This is the accepted version of the paper published in *Structural Change and Economic Dynamics*,
https://doi.org/10.1016/j.strueco.2017.07.001

Leonardo Costa Ribeiro(*)
Leonardo Gomes de Deus(**)
Pedro Mendes Loureiro(***)
Eduardo da Motta e Albuquerque(**)

A network model for the complex behavior of the rate of profit:
exploring a simulation model with overlapping technological revolutions(***)

(4 May 2017)

Abstract:
This article proposes a network model to replicate the behaviour of the profit rate in the long run. Specifically, it accounts for the results of an empirical investigation of the profit rate in the US, which show that it has fractal properties and its complexity changes over time. The starting point of the model is Marx's insights on the interplay between the tendency of the rate to fall and its countertendencies. It combines these insights with the persistent generation of new commodities – inventions – and a specific set of new branches of production that triggers technological revolutions. A simulation running this network model successfully replicates historical features of the system.

Key Words: rate of profit; technological revolutions; Marx; complex systems; metamorphoses of capitalism; simulation models.

JEL Codes: P160, O33, B510.

(*) Inmetro, Rio de Janeiro, Brazil
(**) Cedeplar-UFMG, Belo Horizonte, Brazil
(***) SOAS, London, UK
(**) Cedeplar-UFMG, Belo Horizonte, Brazil
(***) This manuscript is funded by CNPq (Processes 486828/2013-1, 459627/2014-7 and 302857/2015-0) and CAPES (BEX 0840/14-9). This version benefited from comments and criticisms from Fred Moseley, Adalmir Marquetti and João Machado in a Workshop on the Rate of Profit – during the 2016 Seminário de Diamantina (2 September 2016). The usual disclaimers hold.

INTRODUCTION

The opening sentence of *Das Kapital* says that "the wealth of societies in which the capitalist mode of production prevails appears as an 'immense collection of commodities'" (Marx, 1867, p. 126). As Marx shows in the following pages, chapters and volumes, this "immense collection of commodities" grows over time. It might be difficult to guess how many commodities might have existed in 1867, but today we can speculate that they number above 100 million. There are two sources for this number. The first is the ThomasRegister of American Manufactures, which looks mainly to suppliers and producers. This website claims that "our engineers consolidated over 100 million items."[1] The second is a website that deals with barcode information, mainly consumer goods, which indicates that "currently the database includes over 109.162.000 entries, but that is still only a fraction of all the European Article Numbers (EANs) ever issued."[2]

The intrinsic logic for this boundless expansion of the "immense collection of commodities" is developed in Marx's theory. In *Das Kapital*, the commodity contains the "germ of the money-form" (Marx, 1867, p. 163). Money is transformed into capital (p. 252), whose movement is "limitless" (p. 253) and whose aim is "the unceasing movement of profit-making." Capital and its valorisation comprise the accumulation of capital (p. 725), capital has the "power of expansion" (p. 752) and the credit system (p. 777) fuels this "power of expansion." As an empirical verification of this "power of expansion," Figure 1 shows the growth of the wealth of the United States between 1870 and 2008.

****

FIGURE 1

****

Marx also laid a roadmap to study the role of particular commodities and their interactions in these expansionary movements. Certain new commodities – certain inventions – open new branches of production, some of which have large impacts on the system's dynamics and trigger technological revolutions. The anatomy and dynamics of a

---

[1] See https://www.youtube.com/watch?v=G97zT9P6TUk and stop at 1m42s. For other academic uses of data from the ThomasRegister of American Manufactures, see Agarwall and Gort (1996).

[2] See https://www.ean-search.org/. You may copy the barcode of your book *Das Kapital*, search the site https://www.ean-search.org/, and see that Marx himself contributed to the expansion of this "immense collection of commodities."

technological revolution are discussed in chapter 15, Volume 1, of *Das Kapital*, where Marx (1867) describes the emergence of "machinery and large-scale industry" – the industrial revolution, capitalism's first technological revolution (Perez, 2010, p. 190). Technical change is pervasive, since for Marx "the additional capitals formed in the normal course of accumulation serve above all as vehicles for the exploitation of new inventions and discoveries" (1867, p. 780). This affects the entire system, since "the intermediate pauses in which accumulation works as a simple extension of production on a given technical basis are shortened" (p. 782).

The resulting dynamics of capital accumulation, according to Marx, is not a continuous or smooth expansion, but rather "takes the form of a decennial cycle (interrupted by smaller oscillations) of periods of average activity, production at high pressure, crisis and stagnation" (Marx, 1867, p. 785). In the third section of the third volume of *Das Kapital*, Marx combines those contradictory movements into a discussion of the long-term movements of the profit rate. There, Marx presents the interplay between the tendency of the rate of profit to fall – a consequence of the nature of capital accumulation – and its countertendencies (Callinicos, 2014, p. 270). The presentation of this contradictory interaction between the tendency of the rate of profit to fall and its countertendencies is Marx's major contribution to the subject (Ribeiro et al, 2017).

Figure 2 offers an empirical verification of the long-term dynamics of the profit rate. It shows, once more for the United States, the ups and downs of "the unceasing movement of profit-making" between 1870 and 2011.

****

FIGURE 2

****

Figures 1 and 2 summarise two key features of capitalism, namely the presence of intense capital accumulation and the turbulent long-term dynamics of the profit rate. In other words, there is a strong expansionary logic combined with crises, or downward movements of the profit rate. The "immense collection of commodities" grows over time, and it grows exponentially.

Those two long-term movements show a system that never repeats itself – this is the cyclical behaviour of an expansionary system, which not only is complex (see Ribeiro et al,

2017) but whose complexity, as argued below, also changes.[3] Back of the envelope calculations show that, in 1880, when the rate of profit was above 0.30 (see Figure 2), the mass of profits in the US was close to US$ 58 billion (Maddison's 1990 International Geary-Khamis dollars, Figure 1). In 2008, when the rate of profit was around 0.18, the mass of profits was approximately US$3,699 billion.[4] As a comparison, between 1880 and 2008, the US population grew 6.0 times, the mass of profits 64.2 times and GDP 61.5 times (Maddison, 2010; ILO and OECD, 2015). These data show increases in the size of the economic system – a reflection of the growth of the "immense collection of commodities." These changes of scale have important implications, related to complexity, because "more is different" (Anderson, 1972).

To study this complex and expanding system, this article proposes a network model that replicates the behaviour of the profit rate in the long run. The focus on the profit rate is based on the interpretation that this is a key variable, a "synthesis of multiple determinations" that summarises complex movements of an ever-changing system. A network model in turns allows for the permanent inclusion of novelty – new commodities, inventions – constantly generated by the workings of the system in search for profits.

This article is organised in five sections. The first discusses the theoretical framework, integrating new commodities – inventions – and a specific set of new branches of production that triggers technological revolutions. The second section empirically evaluates the long-term behaviour of profit rate, applying techniques from the Physics of Complexity to investigate its non-linearity and complexity. The third section combines the theoretical discussion to the empirical findings to suggest a model that replicates the workings of this system. The fourth section runs a simulation and presents its results. The fifth and final section discusses the implications of the proposed model.

---

[3] Throughout this article, complex systems and complexity are used as in Physics. The economy is thus a complex system, not a complicated one. A system or problem is complicated if it is difficult to solve, but in general it can be split into small parts which are the solved independently or through small alterations of simpler systems/problems. Complex systems are in turn formed by a large number of interacting components and exhibit different organisations at different scales of observation, which lead to different behaviours at different scales (Goldenfeld and Kadanoff, 1999).

[4] Mass of profits = capital share of the economy times GDP. Data for capital share come from the ILO and OECD (2015) and for GDP from Maddison (2015). For the definiton of profits, see Duménil and Lévy (1993, p. 29).

I- THE TENDENCY FOR THE RATE OF PROFIT TO FALL, ITS COUNTERTENDENCIES, AND CHANGES IN THEIR INTERPLAY

Why the focus on profits? Profits are a key variable in capitalism, and the profit rate may summarise the behaviour of the whole system as it is determined by the influence, interaction and manifestation of multifarious factors. From Adam Smith (1776), to Schumpeter (1911) and to modern economics of industry and innovation (Nelson and Winter, 1982; Schamelensee, 1989), the behaviour of the profit rate – its ups and downs, its differences across industries and sectors, and, most importantly, the search for higher profits – is a key concern of economics.

What is Marx's specific contribution to a theory of profits? Marx, between 1863 and 1865, drafted a special section in the manuscripts for Volume III of *Das Kapital* – edited by Engels as Section III – on the long-term behaviour of the profit rate. His insights were presented as an interplay between a tendency for the rate of profit to fall and countertendencies to the latter. Didactically, Marx first presented the logic that pulls the profit rate down, then developed the factors that could push it up. Finally, as his most distinctive contribution, he elaborated a specific struggle between the tendencies and countertendencies, stressing the variegated interactions and influences between those two phenomena (Marx, 1894, chapters 13, 14 and 15).

This struggle between the tendency for the rate of profit to fall and its countertendencies, which became chapter 15 in Engels's edition of Volume III, is what set Marx apart from the previous developments in Classical Political Economy. His forbears always saw the fall of the rate of profit as an inevitable consequence of the system's dynamics. After Marx, with this struggle between the tendency and the contertendencies, the result became an open-ended process, summarised with a short sentence that concludes the third section of volume three: "hence the crises" (Marx, 1894, p. 375). Crises furthermore trigger countertendencies, as they devaluate masses of capital, and, as discussed elsewhere (Paula et all, 2016), they are moments for the reconfiguration of capitalism – metamorphoses of capitalism.

The starting point for this article is this synthetic and insightful Section three of Volume III, which indicates the possibility of understanding capitalism as a complex system. As indicated in the introduction, Marx could only discuss the profit rate at this late stage, because this presupposes all previous concepts and understandings – it is, as Marx would say, "synthesis of multiple determinants." Of key importance for the present elaboration is how Marx presents the "simultaneous and contradictory operation" of the movements that pull the profit rate down and up, suggesting the dynamics of "reciprocal effects" and thus a process in the realm of complexity (for a broader presentation of those arguments, see Ribeiro et al, 2017, section 2).

If capitalism is a complex system, it can be further seen whether its complexity changes over time. Elements supporting this conjecture may be illustrated in Figures 1 and 2: the growth of wealth is illustrated in Figure 1, whilst its drivers are shown in Figure 2. In other words, metamorphoses of capitalism mean a system with a growingly "immense collection of commodities," more actors, more institutions, more innovations, more firms, more competitors, new patterns of competition and new interactions between all those factors. Therefore, the intensity of existing factors responsible for the tendency or the countertendency of the rate of profit to fall may change over time, and new factors that affect the interplay between them may also be created.

Once more, Marx left hints that might motivate an investigation of the changing complexity of capitalism. Elaborating on the interplay between the tendency and the countertendencies, Marx wrote a paragraph that shows how he might have sensed the creation of more and new actors. He is commenting a passage from Richard Jones, concurring that "despite the falling rate of profit, the 'inducements and faculties to accumulate' increase," and then lists six reasons for this: for instance, the "increasing diversity of branches of production," the "development of the credit system," the "growth in needs and desire for enrichment," and the "growing mass of investment in fixed capital" (Marx, 1894, p. 375). In the framework proposed here, the first two reasons would exemplify the creation of new factors, whereas the last two show changes of existing factors.

Although changes happen in multifarious dimensions, this article focuses solely on the development of technology, due to its contradictory effects on the interplay between the tendency and the countertendencies.[5] On the one hand, the system changes all the time

[5] Our focus on technological innovation is a simplification that minimises other institutional changes in the economy. This is nevertheless valid, because technological innovations are directly related to various

through the appearance of new commodities – inventions – that may open new branches of production. Some of the latter are related to technological revolutions, in a dynamic closely connected capital accumulation (chapter 23, volume I). On the other hand, Marx highlights the "increasing diversity of branches of production" and connects it to movements of the profit rate. In a list of countertendencies, he mentions several influences of technological progress as countertendencies to the fall of the rate of profit (organised as chapter 14 by Engels): the "use of inventions" (p. 341), the "devaluation of existing capital ... that goes hand in hand with the development of industry" (p. 343), and "new branches of production" (p. 344). In other words, innovation is of paramount importance for the behaviour of the profit rate.

Marx's works are part of any systematic elaboration on technical change (Rosenberg, 1976). Returning to a very basic starting point, it has been shown that the "immense collection of commodities" grows all the time. New commodities – inventions and improvements in existing commodities – change the system all the time, and a feature of complex systems is that they continuously change. Furthermore, Marx showed how changes in one point affect other parts of the system. A new commodity – a new machine, for instance – not only increases the "immense collection of commodities," but also pushes the devaluation of existing ones, which become obsolete. A new commodity triggers a chain of events that "reacts back on all the old commodities of the same type" (Marx, 1867, p. 318), it causes "moral depreciation" of existing machines (Marx, 1884, p. 264). Marx's view of a continuously changing system, wherein new commodities have a cascading impact on several planes, suggests the presence of network effects.

This view that can be found in Marx, according to which the development of certain key commodities might have systemic impacts, is the basis to study technological revolutions. To reiterate, new commodities are invented all the time and a parcel of them may open new branches of production (NBoP), some of which will have a strong impact on the whole system and thus be related to technological revolutions. Marx's works on the industrial revolution may in fact be read as a pioneer in this, as he shows how a change in a

institutional innovations regarding finance, the state, education, scientific activities, nature, the scope of leading firms etc.

key "new branch of production" – the mechanisation of textiles – spread to forward and backward sectors to revolutionise the whole economy (Marx, 1867, chapter 15).

Kondratiev takes up the investigation of how certain key technologies have a strong effect on economic dynamics – i.e. he can be seen as a researcher of technological revolutions. Kondratiev (1926) stresses a key contribution that "significant changes in the main conditions of economic life" have on the dynamics of "capitalist economic life", the first of which are "far-reaching changes in manufacturing techniques and capacity (which, in turn, are preceded by significant technical inventions and discoveries)" (1926, p. 38). After discussing the industrial revolution, Kondratiev (p. 39) goes on to show a second "series of technical inventions" – from the "improvement of the steam engine (1824)" to the "invention of the cable system (1848)" – and then a third series (in the 1870s-90s), connected to "electricity and chemical knowledge," "a new industrial revolution" (p. 40). Those "technical inventions" would be new factors, which influence different sectors and initiate feedback mechanisms. This leads to changes in behaviour of the profit rate, and hence in its dynamics.

This reference to Kondratiev focuses in his identification of series - patterns of behaviour - of "far reaching changes in manufacturing techniques" that are related to the origin of special NBoP, new factors. The origin of those new factors is a phenomenon that has a dual role in the subject of this section, as it means "countertendencies at work".

In the huge literature derived from Kondratiev's works,[6] two by-products are important for this section: the approach related to General Purpose Technologies (GPTs) (Rosenberg, 1998a) and the systematisation of technological revolutions and their big bangs (Perez, 2010). Perez (2010) discusses big bangs that initiated technological revolutions, which in the current framework can be seen as new branches of production that triggered long chain of economic and technological events. Rosenberg (1998a), a historian of technological revolutions sceptical of long waves (see Rosenberg and Frischtak, 1983), in turn has key contributions for our differentiation between new commodities, new branches of production and special new branches of production.

[6] Schumpeter's (1939) re-elaborated his views on long waves of capitalist development, and later, in the 1970s and 1980s, there was a return of the discussion on long waves (Freeman and Louçã, 2001).

As Rosenberg (1998a) argues, some new branches of production may become so-called GPTs. He shows how different GPTs – or how different special new branches of production – have huge effects upon the economy and, furthermore, how they may interact and reinforce each other. This elaboration that focuses on GPTs and their eventual overlapping is very important to this section, since suggests that, as economies develop, they may have more technological centres of dynamism which interact among themselves. Thus, not only would new factors be created and old ones transformed, but they would also interact among themselves with network effects, adding to the list of possible "countertendencies at work".

Perez (2010, p. 190) summarises a broad view of five technological revolutions in the history of capitalism, describing their main features and the "big bangs initiating the revolution." The "five technological successive technological revolutions" and their big bangs are: 1) in 1771, "Arkwright's mill opens in Cromford;" 2) in 1829, "test of the Rocket steam engine;" 3) in 1875, "the Carnegie Bessemer plant opens in Pittsburg;" 4) in 1908, "first Model-T comes out in Detroit;" and 5) in 1971, "the Intel microprocessor is announced in Santa Clara."[7] This scheme summarises how very special NBoP, truly GPTs, were regularly created throughout the history of capitalism.

These big bangs are, moreover, interconnected. They are related to each other and to other important technological innovations; they are factors that interact amongst themselves. In this vein, Kondratiev (1926) presented long lists of new technologies related to the first, second and third industrial revolutions, respectively "spinning and weaving, the chemical industry, the metallurgic industry, and so on," including "techniques of communication" (p. 39); the steam engine, the turbine, Portland cement, the harvester reaping-machine, the first car, electromagnetic telegraphy, the first wheeled steam engine, the rotatory press, the sewing machine, and the cable system (p. 39); and the dynamo, the vacuum pump, the drilling machine, the electric telephone, new methods for producing steel, the electric railway, petrol engines, AC power transmission, wireless telegraphy, the Diesel engine, and airplanes (p. 40). Perez (2010, p. 190) recognises the interrelatedness of

[7] Figure 2, in the introduction, shows data on the profit rate for the US, spanning the time (1870-2010) during which three big bangs were triggered in the US, according to Perez (2010) (1875, 1908 and 1971).

such technologies, in that, for instance, she calls the fourth industrial revolution "the age of oil, the automobile and mass production."

One of the most important contributions of the GPTs approach is precisely its view on the interaction between different technologies, through the analysis of positive feedback loops between them. Rosenberg's (1998a) investigation on chemical engineering as a GPT elaborates on this kind of interaction – in this case, between the automobile industry, petroleum and chemical industries. For Rosenberg, "after 1920 the history of chemical engineering simply became inseparable from the history of petroleum refining" (p. 180), as the consequence of a "spectacular growth in the automobile industry"(p. 180). The connections between them are now seen as "natural and inevitable," but this was not by any means the case in the 1920s: "before the invention of the internal combustion engine, petroleum was valued primarily as an illuminant and a lubricant" (p. 181). Petroleum "became a major source of fuel for transportation purposes only as a result of the invention of the automobile" (p. 181). Rosenberg shows, furthermore, how those developments lead to other NBoP: "skills that were initially acquired in petroleum refining were later transferred to the much larger canvas of the emerging petrochemical industry, including major new product categories such as plastics, synthetic fibers, and synthetic rubber" (p. 180).[8]

This short review of selected contributions from Marx, Kondratiev, Rosenberg and Perez shows how the creation of factors (new commodities –inventions – NBoP, GPTs, radical technologies) is recurrent in the history of capitalism, consistently altering the behaviour and the dynamics of the profit rate. Marx, in his unedited notes on the rate of profit (1894, chapters 13, 14 and 15), mentions how the introduction of NBoP may impact the economy as countertendencies to a decline of the profit rate, whilst Kondratiev (1926, pp. 38-40) differentiates between those new branches to highlight a small subset with great impact on the dynamics of the overall system. Rosenberg elaborates upon this

[8] This interrelatedness – interaction – described by Rosenberg certainly is present in numerous other combinations of technologies. Two other fascinating essays explore steam power (Rosenberg and Trajtenberg, 2004) and electricity (Rosenberg, 1998b) as GPTs. Rosenberg and Frischtak (1983, pp. 149-150), probably taking up the way Marx investigated the industrial revolution, had earlier also stressed "forward" and "backward linkages": "the interindustry flow of new materials, components and equipment may generate widespread product improvement and cost reduction throughout the economy. This has clearly been the case in the past among a small group of producer goods industries – machine tools, chemicals, electrical and electronic equipment" (p. 150).

differentiation by identifying special NBoP that he terms GPTs (1998a, 1998b and 2004), because they present several interrelations – interactions – with other NBoP or technologies. Due to these interactions, the creation and change of factors causes perturbations in the system, more specifically in the factors that interact directly with those created or transformed in the first moment. These impacted factors then perturb those that that interact with them and so on, establishing a chain of events that may reinforce changes through positive feedbacks and cause a broader effect on the system as a whole. Therefore, if these highly connected GPTs are perturbed there is a higher probability of a global reconfiguration of the system to occur. Changes in the behaviour of capitalism can thus be interpreted as an ongoing sequence of perturbations and reconfigurations towards new self-organised states.

This review of technological innovation puts forward a question about the third section of Volume III of *Das Kapital*: the countertendencies do not remain the same over time, and the interplay between them and the tendency for the rate of profit to fall is not static. Therefore, we must empirically investigate whether or not the interplay between the tendency and the countertendencies also changes over time. How those perturbations affect the dynamics and configuration of this complex system will be investigated empirically in section II and modelled in sections III and IV.

II- EMPIRICAL TESTS TO UNCOVER A DYNAMIC INTERPLAY BETWEEN TENDENCY AND COUNTERTENDENCIES

This section has two main goals. First, it investigates whether or not the movements of profit rate (shown in Figure 2) are in the realm of complexity. Second, it determines whether this complexity changes over time or not.

The first question may be answered through the decomposition of the Fourier transform of the movements shown in Figure 2. Ribeiro et al (2017, pp. 13-15) did so and found a power law regression exponent close to -1, which indicates that the profit rate displays fractal behaviours. This answers the first question: the system is complex.

Does this complexity change over time, however? In other words, does the interplay between the tendency of the rate of profit to fall and the countertendencies change over time? How to investigate this? The strategy to explore this involves two connected questions: first, does the result of the Fourier transform present dynamic properties?; second, does the exponent of the power law change? Those questions are important, because if there is no dynamic properties in the result of the Fourier transform and the exponents do not change, there would be no change in the interplay discussed by Marx in Chapter 15, Volume III, or in the complexity of the system over time.

To assess whether the Fourier transform (FT) displays a dynamic behaviour, the time series for the US profit rate (Figure 2) was analysed by calculating the FT in a moving window shorter than the total period of the time series. Information about longer waves is lost, given the shorter time period of the analysis, in exchange for observing local time-variations of the behaviour of the profit rate.

The main interest here lies in the movement of the PL exponent, which conveys information about the complexity of the system. The coupling between the tendency and the countertendencies is the basis for the system's complexity, which in turn determines the temporal and spatial correlations measured in the FT decomposition. The PL exponent then summarises this information, serving as an indicator of the level of complexity. When it is equal to -2, the analysed behaviour corresponds to Brownian motion, which is a random pattern of motion. Therefore, as the exponent approaches -2 the system's behaviour becomes growingly random, i.e. it shows less self-organisation (control). Random fluctuations become more common, and the temporal correlation decreases. With increases in the PL exponent the system thus has a higher probability of randomly breaking with the previous behaviour.

Figure 3 presents the results of the decomposition of the Fourier transform (DFT).

****

FIGURE 3

****

For the results presented in Figure 3, the DFT was performed in a moving 50-year time window. The beginning of the period corresponds to the year presented in the x-axis of the graph, whilst the y-axis brings the exponent of the power law (PL) regression of the magnitude against the frequency of the FT components.[9] The exponent modulus shows an

[9] The components we refer to are each term of the Fourier transform. I.e., each *n* of the series below:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L} \right)$$

initial growth up to the maximum value of 1.61, in 1930, and then decreases to 1.00 in the period beginning in 1950, finally growing slightly to 1.09 in the last analysed span (the period beginning in 1960).

Results shown in Figure 3, uncovering changes in the exponent over time, indicate changes in the complexity of the system. They reveal a dynamic coupling between tendency and countertendencies, and a very particular one at that: they are suggestive of a system undergoing a process of perturbation and later self-organisation. There is growing complexity during the first decades of the analysis, as the exponents increase between the periods beginning in 1870 and 1930, and then the obverse, with complexity decreasing for the next two periods. In other words, over time the initial perturbations triggered a self-organising process that led to a system with smaller exponents and hence less complexity. This empirical finding, of exponents in the neighbourhood of -1, indicates a dynamic of perturbation and self-organisation with implications for long-term capitalist dynamics.

Figure 4 reorganises the analysis to highlight changes in the exponents.

****

FIGURE 4

****

Figure 4 presents the result of PL regressions for the intervals of 1880-1929, 1920-1969 and 1960-2010. The results show that, in general, longer-period components (greater than 10 years) contribute more strongly in the time-intervals with higher absolute PL exponents. This behaviour is the clearest for the 24-years component, whose magnitude is 0.018 for the 1960-2010 interval (PL exponent of -1.09), 0.049 for 1920-1969 (PL exponent of -1.60) and 0.033 for 1880-1929 (PL exponent of -0.90).

Those two findings – dynamic properties in the Fourier transform (Figure 3) and changes in the exponents of the PL for different periods (Figure 4) – suggest that there is a dynamic coupling. This is evidence that the interplay between the tendency for the rate of profit to fall and its countertendencies is not constant. Over time, new factors and new countertendencies might have been created and incorporated into the system's dynamics.

III- MARX'S INSIGHTS AND A NETWORK MODEL

This section suggests a model that incorporates the dynamics discussed theoretically in section I and empirically evaluated in section II.

This model builds upon that presented in Ribeiro et al (2017), where Marx's insights presented in section three of Volume III of *Das Kapital* were translated into a three-equation system. Basically, each chapter of this section – as edited by Engels – was translated into one term of the equations. The resulting system summarises the operation of the tendency of the rate of profit to fall (as discussed in chapter 13), the countertendencies to its fall (as discussed in chapter 14) and the interaction between tendencies and countertendencies (as discussed in chapter 15). The system of equations is formalised as follows (Ribeiro et al, 2017, pp. 18-19):

(1) $D(t) = \alpha_D * \xi + \beta_D * I(t\text{-}1) * \xi$;[10]

(2) $I(t) = \alpha_I * \xi + \beta_I * D(t\text{-}1) * \xi$;[11]

(3) $RoP(t) - RoP(t\text{-}1) = I(t) - D(t)$.[12]

The terms $\beta_D * I(t\text{-}1) * \xi$ and $\beta_I * D(t\text{-}1) * \xi$ are the translation of the "simultaneous and contradictory operation" (Marx. 1894, p. 357) of tendencies to the fall of the rate of profit and their countertendencies. They integrate equations (1) and (2). This crossover is called "coupling" in Physics. This translation of Marx's insights "leads us to the world of complexity" (Ribeiro et al, 2017, p. 19).

This basic model can replicate the movements of the profit rate as fractals (Ribeiro et al, 2017, pp. 19-21), but to accommodate the findings of the preceding section it needs to be expanded to deal with dynamics in the coupling. This article then proposes an evolution of the three-equation model to capture these local variations (in time) of the coupling, adding to the longer-term behaviour already captured.

[10] $D(t)$ is the intensity of the factors that pull the profit rate down at time (t); ξ is a random number uniformly distributed between 0 and 1 (generated as many times as used in the equations – i.e., the several terms are not equal). The term $\alpha_D * \xi$ varies between 0 and α_D. That is, α_D is their maximum

[11] $I(t)$ is the intensity of the factors that push the rate of profit up, at that same moment (t); ξ is a random number uniformly distributed between 0 and 1 (generated as many times as used in the equations – i.e., the several terms are not equal). The term $\alpha_I * \xi$ varies between 0 and $\alpha_{D,I}$. That is, α_I is their maximum.

[12] The difference between the intensity of the factors that push the rate of profit up ($I(t)$) and the counteracting factors that pull it down ($D(t)$).

This new model considers in more detail the factors behind the tendency and the countertendencies for the rate of profit to fall. In the previous model, only the end-result of these factors was considered: an increase or decrease in the profit rate. The current model adds a level of concreteness by representing these factors as nodes in a network, with the links between them representing their interaction. The interactions between the factors in this model can be of two types. Suppressor interactions decrease the intensity of the interacting factors, which occurs when the factors are of the same type (both tendency or both countertendency). Excitatory ones, conversely, increase the intensity of the interacting factors, occurring when factors are of different types.

Section I provides examples of those new features of our new model. On the one hand, regardless of their impact, new commodities, new branches of production, GPTs, technologies and all factors that lead to the increase or decrease of the profit rate are translated in our new model into nodes in the network. On the other hand, the mutual feedbacks – the magnitude of each node's impact – are translated by the interaction between two or more nodes in the network.

An illustration of this construction, and of how a small perturbation can lead to a reconfiguration of the whole system, is showed in Figure 5. The size of a circle represents the intensity of its corresponding factor (F_i), a continuous line represents an excitatory interaction between the nodes, and a dashed line a suppressor one. If a node is perturbed, such as by increasing its intensity, the intensity of its neighbours will also change, increasing or decreasing depending on the kind of interaction between them (similar results would hold for the addition of a new node). The neighbours of these first neighbours will then be impacted, and through successive links the initial perturbation will cross the whole network. The differences between the initial and final states are examples of how the system is reconfigured after perturbations triggered by, say, technological change.

****

FIGURE 5

****

The system changes all the time, and this is captured in the model. Returning to the introduction of this paper, the "immense collection of commodities" is under constant change as more powerful productive forces may produce old commodities, thereby affecting all similar ones and those related to them. It also constantly grows, with new commodities – inventions –appearing. Beyond the effect of its own inclusion in the system – quantitative growth – each new commodity, represented as a new node, will impact the network, at least affecting its neighbouring nodes, often by devaluating old commodities. Some will have a greater impact, as new branches of production, represented in a node with larger growth and hence broader effects on other nodes. A subset of these new branches of production will have an even larger impact: they are GPTs, or nodes that become hubs in the network.

This way of modelling the economy – through the introduction of nodes and hubs in a network – combines two important features. First, the system is under constant and pervasive change, with new commodities introduced as nodes in the network. Second, the impact of new commodities is differentiated. Importantly, some create special new branches of production: GPTs, or hubs that affect the whole system. The system that is under constant change sometimes experiences revolutionary transformations – and this network model deals with both.

The type of network used to carry out the simulations was a free scale network, because of its generation rule. It has a small number of nodes (compared to the size of the network) called hubs that have a large number of connections, alongside a large number of nodes with a small number of connections. In order to build this type of network, a small, fully connected sub-graph (e.g. 5 nodes connected to each other) is first established. Then new nodes are added one by one to the initial core, connecting them to a fixed number of other nodes. Each node connects to others with a probability proportional to their number of connections. Therefore, a node that due to statistical fluctuations receives more connections than the others during the initial stages will get increasingly more connections, becoming a hub. Similarly, poorly connected nodes tend to continue with a low level of connections (Barabasi et al, 1999; Albert et al, 2000). This kind of network is found in, or can be used to describe, several systems in nature. The current choice is justified by the hubs, which represent the GPTs according to Rosenberg (1998a). Which is to say, they are factors that interact with a high number of other ones, as reviewed in section I.

Factors with more than one type of interaction may act as either a tendency or a countertendency for the rate of profit to fall. This is the case of innovation, discussed in section I. These factors are the focus of the model because they directly generate the coupling between the tendency and the countertendencies, which, as discussed in section III, lead to the complex behaviour of the system.

The intensity of the factors is given by four components:

a) Due to the excitatory interaction between two factors: in this case, the interacting factors are intensified, as modelled in equation (4) below. To maintain correspondence with the second term of equations (1) and (2), this means the randomly selected value of the ξ that multiplies β is greater than its previous one. The variation of the intensity of both interacting factors is governed by:

$$(4)\ F_i(t) - F_i(t\text{-}1) = \delta Fi = \delta F_j = \beta_I * F_i * F_j$$

An example is given by technologies that, following Schumpeter's theory, in the first moment create a temporary monopoly power that raises the prices and hence the profit rate of the innovator. If the saturation of existing markets led to a decline in the profit rate, new commodities could increase it and therefore act as a countertendency. An intensification of the innovation factor thus leads to a weakening of the market saturation factor, in a suppressive interaction. In a second moment, however, competitors of the innovator can use the technology to develop a similar product, thus competing in the same market and eroding the profits of the innovative company. Consequently, this is an example of an excitatory interaction because the intensification of one factor, innovation, eventually leads to a strengthening of another factor, market saturation. Another example of excitatory interactions are when new technologies, NBoP or GPTs use as inputs other technologies or existing GPTs. This illustrates the interdependences Rosenberg and Perez proposed, discussed in section I.

b) Due to the inhibitory interaction between two factors: in this case, the interacting factors are diminished, as modelled in equation (5) below. To maintain correspondence with the second term of equations (1) and (2), this means the randomly selected value of the ξ that multiplies β is lower than its previous one. The variation of the intensity of both interacting factors is governed by:

$$(5)\ \delta F_i = \delta F_j = -\beta_D * F_i * F_j$$

Although section I focused on the processes expressed by equation (4), the introduction of new hubs that would reinforce the tendency of the rate of profit to fall is not difficult to be illustrated. The impact of growth on the payment of taxes to fund the expansion of the state is an example. Likewise, the bursting of financial bubbles, leading to a depression in the economy, can also be seen as a kind of suppressor interaction.

c) Due to external processes that are not modelled as factors of the network, which positively influence the factor in question. This item corresponds to the first term of equations (1) and (2), when the value randomly selected for the ξ that multiplies α is greater than the previous value:

$$(6)\ \delta F_i = \alpha_I * F_i$$

d) Due to external processes that are not modelled as factors of the network, which negatively influence the factor in question. This item corresponds to the first term of equations (1) and (2), when the value randomly selected for the ξ that multiplies α is greater than the previous value:

$$(7)\ \delta F_i = -\alpha_D * F_i$$

These two types of interactions represented by equations (6) and (7) are exogenous, i.e. they are processes that are not modelled as other factors in the network. They thus do not appear as links, but also influence the dynamics of existing factors. Some examples are parts of the state that are not mapped to other factors, but whose actions may strengthen or weaken network factors (e.g. new laws, institutions, market regulations, etc.). Other examples could be negative externalities, such as high levels of pollution caused by previous industrialisation spurts (Wang et al, 2016), or the discovery of natural resources, which can act as either strengthening or weakening factors. The ageing of technologies is another possibility of weakening factors, which happens when institutions, firms and consumers get used to them and develop mechanisms to bypass them, thus reducing their impact.

The initial impacts on the nodes, represented in equations (4) through (7), then spread through the network, as the intensity of a factor depends on the interactions with its neighbours. By changing the intensity of a node, this perturbation thus affects its neighbours, which in turn changes the intensity of its own neighbours and so on, until it runs through the entire network. A local disturbance can hence change the overall configuration of the network, in a non-linear behaviour. This also means that the distribution of connectivity between sites, and the introduction of new ones in the network, affects the dynamics of the intensities.

This article models the changes to the system's dynamics, empirically shown in section 2, through the insertion of new nodes in the network. The reasons for this regard the theoretical background related to the creation of factors (Marx, Kondratiev, Rosenberg and Perez) discussed in section I, as well as the influence of network connectivity on the system's dynamics. The introduction of NBoP, GPTs, technologies or, more generally, other factors that lead to metamorphoses of capitalism, are thus represented by the introduction of new, highly connected nodes in the network, which can produce changes in the global dynamics of the system.

IV- RESULTS FROM A SIMULATION

For the simulations, a free-scale network containing N_{nodes}=3,980 nodes was generated and the link types (suppressor or excitatory) were randomly distributed with probability p=0.5. Monte Carlo simulations were performed (Metropolis et al, 1949; Swendsen et al, 1992), in which for each simulation step N_{nodes} neighbour pairs are randomly selected and their intensity is updated according to the interaction rules. The simulation waits until the system becomes thermalised to begin – i.e. its configuration becomes independent of the initial one – and consecutive simulations steps wait until the configuration becomes uncorrelated with the previous one. For the perturbations, 10 new nodes were added following the same rules of the free-scale network and the same distribution of link types. They were implemented in the simulation steps 10,000 and 20,000.

The first period (stage 0) corresponds to the simulation without external perturbations (i.e. the addition of new nodes). At the 10,000th step, the simulation was paused and 10 new nodes were added. This addition of new nodes, in reference to Perez's work, was called a "Big Bang." The simulation then continues, following the same rules of the first period, until in the 20,000th step another perturbation occurs with the addition of 10 more nodes. It then continues until the 30,000th step.

Figure 6 shows the evolution of the profit rate evolution during the simulation. A visual difference can be detected in the behaviour of the simulated profit rate during each stage. Stage 2 displays better-defined, longer-term peaks and valleys. These peaks and valleys decrease their period in stages 1 and 0, respectively, becoming more like statistical fluctuations. We quantify these differences more precisely in the analysis of Figure 7.

****

FIGURE 6

****

Figure 7 shows the alteration of the system's behaviour caused by the perturbations. The first part of the graph charts the difference between the profit rate after the first perturbation (first "big-bang") and the equivalent simulation without a perturbation. Therefore, if this first perturbation had not changed the profit rate it would be a straight line at zero. The right-hand-side of the graph is the corresponding analysis for the second perturbation, i.e. it shows the difference between the profit rate with two perturbations and with just the first one implemented. Figure 7 thus shows that the "waves" the Big Bangs generate interfere with the behaviour of the profit rate by overlapping with the previous pattern, and not by destroying it.

****

FIGURE 7

****

Figure 8 presents the same analysis of Figure 4, in which the magnitude of the FT components is shown as a function of their period, but for the simulation results. The three time-windows, in this case, are the three stages of Figure 6. The results show that, in general, greater-period components contribute more strongly after perturbations. This behaviour is clearly visible for the 3,400-steps component.

****

FIGURE 8

****

Therefore, this replicates the behaviour of the US profit rate presented in Figure 3. It should also be noted that this behaviour derives from the dynamics of the coupling between the trends and countertendencies. This arises naturally in this model through the insertion of new nodes (factors) in the network.

V- IMPLICATIONS OF THE NETWORK MODEL

The network proposed here models long-term capitalist dynamics, based on the interplay between the tendency and the countertendencies for the rate of profit to fall. It generates a dynamic coupling between the tendency and its countertendencies, which allows for simulating an economy systematically reconfigured by the creation of "new branches of production" and new GPTs. This is represented in the model by introducing new hubs in the network, as shown in Figure 4. It therefore does not represent time and again the same tendencies and countertendencies interacting, as previous models did, but rather introduces new countertendencies, replicating what the history of capitalism has shown.

This network model produces a dynamic coupling between the tendency and the countertendencies for the rate of profit to fall. Put formally, there are continuous changes to the exponents of the power-law regressions of the magnitude against the frequency of the components of the Fourier Transform of the profit rate. This formalisation of a dynamic coupling introduced the question of how the system works through perturbations caused by the introduction of new hubs – representing "new branches of production" or GPTs – and then undergoes processes of self-organisation. It was shown that, as a non-linear system, perturbations lead to changes in complexity, and through a self-organising process the whole system is reconfigured. Over time, this dynamic may lead to greater or lesser complexity.

These dynamics of perturbation and self-organisation might help organise a dialogue between the literature on long waves and GPTs. If, as suggested by Mandel (1981), long waves may be the expression of long-term movements in the rate of profit, the analysis of the data presented in Figure 2 may introduce an investigation of empirical regularities in those movements. In a previous analysis, using a Fourier transform decomposition, it was found that the most important cycles for the US economy between 1870 and 2010 were, in order, the 23-, the 20-, and the 35-year long ones (Ribeiro et al, 2017, p. 14). This can now be reconciled with the findings of this article.

The long-term dynamics Kondratiev and Schumpeter investigated are not organised by well-behaved and relatively regular long waves. The impact of technological revolutions is more chaotic, with more complex regularities. These more chaotic impacts of "new branches of production" are better illustrated by the elaboration of Rosenberg (1998a and 1998b) and Rosenberg and Trajtenberg (2004) on GPTs. GPTs, by definition, have a greater impact in the economy than other "new branches of production." They, also by definition, affect the global dynamic of the system. They might furthermore overlap, and thus have an even greater impact. These perturbations, in the language of complex systems, might differently affect the profit rate and the overall dynamic of the system. The combination of effects can be, as another insight not fully elaborated by Marx, spatially juxtaposed or intertemporally sequential (with a temporal delay) – with different global effects for the reconfiguration of the system. This uneven impact might cause a more chaotic dynamic in the system, where different cycles co-exist and overlap. Hence, we get the combination of important 23-, 20- and 35-year cycles for the US economy.

REFERENCES

AGARWAL, R.; GORT, M. (1996) The evolution of markets and entry, exit and survival of firms. *The Review of Economics and Statistics*, pp. 489-498.

ALBERT, R.; H. JEONG; ALBERT-LASZLO BARABASI (2000). *Nature*, 406, 378-382.

ANDERSON, P. W. 1972. More is different: broken symmetry and the nature of the hierarchical structure. *Science*, 177(4047), 393-396. doi: 10.1126/science.177.4047.393

BARABASI, A. L.; R. ALBERT (1999) *Science*, 286, 509-512 (1999).

CALLINICOS, A. 2014. *Deciphering Capital*: Marx's Capital and its destiny. London: Bookmarks Publications.

DUMÉNIL, G.; LÉVY, D. (1993) *The economics of the profit rate*. Aldershot: Edward Elgar

DUMÉNIL, G.; LÉVY, D. (2015) The U.S. Economy since the Civil War: Sources and Construction of the Series, CEPREMAP (1994) Retrieved from www.jourdan.ens.fr/levy/uslt4x.txt

FREEMAN, C.; LOUÇÃ, F. (2001) *As time goes by*: from the industrial revolution to the information revolution. Oxford: Oxford University.

GOLDENFELD, N. & KADANOFF, L. P. (1999). Simple lessons from complexity. *Science*, 284, 87-89. doi: 10.1126/science.284.5411.87.

ILO and OECD (2015): *Labour share in G20 economies*. Antalya: Report prepared for the G20 Employment Working Group.

KONDRATIEV, N. D. (1926) Long cycles of economic conjuncture. In: *The works of Nikolai D. Kondratiev*. Edited by N. Makasheva, Samuels, W.; Barnett, V. London: Pickering and Chato (1998), pp. 25-60.

MADDISON, ANGUS (2010) Historical statistics of the World Economy – 1-2008 AD. Groningen Growth and Development Centre. Rtrieved from *www.ggdc.net/maddison/Historical_Statistics/horizontal-file_02-2010.xls*

MANDEL, E. (1981) Explaining long waves of capitalist development. *Futures*, v. 13, n. 4, pp. 332-338.

MARX, K. (1867) *Capital*. Volume I. London: Penguin (1976)

MARX, K. (1884) *Capital*. Volume II. London: Penguin (1978)

MARX, K. (1894) *Capital*. Volume III. London: Penguin (1981)

METROPOLIS, N; S. ULAM (1949) *Journal of the American Statistical Association*, 44, 335.

NELSON, R.; WINTER, S. (1982) *An evolutionary theory of economic change*. Cambridge, Mass; London: The Belknap Press of Harvard University Press.

PAULA, J. A. ; CERQUEIRA, H. E. A. ; DEUS, L. G. ; SUPRINYAK, C. E. ; ALBUQUERQUE, E. M. (2016) Investigating Financial Innovation and Stock Exchanges. Marx`s Notebooks on the crisis of 1866 and structural changes in capitalism. *Beiträge zur Marx-Engels-Forschung*, Neue Folge 2014/5, pp. 194-217

PEREZ, C. (2010) Technological revolutions and techno-economic paradigms. *Cambridge Journal of Economics*, v. 34, n. 1, pp. 185-202.

RIBEIRO, L. C.; DEUS, L. G.; LOUREIRO, P. M., ALBUQUERQUE, E. M. (2017) Profits and fractal properties: notes on Marx, countertendencies and simulation models. *Review of Political Economy* (forthcoming). doi.org/10.1080/09538259.2016.1265823

ROSENBERG, N. (1976) Marx as a student of technology. In: *Inside the black box*: technology and economics. Cambridge: Cambridge University Press, pp. 34-51 (1982).

ROSENBERG, N. (1998a) Chemical engineering as a General Purpose Technology. In: HELPMAN, E. *General Purpose Technologies and economic growth*. Cambridge, Mass./London: The MIT Press, pp. 167-192.

ROSENBERG, N. (1998b) The role of electricity in industrial development. In: ROSENBERG, N. (2010) *Studies on science and the innovation process* – selected works by Nathan Rosenberg. New Jersey/London: World Scientific, pp. 137-151.

ROSENBERG, N.; FRISCHTAK, C. (1983) Long waves and economic growth: a critical appraisal. *American Economic Review*, v. 73, n. 2, pp. 146-151.

ROSENBERG, N.; TRAJTENBERG, M. (2004) A General Purpose Technology at work: the Corliss steam-engine in the late Nineteenth-Century United States. *Journal of Economic History*, v. 64, n. 1, pp. 61-99.

SCHMALENSEE (1989) Inter-industry studies of structure and performance. In: SCHMALENSEE, R.; WILLIG, R.D. (ed). *Handbook of industrial organization*. Amsterdam: Elsevier Science, v. 2, pp. 951-1009.

SCHUMPETER, J. (1911) A teoria do desenvolvimento econômico. São Paulo: Nova Cultural, 1985.

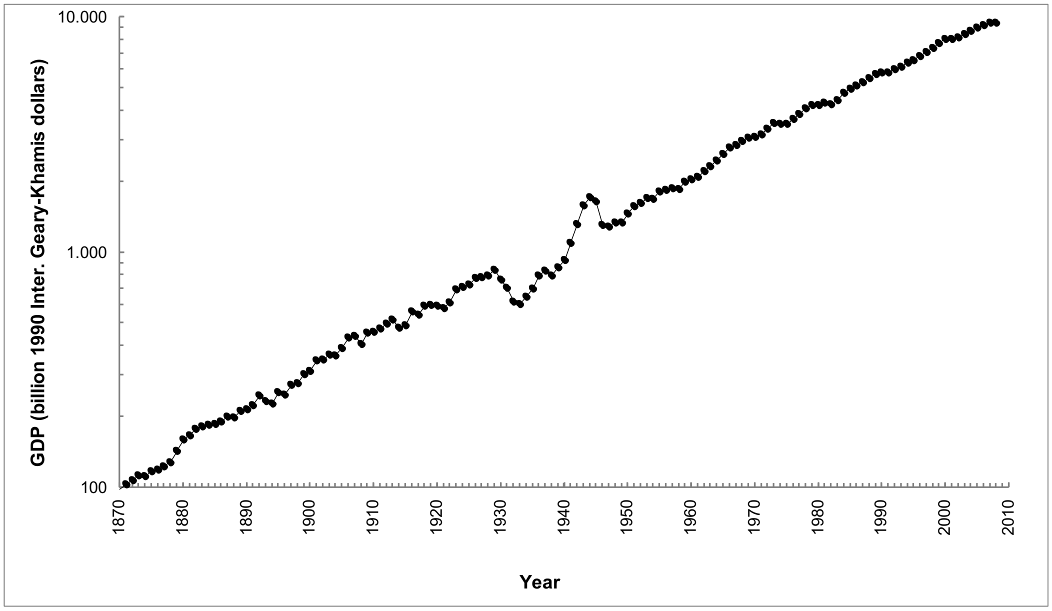
SMITH, A. (1776) *A Riqueza das Nações: investigação sobre sua natureza e suas causas*. São Paulo: Abril Cultural, 1983. Retrieved from http://www.econlib.org/library/Smith/smWN.html

SWENDSEN, R. H.; S. WANG, A. M. FERRENBERG (1992) The Monte Carlo Method. In: *Condensed Matter Physics.* Berlin: Springer-Verlag

WANG, G., et al. (2016), Persistent sulfate formation from London Fog to Chinese haze, *Proceedings of the National Academy of Sciences of the United States of America*, 113(48), 13630–13635.

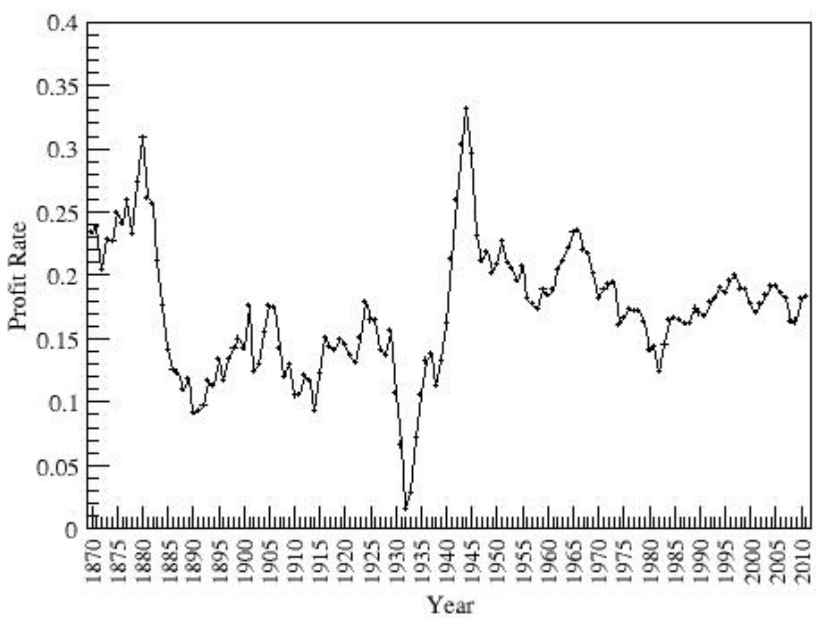
APPENDIX

FIGURE 1
US GDP, in 1990 international Geary-Khamis dollars, (billion)
(1870-2008)



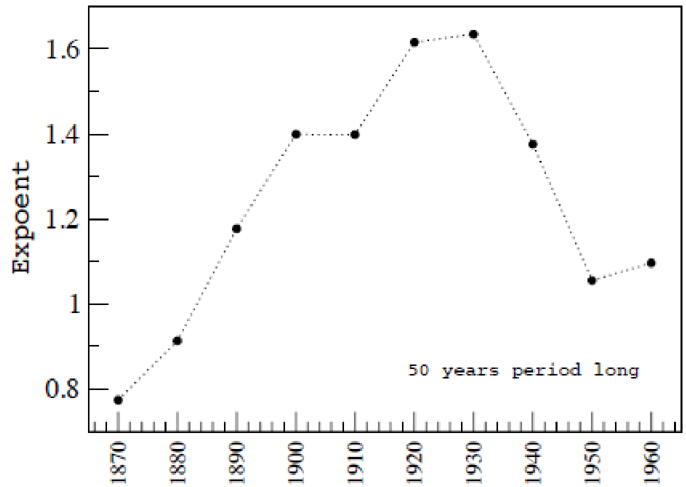
SOURCE: Maddison (2015), authors' elaboration

FIGURE 2
US profit rate data
(1869-2011)



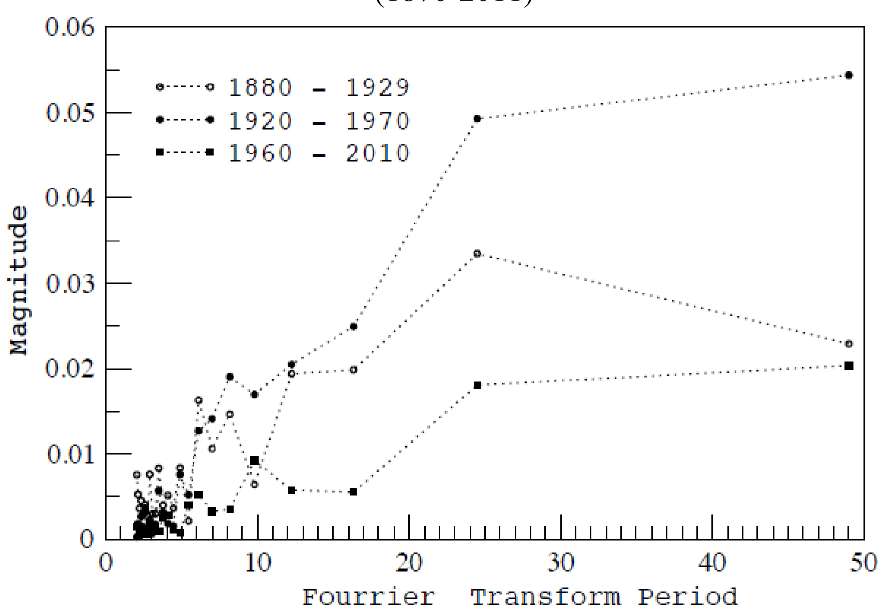
SOURCE: Duménil and Lévy (2015)

FIGURE 3
Fourier Transform - Power Law exponents by moving windows
(1870-2011)



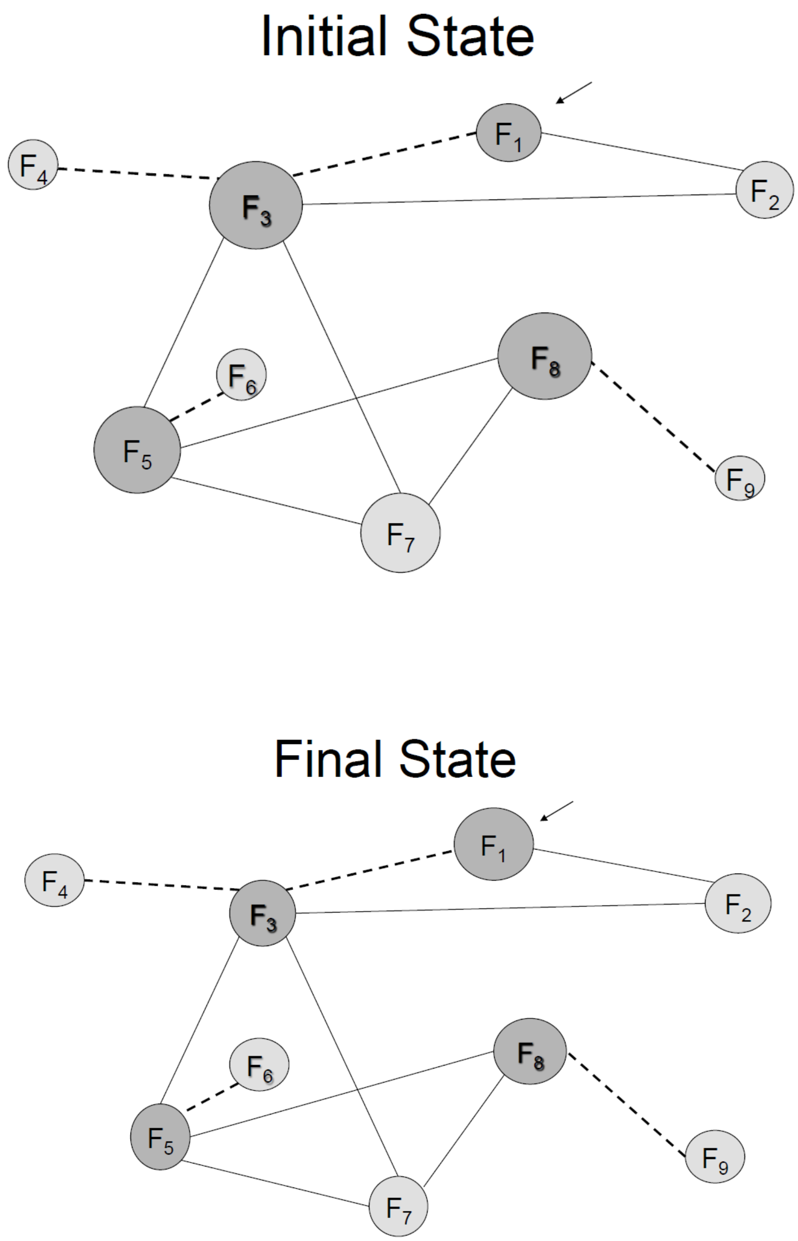
SOURCE: Authors' elaboration, based on data from Figure 1 (Duménil and Lévy, 2015)

FIGURE 4
Magnitudes of Fourier Transform periods by moving windows
(1870-2011)



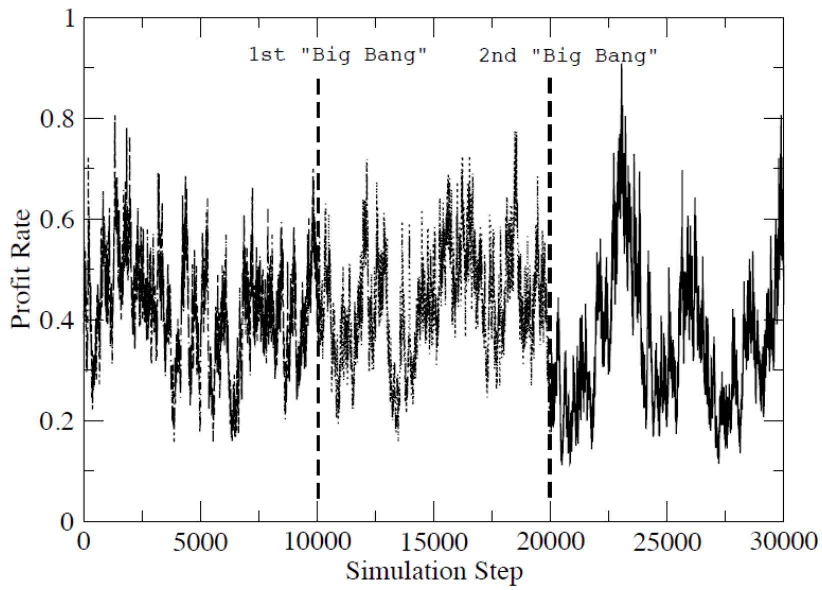
SOURCE: Authors' elaboration, based on data from Figure 1 (Duménil and Lévy, 2015)

FIGURE 5
An illustration of the network's nodes and their interactions



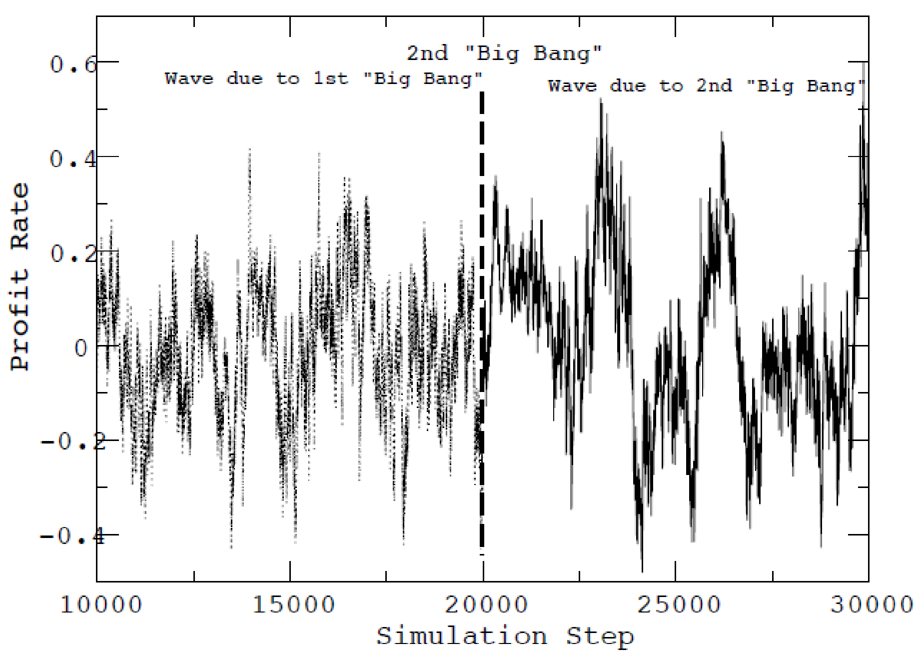
SOURCE: Authors' elaboration

FIGURE 6
Rate of profit from a simulation of the network model with 30,000 steps and performing two perturbations (big bangs)



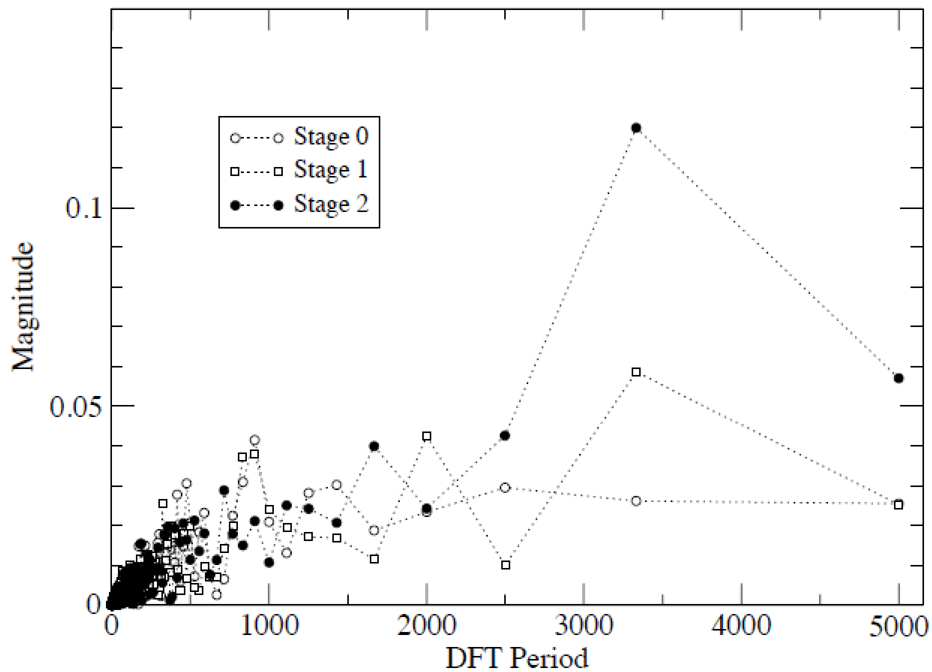
SOURCE: Authors' elaboration

FIGURE 7
Difference of the simulated rate of profit between two sequential stages



SOURCE: Authors' elaboration

FIGURE 8
Magnitudes of Fourier Transform periods of simulated rates of profit by moving windows - three stages



SOURCE: Authors' elaboration