Experimental and analytical studies of solute transport in overland flow over vegetated surfaces

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Abstract:

Solute transport in overland flow is considered as one of the main contributors to water pollution. Although many mechanism models of solute transport from soil to runoff have been proposed, the solute transport mechanism in runoff over vegetated surface still remains unclear. In this study, a series of laboratory experiments were conducted to study the solute transport over vegetated surfaces. Based on the experimental results, the conception of “stationary water layer” in runoff was proposed. Applying the complete mixing layer theory in the stationary water layer, an analytical solute transport model was developed with the assumption that the solute concentration in stationary water layer is equal to that in upper runoff. The results show that the predictions made by the transport model are in good agreement with the measured experimental data. In the vegetated surfaces, the depth of stationary water layer is mostly related to the rainfall intensity, bed slope and vegetation density. The solute transport rate increases from zero to a maximum and then decreases to zero. The analytical solution shows that the maximum solute transport occurs at the time of concentration. This study may advance our understanding of the mechanisms of solute transport in vegetated area, which is particularly useful for the management of water pollution.
1 Introduction

The solute transport in overland flow has been recognized as a major contributor to the deterioration of water quality (Deng et al., 2005; Yang et al., 2016a; Yang et al., 2016b). Understanding the mechanism of solute transport in overland flow is very important for developing effective methods to control water pollution (Dong et al., 2013).

Previous mechanism models have been developed based on the physical processes of solutes transport from soil into runoff. The exchange layer, or mixing layer, framework is the most commonly used approach for models of solute transport into runoff (Ahuja et al., 1981; Ahuja and Lehman, 1983; Ahuja, 1986; Wallach et al., 1988; Wang et al., 1998; Zhang et al., 1997; Zhang et al., 1999; Gao et al., 2004, 2005; Walter et al., 2007; Dong et al., 2013; Yang et al., 2016b; Yang et al., 2016c). The complete-mixing model was proposed by Ahuja et al. (1981), it hypothesizes that rainwater completely mixes with soil water in the effective mixing layer, and the solute concentration in the runoff equals the concentration in the infiltrating water and in the mixing layer. Later, Ahuja and Lehman (1983) found that the concentration in runoff and in infiltration is not the same. The incomplete-mixing model was thus proposed (Ahuja, 1986), it considered that solute concentration in the effective mixing depth was in direct proportion to that in the infiltrating and runoff water. Different from the effective-mixing models, Wallach et al. (1988) developed the effective depth of transfer model and assumed that the solute concentration in the depth was equal to the concentration at the soil surface in the absence of infiltration. Deng et al. (2005)
suggested an active surface layer near the soil surface and developed a one-dimensional model for solute transport over bare soil slopes under constant rainfall. Gao et al. (2004, 2005) and Walter et al. (2007) developed physically based models of solute transport that assumed that chemicals near the surface of the soil were most likely to runoff as a result of raindrop impact and chemicals in the deeper soil diffused into exchange layer. Dong et al. (2013) refined the equivalent model of convection based on Gao et al. (2004, 2005) and Walter et al. (2007), in which the presumed exchange layer was replaced by a mixing depth, the exchange rate was assumed to be controlled by the splashing of raindrops, and the effect of diffusion was neglected. Yang et al. (2016b) proposed a mathematical model that relates to the detachment of soil particles by water flow with the assumption that the mixing depth is an integral of average water flow depth. The well-known Hairsine-Rose model (Hairsine and Rose, 1991) assumed that small particle transfer from soil into overland runoff through a shield layer. Wang et al. (2017) confirmed that the Gao model (Gao et al., 2004, 2005) is identical to the Hairsine-Rose model (Hairsine and Rose, 1991) when simulate the transport of colloidal from soil into overland storm flows. Through these studies, researchers have gained a generally good understanding of the solute transport from soil into overland flow. They all suggested that a “thin layer” (e.g., mixing layer, exchange layer, active layer or shield layer) near the soil surface control the mass transfer between soil and overland flow. However, few studies in the past considered the solute transport in overland flow in vegetated area.

Vegetative Filter Strips (VFS) is widely used in agricultural lands and cities for non-point source pollution control (Deletic, 2001, 2005; Deletic and Fletcher, 2006;
Mekonnen et al., 2015; Muñoz-Carpena et al., 2018), as vegetation has the potential to reduce runoff velocity and water erosion, increase infiltration and retain some pollutants (Deletic, 2001; Pan et al., 2011; Fulazzaky et al., 2013; Lambrechts et al., 2014; Chen et al., 2016). Many experimental and numerical studies have been carried out to investigate the transport of pollutant over grass (Dillaha, 1989; Magette et al., 1989; Muñoz-Carpena et al., 1999; Deletic, 2001, 2005; Deletic and Fletcher, 2006; Kuo and Muñoz-Carpena, 2009; Yu et al., 2012; Yu et al., 2013). However, most of the above studies focused on the transport of sediment particles, for sediment particles, especially fine sediment particles with large specific surface areas (Sabbagh et al., 2009; Lobo et al., 2017), have been regarded as the main source of water pollution during the runoff and the other pollutants (such as nutrients, heavy metals and pesticides) are adsorbed to sediment particles (Deletic, 2001; Herngren et al., 2005; Egodawatta et al., 2007) through electrostatic bonds, ionic and covalent bonds (Bradl, 2004; Wijesiri et al., 2016). Only few studies (Yu et al., 2012; Yu et al., 2013) investigated the bromide (as a conservative tracer) transport in overland flow on vegetated areas. To the knowledge of the authors, so far, there has been no study conducted to investigate the role of vegetation in controlling the solute transport in runoff alone and the mechanism of solute transport in overland flow over vegetated surface is unclear. The objective of this study is to investigate the mechanism of the solute transport over vegetated surfaces under rainfall separated from the soil. To do this, a number of physical simulation experiments are conducted using rainfall simulators and uniform-sloped vegetation catchments. In these experiments, different rainfall intensities, vegetation types and bed
slopes are considered. Based on the experimental results, the solute transport mechanism over vegetated surface is proposed. Then, a simplified mathematical model is developed for the solute transport over vegetated surface. The measured experimental data are used to validate the results predicted by the model developed in this study.

2 Laboratory experiments

All laboratory experiments were conducted in a rainfall-simulation hall. A V-shaped flushing board is used to provide constant rainfall, and it is maintained in an ‘off’ state until a constant rainfall event is achieved. Water nozzles are located 17 m above the model catchment. The rainfall intensity ranges from 10 to 200 mm/h, with the rainfall uniformity of > 0.9 over an area of 15.6 m in length and 12.6 m in width. The rainfall non-uniformity mainly occurs at the boundaries, which can be largely eliminated by placing the catchment at the center of the hall.

As mentioned before, this study focuses on the solute transport in runoff over vegetated surface. In order to increase the measurement accuracy and reduce the number of influencing factors (i.e., vegetation metabolism), we used an artificial turf on the impervious rubber to simulate the vegetated area according to previous study (Deletic, 2005). The ground is considered as an impervious slope in this study. The artificial turf has either dense grass (270 needles /mm) or sparse grass (130 needles /mm), as shown in Fig. 1. It is 2.96 m × 1.46 m and laid on the flat board. For the convenience of collecting samples, the board is placed in a steel flume (3 m × 1.5 m × 0.5 m), as shown in Fig. 1. The slope of the flume is adjustable through a hydraulic
In this paper, the slope was set to 1° or 2°, and four constant rainfall intensities were tested. Each rainfall event lasted for 45 min (dense grass) or 30 min (sparse grass) after the beginning of runoff. Similar to Deng et al. (2005), sodium chloride (table salt) was chosen as the solute pollutant due to its wide availability and ease of use. At the beginning of each experiment, salt was spread uniformly on the artificial turf surface. To ensure a uniform distribution, each artificial turf was divided into 50 (5×10) small squares and 2.5 g of salt was uniformly distributed within each square. For each experiment, the average rainfall intensity was measured. The sampling duration and interval of runoff water are listed in Table 1. Details on sample collection and data recording of the runoff water and solute can be found in Xiao et al. (2017).

3 Results

3.1 Water runoff process

As the transport of solute is closely related to water runoff, the accurate quantification of the overland flow process is essential for predicting the transport of solute. A large quantity of data can be obtained in a relatively short period of time using rainfall and small idealized catchment simulators. Fig. s shows the entire runoff processes under different conditions. In the figure, 22.9 mm/h refers to 22.9 mm/h rainfall intensity and similar notations are applied to others. It shows that (1) since the beginning of a rainfall event, a period of time (defined as the time of generation) is needed to reach the vegetation retention capacity before the overland flow occurs; and
for a constant rainfall event over a long period of time, the runoff rate increases at first after the occurrence of the overland flow and then reaches a constant maximum which is equal to the rainfall input, followed by a sharp drop when the rainfall stops. For a constant rainfall uniformly distributed over an impervious slope, the time needed to reach the equilibrium discharge (a constant maximum runoff rate) is referred to as the time of concentration ($t_c$). In the equilibrium phase, the constant maximum runoff rate can be calculated by Eq. (1).

$$Q_{\text{max}} = LBI$$  \hspace{1cm} \text{(1)}

where $Q_{\text{max}}$ is the maximum flow rate (m$^3$/s); $L$ is the length of the watershed (m); $B$ is the width of the watershed (m); and $I$ is the rainfall intensity (m/s).

The time of concentration $t_c$ can be roughly taken as the time required for the movement of a raindrop from the top of the slope to the outlet (Liang et al., 2015). Accordingly, it is logical to assume that: (1) it takes some time (defined as $t_{cx}$) for the flow at a location $x$ on the catchment to become steady, after which the flow at this location remains unchanged; and (2) the flow is uniform with constant water depth, flow rate $Q(x,t)$ and velocity $v(x,t)$ downstream of the location where the flow reaches a steady state. Therefore, the flow velocity and flow rate can be expressed as:

$$v(x,t) = \frac{dx}{dt} = \frac{dQ(x,t)}{BLdt} \ , \ \ 0 \leq t \leq t_{cx}$$  \hspace{1cm} \text{(2a)}

$$v(x,t) = v(x,t_{cx}) \ , \ \ t_{cx} \leq t$$  \hspace{1cm} \text{(2b)}

$$v(x+a,t_{cx}) = v(x,t_{cx}) \ , \ \ 0 \leq a \leq L - x$$  \hspace{1cm} \text{(2c)}

$$Q(x+a,t_{cx}) = Q(x,t_{cx}) = xBI \ , \ \ 0 \leq a \leq L - x$$  \hspace{1cm} \text{(2d)}

Fig. 2 shows that the runoff rate increases over time almost linearly during the
initial period. Thus, the runoff process can be fitted using a linear equation, as shown
in Fig. 3, where the first 14 min of the hydrographs are presented for clear visualization
of the rising runoff processes. To some extent, the initial runoff processes can be
predicted by the following linear equations:

\[ Q(t) = \alpha BI(t - t_0), \quad t_0 \leq t \leq t_c + t_0 \]  

(3)

where \( Q(t) \) is the runoff rate at the outlet (m\(^3\)/s); \( t \) is the time (s); and \( t_0 \) is the
time of generation (s). It follows from Eq. (2a) that \( \alpha \) is the flow velocity (m/s).

Accordingly, the time of concentration can be expressed as:

\[ t_c = \frac{L}{\alpha} \]  

(4a)

\[ t_{cx} = \frac{x}{\alpha} \]  

(4b)

Table 2 shows \( \alpha \), \( t_0 \) and \( t_c \) values under different conditions. It shows that the
\( t_0 \) values decrease with increasing rainfall intensity and slope for the same catchment,
indicating that increasing rainfall intensity and slope can improve the generation of
overland flow. Given the same rainfall intensity and slope, the \( t_0 \) values for dense
grass (270 needles/mm) are larger than that for sparse grass (130 needles/mm), because
higher vegetation density may lead to higher retention capacity of vegetation and lower
flow velocity, as shown in Table 2. However, there is no significant and consistent
difference in the \( \alpha \) and \( t_c \) values between different rainfall intensities. The \( \alpha \)
values for a given rainfall event are constant in this study, indicating that the flow
velocity at any location of the catchment is equal and does not change throughout the
rainfall. In fact, the \( \alpha \) values for all rainfall events in this study are small. Hence, it is
assumed that the flow velocity is constant throughout the rainfall process, and the
vegetation can effectively reduce the velocity of overland flow. Comparison of the runoff processes between different conditions indicates that a smaller slope and a higher vegetation density may lead to a longer time of concentration and a smaller flow velocity.

3.2 Solute concentration

Solute concentration is the most important parameter for evaluating water quality, and it is essential to better understand its variation during the rainfall runoff. Fig. 4 clearly shows that under each condition, the solute concentration decreases with time and finally approaches zero. In addition, the solute concentration can be affected by rainfall intensity, bed slope and grass density, and it may increase with the decrease of rainfall intensity and bed slope and the increase of grass density. In Fig. 4, it should be noted that $t = 0$ refers to the beginning of overland flow instead of the beginning of rainfall for the sake of comparison between different conditions. For the same catchment, the greater the rainfall intensity and slope, the faster the decrease of the solute concentration. Compared with the dense grass, the solute concentration decreases at a faster rate for the sparse grass, which is largely due to the change in the runoff process as described in the previous section. The solute concentration is closely related to the runoff process.

In this study, the solute is assumed to be uniformly distributed on catchment surface before the start of rainfall. During the time of generation, rainwater accumulates on the catchment to form a stationary water layer, as shown in Fig. 5, and the solute
begin to dissolve in the stationary water layer. After the beginning of overland flow, the solute in the stationary water layer move into the upper runoff and then out of the catchment. For solute transport from soil into runoff, most previous researchers (Ahuja et al., 1981; Wallach et al., 1988; Ahuja and Lehman, 1983; Yang et al., 2016b; Yang et al., 2016c) adopted the concept of mixing layer near soil surface. They hypothesizes that rainwater completely mixes with soil water in the mixing layer and the concentration of runoff is equal to that in the mixing layer. Applying the mixing layer theory, we also assume that upper runoff completely mixes with deeper water in the stationary water layer. The depth of the stationary water layer at the beginning of overland flow \((t = 0)\) is defined as \(h_0\) (mm). As the solute are uniformly distributed on the catchment surface, the initial solute concentration is:

\[
C_0 = \frac{W_0}{BLh_0} = \frac{W_0}{Ah_0}
\]  

(5)

where \(C_0\) is the initial solute concentration (g/L); \(W_0\) is the initial mass of solute on the surface (g); and \(A\) is the area of catchment (m²).

At the time of \(t \ (0 \leq t \leq t_c)\), the raindrops move from the top of the catchment to \(x\) \((x = \alpha t)\) according to Eq. (2), implying that the runoff rate from the top of the catchment \(x = 0\) to \(x\) has reached a steady state. According to Eq. (2), the runoff rate from \(x\) to the bottom of the catchment \(x = L\) is equal to the runoff rate at \(x\). For any control volume on the catchment between \(x\) and \(L\), such as that centering at position A as shown in Fig.5, the incoming flow rate and solute concentration are equal to those at the outflow, and thus the mass of the solute at this volume should not vary. Hence, it can be concluded that the amount of solute at any
position of the catchment remains unchanged until it reaches a steady state. Therefore, the amounts of water and solute flowing out of the catchment at $t$ ($0 \leq t \leq t_c$) depend on what happens over the catchment upstream of $x$. When the raindrops move from the top of the catchment to $x$, the solute is also transported to $x$. Taking into account the effect of diffusion, we propose a hypothesis that the solute concentration from 0 to $x$ is uniform at the time of $t_c$.

Accordingly, the water depth $h$ can be expressed as follows:

$$h(x, t) = h_0 + It, \quad 0 \leq t \leq t_c$$ (6a)

$$h(x, t) = h_0 + It_c, \quad t_c < t$$ (6b)

The water volume from 0 to $x$ ($x = \alpha t$) at the time of $t$ can be calculated by Eq. (7).

$$V(x, t) = xB (h_0 + It / 2)$$ (7)

The outlet solute concentration at time $t$ is defined as $C_r$ (g/L). According to the above hypothesis, the solute concentration from 0 to $x$ ($x = \alpha t$) is equal to $C_r$.

During a tiny interval of $\Delta t$, the runoff moves forward for a distance of $\Delta x$ ($\Delta x = \alpha \Delta t$), indicating that the area from 0 to $x+\Delta x$ has reached a steady state. We define the increase of the solute concentration as $\Delta C_r$ (g/L) during $\Delta t$. During $\Delta t$, the volume of rainfall from 0 to $x+\Delta x$ can be calculated by Eq. (8).

$$V_{rain} = (x + \Delta x) BI \Delta t = \alpha (t + \Delta t) BI \Delta t$$ (8)

According to the law of conservation of mass, we can obtain the following equation during $t$ to $t + \Delta t$.

$$C_r B \alpha t \left( h_0 + \frac{It}{2} \right) + \frac{W_0}{L} \alpha \Delta t = \left( C_r + \Delta C_r \right) B \alpha \left( t + \Delta t \right) \left( h_0 + I \frac{t + \Delta t}{2} + I \Delta t \right)$$
Rounding the second order and converting the above equation into a differential form yield:

\[
\frac{dC_t}{dt} = -C_t \frac{h_0 + 2It}{t \left( h_0 + It / 2 \right)} + \frac{W_0}{At \left( h_0 + It / 2 \right)} \tag{10}
\]

The general solution of the above differential equation can be described as follows:

\[
C_t = \frac{C}{t \left( 2h_0 + It \right)^3} + \frac{2W_0}{3Al} \tag{11}
\]

where \( C \) is a constant which can be determined using the initial condition of solute concentration:

\[
\lim_{t \to 0} C_t = C_0 = \frac{W_0}{Ah_0} \tag{12}
\]

According to Eq. (11), the limit expression of \( C_t \) is:

\[
\lim_{t \to 0} C_t = \lim_{t \to 0} \frac{2W_0It \left( I^2t^2 + 6h_0It + 12h_0^2 \right) + 3AlC + 16W_0h_0^3}{3Al \left( 2h_0 + It \right)^3} \tag{13}
\]

The necessary condition for the existence of the limit is that the constant term \((3AlC + 16W_0h_0^3)\) in the numerator position is equal to 0. Thus:

\[
C = -\frac{16W_0h_0^3}{3Al} \tag{14}
\]

Taking Eq. (14) into Eq. (13), we can verify that Eq. (12) is true. Then, taking Eq. (14) into Eq. (11), we can obtain the expression of \( C_t \):

\[
C_t = \frac{2W_0 \left( I^2t^2 + 6h_0It + 12h_0^2 \right)}{3A \left( 2h_0 + It \right)^3}, \quad 0 \leq t \leq t_c \tag{15}
\]

At time \( t_c \), the raindrops move from the top of the catchment to the outlet, thus indicating that the whole catchment contributes to the flow rate and solute concentration ...
at the outlet. The above hypothesis indicates that the solute concentration at any
position of the catchment is the same, and the flow rate at the outlet is equal to the
rainfall input. According to the law of conservation of mass, we can obtain the follow
equation during \( t \) to \( t + \Delta t \) \((t_c \leq t)\).

\[
C_tBL \left( h_0 + \frac{I_t}{2} \right) = \left( C_t + \Delta C_t \right) BL \left( h_0 + \frac{I_t}{2} + I\Delta t \right)
\]  

(16)

Rounding the second order and converting the above equation into a differential form
yield:

\[
\frac{dC_t}{C_t} = -\frac{Idt}{h_0 + It_c / 2}
\]  

(17)

Integrating Eq. (17) and applying the initial condition yield:

\[
C_t = C_{t_c} e^{\frac{I(t-t_c)}{h_0+It_c/2}}
\]  

(18)

where \( C_{t_c} \) is the solute concentration at \( t_c \) which can be obtained from Eq. (15).

\[
C_{t_c} = \frac{2W_0 \left( I^2t_c^2 + 6h_0It_c + 12h_0^2 \right)}{3A \left( 2h_0 + It_c \right)^3}
\]  

(19)

Taking Eq. (19) into Eq. (18) yields:

\[
C_t = \frac{2W_0 \left( I^2t_c^2 + 6h_0It_c + 12h_0^2 \right)}{3A \left( 2h_0 + It_c \right)^3} e^{\frac{I(t-t_c)}{h_0+It_c/2}}, \quad t_c < t
\]  

(20)

Eq. (15)-(20) have the same parameter \((h_0)\), which is determined by fitting the
experimental results of the solute transport in this study. Then, the values of \( C_0 \) under
each condition can be calculated from Eq. (5). According to the above analysis, the
value of \( h_0 \) may be related to the vegetation retention capacity, and thus it can be
roughly estimated by the time of generation \((t_0)\), which is defined as \( h_0 \) \((\text{mm},
\quad h_0 = It_0)\). Table 3 shows the values of \( h_0 \), \( C_0 \) and \( h_0 \) for different rainfall events.
It is obvious that the value of \( h_0 \) decreases with increasing rainfall intensity and slope.

Given the same rainfall intensity and slope, the value of \( h_0 \) for dense grass is greater than that for sparse grass, which may be related to the vegetation retention capacity. The opposite trend is observed for the values of \( C_0 \). There is a small difference between \( h_0 \) and \( h_{0r} \) values. Hence, we can use the value of \( h_{0r} \) instead of \( h_0 \) in the solute transport equation proposed in this paper.

The comparisons between measured and predicted solute concentrations under different conditions are shown in Fig. 4. Overall, the solute concentration can be well predicted by the above analytical solution. The solute concentration drops rapidly at first and then more slowly towards zero. In previous studies (Yu et al., 2012; Yu et al., 2013), similar results have been found, although the experimental conditions are different from those in this study. It is obvious that both upstream inflow and rainfall will dilute the solute.

### 3.3 Solute transport rate

The solute transport rate is an important indicator to quantify the solute transport.

It is defined as the flow rate of solute transported from the catchment and it can be described by the following equation (Kim et al., 2005):

\[
M_t = C_t Q_t
\]  

(21)

where \( M_t \) is the solute transport rate at time \( t \) (g/s).

Combining Eqs. (2), (3), (15), (20) and (21) yields:
Fig. 6 shows the observed and predicted solute transport rates under different conditions. It is evident that the transport rates of solute under different conditions show a similar single-peak shape consisting of a steep-rising limb at the beginning and a sharp-falling limb later on. The solute transport rate increases from zero to a maximum and then decreases to zero. Overall, the measured solute transport rates are in good agreement with that predicted by the analytical equations. The maximum solute transport rate ($M_{\text{max}}$) for each condition is related to the rainfall intensity, ground slope and grass density. It is reached during or at the end of the initial rising runoff stage, as Eq. (22b) indicates that the solute transport rate decreases gradually at the plateaued runoff process. We can formulate an expression for the maximum solute transport rate from Eq. (22a).

Taking the derivative of Eq. (22a) with respect to time yields:

$$
\frac{dM_t}{dt} = \frac{16\alpha W_0 I h_0^3}{L \left(2h_0 + It\right)^4}, \quad 0 \leq t \leq t_c
$$

(23)

The above equation indicates that the value of $M_t$ increases during the initial rising runoff stage and then reaches a maximum at time $t_c$. The maximum solute transport rate $M_{\text{max}}$ can be written as:

$$
M_{\text{max}} = \frac{2W_0 I \left( I^2 t_c^2 + 6h_0 I t_c + 12h_0^2 \right)}{3 \left(2h_0 + It_c\right)^3}
$$

(24)

The measured and predicted maximum solute transport rates and the time of their
occurrence are listed in Table 4.

Fig. 6 and Table 4 show that the larger the rainfall intensity and ground slope or
the smaller the grass density, the larger the maximum solute transport rate. It shows that
the analytical equation can well predict the value of $M_{\text{max}}$.

3.4 The conservation of model

In the previous sections, we proposed a series of equations to describe the runoff
process and the transport of solute over vegetated surfaces. The analytical results are in
good agreement with the measured ones. However, the mass conservation of the
analytical solution needs to be confirmed. As the solute is a conservative substance, the
total amount of solute flowing out of the catchment should be equal to the initial amount
of solute on the catchment.

The total amount of solute flowing out of the catchment ($W_T$) can be calculated as
follows:

$$W_T = \int_0^\infty M_c dt = \int_0^c M_c dt + \int_c^\infty M_c dt$$  (25)

Combining Eqs. (22) and (25) results in the following expression of $W_T$, which
indicates that the solute transport model proposed in this study satisfies the conservation
of solute mass.

$$W_T = W_0$$  (26)

3.5 Sensitivity analysis for parameters

The solute transport model proposed in this study is in the form of analytical
solution with only three parameters ($\alpha$, $h_0$ and $C_0$) to be determined. These three parameters have clear physical meanings and each can be directly measured. The value of $\alpha$ can be determined by measuring the flow velocity; the value of $C_0$ can be measured at the beginning of overland flow; while the value of $h_0$ can be determined by measuring or roughly estimated by $h_{0'}$ ($I_{0'}$) as described above.

Table 3 shows that the differences between $h_0$ and $h_{0'}$ are less than 0.2 mm and the values of $h_0$ vary from 1.06 to 2.58 mm. Fig. 7 and Fig. 8 shows the effect of $h_0$ on the concentration and transport rate of solute respectively. Fig. 7 shows that $h_0$ has an effect on the solute concentration mainly in the initial stage, and the difference in the later stage is very small. However, Fig. 8 shows that $h_0$ has no obvious effect on the solute transport rate due to the small runoff rate in the initial stage. When the change of $h_0$ value is less than 0.2 mm, it has no significant effects on the solute concentration and solute transport rate. Thus, the replacement of $h_0$ with $h_{0'}$ will not cause a significant deviation of the results. Fig. 9 and Fig. 10 show that rainfall intensity has a great influence on the solute transport. Hence, the value of rainfall intensity should be given exactly in practical application. Similar to previous studies (Gao et al., 2004; 2005), rainfall plays an important role in controlling solute transport.

It should be noted that the good agreement of the curves does not guarantee the good representation of all the physical processes by the proposed model in the complex situations. The model also has some limitations as it is based on the following basis or assumptions: (1) the rainfall intensity remains constant in vegetated surfaces; (2) soil infiltration is negligible; and (3) the solute is uniformly distributed over an idealized
This study is particularly interesting in vegetated surface with strong impervious ground. Pollutant wash-off from an impervious surface (e.g., road and roof surfaces) is commonly modelled by an exponential equation (Egodawatta et al., 2007, 2009; Xiao et al., 2017), and some water quality management models such as US EPA’s SWMM model (Storm Water Management Model) and US Army Corps’s STORM model (Storage Treatment Overflow Runoff Model) (Egodawatta et al., 2007). For a constant rainfall event, the expression of the pollutant concentration at stable stage can be derived from the exponential wash-off model:

\[ C_t = C_c e^{-k(I - t)} \]  

(27)

where \( k \) is the wash-off coefficient (mm\(^{-1}\)).

However, as Egodawatta et al. (2007) noted, the wash-off coefficient \( k \) is an empirical parameter with no direct physical meaning. Comparing Eq. (27) with Eq. (18) shows that \( k \) and \( \frac{1}{h_b + It_c / 2} \) occupy similar locations. In this study, the term \( \left( h_b + It_c / 2 \right) \) means the average water depth of the catchment at the stable stage, and thus the wash-off coefficient \( k \) may be related to water depth and thus has the physical meaning.

According to most previous studies (Ahuja et al., 1981; Ahuja and Lehman, 1983; Wallach et al., 1988; Wang et al., 1998; Zhang et al., 1999; Gao et al., 2004, 2005;
layer theory and exchange theory are the two predominant mechanisms to describe the solute transport from soil into runoff. The mixing layer theory don’t account for solute diffusion from deeper soil and neglects the impact of raindrop. In contrast, the exchange layer theory, proposed by Gao et al. (2004, 2005), considers a soil-water system divided into three vertically distributed, horizontal layers: runoff or surface ponding water, an exchange-layer, and the underlying soil, as shown in Fig. 11 (a). In exchange layer theory, raindrop impact controls the transfer between the exchange layer at the soil surface and runoff and diffusion controls solute transfer between the deeper soil and exchange layer. Comparing with soil water, the water in the stationary water layer is easier to mix with upper runoff water. Hence, we applied the mixing layer theory and developed an analytical model to describe the solute transport over vegetated surface during rainfall runoff. Combining the exchange layer theory and mixing layer theory, the mechanism of solute transport in vegetated area can be described as shown in Fig. 11 (b). After soil saturated, rainwater begin to accumulate and forms a stationary water layer (i.e., mixing layer) over vegetated surface. Meanwhile, the solute transfers from soil into the stationary water layer under the impact of raindrop and solute in deeper soil diffuses into the exchange layer. Then, the solute water in the stationary water layer mix with upper runoff water. It should be noted that the depth of exchange layer in vegetated area may be smaller than that in bare soil area for the vegetation can weaken the impact of raindrop. Similar results can be found in Yu et al. (2013), they suggested that the depth of exchange layer is small without rainfall. But the vegetation also
promotes the formation of mixing layer (the stationary water layer).

Similar to Deng et al. (2005), sodium chloride (table salt) is chosen as the solute pollutant in this study. Of course, different chemicals have different physical and chemical properties. Some chemicals may be easily absorbed to soil particles and the others may be not. This point has been considered in Gao model (Gao et al. 2004; 2005).

We aim to study solute transport in overland flow where only few soil particles exist. Hence, we assume that most dissolved chemicals (such as nutrients) transported into runoff have the same transfer behavior as table salt in overland flow. The model in this study is developed under the assumption that all the solute should be totally removed from the surface after an adequate duration regardless of rainfall intensity. Experiment results proved this assumption for more than 95% of the total solute was collected at the end of experiments.

5 Conclusions

In this study, a series of simplified physical experiments are conducted to study the solute transport over vegetated surfaces, and then an analytical model including solute concentration equation and solute transport rate equation is proposed applying the mixing theory. It shows that the concentration and transport rate of solute can be accurately described by the transport model developed in this study. Our results show that a stationary water layer will be formed before the overland flow occurs. The depth of the stationary water layer referring to the vegetation retention capacity depends largely on the rainfall intensity, bed slope and vegetation density. The solute transport
rate increases at first and then decreases sharply thereafter. In addition, the maximum solute transport rate takes place at the time of concentration, and it is positively correlated with rainfall intensity and bed slope while negatively correlated with vegetation density. Analysis shows that rainfall intensity is the dominant factor in controlling solute transport. Finally, the mechanism of solute transport in vegetated area is described combining the exchange layer theory and the mixing layer theory.

This study aims to investigate the fundamental principles of the solute transport over vegetated surface. A practical vegetated area is much more complicated, with spatially and temporally varied rainfall intensity and ground features. Further study is needed to consider the changing rainfall intensity, slope and soil infiltration in vegetated area.

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References

interaction as determined by $^{32}$P. Water Resources Research. 17(4), 969-974.


from urban road surfaces using simulated rainfall. Water Research. 41, 3025-3031.


Fulazzaky, M.A., Khamidun, M.H., Yusof, B., (2013). Sediment traps from synthetic construction site


Gao, B., Walter, M.T., Steenhuis, T.S., Parlanega, J.-Y., Richardsa, B.K., Hogarthe, W.L. Rose, C.W.,
(2005). Investigating raindrop effects on transport of sediment and non-sorbed chemicals from soil


76, 149-158.


strips to control runoff pollution from phosphate mining areas. Journal of Hydrology. 378, 343-354.

growth and morphology and of sediment concentration on sediment retention efficiency of
vegetative filter strips: Flume experiments and VFSMOD modeling. Journal of Hydrology. 511,


Figure and table captions

Fig. 1 Experimental set-up

Fig. 2 Runoff rates for various conditions (Black, red, blue and green color represents the treatment of dense grass with slope of 1°, dense grass with slope of 2°, sparse grass with slope of 1° and sparse grass with slope of 2°, respectively. The following figures also follow this rule.)
Fig. 3 Close-up of the initial runoff rates for various conditions

Fig. 4 Measured and predicted solute concentrations
**Fig. 5** Conceptual model for solute transport at a given time

**Fig. 6** Solute transport rates for measured and predicted results
**Fig. 7** Solute concentrations for different $h_0$

**Fig. 8** Solute transport rates for different $h_0$
Fig. 9 Solute concentrations for different $I$

Fig. 10 Solute transport rates for different $I$
Fig. 11 Conceptual schematics of the solute transport processes for (a) uncovered soil (from Gao et al., 2004); (b) vegetated area

Table 1 Collection of runoff samples

Table 2 Estimated values of $\alpha$, $t_0$ and $t_c$

Table 3 Estimated values of $h_0$, $C_0$ and $h_{0c}$

Table 4 Measured and estimated maximum solute transport rates