Supply Shocks and the 'Natural Rate of Interest': An Exploration*

Jagjit S. Chadha[†]
Cambridge University

Charles Nolan[‡]
Durham University

1 Introduction

We consider a small, open economy inhabited by an optimising corporate sector and a large number of optimising individuals who must formulate each period, amongst other things, dynamic programmes for investment and consumption following stochastic shifts in total factor productivity. We then ask three questions: What would the marginal productivity of capital, or real interest rate, look like over the course of the cycle in such an economy? How closely does the observed real interest rate in a small, open economy (i.e., the United Kingdom) resemble this hypothetical rate? And can we describe 'inflation and output determination as depending on the relation between a "natural rate of interest" determined primarily by real factors and the central bank's rule for interest rates' (Woodford, 2000, p. 2)?

We are accustomed to the supply side of an economy, as measured by the long-run average rate of productivity growth, as the most natural explanation for long-run real outcomes in growth and welfare (Griliches, 1996). But as economists have recognised that the productivity cycle might be closely related to the propagation or impulses of the business cycle, the cyclical behaviour of productivity has increasingly attracted their attention (Cooley, 1995). Beginning with the seminal papers of Long and Plosser (1983) and Kydland and Prescott (1982), an important strand of macroeconomic research

*Acknowledgements: We are grateful for helpful conversations, constructive criticisms and comments from participants in seminars at which versions of this paper were presented, including the Bank of England, University of Cambridge, University of Leicester and Warwick Business School. In particular we are grateful for the comments of Philip Arestis, Laurence Ball, Sean Holly, Lavan Mahadeva, Anton Muscatelli and Peter Sinclair. Any views expressed, or errors which remain, are solely the responsibility of the authors. We are grateful for research assistance from Wesley Fogel. Part of this research has been funded by Leverhulme Grant no. F/09567/A.

†Department of Applied Economics, Austin Robinson Building, Sidgwick Avenue, Cambridge CB3 9DE. Tel: +44 1223 335280. E-mail: jagjit.chadha@econ.cam.ac.uk

‡Department of Economics, University of Reading, Whiteknights, Reading RG6 6AA. Tel: +44 118 987 5123 x4068. E-mail: C.Nolan@reading.ac.uk.

has attempted to construct small, theoretically coherent models based on optimising agents that capture key cyclical patterns in the actual data. In this paper, we follow the analyses of Backus, Kehoe and Kydland (1989) and Chadha, Janssen and Nolan (2001) and extend the real business cycle model to the case of the open economy. We then analyse the marginal productivity of capital in this economy in response to both the standard measure of productivity growth, the Solow residual, and a measure that attempts to correct for capacity utilisation.

To carry out this experiment we must develop two distinct models of a small, open economy, which nevertheless share many similar characteristics. Both are inhabited by a corporate sector and a large number of individuals (really cohorts) who face finite lives. Both model economies face an exogenous steady-state interest rate determined in a perfectly integrated world capital market, and both economies have access to this world capital market when determining their portfolio allocation decisions.(1) In both economies the factors of production receive their marginal product, and there are otherwise no barriers to price flexibility. However, these economies differ in what makes them grow stochastically through time. In both models the driving process is stochastic variation in a productivity shift term, but in one it is the standard Solow residual while in the other the residual is employed in modified form with a correction for the cyclical variation in capital utilisation (see e.g., Burnside, Eichenbaum and Rebelo, 1996 and Basu and Fernald, 2000).

There are a number of reasons to distrust the standard Solow residual as a measure of total factor productivity (see King and Rebelo, 1999). We shall concentrate on the two we think most pertinent.(2) First, the standard measure of the Solow residual implies an implausibly large probability of technological regress and, concomitantly, an unrealistically high volatility of productivity growth relative to output.(3) Second, cyclical variations in factor inputs, either through labour hoarding or capital utilisation, can lead to mismeasurement of the Solow residual. Barro (1999) summarises many of the possible reasons, in particular where failure to allow for improvements in labour or capital quality tend to lead to an overestimate of growth due to technological change. In a series of very influential papers, Basu (see, for example, 1996, and, with Fernald, 2000) argues that the main reason for the strong procyclicality of TFP is measurement error resulting from variable capital and labour utilisation. He further finds that

[1] The assumption of an exogenous long-run real rate does not affect the dynamic equilibrium path of adjustment of the domestic rate, in so far as the real rate will return to that set by the rest of the world eventually. The question we ask is how that adjustment might vary over the cycle.

[2] Chadha, Janssen and Nolan (2001) show the Solow residual to be exogenous to Hall's (1997) suggested business cycle forcing variable of stochastic shifts in preferences for consumption over leisure.

[3] Burnside et al. (1996) estimate that the quarterly probability of technological regress falls from 37% in US manufacturing to some 10% when variable capacity utilisation is accounted for. We do not find such a stark fall in the probability on economy-wide data in the UK, where the probability of regress falls from 39% to 23%. See Figures 1 and 2 for UK data from 1965 to 1998.

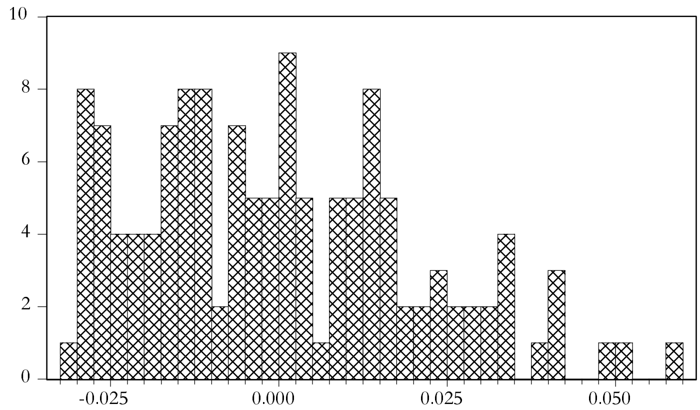

Figure 1: Histogram of Standard Solow Residual

variable utilisation is not merely a bias, but is itself an important propagation mechanism for an economy's primitive driving forces.

King and Rebelo (1999) build on this insight and argue that variable capacity utilisation tends to result in a high- substitution economy amplifying the 'vanishing' productivity shock, in which aggregate hours are very responsive to changes in the real wage or interest rates and the supply of capital responds strongly to changes in aggregate hours. One purpose of this paper is to examine the amplification properties of a standard Solow residual purged of the effects of variable capacity utilisation. Figures 1 and 2 illustrate the impact of this purge on the resulting distribution of shocks, which becomes significantly negatively skewed.

The simulation of these two models, a standard RBC and a high-substitution RBC economy, allows us to obtain a marginal productivity of capital. We find that the marginal productivity of capital is significantly procyclical, thereby confirming Vickers' (2000, p. 11-12) conjecture. We interpret, in our model, the artificial marginal productivity capital as the 'natural rate of interest', which is consistent with constant inflation in the medium run. In some sense we also follow Blinder's (1998, p. 34) suggestion to compute the 'neutral' rate in response to durable shocks. Our aim in this paper, then, is essentially twofold. First, we wish to investigate whether our measure of the corrected productivity shock may still be a plausible candidate for a principal source of the UK business cycle.(4) Second, looking at the difference between the hypothetical marginal productivity of capital and the observed real rate in the UK economy, we ask, in the manner of Wicksell, to what extent the divergence can explain inflation

[4] Chadha, Janssen and Nolan (2001) find that the standard Solow residual captures the main moments of the UK business cycle very well. Basu (1996) finds that the corrected Solow residual cannot produce enough volatility in the business cycle.

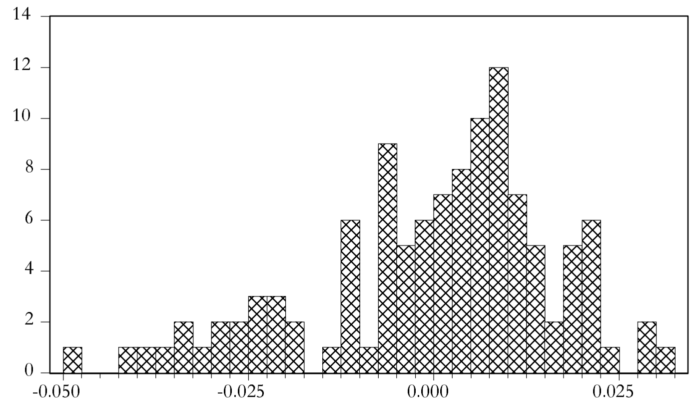

Figure 2: Histogram of Corrected Solow Residual

and output outcomes.

The paper is structured as follows. Section 2 outlines a number of facts relating to the business cycle in the United Kingdom and in particular to the real interest rate. Section 3 develops a more or less canonical dynamic stochastic general equilibrium model driven by supply shocks measured in one of two ways: either as total factor productivity or as total factor productivity corrected for changes in capacity utilisation. Section 4 presents the calibration techniques used to assess the fit of this model. Section 5 provides the main results. Section 6 offers some conclusions on the 'natural rate of interest' hypothesis.

2 The Real Rate of Interest: Some Observations

This section outlines some basic facts about the UK economy. When assessing the models presented, there are two main dimensions along which we will try to map the UK economy: (i) the covariation of the main economic indicators with the economic cycle, measured by the business cycle fluctuation in output per head, and (ii) the relative volatility of each series. Table 1 presents the observed data on the business cycle component of the UK economy obtained with the band pass filter recommended by Baxter and King (1999).[(5)] Investment and the current account are the most volatile series. Output, consumption and real rates display lower, similar levels of volatility. Observed real wages show little cyclical variation and, as suggested by King and Rebelo (1999), the capital stock displays little important variation at the business cycle frequency. Consumption, investment, real wages and hours supplied are procyclical while the current

[5] A wider range of series' moments results are available on request, as are the results of different filtering procedures. The programs we used for implementing the filter are also available, as is a full data annex, but note that all data are sourced from the ONS. All expenditure series are measured in per capita terms.

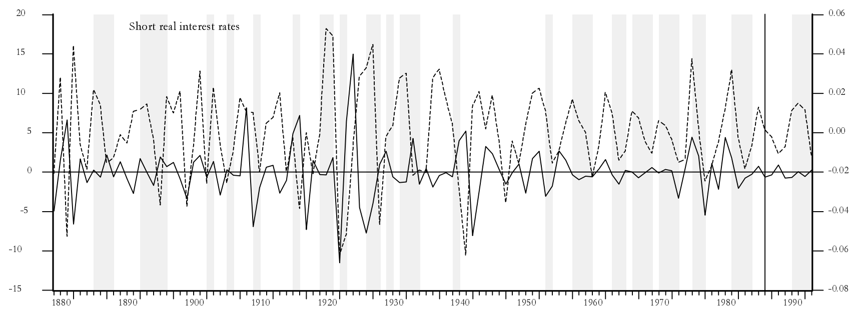

Figure 3: Real Rates and the Business Cycle, 1871-1998

account is countercyclical. Output shows positive leads for consumption, investment and wages, while there are negative leads for the current account. The current account and, to some extent, real wages negatively lead output, while consumption, investment, real rates and labor supply positively lead output.

The behaviour of observed real rates needs further explanation. Figure 3 explores the cyclicality of real interest rates from a somewhat longer perspective. We plot the cyclical component of the real interest rate (dashed line) against both the cyclical component of output per head (solid line) and the phases of expansion (light) and contraction (dark), and find fairly clear evidence that real interest rates have become procyclical in the post-war period and their volatility has fallen to that of output per head. Table 2 quantifies this finding. We find for an annual data set from 1947 that, irrespective of the filter employed, real rates are significantly positively associated with the business cycle and of a similar level of volatility. This finding stands at some variance to the somewhat puzzling and yet standard result in the US literature (see, for example, King and Watson, 1996), which reports countercyclical real rates.[(6)]

In the following section we build two versions of a canonical real business cycle (RBC) model with each of two productivity measures driving an artificial general equilibrium economy. In addition to studying the ability of these two models to explain the key observed moments of real rates, we will examine their ability to explain the main business cycle moments found in expenditures and factors.

[6] See Chadha and Dimsdale (1999) for a discussion of possible reasons for the post-war procyclicality of real rates. They also suggested that acyclicality resulted from high inflation volatility, thereby injecting noise into the real rate cycle and its measured association with the business cycle.

3 An Open Economy Model

In this section we develop our small, open economy RBC model. We employ two versions. The first model is driven by exogenous changes in technological capabilities, while the second is driven by the same changes purged of variable capacity utilisation. The incorporation of variable capital utilisation in the first model requires some important changes to the equations describing optimal corporate behaviour and factor productivities. For ease of exposition, we detail the derivation of the first model only, and then indicate which equations change in the case of the preference shock model.

We model the UK economy as a small, open economy in the sense that it faces an exogenous real interest rate determined in fully integrated world capital markets. This characterisation means that unless the interest rate exactly equals the discount factor, the agent (in our model, the cohort) will be accumulating (if $r > \delta$) or decumulating (if $r < \delta$) net foreign assets such that we strain the small country assumption.(7) We address this issue by adopting the approach of Yaari (1965), assuming that agents in our model face finite lives. In particular we follow Cardia (1991) by adopting a discrete time (open economy) version of Blanchard (1985). The first version of our model is taken from Chadha, Janssen and Nolan (2001). In the exposition below, a number of issues are treated somewhat lightly, particularly issues pertaining to the aggregation over cohorts; we refer the interested reader to Chadha et al. (2000) for further details.

3.1 The Representative Agent

Equation (1) represents the expected lifetime utility of a representative agent.

$$V_0 = E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+\delta}\right)^t \left(\frac{1}{1+\lambda}\right)^t u\left(C_t, L_t\right) \tag{1}$$

The agent gains utility from consumption of a single non-durable good, C_t, and leisure, L_t. Each period, she faces a time-independent probability of death, λ, which lies in the open unit interval. Let δ denote the subjective discount rate. We make the usual assumptions concerning the differentiability of the utility function, which we also assume is concave and increasing in both its arguments. Her maximisation is subject to a sequence of per-period budget constraints,

$$C_t + B_{t+1} = (1+r)(1+\lambda) B_t + \Pi_t + W_t N_t \tag{2}$$

(7) We are not implying, of course, that the underlying optimisation is ill-posed since the budget set remains compact, and in fact bounded. The point is simply that, for reasonable parameter values, $r \neq \delta$ can imply large steady-state net asset holdings. In our discussion of the steady-state properties of our model, we discuss this point further and compare our steady state net asset position with that of a more conventional representative agent model.

plus a transversality condition which we spell out below. That is, total consumption, C_t, plus purchases of net foreign assets, B_{t+1}, is equal to net income, in turn composed of net income from foreign assets, $(1+r)(1+\lambda)B_t$, income from shares in the representative firm, Π_t, and labour income, W_tN_t, which is in turn equal to the product of the per-period wage rate and the amount of time spent working. If an interior optimum exists, then the following two equations, amongst others, will characterise that optimum:

$$u_c^{'}(C_t, L_t) = \left(\frac{1+r}{1+\delta}\right) E_t u_c^{'}(C_{t+1}, L_{t+1}) \tag{3}$$

$$\frac{u_l^{'}(C_t, L_t)}{u_c^{'}(C_t, L_t)} = W_t \tag{4}$$

Equation (3) demonstrates that at the individual (cohort) level, the probability of death entails no tilting of consumption, other than what would normally occur in models with infinitely lived consumers and a discrepancy between market interest rates and the subjective discount factor. Frankel and Razin (1992) suggest an intuitive interpretation of this result. Consider a net creditor who wishes to secure, in expectation, that his advance returns principal plus the risk-free interest rate. To do this he needs to take account of the fact that the borrower may not survive to repay the loan. Assuming a version of the law of large numbers such that the proportion of the population that does survive is given by $(1+\lambda)^{-1}$, then his expected return is given by the product of the proportion of the cohort that survives, and the amount repaid, $(1+r)(1+\lambda)B_{t+1}$. Equation (4) is also familiar, and governs the optimal supply of labour. Equation (3) also makes clear the problem in proxying the dynamics of a small open economy using an infinitely lived representative agent set up. The situation changes radically with respect to aggregate consumption when we integrate over all currently alive cohorts. First we note that the size of the cohort born each period is given by

$$\left(\frac{\lambda}{1+\lambda}\right)\left(\frac{1}{1+\lambda}\right)^t$$

As a result of this, the size of the cohort decreases monotonically with time, and the sum of all currently alive cohorts is equal to unity, that is[8]

$$\frac{\lambda}{1+\lambda}\sum_{j=-\infty}^{t}\left(\frac{1}{1+\lambda}\right)^{(t-j)} = 1 \tag{5}$$

To obtain an expression for aggregate consumption we first need to calculate the agent's present value budget constraint. Iterating on equation (2) in the usual way we get that

[8] We outline in more detail in an appendix the construction of our discrete approximation to the continuous exponential density.

$$\sum_{s=t}^{\infty} E_t \left[\left(\frac{1}{1+r} \right)^{s-t} \left(\frac{1}{1+\lambda} \right)^{s-t} C_s \right]$$
$$= \quad (1+r)(1+\lambda) B_t + \sum_{s=t}^{\infty} E_t \left[\left(\frac{1}{1+r} \right)^{s-t} \left(\frac{1}{1+\lambda} \right)^{s-t} \right] [\Pi_s + W_s N_s] \quad (6)$$

Again equation (6) is a familiar expression, except that both it and the transversality condition now reflect the probability of death:

$$E_t \left(\frac{1}{1+r} \right)^T \left(\frac{1}{1+\lambda} \right)^T B_{t+T+1} \to 0 \quad as \quad T \to \infty \tag{7}$$

Assuming log utility (as we do throughout this paper), we see that a simple stochastic difference equation governs consumption dynamics at the individual level, $E_s C_{s+1} = [(1+r)/(1+\delta)] C_s$. Using this expression in equation (6) successively to substitute for future consumption, we get that

$$\begin{aligned} C_t \quad &= \quad \frac{(1+\lambda)(1+\delta) - 1}{(1+\lambda)(1+\delta)} . \langle (1+\lambda)(1+\delta) B_t \\ &+ E_t \sum_{s=t}^{\infty} \left[\left(\frac{1}{1+r} \right)^{s-t} \left(\frac{1}{1+\lambda} \right)^{s-t} \right] [\Pi_s + W_s N_s] \rangle \end{aligned} \tag{8}$$

Ultimately (the details are contained in Chadha et al., 2000), we can use equation (8) to derive an expression for aggregate consumption dynamics. This is given in equation (9):

$$E_t C_{t+1} = \left(\frac{1+r}{1+\delta} \right) C_t - \lambda (1+r) \beta B_{t+1} \tag{9}$$

where $\beta \equiv \frac{(1+\lambda)(1+\delta)-1}{(1+\lambda)(1+\delta)}$. Here we see that any wedge between the subjective discount factor and the market interest rate need not imply a rising or falling level of net foreign assets. Equations (2) and (4), in aggregated form, along with equation (9), are the three key equations from the representative agent portion of the model.

3.2 The Representative Firm

We now turn to the problem facing the representative firm. First, we note that output is characterized by a Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} \left((1+\gamma)^t N_t \right)^{1-\alpha} \tag{10}$$

where A_t is the Solow residual, consisting of a trend and a stochastic component. We denote the log of the stochastic component by a_t, and we assume that $a_t = \rho a_{t-1} + \epsilon_t$, with $\epsilon \sim (0,\ \sigma_\epsilon^2)$ is indepedently and identically distributed (i.i.d.) $(1+\gamma)$ is a labour-augmenting growth factor, and the common trend among the real magnitudes in our model, including the Solow residual. K_t is the capital stock, and L_t is the labour input. Firms maximise total profits (suitably discounted so that aggregate utility is maximised) subject to a capital evolution equation, which says that the evolution of the capital stock is subject to a constant rate of depreciation, ψ, and a cost of adjustment $\phi(.)$ where ϕ is strictly concave, increasing in investment, I_t, and decreasing in the (predetermined) capital stock. That is,

$$K_{t+1} = (1-\psi) K_t + \phi\left(\frac{I_t}{K_t}\right) K_t \tag{11}$$

Let μ_t denote the marginal utility of aggregate consumption (the marginal utility of consumption is equal across cohorts, so aggregation is straightforward) and Λ_t denote an undetermined multiplier. The optimality conditions for a profit maximising firm (which maximises aggregate utility at the same time), hence the presence of μ_t in these optimality conditions) are given by equation (11) as well as equations (12) and (13):

$$\Lambda_t \phi'(I_t/K_t) = \mu_t \tag{12}$$

$$\begin{aligned}\Lambda_t \quad = \quad & E_t\left\{\alpha\left(\frac{1}{1+\delta}\right)\mu_{t+1}A_{t+1}K_t^{\alpha-1}N_{t+1}^{1-\alpha}\right\} \\ & + E_t\Lambda_{t+1}\left[\phi\left(\frac{I_{t+1}}{K_{t+1}}\right) - \phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}} + (1-\psi)\right]\end{aligned} \tag{13}$$

The preceding two equations are essentially a q model of investment. Roughly speaking, they indicate that investment will be positive when, other things being constant, the marginal utility of consumption is low, adjustment costs are low and the benefits of a higher capital stock (now and in the future) are high. The Cobb-Douglas production function, equation (10), also provides expressions for the marginal product of labour and capital, and these are given, respectively, by equations (14) and (15):

$$W_t = (1-\alpha) A_t K_t^{\alpha}\left[(1+\gamma)^t N_t\right]^{-\alpha}(1+\gamma)^t \tag{14}$$

$$Z_t = \alpha A_t K_t^{\alpha-1}\left[(1+\gamma)^t N_t\right]^{1-\alpha} \tag{15}$$

Two additional equations (or constraints) complete the description of our model economy. First there is a time constraint on the representative agent such that the total time available in any period for leisure and labour is normalised to unity.

$$N_t + L_t = 1 \tag{16}$$

Also, the economy faces a resource constraint that also determines the evolution of net foreign assets. This is given in equation (17):

$$B_{t+1} - B_t = rB_t + Y_t - C_t - I_t. \tag{17}$$

Equations (16) and (17) complete the description of the economy. The next stage involves making our equations stationary, that is, dividing through by our growth factor. So, for example we use the following change of variables for variables observed at date t: $C_t \equiv C_t/\left(1+\gamma\right)^t$ and $K_{t+1} \equiv (1+\gamma)K_{t+1}/\left(1+\gamma\right)^{t+1}$. We then linearise our model equations. These transformed equations are collected in the next section.

3.3 The Linearised Model

In the equations that follow time-subscripted, lower-case letters refer to percentage deviations from steady state of our model equations in stationary format, and lower-case letters with no subscripts refer to steady-state values, i.e., $c_t \equiv dC_t/c$ where $C_t \equiv C_t/\left(1+\gamma\right)^t$ and c is the steady-state value of consumption:

$$\left(1+\gamma\right)E_t c_{t+1} = \left(\frac{1+r}{1+\delta}\right)c_t - \lambda\left(1+r\right)\left(1+\gamma\right)\beta\left(b/c\right)b_{t+1} \tag{18}$$

$$y_t = \left(c/y\right)c_t + \left(i/y\right)i_t - \left(1+r\right)\left(b/y\right)b_t + \left(1+\gamma\right)\left(b/y\right)b_{t+1} \tag{19}$$

$$c_t - l_t = w_t \tag{20}$$

$$nn_t + ll_t = 0 \tag{21}$$

$$\left(1+\gamma\right)k_{t+1} = \left(1-\psi\right)k_t + \left(i/k\right)i_t \tag{22}$$

$$-(1+\gamma)\lambda_t = \frac{(i/k)}{(1+\delta)\zeta} E_t(i_{t+1}-k_{t+1}) + \frac{r+\psi}{1+\delta} E_t(c_{t+1}-z_{t+1}) - \left(\frac{1-\psi}{1+\delta}\right) E_t\lambda_{t+1} \quad (23)$$

$$y_t = \alpha k_t + (1-\alpha) n_t + a_t \quad (24)$$

$$w_t = \alpha(k_t - n_t) + a_t \quad (25)$$

$$z_t = (\alpha - 1)(k_t - n_t) + a_t \quad (26)$$

$$i_t = -\zeta(c_t + \lambda_t) + k_t \quad (27)$$

where $\zeta \equiv \frac{\phi'(.)}{\frac{i}{k}.\phi''(.)}$ is the slope of the investment demand function. We have ten endogenous variables, two of which, b_{t+1} and k_{t+1}, are predetermined, such that $E_t b_{t+1} = b_{t+1}$, and $E_t k_{t+1} = k_{t+1}$. The model has two stable roots, which can be associated with the two predetermined variables, and it otherwise meets the Blanchard-Kahn criteria for a unique bounded rational expectations solution.

3.4 The Model with Variable Capacity Utilisation

The above model can be relatively easily adapted to incorporate the effects of variable capacity utilisation. Here we follow the approach of King and Rebelo (1999). We do not incorporate an analogous effect into the labour market, hence the equations characterising aggregate consumption and asset accumulation, and labour supply given the real wage (where the real wage *is* affected by variable capacity utilisation), do not change. See King and Rebelo (1999)) for more discussion of this point. Let v_t denote the extent of capacity utilisation.[9] We rewrite the Cobb-Douglas production function in this case as

$$Y_t = A_t v_t K_t^{\alpha}\left((1+\gamma)^t N_t\right)^{1-\alpha} \quad (28)$$

where, as before, A_t is the Solow residual and the log of the stochastic component is denoted a_t, and where $a_t = \rho a_{t-1} + \epsilon_t$, with $\epsilon \sim (0,\ \sigma_\epsilon^2)$ is i.i.d.. Although the

[9] We measure v_t from the Quarterly European Commission Business Consumer Surveys 'Directorate General for Economic and Financial Affairs', July 2000, page 4, Table 2.

driving process is formally identical to the case analysed above, the stochastic process, under variable utilisation, is not as volatile and is less persistent. All variables have the same description as before, so K_t is the capital stock, and L_t is the labour input. As before, firms maximise total profits; however, now we assume that there are no adjustment costs and that the depreciation rate is an increasing function of the level of capacity utilisation: $\psi(v_t)$. That is,

$$K_{t+1} = (1 - \psi(v_t))\, K_t + I_t \tag{29}$$

In addition to the choices it faced in the previous problem, the firm now has to choose the level of capacity utilisation. In other words v_t is a choice variable. The optimality conditions are given by equations (30) to (32):

$$\Lambda_t = \mu_t \tag{30}$$

$$\begin{aligned}\Lambda_t \quad = \quad & E_t\left\{\alpha\left(\frac{1}{1+\delta}\right)\mu_{t+1}A_{t+1}K_t^{\alpha-1}[(1+\gamma)^{t+1}N_{t+1}]^{1-\alpha}\right\} \\ & + E_t\Lambda_{t+1}\left[(1-\psi(v_t)\right]\end{aligned} \tag{31}$$

$$\psi'(v_t)K_t = \alpha A_t(v_tK_t)^{\alpha-1}K_t[(1+\gamma)^t N_t]^{1-\alpha} \tag{32}$$

where in the final expression we have used the fact that $\Lambda_t = \mu_t$. The first two expressions are familiar, and reflect optimal investment behaviour in the absence of adjustment costs. This final expression is the efficiency condition characterising the optimal level of capacity utilisation. It indicates that increased capacity utilisation results in increased benefits in the form of higher output and hence higher profits in our set-up. It also raises costs, however, in the form of increased replacement investment. That is, increased utilisation results in a higher period depreciation rate. In addition to these changes, the expressions describing factor productivities also change:

$$W_t = (1-\alpha)\, A_t(v_tK_t)^{\alpha}\left[(1+\gamma)^t N_t\right]^{-\alpha}(1+\gamma)^t \tag{33}$$

$$Z_t = \alpha A_t(v_tK_t)^{\alpha-1}\left[(1+\gamma)^t N_t\right]^{1-\alpha} v_t \tag{34}$$

Therefore, equations (3), (9), (16), (17) and (28) through (34) describe our economy in the presence of variable capacity utilisation. As before, we study a linear approximation to these equations in stationary form.

3.5 The Linearised Equations

As above, lower case letters refer to percentage deviations from steady state. Equations (18) to (21) remain the same. However, we now replace equations (22) through (27) with equations (35) to (40), and we have an additional equation approximating optimal capacity utilisation (41):

$$(1+\gamma)\,k_{t+1} = (1-\psi(v))\,k_t + (i/k)\,i_t - \psi'(v)vv_t \tag{35}$$

$$-c_t = \lambda_t \tag{36}$$

$$((1-\psi(v))E_t\lambda_{t+1} - (r+\psi(v))E_t\,(c_{t+1}-z_{t+1}) + vv_{t+1} = (1+\gamma)\lambda_t \tag{37}$$

$$y_t = \alpha k_t + (1-\alpha)\,n_t + \alpha v_t + a_t \tag{38}$$

$$w_t = \alpha(k_t - n_t) + \alpha v_t + a_t \tag{39}$$

$$z_t = (\alpha-1)\,(k_t - n_t) + \alpha v_t + a_t \tag{40}$$

$$v_t = \frac{1}{1+\sigma-\alpha}a_t + \frac{(\alpha-1)}{1+\sigma-\alpha}(k_t - n_t) \tag{41}$$

where $\sigma \equiv \frac{v.\psi''(.)}{\psi'(.)}$.

This version of our model also has two stable roots that are associated with the two predetermined variables. Both our models may be specified as a singular linear difference model under rational expectations. In both cases a unique bounded R.E. solution is verified. The solution specifies the vector of non-predetermined endogenous variables as a function of predetermined variables and on the exogenous driving processes. The predetermined variables are expressed as functions of their state in the previous time period and of the previous period's driving processes. This recursive solution allows us to simulate easily the behaviour of our model economies under what we hope are realistic driving processes, and then compare the generated moments with those observed during the UK business cycle.[(10)]

[10] Full solution results, including Markov decision rules, are available on request.

4 The Steady State of the Model

We discuss only briefly the calibration of our models, and more details can be found in Chadha, Janssen and Nolan. (2001).[11] For our 'free' variables we adopt standard parameter values. Table 3 lists these fundamental parameters, and we explain the derivation of the other steady values. The annual probability of death, λ, is taken to be $\frac{1}{67}$. For quarterly data this translates into 0.373%. The quarterly real return on capital, r, in the United Kingdom is taken to be 1.25%. The share of capital, α, in the production function is taken to be 38%. We assume, in line with King and Rebelo (1999), that the quarterly rate of capital depreciation, ψ, is 2.5%. Per capita income growth, γ, in the UK is 0.5%. And the quarterly rate of time preference is assumed to be around the level implied by market real interest rates, 0.75%. We explain the remaining parameters, $\beta, b, c, y, i, l, N, L$ and θ in turn. In the steady state we have that

$$z = r + \psi = \alpha \left(\frac{K}{N} \right)^{\alpha - 1} \tag{42}$$

where we have used the steady state analogues of equations (11), (13) and (14). We solve for K/N and find 41.9. From equation (24) we note that $Y/N = K/N^{\alpha} = 4.135$ and thus $K/Y = 10.133$. From equation (22) we see $i/k = \gamma + \psi$ and $i/y = (\gamma + \psi)\, k/y$ which are calculated to be 10.13 and 0.304, respectively. Now note:

$$w_t = (1 - \alpha) \left(\frac{K}{N} \right)^{\alpha} \tag{43}$$

which is equal to 0.513. We shall assume that we spend one fifth our time working and so $N/L = 0.25$ and y, k and i equal 0.827, 8.379 and 0.251, respectively. We now solve for b and c simultaneously using equation (18) and the current account identity:

$$y - c - i = (\gamma - r)\, b \tag{44}$$

from which we calculate that b equals -0.501 and c equals 0.571. Finally, θ is given by the intratemporal efficiency condition given by equation (20) and is found to be 0.535.

5 Results

The main results are presented in Tables 4-6, as direct analogues of Table 2 on the observed data. The results in Table 4, from the model driven by

[11] A spreadsheet calculating all our parameter values and steady state values (including extensive sensitivity calculations) is available on request. There are also some quantitatively minor differences in the steady state of our two models. We do not discuss these differences further.

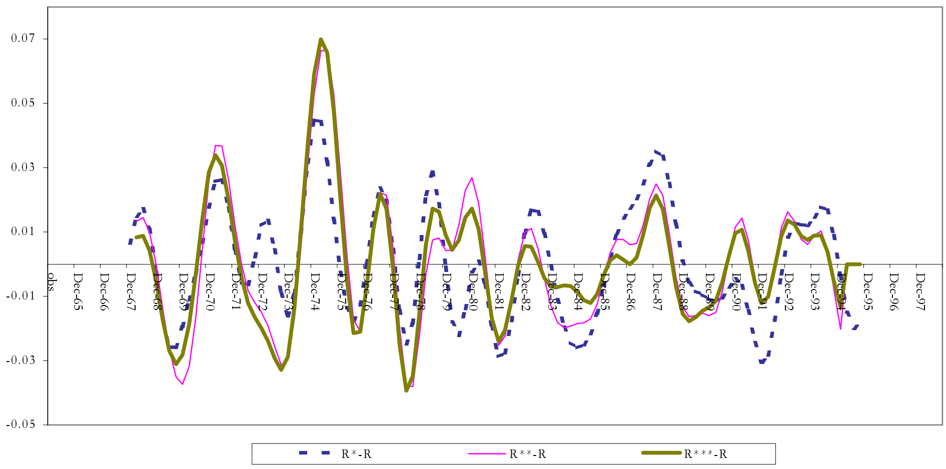

Figure 4: Wicksell Gap for TFP and corrected TFP

productivity shocks, are familiar from many studies in the RBC literature (e.g., see the simulation results reported in King and Rebelo, 1999), although recall that here we are working in an overlapping generations framework. The model basically ranks variables, in terms of their standard deviations, in a manner consistent with the data (although notably it exaggerates the smoothness of consumption). In terms of the contemporaneous correlations with output per head, all of our results are consistent with the data, although the degree of these correlations is high, again a familiar result. We note, in particular, that the productivity-driven model captures both the fact that the current account is a relatively volatile countercyclical variable and that the real rate is procyclical but relatively smooth. The striking aspect of this result is the extent to which the artificial data captures the dynamic, as well as contemporaneous, correlations of the observed data, positive leads for consumption, investment, real interest rates, hours worked, real wages and the capital stock and negative leads for the current account. In each of the two models we consistently find that the marginal productivity of capital is procyclical. That is, the productivity shock, in either case, raises the marginal productivity of capital and leads to consumption growing while the real rate is above the rate of time preference. Figure 4 shows the difference between the business cycle component of the simulated real rate series and observed real rates in the UK.[(12)]

Tables 5 and 6 reflect the model of the high-substitution economy driven by the correct TFP shock with persistence set at 0.99 and 0.70, respectively. Overall, the fit seems less impressive. There are a number of striking issues.

[12] R* refers to the standard TFP model, R** to the high-substitution economy with a highly persistent shock and R*** to the high-subsitution economy with a less persistent shock.

First, output appears about twice as volatile as the observed data, but more worrying is the variability in the generated investment and current account data, some 14 to 15 times more volatile as output compared to less than twice as volatile in observed data. The model also generates counterfactually signed contemporaneous correlations between the current account and output, but the current account is found to have consistent negative leads for output. The main failures of the high-substitution economy, when confronted with the data, are in the volatility of the main expenditure series; consumption is far too smooth and too small a fraction of output, while investment, the current account and the capital stock are too highly volatile. This is the result of the high-substitution economy amplifying what are in fact less volatile supply shocks, which we discover when we strip out the effects of capacity utilisation.[13] The high-substitution economy produces more elastic output responses (see equations 39 to 25) from which, in the case of non-separable utility, little is imparted to consumption and results in highly volatile investment and, in this model's complete market, current account balances.[14]

The results presented in this section would suggest that a model driven by productivity innovations, corrected for capacity utilisation, seems a reasonable approximation to the covariation of aggregate data with business cycle fluctuations and the dynamics of the business cycle.

5.1 Wicksellian Flavours

There has been much interest recently in the Wicksellian interpretation of monetary policy (see Woodford, 2000) as an interest rate feedback rule. The suggestion has been made that the evolution of the real sector of an economy sets the correct target for monetary authorities wishing to stabilise nominal and real variables. It is assumed that any deviation from this 'natural rate' of interest will affect inflation and growth outcomes.[15] We therefore ask to what extent the difference between the dynamic equilibrium real rate of interest in our purely real economy and the observed real rate in the UK is related to developments in nominal income. Figure 5 plots this difference, $R{*}-R$ which we call the Wicksell gap, for three of our four sets of calibrations: the TFP model, and the capacity-stripped model with high and low persistence. The coincident pattern is striking.

Table 8 gives the correlations of the Wicksell gap and nominal income, inflation and output growth for the whole sample and for samples before and after the second oil-price shock. We find that each of the macroeconomic variables are closely matched by developments in the Wicksell gap. We find

[13] Note that the standard devation of the TFP shock is 2.07%, compared to 1.62% in the case of stripped capacity utilisation.

[14] In future work we will modify this effect by lowering the elasticity of intertemporal substitution to below 1. Table 7 presents the results of the high-substitution economy shocked with TFP stripped of capacity utilisation by Kalman filter, rather than proxy. This changes little.

[15] See Fischer (1972) for an earlier exposition along these lines.

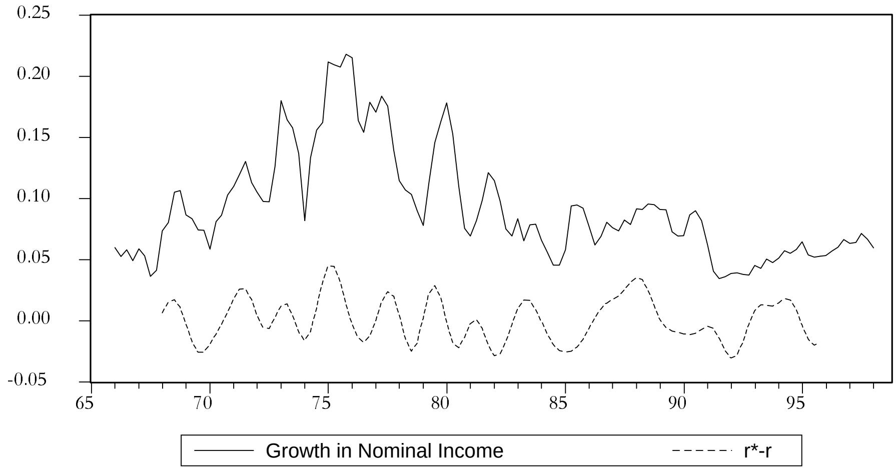

Figure 5: Nominal Income Growth and the UK Wicksell Gap (1965-98)

evidence to suggest that in the first half of our sample the Wicksell gap was inflationary, whereas later it seemed more closely related to output growth. The flattening of the short-run Phillips curve is an issue we leave to future research. We do find, however, that the Wicksell gap seems to be exogenous to macroeconomic developments: in not one case does either output or inflation Granger cause the Wicksell Gap.

6 Concluding Remarks

This paper has examined the hypothesis that in a model driven by supply shocks, defined in one of two ways, the marginal product of capital will be procyclical. We have found evidence to support this hypothesis. Both the economy with low substitutability and the economy with high substitutability produce relatively stable real rates, but the low-substitution economy tends to experience significantly greater procyclicality of real rates. This excessive cyclicality in generated data is often a poor diagnostic test of real business cycle models, but in this case it leads to a Wicksell gap that seems well placed to explain macroeconomic outcomes. We might therefore be tempted to suggest that however we define supply shocks, it would appear that a monetary authority concerned with nominal income volatility would seek to adopt an aggressively procyclical attitude to the setting of a short-run interest rate.

In future work we will extend these results to incorporate a policy rule and a fully specified nominal sector so that, *inter alia*, alternate policy rules can be properly evaluated.[16] But we suggest that the basic insight will remain that a monetary authority concerned about stabilising inflation and output over the course of the business cycle should not fail to account for the impulses to inflation and output generated by the more primitive process of economic growth itself. That care must be taken in measuring the process of economic growth (see Crafts, 2000) does not alter the strength of this message.

[16] Naturally, as the real world world data contains monetary policy shocks we would like to incorporate an explicit derivation for our explanation of inflation and output outcomes within this framwork.

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Tables

Table 1. Observed Data - Summary UK facts

Variable	σ	y_{t-4}	y_{t-3}	y_{t-2}	y_{t-1}	y_t	y_{t+1}	y_{t+2}	y_{t+3}	y_{t+4}
y	1.59									
c	1.52	0.491	0.598	0.706	0.785	0.805	0.749	0.624	0.459	0.284
i	3.76	0.430	0.563	0.671	0.736	0.744	0.690	0.585	0.453	0.315
w	0.91	0.310	0.220	0.182	0.177	0.163	0.108	0.013	-0.093	-0.179
ca	3.83	-0.385	-0.469	-0.532	-0.565	-0.561	-0.513	-0.414	-0.272	-0.113
n	1.67	0.520	0.646	0.769	0.840	0.817	0.691	0.491	0.270	0.075
k	0.10	0.180	0.119	0.063	0.017	-0.014	-0.034	-0.044	-0.056	-0.075
r	1.69	-0.076	0.043	0.156	0.244	0.300	0.320	0.310	0.274	0.210

Notes: The presented moments refer to the bandpass-filtered data with 12 quarters' weights; see Baxter and King (1999) for further details. The full sample for all data is 1963:1 to 1998:4.

Table 2. Real Rates and the Cycle

Panel A:

Annual Postwar	**B-K**	**C-F**	**H-P**
Corr.(y,r)	0.47	0.48	0.46
s.d.(y)	1.41	1.43	1.10
s.d. (r)	1.82	1.85	1.90

Notes: y=GDP per head; r=real interest rate; s.d.=standard deviation in percentage terms; and Corr=correlation coefficient. Results for Panel A, from Chadha et al. (2000), refer to an annual UK study over the period 1871-1997 and to three different filtering procedures: Baxter-King, Christiano-Fitzgerald and Hodrick-Prescott.

Table 3. Quarterly percentages

λ	r	α	ψ	γ	δ
0.373	1.25	38	2.5	0.5	0.75

Notes: The presented moments refer to the bandpass-filtered data with 12 quarters' weights; see Baxter and King (1999) for further details. The full sample for all data is 1963:1 to 1998:4.

Table 4. Model Output - Total Factor Productivity

Variable	σ	y_{t-4}	y_{t-3}	y_{t-2}	y_{t-1}	y_t	y_{t+1}	y_{t+2}	y_{t+3}	y_{t+4}
y	1.59									
c	0.45	0.120	0.336	0.592	0.824	0.960	0.940	0.801	0.605	0.416
i	7.28	-0.152	0.072	0.354	0.632	0.829	0.877	0.798	0.651	0.499
w	0.78	0.237	0.445	0.679	0.882	0.990	0.944	0.791	0.589	0.396
ca	6.40	0.271	0.051	-0.237	-0.529	-0.747	-0.819	-0.765	-0.637	-0.50
n	0.82	0.478	0.652	0.827	0.955	0.990	0.896	0.720	0.515	0.327
k	0.99	0.879	0.889	0.837	0.719	0.552	0.355	0.174	0.025	-0.092
r	1.33	-0.178	0.045	0.329	0.610	0.812	0.865	0.793	0.650	0.501

Notes: The presented moments refer to the bandpass-filtered data with 12 quarters' weights; see Baxter and King (1999) for further details. The full sample for all data is 1963:1 to 1998:4.

Table 5. Model Output - Capacity Utilisation Stripped TFP Shock, $\rho = 0.99$

Variable	σ	y_{t-4}	y_{t-3}	y_{t-2}	y_{t-1}	y_t	y_{t+1}	y_{t+2}	y_{t+3}	y_{t+4}
y	4.43									
c	0.642	-0.211	0.053	0.396	0.732	0.958	0.975	0.789	0.463	0.101
i	69.84	-0.566	-0.725	-0.697	-0.438	-0.010	0.419	0.706	0.771	0.629
w	1.61	-0.122	0.187	0.540	0.841	0.994	0.931	0.686	0.339	-0.006
ca	70.17	0.561	0.738	0.729	0.486	0.065	-0.368	-0.670	-0.755	-0.632
n	2.83	-0.037	0.302	0.654	0.915	0.998	0.864	0.571	0.216	-0.103
k	4.36	0.017	0.366	0.711	0.946	0.989	0.818	0.502	0.143	-0.161
r	0.66	-0.565	-0.662	-0.569	-0.264	0.178	0.579	0.809	0.805	0.603

Table 6. Model Output - Capacity Utilisation Stripped TFP Shock, $\rho = 0.70$

Variable	σ	y_{t-4}	y_{t-3}	y_{t-2}	y_{t-1}	y_t	y_{t+1}	y_{t+2}	y_{t+3}	y_{t+4}
y	5.67									
c	0.04	0.352	0.547	0.720	0.807	0.751	0.536	0.212	-0.139	-0.432
i	82.83	-0.579	-0.725	-0.678	-0.404	0.031	0.452	0.723	0.770	0.616
w	1.34	-0.076	0.254	0.609	0.888	0.999	0.895	0.621	0.271	-0.057
ca	82.84	0.574	0.723	0.720	0.465	0.038	-0.391	-0.681	-0.752	-0.620
n	4.33	-0.070	0.259	0.613	0.890	1.00	0.892	0.617	0.265	-0.064
k	5.14	0.033	0.385	0.726	0.953	0.985	0.804	0.484	0.126	-0.172
r	1.08	-0.509	-0.456	-0.231	0.143	0.561	0.851	0.927	0.775	0.469

Table 7. Model Output - Capacity Utilisation Stripped (by Kalman Filter) TFP Shock, $\rho = 0.99$

Variable	σ	y_{t-4}	y_{t-3}	y_{t-2}	y_{t-1}	y_t	y_{t+1}	y_{t+2}	y_{t+3}	y_{t+4}
y	3.98									
c	0.57	0.223	0.415	0.633	0.839	0.975	0.981	0.859	0.650	0.415
i	49.74	-0.516	-0.568	-0.515	-0.305	0.030	0.370	0.597	0.657	0.589
w	1.45	0.286	0.496	0.718	0.904	0.997	0.956	0.801	0.583	0.357
ca	49.88	0.537	0.605	0.568	0.370	0.040	-0.305	-0.544	-0.691	-0.567
n	2.54	0.339	0.562	0.783	0.946	0.999	0.916	0.7351	0.512	0.299
k	3.88	0.380	0.604	0.819	0.965	0.993	0.885	0.687	0.460	0.250
r	0.48	-0.416	-0.418	-0.318	-0.076	0.263	0.575	0.752	0.756	0.637

We measure capacity utilisation in the final model using the Kalman filter to proxy for cyclical movements in capacity utilisation.

Table 8. Wicksell Gap and Inflation and Output Outcomes

X		$R^* - R$	$R^{**} - R$	$R^{***} - R$	$X \rightarrow R^* - R$	$R^* - R \rightarrow X$
Nominal Income	*Full*	0.366	0.340	0.344	0.82 (0.59)	3.33 (0.00)
	$Pre - 1980$	0.531	0.589	0.564	1.13 (0.36)	2.83 (0.04)
	$Post - 1980$	0.028	-0.061	-0.108	1.28 (0.29)	2.46 (0.06)
Inflation	*Full*	0.180	0.420	0.442	0.68 (0.70)	3.21 (0.00)
	$Pre - 1980$	0.475	0.635	0.652	2.77 (0.04)	1.20 (0.33)
	$Post - 1980$	-0.366	0.058	0.005	1.39 (0.25)	5.49 (0.00)
Output Growth	*Full*	0.292	-0.210	-0.244	1.17 (0.33)	0.76 (0.64)
	$Pre - 1980$	-0.075	-0.298	-0.376	0.59 (0.67)	1.17 (0.34)
	$Post - 1980$	0.532	-0.142	-0.118	2.28 (0.07)	4.45 (0.00)

We present simple correlations of the difference between the calibrated real rate from the standard model (R-R*) and the two versions of the high-substitution economy (R-R^{**} and R-R^{***}). The final two columns present the Granger causality tests for the null of the variable in column 1 (X) not causing the R-R^* and R-R^* not causing X, respectively. Note that we employ 8 lags for the full sample tests and 4 lags for the subsamples. The results of Granger causality tests for the two versions of the high-substitution economy are available on request.