

Context and Implications Document for: Secondary students' proof constructions in mathematics: The role of written versus oral mode of argument representation

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Author's Introduction

The concept of 'proof' is fundamental to deep learning in mathematics and in various countries it is considered to be important for students' mathematical experiences across all the levels of education, as early as the primary school (e.g. National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA & CCSSO], 2010; Department for Education, 2013). The concept of proof is also hard-to-teach and hard-to-learn, and thus over the past few decades it has attracted significant attention internationally by researchers in the field of mathematics education (for a review of the state of the art in this area, see Stylianides, Stylianides, & Weber, 2017).

A main research strand has focused on secondary (i.e. post-primary) students' constructions of mathematical arguments, showing that many secondary students fail to produce arguments that meet the standard of proof. However, the studies in this strand have tended to only consider secondary students presenting their perceived proofs in written form, primarily in the context of survey studies. The lack of consideration by these studies of secondary students presenting their perceived proofs orally—in tandem with students' written proofs for the same claims—might have resulted in an incomplete or a skewed picture of the potential of students' constructed proofs, and this raises concern about the validity of research findings.

In this article I aimed to contribute to this area by exploring the role of the mode of argument representation in secondary students' proof constructions. Using classroom data to compare the written arguments (perceived proofs) constructed by students in two secondary mathematics classrooms with the oral arguments that the students presented in the front of their class for the same claims, I derived findings that suggest

the oral mode of representation is more likely than the written mode to be associated with the construction of student arguments that meet the standard of proof.

Implications for Practice

All of the oral arguments that the secondary students in this study presented in front of their class and perceived to be proofs approximated the standard of proof to the same or higher degree than the corresponding written arguments that the students had produced previously for the same claims. For teachers, an important message is that the way they assess their students' argument constructions—in writing or orally—can lead to different conclusions about their students' potential to construct arguments that meet the standard of proof. Specifically, the findings suggest that if a teacher had assessed the students' written arguments only, s/he would have derived a less favourable picture of the potential of students' constructed proofs than if s/he had assessed only students' oral arguments.

A related message for teachers is that, by considering only one mode of representation and ignoring the other, each assessment individually would have offered an incomplete picture of students' constructed proofs, for apparently it matters whether students present their perceived proofs orally or in writing. A balanced assessment of students' proof constructions would require the use of multiple assessment methods, both written and oral.

The latter is also a message for policy makers. In England and other countries students' mathematical knowledge is assessed primarily, if not exclusively, through students' response to tasks in written tests. The findings suggest that the lack of consideration by national or high stakes assessments of students presenting their solutions to proving or other kind of mathematics tasks orally—in tandem with students' written solutions to the same tasks—might be yielding an inaccurate picture of students' mathematical potential.

References

- Department for Education. (2013) *Mathematics: Programmes of study: Key Stages 1–2* (National Curriculum in England). Retrieved from https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/239129/PRIMARY_national_curriculum_-_Mathematics.pdf.
- National Governors Association Center for Best Practices & Council of Chief State School Officers (NGA & CCSSO). (2010) *Common core state standards for mathematics* (Washington, DC, The Author).
- Stylianides, G. J., Stylianides, A. J. & Weber, K. (2017) Research on the teaching and learning of proof: Taking stock and moving forward, in: J. Cai (Ed.) *Compendium for research in mathematics education* (Reston, National Council of Teachers of Mathematics).