Improving the estimates of the risk premia - application in the UK financial market

Marios Pitsillis*and Steve Satchell
Faculty of Economics and Politics, Cambridge University

June 11, 2001

1 Introduction

Our purpose in this paper is to develop a methodology for improving the estimates of the risk premia (lamda) calculated jointly with the asset sensitivities (beta) using the McElroy and Burmeister (1988) representation of the Arbitrage Pricing Theory (Ross 1976) as a restricted nonlinear multivariate regression model using observed macroeconomic risk factors.

A serious difficulty with the McElroy and Burmeister (1988) methodology is that as the number of assets used to estimate the model is increased, there may be problems in defining the variance-covariance matrix of the estimated parameter. Moreover, increasing the number of assets makes the estimation computationally very demanding as the variance-covariance matrix of the estimated parameter becomes big. Indeed, the largest number of stocks used in the APT non-linear estimation literature is of the order of 70 (McElroy and Burmeister 1988).

In this paper, we first obtain a simpler expression for the variance-covariance matrix of the estimated parameter which allows easier estimation and testing of the risk premium parameter. In the empirically relevant case of a large number of stocks and a small number of observations, we use different samples of stocks to estimate the vector of the risk premia and combine our different estimates in a way that gives a final improved estimate of the risk premium vector. We also derive the variance-covariance matrix of the final estimate of the risk premium. Our improved estimator can be interpreted as the "asymptotically unbiased minimum variance portfolio" among the set of available estimators. We apply the methodology to UK data, using FTSE-350 assets and observed macroeconomic risk factors. This extends existing research on the APT using UK data by Antoniou, Garrett and Priestley (1998), who use different samples

*Financial support from the Association of Commonwealth Universities is greatfully acknowledged. Address for correspondence: Selwyn College, Cambridge, CB3 9DQ, United Kingdom. E-mail: mp227@cam.ac.uk

of UK stocks as we do, to arrive at different estimators of the risk premium. Other papers that compute UK risk premia based on a linear factor model which satisfies the APT include Clare and Thomas (1994), Poon and Taylor (1991), Beenstock and Chan (1988) etc.

The paper is organised as follows: Section 2 explains the methodology, section 3 discusses the data used and presents the empirical results and section 4 concludes. Section 5 is an appendix with mathematical proofs of the propositions we develop.

2 Methodology

The McElroy and Burmeister (1988) methodology involves writing the APT as a multivariate non-linear seemingly unrelated regression model for a sample N of the universe n of assets ($N \leq n$). By considering T time periods, the following system of N non-linear regressions is obtained:

$$r_i(t) - \lambda_0(t) = \sum_{j=1}^{K} b_{ij}[f_j(t) + \lambda_j(t)] + e_i(t)$$

where $i = 1, ..., N, t = 1, ..., T, j = 1, ..., K$, and definitions follow below. $\lambda_j(t)$ is the risk premium associated with the macroeconomic factor j and it is assumed initially that it is time-varying. For the purposes of our estimation we treat it as constant in line with McElroy and Burmeister (1988). We can justify this based on the claims by Cochrane (1999) and Campbell (1999) that over low data frequencies variation in the risk premium is of secondary importance.

Rewriting the system we get:

$$\rho_i = r_i - \lambda_0 = \sum_{j=1}^{K} (\lambda_j i_T + f_j) b_{ij} + e_i \qquad (1)$$

where $i = 1, ..., N$, i_T is a vector of T ones and the following $Tx1$ vectors are defined:

$r_i = (r_i(1), ..., r_i(T))'$ $\quad$ $i = 1, ..., N$, a vector of asset i's returns

$\lambda_0 = (\lambda_0(1), ..., \lambda_0(T))'$ a vector of a risk-free asset's returns

$f_j = (f_j(1), ..., f_j(T))'$ $\quad$ $j = 1, ..., K$, a vector of realisations on the macroeconomic risk factor j

$e_i = (e_i(1), ...e_i(T))'$ $\quad$ $i = 1, ..., N$, the error term

The dependent variable in (1) is the excess rate of return which requires that λ_0 is observed. Equation (1) is re-written as

$$\rho_i = [(\lambda' \otimes i_T) + F]b_i + e_i$$

$$\rho_i = X(\lambda)b_i + e_i \tag{2}$$

where $i = 1, ..., N$, $\otimes$ denotes a Kronecker product and

$\lambda = (\lambda_1, ..., \lambda_k)'$ is a $Kx1$ risk premia vector,

$F = [f_1, ..., f_k]$ is a TxK macroeconomic risk factors matrix,

$b_i = (b_{i1}, ..., b_{iK})', i = 1, ..., N$, a $Kx1$ sensitivities of asset returns to macroeconomic risk factors vector

$X(\lambda) = (\lambda' \otimes i_T) + F$, is a TxK matrix

By definition the λ's are common to all securities that identify them. The matrix $X(\lambda)$ is invariant across securities. If λ were known, equation (1) would be a system of seemingly unrelated linear regressions; since λ is unknown we have a system of seemingly unrelated non-linear regressions with $(N-1)K$ cross-equations restrictions that the λ's are the same for each of the N securities.

Finally, stacking the N equations yields

$$\begin{pmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_N \end{pmatrix} = \begin{pmatrix} X(\lambda) & 0 & \dots & 0 \\ 0 & X(\lambda) & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & X(\lambda) \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{pmatrix}$$

or in obvious notation,

$$\rho = [I_N \otimes X(\lambda)]b + e \tag{3}$$

We assume that the TxK factor matrix F, as well as the NxK sensitivities matrix $B = [b_{ij}]$, are of full column rank. We also assume that $T > N > K$ and that $NT > K(N+1)$, the last condition ensuring that the system represented by (3) has more equations than unknowns. Consequently, the necessary conditions for the NLSUR estimators to exist is satisfied and the Jacobian in equation (4) is of full column rank

$$J(\lambda', b') \equiv \left(\frac{\partial}{\partial(\lambda', b')}\right)\{[I_N \otimes X(\lambda)]b\} = [B \otimes i_T, I_N \otimes X(\lambda)] \tag{4}$$

The joint estimates of b and λ (NLSUR estimators) can be obtained in three steps:

Step 1: Estimate (2) for each asset by ordinary least squares. In this step $(\lambda_1, ..., \lambda_K)$ is not identified, so we replace $\lambda' b_i$ with an intercept α_i and for each $i = 1, ..., N$ select $\hat{\theta}_i$ to minimise $(\rho_i - Z\theta_i)'(\rho_i - Z\theta_i)$, where $\theta_i = (a_i, b_{i1}, ..., b_{ik})$ and $Z = [i_T, F]$. This first step coincides with the first step in the common two-step procedures of Fama and McBeth (1973). The output used from this step is not $\hat{\theta}_i$ but the residuals.

Step 2: Use the residual vector $\hat{e}_i = \rho_i - Z\hat{\theta}_i$ for $i = 1, ..., N$ to estimate Σ as $\hat{\Sigma} = [\hat{\sigma}_{ij}] = [T^{-1}\hat{e}_i'\hat{e}_j]$.

Step 3: Choose the nonlinear seemingly unrelated estimator ($\hat{\lambda}$, $\hat{b}$) to minimise the quadratic form

$$Q(\lambda, b; \hat{\Sigma}) = \{\rho - [I_N \otimes X(\lambda)]b\}' \times (\hat{\Sigma}^{-1} \otimes I_T) \times \{\rho - [I_N \otimes X(\lambda)]b\} \quad (5)$$

These NLSUR estimators are, even in the absence of normality of the error distribution, strongly consistent and asymptotically normal (Gallant 1987), and they form the basis for classical asymptotic hypothesis testing. Finally, with some additional regularity conditions given by Gallant (1975) in the context of non-linear estimation, $\sqrt{T}[(\hat{\lambda}', \hat{b}') - (\lambda', b')]$ is asymptotically normal with mean zero and covariance matrix $\hat{\Omega}^{-1}$ which is consistently estimated by

$$\hat{\Omega} = T^{-1}[J(\hat{\lambda}', \hat{b}')]'(\hat{\Sigma}^{-1} \otimes I_T)J(\hat{\lambda}', \hat{b}')$$

In the McElroy and Burmeister (1988) restricted seemingly unrelated multivariate regression model described above, there is a trade-off between how many time series observations (T) and how many assets (N) are used. There are problems with the covariance matrix if $N > T$. However, for small N, the risk premia will have very large standard errors. McElroy and Burmeister use $T = 72$ and $N = 50$ in most of their analyses. They conjecture that this represents a compromise between computational feasibility and a sufficiently large sample for approximately valid asymptotic properties. Indeed, there are computational problems when the number of assets is large, because the variance covariance matrix of the estimated parameter becomes very big, a problem which we address in this paper.

In an effort to overcome these problems, we develop a simpler expression for the variance covariance matrix of the estimated risk premia. The simplification of the risk premium variance-covariance matrix allows us easier estimation and testing of the risk premia when the number of assets is large. This facilitates the estimation of the risk premium parameter using different samples of stocks, which can be combined in a final improved estimate of the risk premium parameter. We also derive the variance-covariance matrix of this final improved estimate.

We consider a selection matrix A_i of dimension mxN, which selects m stocks out of the sample of N stocks. We vary the A matrix p times to get p different samples of stocks which we use to estimate the risk premia. The expression for the variance-covariance matrix of the risk premium vector estimated using the selection matrix A_i is given in Proposition 1 below which is derived fully in the appendix.

Proposition 1 *The asymptotic variance-covariance matrix of the risk premium vector estimated using the selection matrix* A_i *is given by*

$$var(\hat{\lambda}_{A_i}) = (cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*)] = (B' A_i \Sigma_{A_i}^{-1} A_i B)^{-1}$$

The expression for the asymptotic covariance between different estimates of the vector of risk premia obtained using two different selection matrices, A_i and A_j is given in Proposition 2, also fully derived in the appendix.

Proposition 2 *The asymptotic covariance matrix between two risk premia vectors based upon selection matrices A_i and A_j is given by*

$$cov(\hat{\lambda}_{A_i}, \hat{\lambda}_{A_j}) = cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*), \sqrt{T}(\hat{\lambda}_{A_j} - \lambda^*)]$$

$$= (B' A_i \Sigma_{A_i}^{-1} A_i B)^{-1} (B' A_i \Sigma_{A_i}^{-1} A_i \Sigma A_j \Sigma_{A_j}^{-1} A_j B)(B' A_j \Sigma_{A_j}^{-1} A_j B)^{-1}$$

where $\hat{\lambda}_{A_i}$ is the estimate of the risk premium vector on the basis of stock returns selected from the universe of N stocks using the matrix A_i, λ^* is the true risk premium vector, and Σ_{A_i} is the variance-covariance matrix of the stock returns selected by the A_i matrix.

We estimate the risk premium vector using different random samples of stocks from the FTSE-350. We combine our estimates to get the weighted average "minimum-variance" estimate of the risk premium vector denoted by $\hat{\lambda}^*$ $(Kx1)$. For each macroeconomic risk factor j, $j = 1, ..., K$, and subject to the constraint that the weights sum to 1, the set of weights w_j $(Kx1)$ that minimise the asymptotic variance of the final estimator is given by the expression:

$$w_j = \frac{S_j^{-1} e}{e' S_j^{-1} e}$$

where S_j (KxK) is the variance-covariance matrix of the risk premia estimated over all samples and associated with the jth macroeconomic risk factor and e $(Kx1)$ is a vector of ones. Then the expression $\sqrt{T}(\hat{\lambda}^* - \lambda^*)$ is consistent and asymptotically normal and its asymptotic variance is given by $var(\hat{\lambda}_j^*) = \dfrac{1}{e' S_j^{-1} e}$.

The expression for S_j can be obtained by combining the relevant blocks of the matrices in Propositions 1 and 2 and is given below in Remark 3 for four selection matrices A_1, A_2, A_3 and A_4 corresponding to each of the four samples of stocks we use in this study.

Remark 3 *The variance-covariance matrix of risk premia estimated over all samples associated with each macroeconomi risk factor j, $j = 1, ..., k$*

$$S_j = \begin{pmatrix} var(\hat{\lambda}_{A_1})[j,j] & cov(\hat{\lambda}_{A_1}, \hat{\lambda}_{A_2})[j,j] & \cdots & cov(\hat{\lambda}_{A_1}, \hat{\lambda}_{A_4})[j,j] \\ cov(\hat{\lambda}_{A_2}, \hat{\lambda}_{A_1})[j,j] & var(\hat{\lambda}_{A_2})[j,j] & \cdots & cov(\hat{\lambda}_{A_2}, \hat{\lambda}_{A_4})[j,j] \\ cov(\hat{\lambda}_{A_3}, \hat{\lambda}_{A_1})[j,j] & cov(\hat{\lambda}_{A_3}, \hat{\lambda}_{A_2})[j,j] & \ddots & cov(\hat{\lambda}_{A_3}, \hat{\lambda}_{A_4})[j,j] \\ cov(\hat{\lambda}_{A_4}, \hat{\lambda}_{A_1})[j,j] & cov(\hat{\lambda}_{A_4}, \hat{\lambda}_{A_2})[j,j] & \cdots & var(\hat{\lambda}_{A_4})[j,j] \end{pmatrix}$$

where $[j, j]$ indicates the element that lies at the intersection of the jth row and jth column of a matrix.

For example, if we consider the "unexpected-shifts-in-the-term-structure" risk factor, the associated matrix S_j will comprise on the main diagonal the variances of the term structure risk premia calculated using each of the random samples of stocks. The elements off the main diagonal will then be the covariances of these risk premia.

3 Data description and results

Data on total monthly logarithmic returns for UK stocks are obtained from Datastream. Our sample spans a period of ten years, starting from the end of October 1988 until the end of September 1998. After the ARIMA modelling to generate innovations in the risk factors T equals 108. The sample comprises 66 stocks from FTSE100 and 146 stocks from FTSE250 on which data are available throughout the sample period. We used only the 50 smallest stocks (on the basis of market capitalisation at the beginning of the sample period) in the FTSE100 in an effort to isolate the local risk premium effects, as largest stocks tend to be stocks of multinational companies that are influenced significantly by global factors. In total, 196 stocks were used to form four random samples, three comprising 50 stocks and one comprising 46 stocks. In line with McElroy and Burmeister (1988), the use of $N = 50$ is deemed to be reasonable for asymptotic properties to be approximately valid. This assertion, however, needs to be validated. The estimates of the risk premia for each sample were then combined to obtain an improved estimate of the risk premium vector.

Our use of several observed macroeconomic risk factors to explain asset returns can be justified by the newest generation of empirical research, which has revised the simple view of the investment world that expected returns can be explained solely by their tendency to move with the market as a whole. As summarised by Cochrane (1999) we now know that there are assets whose average returns cannot be explained by their beta. Rather, multifactor models dominate the description, performance attribution and explanation of average returns. Such models associate high average returns with a tendency to move with additional risk factors rather than only to movements in the market as a whole. In fact, asset pricing theory recognised at least since Merton (1971a,b) the theoretical possibility that we should need factors, state variables or sources of priced risk, beyond movements in the market portfolio in order to explain why some average returns are higher than others.

One of the earliest examples of the analysis applying prespecified macroeconomic risks in the APT is the paper by Chen et al. (1986) analysing the pricing of such factors in the US market. Recognising the ability of investors to diversify and the co-movements of asset prices, the authors suggest the presence of pervasive or systematic influences as the likely source of investment risk. To illustrate the effect of macroeconomic factors on asset prices, they express them as follows:

$$p = \frac{E(c)}{k}$$

where c is the dividend stream and k is the discount rate. Then, actual returns in any period are given by:

$$\frac{dp}{p} + \frac{c}{p} = \frac{d[E(c)]}{E(c)} - \frac{dk}{k} + \frac{c}{p}$$

and it follows that the systematic forces that influence returns are those that change the discount factor k and the expected cash flows $E(c)$.

The discount rate is an average of rates over time and also depends on the risk premium. It changes with both the level of rates and the term-structure spreads across different maturities so that unanticipated changes in the riskless interest rate will influence pricing, and through their influence on the time value of future cash flows will influence returns. Unanticipated changes in the risk premium will also influence returns. On the demand side, changes in the indirect marginal utility of real wealth, perhaps as measured by real consumption changes, will influence pricing and will also show up as unanticipated changes in risk premia.

Expected cash flows change because of both real and nominal forces. Changes in the expected rate of inflation would influence nominal expected cash flows as well as the nominal rate of interest. To the extent that pricing is done in real terms, unanticipated price-level changes will have a systematic effect, and to the extent that relative prices change along with general inflation, there can also be a change in asset valuation associated with changes in the average inflation rate. Finally, changes in the expected level of real production would affect the current real value of cash flows. Insofar as the risk premium does not capture industrial production uncertainty, innovations in the rate of productive activity should have an influence on stock returns through their effect on cash flows.

Chen, Roll and Ross find that the following sources of risk are significantly priced in the US market:

1. Risk stemming from unanticipated changes in the expected level of real production (reflecting the changing state of the economy).

2. Risk stemming from unanticipated shifts in the shape of the term structure.

3. Risk stemming from changes in default premiums, and somewhat more weakly,

4. Risk stemming from unexpected inflation and changes in expected inflation, when these variables were highly volatile.

By contrast, risks stemming from unanticipated changes in the market portfolio, aggregate consumption and oil prices are not priced.

The choice of candidate macroeconomic factors in this study is largely inspired by Chen, Roll and Ross (1986) and is a subset of the factors presented in Antoniou et al. (1998). All data for measuring the macroeconomic factors are obtained from Datastream. The correlation patterns in factors do not seem large enough to posit a problem in the econometric estimation. Apart from spanning the space of returns, the most important property required of appropriate factor measures is that they cannot be predictable from their own past. Chen, Roll and Ross (1986) rely on the fact that because the factor measures used are defined in first differences, they can be employed as innovations without alteration. They state that one could identify a vector autoregression model in an attempt to use its residuals as the unanticipated innovations in the economic factors, but that would be antithetical to the spirit of their investigation. However, when calculating autocorrelations for the macroeconomic factors over their entire sample period, Chen, Roll and Ross (1986) found that the factors generally display mild autocorrelations. The autocorrelation in the state variables implies the existence of an errors-in-variables problem that biases estimates of the loadings of the stock returns on these variables and also biases downward estimates of statistical significance.

For this reason, the methodology used by CRR (1986) was criticised by Poon and Taylor (1991), who pre-whiten all series to make sure that only the unexpected components are analysed, as failure to adequately filter the various series may create a spurious relationship and introduce an errors-in-variables problem. The pre-whitening process is carried out by fitting a univariate ARIMA model to each series.

Clare and Thomas (1994) also conjecture that from the autocorrelations properties of CRR's surprises it is evident that there is highly significant lagged information omitted from the generation of their innovations, which is not consistent with the interpretation of these variables as "surprises". They also prefer to use single equation autoregressive models with a careful examination of residuals and model stability to ensure no systematic errors are present wherever possible.

To avoid the problems caused by the potential presence of autocorrelation in the variables, simple ARIMA models were fitted to pre-whiten the series. It has to be noted, however, that although this procedure is designed to avoid spurious correlation, it carries a danger of possible misspecification. Given finite samples, the fitted ARIMA models can only be approximations to the true data generating process. The measurement of the risk factors used in our study is explained below.

3.1 Industrial production

In line with Chen, Roll and Ross, we use the monthly growth rate in industrial production to capture the effect of the production risk factor. This is defined as:

$$MP(t) = \ln IP(t) - \ln IP(t-1)$$

where IP denotes industrial production. We use the UKINPRODG series defined as the 'UK industrial production - total production voln'. An AR(1) model was used to derive the innovations in industrial production.

Even if no complete theoretical foundations have been developed for the signs of the risk premia corresponding to each macroeconomic risk factor, some propositions can still be made based on economics. CRR (1986) find a positive sign for the risk premium of the MP variable with US data and they argue that it reflects the value of insuring against real systematic production risks. However, Antoniou et al. (1998) find a negative value[1] for the UK. It is quite likely that a detailed investigation into the decline of UK manufacturing relative to the US could explain empirically and possibly theoretically why the risk premia could change sign.

3.2 Inflation

We use the difference in the logarithm of the consumer price index (CPI) to capture the effect of the inflation factor, as follows:

$$IR(t) = \ln CPI(t) - \ln CPI(t-1)$$

We use the series UKRP....F defined as the 'UK Retail Price Index NADJ'. An ARMA model was used to derive the unexpected component of this series.

Chen, Roll and Ross (1986) and other studies use a measure of unexpected inflation calculated as the difference between the realised inflation rate and the inflation for the period expected at the beginning of the period. The construction of the expected inflation series is itself a significant task that was carried out by Fama and Gibbons (1984). Another inflation variable that is unanticipated and that might have an influence separable from unexpected inflation is the change in expected inflation. This series has also been used in the literature.

The sign of the risk premium corresponding to the IR variable may be ambiguous because of the fact that changes in inflation generally shift wealth among investors. In the literature, CRR(1986) find a negative risk premium for unexpected inflation. They interpret this finding as evidence that over their sample period US stocks consituted a hedge against the adverse influence on inflation on other assets with relatively more fixed nominal returns. On the other hand, Clare and Thomas (1994) find that in the UK the inflation risk premium is positive, so that UK stocks were not considered to be a hedge against inflation over their sample period.

[1] It is also worth noting that although a factor has a negative price of risk, in terms of the expected return it is the sum of the prices of risk and the sensitivity of the stock to the factors which generates the risk premium that particular assets carry. Both the price of risk and the sensitivity to a factor could be negative, thereby yielding a positive premium for that factor. Thus, a negative price of risk for a factor may not be an unreasonable finding.

3.3 Market risk premium

To capture the effect of the market risk premium factor, we use the difference in the returns on the equity market (EM) and the government bond market (BM) in line with the definition of the risk premium in Datastream:

$$RP(t) = [\ln EM(t) - \ln EM(t-1)] - [\ln BM(t) - \ln BM(t-1)]$$

We use the FTALLSH(RI) series defined as the 'FTSE All Share - Total Return Index' and the series FTAGOVT(RI) defined as the 'FTA Government All Stocks - Total Return Index'. An AR(1) model was used to derive the innovations in this series.

The market risk premium variable is expected to carry a positive risk premium to reflect the fact that investors would want to hedge against unexpected increases in the aggregate market risk premium as a result of an increase in uncertainty. Indeed, Chen, Roll and Ross (1986) and Clare and Thomas (1994) find significantly positive risk premia in US and UK respectively. Antoniou et al. (1980) also find that the price of risk related to excess returns on the UK stock market is positive and significant.

3.4 Term structure

To capture the effect of unanticipated shifts in the term structure, in line with the approach followed in the literature, we use the spread between long-term (LTR) and short-term interest rates (STR):

$$TS(t) = LTR(t) - STR(t)$$

We use the first difference in the logarithm of the series BMUK30Y(RI) defined as the 'UK Benchmark 30 Year DS Government Index - Total Return Index' and the series LDNTB3M defined as the 'UK Treasury Bill Discount 3 month - Middle Rate'. The $TS(t)$ series appears uncorrelated so it can be employed directly in the econometric estimation.

The risk premium associated with the term structure variable may also be ambiguous in sign. CRR find a negative risk premium indicating that stocks whose returns are inversely related to increases in long rates over short rates are, ceteris paribus, more valuable. They argue that one way to interpret these results is that the TS variable measures a change in the long-term real rate of interest, having accounted for the effect of inflation in the IR variable. When long-term real rates decrease there is a lower real return on any form of capital, so investors who want to hedge against this possibility will place a relatively higher value on assets whose price increases when long-term real rates decline, and such assets will carry a negative risk premium. McElroy and Burmeister (1988) find a positive risk premium for the TS variable using US data while Clare and Thomas (1994) find that the TS variable was not significant in the UK market over their sample period.

Sample means, standard deviations and correlations between the macroeconomic factors are shown in Tables 1 and 2. All tables appear at the end of the paper.

The different estimates of the risk premium vector, the combined improved estimate and their t-statistics are shown in Table 3 below. Full results from our estimations with the sensitivities and full variance-covariance and correlation matrices are reported in the appendix. It is evident from Table 3 that the risk premia are not consistently estimated across samples, in the sense that in some cases both their signs and statistical significance differ. This inconsistency has also been documented in Antoniou et al. (1998). Nevertheless, considering the estimates obtained from the four random samples, most of the risk premia appear to be significant. However, considering the minimum-variance estimate, only the inflation risk premium appears to be significant. The production, market risk premium and term structure risk premia appear to be insignificant despite the fact that in the individual samples they appear to be significant most of the time. The minimum-variance estimate all risk premia appear to have plausible signs even if this was not the case in each of the individual estimates.

This phenomenon is due to the fact that the risk premia appear with different signs across the samples, so when they are weighted to obtain the minimum variance estimate, they tend towards zero. To examine the validity of this hupothesis, we repeated the same exercise of combining our estimates considering only samples 2 and 4 in which the risk premia have consistent signs. Because we expect the production risk to carry a positive risk premium we use the one estimated using sample 3. The results of this exercise appear in Table 4 below. In this case, the minimum variance risk premia appear larger in magnitude and they are all statistically significant. Even if we adjust the risk premia by adding back the constant factor means from Table 2, none of the signs are changed.

4 Conclusions

It is well-known that the estimation of the APT using non-linear methods is difficult for numerical reasons, so that it is hard to combine more than about 50 stocks. Antoniou et al. (1998) have presented a range of UK estimates of risk premia based on different samples.

Our contribution is to derive results for the asymptotic covariance matrix of a set of estimators of the risk premium based on different samples of stocks. These are given in Propositions 1 and 2.

Armed with this information we can then combine the different estimates of the risk premia to construct a better estimator, in the sense that it will have minimum-variance among the class of available "consistent estimators" ie. those combinations whose weights add to one. Good combinations only occur if we use information about the signs of the associated factor risk premia.

5 Appendix

Our ultimate purpose is to combine different estimates of the risk premia to obtain an improved estimate. To facilitate this task we find a simpler expression for the variance-covariance matrix of the estimated parameter in each of the samples. In particular, we are mainly interested in that block of the estimated parameter variance-covariance matrix which refers to the risk premia. We repeat some of the definitions to make the appendix more self-contained.

The McElroy and Burmeister (1988) recast of the APT as a restricted non-linear multivariate regression model is:

$$\rho = [I_N \otimes X(\lambda)]b + e \qquad e \sim (0, \Sigma \otimes I_T) \tag{6}$$

and $X(\lambda) = (\lambda' \otimes i_T) + F$

where ρ is $NTx1$

I_N is NxN

I_T is TxT

$X(\lambda)$ is TxK, hereon suppressed to X to simplify the mathematical exposition

$b = vec(B)$ where B is NxK

λ is $Kx1$

We consider a selection matrix A_i of dimension mxN, which selects m stocks out of the universe of N stocks. We vary the A matrix p times to get p different samples of stocks which we use to estimate the risk premia.

Applying the selection matrix to equation (6) we get:

$\rho_{A_i} = (A_i \otimes I_T)\rho$,a vector of dimension $mTx1$ and substituting for ρ from equation (6) we get:

$\rho_{A_i} = (A_i \otimes I_T)\{[I_N \otimes X]b + e\}$

$\rho_{A_i} = (A_i I_N \otimes I_T X)b + (A_i \otimes I_T)e$

$\rho_{A_i} = (A_i \otimes X)b + (A_i \otimes I_T)e$

$\rho_{A_i} = (A_i \otimes X)b + e_{A_i}$ where $e_{A_i} \sim (0, \Sigma_{A_i} \otimes I_T)$ and $\Sigma_{A_i} = A_i \Sigma A_i$ and is of dimension mxm

Σ here refers to the variance-covariance matrix of all N stocks in the universe.

Re-writing the expression above we get:

$$e_{A_i} = \rho_{A_i} - (A_i \otimes X)b \tag{7}$$

where e_{A_i} is of dimension $mTx1$

We combine the unknown parameters λ and b into one vector θ so that $\theta = \begin{pmatrix} \lambda \\ b \end{pmatrix}$, where θ is of dimension $(K + NK)x1$.

The first derivative of expression (7) with respect to θ is $\frac{\partial e_{A_i}}{\partial \theta} = -\frac{\partial (A_i \otimes X)b}{\partial \theta}$

$$\frac{\partial e_{A_i}}{\partial \theta} = -(A_i B \otimes i_T, A_i \otimes X) \tag{8}$$

Equation (8) represents a matrix of dimension $mTx(K + NK)$.

Setting up the Seemingly Unrelated Regression (equivalent to the quadratic form Q_{A_i}) we get:

$$Q_{A_i} = e'_{A_i}(\Sigma^{-1}_{A_i} \otimes I_T)e_{A_i}$$

$$Q_{A_i} = [\rho_{A_i} - (A_i \otimes X)b]'(\Sigma^{-1}_{A_i} \otimes I_T)[\rho_{A_i} - (A_i \otimes X)b] \tag{9}$$

The first derivative of the quadratic form is:

$\frac{\partial Q_{A_i}}{\partial \theta} = 2e'_{A_i}(\Sigma^{-1}_{A_i} \otimes I_T)\frac{\partial e_{A_i}}{\partial \theta}$ and substituting for $\frac{\partial e_{A_i}}{\partial \theta}$ from equation (8) we get:

$$\frac{\partial Q_{A_i}}{\partial \theta} = 2e'_{A_i}(\Sigma^{-1}_{A_i} \otimes I_T)[-(A_i B \otimes i_T, A_i \otimes X)]$$

$$\frac{\partial Q_{A_i}}{\partial \theta} = -2e'_{A_i}(\Sigma^{-1}_{A_i} A_i B \otimes I_T i_T, \Sigma^{-1}_{A_i} A_i \otimes I_T X)$$

$$\boxed{\frac{\partial Q_{A_i}}{\partial \theta} = -2e'_{A_i}(\Sigma^{-1}_{A_i} A_i B \otimes i_T,\ \Sigma^{-1}_{A_i} A_i \otimes X)} \tag{10}$$

The second derivative of the quadratic form is:

$$\frac{\partial^2 Q_{A_i}}{\partial \theta \partial \theta'} = -2\frac{\partial e'_{A_i}}{\partial \theta}(\Sigma^{-1}_{A_i} A_i B \otimes i_T, \Sigma^{-1}_{A_i} A_i \otimes X)$$

$$\frac{\partial^2 Q_{A_i}}{\partial \theta \partial \theta'} = -2\begin{pmatrix} B' A_i \otimes i'_T \\ A_i \otimes X' \end{pmatrix}(\Sigma^{-1}_{A_i} A_i B \otimes i_T, \Sigma^{-1}_{A_i} A_i \otimes X)$$

$$\frac{\partial^2 Q_{A_i}}{\partial \theta \partial \theta'} = -2\begin{pmatrix} TB' A_i \Sigma^{-1}_{A_i} A_i B & B' A_i \Sigma^{-1}_{A_i} A_i \otimes i'_T X \\ A_i \Sigma^{-1}_{A_i} A_i B \otimes X' i_T & A_i \Sigma^{-1}_{A_i} A_i \otimes X' X \end{pmatrix}$$

$$\boxed{\frac{\partial^2 Q_{A_i}}{\partial \theta \partial \theta'} = -2\begin{pmatrix} TB' A_i \Sigma^{-1}_{A_i} A_i B & (A_i \Sigma^{-1}_{A_i} A_i B \otimes X' i_T)' \\ A_i \Sigma^{-1}_{A_i} A_i B \otimes X' i_T & A_i \Sigma^{-1}_{A_i} A_i \otimes X' X \end{pmatrix}} \tag{11}$$

To obtain an expression involving the difference of the estimated parameter from its true mean we use the Taylor series expansion:

$$\frac{\partial Q_{A_i}(\hat{\theta})}{\partial \theta} = 0$$

$$\frac{\partial Q_{A_i}(\theta_{A_i})}{\partial \theta} + \frac{\partial^2 Q_{A_i}(\theta_{A_i})}{\partial \theta \partial \theta'}(\hat{\theta}_{A_i} - \theta) \approx 0$$

where θ is the true value and $\hat{\theta}_{A_i}$ is the estimated value of the parameter on the basis of selection A_i. Re-arranging gives:

$$\hat{\theta}_{A_i} - \theta \approx -(\frac{\partial Q_{A_i}}{\partial \theta})(\frac{\partial^2 Q_{A_i}}{\partial \theta \partial \theta'})^{-1} \tag{12}$$

We define the inverse of the second derivative (11) as follows:

$$(\frac{\partial^2 Q_{A_i}}{\partial \theta \partial \theta'})^{-1} = \begin{pmatrix} V_{11}^{A_i} & V_{12}^{A_i} \\ V_{21}^{A_i} & V_{22}^{A_i} \end{pmatrix}$$

and we use the formula for the partitioned inverse to obtain the required blocks

The general formula for the inverse of a partitioned matrix $M = \begin{pmatrix} E & F \\ G & H \end{pmatrix}$ is $M^{-1} = \begin{pmatrix} K & L \\ P & Q \end{pmatrix}$ where

$K = (E - FH^{-1}G)^{-1}$, $L = -KFH^{-1}$, $P = -H^{-1}GK$ and

$Q = H^{-1} + H^{-1}GKFH^{-1}$

So calculating the blocks of the inverse matrix that we need to proceed with our calculation of the estimated variance covariance matrix we get:

$$K = V_{11}^{A_i} = \{TB'A_i\hat{\Sigma}^{-1}A_iB - [B'A_i\hat{\Sigma}^{-1}A_i \otimes i_T'X]$$
$$[A_i\hat{\Sigma}^{-1}A_i \otimes X'X]^{-1}[A_i\hat{\Sigma}^{-1}A_iB \otimes X'i_T]\}^{-1}$$

$$K = V_{11}^{A_i} = \{TB'A_i\hat{\Sigma}^{-1}A_iB - [B'A_i\hat{\Sigma}^{-1}A_i \otimes i_T'X][(A_i\hat{\Sigma}^{-1}A_i)^{-1} \otimes (X'X)^{-1}]$$
$$[A_i\hat{\Sigma}^{-1}A_iB \otimes X'i_T]\}^{-1}$$

$$K = V_{11}^{A_i} = \{TB'A_i\hat{\Sigma}^{-1}A_iB - [B'A_i\hat{\Sigma}^{-1}A_i(A_i\hat{\Sigma}^{-1}A_i)^{-1} \otimes i_T'X(X'X)^{-1}]$$
$$[A_i\hat{\Sigma}^{-1}A_iB \otimes X'i_T]\}^{-1}$$

$$K = V_{11}^{A_i} = \{TB'A_i\hat{\Sigma}^{-1}A_iB - [B'A_i\hat{\Sigma}^{-1}A_iB \otimes i_T'X(X'X)^{-1}X'i_T]\}^{-1}$$

$$K = V_{11}^{A_i} = \{TB'A_i\hat{\Sigma}^{-1}A_iB - B'A_i\hat{\Sigma}^{-1}A_iB \otimes i_T'X(X'X)^{-1}X'i_T\}^{-1}$$

$$K = V_{11}^{A_i} = (TB'A_i\hat{\Sigma}^{-1}A_iB - cB'A_i\hat{\Sigma}^{-1}A_iB)^{-1}$$

$$K = V_{11}^{A_i} = \frac{1}{T-c}(B'A_i\hat{\Sigma}^{-1}A_iB)^{-1}$$

where c is a scalar and is equal to $i_T'X(X'X)^{-1}X'i_T$.

$$P = V_{21}^{A_i} = -[A_i\Sigma_{A_i}^{-1}A_i \otimes X'X]^{-1}[A_i\Sigma_{A_i}^{-1}A_iB \otimes X'i_T]V_{11}^{A_i}$$

$$P = V_{21}^{A_i} = -[(A_i\Sigma_{A_i}^{-1}A_i)^{-1} \otimes (X'X)^{-1}][A_i\Sigma_{A_i}^{-1}A_iB \otimes X'i_T]V_{11}^{A_i}$$

$$P = V_{21}^{A_i} = -[B \otimes (X'X)^{-1}X'i_T]V_{11}^{A_i}$$

Equation (13) then becomes:

$$\begin{pmatrix} \hat{\lambda}_{A_i} - \lambda^* \\ \hat{b}_{A_i} - b \end{pmatrix} \approx e_{A_i}'(\Sigma_{A_i}^{-1}A_iB \otimes i_T, \Sigma_{A_i}^{-1}A_i \otimes X)\begin{pmatrix} V_{11}^{A_i} & V_{12}^{A_i} \\ V_{21}^{A_i} & V_{22}^{A_i} \end{pmatrix}$$

and considering the risk premium parameter only we get:

$$\hat{\lambda}_{A_i} - \lambda^* \approx e_{A_i}'(\Sigma_{A_i}^{-1}A_iB \otimes i_T)V_{11}^{A_i} + e_{A_i}'(\Sigma_{A_i}^{-1}A_i \otimes XV_{21}^{A_i})$$

Performing the algebra:
$\hat{\lambda}_{A_i} - \lambda^* \approx e'_{A_i}(\Sigma_{A_i}^{-1} A_i B \otimes i_T) V_{11}^{A_i} + e'_{A_i}\{\Sigma_{A_i}^{-1} A_i \otimes X[-[B \otimes (X'X)^{-1} X' i_T] V_{11}^{A_i}]\}$

$\hat{\lambda}_{A_i} - \lambda^* \approx e'_{A_i}(\Sigma_{A_i}^{-1} A_i B \otimes i_T) V_{11}^{A_i} + e'_{A_i}\{\Sigma_{A_i}^{-1} A_i \otimes X[-[B \otimes (X'X)^{-1} X' i_T] V_{11}^{A_i}]\}$

$\hat{\lambda}_{A_i} - \lambda^* \approx e'_{A_i}[(\Sigma_{A_i}^{-1} A_i B \otimes i_T - (\Sigma_{A_i}^{-1} A_i \otimes X)(B \otimes (X'X)^{-1} X' i_T)] V_{11}^{A_i}$

$\hat{\lambda}_{A_i} - \lambda^* \approx e'_{A_i}[(\Sigma_{A_i}^{-1} A_i B \otimes (i_T - X(X'X)^{-1} X' i_T)] V_{11}^{A_i}$

$\hat{\lambda}_{A_i} - \lambda^* \approx e'_{A_i}[\Sigma_{A_i}^{-1} A_i B \otimes (i_T - X(X'X)^{-1} X' i_T)] V_{11}^{A_i}$

$\hat{\lambda}_{A_i} - \lambda^* \approx e'_{A_i}[\Sigma_{A_i}^{-1} A_i B \otimes (i_T - X(X'X)^{-1} X' i_T)] V_{11}^{A_i}$

$\hat{\lambda}_{A_i} - \lambda^* \approx e'_{A_i}[\Sigma_{A_i}^{-1} A_i B \otimes M_x i_T)] V_{11}^{A_i}$

Substituting for $V_{11}^{A_i}$ the expression becomes:

$$\boxed{\hat{\lambda}_{A_i} - \lambda^* \approx e'_{A_i}[\Sigma_{A_i}^{-1} A_i B \otimes M_x i_T)] \frac{(B' A_i \Sigma_{A_i}^{-1} A_i B)^{-1}}{T^*}}$$

where $T^* = T - i'_T X(X'X)^{-1} X' i_T = i'_T M_x i_T$
Similarly, for a different selection matrix A_j the expression above would be:
$\hat{\lambda}_{A_j} - \lambda^* \approx e'_{A_j}(\Sigma_{A_j}^{-1} A_j B \otimes i_T) V_{11}^{A_j} + e'_{A_j}(\Sigma_{A_j}^{-1} A_j \otimes X V_{21}^{A_j})$
and after performing the algebra we end up with a similar expression

$$\boxed{\hat{\lambda}_{A_j} - \lambda^* \approx e'_{A_j}[\Sigma_{A_j}^{-1} A_j B \otimes M_x i_T)] \frac{(B' A_j \Sigma_{A_j}^{-1} A_j B)^{-1}}{T^*}}$$

The expression for the variance-covariance matrix of the risk premium vector estimated using the selection matrix A_i is:

$\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*) = \sqrt{T}(e'_{A_i} W_{A_i})$ where $W_{A_i} = [\Sigma_{A_i}^{-1} A_i B \otimes M_x i_T)] \dfrac{(B' A_i \Sigma_{A_i}^{-1} A_i B)^{-1}}{T^*}$
, a $TmxK$ matrix

$cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*)] = T W'_{A_i} E(e_{A_i} e'_{A_i}) W_{A_i}$

$cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*)] = T W'_{A_i} (\Sigma_{A_i} \otimes I_T) W_{A_i}$

$cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*)] = T \dfrac{(B' A_i \Sigma_{A_i}^{-1} A_i B)^{-1}}{(T^*)^2} (B' A_i \Sigma_{A_i}^{-1} \otimes i'_T M_x)(\Sigma_{A_i} \otimes I_T)$
$(\Sigma_{A_i}^{-1} A_i B \otimes M_x i_T)(B' A_i \Sigma_{A_i}^{-1} A_i B)^{-1}$

$cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*)] = T \dfrac{(B' A_i \Sigma_{A_i}^{-1} A_i B)^{-1}}{(T^*)^2} (B' A_i \Sigma_{A_i}^{-1} A_i B) T^* (B' A_i \Sigma_{A_i}^{-1} A_i B)^{-1}$

$cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*)] = \dfrac{T}{T^*}(B' A_i \Sigma_{A_i}^{-1} A_i B)^{-1}$

and since $i'_T f \kappa = 0$ as the factors have sample means of zero it follows that $\dfrac{T}{T^*} = 1$ and the expression simplifies to:

$$\boxed{var(\hat{\lambda}_{A_i}) = (cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*)] = (B' A_i \Sigma_{A_i}^{-1} A_i B)^{-1}} \quad (13)$$

The expression for the covariance between different estimates of the vector of risk premia is:
$cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*), \sqrt{T}(\hat{\lambda}_{A_j} - \lambda^*)] = T cov(e'_{A_i} W_{A_i}, e'_{A_j} W_{A_j})$
$cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*), \sqrt{T}(\hat{\lambda}_{A_j} - \lambda^*)] = T\ W'_{A_i} cov(e_{A_i}, e'_{A_j}) W_{A_j}$
$cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*), \sqrt{T}(\hat{\lambda}_{A_j} - \lambda^*)] = T\ W'_{A_i}\ E[(A_i \otimes I_T)ee'(A_j \otimes I_T)] W_{A_j}$
$cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*), \sqrt{T}(\hat{\lambda}_{A_j} - \lambda^*)] = T\ W'_{A_i}\ [A_i \Sigma A_j \otimes I_T] W_{A_j}$
$cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*), \sqrt{T}(\hat{\lambda}_{A_j} - \lambda^*)] = T\ W'_{A_i}\ [A_i \Sigma A_j \otimes I_T] W_{A_j}$
$cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*), \sqrt{T}(\hat{\lambda}_{A_j} - \lambda^*)] =$
$\frac{T}{(T^*)^2}(B'A_i\Sigma_{A_i}^{-1}A_iB)^{-1}(B'A_i\Sigma_{A_i}^{-1}\otimes i'_T M_x)(A_i\Sigma A_j \otimes I_T)(\Sigma_{A_j}^{-1}A_jB \otimes M_x i_T)(B'A_j\Sigma_{A_j}^{-1}A_jB)^{-1}$
$cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*), \sqrt{T}(\hat{\lambda}_{A_j} - \lambda^*)] =$
$\frac{T}{(T^*)}(B'A_i\Sigma_{A_i}^{-1}A_iB)^{-1}(B'A_i\Sigma_{A_i}^{-1}A_i\Sigma A_j\Sigma_{A_j}^{-1}A_jB)(B'A_j\Sigma_{A_j}^{-1}A_jB)^{-1}$
and using the fact that $\frac{T}{T^*} = 1$ since the factors have sample means equal to zero, the expression simplifies to:

$$cov(\hat{\lambda}_{A_i}, \hat{\lambda}_{A_j}) = cov[\sqrt{T}(\hat{\lambda}_{A_i} - \lambda^*), \sqrt{T}(\hat{\lambda}_{A_j} - \lambda^*)] =$$

$$= (B'A_i\Sigma_{A_i}^{-1}A_iB)^{-1}(B'A_i\Sigma_{A_i}^{-1}A_i\Sigma A_j\Sigma_{A_j}^{-1}A_jB)(B'A_j\Sigma_{A_j}^{-1}A_jB)^{-1} \quad (14)$$

6 Bibliography

Antoniou, A., Garrett, I., and Priestley, R. (1998) 'Macroeconomic variables as common pervasive risk factors and the empirical content of the arbitrage pricing theory', *Journal of Empirical Finance*, 1998, 221-240

Beenstock, M., and Chan, K. (1988) 'Economic forces in the London stock market', *Oxford Bulletin of Economics and Statistics* 50 (1), 27-39

Campbell, J. Y. (1999) 'Asset prices, consumption and the business cycle', in John Taylor and Michael Woodford eds. *Handbook of Macroeconomics*, Vol. 1 (North-Holland, Amsterdam)

Chen, N. F., Roll, R., and Ross, S. A. (1986) 'Economic forces and the stock market', *Journal of Business*, Vol. 59, no. 3, 383-403

Clare, A. D., and Thomas, S. H. (1994) 'Macroeconomic factors, the APT and the UK stock market', *Journal of Business Finance and Accounting*, 21(3), April, 309-330

Cochrane, J. H. (1999) 'New facts in finance', *Economic Perspectives Federal Reserve Bank of Chicago* 23, 36-58

Fama, E.F., and McBeth, J.D. (1973), 'Risk, return and equilibrium: empirical tests', *Journal of Political Economy* 71, 607-636

Gallant, A.R. (1975) 'Seemingly unrelated nonlinear regressions', *Journal of Econometrics*, 3, 35-50

Gallant, A.R. (1987) 'Nonlinear statistical models', Wiley, New York

McElroy, M. B., and Burmeister, E. (1988) 'Arbitrage Pricing Theory as a Restricted Nonlinear Multivariate Regression Model', *Journal of Business and Economic Statistics*, January, Vol. 6, No. 1, 29-42

McElroy, M. B., and Burmeister, E. (1988) 'Joint Estimation of Factor Sensitivities and Risk Premia for the Arbitrage Pricing Theory', *Journal of Finance*, 721-735

McElroy, M. B., Burmeister, E., and Wall, K.D. (1985) 'Two estimators for the APT model when factors are measured', *Economics Letters* 19, 271-275

Merton, R. C. (1971a) 'Optimum consumption and portfolio rules in a continuous time model', *Journal of Economic Theory* III, 373-413

Merton, R. C. (1971b) 'An intertemporal capital asset pricing model', *Econometrica* 41, 867-887

Poon, S., and Taylor, J. (1991) 'Macroeconomic factors and the UK stock market', *Journal of Business Finance and Accounting*, 18(5), September, 619-636

Ross, S. A. (1976) 'The Arbitrage Theory of Capital Asset Pricing', *Journal of Economic Theory* 13, 341-360

Table 1: Means and standard deviations for the factors

	Mean	Std. dev.
Industrial production	-0.0001	0.0075
Inflation	0.00003	0.0025
Market risk premium	-0.0012	0.0356
Term structure	0.0053	0.0301

Table 2: Correlation structure of macroeconomic factors

	Ind. Prod.	Infl.	Mkt. risk prem.	Term str.
Industrial production	1.00			
Inflation	0.37	1.00		
Market risk premium	0.02	0.02	1.00	
Term structure	-0.07	-0.09	0.08	1.00

Table 3: UK risk premia with respect to UK macroeconomic risk factors

	MP	IR	RP	TS
Sample 1 risk premia	-0.0017	-0.0009	-0.0024	0.0067
t-statistic	-2.1253	-3.6229	-1.4668	2.6769
Sample 2 risk premia	-0.0016	0.0020	0.0033	-0.0014
t-statistic	-1.8752	6.9393	2.0409	-0.5617
Sample 3 risk premia	0.0048	-0.0019	-0.0117	0.0131
t-statistic	6.7539	-7.2098	-5.7472	5.0699
Sample 4 risk premia	-0.0104	0.00002	0.0065	-0.0185
t-statistic	-8.7382	0.0617	3.7488	-6.3392
Combined minimum variance risk premia	-0.00010	-0.00027	0.00058	-0.00071
t-statistic	-0.2055	-2.0925	0.5249	-0.4740

Table 4: UK sign-consistent risk premia with respect to UK macroeconomic risk factors

	MP	IR	RP	TS
Sample 2 risk premia	-	0.0020	0.0033	-0.0014
t-statistic	-	6.9393	2.0409	-0.5617
Sample 3 risk premia	0.0048	-	-	-
t-statistic	6.7539	-	-	-
Sample 4 risk premia	-	0.00002	0.0065	-0.0185
t-statistic	-	0.0617	3.7488	-6.3392
Combined minimum variance risk premia	0.0048	0.0011	0.0048	-0.0089
t-statistic	6.7539	5.2245	3.6279	-4.7496