ORIGAMI-SCISSOR hinged geometry method

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Abstract: *The ancient art of origami has produced some origami patterns that can expand and contract. In order to make folding structures such as roofs, and solar panels, a thickness of the material is required, which has resulted in the research of thick origami. This paper proposes a geometry method in order to make thick origami with a transformable technology: scissor-hinged deployable structures. This new type of origami endows the origami with new kinetic properties: it enables origami patterns that had one degree of freedom, to have two degrees of freedom.*

1 Introduction

Folding origami has inspired ideas for applications across many fields including transformable roofs for architecture and aerospace applications such as solar panels. In order to enable these applications a rigidity and thickness of the material is required. In thick origami, the creases are not determined by sheets of paper intersecting, but by the edges of the thicknesses of the panels.

The author of this manuscript has developed a new type of thick origami in which, for first time, not only can the panels fold and unfold, but the creases can expand and contract. This has been done by enabling the thick panels to be made of a transformable technology: scissor-hinged deployable structures. There are many types of deployable structures [Rivas-Adrover 15], and this new type of origami, origami-scissor hinged, also signifies the birth of a hybrid new type of deployable and transformable technology [Rivas-Adrover 18].

2 Thick Origami

The BYU designed a solar array inspired on the origami flasher pattern made with thick silicon panels glued to a flexible film [Zirbel et al 13]. A rigid-foldable curved origami structure was designed through curved folding of a tessellation of cellular structures which provides an envelope with virtual thickness [Tachi 13].

Thick origami patterns had so far been designed with a kinematic model of a zero-thickness sheet, with the creases intersecting at vertices; however the thickness of the panels causes the panels to collide when folding. Various methods have been suggested using the zero-thickness model in order to create thick origami that can deploy, such as tapering the materials to the zero-thickness planes [Tachi 11] and offsetting panels away from the creases [Edmondson et al. 14], however these create surfaces that are not flat or have voids. Other methods have been proposed [Hoberman 88] [De Temmerman et al. 07], where not all the fold lines meet at a point, therefore the origami surface is absent of vertices. 'Origami of thick panels' [Chen et al. 15] provides a comprehensive method for making thick origami, which demonstrates that thick origami can be devised by mechanism theory alone without referring to its parent zero-thickness kinematic model, and that is effective for different types of origami. Figure 1(a) displays a zero thickness paper model of the diamond pattern. Figure 1(b) displays the deployment of the diamond origami pattern made with thick panels by Chen, Peng and You.

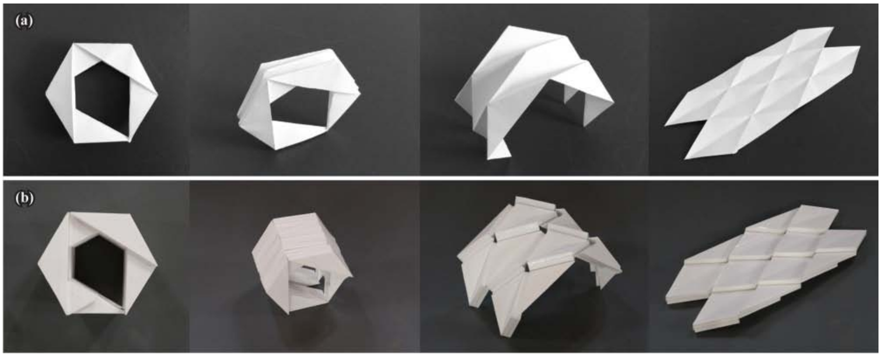

Figure 1: *Diamond pattern. Hexagonal elevation and unfolding sequence. (a) Zero thickness paper model. (b) Origami of thick panels; diamond pattern.*

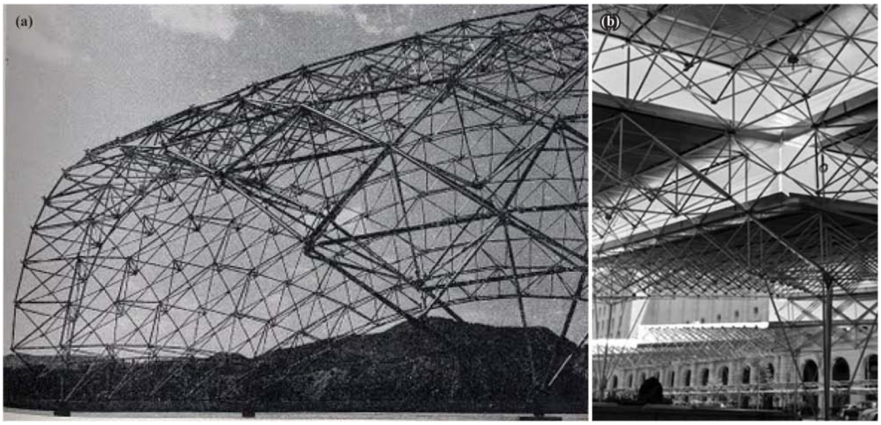

Figure 2: *Scissor-hinged deployable structures. (a) Three-bar scissor spherical prototype. (b) Four-bar scissor transportable pavilion for exhibitions, Madrid.*

3 Scissor-hinged deployable structures

Scissor-hinged deployable structures are made by units of bars joined by a pivot; these units are then replicated and joined by hinges. Pérez Piñero developed scissor structures made with units of three bars [Pérez Almagro 13] and also four bars such as the transportable pavilion for exhibitions [Escrig 93], both shown in Figure 2. Two-bar scissor structures include the swimming pool cover in Seville by Escrig, Valcárcel and Sanchez [Escrig et al. 96], and the Iris Dome by Hoberman [Hoberman 13]. Most scissor structures have been made by replicating units of two bars in grids that make triangles or squares. In this field an advancement is made when a new structure is created with a new geometry that can achieve optimal deployment. The 'form generation method of relative ratios for two-bar scissors' [Rivas-Adrover 17] can be applied to infinite combinations of lines, and therefore allows for infinite scissor structures to be made with optimum deployment.

4 Diamond origami-scissor hinged pattern

Figure 3(a) displays the zero thickness diamond origami pattern next to its thick panel counterpart [Chen et al. 15]. Figures 3(b) and (c) display the diamond origami-scissor hinged pattern developed by the author of this manuscript [Rivas-Adrover 18] next to the thick panel counterpart [Chen et al. 15]; all models are made of six origami faces. Figure 3(b) displays how the origami-scissor hinged pattern can fold as the origami of thick panels, and Figure 3(c) demonstrates how the origami-scissor hinged pattern can further transform due to its expandable creases. This has been made by applying the 'form generation method of relative ratios' (FGMORR) [Rivas-Adrover 17] with translational scissor units to the origami of thick panels [Chen et al. 15]. Where in the diamond thick origami, one triangulated face would be made of two panels, in the diamond origami-scissor structure one triangulated face is made of 77 bars and 124 nodes. The prototype is made of six origami faces made of 744 nodes and 462 bars (with only six different types of bars). The location of the creases is identical, however while in the diamond of thick panels the creases are defined by the continuous length of the edge of the thicknesses, in the diamond origami-scissor hinged pattern the creases are determined by the end nodes of the scissor pantographs. The acrylic bars were laser cut with names in order to identify them. Where two coplanar bars met they were joined by pivots; where non-coplanar or three or more bars met they were joined by elasticated nodes to allow a small margin of deformation. In the theoretical model all nodes meet in one point in space; in the build prototype there are up to 1.5 mm discrepancies in each vertex of the origami triangulated face, which overall cancel each other out and were considered negligible. The prototype is structurally stable and has a fluid motion. While the diamond origami of thick panels has one degree of freedom, the diamond origami-scissor structure has two degrees of freedom because the creases can expand and contract at any stage of the folding or unfolding sequence of the origami.

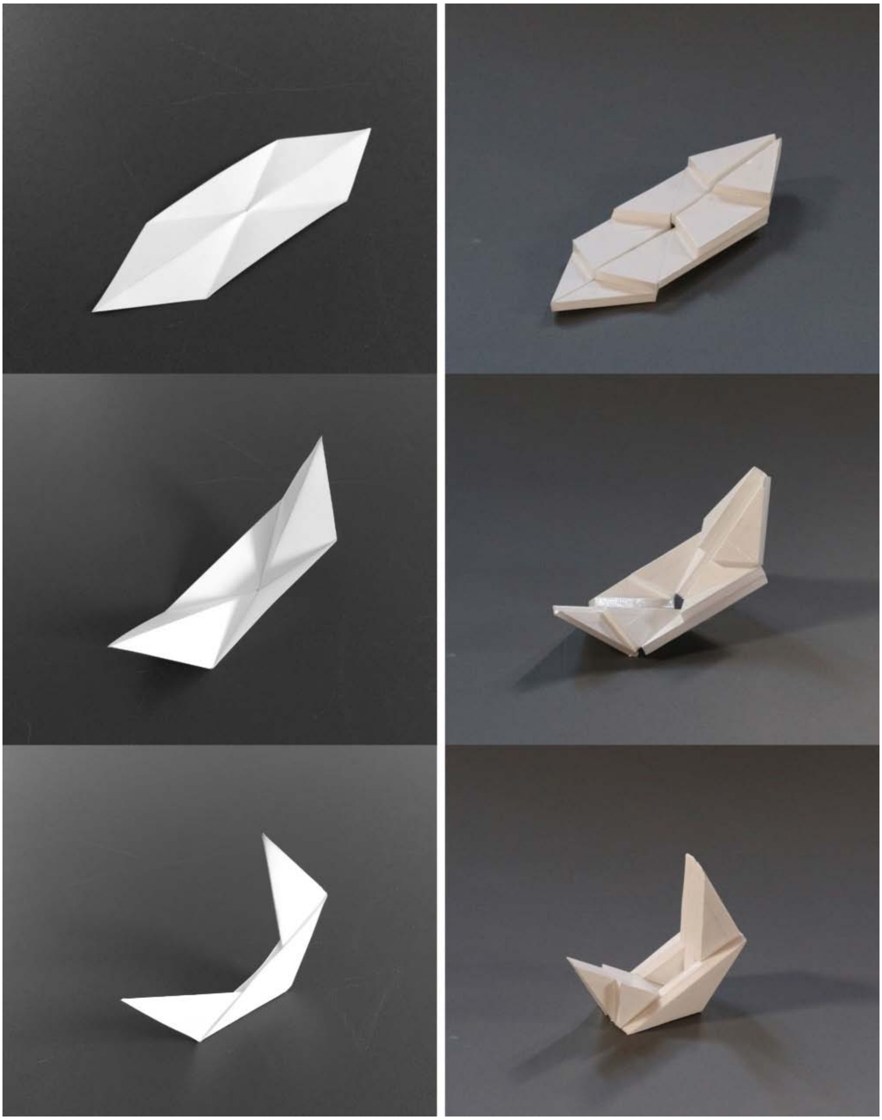

Figure 3 (a): *Diamond pattern. The folding sequence of the diamond origami of thick panels is displayed next to its zero thickness paper model counterpart.*

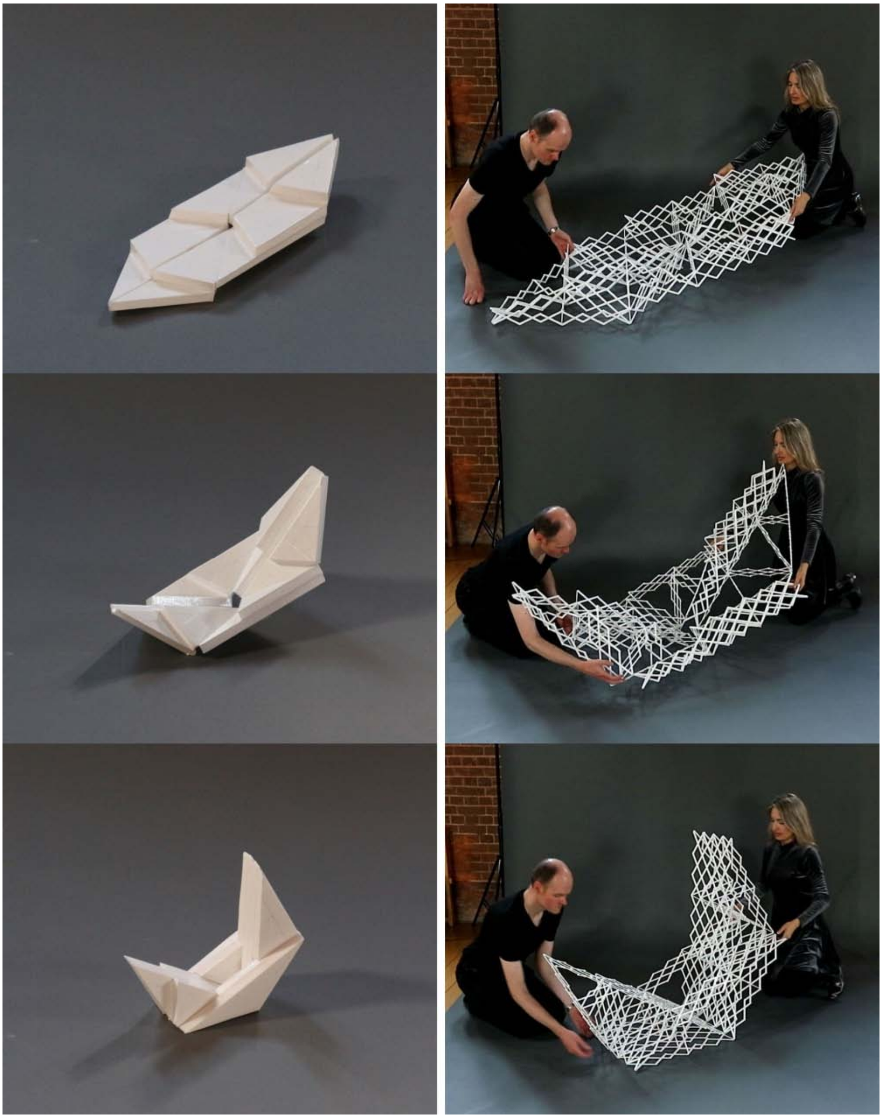

Figure 3 (b): *Diamond pattern. The folding sequence of the diamond origami-scissor hinged pattern is displayed next to its origami of thick panels counterpart.*

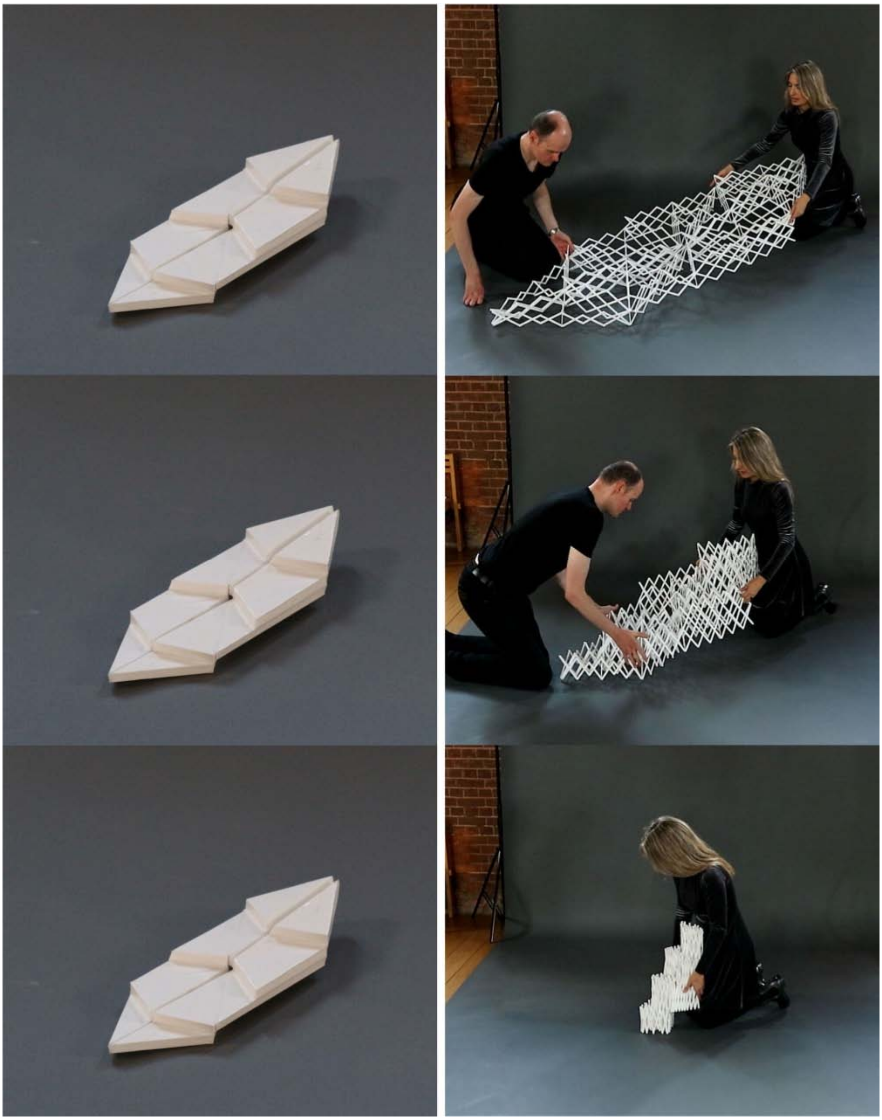

Figure 3 (c): *Diamond pattern. The diamond origami-scissor hinged pattern is displayed next to its origami of thick panels counterpart; while the origami of thick panels remains static, the diamond origami-scissor hinged pattern can further transform due to the scissor pantographs which allow the creases to expand and contract.*

5 Form generation method of relative ratios (FGMORR) applied to thick origami

The first step is to determine the geometry of the origami of thick panels, its thicknesses, vertices, the segments that determine its geometry and its creases. The segments are then made into scissor-hinged pantographs with the 'form generation method of relative ratios' (FGMORR) [Rivas-Adrover 17]. Therefore the thick panels will now be made of virtual thicknesses defined by the scissor pantographs and the creases made by joining the end nodes of the pantographs.

The FGMORR states: in any given combination of lines, a ratio for a scissor unit (or various ratios for different sizes of scissor units with equal angles of motion) can be found as the relation between segments and sub-segments, with respect to the number of times the ratios are contained in the segments and sub-segments. The FGMORR can make infinite scissor hinged surfaces from infinite combinations of lines, therefore the combination of lines that defines the creases of the thick origami can be made into scissor pantographs. This method enables the scissor structure to be made with the minimum number possible of different sizes of bars, as well as achieving an optimal expansion and contraction. In order to apply this method to any given combination of lines, the first step is to identify the smaller sub-segment and to divide it in a series of equal ratios for scissor units; the first ratio is denominated C1. In some occasions the smaller sub-segment can be made of one single ratio for one scissor unit, however when the combination of lines has a certain level of complexity, dividing the smaller sub-segment in three ratios (3 x C1) for instance allows a greater manoeuvrability when transferring this initial ratio to the rest of the segments and sub-segments. The smaller sub-segments dictate the mobility of the structure, therefore this first operation is critical. The following step is to transfer this initial ratio C1 throughout the segments and sub-segments of the structure, and by doing this, other ratios for other scissor units can be generated in order to develop a scissor hinged deployable structure with optimal deployment (a scissor structure has an optimal deployment when the geometry of all the bars allows the structure to fully expand and fully contract). The FGMORR always seeks to find ratios in which the smaller bars are at least half (and ideally greater than half) the size of the original bars in the first ratio C1; this will allow an optimal deployment of the structure made with a reasonable ratio of thickness and length of bars. All these stages are described step by step through figures in the following section, in its application to the waterbomb pattern. For translational scissor units, the bars of all scissor units from the central node to the end nodes must mirror another scissor unit with equal lengths to guarantee an optimum deployability, by doing this the geometric deployability constrain is guaranteed [Escrig 96]. Another property of the FGMORR is that it is not restricted to any given combination of lines, but one could add (or remove) segments or sub-segments; this is particularly relevant when seeking to reinforce and optimize a structure. This versatility of the FGMORR also allows for different configurations of the creases, as it is critical that the creases have equal morphology and bilateral symmetry.

6 Waterbomb origami-scissor hinged pattern

The first step of the origami-scissor hinged geometry method is to determine the geometry, the creases and the relative thicknesses of the origami of thick panels in order to obtain the segments that will be then made into scissor pantographs. If the FGMORR [Rivas-Adrover 17] could indeed make scissor hinged surfaces from infinite combinations of lines, then the segments that determine the geometry of the waterbomb origami or thick panels could be made into a scissor structure. Figure 4(b) displays the deployment of the waterbomb origami of thick panels [Chen et al. 15].

Figure 4: *Waterbomb pattern.(a) Paper model. (b) Origami of thick panels.*

Figure 5(a) displays the geometry and the creases of the waterbomb origami pattern that it is determined by a square; the mid points of the four segments are then joined resulting in an inscribed square within the square generating vertices 1 to 9. Figure 5(b) illustrates segments S6_7, S7_8 and S6_8 that define triangle T6_7_8, and segments S6_2, S2_7 and S7_6 define triangle T6_2_7. By joining vertex 2 to vertex 9 another segment is obtained that intersects segments S6_7 in point 10, which is also the midpoint of the later segment, which generates segment S2_10 and which determines the geometry for a change of thickness: triangle T2_7_10 will have twice the thickness and will be added to triangle T6_2_7 creating three layers of thicknesses. The relative thicknesses of T6_2_7 and T6_7_8 are proportional. Figure 5(c) and 5(d) display both triangles, origami faces, three dimensionally and their creases A, B, C and D defined by the segments. Figures 5(e), 5(f) and 5(g) display the folding sequence of the waterbomb origami of thick panels. In the waterbomb origami of thick panels, the

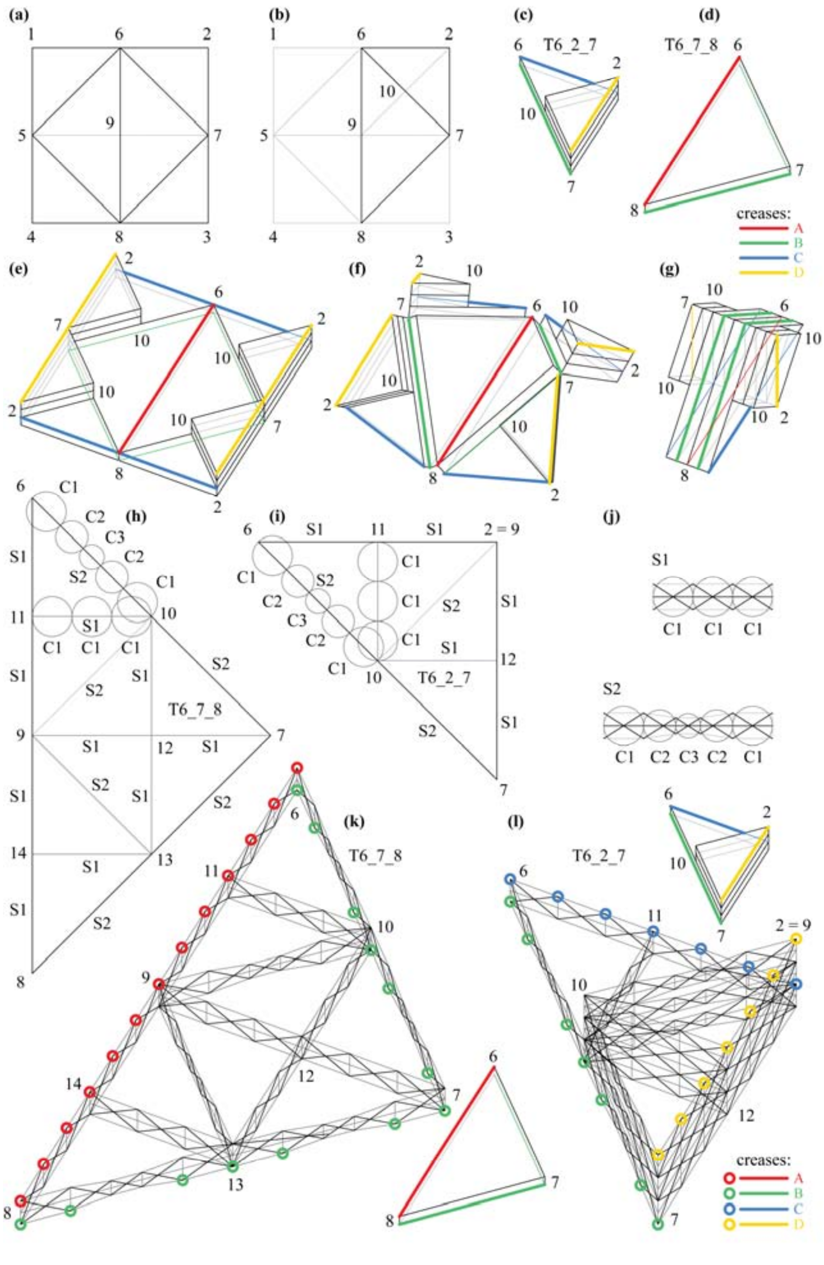

Figure 5: *Origami-scissor hinged geometry method applied to the waterbomb origami of thick panels. (a-g) Determination of the geometry, creases, segments and thicknesses. (h-j) Applying the FGMORR to the segments. (k & l) 3D view of origami faces made with scissor pantographs and its expandable creases.*

different thicknesses have to be proportional; furthermore, the origami face defined by triangle T2_7_10 has to be twice the thickness of T6_2_7 and T6_7_8.

Figure 5(h) and 5(i) display the determination of the geometry of the origami faces T6 _7_8 and T6_2_7 and how the segments and made into scissor pantographs by applying the FGMORR. In order to make the triangle T6_7_8 out of scissor pantographs, further segments are added for reinforcement by triangulation: from point10, vertices 11 to 14 are added from vertex to midpoint and perpendicular to opposite segments. Therefore all the segments and sub-segments that make triangle T6_7_8 are segments: S6_7, S7_8, S6_8 and their sub-segments: S6_10, S10_7, S7_13, S13_8, S6_11, S11_9, S9_14 and S14_8, together with the segments for reinforcement: S9_10, S10_11, S10_12, S9_13, S13_12, S13_14. From all these segments and sub-segments that make triangle T6_7_8, there are only two different types of segments, S1 and S2:

S1 = S6_11 = S11_9 = S9_14 = S14_8 = S10_11 = S10_12 = S13_12 = S13_14
S2 = S6_10 = S10_7 = S7_13 = S13_8 = S9_10 = S9_13 (1)

Therefore, only two different types of pantographs have to be calculated for ratios, S1 and S2, as shown in Figure 5(j); and as long as their ending bars match, S1and S2 can then be replicated throughout in their respective positions. First, the small segment S1 is calculated, which is divided in three ratios denominated C1. This C1 ratio is then transferred to segment S2, and two more ratios C2 and C3 are generated. The geometry of the segments S1 and S2 is defined by the relative ratios C1, C2 and C3, which are described by the following geometrical relationships described by the following set of equations:

$$S1 = 3 \times C1$$
$$S2 = (2 \times C1) + (2 \times C2) + C3$$
$$S2 = (S1 - C1) + (2 \times C2) + C3 \quad (2)$$

Figure 5(k) and 5(l) display a three-dimensional view of the triangulated origami faces T6_7_8 and T6_2_7 made with the scissor pantographs with the previously calculated segments S1 and S2 and with the creases at the end nodes highlighted: A, B, C and D. Figure 6 displays six faces of the waterbomb origami-scissor hinged pattern next to its origami of thick panels counterpart. While creases A, B, C and D in the waterbomb of thick panels are continuous segments, in the waterbomb origami-scissor hinged pattern the creases are determined by points along the segments; this allows the creases to expand and contract. Figure 7(a) displays the origami fold of the waterbomb origami-scissor hinged pattern next to its origami of thick panels counterpart. Figure 7(b) displays the further transformation of the waterbomb origami-scissor hinged pattern due its expandable creases. While the waterbomb origami or thick panels has one degree of freedom, the waterbomb origami-scissor hinged structure has two

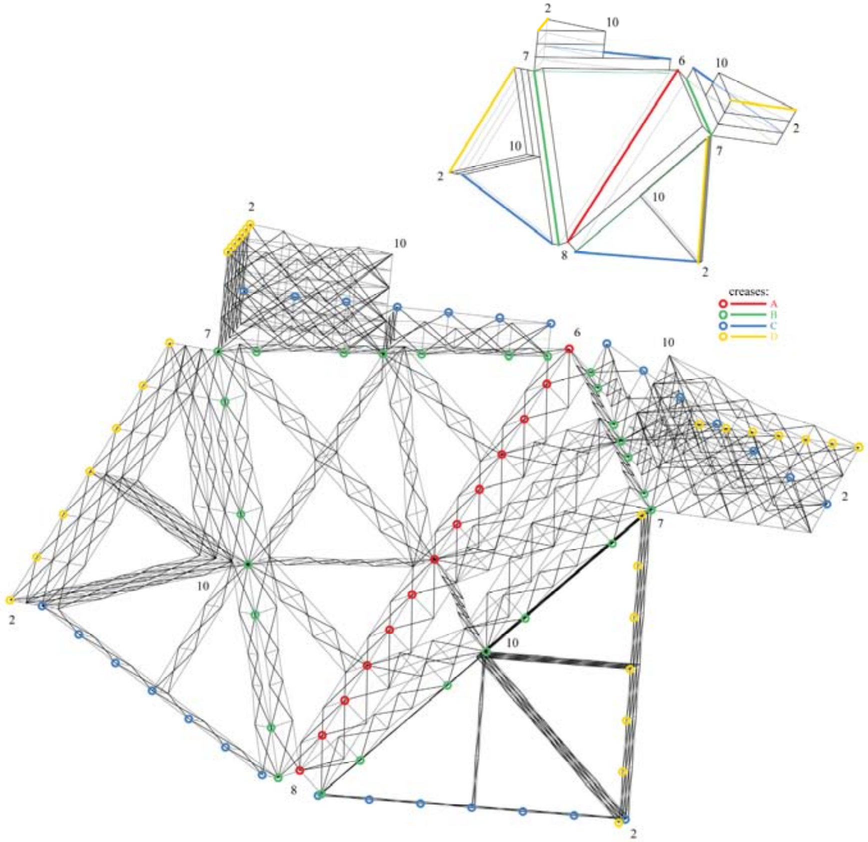

Figure 6: *Waterbomb origami-scissor hinged pattern displayed next to its origami of thick panels counterpart. While creases A, B, C and D in the waterbomb of thick panels are continuous segments, in the waterbomb origami-scissor hinged pattern the creases are determined by points along the segments, this allows the creases to expand and contract.*

degrees of freedom, as the creases can not only fold and unfold, but they can expand and contract. The expansion and contraction of the creases could happen at any stage of the folding sequence of the origami-scissor hinged pattern.

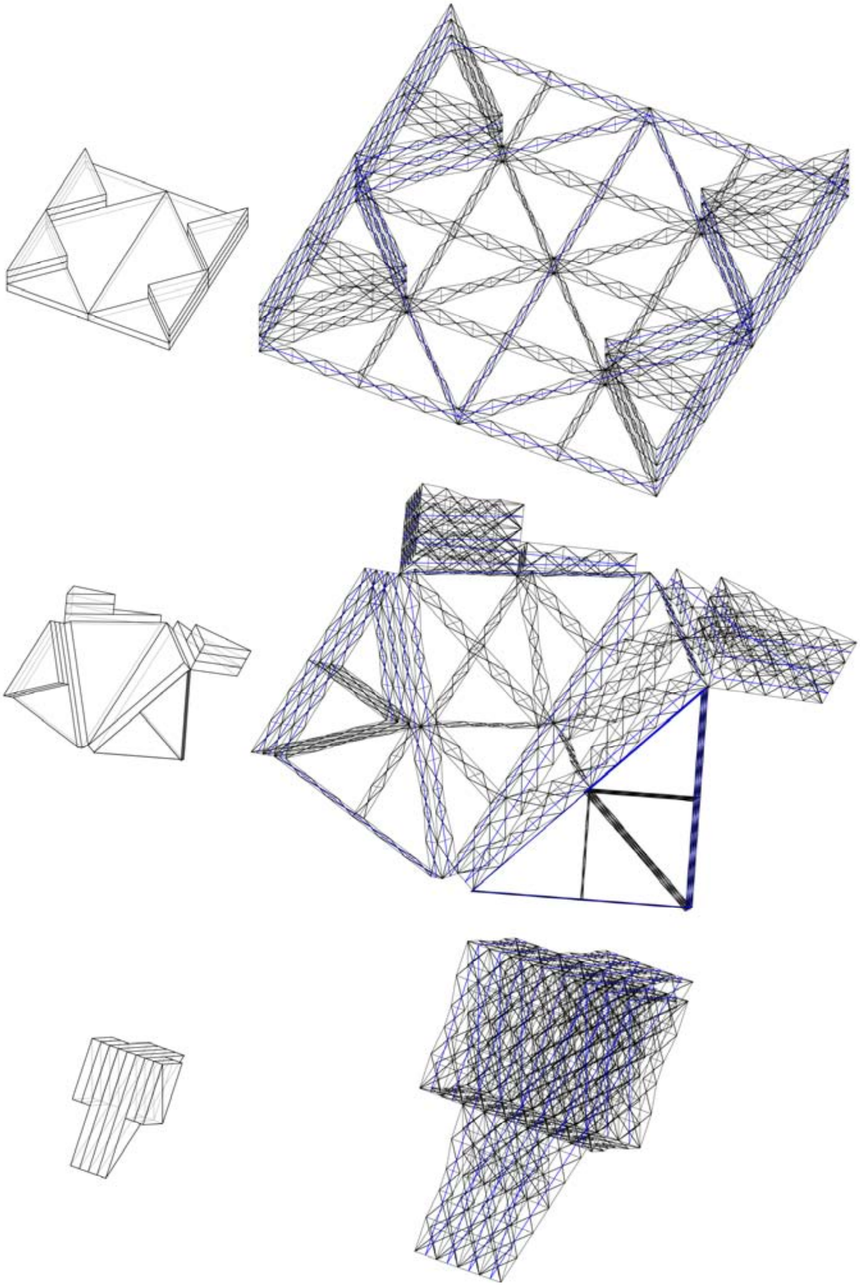

Figure 7(a): *Waterbomb pattern. The waterbomb origami-scissor hinged pattern is displayed next to its origami of thick panels counterpart; origami fold.*

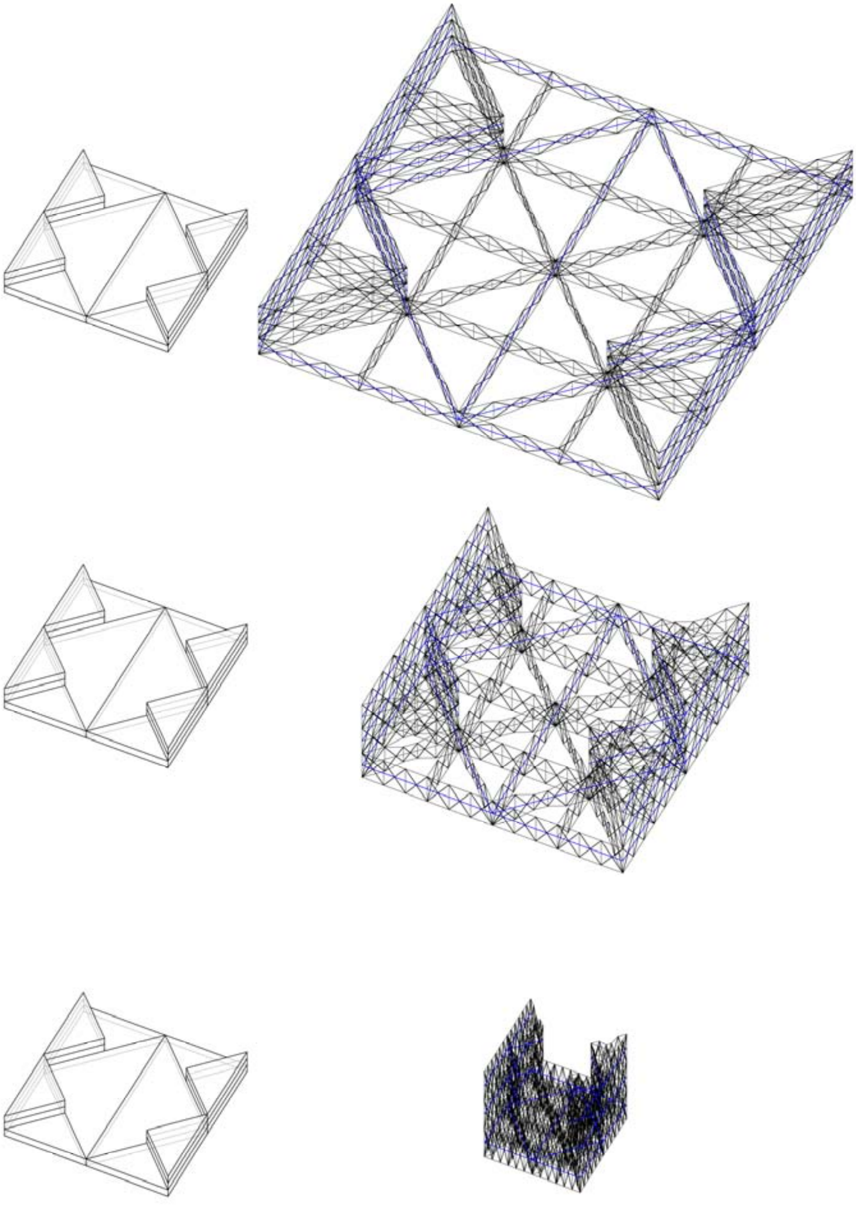

Figure 7(b): *Waterbomb pattern. The waterbomb origami-scissor hinged pattern is displayed next to its origami of thick panels counterpart; while the origami of the thick panels remains static, the waterbomb origami-scissor hinged pattern can further expand and contract due to the scissor pantographs.*

7 Conclusions

The diamond origami-scissor hinged pattern marks a new type of thick origami in which the creases can expand and contract, and the panels are made by virtual thicknesses defined by the scissor-hinged deployable pantographs. Here the origami-scissor hinged geometry method is enunciated, which consists of applying the 'form generation method of relative ratios' (FGMORR) [Rivas-Adrover 17] to the thick origami, and has been applied to the diamond and waterbomb patterns. This new type of origami, origami-scissor hinged, is also a new type of transformable and deployable technology that endows the origami with remarkable kinetic properties: origami patterns that had one degree of freedom, have now two degrees of freedom, because the creases can not only fold and unfold, but also expand and contract.

The 'form generation method of relative ratios' (FGMORR) [Rivas-Adrover 17] has been applied to the 'origami of thick panels' [Chen et al. 15] because this method to make thick origami can be extended and generalized to different types of origami, and therefore the origami-scissor hinged geometry method can also be applied to all these different types of origami. A critical condition is that the thick origami has to be made of equal or proportional thicknesses so that when translating that geometry with scissors the end nodes match. Another condition is that the pantographs that mark the creases and join different origami faces must have an equal morphology and bilateral symmetry. Automation of this method will be investigated with Grasshopper for Rhinoceros.

Origami-scissor hinged patterns provide an extra degree of freedom, origami patterns that could be folded can now also contract and occupy much smaller volumes. This would be useful in applications where a high ratio of deployed-to-stowed volume is required such as space applications, earthbound transportable applications, and to create adaptable spaces and transformable environments in permanent architecture.

Image credits

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