Abstract:

This paper sets out to address the issue of equity duration, one of several risk measures available for asset and liability management. Equity duration, as derived from the use of traditional dividend discount models, results in extremely long duration estimates for equities - often in excess of 50 years for growth stocks. Leibowitz, in his seminal paper (1986), identified an alternative framework for assessing equity duration empirically. This methodology yields equity duration measures more consistent with the experience of practitioners, implying that equities behave as if they are much shorter duration instruments. In our paper, based on an application to UK data, we develop the intuition behind the Leibowitz approach to generate equity duration as a by-product of asset pricing. Our analysis suggests that the equity premium puzzle may comprise an important element in reconciling the Leibowitz approach to equity duration, with the more traditional dividend discount model alternative.

THE DERIVATION OF A NEW MODEL OF EQUITY DURATION

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1. Introduction:

Our paper considers an extension of duration, a standard measure of the price volatility of a bond as defined in the fixed income literature, to equity securities. Fund managers typically use duration in their investment schedules as a method of matching fixed income instruments against known contractual liabilities. Thus immunisation, as first recognised by Redington (1952), was defined as "the investment of the assets in such a way that the existing business is immune to a general change in the rate of interest". Immunisation corresponds not only to the matching of assets with the present value of liabilities, as formalised by Fisher and Weil (1971), but also to the replication of the interest rates sensitivities themselves, as discussed in Bierwag, Kaufman and Toevs (1983). This creates an ostensibly immunised frontier position, or fund surplus, against stochastic changes in interest rates, which consequently have no effect on final valuation. Matching liabilities in this manner adds value directly, by allowing corporate management to focus on their core business, while simultaneously allowing shareholders and future pension recipients to monitor the level of available funds more precisely. International accounting standards enforce a mark-to-market convention for pension fund reporting, thereby exacerbating the need for an improved methodology for quantifying equity duration risk.

In the fixed income literature, Macaulay (1938) first defined what subsequently became known as Macaulay duration[2], as the weighted average term to maturity of the cash flow of a bond. The weights are simply the present values of each of the anticipated cash flows, both coupons and principal, as a percentage of the price of the bond. Macaulay chose each cash flow's contribution to the total price as its weight, which although somewhat arbitrary, remains extremely close to the measure in use today. Duration has been subsequently defined by Fabozzi (1997) as "the approximate percentage change in the price of a bond or bond portfolio to a 100 basis point (1%) change in yields". Hence, the concept of duration expresses the amount by which price fluctuates with respect to changes in underlying bond yields[3].

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[2] Independently, Hicks (1939) contemporaneously derived a similar measure to Frederick Macaulay.
[3] Elton and Gruber (1999) identify the mapping between Macaulay's 'average maturity' duration and Fabozzi's 'elasticity' duration measures.

This principally depends on the term to maturity and associated coupon rate, as well as any embedded options that may be present in more complex fixed income instruments. More generally these combined effects may be captured by a single summary measure of duration[4], which is the differential of the price yield function. Although duration is considered an invaluable tool for gauging the sensitivity of fixed income portfolios to movements in underlying interest rates, fund managers essentially require the derivation of a similar model for their equity portfolios. Such a tool could then have further application to other asset classes held by pension fund managers, to provide for a more complete measure of interest rate risk. The standard Dividend Discount Model (hereinafter referred to as the DDM), as reviewed by Bodie, Kane and Marcus (1996), provides extremely long duration estimates for equity securities. This methodology has been criticised by practitioners for its unrealistic treatment of the observed pricing elasticity of equity securities with respect to changing discount rates.

This problem is particularly acute in the pension industry, where trustees are typically faced with very clear liability schedules arising from the nature of defined benefit packages on offer. It is the trustee's irrefutable responsibility to offset such liabilities, and hence duration matching has become a recognised procedure to immunise pension assets against interest rate risk. Thus, the pension industry would particularly benefit from a more intuitive theoretical framework within which to match specific contractual liabilities against the overall duration of a multi-asset class portfolio. In the absence of a general formula for equity duration, the assets available for stringent portfolio dedication need to be restricted to those of the fixed income variety. Pension mandates, however, are characterised by the recent tendency to increase equity investment, to underpin the long-term growth in assets as interest rates have fallen, leaving the industry vulnerable to the adoption of an unsatisfactory hybrid arrangement of partially immunised assets and liability schedules. The absence of a suitable theoretical solution to the problem of quantifying equity investment interest rate exposure has become a more urgent issue as global competition for international pension mandates intensifies. Indeed according to UK figures compiled by the Office of National Statistics in 2000, equity securities accounted for less than 50% of pension fund assets in 1965, rising to represent very nearly 80% by the end of the 20th century. The issues raised by an ageing population, coupled with increased flexibility in western labour markets, has exacerbated the requirement for greater equity holdings in the typical pension fund, to sustain benefits for an expanding occupational and private pension sector. Indeed a significant fraction of a country's population now undertakes a variety of jobs over a lifetime, so that portability in pension design allowing for disrupted contribution schedules is now an essential consideration. The onus has fallen back on providers to maximise the return schedules on defined contribution schemes, exacerbating the reliance on the equity component of investment returns. It is the rapidity and magnitude of these combined changes, which are causing trustees to focus attention on better ways of measuring asset portfolio sensitivities to macroeconomic changes in the economy.

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[4] The interested reader is referred to Bierwag (1987) for a discussion of different duration measures.

Because anticipated demographic trends reinforce the need for higher levels of funding to ensure sustainability in future pension provision, fund managers have adopted increasing equity exposure in ongoing schemes. Equities are considered well suited to long-term investment planning, providing considerable financial growth as well as acting as a natural hedge against inflation, as discussed by Bodie (1976). Indeed, equity investment now plays such a dominant role in most funded pension schemes that a better theoretical understanding of the behaviour of such instruments is of real economic significance. Any structural mismatch between assets and liabilities will necessarily increase the volatility in pension fund earnings, and hence affect the overall future pension fund surplus of existing schemes. Even the historically unfunded UK State Earnings-Related Pension Scheme (SERPS) is tentatively moving towards a more funded basis, with the introduction of the new Individual Savings Account (ISA). The Myners' report into pension reform and the Minimum Funding Requirement places greater emphasis on the accountability of trustees for selecting suitable investments in pension arrangements.

The recent tilt in asset allocation decisions towards more substantial emphasis on equity investment has created additional risks among pension fund providers. These issues need to be quantified through a better understanding of equity duration, which we proceed to investigate in Section 2. In this section, we analyse the traditional DDM approach to derive a suitable measure of equity duration. We complement this investigation with an examination of an alternative intuitive framework for calculating equity duration empirically. This concept was first proposed by Leibowitz (1986) and subsequently considered in relation to the traditional DDM approach by Arnott, Hanson, Leibowitz and Sorensen (1989), (herein after referred to as Leibowitz *et al*. (1989)). The initial Leibowitz model provided equity duration estimates for the stock market of around two to six years, whereas the traditional DDM methodology often results in much longer equity duration estimates - in excess of fifty years for high growth stocks. The theory behind our asset-pricing model investigating this apparent paradox is subsequently derived in Section 3. It is important to note at this juncture that Johnson (1989) has also presented an alternative explanation for this anomaly, in which he emphasises that the differences arise from estimating duration from price, as opposed to return sensitivity. Whilst we recognise this important contribution, which examines the separate issues of price and reinvestment risk, such an approach does not form an integral part of our own analysis.

For completeness, however, it ought to be stressed that our paper considers equity duration as pertinent to the pension fund management industry, and not to the internal asset and liability measurement referred to in the banking literature as 'equity duration'. In fact, in the two articles by Idol (1997), the term 'equity duration' is used, erroneously in our view, in reference to the effective asset duration of a bank's corporate assets and internal funding sources. Simonson (1993) also examined the interest rate exposure of a banking institution's equity capital base, as derived from the duration of its aggregate loan and deposit portfolios, to calculate yet another, albeit distinct misnomer, once again referred to as 'equity duration'.

It is important to note that whilst such additional concepts exist in the broader literature, they are quite specific and distinct from the definitions we intend to use in our asset pricing approach, in modelling the interest rate features of listed equity securities. Our empirical equity duration calculations, based upon an application to UK market data, are reported in Section 4. Additionally we include a further subset of results arising from an extension of our work towards resolving an additional practical investment consideration, the question of a finite survival horizon for equity securities. The real possibility of bankruptcy should undoubtedly play an important part in regard to formulating long-term equity investment strategies, as applicable to the pension's industry. In our subsequent Addendum section, we therefore further explore some of the pertinent fiscal and insolvency issues that we consider may be of importance when underpinning any reconciliation between the differing equity yields observable across global markets.

This is an important secondary issue with respect to our main paper, since any subsequent empirical calculations for equity duration will be materially influenced by divergent yield characteristics and observed corporate failure rates among listed companies, across the segmented international investor markets. In Section 5, we conclude our article by discussing the further implications of our study and the scope for future research, as well as framing our analysis within the context of global portfolio management. The unsatisfactory resolution of the legitimacy of the inclusion of equity instruments into what are otherwise ostensibly dedicated pension portfolios, clearly merits such further investigation. The need for higher returns, as pension liabilities become more onerous due to demographic effects, have steered investment committees towards placing higher contributions into equity holdings. But this situation appears precarious unless supported by a fundamental quantitative assessment of equity portfolio behaviour to macroeconomic events. The traditional DDM literature provides ambiguous evidence, given the practical experience of investment managers. Our work thus aims to synthesize an alternative approach, by reconciling the needs of the investment community with academic integrity proffered by the traditional DDM approach.

2. Alternative Methodologies for Estimating Equity Duration:

In the academic literature, aside from the traditional DDM approach, there have been other proposals for constructing measures of equity duration for use in pension management, in particular we focus on the seminal work of Leibowitz (1986). The computation of equity duration in this paper is based upon the observable historic correlation between stocks and bonds. This elegant and intuitive formulation relies upon the variance parameters found in conventional asset allocation procedures, and provides for considerably shorter equity duration measures than those usually obtained from utilising the more orthodox DDM approach. We will therefore begin our work by outlining this methodology, where the correlation between stocks and bonds is used in conjunction with a measure of bond market duration to derive an estimate for stock market duration. In combination with the typical portfolio allocation decision between stocks and bonds, the beta of the stock component of any given portfolio can readily be used to obtain a measure of total portfolio duration to assist in asset and liability matching. This is an appealing intuitive solution to the equity duration problem, pertaining to a CAPM style framework. This requires the estimation of *ex ante* values for the variance of stock market returns, the variance of bond market returns and their historic correlation as follows:

$$D_E = \left(\frac{\sigma_E}{\sigma_B} \right) \rho(E,B) D_B, \qquad \text{[Leibowitz (1986), equation (1)]}. \qquad (1.0).$$

[Where D_B is the duration of a broad based measure of the bond market, D_E is the estimated duration for the equity market, σ_B is the standard deviation of the bond market index returns, σ_E is the standard deviation of the equity market returns, and $\rho(E,B)$ is the correlation between the bond and equity market returns].

The co-movement of equity returns R_E can then be expressed with respect to bond returns R_B:

$$\widetilde{R}_E - R_F = A_1 + B(\widetilde{R}_B - R_F) + \widetilde{\varepsilon}, \text{ [Leibowitz (1986), equation (A1)]} \qquad (1.1a).$$

[Where R_F represents the risk-free rate and $\widetilde{\varepsilon}$ represents non-bond market factors affecting equity returns, given that $E(\widetilde{\varepsilon}) = 0$ and $E(\widetilde{\varepsilon}, \widetilde{R}_B) = 0$].

The regression coefficient, relating D_E and D_B in equation (1.0), when combined with stock market returns, can thus be expressed as a linear function of $\widetilde{\delta}$, the change in any given long-term benchmark yield[5].

$$R_E = A - D_E \widetilde{\delta} + \widetilde{\varepsilon}, \quad \text{[Leibowitz (1986), equation (2)]}. \qquad (1.1b).$$

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[5] The reader is referred to Appendix A, in Leibowitz (1986), for the complete derivation of his model.

In the Leibowitz paper (1986), $\tilde{\delta}$ could be specified as any long-term benchmark bond index relevant to pension fund management, and thus the model relies on the effective duration of the bond market relative to yield shifts in any given benchmark. The final step necessary in combining these intuitive concepts allows for the establishment of a direct statistical relationship between the returns on the representative equity market index and changes in the representative benchmark yield. The resulting stock market durations are thus simply empirical estimates of actual stock price sensitivity to underlying changes in interest rates. These duration measures are by their very nature purely statistical relationships and would be subject to randomness. However, they retain econometric credibility in relating stock market returns directly to variations in long-term interest rates. In the follow-up paper, Leibowitz *et al*. (1989), the authors build upon the original conceptual development in Leibowitz (1986) to derive a measure of equity duration through price sensitivity to the real and inflationary components of nominal interest rates. We derived our approach by examining the models proposed by Leibowitz *et al*. (1989), which attempt to incorporate the impact of real interest rates and inflation within the DDM approach. Leibowitz *et al*. (1989) reformulates the DDM to obtain a total differential for stock price, or mean adjusted DDM duration (hereinafter referred to as D_{DDM}), as follows:

$$\frac{d\,P}{P} = -D_{DDM}\left(1 - \gamma + \frac{\partial\,h}{\partial\,r}\right)dr - D_{DDM}\left(1 - \lambda + \frac{\partial\,h}{\partial\,I}\right)dI \equiv -D_{DDM}dk\,.^{[6]} \qquad (1.2).$$

[Where the nominal discount rate for equities, k*,* is split into an inflation rate I, and a real return r, h is an equity premium, γ represents growth rate sensitivity to real interest rates and λ is an inflation flow-through parameter.]

Although our research is motivated by the concepts within these earlier papers, our own analysis differs from this previous work, by incorporating real changes in consumption and dividend growth directly into our model, to derive equity duration from stock market data as a by-product of asset pricing. We have re-examined the derivation of the earlier equity duration measures, such as the DDM, and instead have chosen to combine the variance parameters used in asset allocation decisions, to emphasise the importance of the covariance between changes in real consumption and real dividend yields, for subsequent portfolio immunisation strategies. The use of such an approach enables us to overcome one of the standard criticisms levelled at the DDM, and cited by Hurley and Johnson (1995), that of failing to incorporate within the model realistic patterns of future dividend growth. The appropriateness of the traditional DDM approach has been further challenged by the anecdotal evidence presented by the persistent and considerable divergence in prospective broker forecasts for individual stocks. If opinion within the brokerage profession is itself divided, when it comes to analysing and forecasting the fundamental earnings and dividend growth rates of firms, what useful conclusions can fund managers draw?

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[6] The reader is referred to equation 8 in Leibowitz *et al*. (1989) for the full derivation of this model.

This highlights a major practical caveat in any application of the DDM, since the more complicated task undertaken by fund managers is that of continuously reassessing a company's longer-term performance, which would be confounded by long-term interest rate volatility itself. This is often masked by the mechanistic solutions provided for equity duration, as inculcated in the traditional DDM, since the value of the underlying corporate earnings will be implicitly affected by real changes in aggregate consumption. Leibowitz *et al*. (1989) modified the traditional DDM techniques at this critical stage; by assuming that real interest and inflation rates are in fact the underlying variables that "relate changes in the interest rate and the equity risk premium to equity duration". Their approach re-examined the equity discount rate, dissecting it into a real interest rate, an inflation rate and an equity premium component.

The verification of such a method of analysis is of critical importance, as any immunisation strategy employed against the current liability structure of portfolios (where they will inevitably include an equity component) fundamentally relies upon the confidence pertaining to the duration values thereby assigned. Of course, these duration measures imply some relationship between duration and the established view of market participants in the pension community, as explored for the interested reader in our Appendix 3. By considering equities as part of the financial assets available for pension fund investment, we recognise that the composition of a scheme is an individual investment decision and thus a utility-based model appears an appropriate foundation. Therefore, we adopt a utility framework, wherein the equity risk-premium can be related to aggregate consumption. This in turn implies a link between the equity premium puzzle and reconciling the traditionally long duration estimates for equities. We consider that fund managers, as a central tenet of their investment strategy, are continually assessing and revising the risk premium for specific companies. We believe that this is not the same concept as purely mean adjusting the level of a constant risk premium term structure, as advocated in the Leibowitz *et al*. (1989) paper. The majority of the risk premium models in finance are single period models of one form or another, and so it is natural for us to start from the original single-period DDM approach. The work of Damant and Satchell (1995) will therefore be used as the basis for incorporating the important macroeconomic variable of real consumption into the traditional DDM framework. When considering common stocks, we begin by assuming that the mandate of the incumbent management team is to maximise shareholders' wealth in the context of the current business-operating environment.

This notion is justified by management's vulnerability to potential replacement by shareholders' votes, cast at the annual general meeting, in accordance with *pro rata* interest in the firm's equity capital. However, the dispersed nature of individual and even institutional shareholdings may dilute the ready exercise of these ultimate powers of administrative veto. Nonetheless, shareholders' wealth, or the price of a common stock, can be evaluated using the firm's expected profitability or more specifically the discounted expected after-tax cash flows that could be proportionately distributed to shareholders as dividend payments:

$$P = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t} \ , \qquad (1.3).$$

This is the generalised form of Williams' (1938) seminal dividend discount model:

$$P = \sum_{t=1}^{\infty} \frac{D_t}{(1+\hat{r}_t)^t} \ , \qquad (1.4).$$

      This multi-period model assumes that the future cash flows are known with certainty and that the market determined interest rate is non-stochastic and constant over all time-periods. In practical terms, investors are faced with a conundrum as to which discount rate to apply across such future investment horizons. This is due to the continual re-shaping and twisting of whatever benchmark yield curve is used. Such an obvious dilemma suggests that perhaps we should be thinking of the problem in terms of an equity risk premium, implicit within the discount structure. In the following generalisation, $\hat{r}_t$ is taken as the yield on a risk free bond over the period and Π_t can be considered as a per unit-time risk premium.

$$P = \sum_{t=1}^{\infty} \frac{D_t}{(1+\hat{r}_t+\Pi_t)^t} \ , \qquad (1.5).$$

However, the more traditional approach leads us to the following generalised formula:

$$P = \sum_{t=1}^{\infty} D_0 \frac{(1+g)^t}{(1+k)^t} \ , \qquad (1.6).$$

This can be rearranged thus: $P = \dfrac{D_0(1+g)}{(k-g)} \ , \qquad (1.7).$

And taking logs of equation (1.7) we arrive at:

$$\ln P = \ln D_0 + \ln(1+g) - \ln(k-g), \quad (1.8).$$

By differentiating this result, we obtain the standard DDM duration formulation:

$$\mathrm{D}_{\mathrm{DDM}} = -\frac{\mathrm{d}\ln P}{\mathrm{d}\,k} = \frac{1}{(k-g)} \ , \quad \text{[Leibowitz \textit{et al.}, equation 4]} \qquad (1.9).$$

      The preceding equation will be recognised as the DDM of Gordon (1962) that has been criticised for its lack of practical considerations, most notably the assumption that dividend growth is modelled as an infinite geometric progression.

3. Calculating Equity Duration from the Rubinstein Model:

In this paper, we have elected to build upon the more recent work of Damant and Satchell (1995) and apply a variation of that work in addressing the classic equity duration problem. This should allow us to circumvent what we consider as the underlying practical limitations of the Leibowitz *et al*. (1989) methodology. Thus, we are able to include an equity premium directly into our revised formula, with our discrete-time inter-temporal model echoing the earlier work of Rubinstein (1975):

$$P_t = \sum_{i=1}^{\infty} \frac{E_t(D_{t+i}Y_{t+i})}{(1+r_{t,t+i})^i E_t(Y_{t+i})} \quad , \tag{2.0}$$

While the above equation is extremely general, further assumptions can be made such as those presented in Rubinstein (1975) Theorem 2, which imply that the state variable Y_t is the marginal utility of consumption of the representative agent, i.e. $Y_t = U'(C_t)$.

Thus we can begin with an equation of the following form:

$$P_t = \sum_{i=1}^{\infty} \frac{E_t(D_{t+i}Y_{t+i})}{(1+r_t)^i E_t(Y_{t+i})} \quad , \tag{2.1}$$

As implied in equation (1.5), r_t can be split into a real yield component, defined here as k, and an equity premium related to consumption.

$$P_t = \sum_{i=1}^{\infty} \frac{E_t(D_{t+i}Y_{t+i})}{(1+k)^i E_t(Y_{t+i})} \quad , \tag{2.2}$$

Thus, by taking logs of (2.2):

$$\ln P_t = \ln \sum_{i=1}^{\infty} \frac{E_t(D_{t+i}Y_{t+i})}{(1+k)^i E_t(Y_{t+i})} \quad , \tag{2.3}$$

In addition, by differentiating (2.3), we can now define duration as follows:

$$\frac{d \ln P_t}{d\,k} = \sum_{i=1}^{\infty} i \frac{E_t(D_{t+i}Y_{t+i})}{(1+k)^{i+1} E_t(Y_{t+i})} \Big/ P_t \quad , \tag{2.4}$$

We will now add an extra set of assumptions, in similar manner to the treatment pursued in Damant and Satchell (1995). Thus, the logs of UK aggregate consumption and UK aggregate dividends are defined as being random walks with drift, such that they will jointly follow a bivariate random walk with drift. We further define our representative agent to have a power utility function, consisting of the following assumption:

Assumption 1:

$$U(C_t) = \frac{C_t^{1-\beta}}{1-\beta}, \qquad U'(C_t) = C_t^{-\beta} = Y_t, \qquad U''(C_t) = -\beta C_t^{-(1+\beta)},$$

By substituting Assumption 1 into equation (2.4), we arrive at the following representation, where d represents duration:

$$d = \frac{d \ln P_t}{d k} = \sum_{i=1}^{\infty} i \frac{E_t(D_{t+i} C_{t+i}^{-\beta})}{(1+k)^{i+1} E_t(C_{t+i}^{-\beta})} \bigg/ P_t \quad, \tag{2.5}$$

Now making the assumption that $\beta = 0$, which is the risk neutral case, then (2.5) will become:

$$d = \sum_{i=1}^{\infty} i \frac{E_t(D_{t+i})}{(1+k)^{i+1}} \bigg/ P_t \quad, \tag{2.6}$$

By assuming that $E_t(D_{t+i}) = D_t(1+g)^i$, where g is the expected growth rate in dividends, we can solve for $P_t = \frac{D_t(1+g)}{(k-g)}$, to find again that $d = \frac{1}{(k-g)}$, as in (1.9).

More generally however, we need to specify certain assumptions about the joint probability density function (pdf) of D_t and C_t, which we will now assume follows a bivariate lognormal random walk.

Assumption 2:

$$\ln(D_t) = \alpha_d + \ln(D_{t-1}) + \varepsilon_{dt}, \tag{2a}$$
$$\ln(C_t) = \alpha_c + \ln(C_{t-1}) + \varepsilon_{ct}, \tag{2b}$$

$$\text{where } (\varepsilon_{dt}, \varepsilon_{ct})' \sim N\left[(0,0)', \begin{pmatrix} \sigma_{dd} & \sigma_{dc} \\ \sigma_{cd} & \sigma_{cc} \end{pmatrix}\right].$$

Proposition 1:

If Assumptions 1 and 2 hold, then the price of an asset P_t is given by:

$$\ln(P_t) = \ln(D_t) - \ln(1 + k - \phi) + \ln \phi \quad, \tag{3.1}$$

$$\text{where } \phi = \exp\left[\alpha_d + \frac{1}{2}\sigma_{dd} - \beta\sigma_{cd}\right],$$

$$\text{Thus, } d = -\frac{d\ln(P_t)}{d k} = \frac{1}{1+k-\phi} \quad, \tag{3.2}$$

where k represents real yields, and $\phi < 1 + k$. [Proof: see Appendix 1].

The above calculations can be considered a form of the Gordon Growth Model, where ϕ corresponds to the $(1+g)$ term, and represents the expected per period risk-adjusted relative dividend growth. Since $\beta \geq 0$ we see that if $\sigma_{cd} > 0$, which we would expect, an increase in relative risk aversion leads to a decrease in ϕ and hence a decrease in equity duration. However, we must be aware of the equity premium puzzle, as identified by Mehra and Prescott (1985), which suggests that for power utility, risk premia are far too low to explain empirical values. Alternatively, β needs to be quite large to generate appropriate risk premia of the magnitude estimated in mature financial markets. Accordingly, we select to use an alternative linear risk-tolerance utility function, the HARA specification, whereby we redefine $U(C_t)$ as follows:

Assumption 3:
$$U(C_t) = (C_t - \overline{C}_t)^{1-\beta} / 1 - \beta; \ \beta > 0, \ C_t > \overline{C}_t; \ \overline{C}_t > 0 .^{7}$$

Here $\overline{C}_t$ represents base consumption, or the minimum subsistence level of consumption required to provide the necessities in life. This is analogous to what Marx considered the proletariat's starvation bundle. We will therefore require a procedure such that C_t will always be greater than the $\overline{C}_t$ subsistence level in all periods. We need to choose $\overline{C}_t$ in a systematic manner, and so we refer to the earlier work on external-habit models by Abel (1990) in formulating a difference approach. We thus define base consumption as an auto-regressive function of C_t, such that $\overline{C}_t = \theta C_{t-1}$, where values for θ are taken from a grid of values where $1 > \theta > 0$. This approach is consistent with other empirical procedures in the consumption literature as adopted by Constantinides (1990) and Sundaresan (1989). Before we evaluate this model empirically, we should consider respecifying the data generating process of C_t. Accordingly, we will assume C_t is defined as follows:

Assumption 4:
$$(C_t - \overline{C}_t) \ = (C_0 - \overline{C}_t) \ \exp\left(\alpha_c' t + \sum \varepsilon_{ct+j}'\right).$$

In addition, $(C_t - \overline{C}_t)^{-\beta} = (C_0 - \overline{C}_t)^{-\beta} \exp\left(\beta\alpha_c' t + \beta\sum \varepsilon_{ct+j}'\right),$ (3.3).

where $\varepsilon_{ct}' \overset{d}{\sim} N(0,\sigma_{cc}')$, and σ_{cd} will become σ_{cd}'.

Reformulating proposition 1 with Assumptions 3 and 4, we expect α_c' to be reduced relative to α_c and σ_{cc}' to be larger than σ_{cc}.

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[7] Assumption 3 becomes Assumption 1, and likewise Assumption 4 will become Assumption 2b if $\overline{C} = 0$.

Proposition 2:

If Assumptions 3 and 4 hold, then the net effect of these changes on equation (2.9), (3.0) and ϕ in particular, is that ϕ becomes ϕ'.

$$\text{where } \phi' = \exp\left[\alpha_d + \frac{1}{2}\sigma_{dd} - \beta\sigma'_{cd}\right] \tag{3.4}.$$

$$\text{whilst } d' = \frac{1}{1+k-\phi'} \tag{3.5}.$$

Thus if σ'_{cd} becomes larger for any given β, ϕ' becomes smaller and d' becomes smaller. The above argument effectively establishes a link between the equity premium puzzle and the large values found for equity duration. Therefore, to reinforce this point, we will adopt the following numerical example in our work. If $\beta = 2$, $\alpha_d = 0.04$ (ignoring inflation), $\sigma_{dd} = 0.001$, $\sigma_{cd} = 0.002$ and $k = 0.05$.

$$\therefore d = \frac{1}{1.05 - \exp(0.04 + 0.0005 - 0.004)}$$

$$= \frac{1}{1.05 - \exp(0.0365)} = \frac{1}{0.0128} \approx 78 \text{ years}$$

Thus supposing the average pension fund is invested in 80% equities and 20% bonds, and further assuming the duration of the diversified bond component would be at most 15 years, we can estimate the duration of the average fund's portfolio. Hence, total portfolio duration equals (0.2) * 15 years + (0.8) * 78 years, or 65.4 years, providing for a portfolio duration in excess of the retirement age in many countries. This would be exacerbated in the US market, where typically $\alpha_d(US) \cong \frac{1}{2}\alpha_d(UK)$ is regarded as a fair assumption[8]. Emphasising the significance of the equity premium puzzle to our previous numerical example, if $\beta = 20$, which is generally thought to be too large, we might find $d \approx 20$ years, giving equity the same duration as long-term bonds. We therefore proceed to compute σ'_{cd}, α_d and σ_{dd} from our UK data series, to shed light on this anomaly. From our empirical analysis of the UK market we may conclude reasonable values for our parameters to be taken as $k = 4$, $\beta = 4$ and where $\theta = 75\%$. Hence, according to our empirical calculations in Table 2, an average UK pension fund would have a total portfolio duration equalling (0.2) * 15 years + (0.8) * 28.5 years, or 25.8 years.

This provides for a total portfolio duration that is consistent with the majority of pension funds contractual liabilities. Our findings therefore reinforce the traditional asset allocation decisions of pension fund providers, since the structure of the majority of pension schemes appears consistent with suitable matching strategies for their underlying assets and liabilities. This is in contrast to earlier conclusions on immunisation using the traditional DDM framework.

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[8] The divergent yield characteristics across the US and UK markets are considered in more detail in our Addendum section, which contains a discussion of dividend payments and related fiscal and bankruptcy issues affecting international equity yields.

In the empirical section that follows, we examine the relationship between our equity duration estimates and the underlying parameters in allowing for variations in θ, β and k. We obtained dividend and consumption data for the UK market, as specified in Section 4. We are of course conscious of the important role derivatives play in the context of global fund management, and thus direct the interested reader to an extension of our work to computing equity option duration, as contained in our Appendix 2. One area of empirical research that we have not addressed in our paper is the manner in which one might proceed to estimate reasonable values for β from observable market data. Instead, we refer the interested reader to Damant, Hwang and Satchell (2000) for a discussion of the obstacles involved in such empirical calculations.

Another controversial issue in the area of empirical research concerns the average expected lifetime of equity securities. The traditional DDM approach logically implies equities are infinitely lived financial assets in regard to any prospective investment horizon, thus generating excessively long equity duration estimates. This stands in contrast to the bond markets, where ultimate available redemption periods are generally shorter. For example, in fixed income markets investors are seldom faced with liquid bond issues with maturities in excess of 30 years. More recently, however, some innovative international corporate bond issues have begun to appear with 50 year maturities; for example the British Gas 7.25% 1994 issue, redeemable in 2044. We have thus considered whether it is reasonable to treat equities as undated instruments, when in fact there is clearly an observable corporate failure rate across all companies in the UK, listed and otherwise. This has been shown to fluctuate with the economic cycle, between 1% and 2% per quarter, by Joyce and Lomax (1991). In terms of the pension fund industry, investments are predominantly in listed companies, but undoubtedly even the most risk-averse blue chip investors are exposed to potential bankruptcy risks. Former FTSE 100 companies have failed spectacularly, as exemplified by Polly Peck, British & Commonwealth or Coloroll, and given the volatility in new technology constituents of the FTSE 100 in 2000, bankruptcy remains a real issue with respect to pension fund portfolios. Even the most conservative of fund managers will undoubtedly retain some real residual exposure to potential insolvency, and so we pose this legitimate concern for future research, but present our own initial contribution to the debate. We therefore include a modified form of our equity duration results in table 3, where we adopt an estimate of average equity horizon as 100 years – assuming a one percent annual failure rate; a binomial distribution implies that this is not unreasonable.

Proposition 3:

If the life of the average firm is m years, where we consider 100 years as a reasonable assessment of average equity life expectancy, we can rework our theorems to calculate equity duration for finite-life companies, adjusting equation (2.5) accordingly:

$$d = \frac{d \ln P_t}{d k} = \sum_{i=1}^{m} i \frac{E_t(D_{t+i} C_{t+i}^{-\beta})}{(1+k)^{i+1} E_t(C_{t+i}^{-\beta})} \bigg/ P_t \;, \qquad (3.6).$$

We similarly extend our equity duration analysis, by making use of our earlier Assumptions 1 and 2, to derive the general conditions for any m-year horizon.

Thus, from our m year survival horizon, $\qquad d = D_t \sum_{i=1}^{m} i \frac{\phi^i}{(1+k)^{i+1}} \Big/ P_t$, $\qquad\qquad$ (3.7).

Therefore, correspondingly, we also find that: $P_t = D_t \sum_{i=1}^{m} \left(\frac{\phi}{1+k} \right)^i$, $\qquad\qquad$ (3.8).

Using the result that $\sum_{i=1}^{m} b^i = \frac{b - b^{m+1}}{1 - b}$, thus $P_t = D_t \left(\dfrac{\dfrac{\phi}{1+k} - \left(\dfrac{\phi}{1+k} \right)^{m+1}}{1 - \dfrac{\phi}{1+k}} \right)$

And so simplifying, $P_t = D_t \left(\dfrac{\phi - \dfrac{\phi^{m+1}}{(1+k)^m}}{1+k-\phi} \right) = D_t \left(\dfrac{\phi \left(\dfrac{(1+k)^m - \phi^m}{(1+k)^m} \right)}{1+k-\phi} \right)$

Where $\phi = \exp\left[\alpha_d + \frac{1}{2}\sigma_{dd} - \beta\sigma_{cd} \right]$, as before, in equation (3.1),

Now, however, $\ln(P_t) = \ln(D_t) - \ln(1+k-\phi) + \ln(\phi) - m\ln(1+k) + \ln\left((1+k)^m - \phi^m\right)$,

Thus, duration $= -\dfrac{d\ln(P_t)}{d\,k} = \dfrac{1}{1+k-\phi} + \dfrac{m}{(1+k)} - \dfrac{m(1+k)^{m-1}}{(1+k)^m - \phi^m}$, $\qquad\qquad$ (3.9).

We note for convergence $\phi < 1 + k$, as $m \to \infty$, but for finite m, no such condition is required.

Under Assumption 3, ϕ will again become ϕ' as in equation (3.4). Likewise, substituting this into equation (3.9) will now allow us to recalculate our equity duration estimates to be consistent with any given m year average survival horizon for our listed equity securities. Thus, our general formulation could allow pension managers to define their own equity duration estimates based on any empirically identified value for m. An indication of the anticipated life span of a company, or portfolio of securities, could of course be determined empirically from an examination of the risk premium in corporate debt. The default probabilities, implicit in the pricing of corporate debt instruments, would thus provide a useful proxy to the expected time to default. In any event, howsoever our reader may choose to derive an estimate of the expected lifetime of listed companies; we provide an efficient method for generating subsequent equity duration estimates. We thus illustrate an application of this approach within our results, by including comparative recalculations of the corresponding equity duration parameters in Table 3, where we have taken $m = 100$ years as a plausible illustrative assumption.

4. Empirical Estimation of Equity Duration in the UK Stock Market:

Our empirical work is based on a time series of quarterly data derived from ICV Datastream, spanning a 37-year period from 1963 to 1999. Our market data has been generated from returns on the FT-All Share Index, which we have taken as representative of the broader UK equity market. Real yields have been computed from UK Treasury bond data, both conventional and index-linked, to provide an empirical insight into the historical bounds for real yields. We also examined the index-linked yield gap between gilt issues over the period, which vindicates empirical bounds for k of around three to eight percent; see Barr and Campbell (1997). During 1998 and 1999, conventional gilts yielded barely more than 3% over comparable index-linkers. This historically low real yield suggests that investors believe the recently independent Bank of England will, over the long run, succeed in maintaining inflation within the Government's self-imposed target range of 2.5%. Our historical sample period, however, has witnessed successive political regimes that consistently failed to achieve anything close to this somewhat ambitious target - indeed real rates again almost reached 8% as recently as 1990. These low current yields appear consistent with an overriding expectation that the UK will enter the European single currency in the imminent future, which may yet allow conventional gilt yields to fall further in line with their European counterparts. Global interest rates for both conventional and index-linked bonds are also at historically low levels presently, increasing their vulnerability to any resurgence in commodity or wage price inflation, which could provoke a co-ordinated international rate rise over the longer term.

Building upon the conventional approach in the empirical literature, our consumption measure has been derived from non-durable and service industry expenditure in the UK. This implicitly assumes that utility is separable across this form of consumption and other sources of utility. To allow for the non-separability in utility over time, we have incorporated a simple adjustment to allow for habit formation, or the positive effect of today's consumption on tomorrow's marginal utility of consumption. Our per capita adjustment factor makes use of the UK population series as compiled by the Office of National Statistics (ONS). We have made use of an implicit price deflator series to obtain our inflation measures, and to further adjust our data to reflect our desire to work in real terms. By use of equation (3.1), (3.2) and (3.3), we can derive estimates for σ'_{cd}, α_d and σ_{dd} from our UK market data. We then use our theoretical framework to empirically estimate equity duration parameters directly from UK market data, using equation (3.5) and (3.9). The characteristics of our underlying empirical data set are summarised by the following Table 1 and accompanying Figure 1:

Table 1:   **Empirical Datasheet Summary Statistics:**

Quarterly Data 1963-99	FT-All Share Yield	Inflation Rate	FT-All Share Real Return
Mean	0.0466	0.0667	0.0740
Standard Deviation	0.0128	0.0469	0.2260
Skewness	1.6774	1.4307	-1.5056
Kurtosis	9.8439	4.5766	8.8012
Minimum	0.0239	0.0121	-0.9559
Maximum	0.1171	0.2205	0.7495

Quarterly Data 1963-99	Real Dividend Growth	Real Treasury Yield	Real Consumption Growth
Mean	0.0137	0.0298	0.0216
Standard Deviation	0.0614	0.0408	0.0192
Skewness	-0.1766	-0.7824	0.2352
Kurtosis	2.3589	4.3458	2.7037
Minimum	-0.1390	-0.1069	-0.0188
Maximum	0.1348	0.1165	0.0677

Figure 1:

Real Consumption versus Real Dividends (1963-1999)
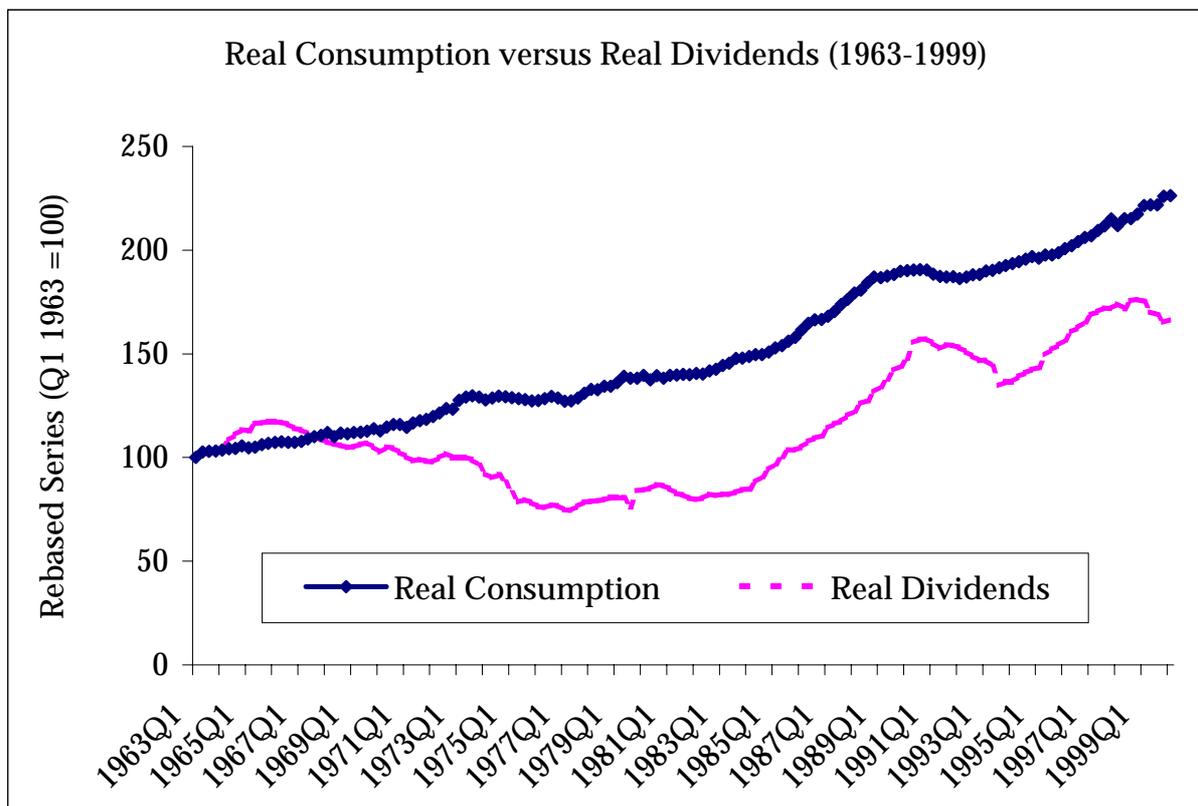
The preceding illustration of real dividend and real consumption growth in Figure 1 highlights the divergence in dividend growth during the 1970s and 1980s. Real dividends fell in response to the ravages of inflation in the 1970s, followed by a period of rapid real dividend growth in the 1980s, partly in response to the subsequent market reforms introduced under the Thatcher administration. The graph illustrates that re-based real consumption has been far less volatile than dividends, at least from a historical perspective. The dividend series illustrates the impact of the successive dividend control regimes applied during the 1970s. Dividend controls were first introduced as policy instruments in the late 1960s and were continuously employed by successive governments from December 1972 until July 1979. The rationale behind imposing curbs on dividend payments assumes a degree of stock market inefficiency in the allocation of funds, and thereby seeks to increase retained earnings in order to improve the supply of organic company financing for future investment opportunities. An important factor that contributed to the longevity of these mechanisms was their inherent consistency with the macroeconomic objectives of the day. These were essentially based on income policies that acted as restraints on the rising level of earned incomes, with dividends controls tantamount to restraints on unearned income, and thus a natural ideological complement towards formulating consistent economic policy. Here we highlight the implications of these controls on our own dividend series, but refer the interested reader to a more complete account of these effects as described in Goudie and Hansen (1988).

The next section contains our summary results in Tables 2 and 3, calculated using equation (3.5) and (3.9) respectively. In these tables we present empirical results for real yields of $k = 4\%$, for both equity duration models, where we have taken $m = 100$ years, to illustrate differences between estimates for finite and infinite horizon equity securities. Our comparative static illustrations that appear in Figures 2 through 5 provide an additional insight into the effects of changing the parameter values for β, k, θ and m respectively. Clearly σ'_{cd} has a pronounced effect on equity duration values, as we vary the level of base consumption via changes in θ. Whatever value the reader may select as most appropriate can then be followed through Tables 2 and 3, to illustrate the effects of varying β in our example. Higher real yields have the effect of reducing equity duration estimates, as predicted from our theory. This is readily highlighted in Figure 2, providing comparative statics for k. This is again consistent with findings on the effects of real interest-rate changes, as presented in the Leibowitz *et al*. (1989) paper. Again in the second comparative static illustration, shown in Figure 3, β plays an unambiguous role in altering our equity duration estimates. This further establishes our theoretical link between the historically high measures of equity duration attributable to the DDM approach, and the underlying equity premium puzzle. The importance of m is emphasised in Figure 4, portraying differences in equity duration estimates obtained from both models. This reveals the practical significance of considering the longevity of listed equity securities themselves, by changing values for m, our chosen measure of the average life span for equity securities. Finally, Figure 5 explores changes in θ, and thereby the level of base consumption upon our equity duration estimates.

Table 2: **FT All Share [1963-98] Equity Duration Estimates for 4% Real Yields**

θ Value	σ'_{CD}	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 7$	$\beta = 8$	$\beta = 9$	$\beta = 10$
0.95	0.01282	35.99	26.69	21.25	16.75	13.85	11.83	10.34	9.20	8.29	7.56
0.90	0.00922	44.85	34.12	27.57	21.99	18.32	15.71	13.77	12.27	11.08	10.10
0.85	0.00783	49.57	38.26	31.18	25.04	20.95	18.03	15.84	14.14	12.77	11.66
0.80	0.00708	52.55	40.92	33.54	27.07	22.72	19.59	17.24	15.40	13.92	12.72
0.75	0.00661	54.60	42.80	35.22	28.52	23.99	20.72	18.25	16.32	14.76	13.49
0.70	0.00629	56.11	44.18	36.47	29.61	24.95	21.57	19.02	17.01	15.40	14.08
0.65	0.00606	57.26	45.25	37.43	30.46	25.70	22.24	19.62	17.56	15.90	14.54
0.60	0.00588	58.17	46.10	38.21	31.14	26.30	22.78	20.11	18.00	16.31	14.91
0.55	0.00574	58.90	46.79	38.84	31.69	26.79	23.22	20.51	18.37	16.64	15.22
0.50	0.00563	59.51	47.37	39.36	32.16	27.21	23.59	20.84	18.67	16.92	15.48
0.45	0.00553	60.02	47.85	39.81	32.55	27.56	23.91	21.13	18.93	17.16	15.70
0.40	0.00546	60.46	48.26	40.19	32.89	27.86	24.18	21.37	19.16	17.37	15.89
0.35	0.00539	60.83	48.62	40.52	33.18	28.12	24.41	21.58	19.35	17.55	16.06
0.30	0.00533	61.16	48.93	40.81	33.44	28.35	24.62	21.77	19.52	17.71	16.21
0.25	0.00528	61.45	49.21	41.06	33.67	28.55	24.80	21.94	19.68	17.85	16.33
0.20	0.00524	61.70	49.45	41.29	33.87	28.73	24.96	22.08	19.81	17.97	16.45
0.15	0.00520	61.93	49.67	41.49	34.05	28.89	25.11	22.22	19.93	18.08	16.55
0.10	0.00517	62.13	49.87	41.67	34.21	29.04	25.24	22.34	20.04	18.18	16.65
0.05	0.00513	62.32	50.04	41.84	34.36	29.17	25.36	22.44	20.14	18.27	16.73
0.00	0.00511	62.48	50.20	41.99	34.49	29.29	25.47	22.54	20.23	18.36	16.81

Table 3: **Recalculation of Equity Duration Estimates for $m = 100$ years and $k = 4\%$**

θ Value	σ'_{CD}	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 7$	$\beta = 8$	$\beta = 9$	$\beta = 10$
0.95	0.01282	29.12	24.17	20.30	16.48	13.78	11.81	10.33	9.20	8.29	7.56
0.90	0.00922	32.42	28.27	24.73	20.88	17.88	15.54	13.71	12.25	11.07	10.10
0.85	0.00783	33.78	30.07	26.79	23.09	20.07	17.63	15.66	14.05	12.73	11.64
0.80	0.00708	34.54	31.09	27.99	24.42	21.43	18.96	16.93	15.25	13.85	12.68
0.75	0.00661	35.02	31.75	28.78	25.30	22.35	19.88	17.82	16.09	14.65	13.43
0.70	0.00629	35.35	32.20	29.33	25.93	23.02	20.56	18.48	16.73	15.25	13.99
0.65	0.00606	35.60	32.54	29.74	26.40	23.53	21.07	18.98	17.22	15.71	14.44
0.60	0.00588	35.78	32.80	30.05	26.77	23.92	21.47	19.38	17.60	16.09	14.79
0.55	0.00574	35.93	33.01	30.30	27.07	24.24	21.80	19.71	17.92	16.39	15.08
0.50	0.00563	36.05	33.17	30.51	27.31	24.50	22.07	19.98	18.19	16.65	15.32
0.45	0.00553	36.15	33.31	30.68	27.51	24.72	22.30	20.20	18.41	16.86	15.53
0.40	0.00546	36.23	33.43	30.82	27.67	24.90	22.49	20.40	18.60	17.05	15.71
0.35	0.00539	36.30	33.53	30.94	27.82	25.06	22.65	20.56	18.76	17.20	15.86
0.30	0.00533	36.36	33.61	31.05	27.94	25.20	22.80	20.71	18.90	17.34	15.99
0.25	0.00528	36.42	33.69	31.14	28.05	25.32	22.92	20.84	19.03	17.47	16.11
0.20	0.00524	36.46	33.75	31.22	28.15	25.42	23.03	20.95	19.14	17.57	16.22
0.15	0.00520	36.50	33.81	31.29	28.24	25.52	23.13	21.05	19.24	17.67	16.31
0.10	0.00517	36.54	33.86	31.36	28.31	25.60	23.22	21.14	19.33	17.76	16.39
0.05	0.00513	36.57	33.91	31.42	28.38	25.68	23.30	21.22	19.41	17.84	16.47
0.00	0.00511	36.60	33.95	31.47	28.44	25.75	23.37	21.30	19.49	17.91	16.54

Figure 2:

Comparative Statics for *k*

Figure 3:

Comparative Statics for *β*

Figure 4:

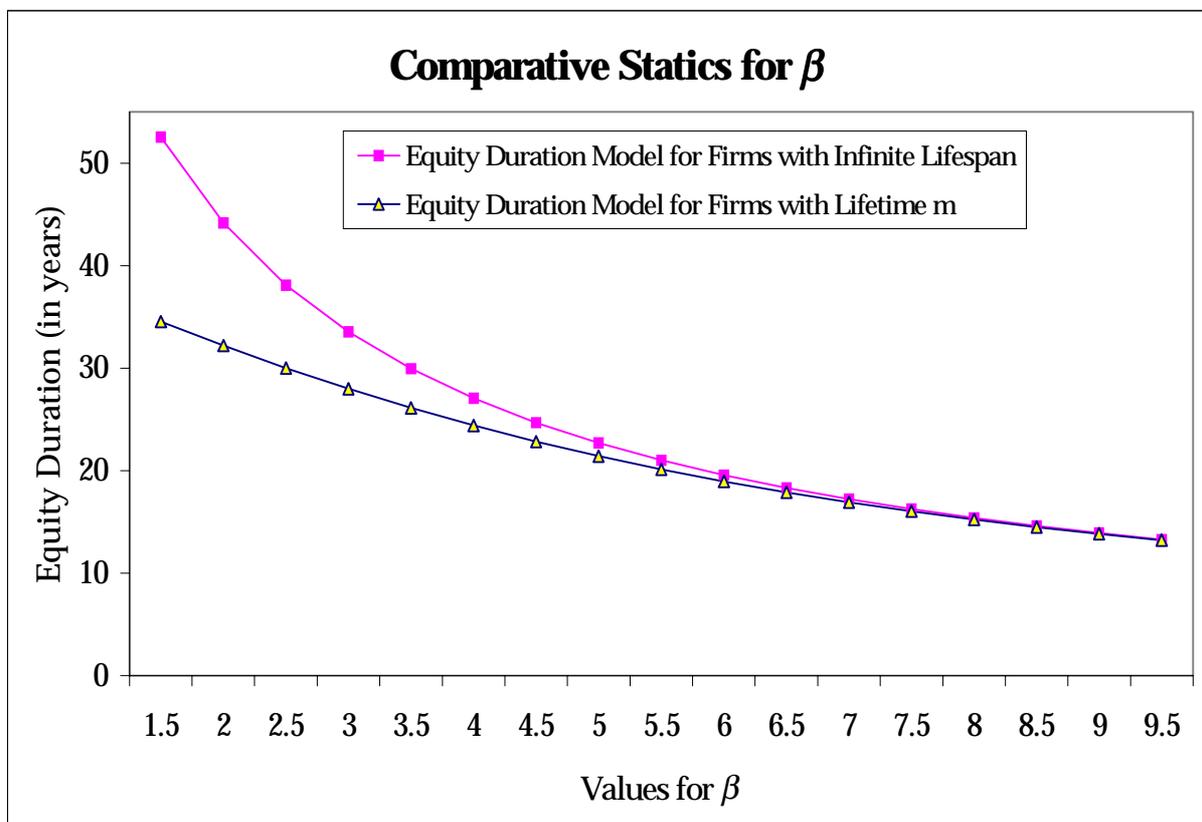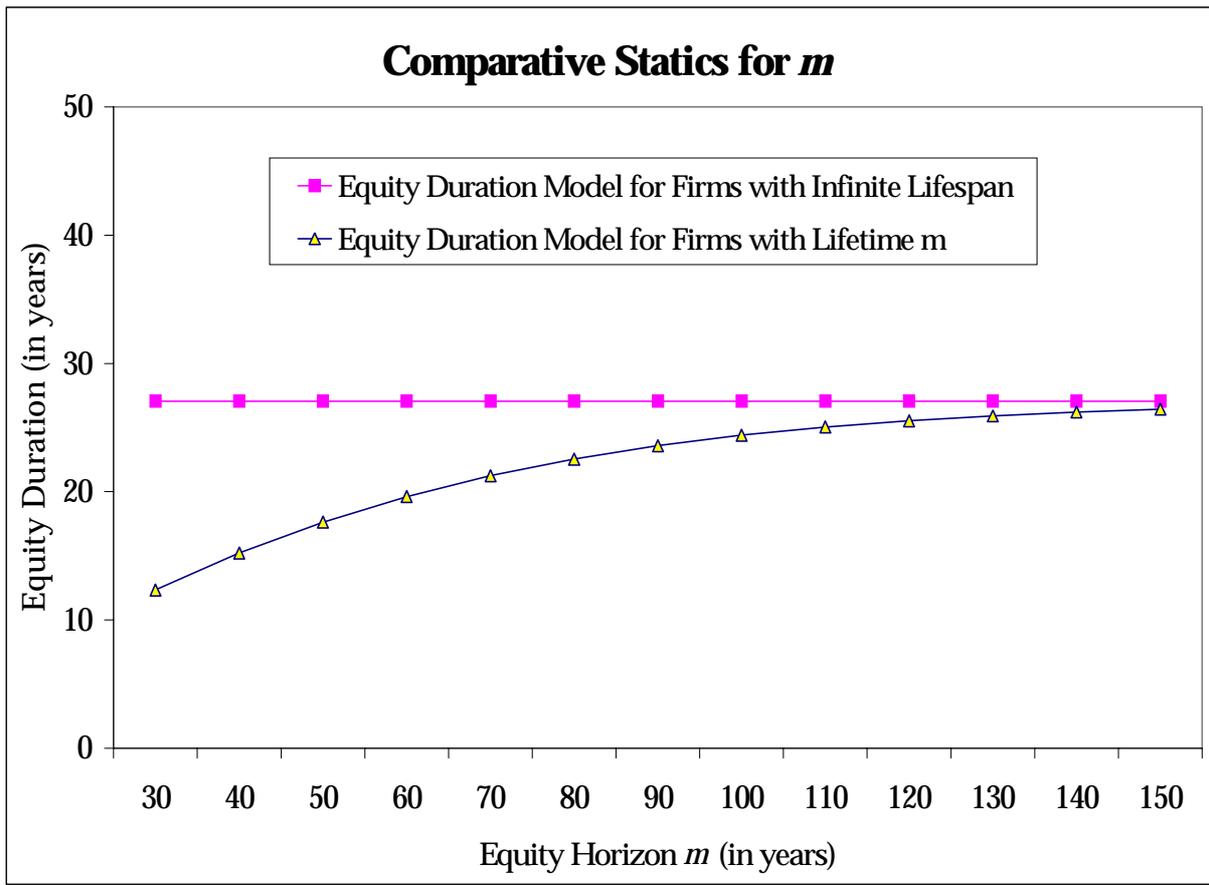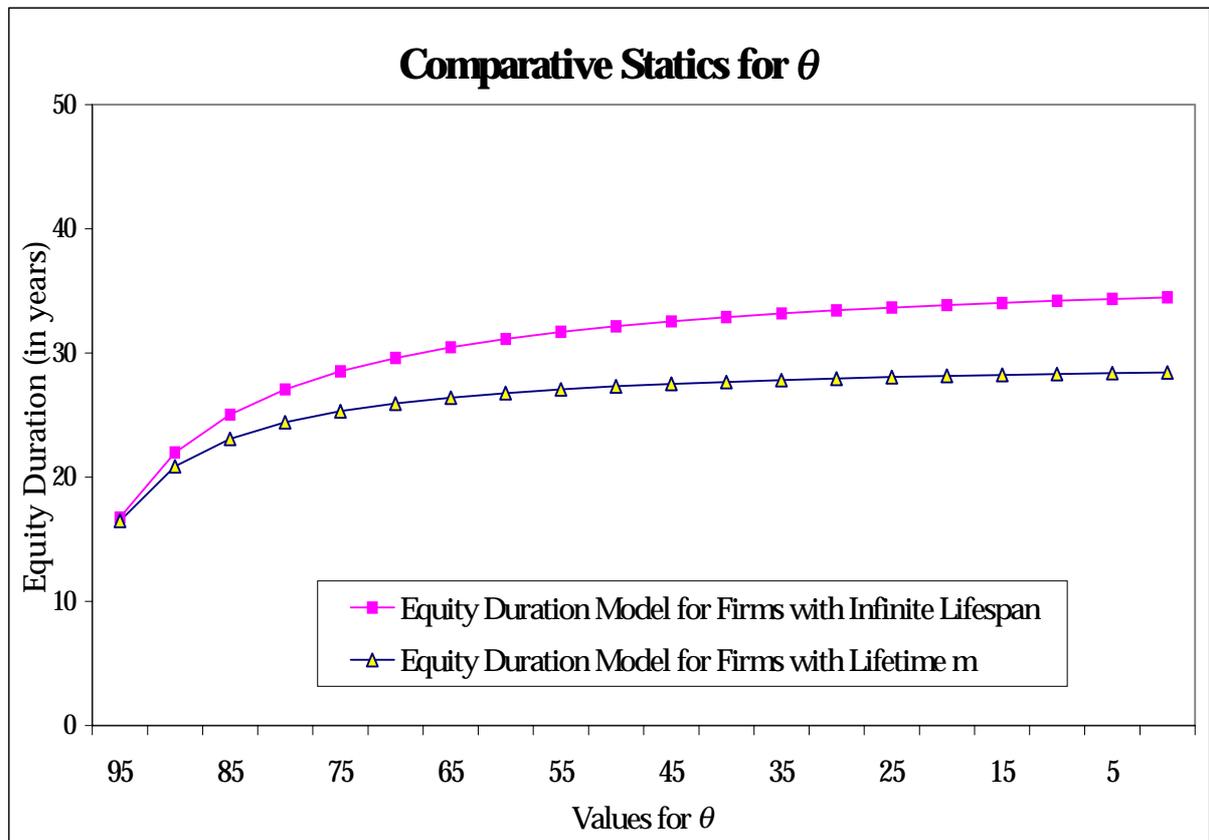Comparative Statics for *m*

Figure 5:

Comparative Statics for *θ*
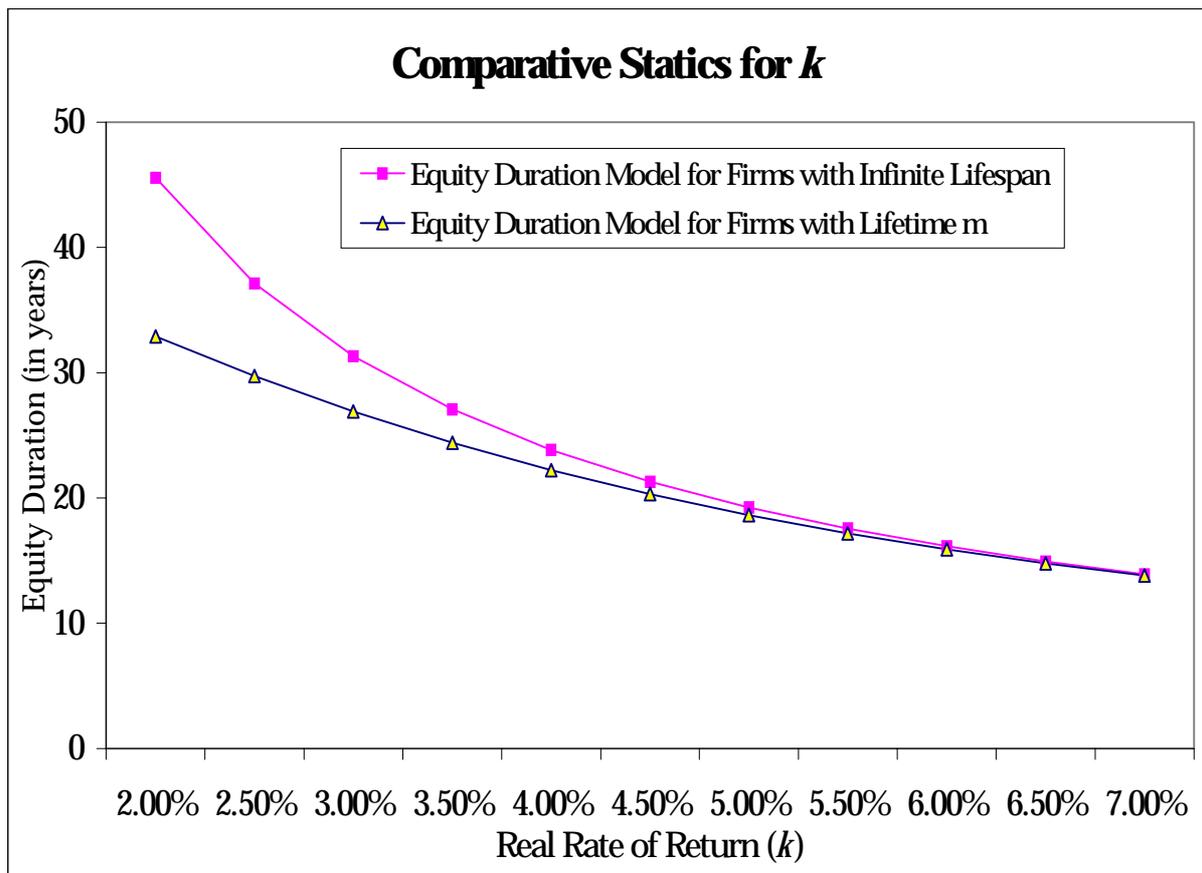
5. Conclusions:

We began our paper by acknowledging a criticism of the traditional dividend discount model, which provides for extremely long equity duration estimates, often in excess of 50 years for high growth stocks. Empirical estimates derived from the intuitive approach of Leibowitz (1986), which relies on standard regression techniques to estimate actual stock price sensitivity, result in duration estimates of between two and six years. Our work provides an alternative methodology to address this apparent paradox, and establishes equity duration estimates within these polarised bounds. We proceeded to demonstrate the flexibility in our approach, by considering the impact of corporate failure rates on equity duration calculations for the UK equity market. Listed company failure rates can average as high as 1% to 2% per annum in the UK, suggesting that there needs to be a reasonable approximation to the average equity survival horizon. We thus calibrated our model to reflect the real investment evidence of equities behaving as more finite horizon securities, choosing 100 years as an average lifetime benchmark. We argued that the resultant calculations are consistent with the liability structure of most pension funds, which have upper liability durations considerably below the 49-year maximum contribution period for a male worker in the UK. Both average contribution and redemption periods across all policyholders are almost certain to be of a considerably shorter duration characteristics and our duration estimates suggest a far closer correspondence in asset matching.

The degree to which pension funds can immunise a multi-asset portfolio relies more generally upon the definition of equity duration used, and hence how we intend to model total portfolio duration to thereby control a fund's overall interest rate risk. From our empirical work, we have concluded that a portfolio's total duration, as a measure of its sensitivity to interest rates, may well be substantially different from that implied solely by calculating the duration of its fixed income components. This has profound implications for fund managers attempting to implement secure immunisation strategies for mixed asset classes, as they may be potentially overlooking a fundamental risk factor, that of equity duration. Inevitably, a significant portion of portfolio risk relies on the accuracy of the equity duration measures used, and so our work retains particular relevance to the investment community in improving such estimates. This is paramount to the trustee context of overall pension liability frameworks, around which most pension funds operate. The introduction of the minimum funding requirements under the *Pensions Act 1995*, has further increased the need to adopt matched investment strategies to prevent cash calls on plan sponsors during downturns in the equity markets.

Our paper has relied on UK market dividend data, but we acknowledge its application in a global asset management context. The dividend yield on the UK market is dissimilar to that available in other international markets, demonstrably the US and Japan. We consider the demand for dividend payments as well as more fully exploring the apparent dissimilarities across investor markets within our Addendum section. Here we draw on the parallels between the UK and the US investor markets, to illustrate how differences in dividend yields are influenced by specific fiscal and bankruptcy legislation.

Historically, the central objection to adopting the traditional DDM framework as a useful tool for pension and asset management was that it tended to provide extremely long measures of equity duration, as distinct from practitioner observations. This suggested, contrary to asset management convention, that the fixed income portion of a fund manager's portfolio might well be used as a means of actually shortening the overall duration of the asset pool. This puzzling result stood in sharp contrast to fund management practice, where the equity component of a portfolio tended to be thought of at least in terms of representing much shorter durations than those implied by traditional DDMs. We thus set out to derive a new theoretical model, which whilst retaining the intuitive appeal of the Leibowitz (1986) methodology, crucially maintains the authenticity of the more traditional approach. Our work allowed us to compute more intuitively acceptable values for UK equity duration, consistent with actual asset allocation decisions undertaken by the major pension funds.

Moreover, our approach provides an inherent consistency with the actuarial assessment of equity fund valuation. This has become increasingly important through recent government initiatives to review the pension legislation with reference to the Myners committee inquiry recommendations, and the report from the Institute of Actuaries. Pension fund actuaries assess the value of an equity investment portfolio by projecting future dividends, based upon long-term real dividend growth and real interest rate assumptions, as defined in Griffin (1998). Our model captures these features comprehensively, aligning the risk management tools of pension fund trustees directly with the mechanical actuarial assumptions used in monitoring their performance. We therefore anticipate considerable further investigation in this area, involving the computation of equity duration by sectors, to allow for portfolio construction and subsequent immunisation strategies. The propensity for additional funded pension provision should ensure that this area of research continues to expand in relation to the needs of a burgeoning pension fund industry. Furthermore, the prolific growth in pension funds since the 1960s, coupled with the ageing demographic profile in the developed world, have eroded the traditional advantages of unfunded pay-as-you-go schemes, exacerbating the need to monitor and improve pension fund performance.

Appendix 1: Derivation of the Equity Duration Model

Under **Assumptions 1 and 2,** it follows that:

$$D_t = D_{t-1} \exp(\alpha_d + \varepsilon_{dt})$$

$$\text{and} \quad D_{t+i} = D_t \exp(\alpha_d i + \sum_{j=1}^{i} \varepsilon_{dt+j})$$

$$E_t(D_{t+i}) = D_t \exp(\alpha_d i + \frac{1}{2}\sigma_{dd} i) \quad (2.7)$$

$$C_{t+i} = C_t \exp(\alpha_c i + \sum_{j=1}^{i} \varepsilon_{ct+j})$$

$$C_{t+i}^{-\beta} = C_t^{-\beta} \exp(-\beta\alpha_c i - \beta \sum_{j=1}^{i} \varepsilon_{ct+j})$$

$$E_t\left(C_{t+i}^{-\beta}\right) = C_t^{-\beta} \exp(-\beta\alpha_c i + \frac{1}{2}\beta^2\sigma_{cc} i) \quad (2.8)$$

$$\text{so that} \quad D_{t+i}C_{t+i}^{-\beta} = D_t C_t^{-\beta} \exp\left[(\alpha_d - \beta\alpha_c)i + \sum_{j=1}^{i} \varepsilon_{dt+j} - \beta \sum_{j=1}^{i} \varepsilon_{ct+j} \right] \quad (2.9)$$

Using moment generating functions, the right hand side of (2.7), can be calculated so that:

$$E_t\left(D_{t+i}C_{t+i}^{-\beta}\right) = D_t C_t^{-\beta} \exp\left[(\alpha_d - \beta\alpha_c)i + \frac{i}{2}\left(\sigma_{dd} - 2\beta\sigma_{cd} + \beta^2\sigma_{cc}\right) \right], \quad (3.0)$$

Equation (2.7) and (2.8) can be substituted into equation (3.0), thus, we find that:

$$d = D_t \sum i \frac{\phi^i}{(1+k)^{i+1}} \Big/ P_t, \quad \text{where } P_t = D_t \sum_{i=1}^{\infty} \left(\frac{\phi}{1+k}\right)^i = \frac{\phi D_t}{1+k-\phi},$$

$$\text{and where } \phi = \exp\left[(\alpha_d - \beta\alpha_c) + \frac{1}{2}\left(\sigma_{dd} - 2\beta\sigma_{cd} + \beta^2\sigma_{cc}\right) + \beta\alpha_c - \frac{1}{2}\beta^2\sigma_{cc} \right],$$

$$= \exp\left[\alpha_d + \frac{1}{2}\sigma_{dd} - \beta\sigma_{cd} \right] \quad (3.1)$$

$$\text{Hence } \ln(P_t) = \ln(D_t) - \ln(1+k-\phi) + \ln\phi,$$

$$\text{and } d = -\frac{d\ln(P_t)}{d k} = \frac{1}{1+k-\phi} \quad (3.2)$$

We note that for convergence $\phi < 1 + k$.

Appendix 2: Extensions to Compute Option Duration

An advantage of our procedure is that we can apply this directly to pricing options consistently with Black and Scholes (1973). Thus duration can be computed for derivatives as well, in similar treatment to Garman (1985), where this can be thought of as the usual "*Greek*", rho (ρ) with respect to interest rates, where O_t is the option price and where $d = \rho/O_t$. Assuming Black and Scholes holds, which we have not shown in our paper:

$$\text{Duration} = \frac{\tau x (1+k)^{-\tau}\Phi(d_2)}{P_t\Phi(d_1) - x(1+k)^{-\tau}\Phi(d_2)}, \qquad (4.1).$$

Where x is the exercise price of the call, $\Phi(.)$ is the standard normal cumulative distribution function τ is the maturity, and d_1 and d_2 are defined as follows[9]

$$d_1 \equiv \frac{\log(P_t/x) + \left(k + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}},$$

$$d_2 \equiv \frac{\log(P_t/x) + \left(k - \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}},$$

That Black and Scholes holds follows from the fact that we can assume that the option pays out $P_{t+\tau} - x$ at maturity if $P_{t+\tau} > x$, and pays zero in all other periods.

$$\text{Therefore,} \quad O_t = \frac{E_t[\max(P_{t+\tau} - X, 0)Y_{t+\tau}]}{(1+k)^{\tau}E_t(Y_{t+\tau})}, \qquad (4.2).$$

The above, under our assumptions, is equivalent to Rubinstein's model[10]. It is straightforward to extend our option price to Assumptions (3) and (4). In fact, the formula turns out to be the same. To show the above, we would need to allow a general pricing formula to hold for all assets, in particular riskless assets. This would allow us to "endogenise" interest rates and consequently "eliminate" the terms in σ_{cd} or σ'_{cd} from our pricing formula.

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[9] In similar treatment to Campbell, Lo and MacKinlay (1997), pp. 352.
[10] As set out in Chapter 5 of Huang and Litzenberger (1988).

Appendix 3: Extensions of the CAPM to Include a Duration Parameter

    In this Appendix, we consider an extension of the capital asset pricing model (CAPM) to incorporate duration, as a general measure of the price volatility of a bond or equity. Duration, as discussed previously, is a widely used measure to quantify and control interest rate risk exposure in asset management. An outline of the theory behind deriving an extended CAPM framework, to account for duration explicitly, is therefore included within the following section. Before we begin this endeavour, we consider some data issues relating to an empirical verification of any such model. We could initially look to take some 5 years or 60 months (in either weeks or days) of data, taking equity and bond indices weighted for the market, for example by 0.8 and 0.2 to reflect standard institutional weightings. Bond data characteristics could then be assumed to mimic the behaviour of bullet bonds. The measure of duration for pure discount bonds, in the absence of coupon payments, is equivalent to their term to maturity. We could subsequently specify our set of assets, where $i = 1…N$, and further assume from the above condition that our N assets have a well-defined duration d_i , (again $i = 1....N$), mimicking the behaviour of zero coupon bonds. We can then estimate μ_i - r_f , β_{im} and d_i from available data, where μ_i , r_f, β_{im} and d_i are defined as the mean return on asset i, the risk free rate, the covariance of asset i with the market m, and the duration of asset i respectively.

    The duration measures we derive still of course imply some relationship between duration and the established view of market participants, as alluded to in our earlier discussion.   Our contribution here is to consider the reformulation of these measures to directly test such hypotheses. Thus, we might aim to rationalise some of the clientele effects observed in financial markets, as well as conducting research into the underlying reasons behind different institutional requirements for specific target-portfolio durations. The particular desired duration for any individually specified portfolio is likely to be either longer or shorter than the duration, ordinarily implied when considering a traditional market equilibrium position. For this to be consistent with market equilibrium, we can invoke a separation theorem argument to provide for a duration-based approach to asset management.   In examining such evidence empirically, we might necessarily discover the institutional clientele effects required for holding differing duration-based portfolio positions in an equilibrium framework.   As is usual for such CAPM type models, we could then assume a representative agent and define an indirect expected utility function V, which is dependent upon expected excess returns, volatility and duration, as defined by μ_m- r_f, σ_m and d_m (where subscript m denotes the market portfolio).

$$\text{Let utility } (V) \ = \ V\!\left(\mu_m - r_f, \sigma_m, d_m\right),$$

where $\bar{\mu}_m = \mu_m - r_f = \displaystyle\sum_{j=1}^{N} x_j (\mu_j - r_f)$, for N risky assets, the riskless asset of zero duration,

$$\text{with } \sigma_m^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}, \text{ and duration, } d_m = \sum_{j=1}^{N} x_j d_j \, .$$

Here we have a representative agent with three-fund money separation.

At the optimum $\mathrm{d}V = 0$, correspondingly, $\dfrac{\mathrm{d}V}{\mathrm{d}x_i} = 0$.

So, $\dfrac{\mathrm{d}V}{\mathrm{d}x_i} = \dfrac{\mathrm{d}V}{\mathrm{d}\bar{\mu}_m}\cdot(\mu_i - r_f) + \dfrac{\mathrm{d}V}{\mathrm{d}\sigma_m}\left(\dfrac{1}{2\sigma_m}2\sum X_i\sigma_{ij}\right) + \dfrac{\mathrm{d}V}{\mathrm{d}\,d_m}(d_i) = 0$,

Therefore, rearranging, $\mu_i - r_f = \left(\dfrac{\dfrac{-\mathrm{d}V}{\mathrm{d}\sigma_m}}{\dfrac{\mathrm{d}V}{\mathrm{d}\bar{\mu}_m}}\right)\cdot\left(\dfrac{\sigma_{im}}{\sigma_m}\right) + \left(\dfrac{\dfrac{-\mathrm{d}V}{\mathrm{d}d_m}}{\dfrac{\mathrm{d}V}{\mathrm{d}\bar{\mu}_m}}\right)(d_i)$,

Finally, we arrive at, $\mu_i - r_f = \left(\dfrac{\mathrm{d}\bar{\mu}_m}{\mathrm{d}\sigma_m}\sigma_m\right)\beta_{im} + \left(\dfrac{\mathrm{d}\bar{\mu}_m}{\mathrm{d}d_m}d_m\right)\cdot\left(\dfrac{d_i}{d_m}\right)$, (5.0)

Since at the optimum, $\left[\dfrac{\dfrac{-\mathrm{d}V}{\mathrm{d}\,a}}{\dfrac{\mathrm{d}V}{\mathrm{d}\,b}} = \dfrac{\mathrm{d}\,b}{\mathrm{d}\,a}\right]$ for any a, b.

We present the above result in equation (5.0) as Theorem 1.

Theorem 1:

In a representative agent equilibrium, given a one period world with returns, duration and volatility defined for each asset, equilibrium expected excess returns are linear in covariance β_i, and in duration d_i, where α_1 and α_2 are defined in the proof accordingly.

i.e. $\mu_i - r_f = \alpha_1\beta_{im} + \alpha_2 d_i$, where we let $\alpha_1 = \dfrac{\mathrm{d}\bar{\mu}_m}{\mathrm{d}\sigma_m}\cdot\sigma_m$ and $\alpha_2 = \dfrac{\mathrm{d}\bar{\mu}_m}{\mathrm{d}d_m}d_m\cdot\dfrac{1}{d_m}$.[11]

The testable implications of this model concern the signs of α_1 and α_2. From the above we expect $\alpha_1 > 0$, since $\mathrm{d}\mu_m/\mathrm{d}\sigma_m > 0$. However, the sign of α_2 is ambiguous. One consideration here is yield curve related, where we might argue that $\alpha_2 > 0$ if the yield curve is upward sloping, but $\alpha_2 < 0$ if it is inverted. So in fact, investor's preferences for any duration component within their portfolios will be related to the envisaged shape of the yield curve, as well as their respective investment horizon. Of course, should our representative agent be a short-term investor, we might expect $\mathrm{d}\mu_m/\mathrm{d}d_m < 0$, but if he were a longer-term investor, we might equally expect $\mathrm{d}\mu_m/\mathrm{d}d_m > 0$. In our example, the representative agent's utility specification will depend upon the expected excess returns, the implied volatility and the duration. The typical UK investor could be thought of as a large pension fund whose liabilities are relatively long-term. Thus, we might envisage a preference for assets characterised by longer durations.

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[11] Given our equation (5.0), this result follows upon substitution and rearrangement.

There is, however, another alternative and more populist literature, exemplified by Pickens (1986) that refutes the preceding argument, advocating that investors are in fact particularly short-termist in their approach. For a rigorous academic version that adds considerably to this area of debate, as well as providing a useful UK interpretation, see Miles (1993) and the related notes appearing in Damant and Satchell (1995). For the details of a parliamentary discussion, which appears to refute itself with its own shallowness in the treatment of this important topic, see the available references in Hansard[12]. For a more journalistic concoction of the same gruel, the reader is referred to the arguments presented in Hutton's books (1996) and (1997). Irrespective of the manner in which our reader may elect to interpret the short-termist claims, as considered in any of these sources, our model permits these issues to be addressed objectively, by associating them with a value for α_2.

In order to extend the equilibrium conditions for our representative agent however, we would have to invoke a three-fund separation theorem. Our theory maintains that, cross-sectionally the risk premium will be related to beta as well as the duration of the asset relative to the duration of the market. We can therefore estimate β_{im} from historical market data. However, the duration of the market d_m presents more of a problem. In our three-funds CAPM framework, we would require to know three rates of return in order to determine a unique structure. We would need to know the return on the market portfolio, the rate of return on our riskless asset and the rate of return on any portfolio with a zero β, conditional upon having an associated non-zero duration.

If Theorem 1 applies to the entire market, taking $i = m$, $\beta_{mm} = 1$, and $d_i = d_m$, will allow for the following expression:

$$\mu_m - r_f = \alpha_1 + \alpha_2 d_m \ .$$

Thus, $(\mu_m - r_f) = \alpha_1 \beta_{im} + ((\mu_m - r_f) - \alpha_1)\dfrac{d_i}{d_m} ,$

$$= \alpha_1\left(\beta_{im} - \frac{di}{dm}\right) + (\mu_m - r_f)\frac{d_i}{d_m} \ , \tag{5.1}$$

Therefore, by equation (5.1), we have arrived at an alternative representation.

Corollary 1: (for $i = 1.... m$.)

The empirical consequences of equation (5.1) are that there should be some cross-sectional relationship between risk premia, the betas and the relative duration, d_i/d_m. Indeed, the sum of the two coefficients should equal the excess returns and will have testable consequences, suitable for empirical investigation.

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[12] 'Budget Resolutions and Economic Situation', *Hansard*, 7th July 1997, pp.688-690, and the Finance Bill, *Hansard*, 16th July 1997, pp.475-476.

Theorem 2:

Here we now consider what a minimum variance frontier might look like for a duration-constrained portfolio. This analysis is reminiscent of early work by Jean (1971) and (1973), as well as Ingersoll (1975). As before, we have N assets with rates of return r, where $r \sim (\mu, \Sigma)$ and the duration of the assets is denoted by d. Let the constrained expected returns be Π and constrained duration d_0. Our frontier is the locus of points (Π, d_0, σ^2) where $\sigma^2 = x' \sum x$ and so we solve the following optimisation problem (where e is a vector of ones):

$$\underset{x}{Min} \left((x' \sum x) - \lambda_1 (x'e - 1) - \lambda_2 (x'\mu - \Pi) - \lambda_3 (x'd - d_0) \right).$$

Differentiating, we see again three-fund separation. Alternatively, this can be written as,

$$\underset{x}{Min} \left(\frac{1}{2} x' \sum x - \lambda' (E'x - E_0) \right),$$

where $E \text{ (an } n \times 3 \text{ matrix)} = [e, \mu, d]$ and $E_0 = (1, \Pi, d_0)'$.

The first order conditions imply that:

for $\hat{x}$, the solution becomes $\Sigma \hat{x} = E\lambda$ and $E'\hat{x} = E_0$.

Therefore, $\hat{x} = \Sigma^{-1} E \hat{\lambda}$, and $E'\hat{x} = E_0 = (E'\Sigma^{-1}E) + \hat{\lambda}$.

So $\hat{\lambda} = (E'\Sigma^{-1}E)^{-1} E_0$, and therefore $\hat{x} = \Sigma^{-1} E (E'\Sigma^{-1}E)^{-1} E_0$.

If we denote (the 3 by 1 matrix) $\theta = (\theta_1, \theta_2, \theta_3)' = (E'\Sigma^{-1}E)^{-1} E_0$,

$$\text{then } \hat{x} = \theta_1 \Sigma^{-1} e + \theta_2 \Sigma^{-1} \mu + \theta_3 \Sigma^{-1} d \ . \tag{5.2}$$

Remarks:

It is possible to compute expressions for θ_i, by computing the inverse of the (3 by 3) matrix $(E'\Sigma^{-1}E)^{-1}$. Equation (5.2) shows that the minimum variance portfolio satisfies three-fund separation. As we vary Π and d_0, we will trace out a minimum variance frontier in three dimensions, $((\sigma, \Pi, d) \text{ space})$. The equation for the efficient frontier will thus be given by,

$$\sigma^2 = E_0' (E'\Sigma^{-1}E)^{-1} E_0. \tag{5.3}$$

Equation (5.3) is clearly quadratic in $\sigma, \Pi,$ and d_0. Whilst it is valid to assume that $d\sigma / d\Pi$ is positive, it may not necessarily be the case that $d\sigma / d d_0$ can be signed. Thus, there is no geometrically determined market portfolio, but rather a region. We could extend this analysis by assuming we have two types of fund manager as follows:

(i) Duration neutral, but return-loving investor.
(ii) Return neutral, but duration-loving pension fund.

These assumptions would readily allow us to generate the required equilibrium position.

Addendum: US & UK Dividend Payments, Fiscal and Bankruptcy Issues.

We now embark on a brief discussion of the comparability of our UK study to articles that have examined US stock market data. Although our approach focuses on the UK perspective, it is useful to highlight some of the basic differences between the US and UK jurisdictions. This addendum section begins by exploring the academic literature concerning the existence of dividend payments. We then move on to explore some of the economic consequences that are reflected in the differing dividend yields observable across the US and UK markets. Our discussion advances two plausible explanations to account for differences in the observed equity yields, namely fiscal and bankruptcy issues. In particular, the inclusion of dividends as a factor in our model highlights the importance that any discrepancy between the historical yields available in the US and UK equity markets would have on our equity duration calculations.

An important question in relation to our work is the existence and payment of dividends by corporations. Famously Modigliani and Miller (1958) set forth an irrelevance proposition regarding the valuation of the firm and the subsequent payment of dividends, using a set of restrictive and simplifying assumptions. In essence this concluded, in valuation terms at least, that owners should be indifferent to receiving income or capital gains from an investment. Black (1976) succinctly put the case that dividends present an important economic puzzle, as corporations ought not to pay dividends at all. The explanation for the actual existence of dividends lies in part from investors' differential and unequal tax treatment of capital gains as opposed to income and related timing issues. Tax aside, investors should exhibit indifference to receiving dividends or declaring their own 'dividend' by a proportional sale. Transaction costs are seldom sufficient in themselves to explain the reluctance on the part of shareholders to declare their own dividend. Allen, Bernardo and Welch (2000) explored a relaxation of one or more of these assumptions to challenge this basic premise. It may be argued that agency costs, as first premised by Jensen and Meckling (1976), present another reason that investors still clamour for the payment of a regular dividend stream. The distrust for management in not applying shareholders funds to otherwise oblique personal goals, such as corporate jet ownership or political power, suggests a higher risk aversion to reinvestment and profit retention. Behavioural finance has attempted to explain the need for dividends on the basis of myopia among investors, who are happy to spend out of current income (from earnings, pensions and dividends) but are extremely reluctant - to the point of denial - to release even a small portion of their accumulated capital for consumption purposes. To an economist this should be irrelevant, as in relation to net worth, there is no difference between consuming income or capital, and as such these ought to be perfect substitutes. Shefrin and Statman (1984) rejected the dividend irrelevancy proposition on the grounds that dividends and capital could not be treated as perfect substitutes, even in the absence of taxes and transaction costs. Investors' preference for cash dividends is explained by reference to signalling theory, which holds that the raising or lowering of dividends provides a glimpse of powerful asymmetric management information that is otherwise unavailable to shareholders.

Two alternative explanations have been proffered, the economic theory of self-control by Thaler and Shefrin (1981), and the prospect theory of choice under uncertainty by Kahneman and Tversky (1979). The former concept is incorporated in a theory of individual inter-temporal choice, by modelling the individual as multi-faceted - at the same time being both a farsighted planner and a myopic doer. The resulting conflict is seen to be similar to the agency conflict between the owners and managers of a firm. The latter relies on investor segmentation theory, for if the demographic attributes of investors preferring high and low dividend payout portfolios can be identified, then the theory suggests that some investors would be willing to pay a premium for cash dividends, because of self-control motives, the desire to segregate income and capital, or the wish to avoid regret. Behavioural patterns studied by psychologists seem to suggest a correspondence to the bequest motive in old age, that helps to explain why observation has demonstrated that people dissave much less than ought to be suggested by any simple life-cycle model of consumption, as explored by Modigliani (1986). A considerable empirical literature relates to equity pricing around dividend events, see Michaely and Vila (1995), which suggests that some investors skim distributions around the ex-dividend date, to satisfy an apparent need for actual cash payments, see Lasfer (1995).

Our contribution to this debate is to suggest that perhaps investors demand dividends owing to certain overriding obligations, in the form of opportunity costs that might otherwise be passed up, thereby causing them to prefer a regular stream of income. A pension fund for example can defray the ongoing cash payments to matured policyholders and current annuitants through the application of investment dividend receipts distributed throughout the year. This would necessarily imply a pool of companies reporting and declaring dividends spatially across the calendar year to fulfil such an implicit need. Anecdotally at least, this is supported by the observed dispersion of ex-dividend dates across listed companies. Moreover, the private investor may also see similar opportunity costs in the market place from ongoing private transactions, which without the associated dividend income stream could not be immediately realised. If companies have a specific level of costs associated with their fundraising activities, then there is no reason to doubt that private or institutional clients will not face a similar level of costs. This provides a transaction demand for dividends to obtain additional incremental investment funds, as a direct proxy for alternative capital raising activities for daily operational requirements. Dividend payments throughout much of the developed world are small relative to the capital value of the underlying investment principal. This fact suggests that dividends may be an efficient method for obtaining small ongoing parcels of funding for opportunistic reinvestment, debt repayment or day-to-day liability matching. The power of the institutions in terms of the concentration of corporate ownership dictates that there must be compelling reasons sustaining progressive dividend policies across the investment spectrum. The segmentation between 'capital' and 'income' funds itself demonstrates a real economic imperative to provide dividends to some clients in preference to capital accumulation as required by others. This situation might suggest an illusion - pertinent to investors - in any 'true' measurement of investment fund performance comparing returns on a purely compound basis.

This implies a policy of perpetual reinvestment that appears abstract and irrelevant in regard to actual investor requirements. Corporate capital structures are themselves principally determined by the trade-off between the tax deductibility of interest payments on debt instruments, and the increased risk and inherent costs associated with bankruptcy, as recognised by Kraus and Litzenberger (1973), and Gordon and Malkiel (1981). This prevents the level of borrowings from increasing indefinitely beyond a certain point contrary to the original Modigliani and Miller hypothesis, which includes the feasibility of exclusively debt-financed organisational structures that are not ordinarily observable. The appropriate gearing level for any organisation is further influenced by the nature of its investment opportunities and expected cash flows, as discussed by Leland (1994), which determines an individual firm's optimum debt to equity ratio. Bankruptcy proceedings in the US and insolvency practices in the UK derive from an economic need to reconcile the microeconomic exit condition, of net realisable value exceeding present value, with the bankruptcy principle of liabilities exceeding assets. There is no legal mechanism in either jurisdiction to enforce the redistribution of assets of uneconomic firms that are not bankrupt, which might otherwise continue indefinitely in the absence of any active market for corporate control. A firm's management may not accurately ascertain the extent of creditor-losses when determining whether or not to file for bankruptcy, so that not all inefficient firms end up in bankruptcy *per se*, while those that do may not necessarily be the most inefficient ones. However, directors are legally obliged to avoid wrongful trading, and are expressively prohibited from deliberately causing companies to trade at a loss.

In relation to issues of bankruptcy, a risk explicitly included in our model, UK insolvency practices tend to accentuate the position of creditors, with the overreaching legal objective to facilitate the immediate dissolution of a company to maximise creditor recovery rates. This leaves little scope for welfare considerations, such as the impact of liquidation on the wider community. The US legal system operates a more sympathetic underlying philosophy, contrasted by way of Chapter 11 versus Chapter 7 bankruptcy proceedings. While Chapter 7 remains closer to the UK-style insolvency proceedings, the more common Chapter 11 provides a vital economic halfway house for insolvent companies, affording several economic advantages, and crucially according to Altman (1984), allowing companies' breathing space for reorganisation over an extended period. Major US airlines have used this as a means to circumvent onerous employment contracts, and to renegotiate improved terms with their creditors. One economic consequence of differing bankruptcy legislation could be the reduction of the requisite returns on US companies relative to their UK counterparts; although historically real returns have been broadly similar at around 8% according to Dimson and Marsh (2001). The more draconian insolvency procedures applicable to ailing UK firms might cause equity investors to be short-termist, requiring higher immediate returns and raising dividend yields to compensate them for the perceived risks associated with higher expected liquidation rates. Conversely, US Chapter 11 regulations raise serious issues of market efficiency in the overprotection of ailing US companies against their creditors, as related by Franks and Torous (1989), thereby lowering corresponding dividends and (possibly) the cost of capital.

If long-term equilibrium prevails in capital markets and surviving organisations maximise profits, equating marginal costs and revenue, goods and services will be produced at an efficient minimum cost in the economy. A variation on Darwin's theory of natural selection has already been extended to corporate economics by Winter (1964), favouring firms who profit maximise, whilst eliminating other 'deviant' species. The impact of capital market imperfections on the natural selection mechanism has also been recently reviewed by Zingales (1998), to analyse the effect of leverage on survival in the US trucking industry surrounding a period of deregulation. This study was used to gauge the extent to which the legal frameworks of the US and UK might engender economic efficiency in capital markets by eliminating inefficient firms. It concluded that differing legal systems discourage Pareto-efficient outcomes, and even in the presence of perfect managerial insight would have a latent effect on returns in the equity market.

Ideologically, a legal system should probably complement an efficient capital market, by assisting the eradication of uneconomic concerns, and thereby reallocating scarce capital-resources in the most Pareto-efficient way. White (1989) argues that bankruptcy legislation results in the underpayment of outstanding liabilities, with significant redistributional effects. The paramount economic benefits of entering US Chapter 11 bankruptcy are not only protection from creditors but also other tax advantages that arise concurrently. Any previous tax losses are automatically carried forward, to shield any resumption in profits. Uncovered pension contributions are immediately waived as the US Government sponsored Pension Benefit Guaranty Corporation automatically subsumes any remaining outstanding liabilities. This acts as a huge incentive for labour-intensive firms to enter Chapter 11 and thereby discharge any unfunded pension obligations. Interest on outstanding loans is also waived, so that creditors can no longer compound outstanding liabilities, the first step towards debt-forgiveness in the US. In combination, these factors encourage US firms (who would otherwise have failed in the UK) to survive, with the fixed costs of their operations drastically reduced as a by-product of US bankruptcy proceedings. Indeed this was typified by US steel company LTV, which successfully reduced steel making costs from $460 to $380 per ton, estimated to be some $60 per ton below average steel industry costs, simply by filing to reorganise under Chapter 11[13].

Overall, neither US bankruptcy nor UK insolvency procedures ensure purely economically efficient exit. The law in each jurisdiction is not prescriptive on economic issues but essentially procedural, with UK insolvency law lying somewhere between Chapter 7 and Chapter 11 in the US. In the US, bankruptcy is associated with certain subsidies, which add significant friction to the exit condition of inefficient firms, hindering the transition of the economy towards long-run efficiency and lowering return expectations of shareholders. In contrast, the UK procedures often eradicate businesses that are economically viable but whose capital structure, perhaps by virtue of macroeconomic instability, becomes incompatible. This lends support to the case for an independent Bank of England, with an impartial inflation mandate to foster a more stable macroeconomic environment for investment.

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[13] 'LTV is Healthier Under Chapter 11, But Not Cured', *Wall Street Journal*, July 24, 1987, Page 5.

Therefore, we can reasonably expect that the US equity market could yield significantly less than the UK, due in part to insolvency issues, which may reduce the risk of US equity investment. Different investor tax treatment regarding income and capital gains also alter the return incentives for investors, according to Fama and French (2001), resulting in a divergence in dividend yields. This is exemplified in the UK by another major fiscal distortion in respect to the payment of company dividends, the imposition of Advanced Corporation Tax (ACT) on all qualifying distributions. Historically tax-exempt sections of the investment community, including pension funds, were able to reclaim up to 25% tax on the dividends they received from their investments. This rebate represented the tax paid by UK companies in the form of ACT on behalf of all recipients of cash dividends and all other qualifying distributions. ACT paid by companies on dividend payments could then be offset against any taxable UK profits. This caused both distortions to investors' preferences for dividends, from the enhanced return arising out of the additional tax credit, and materially influenced corporate distribution policies. For example, firms deriving the majority of their profits from abroad might have insufficient taxable UK earnings to offset more than a modest portion of their ACT liabilities on dividend distribution. This may be a residual effect of the earlier dividend controls of the 1970s, in that it continued to penalise UK companies who imprudently paid out dividends to shareholders in excess of their UK earnings. Secondly, ACT was payable on the date of the dividend distribution, whereas corporation tax is collected retrospectively, resulting in a significant mismatch between timing and funding, which also affects corporate cash flows. The Revenue regarded this secondary punitive funding burden as an essential pre-condition to ensure that tax credits, subsequently allotted to individuals and qualifying institutions, had already been received. An alternative company distribution policy to mitigate this tax charge was the option to issue script dividends, saving the company ACT, but incrementally diluting any non-participating shareholders' equity interests.

From the corporate standpoint, successive years of losses, exacerbated by the lengthy recessions of the early 1980s and 1990s, effectively prevented companies from offsetting this fiscal burden, making ACT payments virtually irredeemable. As an additional obstacle, the qualifying amount of ACT-offset against mainstream corporation tax was itself subject to some important and complicated restrictions. The offset could be no greater than the ACT which *would* have been paid by the company, had it distributed an amount which, together with the ACT imputed on it, *equalled* the company's profits including any capital gains – full details of these calculations appear in Farrar, Furey and Hannigan (1991). It was therefore quite possible for a company to accumulate unrelieved ACT liabilities, despite having surplus taxable profits, in consequence of its distribution policy alone. This is particularly acute in instances where the underlying corporate earnings stream is highly cyclical, such as in the construction industry, and the corresponding dividend streams paid out by companies have been artificially smoothed. From the investor's standpoint, dividend streams became even more valuable due to the additional tax credit, with a smoothed dividend stream even more desirable from the perspective of pension fund asset and liability matching, amplified by the rise in collective investment vehicles.

The institutional pressure resulted in fluctuating periods of unrelieved ACT liabilities for many UK companies. Managers unwilling to suffer the wrath of investors by continually adjusting dividends to reflect actual earnings, instead resorted to creative accounting in annual statements, to conceal the implicit ACT penalties incurred. In 1986, the ACT charge was formally tied in with basic rate of tax. Revenue neutrality ensured that, for the basic rate taxpayer at least, there would be no additional tax burden on the receipt of ordinary dividends. To the economist, however, there are noticeable incentives for each party, which distort the traditional dividend payment schedule. High-rate taxpayers have an incentive to opt for growth stocks, to reduce their annual income tax bill, whilst receiving more of their returns in the form of deferrable capital gains, exacerbated by the more recent introduction of capital gains tax (CGT) taper relief. Lower rate taxpayers would prefer to receive dividends taxed as income at 20-25%, rather than CGT, payable on proceeds in excess of the annual personal allowance, at 40%. For non-taxpayers, the Inland Revenue refunded the applicable tax credits on dividend payments through annual tax returns. These clientele effects were magnified within the investment community, since a major slice of the income received by the tax-exempt organisations traditionally comprised these attached tax credits.

Institutional pressure was thus firmly entrenched against any reductions in dividend streams during recessions, vindicating political concerns of insufficient profit retention within UK companies. Hence, the motivation behind the recently phased abolition of the ACT credit and the corresponding introduction of taper relief, as a political prerequisite to enhance long-term domestic equity investment. This has already prompted a fall in the average UK dividend yield, which has become more aligned towards US levels. The abolition of ACT also provides an inducement to return cash directly to shareholders via share buy-backs rather than through dividend payments, as interest on company borrowing is tax deductible, see Talmor and Titman (1990). This affects the optimal corporate gearing level, as debt can be increased to finance the repurchase of a company's own shares, which is earnings enhancing as well as providing a fairer mechanism for returning value to stockholders, see Miller (1986). We have already seen a reduction in the number of companies offering script dividend alternatives, as well as a corresponding rise in the number of share buy-back programmes. If this trend continues in the UK, investors may be faced with an overall reduction in the total outstanding equity in issue by around 2% or 3% per annum. This could reduce UK investment opportunities, although the recent boom in technology, media and telecom companies coupled with an explosion in initial public offerings in these areas have probably more than compensated for any physical reductions in outstanding 'old economy' equity. Increased globalisation across the asset management community may lead to further financial innovation, taking advantage of arbitrage opportunities existing between separate legal and fiscal jurisdictions. In the future therefore, we would expect to see progressive co-ordination between international jurisdictions aimed at the harmonisation of both the legal and tax consequences of global fund management, leading to a gradual convergence in global equity market yields.

Bibliography of References:

Abel, **A**. **B**., "Asset Prices under Habit Formation and Catching up with the Joneses", *American Economic Review*, Vol. 80, No.2, May 1990, pp.38-42.

Abel, **A**. **B**., "The Equity Premium Puzzle", *Business Review - Federal Reserve Board of Philadelphia*, No.5, September/October 1991, pp.3-14.

Allen, **F**., **Bernardo**, **A**. **E**., **Welch**, **I**., "A Theory of Dividends Based on Tax Clienteles", *Journal of Finance*, Vol.55, No.6, December 2000, pp.2499-2536.

Altman, **E**. **I**., "A Further Empirical Investigation of the Bankruptcy Cost Question", *Journal of Finance*, Vol.39, No.4, September 1984, pp.1067-1089.

Arnott, **R**. **D**., **Hanson**, **H**. **N**., **Leibowitz**, **M**. **L**., **Sorensen**, **E**. **H**., "A Total Differential Approach to Equity Duration", *Financial Analysts Journal*, Vol. 45, No. 5, September/October 1989, pp.30-37.

Barr, **D**. **G**., **Campbell**, **J**. **Y**., "Inflation, Real Interest Rates, and the Bond Market: A Study of UK Nominal and Index-Linked Government Bond Prices", *Journal of Monetary Economics*, Vol.39, No.3, 1997, pp.361-383.

Bernheim, **B**. **D**., "Tax Policy and the Dividend Puzzle", *Rand Journal of Economics*, Vol.22, No.4, winter 1991, pp.455-476.

Bernstein, **R**., **Tew**, **B**., "The Equity 'Yield' Curve", *Journal of Portfolio Management*, Vol.18, No.1, Fall 1991, pp.35-39.

Betker, **B**., "Management's Incentives, Equity's Bargaining Power, and Deviations from the Absolute Priority in Chapter 11 Bankruptcies", *Journal of Business*, Vol.68, No.2, 1995, pp.161-183.

Bierwag, **G**.**O**., Duration Analysis, Ballinger, Cambridge, Massachusetts, 1987.

Bierwag, **G**.**O**., **Kaufman**, **G**., **Toevs**, **A**., "Immunisation Strategies for Funding Multiple Liabilities", *Journal of Financial and Quantitative Analysis*, Vol.18, No.1, 1983, pp.113-123.

Black, **F**., "The Dividend Puzzle", *Journal of Portfolio Management*, Vol.2, No.2, winter 1976, pp.5-8.

Black, **F**., **Scholes**, **M**., "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy*, Vol.81, No.2, May/June 1973, pp.637-654.

Bodie, **Z**., "Common Stocks as a Hedge Against Inflation", *Journal of Finance*, Vol.31, No.2, May 1976, pp.459-470.

Bodie, **Z**., **Kane**, **A**., **Marcus**, **A**., Investments (*Third edition*), Dow Jones-Irwin, Homewood, Illinois, 1996.

Campbell, **J**. **Y**., **Lo**, **A**. **W**., **MacKinlay**, **A**. **C**., The Econometrics of Financial Markets, Princeton University Press, 1997.

Casabona, **P**. **A**., **Fabozzi**, **F**. **J**., **Francis**, **J**. **C**., "How to Apply Duration to Equity Analysis", *Journal of Portfolio Management*, Vol.10, No.2, winter 1984, pp.52-58.

Chan, **L**. **K**. **C**. "Consumption, Inflation Risk, and Real Interest Rates: An Empirical Analysis", *Journal of Business*, Vol.67, No.1, 1994, pp.69-96.

Chapman, **D**. **A**., "Habit Formation and Aggregate Consumption", *Econometrica*, Vol.66, No.5, September 1998, pp.1223-1230.

Christie, **W**. **G**., "Dividend Yield and Expected Returns", *Journal of Financial Economics*, Vol.28, No.1, 1990, pp.95-125.

Constantinides, **G**. **M**., "Habit Formation: A Resolution of the Equity Premium Puzzle", *Journal of Political Economy*, Vol.98, No.3, 1990, pp.519-543.

Constantinides, **G**. **M**., **Ferson**, **W**. **E**., "Habit Persistence and Durability in Aggregate Consumption: Empirical Tests", *Journal of Financial Economics*, Vol.29, No.2, October 1991, pp.199-240.

Cornell, **B**., "Equity Duration, Growth Options, and Asset Pricing", *Journal of Portfolio Management*, Vol.26, No.3, spring 2000, pp.105-111.

Damant, **D**.**C**., **Satchell**, **S**. **E**., "Testing for Short Termism in the UK Stock Market: A Comment", *Economic Journal*, Vol.105, No.432, September 1995, pp.1218-1223.

Damant, **D**.**C**., **Hwang**, **S**., **Satchell**, **S**. **E**., "Using a Model of Integrated Risk to Assess U.K. Asset Allocation", *Applied Mathematical Finance*, Vol.7, No.2, 2000, pp.127-152.

Dimson, **E**., **Marsh**, **P**., "U.K. Financial Market Returns, 1955-2000", *Journal of Business*, Vol.74, No.1, January 2001, pp.1-31.

Elton, **E.J**., **Gruber**, **M.J**., Modern Portfolio Theory and Investment Analysis, 5th Edition, Wiley, 1995.

Evans, **M**. **D**. **D**., "Real Rates, Expected Inflation, and Inflation Risk Premia", *Journal of Finance*, Vol.53, No.1, February 1998, pp.187-218.

Fabozzi, **F**. **J**., The Handbook of Fixed Income Securities (*Fifth Edition*), Irwin Professional Publication, 1997.

Fama, **E**. **F**., **French**, **K**. **F**., "Disappearing Dividends: Changing Firm Characteristics or Lower Propensity to Pay", *Journal of Financial Economics*, Vol.60, No.1, 2001, pp.3-41.

Farrar, **J**. **H**., **Furey**, **N**. **E**., **Hannigan**, **B. M**., Farrar's Company Law, 3rd Edition, Butterworths, 1991.

Fisher, **L**., **Weil**, **R**., "Coping with Risk of Interest Rate Fluctuations: Returns to Bondholders from Naïve and Optimal Strategies", *Journal of Business*, Vol.44, 1971, pp.410-431.

Franks, **J**. **R**., **Torous**, **W**. **N**., "An Empirical Investigation of US Firms in Reorganization*", Journal of Finance*, Vol.44, No.3, July 1989, pp.747-769.

Franks, **J**. **R**., **Torous**, **W**. **N**., "A Comparison of Financial Recontracting in Distressed Exchanges and Chapter 11 Reorganizations", *Journal of Financial Economics*, Vol.35, No.3, 1994, pp.349-370.

Franks, **J**. **R**., **Nyborg**, **K**. **G**., **Torous**, **W**. **N**., "A Comparison of US, UK, and German Insolvency Codes", *Financial Management*, Vol.25, No.3, autumn 1996, pp.86-101.

Garman, **M**. **B**., "The Duration of Option Portfolios", *Journal of Financial Economics*, Vol.14, No.2, June 1985, pp.309-315.

Goetzmann, **W**. **N**., **Jorion**, **P**., "A Longer Look at Dividend Yields", *Journal of Business*, Vol.68, No.4, 1995, pp.483-508.

Gordon, **M**. **J**., The Investment, Financing, and Valuation of the Corporation, Homewood Irwin, 1962.

Gordon, **R**. **H**., **Malkiel**, **B**. **G**., "Corporation Finance", in How Taxes Affect Economic Behaviour, Aaron, H. J., Pechman, J.A., *Studies of Government Finance*, Brookings Institution, 1981, pp.131-198.

Goudie, **A**. **W**., **Hansen**, **S**. **L**., "The Effect of Dividend Controls on Company Behaviour", *Applied Economics*, Vol. 20, No.2, February 1988, pp.143-164.

Gould, **J**. **B**., **Sorensen**, **E**. **H**., "Duration: A Factor in Equity Pricing", *Journal of Portfolio Management*, Vol.13, No.2, fall 1986, pp.38-43.

Griffin, **M**. **W**., "A Global Perspective on Pension Fund Asset Allocation", *Financial Analysts Journal*, March/April 1998, pp.60-68.

Hevert, **K**. **T**., **McLaughlin**, **R**. **M**., **Taggart**, **R**. **A**., "Growth Options and Equity Duration", *Journal of Portfolio Management*, Vol.25, No.1, fall 1998, pp.43-50.

Hicks, **J**. **R**., Value and Capital, Oxford, 1939.

Huang, **C-F**., **Litzenberger**, **R**. **H**., Foundations for Financial Economics, Prentice-Hall, 1988.

Hurley, **W**. **J**., **Johnson**, **L**. **D**., "A Note on the Measurement of Equity Duration and Convexity", *Financial Analysts Journal*, Vol.51, No.3, May/June 1995, pp.77-79.

Hutton, **W**., The State We're In, Vintage Publications, London, 1996.

Hutton, **W**., The State To Come, Vintage Publications, London, 1997.
Idol, **C**. **R**., "Equity duration to assess interest-rate risk", *Credit Union Executive*, Vol.37, No.4, July/August 1997, pp.32-36.

Idol, **C**. **R**., "ALM III: Practical issues with equity duration", *Credit Union Executive*, Vol.37, No.5, September/October 1997, pp.16-19.

Ingersoll, **J**., "Multidimensional Security Pricing" *Journal of Financial and Quantitative Analysis*, Vol.10, No.5, 1975, pp.785-798.

Jean, **W**. **H**., "The Extension of Portfolio Analysis to Three or More Parameters", *Journal of Financial and Quantitative Analysis*, Vol.6, No.1, January 1971, pp.505-515.

Jean, **W**. **H**., "More on Multidimensional Portfolio Analysis", *Journal of Financial and Quantitative Analysis*, Vol.8, No.3, June 1973, pp.475-490.

Jensen, **M**. **C**., **Meckling**, **W**. **H**., "Theory of the Firm: Managerial Behaviour, Agency Costs and Ownership Structure", *Journal of Financial Economics*, Vol.3, No.4, 1976 pp.305-360.

Johnson, **L**. **D**., "Equity Duration: Another Look", *Financial Analysts Journal*, Vol.45, No.2, March/April 1989, pp.73-75.

Johnson, **L**. **D**., "Convexity for Equity Securities: Does Curvature Matter?", *Financial Analysts Journal*, Vol. 46, No.5, September/October 1990, pp.70-73.

Johnson, **L**. **D**., "The Role of Convexity in Equity Pricing", *Financial Analysts Journal*, Vol.48, No.5, September/October 1992, pp.69-74.

Joyce, **M**., **Lomax**, **J**., "Patterns of Default in the Non-Financial Private Sectors", *Bank of England Quarterly Bulletin*, November 1991, pp.534-537.

Kahneman, **D**., **Tversky**, **A**., "Prospect Theory: An Analysis of Decision Under Risk", *Econometrica*, Vol.47, March 1979, pp.263-291.

Kadiyala, **P**.,*"The Relation between the Magnitude of Growth Opportunities and the Duration of Equity"*, *The Journal of Financial Research*, Vol. 23, No. 3, Fall 2000, pp.285-310.

Kraus, **A**., **Litzenberger**, **R**. **H**., "A State-Preference Model of Optimal Financial Leverage", *Journal of Finance*, Vol.28, September 1973, pp.911-922.

Lanstein, **R**., **Sharpe**, **W**. **F**., "Duration and Security Risk", *Journal of Financial and Quantitative Analysis*, Vol.13, No.4, November 1978, pp.653-668.

Lasfer, **M**. **A**., "Ex-day Behavior: Tax or Short-Term Trading Effects, *The Journal of Finance*, Vol. 50, No. 3, Jul 1995, pp.875-898.

Leibowitz, **M**. **L**., "Total Portfolio Duration: A New Perspective on Asset Allocation", *Financial Analysts Journal*, Vol.42, No.5, September/October 1986, pp.18-29 and page 77.

Leibowitz, **M**. **L**., "Spread-driven Dividend Discount Models", *Financial Analysts Journal*, Vol.56, No.6, November/December 2000, pp.64-81.

Leland, **H**. **E**., "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance*, Vol.49, No.4, 1994, pp.1213-1252.

Leland, **H**. **E**., **Toft**, **K**. **B**., "Optimal Capital Structure, Endogenous Bankruptcy and the Term Structure of Credit Spreads", *Journal of Finance*, Vol.51, No.3, July 1996, pp.987-1019.

Macaulay, **F**. **R**., "Some Theoretical Problems Suggested by the Movement of Interest Rates, Bond Yields, and Stock Prices in the United States Since 1856", *National Bureau of Economic Research*, New York, 1938.

Mehra, **R**., **Prescott**, **E**. **C**., "The Equity Premium: A Puzzle", *Journal of Monetary Economics*, Vol.15, 1985, pp.145-161.

Michaely, **R**., **Vila**, **J-L**., "Investors' Heterogeneity, Prices, and Volume around the Ex-dividend Day", *Journal of Financial and Quantitative Analysis*, Vol. 30, No. 2, June 1995, pp.171-199.

Miles, **D**., "Testing for Short Termism in the UK Stock Market", *Economic Journal*, Vol.103, No.421, November 1993, pp.1379-1393.

Miles, **D**., "Testing for Short Termism in the UK Stock Market: A reply", *Economic Journal*, Vol. 105, No. 432, September 1995, pp.1224-1227.

Miller, **M.H**., "Behavioral Rationality in Finance: The Case of Dividends", *Journal of Business*, Vol.59, No.4, 1986, pp.451-468.

Miller, **M**.**H**., **Modigliani**, **F**., "The Cost of Capital, Corporation Finance, and the Theory of Investment", *American Economic Review*, Vol.48, No.3, June 1958, pp.261-297.

Miller, **M**. **H**., **Shiller**, **R**. **J**., "Behavioral Rationality in Finance: The Case of Dividends" *The Journal of Business*, Vol.59, No.4, October 1986, pp. 451-472.

Modigliani, F., "Life Cycle, Individual Thrift, and the Wealth of Nations", *The American Economic Review*, Vol.76, No.3, June 1986, pp.297-323.

Pickens, **T**. **B**., "Professions of a Short-Termer" *Harvard Business Review*, Vol.64, Issue 3, May-June 1986, pp.75-79.

Redington, **F**. **M**., "Review of the Principle of Life-Office Valuations", *Journal of the Institute of Actuaries*, Vol.78, 1952, pp.286-340.

Rubinstein, **M**., "The Valuation of Uncertain Income Streams and The Pricing of Options", *The Bell Journal of Economics*, Vol. 7, 1975, pp.407-425.

Schwartz, **A**., "Security Interests and Bankruptcy Priorities: A Review of Current Theories", *Journal of Legal Studies*, Vol.10, January 1981, pp.1-37.

Shefrin, **H**. **M**., **Statman**, **M**., "Explaining Investor Preference for Cash Dividends", *Journal of Financial Economics*, Vol.13, No.2, June 1984, pp.253-282.

Shefrin, **H**. **M**., **Thaler**, **R**. **H**., "An Economic Theory of Self-Control", *The Journal of Political Economy*, Vol. 89, No.2, April 1981, pp.392-407.

Shiller, **R**. **J**., "Comovement in Stock Prices and Comovements in Dividends", *Journal of Finance*, Vol.44, No.3, July 1989, pp.719-729.

Simonson, **D**. **G**., "How One Leader Manages A/L Risk", *United States Banker*, Vol.103, No.2, April 1993, pp.56-57.

Sundaresan, S., "Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth", *Review of Fiscal Studies*, Vol. 2, 1989, pp.73-88.

Talmor, **E**., **Titman**, **S**., "Taxes and Dividend Policy", *Financial Management*, Vol.19, No.2, summer 1990, pp.32-35.

Webb, **D**. **C**., "An Economic Evaluation of Insolvency Procedures in the United Kingdom: Does the 1986 Insolvency Act Satisfy The Creditors' Bargain?", *Oxford Economic Papers*, Vol.43, No.1, January 1991, pp.139-157.

White, **M**. **J**., "The Corporate Bankruptcy Decision", *Journal of Economic Perspectives*, Vol.3, No.2, spring 1989, pp.129-151.

White, **M**. **J**., "Insolvency and Bankruptcy", in Eatwell, J., Milgrom, M., Newman, P., The New Palgrave Dictionary of Money & Finance, Vol.2, Macmillan (London), 1992, pp.419-422.

Williams, **J**. **B**., "Evaluation By the Rule of Present Worth", The Theory of Investment Value, Harvard University Press, 1938, pp.55-75.

Winter, **S**. **G**., "Economic 'Natural Selection' and the Theory of the Firm", *Yale Economic Essays*, 1964, pp.225-271.

Woodward, **G**. **T**., "The Real Thing: A Dynamic Profile of the Term Structure of Real Interest Rates and Inflation Expectations in the United Kingdom, 1982-89", *Journal of Business*, Vol.63, No.3, 1990, pp.373-398.

Zingales, **L**., "Survival of the Fittest of the Fattest? Exit and Financing in the Trucking Industry", *Journal of Finance*, Vol.53, No.3, June 1998, pp.905-938.