Investigating fast flow of the Greenland Ice Sheet
Characterising the internal and basal environment of Store Glacier using phase-sensitive radar

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This dissertation is submitted for the degree of
Doctor of Philosophy

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Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. This dissertation contains less than 275 numbered pages, of which not more than 225 pages are text, appendices, illustrations, tables, equations, and bibliography.

Young Tun Jan
July 2018
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Abstract

The dynamic response of a faster-flowing Greenland Ice Sheet to climate change is modulated by feedbacks between ice flow and surface meltwater delivery to the basal environment. While supraglacial melt processes have been thoroughly examined and are well constrained, the response of the englacial and subglacial environment to these seasonal perturbations still represent the least-studied, understood, and parameterised processes of glacier dynamics due to a paucity of direct observation. To better understand these processes in the wake of a changing climate, novel in-situ geophysical experiments were undertaken on Store Glacier in west Greenland to quantify rates of englacial deformation and basal melting. The records produced from these experiments yield new insights into the thermodynamic setting of a major outlet glacier, and the physical mechanisms underlying and resulting from fast glacier motion. The deployment of autonomous phase-sensitive radio-echo sounders (ApRES) 30 km from the calving terminus of Store Glacier between 2014 and 2016 revealed high rates of both englacial deformation and basal melting, driven primarily by the dynamic response of the basal hydrological system to seasonal surface meltwater influxes. Thermodynamic modelling of this process revealed a convergence of large-scale basal hydrological pathways that aggregated large amounts of water towards the field site. The warm, turbulent water routed from surface melt contained and dissipated enough energy at the ice-bed interface to explain the observed high melt rates. Simultaneously, changes in the local strain field, involving seasonal variations in the morphology of internal layers, were found to be the result of far-field perturbations in downstream ice flow which propagated tens of kilometres upglacier through longitudinal stress coupling. When observed in multiple dimensions, the layer structure revealed complex internal reflection geometries, demonstrating ApRES as not just a monitor of depth changes in ice thickness, but also as an imaging instrument capable of characterising the subsurface environment within and beneath ice sheets. Altogether, the observations and analyses comprising this thesis provide new and significant insight and understanding into the structural, thermal, and mechanical processes tied to Store Glacier and its fast, complex, and dynamic ice flow.
# Table of contents

List of figures xiii  
List of tables xvii  
Nomenclature xix  

1 Introduction 1  
1.1 Response of the Greenland Ice Sheet to climate change 1  
1.2 Aim & objectives 4  
1.3 Thesis structure 5  

2 Background 7  
2.1 Mass balance of the Greenland Ice Sheet 7  
2.2 Glacier and ice sheet hydrology 8  
2.2.1 Subglacial controls on ice sheet flow 10  
2.3 Thermomechanical regimes under ice sheets 19  
2.3.1 Basal heat budget 19  
2.3.2 Enthalpy of temperate ice 21  
2.3.3 Viscous heat dissipation 22  

3 Development and application of ice-penetrating radar 23  
3.1 Methodologies of investigations 23  
3.2 Application of radar to the Greenland and Antarctic Ice Sheets 25  
3.2.1 Development of stationary phase-sensitive radar 30  

4 Theory 33  
4.1 Changes in ice thickness 33  
4.1.1 Principles of stress and strain 33  
4.1.2 Mass conservation of an ice column 36
4.2 Principles of radar signal propagation ........................................... 38
  4.2.1 The radar equation ............................................................. 39
  4.2.2 Propagation of electromagnetic waves in ice ......................... 40
4.3 Phase-sensitive FMCW radar signal processing ............................... 44
  4.3.1 Coarse range equations ...................................................... 45
  4.3.2 Fine range equations ......................................................... 46
  4.3.3 Determination of total range ............................................. 47
4.4 Antenna array theory ............................................................. 47
  4.4.1 Derivation of array factor .................................................. 47
  4.4.2 Fourier analysis of the array factor ..................................... 52

5 Study area and radar deployment ................................................... 55
  5.1 Introduction ............................................................................. 55
  5.2 The Subglacial Access and Fast Ice Research Experiment (SAFIRE) ... 56
  5.3 Study area .............................................................................. 59
    5.3.1 Store Glacier .................................................................. 59
    5.3.2 S30 ............................................................................... 60
5.4 ApRES and antenna instrumentation and deployment ................. 64
    5.4.1 ApRES instrumentation and operation .................................. 64
    5.4.2 ApRES array deployment .................................................. 65
    5.4.3 Antenna instrumentation and design .................................... 68

6 Seasonal evolution of vertical deformation ........................................ 71
  6.1 Summary ............................................................................... 72
  6.2 Introduction ............................................................................ 73
  6.3 Methods ................................................................................ 75
    6.3.1 ApRES measurements of vertical deformation ..................... 75
    6.3.2 GPS measurements of surface ice motion ......................... 90
    6.3.3 Borehole measurements of basal conditions ...................... 90
  6.4 Results ................................................................................ 92
  6.5 Possible controls over variations in vertical strain ..................... 98
    6.5.1 (a) Are variations in vertical velocity governed by local variations in topography? ........................................... 100
    6.5.2 (b) Are variations in vertical velocity governed by local variations in basal traction? ........................................... 102
    6.5.3 (c) Are variations in vertical velocity governed by far-field variations in glacier dynamics? ................................... 103
## Table of contents

6.6 Discussion ................................................................. 106
6.7 Conclusions ............................................................... 109
A Appendix: Temperature dependence of signal propagation .............. 112

7 Sustained melting under the Greenland Ice Sheet ......................... 115
7.1 Summary ................................................................. 116
7.2 Introduction ............................................................. 116
7.3 Methods ................................................................. 118
  7.3.1 Radar measurements of ice vertical motion and ablation .......... 118
  7.3.2 Decomposing the basal heat budget ................................ 120
7.4 Results ................................................................. 125
  7.4.1 Measurements of vertical strain rates and basal melt rates .... 125
  7.4.2 Discrepancies between measured and modelled results ......... 126
  7.4.3 Viscous heat dissipation as a missing heat source ............. 129
  7.4.4 Spatial distribution of basal melt rates ......................... 130
7.5 Discussion and conclusions ........................................... 131
A Appendix: Extended methods ............................................ 135
B Appendix: Kinetics of glacial conduit heat exchange .................... 138

8 Imaging glacier internal and basal geometry ................................ 141
8.1 Summary ................................................................. 141
8.2 Introduction ............................................................. 142
8.3 Methods ................................................................. 144
  8.3.1 Radar and antenna array architecture .......................... 144
  8.3.2 Digital signal processing ........................................ 144
  8.3.3 2-D and 3-D vertical section processing ........................ 147
8.4 Identification and interpretation of internal layers .................... 148
  8.4.1 Vertical stratigraphy of the ice column ......................... 148
  8.4.2 Simulation of the vertical section ............................... 150
  8.4.3 Interpretation of internal layer slopes .......................... 153
8.5 Resolving the basal layer ............................................. 156
8.6 Suggestions for future deployments .................................. 157
8.7 Summary and conclusions ............................................. 158
A Appendix: Extended methods ............................................ 159
B Appendix: Simulations of the antenna radiation pattern ................ 161

9 Conclusions .................................................................. 167
9.1 Synthesis of research findings ............................................. 167
9.2 Perspectives on future research ........................................... 169
  9.2.1 Identification of and deformation in pre-Holocene ice ............ 169
  9.2.2 The basal hydrological and thermal regime .......................... 170
  9.2.3 Influence of surface crevasses on horizontal strain rates .......... 171
  9.2.4 Resilience of ApRES deployments to adverse conditions ........ 174
  9.2.5 Imaging the basal environment ...................................... 175
9.3 The RESPONDER project .................................................. 177
9.4 Concluding remarks ..................................................... 177

References 179

Appendix A GPS measurements of horizontal deformation 207
  A.1 Calculation of horizontal strain rates .............................. 208
  A.2 Comparison of horizontal and vertical strain rates ............... 210
List of figures

1.1 Mass change of the Greenland and Antarctic Ice Sheets 2
1.2 Topographic controls on marine ice sheet instability 4
2.1 Dynamic changes in mass and flow of the Greenland Ice Sheet 9
2.2 Possible configurations of subglacial hydrological systems 12
2.3 Seasonal development of melt-induced ice velocity variations 14
2.4 Evolution of the subglacial hydrological system 16
2.5 GPS observations of direct coupling between melt and ice flow 18
2.6 Amplified melt and flow driven by an intense rainfall event 20
3.1 SPRI-NSF-TUD radargram of the Antarctic Ice Sheet 24
3.2 Internal layering classification from Antarctic Ice Sheet radargrams 27
3.3 Bed topography of the Antarctic and Greenland Ice Sheets 28
3.4 3-D bed topography at the GRIP ice core study site 29
3.5 Generations of the ApRES instrument 31
4.1 Mass conservation of the ice column 38
4.2 Variation of ice permittivity with temperature and frequency 42
4.3 Wave retrieval of a linear antenna array 49
4.4 Controls of array factor parameters on antenna radiation pattern 53
5.1 Surface velocities and bed topography of Store Glacier 57
5.2 Map of study area (S30) 59
5.3 Surface and bed topography profiles at S30 60
5.4 2014 and 2016 meteorological and borehole records at S30 62
5.5 Temperature profile of S30 63
5.6 Surface conditions of ApRES array deployments 66
5.7 Antenna array deployment configurations 67
5.8 Radiation patterns for a cavity-backed bowtie antenna 69
<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.9</td>
<td>Assembly of the bowtie antenna</td>
<td>69</td>
</tr>
<tr>
<td>6.1</td>
<td>ApRES and GPS locations at S30</td>
<td>75</td>
</tr>
<tr>
<td>6.2</td>
<td>Summary of phase-sensitive FMCW range processing steps</td>
<td>77</td>
</tr>
<tr>
<td>6.3</td>
<td>Phase processing of two sequential ApRES samples</td>
<td>81</td>
</tr>
<tr>
<td>6.4</td>
<td>Modelling ApRES-measured vertical velocity profiles</td>
<td>85</td>
</tr>
<tr>
<td>6.5</td>
<td>Internal reflector pair cross-correlation coherence strength</td>
<td>89</td>
</tr>
<tr>
<td>6.6</td>
<td>ApRES-observed vertical velocities</td>
<td>94</td>
</tr>
<tr>
<td>6.7</td>
<td>ApRES-observed vertical velocity errors</td>
<td>95</td>
</tr>
<tr>
<td>6.8</td>
<td>ApRES-modelled vertical velocities</td>
<td>96</td>
</tr>
<tr>
<td>6.9</td>
<td>ApRES-modelled vertical strain rates</td>
<td>97</td>
</tr>
<tr>
<td>6.10</td>
<td>Time evolution of monthly-averaged ice column vertical strain</td>
<td>99</td>
</tr>
<tr>
<td>6.11</td>
<td>Correlation between vertical velocities and basal parameters</td>
<td>101</td>
</tr>
<tr>
<td>6.12</td>
<td>Downstream 2014–2015 TerraSAR-X surface velocities</td>
<td>104</td>
</tr>
<tr>
<td>6.13</td>
<td>Surface velocities and melt rates at S30</td>
<td>106</td>
</tr>
<tr>
<td>7.1</td>
<td>Phase processing of two sequential ApRES samples</td>
<td>119</td>
</tr>
<tr>
<td>7.2</td>
<td>Vertical strain and basal melt time series</td>
<td>125</td>
</tr>
<tr>
<td>7.3</td>
<td>Time series of supraglacial and subglacial records</td>
<td>127</td>
</tr>
<tr>
<td>7.4</td>
<td>Comparison of modelled and measured basal melt rates</td>
<td>128</td>
</tr>
<tr>
<td>7.5</td>
<td>Spatial distribution of water accumulation and basal melt rates</td>
<td>132</td>
</tr>
<tr>
<td>7.A</td>
<td>Time-averaged vertical displacement and strain rate at S30</td>
<td>137</td>
</tr>
<tr>
<td>8.1</td>
<td>Exact geometrical correction based on path lengths (EGCPL)</td>
<td>145</td>
</tr>
<tr>
<td>8.2</td>
<td>Nomenclature of 2-D ApRES cross-sections</td>
<td>146</td>
</tr>
<tr>
<td>8.3</td>
<td>2-D vertical sections beneath S30 field ApRES deployments</td>
<td>149</td>
</tr>
<tr>
<td>8.4</td>
<td>Radiation patterns of S30 field ApRES deployments</td>
<td>151</td>
</tr>
<tr>
<td>8.5</td>
<td>Simulation of synthetic vertical section using deployment parameters</td>
<td>152</td>
</tr>
<tr>
<td>8.6</td>
<td>Scattering properties for internal and basal layers</td>
<td>153</td>
</tr>
<tr>
<td>8.7</td>
<td>3-D representation of internal layers beneath S30 field deployments</td>
<td>154</td>
</tr>
<tr>
<td>8.8</td>
<td>Reconstructed glacier vertical section emphasising bed clarity</td>
<td>157</td>
</tr>
<tr>
<td>8.A</td>
<td>Array factor and reconstructed sections varying HPBW</td>
<td>162</td>
</tr>
<tr>
<td>8.B</td>
<td>Array factor and reconstructed sections varying element separation</td>
<td>163</td>
</tr>
<tr>
<td>8.C</td>
<td>Array factor and reconstructed vertical varying number of elements</td>
<td>164</td>
</tr>
<tr>
<td>9.1</td>
<td>GPS- and ApRES-derived near-surface vertical strain rates</td>
<td>172</td>
</tr>
<tr>
<td>9.2</td>
<td>Lab testing of ApRES system temperature effect on reflector range</td>
<td>175</td>
</tr>
<tr>
<td>9.3</td>
<td>Field testing of ApRES system temperature effect on reflector range</td>
<td>176</td>
</tr>
</tbody>
</table>
9.4 Proposed study sites for the RESPONDER project. . . . . . . . . . . . . 178
A.1 Location of GPS deployments . . . . . . . . . . . . . . . . . . . . . . . . . . 207
A.2 Calculation of horizontal strain rates . . . . . . . . . . . . . . . . . . . . . . 209
List of tables

5.1 Metadata on deployed borehole instruments . . . . . . . . . . . . . . . . . . . . . . . 58
5.2 Metadata on deployed ApRES instruments . . . . . . . . . . . . . . . . . . . . . . . 64
7.1 Parameters used in basal melt rate models . . . . . . . . . . . . . . . . . . . . . . . 121
# Nomenclature

## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADC</td>
<td>Analogue-to-digital converter</td>
</tr>
<tr>
<td>AWS</td>
<td>Automatic weather station</td>
</tr>
<tr>
<td>BAS</td>
<td>British Antarctic Survey</td>
</tr>
<tr>
<td>BMR</td>
<td>Basal melt rate</td>
</tr>
<tr>
<td>COF</td>
<td>Crystal orientation fabric</td>
</tr>
<tr>
<td>CTS</td>
<td>Transition between cold and temperate ice</td>
</tr>
<tr>
<td>DDS</td>
<td>Direct digital synthesis</td>
</tr>
<tr>
<td>DOY</td>
<td>Day of year</td>
</tr>
<tr>
<td>DTU</td>
<td>Technical University of Denmark</td>
</tr>
<tr>
<td>EGCPL</td>
<td>Exact geometrical correction based on path lengths</td>
</tr>
<tr>
<td>ELA</td>
<td>Equilibrium line altitude</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier transform</td>
</tr>
<tr>
<td>FMCW</td>
<td>Frequency-modulated continuous-wave</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>GRIP</td>
<td>Greenland Ice Core Project</td>
</tr>
<tr>
<td>HF</td>
<td>High frequency</td>
</tr>
<tr>
<td>HPBW</td>
<td>Half-power beamwidth</td>
</tr>
<tr>
<td>HWT</td>
<td>Holocene-Wisconsin transition</td>
</tr>
<tr>
<td>KU</td>
<td>University of Kansas</td>
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</tbody>
</table>
MIMO  Multiple-input multiple-output
NERC  National Environment Research Council [United Kingdom]
NSF  National Science Foundation [United States of America]
PCB  Printed circuit board
PMP  Pressure melting point
RADAR  Radio Detection and Ranging
RCP  Representative concentration pathway
RES  Radio-echo sounding
RESPONDER  Resolving subglacial properties, hydrological networks and dynamic evolution of ice flow on the Greenland Ice Sheet
RF  Radio frequency
RMS  Root-mean-square
SAFIRE  Subglacial Access and Fast Ice Research Experiment
SAR  Synthetic aperture radar
SNR  Signal-to-noise ratio
SPRI  Scott Polar Research Institute [United Kingdom]
Tx/Rx  Transmitting/Receiving [antenna]
UHF  Ultra high frequency
UTC  Coordinated Universal Time
VHD  Viscous heat dissipation
VHF  Very high frequency
VSR  Vertical strain rate

Universal Symbols

\(<\vec{r}, \theta, \varphi>\)  Components of spherical coordinate system
\(<x, y, z>\)  Components of Cartesian coordinate system \((x\ \text{aligned to flow})\)
\(\sigma\)  Standard deviation
\(c\)  Speed of light
### Symbols for Glaciological Applications

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Flotation fraction at ice bed</td>
<td></td>
</tr>
<tr>
<td>$\dot{\epsilon}_{ij}$</td>
<td>Strain tensor along the $ij$ direction</td>
<td></td>
</tr>
<tr>
<td>$\dot{m}_b$</td>
<td>Vertical mass flux at glacier bed (rate of basal freeze-on/melt)</td>
<td></td>
</tr>
<tr>
<td>$\dot{m}_s$</td>
<td>Vertical mass flux at glacier surface (rate of surface accumulation/ablation)</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>Magnetic permeability</td>
<td>N A $^{-2}$</td>
</tr>
<tr>
<td>$\nu_b$</td>
<td>Effective porewater thickness at the ice-bed interface</td>
<td>m</td>
</tr>
<tr>
<td>$\omega_w$</td>
<td>Water moisture fraction of basal ice</td>
<td></td>
</tr>
<tr>
<td>$\phi_h$</td>
<td>Hydropotential</td>
<td>Pa</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Density of ice</td>
<td>917 kg m $^{-3}$</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>Density of water</td>
<td>1000 kg m $^{-3}$</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Conductivity</td>
<td>S m $^{-1}$</td>
</tr>
<tr>
<td>$\sigma_{ij}$</td>
<td>Stress tensor along the $ij$ direction</td>
<td></td>
</tr>
<tr>
<td>$\tau_b$</td>
<td>Basal shear stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$\Theta_i$</td>
<td>Englacial ice temperature</td>
<td>°C</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Friction angle</td>
<td>°</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Principal axis of longitudinal strain tensor</td>
<td>°</td>
</tr>
<tr>
<td>$\vec{u}$</td>
<td>Ice velocity vector</td>
<td>m s $^{-1}$</td>
</tr>
<tr>
<td>$A$</td>
<td>Creep parameter</td>
<td></td>
</tr>
<tr>
<td>$c_0$</td>
<td>Apparent cohesion</td>
<td></td>
</tr>
<tr>
<td>$C_i$</td>
<td>Heat capacity of ice</td>
<td>2009 J kg $^{-1}$ K $^{-1}$</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific heat of water</td>
<td>4184 J kg $^{-1}$ K $^{-1}$</td>
</tr>
<tr>
<td>$C_T$</td>
<td>Clausius-Clapeyron slope</td>
<td>$8.6 \times 10^{-8}$ K Pa $^{-1}$</td>
</tr>
<tr>
<td>$d$</td>
<td>Path depth</td>
<td>m</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s Modulus</td>
<td>Pa</td>
</tr>
<tr>
<td>$H$</td>
<td>Enthalpy</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
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<td>--------</td>
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<td>-------</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Thermal conductivity of ice</td>
<td>$2.14 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$</td>
</tr>
<tr>
<td>$k_w$</td>
<td>Thermal conductivity of water</td>
<td>$0.561 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$</td>
</tr>
<tr>
<td>$l$</td>
<td>Path length</td>
<td>m</td>
</tr>
<tr>
<td>$L_f$</td>
<td>Latent heat of fusion</td>
<td>$3.34 \times 10^5 \text{ J kg}^{-1}$</td>
</tr>
<tr>
<td>$n_G$</td>
<td>Creep exponent [Glen’s flow law]</td>
<td></td>
</tr>
<tr>
<td>$q_f$</td>
<td>Frictional heat at the ice-bed interface $[\tau_b \bar{u}_b]$</td>
<td>W m$^{-2}$</td>
</tr>
<tr>
<td>$q_G$</td>
<td>Geothermal (lithospheric) heat flux</td>
<td>W m$^{-2}$</td>
</tr>
<tr>
<td>$q_i$</td>
<td>Convective heat flux into ice $[k_i \frac{\partial \Theta_i}{\partial z}]$</td>
<td>W m$^{-2}$</td>
</tr>
<tr>
<td>$q_{VHD}$</td>
<td>Heat flux from viscous heat dissipation</td>
<td>W m$^{-2}$</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>°C, K</td>
</tr>
<tr>
<td>$w$</td>
<td>Regression weighting factor</td>
<td></td>
</tr>
<tr>
<td>$W_b$</td>
<td>Energy balance at glacier bed</td>
<td>W m$^{-2}$</td>
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**Symbols for Radar Applications**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{fg}$</td>
<td>Cross-correlation coherence of two spectra $f$ and $g$</td>
<td></td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Cross-correlation lag</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
<td>m</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>Wavelength at centre frequency</td>
<td>m</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Angular centre frequency of the deramped chirp</td>
<td>rad s$^{-1}$</td>
</tr>
<tr>
<td>$\omega_d$</td>
<td>Angular frequency of the deramped chirp</td>
<td>rad s$^{-1}$</td>
</tr>
<tr>
<td>$\phi_d$</td>
<td>Signal deramped phase</td>
<td>rad</td>
</tr>
<tr>
<td>$\phi_r$</td>
<td>Signal deramped phase at bin centre</td>
<td>rad</td>
</tr>
<tr>
<td>$\phi_{R_x}$</td>
<td>Signal deramped phase at receiving element</td>
<td>rad</td>
</tr>
<tr>
<td>$\phi_{T_z}$</td>
<td>Signal deramped phase at transmitting element</td>
<td>rad</td>
</tr>
<tr>
<td>$\sigma^0$</td>
<td>Target scattering coefficient</td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>Two-way travel time from transmitter to receiver</td>
<td>s</td>
</tr>
<tr>
<td>$\tau_r$</td>
<td>Relaxation time</td>
<td>s</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Relative complex dielectric permittivity ($\varepsilon = \varepsilon' + j\varepsilon''$)</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
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<td>-------</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>Dielectric constant of co-axial cable</td>
<td>2.2</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>Dielectric constant of ice</td>
<td>3.10–3.18</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>Dielectric constant of water</td>
<td>80</td>
</tr>
<tr>
<td>$\zeta_c$</td>
<td>Physical cable length</td>
<td>m</td>
</tr>
<tr>
<td>$\zeta_i$</td>
<td>Ice-equivalent cable length</td>
<td>m</td>
</tr>
<tr>
<td>$B$</td>
<td>System bandwidth</td>
<td>Hz</td>
</tr>
<tr>
<td>$c_i$</td>
<td>Speed of light within ice</td>
<td>$3 \times 10^8$ m s$^{-1}$</td>
</tr>
<tr>
<td>$f$</td>
<td>Periodic frequency</td>
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<td>$f_d$</td>
<td>Deramped frequency</td>
<td>Hz</td>
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<td>$K$</td>
<td>Chirp gradient</td>
<td>rad s$^{-2}$</td>
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<tr>
<td>$L_a$</td>
<td>Attenuation losses within medium</td>
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<tr>
<td>$L_s$</td>
<td>System losses within radar</td>
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<tr>
<td>$p$</td>
<td>Pad factor</td>
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<tr>
<td>$P_r$</td>
<td>Received power</td>
<td>W</td>
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<tr>
<td>$P_t$</td>
<td>Transmitted power</td>
<td>W</td>
</tr>
<tr>
<td>$R$</td>
<td>Total range to target</td>
<td>m</td>
</tr>
<tr>
<td>$R_c$</td>
<td>Coarse range to target</td>
<td>m</td>
</tr>
<tr>
<td>$R_f$</td>
<td>Fine range to target (within coarse range bin)</td>
<td>m</td>
</tr>
<tr>
<td>$T$</td>
<td>Pulse (total duration)</td>
<td>s</td>
</tr>
<tr>
<td>$t$</td>
<td>Instantaneous time</td>
<td>s</td>
</tr>
</tbody>
</table>

**Symbols for Antenna Applications**

$<\theta, \psi>$ Components of angular swath (in relation to the flow direction) $^\circ$

$<k,l>$ Planar element number of $K \times L$ elements

$\delta$ Element spacing m

$A$ Signal amplitude dB

$F$ Received signal of overall array

$F_a$ Array factor
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_e$</td>
<td>Element factor</td>
<td></td>
</tr>
<tr>
<td>$G$</td>
<td>Antenna gain</td>
<td>dB</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Free space wave number</td>
<td></td>
</tr>
<tr>
<td>$l$</td>
<td>Path length</td>
<td>m</td>
</tr>
<tr>
<td>$S_k$</td>
<td>Linear array vector to element number $k$</td>
<td></td>
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</tbody>
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Chapter 1

Introduction

1.1 Rationale: Response of the Greenland Ice Sheet to contemporary climate change

Since the early 1900s, the global mean (eustatic) sea level began to dramatically rise above the late Holocene background rate (Gehrels and Woodworth, 2013; Lambeck et al., 2014) at a rate of \( \sim 1.7 \text{ mm a}^{-1} \) (Masters et al., 2012), and is currently \( \sim 120 \text{ m} \) higher than the predicted minimum mean sea level during the end of the last Ice Age (\( \sim 21000 \text{ years ago; Church et al., 2013} \)). Thermal expansion of the oceans and melting of glaciers are the two main contributors to eustatic sea level rise over the past century, with each accounting for about half the observed sea level rise, and each consistently increasing with the steady rise in the global mean temperature (Church et al., 2013). While the contributions from ocean thermal expansion are well-constrained through good agreements between observed and modelled values (Kirtman et al., 2013), the contributions from melting glaciers are less certain, due to a combination of a lack of long-term continental-scale observations of glacier mass balance and the complex, stochastic response of glaciers to climatic and oceanic forcings (Bamber and Aspinall, 2013).

Greenland and Antarctica, where the Earth’s two largest ice sheets are located, make up the vast majority of the Earth’s ice and hold nearly 80% of the fresh water in the world. These ice sheets consist of vast amounts of slow-moving ice drained by fast-
moving “ice-walled ice streams” and “rock-walled outlet glaciers” that transport ice from the continental interior directly to the surrounding coastline or directly into the ocean (Vaughan et al., 2013). If these ice sheets completely melted, they would contribute 7.4 m and 58.3 m of eustatic sea level rise respectively (Church et al., 2013).

Since the advent of continental-scale observations of ice sheet mass balance from spaceborne satellites in 1992, it was discovered that both the Greenland and Antarctic ice sheets were losing mass at an ever-increasing rate (Bamber and Aspinall, 2013; Rignot et al., 2011; Shepherd et al., 2012; Shepherd and Wingham, 2007; Zwally et al., 2005). While the regional response of these ice sheets to changes in near-surface meteorology have been well-documented (e.g. Drewry and Morris, 1992; Huybrechts and de Wolde, 1999; Noël et al., 2017; Steig et al., 2009), there is growing evidence that warming oceans also induce high rates of melting and iceberg production around the ice sheet margins (e.g. Holland et al., 2008; Jenkins, 2011; Seale et al., 2011).

![Cumulative changes in the mass of the Greenland and Antarctic Ice Sheets](image)

**Figure 1.1** Cumulative changes in the mass of the Greenland and Antarctic Ice Sheets, determined from a reconciliation of multiple measurements acquired using multiple independent techniques. From Shepherd et al. (2012).

Although the average accumulation rate over the Greenland Ice Sheet (~0.3 m a⁻¹) is roughly twice that of Antarctica (Cullather and Bosilovich, 2012; Koenig et al., 2016), consistent summer melting regularly occurs over half of the ice sheet surface, where much
of this melting is transported to the surrounding ocean (Rignot and Thomas, 2002). Consequently, the Greenland Ice Sheet is losing mass and contributing to sea level rise at a significantly faster rate (Figure 1.1; Vaughan et al., 2013), with the majority of mass loss attributed to low-level marine-terminating glaciers (Jensen et al., 2016; Luthcke et al., 2006; Pritchard et al., 2009) with flux rates higher than 200 Gt a$^{-1}$ (Bevan et al., 2012; Mouginot et al., 2013; Rignot and Kanagaratnam, 2006). At a finer scale, the local response of these glaciers to atmospheric and marine forcings is highly variable, where adjacent glaciers often exhibit asynchronous behaviour on multiple time scales (Carr et al., 2013; Felikson et al., 2017; Moon et al., 2012, 2014). This variability can often be attributed to the local bed topography that designates the locations where glacier ice fronts, or grounding lines, are stable to atmospheric and oceanic perturbations (Morlighem et al., 2016). If the grounding line rests on a reversed bed slope, where the bed elevation becomes deeper inland, the glacier may undergo rapid and runaway retreat (Figure 1.2; Vaughan and Arthern, 2007).

While detailed surface observations are becoming routine, conditions at the glacier bed remain far more difficult to observe (Joughin et al., 2009; MacGregor et al., 2016). Hence, there is a paucity of data characterising basal processes, which include (i) sliding of ice over soft (sediment) as well as hard (bedrock) glacier beds; (ii) the deformation of basal ice; (iii) hydrological controls on ice flow; and (iv) the effect of water flowing in and within different types of hydrological systems. In particular, the basal motion characterising Greenland’s fast-flowing outlet glaciers is virtually unknown as very few studies have attempted to directly observe the basal environment (Andrews et al., 2014; Doyle et al., 2018; Fountain and Walder, 1998; Lüthi et al., 2002; Meierbachtol et al., 2013; Ryser et al., 2014b). While it is well known that basal drag exerts a fundamental control on the flow of these highly dynamic glaciers, the nature of this resistive force is still not well-understood (Morlighem et al., 2010). Therefore, it is imperative to better understand the roles that englacial ice deformation and basal sliding play, with regard to the development of parameterisations needed in order to improve numerical ice sheet models (Nowicki et al., 2013).
Figure 1.2 Schematics showing the importance of the basal topography on the stability of marine-terminating glaciers. In the top panel, the ice sheet is in equilibrium; influx from snowfall ($q$) is balanced by outflow. In the lower panel, a small retreat provokes changes in both the influx and the outflow, promoting further retreat by the instability of the grounding line. The profile shown is based on Thwaites Glacier, West Antarctica. From Vaughan and Arthern (2007).

1.2 Aim & objectives

The fundamental aim of this thesis is to quantify englacial ice deformation rates and basal melting for a fast-flowing outlet glacier of the Greenland Ice Sheet. To achieve this aim, this thesis employed the use of an autonomous phase-sensitive radio-echo sounder (ApRES; Chapter 5.4), which offers quasi-monostatic applications in 1-D as well as a multiple-input multiple-output (MIMO) configuration for 2-D and potentially 3-D radar imaging. Both types of configurations were used in this study through three deployments, which obtained high-resolution records between 2014 and 2016 to shed new light on the physical processes and thermodynamics underlying fast glacier flow. In addition, deployment of
ApRES units was contemporaneous with the installation of supraglacial, englacial, and subglacial sensors, which provided a rare opportunity to directly and comprehensively link radar observations to environmental forcings. As englacial and basal processes represent some of the least-studied and least-understood processes of glacier motion, the insights gained from this project provide new observational understanding of fast glacier flow in outlets draining the Greenland Ice Sheet. The continued development of the ApRES over the duration of this project furthermore provided an opportunity to investigate the deployments’ design from an antenna engineering perspective. Therefore, the overall aim can be separated into the following component objectives:

1. To test the overall capability of ApRES on challenging (i.e. fast-flowing, crevassed, high-melt) regions of ice sheets;
2. To use ApRES to estimate englacial ice deformation rates in a region of fast glacier flow;
3. To use ApRES to quantify rates of basal melting in a hydrologically-active region within the ablation zone;
4. To use ApRES and models to quantify the basal energy budget of a fast-flowing outlet glacier, including energy transferred from surface to the bed;
5. To investigate the relevant environmental forcings that drive variations in englacial and basal ApRES observations;
6. To image the englacial and basal environments of ice sheets in multiple dimensions;
7. To evaluate potential configurations of multiple-element antenna arrays to best image the ice sheet subsurface; and
8. To further knowledge of the glacial environment and glaciers affected by climate change.

1.3 Thesis structure

This thesis is broken into nine chapters, including this introduction. Specifically:
Chapter 1 frames and poses the questions that this thesis aims to address;

Chapter 2 synthesises the current knowledge regarding ice sheet flow, with an emphasis on studies conducted on the Greenland Ice Sheet;

Chapter 3 reviews the development and application of ice-penetrating radar;

Chapter 4 provides the relevant theory regarding force mechanics and behind the application of ApRES to achieve the project objectives;

Chapter 5 introduces the study area, Store Glacier and S30, as well as the overarching experiment design that laid the basis for the following 3 chapters (6-8);

Chapter 6 investigates the spatial and temporal variability seen in force balance, particularly variations in full-column vertical velocities at the study area and its mediation from dynamics occurring across the wider catchment;

Chapter 7 contextualises records of high basal melt with supraglacial, englacial, and subglacial measurements at the study area and across the wider catchment;

Chapter 8 investigates the 2- and 3-dimensional imaging capabilities of the ApRES through simulation of the deployment configurations; and

Chapter 9 concludes the thesis with final reflections and offers suggestions into future research using ApRES.
Chapter 2

Background

2.1 Mass balance of the Greenland Ice Sheet

The Greenland Ice Sheet is the largest reservoir of freshwater in the Northern Hemisphere. Since the early 1990s, mass loss from the Ice Sheet has contributed approximately 8 mm, or 10%, of the global mean sea level rise (Vaughan et al., 2013). Within this period, the majority of this loss occurred after 1998, when the Ice Sheet recorded a persistently negative mass balance, even despite a temporary rebound in 2013 (van den Broeke et al., 2016). In Greenland, the ice mass loss can be approximately partitioned into equal amount between surface mass balance and discharge at the grounding line (van den Broeke et al., 2009). However, recent warmer atmospheric conditions, including several exceptionally hot summers (Fettweis et al., 2013; Hanna et al., 2008, 2013) have triggered extensive thinning of the ice sheet via surface melt runoff, and as a result, between 2009 and 2012, surface mass balance has increased its contribution up to 70% of the total ice sheet imbalance (Enderlin et al., 2014; McMillan et al., 2016).

The majority of mass loss within the Greenland Ice Sheet has been found to occur at lower altitudes along the coastline (Vaughan et al., 2013), with the greatest losses dominating within the southeast and central west regions. More recently, this trend also includes the northwest margin, where glaciers are now undergoing sustained dynamic thinning due to regional warming (Khan et al., 2015, 2010; Velicogna et al., 2014). Here, the relatively
fast-flow of marine-terminating glaciers, which discharge significant amounts of ice into
the surrounding oceans, negates the marginally positive mass gain observed through
surface accumulation at high altitudes (Figure 2.1; Rignot and Kanagaratnam, 2006).
In contrast to the rapidly changing marine margins of the ice sheets, land-terminating
regions of the Ice Sheet are changing more slowly, with some regions even experiencing a
slowdown in ice flow due to the evolution of an increasingly efficient subglacial drainage
system (Tedstone et al., 2015).

At a finer scale, the dynamics of marine-terminating glaciers show spatial and temporal
heterogeneity in terminus retreat, surface elevation, and modulation in flow that ultimately
influence when and where mass is lost from the ice sheet (Felikson et al., 2017; Moon et al.,
2012; van den Broeke et al., 2016). Often, neighbouring glaciers exhibit asynchronous
dynamic behaviour in retreat or acceleration (Moon et al., 2012). While the observed
differences in terminus stability have largely been attributed to the fjord geometry
surrounding each glacier front (Bartholomaus et al., 2016; Morlighem et al., 2015; Todd
and Christoffersen, 2014), variations in flow velocities appear to be controlled by the
evolution of glacier-specific subglacial drainage networks in response to surface melt
(Moon et al., 2014).

2.2 Glacier and ice sheet hydrology

Significant amounts of surface melting occur over extensive areas of the Greenland Ice
Sheet, often forming vast supraglacial lakes (e.g. Hoffman et al., 2011) or entering the
subsurface firn aquifer (e.g. Koenig et al., 2016). During the melt season, these reservoirs
of stored water, along with additional melt from increased temperatures, often propagate
to the subglacial environment via fractures and moulins, modulating ice sheet velocity
(Bell, 2008). Until recently (up to and including the Third Assessment Report of the
Intergovernmental Panel on Climate Change (Church et al., 2001)), it was assumed
that the Greenland and Antarctic Ice Sheets were insensitive to climate warming, with
response time scales on the order of 10,000–100,000 years. More recent studies have
counteracted this argument, and have instead suggested that the Earth’s major ice sheets
respond much more rapidly to changes in both the surrounding atmosphere (Doyle et al., 2015; Noël et al., 2017; Van Tricht et al., 2016) and ocean (Cook et al., 2016; Howat et al., 2010; Seale et al., 2011). In 2002, Zwally et al. first suggested that the flow of ice sheets was modulated by surface meltwater routed through the entirety of the ice column within a time scale of a few hours, lubricating the glacier bed, shattering previous assumptions that the basal environment was completely isolated from outside forcings. While this phenomenon has been well-documented on smaller Alpine valley glaciers (e.g. Hubbard and Sharp, 1995; Nienow et al., 2005), Zwally et al.’s observations of a direct correlation between surface melt intensity and acceleration of summer flow velocities up to 25% above the winter mean at Swiss Camp, located 35 km from the margin of the Greenland Ice Sheet (69°34’ N, 49°19’ W), brought to light the direct and almost-instantaneous response of larger ice sheets to climate warming.

Although the dynamics of slow-moving ice sheets and of ice shelves are relatively well-known through both observations and models, there still remain significant gaps in
understanding the dynamics of fast-moving ice streams and outlet glaciers, particularly regarding their basal properties, which are either completely unknown or poorly constrained still to this day (Church et al., 2013). Considering that marine-terminating outlet glaciers drain 88% of the Greenland Ice Sheet and are responsible for at least half of the ice sheet’s net annual mass loss, which at present is ~200 km$^3$ a$^{-1}$ (Rignot and Mouginot, 2012), it is imperative to better characterise the basal control on fast ice flow, as well as the sensitivity of these glaciers to atmospheric and oceanic forcings.

While the characterisation of ice flow near the calving front of a marine-terminating glacier is complicated by the glacier’s response to oceanic forcings, the dynamics occurring sufficiently inland from the glacier terminus (i.e. ≥ 30 km) are instead regulated by the nature of the subglacial environment. Although a combination of modelling and borehole-access experiments have provided some geophysical insights beneath the glacier surface, the mechanisms of how water pressure and the nature of the underlying bed control ice velocity still remain vaguely understood (Bougamont et al., 2014; Cowton et al., 2016; Harper et al., 2007; Sugiyama and Gudmundsson, 2004). Notwithstanding this, the majority of our understanding of ice sheet flow dynamics is based upon theoretical linkages between meltwater discharge, drainage system morphology and subglacial water pressure (e.g. Kamb, 1987; Röthlisberger, 1972; Schoof, 2010; Zwally et al., 2002), which have been applied in contemporary studies to observe and interpret variations in intra- and inter-annual ice sheet motion (Andrews et al., 2014; Bartholomew et al., 2010; Doyle et al., 2018; Hoffman et al., 2016). At the ice sheet surface, water produced from melting flows over the ice along fluvial pathways, disappearing into crevasses and moulins, or collecting in surface lakes, which have the potential to drain late into the melt season (Bell, 2008). Within the heavily-crevassed ablation zone, some of this surface meltwater can potentially access the base of the ice sheet.

### 2.2.1 Subglacial controls on ice sheet flow

Recent research within this realm have not only confirmed that both land- and marine-terminating glaciers exhibit seasonal flow variability, indicating widespread melt-induced acceleration (Howat et al., 2010; Joughin et al., 2008), but also have examined the
evolution of subglacial drainage systems that modulates the amount of water pressure exerted to reduce basal traction between the ice sheet and the underlying bed (e.g. Andrews et al., 2014; Doyle et al., 2018; Meierbachtol et al., 2013). The movement of ice is driven principally by the gravitational driving stress, where the underlying subglacial environment exerts a strong influence on the glacier’s velocity. This influence is quantified by the frictional resistance exerted from the bed surface onto the glacier base, which, due to the visco-elastic nature of ice, induces internal (shear) deformation within the ice column. If the underlying glacier bed is comprised of sedimentary material, the exposure of the bed to the same frictional stresses also results in similar deformation of subglacial material. It is, therefore, these three processes—the sliding of the glacier bed across the underlying bed, the englacial deformation of ice, and the deformation of underlying sediments—that dictate the movement of glaciers (Cuffey and Paterson, 2010).

The nature of the subglacial hydrological system and environment that exists underneath glaciers have a profound influence on ice dynamics. Specifically, the type of system has a categorical impact over the basal velocity by modulating the effective pressure between the underlying bed and the overlying ice sheet, which then depends on the pressures exerted from the subglacial water and the ice thickness (Willis, 1995). In general, as the basal water pressure approaches the overburden pressure across extensive regions, the effective pressure decreases and accelerates ice sheet velocities through the reduction of friction between the two facies (Bell, 2008).

Subglacial systems can be broadly categorised into distributed systems, where water flows at slower speeds over larger proportions of the bed, and channelised systems, where the flow of water is rapid and confined to relatively narrow conduits (Figure 2.2; Fountain and Walder, 1998; Humphrey and Raymond, 1994).

**Distributed systems**

Beneath ice sheets underlain by porous sediment, water can percolate between the grains of the underlying till through Darcian flow (groundwater flow; Boulton and Jones, 1979), or if water supply exceeds the capacity of the subglacial aquifer due to increases in pore water pressure, the excess water may flow along the ice-bed interface as a thin distributed
sheet. The latter process is likely to occur only during the summer months of the melt season, as the aquifer provided by porous basal sediment can only accommodate small amounts of meltwater (Boulton and Jones, 1979; Fountain and Walder, 1998).

Otherwise, for ice sheets resting on hard bedrock, the influx of water accumulates between the bed and basal ice, and is relieved by slow, downstream, distributed flow. When more water enters the basal environment than is drained, the sheet of water is sustained, and the volume and pressure of water at the ice-bed interface increases, speeding up flow (Cuffey and Paterson, 2010). However, the decoupling of ice from the underlying bed is predicted to be unstable beyond a critical thickness, estimated at ~1 mm, and will likely break up into anastomosing networks of smaller channels if this threshold is exceeded (Cuffey and Paterson, 2010; Nye, 1976; Walder and Fowler, 1994).

The distribution of subglacial water also depends largely on the distribution of effective pressure across the bed, which is governed by the subglacial topography of the underlying bedrock (Lliboutry, 1968). Water-filled cavities will open up where basal water pressures exceed the local ice overburden pressure, which is more likely to be on the downglacier
side of bumps where the normal ice pressure relative to the glacier bed is at a minimum \citep{Cuffey2010, Weertman1957}. The sustained balance between the ice overburden pressure and the water pressure prevents the closure of these \textit{linked cavities}, resulting in the slow and inefficient discharge of water downglacier beyond the end of the melt season into winter.

\textbf{Channelised systems}

Alternatively, given the right conditions of basal velocity and sustained meltwater input into the subglacial system, pipe-like conduits can be carved upwards into the ice (\textit{Röthlisberger (R-) channels}; \citeauthor{Rothlisberger1972}, \citeyear{Rothlisberger1972}), as well as downwards into the bedrock or sediment (\textit{Nye (N-) channels}; \citeauthor{Nye1976}, \citeyear{Nye1976}). These conduits efficiently transport water towards the ice margins, and are most evident at the base of mountain glaciers. The turbulent, fast flow of water within the confines of the channels maintain the openings through the viscous dissipation of heat that counteract the inward creep of the overlying ice \citep{Bell2008}.

\textbf{Feedbacks between subglacial hydrology and ice sheet flow}

The relationship between basal water pressure and the separation of the overlying ice body from the bed modulates the variations observed in seasonal ice velocities. A greater separation reduces the amount of frictional traction by the creation of basal cavities, reducing the roughness of the bed \citep{Lliboutry1968}. As a result, the traction between the ice and bed is then concentrated in localised areas, enhancing both the rate of deformation of basal ice and the velocity of the overlying ice \citep{Iken1981}. 
Figure 2.3 24 h horizontal velocity (blue) and surface height (green) at sites (a) 7 km (395 m a.s.l); (b) 12 km (618 m a.s.l); (c) 21 km (795 m a.s.l); and (d) 39 km (1063 m a.s.l) from the terminus of Russell Glacier, southwestern Greenland. The dashed lines show winter background velocity (black) and velocities from periods with sparse data (blue). The shaded sections identify periods of ice acceleration associated with surface uplift (red) and ice deceleration associated with a decrease in surface height (blue). (e) Temperature records at GPS deployments in (a; magenta), (c; red) and (d; blue). From Bartholomew et al. (2010).

At the onset of the melt season, initial meltwater fluxes enter the subglacial environment, which induce a net increase in ice velocity. The speed-up is detected first at the glacier terminus, cascading progressively further inland through the course of the melt season due to decreasing air temperatures with increasing surface elevation (Figure 2.3). The gradual build-up of water at the ice-bed interface leads to an increase in water pressure,
enhancing sliding by reducing the frictional drag exerted from the ice onto the underlying bed (e.g. Fountain and Walder, 1998; Iken, 1981; Kamb, 1987; Walder, 1986). It is thought that when a level of critical discharge is reached in the distributed system, heat dissipation within the water flow enables cavity growth, forming discrete, efficient channels (Hewitt, 2011; Schoof, 2010; Walder, 1982, 1986). The presence of such channels efficiently evacuates water from the subglacial environment, causing the flow of glaciers to decelerate despite sustained meltwater inputs, eventually reverting to its annual minimum velocity following the cessation of seasonal melting and consequent channel collapse (Figure 2.4b; Flowers, 2015; Hewitt, 2011; Hoffman et al., 2011; Schoof, 2010).

During the melt season, surface meltwater directly accesses the subglacial environment almost immediately from its production. From global positioning system (GPS) and automatic weather station (AWS) observations on Leverett Glacier, southwestern Greenland, diurnal variations in ice sheet motion reach a maximum between 2–4 h after peak daily melting caused from peaks in local daily temperatures (Figure 2.5c). In cases of extreme surface melt or sudden lake drainage, such transient speedups may be exacerbated by decoupling the ice from the underlying bed, as evidenced by peak surface uplift observations at similar time scales relative to peaks in horizontal velocity (Figure 2.5a,b). On longer time scales, velocity increases in Greenland’s fast-flowing marine-terminating glaciers contrast with stable velocities on ice-shelf-terminating glaciers and slow velocities on land-terminating glaciers. Spatially, glaciers in the northwest accelerated steadily over the course of a decade, with more variability in the southeast and relatively steady flow elsewhere (Moon et al., 2012).
Figure 2.4 (a,b) Conceptual model of the traditional two-component subglacial hydrological system, and the (c) additional third component described in Andrews et al. (2014) and Hoffman et al. (2016). From Hoffman et al. (2016).

Although the two-component subglacial hydrological system is now commonly accepted within the glaciological community, increasing evidence suggests that this conceptual...
model overemphasises the role of channelisation in controlling subglacial drainage system capacity and water pressure, and neglects the complexity in unchannelised regions of the bed underneath the Greenland Ice Sheet (Andrews et al., 2014; Hoffman et al., 2016). Following the seasonal maximum in ice flow, velocities decrease at and well beyond the end of the melt season (Hoffman et al., 2011; Van De Wal et al., 2015), contrasting with the markedly shorter lifespan of channel collapse of at most several days (Dow et al., 2014). As well as intra-annual variability in glacier flow, observations on land-terminating glaciers suggest a multiyear response of annual ice motion, modulated by summer melt volume from the previous 1 to 4 years due to gradual net evacuation of stored meltwater in unchannelised regions between melt seasons (Sole et al., 2013; Tedstone et al., 2015). It has therefore been suggested that an additional hydrological system intermediate to distributed sheet and channelised drainage systems should be incorporated into the current two-component hydrological system to address the mismatch between modelled and observed flow rates, and to account for the multiyear trends seen in ice sheet velocities (Figure 2.4c Andrews et al., 2014; Hoffman et al., 2016).

At the same time, by focusing explicitly on the morphology of the hydrological system, the majority of previous research has implicitly assumed that ice sits on hard bedrock. Increasingly, seismic surveys and borehole studies have discovered large swathes of the Greenland Ice Sheet underlain by soft sediments (Christianson et al., 2014; Cowton et al., 2012; Dow et al., 2013; Hofstede et al., 2018; Kulessa et al., 2017; Walter et al., 2014), which modulate basal traction and ice flow not by channelisation, but instead through the relative strength of the subglacial material via fluctuations in basal water pressure that drive water in and out of the sediment layer. This mechanism is likely enhanced during the summer months of the melt season when large quantities of meltwater are discharged rapidly into the subglacial environment through catastrophic drainage of supraglacial lakes (Bougamont et al., 2014). The subsequent strengthening of the till layer from high basal water pressure gradients is thought to increase basal traction, suggesting an alternative hypothesis to explain the observed decrease in ice flow from summer to autumn (Kulessa et al., 2017).
Figure 2.5 2010 GPS records showing (a) surface velocity and (b) surface height profile at 83 km from the terminus of Russell Glacier, southwestern Greenland during a lake drainage event, the duration marked by two black lines. (c) Contemporaneous records during 2010 highlighting diurnal cycles in ice velocity, air temperature, and proglacial discharge 12 km from the terminus of Russell Glacier. Daily peaks and troughs are marked by coloured dots. Winter background ice velocity indicated by a black dashed line. Adapted from Bartholomew et al. (2012).

Separately, increases in longwave radiative and turbulent heat fluxes brought by moist air masses are projected to becoming more frequent through the 21st century (Boisvert and Stroeve, 2015; Schuenemann and Cassano, 2010). These trends have been attributed
in response to a warmer and cloudier regional climate, a northward shift in stormtracks, and decreases in regional sea ice (Schuenemann and Cassano, 2010; Van Tricht et al., 2016). The implications of higher precipitation rates over the Greenland Ice Sheet were observed by Doyle et al. (2015), who captured the wide-scale effects of an intense rainfall event at Russell Glacier (67°N, 50°W) late into the melt season. With regards to the mass balance of the ice sheet, the large amount of precipitation brought by these events triggered intense ice sheet surface melting and induced glacier-wide accelerations in ice flow observed as far as 140 km into the ice-sheet interior (Figure 2.6).

2.3 Thermomechanical regimes under ice sheets

2.3.1 Basal heat budget

Although the surfaces of glaciers are exposed and respond rapidly to changes in near-surface temperatures, their basal thermal environments are effectively isolated from the Earth’s climate, with the ice itself acting as an effective thermal insulator (Cuffey and Paterson, 2010). Outside the summer melt season, when crevasses, moulins, and other englacial features link the supraglacial and subglacial environments and water is transferred through the ice column, any water formed is the result of only processes occurring at the glacier bed, which in turn controls the movement of the overlying ice (Cuffey and Paterson, 2010).

If the basal temperatures of the glacier bed and its underlying bedrock or sediment are all below the pressure melting point (PMP), the high adhesive strength of the frozen glacier bed interface inhibits basal sliding (Echelmeyer and Wang, 1987) and potentially induces basal freeze-on (Christoffersen and Tulaczyk, 2003; Tulaczyk et al., 2000a). At the other end of the continuum, where the basal environment is at or above the PMP, the warm temperatures induce basal melting resulting in a film of water between the ice-bed interface, facilitating decoupling and enhancing ice flow (Weertman, 1964).
Figure 2.6 Amplified melt and flow at a site 13 km from the terminus of Russell Glacier during an observed late summer rainfall event, shaded in grey. (a) The surface energy budget, including net short-wave ($SW_{\text{net}}$) and net long-wave ($LW_{\text{net}}$) radiation, the sensible (SHF) and latent (LHF) heat fluxes and the ground flux (GF). Fluxes are positive when they add heat to the surface. (b) Surface melt rates and total daily melt. (c) Borehole water pressure, ice surface velocity, and ice surface uplift. From Doyle et al. (2015).

For the majority of ice sheets, the primary sources of subglacial heat come from:

1. frictional heat from basal motion and from flowing water at the ice-bed interface; and

2. geothermal heat arising from the Earth’s interior
Conversely, conductive heat fluxes into the overlying ice represent the primary sink of subglacial heat, although within cold-based glaciers the positive temperature gradient can potentially supply an additional, albeit small amount of heat. The overall energy balance at the glacier base from thermomechanical processes can then be represented by (Equation 8, Alley et al., 1997):

\[
W_b = q_G + \tau_b \vec{u}_b + k_i \frac{\partial \Theta_i}{\partial z}
\]  

(2.1)

Here, the net energy supply \( W_b \) [W m\(^{-2}\)] is determined by the various sources and sinks mentioned: the geothermal heat flux originating from the underlying substrate \( q_G \), the heat by frictional dissipation at a rate equal to the product of the basal slip of ice over its bed \( \vec{u}_b \) [m s\(^{-1}\)] and the shear stress exerted on the bed \( \tau_b \) [Pa], and the heat conduction into the ice \( \frac{\partial \Theta_i}{\partial z} \) [K m\(^{-1}\)] scaled by the thermal conductivity of ice \( k_i \) [W m\(^{-2}\)], which supplies or removes heat depending on the sign of the englacial temperature (\( \Theta_i \)) gradient. If \( W_b > 0 \), the surplus energy is then used to melt the basal ice, while if \( W_b < 0 \), basal ice accumulation occurs.

The mass balance at the glacier base \( \dot{m}_b \) [m s\(^{-1}\)] can then be calculated by scaling the net energy supply \( W_b \) by the density of ice \( \rho_i \) [kg m\(^{-3}\)] and the latent heat of fusion \( L_f \) [J kg\(^{-1}\)]:

\[
\dot{m}_b = \frac{W_b}{\rho_i L_f}
\]  

(2.2)

### 2.3.2 Enthalpy of temperate ice

At the base of a fast-flowing glacier, ice is typically at or near the PMP, and any surrounding heat can only be used for melting (Blatter and Hutter, 1991; Fowler, 1984). Therefore, temperate ice often is a mixture of solid ice (95–99 %) and liquid water (1–5 %), as well as small amounts of air and minerals that increase its electrical conductivity (Lliboutry and Duval, 1985). Because thermomechanical ‘cold-ice’ models (Bueler et al., 2007; Christoffersen and Tulaczyk, 2003; Payne et al., 2000) approximate the basal energy
budget through a differential equation for the temperature variable, such models do not account for the energy content of temperate ice, which, due to the increased liquid water fraction, can soften the ice by a factor of ~3 for every percent increase in water content (Lliboutry and Duval, 1985) and can enhance strain heating and increase ice flow (Aschwanden et al., 2012). Therefore, in addition to standard thermomechanical sources, heat also arises from:

3. the enthalpy of latent heat of liquid water within temperate ice.

Newer models (e.g. Aschwanden and Blatter, 2009; Aschwanden et al., 2012) incorporate the enthalpy of temperate ice by describing temperature and water content in a consistent and energy-conserving formulation. Any changes in the enthalpy are encapsulated by changes of temperature in cold ice and by changes of the liquid water fraction in temperate ice (Kleiner et al., 2015). When compared to cold ice schemes, these models do a better job of conserving energy (Aschwanden et al., 2012).

### 2.3.3 Viscous heat dissipation

So far, the thermomechanical models described above only consider heat sources originating from within the ice-bed interface. During the melt season, surface meltwater penetrates the bed through moulins and crevasses, modulating ice velocities on hourly to decadal timescales (Bartholomew et al., 2010; Doyle et al., 2015, 2014; Joughin et al., 2008; Mikkelsen et al., 2016; Rignot and Kanagaratnam, 2006; Shepherd et al., 2009; Tedstone et al., 2015; Van De Wal et al., 2015). As well as lubricating basal flow, the influx of supraglacial water generates an additional source of subglacial heat:

4. viscous heat dissipation (VHD) from surface runoff routed to the bed.

Specifically, VHD is an important process in the creation and retention of R-channels through continued flow unevenly melting the overlying ice within specific pathways (Flowers, 2015). While VHD has minimal impact within the ice sheet interior (<75 km inland), near the margin, the energy produced by VHD regularly exceeds that of both geothermal and frictional heat combined (Mankoff and Tulaczyk, 2017).
Chapter 3

Development and application of ice-penetrating radar

3.1 Methodologies of englacial and subglacial investigations

Several methods exist to observe the englacial and basal conditions of an ice sheet. Of these, only ice coring and access-borehole drilling provide direct access and observation to the interior of a glacier, the underlying bed or the sub-ice-shelf environment. Typically, sensors are directly installed at various depths below the ice surface via long cables, thereby obtaining extremely precise empirical measurements such as internal deformation, temperature, and subglacial water pressure, and combined can yield valuable insight into the thermomechanics and the hydrological structure of the subglacial environment (Benn and Evans, 2010). However, such measurements come at an extremely high burden financially, temporally, and environmentally through the use of significant amounts of diesel to melt thick amounts of ice. Practically, directly accessing the subglacial environment is often a difficult endeavour due to rapidly-freezing boreholes and complex flow regimes that can stretch and break instrumentation cables (Lüthi, 2013). Lastly, these methods only constitute point measurements and are not representative of wider areal coverage.
Seismic exploration studies provide a slightly more cost-effective method to profile subsurface conditions, adding a second dimension to observations by towing an array of geophones across an ice sheet to capture seismic waves propagating along and beneath the ice sheet surface. Seismics represent the only method that enables investigation of sub-basal conditions to classify the type of substrate beneath glaciers, which is an important parameter in many ice sheet flow models. However, such studies must be ground-based, and therefore subject to surface conditions being flat enough to efficiently move the array, either by snowmobile or physical labour.

Radio-echo sounding (RES, where the term “radar” is synonymous), on the other hand, provides the ability to efficiently measure the properties beneath the ice sheet surface in multiple dimensions over a comparatively short temporal period without the constraints posed by both drilling and seismic techniques. Due to the portable nature of the majority of radar instruments, they can be deployed either directly on the ice surface or from aircraft to obtain continuous profiles of both ice sheet surface elevation and thickness, thereby yielding the shape of the glacier bed beneath the ice mass and producing the first three-dimensional shape of large ice masses for the first time in 1983 (Dowdeswell and Evans, 2004). Most recently, radar has been mounted on spaceborne satellites to collect elevation data across the Earth’s surface, with missions such as CryoSat (Drinkwater et al., 2004) specifically designed for the measurement and monitoring of the Earth’s polar regions.

Figure 3.1 Original radargram of the Antarctic Ice Sheet, obtained using the 60 MHz SPRI-NSF-TUD pulsed instrument. From Dowdeswell and Evans (2004).
3.2 Application of radar to the Greenland and Antarctic Ice Sheets

Arguably for glaciologists, the primary motivation behind the development and application of ice-penetrating radar is to quantify the subglacial topography of ice sheets and glaciers to provide a realistic representation of the underlying bed morphology. This goal is normally achieved by using radar to measure the ice thickness, which is then subtracted from the surface elevation to provide an estimate of the bed elevation above sea level. Data collection usually involves the movement of the radar equipment and its associated antennae following a series of planned transects designed to give maximum and equal coverage of the survey area. This can be conducted from the ice surface using snowmobiles or tractors, from specially designed aircraft, or, more recently, from satellites. The concept for this approach can be traced independently to Stern (1930) and to Admiral Byrd’s base at Little America (later known as McMurdo Station), both of which suggested that snow and ice are transparent to high frequency (HF) and very high frequency (VHF) radio waves. Gogineni et al. (1998) and Dowdeswell and Evans (2004) give a comprehensive and valuable summary and a comprehensive list of references to the development of ice-penetrating radar; the following sections summarise these developments and compiles an additional decade’s worth of recent technical developments since these review publications.

It was not until the 1960s that investigations into the apparent unreliability of military radio altimeters over deep snow and ice by American researchers at the U.S. Army Signals Research Laboratory prompted the development of dedicated depth-sounding instruments (Waite and Schmidt, 1962). To resolve this issue, Amory Waite and colleagues successfully applied and deployed the SCR718 440 MHz altimeter to measure ice depths to 250 m on the Northwest Greenland Ice Sheet and to 150 m on the Ross Ice Shelf in Antarctica (Waite, 1959). Similarly, from the discovery at Halley Bay on the Brunt Ice Shelf, Antarctica, by Piggott and Barclay (1961) that dry snow and ice were transparent to radio waves between 1–20 MHz, Stanley Evans at the Scott Polar Research Institute (SPRI) designed and deployed the first ionosonde at these frequencies to detect the
depth of various ice shelves across Antarctica to estimate the dielectric properties of ice through the process (Evans, 1961). Further trials with these airborne systems (e.g. Evans, 1963; Evans and Robin, 1966; Evans and Smith, 1969; Gudmadsen, 1969; Luchininov and Macheret, 1975; Walford, 1964; Weber and Andrieux, 1970) used progressively lower frequencies into the VHF range to maximise penetration depth, and incorporated significant improvements in transmitting power and automatic recording (Plewes and Hubbard, 2001). Following these demonstrations, the successful deployment of a 60 MHz pulsed instrument by SPRI and the Technical University of Denmark at Lyngby (DTU) on an extensive series of flights funded by the US National Science Foundation (NSF) in both Antarctica and Greenland from 1967 to 1979 allowed researchers from these institutions to determine the ice thickness and internal stratigraphy of these large ice bodies (Figure 3.1; Evans and Smith, 1969; Robin et al., 1977).

Further innovations both in hardware development and signal processing techniques enabled radar systems to improve the quality of measurements and discern additional properties at the bed of ice sheets. By investigating the properties of the returned echo power, researchers were able to speculate about the nature of the ice-bed interface, as well as detect its spatial variation across entire glaciers and ice sheets (MacGregor et al., 2016; Neal, 1976; Oswald and Gogineni, 2012). Comparative observations of unusually smooth, highly reflective basal echoes within the SPRI-NSF-DTU datasets revealed the presence of large and active subglacial lakes beneath the Antarctic continent (Carter et al., 2007; Jamieson et al., 2016; Robin et al., 1977; Siegert, 2000; Siegert and Kwok, 2000; Smith et al., 2009). While internally-reflecting layers have been observed since the first radar investigations (e.g. Drewry and Meldrum, 1978; Gudmadsen, 1975), researchers began to speculate as to their presence and absence within the ice sheet, attributing possible causes to variations in ice crystal fabric and anisotropy (e.g. Eisen et al., 2007; Fujita et al., 1993; Harrison, 1973; Matsuoka et al., 2003), density (e.g. Clough, 1977; Robin et al., 1969), and/or conductivity (Figure 3.2; Björnsson et al., 1996; Clarke et al., 1989; Gow, 1968; Hempel et al., 2000). Cross-referencing the vertical locations of internal layers with direct englacial measurements revealed isochrones that may illuminate many aspects of ice dynamics, such as their strain history and their response to climatic and subglacial forcings (e.g. Arcone et al., 2005; Carter et al., 2009; Catania and Neumann,
3.2 Application of radar to the Greenland and Antarctic Ice Sheets

2010; Catania et al., 2006; Cavitte et al., 2016; Christianson et al., 2013; Conway et al., 2002; Corr and Vaughan, 2008; Drews et al., 2009; Fahnestock et al., 2001a,b; Jacobel and Welch, 2005; Keisling et al., 2014; Kingslake et al., 2016; Leysinger Vieli et al., 2011; MacGregor et al., 2009, 2015; MacGregor et al., 2009; Matsuoka et al., 2003; Medley et al., 2013; Nereson et al., 1998; Siegert, 1999; Waddington et al., 2007; Wang et al., 2002; Whillans, 1976).

![Figure 2](image)

**Figure 2.** Z-scope RES data and the classification of RES layers. (a) Continuous layering from line 148 collected in the 1974/75 season. (b) Buckled layering from line 005 taken during the 1977/78 season. (c) Absent layering from line 138 acquired in the 1974/75 season.

In Antarctica, the gridded survey tracks flown as part of the SPRI-NSF-TUD collaborative program in the 1970s enabled the mapping of the 3-dimensional bed topography of the

![Figure 3.2](image)

**Figure 3.2** Radargrams of the Antarctic Ice Sheet showing (a) continuous layering (collected 1974/1975); (b) buckled layering (collected 1977/1978); and (c) absent layering (collected 1974/1975). From Siegert et al. (2005).
The majority of the Antarctic Ice Sheet by interpolating between survey lines (e.g. Drewry, 1983; Lythe et al., 2001; Robin et al., 1977). Similarly, in Greenland, much of the $1.71 \times 10^6 \text{km}^2$ ice sheet has been mapped primarily through the numerous campaigns led by the University of Kansas, using various versions of the Coherent Radar Depth Sounder (e.g. Gogineni et al., 1998, 2001; Legarsky et al., 2001; Li et al., 2013; Rodriguez-Morales et al., 2010; Shi et al., 2010). With subsequent radar surveys “filling in” previously-unmapped areas of both major Ice Sheets (e.g. Christensen et al., 2000; Lindbäck et al., 2014; Nixdorf et al., 1999; Peters et al., 2007; Ross et al., 2012; Vaughan et al., 2006), and aided by modern statistical and modelling techniques, complete maps of both the Antarctic (Figure 3.3a; Fretwell et al., 2013; Lythe et al., 2001) and Greenland (Figure 3.3b; Bamber et al., 2013;Morlighem et al., 2015) Ice Sheets were rigorously computed for the first time.

![Figure 3.3](image-url) Map of interpolated bed topography of (a) Antarctica (Fretwell et al., 2013) and (b) Greenland (Morlighem et al., 2015).

Separately, the development of synthetic aperture radar (SAR) principles enabled the generation of fine-resolution bed topography of smaller areas. While SAR has been used to map the grounding line of Antarctic ice shelves (Musil and Doake, 1987) and the internal structure of Norwegian glaciers (Hamran and Aarholt, 1993), the first fully
3-dimensional reconstruction of the bed topography was achieved by John Paden et al. (2010) at the University of Kansas (Figure 3.4). By applying tomographic techniques to data collected around the GRIP ice core study site using a linear antenna array, the project was able to generate fine-resolution bed topography from single-pass data and estimate the basal roughness and bed conditions from the backscattered signal with an accuracy of up to 10 m in depth.

Beginning in the 1990s, a new generation of specialised radar systems employing a range of frequencies was developed to increase the signal bandwidth and enhance the spatial resolution, and to increase the echo’s signal-to-noise radio (SNR). These comprised both stepped-frequency and, more recently, frequency-modulated continuous-wave (FMCW) systems capable of being successfully deployed for specific glaciological applications. These systems proved effective at handling the additional problems often encountered on temperate glaciers (Arcone and Yankielun, 2000; Dowdeswell and Evans, 2004) and profiling shallow snow stratigraphy (Arthern et al., 2013; Ellerbruch and Boyne, 1980; Holmgren et al., 1998; Koh et al., 1996; Machguth et al., 2006), due to the range of wave frequencies being able to penetrate and reflect internal layers at various depths (Rodriguez-Morales et al., 2014). In 2001, the development and application of an ultra-wideband FMCW radar with a frequency sweep of 170 MHz to 2 GHz by researchers at the University of Kansas (KU) to map the near-surface internal layers at the North Greenland Ice Core Project reduced the vertical resolution to <1 m resolution for the first time, greatly improving the precision and performance of ice-penetrating radars.
Development and application of ice-penetrating radar (Kanagaratnam et al., 2001). As the radar systems by KU improved, much of the interior of the Greenland Ice Sheet was successfully mapped. Subsequent surveys then gravitated towards the southwestern margin of the Greenland Ice Sheet where ice thickness is harder to measure: in these regions, ice tends to be warmer, flow faster, and exhibit complex flow relative to the ice sheet interior (MacGregor et al., 2015; Moon et al., 2012). Additionally, the presence of crevasses, englacial water storage, and highly-variable temperature profiles through the ice column, inhibited detection of deep internal reflections (MacGregor et al., 2015). Combined, these features present the remaining challenges to sounding the Greenland Ice Sheet.

3.2.1 Development of stationary phase-sensitive radar

While traditional kinematic radio-echo sounding provides accurate measurements in the horizontal (X-Y) plane and maps the basal topography through quantifying areal ice thickness, it is insensitive to changes in the vertical displacement, which would provide insights into changes in the thickness of ice sheets. Nye et al. (1972) first proposed using a stationary phase-sensitive radar to measure the changes in ice thickness within a Eulerian framework by studying the phase of returned bursts reflected from the bed. This thought experiment was finally realised in 2002 by researchers at the British Antarctic Survey (BAS), who pioneered the use of a phase-sensitive Radio Echo Sounder (pRES) that was able to directly measure the change in thickness of ice shelves, thereby quantifying the basal melt rate to millimetre precision. First demonstrated by Corr et al. (2002), who provided a detailed description of the technique and its application to a site on the George VI Ice Shelf, Western Antarctic Peninsula, it has since been used to quantify vertical strain rates on ice sheets (Gillet-Chaulet et al., 2011; Kingslake et al., 2014, 2016; Nicholls et al., 2015) and basal melt rates on ice shelves (Brennan et al., 2014; Dutrieux et al., 2014; Jenkins et al., 2006, 2010; Marsh et al., 2016; Nicholls et al., 2015; Stewart, 2018) in both Greenland and Antarctica.

Phase-sensitive radar measures the vertical strain and basal melt of ice sheets by directly profiling the incident ice column thickness (the area between an upper reference horizon and the ice column base) through time. The change in ice column thickness is a result of
3.2 Application of radar to the Greenland and Antarctic Ice Sheets

The prototype radar system is pictured in Fig. 7. It is based on an elaboration of Fig. 1, including additional front-end filtering and digital clock generation and synchronisation. The system makes use of an Analog Devices AD9910 DDS synthesiser, generating a 200–400 MHz chirp signal with a 1 GHz clock. A pair of Mini-Circuits ZX76-31-PP + digital step attenuators are employed to sequentially switch the receiver RF gain between values of 4, 16, 28 and 40 dB, on successive chirps, in order to provide a range of receiver gain settings to allow good performance with both near and far scatterers. A second-order high-pass filter is incorporated, with a gain of 0 dB at frequencies below 50 Hz, rising to a peak of 80 dB at 5 kHz, in accordance with the predicted echo power levels indicated in Fig. 3. The instrument consumes 750 mA at 6 V when operating and 0.24 mA in standby, thus fulfilling the low-power consumption requirement (in relation to the predecessor ‘pRES’ system that consumed hundreds of Watts in operation and required a petrol generator). In addition, the much reduced noise figure allows much faster signal capture, further reducing the energy consumption.

A system model has been carefully constructed, based on the parameters listed in Table 1, in particular to assess phase-sensitive FMCW performance at low signal levels. Monte Carlo simulation, with 1000 trials, has been used to numerically estimate the RMS range measurement error as a function of signal strength for a test signal emulating a discrete target at 1800 m range in the presence of additive white Gaussian noise to simulate thermal noise. The processing includes peak detection in the vicinity of the known target, to establish the likely closest range bin. The modelled results, given in Fig. 8, show that the RMS range error increases from 1.3 mm for a −140 dBm receiver signal level to 14 mm for a −160 dBm signal level. The equivalent SNR (in the 1 Hz FFT bandwidth) over this range is 27.8–7.8 dB. These results are plotted alongside the simple analytic SNR expression of (8), and it is clear that there is very good agreement, even down to very low signal and SNR levels, significantly below the −154 dBm and 14 dB worst-case values expected at maximum range (2 km). It is observed that, at very low signal levels, occasional results are produced that are in error by approximately half a wavelength (284 mm). These are

**Figure 3.5** Photograph of the (a) prototype ApRES instrument (Brennan et al., 2014) and (b) the first version of the ApRES instrument, which was used in this thesis.

surface snow accumulation and melt, firn compaction, vertical strain, and basal melt. The former two components are determined independently by surveying surface markers and the reference horizon, and the latter two components are identified using the mentioned radar methodology (Corr et al., 2002). A key disadvantage of the system used to date, however, is that the instrument is relatively heavy, expensive, and power-consuming; the net effect is that the instrument is restricted to collecting only snapshots of data during Austral summers (Rahman et al., 2013).

In response, a new instrument has been developed that addresses these constraints, and most importantly reduces power consumption enough to be capable of running unattended for over a year (Brennan et al., 2014). The radar instrument, a collaboration between BAS and University College London, is comprised of a frequency-modulated continuous-wave autonomous phase-sensitive radio echo sounder (ApRES) that is similar to the previous pRES system, but extends the normal operating distance up to 1800 m. The FMCW radar addresses all the shortcomings of the previous system, and crucially, is capable of continuous unattended operation for over a year by reducing power consumption to 20 W during operational and 10 mW during standby modes.
Chapter 4

Theory

4.1 Changes in ice thickness

Within an ice column, changes in ice thickness can occur as a result of ice accumulation or ablation at the upper and lower ice boundaries, or from deformation processes occurring within the ice body from lateral and basal frictional stresses opposing flow (Benn and Evans, 2010).

4.1.1 Principles of stress and strain

As this dissertation primarily investigates the strain response to flow dynamics, the principles of strain are briefly covered with only necessary reference to its relationship to stress; various textbooks on glacier physics (Cuffey and Paterson, 2010; van der Veen, 1999) cover stress-strain relations in ice dynamics in greater detail.

The internal deformation of ice as it flows down a glacier is often characterised using continuum mechanics, which dictate that glaciers can be treated as a continuous distribution of matter. As such, the deformation of ice when subjected to applied forces can be translated into its stress and strain components.

In continuum mechanics, stress is a measure of the strength of a material when subjected to applied forces, while strain is a measure of deformation as a result of stress. Stress,
defined as the force per unit area, is often described by a tensor with independent components $\sigma_{ij}$ (Pfeffer et al., 2000). These components can be separated into two categories: those acting parallel to the surface (the shear stress), and those acting perpendicular to the surface (the normal stress).

All materials undergo strain when they are subject to stress; their relationship approximated through Hooke’s Law:

$$\sigma_{ij} = \dot{\varepsilon}_{ij}/E$$

(4.1)

where $\sigma_{ij}$ and $\dot{\varepsilon}$ are respectively the stress and strain tensor along the $ij$ direction, and $E$ is the Young’s modulus, a measure of the stiffness of a linear elastic material.

**Strain**

Consider first a deforming body of ice that flows at a rate $\vec{u}$, with the three components of the strain tensor $\dot{\varepsilon}$ within a 2-dimensional space given as:

$$\dot{\varepsilon}_{ii} = \frac{\partial \vec{u}_i}{\partial i}; \quad \dot{\varepsilon}_{jj} = \frac{\partial \vec{u}_j}{\partial j}; \quad \dot{\varepsilon}_{ij} = \frac{1}{2} \left( \frac{\partial \vec{u}_i}{\partial j} + \frac{\partial \vec{u}_j}{\partial i} \right)$$

(4.2)

Applying Equation 4.2 to the Cartesian (X-Y) coordinate system with the X-axis aligned with the principal flow direction, the rate of flow can be given as $\vec{u}_x$, and the strain tensor components as:

$$\dot{\varepsilon}_{xx} = \frac{\partial \vec{u}_x}{\partial x}; \quad \dot{\varepsilon}_{yy} = \frac{\partial \vec{u}_y}{\partial y}; \quad \dot{\varepsilon}_{xy} = \frac{1}{2} \left( \frac{\partial \vec{u}_x}{\partial y} + \frac{\partial \vec{u}_y}{\partial x} \right)$$

(4.3)

where the indices $i$ and $j$ respectively represent $x$ and $y$ as the two orthogonal tensor directions. Here, the diagonal components of the strain rate $\dot{\varepsilon}_{xx}$ and $\dot{\varepsilon}_{yy}$ represent strain thickening or thinning parallel to the coordinate axes, while $\dot{\varepsilon}_{xy}$ represents the shear strain of ice.
4.1 Changes in ice thickness

Note that the denominator \( \partial x \) represents the distance between two points. Because the component velocities can be represented as the change in position of these two given points with respect to the time elapsed, this allows the strain rate to alternatively be written in terms of the component change in length, for example, in the flow direction:

\[
\dot{\varepsilon}_{xx} = \frac{\partial \vec{u}_x}{\partial x} = \frac{\vec{u}_{xb} - \vec{u}_{xa}}{x_b - x_a} = \frac{1}{l} \frac{\partial l}{\partial t} \tag{4.4}
\]

where \( l \) represents the length between two positions \( x_a \) and \( x_b \), so that \( l = x_b - x_a \). This form is useful for glaciological calculations, as measurements of displacement on ice sheets are largely conducted with GPS stations, which record absolute position rather than velocity. Within this study, we instead calculate strain and strain rates using downward-sounding stationary radar, which is oriented in the vertical (Z) axis. Therefore, expanding into three-dimensions, the third strain and strain rate component is similarly given:

\[
\dot{\varepsilon}_{zz} = \frac{\partial \vec{u}_z}{\partial z} = \frac{\vec{u}_{zb} - \vec{u}_{za}}{z_b - z_a} = \frac{1}{d} \frac{\partial d}{\partial t} \tag{4.5}
\]

where, instead of horizontal length, the change in depth of specific features (e.g. internal layers) are measured through time.

Lastly, assuming incompressibility and a constant density of ice:

\[
\nabla \cdot \vec{u} = \frac{\partial \vec{u}_x}{\partial x} + \frac{\partial \vec{u}_y}{\partial y} + \frac{\partial \vec{u}_z}{\partial z} = \dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} + \dot{\varepsilon}_{zz} = 0 \tag{4.6}
\]

which is the condition satisfied by the normal strain rates in an incompressible material.

**Stress and strain in ice sheets**

With regard to ice sheets, the flow of glaciers is a response to their driving stresses from the weight of the ice and its surface slope. At the bed of the glacier, the driving stresses are resisted by basal friction between the sole of the ice and subglacial material; similarly,
valley walls can also exert lateral drag to resist glacier movement. The amount of basal shear stress is dependent on the bed roughness, as well as the type of subglacial material. Englacially, the ice body experiences deformation as a result of basal and lateral shear stresses. Such combinations, especially in areas of fast flow, create vertical shear stress gradients where there is significant shearing near the ice bed and limited shearing near the ice surface.

At high stresses typical in normal glacier flow, deformation within glaciers exhibits plastic flow, and the relationship between a dominant shear stress $\tau_{xz}$ and the corresponding shear strain rate $\dot{\varepsilon}_{xz}$ can be characterised by Glen’s flow law (Glen, 1955):

$$\dot{\varepsilon}_{xz} = A\tau_{xz}^{n_G}$$

(4.7)

Here, the creep exponent $n_G$ is normally assumed to be constant, and $A$ represents the flow parameter, which depends strongly on the temperature and crystal fabric of ice (Cuffey and Paterson, 2010).

### 4.1.2 Mass conservation of an ice column

This section summarises the quantitative methods founded and developed by Corr et al. (2002) and Jenkins et al. (2006); a complete description and derivation of these methods can be found in these two papers.

Most glaciological observations made at the ice surface are inherently Lagrangian in nature, where marked points at the surface are followed in space through time (Jenkins et al., 2006). Therefore, the total thickness change measured is the compilation of various factors at the upper surface: snow accumulation and/or ablation occurring at the upper surface, compaction and conversion of the upper snow layer and firn into solid ice, vertical compression of the entire ice column through divergence of the horizontal ice flow; and at the lower surface: ablation or freeze-on (Figure 4.1). The dynamics occurring between the various factors can be summarised using the generalised equation for the conservation of mass:
4.1 Changes in ice thickness

\[ \nabla \cdot \vec{u} = \dot{m}_s \frac{\partial}{\partial z} \left( \frac{1}{\rho_i} \right) \]  

(4.8)

where \( \vec{u} \) is the three-dimensional ice velocity vector with components \( <x,y,z> \), \( z \) in the vertical direction; \( \dot{m} \) refers to the vertical mass flux through the upper surface (subscript \( s \); defined as negative for a downwards flux), and \( \rho_i \) is the ice or firn density. Equation 4.8 is derived from the assumption that the density-depth ice column profile is both temporally and spatially constant (Corr et al., 2002) and that within the firn, vertical compaction is independent of the viscous responses to deviatoric stress (Jenkins et al., 2006).

A depth-integration of Equation 4.8 between the upper radar reflector (subscript \( u \)) and the glacier basal boundary (subscript \( l \)) provides the expression for the Lagrangian conservation of mass within a moving column of ice:

\[ \frac{\partial}{\partial t} (z_u - z_l) + \frac{\partial}{\partial x} [\vec{u}_x (z_u - z_l)] + \frac{\partial}{\partial y} [\vec{u}_y (z_u - z_l)] = \frac{\dot{m}_s}{\rho_i} + \frac{\dot{m}_u}{\rho_u} - \frac{\dot{m}_l}{\rho_l} \]  

(4.9)

where \( t \) denotes time and \( \rho_{u-l} \) represents the density of the measured layer. The glacier basal melt rate (\( \dot{m}_b \), equivalent to \( \dot{m}_l \) in Equation 4.9) is inferred from the residual of measurements of the ice flux within a specified area of the glacier (second two terms of the left-hand side) and the net surface mass balance over the area (first term on the right-hand side; Corr et al., 2002):

**Application of mass conservation with ApRES**

Using stationary radar, such as the autonomous phase-sensitive radio-echo sounder (ApRES), the englacial vertical strain and subglacial melt within and beneath ice sheets can be inferred by repeat radar sounding using Equation 4.9, tracking the displacement of internal reflectors and the basal reflector in a Lagrangian coordinate system (within the same ice column through time). This technique has been conducted previously on ice shelves, where the plug flow regime of these ice bodies allows for robust measurements to be separated by as far as a year in time (e.g. Corr et al., 2002; Jenkins et al., 2006). However, as glaciers situated on the Greenland Ice Sheet exhibit gravity-driven creep...
flow and deformation, long-term radar measurements on these glaciers will encapsulate inaccuracies by measuring increasingly different ice through depth. As such, measurements of vertical strain and basal melt can be measured directly with this technique, but only with a sufficiently short temporal interval $t$ to minimise errors due to differential horizontal velocities through depth (Jenkins et al., 2006).

4.2 Principles of radar signal propagation

This section summarises the principles governing radar signal models and processing methods from Dowdeswell and Evans (2004) and Richards (2014); a complete description and derivation of these methods can be found in the mentioned references.
4.2 Principles of radar signal propagation

4.2.1 The radar equation

The well-known radar equation calculates the cumulative power measured at the receiving antenna \( (P_r) \), given a pulse of power emitted by the transmitting antenna \( (P_t) \):

\[
P_r = \frac{P_t G^2 \lambda^2 \sigma_0}{4\pi R^2} L_a L_s
\]

Here, Equation 4.10 is represented in terms of system design, medium, and target parameters, with the components comprising this equation described below.

The first term represents the isotropic power density of the transmitted waveform at a specified target. Here, assume that the waveform travels in a lossless medium. Then, the power density at the target range \( R \) (which is the average distance between the source and the transmitting and receiving element) is the total power \( P_t \) divided by the surface area of a sphere of radius \( R \) (where the radar is located at the 3-dimensional centre of the sphere). \( R \) therefore represents both the target range and the spherical radius:

\[
\text{Isotropic transmitted power density} = \frac{P_t}{4\pi R^2}
\]

Here, \( R \) is both the target range and the spherical radius; as the signal source is emitted isotropically, the signal travels equally in all directions, hence depicting the surface area of a sphere (the denominator term in Equation 4.11).

In practice, directive antennas are often preferred over isotropic radiators to focus the outgoing waveform. The second term, \( G \), represents the antenna gain, which is the ratio of maximum power density compared to an isotropic density. This term is squared due to the waveform being transmitted and received, interacting with the antenna twice.

The periodic frequency \( (f) \) and length \( (\lambda) \) of the waveform (where \( \lambda = c_i / f \), \( c_i \) being the speed of light in ice) play a major role in determining the sensitivity and the range resolution of the radar. While radar systems operate at frequencies ranging from 2 MHz to 220 GHz, most radars operate in the microwave frequency region of between 200 MHz to 95 GHz (0.67 m to 3.16 mm). In general, lower frequencies are able to sound
through extensive (>3 km) depths of the ice sheet due to their low attenuation and high available power, while high frequencies achieve finer resolution at the expense of reduced range.

Regarding the fourth term, the effectiveness of the specified target at absorbing and scattering radiation (relative to the radar) can be quantified through its scattering coefficient, $\sigma^0$, considering the gain of the scattered energy re-radiated towards the receiving element. Although the scattering coefficient is often represented through either a Gaussian or sinusoidal function with respect to the angle of incidence, the inability to directly observe the englacial and subglacial properties of an ice sheet coupled with the variation of the materials observed results in uncertainty within this term, to be investigated further in Chapter 8.

The last two terms $L_a$ and $L_s$, represent losses due to attenuation of the waveform within the travelling medium and within the radar system, respectively. Again, due to the same restrictions associated with $\sigma^0$, the medium losses are difficult to accurately quantify, while the system losses are dependent on the radar instrument. Otherwise, Brennan et al. (2014) estimate $L_s$ to be typically $0.01$–$0.02$ dB m$^{-1}$ in glacier ice at frequencies below 600 MHz. The following section explores the various conditions encapsulated within $L_a$ and $L_s$ that affect the propagation of radar waves within ice.

### 4.2.2 Propagation of electromagnetic waves in ice

#### Permittivity of ice

Within dielectric materials, permittivity is an all-encapsulating measure of the electric properties within a given medium. Above all other factors, the complex dielectric permittivity of ice is the dominant controlling factor for radiowave propagation, reflection, and attenuation within ice (Fujita et al., 1999). In polar environments, radar surveys often distinguish ice stratification through identifying abrupt changes in the dielectric permittivity of sequential ice layers. Additionally, because the water content of glacier ice and its distribution exerts a strong control on radar propagation velocity and attenuation
by increasing its relative permittivity, characterising the variability of ice permittivity through various parameters is of utmost importance.

Permittivity often represented by the ratio of its absolute permittivity to the electric constant, in which the dimensionless constant is referred to as the medium’s relative permittivity, or the dielectric constant. Hereafter, all references to the permittivity $\varepsilon$ and its derivatives refer to the term in its relative sense.

The relative complex dielectric permittivity $\varepsilon$ is described by the Debye expression (Glen and Paren, 1975):

$$\varepsilon = \varepsilon' + j\varepsilon'' = \varepsilon'_\infty + \frac{\varepsilon'_s - \varepsilon'_\infty}{1 + j\omega\tau_r}$$

(4.12)

Here, Equation 4.12 can be partitioned into a real and imaginary component $\varepsilon'$ and $\varepsilon''$, and decomposed further into the relative high-frequency-limit dielectric constant $\varepsilon'_\infty$, the relative static dielectric constant $\varepsilon'_s$, the relaxation time $\tau_r$, and the angular frequency $\omega$.

The relative permittivity is also related to attenuation by the following equation (e.g. Scott et al., 1967):

$$L\alpha = \omega \left\{ \frac{\mu\varepsilon}{2} \left[ \left( 1 + \frac{\sigma_c^2}{\omega^2\varepsilon^2} \right)^\frac{1}{2} - 1 \right] \right\}^{\frac{1}{2}}$$

(4.13)

where $L\alpha$ is the attenuation constant, $\mu$ is the magnetic permeability, and $\sigma_c$ is the conductivity.

From laboratory experiments, within the microwave and millimetre-wave range, the real part of Equation 4.12 is in general proportional with temperature (Figure 4.2a). It follows that the speed of electromagnetic waves decreases in warmer ice from the following equation (Fujita et al., 2000):

$$c_i = \frac{c}{\sqrt{\varepsilon_r}}$$

(4.14)
Figure 4.2 Variation of the real part of the permittivity of ice hexagonal crystals ($\varepsilon'$) in the megahertz and microwave range from laboratory experiments with changes in (a) temperature and (b) frequency. From Fujita et al. (2000).

where $c_i$ is the speed of light within ice and $\varepsilon' = \varepsilon_r$ the real component of the relative permittivity in ice. Similarly, $\varepsilon_r$ decreases with increasing frequency in the megahertz range, showing a drop of 0.04 at 252 K (Figure 4.2b). Accordingly, given the temperature range of the cryosphere and frequency range of RES instruments available, $\varepsilon_r$ is often assumed to be between 3.10–3.18 (Dowdeswell and Evans, 2004).

Radar wave attenuation

Due to the combined properties of radar wave propagation and impure ice, signal attenuation occurs as a result of three main processes: (i) volume scattering; (ii) dielectric dissipation; and (iii) geometrical spreading (Plewes and Hubbard, 2001).

Volume scattering

Generally, the bulk of signal attenuation within ice occurs through scattering, which broadly encompasses a variety of processes leading to the loss of energy, including reflection, refraction, and diffraction. Desirable scatter is produced by direct wave reflection from the target of interest, such as an internal layer, while unwanted scatter is often regarded as clutter, or noise (Plewes and Hubbard, 2001). The latter often
arises through impurities within the ice column, such as air bubbles and/or ice lenses, a firn aquifer, meltwater on the top surface of the ice, or brine in an ice shelf from basal freeze-on. Scattering losses to the received signal are dependent on the number, size, and type of scattering bodies within the ice, as well as their electrical and geometrical properties.

By using detailed numerical models of ice, fresh and salt water, and air, Smith and Evans (1972) showed the potential effect of these scatterers on the signal-to-noise ratio (SNR) can be potentially intolerable; however the total loss to the received signal is not likely serious in practice. Dowdeswell and Evans (2004) states that, because the cross-section of scatterers are often less than the radio wavelength in the medium and therefore follows Rayleigh’s $f^4$ law, the SNR can be reduced by either reducing the frequency of the carrier signal, or to reduce the transmitted pulse duration (and hence increasing the radio frequency) that reduces the total amount of echo generated.

### Dielectric dissipation

The dielectric absorption occurs in ice through two main processes: the conduction of free-charge electrons and relaxation that causes the oscillation of water molecules within their lattice structure (Plewes and Hubbard, 2001). In general, dielectric loss is generally characterised either by the imaginary component of relative permittivity ($\varepsilon''$), or by rewriting permittivity in terms of conductivity $\sigma$ (Equation 4.13; Dowdeswell and Evans, 2004). The density and mobility of electrons within ice, and hence the dielectric absorption, is dependent on the impurity content and temperature of ice, the former directly linked to conductivity (Equation 4.13) and the latter directly influencing $\tau_r$, the relaxation time (Equation 4.12; Evans, 1965).

### Geometrical spreading

As a radar wave propagates away from the source, energy is lost as it spreads out across the volume of the subsurface. Although part of the loss of energy is dictated by the array factor of the antenna (Section 4.4), the geometrical spreading of the signal is primarily
a function of the radar wave travel path. The geometric power fall-off is dependent on
the radar equation and the different reflector surface types, but is usually represented
as a loss of energy at a rate of $1/r^2$, where $r$ is the radius from source (Haynes et al.,
accepted).

### 4.3 Phase-sensitive FMCW radar signal processing

This section summarises the quantitative methods founded and developed by Brennan
et al. (2014) and Stewart (2018); a complete description and derivation of these methods
can be found in the mentioned references.

First, consider the instantaneous phase of the deramped FMCW signal, which is the
difference between the transmit and receive signal phases:

$$
\phi_d(t) = \phi_T - \phi_R
$$

$$
= \left[ \omega_c t + \frac{K t^2}{2} - \frac{K T t}{2} + C_1 \right] - \left[ \omega_c (t - \tau) + \frac{K (t - \tau)^2}{2} - \frac{K T (t - \tau)}{2} + C_2 \right]
$$

$$
= \omega_c \tau + \frac{K \tau (t - T)}{2} - \frac{K \tau^2}{2} + C
$$

(4.15)

where $\phi_T$ and $\phi_R$ represent the instantaneous phase of the transmit and receive signals
in s, $\omega_c$ is the chirp centre frequency in terms of angular velocity [rad s$^{-1}$], $t$ the time
elapsed, again in s, and $C$ a constant offset term. $K = 2\pi B/T$ is the chirp gradient
[rad s$^{-2}$] that represents the frequency-modulated sweep rate, where, within the equation,
$B$ is the system bandwidth (200–400 MHz), and $T$ is the total pulse duration. $\tau$ is the
round-trip delay from the radar transmitter to the receiver, and is given as:

$$
\tau = \frac{2\pi f_d}{K}
$$

(4.16)

where $f_d$ is the deramped frequency.
4.3.1 Coarse range equations

$f_d$ can be obtained by differentiating Equation 4.15 with respect to the instantaneous time $t$:

$$f_d = \frac{2BR_c}{c_i T}$$  \hspace{1cm} (4.17)

Here, $R_c$ is the “coarse range” estimate to a reflector determined by the two-way travel time of the pulse, commonly estimated by:

$$R_c = \frac{\tau c_i}{2}$$  \hspace{1cm} (4.18)

Note also that the deramped frequency is scaled to the angular frequency of the deramped chirp by: $f_d = \omega_d / 2\pi$. With all the above, rearranging Equation 4.17 will then give the target coarse range, now in terms of the deramped frequency:

$$R_c = \frac{f_d c_i T}{2B}$$  \hspace{1cm} (4.19)

Equation 4.19 is normally determined using Fast Fourier Transform (FFT) processing of the deramped pulse of duration $T$, resulting in a range profile with a frequency resolution $\Delta f_d = 1/T$ and a range resolution $\Delta R_c$:

$$\Delta R_c = \frac{c_i}{2B}$$  \hspace{1cm} (4.20)

Therefore, the coarse range to the $n^{th}$ range bin can be represented by:

$$R_c(n) = \frac{nc_i}{2B}$$  \hspace{1cm} (4.21)
4.3.2 Fine range equations

Equation 4.21 provides the range of a specified reflector to the closest bin centre; however, it does not provide any fine-scale quantification of the distance between the reflector and the nearest bin centre. This measurement, known as the “fine range”, can be estimated from the phase of the deramped signal (Equation 4.15). Here, the final equation is composed of four components: the first term \( \omega_c \tau \) representing the phase of the deramped signal at the FFT bin centre; the second term \( K \tau \left( t - T \right)/2 \) being a time-linear phase term from the signal frequency; the third term \(-K\tau^2/2\) representing a very small phase offset; and a final constant term \( C \). Disregarding the constant, evaluating Equation 4.15 at the centre of the chirp \( t = T/2 \) removes the second term, leaving the measured phase and the phase offset:

\[
\phi_d = \omega_c \tau - \frac{K \tau^2}{2} \quad (4.22)
\]

The second term can be ignored for small ranges (i.e. smaller than each range bin); leaving just the first term. Substituting Equation 4.16 and the centre frequency wavelength in ice \( \lambda_c = \frac{2 \pi c}{\omega_c \sqrt{\varepsilon_r}} \) into Equation 4.22 gives the fine range to the reflector of each coarse range bin:

\[
R_f (n) = \frac{\phi_d \lambda_c}{4\pi} \quad (4.23)
\]

Note that Equation 4.23 is not yet normalised to each bin centre; this process requires subtracting the FFT-processed deramped waveform by the expected phase at the centre of each range bin. This phase shift is essential if offsets are calculated using phase differencing procedures to determine changes in range to millimetre precision. Mathematically, the amount of phase shift is represented by weighting the FFT-processed deramped waveform by a reference array equal to the phase conjugate of the expected phase at the centre of the range bin. The reference array is described by (Equation 17, Brennan et al., 2014):
\[ \phi_{ref}(n) = \exp \left[ -i \left( \frac{n \omega_c}{Bp} - \frac{n^2 K}{2B^2 p^2} \right) \right] \] (4.24)

The exponent in Equation 4.24 was derived by substituting the round-trip delay (\(\tau_n\); Equation 4.16), into Equation 4.22, given that the coarse range of the \(n^{th}\) range bin is \(n\Delta R\).

Finally, the referenced phase of the \(n^{th}\) range bin relative to the bin centre is obtained:

\[ R_f(n) = \frac{\phi_r \lambda_c}{4\pi} \] (4.25)

Note that to obtain an unambiguous measurement of Equation 4.25, \(\phi_r\) must lie within the bounds of \([-\pi, \pi]\), which constrains observations to \(|R_f(n)| < \lambda_c/4\).

### 4.3.3 Determination of total range

Therefore, the total range to a specified reflector \(n\) is given by the sum of the coarse and fine range:

\[ R(n) = R_c(n) + R_f(n) \] (4.26)

### 4.4 Antenna array theory

#### 4.4.1 Derivation of array factor

Note that entire textbooks and manuals have been written on antenna and array design (e.g. Haupt, 2010; Mulligan, 2005; Visser, 2005). The necessary theory for the purposes of this project is covered here, following the examples of the above references.
Phased array pulse generation and retrieval

In antenna theory, a phased array is an electronically directive array made up of a number of individual real antennas, or radiating real elements (Mulligan, 2005). Arguably the main advantage of phased arrays is their ability to electronically steer the overall beam by controlling the phase of the transmitted signal produced by each element (Skolnik, 2001). By scanning the beam across a specified range of angles, data from the series of beams can be synthesised to generate a multidimensional image of multiple reflectors with depth (Lok et al., 2015).

The antenna elements comprising a phased array are often arranged in one of several ways: as a linear array, in which elements are arranged in a straight line in one dimension; as a planar array, where real elements form a 2-dimensional grid within a plane; and as a circular array, where elements are arranged in a circle around an origin. The midpoints of combinations of real antenna pairs are known as virtual antennas, or virtual elements. This paper focuses on the linear and planar array as potential arrangements for the ApRES unit.

Radiation pattern and the array factor of phased arrays

Due to its modular configuration, the overall beam of a phased array is composed of radio waves from each individual antenna. The received signal of the total array ($F$) is thus the combination of the radiation pattern of a single antenna element (element factor; $F_e$) and the radiation pattern of the overall array (array factor; $F_a$), which are all dependent on the angles $\theta$ (relative to nadir) upon which the wave is received:

$$F (\theta) = F_e (\theta) F_a (\theta) \quad (4.27)$$
Consider first a linear antenna array $S$ of $K$ elements equally spaced $\delta$ m apart, with the positions of each element $k$ in the array referenced relative to the first element (Figure 4.3):

$$S_k = (k - 1) \delta u \quad (4.28)$$

where $u$ is the unit vector in the direction of the array elements. The overall received signal along one axis is then expressed as (e.g. Visser, 2005):

$$F(\theta) = \sum_{k=1}^{K} F_k(\theta) = \sum_{k=1}^{K} \exp \left[ -j (K - k) k_0 \delta \sin(\theta) \right] \quad (4.29)$$

where $j^2 = -1$ is the imaginary unit and $k_0$ is the free space wave number, which is inversely proportional to the signal wavelength within a specified medium.
Note that the phase of the received signal $F(\theta)$ is manifested in Equation 4.29 as the imaginary portion of the equation.

The path-length difference between two consecutive elements in the array antenna, $\Delta l$, can be found through simple trigonometry (Figure 4.3):

$$\Delta l = \delta \sin (\theta)$$ (4.31)

The phase difference between two adjacent elements along $x$ is then the path length difference converted into units of phase with the free space wave number:

$$\Delta \phi = k_0 \Delta l = k_0 \delta \sin (\theta)$$ (4.32)

The phase of element $k$ relative to element 1, $\phi_k$, is then:

$$\phi_k = \phi_{k-1} \Delta \phi = k_0 (k - 1) \delta \sin (\theta)$$ (4.33)

which forms the imaginary component of Equation 4.29. The complex signals received by each element of the antenna array are then:

$$F_k (\theta) = F_e A_k \exp [- j (K - k) \Delta \phi] \text{ for } k = 1, 2, ..., K$$ (4.34)

where $A_k$ is the signal amplitude received by the $k^{th}$ element, which, here, is uniform across all elements due to equal element spacing $\delta$. The overall received signal is then the summation of the signals received by each element of the array antenna, where the array factor is the component influenced by the array configuration:

$$F_a (\theta) = \sum_{k=1}^{K} \exp [- j (K - k) \Delta \phi]$$ (4.35)
Array factor for planar arrays

A planar array can be constructed by stacking linear arrays in the orthogonal direction. Given the original array $S$ (Equation 4.28), now with the element positions rewritten into two dimensions $S_k = S_{k,1}$ and with element spacings $\delta = \delta_x$, additional elements $S_{k,l}$ are placed at spacings $\delta_y$ below the corresponding element in the previous row $S_{k,l-1}$ until $L$ number of additional arrays are constructed. We now have a system of $K \times L$ identical, directive elements located at equidistant positions along two axes (i.e. arranged on a regular, rectangular grid), and can represent the position of each element $(k,l)$ in the array $S_{k,l}$ within the system plane relative to the first element as:

$$S_{k,l} = (k - 1) \delta_x u_x + (l - 1) \delta_y u_y \quad (4.36)$$

Therefore, the path-length difference $l$ between element $(k,l)$ with respect to the $(1,1)^{th}$ element is now given by:

$$\Delta l_{k,l} = S \cdot r_{k,l}$$
$$= (k - 1) \delta_x S \cdot u_x + (l - 1) \delta_y S \cdot u_y$$
$$= (k - 1) \delta_x \sin (\theta) \cos (\psi) + (l - 1) \delta_y \sin (\theta) \sin (\psi) \quad (4.37)$$

and the element’s corresponding phase (again, relative to the $(1,1)^{th}$ element) is given by:

$$\phi_{k,l} = k_0 (k - 1) \delta_x \sin (\theta) \cos (\psi) + k_0 (l - 1) \delta_y \sin (\theta) \sin (\psi) \quad (4.38)$$

where the cross-angle is given by $\psi$ and runs perpendicular to the swath of $\theta$. The received component (Equation 4.27) is then separated into one component from a single radiator and two components due to the array configuration:

$$F(\theta, \psi) = F_x(\theta, \psi) F_{a_x}(\theta, \psi) F_{a_y}(\theta, \psi) \quad (4.39)$$
Theory

where:

\[
F_{ax}(\theta, \psi) = \sum_{k=1}^{K} \exp \left[ -jk_0 (K - k) \delta_x \sin(\theta) \cos(\psi) \right] \tag{4.40}
\]

\[
F_{ay}(\theta, \psi) = \sum_{l=1}^{L} \exp \left[ -jk_0 (L - l) \delta_y \sin(\theta) \sin(\psi) \right] \tag{4.41}
\]

4.4.2 Fourier analysis of the array factor

The overall antenna behaviour is dominated by the shape of the array factor, which, in addition to the majority of the power transmitted and received from within the confines of the main lobe, is influenced by residual and often undesired power from side and grating lobes (Visser, 2005). Equations 4.40 and 4.41 also shows that changing the element separation \((\delta_x, \delta_y)\) and the number of antennas \((K, L)\) will alter the resulting array factor.

Element separation

Figure 4.4a shows the effects of increasing the element separation length on the resulting array factor. Here, the array factor using an element spacing of \(\delta = \lambda/2\) has the widest main beam with a half-power beamwidth (HPBW) of \(\pm 4.33^\circ\), but by doubling the element spacing to \(\delta = \lambda_c\), the HPBW is halved to \(2.17^\circ\). The angular range of the array factor has been compressed by a factor of 3 (due to the sine function in Equation 4.29). However, by doing so, grating lobes are introduced at \(\theta = \pm 90^\circ\) due to the array factor being a periodic function, with maxima occurring at:

\[
\frac{\pi \delta}{\lambda_c} \sin(\theta) = n\pi \text{ for } n = 1, 2, 3, ...
\]

Beyond this critical element distance, the grating lobes approach closer to the main beam and aliasing is observed by means of pattern replication, as displayed when we use an element spacing of \(\delta = 2\lambda_c\) (Figure 4.4a). The presence of grating lobes within the desired range of scanning angles, therefore, introduces a significant problem in that the
Figure 4.4 (a) Array factors of $16 \times 1$ element array for 3 different spacings relative to $\lambda_c$. (b) Array factors at $1 \times \lambda_c$ spacing for 3 different element array lengths. (c) Influence of antenna element pattern on overall array radiation pattern for a $16 \times 1$ element array at $1 \times \lambda_c$ spacing.
direction of the signal cannot be determined due to the angle of arrival being ambiguous (Haupt, 2010). Consequently, we can limit the range of element distances to ensure only one maximum within the range $\theta_{\text{min}} \leq \theta \leq \theta_{\text{max}}$:

$$\frac{\delta}{\lambda_c} = \left| \frac{1}{\sin(\theta)} \right|$$  (4.43)

Equation 4.43 will thus give the angle at which the grating lobes will display aliasing of the internal layers.

**Array size**

In Figure 4.4b, the effects of increasing the number of elements $K$ along a linear array $S$ can be seen. By doubling the number of elements from 8 to 16, the HPBW can be decreased from $\pm 4.35^\circ$ to $\pm 2.17^\circ$, which lowers the power of the sidelobes by $\sim 3$ dB (roughly half of the original power) and effectively increasing both the gain and the angular resolution. Simultaneously, the number of sidelobes also doubles as a result.

As a sidenote, increasing the array dimension by additional linear arrays of dimension $L$ does not change the array factor $F_{ax}$ of the linear array $S$, but instead will introduce an additional array factor $F_{ay}$ for the orthogonal direction.

**Element factor**

From Equation 4.27 and Figure 4.4c, we can see that the shape of the element factor from a single antenna can influence the overall array factor by acting as an angular filter that reduces the radiated power of the array antenna for angles away from the direction of the antenna main beam. As with all antennas, there exists a tradeoff between gain and beamwidth (Visser, 2005). High-gain narrow-beam antennas (e.g. Yagi-Uda and parabolic antennas) may overly restrict the array’s field of vision, while wide-beam antennas (e.g. horn, patch, and bowtie antennas) lack angular precision and, if the array design is not optimised, may have unwanted side or grating lobes. The choice of antenna type therefore has large implications on the angular extent of the recreated profile.
Chapter 5

Study area and radar deployment

Author contributions

As with all collaborative projects, several authors have contributed to the fieldwork and methods presented within this chapter. All fieldwork was conducted in conjunction with the Subglacial Access and Fast Ice Research Experiment (SAFIRE; Section 5.2) funded by the Natural Environment Research Council. The SAFIRE team comprised Poul Christoffersen (Lead PI), Bryn Hubbard (Co-PI), Samuel Doyle, Alun Hubbard, and the author (T.J. Young). Regarding the radar hardware: Paul Brennan, Lai Bun Lok, Keith Nicholls, and Hugh Corr designed the ApRES. Lai Bun Lok and Keith Nicholls designed the cavity-backed bowtie antennas. Keith Nicholls and the author designed the antenna array, with guidance from Lai Bun Lok. The author constructed the bowtie antennas and deployed the antenna array, with help from the SAFIRE team and Leo Nathan.

5.1 Introduction

The main aim of this thesis is to investigate the englacial and basal processes influencing fast glacier motion occurring beneath the Greenland Ice Sheet. Therefore, for reasons outlined in Chapter 1.2, the thesis is primarily based upon field measurements obtained by autonomous phase-sensitive radio-echo sounding (ApRES).
This chapter introduces the SAFIRE project (Section 5.2), the larger study area (Store Glacier, Section 5.3.1), and the specific field site of the thesis (S30, Section 5.3.2). Additionally, this chapter describes the field experimental design of the ApRES deployments (Section 5.4).

5.2 The Subglacial Access and Fast Ice Research Experiment (SAFIRE)

The studies outlined within this thesis supplement research conducted as part of the Subglacial Access and Fast Ice Research Experiment (SAFIRE), led by Professors Poul Christoffersen and Bryn Hubbard. SAFIRE is funded by the United Kingdom’s National Experimental Research Council (NERC), and is a joint project between the University of Cambridge’s Scott Polar Research Institute and Aberystwyth University’s Centre for Glaciology. The primary goal of SAFIRE was to identify and characterise the mechanical and hydrological conditions at the bed of a large marine-terminating glacier in Greenland, which was achieved by a number of englacial and subglacial instruments installed in access-boreholes drilled to the glacier bed. Therefore, the datasets obtained through SAFIRE provide environmental context for ApRES findings, and will be summarised below. Further details of the study area, particularly the interpretations regarding the underlying subglacial environment, can be found in Doyle et al. (2018).

In late July and early August of 2014, the SAFIRE team drilled four boreholes to the glacier bed at S30 using a pressurised hot-water drill (Doyle et al., 2018) (Figures 5.1, 5.2). All four boreholes instantly connected with a basal water system upon reaching the bed, and were instrumented to monitor basal water pressure, temperature, electrical conductivity and turbidity along with englacial ice temperature and deformation (Table 5.1). Additionally, an autonomous weather station (AWS) and several global positioning system (GPS) receivers were deployed at and surrounding the drill site (Figure 5.2). The results from these deployments are summarised below (Section 5.3.2), with a thorough discussion and interpretation given in Doyle et al. (2018).
Figure 5.1 (a) InSAR-derived surface velocities of Store Glacier (Rignot and Mouginot, 2012); and (b) bed topography inferred from mass conservation, with the central flowline marked (Morlighem et al., 2015). S30 (Section 5.3.2) is shown with a white star. GPS records were referenced to the STNN base station (white triangle). Figures (a) and (b) were adapted from Morlighem et al. (2016). (c) Surface and bed topography of Store Glacier along the central flowline.
Table 5.1 Metadata on the borehole instruments and settings deployed at S30. Sensors included a string of 11 thermistors (T) and 5 inclinometers (A), as well as 3 multiprobes (M). Adapted from Doyle et al. (2018).

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Borehole</th>
<th>Depth [m]</th>
<th>Date installed [UTC]</th>
<th>Date stopped [UTC]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>14b</td>
<td>601.5</td>
<td>2014-07-26 23:01</td>
<td>2014-10-11 03:00a</td>
</tr>
<tr>
<td>T2</td>
<td>14b</td>
<td>600.5</td>
<td>2014-07-26 23:01</td>
<td>2014-10-18 01:00a</td>
</tr>
<tr>
<td>T3</td>
<td>14b</td>
<td>596.5</td>
<td>2014-07-26 23:01</td>
<td>2014-10-16 15:00a</td>
</tr>
<tr>
<td>T4</td>
<td>14b</td>
<td>591.5</td>
<td>2014-07-26 23:01</td>
<td>2014-10-12 23:00a</td>
</tr>
<tr>
<td>T5</td>
<td>14b</td>
<td>551.6</td>
<td>2014-07-26 23:01</td>
<td>2014-10-27 16:00a</td>
</tr>
<tr>
<td>T6</td>
<td>14b</td>
<td>501.9</td>
<td>2014-07-26 23:01</td>
<td>2015-07-04 14:00a</td>
</tr>
<tr>
<td>T7</td>
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<td>2014-07-26 23:01</td>
<td>2015-07-04 14:00a</td>
</tr>
<tr>
<td>T8</td>
<td>14b</td>
<td>401.9</td>
<td>2014-07-26 23:01</td>
<td>2015-07-04 14:00a</td>
</tr>
<tr>
<td>T9</td>
<td>14b</td>
<td>302.0</td>
<td>2014-07-26 23:01</td>
<td>b</td>
</tr>
<tr>
<td>T10</td>
<td>14b</td>
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<td>2014-07-26 23:01</td>
<td>b</td>
</tr>
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<td>T11</td>
<td>14b</td>
<td>101.7</td>
<td>2014-07-26 23:01</td>
<td>b</td>
</tr>
<tr>
<td>A1</td>
<td>14b</td>
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<td>2014-07-26 23:01</td>
<td>2014-09-29 12:00a</td>
</tr>
<tr>
<td>A2</td>
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<td>597.3</td>
<td>2014-07-26 23:01</td>
<td>2014-09-29 12:00a</td>
</tr>
<tr>
<td>A3</td>
<td>14b</td>
<td>592.3</td>
<td>2014-07-26 23:01</td>
<td>2014-09-29 12:00a</td>
</tr>
<tr>
<td>A4</td>
<td>14b</td>
<td>552.5</td>
<td>2014-07-26 23:01</td>
<td>2014-09-29 12:00a</td>
</tr>
<tr>
<td>A5</td>
<td>14b</td>
<td>401.9</td>
<td>2014-07-26 23:01</td>
<td>2014-09-29 12:00a</td>
</tr>
<tr>
<td>M1c</td>
<td>14c</td>
<td>603.3</td>
<td>2014-07-29 19:45</td>
<td>2014-10-21 10:00a</td>
</tr>
<tr>
<td>M2c</td>
<td>14d</td>
<td>615.9</td>
<td>2014-07-31 19:56</td>
<td>2014-08-29 16:00a</td>
</tr>
<tr>
<td>M3d</td>
<td>16b</td>
<td>619.2</td>
<td>2016-07-12 10:00</td>
<td>2016-09-07 08:00a</td>
</tr>
</tbody>
</table>

aTermination due to cable failure.
bThese thermistors were still working on the final download at 2015-07-04 20:00 UTC, albeit with slight errors in temperature due to cable strain.
cBasal multiprobe measuring water pressure, temperature, conductivity.
dSame as c, but also measuring turbidity.
In July of 2016, the SAFIRE team drilled an additional three boreholes at a site located 50 m to the northeast of the 2014 drill site (Figure 5.2b), installing one multiprobe monitoring basal water pressure, temperature, electrical conductivity, and turbidity (Table 5.1; Doyle et al., 2018). Although the data gleaned from this instalment were not used in this thesis, they nevertheless contextualize the complexities of studying the geodynamics of fast flow.

**Figure 5.2** (a) Study area (S30) showing the locations of instruments deployed during May 2014 (red), July/August 2014 (yellow), July 2015 (blue), and July 2016 (green). The local ice thickness is interpolated from quasi-monostatic (1 Tx / 1 Rx) ApRES measurements. Map is superimposed on a WorldView-2 image at 2 m resolution (27 July 2008). The two transects (1 to 1'; 2 to 2') indicate location of topographic profiles (Figure 5.3). Locations for Figure 6.1 and Figure A.1 are outlined. (b) Inset of borehole instrument deployment locations.

### 5.3 Study area

#### 5.3.1 Store Glacier

Store Glacier (*Qarassap Sermia*) is a fast-flowing, marine-terminating glacier in the Uummannaq region in western Greenland (*Qaasiutsup Kommunia*; Figure 5.1). The glacier has a catchment of 35 000 km² and is 5 km wide at the terminus, where the glacier flows at 6300 m a⁻¹ (Rignot and Mouginot, 2012) and discharges 14–18 km³ of ice onto the ocean annually (Weidick, 1995). While many surrounding glaciers have recently experienced dynamic thinning due to acceleration and retreat of their termini, Store
Study area and radar deployment

Glacier has been observed to be stable in both mass balance and terminus position since 1968 with a 200 m seasonal oscillation in the terminus position (Box and Decker, 2011; Howat et al., 2010; Weidick, 1995). This stability is underpinned by topographic narrowing and grounding on a sill that is ~450 m below sea level, located near the glacier’s terminus (Figure 5.1a,b; Morlighem et al., 2016; Todd and Christoffersen, 2014). However, past this sill, the bed is retrograde with depths 700–800 m below sea level and extends 30 km inland from the margin (Figure 5.1c; Morlighem et al., 2016). Therefore, Store Glacier may experience rapid and prolonged retreat if the terminus stability is undermined by climate change impacts (Morlighem et al., 2016).

5.3.2 S30

The study site, S30, is approximately 30 km inland of Store Glacier’s calving front near the central flowline (~1 km), with an ice thickness between 600–650 m (70° 31’ N, 49° 56’ W; Figure 5.2a) resting on a ~50 m thick layer of unconsolidated sediment (Hofstede et al., 2018). In general, the local ice thickness increases towards the north (Figure 5.2a) and drops consistently with the bed along-flow (Figure 5.3b), while to the south, the flow is constrained by increasing bed elevation (Figure 5.3a). Qualitatively, crevasses bound
5.3 Study area

the drill site on all sides, particularly to the west, where a 50 m-high icefall is located approximately 2 km from the drill site (Hofstede et al., 2018).

S30 is located well within the ablation zone of the Greenland Ice Sheet. During the summer melt season, temperatures ranged between \(-8\)–\(-6^\circ\) (Figure 5.4a,f), and, together with periodic high rates of recorded precipitation, generated transient bursts of surface melt of up to 60 mm d\(^{-1}\) (Figure 5.4b,g). Subglacially, with water pressures persistently close to overburden (93–95\% of \(p_i\); Figure 5.4c,h), the basal hydrological system is thought to be inefficient with water flowing both at the ice-sediment interface and within the basal sediment layer itself, as evidenced by fluctuations in the electrical conductivity of subglacial water (Figure 5.4d,i). Here, the glacier exhibits a moderate response to seasonal changes in ice velocities, where small variations in basal water pressures were concomitant with large fluctuations in surface ice velocity and uplift (Figure 5.4e,j). These observations indicate that basal motion at S30 is sensitive to the influx of surface meltwater into the subglacial environment.

Englacially, ice temperatures varied considerably with depth, from \(-21.25^\circ\)C at 302 m below the surface to near the pressure melting point (PMP) approaching the glacier bed (Figure 5.5a,d), producing a temperature profile indicative of fast flow. The cause of the kink seen in the temperature profile at 301–451 m in depth, with temperatures \(-1\)–\(-2^\circ\)C warmer than expected, is unknown, but could be the due to englacial heat sources from refreezing water or strain heating. Temperatures recorded by T1, the lowest thermistor in one of the borehole deployments, persistently fluctuated above PMP, and is thought to have remained in relatively warm liquid water or unfrozen sediment for the duration of its operation (Figure 5.5e).

Analysis of the inclinometer measurements revealed enhanced deformation in the lower 50–100 m, producing a vertical flow profile similar to that of fast ice flow. Specifically, 70\% of the deformation was found to occur in the lower 100 m of the ice column, with 40\% in the lowest 50 m (Figure 5.5b). Supported by seismic studies conducted over the same area (Hofstede et al., 2018), this zone of enhanced deformation is thought to be comprised of softer, Wisconsin-age ice, with the upper boundary estimated at 528–566 m depth (86\% of the ice thickness). The two lowest sensors (A1, A3) recorded anomalous
Study area and radar deployment

Figure 5.4 Time series of (a) near-surface air temperature and surface melt rate; (b) precipitation rate and relative humidity; (c) subglacial water pressure ($\rho_w$); (d) subglacial electrical conductivity; and (e) surface velocity ($u_s$) and linearly-detrended vertical surface uplift ($z$). Subplots (f) to (j) are the same as (a) to (e) for 2016. From Doyle et al. (2018).
5.3 Study area

Figure 5.5 Depth profiles at S30 of (a) temperature, (b) internal deformation, and (c) velocity. In (a), the purple line shows the approximate depth of the Holocene-Wisconsin Transition (HWT) as estimated from (Hofstede et al., 2018). The green box shows the extent of the inset profile (d) for thermistors near the bed. The grey shade indicates the estimated depth of the transition surface between cold and temperate ice (CTS). Dashed lines indicate the pressure melting point between pure ice air-saturated water (red) and between pure ice and pure water (blue). In (b) and (c), the horizontal velocity gradients $du/dz$ were modelled using parameters suggested in Cuffey and Paterson (2010) $\pm 1^\circ$ (blue shade). (e) Temperature time series for the thermistors near the bed, with dashed vertical lines showing the installation of the latter two thermistors. Adapted from Doyle et al. (2018).
Table 5.2 Metadata on the three ApRES instruments and settings deployed at S30.

<table>
<thead>
<tr>
<th>Deployment</th>
<th>14a</th>
<th>14b</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start location [Latitude]</td>
<td>70° 31’ 2” N</td>
<td>70° 31’ 5” N</td>
<td>70° 31’ 7” N</td>
</tr>
<tr>
<td>Start location [Longitude]</td>
<td>49° 55’ 46” W</td>
<td>49° 55’ 9” W</td>
<td>49° 55’ 37” W</td>
</tr>
<tr>
<td>Start Altitude [m a.s.l.]</td>
<td>961</td>
<td>981</td>
<td>961</td>
</tr>
<tr>
<td>Stop date/time [UTC]</td>
<td>2014-07-16 12:42</td>
<td>2014-12-04 02:05</td>
<td>2016-06-06 08:25</td>
</tr>
<tr>
<td>Duration [days]</td>
<td>72</td>
<td>124</td>
<td>338</td>
</tr>
<tr>
<td>Distance travelled [m]</td>
<td>121</td>
<td>213</td>
<td>570</td>
</tr>
<tr>
<td>Burst mode</td>
<td>MIMO</td>
<td>MIMO</td>
<td>Alternating</td>
</tr>
<tr>
<td>Burst interval [hours]</td>
<td>1</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Total bursts</td>
<td>1668</td>
<td>734</td>
<td>1011</td>
</tr>
<tr>
<td>Chirps per burst</td>
<td>64</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>Chirp sampling rate [Hz]</td>
<td>40000</td>
<td>40000</td>
<td>40001</td>
</tr>
<tr>
<td>Tx₁-Rx₁ separation [mm]</td>
<td>2350</td>
<td>2700</td>
<td>2480</td>
</tr>
<tr>
<td>Orientation [°]</td>
<td>+12</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<sup>a</sup>Termination due to instrument hardware failure.
<sup>b</sup>Alternate bursts between quasi-monostatic and MIMO mode every 4 hours.
<sup>c</sup>per burst mode
<sup>d</sup>Relative to the principal flow direction (262°).

tilt measurements deviating three times lower than theoretical values (Figure 5.5b); however, these findings are similar to those reported in other Greenland-based borehole studies (Lüthi et al., 2002; Ryser et al., 2014b). Integrating through the ice column, we found that over the stable periods of its operation (03–26 September 2014), basal motion averaged 428 m a⁻¹ (72% of averaged surface motion, estimated to be 592 m a⁻¹; Figure 5.5c).

5.4 ApRES and antenna instrumentation and deployment

5.4.1 ApRES instrumentation and operation

The first generation of the ApRES (Figure 3.5b) was used within the following three chapters of this dissertation (Chapters 6, 7, 8). This version of the ApRES comprises
a direct digital synthesis (DDS) board that generates a linear frequency chirp signal over a period of 1 s, giving a centre frequency of 300 MHz and a sweep bandwidth of 200–400 MHz. The modulation of frequency allows for ice penetration at sufficient depths, while its phase sensitivity allows the system to achieve millimetre-level depth precision. The radio frequency subsystem comprises a single-channel transmitter and homodyne receiver, which is capable of switching the transmitting chirp signal and its echo through 8 transmitting (Tx) and 8 receiving (Rx) antennas for a total of 64 possible antenna pair combinations. The received deramped signal was sampled at 40 kHz by an analogue-to-digital converter (ADC). The ADC and DDS board are both clocked at 1 GHz with a high-performance crystal oscillator, which synchronises the transmitted and received signals allowing for phase-coherent operation and achieving high precision measurements. The design and technical details for the instrument’s radar board are described in detail in Brennan et al. (2014) and Lok et al. (2015) and the practical aspects and limitations of its deployment in a quasi-monostatic configuration are presented in Nicholls et al. (2015).

Operation of the ApRES instrument was conducted in both attended and unattended modes (Table 5.2). In both cases, the ApRES was configured to transmit a series of chirps, known as a burst, periodically throughout the duration of its deployment.

### 5.4.2 ApRES array deployment

Over three field campaigns (Table 5.2), three frequency-modulated continuous-wave (FMCW) ApRES arrays were deployed in an 8 Tx × 8 Rx gridded arrangement at S30 (Figure 5.6). Crucially for this thesis, the power consumption of the system is on the order of 5 W during operation and 1 mW while in standby mode, which, with an on/standby duty cycle of ~1 minute every 6 hours, gives a mean power consumption of ~31 mW (Brennan et al., 2014). With each radar deployment configured to sample 64 chirps per burst, a continuous year-long autonomous operation with a burst frequency of 6 bursts per day can be achieved using a 6 V180 Ah battery, after accounting for drainage that occurs in low temperatures.
Figure 5.6 Setup configurations of ApRES array deployments (a) 14a; (b) 14b; and (c) 15. The principal flow direction is oriented west-southwest (262°).
Figure 5.7 (a) Schematic diagram (along the X-Y plane, where the Z-axis points into the diagram) showing the field configuration of the planar phased-array antenna, composed of 8 transmitting (Tx) and 8 receiving (Rx) bowtie antennas oriented in two orthogonal linear arrays, producing 64 virtual elements. The \( T_{x1}-R_{x1} \) separation (*) is given in Table 5.2. (b) Dimensions of a single bowtie antenna. (c) Conceptual diagram showing the footprint of the deployed antenna array. All measurements shown are in mm.
The coherent multiple-input multiple-output (MIMO) ApRES array deployed in the field experiments consisted of two linear arrays oriented orthogonally to each other resting on the ice surface. Each array was composed of 8 cavity-backed bowtie antennas (real elements) functioning in either transmit (Tx) or receive (Rx) mode, therefore synthesizing a planar grid of 64 virtual elements (Figure 5.7a; Lok et al., 2015). All real elements were connected to the central radar unit by a series of 5 m or 10 m TNC-type coaxial cables. Each array was mounted on a wooden frame and loosely anchored into the ice with thick (~50 mm) bamboo poles drilled to 3 m depth (Figure 5.6), to both stabilise the array against differential ablation on the ice sheet surface and to prevent the array from being blown away by surface katabatic winds. The radar unit and an attached 6 V 180 Ah battery (Marathon XL series) were placed at the beginning of the Tx array frame in an additional box identical to those housing each bowtie antenna. A GPS and an Iridium signal receiver were attached on an elevated [+1 m] pole to remotely monitor the radar’s position and configuration.

### 5.4.3 Antenna instrumentation and design

Each bowtie antenna was comprised of a pair of triangular copper sheets with bow dimensions of length 350 mm and width 588 mm, resulting in a wide bow angle of 80° (Figure 5.7b). The bow separation was set at 10 ± 1 mm and connected by a 2-layer printed circuit board (PCB) balun. All bowtie antennas were constructed in-house for a centre frequency 300 MHz, resulting in a predicted gain of 6.6 dB and a half-power beamwidth (HPBW) of 115° (Figure 5.8). The bowtie dimensions used in this study were designed for in-air performance and were housed at the bottom of a square corrugated plastic box of dimensions 820 × 820 × 300 mm (Figure 5.9). An aluminium reflector with the same planar dimensions 820 × 820 × 1.3 mm was positioned at the top of the box to redirect all energy back downwards into the ice (Figure 5.9). The plastic box, as well as the supporting frame, were specifically designed using non-conducting material to prevent the occurrence of in-band resonances that degrade the frequency response of the antenna (L. B. Lok, personal communication).
5.4 ApRES and antenna instrumentation and deployment

\begin{figure}[h]
\centering
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{simulated_far_field_azimuth}
\caption{Azimuth radiation pattern}
\end{subfigure}
\hfill
\begin{subfigure}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{simulated_far_field_elevation}
\caption{Elevation radiation pattern}
\end{subfigure}
\caption{Simulated far-field (a) azimuth ($\phi$) and (b) elevation ($\theta$) radiation pattern at 300 MHz for the cavity-backed bowtie antenna.}
\end{figure}

III. ANTENNA CONSTRUCTION

The antenna was constructed using commonly available engineering materials and components. The weight of the antenna is \(N\) kg and the total cost of the prototype is around £100. Table 1 gives a breakdown of the material costs for constructing the antenna prototype.

The triangular-bow antenna elements were formed using 35 µm thickness copper foil (3201, Holland Shielding Systems, B. V.) which contains an adhesive backing. These copper bows were attached onto a 10 mm thick twin-ply polycarbonate sheet which serves as a lightweight yet rigid base for the antenna. A 20 mm $\times$ 25 mm $\times$ 1.6 mm FR4 printed circuit board (PCB), containing a surface-mount balun and SMA connector, was soldered onto the inner tips of the bows as shown in Fig. 6. An RG316 coaxial cable of 500 mm length was connected from the SMA connector on the PCB to an N-type bulkhead connector which was fixed in the centre of a conducting backplane. The backplane is an Aluminium sheet of 82 cm $\times$ 70 cm dimension and 1.3 mm thickness which rest on the cavity walls. Expanded polystyrene boards of 25 mm thickness were cut to form the antenna cavity walls. These polystyrene sidewalls are attached to plastic side flashings which are clipped firmly onto the edge of the polycarbonate sheet base. Fig. 2 shows an assembly view of the proposed antenna, the whole structure is flat-packable within a volume of approximately \(N^3\) m$^3$.

\begin{figure}[h]
\centering
\begin{subfigure}{0.5\textwidth}
\centering
\includegraphics[width=\textwidth]{assembly_view}
\caption{Assembly of the proposed flat-packable antenna design. (Not to scale)}
\end{subfigure}
\hfill
\begin{subfigure}{0.5\textwidth}
\centering
\includegraphics[width=\textwidth]{prototype_cavity_backed}
\caption{Photograph of the prototype cavity-backed bowtie antenna. Front cavity-wall has been removed for clarity.}
\end{subfigure}
\caption{(a) Assembly of the cavity-backed bowtie antennas used in this study, and (b) photograph of the prototype cavity-backed bowtie antenna, with the front cavity wall removed for clarity. The balun and walls used in this figure are different from the one used in this project. From Lok et al. (in prep.).}
\end{figure}

<table>
<thead>
<tr>
<th>Component</th>
<th>Material/ Description</th>
<th>Unit cost</th>
<th>Total cost (approx.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna base</td>
<td>Polycarbonate sheet 10 mm thick</td>
<td>£5</td>
<td>£5</td>
</tr>
<tr>
<td>Cavity walls ($\times$ 4)</td>
<td>Polystyrene board 25 mm thick</td>
<td>£8</td>
<td>£32</td>
</tr>
<tr>
<td>Side flashing ($\times$ 2)</td>
<td></td>
<td>£4</td>
<td>£8</td>
</tr>
<tr>
<td>Backplane</td>
<td>Aluminium 1.3 mm thick</td>
<td>£15</td>
<td>£15</td>
</tr>
<tr>
<td>Triangular bow ties ($\times$ 2)</td>
<td>Copper foil with adhesive backing</td>
<td>£50*</td>
<td>£100*</td>
</tr>
<tr>
<td>Surface-mount balun</td>
<td></td>
<td>£1 - £2</td>
<td>£1 - £2</td>
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<tr>
<td>Surface-mount SMA connector</td>
<td>SMA female</td>
<td>£2</td>
<td>£2</td>
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<tr>
<td>N-type bulkhead connector</td>
<td>SMA female to N-type female</td>
<td>£6</td>
<td>£6</td>
</tr>
<tr>
<td>SMA cable assembly</td>
<td>50 cm length, RG316, male-male</td>
<td>£10</td>
<td>£10</td>
</tr>
</tbody>
</table>

*Can reduce this by cutting the bows out efficiently!

IV. EXPERIMENTAL RESULTS

A prototype of the proposed antenna was measured across a frequency range of 50 MHz to 1000 MHz using a vector network analyser (VNA). For testing purposes, the antenna was pointed upwards in the UCL radar laboratory as shown in Fig. 6. The antenna was raised off the ground in order to access the N-type input port during the measurements. The antenna was tested, in turn, with different balun-feeds soldered onto the copper bow-ties. The baluns under consideration are summarized in Table 2. Fig. 7 shows a photograph of the different balun-feeds mounted on their PCB which are used in this paper.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Ref. & Part Number & Manufacturer & Balun Type & Specified frequency range \\
\hline
#1 & ADT4-1W & Mini-Circuits & Ferrite, wire-wound & 2 MHz to 775 MHz \\
#2 & TCM4-1W & Mini-Circuits & Ferrite, wire-wound & 3 MHz to 800 MHz \\
#3 & TC4-6TX & Mini-Circuits & Ferrite, wire-wound & 1.5 MHz to 600 MHz \\
#4 & B0205F50200AHF & Anaren & Non-ferrite, multilayer & 200 MHz to 500 MHz \\
#5 & DXP18BN5014T & Murata Guanella?, film & 50 MHz to 870 MHz \\
\hline
\end{tabular}
\caption{List of the 1:4 baluns considered in the experiments}
\end{table}
Within each linear array, the antennas were arranged end-to-end with no additional separation, thus giving a virtual element separation of \( \delta = 410 \text{ mm} \) (equivalent to \( 0.74 \times \lambda_c \) assuming a dielectric constant of ice of \( \varepsilon_r = 3.18 \)).
Chapter 6

Resolving seasonal patterns of ice deformation in a fast-flowing Greenland outlet glacier using phase-sensitive radar

Author contributions

In addition to the design and deployment of the antenna array (Chapter 5), several authors contributed to the research presented in this chapter, which is structured and written in a style suitable for publication. Lai Bun Lok and Paul Brennan developed the cable correction algorithms. Craig Stewart provided a range processing script for Antarctic ice shelves, which the author (T.J. Young) adapted for a fast-flowing outlet glacier within the Greenland Ice Sheet. Samuel Doyle and Alun Hubbard collected and processed GPS records, which the author used to derive surface velocities. Basal velocities were calculated using borehole tiltmeter records collected by the SAFIRE team, which included Poul Christoffersen, Bryn Hubbard, Alun Hubbard, Samuel Doyle, and the author. Adrian Luckman and Doug Benn provided TerraSAR-X surface velocity in high temporal and spatial resolution. The author built the antennas, installed the ApRES
Seasonal evolution of vertical deformation

systems, and processed all radar data presented in this chapter. The text was written by the author with minor contributions by Poul Christoffersen and Samuel Doyle.

6.1 Summary

The internal layers within ice sheets and glaciers contain valuable records from which the history of past ice flow and deformation can be deduced. These records are typically used to infer large-scale changes in ice flow on centennial to millennial timescales. Shorter-term changes in the morphology of internal layers require very accurate measurements, which means that potentially important aspects of glacier dynamics remain largely unexplored. Here we use autonomous phase-sensitive radio-echo sounding (ApRES) to track the daily displacement of internal layers on Store Glacier in West Greenland to millimetre accuracy. At a study site located 30 km from the calving ice front, where the ice is \( \sim 600 \text{ m} \) thick and ice flows at \( \sim 700 \text{ m} \text{ a}^{-1} \), we measure distinct seasonal variations in vertical velocities and vertical strain rates over a two-year period. Prior to the melt season (March – June), we observe increasingly non-linear englacial deformation, with negative vertical strain rates in the upper half of the ice column of \( \sim -0.03 \text{ a}^{-1} \) causing strain thinning within this section, whereas the ice below thickens under vertical strain that occasionally can be as high as \( 0.16 \text{ a}^{-1} \). Early in the melt season (June – July), vertical thinning ceases as the glacier starts to thicken throughout the ice column. During late summer to midwinter (August – February), vertical thickening occurs linearly throughout the ice column with a strain rate that averages \( 0.016 \text{ a}^{-1} \). We show that these complex variations are unrelated to topographic setting and localized basal slip, and we hypothesize that the seasonality is driven by far-field perturbations in the glacier’s force balance, in this case generated by variations in basal hydrology near the glacier’s terminus and propagated tens of kilometres upstream through longitudinal coupling.
6.2 Introduction

Conditions at the ice-bed interface exert a major control on the force balance of glaciers and ice sheets. Spatially, the flow of ice masses is modulated by variations in basal traction, which arise from variations in: bed topography (Gudmundsson, 1997, 2003), hydrology (Hoffman et al., 2016), the presence and properties of sediments (Bougamont et al., 2014), and the basal thermal regime (Christoffersen and Tulaczyk, 2003; Tulaczyk et al., 2000b). In places where soft subglacial sediments provide limited frictional resistance to ice flow, the gravitational driving stress may be countered predominantly by lateral stresses along the glacier’s side walls (Raymond, 1996; Rippin et al., 2005), or by membrane-like stresses (Hindmarsh, 2006), consisting of longitudinal and transverse stress gradients within the ice mass itself (Kamb and Echelmeyer, 1986; Price et al., 2008).

Variations in basal traction can also be temporal, with fluctuations in subglacial water pressures causing ice flow to vary on timescales from hours to months (e.g. Bartholomew et al., 2008; Doyle et al., 2015; Hooke et al., 1989; Iken and Bindschadler, 1986; Iken and Truffer, 1997; Kamb, 1987; Meierbachtol et al., 2016). Such changes in local and regional water pressures can occur when there is a sudden influx of water into the basal environment, either from the surface where meltwater is produced with diurnal variations (Bartholomew et al., 2012) or released from surface lake drainage (Doyle et al., 2014; Hoffman et al., 2011), or at the bed from an active subglacial lake (Fricker and Scambos, 2009; Palmer et al., 2015; Wingham et al., 2006).

Although the concept of force balance is well established in the geophysical and glaciological literature, it is rarely measured and mostly applied within numerical modelling studies (Kamb and Echelmeyer, 1986; Meierbachtol et al., 2016; Price et al., 2008). Direct measurements of depth-dependent englacial strain are important because they inform how ice deforms internally when glaciers flow. These measurements are, however, sparse as they typically require installation of strain gauges and inclinometers in boreholes, which are both costly and time-consuming (Gudmundsson, 2002; Keller and Blatter, 2012; Perutz, 1952; Raymond et al., 1994; Ryser et al., 2014b; Sugiyama and Gudmundsson, 2003).
Phase-sensitive radio-echo sounders (pRES) provide the first non-invasive, direct, and continuous measurements of vertical velocity and englacial strain (Brennan et al., 2014). Pioneered by Corr et al. (2002), the instrument has primarily been used to measure basal melt rates of Antarctic ice shelves with its ability to detect range with millimetre precision by repeat stake surveys (Jenkins et al., 2006, 2010; Marsh et al., 2016; Stewart, 2018). Jenkins et al. (2006) examined and discussed spatial and temporal variations in vertical strain rates in detail, observing force balance effects of tidal bending, while within other studies, measurements of vertical strain rates were but a means to an end to obtain basal melt rates beneath Antarctic ice shelves. Of the few studies conducted on grounded ice sheets, focus on the vertical movement of internal layers was limited to ice divides, measuring strain rates on the order of $1 \times 10^{-4} \text{a}^{-1}$ as a result of slow flow. Gillet-Chaulet et al. (2011) tracked englacial layers along two 20 km survey lines crossing ridges near the GRIP and NEEM ice-core sites to quantify the rheological properties of ice near divides. Kingslake et al. (2014) and Kingslake et al. (2016) used a similar survey design to investigate the Raymond Effect near ice divides (Raymond, 1983). In 2014, the development of a low-power autonomous pRES (ApRES) allowed long-term, unattended monitoring of ice-shelf and ice-sheet thinning (Nicholls et al., 2015). Importantly, this new instrument allowed measurement of vertical strain rates on fast-flowing glaciers and ice streams due to its ability to track unambiguously and continuously the vertical movement of deforming internal layers at sufficiently short time steps (Young et al., accepted).

In this study, we used ApRES to measure daily displacements of internal reflectors within Store Glacier, West Greenland. The records, which have millimetre accuracy and cover the majority of a two-year period, are the first to resolve the fine-scale change in englacial strain in a fast-flowing outlet glacier of the Greenland Ice Sheet. We observe the seasonal evolution of the vertical strain profile, and investigate the causes of these variations using complementary borehole data acquired in the close vicinity of the ApRES deployments.
6.3 Methods

6.3.1 ApRES measurements of vertical deformation

During three field seasons (May 2014, July/August 2014, and July 2015), we deployed three ApRES arrays (labelled 14a, 14b, 15, respectively) at site S30 on Store Glacier (Figure 5.1a,b). At each deployment site, the ApRES arrays advected downglacier at a rate of ~600 m⋅a⁻¹, obtaining profiles of radar reflectivity beneath the radar at the ice-bed interface and through the incident ice column within a Lagrangian reference frame (i.e. measurements spatially referenced to the array). As the ice ablated during the summer melt season, the frame of the ApRES array (Figure 5.6) maintained continuous contact with the surface by lowering under its own weight down bamboo canes drilled 3 m into the ice. Data collected from each deployment were collapsed into 1-dimension. The three array sites were separated from each other by up to 450 m and made measurements at a sampling interval ranging from 1 to 8 hours (Figure 6.1; Table 5.2). Deployment 15 (blue pentagon, Figure 6.1) operated continuously for 338 days from 05 July 2015 to 06 June 2016 and travelled 570 m downglacier, while deployments 14a and 14b (red and yellow pentagons respectively, Figure 6.1) operated for 72 and 124 days and travelled 121 m and 213 m respectively. Deployment 14b ended abruptly on 04 December 2014 when the
array was damaged by strong winds. Further information of the array design used in the field deployments are detailed in Chapter 8.

From these records, we measured empirically and modelled the vertical displacement and velocity of these reflectors relative to the starting location of the ApRES antennas, and calculated vertical strain-rate profiles from time series of the vertical velocity of internal reflectors. Through this process, we standardized the time step of each time series by aggregating consecutive records, resulting in continuous daily measurements of ice column vertical deformation.

After pre-processing each burst to remove noisy records, and Fourier-transforming the data into observations of range from the ice surface, we follow the (i) phase processing procedures described by Brennan et al. (2014) (Figure 6.2a) and the (ii) phase differencing procedures described by Stewart (2018) (Figure 6.2b), thereby producing time series of 1-dimensional depth profiles of vertical velocity spanning the majority of a two-year period (2014–2016).

**Pre-processing procedure**

Occasionally, the noise level of an individual chirp was significantly elevated due to user error, external interference, significant variations in the temperature of the network analyser during measurement, or some combination of these factors (Kingslake et al., 2014). Contaminated chirps were removed prior to phase processing following Stewart (2018), which uses an automated iterative process to determine outliers based on their root-mean-square (RMS) difference from the burst-mean signal.

**Phase processing procedure**

Phase processing of the received deramped chirp signal followed the steps outlined in Brennan et al. (2014). The phase processing procedure (Figure 6.2a) consists of: (i) weighting the deramped waveform in the time domain using a Blackman window to reduce spectral leakage (especially from strong internal and basal reflectors); (ii) increasing the series to a multiple of its original length by appending trailing zeros (“zero-padding”);
6.3 Methods

Figure 6.2 Schematic of ApRES processing steps to measure and model vertical velocities of internal layers. (a) Processing steps developed in Brennan et al. (2014). (b) Processing steps developed in Stewart (2018). The rest of the processing steps were developed in this study.
(iii) time-shifting the signal so that the phase centre aligns with the start of the sample; and (iv) Fast Fourier-Transforming (FFT) the modified time series to transform the waveform into the frequency domain.

Zero-padding the signal greatly improves the resulting range resolution by effectively interpolating between coarse range bin centres at a frequency $p$. Mathematically, zero-padding the signal modifies the coarse range and fine range equations (Equations 4.20 and 4.25) to:

\[
R_c(n) = \frac{nc}{2B\sqrt{\varepsilon_r}p} \quad (6.1)
\]

\[
R_f \left(n, \text{mod} \frac{\lambda_c}{2}\right) = \frac{\lambda_c \phi}{4\pi} \quad (6.2)
\]

where $p$ is the pad factor. Unless stated otherwise, all data processed within this thesis used pad factors of either 2, 5, or 10 and a Blackman window, following the recommendations in Brennan et al. (2014). Variation of the pad factor did not significantly alter results.

The signal was then re-centred to ensure that the instantaneous phase is evaluated at the centre of the chirp rather than at the antenna ($t = T/2$), a requirement described in Section 4.3.2. This step is essential to the cross-correlation methods detailed later in this section. The chirp spectrum was then calculated through a Fast Fourier Transform (FFT) processing step in which the FFT-processed signal was weighted by a reference array equal to the phase conjugate of the expected phase at the centre of each range bin. By applying a phase shift to the spectrum to account for the expected signal phase at the bin centre, the fine range of the signal (Equation 4.25) is zero at the bin centre rather than at the antenna (Stewart, 2018).

The normalisation of the phase of the FFT-processed signal to zero can be represented by (Equation 17, Brennan et al., 2014):
\[ f \ast f_{\text{ref}} = f \ast \exp \left[ -j \left( \phi_d(t) - K \tau (t - T) / 2 \right) \right] \]
\[ = f \ast \exp \left[ -j \left( \omega_c \tau - K \tau^2 / 2 \right) \right] \]
\[ = f \ast \exp \left[ \frac{n \omega_c}{Bp} - \frac{n^2 K}{2B^2 p^2} \right] \] (6.3)

where \( \tau \) (Equation 4.16) is now referenced to the \( n^{\text{th}} \) bin centre:

\[ \tau_n = \frac{2 \sqrt{\varepsilon_r}}{c} n \Delta R \]
\[ = \frac{2 \sqrt{\varepsilon_r}}{c} \frac{n c}{2B \sqrt{\varepsilon_r} p} \]
\[ = n/Bp \] (6.4)

Note that Equation 6.3 substitutes Equation 4.20 for the reference array \( f_{\text{ref}} \); in doing so, the phase offset \((-K \tau^2 / 2)\) is removed.

**Cable correction**

During operation, the ApRES may experience delays in burst transmission either internally within the system or within the cables connecting the ApRES to the antennas. As two different cable lengths of 5 and 10 m were used within each array, each resulting antenna pair will use one of three different cable combinations, where pairs with longer combinations (i.e. 5 m/10 m and 10 m/10 m) exhibited a positive offset in range in the received signal (+5 or +10 m) relative to pairs using the shortest possible combination (i.e. 5 m/5 m).

The cable delays were removed by shifting each chirp profile in the range domain to align the direct breakthrough with all antenna pair combinations. Computationally, this shift was achieved by applying a phase gradient in the frequency domain equivalent to the range offset generated by the additional cable lengths—essentially, an electrical delay. Mathematically, this shift was implemented by recalculating the phase offset in...
Equation 6.3 \((-K\tau^2/2\) to instead equal the range offset generated by the different cable lengths.

Specifically, this is shifted in the coarse and fine range:

\[
R_{c,o}(n) = \frac{\zeta_ip}{2\Delta R_c} \quad (6.5)
\]

\[
R_{f,o} = \frac{2\pi\zeta_if_c}{c} \quad (6.6)
\]

where \(c\) is the speed of light, and \(\zeta_i\) is the ice-equivalent cable length proportional to the addition in physical cable length \(\zeta_c\) and the dielectric permittivities of co-axial cable \((\varepsilon_c = 2.20)\) and ice \((\varepsilon_r = 3.10)\):

\[
\zeta_i = \zeta_c\sqrt{\varepsilon_c/\varepsilon_r} \quad (6.7)
\]

Note that Equations 6.5 and 6.6 are subject to the same bounds as their counterparts (Equations 4.20 and 4.25, respectively).

**Internal reflector displacement**

The vertical displacement of internal reflectors were calculated following established methods detailed in Stewart (2018), Corr et al. (2002), Jenkins et al. (2006), and Rosen et al. (2000). For the phase differencing procedure (Figure 6.2b), burst pairs from each range-processed spectrum (profile) were then processed iteratively with a daily (24-hour) time step through complex cross-correlation of the burst spectra pair. Only internal layers greater than 40 m in depth and passing certain threshold criteria were used within the processing steps. Phase differences were calculated by comparing bursts sampled 24 h apart using short (5 m) segments along the profile, where each segment \(m\) represents the internal reflectors residing within the segment depth range. With sampling intervals averaging from 1–8 h, this provided 3–24 estimates of reflector displacement per day, and daily average velocities were calculated from the output.
Figure 6.3 Phase processing of two paired ApRES samples (profiles obtained 19:34 04 November 2014 UTC and 19:34 05 November 2014 UTC). (a) Raw time series of averaged burst. (b) Fourier transform of first (red) and second (blue) bursts. (c) Coherence of cross-correlation of segments, representing internal layers. Only layers above the coherence threshold of 0.925 (Figure 6.5b) were used in subsequent analyses. (d) Displacement estimates of segments (internal layers, with corresponding standard error) and basal layer (magenta star), with the modelled vertical velocity fitted as a linear curve. Green dots represent layers not meeting the minimum coherence threshold for strain rate calculation.
The amount of vertical displacement for each segment (Figure 6.3c) was calculated by processing the complex cross-correlation of the segment in the first profile \( f \) with the corresponding segment in the second profile \( g \) (Eq. 4.18 from Stewart, 2018):

\[
f \star g (m, n, \Lambda) = \frac{\sum f^* (k(m, n)) g (k(m, n) + \Lambda)}{\sqrt{\sum |f(k(m, n))|^2} \sqrt{\sum |g(k(m, n) + \Lambda)|^2}}
\]  

(6.8)

Here, the complex correlation \( f \star g \) is the multiplication of the profile \( g \), lagged with the vector \( \Lambda \), with the complex conjugate of profile \( f \), indicated by \( * \). \( k(m, n) = [k_{-n} \cdots k_{-2} k_{-1} k_0 k_1 k_2 \cdots k_n] \) represents an array of range bin indices that surround the centre of the specified range segment \( m = k_0 \). While the length of \( k \) has no impact on the performance of the complex correlation, \( k \) should be sufficiently long (i.e. containing enough sample reflectors for accurate cross-correlation) to correctly identify the amount of displacement between reflector pairs. The magnitude of \( f \star g \) is the coherence, which indicates the similarity of the segments to each other, and is scaled from 0 for incoherent segments to 1 for segment pairs with a constant phase difference. The phase of \( f \star g \) is the amplitude-weighted (vector-averaged) phase difference between the segments.

In cross-correlation, the goal is to maximize the coherence, which then identifies the most-likely amount of displacement: \( \hat{\gamma}_{fg}(n, \hat{\Lambda}) = \arg \max_{\Lambda} \gamma_{fg}(n, \Lambda) \). Here, the maximization of the coherence and its corresponding lag vector is represented by the hat operator \( \hat{\cdot} \). Therefore, the coarse-range displacement of the segment \( n \) is the amount of lag \( \hat{\Lambda} \) that corresponds to the maximum coherence \( \hat{\gamma}_{fg} \), scaled by the coarse range resolution (Equation 4.19):

\[
\delta_c(n) = \Delta R_c \Lambda(n)  
\]  

(6.9)

where the maximum coherence \( \hat{\gamma}_{fg} \) is:

\[
\hat{\gamma}_{fg}(n, \hat{\Lambda}) = \arg \max_{\Lambda} \gamma_{fg}(n, \Lambda) = \arg \max_{\Lambda} |f \star g (m, n, \Lambda)|  
\]  

(6.10)
Similarly, the fine-range displacement of the segments is calculated by substituting $\phi_c$ for $\hat{\Lambda}$ in the equation calculating the referenced and centred phase within the coarse range bin (Equation 6.2):

$$
\delta_f (n) = \frac{\lambda_c \hat{\Lambda}}{4\pi}
$$

(6.11)

Finally, the total displacement of the segments is the sum of the coarse- and fine-range displacements (Figure 6.3d):

$$
\delta_t (n) = \delta_c (n) + \delta_f (n)
$$

(6.12)

**Basal Topography**

Using Equations 6.1, 6.2, and 4.26, the total ice thickness at any point in time can be determined by setting the coarse range bin centre to the basal reflector (thick dashed line, Figure 6.3b-d). The bed elevation can then be determined from subtracting the ice thickness from the surface elevation (GIMP DEM; Howat et al., 2014). Repeating this procedure for all bursts (Table 5.2) produces profiles of basal topography for each radar deployment.

**Modelling full-column vertical strain**

Depth-dependent profiles of vertical strain were calculated by fitting a model to each internal layer’s vertical velocity profile. As these profiles vary seasonally and on timescales as short as a single day, we used quadratic best-fit models (Figure 6.4a,b) to describe englacial strain rates in the form:

$$
\delta_m (R) = aR^2 + bR + c
$$

(6.13)

Here, $\delta_m (R)$ is the modelled displacement, and the model parameters $a$, $b$, and $c$ were solved using a robust non-linear least-squares search. The use of a quadratic model,
though simple, easily identifies the degree of nonlinearity within vertical velocity profiles through the first model parameter \( a \), which represents the curvature of the model with depth. When the value of \( a \) approaches zero, the model depicts an increasingly linear strain regime through depth (e.g. Fig. 6.4a); conversely at non-zero values, the model reflects quadratic behaviour (e.g. Fig. 6.4b).

Calculating the depth-dependent derivative of Equation 6.13 will generate the vertical strain rate:

\[
\frac{\partial \delta_m}{\partial R} = 2aR + b
\]  

which is linear or constant depending on the value of \( a \). In the non-linear case, it also informs the polarity point of strain reversal (i.e. the depth at which strain thinning switches to strain thickening, and vice versa) when Equation 6.14 equals 0.

The ApRES range error from phase noise increases with increasing depth, due to progressive attenuation in the received signal power with increasing depth. Therefore, to address the heteroscedastic behaviour in range, observations were weighted (Holland and Welsch, 1977) using the inverse of the measurement variance of each of the internal reflector’s daily vertical velocity as a weighting factor (Equation 6.15; Figure 6.4):

\[
w_n = \frac{1}{\sigma_n^2}
\]

where \( \sigma_n \) is the standard deviation of each measured segment \( n \). This ensures that prominent internal layers influence the fitted curve more than weak internal layers.

To remove the effects of surface ablation and occasional delays in signal transmission, all profile pairs were shifted vertically prior to modelling the full-column vertical strain, but after calculating the daily vertical velocity profiles. This process was done using the same procedures to determine internal reflector displacement (Equation 6.8), but instead of 5 m segments, we used a large section of the profile with consistently high cross-correlation values at a shallow depth (40–100 m). Using a weighted robust linear regression, we determined the overall delay by extrapolating the regression to the ice surface, and
Figure 6.4 (a,b) Two examples of two ApRES-measured vertical velocity profiles fitted with a weighted robust quadratic regression (green; Equation 6.13), with the model error a function of curve fitting. A weighted robust linear regression (dashed blue) is underlain as comparison. Each marker represents one englacial reflector; all reflectors are relative to the initial surface of the measured ice column. (c) and (d) respectively show the amplitude and phase of the specified internal reflector identified in (a); (e) and (f) show equivalent plots for the specified internal reflector identified in (b).
standardised the initial depth of all profiles to 0 m by applying an appropriate offset to the resulting displacement profile to negate the measured delay.

Noisy chirps below a certain cross-correlation coherence threshold, which represent poor tracking of internal reflectors between measurements, were removed, causing gaps and missing data in the time series (dotted red line, Figure 6.5). The processed layer vertical velocity profiles were subsequently averaged into daily bins through weighted means, using the standard error of each internal reflector’s measurement as the weighting factor (Equation 6.15). In doing so, we also removed potential temperature biases that affect the signal propagation speed within the ApRES unit from sampling at different times throughout the day.

**Estimation of error**

Excluding errors arising from within the ApRES unit, which are detailed in Brennan et al. (2014), the overall error in ApRES-measured vertical velocities and strain rates broadly arise from assumptions made in characterising the underlying ice column and statistical variability within the phase processing procedure, which in turn can be categorised into four sources: (i) variations in the complex dielectric permittivity of ice due to temperature variations within the ice column (Fujita et al., 2000); (ii) phase noise from the internal reflector; (iii) compaction of the vertical ice column from near-surface firn, if present; and (iv) discrepancies between observed and modelled vertical strain from inappropriate model choice. Below, we examine the effects of these errors.

**i. Dependence of signal propagation to relative permittivity**

Being located within the ablation zone, S30 experiences drastic seasonal fluctuations in temperature ranging from $-40^\circ$C in the quiescent winter months to more than $10^\circ$C at the peak of the summer melt season, where the presence of surface meltwater drastically alters the physical characteristics of the incident ice column. As both changes in the temperature and composition of ice drastically affects its relative permittivity, and hence the speed of signal propagation (Section 4.2.2), such changes will undoubtedly contribute
most to uncertainties within measured displacements of internal reflectors. Appendix 6A explores the various impacts that changes to the relative permittivity of the incident ice column have on measured results, and details how these issues have been addressed within data processing.

ii. Phase noise from internal reflectors

Following Rosen et al. (2000) and Stewart (2018), phase noise from a specified internal reflector \( (\sigma_n) \) was calculated from the maximum coherence between two sequential cross-correlated bursts:

\[
\sigma_n^2 = \frac{1}{2N_L} \frac{1 - \tilde{\gamma}_{fs}(n, \hat{\Lambda})^2}{\tilde{\gamma}_{fs}(n, \hat{\Lambda})^2}
\] (6.16)

Here, \( N_L \) is the number of elements within the segment \( n \). These estimates of error were then used as weights for robust curve fitting to obtain modelled vertical strain rates (Equation 6.15).

iii. Firn compaction

As ApRES arrays were deployed directly on the ablating ice surface, snow and firn beneath the antennas do not contribute towards error in this study. The downward displacement of the ApRES caused by ablation was taken into consideration in the model fitting process, as described above.

iv. Discrepancies between observed and modelled vertical strain

The statistically automated selection of models to fit the depth profiles of vertical velocity gave high significance for the large majority of the time series (Figure 6.4). The quadratic model used approximately characterise variations in the shape of the velocity profiles, but occasionally fails to capture trends in the movement of internal reflectors at depth (>2/3 of the total ice thickness). Furthermore, we were unable to consistently identify
and track the movement of the deepest internal reflectors (Figure 6.5), and as a result, had to extrapolate through this range to yield full-depth vertical velocities. Although at present there exist no known studies, modelled or observed, that suggest a polarity reversal of vertical movement or deformation in this lowest section of the ice column, we cannot definitely preclude the possibility of deformation within this section that deviates from the model extrapolation. In Section 9.2.1, we further investigate the possible causes for, and ramification of, this limitation.

Quality assessment

Because the vertical displacements of internal reflectors were assessed through cross-correlation of short segments of paired profiles, the accuracy of a reflector’s determined vertical displacement wholly depends on the coherence of the correlation (Equation 6.16). In general, the coherence of the cross-correlation should decrease with increasing depth due to the englacial attenuation of the transmitted signal.

Occasionally, burst pairs produce either very few measurements of vertical displacement or undulating profiles with very steep velocity gradients inconsistent with neighbouring displacement profiles in time. These erroneous measurements were found to be either coincident with environmental influences (e.g. pooling of surface meltwater near the radar) or some form of instrument malfunction, both of which greatly reduced the correlation strength when internal reflectors were matched. The low coherence of numerous internal reflectors within a single burst pair resulted in a low usable fraction of segments useful for fine-scale analysis. In these instances, the entire burst pair was automatically excluded if the fraction of segments through the entire ice column exceeding a specified minimum amplitude correlation did not meet a specified fractional threshold (dashed red; Figure 6.5). Removal of an entire burst pair, as a result, presented occasional gaps in the time series.

Within the three deployments, the instrument operated either in MIMO, where 64 chirps were cycled through the combinations of virtual transmitting and receiving antenna pairs within one burst, or in a quasi-monostatic mode, where all 64 chirps were transmitted and received between one transmitting and one receiving antenna. While deployments
**Figure 6.5** Coherence strength from cross-correlation of internal reflector pairs and the fraction of internal layers within each profile used in subsequent analyses, measured with ApRES at S30 at sites (a) 14a; (b) 14b; and (c) 15. All reflector pairs falling below the required correlation threshold (lower limit of the colour bars) were excluded from further analysis. The depth range used for analyses is indicated in grey. The threshold for overall profile inclusion is indicated in dashed red, where, if the fraction of points fitted falls below this threshold, the entire profile is excluded altogether from further analysis. Note the different colour scales used.
14a and 14b operated solely in MIMO, deployment 15 alternated between MIMO and a quasi-monostatic operation (Table 5.2). Because all chirps in deployment 15 were stacked upon one virtual antenna, rather than interspersed between the gridded locations of the synthetic aperture, the phase noise is reduced and a higher threshold was used (Figure 6.5c). On the other hand, due to the data quality being hampered by intense surface melt in August of 2014, a lower threshold was implemented to reduce gaps in the time series of deployment 14b (Figure 6.5b).

### 6.3.2 GPS measurements of surface ice motion

Within this study, surface velocities ($\vec{u}_s$) were measured during 2014 using dual-frequency GPS receivers (Trimble 5700 and R7) installed on 4.9 m-long poles drilled 3.9 m into the ice surface. The GPS receivers were powered by a 50–100 Ah battery, solar panels and a wind generator, and sampled at a 10 s interval. However, despite the multiple power options, some data gaps exist due to power outage. Additionally, entire records from several GPS instalments were rendered unusable due to continuous data gaps of multiple months long. Data from the receivers were processed kinematically (King, 2004) using Track v. 1.28 (Chen, 1998) relative to bedrock-mounted reference receivers using precise ephemeris from the International GNSS Service (Dow et al., 2009) and IONEX maps of the ionosphere (Schaer et al., 1998). Reference GPS receivers were located near the glacier terminus (STNN; Trimble NetRS; Figure 5.1) and farther afield at Qaarsut (QAAR). Preference was given to STNN over QAAR due to its 30 km baseline length compared to QAAR’s 105 km.

### 6.3.3 Borehole measurements of basal conditions

As part of the Subglacial Access and Fast Ice Experiment (SAFIRE; Section 5.2), four boreholes were drilled by hot water to the bed at 605–615 m depth and instrumented with several englacial and subglacial sensors at site S30, where the ApRES units were deployed (Figure 5.2a; Doyle et al., 2018). Of the datasets obtained from instrumentation

---

1Data from the QAAR reference station are provided by UNAVCO at a 30 s interval and are available via anonymous FTP: [ftp://garner.ucsd.edu/pub/rinex](ftp://garner.ucsd.edu/pub/rinex)
of boreholes, we investigate herein the time series of tilt sensor readings to obtain a two-month-long record of basal velocity, derived by subtracting the difference between surface velocity measured by GPS and horizontal deformation rates calculated from the installed tilt sensors.

Tilt sensor time series data were processed assuming all angle variations are in the direction of flow and occur solely due to vertical variations in horizontal shear (Keller and Blatter, 2012). This method determines the vertical gradients of horizontal velocity ($\partial u_x/\partial z$), where $\partial u_x$ is the direction of flow. We interpolated through these sparse measurements using a temperature-dependent flow model (Cuffey and Paterson, 2010) at specific depth points (Doyle et al., 2018).

We integrated ($\partial u_x/\partial z$) cumulatively over the entire ice column to calculate the deformational velocity time series $u_d$ for the period 04 August – 25 September 2014 when the tilt sensor string was operational. Subtracting $u_d$ from the time-varying surface velocity $u_s$ (Section 6.3.2) generates a time series of basal motion $u_b$:

$$u_b = u_s - u_d \quad (6.17)$$

To match the time steps of other measurements in this study, the resulting time series was filtered with a two-pole, low-pass Butterworth filter with a 72 h cut-off period, and then binned into daily averages, matching the time steps of other measurements in this study.

The basal traction $\tau_b$ was calculated using the Mohr-Coulomb failure criterion that describes the plastic shear strength of the porous layer that underlies Store Glacier (Hofstede et al., 2018) and several other glaciers in Greenland (Booth et al., 2012; Bougamont et al., 2014; Christianson et al., 2014; Dow et al., 2013; Kulessa et al., 2017; Walter et al., 2014). The Mohr-Coulomb failure criterion is expressed as: (Equation 12, Christoffersen and Tulaczyk, 2003):

$$\tau_b = c_0 + \sigma_n \tan \mu \quad (6.18)$$
where \( c_0 \) is the apparent cohesion (0), \( \mu \) is the friction angle (30°), and \( \sigma_n \) is the normal stress of the slip plane, which we substituted with the effective pressure \( p_e \), obtained by subtracting the subglacial water pressure \( (p_w) \) from the overburden pressure \( (p_i) \):

\[
\sigma_n \equiv p_e = p_i - p_w = \left[ \rho_i g D \cos \beta \right] - p_w \tag{6.19}
\]

\( p_i \) was obtained by scaling the calculated ice thickness \( D \) by the ice density \( \rho_i \), gravity \( g \), and the surface slope \( \beta \). \( p_w \) was recorded empirically by two vibrating-wire piezometers installed at the ice-sediment interface (Doyle et al., 2018). The value of \( \beta \) was input as 3.7°, determined through GPS measurements along a 2 km transect aligned East to West (Hofstede et al., 2018), and values of \( c_0 \) and \( \mu \) were set to those of Trapridge till (Clarke, 1987), following Bougamont et al. (2014).

### 6.4 Results

The ApRES, when left in unattended mode, measures the vertical motion (the vertical velocity) of individual internal reflectors relative to the radar antennas by comparing their internal phase between successive bursts (Figure 6.4; Nicholls et al., 2015). Then, to examine how vertical strain varies through time, we applied best fits with quadratic models (Figure 6.4) to each daily velocity profile (i.e. Figure 6.6 to Figure 6.8) and calculated vertical strain rates (i.e. Figure 6.8 to Figure 6.9) through taking the derivative along each modelled profile (Equation 6.14).

The resulting vertical velocities during spring (deployment 14a), summer and autumn (deployment 14b), and a whole year (deployment 15) varied substantially with depth and over time at and between each site (Figure 6.6). Within each deployment, barring occasional erroneous records affecting the entire measurement of the ice column for short (1–3 day) periods, vertical velocities were relatively constant (Figure 6.7). While the upper 400 m of the measured ice column during the Spring of 2014 at deployment 14a revealed negative vertical velocities, where internal reflectors are moving increasingly closer to the antennas located at the ice surface (Figure 6.6a), deployments 14b and 15
during the summers and autumns (July – December) of 2014 and 2015 respectively record positive vertical velocities, where internal reflectors move in the opposite direction away from the ice surface (Figure 6.6b,c). As well as exhibiting a change in sign with depth within a single profile for extended periods, as observed in deployments 14a (06 May – 22 June; Figure 6.6a) and 15 (January/February 2016 – ; Figure 6.6c), the vertical velocities of internal layers also underwent temporal evolution within the same deployments. In the former (deployment 14a), vertical velocities became increasingly positive between mid-June and mid-July of 2014, with the deeper movement of deeper individual reflectors occurring earlier than those located at shallower portions of the ice column (Figure 6.6a). The opposite trend was observed in deployment 15 from January 2016 onwards, where shallow internal layers begin to show movement towards the ice surface earlier than those at depth.

In deployment 14a, we first observed complex strain behaviour within the ice column in May 2014, where strain thinning in the upper section of ice is counteracted by strain thickening at depth. The former increased progressively from May to late June (Figure 6.9a) when the point of zero strain changed from 200 m to 400 m below surface. This pattern changed abruptly on 18 June, when vertical strain thickening in the lower part of the glacier instead started to increase significantly in a transition that ultimately led to a new strain regime in which most of the ice column thickened. This strain thickening regime dominated deployment 14b (Figure 6.9b), which ceased on 04 December 2014 when the antenna array was damaged by strong winds (Table 5.2). During this period (leading up to 04 December), vertical strain rates showed consistent and uniform strain thickening through the ice column with rates averaging $0.0165 \text{a}^{-1}$. Redeployment of the ApRES in July 2015 (deployment 15) confirmed that strain thickening dominated the ice column from July to December; it also confirmed that the vertical strain regime subsequently returned to one where thinning dominates the ice column, as observed at the beginning of deployment 14a. This transition was gradual and took place between January and June (Figure 6.9c), whereas the transition from thinning to thickening during June/July 2014 was much more rapid and lasted only a few weeks (Figure 6.9a).

Further examination of the curvature of the model fits reveal seasonal patterns and trends in strain within the ice column as it evolves through time. During the late summer
Figure 6.6 Vertical velocity time series relative to the transmitting/receiving antenna for internal reflectors within the entire ice column measured using ApRES at S30 from deployments (a) 14a; (b) 14b; and (c) 15. Here, blue represents strain thinning and red strain thickening. The effects of surface ablation on internal reflector vertical velocities were removed. Note the different scales used for time between subplots.
Figure 6.7 Same as Figure 6.6, but showing vertical velocity standard errors. Note the different scales used for error magnitude and time between subplots.
Figure 6.8 Time series of modelled internal layer vertical velocity profiles within the ice column measured using ApRES at S30 at sites (a) 14a; (b) 14b; and (c) 15. All profiles were automatically fitted with a quadratic model (Equation 6.13). Here, blue represents upward movement of an internal reflector, and red downward movement. Note the different scales used for time and distance between subplots.
Figure 6.9 Time series of modelled internal layer vertical strain rate profiles within the ice column measured using ApRES at S30 at sites (a) 14a; (b) 14b; and (c) 15. The horizontal bars at the top of each plot show the curvature of the displacement (parameter ‘α’, white – blue colour scale) and the goodness of fit ($R^2$, white – green colour scale) of the output model for each day of observation. The location of black dots signify the depth at which the ice column switches in polarity from a vertically-compressive (i.e. vertical strain thinning, in blue) to a vertically-extensive (i.e. vertical strain thickening, in red) regime. The bed elevation for each transect was calculated by subtracting the local ice thickness (measured by ApRES) from the surface DEM (GIMP DEM; Howat et al., 2014). Note the different scales used for time and distance between subplots.
Seasonal evolution of vertical deformation

(August – October), the curvature of the strain profile \( a \) is positive but small \((4 \times 10^{-12} \text{ to } 3 \times 10^{-8})\), suggesting almost-linear strain-thickening along the ice column (Figure 6.10). Indeed, this was found in the majority of the data acquired during late summer and autumn in 2014 (deployment 14b; Figure 6.9b) as well as during the same seasons in 2015 (deployment 15; Figure 6.9c). After this period, \( a \) gradually increases with time up to \( 1 \times 10^{-7} \), reflecting increasing vertical strain with increasing depth. This trend in curvature eventually introduces strain thinning near the ice surface at the peak of mid-winter in January/February, when the identified point of the polarity reversal (where the strain regime switches from strain thinning to thickening) begins to descend in depth through the rest of the winter and into spring (Figure 6.9c). Only from June does \( a \) wane and revert back to a purely-thickening ice column through the course of the summer season (Figure 6.10).

While the quadratic models of vertical strain provided a high goodness-of-fit \((R^2 > 0.90)\) outside the melt season, they were unable to capture fully the nonlinearity in strain behaviour during late spring and early summer, as evidenced by the markedly lower \( R^2 \) values through this period (Figure 6.10). This deviation was especially apparent in mid-June, when the directional switch of the polarity point upwards through the ice column caused difficulty with the quality of the model fitting (Figure 6.9a). At this point in time, the vertical displacement of internal reflectors deviated away from the prescribed quadratic function within deeper sections of the ice column (>420 m). Instead, the profile resembles a piecewise linear function, with slight strain thinning above 420 m, and much more substantial strain thickening below this depth (Figure 6.10).

6.5 Possible controls over variations in vertical strain

Variations in the vertical strain rate within an ice column are often attributed to spatial and temporal variations in processes occurring at the base of the glacier. These can consist of changes in (i) basal topography, (ii) basal mass balance, influenced by material properties and/or presence and pressure of meltwater, and (iii) basal traction (Hindmarsh et al., 2006; Holschuh et al., 2017; Panton and Karlsson, 2015; Rippin et al., 2005; Ryser...
Figure 6.10 Time evolution of monthly-averaged vertical velocity profiles, with curves showing strain behaviour of the ice column. Marker size represents the relative weight of each reflector towards robust fitting, where larger markers carry more weight than smaller markers. The May, June, and July profiles (last row) were collated from deployment 14a; the rest of the profiles (first three rows) were collated from deployment 15.
et al., 2014a; Weertman, 1964; Wolovick and Creyts, 2016). Such stress variability can be generated either locally or transferred from farther afield by longitudinal and lateral coupling. There are therefore at least three scenarios with the capacity to drive the spatiotemporal patterns in vertical strain that we measured at Store Glacier:

(a) local variations in topographic setting;

(b) local variations in basal traction; and

(c) far-field variations in glacier dynamics.

We therefore supplement the ApRES data with local observations of GPS- and satellite-derived topographies and velocities in an attempt to partition the effects of these processes on the measured strain field. To this end, we discuss each scenario in turn below.

6.5.1 (a) Are variations in vertical velocity governed by local variations in topography?

Topographically, S30 (30.3 km from the glacier terminus) is located over a local 3 km-wide depression within a regional bedrock high (Figure 5.1c). This regional bedrock high is composed of three local maxima located 28.8 km, 32.4 km, and 35.6 km from the terminus respectively, the last maximum descending 150 m into this depression. Together, they represent a bottleneck with respect to the orientation of glacier flow. Beyond the first subglacial peak 1.5 km downstream of S30, the glacier flows over a 70 m-high subglacial cliff, after which it enters a deep trough that extends to depths of ~600 m below sea level (Figure 5.1c).

Measurements of ice thickness and the resulting profiles established along each of the three ApRES deployments are consistent with ice flowing over a local depression (Figure 6.9). The eastern extent of this local depression is captured by deployments 14a and 14b, which show that the glacier at those sites flows downhill (Figure 6.9a,b). Deployment 15, which was operational for longer and therefore profiled a greater distance, showed that the glacier flowed 50 m downhill for approximately one month, after which it traversed relatively flat terrain before flowing 150 m uphill during the last three months of its operation.
6.5 Possible controls over variations in vertical strain

Although the transition from vertical thickening to vertical thinning during this three-month period was spatially coincident with the glacier accelerating across a bedrock ridge (Figure 6.9c), this interpretation cannot explain why a similar strain regime was also observed in deployment 14a, which acquired data when the glacier flowed into a depression. This interpretation also cannot explain why the strain regime of deployment 14b, which also flowed downhill, recorded an altogether different strain regime. A comparison between daily depth-integrated vertical velocities and underlying bed slope, which was acquired in high spatial and temporal resolution with the ApRES system, further revealed no obvious relationship ($R^2 = 0.045$; Figure 6.11a). Therefore, given the available data, local variations in bed topography cannot alone explain the measured variations in the vertical velocities of internal layers.

Figure 6.11 Scatterplot correlation comparing (a) total-column vertical strain of modelled profiles in all deployments with the underlying bed slope, (b) total-column vertical strain of modelled profiles in deployment 14b and basal velocity, and (c) total-column vertical strain of modelled profiles in deployment 14b and basal traction. (d) Time series of surface and basal velocities inferred from kinematic GPS deployments and borehole-installed inclinometers at S30. (e) Time series of basal traction.
6.5.2 (b) Are variations in vertical velocity governed by local variations in basal traction?

Basal conditions at S30 have been examined through seismic analysis by Hofstede et al. (2018), who observed the ice to be generally underlain by ~45 m of unconsolidated sediment. However, at a smaller (~100 m) scale within our study area, there exists significant spatial variation in the observed polarity of the ice-bed reflection, suggesting that these heterogeneous patches of basal slipperiness is associated with variable amounts of water. We are also fortunate to have detailed information about basal and englacial conditions through instrumented boreholes drilled contemporaneously in close vicinity (~30 m west) to deployment 14b in July 2014 (Figure 6.1), where Doyle et al. (2018) reported subglacial water pressures persistently close to overburden (93–95% of $p_i$). Together with GPS measurements at the ice surface, the glacier exhibits a moderate response to seasonal changes in ice velocities at S30, where small variations in basal water pressure were concomitant with large fluctuations in surface ice velocity and uplift. These observations suggest an inefficient basal hydrological system with water flowing both at the ice-sediment interface and within the basal sediment layer itself, resulting in a situation where basal motion at S30 is sensitive to the influx of surface meltwater to the subglacial environment.

Discrete ice flow acceleration events occurred during the melt season, when the influx of surface meltwater to the subglacial environment generated fluctuations in surface ice velocities and surface uplift (Doyle et al., 2018). This induced high variability in the coupling between basal and surface motion, for example as seen during a recorded rainfall event between 17 and 20 August 2014 when basal motion accounted for as much as 81.9% and as little as 32.1% of surface motion (Figure 6.11d). When compared to the record of full-column vertical strain measured from ApRES, however, the contemporaneous record of basal velocity did not produce any statistically significant correlation ($p = 0.360$; Figure 6.11b). Similarly, the comparison between full-column vertical strain and basal traction (calculated from measurements of subglacial water pressure) produced no obvious correlation ($p = 0.430$; Figure 6.11c). Although the time series of basal traction shows a consistent decrease through time, which Doyle et al. (2018) suggested...
reflects the progressive isolation of the subglacial hydrological system from influent water (Figure 6.11d), the strain profile of the ice column measured concurrently shows little variation in either curvature or magnitude (Figure 6.9b). We therefore conclude that variations in local basal slip are unable to explain the observed variations in vertical strain through space and time.

6.5.3 (c) Are variations in vertical velocity governed by far-field variations in glacier dynamics?

Temporal variations in basal conditions can generate membrane-like stresses consisting of longitudinal and transverse stress gradients in the overlying ice. These gradient stresses can develop across areas that are much larger than the area over which basal conditions changed in the first place (Hindmarsh, 2006). In fast-flowing glaciers, gradient stresses can be transmitted over horizontal distances extending up to $10 \times 20 \times$ the local ice thickness away from the region where the change in basal conditions occurred (Kamb and Echelmeyer, 1986; Price et al., 2008). To examine this effect, we analysed contemporaneous TerraSAR-X surface velocity datasets (Figure 6.12a) covering the fastest flowing portion of Store Glacier, including our field site, S30. The satellite velocity data, acquired from May 2014 to June 2015 at a temporal resolution of 11 days, show a distinct seasonal pattern of ice velocity. This variation is especially conspicuous near the calving terminus where surface motion, following a short-lived increase in velocities in June 2014 to a high of $15.8 \text{ m d}^{-1}$, decelerated rapidly thereafter to reach an annual minimum ($12.4 \text{ m d}^{-1}$) in late July 2014, after which surface velocity increased gradually through the winter and spring to peak once again, averaging $15 \text{ m d}^{-1}$ between March and June 2015 (Figure 6.12a). These seasonal variations are observed to propagate at least $30 \text{ km}$ up-glacier from the terminus. Although minor compared to the absolute velocity, the pronounced deceleration commences during the melt season (Figure 6.13), and could not be caused by calving and iceberg formation alone (Todd et al., 2018). Instead, we hypothesize that this decrease in ice flow, which is a persistent characteristic of Store Glacier (Ahlstrøm et al., 2013; Howat et al., 2010), is most likely a glaciological response to the sudden injection of vast amounts
Figure 6.12 (a) Time series of surface velocity sampled every 1 km along the central flowline of Store Glacier (Figure 5.1b), extracted from sequential TerraSAR-X imagery obtained between 2014 and 2015. The surface velocity at S30 (30.3 km from the glacier terminus) is shown in thick black. (b) Time series of the location of polarity switch between vertical thinning (blue) and thickening (red, Figure 6.9) as a fraction of the total ice column thickness for deployments 14a and 14b (May – December 2014). (c) Same as (b) but for deployment 15 (July 2015 – May 2016). The duration of seasonal phases of the movement of strain polarity are marked in (a) as either descending down (blue) or ascending up (pink) the ice column. Within the latter, the observations of surface meltwater influences on the flow velocity are superimposed over the summer slowdown (“H”).
of surface meltwater to the bed of the ice sheet. Although this response is similar to those
inferred along the land-terminating ice margin (Bartholomew et al., 2010; Chandler et al.,
2013; Van De Wal et al., 2015), our local observations of pressurised meltwater within
an inefficient subglacial drainage system, coupled with remote sensing measurements of
glacier-wide variations in ice flow velocities is akin to the processes underlying glaciers
in surge (e.g. Kamb et al., 1994; Meier et al., 1994). At Store Glacier, the build-up
of ice volume over the quiescent winter period is suddenly released by the concurrent
breakup of the proglacial ice mélange, removing a vital source of stabilising backstress
(Todd et al., 2018). The initial acceleration in flow is thought to be driven by enhanced
submarine melting and calving at the glacier front (Morlighem et al., 2016), as well as
the presence of widespread pressurised basal water residing in small cavities within an
inefficient subglacial drainage system (Doyle et al., 2018). As discharge reaching the small
cavities at the ice-bed interface becomes critically high, turbulent heat dissipation from
the incoming water flux encourages conduit growth and allows for greater discharge fluxes
and yet more energy dissipation (Kamb, 1987). With meltwater increasingly captured
by the growing channels, and with water pressure falling in those channels, frictional
resistance along the bed increases and the glacier eventually experiences a slowdown in
flow (Kamb, 1987). Outside the melt season, the gradual increase in satellite-observed
ice flow during the autumn and winter from \(12.4 \text{ m d}^{-1}\) in August 2014 to the mean
maximum of around \(15 \text{ m d}^{-1}\) in March 2015 is consistent with ice flowing gradually faster
as subglacial water pressure builds up in an inefficient basal water system consisting of
small cavities, which accommodates a continuous production of basal meltwater while
subject to creep closure due to the weight of the ice above.

The characteristic seasonal velocity of Store Glacier coincides with the observed seasonal
variations in vertical velocity and strain reported above. From mid-June to late July, when
peak summer melting occurred (Doyle et al., 2018), we infer a rapid slowdown extending
from the terminus to S30. Although the slowdown is very small in our GPS record
from S30 (Figure 6.13), we note that the longitudinal and transverse stress gradients
theoretically extend well beyond the area over which basal conditions has changed. This
inference is consistent with the strain regime observed within the ice column at S30
during this period, where vertical thickening at depth rapidly overtook the previously
permeating vertical thinning in the upper portion of the ice (Figure 6.12b). As the summer melt season comes to a close, the ice velocity returns toward its balanced, winter values, causing longitudinal stress gradients to wane. Here, we infer the basal hydrological system to again consist of small cavities distributed over a large area, which grow in size as the flow of the glacier gradually increases during winter and spring. At S30, we measure relatively constant vertical thickening until December, after which vertical thinning gradually and progressively increases in the upper part of the glacier propagating through depth, culminating with concurrent half-column thinning and half-column thickening in early June the following year (Figure 6.12c).

6.6 Discussion

The combination of field and satellite data leads us to hypothesize that longitudinal coupling provides the most likely explanation for the spatiotemporal variations observed in the vertical velocity of englacial reflectors and the vertical strain rates at S30. At Store Glacier, concomitant seasonal variations in both the vertical strain regime and ice velocities are likely far-field glaciological responses superimposed on localized variations in ice flow.

While most of the glacier front experienced rapid acceleration followed by dramatic deceleration at the onset of summer melt in June (Figure 6.12a), surface velocities
at S30 instead exhibited a gradual increase through the course of the melt season. Superimposed on this dampened seasonal effect are large fluctuations in velocities, driven by injection of locally-produced meltwater to the bed at S30 (Figure 6.13). At S30, these synchronous patterns between daily velocities and surface melt rates, combined with coincident measurements of borehole water pressure, all support a spatially and temporally homogeneous and inefficient basal hydrological system underlying the drill site (Doyle et al., 2018), that, in contrast to the channelized water systems developing closer to the ice front, remain relatively inefficient and unchanged throughout the year. The longitudinal coupling effect observed here may be part of a catchment-wide phenomenon that is governed by melt-induced perturbations from surface water forming and reaching the bed, which propagate upglacier through the course of the summer melt season. A detailed numerical analysis of the longitudinal coupling effect by Christoffersen et al. (2018) is consistent with the vertical thickening observed throughout the ice column at S30, starting in July when surface melt is propagating farther inland and lubricating the bed there, while ice flow at lower elevations becomes markedly slower in response to the formation of an efficient basal drainage system. The inferred presence of distributed basal drainage at higher elevation is consistent with borehole observations (Doyle et al., 2018), which support inefficient basal drainage at S30 throughout the melt season. This regime of vertical thickening persist throughout autumn and winter (Figure 6.12b,c), which shows that it takes several months to build up high water pressure in the distributed basal drainage system forming at elevations below S30 after the summer melt season. Only in March does the ice column at S30 begin to experience vertical thinning in response to ice flow acceleration induced by the re-pressurisation of the distribution basal drainage system at lower elevations, where an efficient system develops in summer (Figure 6.12c).

Although S30 is located upstream from the region where basal hydrology induces large seasonal variations in ice flow (“H” in Figure 6.12a), our observations show that vertical strain nevertheless undergoes distinct seasonal transformations. These transformations involve: (i) a gradual transition with vertical thinning extending approximately halfway through the ice column, resulting in a distinctly non-linear vertical strain profile (March–June); (ii) a shorter period during which vertical thinning vanishes, but with vertical
strain remaining distinctly non-linear (June – July); and (iii) an extended period with vertical thickening throughout the full column and a linear vertical strain regime (August-March). Given that the non-linearity arises from the high strain rates observed in the lower half of the ice column, the latter is most likely linked to the high basal strain observed in boreholes, with approximately 70% of the deformation occurring in the lowermost 100 m of the ice column, and 40% in the lowermost 50 m (Doyle et al., 2018). Supported by coincident seismic studies (Hofstede et al., 2018), this warm basal zone of enhanced deformation has been interpreted to consist of “softer”, potentially pre-Holocene ice (Doyle et al., 2018). With an ice-column consisting of Holocene as well as softer Wisconsin aged ice, enhanced deformation in the latter should be expected for a glacier flowing over a relatively strong bed. The vertical velocity profiles observed in this study are consistent with a high concentration of strain within this highly-deforming layer, with colder ice higher up the ice column resembling a “stiff beam”, resisting these stresses (Ryser et al., 2014a). Therefore, the compressional effect from strong longitudinal coupling observed in the summer melt season, triggering the rapid transition from vertical strain thinning to thickening, may manifest first within the lowermost internal layers, before gradually propagating up the ice column into colder ice, which, because of its increased stiffness from low temperatures, is less prone to deformation.

The degree of influence that basal slip has on englacial ice deformation likely depends on the period of local stick-slip behaviour imposed by the basal boundary condition. At the scale of 5–10× the local ice thickness, both Ryser et al. (2014a) and Holschuh et al. (2017) attributed variable surface velocities and ice deformation patterns in both space and depth to periodic patches of different basal slipperiness, while at a finer scale (≤ 1× the local ice thickness), other studies (Balise and Raymond, 1985; Kamb and Echelmeyer, 1986; Mair et al., 2001) do not observe any clear relationship between basal drag and glacier motion due to horizontal coupling with surrounding areas. Given the fine spatiotemporal resolution within our study (< 1× the ice thickness), it is therefore not surprising that we do not find any significant correlation between variations in basal and vertical velocities (Figure 6.11b). Similarly, we also find variations in fine-scale basal topography unable to explain variations seen in the vertical velocity profiles at the same spatial scale (Figure 6.11a). However, at sufficiently long wavelengths (5–10× the
ice thickness), internal layering is expected to drape along the subglacial topography (Hindmarsh et al., 2006). Therefore, at this scale, significant subglacial obstacles can exert marked hypsometric control over the morphology of internal layers (Bingham et al., 2015).

We successfully adjusted the curvature term \(a\) in a quadratic model (Equation 6.13) to match temporal variations in vertical strain at S30 (Figure 6.4). While the output models produced had a high goodness-of-fit for all deployments \(R^2 > 0.75\), we acknowledge that, occasionally (e.g. May and June, Figure 6.10), our automated model was unable to completely encapsulate the vertical velocity distribution within the ice column. On these occasions, the modelled profiles were unable to fully capture the nonlinearity in strain behaviour at the deepest sections of the ice column (>420 m), most likely due to the enhanced deformation suggested by coincident borehole-installed tiltmeters (Doyle et al., 2018) and seismic studies (Hofstede et al., 2018). However, given the highly variable basal velocities observed below S30 (Figure 6.11c), small fluctuations in the basal traction at the daily scale may transiently perturb the soft Wisconsin-aged ice. Given that these ephemeral perturbations occur at the bed, most of the deformation would be accommodated by deformation in the immediately-overlying Wisconsin-aged ice. Therefore, these perturbations would not have enough time to propagate through the entire column, and hence non-linear models would adequately reflect the anomalously high strain in deeper ice. We hypothesize that this state is not always present, but is only activated when basal conditions at S30 experience substantial local perturbation.

### 6.7 Conclusions

We have presented time series of vertical velocity and strain in a fast-flowing, marine-terminating outlet glacier in West Greenland. The time series, which was derived using autonomous phase-sensitive radio-echo sounding to track day-to-day displacements of englacial layers with millimetre accuracy, provide a novel insight into the mechanics of fast glacier flow. The data, which were captured with unprecedented resolution and detail, revealed variations in vertical velocity and strain that are complex and far more
pronounced than previously reported. From the second half of the melt season into the quiescent winter months, the ice was found to exhibit a full-column vertical thickening averaging $0.0165 \text{a}^{-1}$ over a largely linear strain regime. Nevertheless, between March and June, we observed minor vertical thinning encroaching from the ice sheet surface downwards of around $0.03 \text{a}^{-1}$ gradually offsetting the aforementioned vertical thickening. Here, the strain regime becomes increasingly non-linear, with rates reaching up to $0.1 \text{a}^{-1}$ near the bed of the ice column, until, with the onset of surface melting, the vertical thinning rapidly gives way to full-column vertical thickening once more.

We show that these seasonal variations in velocity are unlikely to be caused by the local setting of the glacier at site S30 where the data were collected. On the basis of borehole observations made in the close vicinity of the installation of ApRES, we also find little empirical support that the velocity variations result from heterogeneities in local basal conditions. Instead, we found that the effects of hydrologically-driven far-field horizontal stress transfer are able to explain the variations seen in vertical strain. Further work investigating the causes and consequences of longitudinal stress coupling is required to conclusively determine the mechanisms that govern the local deformational regime.

By observing the corresponding patterns of radar-derived vertical ice velocities and far-field glacier dynamics observed in TerraSAR-X satellite imagery, we hypothesize that variations in vertical velocity at site S30 are primarily driven by longitudinal coupling, and that concomitant seasonal variations in both the vertical strain regime and ice velocity are likely far-field glaciological responses superimposed on localized variations in ice flow. During the peak of the summer melt season between June and July, the rapid slowdown near the terminus of Store Glacier induced compressive longitudinal stress gradients cascading upglacier to S30, quickly replacing the vertical thinning in the upper portion of the ice with consistent vertical thickening at depth. As surface meltwater production ceases past the melt season in September, the basal hydrological system becomes increasingly inefficient, and the entire frontal region of the glacier gradually increases in speed until reaching a steady maximum velocity in March, averaging $15 \text{ m d}^{-1}$ at the glacier terminus. Only at this time does the ice column begin to re-experience the vertical thinning in response to the change in longitudinal stress gradients, gradually
offsetting the vertical thickening at depth until the onset of surface melt the following June.

The distinct response of local strain at site S30 to temporal variability of ice flow taking place tens of kilometers away indicates that transport of melt-water along subglacial drainage paths prolong the longitudinal coupling length, connecting site S30 with the fastest flowing portion of the glacier. From mid-June to late July, when the basal water system accommodates the melt season’s peak production of surface water, efficient channels form a network, which drains the glacier more effectively and causes a pronounced slowdown at elevations extending up to, but not beyond site S30. Although the glacier flows slower at higher elevations, there is no slow-down due to spatial limitations in the ability of channels to form. Strain at site S30 therefore becomes compressional, with thickening taking place throughout the ice column. The strain rate of this seasonal flow regime can be approximated well with a linear model, and the goodness of fit of this model ($R^2 > 0.75$) persist from July to late January or early February the following year. In late February or early March, we find vertical velocity to gradually induce vertical thinning in the upper half of the ice column, while the lower half continues to thicken. At this point the best-fit model for the observed strain becomes quadratic with a goodness-of-fit averaging $R^2 > 0.85$. While we cannot fully explain this transition, we infer that it is associated with a relatively thick layer of Wisconsin-age ice present in the lowest ~100 m of the ice column at S30 (Doyle et al., 2018; Hofstede et al., 2018). The higher degree of viscosity attributed to Wisconsin ice facilitates enhanced deformation, in contrast to the Holocene ice residing above this layer. The latter, being less viscous, may thus act as a stiff beam, which accommodates the far-field transfer of stresses via longitudinal coupling (Christoffersen et al., 2018). The latter plays a crucial role in the force balance of the ice sheet in summer, but may also be important in winter and spring, when a significant increase in the water pressure inside small basal cavities reduces the basal traction, resulting in faster glacier motion. The hydrological control on ice flow indicates that Store Glacier poses physical traits that are similar to those of surging glaciers.
A Appendix: Temperature dependence of signal propagation

Section 4.2.2 details the propagation and attenuation of electromagnetic waves within ice. While the attenuation of the radar signal will simply result in the inability to detect and track the movement of internal reflectors, particularly at depth, the velocity of the ApRES signal propagation through ice is particularly sensitive to ice temperature.

The temperature profile within the ice column is heavily influenced by englacial and basal heat sources and sinks and, for S30, varies from temperatures near the pressure melting point just above the ice-bed interface to $-21.24\pm0.05^\circ$C near the centre of the ice column (Figure 5.5). As such, the signal propagation speed through the vertical ice column at S30 is highly heterogeneous. This would present a significant issue if the radar was used to quantify absolute ice thickness to high precision and did not account for these temperature variations. The velocity of the radar signal in ice is dictated by the relative dielectric permittivity of the medium, and is often assumed to be between 3.10–3.18 (equivalent to 168–171 m $\mu$s$^{-1}$), with lower velocity values for lower ice temperatures (Fujita et al., 2000). Within this thesis, we use a relative dielectric permittivity value of $\varepsilon_r = 3.10$, which corresponds to wave propagation speeds in cold ($<200$K) ice. Resulting output values are therefore likely to overestimate the absolute ice thickness: given a nominal range of 600 m using $\varepsilon_r = 3.10$, increasing $\varepsilon_r$ to 3.18 will result in a 7.70 m decrease in ice thickness. Although we do measure absolute ice thickness within this chapter (e.g. Figure 6.9), its primary purpose is to show relative variations in basal topography along the flowline, and not to report absolute measurements.

Similarly, temporal fluctuations in temperature within the ice column will also have a pronounced impact on signal wave propagation (~23 000 m s$^{-1}$ K$^{-1}$). Specifically, there is a ~9 mm change over 600 m given a temperature increase in 0.1 $^\circ$C. As variations in englacial ice temperature occur over timescales of decades and longer (Cuffey and Paterson, 2010; Pettersson et al., 2007), and as we are measuring relative change instead of absolute thickness, we consider this source of error to be very small. Nevertheless, any anomalous movement in internal layers would have been already assumed into the
total observed depth of each internal reflector and their errors accumulated through daily averaging, with larger errors accounted for through weighting (Equation 6.15) and with hourly and daily outliers removed altogether from model fitting (Section 6.3.1; Figure 6.5).

Fluctuations in the ambient air temperature immediately surrounding the ApRES may also delay the breakthrough from the transmitting antenna, where warmer temperatures increase the signal propagation speed. This can be avoided by burying the ApRES in snow to provide insulation against temperature fluctuations, by restricting samples to the same time each day, or by incorporating the temperature effect into the cable delay (Section 6.3.1). Because the ApRES arrays were installed directly on the ice surface in the ablation area, the first option can be excluded, and therefore we address this issue through the latter two options, with each burst pair $f$ and $g$ separated by a period of 24 h. The influence of temperature on ApRES system operation is discussed further in Section 9.2.4; however, in general, observations both from laboratory and field experiments conclude that the influence of temperature on internal reflector range detection is manifested as a simple time delay.

Lastly, transient environmental changes, such as the generation of a firn aquifer from summer meltwater, are likely to affect the dielectric properties of the upper portions of the ice column. Recent studies using the same ApRES datasets have estimated the firn aquifer to extend down to 41 m below the ice surface, impounding substantial surface meltwater beyond the summer melt season into winter, when it is either released or refrozen (Kendrick et al., submitted). Aside from markedly attenuating the received signal, the seasonal presence of meltwater will increase the signal velocity (Equation 4.14) within the affected section of ice due to the differences in the dielectric permittivities between pure water and pure ice (approximately $\varepsilon_w = 80$ and $\varepsilon_r = 3.10$, respectively; Evans, 1965). Given a sudden increase in the water content, internal layers within this zone would be detected at increasingly shallower depths, causing the entire zone to falsely experience heightened strain thinning, with layers falling below the lower boundary experiencing a negative bulk offset. Accordingly, the upper boundary of the identification and tracking of internal reflectors has been set to 40 m, and any bulk offset in internal
layers (i.e. the parameter $c$ in Equation 6.13) removed through taking the derivative to obtain the vertical strain rate.
Chapter 7

Surface runoff drives sustained melting at the base of the Greenland Ice Sheet

Author contributions

In addition to the design and deployment of the antenna array (Chapter 5) and collection and processing of GPS records of surface (Section 6.3.2) and basal velocities (Section 6.3.3), several authors contributed to the research presented in this chapter, which is structured and written in a style suitable for publication. Kenneth Mankoff and Marion Bougamont modelled flow routing and viscous heat dissipation across the Store Glacier catchment area (Figure 7.5). Slawek Tulaczyk advised on the kinetics of glacial conduit heat exchange (Appendix 7B), and Bryn Hubbard and Samuel Doyle advised on the interpretation of the subglacial hydrological system. The author (T.J. Young) built the antennas, installed the ApRES systems, and processed all radar data presented in this chapter, with guidance from Keith Nicholls and Craig Stewart. The author also wrote the text, with minor contributions by Poul Christoffersen and Samuel Doyle.
7.1 Summary

Basal melt exerts significant control over the flow of ice sheets, yet there still exists no empirical observations of this crucial parameter beneath grounded ice. Here, we present the first measurements of basal melt rates beneath a fast-flowing outlet glacier of the Greenland Ice Sheet. With a continuous daily record spanning four months, we observed high rates of basal melt of up to 57 mm d\(^{-1}\), modulated by surface water runoff into the subglacial environment of Store Glacier, West Greenland. Measurements of basal melt surpassed predictions from traditional thermomechanical models by an order of magnitude, suggesting a crucial, yet missing parameter not yet accounted for within these models. Through quantifying the potential energy available from surface runoff converted to and released as viscous heat dissipation, we discover large amounts of basal water directly routed through the study site, with enough energy to fully explain the observed high rates of basal melting. We attribute viscous heat dissipation to be the main driver of high summer basal melt rates and show that warming of subglacial water and its subsequent storage induces high melt rates to persist outside the melt season. Given predictions of a warmer and wetter Greenland in the near future, our findings suggest that viscous heat dissipation, and therefore basal melt production, is a crucial, yet so far overlooked process to be included in future ice sheet models.

7.2 Introduction

Sources and sinks of water and heat are an integral part of ice sheet dynamics, which involve complex interplay among mechanical, hydrological and thermodynamic processes (Bartholomew et al., 2010; Christoffersen and Tulaczyk, 2003; Fahnestock et al., 2001a; Iken et al., 1993; Zwally et al., 2002). In general, the gravity-driven flow of glaciers are enhanced when lubrication from surface meltwater entering the basal environment induces sliding at the ice-bed interface, where energy from strain, frictional, and geothermal sources contribute to the melting of basal ice (Andrews et al., 2014; Bougamont et al., 2015; Christoffersen et al., 2014; MacGregor et al., 2016; Tulaczyk et al., 2000a,b; van de...
Wal et al., 2008). Although basal melting critically influences the flow of ice sheets, all
former quantification of basal melt to this date have been derived either from models
integrating local borehole and radar observations in areas of slow flow (e.g. Beem et al.,
2010; Dahl-Jensen et al., 2003; Fahnestock et al., 2001a; Rogozhina et al., 2016) or
from regional or continental-scale inversions of surface velocities (e.g. Buchardt and
Dahl-Jensen, 2007; Joughin et al., 2004, 2009, 2003). Hence, there is only a limited
understanding regarding the production of basal meltwater.

Phase-sensitive radio-echo sounding (pRES), capable of detecting changes in range with
a precision of up to 1.8 mm over 3 km, comprises a new technique whereby basal melting
can be detected from extremely precise measurements of ice thickness and vertical strain,
which are established from the displacement of internal and basal reflectors (Brennan
et al., 2014; Corr et al., 2002; Nicholls et al., 2015). So far, pRES systems have been
used successfully to estimate melting at the ice-ocean interface of ice shelves (Arthern
et al., 2013; Corr et al., 2002; Dutrieux et al., 2014; Jenkins et al., 2006, 2010; Marsh
et al., 2016; Nicholls et al., 2015), but so far there have been no attempts to measure
basal melting beneath fast flowing glaciers and ice streams.

Here, we report the first time series of directly-measured rates of basal melting beneath
the Greenland Ice Sheet. Using an autonomous phase-sensitive radar-echo sounder
(ApRES) to continuously track daily displacements of internal reflectors and the ice-bed
interface, we produced high resolution time series of vertical strain and basal melt rates
beneath a fast-flowing section of Store Glacier, West Greenland. The records reveal not
only sustained vertical strain at rates of 15 mm d$^{-1}$ (5 m a$^{-1}$) across the ice column, but
also show basal melting at a rate of 10 mm d$^{-1}$ (3 m a$^{-1}$) during winter and increasing up
to 57 mm d$^{-1}$ (20 m a$^{-1}$) during the peak of summer melt when surface water is delivered
to the bed of the glacier. Using data from a 610 m-deep borehole drilled to the bed of the
glacier, we quantify the basal heat budget, which alone accounts for only a small fraction
of the observed high melt rates. To explain the latter, we consider the gravitational
potential energy of water produced on the surface and we show that its transfer as viscous
heat dissipation (VHD) along the bed, when water penetrates the ice column, can explain
our observations. The discovery that VHD is by far the largest source of energy for basal
melting shows that the basal production of meltwater in Greenland comprises a crucial,
but so far overlooked, freshwater flux to the ocean.

7.3 Methods

7.3.1 Radar measurements of ice vertical motion and ablation

In this study, we deployed an ApRES system at site S30 on Store Glacier in West
Greenland (70° 31’ 5” N 49° 55’ 9” W, 981 m a.s.l.; Figure 5.2a). Site S30 is located
~30 km from the calving terminus, near the glacier’s central flowline (Figure 5.1). The
ApRES system was deployed in unattended mode (Figures 5.7, 5.6b) and continuously
collected radar reflection data at 4-hourly intervals from 26 July to 04 December 2014,
after which the antennas were damaged and data collection stopped\(^1\). Contemporaneous
measurements of ice temperature, tilt and basal water pressure were made in boreholes
drilled 30 m east of the ApRES (Figure 5.2b; Doyle et al., 2018).

To calculate the vertical strain rate (VSR), we used a best-fit robust linear regression
model on data collected in the upper ~520 m (~83% of the total ice depth; Figure 7.1d)
where correlations were persistently high (Section 6.3.1; Figure 7.1c) and where \( F \)-
tests consistently show a constant vertical strain rate through the measured ice column
(Figure 6.6b). The total amount of strain throughout the ice column was obtained
by extrapolating the fitted curve to a strong and well-defined basal reflector, where
temporally-adjacent bursts coalesce and clearly define the ice-bed interface (Figure 7.1e,f).
Although we cannot rule out the possibility that strain rates in the lowermost ice (>520 m)
may be non-linear, a linear VSR is a simple and common assumption (Cuffey and Paterson,
2010) supported by previous observation (Gillet-Chaulet et al., 2011; Nicholls et al.,
2015). The daily VSR was then obtained by averaging six measurements acquired with
a separation of 4 h. This procedure was repeated at a daily timestep for data collected
from 03 August to 04 December 2014, when the radar hardware malfunctioned under
katabatic winds that damaged the antenna array.

\(^1\)The resulting time series produced is analogous to the data collected from deployment 14b (Table 5.2).
Figure 7.1 Phase processing of two ApRES samples within time series (profiles obtained 18:39 27 September 2014 UTC and 18:39 28 September 2014 UTC). (a) Raw time series of averaged bursts. (b) Fourier-transformed range spectra of first (red) and second (blue) burst. (c) Coherence of cross-correlation of segments, representing internal layers. Only layers above the coherence threshold (92.5%; dotted line) were used in subsequent analyses. (d) Displacement estimates of segments (internal layers) and basal layer (magenta star), with estimated vertical strain rate fitted as a linear curve. Green dots represent layers not meeting the minimum coherence threshold for strain rate calculation. Zooming in to the basal layer, (e) and (f) respectively represent the amplitude and phase of the two bursts (thick red and blue), with adjacent bursts (14:39 and 22:39 27–28 September 2014 UTC) marked in thin red and blue.
Basal melt rates (BMR) were calculated by differencing the change in range of the basal reflector from the amount of vertical ice deformation, the latter calculated by extrapolating the linear regression to the depth of the basal reflector (Figure 7.1d; Appendix 7A; Brennan et al., 2014; Kingslake et al., 2014; Marsh et al., 2016). To vertically align the measurements, we used distinct internal layers near the surface (40–100 m) as reference points, eliminating the effect of surface ablation, which slowly lowered the elevation of the radar during the summer melt period (Section 6.3.1). Internal layers within the ice column (20–600 m) were captured with better than 92.5% phase coherence in returned power (Figure 6.5b), and by tracking the vertical movement of these layers through depth at a daily interval (Chapter 6), we calculated the vertical velocity of the ice column at a 5 m depth step (Figure 6.6b). The cross-correlation coherence of the tracked internal layers was high (>0.925–0.99) in the upper 500 m, and dropping (>0.85–0.925) at greater depths where ice rapidly deforms (Figure 7.1d).

ApRES measurement errors are primarily due to phase noise, which are combined with root-mean-square (RMS) values from strain fitting to provide overall errors of vertical strain rate and basal melt rate (Section 6.3.1). Similar to other studies involving pRES and ApRES (e.g. Kingslake et al., 2014), instrument malfunctions and environmental influences (e.g. accumulation of surface meltwater beneath antennas) were occasionally found to cause erroneous measurements. These errors were automatically filtered out if the total number of usable internal reflectors (i.e. >92.5% phase coherence) within a single burst measurement falls below 20% (Figure 6.5b), resulting in minor gaps in the time series.

### 7.3.2 Decomposing the basal heat budget

To constrain and verify estimates of basal melt rates obtained from ApRES, we ran several heat budget models encapsulating sources and sinks within the ice-bed interface (Section 2.3). Values of physical constants and parameters for all models are summarised in Table 7.1.
Table 7.1 Parameters used in models of basal melt rates. All values are constants except for those indicated by *, for which they represent averaged values over the time series considered.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Flotation fraction</td>
<td>0.9</td>
<td>a</td>
</tr>
<tr>
<td>ηb</td>
<td>Effective porewater thickness</td>
<td>0.5 m</td>
<td>b</td>
</tr>
<tr>
<td>∂Θb/∂z</td>
<td>Temperature gradient of basal ice</td>
<td>−0.0286 K m⁻¹</td>
<td>c, d</td>
</tr>
<tr>
<td>ϕ</td>
<td>Friction angle</td>
<td>30°</td>
<td>e</td>
</tr>
<tr>
<td>ρi</td>
<td>Density of ice</td>
<td>917 kg m⁻³</td>
<td>f</td>
</tr>
<tr>
<td>ρw</td>
<td>Density of porewater</td>
<td>1000 kg m⁻³</td>
<td>f</td>
</tr>
<tr>
<td>τb</td>
<td>Basal traction</td>
<td>2.401 × 10⁶ Pa*</td>
<td>c</td>
</tr>
<tr>
<td>ωw</td>
<td>Water moisture fraction of basal ice</td>
<td>0.029</td>
<td>b</td>
</tr>
<tr>
<td>Ci</td>
<td>Heat capacity of ice</td>
<td>2009 J kg⁻¹ K⁻¹</td>
<td>b</td>
</tr>
<tr>
<td>CT</td>
<td>Clausius-Clapeyron slope</td>
<td>8.6 × 10⁻⁸ K Pa⁻¹</td>
<td>a</td>
</tr>
<tr>
<td>c₀</td>
<td>Apparent cohesion</td>
<td>0</td>
<td>e</td>
</tr>
<tr>
<td>cₚ</td>
<td>Specific heat of water</td>
<td>4184 J kg⁻¹ K⁻¹</td>
<td>a</td>
</tr>
<tr>
<td>kᵢ</td>
<td>Thermal conductivity of ice</td>
<td>2.14 J s⁻¹ m⁻¹ K⁻¹</td>
<td>f</td>
</tr>
<tr>
<td>kₘw</td>
<td>Thermal conductivity of water</td>
<td>0.561 J s⁻¹ m⁻¹ K⁻¹</td>
<td>g</td>
</tr>
<tr>
<td>K₀</td>
<td>Thermal constant</td>
<td>1.045 × 10⁻⁴</td>
<td>a</td>
</tr>
<tr>
<td>Lf</td>
<td>Latent heat of fusion</td>
<td>3.34 × 10⁵ J kg⁻¹</td>
<td>f</td>
</tr>
<tr>
<td>pₘb</td>
<td>Porewater pressure at bed</td>
<td>4.965 × 10⁶ Pa*</td>
<td>c</td>
</tr>
<tr>
<td>qG</td>
<td>Geothermal heat flux</td>
<td>0.06 W m⁻²</td>
<td>b, d</td>
</tr>
<tr>
<td>〈ub〉</td>
<td>Basal ice velocity</td>
<td>1.125 × 10⁻⁵ m s⁻¹*</td>
<td>c, d</td>
</tr>
</tbody>
</table>

a Mankoff and Tulaczyk (2017)
b Aschwanden et al. (2012)
c This study
d Doyle et al. (2018)
e Bougamont et al. (2014)
f Christoffersen and Tulaczyk (2003)
g Ramires et al. (1995)
h Rogozhina et al. (2012)
Thermomechanical sources of basal heat

We begin by comparing our results with the well-known equation specifying the rate of basal melting from thermomechanical heat sources and sinks of the system (Section 2.3.1; Alley et al., 1997; Christoffersen and Tulaczyk, 2003; Cuffey and Paterson, 2010):

\[
\dot{m}_b = \frac{q_G + q_f + q_{ib}}{\rho_i L_f}
\]  

(7.1)

where \(q_G\) is the geothermal heat flux, \(q_f = \tau_b u_b\) is the frictional heat flux, and \(q_{ib} = k_i \frac{\partial \Theta_i}{\partial z}\) is the upwards conductive heat flux into the ice. At S30, \(q_G\) was approximated through modelling to be 0.06 W m\(^{-2}\) (Rogozhina et al., 2016). The remaining sources and sinks were quantified using observational data acquired from borehole-installed sensors. \(\frac{\partial \Theta_i}{\partial z}\) is the temperature gradient of basal ice, which was calculated by fitting a robust linear curve with the undisturbed ice temperatures recorded by the bottommost three thermistors (M1, T2, and T3; Table 5.1, Figure 7.3b), and \(k_i\) is the thermal conductivity of ice (2.14 J s\(^{-1}\) m\(^{-1}\) K\(^{-1}\)). Additionally, \(\rho_i\) is the density of ice (917 kg m\(^{-3}\)) and \(L_f\) is the latent heat of fusion (3.34 \times 10^5 J kg\(^{-1}\)). The basal velocity \((u_b)\) was calculated by subtracting the depth-integrated deformational velocity (calculated from borehole-installed tilt sensors) from the measured surface velocity (calculated from GPS measurements) following (Ryser et al., 2014b). Lastly, \(\tau_b\) was calculated using the Mohr-Coulomb failure criterion, which includes inputs from the overburden ice pressure \(p_i\) and the subglacial water pressure \(p_w\), the former derived from ApRES measurements of absolute ice thickness and the latter recorded empirically by two vibrating-wire piezometers installed at the ice-sediment interface (Doyle et al., 2018). The calculations for both \(u_b\) and \(\tau_b\) are detailed in Section 6.3.3.

Enthalpy of basal ice

We also compare our results with a 1-dimensional representation of the jump equation for enthalpy at the ice base (Equation 50, Aschwanden et al., 2012):
\[ \dot{m}_b = \frac{q_f - (q_{ie} - q_G) - \rho_w \eta_b \left( \frac{\partial H(p_b)}{\partial p_b} \right) \left( \frac{dp_b}{dt} \right)}{H - H_l(p_b)} \]  

(7.2)

which expresses the non-advective heat flux into temperate ice, \( q_{ie} \), in terms of enthalpy \( H \) and pressure \( p \); Equation 61, Aschwanden et al., 2012):

\[ q_{ie} = \begin{cases} 
- \left( \frac{k_i}{C_i} \right) \nabla H, & \text{for cold ice} \\
- (k \nabla T_m(p) + K_0 \nabla H), & \text{for temperate ice} 
\end{cases} \]  

(7.3)

where \( k(H, p) = (1 - \omega_w(H, p)) k_i(H) + \omega_w(H, p) k_w \) (Equation 57, Aschwanden et al., 2012) as a function of both enthalpy and pressure, \( k \) and \( \omega_w \), respectively being the mixture conductivity (composed of \( k_i = 2.14 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1} \) and \( k_w = 0.561 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1} \), the thermal conductivities of ice and water) and water fraction of temperate ice. Additionally, \( \eta_b \) represents the thickness of the subglacial water layer, set to 0.5 m as in Aschwanden et al. (2012), and \( C_0 = 2009 \text{ J kg}^{-1} \text{ K}^{-1} \) represents the specific heat capacity of ice, which was set corresponding to the pressure melting point (PMP = -0.53°C) for pure water and saturated ice at 611 m depth (Equations 9, 10, and 75, Aschwanden et al., 2012).

**Viscous heat dissipation**

Parameters within the thermomechanical and enthalpy models previously mentioned only consider heat sources at the ice-bed interface and neglect additional heat generated from the influx of supraglacial meltwater into the subglacial environment during the summer melt season. Therefore, we also incorporate this heat source by quantifying the amount of energy available for viscous heat dissipation (VHD) through the melt season, both at S30 and across the wider Store Glacier catchment area (Mankoff and Tulaczyk, 2017).

Specifically, the model tracks the flux of surface meltwater from source (i.e. surface melt or precipitation) to sink (i.e. into outlet fjords) and calculates the amount of energy produced from pressure and elevation changes, and lost through VHD. This was done using runoff from the RACMO regional climate model at a 1 km resolution (Noël et al.,...
Runoff is transferred to the bed and routed using the D8 (deterministic eight-node) single-flow-direction algorithm, which directs flow from each grid cell to one of either nearest neighbours based on the gradient of the hydropotential (Equation 1, Mankoff and Tulaczyk, 2017):

$$\nabla \phi_h = \nabla \phi_{h_z} + \nabla \phi_{h_p} = \rho_w g \nabla z_b + \alpha \rho_i g (\nabla z_s - \nabla z_b)$$ (7.4)

where the hydropotential ($\phi_h$) can be decomposed into an elevation component ($z$) and a pressure component ($p$) in Pa. $\rho_i$ and $\rho_w$ represent the densities of ice (917 kg m$^{-3}$) and water (1000 kg m$^{-3}$) respectively, $g$ is the gravity [m s$^{-2}$], $z_s$ and $z_b$ the surface and bed elevation, and $\alpha$ the flotation fraction (0.9, due to the subglacial system being often slightly less than the ice-overburden pressure; Engelhardt and Kamb, 1997; Fountain, 1994; Meierbachtol et al., 2013). Surface and subglacial flow routing and subglacial pressures were calculated using high-resolution surface and bed digital elevation models (DEMs) at 150 m resolution (IceBridge BedMachine Greenland, version 2, Morlighem et al., 2015). The model output was gridded at both a 1 km and 500 m spatial resolution.

At the ice-bed interface, energy is assumed to dissipate as heat within the grid cell where the energy transfer occurs ($q_{VHD}$), which is determined from the available volume of meltwater ($V$), the change in hydropotential ($\nabla \phi_h$; Equation 7.4), and the change in the pressure melting point (Equation 2, Mankoff and Tulaczyk, 2017):

$$q_{VHD} = V \left( \nabla \phi_h - C_T c_p \nabla \phi_{h_p} \rho_w \right)$$ (7.5)

where $C_T$ is the Clausius-Clapeyron slope (8.6 $\times$ 10$^{-8}$ K Pa$^{-1}$; Hooke, 2005) and $c_p$ the specific heat of water (4184 J kg$^{-1}$ K$^{-1}$). Here, the energy from VHD is used to induce basal melting, releasing latent heat that gets transported and dissipated further downstream. The model assumes that all energy generated by viscous dissipation during water flow is instantaneously used in melting (Isenko et al., 2005).
We investigated the amount of basal melt produced through VHD in the entire catchment of Store Glacier, as well as within the vicinity of S30 where our observations were made.

### 7.4 Results

#### 7.4.1 High measurements of vertical strain rates and basal melt rates

![Figure 7.2](image)

**Figure 7.2** (a) Vertical strain rate time series extrapolated to the bed. (b) Concurrent basal melt rate time series.

The VSR of the ice column\(^2\) showed persistent strain thickening through the entire time series, averaging 15.3 ± 0.3 mm d\(^{-1}\) when extrapolated to the ice bed, with a transient 3-day increase in values up to 23.4 ± 2.9 mm d\(^{-1}\) between 18 and 20 August 2014 (Figure 7.2a). The mean values observed during the summer melt season were 14.8 ± 0.7 mm d\(^{-1}\), which were not statistically different to the averaged values in the winter season (15.4 ± 0.4 mm d\(^{-1}\)). The displacement of internal reflectors appeared to be a linear function of depth (Figure 7.1d), implying a constant VSR through the ice column.

\(^2\)see Section 6.4 for a more in-depth analysis on the spatiotemporal variability of vertical strain at S30
In contrast to the VSR of the ice column, which remained mostly constant through time, the BMR varied markedly and can be separated into two distinct periods, with higher and more variable basal melting during the summer melt season (20.5 ± 2.5 mm d$^{-1}$), and smaller, less variable melt rates in winter (9.8 ± 0.9 mm d$^{-1}$) when there exists no hydrological connections between the ice surface and bed (Figure 7.2b). The use of a linear regression model to estimate VSR may influence BMR in that strain below 520 m may differ from the applied linear approximation. However, although the vertical strain may not be linear, we note that the use of a non-linear model increases VSR and therefore also the estimated BMR, i.e. if we underestimate the quantity of positive strain in the ice column, we simultaneously underestimate the BMR (Appendix 7A). Our record of basal melting can therefore be interpreted as conservative.

The highest melt rates occurred between 18 and 20 August 2014, when BMR reached values up to 56.7 ± 4.6 mm d$^{-1}$. This period of enhanced basal melting temporally coincided with a week-long rainfall event, during which surface runoff was significantly increased by the advection of warm, moist air masses over S30 that brought heavy precipitation to the study area (Doyle et al., 2018). The cumulative observed rainfall during the event accounted for 35% (80.0 mm in accumulation), which, when superimposed with concurrently measured surface melt rates (288 mm w.e.), resulted in the vertical transport of large amounts of surface water to the bed, adding significant heat to the subglacial environment and modulating ice flow (Figure 7.3c).

7.4.2 Discrepancies between measured and modelled results

We compared empirical measurements with theoretical estimates from: (i) a thermo-mechanical model of the basal heat budget (Equation 7.1; Christoffersen and Tulaczyk, 2003), and (ii) an enthalpy-based formulation of the lower ice boundary (Equation 7.2; Aschwanden et al., 2012). Within these two models, heat sources and sinks are partitioned into components: frictional heat from ice sliding over the bed ($q_f$); upwards conductive heat loss into the ice ($q_i$); and geothermal heat flux ($q_G$). Additionally, the enthalpy formulation includes additional parameters describing how pore-water pressure changes
Figure 7.3 Time series of (a) borehole-measured basal pressure as a percentage of the ice overburden pressure; (b) temperature of the subglacial water layer measured from thermistor T1, as well as the three lowest thermistors (M1, T2, T3) installed in basal ice (Table 5.1) and the pressure melting point (PMP) for pure water and saturated water (dashed black); and (c) surface melt from AWS measurements and precipitation from NCEP/NCAR reanalysis data (Section 5.3.2), overlain by surface and basal velocity inferred from GPS and borehole tilt measurements (Sections 6.3.2, 6.3.3).
Figure 7.4 (a) Modelled basal melt rates at a daily time step. The components: basal heat budget from thermomechanical forcing (green, Christoffersen and Tulaczyk, 2003); additional heat generated from enthalpy (yellow, Aschwanden et al., 2012); VHD from surface water routed to the bed calculated at 500 m resolution (purple, Mankoff and Tulaczyk, 2017). (b) Magnification of (a) at low BMR values. (c) Concurrent record of ApRES-measured BMR (Figure 7.2b).
(\frac{dp_b}{dt}) are related to energy fluxes into the subglacial layer and the “wetness” of basal ice (\omega_w).

Of the mentioned heat sources and sinks, those generated from frictional heat alone (averaging 2.7 W m\(^{-2}\) through the time series) contributed to \(-97\%\) of the total estimated heat produced at the base of the glacier, with smaller, negligible fluxes stemming from geothermal sources (\(6 \times 10^{-2}\) W m\(^{-2}\)) and convective sinks (\(-2.8 \times 10^{-1}\) W m\(^{-2}\)). Therefore, the temporal variation of the modelled basal melt rates (Figure 7.4b) closely mimics that of the calculated basal velocity (Figure 7.3c). The incorporation of enthalpy generates on average an additional \(3 \times 10^{-2}\) W m\(^{-2}\) of heat flux; however, the combination of these terms only amount to at most 1.3 mm d\(^{-1}\) of basal melt, which is still an order of magnitude lower than BMR derived from radar observations. There is little difference in the rate of heat loss estimated from thermomechanical and enthalpy-based formulations, and neither frictional heat nor the geothermal heat flux can produce energy sufficient to explain observed melting at rates up to 56.7 \pm 4.6\mathrm{\,mm\,d^{-1}}. Even if sliding along the bed occurred at the same rate observed on the surface, the amount of frictional heat flux produced would only increase at most up to 5.3 W m\(^{-2}\), which, although doubled, still contributes little difference to the overall modelled result. Additionally, the uncertainty of geothermal heat fluxes across the Greenland Ice Sheet, due to a paucity of observations, cannot explain the discrepancy between theory and observation due to its relatively low flux rates. We therefore conclude that a crucial source of heat is missing in the models of BMR so far used.

### 7.4.3 Viscous heat dissipation as a missing heat source

Our theoretical estimates of BMR have so far only considered heat sources and sinks originating from within the basal environment. Given the significant amounts of supraglacial meltwater that enters the subglacial environment during the melt season, the missing heat source may therefore stem from energy produced on the surface and transported to the bed. To explore this possibility, we quantify the amount of energy available for the viscous heat dissipation (VHD) at S30 over the course of the melt season. This energy, combined with the turbulent flux of basal meltwater flowing through large-scale
subglacial conduits, is then dissipated as heat, inducing significant amounts of basal melt (Mankoff and Tulaczyk, 2017).

Importantly, the incorporation of VHD produces an estimate of basal melting that, during the melt season, is comparable with our radar-derived observations (Figure 7.4a,c). Due to its dependence on surface meltwater availability, VHD-induced melt rates correspond to temporal trends in the amount of surface runoff generated at S30 through surface melt and precipitation (Figure 7.3c). For instance, we observe high melt rates of up to 53.5 mm d\(^{-1}\) (19.5 m a\(^{-1}\)) on 18 August 2014 (day of year (DOY) 230), deviating from the ApRES-observed melt rate by only 3.2 mm d\(^{-1}\) (~95% of the total melt rate).

### 7.4.4 Spatial distribution of basal melt rates

We found sustained rates of high BMR to be concentrated within the large-scale drainage network under Store Glacier (Figure 7.5b,d). The drainage paths were often spatially coincident with the location of subglacial troughs that, when tracked downglacier, decreases in elevation, generating more potential energy towards VHD.

We find that the amount of basal melt predicted along major drainage paths increases significantly when the resolution of the model is improved. This illustrates how the concentration of water in a smaller area increases the magnitude of basal melting caused by VHD, indicating that rates of VHD depend on the dimensions of the drainage system. Specifically, on 18 August 2014 (DOY 230), we observe a two-fold increase from 25 mm d\(^{-1}\) at 1000 m spatial resolution (Figure 7.5b) to 54 mm d\(^{-1}\) at 500 m spatial resolution (Figure 7.5d). However, as the simulation predicts an instantaneous termination of VHD processes at the culmination of the melt season (Figure 7.4c), the upper bound of the rate of predicted basal melt should theoretically be limited by the total available volume of water rather than the grid resolution.

At the ice-bed interface, water is preferentially routed on the basis of spatial hydropotential gradients, which then depends on the accuracy of the basal DEM. S30 lies in the immediate vicinity (~2 km) of a major hydrological pathway, which we believe is the source of the VHD-generated high amounts of melt observed. The choice of pixel in the modelled
spatial output that is attributed to the ApRES-derived melt rates is important—while the pixel directly coincident with S30 generates \( \sim 4 \text{ mm d}^{-1} \) on DOY 230 regardless of pixel size (Figure 7.5a,b), melt rates at the nearby channel were found to be more than 10\( \times \) higher than calculated at the exact location of the ApRES. However, as current bed DEMs still contain a large degree of uncertainty, and have been shown to produce mismatches at the fine-scale in the basal thermal regime (Bell et al., 2014; Mankoff and Tulaczyk, 2017), we attribute the spatial offset between S30 and the underlying subglacial conduit to the uncertainty in geometrical constraints used to produce these bed DEMs.

We also observe several patches, including several located directly upstream of S30, where basal freeze-on (i.e. \( \dot{m}_b < 0 \)) is predicted to occur. These patches identify the far end of overdeepened troughs where water flow is uphill (expending excess potential energy) and the glacier experiences sustained thinning (increases the PMP), both of which promote low hydraulic gradients and basal freezing. Ice flow across such areas is particularly sensitive to rerouting (Chu et al., 2016a), and as such, water piracy by neighbouring conduits may markedly alter flow paths and the overall hydraulic activity surrounding these areas. Furthermore, the surrounding topography of these areas match the descriptions of likely areas of winter subglacial water storage, hypothesised to exist in areas with nearby flat hydraulic potential gradients that favour slow water flow and subglacial ponding (Chu et al., 2016b).

**7.5 Discussion and conclusions**

Through using ApRES deployed over a continuous period of four months, we observed high rates of basal melting (peaking at 57 mm d\(^{-1}\) on 18 August 2014) beneath a major outlet glacier of the Greenland Ice Sheet. The rates surpass predictions from thermodynamic models by an order of magnitude. This discrepancy can only be explained by the convergence of basal hydrological pathways towards the field site, where the viscous heat dissipation of turbulent water originating from the surface of the glacier fully explain the observed high melt rates during the summer melt season (averaging 21 mm d\(^{-1}\); Mankoff
Figure 7.5 Estimated (a) water accumulation through flow routing and (b) basal melt produced through viscous heat dissipation using a spatial resolution of 1 km on DOY 230 (18 August 2014). The inset in (a) shows the principal direction of water flow and the location of S30 (white star). (c,d) Same as (a,b) except using a spatial resolution of 500 m. The white cross in (b) and (d) shows the pixel used to calculate basal melt rates from VHD.
and Tulaczyk, 2017). While previous studies have modelled basal melt rates from surface or borehole-based observations (e.g. Dahl-Jensen et al., 2003; Fahnestock et al., 2001a; Joughin et al., 2009; Rogozhina et al., 2016), our observations, to the best of our knowledge, are the first empirical measurements of basal melt rates beneath a grounded ice sheet. Additionally, the congruence of empirical observations and theoretically-underpinned models imply the importance of including supraglacial runoff in the basal heat budget of the subglacial environment.

Our high observed BMR is similar to that of Marsh et al. (2016), who observed high basal melting of up to 22 m a^{-1} (60 mm d^{-1}) in a nascent channel at the grounding line of the Ross Ice Shelf. Here, they attribute the growth of the subglacial channel to enhanced mixing from a buoyant meltwater plume caused by outflow induced by surrounding subglacial lake outflow, whereas we find similarly high BMR along turbulent arborescent drainage pathways. Whereas melt rates on an ice shelf are primarily driven by warm oceanic currents (Stewart, 2018), we observe a scenario where, during the peak of summer melt and precipitation, the aggregation of large amounts of water from an extensive drainage basin into concentrated drainage pathways presents turbulent hydrological flow that induces high basal melting at S30 (Figure 7.5a,c).

The relatively high melt rates observed in autumn and winter (averaging 9.8 \pm 0.9 \text{mm d}^{-1}; Figure 7.2b) is at odds with theoretical modelling (Figure 7.4a,b), which shows that enhanced melting through VHD should cease at the end of the melt season when the bed no longer receives surface runoff. This discrepancy may indicate that all energy routed to the bed of the ice sheet is not instantaneously converted to and dissipated as heat, as assumed in the VHD model. Instead, some of the energy is also used to keep the temperature of basal water at the pressure melting point (PMP). At S30, borehole observations show anomalous high temperatures consistently +0.6–1.2°C above the pressure melting point (PMP) at the lowest thermistor through the entirety of its 76-day operation (T1, Figure 7.3b), which Doyle et al. (2018) assume to represent relatively warm liquid water or unfrozen sediment during this period. Following this, we further interpret that the high temperature record reflects the temperature of subglacial water that is flowing and being warmed from surface energy while losing heat to the ice base (e.g. Isenko et al., 2005) and causing the high basal melt rates observed independently with ApRES.
The extension of the temperature time series outside the melt season further indicates a sustained release of energy even after surface melting ceases (Appendix 7B).

We consider the possibility of winter subglacial water storage upstream of S30 as a potential energy source for additional basal melt outside the summer melt season. The locations of winter water storage, usually coinciding with high-elevation bedrock ridges (Chu et al., 2016b), correspond to areas of predicted basal freeze-on in the spatial representation of our model output (Figure 7.5b,d). The slow release of subglacially-stored water upstream may sustain high basal temperatures and melt rates through autumn and likely continuing into the winter season, potentially explaining the discrepancy between measured and modelled results. Indeed, borehole records of high subglacial water pressures (Figure 7.3a), together with stable conductivity readings (Figure 5.4d) suggest prolonged horizontal transport of water through and past the melt season due to a combination of low hydraulic transmissivity and the sustained replacement of transported water from higher elevations (Doyle et al., 2018). Similarly, in a separate study, borehole instruments installed at a similarly-fast-flowing section of Jakobshavn Isbræ (Lüthi et al., 2002) measured hydraulic transmissivity of the surrounding inefficient basal drainage system to be between $1 \times 10^{-5}$ m$^2$ s$^{-1}$ and $1 \times 10^{-4}$ m$^2$ s$^{-1}$ ($\sim$0.86–8.6 m$^2$ d$^{-1}$).

As inefficient systems exhibiting sheet or Darcian flow dissipate heat more effectively than in a channelised system due to a higher surface area to volume ratio (e.g. Flowers, 2015), VHD may also be an important process active not only during, but also outside the melt season.

With climate predictions portending a warmer and wetter Arctic and Greenland (e.g. Boisvert and Stroeve, 2015), the projected increase in surface melt production and meltwater availability will undoubtedly influence future VHD and melt processes at the ice sheet bed. Given various IPCC AR5 representative concentration pathway (RCP) scenarios of climate change (Vaughan et al., 2013), Mankoff and Tulaczyk (2017) observed up to a sevenfold increase in additional VHD and basal melt over the entire Greenland Ice Sheet, which they attribute to the gradual shift of the equilibrium line altitude (ELA) to higher elevations. The impact of increased VHD may be most strongly manifested in marine-terminating glaciers, which may accelerate when basal melting increases beyond the sustained rates observed in this study (Moon et al., 2012, 2014). The resulting
increases in basal melt, sustained well outside the melt season, may also underlie the reasons for the observed interannual acceleration of these glaciers (Moon et al., 2012; Walsh et al., 2012). While the high amounts of observed basal melt rates and the overall abundance of water at the ice-bed interface are thought to contribute to and sustain the already-fast flow of Store Glacier (Doyle et al., 2018), further increases in future basal melt rates may propagate enhanced flow, especially into inland regions approaching the ELA, where the production of basal meltwater is still nascent.

A Appendix: Extended methods

Range shift error in the basal reflector

In theory, the change in the range of the basal reflector through time can easily be calculated by the difference in total range between successive measurements of the basal reflector (peaks), where the base was identified as the maximum amplitude reflector within a specified depth range. However, this method only works if the basal reflector can easily be identified through cross-correlation or similar techniques, i.e. the shape of the imaged bed does not change significantly through time. In the case of the Greenland Ice Sheet, the shape of the basal reflector is dictated by the topography of the underlying bed layer, and the rate of this change is dictated by the glacier’s basal velocity, and compounded by the internal deformation rate.

As the radar system is resolution limited with a footprint of radius $r = \sqrt{2R\Delta R}$ (Equation 2, Brennan et al., 2014), where $R$ is the total range from source to reflector (Equation 4.26) and $\Delta R_c$ is the range resolution (Equation 4.20), the current setup gives a footprint radius of 22.716 m and area of 1621.062 m$^2$, assuming a depth of 600 m. With this in mind, range change was conducted incrementally at an elapsed time of 24 h to minimise the change in basal reflector shape, and at a maximum surface velocity of 670 m a$^{-1}$, will result in a maximum offset in footprint radius and area of 1.836 m (8.1%) and 167.144 m$^2$ (10.3%) respectively. Therefore, a sufficiently short (i.e. 24 h) separation between bursts
is required to retain a similar shape of the bed peak to track its movement through range and phase.

The errors presented in the calculation of basal melt rates (Figure 7.2b) are under assumption that the basal echo identified and tracked is indeed the correct reflector, and not instead from off-nadir effects. Although there is no current method or technique to validate this assumption, the convergence of sequential bursts at a precise depth (Figure 7.1e,f) strongly suggests the presence of a strong reflector with a high dielectric contrast (i.e. the ice-water interface) immediately following a largely-transparent basal ice layer (Section 9.2.1).

**Range ambiguity resolution**

Signals with low signal-to-noise ratios may theoretically induce an integer half-wavelength range ambiguity when the estimate of $\delta_c$ from cross-correlation exceeds $\lambda_c/4$ of the true displacement. This would cause erroneous phase wrapping, resulting in $\delta_f$ and $\delta_t$ to be in error by an integer multiple of $\lambda_c/2$. However, to reduce the likelihood of an integer half-wavelength range ambiguity occurring, “split-spectrum” estimation algorithms were employed (Stewart, 2018), which effectively increased the phase wrapping range to 2.13 m. We also induced hysteresis by lagging the coarse range bin to prevent unwanted oscillatory behaviour between range bins if an abrupt range shift (due to range ambiguity) was detected. The implementation of hysteresis was able to reduce ~75% of observed ambiguities in the time series, and was able to obtain a coherent time series of change.

**Linearly-varying vertical strain rates**

We previously confirmed through statistical means the validity in using a linear model to fit profiles of internal layer displacement (Section 6.4; Figure 6.6b). However, to further justify the linear fitting of internal layer displacements, we collated the entirety of the time series (788 bursts) and obtained the weighted mean of the displacement of

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3See Chapter 8 for a quantification of the identification of off-nadir reflectors.
Figure 7.A Individual (greyscale) daily layer displacement of all internal layers over the entire temporal record of ApRES deployment (03 August – 04 December 2014) with darker points representing higher coherence when tracking layers between successive bursts. All layers within each depth bin were averaged (green points) and were fitted with a linear (blue) and quadratic (cyan) function. To estimate the total amount of vertical strain within the total ice column, fits were extrapolated to the average bed depth (~615 m; marked with stars); additionally, the 4 identified internal layers closest to the bed layer were averaged to obtain observed strain just above the bed layer (green star).

Each internal layer relative to the surface after negating the effects of surface ablation (Section 6.3.1). The total amount of strain to the basal layer was estimated by fitting both a linear and a quadratic function to the averaged internal layer displacement.

We find that allowing for a quadratic fit to internal layer displacement marginally increases the mean VSR to full depth over the 4-month-long time series to 5.877 m a\(^{-1}\), from 5.256 m a\(^{-1}\) when fitted with a linear curve (blue and cyan stars; Figure 7.A). Although both linear and quadratic fits accurately fitted the data to ~460 m depth, the differences between the two fits were not statistically significant (\(F = 1.045, p = 0.428\)).

The deviation of averaged internal layer displacements from both fits beyond ~460 m was observed to be non-linear, where the points oscillate in a sinusoidal pattern about the fitted data, dipping below the fitted data between 520–560 m. We are inclined to believe that the lower internal layer displacement between 520–560 m are an artefact of the post-processing algorithm, stemming from lower cross-correlation strength resulting in phase wrapping errors.

When observing the vertical displacement of the deepest internal layers immediately above the bed layer, we obtain a mean VSR to full depth of 8.385 m a\(^{-1}\) (green star;
Figure 7.A). However, reflectors near the ice sheet bed are frequently contaminated by spectral leakage from the much stronger basal return (Stewart, 2018). Therefore, we are inclined to ignore the values reported from the displacement of these layers.

B Appendix: Kinetics of glacial conduit heat exchange

The theory of subglacial meltwater flow grew out of an assumption of a thermodynamic equilibrium in which (i) temperatures of the fluid and the conduit walls are taken to be fixed at the pressure-dependent freezing point of freshwater and (ii) heat generated by viscous dissipation during water flow is either instantaneously used in conduit wall melting or to keep water temperature at the pressure-dependent freezing point (e.g. Röthlisberger, 1972; Shreve, 1972). This production-limited approach takes the view that the rate of melting of conduit walls is limited only by the rate of viscous heat dissipation, with implicit assumption of infinitely rapid heat transfer from the flowing water to conduit walls. Whereas this simplifying assumption of thermodynamic equilibrium was helpful in formulating analytical expressions for steady-state conduit geometry, there is a direct contradiction between assuming that the bulk temperature of subglacial meltwater and the surface temperature of conduit walls are equal while implying that viscous heat dissipation in the flowing fluid leads instantaneously to melting of conduit walls.

As expressed in the so-called Newton’s law of cooling, the rate of heat transfer between a fluid and a solid surface is proportional to the temperature difference between the two (e.g. Prandtl, 1952):

\[ q = h (T_w - T_i) \]  

(7.A)

where \( q \) is the heat flux per unit surface area [W m\(^{-2}\)] (here, positive heat flux is from the fluid to the solid), \( h \) is the heat transfer coefficient [W m\(^{-2}\) °C\(^{-1}\)], \( T_w \) is the bulk water temperature (‘free stream temperature’ in heat transfer literature) [°C], and \( T_i \) is the surface temperature of conduit ice walls [°C]. Hence, a flowing subglacial water
can either cause melting of conduit walls or have its bulk temperature be equal to the
temperature of the walls, but not both at the same time. In the classical theory of water
drainage beneath glaciers, melting is required to counteract creep closure of subglacial
conduits (e.g. Röthlisberger, 1972; Shreve, 1972).

The feasibility of such melting is supported by the fact that viscous heat dissipation
(VHD) represents a large source of heat under most of the ablation zone of Greenland Ice
Sheet (Mankoff and Tulaczyk, 2017). In fact, the magnitude of VHD calculated by these
authors is large enough so that subglacial water could emerge at ice sheet margins with
temperature of 2–3 °C in the absence of heat losses, such as these accompanying melting
of conduit walls. This estimate does not even account for the possibility that surface
meltwater entering the subglacial drainage system through moulins may be initially
warmer than the freezing point of freshwater. Our lowermost borehole temperature
sensor records variable temperature values ranging mostly between 0.2 and 0.8 °C over
the course of a few months (Figure 7.3b). These values are 0.6 to 1.2 °C above the
pressure-dependent freezing point of freshwater, estimated to be −0.4 °C at this location
based on 600 m of ice thickness and borehole temperature observations. We interpret
that the temperature record in Figure 7.3b reflects the temperature of subglacial water
that is flowing and being warmed up by VHD while losing heat to the ice base and
causing the high basal melt rates observed independently with ApRES. Isenko et al.
(2005) demonstrated that water flowing through a glacial conduit can sustain temperature
well above the freezing point of freshwater through dynamic balance between VHD and
conduit wall melting. Application of their equation for equilibrium water temperature
(Equation 17, Isenko et al., 2005) yields values similar to the ones observed by our sensor
for subglacial water conduits with hydraulic radii of about 1 m, given the relatively high
hydraulic slope of the bed in the study area of about 12°.

We have sufficient observational constraints to use Equation 7.A to estimate the range of
values for the heat transfer coefficient, $h$, consistent with the observed basal melt rates,
which vary between about 10 and 60 mm d$^{-1}$ over the observation period (Figure 7.4c).
Making the assumption that the higher basal melt rates are associated with the higher
water temperatures (0.06 m d$^{-1}$ and 1.2 °C), and vice versa (0.01 m d$^{-1}$ and 0.6 °C), the
estimated range of heat transfer coefficients is between ca. 175 and 60 W m$^2$ °C$^{-1}$. To
evaluate whether such heat transfer coefficients are reasonable, we turn to an empirical
relationship developed for heat transfer to river ice covers (Ashton, 1979):

\[ h = B \left( \frac{U^{0.8}}{D^{0.2}} \right) \]  

(7.B)

where \( B \) is an empirical constant, here taken to be the value for smooth conduit walls
given by Ashton (1979) as \( 1622 \text{ W m}^{-2.6} \text{s}^{0.8} \text{°C}^{-1} \), \( U \) is the mean flow velocity, and \( D \) is
the water depth. Equation 7.B suggests that the heat transfer coefficient is relatively
insensitive to the water depth and the main control comes from flow velocity. Equation 7.B
yields \( h \) in the range of 30 to 180 \( \text{W m}^{-2} \text{°C}^{-1} \) when water velocities are at the level of
centimetres per second for a large range of water depths between 0.01 and 10 m. Isenko
et al. (2005, modified from their Equation 1) use an expression for the heat transfer
coefficient that assumes simple linear dependence on flow velocity:

\[ h = cU \]  

(7.C)

where \( c \) is an empirical constant equal to \( 2640 \text{ J m}^{-3} \text{°C}^{-1} \). The values of the heat
transfer coefficient are equal to the ones implied by our observations for flow velocities
between 0.04 and 0.06 m s\(^{-1}\), which are in good agreement with the ones calculated from
Equation 7.B.
Chapter 8

Resolving the internal and basal geometry of ice masses using imaging phase-sensitive radar

Author contributions

In addition to the design and deployment of the antenna array (Chapter 5), several authors contributed to the research presented in this chapter, which has been accepted for publication in the Journal of Glaciology as Young et al. (accepted). Lai Bun Lok developed the EGCPL algorithm for processing the array datasets. TJ Young processed the ApRES data, developed the simulation, and wrote the chapter with Dustin Schroeder and Keith Nicholls advising on the text. In addition, two anonymous reviewers provided helpful comments to improve the quality of the overall manuscript, which have been incorporated into this chapter.

8.1 Summary

The autonomous phase-sensitive radio-echo sounder (ApRES) is a powerful new instrument that can measure the depth of internal layers and the glacier bed to millimetre
accuracy. We use a stationary 16-antenna ApRES array on Store Glacier in West Greenland to measure the 3-dimensional orientation of dipping internal reflectors, extending the capabilities of ApRES beyond conventional depth sounding. This novel technique portrays the effectiveness of ApRES in deriving the orientation of dipping internal layers that may complement profiles obtained through other geophysical surveying methods. Deriving ice vertical strain rates from changes in layer depth as measured by a sequence of ApRES observations assumes that the internal reflections come from vertically beneath the antenna. By revealing the orientation of internal reflectors and the potential deviation from nadir of their associated reflections, the use of an antenna array can correct for this assumption. While the array configuration was able to resolve the geometry of englacial layers, the same configuration could not be used to accurately image the glacier bed. Here, we use simulations of the performance of different array geometries to identify configurations that can be tailored to study different types of basal geometry for future deployments.

8.2 Introduction

Over the last 50 years, the Greenland and Antarctic ice sheets have been extensively surveyed using radio-echo sounding (radar), allowing researchers to determine ice thickness and internal stratigraphy of these large ice bodies (e.g. Bamber et al., 2013; Dahl-Jensen et al., 1997; Drewry and Meldrum, 1978; Evans and Smith, 1969; Harrison, 1973; Keisling et al., 2014; Lythe et al., 2001; MacGregor et al., 2015; Paden et al., 2010; Robin et al., 1977), and revealing insights into their past and present flow dynamics (e.g. Bingham et al., 2015; Cavitte et al., 2016; Christianson et al., 2016; King et al., 2009; MacGregor et al., 2015; Sime et al., 2014; Winter et al., 2015). While the majority of glaciological radar studies were conducted using airborne surveys or ground-based traverses, stationary phase-sensitive radio echo sounders (ApRES) have recently emerged as an important tool to measure 1-dimensional vertical strain on ice sheets (Kingslake et al., 2014, 2016; Nicholls et al., 2015) and basal melting on ice shelves (Corr et al., 2002; Dutrieux et al., 2014; Jenkins et al., 2006; Marsh et al., 2016) with both high accuracy and precision.
Nicholls et al. (2015) and Lok et al. (2015) briefly described the potential of ApRES to be deployed in an imaging mode using a multiple-input multiple-output (MIMO) array system, which has the capability to sequentially switch between up to eight transmitting (Tx) and receiving (Rx) antennas (yielding up to 64 virtual antenna pairs) with the aim to image the basal topography of ice sheets. Similar to phased arrays, a MIMO system involves the transmission and reception of its signals (and combinations thereof) from multiple transmitting and receiving antennas, arranged in such a way to create a gridded synthetic aperture from the midpoints of each virtual antenna pair. The combined signals from the antennas are then manipulated to electronically steer the array’s radiation pattern in desired directions. Although the use of phased and MIMO arrays for through-air imaging are developing traction due to their wide virtual aperture and high accuracy in detecting and tracking moving targets (e.g. Li et al., 2008), its application to subsurface imaging through ice is rare. While previous studies have implemented synthetic aperture experiments to image englacial and subglacial waterways and geometries by manually moving the antenna across gridded points on the glacier surface (Kennett, 1989; Walford and Harper, 1981; Walford and Kennett, 1989), only recently have radars been able to successfully and instantaneously image the full 3-dimensional subglacial topography using multiple antennas (Jezek et al., 2011; Paden et al., 2010; Wu et al., 2011).

Here, we use a MIMO ApRES array with 16 real antennas on Store Glacier in West Greenland to explore its imaging capabilities. We investigate the geometry of internal layers and the glacier bed obtained from 3 field sites (Figure 5.2a; Table 5.2). Additionally, by investigating the effects of antenna array parameters that control the radiation patterns, we demonstrate the importance of the array configuration on the resulting imagery. Finally, we highlight key array parameters to be considered in future ApRES system deployments.
8.3 Methods

8.3.1 Radar and antenna array architecture

Over three field campaigns (Table 5.2), three identical ApRES instruments were deployed in an 8 × 8 MIMO array arrangement within S30 (Figures 5.2a, 5.6; Chapter 5.3.2). The ApRES instrument operates over the frequency range 200–400 MHz and is centred at 300 MHz, which allows for sufficient ice penetration while its phase sensitivity allows it to achieve millimetre-depth precision. The design and technical details for the instrument’s radar board are described in detail in Brennan et al. (2014) and the practical aspects and limitations of its deployment in a quasi-monostatic (1 Tx / 1 Rx) configuration are presented in Nicholls et al. (2015).

The MIMO array used in the field experiments consisted of two rows of antennas on the ice surface oriented orthogonal to each other (Figure 5.7a) and mounted on a wooden frame to stabilise the arrangement against differential ablation on the ice sheet surface (Figure 5.6; Section 5.4.2). Each row was comprised of 8 cavity-backed bowtie antennas functioning in either transmit (Tx) or receive (Rx) mode, therefore synthesising a planar grid of 64 virtual antenna pairs (Lok et al., 2015), each recording 1 chirp for a burst total of 64 chirps (~1 minute duration). All bowtie antennas were constructed in-house for a centre frequency $f_c = 300$ MHz with a bandwidth of 200 MHz. The dimensions of the antenna used in this study were tuned to operate on the ice surface and each was housed in a square corrugated plastic box of dimensions $820 \times 820 \times 300$ mm (Figure 5.9; Section 5.4.3). Each row of antennas were arranged end-to-end, thus giving a virtual antenna pair separation of $\delta = 410$ mm (equivalent to $0.74 \times \lambda_c$, the central wavelength, assuming a dielectric constant of ice of $\varepsilon_r = 3.18$).

8.3.2 Digital signal processing

In addition to the processing steps described in Brennan et al. (2014), we processed each of the 64 virtual antenna pairs using exact geometrical correction based on path lengths
(Equation 8.1). This method is a direct and convenient way of processing the image in horizontal depth layers and sequentially building a 2- to 3-D depth image. Here, the beam is digitally steered by manipulating the received phase signals of each virtual antenna pair from a desired angle of incidence through combinations of constructive and destructive interference. Through this technique, the overall antenna array can scan rapidly through both the along- and cross-range of the incident ice column (Figure 8.1).

Denoting the range profile of the \((x_m, y_n)\) pixel by \(E(x_m, y_n, r)\), the \((X, Y)\) pixel amplitude at a specified depth beneath the ice surface \(R\) is:

\[
P(X, Y) = \sum_{m=1}^{M} \sum_{n=1}^{N} E(x_m, y_n, d(X, Y, x_m, y_n)) \exp \left[ -j \frac{4\pi d(X, Y, x_m, y_n)}{\lambda_c} \right]
\]

\[(8.1)\]

where \(\lambda_c\) is the wavelength corresponding to the ApRES chirp’s centre frequency and where \(d\) is the trigonometric distance from \((x_m, y_n)\) to \((X, Y)\):

\[
d(X, Y, x_m, y_n) = \sqrt{(X - x_m)^2 + (Y - y_n)^2 + R^2}
\]

\[(8.2)\]

so that when the angle of incidence is 0, \(d = R\).
Figure 8.2 Nomenclature of 2-D cross-sections. Within each vertical section, peaks in returned backscatter power are identified and can be traced with increasing depth. At specific depths, the location of these peaks were used to triangulate the peak within the corresponding horizontal section.

The exponent in Equation 8.1 encompasses the phase of the up-chirp:

\[ \phi = \frac{+4\pi d}{\lambda_c} \]  

(8.3)

and so the compensation is the negative of this term.

For the experiments presented herein, we use \( M = N = 100 \) and a depth range of 10–650 m with a depth step of 1 m.
8.3 Methods

8.3.3 2-D and 3-D vertical section processing

Using the resulting 3-D images, we extracted 2-D horizontal and vertical cross-sections for analysis (Figure 8.2). Horizontal sections are aligned parallel (i.e. along the X-Y plane) and vertical sections are aligned orthogonal to the ApRES synthetic aperture (i.e. along the X-Z and the Y-Z plane; Figure 8.2). To minimise the effect of instrumental and environmental noise, all horizontal and vertical sections were subjected to first a 2-D median filter consisting of a $4 \times 4$ matrix moved over the image, and then a 2-D peak convolution using a Gaussian low-pass filter with the same moving matrix dimensions. While the post-filtering location and shape of internal layers did not significantly change when varying the window size, filtering reduced the amount of erroneous internal layers (identified through an automated peak-detection function) by roughly 85%.

With the exception of the deployment at site 14a, which was aligned to true north resulting in a slight offset of $12^\circ$ relative to the principal flow direction ($262^\circ$), all arrays were oriented relative to flow (Table 5.2). Therefore, while the vertical images obtained beneath sites 14b and 15 depict the ice column oriented parallel and perpendicular to flow, the image obtained beneath site 14a respectively depict the ice column oriented north to south and east to west.

Within each vertical section, we identified the spherical location of peaks $(r, \theta, \varphi)$ with a returned backscatter power above $-50 \text{ dB } V_{\text{RMS}}$. Doing so revealed a series of peaks traceable through increasing depth (Figure 8.2). Although the radial distance of the peak to the antenna array $(r)$ and the angle from nadir $(\theta)$ can be accurately determined within each vertical section, the azimuth $(\varphi)$ is not yet constrained due to the pre-set orientation of the vertical sections $(\varphi = 0, 90, 180, \text{ or } 270^\circ)$; Figure 8.2). Therefore, using the traced peaks within each vertical section as a guide, we then located the identified peaks in 3-D through sequential horizontal sections, resolving the third spherical coordinate $\varphi$ (Figure 8.2).

Range migration begins to have a significant effect when the number of linearly-spaced elements is at least $1 + 4/B_f$, which is 7 for the parameters of the ApRES system given $B_f = 2/3$ (a fractional bandwidth of 200/300 MHz; Brennan et al., 2015). The
configuration of the $8 \times 8$ planar MIMO antenna array (Figure 5.7a) exceeds this limit very slightly, and therefore the effect of range migration is minimal and only marginally affects the outer limits of the scan angle range. Therefore for simplicity, we do not apply range migration correction.

### 8.4 Identification and interpretation of internal layers

#### 8.4.1 Vertical stratigraphy of the ice column

All ApRES vertical sections acquired on Store Glacier show similar internal stratigraphy through depth, with the sections divided into 4 distinct bands (Figure 8.3). Through the majority of the vertical sections, a series of peaks, which we interpret to be dominant internal layers, can be traced from the ice surface (black dots, Figure 8.3). The traced peaks all curve away from nadir with depth, and extend to considerable (~300 m) depths, particularly those aligned perpendicular to flow (i.e. along the Y-Z plane). Besides the identified peaks, there exists some clustering in reflections away from the traced peak trajectory, particularly between 300–500 m depth.

The upper 30–80 m of the vertical section is dominated by closely-spaced high ($-40$–$-20 \text{dB V}_{\text{RMS}}$) signal strength reflections. We do not attribute the source of the observed near-surface clutter without further field investigations; however, these near-surface inhomogeneities could be due to any or a combination of i. Fresnel diffraction within the near-field region (Yamaguchi et al., 1992); ii. englacial heterogeneities (e.g. water-filled pockets, cracks, veins, conduits) within the upper glacier surface (Kanagaratnam et al., 2004; Smith and Evans, 1972; Watts and England, 1976); and iii. presence of surface crevasses (Colgan et al., 2016; Rignot et al., 2013), of which the latter two are abundantly present at S30 (Hofstede et al., 2018).

Beyond depths of 500 m, the returned signal strength drops markedly, indicating a ~100 m thick transparent region of ice above the ice-bed interface. This region in our
8.4 Identification and interpretation of internal layers

Figure 8.3 2-D vertical sections within S30 acquired along (left column) and across (right column) the ice flow direction (262°). Black dots show identified peaks in returned backscatter power (>−50 dB V_{RMS}) traceable through depth, and vertical bands indicating types of layering are shown in (a). Sections were acquired at (a) site 14a on 06 May 2014 (offset by +12°; Table 5.2); (b) site 14b on 03 August 2014; and (c) site 15 on 05 July 2014. Figure 5.2 and Table 5.2 shows the location and orientation of each deployment within S30.
study is located close to an observed change in crystal fabric orientation transitioning to anisotropic Wisconsin-aged ice detected from seismic experiments (Hofstede et al., 2018) and the onset of high deformation inferred from borehole-installed tilt sensors both conducted within S30 (Doyle et al., 2018). While the enhanced shearing observed at S30 may further deteriorate the presence of layers within this region, we are still able to detect some deep reflectors (e.g. at 580 m at site 14b), implying that the reduction in signal strength is caused by an attenuation in the power of the received signal. Further investigations are needed to provide definitive conclusions as to whether the lack of internal layers depth range represents these changes in ice properties.

The ice-bed interface, located between 600 and 640 m depth, is consistently identified in all sections as an abrupt increase in signal strength (+60 dB V_{RMS}). The signal strength is sustained beyond this range to at least 650 m, which is the lower boundary applied to our images. For reasons discussed below, we discount the curved bed geometry seen in all sections as an artefact of processing, and instead focus on the traceable peaks and their representation as internal layers.

8.4.2 Simulation of the vertical section

Although at first glance the traced peaks identified in all vertical sections within S30 (Figure 8.3) may visually resemble surface crevasses or large moulins (e.g. Arcone and Yankielun, 2000), the minimum depths to which the traced peaks all at least propagate to (>100 m) are all an order of magnitude beyond field observations of crevasses, and likely would not penetrate deeper than ~30 m without the presence of meltwater due creep closure from the stress supplied by the surrounding ice (Colgan et al., 2016). Therefore, to determine the cause of these traced peaks, we conducted a series of simulations that reconstructs a synthetic 2-D glacier vertical section (Figure 8.5) varying the parameters of the array factor of the antenna array deployed in the field (Figure 8.4; see Appendix 8A). The array factor (F_a) characterises how the power of a radar transmitted signal is spatially distributed (i.e. the height and width of the main lobe and sidelobes, and the presence of grating lobes) across the swath of scanning angles (θ), and combined with the radiation
Figure 8.4 Radiation patterns of the overall array \( F \); solid blue line), the element factor \( F_e \); dashed black line), and the array factor \( F_a \); dotted blue line) using the setup as described in Figure 5.7a. The half-power beamwidth (HPBW) of both \( F \) and \( F_a \), indicated by arrows, is ±6°.

Pattern of a single virtual antenna pair (element factor; \( F_e \)) produces the transmitted and received signal of the overall antenna array \( F \):

\[
F(\theta) = F_e(\theta) F_a(\theta) = F_e(\theta) \sum_{k=1}^{K} \exp \left[ -j (K - k) k_0 \delta \sin(\theta) \right]
\]

for a linear system of \( K \) virtual antenna pairs uniformly spaced \( \delta \) m apart (Huang et al., 2011), where \( k_0 = 2\pi/\lambda_c \) is the free space wave number (Section 4.4). The antenna array’s performance can be optimised by manipulating Equation 8.4’s key parameters, namely:

(i) the element factor \( F_e \);

(ii) the number of linear virtual antenna pairs \( K \); and

(iii) the separation between virtual antenna pairs \( \delta \).

Accordingly, the series of simulations (Appendix 8B) varied these three parameters.

Specifically, the simulations track the transmission and reflection of a signal beam (Equation 8.4) across a range of angles through depth, and reconstructs a vertical glacier along-flow section by vertically stacking the received signals through depth, as described
Figure 8.5 (a) Synthetic glacier vertical section as input to model simulation. The location of the array is at (0, 0), indicated by a black square. (b) Corresponding reconstructed glacier vertical section parameterised using the setup as described in Figure 5.7a.

in Appendix 8A. The signal beam was generated from an 8-antenna linear array using the array factor derived from the field configuration (Figure 5.7a). The synthetic glacier vertical section (synthetic section) used as input to all simulations (Figure 8.5a) included several artificial features typical of radargrams of ice sheets, notably 4 internal layers at various slope angles and a flat, subglacial layer with an undulating bed and a small topographic protuberance 20 m high, 50 m wide, with its apex located 100 m left of nadir (equivalent to $-9.46^\circ$; Appendix 8A). By assuming internal layers to be specular reflectors (Appendix 8A; Cavitte et al., 2016; Drews et al., 2009; Holschuh et al., 2014), the transmitted signal is only reflected from the internal layer when the incident ray beam path is at or close to normal relative to the layer plane (Figure 8.6). The incorporation of this assumption into the simulation importantly reconstructs the synthesised internal layers (Figure 8.5a) into a peak of concentrated returned backscatter visually similar to field-acquired sections at S30 (Figure 8.3). Geometrically, the centre of the simulated peak, where returned backscatter power is highest, is normal to the slope of the synthesised internal layer, and the depth of the peak corresponds to the respective layer’s mean depth (Figure 8.5b). It then follows that the slope of the synthesised internal layers correspond to the peak’s angle from nadir (Figure 8.6).
8.4 Identification and interpretation of internal layers

8.4.3 Interpretation of internal layer slopes

Based on the results from the simulations, we plot the peaks identified as internal layers in Figure 8.3 in 3-D to better illustrate the dipping layer (Figure 8.7). Most layers within all three radargrams slope from South to North, and the azimuth and direction of these layers agree closely with the basal topography (Figure 5.3). This agreement is predominantly the case in the cross-flow direction, where the bed elevation rises to the south (Figure 5.3a), but also to a lesser extent along-flow where the glacier is flowing downhill (Figure 5.3b). The magnitude of the slopes observed were small overall (~2–6°) and generally constant in tilt through depth, with the exception of more steeply-dipping layers at ~100 m depth with slopes as high as 15° beneath site 14a (Figure 8.7). Although it is unlikely that moulins, water pockets, or other point scattering sources would have caused these dipping layers, as their scattering functions (spread in the 2-D image) will be wider than a specular layer, larger features such as cross-cutting crevasses or refrozen englacial water can potentially reflect anomalously high slopes if they are wide enough (a significant fraction of a Fresnel zone).
The stratigraphy of internal layers is considered representative of past and present variations in ice flow. Regions of stable and/or slow flow often develop reflector slopes that monotonically change with depth, while spatial variations in ice flow disrupt regular internal layering (Holschuh et al., 2017; Ng and Conway, 2004). These variations mostly result from changes to the local strain field as the ice transitions between slow and fast flow regimes; alternatively, the strain field can also be disrupted by changes to the roughness of the underlying subglacial topography (Bingham et al., 2015; Hindmarsh et al., 2006; Hubbard et al., 2000; Siegert et al., 2003). Indeed, prior seismic analysis of S30 conducted by Hofstede et al. (2018) revealed variations in the internal layering and the nature of the ice-bed interface within the study area, including a steep symmetric syncline located approximately 400 m north of array 14a (Figure 5.3a) that matches the primary orientation of the identified sloping internal layers (Figure 8.7). The syncline is thought to have formed in response to a combination of converging flow and basal melting. This observation, together with small-scale variations in subglacial properties, suggest that the presence of patches of different basal slipperiness is associated with variable amounts of water at the ice-bed interface. While quantitative analysis of internal layering typically requires numerical modelling of ice flow, given our knowledge of S30’s subglacial environment, we believe that the spatial variability of observed englacial layer slopes through depth (Figure 8.7) could be attributed to the patchy nature of the underlying
bed that results in local stress variations dynamically deforming the overlying ice (Ryser et al., 2014a). Such variations are able to induce anomalously steep layer slopes in the middle of the ice column, allowing slopes to deviate from expected monotonic trends (Holschuh et al., 2017). Further analysis of the subglacial conditions at the local study site, together with additional imaging ApRES deployments, are needed to conclusively explain the contrasts in slope gradients, especially within the upper 100 m between the three sites at S30.

Studies of englacial layers, and their significance within the context of past and present ice flow dynamics, have so far only been based on 2-D profiling, with the 3rd dimension, if present, projected through interpolation (e.g. Bingham et al., 2015; Winter et al., 2015). Such studies usually require ground- or air-based radar traverses that typically demand extensive logistical support. The advent of an instrument to measure the 3-D variability of internal layers with depth represents an additional method to investigate the ice flow structure over a large spatial area. In particular, the use of multiple imaging ApRES arrays may complement traditional surveys in identifying potential ice-coring and/or subglacial access hot-water drilling sites, both of which require prior knowledge of the internal layer geometry to understand the local ice flow regime.

Past studies using ApRES have tracked the vertical movement of internal layers through time, and have used this information to extract basal melt rates beneath Antarctic ice shelves (Corr et al., 2002; Dutrieux et al., 2014; Jenkins et al., 2006; Marsh et al., 2016; Nicholls et al., 2015). As the instrument detects internal layers only when the antenna beam is near perpendicular relative to the layer plane (Figure 8.6a), tracking the englacial vertical velocity of dipped layers through time in a single depth dimension, similar to previous studies using a quasi-monostatic configuration (e.g. Kingslake et al., 2014, 2016), may produce erroneous vertical strain values. Specifically, if the ice column contains steep or highly variable layering, the measured vertical strain may be inflated due to the ApRES partially capturing the horizontal movement of internal layers in addition to their vertical advection. Application of the ApRES in MIMO configuration therefore enables the correction of off-nadir effects in the apparent vertical strain signal through its ability to partition the horizontal and vertical components of internal layer advection through time.
8.5 Resolving the basal layer

So far, we have considered only the ability of ApRES to investigate the geometry of internal layers without discussion of the curved bed layer observed in the field-acquired vertical sections (Figure 8.3). To determine the cause of this artefact, we return to the output image of the simulation (Figure 8.5b), which visually reproduced the same characteristic curve despite using an overall flat bed as input (Figure 8.5a). At a smaller scale, the simulation was also unable to recreate the sinusoidal rough bed: the subglacial topographic protuberance in the synthetic profile was reconstructed as a shallow (~20 m), but broad (~150 m) protuberance superimposed above the curved bed roughly at the correct location (~9.46° from nadir), and repeated across the curved bed (Figure 8.5b). Considering the array’s radiation pattern (Figure 8.4, Equation 8.4), the ability of the antenna array to resolve spatial wavelengths at the ice-bed interface is directly related to the angular acuity produced by the overall array. The high sidelobe power of the overall field-based array radiation pattern, spaced at 12.5° angular intervals, is primarily responsible for the horizontal repetition of the protuberance, while its half-power beamwidth (HPBW) at ±6° is too wide to significantly resolve the ~80 m (4°) spatial wavelength undulations at the bed. Because the single protuberance at the bed is not a repeating signal, it has a range of spatial wavelengths, of which only the longer of these wavelengths is resolved by the array signal. In other words, the HPBW of the overall array factor determines the angular resolution of the output image, while its gain amplifies features within the ice column over unwanted clutter.

Given the importance of the array’s radiation pattern on imaging the ice column and the basal topography, we modified the simulation’s array factor to best reconstruct the input basal conditions by increasing the virtual antenna separation (δ) from 0.74 to 1 × λc and increasing the number of virtual antenna pairs (K) to 32 elements (Figure 8.8). As the directivity of the beam increases with both δ (Figure 8.B) and K (Figure 8.C), the reconstructed section was able to resolve the undulations in basal topography with a significantly higher angular resolution, and better characterise the size and shape of the subglacial protuberance.
8.6 Suggestions for future deployments

As with all antenna systems, there exists a tradeoff between gain and beamwidth (Visser, 2005); however, correct modifications to the element separation distance ($\delta$) and the number of linear elements ($K$) will likely improve the imaging quality of both internal layer slopes and the basal topography in future deployments. From our study, we found that $\delta$ is itself secondary compared to $K$. Given the angular swath used in the simulations within this study ($\pm 30^\circ$), we suggest $\delta$ to be roughly equivalent to the central wavelength of the transmitted signal. Doing so will increase the likelihood of constraining internal layer slopes and resolving irregularities in the basal topography. Having the elements too widely spaced risks aliasing the signal returns via the presence of grating lobes (Figure 8.Bd), while too narrow an element separation reduces the directivity and gain of the signal (Figure 8.Ba). Similarly, a higher $K$ value increases the angular acuity and therefore the resulting image’s angular resolution (Figure 8.C), effectively resolving smaller wavelengths manifested within the bed.

In our simulations (see Appendix 8A), we simplify the scenario by using a linear array with a corresponding bed varying in one direction to reproduce the along-flow section of the ice. As a planar array, similar to our field configuration (Figure 5.7a), is simply a

Reconstructed glacier vertical section, using a linear 32-element broadside array setup with the separation of virtual antenna pairs set at $\delta = 1 \times \lambda_c$, and a theoretical directive beam (HPBW = $\pm 30^\circ$).
series of stacked linear arrays, the number of virtual antenna pairs will either need to be multiplied or the array be towed to create a planar synthetic aperture (e.g. Paden et al., 2010; Walford and Kennett, 1989) in order to image the bed in two directions to accurately resolve a 3-D representation of the bed roughness. The current version of the ApRES unit, which has the capability of switching between up to eight Tx and eight Rx antenna pairs, limits the maximum number of pairs to 64 elements in a planar arrangement, or 32 pairs in a linear arrangement. Therefore, given the current instrument capabilities, there exists a tradeoff between 3-D imaging of internal layer slopes and resolving the basal topography in 2-D. Characterising internal layers in 3-D with a planar setup (e.g. 8 × 8 virtual antenna pairs) can be traded for a maximum-element linear array (32 × 1 virtual antenna pairs) with higher 2-D resolution to better resolve the bed.

As instrument costs often play a significant factor in the choice of materials used, the ideal antenna would have to be low cost, easily constructed, transportable, and robust to withstand the harsh conditions of the polar regions. Additional simulations varying the element factor (Figure 8.A) show that the choice of antenna merely limits the width of the usable swath, which is approximately equivalent to its HPBW and exerts no influence over the array factor (Equation 8.4). The current size of the physical bowtie antennas used in this study (820 × 820 mm; Figure 5.7a), in addition to their simplicity and low associated costs, have a HPBW of ±80° and were optimised to operate at the air-ice interface. If the antenna size was reduced, a higher frequency would be required to maintain the current antenna properties, which ultimately reduces the range of the system from the internal losses from englacial signal scattering.

8.7 Summary and conclusions

By tracking the relative range change of internal and basal reflectors, studies involving ApRES have demonstrated its potential for measuring vertical strain within ice sheets (Kingslake et al., 2014, 2016) and basal melt rates beneath ice shelves (Corr et al., 2002; Dutrieux et al., 2014; Jenkins et al., 2006; Marsh et al., 2016; Nicholls et al., 2015). Here, we describe the use of the instrument with a 16-antenna MIMO array to image internal
layers in a fast-flowing outlet glacier that drains a 35,000 km$^2$ catchment on the Greenland Ice Sheet. The capability of the instrument to measure and partition the movement and slope of internal layers in three dimensions sheds new light on the vertical stratigraphy in a region of complex ice flow, with the potential to greatly improve estimates of vertical velocities and strain rates within such regions. We use a forward model to optimise and improve the angular resolution of the bed echo layer, demonstrating the importance of the array factor to reflect on the specific purposes of the deployment. In addition to its lower power consumption and light weight, ApRES offers novel possibilities when applied in a MIMO configuration.

A Appendix: Extended methods

Synthesised vertical ice section

The model input for our simulation is a synthetic vertical ice section representing an along-flow section of an ice sheet of 800 × 700 m. Within the vertical section, we introduced several artificial features:

(i) 4 internal layers situated at 100, 200, 300, and 400 m depth along the cross-range centre ($x = y = 0$ m) with slopes of 0, 5, 10, and 15 degrees respectively, with a uniform reflectivity of $-30 \, \text{dB} \, V_{\text{RMS}}$ at nadir;

(ii) a sinusoidal bed situated at a mean depth of 600 m, with $\pi$ amplitude and spanning 10 wavelengths across the domain, with a uniform reflectivity of $-20 \, \text{dB} \, V_{\text{RMS}}$ at nadir;

(iii) A small subglacial topographic feature of 20 m in height and 50 m in width located 100 m off-nadir (equivalent to 9.46$^\circ$); and

(iv) presence of a thick layer of subglacial material situated immediately below the bed layer to the bottom of the section, with an isotropic reflectivity of $-30 \, \text{dB} \, V_{\text{RMS}}$ below the bed layer.
We assume the ice within this section to be homogeneous in composition, resulting in the dielectric properties of ice to be constant through depth and space. Though this is rarely the case, especially in areas of accumulation and/or fast flow, there exists only several profiles of ice composition from ice core records (e.g. Dahl-Jensen et al., 2003) and this assumption is often necessary, especially when the purpose of the experiment is exploratory, as in this simulation.

**Backscattering coefficient**

In order to make effective decisions regarding radar design, we need an expectation of the target’s scattering characteristics. Although radar signal propagation through ice incurs increasing attenuation through depth as a result of radiation scattering and absorption (e.g. Plewes and Hubbard, 2001), this simulation assumes no power loss unless the signal interacts with designated layers within the vertical section for simplicity. Here, we characterise the backscattering coefficient \( \sigma^0 \) using a Gaussian distribution with respect to the angle of incidence \( \alpha \), with an incoming beam angle normal to the layer reflecting all power back to the source, and a beam angle perpendicular to the layer reflecting no power back to the source:

\[
\sigma^0 = \beta^0 \gamma f(\alpha)
\]  

(8.A) where \( \beta^0 \) is the radar brightness of the layer, \( f(\alpha) \) the distribution as a function of the angle of incidence \( \alpha \), and \( \gamma \) an adjustment factor dictating the spread (variance) of the distribution.

While internal layers are known to be specular reflectors (Cavitte et al., 2016; Drews et al., 2009; Holschuh et al., 2014, Figure 8.6a), the ice-bed interface is in most places a diffuse reflector (Drews et al., 2009, Figure 8.6b) and is often highly variable in specularity (Schroeder et al., 2015; Young et al., 2016). Therefore, for the simulations within this paper, the value of \( \gamma \) was set at 1 with respect to receiving the bed echo in the absence of definitive knowledge of the bed, while \( \gamma \) was reduced to 0.025 for internal layers.
due to their high specularity (Cavitte et al., 2016; Drews et al., 2009; Holschuh et al., 2014).

### Antenna beam generation and synthesis

For the purposes of the simulation, the antenna array was treated as a point located at the centre of the domain resting on the ice surface \((0,0)\) from where the beams are transmitted and received (Section 4.2.1). A virtual beam is emitted from the array point into the ice vertical section as a function of the scanning angle, producing an array factor \(F_a\) with a field of vision constrained by the antenna’s radiation pattern (Equation 8.4). The beam then travels through the section at the designated steering angle, of which at each depth step, a proportion of the total energy is reflected back and received by the array through backscatter as a function of the two-way distance from the array.

The received power \(P_r\) for each pixel at a specific range \(R\) and angle \(\theta\) from the array is then a combination of the transmitted and received signals \(F^2\) due to reciprocity and the resulting backscattering coefficient \(\sigma^0\):

\[
P_r(\theta, d) = F(\theta, R)^2 \sigma^0(\theta, R)
\]  

(8.B)

Pixels were then synthesised through distance and angle to reproduce the input ice profile.

### B Appendix: Simulations of the antenna radiation pattern

In addition to the simulation that reconstructs a synthetic glacier vertical section using the array factor of the field deployments (Figure 8.5), we produced 3 sets of simulations by varying 3 key parameters that control the overall antenna radiation pattern (Equation 8.4): the half-power beamwidth (HPBW, Figure 8.A), the separation between virtual antenna pairs \(\delta\) (Figure 8.B), and the number of linear virtual antenna pairs \(K\) (Figure 8.C).
Figure 8.A Array factor and respective reconstructed ice vertical sections using a linear 32-element broadside array setup with an antenna separation of δ = λ_c, using a directive beam with HPBW = (a) ±10°, (b) ±20°, (c) ±30°, and (d) ±45°. The section marked with a star (⋆) is run with the same configurations in all 3 sets of simulations, and is the same as shown in Figure 8.8.
Figure 8.B Array factor and respective reconstructed ice vertical sections using a linear 32-element broadside array setup with an antenna separation of $\delta = (a) \frac{1}{4} \times \lambda_c$, (b) $\frac{1}{2} \times \lambda_c$, (c) $1 \times \lambda_c$, (d) $2 \times \lambda_c$, using a directive beam with HPBW = $\pm 30^\circ$. The section marked with a star ($\star$) is run with the same configurations in all 3 sets of simulations, and is the same as shown in Figure 8.8.
Figure 8.C Array factor and reconstructed ice vertical sections using a linear broadside array setup of (a) 32, (b) 24, (c) 16, and (d) 8 elements, with an antenna separation of $\delta = \lambda_c$ and using a directive beam with HPBW $= \pm 30^\circ$. The section marked with a star (⋆) is run with the same configurations in all 3 sets of simulations, and is the same as shown in Figure 8.8.
Overall, we observe that the imaging resolution of the reconstructed vertical sections and system design are conflicting parameters: although increasing $K$ may result in sharper directivity and angular resolution across the beam swath (Figure 8.Ca), doing so will result in increasing costs and power consumption, both of which are limited by the current radar hardware available. Similarly, increasing $\delta$ results in increased directivity and gain of the beam resolving features with greater clarity (Figure 8.B); however, unwanted grating lobes are introduced after an optimal distance (Section 4.4.2). Lastly, all simulations produced within this paper only examine the along-flow section through using a linear array arrangement, and neglect an additional dimension that would be produced by the addition of a cross-steering angle.

For purposes of simplicity, we did not apply range migration processing to the output ice vertical sections. However, as the maximum permissible number of antennas for the ApRES system to avoid significant range migration is seven (Brennan et al., 2015), applying range migration correction to the output simulations, particularly those with higher $K$, would markedly improve the definition of layers within the output section. We therefore recommend applying range migration correction to sections obtained from arrays with more than seven linear antennas.
Chapter 9

Conclusions

9.1 Synthesis of research findings

As originally stated in Section 1.2, the fundamental aim of this thesis was “to quantify englacial ice deformation rates and basal melting for a fast-flowing outlet glacier of the Greenland Ice Sheet”, this aim then being partitioned into eight component objectives. To achieve this aim and these objectives, we investigated the structural and temporal evolution of the ice column impacted by fast glacier motion using autonomous phase-sensitive radio-echo sounders deployed in the ablation zone (30 km from the terminus) of the Greenland Ice Sheet (Objective 1; Chapter 5). From these deployments, we obtained high resolution measurements of internal deformation and basal melting that, to the best of our knowledge, are the first empirical measurements that quantify these processes on the Greenland Ice Sheet (Objective 8).

An observation that proved crucial in contextualising our findings was that our measurements of vertical strain at S30, located 30 km inland, were largely insensitive to local variations in basal conditions, but instead were strongly driven by far-field longitudinal effects that were coupled with the glacier’s near-terminus stress regime tens of kilometres downstream (Chapter 6). The seasonal evolution of the hydrological regime across the glacier catchment, induced by summer surface meltwater routed to the basal environment, triggered dynamic changes occurring at the bed near the glacier terminus, propagating
perturbations in the force balance to S30. At S30, we observed complicated vertical velocity and strain profiles, including those that changed polarity within a single ice column, suggesting uneven stress transfer across depth (Objective 2). Although we did not initially expect such dynamic strain behaviour, which involved the migration of the polarity point up and down the vertical ice column, we were able to explain the observed seasonal trends through investigating various processes occurring at the ice-bed interface that impact the local and catchment-wide force balance. From these investigations: (i) spatially-variable conditions in basal traction at the mesoscale (on the order of several ice thicknesses) induce horizontal stress transfer between heterogeneous patches (Objective 5); and (ii) enhanced shear within softer, pre-Holocene ice allows for dipping internal velocity slopes and velocity profiles that change polarity within a single ice column (Objective 6).

Similarly, we found that the potential energy within routed surface runoff dissipated as viscous heat at the bed of the ice sheet plays a crucial role in explaining high rates of basal melting measured up to 57 mm d$^{-1}$ at S30 (Objective 3; Chapter 7). Viscous heat dissipation (VHD) was found to dominate other basal heating terms considered in other models of heat transfer, a finding consistent with theoretical results found in Mankoff and Tulaczyk (2017), which found VHD to deliver the greatest impact in areas near the ice sheet margin (Objective 4). Given the current trends in surface mass balance and velocities across the Greenland Ice Sheet, these observations as a whole portend a future where the impact of VHD will move farther inland, inducing higher volumes of basal ice melt that may markedly influence the interaction between subglacial water pressure, ice velocities, and the overall mass balance of the glacier and catchment (Objective 5).

Operation of ApRES units with multiple transmitting and receiving antennas provide the ability to image the internal and basal stratigraphy of ice sheets. By deploying these radars in a planar (8×8) array at S30, we demonstrated their capability to instantaneously identify, image, and quantify the slope of internal layers in two and three dimensions, shedding new light on the vertical stratigraphy in a region of fast and complex ice flow (Objective 6; Chapter 8). By partitioning the movement and slope of these layers through time, we also show the potential to greatly improve estimates of vertical velocities and strain rates within such regions by accounting for the full three-dimensional deformation.
of the ice column. In addition, the overall array’s beam pattern (the array factor) can be strategically manipulated by utilising various configurations of transmitting and receiving antennae to tailor to specific purposes of their deployment (Objective 7).

As demonstrated within this project, the ApRES offers novel possibilities whether applied in depth-ranging or imaging configurations. Accordingly, the overarching challenge in future investigations involving ApRES is to effectively and efficiently optimise the system to tackle the remaining questions in modern glaciology.

9.2 Perspectives on future research

We have successfully addressed the eight component objectives of the project (Section 1.2). However, several assumptions needed to be made due to the current limitations in the applied methods and experimental designs mentioned in Chapters 5, 6, 7, and 8. Similarly, several tangential avenues of research were briefly examined, but not expanded on. Here, we provide a personal perspective into these limitations and areas worth future investigation.

9.2.1 Identification of and deformation in pre-Holocene ice

Studies employing traditional kinematic radio-echo sounding traverses often reveal a zone in the lowest hundreds of metres above the ice bed largely free of radio echoes. Known as the echo-free zone (EFZ; Drewry and Meldrum, 1978), its direct cause is so far unclear, but from previous studies, could be due to one or a combination of several factors, including changes in the crystal orientation fabric (COF), dielectric properties, enhanced liquid fraction, or temperature effects on signal attenuation (Fujita et al., 2000; Hubbard and Sharp, 1995; Matsuoka et al., 2003; Siegert and Kwok, 2000). Therefore, the inability to quantify, image, and accurately characterise basal ice extends to the entirety of all RES instruments. This limitation was particularly evident in Chapters 6 and 7, where we were neither able to consistently identify nor track the movement of the deepest internal reflectors (~83%–100% of the ice column) through successive bursts,
and had to utilise extrapolation techniques in line with results from recent observational and modelling studies to yield full-column vertical strain and basal melt rates.

To this date, only ice coring and access-borehole experiments provide direct access and observation of basal ice, with the latter also able to observe deformation at specific depths with the installation of embedded inclinometers and magnetometers (Keller and Blatter, 2012). Using these methods, enhanced, and often non-linear deformation of old (pre-Holocene) ice have been reported at multiple sites in Greenland (Dahl-Jensen and Gundestrup, 1987; Doyle et al., 2018; Lüthi et al., 2002; Shoji and Langway, 1988; Thorsteinsson et al., 1999). Similar to theories regarding the EFZ, this phenomenon is thought to arise from a combination of the high dust content of pre-industrial ice and the rapid recrystallisation that concentrates impurities into boundaries (e.g. Fisher, 1987). However, regardless of whether the oldest and deepest ice does indeed exhibit nonlinear deformation and irregular rheology, more measurements at the basal zone of the Greenland Ice Sheet are required to provide enough robust data to accurately model deformation in this zone, especially within regions of fast flow.

### 9.2.2 The basal hydrological thermal regime underlying areas of fast glacier flow

Studies involving hot-water drilling have largely been conducted within easily-accessible areas at topographic lows, which are often subject to a longitudinally-compressional flow regime (Lüthi, 2013); in contrast, the Subglacial Access and Fast Ice Experiment (SAFIRE; Section 5.2) is one of only two known borehole studies in Greenland conducted in an area of fast flow and therefore majority-extensive flow, with the other being Lüthi et al. (2002). Observations of englacial temperatures and basal water pressures from SAFIRE (Doyle et al., 2018) contrasted with all borehole experiments conducted on slower-flowing (i.e. 100–150 m a\(^{-1}\)), land-terminating glaciers (e.g. Andrews et al., 2014; Harrington et al., 2015; Lüthi et al., 2015; Meierbachtol et al., 2013; Van De Wal et al., 2015; Wright et al., 2016), instead revealing an inefficient subglacial hydrological system with high basal water pressures modulated by surface meltwater input. These findings
were instrumental in contextualising the seasonal evolution of internal layer vertical velocities and high rates of basal melt discovered within this thesis.

Obviously, a large number of boreholes spread over the entire Greenland Ice Sheet is required to better contextualise these findings and their variation on a continental scale. But, while coordinated efforts to drill and instrument boreholes in conditions similar to SAFIRE are urgently needed to gain further insights into processes driving fast glacier motion, simply repeating experiments is unlikely to reveal these processes, which have been hypothesised to be spatially heterogeneous and temporally transient at multiple scales due to the uneven roughness of the bed and its seasonal modulation from surface meltwater influxes.

In SAFIRE, sensors installed within and beneath basal ice were operational for at most two months due to stretched and snapped instrument cables from enhanced shear strain within this zone of enhanced deformation (Doyle et al., 2018). To obtain observations within this zone beyond the melt season, it is imperative that experiments move away from wired sensors, and explore wireless options. With the increasing miniaturisation of electronic components (i.e. Moore’s Law), such options are starting to be realised (e.g. Bagshaw et al., 2014; Chong and Kumar, 2003; Hart and Martinez, 2006).

### 9.2.3 Influence of surface crevasses on horizontal strain rates

In Chapter 6, we discovered seasonal variations in the vertical velocities of internal reflectors and the overall vertical strain regime beneath S30, and attribute the majority of their variation to far-field glacier dynamics. Given that our measurements encapsulate the vertical deformation of the ice column, it would be logical to complement these measurements with records of horizontal deformation. Therefore, we calculated horizontal strain rates by installation of three kinematic GPS units (Trimble 5700), operational between 07 May 2014 and 13 July 2014 (Appendix A).

Horizontal strain rates obtained from the centroid of three kinematic GPS deployments during Spring of 2014 reported values roughly an order greater than concomitant near-surface (<30% of the ice column) vertical strain rates obtained from ApRES deployment
172

Conclusions

(a) Comparison of near-surface (<30% of the ice column) vertical strain rates (VSR) obtained from ApRES at site 14a and inferred from GPS measurements at centroid WNE. (b) Surface velocities from GPS deployment SAGC, with surface melt rates from AWS measurements. A comparison of the two time series are also shown as (c) residual vertical strain rates and (d) correlation through time.

Figure 9.1 (a) Comparison of near-surface (<30% of the ice column) vertical strain rates (VSR) obtained from ApRES at site 14a and inferred from GPS measurements at centroid WNE. (b) Surface velocities from GPS deployment SAGC, with surface melt rates from AWS measurements. A comparison of the two time series are also shown as (c) residual vertical strain rates and (d) correlation through time.

14a, located ~150 m south of the GPS centroid (Figure A.1). Temporally, GPS-inferred vertical strain rates revealed constant vertical compression ~0.05 a\(^{-1}\) greater than ApRES-inferred vertical strain rates until 15 June, a week after the the onset of melt-induced speed-up on 09 June. Thereafter, vertical strain rates obtained by ApRES captured increasing vertical extension through the upper ice column while GPS-inferred vertical strain rates continued to experience higher rates of vertical compression (Figure 9.1a). During this period, the residual between the two vertical strain rate measurements increased linearly until the end of the time series (Figure 9.1c), with variations to this trend up to 0.16 a\(^{-1}\) coinciding with transient peaks in surface velocities and surface melt generation (e.g. 15–16 June, 24–26 June, 8–9 July; Figure 9.1b). Given our knowledge of hydrological coupling (Section 2.2), it is likely that the generated surface melt at S30 connected to the basal hydrological system on 15 June, and periodic melt fluxes into the system modulated faster flow downstream, influencing both the GPS- and ApRES-inferred vertical strain rates.
These comparisons clearly show a discrepancy between ApRES- and GPS-inferred vertical strain rates, suggesting that the continuity condition (Equation A.7) is not a valid assumption at S30, possibly also extending to glaciated areas experiencing large amounts of surface melt. While the former directly measures the displacement of internal reflectors through the two-way travel time of the emitted beam (Section 4.2.1), the latter instead indirectly infers vertical strain through measuring horizontal divergence and assuming mass conservation through the ice column (Appendix A). Consequently, horizontal strain values may be inflated, and even misleading, when directly converted to vertical strain measurements. This result is not surprising, given that crevasses are one of many secondary structures on glaciers that result from high levels of strain (Colgan et al., 2016). However, to determine whether spatial and temporal variations in glacier vertical velocities are controlled by near-field basal drag or far-field longitudinal stress coupling, it is essential to refer to measurements of GPS velocities and strain across the horizontal scale, from which spatial variations in subglacial roughness can be identified (Mair et al., 2001). Therefore, within the study, we restrict ourselves to only reporting qualitative observations that are consistent across several spatial or temporal steps.

Separately, although not within the scope of this thesis, this deviation revealed the importance of surface crevasses in exacerbating surface strain rates, and the influence of surface melt-induced velocities in this exacerbation. The morphology of crevasses is well-studied (e.g. Colbeck and Evan, 1971; Colgan et al., 2016; Harper et al., 1998, 1996), many of them characterising the local stress and strain field to yield critical threshold estimates required for crevasse formation. Although these estimates still vary greatly, the critical strain rate threshold has been found to vary consistently with temperature (Colgan et al., 2016).

Additionally, strain rate values are highly dependent on the distance between markers, with measurements made at the metre scale yielding rates an order of magnitude larger than rates measured at the 100 m scale (Colbeck and Evan, 1971; Colgan et al., 2016). However, with the advent of ultra-high resolution satellite imagery (e.g. QuickBird, IKONOS, WorldView) and unmanned aerial vehicles (UAVs), measurements of horizontal strain can now be made on a much larger spatial scale (Ryan et al., 2015), yielding orders of more data. Point clouds generated from applying photogrammetric techniques to
such imagery have huge potential to resolve this limitation, in addition to quantifying the 3-dimensional flow of sizeable areas at sub-metre resolution. Therefore, with these findings to date, and with the potential of more data accessibility from new technologies, we now have the ability to attempt to resolve the disparities between horizontal and vertical strain rates.

9.2.4 Resilience of ApRES deployments to adverse conditions

As the instrument temperature affects both the system background noise and the relative phase variations within the components comprising the radio frequency (RF) chain\(^1\), and, coupled with the stringent phase stability required to form the transmitting and receiving RF beams, fluctuations in the instrument temperature will have a marked impact on the detectable range to a specified reflector. Through a series of loop tests using a 240 m-long cable, Rahman (2016) observed a majority-linear inverse dependence between system temperature and range to the reflector, with a 2 mm increase in range over a 35 °C temperature difference (Figure 9.2).

Examining the time series obtained through field deployments within this study, there similarly exists a linear correlation between the temperature of the ApRES unit\(^2\) and the change in phase (Figure 9.3d). Here, the observed phase in five selected internal reflectors (Figure 9.3a) were de-meaned and de-trended to remove effects of vertical displacement and strain thinning, respectively. Temporally, variations in the unit temperature inversely affected the signal propagation speed within the instrument but not within the englacial environment, as evidenced by the synchronous oscillations of the specified internal layers with system temperatures (Figure 9.3b,c), and by the relative stability of englacial temperatures on the order of at least decades (Section 6.3.1; Pettersson et al., 2007). In other words, given the majority-linear dependence in its phase response, the temperature sensitivity in the ApRES is largely manifested as a simple time delay. Therefore, by referencing internals between successive bursts (Section 6.3.1), the temperature sensitivity

---

\(^1\)In electronic engineering, the RF chain often comprises the subsystem of various cables, circuits, antennas, signal modulators, etc. that control the nature of the generated and received radio beam

\(^2\)The ApRES has two temperature sensors located on different areas of the PCB: one is beside the crystal clock and the other is beside the active filter.
9.2 Perspectives on future research

Conventional processing. So, a step by step method was developed which would take into account all the factors regarding the radar system parameters and the experimental scenario. The beamforming technique with the 3D geometry, range migration correction requirements, surface curvature correction and sidelobe suppression; all of these are sequentially applied to obtain the final image. This algorithm was not only successfully applied on the Antarctic ice shelf data, but also on the Greenland ice sheet data to visualize the image of the base.

6.2 Future Works

The phase-sensitive radar system requires very rigorous stability in phase. In practice, it was observed that the temperature variation within the components in the RF chain has effect on eventual range values.

Figure 6.1: Plot of change in range values with respect to the temperature change in the radar system. The above figure shows the variation in range values when temperature of the radar unit is varied. This plot was created by using the data from one of the loop tests with

Figure 9.2 Variation in reflector range with increasing temperature of the ApRES system through a loop test using a 240 m-long cable. From Rahman (2016).

was removed. As an additional precaution, we restricted ourselves to short, circadian (24 h) time steps when comparing bursts, further limiting the variability in temperature and phase.

However, if the time steps are sufficiently large, where seasonal trends in temperature are evident, the errors caused by system temperature fluctuations will increase, particularly at temperature extremes. Future ApRES versions should therefore exploit the linear relationship between system temperature and range measurements to examine the possibility of an internal calibration process within the radar hardware.

9.2.5 Imaging the basal environment

Although one of the component aims of this project was to image the subglacial environment, we were only able to quantify the orientation of dipping englacial reflectors; accordingly, we employ forward models in in Section 8.5 to explore potential antenna array configurations to image the basal layer, and in Section 8.6, evaluate the feasibility of alternative configurations given the tradeoff between array gain and beamwidth. While the straightforward conclusion is to increase the number of virtual antennas ($K$) in
one direction and to optimise the separation between virtual antennas ($\delta$), a significant variable that we are not able to either predict or control is the overall specularity of the ice-bed interface. In our analysis, we use a probability distribution suggestive of a wet bed; however, a multitude of parameters that control the specularity content (including the roughness of the ice/water interface) thickness and purity of the subglacial water layer, and size of the subglacial water body may result in a bed-echo specularity that is both spatially and temporally variable, hampering the possibility of basal imaging (Christianson et al., 2016; Pettersson et al., 2011; Schroeder et al., 2015; Young et al., 2016). In addition to better characterising the subglacial environment through future access-borehole experiments, the current forward simulation (Section 8.4.2) will need to incorporate these complex aspects into producing a more realistic distribution of the specularity of the ice-bed interface.

**Figure 9.3** Effect of ApRES system temperature fluctuations in range to five specified reflectors, within time series of deployment 14a (Table 5.2). (a) Depth and (b) temporal variation in phase of five specified reflectors. The total range to all reflectors were de-meaned and de-trended to remove effects of vertical displacement and strain thinning, respectively. (c) Temperature of the ApRES system measured at two locations on the radar PCB: beside the crystal clock (black) and beside the active filter (blue). (d) Comparison of internal reflector phase and temperature.
9.3 Resolving subglacial properties, hydrological networks and dynamic evolution of ice flow on the Greenland Ice Sheet (RESPONDER)

In 2016, commencement of another major project, again a collaboration between the University of Cambridge’s Scott Polar Research Institute and Aberystwyth University’s Centre for Glaciology, enabled further research to be conducted on Store Glacier until 2021. The project—Resolving subglacial properties, hydrological networks and dynamic evolution of ice flow on the Greenland Ice Sheet (RESPONDER)—extends the work conducted through the SAFIRE project (Section 5.2) by investigating how hydrological networks at the base of the Greenland Ice Sheet evolve over seasons and multiple years, and how this evolution impacts on ice flow in the interior and at the margins. In the summer of 2017, two ApRES units were deployed—one at Low Camp another at High Camp—and hot-water borehole drilling has been planned for the summer of 2018 (Figure 9.4). Therefore, due to the congruence of the goals within RESPONDER and the findings produced from this thesis, RESPONDER offers an opportunity for the majority of the areas listed above worth further investigation.

9.4 Concluding remarks

Tidewater outlet glaciers drain 88% of the Greenland Ice Sheet (Rignot and Mouginot, 2012), but the dynamics that underlie and result from their fast flow still to this day remain one of the least-studied, least-understood, and most poorly-parameterised processes of glacier motion. By resolving field-based observations with theoretical linkages underpinning fast glacier motion, we provide empirical evidence of the various links between supraglacial, englacial, and subglacial processes, the majority of which have so far only been theorised through model simulations.

Clarke (1987) theorised that technological advances, rather than revolutions in thinking, have often been the impetus to advancement within the field of glaciology. Thirty years
forward, this is still largely the case: while instruments and satellites, primarily spurred by an exponential increase in processing power coupled with the invention of innovative processing techniques, are now able to capture the glacier subsurface at up to millimetre resolution, the overall methodology of investigation and the target environments remain fundamentally unchanged. Further innovations in remote sensing will increasingly reveal detailed observations of poorly-characterised areas within the ice sheet. At the same time, there is still an inherent need for ice coring or borehole drilling studies, which still remain the only methods to directly access the ice sheet subsurface for the foreseeable future. Continued investigation in these environments using an integration of methods and technological innovations should therefore be prioritised to help answer the remaining questions in the dynamics of the Greenland Ice Sheet.
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Appendix A

GPS measurements of horizontal deformation

To complement vertical velocity and strain measurements from ApRES, horizontal strain rates were measured in 2014 by installation of three kinematic GPS units (Trimble 5700) operational between 07 May and 13 July 2014 (Figure A.1).

Figure A.1 Locations of deployments, and the resulting strain centroid (WNE) from the triangle created by deployments SAGN, SAGW, and SAGE. Arrows show displacements of the ApRES unit (deployment 14a) and centroid over their respective operational periods. Figure 5.2a shows the approximate location of this figure within S30.
A.1 Calculation of horizontal strain rates

In the simplest of configurations, strain can be calculated using a minimum of 3 points in 2-dimensions (or, alternatively, a minimum of 4 points in 3-dimensions). We follow the methods of Nye (1959) and van der Veen (1989), which calculates strain rates from measurements of the relative displacement of markers.

Here, we consider a set of points, \((A, B, C)\), that move downglacier over a period of time \(\Delta t\), resulting in a set of start and end positions \((A_0, B_0, C_0)\) and \((A_1, B_1, C_1)\), respectively (Figure A.2). Following Equation 4.4, the three strain rates that can be determined from the displacements are the sides of the triangle created from the 3 points:

\[
\dot{\varepsilon}_{AB} = \frac{\Delta AB}{|AB| \Delta t}; \quad \dot{\varepsilon}_{AC} = \frac{\Delta AC}{|AC| \Delta t}; \quad \dot{\varepsilon}_{BC} = \frac{\Delta BC}{|BC| \Delta t};
\]  

(A.1)

Working in the XY-plane, with \(y\) parallel to true North, Equation A.1 can be decomposed into coordinate form using Pythagoras’ Theorem:

\[
\dot{\varepsilon}_{ij} = \frac{T_{ij} - T_{ij}^0}{|IJ| \Delta t} = \frac{\sqrt{(J_{x1} - I_{x1})^2 + (J_{y1} - I_{y1})^2} - \sqrt{(J_{x0} - I_{x0})^2 + (J_{y0} - I_{y0})^2}}{\sqrt{(J_{x0} - I_{x0})^2 + (J_{y0} - I_{y0})^2} \Delta t}
\]  

(A.2)

where \(I\) and \(J\) represent combinations of the set of points \((A, B, C)\).

To rotate the coordinate axes by \(\vartheta\) to align to the Cartesian coordinate system \((<x,y,z>)\), the measured strain rates in Equation A.2 can be transformed through trigonometry (Figure A.2):

\[
\dot{\varepsilon}_{AB} = \dot{\varepsilon}_{xx} \quad \dot{\varepsilon}_{AC} = \dot{\varepsilon}_{xx} \cos^2 \alpha + \dot{\varepsilon}_{yy} \sin^2 \alpha + \dot{\varepsilon}_{xy} \sin 2\alpha \quad \dot{\varepsilon}_{BC} = \dot{\varepsilon}_{xx} \cos^2 \beta + \dot{\varepsilon}_{yy} \sin^2 \beta + \dot{\varepsilon}_{xy} \sin 2\beta
\]  

(A.3a, b, c)

which then can be rewritten as:

\[
\dot{\varepsilon}_{xx} = \dot{\varepsilon}_{AB}
\]  

(A.4a)
A.1 Calculation of horizontal strain rates

\[ \dot{\varepsilon}_{yy} = \frac{1}{\tan \beta - \tan \alpha} \left[ \dot{\varepsilon}_{AB} \left( \cot \alpha - \cot \beta \right) - \dot{\varepsilon}_{AC} \left( \csc \alpha \sec \alpha \right) + \dot{\varepsilon}_{BC} \left( \csc \beta \sec \beta \right) \right] \quad (A.4b) \]

\[ \dot{\varepsilon}_{xy} = \frac{1}{2 (\cot \beta - \cot \alpha)} \left[ \dot{\varepsilon}_{AB} \left( \cot^2 \alpha - \cot^2 \beta \right) - \dot{\varepsilon}_{AC} \csc^2 \alpha + \dot{\varepsilon}_{BC} \csc^2 \beta \right] \quad (A.4c) \]

where \( \vartheta \) is the counter-clockwise amount of rotation achieved from the X-axis (in this case, true East) to the direction of \( \overline{AB} \) (Figure A.2).

Further rotation of the coordinate axes, for example, to align with the primary flow direction, can be achieved by applying a similar transformation (Figure A.2):

\[ \dot{\varepsilon}_{pp} = \dot{\varepsilon}_{xx} \cos^2 \varpi + \dot{\varepsilon}_{yy} \sin^2 \varpi + \dot{\varepsilon}_{xy} \sin 2\varpi \quad (A.5a) \]

\[ \dot{\varepsilon}_{qq} = \dot{\varepsilon}_{xx} \sin^2 \varpi + \dot{\varepsilon}_{yy} \cos^2 \varpi - \dot{\varepsilon}_{xy} \sin 2\varpi \quad (A.5b) \]

\[ \dot{\varepsilon}_{pq} = \frac{1}{2} (\dot{\varepsilon}_{yy} - \dot{\varepsilon}_{xx}) \sin 2\varpi + \dot{\varepsilon}_{xy} \cos 2\varpi \quad (A.5c) \]

where \( \varpi \) is the clockwise amount of rotation from \( \overline{AB} \) to the desired direction, and \( <p,q,r> \) is the coordinate system aligned along-flow (\( r \equiv z \); Figure A.2).
Lastly, to determine the principal axis of strain $\nu$, $\dot{\varepsilon}_{pq}$ should equal 0, in which Equation A.5c can then be rewritten as:

$$\tan 2\nu = \frac{2\dot{\varepsilon}_{xy}}{\dot{\varepsilon}_{xx} - \dot{\varepsilon}_{yy}}$$

(A.6)

### A.2 Comparison of horizontal and vertical strain rates

We can compare the radar-measured vertical strain rates with contemporaneous measurements of GPS-measured horizontal strain rates conversion of strain rate components under the continuity condition (e.g. Meierbachtol et al., 2016), producing two independently-obtained measurements of vertical strain rates:

$$\dot{\varepsilon}_{pp} + \dot{\varepsilon}_{qq} + \dot{\varepsilon}_{rr} = 0$$
$$\dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} + \dot{\varepsilon}_{zz} = 0$$

(A.7)

As we are in the ablation zone, we can assume a relatively constant ice density through the majority of the ice column (917 kg m$^{-3}$).