Peristaltic elastic instability in inflated cylindrical channel: Numerical Supplement

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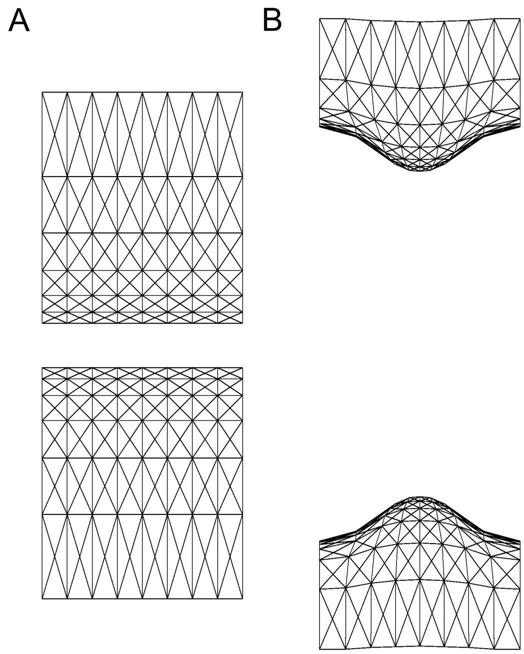

FIG. 1. The mesh used in 2-D numerical simulation (A) sable and (B) unstable states. For clarity of visualization, we here show a sparse 8×6 mesh with dramatic mesh coarsening: our actual calculations used a much finer mesh.

Our 2-D numerical calculations use an explicit finite-element method, based on the same cpp code used in [1–4] as modified for $r - z$ cylindrical coordinates in [3, 4]. The code constructs an elastic hollowed cylinder from constant-strain triangular elements in the $r - z$ plane, each representing a triangular cross-sectioned torus of the body. The triangles form a rectangular mesh, spanning from the inner radius $r = a$ to the outer radius $r = b$, and from $z = 0$ to $z = L$, with the periodic boundary conditions at the two ends $z = 0, L$, which makes the simulation domain contain a wavelength of the instability, as shown in fig 1. The mesh is not uniform but is finer close to the inner surface and coarsens by 2% per element in the r-direction.

Each constant-strain element in the mesh is assigned a compressible neo-Hookean elastic energy (within a quasi-incompressible nodal pressure formulation) with shear modulus μ with a bulk modulus $K = 10^3\mu$. The force on each node is calculated as the gradient of the total energy with respect to nodal position, and the nodes are moved according to damped Newtonian dynamics. In our calculations, we change the cavity pressure P_{in} while maintaining a constant wavelength, L. In each calculation the changes were imposed slowly enough as to be quasi-static, so although the simulation uses Newtonian dynamics, the states reported in the paper are all converged energy minima. The good agreement between the predicted and observed thresholds and wavelengths verify that the the bulk modulus was high enough, the mesh was fine enough and the simulations were slow enough to mimic an incompressible, equilibrated continuum hollowed cylinder.

We summarize the numerical parameters for each figure in the table below.

Figure	P_{in}/μ	L/a	b/a	Mesh
1	0, 1.9, 2.2	12.278	60	40×100
3a,b and d	0–2.75	12.278	60	40×100
3c	2.0–3.1	1–15	100	50×100
num. SI 1	0, 2.2	12	10	8×6
theory SI 1	1.4–3.1	2–16	5–100	100×50
theory SI 2	2.1	200	48	450×60
theory SI 3	2.05–2.2	12–20	60	60×100
video N1	0–2.3	12.278	60	60×100

[1] T. Tallinen, J. Y. Chung, J. S. Biggins, and L. Mahadevan, PNAS **111**, 12667 (2014).
[2] T. Tallinen and J. S. Biggins, Physical Review E **92**, 022720 (2015).
[3] C. Xuan and J. Biggins, Physical Review E **94**, 023107 (2016).
[4] C. Xuan and J. Biggins, Physical Review E **95**, 053106 (2017).