L_D < L(\frac{V_s}{V_o}+s)) \) shown in Figure 9(v), increasing \( Q_D \) will increase the stable operating velocity. This is because the additional downcomer air decreases the weight of the riser leg more than it does the downcomer leg, and hence the driving force increases. At the higher values of \( L_D \) \( (L_D > L(\frac{V_s}{V_o}+s)) \) the reverse is true, i.e. the weight of the downcomer leg decreases more than that of the riser leg, so the driving force decreases and the velocity falls. That this is true can also be seen by substituting \( L_D = L(\frac{V_s}{V_o}+s) \) into equation 3.6.

(iv) When riser air only is used there is only one root of equation 3.6, and this always represents a stable operating point: see Figures 9(iii) and (iv).

(v) Increasing \( L_D \) decreases the stability of the system and also decreases the stable operating velocity.

(vi) Increasing \( L_R \) has the same effects as noted in (v).
3.4 Hydraulic Losses and Effective Slip Velocity in the Circuit

3.4.1 Estimation of Slip Velocity

Equation 3.8 which was derived using the simple steady-state theory, gives a relationship involving the number of velocity heads lost, the apparatus geometry, the riser and downcomer air flowrates and the bubble slip velocity in the circuit. It shows that if the apparatus geometry is specified then a plot of \( N \) against \( Q_R \) for a given value of \( Q_D \) should result in a constant value for \( N \), provided that the correct bubble slip velocity figure is specified.

The procedure adopted with data from runs on the large apparatus was as follows. A value of the slip velocity was guessed, and the experimental values of \( Q_D \), \( Q_R \), apparatus geometry, and \( V_o \) were now used with this to calculate \( N \) from equation 3.8. The values of \( Q_D \) and \( Q_R \) used here represented the average of values between the injectors and the liquid surface, in order to give an average air flowrate figure with some compensation for the variation of hydrostatic head with depth. The values of \( V_o \) were given by measurements using both the centre and the side calibrated pitot tubes, described earlier. Thus plots of the calculated value of \( N \) against \( Q_R \) for various guessed values of \( V_s \) could be prepared. Figure 10 shows such plots which were obtained from runs on one apparatus geometry, and with slip velocity figures of 0.3, 0.5 and 0.7 m/s. The effect of apparatus geometry on these plots will be discussed later.

The first point to note is that the data for riser air only appears to give fairly constant values for \( N \) regardless of the value of \( V_s \) chosen. In contrast to this, when any downcomer air is injected the data appears to be very sensitive to the value of \( V_s \) chosen. Since \( V_s \) is of the same order as \( V_o \) and the voidage in the downcomer depends upon \( (V_o - V_s) \), whereas riser voidage depends upon \( (V_o + V_s) \),
the voidage in the downcomer is much more sensitive to the value of \( V_s \) chosen. This in turn affects the density driving force and thus the liquid velocity and \( N \).

The value of \( V_s \) which clearly gives the most constant value of \( N \) for all values of \( Q_R \) and \( Q_D \) is found to be 0.5 m/s, see Figure 10(ii). This is considerably higher than the 0.25 m/s which is given as the rise velocity of single 3 to 8 mm diameter bubbles by Habermann and Morton (1953) and by Peebles and Garber (1953). In this respect our figure agrees with an enhanced value as proposed by Hills (1976), Kubota et al. (1978), and Davidson and Harrison (1966), see section 1.2, rather than a lower value as proposed by other workers. This enhancement is believed to be due to the presence of larger bubbles leading to high slip velocities, as observed by Hills and Darton (1976).

### 3.4.2 Voidage Variation with Depth

The variation of voidage with depth may be taken into account when calculating \( N \) and \( V_s \) by using equation A3.9 from Appendix 3, in place of equation 3.8. In this case the values of \( Q_R \) and \( Q_D \) to be substituted into equation A3.9 are the values of the air flowrates at the riser and downcomer injection points.

Figure 10(iv) shows a plot of \( N \) vs \( Q_R \) with hydrostatic effects allowed for and with a guessed value of the slip velocity of 0.3 m/s used. This plot shows the features already described in the last section, namely that the value of \( N \) found is much more sensitive to downcomer air than to riser air only. Figure 10(iv) should be compared with Figure 10(i), which uses average values of air flowrates to take into account hydrostatic effects but has the same guessed value of \( V_s \). It can be seen that these two plots are very similar, having nearly the
same best values of \( N \) and with much the same pattern of data scatter. These similarities were also found for the other \( V_s \) values when comparing plots prepared on the basis of average air flowrates with those prepared allowing for voidage variation with depth.

Thus for the large apparatus it seems that no significant benefit is gained by using a model which takes into account voidage variation with depth over one which uses average air flowrates to account for hydrostatic effects. This can be explained by the fact that the columns are relatively shallow in hydrostatic terms, and so average air flowrates are still fairly meaningful. Based on these results it was decided that the large apparatus may be modelled adequately, for both steady and unsteady state operation, using average air flowrates. However, in much taller systems hydrostatic effects are much more important and this approximation cannot be made: see Appendix 3 and Appendix 4 for full derivations including hydrostatic effects.

### 3.4.3 Effect of Apparatus Geometry on Hydraulic Losses

It has already been pointed out that only one apparatus injector geometry was used to prepare the plots shown in Figure 10. However, the same trends were found to occur in plots for the other three apparatus geometries used. In each case the value of \( V_s \) which gave the most constant value of \( N \) was found to be 0.5 m/s. The best values of \( N \) (\( N_{BEST} \)) obtained from different apparatus geometries, with \( V_s = 0.5 \) m/s did vary somewhat as is shown:

(i) \( L_R = 3.65 \) m, \( L_D = 2.25 \) m, \( N_{BEST} = 3.98 \).
(ii) \( L_R = 3.65 \) m, \( L_D = 3.47 \) m, \( N_{BEST} = 4.15 \).
(iii) \( L_R = 5.18 \) m, \( L_D = 2.25 \) m, \( N_{BEST} = 4.68 \).
(iv) \( L_R = 5.18 \) m, \( L_D = 3.47 \) m, \( N_{BEST} = 4.50 \).

It can be seen that the largest variation in \( N_{BEST} \) is between
the upper and the lower riser injector positions. It was observed that there was a tendency for air injected into the riser to "jet" out of the tail of the riser aerofoil injector and to take some distance to become radially distributed. Now since the density driving force is based upon an assumed homogeneous voidage, this jetting length will cause the driving force to decrease, and so the liquid velocity is lower than would have been expected leading to a higher experimental $N$ value. This will affect the upper riser position more, since in this position the ratio of the jetting length to the evenly distributed length is greater for the riser air. This effect is most noticeable at the highest riser air rates.

It would be expected that by increasing $L_D$ the experimental value of $N$ would increase slightly, since more of the circuit is then in two-phase flow which is generally found to lead to greater hydraulic losses. This does appear to be the case when changing from geometry (i) to geometry (ii). However, this trend is not observed when changing from geometry (iii) to geometry (iv), and there is no obvious reason for this discrepancy except that the differences in $N$ are small and subject to experimental errors (some evidence does support the increase of $N$ with $L_D$ at high $Q_D$, see 3.5).

These results show that the apparatus injector geometry can have an effect on the hydraulic losses in the circuit. Thus in this dissertation both the value of $N_{BEST}$ found for geometry (i) and that found for the actual geometry are used when comparing theories with experimental data. The results illustrate the problems of trying to evaluate hydraulic losses without experimental data, though a rough theoretical calculation of hydraulic losses is given in the next section.
3.4.4 A Theoretical Calculation of the Hydraulic Losses

We have previously derived a steady-state circulation model for the system, based on a balance between the density driving force and the number of velocity heads lost in the circuit, \( N \). \( N \) has been found experimentally to be between 4 and 4.7 dependent upon the apparatus geometry, see 3.4.3. It is of interest to see whether a figure for \( N \) can be predicted by a simple series of calculations on the various points in the system where hydraulic losses can occur.

The various possible major sources of hydraulic losses are shown in Figure 11. Proceeding around the circuit we see the following:

(i) Entry losses from the disengagement tank into the downcomer.

(ii) Wall friction contributions around the circuit.

(iii) At the downcomer injector, the liquid is accelerated from \( V \) to \( V_D^* \) due to the injection of air, this leads to an hydraulic loss.

(iv) At the bend, phase separation leads to the setting up of a standing bubble, blocking off some of the flow area and leading to further losses.

(v) Additional air injected into the riser causes acceleration of the liquid from \( V_R \) to \( V_R^* \) leading to an hydraulic loss.

(vi) At the top of the riser some of the kinetic energy of the fluid will be converted into pressure energy, see 2.2.3: the rest will be lost.

(vii) Losses within the disengagement tank itself.

It is interesting to note the views of other authors who have tried to evaluate hydraulic losses in similar systems. Hills (1976) and Kubota et al. (1978) both used single phase friction factors when evaluating wall friction, the latter also approximated the effects of the U bend by 75 pipe diameters of equivalent single-phase flow. Hsu and Dudukovic (1980) have found experimentally that the two-phase
friction factor in an air lift fermenter is \( f_{TP} = 0.02 \), though their pipe diameters were very small \((0.02 - 0.045 \text{ m})\), this is four times the figure quoted by the other authors. Hsu and Duduković also allowed for the effect of the bend and entry and exit losses, they also realised that an allowance should be made for the increase in liquid velocity at the injector which others had not.

In the system under consideration, the losses were evaluated as follows:

(i) Due to the use of a diffuser between the disengagement tank and the downcomer, see 2.2.3, nearly all the pressure energy in the disengagement tank is converted into kinetic energy.

(ii) Following the recommendation of others, see 1.5.1, a single-phase friction factor of 0.005 was used, though the experimental evidence of Hsu and Duduković (1980) suggests that this is an underestimate. An allowance of 70 pipe diameters of equivalent length for the U bend was made following Coulson and Richardson (1964). The equation for the number of heads lost due to wall friction is

\[
N = 4 f_e \frac{L_e}{D}
\]

Substituting in for \( f_e \), the total equivalent pipe length \( L_e \) (which is equal to 20 metres + 70 D), and the tube diameter \( D \) (0.24 m), gives \( N = 3.07 \) heads lost.

(iii) The liquid velocity below the downcomer injector is given by a liquid mass balance as \( V_D^* = V/(1 - \varepsilon_1) \). The force needed to accelerate the liquid from \( V \) to \( V_D^* \) is given by its rate of change of momentum, which in turn may be found from the product of the mass flowrate of liquid \((VAp)\) and its velocity change \((V_D^* - V)\). This gives the number of velocity heads lost as
In order to find $N$, one must also know the downcomer voidage $\varepsilon_1$, which is assumed invariant with depth. A mass balance on the air injected at the downcomer injector gives

$$Q_D = A\varepsilon_1 (V_D^* - V_s)$$  \hspace{1cm} (3.11)

Substituting in for $V_D^*$ gives an expression for $\varepsilon_1$, which must be positive, of the form

$$\varepsilon_1 = \frac{-[\frac{V}{V_s} - 1 + \frac{Q_D}{AV_s}] + \sqrt{\left(\frac{V}{V_s} - 1 + \frac{Q_D}{AV_s}\right)^2 + \frac{4Q_D}{AV_s}}}{2}$$  \hspace{1cm} (3.12)

Thus for a given downcomer air rate and liquid velocity, $\varepsilon_1$ and $N$ may be evaluated. The slip velocity already found of 0.5 m/s, the highest gas rate used $Q_D = 0.0042$ m$^3$/sec, and a low experimental liquid velocity $V = 1.14$ m/s were used to evaluate $N$, giving a value of 0.26 velocity heads.

(iv) At the bend, see enlarged view of Figure 11, the flow area for the liquid is reduced from $A$ to $a_B$ by the standing bubble. Let us assume that the liquid entering the bend is bubble free and has a velocity $V_B^*$, its velocity is increased to $V_B^*$ at the bottom of the bend. (Hydraulic losses at the bend are assumed irreversible.) Again this increased velocity may be calculated from a liquid mass balance which gives $V_B^* = V(A/a_B)$. Using a momentum balance to calculate $N$ gives:

$$N = 2\left(\frac{A}{a_B} - 1\right)$$  \hspace{1cm} (3.13)

It was found experimentally that the bubble height, $x_B$ in Figure 11, was about 0.07 m. This corresponds to a value of $a_B$ of $3A/4$, the number of heads lost being 0.67.
(v) The liquid velocity below the riser injector is \( V_R = \frac{V}{1 - \varepsilon_2} \).

It increases above the riser injector to \( V_R^* = \frac{V}{1 - \varepsilon_2 - \varepsilon'} \) due to the injection of riser air, voidages being assumed invariant with depth.

\( N \) may be found, as for the downcomer injector, by a momentum balance giving

\[
N = \frac{2\varepsilon'}{(1 - \varepsilon_2 - \varepsilon')(1 - \varepsilon_2)}
\]

(3.14)

Equation 3.14 shows that the higher the values of \( \varepsilon' \) and \( \varepsilon_2 \), the higher are the hydraulic losses. A balance on the riser air gives

\[
Q_R = A\varepsilon'(V_R^* + V_s)
\]

(3.15)

Substituting in for \( V_R^* \) gives an expression for \( \varepsilon' \) which can be solved by repeated substitution

\[
\varepsilon' = \frac{Q_R}{A(V_R^* + V_s)}
\]

(3.16)

Since \( \varepsilon_1 \) is known from equation 3.12, \( \varepsilon_2 \) may be found approximately from

\[
\varepsilon_2 = \frac{(V - V_s)}{(V + V_s)} \varepsilon_1
\]

(3.17)

If the following values are used \( V_s = 0.50 \text{ m/s} \), \( V = 1.40 \text{ m/s} \), \( Q_R = 0.0035 \text{ m}^3/\text{s} \), \( Q_D = 0.0042 \text{ m}^3/\text{s} \): the number of velocity heads lost is found to be 0.09.

(vi) It is assumed that 80% of the kinetic energy at the top of the riser is converted into pressure energy through the use of the riser diffuser: this being a commonly found efficiency in the laboratory. Thus if the effects of the riser and downcomer diffusers are taken together, we assume that there is an overall hydraulic loss of 0.2 velocity heads.

(vii) Due to its size liquid velocities in the disengagement tank
are small, and hence losses within the tank itself are also assumed to be small.

If all of these contributions are summed up, a total figure of 4.3 velocity heads is found compared to the experimental values of 4 to 4.7. Though this figure is obtained from "worst possible case" analysis of some of the losses, and by neglecting other losses, the overall result shows that the experimental figure is at least meaningful.
3.5 Discussion of the Circulation Model

It has already been decided that a combination of \( V_s = 0.50 \) m/s together with either \( N = 3.98 \) velocity heads or the value of \( N \) appropriate to the apparatus geometry may be used in the simple steady-state circulation theory. Substitution of these figures into equation 3.8 results in a series of plots of steady-state circulation velocity against riser air rate for the various apparatus geometries at a fixed value of \( Q_D \). These plots are shown on Figure 12, together with the experimental data from this work. The presentation of plots in this form brings out certain points which cannot be seen from the single apparatus geometry plots of Figure 10.

It can be shown, at a particular value of \( Q_R \), that generally equation 3.8 will predict two meaningful operating velocities for fixed values of the other variables. The lower operating velocity is unstable, see section 3.3.1, and is shown in Figure 12(ii) as the dashed extension to curve "A". In the rest of Figure 12 only the stable velocity curves are shown for clarity.

Curve "A" on Figure 12(ii) also serves to demonstrate another feature. It shows that for most values of \( Q_D \) and apparatus geometry, equation 3.8 also predicts that below a certain value of \( Q_R \) circulation will cease. This value is \( Q_R = 0.0011 \) m\(^3\)/sec in the case of curve "A". (The low limit on \( Q_R \) increases with increasing \( L_R \) and more strongly with increasing \( L_D \).)

The theoretical curves for \( N = 3.98 \) may be compared with those using \( N_{BEST} \) for the particular apparatus geometry as follows.

(i) Generally using \( N_{BEST} \) for that geometry does seem to account for the effects of increasing \( Q_R \) better than does \( N = 3.98 \).

(ii) It was shown earlier that for all air flowrates the overall effect of increasing \( L_D \) on \( N \) was unclear. However, it may be seen
that at higher values of $Q_D$, the data for high $L_D$ falls below the appropriate $N_{BEST}$ curves; see Figure 12(vi) and 12(vii). This shows that for high values of $Q_D$, increasing $L_D$ does increase the losses in the circuit as had previously been argued; see 3.4.3.

(iii) As would be expected riser air only data is well fitted by taking into account the effect of increasing $L_R$ on $N$, and not worrying about $L_D$; see Figure 12(i).

In conclusion it may be said that the simple steady-state circulation theory does provide reasonable agreement with the experimental data. The agreement is better if the hydraulic losses appropriate to the apparatus geometry are used. The potential effect of mass transfer on circulation is examined in Appendix 2 and is shown to be small.
CHAPTER FOUR: THE STABILITY OF THE SYSTEM

4.1 Unstable Modes of Operation

4.1.1 The Downcomer Slug

Several authors have commented upon the characteristics of slugs which are present in a downward flowing liquid. They agree that slugs in downflow are irregularly shaped and tend to "ride" the tube wall in an attempt to avoid the faster flowing liquid at the centre of the tube. An exception to this was pointed out by Davidson and Kirk (1969), who succeeded in holding a slug stationary in the centre of a tube through which liquid was flowing downwards by modifying the liquid velocity profile.

Nicklin et al. (1962) and Golan and Stenning (1969-70) commented that in downward flow slugs will always tend to rise as quickly as possible, and that slugs in small downflows are also subject to continual changes in shape. Martin (1976) made another comment based on his own data, which may well have relevance to this work, when he stated that above a certain pipe diameter in downflow the slug rise velocity becomes independent of the tube diameter as the bubble rides the tube wall.

In this work the following qualitative observations were made on the "downcomer slugs" sometimes present in the downcomer:

(i) They have much more pointed noses than do conventional axisymmetric slugs in upflow.

(ii) They always cling to the tube walls.

(iii) They are irregularly shaped as is shown for small downcomer slugs in Plate 3 and their larger relatives in Plate 4.

(iv) That as they climb up the tube walls air is ripped from them by the passing liquid: see Plate 3 and Plate 4.
(v) That they form in different sizes and larger ones can be broken up to form smaller ones.

(vi) That they can "breakthrough", as defined in section 4.1.2, and climb up the downcomer when the liquid circulation velocity is reduced below a certain figure: see section 4.3.

Authors have discussed the breakup of large bubbles like that noticed in this work: see observation (v). They cite the mechanism proposed by Taylor (1950) as a means whereby larger bubbles may become unstable and split into smaller ones: see section 1.3.1. This instability manifests itself as an indentation at the upper surface of the bubble which grows deeper as time advances. Splitting tends to occur if the disturbance grows sufficiently quickly relative to the velocity at which it is swept around to the bubble equator by tangential movement along the interface.

It may be shown that below a certain wavelength, \( \lambda_{cr} \), Taylor instability cannot occur. This result is quoted by Clift et al. (1978c)

\[
\lambda_{cr} = \frac{2\pi\sqrt{\sigma}}{g(\rho - \rho_G)}
\]  

(4.1)

It has been noticed by Grace et al. (1978), amongst others, that the motion of a growing indentation along the interface is greatly retarded once the disturbance has reached an appreciable size. Hence splitting can be assumed to occur if the first exponential stage of growth is complete before the disturbance reaches the bubble equator. This argument leads to an upper limit on the stable bubble size. This is quoted for air bubbles in water as an equivalent diameter of 4.9 cm by Grace et al. (1978).
4.1.2 Unstable Modes

In Chapter 3 one mode of operation of the apparatus was described. In this stable mode (mode (i)) air injected into the downcomer is swept straight down and around the U bend, it then joins air injected into the riser and travels upwards to break surface in the disengagement tank.

Other modes of operation are also possible which may lead to the cessation of stable operation as defined above. In describing these other modes it is necessary to introduce some new terms, defined as follows:

"Breakthrough" --- This is when a slug initially present at the downcomer injector succeeds in rising further up the downcomer against the downward flowing liquid.

"Continued Circulation" --- If downcomer slug "breakthrough" has occurred, but the system is still circulating with liquid flow down the downcomer and up the riser, then we have "continued circulation".

"Total Flow Reversal" --- If downcomer slug "breakthrough" has occurred, and as a result the system is circulating with a reversed liquid flow, i.e. down the riser and up the downcomer, then a "total flow reversal" has taken place.

Using these definitions it is possible to describe three more modes of operation of the apparatus:

(ii) The second mode involves the continuous "breakthrough" of small slugs from the downcomer injection point, when they move upwards through the descending liquid. In this mode these slugs do not succeed in breaking surface at the top of the downcomer, though they may climb three or four metres up it. They are gradually broken up by the descending liquid, and the air from them is swept back down the downcomer. This may be referred to as "downcomer slug breakthrough"
with continued circulation". Plate 3 shows four photographs of this type of downcomer slug, taken using an exposure time of 1/1000 second (the scale is in centimetres). Large indentations may be seen in the upper surface of these slugs as would be expected if they are splitting by Taylor instability. Equation 4.1 predicts that for air bubbles in water $\lambda_{cr} = 1.7$ cm. In all cases the wavelength of the instability may be seen from Plate 3 to be larger than this figure, and so Taylor instability is indeed possible.

(iii) In the third mode, "breakthrough" of a large downcomer slug occurs. In this case it manages to break surface at the top of the downcomer and is followed by continuously generated smaller slugs and bubbles. This means that the driving force for circulation has disappeared, since all of the apparatus is now full of a bubbly liquid. Thus circulation ceases and the only movement of liquid is a damped oscillation as the portions of the riser and downcomer above their respective injectors fill and empty alternately with small slugs and bubbles. This is similar to the phenomenon described by Garland and Davidson (1975).

(iv) The fourth mode is very similar to the third in that following slug "breakthrough", circulation ceases, as that part of the downcomer above the injector is full of bubbly liquid. However, in this case sufficient air is now present in the downcomer for the density driving force to reverse, since the riser is now much heavier than the downcomer. The end result of this is a "total flow reversal".

Plate 4 shows four photographs of the "breakthrough" of a large downcomer slug. These photographs are from a cine film by Söderberg and Pike (1980), which was made to show the characteristics of the apparatus, and represent $\frac{1}{4}$ second steps in time. It is not easy to measure the slip velocity of these large slugs, since the liquid
velocity in the system is continually decreasing with time after slug "breakthrough".
4.2 Experimentally Determined Stable and Unstable Regions

In order to investigate the stability of circulation of the system under consideration, a series of experiments were performed at various values of the riser and downcomer air supply rates for each of the four apparatus geometries described in section 3.4.3.

An experiment consisted of injecting a large amount of air into the riser, to establish stable circulation in the system, and then gradually injecting downcomer air up to the chosen value of $Q_D$. The riser air was now gradually shut off and at each riser setting the system stability or instability was noted, after allowing five minutes for any instability to make itself obvious. The various stable or unstable modes of operation have been discussed in section 4.1.2; they may be summarised as:

(i) Stable operation.
(ii) Downcomer slug breakthrough with continued circulation.
(iii) No circulation and both limbs partially filling and emptying with air.
(iv) Total flow reversal.

Figure 13 shows these experimental results for the four different $L_D/L_R$ combinations used. It should be noted that one symbol is used to represent one mode of operation at given values of $Q_R$ and $Q_D$; another symbol at these values indicates that a change in the mode of operation occurred on repeating the experiment. It should also be pointed out that a stable point is only stable if it is approached gradually, with an initial excess of riser air being slowly removed. The rapid removal of the riser air supply is destabilising to the system as might be expected.

It is now possible to draw lines on these graphs which show the regions of stable and unstable operation of the apparatus based on the
experimental points. It can be seen from Figure 13 that two distinct boundary lines, AB and BC, may be drawn. The line AB represents the transition between stable operation (mode (i)) and "downcomer slug breakthrough with continued circulation" (mode (ii)), and is described in section 4.3. The line BC represents the transition between stable operation (mode (i)) and either "total flow reversal" (mode (iv)) or circulation ceasing and both limbs partially emptying and filling with air (mode (iii)). The latter instability, which is discussed in detail in Chapter 5, will be shown to be different in nature from the former.

Figure 13 shows certain features which are common to all apparatus geometries.

(i) "Downcomer slug breakthrough with continued circulation" generally occurs for low values of the downcomer air rate and high values of the riser air rate. Thus the riser air keeps the system circulating for low downcomer air rates, regardless of slug breakthrough.

(ii) "Total flow reversal" or "No circulation and both limbs partially filling and emptying with air" can be seen to occur at higher values of the downcomer air rate. This might be expected since a large amount of downcomer air is needed to cancel out the density driving force or to make it reverse and pull the air back down the riser.

Figure 13 also shows some features which depend upon the apparatus geometry.

(iii) As the position of the downcomer injector is moved higher up, the apparatus becomes prone to instabilities at higher values of $Q_R$, and especially to both limbs partially filling and emptying with air. This observation is in agreement with a feature of the steady-state circulation theory which has already been mentioned in section 3.5. This is that the lower limit on $Q_R$ for continued circulation increases as the downcomer injector is moved higher up.
(iv) As the position of the riser injector is moved higher up, the system becomes more and more unstable. This is because more riser air is required for a given density driving force, and hence to maintain the circulation velocity.

(v) With the downcomer injector in its lower position, \( L_D = 2.25 \text{ m} \) see Figure 13(i) and (ii), the boundary line BC represents a transition from stable circulation to a total flow reversal. In contrast for the downcomer injector in its upper position, \( L_D = 3.47 \text{ m} \), see Figure 13(iii) and (iv), the boundary line BC represents a transition from stable circulation to no circulation and both limbs partially filling and emptying with air.

The reason for this change is that in the lower position, \( L_D = 2.25 \text{ m} \), a downcomer slug breakthrough can generate a larger reverse density driving force, due to the greater length of downcomer tube above the injector which can be filled with bubbles. This greater reverse driving force can easily pull the riser air back down and around the U bend giving a flow reversal.
4.3 Instability Initiated by Slug Breakthrough

In this section an attempt is made to quantify the instability referred to as "downcomer slug breakthrough with continued circulation", the boundary of which is given experimentally by the lines marked AB in Figure 13.

It is possible on plots of $Q_R$ against $Q_D$ to superimpose lines of constant circulation velocity. This may be done by calculating $V_o$ from equation 3.8 using the known apparatus geometry, $V_s = 0.50 \text{ m/s}$, and the experimentally determined value for the number of velocity heads lost in the circuit, $N$. Figure 14 shows lines of constant $V_o$, which are in fact straight lines, on a plot of $Q_R$ against $Q_D$ for the various apparatus geometries used: the unstable boundary lines from Figure 13 are also shown. Two sets of constant velocity lines are shown. The first was produced by using $N = 3.98$, i.e. $N_{\text{BEST}}$ for the original apparatus geometry used (see section 3.4.3): these lines are shown dashed. The second set, shown as full lines, was produced by using $N_{\text{BEST}}$ for that particular apparatus geometry. The two sets of lines obviously coincide for the original geometry shown in Figure 14(i).

It may be seen that if the full lines are used, the line AB always lies in between $V_o = 1.0$ and $V_o = 1.2 \text{ m/s}$ for all apparatus geometries. Thus an obvious conclusion is that downcomer slug breakthrough with continued circulation is initiated if the liquid circulation velocity falls below about $1.1 \text{ m/s}$.

It is interesting to compare this "breakthrough" figure of $1.1 \text{ m/s}$ with the predicted rise velocity of a slug as given by the Dumitrescu (1943) equation, i.e. $V_s = 0.35 \sqrt{gD}$: see section 1.2.1. This predicts that $V_s = 0.54 \text{ m/s}$ for a slug rising in an apparatus with a column diameter as used in this work. It predicts that a
"wall slug", i.e. a slug which has its nose on the tube wall and behaves as if it is in a tube twice the diameter, will have a rise velocity of 0.76 m/s in this apparatus. It must be pointed out that the figure of 1.1 m/s is an average liquid velocity across the tube, whereas the velocity experienced by the slug nose near the wall will be less due to the nature of the velocity profile. Nevertheless, it is obvious that the behaviour of slugs rising against a liquid downflow is considerably different to their upflow relations: see section 4.1.1.

When the steady-state circulation theory was derived in section 3.3, the existence of multiple steady states and their relative stabilities was also discussed: see section 3.3.1. These multiple steady states may be seen in Figures 14(iii) and 14(iv). Here points like "X" and "Y" apparently show that two circulation velocities $v_1$ and $v_2$ may exist at a particular combination of $Q_R$ and $Q_D$. It is, however, quite easy to show numerically, as follows, that only the higher velocity $v_2$ is stable. Figure 14(v) shows the two velocities $v_1$ and $v_2$ which are possible at the point "X" in Figure 14(iii). It may be seen that a small decrease in $v_1$ causes resisting forces to dominate, whereas a small increase causes the driving forces to dominate: thus $v_1$ is unstable. Conversely, $v_2$ is stable, and where two constant velocity lines cross the higher velocity is stable.
4.4 Compressor/Bubble Column: Stability

In this section a simple theory is developed to investigate the possible interaction between a compressor air supply to the riser and the circulating system itself: see Figure 15(b). The air is considered to be available at a constant pressure $P_o$, the only limit to the supply rate, $Q_R$, being the pressure drop $(P_o - P_s)$ available across the riser air injector: see Figure 15(a). The pressure at which air is delivered into the riser, $P_s$, is then compared with the hydrostatic pressure above the riser injector, $P_B$, since when these two are equal an operating point has been reached. The nature of the stability of this operating point is then discussed and a simple numerical example is given.

Let us firstly consider the situation at the riser injector as shown in Figure 15(a). We shall assume that the density of the air changes little between its supply pressure, $P_o$, and the pressure at which it enters the riser, $P_s$. We shall also assume that the air velocity in the sparger pipe is small and that the velocity of the air travelling through the holes in the sparger is $v_s$. Thus applying Bernoulli's equation to the sparger we obtain

$$P_s + \frac{1}{2} \rho_G v_s^2 = P_o$$  \hspace{1cm} (4.2)

This may be rearranged by noting that $v_s = Q_R/A_s$ where $A_s$ is the area of the sparger holes, and also by defining a sparger constant $K_s = A_s \sqrt{2/\rho_G}$. This gives an equation for the delivery pressure of air into the riser of the form

$$P_s = P_o - \frac{Q_R^2}{K_s^2}$$  \hspace{1cm} (4.3)

We shall now consider the circulating system shown in Figure 15(b), making the following assumptions as was done in deriving the steady-state
circulation theory: see section 3.3.

(i) That the voidage in the riser is invariant with depth.
(ii) That the bubbles present have a constant slip velocity $V_s$.
(iii) That the resisting forces in the circuit have the form $KV_o^2$ at steady-state.
(iv) That mass transfer effects are small, and that the gas density is small compared to the liquid density.

Considering a mass balance at the riser injector gives

$$Q_R = A\epsilon_R (V_o + V_s) \quad (4.4)$$

The equation of motion of the system is given by $M_e(dV/dt) =$ Density driving force minus Resisting forces. Since we are dealing with steady-state circulation, when a balance is achieved between the air injection pressure and the hydrostatic pressure above the riser injector, the acceleration term is zero.

Since voidage is assumed invariant with depth, the density driving force is given by

$$\rho gA\delta - \rho gA(L-\delta) - \rho gA\delta(1-\epsilon_R) \quad (4.5)$$

Thus the equation of motion is given by

$$V_o = \sqrt{\frac{\rho gA\delta \epsilon_R}{K}} \quad (4.6)$$

The riser voidage, $\epsilon_R$, may be found by substituting for $V_o$ into equation 4.4 to give an equation which may be solved for $\epsilon_R$ by repeated substitution

$$\epsilon_R = \frac{Q_R}{A(V_s + \sqrt{\frac{\rho gA\delta \epsilon_R}{K}})} \quad (4.7)$$

The hydrostatic pressure above the riser injector, $P_{BC}$, is given by

$$P_{BC} = P_A + \rho g(1-\epsilon_R) \quad (4.8)$$
Thus using equations 4.7, 4.8 and 4.3, a plot of both the hydrostatic pressure, $P_{BC}$, and the air injection pressure $P_s$ against $Q_R$ can be made. When $P_s = P_{BC}$ an operating point has been found.

From equation 4.3 it can be seen that

$$\frac{dP_s}{dQ_R} = -\frac{2Q_R}{K_s^2}$$

(4.9)

Noting that $(dP_{BC}/dQ_R) = (dP_{BC}/d\varepsilon_R)(d\varepsilon_R/dQ_R)$ it can be shown that

$$\frac{dP_{BC}}{dQ_R} = \frac{-\rho g \delta}{A(V_s + \frac{3}{2}\sqrt{\frac{\rho g A \delta e_R}{K}})}$$

(4.10)

Now since $\varepsilon_R$ increases as $Q_R$ increases, $|dP_{BC}/dQ_R|$ must decrease as $Q_R$ increases. This is in contrast to the behaviour of $|dP_s/dQ_R|$ which does the opposite. When no riser air is supplied $P_s$ must be greater than $P_{BC}$ for the startup of the apparatus. Using this fact in conjunction with the behaviour of $(dP_{BC}/dQ_R)$ and $(dP_s/dQ_R)$, it is obvious that only one possible operating point will exist.

Figure 15(c) shows the resultant curves of $P_{BC}$ and $P_s$ against $Q_R$ for the typical numerical values given. It shows only one operating point "X" at a very high value of $Q_R$. That this single operating point is stable may be shown by considering the effect of a small change in $Q_R$ from its value at point "X". A small increase in $Q_R$ means that the hydrostatic pressure dominates over the air supply pressure and hence $Q_R$ must fall. Conversely, a small decrease in $Q_R$ means that the air supply pressure dominates over the hydrostatic pressure and $Q_R$ must rise: thus the point "X" is stable.

The reason why a very high operating value of $Q_R$ is obtained in this example is because of the nature of equation 4.3. This simple equation is really inadequate for large riser air flowrates, and a
more accurate relationship for the injection pressure, $P_s$, would result in a much lower operating value for $Q_R$. 
Deceleration of the System

Let us firstly consider the system circulating with riser air only, as described in section 3.1. Suppose now that the riser air supply is turned off. Bubbles are still leaving the system from the disengagement tank though no new bubbles are being injected: thus the density driving force falls and the circulation velocity decreases. After a short time all of the bubbles will have left the system and no density driving force remains.

In this section a simple theory is given to describe the "slowdown" of the system when the last of the bubbles has left. Using data from experimental runs with and without an orifice plate present, it is possible to obtain estimates of the single-phase hydraulic losses and the effective mass of the system.

In section 3.3 it was stated that the equation of motion of the system was given by $M_e (dV/dt) = \text{Density driving force minus Resisting forces}$. Since we are now considering the system after all the bubbles have been disengaged, the Density driving force is zero, and the Resisting forces are given by $K_{SF} V^2$. Thus the equation of motion becomes

$$M_e \frac{dV}{dt} = -K_{SF} V^2 \quad (4.11)$$

This equation may be integrated to give

$$V = \frac{1}{\left(\frac{K_{SF}}{M_e} t + \frac{1}{V_i}\right)} \quad (4.12)$$

where $V_i$ is the initial system velocity, and $V$ is the velocity after a time $t$, i.e. $V < V_i$. It is useful when considering equation 4.12 to take steps backwards rather than forwards in time. Thus equation 4.12 may be rewritten as
\[ \frac{1}{V} = \frac{1}{V_f} + \frac{K_{SF}}{M_e} t \]  

(4.13)

where \( V_f \) is a chosen final velocity at \( t = 0 \), and \( V \) is now defined as the system velocity at a time \( t \) earlier, i.e. \( t < 0 \) and \( V > V_f \).

Thus a plot of \( \frac{1}{V} \) against \( (-t) \) for experimental runs with the same chosen value of \( V_f \) should result in a single straight line of gradient \( (-K_{SF}/M_e) \) once all the bubbles have left the system. Figure 16(a) gives such a plot for one set of data with the orifice plate present. Data from runs where the system was allowed to slow down were plotted in this way, and \( (K_{SF}/M_e) \) values were obtained using a "least squares fit" of straight lines to the data. The results were as follows:

(i) For "slowdown" runs where the orifice plate was present, the average of five "least squares" values of \( (K_{SF}/M_e) \) obtained was 0.1166. Details of the orifice plate itself have previously been given in section 3.2, and the velocities in these runs were found from the orifice plate pressure drop.

(ii) For "slowdown" runs where the orifice plate was not present, the average of three "least squares" values of \( (K_{SF}/M_e) \) obtained was 0.0780. In these runs the velocities were measured by using the calibrated pitot tubes.

Figure 16(b) shows a plot of \( V \) against \( (-t) \) for data from the "slowdown" runs. The experimental data may be compared with the curves for \( V \) as a function of \( (-t) \) using \( (K_{SF}/M_e) = 0.1166 \) and \( (K_{SF}/M_e) = 0.0780 \) in equation 4.13. It may be seen from Figure 16(b) that the theoretical curves A and B provide a reasonable fit to the data.

The data from one run with the orifice plate absent and \( t < -15 \)
are not shown as the expanded (-t) scale required would spoil the clear presentation of the other data points. The missing data generally lie slightly below curve B, which explains why the rest of the data with the orifice plate absent lie slightly above the curve since the curve is based on all the available data.

We know that with the orifice plate present $K_{SF}$ is the sum of the system and orifice plate losses. On the other hand, the value of $K_{SF}$ with the orifice plate absent is purely due to the system losses. Thus we have

$$\frac{K_{SYSTEM} + K_{ORIFICE}}{M_e} = 0.1166$$

and

$$\frac{K_{SYSTEM}}{M_e} = 0.0780$$

BS 1042 (1964) shows that for the orifice plate used with an area ratio, $m$, of 0.6, the overall system head loss caused by its presence is 40% of the measured head drop across the orifice plate. The British Standard also shows that this orifice plate, described in section 3.2, will produce a measured head drop in metres of water of 0.24425 $V^2$. Thus the head loss caused by its presence will be 0.0977 $V^2$ metres of water. Since the pressure drop, $\Delta P$, caused by the orifice plate is thus 0.0977 $\rho gV^2$, the $K_{ORIFICE}$ value may be found from equation 3.7 knowing the tube area

$$K_{ORIFICE} = \frac{\Delta P A}{V^2} = \frac{0.0977 \rho g V^2 A}{V^2} = 43.83$$

Substituting for $(K_{SYSTEM}/M_e)$ from equation 4.15 together with $K_{ORIFICE}$ from equation 4.16 into equation 4.14 gives $M_e = 1135$ kg, and $K_{SYSTEM} = 88.6$ (which from equation 3.7 gives $N_{SYSTEM} = (2K_{SYSTEM}/A_P) = 3.87$ velocity heads lost).
This estimate for $M_e$ compares with 915 kg for a 20 m length of 0.24 m diameter tube. In the rest of this work a value of $M_e = 1000$ kg was taken, since (a) it lies between these two estimates, and (b) it is not obvious how to calculate $M_e$ with bubbles present in the system. The estimate of $N_{\text{SYSTEM}}$ compares with the two-phase experimental values of 4.0 - 4.7 velocity heads lost given in section 3.4.3: being a single-phase value it would be expected to be somewhat lower.
In this chapter the possibility of a growing perturbation in the steady-state circulation velocity is discussed. A criterion is presented as to whether or not this perturbation will magnify with time, and hence whether the system remains stable or not. It is shown experimentally that perturbations can lead to oscillations in the circulating liquid velocity which may be damped or undamped. The experimental results are then compared with a circulation theory which predicts the existence of such oscillations.

Other workers have proposed theories for the occurrence of oscillations in similar systems. However, these are not of the same nature as those described in this dissertation. Hjalmars (1973) describes an instability of the airlift pump. However, when the analysis given in section 5.3 was applied to this system it showed that it was stable and hence the oscillations described are not the same. Garland and Davidson (1975) describe self-excited oscillations in a U-tube, but there is no net circulation of the liquid, and thus again the oscillations differ in type.
5.1 A Simple Explanation for the Existence of Oscillations

In this section the effect of a small velocity perturbation, \( dV \), on the steady-state liquid circulation velocity, \( V_0 \), is discussed. It is shown that under certain conditions the perturbation will grow indefinitely.

Certain assumptions are made to simplify the analysis, these follow those made in section 3.3 and are briefly:

(i) Voidage invariant with depth.

(ii) Constant bubble slip velocity \( V_s \).

(iii) Resistance forces in the circuit of the form \( KV^2 \), where \( V \) is the liquid circulation velocity at any time.

(iv) Air supplied at rates \( Q_R \) and \( Q_D \) to the riser and downcomer respectively.

(v) Voidages are assumed small enough so that the circulation can be adequately described by a single liquid velocity, \( V \), around the circuit.

(vi) Mass transfer is small.

(vii) The gas density is much less than the liquid density.

The system is shown in Figure 8(a), with voidages \( \varepsilon_1 \) (below the downcomer injector), \( \varepsilon_2 \) (below the riser injector), and \( \varepsilon_3 \) (above the riser injector). The relationship between \( \varepsilon_2 \) and \( \varepsilon_1 \) is given at any time by

\[
\varepsilon_2 = \varepsilon_1 \frac{(V-V_s)}{(V+V_s)} \quad (5.1)
\]

A change in the liquid velocity of \( dV \) will produce changes of voidage at the downcomer and riser injectors of \( d\varepsilon_1 \) and \( d\varepsilon_3 \) respectively. Hence a balance on the air injected at the downcomer injector before and after this velocity change gives

\[
\frac{Q_D}{A} = (V_0-V_s)\varepsilon_1 = (V_0+dV-V_s)(\varepsilon_1+d\varepsilon_1) \quad (5.2)
\]
This may be rearranged to give the change in downcomer injector voidage, $d\varepsilon_1$, as

$$d\varepsilon_1 = -\frac{Q_D dV}{A(V_o - V_s)^2} \quad (5.3)$$

A similar "before and after" balance at the riser injector gives the change in the riser injector voidage, $d\varepsilon_3$, as

$$d\varepsilon_3 = -\frac{(Q_D + Q_R) dV}{A(V_o + V_s)^2} \quad (5.4)$$

The resisting forces at any time are given by $KV^2$ and the driving force is given by equation 3.4.

It may be seen from equations 5.3 and 5.4 that $d\varepsilon_1$ and $d\varepsilon_3$ both represent "plugs" of negative voidage: see Figure 17(a). After a time, $dt$, these plugs of negative voidage will have moved a distance of $(V_o - V_s)dt$ and $(V_o + V_s)dt$ down from the downcomer injector and up from the riser injector respectively. The resultant change in the system density driving force is given by the new density driving force (after a time $dt$) minus the old density driving force (before the change in liquid velocity), i.e.:

$$\rho gA[(L - L_D) + (V_o - V_s)dt(1 - \varepsilon_1 - d\varepsilon_1) + (L_D - (V_o - V_s)dt)(1 - \varepsilon_1) - L_R(1 - \varepsilon_2) - (L - L_R)(V_o + V_s)dt(1 - \varepsilon_3 - d\varepsilon_3)]$$

$$-\rho gA[(L - L_D) + L_D(1 - \varepsilon_1) - L_R(1 - \varepsilon_2) - (L - L_R)(1 - \varepsilon_3)] \quad (5.5)$$

This is obviously only true if $dt$ is short enough such that the plug of negative voidage at the downcomer injector has not reached the U-bend, and the plug of negative voidage at the riser injector has not reached the top of the riser. At longer times the plug of negative voidage from the downcomer will enter the riser, lowering the driving force and slowing the system down. If the perturbation is large
enough, this decrease in velocity as the negative voidage plug enters the riser will cause a slug "breakthrough" to occur at the downcomer injector (as described in section 4.1.2). It should, however, be noted that although a slug "breakthrough" is the final event in this sequence of events, it is the result and not the cause of this instability.

The change in the resisting forces in the system due to the velocity change \( dV \), is given by \( 2KV_0 dV \), neglecting second order terms. We are interested in the condition that a perturbation can amplify with time, for this the change in the density driving force must be greater than the change in the resisting forces, i.e.:

\[
\rho g Ad \left[ \varepsilon_3 (V_o + V_s) - \varepsilon_1 (V_o - V_s) \right] > 2K V_0 dV
\]  

Substituting in for \( \varepsilon_3 \) from equation 5.4, \( \varepsilon_1 \) from equation 5.3, and noting that \( K = NAp/2 \) from equation 3.7, we finally obtain

\[
\frac{Q_D}{V_o (V_o - V_s)} - \frac{(Q_D + Q_R)}{V_o (V_o + V_s)} > \frac{NA}{g}
\]  

Thus if the term in brackets is positive, this is a necessary but certainly not sufficient condition for a perturbation in the liquid velocity to magnify, which would lead to instability in the system.

5.1.1 Comparison with Experimental Results

In Chapter 4 it has been shown that one stability/instability boundary is given by \( V_o = 1.1 \text{ m/s} \), since below this velocity slug "breakthrough" occurs. The other experimental stability boundary, shown by lines BC on Figure 13, will now be compared with the theory proposed in section 5.1.

The necessary but not sufficient condition for a perturbation to magnify is given when the term in brackets in equation 5.7 is zero, i.e.:

\[
Q_R = \frac{2Q_D V_s}{(V_o - V_s)}
\]  

(5.8)
This expression for $Q_R$ may be substituted into equation 3.8 for the steady-state circulation velocity of the system. Rearranging this gives

$$V_o = \sqrt{\frac{2gQ_D}{N(A(V_o - V_s)(V_o - V_s)) - L_D(V_o + V_s) - L_s(L - L_R)}}$$  \hspace{1cm} (5.9)

This can be solved for $V_o$ by trial and error if $V_s$, $Q_D$, $N$ and the apparatus geometry are known.

Figure 17(b) shows the predicted boundary of the transition from stability to potential instability. It was constructed for each apparatus geometry and using both $N = 3.98$ and $N = N_{BEST}$ for the particular apparatus geometry, as follows:

(i) Set $Q_D$.

(ii) Find $V_o$ from equation 5.9 by trial and error.

(iii) Use this value of $V_o$ to calculate $Q_R$ from equation 5.8.

Figure 17(b) also shows the previously determined experimental stability/instability transition lines (see section 4.2). The following points should be noted from Figure 17(b):

(i) The theoretical "voidage plug" boundary curve is a very conservative estimate of the stability of the system, especially for $L_D = 2.25$ m.

(ii) The theoretical curves produced using $N_{BEST}$, noting that $N_{BEST} > N$, are even more conservative than those produced by using $N = 3.98$.

Hence the "voidage plug" instability line may be used as a very conservative first estimate of where the system should be operated to avoid instabilities of the type represented by experimental lines BC.
5.2 Experimentally Observed Oscillations

A series of experiments was performed, using the four apparatus geometries described in section 3.4.3, to investigate in more detail the transition from stable operation, mode (i), to either no circulation and both limbs partially filling and emptying with air, mode (iii), or a total flow reversal, mode (iv). The transition from stable circulation, mode (i), to downcomer slug breakthrough with continued circulation, mode (ii), was also investigated. These modes have already been described in section 4.1 and the experimental stability/instability transitions have already been shown on Figure 13 by the lines AB and BC.

The procedure adopted for these experiments was similar to that originally used in the experimental determination of the stable and unstable regions of operation, see section 4.2. In this case a longer time was allowed at each \( Q_R/Q_D \) setting to allow instabilities time to develop: the procedure adopted may be summarised as follows:

(i) Initiate stable circulation with a large amount of riser air, \( Q_R \).
(ii) Gradually inject downcomer air, \( Q_D \), up to the chosen value.
(iii) Gradually turn the riser air down, allowing about 40 minutes at each \( Q_D/Q_R \) setting to observe the stability/instability of the system.

It was found that at each of the \( Q_D/Q_R \) settings near the stability boundaries AB and BC, one of the following five states was observed:

(i) System stable with no velocity perturbations.
(ii) System stable with oscillations observed in the liquid velocity.
(iii) Oscillations observed in the liquid velocity leading to
both limbs partially filling and emptying with air after downcomer slug breakthrough \((L_D = 3.47 \text{ m}, \text{ see section 4.2})\).

(iv) Oscillations observed in the liquid velocity leading to a total flow reversal after downcomer slug breakthrough \((L_D = 2.25 \text{ m}, \text{ see section 4.2})\).

(v) No oscillations observed, but downcomer slug breakthrough with continued circulation occurs.

It should be pointed out that in (ii) the system was only stable for as long as observations were made, i.e. about 40 minutes, and there is no guarantee that stability would have continued beyond this time.

The oscillations observed in the four geometries used are tabulated in Table 1. Their position on the stability plot of \(Q_R\) against \(Q_D\) is shown in Figure 17(b), for comparison with stability boundaries AB and BC: actual pen traces of four typical oscillations (one from each \(L_D/L_R\) combination used) are shown in Figure 18.

The following points should be made about these observed oscillations:

(i) It can be seen from Figure 17(b) that the observed oscillations, some of which appear stable (see Table 1), generally lie below the experimental stability boundary BC. The reason why apparently stable oscillations can exist in what has been defined as an unstable area is because such a long time was taken to approach these operating points. This means that the system was given a lot longer to settle down after changes in \(Q_R\), compared to the conditions under which the line BC was originally determined (see section 4.2): this enhances the stability of these points.

(ii) The operating points where oscillations were observed were always near the line BC and never near the line AB. This shows that the occurrence of oscillations is a separate phenomenon to downcomer
slug breakthrough which occurs at a constant velocity of 1.1 m/s: see section 4.3.

(iii) The right hand end of Table 1 shows that the theoretical values of $V_0$ (based on $N$, $Q_D$ and $Q_R$ values) can vary quite a lot from the experimentally measured pitot tube velocities $V_C$ and $V_{SI}$. Quite a lot of this variation can be ascribed to the large amplitude of the oscillations involved, leading to errors in obtaining mean values for $V_C$ and $V_{SI}$.

Table 1 shows that $V_0$ (theoretical) can either be lower or higher than $V_C$ or $V_{SI}$ (experimental) for all geometries used, except for $L_R = 5.18/L_D = 3.47$ m where it is always higher. It has already been shown for this geometry that experimental steady-state velocities lie somewhat below the theoretical circulation curves: see Figure 12(vii). This is due to (a) riser air "jetting" from the aerofoil tail at high $Q_R$ and $L_R$ (see section 3.4.3), (b) increased bubble carryover into the downcomer, at high $Q_R$ and $Q_D$, leading to a decreased density driving force and lower experimental velocities.

Thus all predictions of oscillations using $V_0$ (theoretical) for this geometry should be treated with caution: see section 5.5.

(iv) Table 1 shows that the observed oscillations have periods from 42 to 82 seconds and amplitudes of between 0.14 and 0.26 m/s.

(v) Table 1 shows that the effect of increasing $L_D$ is a very definite increase in the period of the oscillations.

(vi) The effect of increasing $L_R$ seems to be a slight decrease in the period of the observed oscillations: i.e. (v) and (vi) taken together mean that $L_R = 5.18/L_D = 2.25$ m has the shortest periods observed, whereas $L_R = 3.65/L_D = 3.47$ m has the longest.

(vii) Table 1 shows that at the same $Q_R/Q_D$ setting, oscillations of differing periods can occur in the same experimental run. There is
no obvious explanation of this phenomenon.

(viii) Figure 18 shows that the oscillations affect the central and side pitot tubes at the same time, confirming that the oscillations affect the whole of the liquid velocity profile simultaneously.

(ix) Figure 18 also shows that the turbulent noise not cut out by the smoothing capacitors on the pressure transducer amplifiers, see section 2.2.5 and Figure 6(a), does not mask the obvious liquid oscillations.

The experimental oscillations described in this section are compared with the predictions of a theory, which predicts oscillations, in section 5.5: the theory itself is described in section 5.3.
5.3 Equations of Oscillatory Behaviour

In this section a simple theory is derived which predicts that oscillations in the liquid velocity can occur in the system. This theory will be compared in section 5.5 with the experimentally observed oscillations, which were described in section 5.2.

The assumptions made in this analysis are summarised below: basically they are the same as were made for the Steady-State Circulation Theory in section 3.3 excepting that voidage is now considered to be a function of time.

These assumptions are:

(i) The effect of hydrostatic head on system voidages is ignored (Appendix 4 gives a full derivation of this theory where this assumption is not made).

(ii) Constant bubble slip velocity $V_s$.

(iii) Resistance forces $KV^2$ in the circuit.

(iv) Air supplied at rates $Q_R$ to the riser and $Q_D$ to the downcomer. ($Q_R$ and $Q_D$ again represent the average of values between the injector positions and the liquid surface, to give some compensation for the varying hydrostatic head.)

(v) Voidages are assumed small, such that $V$ does not change significantly around the circuit.

(vi) Mass transfer effects (see Appendix 2) are considered small and the gas density is negligible compared to the liquid.

(vii) Voidages around the system vary with time as will be described.

The system under consideration is shown in Figure 19(a), with the downcomer injection voidage being $\varepsilon_{1I}(t)$, and the extra voidage due to riser air at the riser injector being $\varepsilon_{1R}(t)$. It is obvious that the voidage $\varepsilon_1$ at a position $x$ below the downcomer injector is the
voidage that existed at the downcomer injector at a time \( x/(V-V_s) \) earlier. Thus \( \varepsilon_1 \) at a position \( x \) is a function of time of the form

\[
\varepsilon_1(t) = \varepsilon_{1I}(t - \frac{x}{V-V_s}) \quad \text{for} \quad 0 < x < L_D
\]

(5.10)

Similar expressions may be written down for \( \varepsilon_2 \) in the riser, noting the effect of additive bubble slip velocity, and \( \varepsilon' \) above the riser injector (where \( \varepsilon' \) is the additional voidage due to the injection of riser air). These relationships have the form:

\[
\varepsilon_2(t) = \varepsilon_{1I}(t - \frac{L_D}{V-V_s} - \frac{x-L_D}{V+V_s}) \times \frac{V-V_s}{V+V_s} \quad \text{for} \quad L_D < x < L_D+L
\]

(5.11)

and

\[
\varepsilon'(t) = \varepsilon'_I(t - \frac{y}{V+V_s}) \quad \text{where} \quad y = x - (L_D+L_R) \quad \text{for} \quad 0 < y < L_R
\]

(5.12)

The density driving force in the system is given by the right hand side of equation A3.7 as

\[
\rho g A \left[ \epsilon_2 dx - \int_0^{L_D} \varepsilon_1 dx + \int_0^{L_R} \varepsilon' dy \right]
\]

The resisting forces are given by \( KV^2 \), and the resultant forces by \( M_e \frac{dV}{dt} \).

We shall now assume that the liquid circulation velocity, \( V \), is the sum of the steady-state velocity, \( V_o \), and a small perturbation velocity \( v \) (i.e. \( v \ll V_o \)). We shall furthermore assume that the perturbation velocity is exponential in nature and of the form \( D^* e^{at} \). Thus if \( \alpha \) is positive the system will be unstable, if \( \alpha \) is negative it will be stable, and if \( \alpha \) is zero it will be marginally stable.

If this form of the velocity is substituted into equation 3.1 instead of \( V_o \), then a balance on the air injected at the downcomer injector gives
voidage that existed at the downcomer injector at a time \( x/(V-V_s) \) earlier. Thus \( \varepsilon_1 \) at a position \( x \) is a function of time of the form

\[
\varepsilon_1(t) = \varepsilon_1(t - \frac{x}{V-V_s}) \quad \text{for} \quad 0 < x < L_D
\] (5.10)

Similar expressions may be written down for \( \varepsilon_2 \) in the riser, noting the effect of additive bubble slip velocity, and \( \varepsilon' \) above the riser injector (where \( \varepsilon' \) is the additional voidage due to the injection of riser air). These relationships have the form:

\[
\varepsilon_2(t) = \varepsilon_2(t - \frac{L_D}{V-V_s} - \frac{x-L_D}{V+V_s}) \times \frac{V-V_s}{V+V_s} \quad \text{for} \quad L_D < x < L_D+L
\] (5.11)

and

\[
\varepsilon'(t) = \varepsilon'(t - \frac{y}{V+V_s}) \quad \text{where} \quad y = x - (L_D+L_R) \quad \text{for} \quad 0 < y < L-L_R
\] (5.12)

The density driving force in the system is given by the right hand side of equation A3.7 as

\[
\rho g A \left[ \int_{L_D}^{L+L_D} \varepsilon_2 \, dx - \int_{0}^{L_D} \varepsilon_1 \, dx + \int_{0}^{L-L_R} \varepsilon' \, dy \right]
\]

The resisting forces are given by \( KV^2 \), and the resultant forces by \( M_e \, dV/dt \).

We shall now assume that the liquid circulation velocity, \( V \), is the sum of the steady-state velocity, \( V_o \), and a small perturbation velocity \( v \) (i.e. \( v \ll V_o \)). We shall furthermore assume that the perturbation velocity is exponential in nature and of the form \( D e^{\alpha t} \). Thus if \( \alpha \) is positive the system will be unstable, if \( \alpha \) is negative it will be stable, and if \( \alpha \) is zero it will be marginally stable.

If this form of the velocity is substituted into equation 3.1 instead of \( V_o \), then a balance on the air injected at the downcomer injector gives
\[ \varepsilon_{11}(t) = \frac{Q_D}{A(V_o - V_s)(1 + \frac{v}{(V_o - V_s)})} \approx \frac{Q_D}{A(V_o - V_s)} \left( 1 - \frac{v}{(V_o - V_s)} \right) = A_D - B_D e^{\alpha t} \] (5.13)

where \( A_D = \frac{Q_D}{A(V_o - V_s)} \) and \( B_D = \frac{Q_D}{A(V_o - V_s)} \). A similar balance on the extra air injected at the riser injector gives

\[ \varepsilon_{11}'(t) = \frac{Q_R}{A(V_o + V_s)} \left( 1 - \frac{v}{(V_o + V_s)} \right) = A_R - B_R e^{\alpha t} \] (5.14)

where \( A_R = \frac{Q_R}{A(V_o + V_s)} \) and \( B_R = \frac{Q_R}{A(V_o + V_s)} \). Thus using the results of equations 5.10 to 5.14, the equation of motion of the system becomes

\[
M \frac{dV}{dt} = -KV^2 + \rho g A \left[ \int_{-L}^{L} [A_D - B_D e^{\alpha (t - \frac{L}{V_s} - \frac{x}{V_s})}] (V - V_s) dx + \int_{-L}^{L} [A_R - B_R e^{\alpha (t - \frac{x}{V_s})}] dy \right] (5.15)
\]

This equation may be integrated, and if the conditions for steady state are substituted in, namely \( D^* = 0 \) and \( V = V_o \), the resultant equation is the same as equation 3.6 which was derived for the steady state.

If the steady-state solution is subtracted out of the integrated version of equation 5.15, the eventual result after putting \( V = V_o \) and dividing both sides by \( V (= D^* e^{\alpha t}) \), is given by

\[
M \alpha + 2KV_o = \frac{\rho g Q_D}{\alpha (V_o - V_s)} \left[ 1 + e^{-\alpha \left( \frac{L_D}{V_o - V_s} + \frac{L}{V_o + V_s} \right)} - \frac{\alpha L_D}{V_o - V_s} \right] - \frac{\alpha L_D}{V_o - V_s} \\
+ \frac{Q_R}{Q_D} \left( \frac{V_o - V_s}{V_o + V_s} \right) \left( e^{-\alpha \left( 1 - \frac{L_R}{V_o + V_s} \right)} - \frac{\alpha (L - L_R)}{V_o + V_s} \right) (5.16)
\]

If we now assume the most general case for \( \alpha \), i.e. that it is a complex number of the form \((\beta + i \omega)\) we can see that oscillations in
the liquid velocity can occur, since $V$ now has the form $V_0 + D e^{(\beta+i\omega)t}$. Thus if we have a positive value of $\beta$ the oscillations are undamped, whereas a negative value of $\beta$ corresponds to damped oscillations.

The resultant real and imaginary equations obtained by substituting $\alpha = (\beta+i\omega)$ into equation 5.16 are

### REAL

\[
M e^{\beta + 2KV_0} = \frac{\rho g Q_D \beta}{(\beta^2+\omega^2)(V_0-V_s)} \left[ -\beta \left( \frac{L}{V_0-V_s} + \frac{L}{V_0+V_s} \right) \right] 1 + e^{-\beta \left( \frac{L}{V_0-V_s} + \frac{L}{V_0+V_s} \right)} \cdot \cos \omega \left( \frac{L}{V_0-V_s} + \frac{L}{V_0+V_s} \right)
\]

### IMAGINARY

\[
M e^{\omega} = \frac{\rho g Q_D \omega}{(\beta^2+\omega^2)(V_0-V_s)} \left[ -\beta \left( \frac{L}{V_0-V_s} + \frac{L}{V_0+V_s} \right) \right] \cdot \sin \omega \left( \frac{L}{V_0-V_s} + \frac{L}{V_0+V_s} \right)
\]

It should be remembered that these equations are for small oscillations taking place about the steady-state velocity, $V_0$. 

\[
(5.17)
\]

\[
(5.18)
\]
It would be expected that the border between stable and unstable velocity perturbations would be given by the solution to equations 5.17 and 5.18 with $\beta = 0$. This solution is compared with the experimental stability transition line BC in section 5.5.
5.4 Solution of the Resultant Equations

5.4.1 Simplex and the Multiple Root Problem

In section 5.3, equations 5.17 and 5.18 were derived which describe the oscillations in liquid velocity which can occur in the system. The problem now is to find a method of solving these equations together, in order to compare the theoretical predictions with experimentally observed oscillations and with an experimental transition from stable to unstable operation (given by lines BC on Figure 13).

In order to do this we must define a variable which enables roots of these equations to be identified for given values of the other parameters. Thus the variable SUM is introduced as

\[ \text{SUM} = \left( \frac{\text{Left hand side}}{\text{of equation 5.17}} - \frac{\text{Right hand side}}{\text{of equation 5.17}} \right)^2 + \left( \frac{\text{Left hand side}}{\text{of equation 5.18}} - \frac{\text{Right hand side}}{\text{of equation 5.18}} \right)^2 \]

When we have a root to these equations \( \text{SUM} = 0 \), and because of its definition it cannot be negative and has no upper positive limit, i.e. \( \text{SUM} \) tends to \(+\infty\) under certain conditions. Thus we must look for a numerical method which will find the minimum value of \( \text{SUM} \) in order to obtain the values of \( \beta \) and \( \omega \) when this is zero (i.e. at a root).

The first approach adopted was to use a simplex routine on the Departmental computer, a PDP 11, to find \( \beta \) and \( \omega \) given the values of the other parameters involved. It is obvious that the substitution \( \omega = -\omega \) does not affect equations 5.17 and 5.18, and thus complex conjugate roots were to be expected. However, it soon became clear using the simplex routine (which requires a starting value for \( \beta \) and \( \omega \)) that multiple solutions for \( \beta \) and \( \omega \) were present, since different starting values could generate different roots. Thus the simplex method could only be used if a starting value close to the
physically meaningful root could always be specified: this would then lead to convergence onto that root.

Considering the nature of equations 5.17 and 5.18 it is not at all surprising that multiple roots exist. Exponentials and sines and cosines can all be expressed in the form of infinite polynomials, and thus there is no reason why even an infinite number of roots to these equations might not be expected. Obviously a method was required which showed more than one possible root for \( \beta \) and \( \omega \) at a time, so that the root with the most physical significance to the system could be selected.

5.4.2 The Phase Plane Analysis

It was found that the variable \( \text{SUM} \) varied widely, over a range of \( \beta \) and \( \omega \), depending upon the values of \( \beta \) and \( \omega \) used. Thus in order to locate roots across a wide \( \beta-\omega \) "phase-plane", for given values of the other parameters, a new and less sensitive variable was defined as

\[
\begin{align*}
  f &= \ln(1 + \text{SUM}) \\
  (5.20)
\end{align*}
\]

This, like \( \text{SUM} \), is zero at a root, cannot be negative, and can tend to \( +\infty \) under certain conditions. However, unlike \( \text{SUM} \), it varies quite slowly across the \( \beta-\omega \) phase plane and thus is suitable for a wider search.

In order to locate the multiple roots the approach now adopted was to examine the \( \beta-\omega \) phase-plane contours. This was done by using the University IBM computer to solve equation 5.20 for different values of \( \beta \) and \( \omega \), and to plot out the resultant contours of "\( f \)" on the \( \beta-\omega \) plane by using a plotting routine within the system.

Such contour plots are shown in Figure 20(a) and 20(b). The
similar form of these plots, which are for different $L_D/L_R$ values and different values of $Q_R/Q_D$, shows that large scale changes of the phase plane contours do not occur for one piece of apparatus.

The arrows on these plots indicate the direction in which the numerical values of the contours of $f$ are decreasing. Thus it is obvious that each plot shows the existence of several roots which appear as the centres of high concentrations of concentric contours, where $f = 0$. There seem to be an unending number of roots which lie along a band of $\beta$ for increasing $\omega$.

The root which is physically meaningful is the one which is closest to the $\beta$ axis; this can be shown to be the case for two reasons:

(i) All the other roots involve much more negative values of $\beta$, and since the velocity is of the form $V = V_0 + D e^{(\beta+i\omega)t}$ they are inherently very stable and would not show up as experimental oscillations.

(ii) The root nearest the $\beta$ axis has the most meaningful value of $\omega$, since it represents periods of oscillation $> 20$ seconds, whereas all the other roots involve very much shorter periods which are just not observed experimentally.

The "physically" meaningful roots on the $\beta-\omega$ phase planes from Figures 20(a) and 20(b) are shown enlarged in Figures 21(a) and 21(b). These show more clearly that each root is at the centre of decreasing value concentric contours of $f$ as would be expected.

The $f$ values of the contours in the neighbourhood of the meaningful root all decrease towards that root. This leads to the conclusion that a simplex routine can be used to find the meaningful root provided that the starting point is within the neighbourhood of concentric contours. It was found that a starting point close to
\( \beta = 0, \omega = 0 \) would always lead to the meaningful root for all the cases studied. However, the point \( \beta = 0, \omega = 0 \) cannot itself be used as this point is obviously indeterminate from equations 5.17 and 5.18.
5.5 Instability Initiated by Oscillation

It has been shown in section 5.4.2 that the equations of oscillatory behaviour of the liquid velocity can be solved by a simplex method, to give a physically meaningful solution, provided that a starting condition close to $\beta = 0$, $\omega = 0$ is chosen.

This analysis was performed to compare the experimentally observed oscillations, listed in Table 1, with the predictions of the theory. The procedure was as follows:

(i) Take experimental $Q_R$ and $Q_D$ values and together with the apparatus geometry, $V_s = 0.50$ m/s, and $N = 3.98$ or $N = N_{\text{BEST}}$, calculate the theoretical steady-state circulation velocity, $V_0$, from equation 3.8.

(ii) Use this value of $V_0$ to calculate the $\beta$ and $\omega$ values making $\text{SUM} = 0$ in equation 5.19 by using the simplex method (with a starting value close to $\beta = 0$, $\omega = 0$) as outlined before.

(iii) Compare calculated values of $V_0$, $\beta$ and $\omega$ with the experimental observations.

This procedure was used to generate Table 2, which represents a comparison between the experimental oscillations and the predictions of the theory set out in section 5.3. The following comments should be made on the comparison:

(i) The comparison between $V_0$ (theoretical) and $V_{C/V_{SI}}$ (experimental) has already been described in section 5.2. This shows that the prediction of velocity is not exact, but no systematic error is obvious except for $L_D = 3.47/L_R = 5.18$ m.

(ii) The theory does seem to predict that where oscillations are experimentally observed small values of $\beta$ exist, i.e. $\beta \approx 0$. This is in agreement with what would be expected since for $\beta \ll 0$ the oscillations would be too damped to observe, and for $\beta \gg 0$ the
system would be too unstable for even a single cycle to appear before total instability set in.

(iii) The agreement between experimental and theoretical periods of the oscillations is not good. The theoretical periods are always shorter than the experimental periods by a factor of between 1.2 and 3. However, the theory is only intended for use where small perturbations occur, i.e. \( v \ll V \), and since Table 1 shows that the perturbations are of the order of 20% of the liquid velocity, it is not surprising that the agreement is not better. The periods are of the right order which means that a lot of credence must be given to the model.

(iv) The experimental periods of oscillation are generally longer for \( L_D = 3.47 \text{ m} \) (i.e. \( T = 50 \) to \( T = 82 \)) compared to \( L_D = 2.25 \text{ m} \) (i.e. \( T = 42 \) to \( T = 60 \)). This is shown to be the case theoretically in Table 2 if the results for \( L_D = 3.47 / L_R = 5.18 \text{ m} \) are neglected (for the reasons outlined in section 5.2).

5.5.1 Discussion of the Stability Plot

As has been stated in section 5.5, it is possible using the simplex method to generate theoretical \( V_o \), \( \beta \) and \( \omega \) values for given values of \( Q_R \), \( Q_D \), \( N \), \( V_s \) and the apparatus geometry. This was done at many \( Q_R/Q_D \) values for each apparatus geometry used (also using \( V_s = 0.50 \text{ m/s} \) and either \( N = 3.98 \) or \( N = N_{\text{BEST}} \) for that \( L_D/L_R \) combination).

By interpolating between the values obtained, contours of \( \beta \) and \( \omega \) can be plotted onto \( Q_D \) against \( Q_R \) plots. This was done for the four geometries used, for \( N = 3.98 \) and \( N = N_{\text{BEST}} \). These plots are given in Figures 22 to 25(b). Lines of constant \( V_o \) are not shown on these plots, for the sake of clarity of presentation: constant \( V_o \)
lines have already been given on $Q_D$ against $Q_R$ plots in Figure 14.

The following points can be made in general about these plots:

(i) Constant $\beta$ curves radiate outwards from the $Q_R$ axis.

(ii) As $Q_D$ tends to zero, constant $\omega$ lines asymptote upwards to $+\infty$ on the $Q_R$ axis. The $Q_R$ axis itself represents the line of $\omega = 0$.

(iii) The effect of changing the apparatus geometry is to alter radically the $\beta$ and $\omega$ contour positions. This effect will be discussed fully in section 5.5.2.

Using Figures 22, 23(a), 24(a), 25(a) and Figure 14, plots were produced to compare all the stable and unstable regions experimentally observed and theoretically predicted. These plots for all apparatus geometries are given in Figure 26, showing:

(a) The stability/instability boundary for instability initiated by slug breakthrough, i.e. lines $AB$ ($V_o = 1.1$ m/s in theory).

(b) The stability/instability boundary for instability initiated by oscillation ($\beta = 0$ in theory), i.e. lines $BC$. (The theoretical lines $BC$ use $N = N_{\text{BEST}}$, since it was found that these gave better agreement between experiment and theory than did those using $N = 3.98$ only.)

The following points should be made when comparing the expected and observed stable and unstable regions in Figure 26:

(i) There is reasonable agreement between theory and observation for the slug breakthrough boundary, $V_o = 1.1$ m/s (discussed in section 4.3), and also for the oscillatory instability boundary ($\beta = 0$ in theory).

(ii) There is very good agreement between theoretical and experimental changeover points from the slug breakthrough to the oscillatory initiated instability, i.e. point $B$. 

(iii) Figure 26 shows that the theory can be used with some confidence to predict stable and unstable regions of operation. In practice, of course, a safety margin would always be added to the prediction.

5.5.2 Effects of Changing Operating Variables \((Q_R, Q_D, L_R, L_D, N)\)

Now that the \(\beta\) and \(\omega\) contours have been plotted on Figures 22 to 25(b) together with the \(V_0\) contours on Figure 14, it is possible to investigate qualitatively the effect of changing one of the operating variables near the stability/instability boundary BC. This has important consequences for the operation of such a system, since unlike the boundary AB (slug breakthrough at 1.1 m/s) where the steady-state theory enables \(V_0\) to be simply calculated and maintained above 1.1 m/s, the effect of changes in the operating variables are not immediately obvious for the region near BC.

Let us consider the point "P" near the experimental boundary BC on Figure 25(b), and consider in turn the theoretical effect of a change in \(Q_R\), \(Q_D\), \(L_R\), \(L_D\) and \(N\) on the values of \(\beta\), \(\omega\) and \(V_0\). The same conclusions may be drawn for any point near this stability/instability boundary in any geometry.

(i) An increase in \(Q_R\) leads to:

(a) A large decrease in \(\beta\), it becomes more negative and hence more stable.

(b) A large increase in \(\omega\), i.e. the period of oscillation decreases (this is not obvious experimentally: see Table 2).

(c) An increase in the circulation velocity (see Figure 14(iv)).

(ii) An increase in \(Q_D\) leads to:

(a) Not much change in \(\beta\). Either a positive or a negative change in \(\beta\) is possible near BC.
(b) A small increase in \( \omega \) (there is no real experimental evidence for a change in \( \omega \) one way or the other: see Table 2).

(c) A small increase in the circulation velocity (see Figure 14(iv)).

(iii) \( L_R \) is decreased to 3.65 m, i.e. point \( P \) changes from Figure 25(b) to Figure 24(b), leading to:

(a) A decrease in \( \beta \).

(b) An increase in \( \omega \).

(c) An increase in the circulation velocity (see Figure 14(iii)).

(iv) \( L_D \) is decreased to 2.25 m, i.e. point \( P \) changes from Figure 25(b) to Figure 23(b), leading to:

(a) A very large decrease in \( \beta \).

(b) A very large increase in \( \omega \) (this is observed experimentally: see Table 2).

(c) A very large increase in the circulation velocity (see Figure 14(ii)).

(v) \( N \) is increased to 4.50, i.e. point \( P \) changes from Figure 25(b) to Figure 25(a), leading to:

(a) A small increase in \( \beta \), it becomes more positive and hence more unstable.

(b) A small decrease in \( \omega \), i.e. the period of oscillation increases.

(c) A decrease in the circulation velocity (see Figure 14(iv)).

One obvious conclusion to be drawn from these findings is that the values of \( \beta \) and \( \omega \) are directly linked to the circulation velocity \( V_0 \) in that as \( V_0 \) increases:

(i) With one possible exception (see increase in \( Q_D \)), \( \beta \) becomes more negative and the system more stable.
(ii) \( \omega \) increases, i.e. the period of oscillation decreases.

The effects of changing the operating variables on \( \omega \) are not verified experimentally. However, a reason for this has already been proposed in section 5.5, i.e. the assumption of \( v \ll V \) does not hold.

The experimental effects of changing the operating variables on \( \beta \) verify the theory in that changes in \( Q_R, Q_D, L_R \) or \( L_D \) theoretically causing a decrease in \( \beta \), experimentally take the operating point further into the stable region and away from the experimental stability/instability transition line \( BC \). Thus changes in these operating variables can be arranged to make the system more or less stable, if use is made of the stated findings.

In this work \( N \) was not varied in any particular experiment. However, the experimental effect of increasing \( N \) would be to slow the system down which is definitely destabilising.
CHAPTER SIX: CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

1: The Large Apparatus

A "riser-downcomer" circulating system has been built, this is about 10 m tall and has limbs of 0.24 m diameter. The apparatus is suitable for observations in upward and downward co-current gas/liquid flow in the bubbly and slug regimes. The Cambridge apparatus can circulate with air supplied to the downcomer limb only.

2: Steady-State Circulation

The operating procedure of the "riser-downcomer" system has been outlined, and a simple theory proposed for its steady-state circulation. This theory has been compared with the experimental data obtained. Although previous workers have also produced circulation models, they have supplied no data on circulation involving riser and downcomer air.

It has been found that:

(i) The experimental data are well correlated by a figure for the bubble slip velocity of 0.5 m/s. This high value has been explained by the presence of large bubbles in the circuit as observed by other workers such as Hills and Darton (1976).

(ii) The effects of the hydrostatic head on the voidage around the circuit may be accounted for by using air flowrates which are the average between injection values and values at atmospheric pressure.

(iii) The experimental value for the total hydraulic loss around the circuit is in good agreement with a summation of the individual losses given by theory and the literature.

(iv) The hydraulic loss around the circuit has been found to change according to the positioning of the injectors: explanations have been put forward for these changes.
(v) When downcomer air is injected the theory shows the existence of multiple steady-states. It has been shown that only one of these is stable.

3: Experimental Instability

The different modes of operation of the apparatus have been discussed and may be summarised as follows:

(i) Stable operation as described by the steady-state circulation theory.

(ii) Downcomer slug "breakthrough" with continued liquid circulation. ("Breakthrough" is when a slug initially present at the downcomer injector succeeds in rising further up the downcomer against the downward flowing liquid.)

(iii) No liquid circulation with both limbs partially filling and emptying with air.

(iv) "Total flow reversal" where liquid flows down the riser and up the downcomer.

Two separate stable/unstable boundaries have been identified, these are affected by the position of the injectors and by the air supply rates to the riser and downcomer.

Experiments allowing the system velocity to decrease by shutting off the air have been performed: these enable the effective mass and hydraulic losses in the system to be estimated. The value for the effective mass is in agreement with that obtained from a simple consideration of the apparatus geometry. As might be expected the hydraulic losses are found to be slightly less than the experimental figures obtained with bubbles in the circuit.

4: Slug Initiated Instability

Slugs have been observed in the downward flowing liquid in the downcomer. They have the following properties:
(i) They are much more pointed than conventional axisymmetric slugs in upflow.

(ii) They cling to the tube wall.

(iii) They are irregularly shaped.

(iv) As they climb up the tube wall air is ripped from them by the passing liquid.

(v) Downcomer slugs are liable to be broken up by what appears to be the mechanism proposed by Taylor (1950).

(vi) Downcomer slug "breakthrough" with continued circulation has been shown to occur at circulation velocities below 1.1 m/s. This constitutes one stability/instability boundary.

5: Oscillation Initiated Instability

Experimentally it has been found that undamped oscillations can occur in the liquid velocity: these constitute the second type of instability in the system. A theory has been proposed to explain these oscillations: namely that they appear as a small perturbation velocity, $v$, which is superimposed upon the steady-state circulation velocity $V_0$. The form of the perturbation is $v = D^* e^{(\beta + i\omega)t}$, where $D^*$ is a constant and $\beta$ and $\omega$ relate to the oscillation damping and period respectively. Methods of solution of the resultant equations have been discussed.

It has been shown that:

(i) The boundary of the instability is described by a contour $\beta = 0$.

(ii) Oscillations are only observed near the experimental stability boundary where the circulation velocity is above 1.1 m/s, since below this figure downcomer slug breakthrough occurs.

(iii) The theoretically predicted periods of oscillation are smaller than those observed experimentally by a factor of between 1.2 and 3. The theory is based on small perturbations, however experimentally $v = 0.2 V_0$ so this assumption is no longer valid.
(iv) The effects of the operating variables on the stability of the system close to the region of oscillatory initiated instability are as follows:

(a) Increasing the riser air rate stabilises the system.
(b) Increasing the downcomer air rate can either stabilise or destabilise the system.
(c) Increasing the height of the riser injector, \( L_R \), above the U bend destabilises the system.
(d) Increasing the height of the downcomer injector, \( L_D \), above the U bend drastically destabilises the system.
(e) Increasing the hydraulic losses, \( N \), in the circuit destabilises the system.

6: Stability Plot

Overall stability plots of riser air rate against downcomer air rate have been produced to compare the two experimental and theoretical stability boundaries: the following conclusions can be drawn:

(i) The plots show good agreement between the experimental and predicted stability/instability boundaries for both types of instability, and can be used as part of a design method for large scale industrial units.
(ii) The experimental changeover point between the slug initiated instability and the oscillation initiated instability is accurately predicted.
6.2 Future Work

The work contained in this dissertation must be considered as primarily a stability study of the system involved, as is implied by its title: "The Stability of Circulation in Bubble Columns". Nevertheless, certain areas of interest have been revealed during this work, which though not of primary interest here are undoubtedly worthy of further in depth investigation. These are:

(i) The concept of the operation of a "riser-downcomer" system under reduced pressure conditions. This is of interest since the changes in bubble shape and behaviour in an industrial unit of great depth may be observed in the laboratory by this means.

(ii) The downcomer slugs and their "breakthrough" behaviour is interesting not only in this system, but also anywhere where liquid flows downwards through pipes. Though centrally located slugs in upflow have been studied theoretically no such analysis exists for the downcomer slugs described in this work.

(iii) Two-phase pressure drop and void fraction data in large diameter pipes are very sparse, and data for flow around bends is even scarcer as pointed out by Whalley (1979). For this reason it would be of great industrial interest to use the existing apparatus to investigate these areas.

(iv) The form of the voidage distribution profile in downward two-phase flow is still the subject of controversy. It should be possible to examine voidage profiles in downflow over considerable distances in the large apparatus built for this work.
Appendix 1

Al  The Variation of Voidage with Depth

In this Appendix expressions for the variation of voidage with depth are compared and a guide is given as to which expression should be used according to the values of depth (h) and voidage (ε) involved.

It has been shown that in this work voidages in the apparatus may be calculated by using average gas flowrates: see 3.4.2. In equipment where the total depth is greater, an expression is given by Hines et al. (1975) to allow for the effect of the hydrostatic head upon the voidage:

\[ \varepsilon(h) = \varepsilon_0 \frac{a}{a+h} \quad \text{where} \quad a = \frac{p_A}{\rho g} \quad (\text{Al.1}) \]

This equation may be integrated between two depths b and c (c > b) to give the average voidage between them as

\[ \bar{\varepsilon}_{bc} = \frac{\varepsilon_0 a}{(c-b)} \ln\left(\frac{a+c}{a+b}\right) \quad (\text{Al.2}) \]

Equations Al.1 and Al.2 do not allow for the effect of bubbles present upon the values of \( \varepsilon(h) \) and \( \bar{\varepsilon}_{bc} \).

The following analysis allows for the presence of bubbles, providing the following assumptions are made:

(i) The mass of gas per unit depth of the column is constant, and the bubbles are evenly distributed.

(ii) That effects due to liquid movement and bubble-bubble interactions are small, also that the gas density is negligible compared to the liquid.

At a particular depth, see Figure 27(a), the voidage is proportional to the volume of gas present, thus applying Boyle's law at depths h and (h+dh) gives
\[ P_h \varepsilon = (P_h + \rho g d h (1-\varepsilon))(\varepsilon + d\varepsilon) \]  
\hspace{1cm} (Al.3)

This may be rearranged to give the variation of voidage with depth as

\[ \frac{d\varepsilon}{dh} = -\frac{\rho g c (1-\varepsilon)}{P_h} \]  
\hspace{1cm} (Al.4)

Now \( P_h \) is given by \( P_A + \rho gh (1-\varepsilon) \), and \( \varepsilon \) by \( \frac{1}{H} \int_0^H \varepsilon dh \), where \( H \) is the depth up to which the average voidage is to be calculated.

Thus the variation of \( P_h \) with \( h \) is

\[ \frac{dP_h}{dh} = \rho g (1-\varepsilon) \]  
\hspace{1cm} (Al.5)

Noting that \( \left( \frac{d\varepsilon}{dP_h} \right) = \left( \frac{d\varepsilon}{dh} \right) \left( \frac{dh}{dP_h} \right) \) an expression is obtained for \( \frac{d\varepsilon}{dP_h} \), which may be integrated from \( \varepsilon = \varepsilon_o \), \( P_h = P_A \) to \( \varepsilon = \varepsilon \), \( P_h = P_h \) to give

\[ \varepsilon_{oP} = \varepsilon_{P_h} = \text{CONSTANT} \]  
\hspace{1cm} (Al.6)

Substituting for \( P_h \) from equation Al.6 into equation Al.4 and integrating from \( h = 0 \), \( \varepsilon = \varepsilon_o \) to \( h = h \), \( \varepsilon = \varepsilon \) gives

\[ \varepsilon = \frac{\varepsilon_o a}{a + h + \varepsilon_o a \ln \left( \frac{\varepsilon (1-\varepsilon_o)}{\varepsilon_o (1-\varepsilon)} \right)} \]  
\hspace{1cm} (Al.7)

This equation may be solved for \( \varepsilon \) by means of repeated substitution for given values of \( \varepsilon_o \) and \( h \). To find the average voidage between depths \( b \) and \( c \), we make use of the definition

\[ -\varepsilon_{bc} = \int_b^c \frac{\varepsilon dh}{(c-b)} = \int_{\varepsilon_b}^{\varepsilon_c} \frac{\varepsilon d\varepsilon}{(c-b)} \left( \frac{dh}{d\varepsilon} \right) \]  
\hspace{1cm} (Al.8)

Now \( \frac{d\varepsilon}{dh} \) is given by equations Al.4 and Al.6, and substituting this into equation Al.8 gives upon integration

\[ -\varepsilon_{bc} = \frac{a c}{(c-b)} \ln \left( \frac{\varepsilon_b (1-\varepsilon_c)}{\varepsilon_c (1-\varepsilon_b)} \right) \]  
\hspace{1cm} (Al.9)
Thus knowing values of $\varepsilon_0$, $c$ and $b$, and $\varepsilon_b$ and $\varepsilon_c$ from equation Al.7, a value for $\bar{\varepsilon}_{bc}$ may be calculated. Figure 27(b) shows the error involved in calculating $\bar{\varepsilon}_{bc}$ (where $b = 0$) from equation Al.2 rather than from equation Al.9, at given values of $\varepsilon_0$ and $h$. The following points should be made:

(i) At low voidages there is little error in calculating $\bar{\varepsilon}_{bc}$ from equation Al.2.

(ii) At high voidages large errors result from using equation Al.2, as would be expected.

(iii) As the column depth increases the error involved reaches a maximum and then drops: since at large depths the bubbles present are greatly compressed and contribute little to the average voidage thereby bringing results from equation Al.9 closer to those from equation Al.2.

In this work since low voidages were used, equations of type Al.1 were always used to allow for hydrostatic effects: see Appendix 3 and Appendix 4.
Appendix 2

A2 Mass Transfer Effects

In this Appendix, expressions for bubble radius and oxygen mole fraction are derived for a single spherical bubble in a liquid. These expressions are used to show that in the system under consideration, the voidage and hence the density driving force is only slightly affected by mass transfer. The equations also show that for the long bubble residence times encountered in a much larger system, mass transfer effects are important when starting up the system.

In this analysis the following assumptions are made:

(i) The gas phase within the bubble is well mixed all the time and hence the gas side resistance to mass transfer is negligible: this has been shown to be the case by Motarjemi and Jameson (1978).

(ii) The liquid surrounding the bubble does not contain any dissolved oxygen or nitrogen. This is true for a system where pure oxygen bubbles are used, and where there is a high biological oxygen demand, as investigated theoretically by Kubota et al. (1978). It is also true in a non-biological system during the "startup" period.

(iii) There is a constant hydrostatic pressure, \( P_{TOT} \), acting on the bubble, thus the equilibrium concentrations of dissolved oxygen and nitrogen remain constant.

(iv) That the absorption of gases is isothermal.

(v) That the bubble radius is large enough so that internal pressure effects \( (\Delta P = \frac{2\sigma}{r}) \) remain small.

(vi) That Henry's law applies to both of the gases and that

\[ C_{BOX} = C_{OX}^* z \quad \text{and} \quad C_{BNI} = C_{NI}^* (1-z). \]

The spherical bubble under consideration is shown in Figure 28. We must firstly consider a mass balance over the differential time \( dt \).
for the oxygen present. The rate of absorption of oxygen per unit time at time \( t \) is given by \( K_{\text{LOX}} a_s V_{\text{BU}} (C_{\text{BOX}} - C_{\text{LOX}}^*) \) in kmol/sec.

Now using assumptions (ii) and (vi) gives the amount of oxygen absorbed in time \( dt \) as \( 4\pi r^2 K_{\text{LOX}} z C_{\text{OX}}^* dt \) kmoles. A balance on the number of moles of oxygen present before and after the time considered gives the change as \( -4\pi r^2 z \frac{P_{\text{TOT}} \text{dr}}{RT_K} = \frac{4}{3} \pi r^3 \frac{P_{\text{TOT}}}{RT_K} dz \) kmoles (since \( \text{dr} \) is negative).

Equating this to the oxygen absorbed gives

\[
z \frac{dr}{dt} + \frac{r}{3} \frac{dz}{dt} = -z K_{\text{LOX}} C_{\text{OX}}^* \frac{RT_K}{P_{\text{TOT}}} \tag{A2.1}
\]

A similar balance on the absorbed nitrogen gives

\[
(1-z) \frac{dr}{dt} - \frac{r}{3} \frac{dz}{dt} = -K_{\text{LNI}} C_{\text{NI}}^* (1-z) \frac{RT_K}{P_{\text{TOT}}} \tag{A2.2}
\]

Addition of equations A2.1 and A2.2 gives the rate of change of radius as

\[
\frac{dr}{dt} = \frac{RT_K}{P_{\text{TOT}}} \left( K_{\text{LOX}} C_{\text{OX}}^* z + K_{\text{LNI}} C_{\text{NI}}^* (1-z) \right) \tag{A2.3}
\]

Now substituting for \( \frac{dr}{dt} \) in equation A2.1, and noting that

\[
\frac{dr}{dz} = \frac{\frac{dr}{dt}}{\frac{dz}{dt}} = \frac{r(z+F_M)}{3z(1-z)} \tag{A2.4}
\]

where \( F_M = \frac{K_{\text{LNI}} C_{\text{NI}}^*}{(K_{\text{LOX}} C_{\text{OX}}^* - K_{\text{LNI}} C_{\text{NI}}^*)} \). Equation A2.4 may be integrated between \( z = z_o \), \( r = r_o \) and \( z = z \), \( r = r \) to give

\[
r = r_o \left( \frac{z}{z_o} \right)^{\frac{1}{3}} \left( \frac{F_M^+1}{1-z_o} \right)^{\frac{3}{1-z_o}} \tag{A2.5}
\]

Differentiation of equation A2.5 with respect to time, and substituting in for \( \frac{dr}{dt} \) from equation A2.3 gives

\[
G_M r_o = \left( \frac{3RT_K}{P_{\text{TOT}}} \right) \left( \frac{K_{\text{LOX}} C_{\text{OX}}^* - K_{\text{LNI}} C_{\text{NI}}^*}{z_o} \right) \frac{F_M^+1}{3} = \frac{dz}{dt} \left( \frac{3-F_M}{F_M+4} \right) \frac{r_o}{z^{\frac{3}{3}}(1-z)^{\frac{3}{3}}} \tag{A2.6}
\]
where $G_M$ has units $(\text{sec})^{-1}$ and is independent of $r$, $z$ and $t$.

It follows that \( \frac{dr}{dt} \) may be written as

\[
\frac{dr}{dt} = G_M \rho \left( \frac{1-z}{3} \right) (z + F_M)
\]

(Equation A2.7)

Equation A2.6 may be rewritten by expanding binomially \( (1-z)^{-1} \), since $0 < z < 1$. The resultant expression may then be integrated from $t = 0$, $z = z_o$ to $t = t_B$, $z = z$: giving $t$ as a function of $z$

\[
G_M t_B = \left[ \frac{F_M}{3} + \frac{(F_M+4)}{(F_M+3)} \frac{F_M+3}{3} + \frac{(F_M+4)(F_M+7)}{(F_M+6)2!} \frac{F_M+6}{3} \right. \\
+ \frac{(F_M+4)(F_M+7)(F_M+10)}{(F_M+9)3!} \frac{F_M+9}{3^2} + \frac{(F_M+4)(F_M+7)(F_M+10)(F_M+13)}{(F_M+12)4!} \frac{F_M+12}{3^3} \\
+ \left. \frac{(F_M+4)(F_M+7)(F_M+10)(F_M+13)(F_M+16)}{(F_M+15)5!} \frac{F_M+15}{3^4} \right] z \]

(A2.8)

Thus in order to obtain the life history of a bubble from $t = 0$, $z = z_o$, $r = r_o$ to $z = z$ the following procedure is adopted.

(i) Specify $r_o$, $z_o$, Henry’s law constants, system pressure, temperature and mass transfer coefficients.

(ii) Calculate $G_M$ from equation A2.6 and $F_M$ as indicated.

(iii) Calculate $t_B$ at $z = z$ from equation A2.8, using one more term than is shown to gain greater accuracy.

(iv) Calculate $r$ at $z = z$ from equation A2.5.
A2.1 Numerical Values

The values of $K_{LOX}$ and $K_{LNI}$, being for similar sized molecules, were both taken as $3 \times 10^{-4} \text{ m/s}$: this compares with values of between $1 \times 10^{-4}$ and $4 \times 10^{-4} \text{ m/s}$ quoted by Motarjemi and Jameson (1978) for different sized bubbles.

Henry's law constants for oxygen and nitrogen at $20^\circ \text{C}$ were taken from the International Critical Tables (1928). These can be used to express the saturated oxygen and nitrogen liquid concentrations as a function of the system pressure in N/m$^2$, thus:

$$C_{OX}^* = 1.36 \times 10^{-8} \frac{P_{TOT}}{m^3} \text{ (kmol)} \quad (A2.9a)$$

$$C_{NI}^* = 6.81 \times 10^{-9} \frac{P_{TOT}}{m^3} \text{ (kmol)} \quad (A2.9b)$$

However, since the terms involving $C_{OX}^*$ and $C_{NI}^*$ are always either divided by $P_{TOT}$, or in the form of a concentration ratio, it follows that $(\frac{dr}{dt})$ and $(\frac{dz}{dt})$ are independent of $P_{TOT}$. Hence for a bubble, its value of $z$ depends only upon $r_0$, $t$ and $z_0$; conversely its value of $r$ depends only upon $z_0$, $t$ and $r_0$.

We have shown that $K_{LOX}C_{OX}^* = 2K_{LNI}C_{NI}^*$, thus $F_M = 1$. Now from equation A2.3 $(\frac{dr}{dt}) \approx -\left(\frac{RT}{P_{TOT}}\right)K_{LOX}C_{OX}^*$, and hence from equation A2.1, $(\frac{dz}{dt})$ must always be negative. Equation A2.9a gives a value for $C_{OX}^*$ at atmospheric pressure of $0.044 \text{ kg/m}^3$, which is about the value of $0.039 \text{ kg/m}^3$ quoted by Kubota et al. (1978), though they misquote the concentration units by a factor $10^3$.

If these values are substituted into equation A2.6, they give a value of $G_{M,r_o}$ of $-1.06 \times 10^{-5} \text{ m/s}$ and thus $(\frac{dz}{dt})$ is very small.

Figure 28 shows a graphical representation of equations A2.8 and A2.5, for air bubbles at room temperature, i.e. $G_{M,r_o} = -1.06 \times 10^{-5}$ and $z_o = 0.21$. The values obtained using Figure 28 compare well with the
theory of Motarjemi and Jameson (1978) for single air bubbles rising up a column of liquid, though they used a higher mass transfer coefficient.

Using $V_o = 1.1 \text{ m/s}$ and $V_s = 0.5 \text{ m/s}$ together with the apparatus geometry, it can be shown that the maximum expected bubble residence time is about 12 seconds. Figure 28 shows that a 3 mm diameter air bubble after 12 seconds absorption will have a diameter of 2.84 mm, a volume change of 15%: an 8 mm bubble will have a diameter of 7.84 mm after the same time, a volume change of 6%.

In practice the values of $C_{\text{LNI}}$ and $C_{\text{LOX}}$ will not be zero once the system has been started up, and so the mass transfer effect will be even less. Thus for the apparatus used in this work, with 3-8 mm diameter bubbles, mass transfer can be neglected when calculating the circulating velocity once the apparatus has reached steady-state. Mass transfer should be taken into account either when a very large circulating system is being started up, or in biological systems where the oxygen demand is high.
Appendix 3

In a system where the value of $L$ is large, the riser and downcomer voidages will vary considerably due to the hydrostatic head. In this case it is not sufficient to use an average air flowrate between the relevant injector and the liquid surface, as was done in section 3.3, and a full account of pressure effects must be made.

A3 Steady-State Circulation Theory

This Appendix is a more general version of the simple steady-state circulation theory already presented: in it hydrostatic head effects are allowed for. The system is shown in Figure 8(b).

The assumptions inherent in this derivation are the same as have been presented in section 3.3, with the exception of assumption (i), which now becomes:

(i) Following Hines et al. (1975) voidage variation with depth is allowed for as follows:

$$\varepsilon(x) = \frac{\varepsilon_{II}(a+L-L_D)}{(a+L-L_D+x)}$$ for $0 < x < L_D$ \hspace{1cm} (A3.1)

where $\varphi = \frac{P_A}{\rho g}$, the equivalent head of water,

$$\varepsilon(x) = \frac{\varepsilon_{II}(a+L-L_D)}{(a+L+L_D-x)} \times \frac{(V_o - V)}{(V_o + V_s)}$$ for $L_D < x < L_D + L_R$ \hspace{1cm} (A3.2)

$$\varepsilon(x) = \frac{\varepsilon_{II}(a+L-L_D)}{(a+L+L_D-x)} \times \frac{\varepsilon'_I(a+L-L_R)}{(V_o + V_s)} + \frac{\varepsilon'_I(a+L-L_R-y)}{(a+L-L_R-y)}$$ for $L_D + L_R < x < L + L_D$ \hspace{1cm} (A3.3)

where $y = x - (L_D + L_R)$. The definitions of $x$ and $y$ are shown in Figure 8(b): $\varepsilon_{II}$ represents the voidage at the downcomer injector, and $\varepsilon'_I$ represents the additional voidage at the riser injector due to the injection of riser air there.
A mass balance at the downcomer injector gives

\[ Q_D = A\varepsilon_{1 I}(V_o - V_s) \]  
(A3.4)

A mass balance on the additional air injected at the riser injector
gives

\[ Q_R = A\varepsilon_1'(V_o + V_s) \]  
(A3.5)

Since we are operating at steady state, the equation of motion of the
System is of the form Resisting Forces = Density Driving Force. The
density driving force is given by

\[ \rho g A \int_{-L}^{L} dx + \int_{0}^{L_D} (1-\varepsilon(x)) dx - \int_{L_D}^{L+L_R} (1-\varepsilon(x)) dx + \int_{L_D}^{L+L_R} (1-\varepsilon(x)-\varepsilon'(y)) dx \]

Thus the equation of motion of the system becomes

\[ KV_o^2 = \rho g A \int_{L_D}^{L+L_D} \varepsilon(x) dx + \int_{0}^{L-L_R} \varepsilon'(y) dy - \int_{0}^{L_D} \varepsilon(x) dx \]  
(A3.7)

Substituting into equation A3.7 the expressions for voidage variation
with depth from equations A3.1, A3.2 and A3.3, together with the
expressions for \( \varepsilon_{1 I} \) and \( \varepsilon_1' \) from A3.4 and A3.5, gives after
integration

\[ KV_o^2 = \rho g \left[ \frac{Q_D(a+L-L_D)}{(V_o + V_s)} \ln \left[ \frac{a+L}{a} \right] + \frac{Q_R(a+L-L_R)}{(V_o + V_s)} \ln \left[ \frac{a+L-L_R}{a} \right] - \frac{Q_D(a+L-L_D)}{(V_o - V_s)} \ln \left[ \frac{a+L}{a+L-L_D} \right] \right] \]

(A3.8)

Now substituting for the number of velocity heads lost, \( N \), from
equation 3.7 gives

\[ N = \frac{2g}{AV_o^2} \left[ \frac{Q_D(a+L-L_D)}{(V_o + V_s)} \ln \left[ \frac{a+L}{a} \right] + \frac{Q_R(a+L-L_R)}{(V_o + V_s)} \ln \left[ \frac{a+L-L_R}{a} \right] - \frac{Q_D(a+L-L_D)}{(V_o - V_s)} \ln \left[ \frac{a+L}{a+L-L_D} \right] \right] \]

(A3.9)

This equation may be compared with equation 3.8, which neglects the
effect of the hydrostatic head on the voidage.

Equation A3.8 may be reformulated in terms of seven dimensionless groups as follows

\[
\frac{V_0}{V_s}^2 = \frac{\rho g a L}{K V_s^2} \left[ \frac{Q_D}{AV_s} \left( \frac{a}{L} + 1 - \frac{L_D}{L} \right) + \ln \left[ \frac{a + 1}{a/L} \right] + \frac{Q_R}{AV_s} \left( \frac{a}{L} + 1 - \frac{L_R}{L} \right) - \ln \left[ \frac{a + 1}{a/L} \right] \right]
\]

These groups have the following significance:

(i) \( \frac{V_0}{V_s} \) a velocity group.

(ii) \( \frac{\rho g a L}{K V_s^2} \) or \( \frac{2 g L}{N V_s} \) a system geometry group.

(iii) \( \frac{Q_D}{AV_s} \) a downcomer air group.

(iv) \( \frac{Q_R}{AV_s} \) a riser air group.

(v) \( \frac{L_D}{L} \) a downcomer injector group.

(vi) \( \frac{L_R}{L} \) a riser injector group.

(vii) \( \frac{\rho g a L}{K V_s^2} \) a hydrostatic effect group.

Equation 3.6 can be rearranged in terms of the first six groups, since there is no hydrostatic effect, in the form

\[
\frac{V_0}{V_s}^2 = \frac{\rho g a L}{K V_s^2} \left[ \frac{Q_D}{AV_s} \left( \frac{a}{L} + 1 \right) - \frac{Q_D}{AV_s} \left( \frac{a}{L} - \frac{L_D}{L} \right) + \frac{Q_R}{AV_s} \left( \frac{1}{L} - \frac{L_R}{L} \right) \right]
\]  

(A3.11)
Appendix 4

A4 The Theory of Oscillations

In this Appendix a theory which predicts the occurrence of oscillations in the circulating liquid velocity is given. This theory differs from that presented in section 5.3 in that the variation of voidage with the changing hydrostatic head around the circuit is allowed for. Thus this modification should be used where the variation of voidage with depth is large, i.e. large values of $L$. The system itself is shown in Figure 19(b).

The assumptions inherent in this derivation are the same as have been presented in section 5.3, with the exceptions of assumption (i) and assumption (iv) which now become:

(i) Following Hines et al. (1975) voidage variation with depth is allowed for by using equations A3.1, A3.2 and A3.3 from Appendix 3 with $V_0 = V$.

(iv) Air is supplied at rates $Q_R$ to the riser and $Q_D$ to the downcomer. (However, in this case $Q_R$ and $Q_D$ represent the injector values rather than the average values used in section 5.3.)

It is obvious that the voidage $\varepsilon_1$ at a position $x$ below the downcomer injector is the voidage that existed at the downcomer injector at a time $x/(V-V_S)$ earlier, adjusted to take account of the depth. Thus $\varepsilon_1$ at a position $x$ is a function both of position and time of the form

$$\varepsilon_1 = \varepsilon(x,t) = \varepsilon_{1I}(t - \frac{x}{V-V_S}) \times \left(\frac{a+L-L_D}{a+L-L_D+x}\right) \text{ for } 0 < x < L_D \quad (A4.1)$$

where $a = P_A/\rho g$. Similar expressions may be written down for $\varepsilon_2$ in the riser, noting the effect of additive bubble slip velocity, and $\varepsilon'$ above the riser injector (where $\varepsilon'$ is the additional voidage due to
the injection of riser air). These relationships have the form

\[ \varepsilon_2 = \varepsilon(x, t) = \varepsilon_{II}(t - \frac{L_D}{V-V_s} - \frac{x-L_D}{V+V_s}) \times \frac{V-V_s}{V+V_s} \times \frac{a+L-L_D}{a+L+L_D} \]

for \( L_D < x < L + L_D \) \hspace{1cm} (A4.2)

and

\[ \varepsilon' = \varepsilon(y, t) = \varepsilon_{I}(t - \frac{y}{V+V_s}) \times \frac{a+L-L_R}{a+L-L_R} \]

for \( 0 < y < (L-L_R) \) \hspace{1cm} (A4.3)

The density driving force in the system is again given by the right hand side of equation A3.7 as

\[ \rho g A \left[ \int_{L_D}^{L+L_D} \varepsilon_2 \, dx - \int_{0}^{L_D} \varepsilon_I \, dx + \int_{0}^{L-L_R} \varepsilon'_I \, dy \right] \]

The resisting forces are given by \( KV^2 \), and the resultant forces by \( M \frac{dV}{dt} \). As in section 5.3 we assume that the velocity has the form \( V = V_o + v = V_o + D e^{at} \), where \( v << V_o \). Thus balances at the downcomer and riser injectors give \( \varepsilon_{II}(t) \) and \( \varepsilon'_I(t) \) from equations 5.13 and 5.14.

Using the results of equations A4.1, A4.2, A4.3, 5.13 and 5.14 the equation of motion of the system now becomes

\[ M \frac{dV}{dt} = -KV^2 + \rho g A \left[ \int_{L_D}^{L+L_D} A_{D} e^{x} \, dx \right] \times \frac{a+L-L_D}{a+L+L_D} \times \frac{V-V_s}{V+V_s} \]

\[ - \int_{0}^{L_D} A_D e^{x} \, dx \times \frac{a+L-L_D}{a+L+L_D} \times \frac{x-L_D}{V-V_s} \]

\[ + \int_{0}^{L-L_R} A_R e^{y} \, dy \times \frac{a+L-L_R}{a+L-L_R} \times \frac{y}{V+V_s} \]

\[ \varepsilon_I(t) = \varepsilon_{II}(t) = \varepsilon'_I(t) \]

NOW NOTING THAT

\[ \int \frac{e^{px}}{x} = \ln(x) + \frac{p^2 x^2}{2.2!} + \frac{p^3 x^3}{3.3!} + \ldots \]

(A4.5)
where \( p \) is a constant, equation A4.4 can be integrated.

If the conditions for steady-state operation are substituted in, namely \( D^* = 0 \) and \( V = V_o \), the resultant equation is the same as equation A3.8 which was derived for the steady state (allowing for voidage variation with depth).

If the steady-state solution is subtracted out of the integrated version of equation A4.4, the eventual result after putting \( V = V_o \) and dividing both sides by \( v = D e^{\alpha t} \), is given by

\[
M \alpha + 2KV_o = \frac{\rho g Q_D (a+L-L_D)}{(V_o - V_s)^2} \left[ \alpha \left( \frac{a+L-L_D}{V_o - V_s} \right) \ln \left( \frac{a+L}{a+L-L_D} \right) - \frac{\alpha ((a+L)-(a+L-L_D))}{(V_o - V_s)} \right]
\]

\[
+ \frac{\alpha^2((a+L)^2-(a+L-L_D)^2)}{(V_o - V_s)^2 2.2!} \ldots - e^{-\alpha \frac{L_D}{V_o - V_s} + \frac{a+L}{V_o + V_s} V_o - V_s} \frac{\ln(a+L)}{V_o + V_s}
\]

\[
+ \frac{\alpha((a+L)-a) + \alpha^2((a+L)^2-a^2)}{(V_o + V_s)^2 2.2!} \ldots - e^{-\alpha \frac{a+L-L_D}{V_o - V_s} + \frac{a+L}{V_o + V_s} Q_R} \frac{V_o - V_s}{V_o + V_s}^2
\]

\[
\left\{ \ln \left( \frac{a+L-L_D}{a} \right) + \frac{\alpha((a+L-L_D)-a) + \alpha^2((a+L-L_D)^2-a^2)}{(V_o + V_s)^2 2.2!} \ldots \right\}
\]

(A4.6)

Now we can generalise the situation by considering \( \alpha \) as a complex number of the form \((\beta + i\omega)\). We note the three complex series involved in the solution, i.e.

\[
S1 = -(\frac{\beta + i\omega}{V_o - V_s})((a+L)-(a+L-L_D)) + (\frac{\beta + i\omega}{V_o - V_s})^2 \frac{1}{2.2!}((a+L)^2-(a+L-L_D)^2) \ldots
\]

(A4.7)

\[
S2 = (\frac{\beta + i\omega}{V_o + V_s})((a+L)-a) + (\frac{\beta + i\omega}{V_o + V_s})^2 \frac{1}{2.2!}((a+L)^2-a^2) \ldots
\]

(A4.8)

\[
S3 = (\frac{\beta + i\omega}{V_o + V_s})((a+L-L_D)-a) + (\frac{\beta + i\omega}{V_o + V_s})^2 \frac{1}{2.2!}((a+L-L_D)^2-a^2) \ldots
\]

(A4.9)
Now by representing the real parts of these series by \( \text{Re}(S_1), \text{Re}(S_2), \) \( \text{Re}(S_3), \) and their imaginary parts by \( \text{I}(S_1), \text{I}(S_2), \text{I}(S_3), \) the resultant real and imaginary equations obtained by substituting \( a = (\beta + i\omega) \) into equation A4.6 are

**REAL**

\[
M \beta + 2KV_o = \frac{\rho gQ_D(a+L-L_D)}{(V_o - V_s)^2} \left[ e^{\frac{a+L-L_D}{(V_o - V_s)^2}} + \frac{\beta(a+L-L_D)}{V_o - V_s} \right] (\text{ln}(\frac{a+L}{a+L-L_D}) + \text{Re}(S_1))
\]

\[
- \sin(\frac{a+L-L_D}{V_o - V_s})(\text{I}(S_1)) - e^{-\frac{\beta(a+L-L_D)}{V_o - V_s}} \cdot \frac{L_D}{V_o - V_s} \left\{ \cos(\frac{L_D}{V_o - V_s} + \frac{a+L}{V_o - V_s}) \right\}
\]

\[
(\text{ln}(\frac{a+L}{a}) + \text{Re}(S_2)) + \sin(\frac{L_D}{V_o - V_s} + \frac{a+L}{V_o - V_s})(\text{I}(S_2))
\]

\[
- \frac{Q_R(a+L-L_R)}{(a+L-L_D)} \cdot \left\{ e^{-\frac{\beta(a+L-L_D)}{V_o - V_s}} \cdot \left\{ \cos(\frac{L_D}{V_o - V_s} + \frac{a+L}{V_o - V_s}) \right\} \right\}
\]

\[
+ \sin(\frac{a+L-L_R}{V_o - V_s})(\text{I}(S_3)) \right] \]

\[(A4.10)\]

and **IMAGINARY**

\[
M \omega = \frac{\rho gQ_D(a+L-L_D)}{(V_o - V_s)^2} \left[ e^{\frac{a+L-L_D}{(V_o - V_s)^2}} \right] (\text{ln}(\frac{a+L}{a+L-L_D}) + \text{Re}(S_1))
\]

\[
+ \cos(\frac{a+L-L_D}{V_o - V_s})(\text{I}(S_1)) - e^{-\frac{\beta(a+L-L_D)}{V_o - V_s}} \cdot \frac{L_D}{V_o - V_s} \left\{ -\sin(\frac{L_D}{V_o - V_s} + \frac{a+L}{V_o - V_s}) \right\}
\]

\[
(\text{ln}(\frac{a+L}{a}) + \text{Re}(S_2)) + \cos(\frac{L_D}{V_o - V_s} + \frac{a+L}{V_o - V_s})(\text{I}(S_2))
\]

\[
- \frac{Q_R(a+L-L_R)}{(a+L-L_D)} \cdot \left\{ e^{-\frac{\beta(a+L-L_D)}{V_o - V_s}} \cdot \left\{ -\sin(\frac{L_D}{V_o - V_s} + \frac{a+L}{V_o - V_s}) \right\} \right\}
\]

\[
+ \cos(\frac{a+L-L_R}{V_o - V_s})(\text{I}(S_3)) \right] \]

\[(A4.11)\]
The theoretical boundary between stable and unstable operation is the solution of equations A4.10 and A4.11 involving $\beta = 0$.

In the experimental system under consideration in this dissertation, the value of $L$ is not very large. Hence the system can be modelled accurately, and more simply, by assuming voidage to be invariant with depth and using average values of $Q_R$ and $Q_D$: see section 5.3.
REFERENCES


Lapidus L. and Elgin J.C. (1957), A.I.Ch.E.Jl., 3, 63.


Shipley D.G. (1975), Paper presented at the Annual Research Meeting of the Institution of Chemical Engineers (Bradford)


Wallis G.B. (1969a), In "One-Dimensional Two-Phase Flow", 1st Ed., Ch.9, McGraw-Hill.


Whalley P.B. (1979), Personal Communication.

### Table 1: Experimentally Observed Oscillations
(see also Figure 17 and Figure 18)

<table>
<thead>
<tr>
<th>Apparatus Geometry</th>
<th>Average Air Rate, $V_A^{*} * 10^2$ (m$^3$/s)</th>
<th>Average Downcomer Air Rate, $V_D^{*} * 10^2$ (m$^3$/s)</th>
<th>Central Pitot Velocity, $V_{c}$ (m/s)</th>
<th>Side Pitot Velocity, $V_{s}$ (m/s)</th>
<th>Observed Period of Oscillation, $T$ (s)</th>
<th>Approximate Amplitude of Oscillations, $A_{o}$ (m/s)</th>
<th>Comments on Experimentally Observed Oscillations</th>
</tr>
</thead>
<tbody>
<tr>
<td>L = 9.85</td>
<td>44</td>
<td>46</td>
<td>1.08</td>
<td>1.10</td>
<td>~60</td>
<td>~0.23</td>
<td>Four oscillatory periods. System remained stable.</td>
</tr>
<tr>
<td>$L_D = 3.65$</td>
<td>0</td>
<td>46</td>
<td>0.99</td>
<td>1.04</td>
<td>~60</td>
<td>~0.20</td>
<td>Series of oscillatory periods with two large peaks. System went unstable (downcomer slug breakthrough) just after the last large peak.</td>
</tr>
<tr>
<td>$L_D = 2.25$</td>
<td>70</td>
<td>529</td>
<td>1.12</td>
<td>1.17</td>
<td>~55</td>
<td>~0.14</td>
<td>Three oscillatory periods. System remained stable.</td>
</tr>
<tr>
<td>$D = 40$</td>
<td>0</td>
<td>46</td>
<td>1.03</td>
<td>1.11</td>
<td>~50*</td>
<td>~0.22</td>
<td>Long series of sustained oscillations. System remained stable.</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>426</td>
<td>1.09</td>
<td>1.06</td>
<td>~50/55</td>
<td>~0.16</td>
<td>Three oscillatory periods. System remained stable.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L = 9.85</td>
<td>0</td>
<td>46</td>
<td>1.05</td>
<td>1.05</td>
<td>46</td>
<td>~0.17</td>
<td>Five clear oscillatory sections on trace in blocks of 4, 6, 5(*) 2, 4 oscillatory periods. System remained stable.</td>
</tr>
<tr>
<td>$L_D = 5.18$</td>
<td>59</td>
<td>402</td>
<td>1.03</td>
<td>0.99</td>
<td>~52</td>
<td>~0.17</td>
<td>Five oscillatory periods. System went unstable (downcomer slug breakthrough) shortly afterwards.</td>
</tr>
<tr>
<td>$L_D = 2.25$</td>
<td>0</td>
<td>46</td>
<td>1.13</td>
<td>1.09</td>
<td>~42</td>
<td>~0.12</td>
<td>Three oscillatory periods. System remained stable.</td>
</tr>
<tr>
<td>$D = 40$</td>
<td>59</td>
<td>402</td>
<td>1.12</td>
<td>1.07</td>
<td>~50</td>
<td>~0.16</td>
<td>Three oscillatory periods. System remained stable.</td>
</tr>
<tr>
<td></td>
<td>246</td>
<td>412</td>
<td>1.10</td>
<td>1.12</td>
<td>~82</td>
<td>~0.22</td>
<td>Four oscillatory periods. System went unstable (downcomer slug breakthrough) about 6 mins later.</td>
</tr>
<tr>
<td></td>
<td>348</td>
<td>543</td>
<td>1.16</td>
<td>1.17</td>
<td>~75</td>
<td>~0.26</td>
<td>Two oscillatory periods. System went unstable (downcomer slug breakthrough) about 4 mins later.</td>
</tr>
<tr>
<td></td>
<td>313</td>
<td>543</td>
<td>1.16</td>
<td>1.14</td>
<td>~75/70</td>
<td>~0.17</td>
<td>Two clear single cycles observed just before system went unstable (downcomer slug breakthrough)</td>
</tr>
<tr>
<td></td>
<td>241</td>
<td>412</td>
<td>1.06</td>
<td>1.07</td>
<td>~70</td>
<td>~0.24</td>
<td>One clear single cycle observed just before system went unstable (downcomer slug breakthrough)</td>
</tr>
<tr>
<td></td>
<td>246</td>
<td>479</td>
<td>1.11</td>
<td>1.11</td>
<td>~70*</td>
<td>~0.24</td>
<td>Four oscillatory periods before system went unstable (downcomer slug breakthrough)</td>
</tr>
<tr>
<td>L = 9.85</td>
<td>474</td>
<td>543</td>
<td>1.20</td>
<td>1.15</td>
<td>~50/70</td>
<td>~0.17</td>
<td>A burst of six cycles (~50 secs), then two longer cycles (~70 secs). System went unstable (downcomer slug breakthrough) about 6 mins later.</td>
</tr>
<tr>
<td>$L_D = 5.18$</td>
<td>519</td>
<td>543</td>
<td>1.18</td>
<td>1.18</td>
<td>~73/65</td>
<td>~0.17</td>
<td>A burst of four cycles (~73 secs), followed by three cycles (~65 secs). System remained stable.</td>
</tr>
<tr>
<td>$L_D = 3.47$</td>
<td>474</td>
<td>543</td>
<td>1.18</td>
<td>1.18</td>
<td>~37/85</td>
<td>~0.20</td>
<td>A burst of four cycles (~57 secs), followed by three cycles (~65 secs). System went unstable (downcomer slug breakthrough) about 3 mins later.</td>
</tr>
</tbody>
</table>

*See Figure 18, which shows the plot produced by these oscillations.

**These velocities are too low, since the transducer amplifiers were not properly zeroed for this run.

*These represent measured mean velocities about which the oscillations are taking place.

"Stability" in this context means that stable circulation continued as long as observations were made at this setting.
### TABLE 2: COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL OSCILLATIONS

<table>
<thead>
<tr>
<th>Apparatus Geometry</th>
<th>Average Riser Air Rate $Q_R \times 10^5$ (m$^3$/s$\times 10^5$)</th>
<th>Average Downcomer Air Rate $Q_D \times 10^5$ (m$^3$/s$\times 10^5$)</th>
<th>Central Pitot Velocity $V_C$ (m/s)</th>
<th>Side Pitot Velocity $V_{SL}$ (m/s)</th>
<th>Observed Period of Oscillation $T$ (secs)</th>
<th>Theoretical Values Calculated Using $N = 3.98$ #</th>
<th>Theoretical Values Calculated Using $N = N_{BEST}$ (see Fig.12)#</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 9.85$</td>
<td>44</td>
<td>466</td>
<td>1.08</td>
<td>1.10</td>
<td>~60</td>
<td>1.20 $-0.005$</td>
<td>24</td>
</tr>
<tr>
<td>$L_R = 3.65$</td>
<td>70</td>
<td>529</td>
<td>1.12 $^\Delta$</td>
<td>1.17 $^\Delta$</td>
<td>~55</td>
<td>1.32 $-0.06$</td>
<td>20</td>
</tr>
<tr>
<td>$L_D = 2.25$</td>
<td>0</td>
<td>466</td>
<td>1.03</td>
<td>1.11</td>
<td>~50*</td>
<td>1.08 $+0.04$</td>
<td>29</td>
</tr>
<tr>
<td>$L = 9.85$</td>
<td>0</td>
<td>426</td>
<td>1.09</td>
<td>1.06</td>
<td>~50/55</td>
<td>1.00 $+0.05$</td>
<td>33</td>
</tr>
<tr>
<td>$L_R = 5.18$</td>
<td>0</td>
<td>466</td>
<td>1.05</td>
<td>1.05</td>
<td>~45*</td>
<td>1.08 $+0.04$</td>
<td>29</td>
</tr>
<tr>
<td>$L_D = 2.25$</td>
<td>59</td>
<td>402</td>
<td>1.03</td>
<td>0.99</td>
<td>~52</td>
<td>1.08 $0.00$</td>
<td>26</td>
</tr>
<tr>
<td>$L = 9.85$</td>
<td>246</td>
<td>412</td>
<td>1.10</td>
<td>1.12</td>
<td>~82</td>
<td>1.09 $+0.03$</td>
<td>38</td>
</tr>
<tr>
<td>$L_R = 3.65$</td>
<td>348</td>
<td>543</td>
<td>1.16</td>
<td>1.17</td>
<td>~75</td>
<td>1.41 $-0.04$</td>
<td>23</td>
</tr>
<tr>
<td>$L_D = 3.47$</td>
<td>313</td>
<td>543</td>
<td>1.16</td>
<td>1.14</td>
<td>~75/70</td>
<td>1.35 $-0.02$</td>
<td>25</td>
</tr>
<tr>
<td>$L = 9.85$</td>
<td>241</td>
<td>412</td>
<td>1.06</td>
<td>1.07</td>
<td>~70</td>
<td>1.05 $+0.03$</td>
<td>38</td>
</tr>
<tr>
<td>$L_R = 5.18$</td>
<td>246</td>
<td>479</td>
<td>1.11</td>
<td>1.11</td>
<td>~70*</td>
<td>1.14 $+0.02$</td>
<td>31</td>
</tr>
<tr>
<td>$L_D = 3.47$</td>
<td>474</td>
<td>543</td>
<td>1.20</td>
<td>1.15</td>
<td>~50/70</td>
<td>1.43 $-0.05$</td>
<td>23</td>
</tr>
<tr>
<td>$L = 9.85$</td>
<td>474</td>
<td>543</td>
<td>1.18</td>
<td>1.18</td>
<td>~73/65</td>
<td>1.47 $-0.07$</td>
<td>22</td>
</tr>
<tr>
<td>$L_R = 5.18$</td>
<td>474</td>
<td>543</td>
<td>1.18</td>
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<td>1.18</td>
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<td>23</td>
</tr>
</tbody>
</table>

*See Figure 18, which shows the plot produced by these oscillations.*

$^\Delta$These velocities are too low, since the transducer amplifiers were not zeroed properly for this run.

$^w$These represent measured mean velocities about which the oscillations are taking place.

$^*$See Figures 22 to 25 for $\beta$ and $\omega$ contours on a plot of $Q_R$ against $Q_D$. 

$^\#$These velocities are too low, since the transducer amplifiers were not zeroed properly for this run.
**FIGURE 1:** U Tubes For Circulating Gas-Liquid Mixtures (To Scale):
(a) Small, 50.8 mm id tube
(b) Large, 241 mm id tube

**KEY:** © Collar Supports (See Also Figure 2(b))

- Disengagement Tank
- Enlarged View
- 50.8 mm id tube
- 0.25m Injectors
- Diffuser Tubes
- Baffle
- 241 mm Tube Internal Diameter
- Riser Injection Aerofoil
- Riser Injection Assembly
- Pitot Tube Module
- Downcomer Injection Assembly
- Downcomer Pitot Tube
- Gas Disengagement Tank
- Inlet
- Baffle
- Overflow
- Jacking Bolts
- QVF 'U'Bend and Drain Valve
- Floor of First Floor Laboratory
- 995m
- Scale: 0 0.5 1.0 1.5 2.0 m

(a) SMALL
(b) LARGE
**FIGURE 2:** TUBE SECTION CONSTRUCTION AND SUPPORT

- Flange Thickness 12.7 mm
- 10 mm Bolt Holes (12 off per flange) On 318 mm PCD

 Tube, 254 mm od, 241 mm id, Overall Section Length 1.23 m

- "O" Ring Groove And Seal at 279 mm Diameter (One Per Tube Section)

Step in Flange Face, from 241 mm diameter to 254 mm diameter, and 6.35 mm deep (On Rear Face of both Flanges)

**FIGURE 2(a): CONSTRUCTION OF A TUBE SECTION**

- Flange 356 mm od, 241 mm id

- Tube Section

- "Collar" In Two Halves 0.34 x 0.17 x 0.04 m

- Dexion Beams Attached to "Collar" (Beams also Attached to Support Structure at Ends)

- 6mm Thick Rubber Sheet

- Bolts to Hold Two "Collar" Halves Together

- Tube Flanges

**FIGURE 2(b): COLLAR SUPPORT OF A TUBE**
FIGURE 3: THE INJECTORS

FIGURE 3(a): RISER INJECTOR

FIGURE 3(b): DOWNCOMER INJECTOR
FIGURE 4: THE 'U' BEND
FIGURE 5: MORE INJECTOR DETAILS

FIGURE 5(a): VIEWS OF THE AEROFOIL

FIGURE 5(b): DOWNCOMER AIR 'STREAKS'

Enlarged View on X-X

Enlarged View on Y-Y

Direction of Flow

Breakup Into Bubbles

Flow Into Paper

'Streaks' of Air

Tube Wall

Manifold Box

Injector Hole

'Streak' of Air

Tube Wall

Manifold Box

Enlarged View on X-X

SCALE 1:4

SCALE 1:4

Air Supply Pipe

Manifold Box

Air Supply Manifold

2mm Holes

Chord Length 'L' = 0.20m

Maximum Thickness 0.05m

Car Body Filler

Perspex Section

FIGURE 5(a): VIEWS OF THE AEROFOIL

(1) Section on A-A

(2) Front Elevation

(3) Side Elevation

0.85L

0.05m

FIGURE 5: MORE INJECTOR DETAILS
FIGURE 6(a): PITOT MEASURING SYSTEM

KEY:

**Direction of Flow**

- **Static Pitot Line**
- **Side Pitot**
- **Orifice Plate For Calibrating Pitot Tubes** (Removed After Calibration)
- **Central Pitot**

**KEY:**

- $r^* = 0.758r$
- Hydraulic Lines
- Signal from Transducer
- Amplified and Smoothed Signal
- $12 \, \text{k} \Omega$ Resistor for Smoothing
- $100 \, \mu\text{F}$ Capacitor for Smoothing
- Pressure Transducer
- Amplifier
- $r$: Tube Radius
- $r^*$: Radial Position of Side Pitot

--- Tekman 220, 2 Pen Recorder

FIGURE 6: VELOCITY MEASUREMENT AND AIR SUPPLY

FIGURE 6(b): AIR SUPPLY SYSTEM

**KEY:**

- Compressed Air Main: $414 \, \text{kN/m}^2$
- Rotameter Line: $172 \, \text{kN/m}^2$
- Injection Lines (Injection Pressure)
- Bursting Disc: $206 \, \text{kN/m}^2$
- Norgren Pressure Reducing Valve
- Gate Valves
- Diaphragm Valves

1. Downcomer Injector Supply
2. Spare
3. Riser Injector Supply
4. Spare

1. $a_3$: 3 Way Ball Valves (3-1 or 3-2)
3. $a_1$:

R: Rotameters
FIGURE 7(a) Pitot Tubes in Lower Position \( L_D = 2.25 \) m

- Pitot Experiencing \( U_L \) (Av. Liquid Velocity)
- Pitot Experiencing \( 1.1 U_L \)
- Pitot Experiencing \( 1.22 U_L \)

A, B, C Theoretical Curves From \( U = \sqrt{2g\Delta h} \)

FIGURE 7(b) Pitot Tubes in Upper Position \( L_D = 3.47 \) m

- Pitot Experiencing \( U_L \) (Av. Liquid Velocity)
- Pitot Experiencing \( 1.1 U_L \)
- Pitot Experiencing \( 1.22 U_L \)

A, B, C Theoretical Curves From \( U = \sqrt{2g\Delta h} \)
FIGURE 8(a): SIMPLE CIRCULATION THEORY

(1) $\varepsilon = \varepsilon_1$ : $0 < x < L_D$
(2) $\varepsilon = \varepsilon_2$ : $L_D < x < L_D + L_R$
(3) $\varepsilon = \varepsilon_3 + \varepsilon'_3$ : $L_D + L_R < x < L_D + L$

Experimental $Q_D$ and $Q_R$ values are average of injector and atmospheric values. See text.

FIGURE 8: STEADY-STATE CIRCULATION

FIGURE 8(b): CIRCULATION THEORY (WITH HYDROSTATIC EFFECT)

(1) $\varepsilon(x) = \varepsilon_1 \frac{(a+L-L_D)}{(a+L-L_D+x)}$ : $0 < x < L_D$
(2) $\varepsilon(x) = \varepsilon_2 \frac{(V_0-V_S)}{(a+L-L_D-x)} \left( \frac{V_0}{V_S} \right)$ : $L_D < x < L_D + L_R$
(3) $\varepsilon(x) = \varepsilon_3 \frac{(V_0-V_S)}{(a+L-L_D-x)} \left( \frac{V_0}{V_S} \right) + \frac{1}{2} \left( \frac{\varepsilon_3}{(a+L-L_D-x)} \right) \left( \frac{V_0}{V_S} \right) \left( \frac{a+L-L_D}{(a+L-L_D-x)} \right)$ for $L_D + L_R < x < L + L_D$

Experimental $Q_D$ and $Q_R$ values used are injection values.
**FIGURE 9: MULTIPLE STEADY STATES**

**F**: Forces Acting (N)

**V_o**: Liquid Velocity (m/s)

- **1** Resisting Forces
- **2** \( Q_R = 0.000 \text{ m}^3/\text{s} \)
- **3** \( Q_R = 0.005 \text{ m}^3/\text{s} \)
- **4** \( Q_R = 0.010 \text{ m}^3/\text{s} \)

**A**: Unstable

**B**: Stable

(i) \( K=100 \), \( L=10 \), \( L_D=3 \), \( L_R=5 \), \( V_o=0.3 \), \( Q_D=0.005 \)

(ii) \( V_o=0.3, Q_D=0.005 \)

(iii) \( K=100 \), \( L=10 \), \( L_D=3 \), \( L_R=5 \), \( V_o=0.3 \), \( Q_D=0.005 \)

(iv) \( V_o=0.3, Q_D=0.005 \)

(v) \( V_o=0.3, Q_R=0.005 \)

Stable Operating Velocities

6. \( L_D = 5 \text{ ie } L(V_o-V_s)/(V_o+V_s) \)

7. \( L_D = 6 \text{ ie } L(V_o-V_s)/(V_o+V_s) \)

8. \( L_D = 7 \text{ ie } >L(V_o-V_s)/(V_o+V_s) \)

9. \( L_D = 8 \text{ ie } << L(V_o-V_s)/(V_o+V_s) \)
FIGURE 10: HYDRAULIC LOSSES AND EFFECTIVE SLIP VELOCITY IN THE CIRCUIT

N: Number of Velocity Heads Lost, \( V_s \): Slip Velocity (m/s), 
\( Q_D, Q_R \): Average Downcomer and Riser Air Rates (m³/s)

(i) \( V_s = 0.30 \) m/s Simple Theory

Symbol \( Q_D \)
+ : 0.0000 (Riser Air Only)
● : 0.0007
○ : 0.0034
□ : 0.0040
▲ : 0.0047
● : 0.0053

\( N_{BEST} = 4.55 \)
\( L_D = 2.25 \) m, \( L_R = 3.65 \) m

(ii) \( V_s = 0.50 \) m/s Simple Theory

Symbol \( Q_D \)
+ : 0.0000 (Riser Air Only)
● : 0.0007
○ : 0.0034
□ : 0.0040
▲ : 0.0047
● : 0.0053

\( N_{BEST} = 3.98 \)
\( L_D = 2.25 \) m, \( L_R = 3.65 \) m
FIGURE 10 (cont) - HYDRAULIC LOSSES AND EFFECTIVE SLIP VELOCITY IN THE CIRCUIT

N: Number of Velocity Heads Lost, \( v_s \): Slip Velocity (m/s),

\( Q_D, Q_R \): Downcomer and Riser Air Rates (Average Values in (iii) and Injection Values in (iv)) (m³/s)

(iii) \( v_s = 0.70 \text{ m/s} \) Simple Theory.

Symbol \( Q_D \)

- : 0.0000 (Riser Air Only)
- : 0.0027
- : 0.0034
- : 0.0040
- : 0.0047
- : 0.0053

\( N_{BEST} = 3.54 \)

\( L_D = 2.25 \text{ m}, L_R = 3.65 \text{ m} \)

(iv) \( v_s = 0.30 \text{ m/s} \) Voidage Variation With Depth

Symbol \( Q_D \) (Injection Value)

- : 0.0000 (Riser Air Only)
- : 0.0019
- : 0.0025
- : 0.0029
- : 0.0034
- : 0.0039

\( N_{BEST} = 4.46 \)

\( L_D = 2.25 \text{ m}, L_R = 3.65 \text{ m} \)
FIGURE 11: HYDRAULIC LOSSES IN THE CIRCUIT
FIGURE 12: THE CIRCULATION MODEL

Vo: Liquid Velocity (m/s), *: Theoretical Curve Based on N, 
QpQr: Average Downcomer and Riser Air Rates (m³/s)

(i) Riser Air Only, V₅ = 0.50 m/s

(ii) Qᵣ = 0.0020 m³/s, Vₛ = 0.50 m/s

(iii) Qᵣ = 0.0027 m³/s, Vₛ = 0.50 m/s
FIGURE 12 (cont): THE CIRCULATION MODEL

Vo: Liquid Velocity (m/s), *: Theoretical Curve Based on N,
Q_D, Q_R: Average Downcomer and Riser Air Rates (m³/s)

(iv) \( Q_D = 0.0034 \text{ m}^3/\text{s}, V_S = 0.50 \text{ m/s} \)

(v) \( Q_D = 0.0040 \text{ m}^3/\text{s}, V_S = 0.50 \text{ m/s} \)
**FIGURE 12 (cont): THE CIRCULATION MODEL**

\( V_o \): Liquid Velocity (m/s), \( \delta \): Theoretical Curve Based on \( N \),
\( Q_D, Q_R \): Average Downcomer and Riser Air Rates (m\(^3\)/s)

\((vi)\) \( Q_D = 0.0047 \text{ m}^3/\text{s}, V_5 = 0.50 \text{ m/s} \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
L_R & L_D & N & \% & EXP. \\
\hline
3.65 & 2.25 & 3.98 & A & + \\
5.18 & 2.25 & 3.98 & B & C \\
5.18 & 3.47 & 3.98 & D & E \\
3.65 & 3.47 & 3.98 & F & G \\
\hline
\end{array}
\]

\((vii)\) \( Q_D = 0.0053 \text{ m}^3/\text{s}, V_5 = 0.50 \text{ m/s} \)

\[
\begin{array}{|c|c|c|c|c|}
\hline
L_R & L_D & N & \% & EXP. \\
\hline
3.65 & 2.25 & 3.98 & A & + \\
5.18 & 2.25 & 3.98 & B & C \\
5.18 & 3.47 & 3.98 & D & E \\
3.65 & 3.47 & 3.98 & F & G \\
\hline
\end{array}
\]
FIGURE 13: EXPERIMENTAL STABLE AND UNSTABLE REGIONS OF OPERATION

$Q_D, Q_R$: Average Downcomer and Riser Air Rates ($m^3/s$)

(i) $L = 9.85m, L_R = 3.65m, L_D = 2.25m$

+: Stable Circulation.
○: Downcomer Slug Breakthrough with Continued Circulation.
△: Flow Reversal.

(ii) $L = 9.85m, L_R = 5.18m, L_D = 2.25m$

+: Stable Circulation.
○: Downcomer Slug Breakthrough with Continued Circulation.
△: Flow Reversal.
FIGURE 13 (cont): EXPERIMENTAL STABLE AND UNSTABLE REGIONS OF OPERATION

Q_D, Q_R: Average Downcomer and Riser Air Rates (m^3/s)

(iii) L=9.85m, L_R=3.65m, L_D=3.47m

+: Stable Circulation.
•: Downcomer Slug Breakthrough with Continued Circulation.
▲: No Circulation and Both Limbs Partially Filling and Emptying with Air: See 4.1.2.

(iv) L=9.85m, L_R=5.18m, L_D=3.47m

+: Stable Circulation.
•: Downcomer Slug Breakthrough with Continued Circulation.
▲: No Circulation and Both Limbs Partially Filling and Emptying with Air: See 4.1.2.
FIGURE 14: INSTABILITY INITIATED BY SLUG BREAKTHROUGH

Q_D, Q_R: Average Downcomer and Riser Air Rates (m³/s),
Experimental Stability Boundary, On All Plots L = 9.85 m, V_s = 0.50 m/s.

(i) \( L_R = 3.65 \text{ m}, L_D = 2.25 \text{ m}, \quad N_{\text{BEST}} = 3.98. \)

(ii) \( L_R = 5.18 \text{ m}, L_D = 2.25 \text{ m}, \quad N_{\text{BEST}} = 4.68. \)

(iii) \( L_R = 3.65 \text{ m}, L_D = 3.47 \text{ m}, \quad N_{\text{BEST}} = 4.15. \)

--- Constant \( V_0 \) Lines For \( N = 3.98 \)
--- Constant \( V_0 \) Lines For \( N = N_{\text{BEST}} \)

From Equation 38

<table>
<thead>
<tr>
<th>Circum. Velocity</th>
<th>Resisting Force</th>
<th>Driving Force</th>
<th>( Q_D = 0.00463 \text{ m}^3/\text{s}, Q_R = 0.00276 \text{ m}^3/\text{s}, )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_1 = 0.99 )</td>
<td>93.1</td>
<td>91.3</td>
<td>( V_1 = 1.0 \text{ m/s}, V_2 = 1.2 \text{ m/s}, N_{\text{BEST}} = 4.15 )</td>
</tr>
<tr>
<td>( V_1 = 1.00 )</td>
<td>95.0</td>
<td>95.0</td>
<td>Resisting and Driving Forces Given by LHS and RHS of Equation 3.6.</td>
</tr>
<tr>
<td>( V_1 = 1.01 )</td>
<td>96.9</td>
<td>98.4</td>
<td>( V_1 ) is Unstable, ( V_2 ) is Stable: See Section 4.3 In Text.</td>
</tr>
<tr>
<td>( V_2 = 1.19 )</td>
<td>134.5</td>
<td>135.6</td>
<td></td>
</tr>
<tr>
<td>( V_2 = 1.20 )</td>
<td>136.8</td>
<td>136.8</td>
<td></td>
</tr>
<tr>
<td>( V_2 = 1.21 )</td>
<td>139.1</td>
<td>137.8</td>
<td></td>
</tr>
</tbody>
</table>
(a) THE AIR INJECTOR

Sparger Hole Area $A_s$
Sparger Constant $K_s$

(b) THE CIRCULATING SYSTEM

$\delta = (L - L_R)$

Resistance Force $Kv^2$

FIGURE 15: COMPRESSOR / BUBBLE COLUMN: STABILITY.

(c) AN EXAMPLE TO ILLUSTRATE STABILITY

Point 'X' is Stable: See 4.4

Hence $K_s = 5.164 \times 10^4$
FIGURE 16: SLOWDOWN CURVES

V: Liquid Velocity (m/s)

$V_t = 0.39 \text{ m/s} \quad (t=0)$, initial System Velocity $0.92 \text{ m/s} \quad (t=-13)$

ORIFICE PLATE PRESENT

Data Points.

$V = \frac{1}{t}$

\[ \frac{1}{V} = 2.475 + 0.1109t \]

i.e. $K_{SF} = 0.1109$ for this run.

$V_{Ex}$, Experiment Initial $V_t$, System Velocity

$V_{Sy}$, System Velocity at $t=0$

$L_R = 3.65 \text{ m}$.

FIGURE 16(a): TYPICAL $\frac{1}{V}$ VS. $(-t)$ CURVE.

FIGURE 16(b): $V$ VS. $(-t)$ CURVES
FIGURE 17(a): THE VOIDAGE "PLUG"

(i) VOIDAGE PROFILES
AT TIME \( t = 0 \).

(ii) VOIDAGE PROFILES
AFTER TIME \( \Delta t \):
SHOWING NEGATIVE VOIDAGE
"PLUGS" \( \Delta \varepsilon_1 \) AND \( \Delta \varepsilon_3 \).
FIGURE 17(b): VOIDAGE PLUG INSTABILITY

Q_D, Q_R: Average Downcomer and Riser Air Rates (m³/s),
L = 9.85 m, V_g = 0.50 m/s,
--- Experimental Stability Boundaries,
--- Theoretical "Voidage Plug" Stability Boundary
(Given by Equations 5.9 and 5.8),
θ: Experimentally Observed Oscillations: See Table 1.

(i) L_R = 3.65 m, L_D = 2.25 m.
(ii) L_R = 5.18 m, L_D = 2.25 m.
(iii) L_R = 3.65 m, L_D = 3.47 m.
(iv) L_R = 5.18 m, L_D = 3.47 m.
FIGURE 18: EXPERIMENTALLY OBSERVED OSCILLATIONS
FIGURE 18(iii)
L = 9.85 m,
L_D = 3.47 m,
L_R = 3.65 m,
Q_p (av) = 0.0024 m³/s,
Q_o (av) = 0.0047 m³/s,
V = 1.11 m/s,
T = 70 secs,
Amplitude ~ 0.24 m/s
Upper Plot — Central Pitot,
Lower Plot — Side Pitot.

FIGURE 18(iv)
L = 9.85 m,
L_D = 3.47 m,
L_R = 5.18 m,
Q_p (av) = 0.00074 m³/s,
Q_o (av) = 0.0054 m³/s,
V = 1.18 m/s,
T ~ 65 secs,
Amplitude ~ 0.20 m/s
Upper Plot — Central Pitot,
Lower Plot — Side Pitot.

FIGURE 18(cont): EXPERIMENTALLY OBSERVED OSCILLATIONS.
**FIGURE 19:** OSCILLATORY BEHAVIOUR.

**FIGURE 19(a): SIMPLE BEHAVIOUR**

Experimental $Q_D$ and $Q_R$ values used are average of injector and atmospheric; see text.

**TIME DELAYS**

(1) $E_1(t) = E_1(t - \frac{x}{V-V_s})$: $0 < x < L_D$.

(2) $E_2(t) = E_1(t - L_D \frac{L-D}{V-V_s} - \frac{L-D}{V+V_s})$: $L_D < x < L_D + L$.

(3) $E_3(t) = E_1(t - \frac{y}{V+V_s})$: $0 < y < (L-L_R)$.

Where $y = x - (L+L_R)$.

**FIGURE 19(b): BEHAVIOUR ALLOWING FOR HYDROSTATIC EFFECTS.**

Experimental $Q_D$ and $Q_R$ values used are injector values.

**HYDROSTATIC EFFECT**

(1) $E(x,t) = E_1(t)(\frac{x + L - L_D}{a + L - L_D})$: $0 < x < L_D$.

(2) $E(x,t) = E_2(t)(\frac{x + L - L_D}{a + L - L_D})$: $L_D < x < L + L_D$.

(3) $E(y,t) = E_3(t)(\frac{a + L - L_R}{a + L - L_R - y})$: $0 < y < (L - L_R)$.
FIGURE 20(a): THE PHASE PLANE SHOWING MULTIPLE ROOTS

(This Plot is Contours of "f" In The \( \beta-\omega \) Plane, and When "f" = 0 There is a Root to Equation 5.19 )

An Enlargement of The Small Box Shown is Given in Figure 21(a).

\[ V = V_0 + D e^{(\beta + i\omega)t} \]

Period = \( 2\pi / \omega \)

Arrows Indicate Direction of Decreasing "f"

\[ f = \ln(1 + SUM) \]

\( Q_R = 0.0 m^3/s, Q_D = 0.00466 m^3/s, \]

\( V_0 = 1.09 m/s, V_s = 0.50 m/s, \rho = 1000 kg/m^3, \]

\( g = 9.81 m/s^2, L_D = 2.25 m, N = 3.98, L = 9.85 m, \]

\( L_R = 5.18 m, M_e = 1000 kg \), At Root \( f = 0 \).
FIGURE 20(b) : THE PHASE PLANE SHOWING MULTIPLE ROOTS

(This Plot is Contours of $f$ in The $\beta - \omega$ Plane, and When $f^* = 0$
There is a Root to Equation 5.19 )
An Enlargement of The Small Box Shown is Given in Figure 21(b).

\[ V = V_0 + D^* e^{(\beta + i\omega)t} \]
\[ \text{Period} = \frac{2\pi}{\omega} \]
\[ f = \ln(1 + \text{SUM}) \]
Arrows Indicate Direction of Decreasing $f$
\[ G_R = 0.00246 \text{ m}^3/\text{s}, \]
\[ Q_D = 0.00412 \text{ m}^3/\text{s} \]
\[ V_0 = 110 \text{ m}/\text{s}, \]
\[ V_s = 0.50 \text{ m}/\text{s}, \]
\[ \rho = 1000 \text{ kg/m}^3 \]
\[ g = 9.81 \text{ m}/\text{s}^2, \]
\[ L_D = 3.47 \text{ m}, \]
\[ N = 3.98, \]
\[ L = 9.85 \text{ m}, \]
\[ L_R = 3.65 \text{ m}, \]
\[ M = 1000 \text{ kg}, \]
At Root $f = 0$. 

\[ \beta \text{ vs. } (s - 1) \]
FIGURE 20(b) • THE PHASE PLANE • SHOWING MULTIPLE ROOTS

(This Plot is Contours of "f" in The $\beta - \omega$ Plane, and When $\phi = 0$
There is a Root to Equation 5.19.)
An Enlargement of The Small Box Shown is Given in Figure 21(b).

\[ V = \text{Vo} + D^* e^{(\beta + i\omega)t} \]
Period = $\frac{2\pi}{\omega}$
\[ f = \ln(1 + \text{SUM}) \]
Arrows Indicate Direction of Decreasing "f"

$Q_R = 0.00246 \text{ m}^3/\text{s}, Q_O = 0.00412 \text{ m}^3/\text{s},$
\[ v_o = 1.10 \text{ m/s}, V_s = 0.50 \text{ m/s}, \rho = 1000 \text{ kg/m}^3, \]
g = 9.81 m/s$^2$, $L_O = 3.47 \text{ m}$, $N = 3.98$, $L = 9.85 \text{ m},$
L$R = 3.65 \text{ m}$, $M_e = 1000 \text{ kg}$, At Root $f = 0$. 

\[ \beta (\text{s}^{-1}) \]
\[ \omega (\text{s}^{-1}) \]
FIGURE 21(a): ENLARGEMENT OF FIGURE 20(a) SHOWING MEANINGFUL ROOT

Root is $\beta = +0.04, \omega = 0.23$.
(This Plot is Contours of "f" In The $\beta$-$\omega$ Plane, and When "f" = 0 There is a Root to Equation 5.19)

\[ V = V_0 + D e^{(\beta + i\omega)t} \]

Period = $2\pi / \omega$, $f = \ln(1 + \text{SUM})$

Arrows Indicate Direction of Decreasing "f"

$Q_R = 0.0 \text{m}^3/\text{s}, Q_D = 0.00466 \text{m}^3/\text{s}, V_0 = 1.09 \text{m/s}, V_s = 0.50 \text{m/s},$

$\rho = 1000 \text{ kg/m}^3, g = 9.81 \text{ m/s}^2, L_D = 2.25 \text{ m}, N = 3.98, L = 9.85 \text{ m},$

$L_R = 5.18 \text{ m}, \text{ At Root } f = 0, M_e = 1000 \text{ kg}.\]
FIGURE 21(b) • ENLARGEMENT OF FIGURE 20(b) • SHOWING MEANINGFUL ROOT.

Root is $\beta = +0.03, \omega = 0.17$; Note Other Root in Top Left Hand Corner.
(This Plot is Contours of \( t \) in the $\beta$-$\omega$ Plane, and When $\omega = 0$ There is a Root to Equation 5.19.)

Root is $\beta = +0.03, \omega = 0.17$; Note Other Root in Top Left Hand Corner.

Other Root in Top Left Hand Corner.

(Other Plot is Contours of $t$ in the $\beta$-$\omega$ Plane, and When $\omega = 0$ There is a Root to Equation 5.19.)

$V = V_0 + D e^{(\beta + i\omega)t}$, Period $= \frac{2\pi}{\omega}, f = \ln(1 + SUM)$.

Arrows Indicate Direction of Decreasing $t$.

$Q_R = 0.00246 \text{ m}^3/\text{s}, Q_D = 0.00412 \text{ m}^3/\text{s}, V_b = 110 \text{ m/s}$, $V_s = 0.50 \text{ m/s}, \rho = 1000 \text{ kg/m}^3, g = 9.81 \text{ m/s}^2$.

$L_p = 3.47 \text{ m}, N = 3.98, L = 9.85 \text{ m}, L_R = 365 \text{ m}$.

$M_e = 1000 \text{ kg}, \text{At Root } f = 0.$
FIGURE 22: THEORETICALLY PREDICTED CONTOURS OF $\beta$ AND $\omega$ ON THE $Q_R$ VS $Q_D$ STABILITY PLOT FOR THE ORIGINAL APPARATUS GEOMETRY.

$\beta = 0$ Lines Shown Thicker
$L = 9.85$ m, $M_e = 1000$ kg, $V_s = 0.50$ m/s
$Q_D, Q_R$: Average Downcomer and Riser Air Rates (m$^3$/s)

**Stable/Unstable Boundaries:**
- Observed Stable/Unstable Boundaries.
- $\theta$: Experimentally Observed Oscillations (See Table 1).

$L_D = 2.25$ m, $L_R = 3.65$ m, $N = 3.98$. 

Graphical representation showing contours for different values of $\beta$ and $\omega$. The figure includes annotations indicating the values of $\omega$ and $\beta$ at various points on the graph.
FIGURE 23: THEORETICALLY PREDICTED CONTOURS OF $\beta$ AND $\omega$ ON THE $Q_R$ VS $Q_D$ STABILITY PLOT FOR $L_R=5.18\,\text{m}, L_D=2.25\,\text{m}$

$\beta=0$ Lines Shown Thicker, $L=9.85\,\text{m}, M_e=1000\,\text{kg}, V_s=0.50\,\text{m/s}$,

$Q_D, Q_R$: Average Downcomer and Riser Air Rates ($\text{m}^3/\text{s}$),

$\Phi$: Experimentally Observed Oscillations (See Table 1).
FIGURE 24: THEORETICALLY PREDICTED CONTOURS OF $\beta$ AND $\omega$ ON THE $Q_R$ VS $Q_D$ STABILITY PLOT FOR $L_R=3.65\text{m}, L_D=3.47\text{m}$.

- $\beta=0$ Lines Shown Thicker, $L=9.85\text{m}, M=1000\text{kg}, V_s=0.50\text{m/s}$,
- $Q_D, Q_R$: Average Downcomer and Riser Air Rates ($\text{m}^3/\text{s}$),
- Observed Stable/Unstable Boundaries,
- $\Theta$: Experimentally Observed Oscillations (See Table 1)

**FIGURE 24(a):** $N_{\text{BET}}=4.15$
$L_R=3.65\text{m}, L_D=3.47\text{m}$

**FIGURE 24(b):** $N=3.98$
$L_R=3.65\text{m}, L_D=3.47\text{m}$
FIGURE 25 - THEORETICALLY PREDICTED CONTOURS OF $\beta$ AND $\omega$ ON THE $Q_R$ VS $Q_D$ STABILITY PLOT FOR $L_R=5.18\,m$, $L_D=3.47\,m$.

$\beta=0$ Lines Shown Thicker, $L=9.85\,m$, $Me=1000\,kg$, $Vs=0.50\,m/s$,
$Q_R, Q_D$: Average Downcomer and Riser Air Rates ($m^3/s$),
---: Observed Stable/Unstable Boundaries,
θ: Experimentally Observed Oscillations (See Table 1).

FIGURE 25(a): $N_{BEST}=4.50$
$L_R=5.18\,m$, $L_D=3.47\,m$

FIGURE 25(b): $N=3.98$
$L_R=5.18\,m$, $L_D=3.47\,m$
FIGURE 26: COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL STABILITY BOUNDARIES.

L = 9.85 m, Me = 1000 kg, Vs = 0.50 m/s,
Q_D, Q_R: Average Downcomer and Riser Air Rates (m^3/s)

(i) L_D = 2.25 m, L_R = 3.65 m,
N_{BEST} = 3.98.

(ii) L_D = 2.25 m, L_R = 5.18 m,
N_{BEST} = 4.68.

(iii) L_D = 3.47 m, L_R = 3.65 m,
N_{BEST} = 4.15.

(iv) L_D = 3.47 m, L_R = 5.18 m,
N_{BEST} = 4.50.

A, B, C: Observed Stable/Unstable Boundaries.
A-B: Theoretical V_0 = 1.1 m/s Line (Slug Breakthrough).
B-C: Theoretical β = 0 Curve (Oscillation Initiated Instability).
Atmospheric Pressure $P = \rho g A$

Voidage at Atmospheric Pressure $\varepsilon_0$

Voidage at Depth "h" is $\varepsilon$.

Voidage at Depth (h+dh) is $(\varepsilon + d\varepsilon)$.

Pressure at Depth "h" is $P$.

$P = P_A + \rho gh(1-\varepsilon)$

Pressure at Depth (h+dh) is $P + \rho g dh(1-\varepsilon)$

**FIGURE 27(a): The Effect Of Other Bubbles.**

**FIGURE 27(b): Errors In $\varepsilon$ Caused By Neglecting Hydrostatic Effects Of Bubbles Present.**

$(\% \text{ Error} = 100 \times (\varepsilon \text{ from A1-9} - \varepsilon \text{ from A1-2}) / \varepsilon \text{ from A1-9})$
Initially \( r = r_0, z = z_0 = 0.21 \)

After Time \( t_B : r = r, z = z \)

\( t_B \) in seconds, \( r \) and \( r_0 \) in metres.

\( r \): Bubble Radius at Time \( t_B \),
\( r_0 \): Initial Bubble Radius,
\( z \): Oxygen Mole Fraction at Time \( t_B \),
\( z_0 \): Initial Oxygen Mole Fraction.

\( z \): Mole Fraction of Oxygen in Bubble.
PLATE 3: DOWNCOMER SLUG BREAKTHROUGH WITH CONTINUED CIRCULATION

[NOTE INDENTATIONS IN TOP SURFACE OF BUBBLE]
PLATE 4: LARGE SLUG BREAKTHROUGH IN DOWNCOMER

[PHOTOGRAPHS TAKEN AT 1/4 SECOND INTERVALS]
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