Retrofitting of Reinforced Concrete Beams with CFRP Straps to Enhance Shear Capacity

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Declaration

This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. This dissertation is not substantially the same as any that I have submitted or will be submitting for a degree or diploma or other qualification at any other University. The length of this dissertation is approximately 65000 words and it includes 119 figures.

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Whilst only my name appears on the cover of this lengthy tome, it was most certainly not an individual effort. The efforts and support of many people have gone into this thesis. While some are named below, I have most assuredly forgotten several key people, who while they are not named, were nonetheless important parts of this work.

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A technique has been developed that employs thin Carbon Fibre Reinforced Polymer (CFRP) straps for shear enhancement of reinforced concrete beams. Previous studies using these straps have employed steel pads on top of the beam to support the strap but this is problematic from a practical point of view. The initial goal of this research was to develop a method of installing the straps without requiring access to the top surface of the slab that still provided effective shear enhancement, thus reducing the costs associated with repair.

This novel strap installation method was developed through a series of tensile tests on the straps. It was found that the straps must be kept away from the sharp edges of the concrete, that the strap cross section must be kept flat, and that the material supporting the strap must be sufficiently stiff.

Seven T-beam tests were performed to develop a strap configuration that maximized the shear enhancement but could also be installed from under the slab. The chosen installation technique involved drilling holes in the flange and then filling the holes with grout except for a small groove to support the strap. To ensure maximum enhancement, the straps need to fully penetrate the compression flange of the beam and the amount of flange area removed for the straps should be minimized. The shear capacity is also affected by the size of the loading pad.

A Finite Element Analysis determined that holes in the flange act as crack propagators, strap penetration affects the stiffness of the beam and undersized load pads allow shear cracks to form on lower energy paths.

Design equations were developed by comparing the predictions of existing shear models using the straps. The most accurate model, a shear friction approach, was then used to develop a retrofit design procedure for use with the straps.

To evaluate the long-term performance of the straps and the under-slab installation technique two further T-beam tests were undertaken: a sustained load test where the beam was loaded to 80% of its retrofitted capacity for 260 days and a cyclic test where the load was varied between 50 and 80% of the retrofitted capacity for 2.1 million cycles. In both cases the straps withstood the loading. However, the strains in the straps increased over time, an important observation considering the brittle CFRP straps, which led to the development of a model to predict the long-term strap strains.
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Nomenclature

\[ A = \text{area of potential shear failure plane (mm}^2) \]
\[ A_c = \text{area of compressive reinforcement (mm}^2) \]
\[ A_{cf} = \text{effective concrete flange area (mm}^2) \]
\[ A_{cv} = \text{area of concrete strut (mm}^2) \]
\[ A_{cw} = \text{effective concrete web area (mm}^2) \]
\[ A_{\text{element}} = \text{area of FE element (mm}^2) \]
\[ A_{\text{FRP}} = \text{area of CFRP strap (mm}^2) \]
\[ A_{\text{links}} = \text{area of shear links along crack (mm}^2) \]
\[ A_s = \text{area of longitudinal tension reinforcement (mm}^2) \]
\[ A_{sv} = \text{area of steel shear link (mm}^2) \]
\[ A_{sv} = \text{total stirrup area placed over a length } d \text{ (mm}^2) \]
\[ C = \text{limiting compressive force in beam web based on total depth (N)} \]
\[ C_v = \text{force in the compression strut (N)} \]
\[ D_{\text{frp}} = \text{strain distribution factor} \]
\[ E_c = \text{elastic modulus of the concrete (MPa)} \]
\[ E_{\text{FRP}} = \text{elastic modulus of FRP (MPa)} \]
\[ E_s = \text{elastic modulus of tension reinforcement (MPa)} \]
\[ E_{\text{trans}} = \text{elastic modulus of transverse reinforcement (MPa)} \]
\[ E_v = \text{elastic modulus of steel shear links (MPa)} \]
\[ F_{\text{FRP}} = \text{force provided by the CFRP straps (N)} \]
\[ F_{\text{FRPflange}} = \text{force provided by the CFRP straps in the flange (N)} \]
\[ F_{\text{FRPweb}} = \text{force provided by the CFRP straps in the web (N)} \]
\[ F_{\text{FRPn-1}} = \text{force provided by the CFRP straps at iteration } n-1 \text{ (N)} \]
$F_v = \text{force provided by the steel shear links (N)}$

$F_{vn-1} = \text{force provided by the steel shear links at iteration } n-1 \text{ (N)}$

$G = \text{shear modulus (MPa)}$

$G_f = \text{fracture energy (N/mm)}$

$I_{cr} = \text{cracked second moment of area of the transformed section (mm}^4\text{)}$

$I_g = \text{gross second moment of area (mm}^4\text{)}$

$I_{eff} = \text{effective second moment of area (mm}^4\text{)}$

$K_{cone} = \text{concrete shear stiffness (N)}$

$K_{FRP} = \text{FRP shear stiffness (N)}$

$K_{link} = \text{steel link shear stiffness (N)}$

$K_v = \text{total shear stiffness (N)}$

$L_{FRPflange} = \text{length over which the CFRP straps are effective in the flange (mm)}$

$L_{FRPweb} = \text{length over which the CFRP straps are effective in the web (mm)}$

$L_{sflange} = \text{length over which steel shear links are effective in the flange (mm)}$

$L_{svweb} = \text{length over which steel shear links are effective in the web (mm)}$

$M_{ax} = \text{applied moment at a point } x \text{ along the beam (Nmm)}$

$M_{cr} = \text{cracking moment (Nmm)}$

$M_{fk} = \text{flexural capacity at a point } x \text{ along the beam (Nmm)}$

$M_{sx} = \text{moment corresponding to shear failure at a point } x \text{ along the beam (Nmm)}$

$N = \text{axial force applied to the beam (N)}$

$R = \text{normal force acting on a potential shear plane (N)}$

$R_{l} = \text{remaining bonded length over initial bonded length ratio}$

$S = \text{shear force acting along a potential shear plane (N)}$

$T = \text{tensile force in longitudinal reinforcement (N)}$

$T_{FRP} = \text{tensile capacity of the FRP reinforcement (N)}$
\( T_{\text{links}} \) = force in the steel shear links (N)
\( T_v \) = tensile force in a steel shear link (N)
\( V \) = shear force (N)
\( V' = V_{\text{required}} - V_y \)
\( V_{\text{available}} \) = available shear capacity (N)
\( V_{\text{ax}} \) = applied shear at point x along the beam (N)
\( V_c \) = concrete shear contribution (N)
\( V_{\text{difference}} \) = required additional shear capacity (N)
\( V_{\text{frp}} \) = shear contribution of CFRP straps (N)
\( V_f \) = factored shear capacity (N)
\( V_{fs} \) = shear capacity at a point x along the beam (N)
\( V_n \) = nominal shear capacity of the beam (N)
\( V_{\text{required}} \) = required shear force (N)
\( V_{\text{u1}} \) = shear force due to a unit load applied where the deflection is required
\( V_y \) = shear force at yield of steel shear links (N)
\( V_{y\text{web}} \) = shear force at yield of steel shear links in the web (N)
\( a \) = the shear span (mm)
\( a_{\text{agg}} \) = maximum aggregate size (mm)
\( a_w \) = ratio of applied moment to applied shear at a point x along the beam (mm)
\( b \) = effective width (mm)
\( b_f \) = flange width (mm)
\( b_w \) = web width (mm)
\( c_{\text{FRPB}} \) = bottom cover to straps (mm)
\( c_{\text{FRPT}} \) = top cover to straps (mm)
1. $c_v =$ cover to steel transverse reinforcement (mm)
2. $d =$ the effective depth (mm)
3. $d_{FRP} =$ height of FRP sheets (mm)
4. $d_s =$ length of steel shear links (mm)
5. $f'_c =$ compressive cylinder strength of concrete (MPa)
6. $f_{ci} =$ compressive stress across a crack (MPa)
7. $f_{cu} =$ compressive cube strength of concrete (MPa)
8. $f_{FRP} =$ stress in CFRP strap (MPa)
9. $f_{frpul} =$ ultimate strength of CFRP strap (MPa)
10. $f_{frp,e} =$ effective stress in the CFRP strap (MPa)
11. $f_{pc} =$ longitudinal compression force (N)
12. $f_t =$ concrete tensile strength (MPa)
13. $f_{ul} =$ split cylinder strength (MPa)
14. $f_{t2} =$ modulus of rupture strength (MPa)
15. $f_s =$ yield stress of tensile steel (MPa)
16. $f_{yv} =$ yield stress of transverse reinforcement (MPa)
17. $g_i =$ out-of-balance force vector for iteration $i$
18. $g_i^T =$ transpose of out-of-balance force vector for iteration $i$
19. $g_o =$ out-of-balance force vector for initial iteration
20. $g_o^T =$ transpose of out-of-balance force vector for initial iteration
21. $h =$ total depth of the beam (mm)
22. $h_b =$ estimated numerical crack bandwidth (mm)
23. $h_f =$ flange depth (mm)
24. $h_{FRP} =$ unsupported strap length (mm)
\( k = \) coefficient to relate experimental shear strength and normal stress

\( k_f = \) coefficient to relate experimental shear strength and normal stress in the flange

\( k_v = \) coefficient for load sharing between transverse reinforcement and concrete

\( k_{vw} = \) coefficient for load sharing between transverse reinforcement and concrete in the web

\( k_w = \) coefficient to relate experimental shear strength and normal stress in the web

\( n_v = \) number of steel shear links crossing a crack

\( r = \) radial distance away from the centre of the hole (mm)

\( r_{\text{hole}} = \) radius of the hole (mm)

\( s_{\text{FRP}} = \) FRP reinforcement spacing (mm)

\( s_v = \) spacing of steel shear links (mm)

\( t_{\text{FRP}} = \) total thickness of FRP (mm)

\( v = \) shear stress (MPa)

\( v_{ci} = \) average shear stress transferred across a crack (MPa)

\( v_{cimax} = \) maximum shear stress transferred across a crack (MPa)

\( v_{ult} = \) shear friction capacity (MPa)

\( w = \) crack width (mm)

\( w_{\text{crFRP}} = \) crack width in direction of FRP (mm)

\( w_{\text{FRP}} = \) FRP reinforcement width (mm)

\( x = \) depth to neutral axis of linear elastic transformed section (mm)

\( y = \) distance from the centroid to the tension face (mm)

\( z = \) flexural lever arm (mm)

\( \Delta_{rv} = \) displacement of concrete compressive strut (mm)

\( \Delta_{sv} = \) elongation of steel shear links (mm)

\( \alpha = \) angle between the transverse reinforcement and the longitudinal axis
\( \beta = \) shear retention factor

\( \delta_{\text{shear}} = \) deflection due to shear (mm)

\( \delta_{\text{shear,yield}} = \) deflection due to shear at yielding of shear links (mm)

\( \delta_i = \) deflection at time \( t \) (mm)

\( \delta_o = \) initial flexural deflection at time \( t = 0 \) (mm)

\( \delta_{\text{yield}} = \) initial flexural deflection at time \( t = 0 \) after yielding of shear links (mm)

\( \varepsilon_{\text{allowable}} = \) maximum allowable strain in FRP

\( \varepsilon_r = \) crack strain in an element

\( \varepsilon_{cv} = \) strain in the concrete compression strut

\( \varepsilon_{\text{crack}} = \) strain due to crack opening

\( \varepsilon_{\text{ult}} = \) ultimate tensile strain in concrete

\( \varepsilon_{\text{experimenatal}} = \) experimentally determined maximum FRP strain

\( \varepsilon_{\text{FRP}} = \) strain in FRP reinforcement

\( \varepsilon_{\text{max}}^{\text{FRP}} = \) maximum CFRP strap prestressing strain

\( \varepsilon_{\text{min}}^{\text{FRP}} = \) minimum CFRP strap prestressing strain

\( \bar{\varepsilon}_{\text{FRP}} = \) normalized prestressing strain

\( \varepsilon_{\text{frprob}} = \) rupture strain of the CFRP straps

\( \varepsilon_{\text{FRP,LT}} = \) calculated long-term strain in the CFRP straps at the required shear force

\( \varepsilon_{\text{FRPave}} = \) average strain in FRP reinforcement

\( \varepsilon_{\text{FRP,required}} = \) calculated strain in the CFRP straps at the required shear force

\( \varepsilon_{\text{max}} = \) maximum useable CFRP strain

\( \varepsilon_{\text{min}} = \) minimum strain to develop aggregate interlock
\( \varepsilon_{\text{prestress}} = \) strain in CFRP strap due to prestressing force

\( \varepsilon_s = \) strain in the tensile reinforcement

\( \varepsilon_{sv} = \) strain in the transverse reinforcement

\( \varepsilon_{\text{transy}} = \) strain in CFRP straps after the shear links have yielded

\( \varepsilon_{sv} = \) the yield strain of the steel transverse reinforcement

\( \phi = \) creep coefficient

\( \gamma = \) shear rotation

\( \gamma_c = \) material capacity reduction factor for concrete

\( \gamma_{\text{FRP}} = \) material capacity reduction factor for FRP

\( \gamma_m = \) material capacity reduction factor

\( \gamma = \) material capacity reduction factor for steel

\( \gamma_{tot} = \) total shear rotation

\( \gamma_{\text{yield}} = \) shear rotation after yielding of the shear links

\( \theta = \) crack angle

\( \theta_{\text{ave}} = \) average crack angle

\( \theta_f = \) crack angle in the flange

\( \theta_w = \) crack angle in the web

\( \rho_c = \) compressive reinforcement ratio

\( \rho_s = \) tensile reinforcement ratio

\( \rho_{\text{FRP}} = \) FRP reinforcement ratio

\( \rho_{\text{FRPinit}} = \) initial FRP reinforcement ratio

\( \rho_{\text{FRPn}} = \) FRP reinforcement ratio at iteration \( n \)

\( \rho_{\text{FRPn-1}} = \) FRP reinforcement ratio at iteration \( n-1 \)

\( \rho_{\text{trans}} = \) transverse reinforcement ratio
\( \rho_v \) = steel shear link reinforcement ratio

\( \sigma \) = average normal stress on potential shear failure plane (MPa)

\( \sigma_{ave} \) = average stress applied to the specimen away from the hole (MPa)

\( \sigma_{y@x} \) = stress in the y direction at a location x along the cross section (MPa)

\( \xi \) = ACI factor to account for the duration of loading
Chapter 1
Introduction

The cost of retrofitting strength-deficient reinforced concrete bridges in England alone is estimated to be £2.2 billion (Middleton 1997). While not all of this money is earmarked for upgrading shear-deficient bridges, even a small percentage still represents a considerable investment, especially when one considers that this estimate is only for bridges in England. Shear deficiencies are generally the result of increased load requirements or, as Collins and Mitchell (1997) note, the fact that previous design codes were less conservative than current codes. In some climates, corrosion of internal reinforcing steel due to the use of de-icing salts is a further issue that can have a detrimental effect on shear capacity. Considering both the cost, and the number of structures involved, there is a need to find an effective shear retrofitting system for reinforced concrete (RC) structures.

Current research efforts focus primarily on the use of Fibre Reinforced Polymers (FRPs) to enhance the shear capacity of RC beams. FRPs are not susceptible to corrosion and thus have an advantage over more traditional retrofitting techniques that use steel. FRPs are also lighter and will not contribute greatly to the dead weight of the structure. Numerous researchers have investigated the use of FRP laminates or sheets that are bonded to the side of the specimen. The amount of shear force that can be transferred to the beam is dependent on the strength of the bond, the anchorage length, and the FRP strength and stiffness. If the anchorage length is insufficient, the FRP retrofit will delaminate before the ultimate capacity of the FRP is reached.

This type of research has highlighted the need to develop a system that is not susceptible to bond breakdown, loss of anchorage or localized strain concentrations due to crack formation. A shear retrofitting system that meets these criteria was developed by Winistoerfer (1999) and involves wrapping thin CFRP thermoplastic tapes around a beam to form an external reinforcing element. The outermost layer of the tape is then welded to the next outermost layer by melting the thermoplastic matrix of both layers together. Winistoerfer discovered that although only the outermost loop is closed, the inner layers act independently, each one able to reach its maximum tensile strength. In contrast, if all the layers are laminated together,
through-thickness effects reduce the capacity of each layer and the overall capacity of the strap. A further benefit is that the straps can be prestressed.

One possible application of these straps is shear retrofitting of RC beams. The straps are wrapped around the beam and are supported on metal pads on the top and bottom of the beam as illustrated in Figure 1.1. These metal pads serve to maintain a flat cross-sectional profile for the strap as well as keeping the curvature of the strap above the limiting value prescribed by Winistoerfer. The straps act as additional external shear reinforcement, increasing the overall capacity of the beam. Because the straps are not bonded to the beam face, they are not susceptible to failure due to debonding, as is sometimes the case with retrofitting techniques that use epoxy bonded FRP sheets. Finally, because the straps can be prestressed, this can create additional capacity in beam sections.

Experiments conducted by Chan (2000) using the CFRP strap strengthening system investigated the results of two RC T-beams tested in four point bending. She tested an unretrofitted specimen and a retrofitted specimen, discovering that the addition of the straps in the shear span of the retrofitted beam led to a considerable capacity enhancement and changed the failure mode from shear to flexure. Kesse (2003) tested a series of rectangular RC cantilever sections and looked at the effect of varying the strap spacing, strap stiffness and initial prestress. He concluded that with the right combination of parameters, a flexural failure was possible in an initially shear-deficient beam. Work by Stenger (2000) proved that the straps could be used to significantly enhance the capacity of deep beams as well.
Although the previous experimental studies have shown that CFRP straps have potential as a retrofitting system, they have all been limited by the use of the metal support pads. The use of these pads, especially on the top surface of the beam, leads to serviceability issues for the structure. A motorway bridge with these pads, for example, might require an extra topping in order to ensure a smooth running surface for vehicles. The extra weight of the running surface could negate any improvement achieved by using this lightweight material. Another possibility would be to cut grooves into the road surface which the straps could sit in, but this would necessitate the road being closed and the surface repaired. Both approaches would require access to the top surface of the slab, which could lead to significant traffic interruptions and additional costs.

The current research has sought to develop a method of installing these straps from underneath the slab without penetrating the top surface. If such a technique were feasible, it would result in reduced disruptions to the structure since the top surface of the slab would still be useable during retrofitting and also enhance the durability of the retrofit since the metal bearing pads on the exposed top surface would not be required. A considerable amount of testing and verification is required to develop a technique that is both practical and creates a significant amount of shear capacity enhancement. As well, the designer needs some method of predicting the amount of capacity enhancement, preferably a straightforward closed-form approach. The following report attempts to cover the initial work towards developing such an installation technique.

Chapter 2 presents an overview of shear and the significant factors that affect shear capacity. Various current shear models are examined and the most viable approaches for predicting the capacity enhancement due to the straps are chosen for further study. Methods of FRP retrofitting are then introduced with key points from the experimental work highlighted. Previous work involving the CFRP straps will then be examined. Current FRP shear models are investigated and their shortcomings are outlined. The experimental specimens used in this study are presented and several important elements of the design that will impact the results are examined. Finally,
several considerations that need to be addressed to facilitate an effective under-slab installation will be examined.

Chapter 3 looks at the development of the under-slab installation technique. The process began with a series of tests on the CFRP straps to ascertain how they could be placed in the flange and still develop the full strap capacity. Once an effective method of strap installation was developed this method was employed on T-beam specimens. The next section of this chapter looks at the static beam tests that were performed to develop a retrofitting technique that was both easy to implement and provided effective shear capacity enhancement to the beam. The results of these tests proved interesting not only in terms of developing effective enhancement but also in terms of investigating other parameters that affect shear capacity. Shear reinforcement penetration into the compression flange, size of bearing pad, and the presence of intrusions in the compression flange all proved to play a role in terms of the shear capacity.

Chapter 4 examines the results of the Finite Element Analysis (FEA). The role of holes in the compression flange, which were required for the strap installation as discussed in Chapter 3, proved to be a significant factor affecting the beam capacity. The results of compression tests performed on concrete specimens with both holes and grout filled holes are examined. These tests are then modelled using FEA to develop a concrete model that accounts for the presence of the holes for use with the 2-D beam analysis. The results of the static beam tests are then predicted using FEA. The predictions of the models allow for a clearer understanding of the affects of holes, strap penetration and bearing area. An attempt is then made to use FEA to predict the capacity of a specimen configuration that was not tested during the static test series.

Chapter 5 gives details of the models selected in Chapter 2 and uses them to predict the capacity of several series of beam tests performed using the CFRP strap system. The results of tests from the current study as well as Chan (2000), Kesse (2003), and Stenger (2000) are all predicted. A model is selected based on its accuracy. The selected model, a modified version of the shear friction approach (Loov 1998), is then used to develop a design procedure. A design example is presented and compared against the results of a similar beam test.
Chapter 6 deals with the long-term testing of beams retrofitted using the strap installation technique developed in chapter 3. These experiments are important not only in terms of the current work but also for the use of the CFRP straps in general as their long-term behaviour has not been looked at in this context before. Both sustained and cyclic load tests were performed and the results examined. A model is also proposed for calculating the long-term increase in strap strains, which proved to be the most significant issue associated with the long-term testing.

Chapter 7 highlights the most important conclusions drawn from the current work. Recommendations for future work are then presented as well as some suggestions for making the strap system more commercially viable.
Chapter 2
Background

The following chapter outlines the background of the current research. A brief introduction into the nature of shear in concrete is presented. The basic mechanisms of shear transfer will then be dealt with. It is important to be able to predict the capacity enhancement delivered by any retrofitting system and so shear modelling will be examined. Numerous models will be investigated and a select few that seem best suited to model the retrofitted beam behaviour will be chosen for further study. Previous work in the field of shear retrofitting will then be highlighted. Particular attention will be paid to the development of the CFRP straps used in this research and previous tests conducted with these straps. The design philosophy behind the experimental specimens used in the current study will be presented with a specific focus on the impact of beam cross-section. Finally, some initial considerations for the under-slab installation technique will be addressed.

2.1 Shear

The tensile capacity of concrete is much lower than its compressive capacity. As a result, cracks in concrete members form perpendicular to the direction of maximum tensile stresses once the cracking stress is exceeded. In pure tension and flexural members, these cracks form perpendicular to the longitudinal axis. However, for members subjected to pure shear, resolving the principal stress components using Mohr’s circle indicates that the maximum tensile stresses act at 45° to the longitudinal axis and thus cracks form perpendicular to this 45° axis. In an actual beam, the behaviour is more complex as tensile, flexural, and shear stresses can all be present simultaneously in the member. Therefore, flexural cracks, diagonal shear cracks, and flexure-shear cracks can all exist within one specimen.

2.2 Shear Modeling

The CFRP straps used in this research are made from a high strength, linear elastic, and brittle material. These characteristics make them difficult to incorporate into the design equations used in most current codes (BS 8110 1997 and CSA 1994) that
assume the shear reinforcement is ductile and that the yield strength of the material can be used. These equations also assume that plasticity within the beam will allow the shear forces to redistribute, which is an acceptable approach when ductile shear links are considered. This is not the case with the CFRP straps where the exact strain must be known to determine both the force in the strap and whether it has failed. Any model chosen to predict the beam capacity must have some allowance for the brittle nature of the straps. The following section briefly covers the history of shear modeling. Many of the current models are then introduced and three models are chosen as being best suited to model the straps. These models will then be described in greater detail in Chapter 5.

2.2.1 Historical Background

Ritter (1899) developed a model whereby a cracked reinforced concrete beam could be visualized as a truss. The concrete acts as both the compressive struts and the compressive chord of the truss while the steel acts as the tension ties and the tension chord. Morsch (1909) elaborated on Ritter’s model by suggesting the compressive struts were not discrete elements but a continuum of compressive force traveling through the web of the beam. Their analogies are still useful today for envisioning shear transfer, provided that this is the correct mechanism of shear transfer. Morsch also used the flexural stress distribution and longitudinal equilibrium between two points along the span to determine the shear stresses within a beam as illustrated in Figure 2.1. Based on these equilibrium considerations, he suggested that approximately 30% of the shear stress was carried in the compressive region and that the remaining stress was carried below the neutral axis. Using this approach only the longitudinal reinforcement needs to be considered, as the transverse reinforcement is not required for longitudinal equilibrium.
However, the major failing of the Ritter and Morsch models is that without shear links, the shear capacity of the beam would be zero as the models require the tension ties to carry the vertical component of the shear force. This assumption is incorrect since cracks must first form due to tensile stresses in the concrete before failure can occur. The 1910 ACI (NACU 1910) code recognized the concrete contribution by allowing the designer to use 0.28MPa as the concrete’s tensile capacity in contributing to the shear resistance. This method of combining a steel contribution with a concrete contribution is still used by design codes (BS8110 1997 and CSA 1994). These codes do not currently allow for the additional capacity contributed by FRP retrofitting. The addition of an FRP term maybe relatively straightforward as such a term has already been proposed for other types of FRP shear retrofitting as discussed later.

Another flaw with the early truss models was that they assumed a strut angle of 45°. The combination of stresses within a beam generally means that the angle of the cracks will not be 45°. However, using statics there are 3 equations with 4 unknowns making it mathematically impossible to solve for the actual strut angle. Kupfer (1964) developed a method that assumed all material behaviour was linear elastic and used...
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energy methods to solve for the actual crack angle. Another possibility for solving this system of equations is to assume either that the steel yields or that the concrete stress is known and solve two of the equations for the remaining two unknowns. These methods for determining the shear capacity and corresponding crack angles are referred to as plasticity approaches (Nielsen 1984). The notion of a variable angle for the shear crack is also at the heart of the Modified Compression Field Theory and shear friction approach, which will be introduced in the next section.

Researchers in the 1950s such as Bresler and Pister (1958) and Guralnick (1959) suggested that the compression zone carried the majority of shear. Their work was based on beams with no shear reinforcement and the assumption that the concrete in the tension zone was incapable of carrying shear. Or as Guralnick states “because the concrete below the neutral plane cannot sustain shear stresses at ultimate load in view of the presence of numerous flexural tension cracks.” Kani (1964) disputed this, indicating instead that concrete beams behave like a “comb” with each tooth of the comb carrying shear until the diagonal crack breaks the stem of the tooth. After the tooth is fractured, Kani et al. (1979) indicate that reinforcement is needed for the beam to continue carrying load. Work by Fenwick and Paulay (1968) suggested that most of the shear force was carried through aggregate interlock in the web as opposed to the compression flange. This debate has not been resolved as the Modified Compression Field Theory (Vecchio and Collins 1986) and the Compressive Force Path (Kotsovos 1988) are two current approaches to shear that are contradictory since the former uses the web as the dominant method of shear transfer while the later uses the compression zone. Both of these models will be examined in greater detail later in the next section as well as in Chapter 5. The assumed shear transfer mechanism drastically changes the nature of both modeling and design as will be highlighted later.

2.2.2 Current Models

The ASCE-ACI Committee 445 on Shear and Torsion (1998) issued a report on the state of the art in shear design. In this report they grouped the models into three main categories, namely: sectional approaches using compression field theory, sectional approaches using a concrete contribution, and strut and tie models.
There are two main models that form the compression field approach group. One is the Modified Compression Field Theory (MCFT) developed by Vecchio and Collins (1986). The other is the Rotating-Angle Softened Truss Model (RA-STM) developed by Belarbi and Hsu (1994 and 1995). The ASCE-ACI Committee 445 on Shear and Torsion report offers a comparison of both models and shows that they produce similar results except at high reinforcement ratios where the RA-STM is more conservative. The equations developed by the RA-STM involve more terms and thus would provide a greater opportunity for error if integrated into a code design method. Obviously this would be less of an issue if the method were implemented using a computer. However, current codes still depend largely on simple equations that give conservative results to the designer. As such, the MCFT will be used as a representative model from this particular group as it has already been implemented in current codes (CSA 1994).

The next group contains a much wider array of models all developed using the so-called concrete contribution. These are subdivided into three groups: the Modified Sectional Truss Model Approach, Truss Models with Crack Friction, and the Fixed-Angle Softened Truss Model. The Modified Sectional-Truss Model Approach is the one used by Eurocode No. 2 (1991) and suggests that the total shear capacity is the sum of a concrete and a steel contribution. In Eurocode No. 2 the concrete contribution is based on empirical formulas with an upper limit that prevents web crushing. Truss Models with Crack Friction rely on forces being transferred across the cracks for their concrete contribution. The amount of force that can be transferred across the crack depends on the crack width and displacement. Although not mentioned in the report, Loov (1998) has developed a shear friction approach for beam design, which has subsequently been modified for FRP by Deniaud and Cheng (2001 and 2003). Finally, the Fixed-Angle Softened Truss Model (Pang and Hsu 1992, 1996; Zhang, 1995) is similar in approach to the RA-STM except that the crack angle is assumed to be fixed. Because the model by Deniaud and Cheng has already produced promising results for the capacity of FRP retrofitted T-beams, it will be used to represent this group.
The final group contains strut and tie models. These models are generally used to visualize the flow of forces in a member and resolve them using equilibrium. In fact, truss models are one form of strut and tie model. They are especially helpful for the design of members in regions where beam theory does not apply. However, they have three significant disadvantages. One is that they only produce element forces. Thus in order to determine the force in the straps, a trial and error approach may be required to determine the force sharing between the internal stirrups and the straps based on strain compatibility. Another disadvantage is that strut and tie models often involve a complicated series of checks (CSA 1994) which make them cumbersome for everyday design. However, most importantly, strut and ties models are based on a plasticity approach, which is acceptable when ductile materials are used and load can be redistributed. However, since the CFRP straps are brittle, any redistribution could result in failure of the strap. For these reasons, strut and tie models will not be considered for design of the strap system at this time.

The ASCE-ACI report does not directly address some of the available models for shear design. One such model is the Compressive Force Path model developed by Kotsovos (1988). This model suggests that a zone of compression along the beam carries the majority of shear. Mechanisms such as aggregate interlock and dowel action play a much smaller role in the shear capacity than is assumed in other models such as the MCFT. Because of its entirely different approach to the calculation of shear capacity, the Compressive Force Path approach will be considered in greater detail.

Based on the preceding discussion, three models will be investigated further and used to model the retrofitted specimens in Chapter 5: the MCFT, the Compressive Force Path method, and Deniaud and Cheng’s shear friction approach.

2.3 Shear Retrofitting with Fibre Reinforced Polymers

As mentioned in Chapter 1, increases in loading as well as poor design practices and code changes have meant that many existing structures require additional shear capacity in order to remain in service and avoid loading restrictions. Traditionally these capacity increases have been provided by using materials that have well-
understood properties such as steel plates. The disadvantage of steel is that it is heavy and susceptible to corrosion. Fibre Reinforced Polymers (FRPs) on the other hand are light and not susceptible to corrosion. However, their material properties are not as well understood as steel and their lack of ductile behaviour also requires a cautious approach to their use.

2.3.1 Externally Bonded FRP

According to Triantafillou (1998), the first attempt at retrofitting RC beams in shear using FRPs was undertaken by Berset (1992). Berset epoxy-bonded Glass FRPs (GFRPs) to the sides of shear-critical beams. He was able to achieve a 66% increase in capacity over an unretrofitted specimen using these sheets on beams with no internal steel shear reinforcement and a 20% increase in capacity using beams with internal steel reinforcement. Unfortunately the size of his specimens, at just 114mm deep, meant that size affects were an issue as noted by Berset himself. Interestingly, he also stressed the need to ensure that the retrofit tied the compression and tension chords together as effectively as possible, and suggested this could be a problem with sections such as T-beams. Since then a significant amount of research has been undertaken using FRP sheets to retrofit rectangular beams by researchers such as Triantafillou, Pellegrino and Modena (2002), Adhikary et al. (2003) and Li et al. (2003) among others. The amount of shear force that can be transferred to the beam using a bonded system is dependent on the strength of the bond, the anchorage length, and the FRP strength and stiffness. If the anchorage length is insufficient, the FRP retrofit will delaminate before the ultimate capacity of the FRP is reached. Such was the case with the specimens tested by Triantafillou where CFRP fabrics were bonded only to the sides of the beams and failure in each case was in shear following debonding of the fabric. However, his retrofitted specimens still attained capacity enhancements of 65 to 95% over his control specimens. Pellegrino and Modena tested a series of 11 rectangular beams with CFRP fabrics bonded only to the side of the beams to gauge the impact of the presence of internal steel reinforcement on the effectiveness of the FRP enhancement. They found that in beams with internal steel links, the links had a negative impact on the bond between the sheets and the concrete by causing a more elaborate crack pattern to form than would be the case in a section with no internal steel shear links. This elaborate crack pattern caused a greater
proportion of the bond between the sheet and the concrete to break down, resulting in shorter bond lengths and reduced FRP effectiveness. However, anchorage-length issues can be mitigated if the FRP laminates are wrapped around the full cross section. This was investigated by both Adhikary et al. (2003) and Li et al. (2003) who employed a ‘U’ configuration where the sides and bottom of the beams were wrapped and a fully-wrapped configuration as illustrated in Figure 2.2. Adhikary et al. obtained capacity enhancements over unretrofitted specimens of 47% using CFRP fabrics in a ‘U’ configuration where failure resulted from debonding of the fabrics. When the fully wrapped configuration was employed, the capacity enhancement increased to 123%. Similarly, Li et al. obtained capacity increases of 30% using the ‘U’ configuration versus 61% for fully wrapped specimens. Once again, failure of the specimens using the ‘U’ arrangement was the result of debonding whereas rupturing of the CFRP fabric caused failure for the fully wrapped sections. Thus, while bonding to the side of the beam can offer good shear enhancement, a better solution is to fully wrap the section and eliminate potential debonding of the FRP. Unfortunately many typical structures do not use rectangular beams, nor is it very easy to access the top surface of such beams in order to fully wrap them.

A more realistic approach to simulate many structures is to use a T-beam since it more closely resembles slab-on-beam construction. Work has been undertaken to
investigate the potential for shear retrofitting T-beams using bonded FRPs by researchers including Chajes et al. (1995), Bousselham et al (2004), Deniaud and Cheng (2001 and 2003), Melo et al. (2003) and Czaderski (1998). Unlike rectangular sections, access to the full depth of the beam is made more difficult by the presence of the flange. Chajes et al., Bousselham et al. and Deniaud and Cheng wrapped only the web portion of the beams using FRPs. It is interesting to note that almost all of their specimens failed in shear when tensile cracks began to propagate through the flange. An exception to this were specimens tested by Bousselham et al. that contained internal shear links at a spacing of $d/4$ (where $d$ is the effective depth) that penetrated the flange of the beam and potentially prevented failure in the flange. Melo et al. tested T-beam specimens where both the ‘U’ and fully wrapped configurations were used as illustrated in Figure 2.3. Whilst specimens employing the ‘U’ configuration achieved a 17% increase in capacity over an equivalent unretrofitted specimen, the fully wrapped specimens achieved a maximum increase in capacity of 84%. However, they noted that wrapping the web and flange was “quite laborious and messy.”

Research by Czaderski used L-shaped CFRP plates as opposed to wrapping the beams with fabrics. The plates were bonded to the beam surface using structural adhesive and in order to develop the full capacity of the plates, they were anchored into the flange of the beam as illustrated in Figure 2.4. Whilst Czaderski did not test any control specimens, he was able to achieve flexural failure in several beams with minimal internal shear reinforcement. The preceding research suggests that there is a correlation between penetrating the flange of a T-beam specimen, and being able to achieve a full shear enhancement. Whilst it is possible to use CFRP fabrics to fully wrap the section, this would require access to the top surface in order to be able to achieve the required bond.

![Figure 2.3 - 'U' and fully wrapped configuration used by Melo et al. (2003)](image-url)
2.3.2 Near Surface Mounted FRP Rods

De Lorenzis and Nanni (2001) have investigated the use of Near Surface Mounted (NSM) FRP rods. Their installation technique involves cutting a square groove into the side of a concrete beam and in some specimens drilling holes into the flange as illustrated in Figure 2.5. The groove is partially filled with epoxy, the FRP rod is pressed into the epoxy and the groove is then completely filled with epoxy. De Lorenzis and Nanni’s experimental series of 8 T-beam specimens investigated four variables: rod spacing, rod angle, rod length, and use of internal steel stirrups. They found that rod spacing had the least impact on capacity, however, the rod spacings they tested (127 and 178mm) were considerably less than the effective depth of 250mm and so a larger spacing may have had more impact. Changing the rod angle from 90° to 45° created a significant increase in capacity but the largest capacity increase for the specimens without internal stirrups was achieved when the rods were embedded in the flange. Although not embedding the rods in the flange still led to a capacity increase of 41% over the unretrofitted specimen, a 106% increase was achieved when the rods were fully embedded in the flange at the same spacing. Although the NSM FRP rods achieve significant capacity increases, the epoxy takes 15 days to reach full cure, which could lead to delays in terms of facility use. Also, for the rods that were not anchored in the flange, failure was due to splitting of the epoxy.
Even for those rods that were anchored in the flange, there was noticeable slipping of the rods in the epoxy during the tests. The next section describes a retrofitting technique where epoxy bonding issues no longer govern.

![Diagram of beam with FRP rods](image)

Figure 2.5 - Near surface mounted FRP rods

### 2.3.3 CFRP Straps

Most methods of FRP shear reinforcing generally require that the FRP be bonded to the concrete surface using epoxy, however another method of shear retrofitting was proposed by Winistoerfer (1999) who developed a CFRP tape system. Winistoerfer carried out a series of tests whereby CFRP tape was wrapped around metal pins of varying diameters and tested in axial tension. His work helped to establish the ideal CFRP material, the minimum radius through which this material could be bent and a method of welding the tape to itself. The advantage of this system when applied to concrete beams is that it does not need to be bonded to the concrete. Instead, layers of thin CFRP tape are wrapped around the beam. The tape is supported on metal pads placed on the top and bottom faces of the beam as illustrated in Figure 1.1. The outer layer of tape is then fused to the next tape layer by melting the thermoplastic matrix material of the two layers together. This outer loop closes the system and the inner loops move freely until friction between the tape and the pads causes them to tighten. In his work, Winistoerfer discovered that the strains in each layer of tape were quite similar and thus each layer of the tape was carrying the load fairly evenly. Tests using
the same amount of tape but with all the layers completely laminated to one another produced straps with significantly lower capacities. This was due to development of through thickness stresses within the laminate causing premature failure. If the strap is installed so that it bends at a radius greater than or equal to an established minimum, premature rupturing due to stress concentrations can be greatly reduced. In his work, Winistoerfer found that this minimum radius was 12.5mm, and that there was no significant reduction in strap capacity if the radius was kept above this value. Winistoerfer et al. (2001) suggest that up to 50 loops of strap can be used before the force provided by each additional loop no longer provides a linear capacity increase. Additional loops beyond 50, while increasing the capacity, cause excessive through thickness stresses to be developed, which results in a decrease in the capacity provided by additional loops. The tensile capacity provided by a 50 loop strap is 250kN, which would represent a significant contribution to the shear capacity of any beam. A final advantage of these straps is that they can be prestressed. The prestressing process will be described in Chapter 3. It allows for an initial tensile stress to be placed in the strap before the beam is loaded. This ability to prestress the straps is especially advantageous for deep beams as will be discussed in Chapter 5.

2.3.4 Previous Experimental Work using CFRP Straps

This CFRP tape system has been applied to beams by Chan (2000), Lees et al (2002), Stenger (2000) and Kesse (2003). The tape used in these studies, as well as in the present study, was a 12mm wide, 0.16mm thick CFRP tape provided by Switzer, Switzerland. The work performed by both Chan and Lees et al. used these straps to retrofit shear-deficient T-beams. The straps were supported on steel pads on the top and bottom of the beam. The control specimens failed in shear but after strengthening the beams with CFRP straps, the retrofitted specimens attained a significant increase in shear capacity over the control specimen. Chan saw a 48% increase in ultimate load carrying capacity as well as a change in failure mode from shear to flexure. Lees et al. tested 2 strengthened specimens. One failed in flexure due to yielding of the longitudinal steel with an increase in capacity of 38% over the control specimen while the other failed due to strap rupturing with an increase in capacity of 33%. These results suggest that retrofitting concrete beams with CFRP straps is an extremely promising approach.
Kesse (2003) has done the most extensive work on the use of these CFRP straps to date. His research also employed steel pads on the top and bottom of the beam to provide support for the straps. The tests in his program were performed on rectangular specimens, thus avoiding the confusion of the additional capacity provided by the flange of the T-beam as will be discussed later. The main variables in his testing program were the number of strap loops (5 or 10), the spacing of the straps (230 or 345mm) and the prestressing force (5, 25 or 50% of the ultimate strap capacity). It was concluded that even with a strap spacing of 345mm (i.e. one strap), 5 loops, and an initial prestressing force of approximately 11.5kN (i.e. 50% of ultimate), a 55% capacity increase over an equivalent unstrengthened beam was possible. However, in order to change the failure mode from shear to flexure, a strap spacing approximately equal to \( d \) was required. Also, the number of loops played a role since using just 5 loop straps at this spacing, while increasing the capacity, did not prevent a shear failure. The 10 loop straps, on the other hand, were able to provide the right combination of additional prestressing force and stiffness to keep the dominant crack from opening and carry enough shear force resulting in a flexural failure.

2.4 Modeling of Shear Enhancement due to FRPs

There are numerous models available for specimens retrofitted with FRPs but in the words of Triantafillou (1998): “[t]he analytical models proposed in the literature are almost as numerous as the studies from which they came, and are in most cases contradictory.” Indeed, models to predict the shear capacity enhancement due to FRPs have been proposed by Berset (1992), Chajes et al. (1995), Adhikary et al. (2003), Li et al. (2003), Deniaud and Cheng (2001 and 2003), Chen and Teng (2001), Khalifa et al. (1998), De Lorenzis and Nanni (2001), Triantafillou and Antonopoulos (2000) as well as Triantafillou, and this is only a small sample.

Berset bases his model on the truss analogy. The FRP component uses a limiting strain term for the FRP, which Berset indicates is necessary since the full FRP capacity will not be mobilized without excessive deformations in the beam. He then presents a model for ensuring that failure will not occur due to debonding of the FRP. Unfortunately, the critical strain outlined by Berset is based on localized strains in the
FRP, which would not develop in the CFRP straps, as they are not bonded to the surface. For the same reason, the bond strength between the FRP and the beam is not applicable to the straps, making Berset’s model rather impractical for the straps.

Chajes et al. provide a straightforward model that involves an FRP term similar to the steel term in most codes. The FRP contribution is based on a limiting strain term in the FRP of 0.005. Although the model is appealing having been developed for T-beams, it is based solely on their experimental results and they acknowledge the need for a more extensive database for verification.

Triantafillou proposes another method similar to current codes whereby the FRP provides a shear contribution in addition to the concrete and steel. The FRP contribution involves an axial rigidity term expressed as the product of the FRP modulus and an FRP area fraction. The limiting strain is then a function of this axial rigidity. These limiting strain functions were derived from curve fitting of experimental results and apply to both failure due to debonding and rupturing of the FRP. Triantafillou and Antonopoulos have revised this model, acknowledging that linking debonding and tensile failures together was inappropriate. They have also created equations for fully wrapped and ‘U’ wrapped sections. As with other models, when debonding is ignored, the main design parameter becomes a maximum strain value to ensure that aggregate interlock can be developed, which is approximately 0.004. This limiting strain to account for the loss of aggregate interlock capacity has also been proposed by Khalifa et al. and was also assumed to be 0.004.

In general, since bond is not an issue with the CFRP straps, most of these models are essentially truss models with an FRP term limited by a maximum FRP strain of 0.004. Unfortunately it is possible that the straps will not achieve this strain. For example, the strap strains due to crack openings in deep beams remain relatively low as discovered by Stenger (2000) and so assuming a strain of 0.004 may lead to unconservative predictions, as discussed further in Chapter 5. Also, because the straps can be prestressed, they can alter the strain at which aggregate interlock becomes ineffective by forcing the cracks closed and reducing the crack widths. These two factors make it hard to apply this strain limit when using CFRP straps. However, it is difficult to determine the actual strain in the straps and so this strain limit may offer a
useful simplified approach, and is employed in one of the models compared in Chapter 5.

Even Chen and Teng who have developed a model specifically for use with CFRP strap retrofitted beams use a maximum strain term rather than calculating the actual strain. Although they have not calibrated the model against existing test data, it is fairly straightforward and involves a minimal number of terms. As such, it will also be examined further in Chapter 5.

2.5 Experimental Beam Design Philosophy

The goal of the current study is to develop a method of installing the CFRP straps from the underside of the specimen. The specimens used in this study had to have a shape that adequately simulates slab-on-beam construction. Also, in order to develop an experimental program to determine the effectiveness of the CFRP straps, the specimens need a measurable difference between the unstrengthened beam shear and moment capacity. Any enhancement provided by the CFRP straps would then be distinguishable from random variations in strength.

One such way of achieving this difference in capacity is to make deep rectangular beams with short spans. In this case the shear applied to the beam would be more significant than the applied moment. The ratios of span-to-depth required to create shear critical beams were developed by Kani (1964) and are known as Kani’s valley. This valley consists of a dip in the plot of shear strength versus shear span-to-depth ratio where beams with shear span-to-depth ratios greater than 1 but less than 7, reaching a minimum (where the actual flexural capacity of a beam with no shear reinforcement is approximately 50% of the cross sectional flexural capacity) at a ratio just above 2. However, deep rectangular beams are not a realistic model of the slab on beam type construction examined in the current study.

Another way to increase the moment capacity is to use a T-beam section. In this case the flange provides additional compressive capacity, which in conjunction with an increased amount of tension steel will raise the moment capacity. Current design codes (BS 8110 1997 and CSA 1994) would suggest that the shear capacity of the
section is unaffected as it is based only on the web width. Thus, the difference in applied load to cause the two types of failure would be increased. However, the compression flange does enhance the shear capacity of the beam as discussed in the next section and so this will decrease the difference. From the point of view of developing an under-slab installation technique, the T-beam cross section is ideal as it provides a realistic representation of slab on beam type construction.

It was decided that a combination of the two approaches would be used for the experimental specimens in this study. The span-to-depth ratio was kept within Kani’s Valley while also employing the T-beam section required to properly develop the installation technique. The specimen design was based on that used by Chan (2000). A T-beam cross section with a longitudinal reinforcement ratio, ρ, of 4.7% was used as illustrated in Figure 2.6. The beams were loaded in 4 point bending as illustrated in Figure 2.7 with shear spans of 750mm resulting in an a/d ratio of approximately 3.33. The internal steel shear links consisted of 6mm bars placed at 250mm spacing.

![Figure 2.6 - Test specimen cross section](image)
This specimen design resulted in a 48% increase in retrofitted capacity over the unretrofitted control beam in Chan’s tests. This difference in capacities should be more than adequate to determine the effectiveness of the installation techniques developed in the current study. In order to obtain this difference, Chan used 10-loop straps with a prestress of 30kN (50% of the ultimate strap capacity). The same 10-loop straps will be used in the current study, as they should offer the required capacity. The level of prestress will be reduced to 15kN (25% of the ultimate strap capacity) based on the results of Kesse (2003) who found that this level of prestress still forced a flexural failure whilst increasing the amount of strap capacity available for carrying stresses due crack openings. The initial strap spacing will be 250mm, also based on Kesse’s results, which suggested that a spacing equal to the effective depth, \(d\), or less was required to force a flexural failure.

### 2.5.1 Shear capacity of T-beams

One problem with the use of a T-beam is that the shear capacity of such sections is difficult to predict. Whilst design codes suggest only the web of the beam contributes to the shear capacity, work done by Zoheary et al. (1998) has demonstrated that the flange significantly enhances the shear capacity of beams. Their study focused on the effect of the ratio of the flange depth to beam depth, flange width to web width, and also the influence of any flange reinforcement. The results of these tests indicated that the flange developed significant additional shear capacity. For a beam with a flange depth to overall depth ratio of 0.45 and flange width to web width ratio of 5, the
increase in capacity over a rectangular section was approximately 118%. Unfortunately, the study stops short of presenting models for estimating these capacity increases.

Tozser and Loov (1999) addressed this additional T-beam capacity when applying it to a shear friction approach (to be discussed further in Chapter 5). They proposed that the effective flange area for shear was bounded by 45° lines extending from where the flange meets the web to the top surface of the flange as illustrated in Figure 2.8.

![Effective Shear Area](image)

Figure 2.8 – Effective shear area as proposed by Tozser and Loov (1999)

### 2.6 Under-slab Installation Technique – Flange Penetration

In order to install the straps from underneath the slab surface, some method of passing the strap through the beam so that it can form a closed loop must be devised. The easiest way to do this would be to create a hole or groove in the beam horizontally across the web under the flange. However, this method may not offer effective shear enhancement based on the models discussed earlier. The truss analogy, for example, requires that the tension ties connect the tension and compression chords. If the straps do not penetrate the compressive flange, they will not tie the chords together and are likely to be ineffective. Previous research using FRPs has also demonstrated a correlation between flange penetration and the level of shear enhancement.

A report by Kani et al. (1979) also explores this notion of effective shear reinforcement length. It suggests that cracked reinforced concrete beams form a series of tied arches as illustrated in Figure 2.9. It further suggests that vertical stirrups (as
well as inclined stirrups and bent-up bars) serve to connect these arches together. The inner arches must be connected to, or “hung off”, the primary arch that spans between the support points. Using this analogy, failure occurs in compression when not enough arches are connected together to carry the required compression stresses in the beam. According to Kani et al., stirrups placed within a distance equivalent to the effective depth, \( d \), away from either the support points or the load points are ineffective. In the case of stirrups near the support points, they are ineffective because the primary arch loads directly into the supports and does not need to be supported by stirrups. Near the load points, the small arches that form in this area do not carry any compressive force as they lie below the neutral axis. Since these arches do not carry any force, there is no need to connect them into the primary arch.

The report gives details of a series of beam tests performed to determine the optimum placement of stirrups. By looking at beams with only one stirrup, placed at a variable distance away from the load point it was discovered that the shear capacity varied dramatically. A stirrup placed at \( 1.25d \) from the load point had the optimum effect, forcing a flexural failure for these particular specimens with an \( a/d \) ratio of 3. It was suggested that stirrups near the load point were often ineffective because they did not have enough anchorage into the primary arch. To explore this theory, another set of beam tests involving two different stirrup configurations placed within a region of length \( d \) from the load point were performed. Type A stirrups were regular closed loop bars. Type B stirrups consisted of U shaped bars anchored to a steel plate on the beam’s top surface. Type A stirrups produced huge variations in capacity with some specimens failing below the capacity of beams without web reinforcement. The beams
with well-anchored Type B stirrups produced consistent results with increases in capacity between 25 to 50% over unreinforced specimens, depending on the exact stirrup location. This is an important result as it suggests that stirrups, and CFRP straps, near the load point need to be anchored as close to the top of the beam as possible. The required depth of anchorage into the compression flange reduces closer to the support point as the depth of the primary arch increases. This result also has implications for the under-slab strap installation technique as it indicates that the straps will also have to encompass as much of the beam depth as possible near the load points. It also tends to validate the conclusion that attempts to shear retrofit T-beams with FRPs are most effective when the flange is engaged by the retrofit.

Work done by Regan and Reid (2004) looked at the effect of corrosion on the effectiveness of steel stirrups. They proposed that corrosion might well affect the top of the stirrup but leave the vertical legs intact. They tested a series of beams with varying amounts of available stirrups, where in the most extreme case only vertical bars were present. They discovered that even with only vertical bars, the reduction in load carrying capacity was only between 14 to 33%. The beam’s ability to carry shear was largely dependent on the bond available between the concrete and the stirrup above and below the shear crack. This suggests that the straps are an ideal solution as they are mechanically anchorage at both ends, which should allow them to develop their full capacity across the crack. However, this work also indicates that the straps would only be effective if they were able to encompass the shear crack.

Based on these studies it can be seen that strap penetration into the compressive flange will be crucial in terms of developing an effective shear enhancement. In the next chapter these studies will be used as a starting point for the development of an under-slab installation technique. The more practical aspects of this technique will also be dealt with, such as how to form a passage in the beam through which to pass the strap, and how to support the strap within the beam.
Chapter 3
Under-slab Strap Installation: Development and Testing

This chapter summarizes the development of the under-slab installation technique. There were two main aims for this development program: to find a way to pass the strap through the beam whilst maintaining the strap’s tensile capacity and to be able to gain the maximum shear enhancement for the beam using the straps. As such, two testing programs were undertaken. First, a series of tension tests were performed on the CFRP straps. These tests helped to determine which methods of wrapping the straps around the beam were viable both in terms of practicality as well as ensuring sufficient tensile capacity in the straps. The initial background for these tests will be presented, followed by an examination of various aspects of the test set-up. The results of these strap tests are then given and discussed. However, being able to develop the full tensile capacity of the straps will not translate into maximum shear enhancement for beams unless the straps have an effective configuration within the beam. As such, a series of eight RC T-beam tests was undertaken using the results of the strap tests as the starting point. The eight test specimens are introduced along with their specifications and the test set-up. The beam tests were a progressive development process where the results of one beam test led to the strap configuration used in the next beam. This process is summarized while examining some of the critical results of each beam test. A discussion on the stiffness of the specimens and the transverse reinforcement performance follows. The chapter finishes with conclusions on the results of both test series.

3.1 Strap Tests

As discussed previously, in order to form closed loops with the CFRP straps as well as to avoid having to access the top of the beam, some method of passing the strap through the flange of the beam section from underneath was required. The depth of penetration of the strap into the flange is also believed to be important to develop an effective shear enhancement. As such, hole orientations were needed that would tie the beam together effectively and develop the beam’s full shear capacity. Based on the current available technology, the most practical way of forming these holes was to drill them into the flange. The major limitation of this approach is that the holes can
only be drilled in a straight line, so to penetrate the flange effectively these holes must be drilled at angles into the flange. At the same time, having the straps sit directly in the holes with no other means of support may reduce the strap’s capacity, and so methods of supporting the straps also need to be examined. The following sections examine strap tests that were undertaken to develop effective hole configurations and methods of supporting the straps.

In order to gauge the effectiveness of both the hole drilling process as well as the hole orientations, a T-beam that had been used for Chan’s (2000) tests was used for these experiments. The dimensions of this beam were given in Figure 2.2. It was also necessary to establish a baseline capacity for the straps against which the capacities of other strap configurations could be compared. The baseline, or ‘Ideal’ strap configuration was considered to be that used by Chan (2000) and Kesse (2003) whereby the strap was supported on steel pads on both the top and bottom of the beam as illustrated in Figure 1.1. A series of strap tests using both 5 and 10 loop straps were carried out using this configuration, the results of which are presented in section 3.3. As a second point of comparison, the theoretical maximum capacity of each strap was also determined. This capacity was based on the tensile strength as quoted by Winistoerfer (1999) of the Toray T700S CFRP fibres used in the CFRP straps, which in this case was 4900MPa. This strength was multiplied by the area of fibres in the strap, which was calculated by multiplying the area of each tape (12mm by 0.16mm) by the fibre volume fraction of 56%. The capacity of the matrix material, PA 12, was assumed to be negligible. The resulting theoretical capacity per layer of tape was 5.28kN. This value was then multiplied by the number of tape layers (10 or 20) to get the strap capacity, which is also presented in section 3.3.

3.1.1 Hole Orientation

The basic hole diameter was chosen to be 25mm. This was felt to be the minimum diameter that would allow for the threading of the 12mm wide tape through the beam as well as for access to the hole. There were numerous possible drill hole configurations. However, the initial strap tests considered the two hole layouts illustrated in Figure 3.1.
Figure 3.1 – Hole layouts for strap tests

(a) 45° Hole Configuration  
(b) 30° Hole Configuration

Figure 3.1(a) shows the 45° hole configuration. The advantage of this configuration was that it penetrated more of the compression flange than the alternative 30° configuration shown in Figure 3.1(b). This made it more likely that the two chords of the truss would be effectively tied together, and that Kani’s primary arch would be penetrated.

The 30° configuration was developed in an attempt to minimize possible stress concentrations in the strap since the strap would turn through 120° as opposed to 90° angles. A modification of the 30° configuration used in the beam tests (section 3.7) is shown in Figure 3.2. This configuration encompassed a significantly larger portion of the compression flange. Once again, this would be beneficial based on most shear models. However, the major drawback of this design was that the holes reduced the area of the compression flange by 35%, which could significantly reduce the shear capacity.
3.2 Test Program

3.2.1 Drilling the Holes

Holes were drilled into the beam, while it was seated upside down, at 30° and 45° using a 25mm diameter drill bit and a rotary hammer drill. In order to ensure that the holes were drilled at the correct angle a rig was used, as seen in Figure 3.3, which clamped to the web of the beam. The drill bit was passed through a circular tube in the rig that was set to the desired angle.

Figure 3.3 – The drill rig
3.2.2 Inserts
3.2.2.1 Types of Inserts

Because of the need to avoid the strap bending below the minimum radius recommended by Winistoerfer that could lead to premature failure of the strap, the possibility of using inserts or other means of support in the holes was investigated. For the strap tests, the approaches used were: nothing in the hole, PVC pipe in the hole, dental plaster inserts, grout inserts, and steel inserts.

3.2.2.2 Manufacturing the Inserts

The dental plaster inserts were made using a long straight mould with a semi-circular cross section. Once the plaster had hardened, the inserts were cut to the appropriate length for the hole angle. Sanding the ends then created the end radii. Figure 3.4 illustrates the standard shape of the inserts. The end radii of 12.5mm was based on Winistoerfer’s work, which indicated that the strap could still reach its maximum capacity at this curvature. The complex shape of the inserts meant that forming them using a mould would be the easiest manufacturing approach. Since dental plaster was relatively cheap and could easily be poured into a mould, it was selected as a preliminary insert material.

The grout inserts were also manufactured using a long semi-circular mould. However, to achieve the required end radii, steel end pieces were fabricated to sit in the semi-circular groove in the mould. Thus, once the grout had hardened in the mould, the
resulting piece was the required shape. Some sanding was required to remove any imperfections.

The steel inserts were machined to the profile shown in Figure 3.4.

3.2.3 Installing the Inserts

The procedure varied slightly depending on the insert material. In the case of the dental plaster and grout inserts, a thin bead of mastic was placed on the insert and the insert was placed in the hole. The mastic was sufficiently strong to hold the inserts in place after only a few seconds. Plastic padding was used to hold the steel inserts in place, as the mastic did not have enough bond strength to support the weight of the steel inserts.

3.2.4 Installing the Straps

Once the holes were drilled and the inserts installed, a 20mm wide aluminium strip that was tapered and bent at one end was inserted through the hole to act as a guide. An initial 500mm long piece of CFRP tape was then pushed through the hole. The bend on the aluminium strip meant that the tape was forced around the corner where the two holes joined and did not get caught against the concrete. The 500mm piece of CFRP served as a guide that allowed layers of the actual strap to be passed through the holes with relative ease and the aluminium strip could be removed.

With the guide in place, a length of CFRP tape was cut depending on the numbers of loops required for the strap. Five metres of tape were required for five loops while nine metres were required for ten loops. The CFRP tape was then passed through the holes and one end was affixed to the beam using electrical tape. The CFRP tape was then wrapped over the metal pad on the surface of the beam and into the hole on the other side. This process was repeated until the desired number of loops was attained. Unfortunately, since the welding apparatus (Section 3.2.5) had a tendency to crease the CFRP tape, a fair amount of slack had to be left in the system to ensure the tape was not damaged. Although this was not crucial for the tensile strength tests, excessive slack would have been undesirable for actual beam applications and thus
was avoided in the beam tests by modifying the length of the welding apparatus. Also, if the end of the tape in the innermost loop and the weld occurred in the same place, this resulted in a section of the strap where only four or nine layers of strap were present as illustrated in Figure 3.5(B). As such, the strap was actually wound around the beam either six or eleven times to ensure that complete loops were formed.

![Diagram of correct and incorrect strap installation](image)

(A) Correct Installation  
(B) Incorrect Installation  

Figure 3.5 – Minimum number of loops

### 3.2.5 Welding the Straps

With the desired number of loops wrapped around the beam, the outermost tape layer was then welded to the next outermost layer. This closed the system and allowed the strap to develop its tensile capacity. In order to form the joint, the outer two layers of strap were separated from the other loops. A 14mm wide and 98 long strip of thin polymer film was inserted between the two layers of CFRP. The polymer film was made from the same thermoplastic material that composed the matrix of the CFRP tape and was used by Winistoerfer to overcome bond problems that were experienced when attempting to weld the two sections of CFRP tape directly to one another. The lower layer was then placed against the bottom plate of the welding apparatus so that the joint section ran down the middle of the apparatus. The top plate of the apparatus was then placed on top of the top layer and the two plates were held together using bulldog clips as illustrated in Figure 3.6. The welding apparatus was then plugged into
a thermocouple gauge and into the electrical power supply. In his work, Winistoerfer discovered there was very little difference in joint capacity as long as the thermoplastic material, which is present in the matrix of the CFRP as well as in the thin film between the tapes, was heated above 175° Celsius (this particular thermoplastic has a melting temperature between 172 and 178°). However, in order to ensure full bond the apparatus was heated to a temperature of 200° Celsius. The apparatus was allowed to cool down to 180° and then heating recommenced by reconnecting the electricity supply. The process was repeated once more before the entire apparatus was allowed to cool down to 50°, at which point the bulldog clips were removed and the welded strap was taken out of the welding apparatus.

![Diagram](image.png)

**Figure 3.6 – Welding apparatus**

### 3.2.6 Installing the Prestressing/Loading Apparatus

The prestressing/loading apparatus is shown in Figure 3.7. The rounded steel plate that supports the strap sat in a groove in another steel plate which had four tapped holes drilled in the corners. Threaded rods were screwed into these holes and used to connect this plate to an upper plate that had a fifth hole through the centre. A threaded rod was passed through this hole and bolted underneath the top plate. The other end of the rod was screwed into the bottom of a 100kN load cell. A second threaded rod was screwed into the top of the load cell. A steel plate supported on two C-channels served as a reaction frame against the concrete beam. A hydraulic jack was seated on top of
the reaction frame. The threaded rod coming out of the top of the load cell passed through a hole in the reaction frame as well as a hole in the centre of the hydraulic jack. A steel plate with a hole in it was placed on top of the load jack. A bolt was then screwed down the threaded rod until it came in contact with the steel plate. In this way, once fluid was pumped into the hydraulic jack, the piston in the centre would apply load to the threaded rod through the steel plate and bolt. This load would be transferred to the strap through the threaded rods while being read by the load cell.

![Figure 3.7 - Strap loading/prestressing rig](image)

3.2.7 Testing the CFRP Straps

A data acquisition unit attached to the load cell allowed the current and maximum loads to be viewed. The straps were tested until failure and the maximum load was noted.

3.3 Strap Test Results

3.3.1 5 Loop Straps in 30° Holes

A series of strap tests was carried out using the 30° hole configuration seen in Figure 3.1(a). For each test, 5 loops of CFRP tape were wrapped around the beam. The main
variable in each test was the type of support provided to the tape. Three tests were conducted for each configuration. Figure 3.8 gives the results for these strap tests.

The lines above each material represented the bounds on strength. The upper bound was calculated as the average maximum tensile load plus 3 times the standard deviation. Similarly, the lower bound was the average minus 3 times the standard deviation. Statistically, this formed the 99% confidence interval. In other words, only 1% of all strap strengths would fail outside this range.

It should be noted that the theoretical capacity of the strap is considerably higher than even the “Ideal” capacity of the strap. This is due to a combination of material imperfections and stress concentrations caused by the pads that lead to premature failure before the theoretical value is reached as discussed by Winistoerfer.

The easiest way to install a CFRP strap would be to put nothing in the holes to support it. This would require both the minimum amount of labour and materials. However, it can be seen that while having nothing in the hole to protect the strap against the concrete actually gave quite consistent results (as shown by the small bounded area), the average capacity, as indicated by the tick, was very low. This was
because of the sharp edges where the holes met and also where the holes entered the beam. These edges served to tear the strap into two pieces. Thus, the load capacity of the strap was only 28% of the “Ideal” and 15% of the theoretical capacity. This low capacity would have to be offset by either increasing the number of layers or by using a conservative safety factor. Neither of these alternatives would result in a cost effective solution.

Since the main cause of failure when nothing was used to support the strap was tearing of the strap against the sharp concrete edges, the next choice was to put PVC tubing in the hole to protect the strap. PVC tubing was relatively cheap and could be cut to length on-site. Indeed, using PVC tubing to protect the strap from shearing had a beneficial effect in terms of strap capacity. However, while the straps did achieve 80% of the “Ideal” capacity, the effort required for installation made this configuration undesirable. Each strap had to be fed through the PVC tubing using a string attached to the previous loop of the strap. This process was extremely time consuming. Also, it was very difficult to keep the matrix, which holds the CFRP fibres together, from breaking due to the curvature of the tube. The straps tended to fail progressively because they were curved along their transverse axis by the tube. Each individual strand of carbon fibre was subjected to a different strain, experienced a different stress and ruptured at a different load. While this method offered a ‘pseudo’ ductility, the premature failure of some strands meant that the “Ideal” capacity could never be reached. Therefore, an insert should not only protect the strap from the sharp edges of the concrete, but also eliminate transverse curvature in the strap.

The straps supported by the dental plaster inserts reached about 88% of the ideal capacity with an average range of results. However, the dental plaster lacked the stiffness and strength required to support the straps as during the tests the strap would bed down into the dental plaster. This crushing of the dental plaster was beneficial in that it protected the strap from premature failure by giving the strap a smooth bed to react against as opposed to the sharp edges of the concrete. However, it also allowed the strap to extend significantly without straining. On an actual beam, this would allow the shear cracks to open, possibly to the point of failure, before the straps became effective. Cylinder tests on the dental plaster showed that it had a
Compressive capacity of approximately 5MPa with a standard deviation of 37%. The strength was highly dependent upon how much water was used in the mix. A stronger, stiffer material was required.

Grout inserts were an obvious progression from dental plaster. The grout offered a much higher compressive capacity, of the order of 43MPa and yet could still be poured into a mould. The standard deviation of the strength was only 7.8%, which was a direct result of the much more consistent mix proportions. Despite the additional compressive capacity, the grout inserts actually gave lower strap capacities than the dental plaster inserts. The straps only attained 83% of the “Ideal” capacity and 45% of the theoretical capacity. This lower capacity was due to the failure of the insert where it projected outside the edge of concrete beam as seen in Figure 3.1. This overhang, which was necessary to prevent the strap from touching the sharp edges of the concrete and causing a tearing failure of the strap, forced the insert to act as a cantilever. Thus, the tensile stresses caused by bending led to the insert cracking. Once cracked, it was no longer able to keep the strap away from the beam face and failure ensued. The grout did appear to be stiff enough, as there was very little evidence of the strap bedding down into the insert. If the tensile forces could be reduced, or eliminated, the grout showed promise as an insert material.

In order to be sure that the idea of using inserts in the holes had merit in terms of enabling a high strap capacity to be reached, steel inserts were investigated. The possibility of corrosion and the cost of manufacture may preclude the use of steel for actual applications. However, steel’s high strength, both in tension and compression, as well as its stiffness made it ideal for testing the insert concept. The results were quite promising in terms of both the strap capacity and variability of results. The straps achieved 94% of the “Ideal” capacity. Thus, the insert concept had merit, provided the right material could be selected.

3.3.2 Test Comparisons

The results of the 10 loop straps in 30° holes, 5 loop straps in 45° holes, and the 10 loop straps in 45° holes are presented in Figures 3.9 through 3.11. The tests results
were quite similar to the 5 loop straps in 30° holes and will not be discussed in detail. Only the salient points of comparison will be discussed.

Figure 3.9 – Results of 10 loop straps in 30° hole tests

Figure 3.10 – Results of 5 loop straps in 45° hole tests
3.3.3 Insert Materials

The relative performance of each material remained similar throughout the tests. There were slight deviations, which for the most part were probably due to the sample size of each test. In the case of the 10 loop straps in the 30-degree holes, the steel inserts had a higher lower limit value than the "Ideal" configuration. This validates the result presented by Winistoerfer that radii of 12.5mm and above do not have a detrimental effect on strap capacity. Since the pads used in the previous experimental work, as well as in the prestressing/loading apparatus in these tests, had a minimum radius of 20mm, this result had not been validated until this point. Based on the test results, steel would be the ideal insert material while dental plaster and grout could also be used, possibly with more conservative safety factors. However, since the dental plaster crushes significantly, it would not be appropriate for beam applications where minimizing strap displacements would be a concern. If the tensile stresses could be eliminated, grout could work well as an insert material.
3.3.4 5 Loops versus 10 Loops

There was no generally discernible trend between the number of layers and the strap capacities. In some instances the strap capacity per layer increased if the number of layers was increased whereas other times it did not. Part of this inconsistency can be attributed to the size of the statistical sample. However, it was obvious that the materials had similar relative performances with 5 and 10 loops. Indeed, Winistoerfer's findings are validated by the results for the steel inserts and the “Ideal” configuration, since the capacities of the 10 loop straps were almost double that of 5, which suggested that each loop of the strap was taking the same amount of load creating a linear increase. Based on these linear increases, the optimum number of loops will be more dependant on how many loops are required to achieve the desired shear capacity of the beam and not on the hole configuration. However, when designing a beam using a specific number of loops, consideration should be given to whether the beam can support the increased local stresses created by the force in the strap.

3.3.5 30° versus 45° Holes

There was a more consistent trend between the capacity for the 30° holes and the 45° holes. It seemed that the 30° holes offered better strap capacity than the 45° holes. This was certainly true for the PVC tubing, and perhaps more importantly for the steel inserts as well. In the case of the PVC tubing, one possible reason for this was the angle that the strap was required to turn at each corner. In the case the 30° holes, the strap was always required to bend through an angle of 120°. For the 45° holes, the strap had to bend through a much tighter angle of 90° where the two holes met.

When steel inserts were used, the strap capacity as a percentage of the “Ideal” dropped from 94% and 101% for 5 and 10 loops in the 30° configuration to 85% and 96% respectively in the 45° configuration. However, it was more difficult to explain why the hole angle made a difference for the steel inserts. Since the straps rested on the inserts, and not on the concrete, the straps would always have one of two radii. The straps would either have had an infinite radius when sitting on the flat portion of the inserts and when spanning between supports or a radius of 12.5mm when passing
over the curved section of the insert. This would have been the same regardless of the angle of the holes. Thus, there was no decrease in radius to promote stress concentrations between the two configurations. However, the stress concentration problem is quite complex and is dependant upon a number of factors including stress changes due to friction against the support surface and the stress distribution through the layers of the strap. Further research into this area is required, however, the final strap configuration for the beams was further modified as will be discussed later in this chapter, alleviating this potential problem. As such it will not be discussed further here.

3.4 Strap Test Conclusions

The strap tests have shown that the same strap capacities that were obtained using support pads on the top and bottom of the beam can also be achieved when the strap is passed through the flange from the underside of the beam. As long as the strap cross section maintains a flat profile and the strap is kept from bending below its critical radius, significant strap capacity can be developed. These strap configurations can now be implemented on shear deficient beams to gauge their effectiveness in terms of shear enhancement. Because the stiffness of the support pad was critical in terms of minimizing unstressed strap displacements, steel pads will be used for the initial beam tests to ensure maximum stiffness.

3.5 Beam Tests

The beam test series consisted of eight reinforced concrete T-beam specimens using the cross section discussed in Chapter 2. Except for the control beam, B1/25, each specimen has a four part designation given as AA/BB/C/DD where AA is the number of the beam test, BB is the angle of the holes in the beam, C indicates whether the holes were left hollow (H), grouted (G) or cast-in place concrete (C) and DD indicates the cube strength of the concrete used. In the case of the control beam the designation just gives the beam number and the concrete cube strength. Table 3.1 lays out the key test parameters for each specimen indicating whether the beam was retrofitted with straps, the hole angle, the condition of the holes, the concrete cube strength and the
width × length of the bearing pad. The reasons for the changes in testing parameters will be discussed in section 3.8.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Retrofitted</th>
<th>Hole Angle</th>
<th>Holes</th>
<th>$f_{cu}$ (MPa)</th>
<th>Bear. Pad (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1/25</td>
<td>No</td>
<td>-</td>
<td>-</td>
<td>24.8</td>
<td>100 × 100</td>
</tr>
<tr>
<td>B2/45/H/20</td>
<td>Yes</td>
<td>45°</td>
<td>Empty</td>
<td>19.8</td>
<td>100 × 100</td>
</tr>
<tr>
<td>B3/30/H/22</td>
<td>Yes</td>
<td>30°</td>
<td>Empty</td>
<td>22.3</td>
<td>100 × 100</td>
</tr>
<tr>
<td>B3/30/Hb/27</td>
<td>No</td>
<td>30°</td>
<td>Empty</td>
<td>26.8</td>
<td>100 × 100</td>
</tr>
<tr>
<td>B4/30/G/25</td>
<td>Yes</td>
<td>30°</td>
<td>Grouted</td>
<td>24.6</td>
<td>100 × 100</td>
</tr>
<tr>
<td>B5/30/C/27</td>
<td>Yes</td>
<td>30°</td>
<td>Concrete</td>
<td>26.7</td>
<td>100 × 100</td>
</tr>
<tr>
<td>B6/30/C/44</td>
<td>Yes</td>
<td>30°</td>
<td>Concrete</td>
<td>44.0</td>
<td>250 × 140</td>
</tr>
<tr>
<td>B7/30/G/36</td>
<td>Yes</td>
<td>30°</td>
<td>Grouted</td>
<td>36.1</td>
<td>250 × 140</td>
</tr>
</tbody>
</table>

Table 3.1 – Specimen parameters

The strap configurations used for each specimen are illustrated in Figure 3.12. Once again, the development of these configurations will be explained in section 3.8.
3.6 Specimens

3.6.1 Steel Reinforcement

The beams were reinforced with both longitudinal and transverse steel reinforcement. The reinforcement was detailed so that, without external reinforcing, the beam would fail in shear. The longitudinal reinforcement consisted of 16 and 20mm diameter bars in the bottom tensile region and 8mm diameter bars in the flange as seen in Figure 2.2. The transverse reinforcement consisted of 6mm diameter links placed at a spacing of 250mm along the beam.

The steel reinforcement was tested in tension in order to obtain both its strength and stiffness properties. Three tests were performed for each bar diameter. The results for these tests are given in Table 3.2. The stress-strain curves for the three 6mm reinforcement tests are illustrated in Figure 3.13, as these bars did not have a well-defined yield plateau.

<table>
<thead>
<tr>
<th>Bar Diameter (mm)</th>
<th>Yield Strength (MPa)</th>
<th>Yield Strain</th>
<th>Elastic Modulus (GPa)</th>
<th>Ultimate Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>578*</td>
<td>0.00501*</td>
<td>187.4</td>
<td>646</td>
</tr>
<tr>
<td>8</td>
<td>467</td>
<td>0.00233</td>
<td>200</td>
<td>540</td>
</tr>
<tr>
<td>16</td>
<td>505</td>
<td>0.00262</td>
<td>192.9</td>
<td>586</td>
</tr>
<tr>
<td>20</td>
<td>523</td>
<td>0.00263</td>
<td>198.7</td>
<td>633</td>
</tr>
</tbody>
</table>

* using the 0.2% offset method

Table 3.2 – Reinforcement properties
3.6.2 Concrete

Two concrete mix designs were used in the beam tests. The first concrete mix design was selected to give a low 14-day strength of approximately 25MPa that would realistically represent the concrete strengths found in structures on which this retrofitting technique would be used. This mix design was used for specimens B1/25 through B5/30/C/27 and is given in Table 3.3. A new mix design (based on that used by Chan) with a target strength of approximately 40MPa was used for specimens B6/30/C/44 and B7/30/G/36. This mix was used to alleviate the bearing problems described in section 3.8 and is given in Table 3.4.

<table>
<thead>
<tr>
<th>Material</th>
<th>% of Total</th>
<th>Mass required for 1 Beam Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 –10mm Gravel</td>
<td>43</td>
<td>163.4kg</td>
</tr>
<tr>
<td>Medium Sand</td>
<td>35.2</td>
<td>133.8kg</td>
</tr>
<tr>
<td>OPC</td>
<td>12.8</td>
<td>48.7  kg</td>
</tr>
<tr>
<td>Water</td>
<td>9</td>
<td>34.2 l</td>
</tr>
</tbody>
</table>

Table 3.3 – Concrete mix design for B1/25 to B5/30/C/27
Table 3.4 – Concrete mix design for B6/30/C/44 and B7/30/G/36

Table 3.5 presents the material properties for the concrete used in the eight beam tests. The table includes the compressive cube strength, $f_{cu}$, the split cylinder strength, $f_{cl}$, the modulus of rupture strength, $f_{r2}$, and the age at testing of the specimens.

<table>
<thead>
<tr>
<th>Material</th>
<th>% of Total</th>
<th>Mass required for 1 Beam Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>5–10mm Gravel</td>
<td>40.8</td>
<td>155.1kg</td>
</tr>
<tr>
<td>Medium Sand</td>
<td>34.8</td>
<td>132.2kg</td>
</tr>
<tr>
<td>OPC</td>
<td>16</td>
<td>60.8kg</td>
</tr>
<tr>
<td>Water</td>
<td>8.35</td>
<td>31.7 l</td>
</tr>
</tbody>
</table>

Table 3.5 – Concrete properties

### 3.6.3 CFRP straps

The mechanical properties of the CFRP straps are given in Table 3.6.

<table>
<thead>
<tr>
<th>Modulus of Elasticity (GPa)*</th>
<th>Theoretical Tensile Capacity for 10 loops (kN)*</th>
<th>Experimental Tensile Capacity (kN)**</th>
<th>Ultimate Strain (mm/mm)***</th>
</tr>
</thead>
<tbody>
<tr>
<td>121</td>
<td>105.4</td>
<td>59.3</td>
<td>0.0127</td>
</tr>
</tbody>
</table>

* based on material properties given in Winistoerfer (1999) and using the rule of mixtures
** based on the average results of the 10 loop tests using the “Ideal” configuration
*** calculated using the experimental tensile capacity and the modulus of elasticity

Table 3.6 – CFRP strap properties

Ten loop straps with a prestressing force of 15kN (25% of the ultimate capacity) were used as discussed in section 2.5. The strap spacing was 250mm for specimens B2/45/H/20 through B6/30/C/44 as also discussed in section 2.5. The strap spacing was changed to 200mm for specimen B7/30/G/36 based on the strap strain results of
the other specimens as discussed later. The initial strap spacing is illustrated in Figure 3.14.

![Figure 3.14 - CFRP strap spacing](image)

3.6.4 Strain Gauges

Strain gauges were placed on the longitudinal reinforcement at the midpoint of the beam as well as on the shear links within the maximum shear region. Gauges were also placed on the outer face of the CFRP straps on both sides of the beam at the mid-height. The gauges used were TML FLA-6-11 (6mm long). The surface of all the bars was ground flat and prepared with 400 grit emery paper and acetone. In the case of the straps, the surface was prepared using emery paper and acid. The surface was then cleaned with paper towel and neutralizer. The gauges were then attached using cyanoacrylate adhesive. Figure 3.15 shows the locations of the gauges on the steel reinforcement.

![Figure 3.15 - Strain gauge locations on the steel reinforcement](image)
3.6.5 Beam Casting

The concrete was produced at the University of Cambridge Engineering Department. The first batch of concrete was placed in the formwork and then the formwork was vibrated. The second batch of concrete was then placed and subjected to further vibration. A similar process was followed for the material test specimens (cubes, split cylinders and modulus of rupture beams). All specimens were finished approximately 3 hours after initial placing. The specimens were then covered with a plastic sheet. The moulds were then removed from all specimens approximately 48 hours after casting. The specimens were then air cured until they were tested.

3.6.6 Hole Formation and Strap Installation

Before the straps could be installed in B2/45/H/20, two 45° holes had to be drilled as per Figure 3.1(a). For beam B3/30/H/22, the location of the 30° holes as seen in Figure 3.2 made it difficult to drill the holes without hitting either the longitudinal reinforcement or the strain gauge wires. Instead, the holes were cast into the concrete. This was done by placing 22.2mm diameter steel bars wrapped in bicycle inner tubes into the moulds before casting. A layer of grease was placed between the steel bars and the inner tubes to ensure that the bars could be removed after the concrete had hardened. The inner tubes were also coated in mould oil to ensure that they could be removed. The vertical holes were then drilled into the beam 7 days after the concrete was cast. After the holes were drilled, the steel inserts and straps were installed, and then the straps were welded.

The same procedure was used for forming the holes in specimens B4/30/G/25 and B7/30/G/36. However instead of installing steel inserts, the holes were grouted. This was accomplished by first turning the beam upside down. The openings to the 30° holes were then taped over. A 3mm thick polytetrafluoroethylene (PTFE) strip was cut in a tapered shape to a length of approximately 350mm. The taper was such that the wide end was 18mm and the thin end was 15mm wide. This taper made it easier to remove the PTFE strip once the grout had hardened. The strips were then coated in mould oil and placed in the vertical hole, threaded around through the 30° holes and
out through the other vertical hole to create the slot shown in Figure 3.12 for specimens B4/30/G/25 and B7/30/G/36. The holes were then filled with Pyrapatch, a concrete repair material developed by Weber SBD. Pyrapatch (hereafter referred to as grout) was selected because it was shrinkage compensated, ensuring good contact with the concrete in the flange. When mixed using a high shear mixer, it also exhibited good workability, allowing it to be placed in the holes. Table 3.7 gives the material properties (based on compressive cylinder tests) and mix proportions of the grout. Once the grout was mixed it was spooned into one of the vertical holes and the PTFE strap was moved up and down in the hole to help distribute the grout within the hole. A small vibrator was also used to ensure the holes were filled completely. Once the holes were filled, the PTFE straps were held upright and in the correct location by wooden blocks that were clamped to the flange of the beam. The grout took about 30 minutes to set. The PTFE strips were then pulled out of the holes within 24 hours of the grout setting.

<table>
<thead>
<tr>
<th>Modulus of Elasticity (GPa)</th>
<th>Compressive Strength (MPa)</th>
<th>Maximum Compressive Strain</th>
<th>Mix Proportions Pyrapatch/Water (grams, per hole)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>47.4</td>
<td>0.002</td>
<td>500 / 50</td>
</tr>
</tbody>
</table>

Table 3.7 – Pyrapatch properties

In specimens B5/30/C/27 and B6/30/C/44, the PTFE strips were placed into the formwork before the beams were cast. The strips were then removed from the concrete 3 days after casting the beam. As such there was no need to drill holes in these specimens.

The straps were then prestressed using the apparatus shown in Figure 3.7. The method used for B2/45/H/20 was to load the strap until the load cell indicated approximately 25kN of total tension, then metal shims were inserted underneath the strap pad and the load in the jack was released. The load was then reapplied until the strain gauges began to show an increase. The load at which this occurred was considered to be the prestress in the strap. The number of shims was adjusted until this load was 15kN.

The prestressing procedure for B3/30/H/22 and all subsequent retrofitted specimens was modified slightly in an effort to achieve a more accurate prestress. The straps
were loaded to 15kN (the prestressing force) and the strain in each gauge was noted. The straps were then loaded to approximately 25kN and metal shims were installed under the pad supporting the strap. The jack was then released and the strains in the straps were observed. If the strains were within 10% of the strains that were noted when the load cell read 15kN, no adjustment was needed. If the strains were not within 10%, shims were either added or removed in order to obtain the correct strain value.

3.7 Beam Testing

The beams were tested using a specially built test rig in the University of Cambridge structural laboratory. The test rig is shown in Figure 3.16. Figure 3.17 gives a schematic illustration of the loading arrangement for the beams. For specimens B1/25 to B3/30/H/22 the beams were seated on two captured rollers placed 2.5m apart from centre to centre. In order to ensure the reaction was spread properly from the beam to the rollers, a thin bed of dental plaster was placed between the beam and the rollers. The captured rollers were in turn seated on two 30ton sliders, which were placed on two pedestals. One of the 30ton sliders was locked into place to prevent horizontal movement of that slider. For specimens B4/30/G/25 to B7/30/G/36, the supports were changed to one captured roller and one free roller, eliminating the need for sliders.

Two 200kN hydraulic jacks were used to apply the load. The jacks were attached to the support beams of the test rig and were connected to two 200kN load cells that had spherical seats on one end. A 100mm by 100mm plate was dental plastered on top of the beam under each load point and a horizontal slider was attached to each plate. In the case of B6/30/C/44 and B7/30/G/36 the 100mm by 100mm plates were replaced by 250mm wide by 140mm long rollers as will be discussed in section 3.8. Horizontal movement was permitted under both load points. If the beam was restrained horizontally in more than one location, compressive stresses could have built up between those two points. These compressive stresses tend to have a similar effect to prestressing the beam by adding a compression component to the tension region as the beam deflects downwards. This stiffens the response of the beam by delaying the formation and growth of cracks. A spherical ball seat was attached to each slider under the load points. The balls ensured that the load points were free to rotate.
The load was applied by means of a hand pump, which pushed hydraulic fluid into the system. Non-return valves on each hydraulic jack ensured that the applied load was kept relatively constant during the load stages and that any drop in load was solely
due to creep in the beam. The displacements were measured using a series of 11 LRDTs (linear resistance displacement transducers) spaced at 250mm centres along the length of the beam.

For beam B3/30/Hb/27, the distance between supports was decreased to 1.5m by moving one of the support pedestals. One of the loading jacks was also disconnected so that only a single point load was applied. Since the beam length was reduced, only 7 LRDTs were used at 250mm spacing. The two testing set-ups are illustrated in Figure 3.18.

![Four Point Bending Set-up](image1)
![Three Point Bending Set-up (B3/30/Hb/27 only)](image2)

Figure 3.18 – Four and three point loading arrangements

3.8 Beam Test Results

3.8.1 Shear-Deflection Results

The shear force versus mid-span deflection curves for specimens B1/25 through B7/30/G/36 are given in Figure 3.19. The maximum shear force attained by each specimen as well as the maximum mid-span deflection and the failure mode is given in Table 3.8. The maximum mid-span deflection is taken as the maximum deflection...
achieved while the shear force was still within 5% of the maximum shear force. This is to illustrate the ductility of the specimen, if any.

![Shear force-deflection curves for beam tests](image)

Figure 3.19 – Shear force-deflection curves for beam tests

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Ultimate Shear Force (kN)</th>
<th>Maximum Mid-span Deflection (mm)</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1/25</td>
<td>88.2</td>
<td>15.5</td>
<td>Shear in Concrete</td>
</tr>
<tr>
<td>B2/45/H/20</td>
<td>95.4</td>
<td>15.6</td>
<td>Shear in Concrete</td>
</tr>
<tr>
<td>B3/30/H/22</td>
<td>91.4</td>
<td>16.0</td>
<td>Shear in Concrete</td>
</tr>
<tr>
<td>B3/30/Hb/27</td>
<td>82.4</td>
<td>5.6</td>
<td>Shear in Concrete</td>
</tr>
<tr>
<td>B4/30/G/25</td>
<td>105.2</td>
<td>16.9</td>
<td>Shear in Concrete</td>
</tr>
<tr>
<td>B5/30/C/27</td>
<td>111.0</td>
<td>16.9</td>
<td>Shear in Concrete</td>
</tr>
<tr>
<td>B6/30/C/44</td>
<td>140.9</td>
<td>22.1</td>
<td>Shear in Concrete</td>
</tr>
<tr>
<td>B7/30/G/36</td>
<td>134.7</td>
<td>29.1</td>
<td>Flexure</td>
</tr>
</tbody>
</table>

Table 3.8 – Specimen capacities

3.8.2 Specimen B1/25 – Unstrengthened Control Beam

3.8.2.1 Shear-Deflection Results

The maximum shear force resisted by the unstrengthened control specimen B1/25 was approximately 88kN. The failure was quite brittle as exhibited by the sharp drop in shear force after the maximum load was reached.
achieved while the shear force was still within 5% of the maximum shear force. This is to illustrate the ductility of the specimen, if any.

![Shear force-deflection curves for beam tests](image)

**Figure 3.19 – Shear force-deflection curves for beam tests**

<table>
<thead>
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<th>Maximum Mid-span Deflection (mm)</th>
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<tbody>
<tr>
<td>B1/25</td>
<td>88.2</td>
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<td>Shear in Concrete</td>
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<td>B2/45/H/20</td>
<td>95.4</td>
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<td>Shear in Concrete</td>
</tr>
<tr>
<td>B3/30/H/22</td>
<td>91.4</td>
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<tr>
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<td>105.2</td>
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<td>111.0</td>
<td>16.9</td>
<td>Shear in Concrete</td>
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<tr>
<td>B6/30/C/44</td>
<td>140.9</td>
<td>22.1</td>
<td>Shear in Concrete</td>
</tr>
<tr>
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<td>134.7</td>
<td>29.1</td>
<td>Flexure</td>
</tr>
</tbody>
</table>

**Table 3.8 – Specimen capacities**

3.8.2 Specimen B1/25 – Unstrengthened Control Beam

3.8.2.1 Shear-Deflection Results

The maximum shear force resisted by the unstrengthened control specimen B1/25 was approximately 88kN. The failure was quite brittle as exhibited by the sharp drop in shear force after the maximum load was reached.
The ultimate shear capacity prediction using the BS 8110 (1997) code with any safety factors set to unity was 56.9kN and using the CSA (1994) code it was 49.2kN. These results only considered the web to be effective in carrying shear. However, work by Zoheary et al. (1998) showed that in T-beams, the flange also contributes to the shear capacity of the beam as discussed earlier. For the flange width to web width and flange depth to beam depth ratios of the specimens tested in this series (2.38 and 0.39 respectively), linear interpolation of their work suggested an increase in shear capacity of approximately 60% over that of a rectangular beam. This meant that BS 8110 predicted capacity of 56.9kN would increase to 91kN, which was much closer to the actual capacity. Similarly the CSA prediction would increase from 49.2 to 78.7kN. However, it was difficult to make exact comparisons based on the experimental work of Zoheary et al. as the material, reinforcing and cross sectional details are all different from those used in the current study. Further attempts to accurately predict the beam capacity before and after retrofitting are presented in Chapter 5.

3.8.2.2 Failure Mode

The major shear crack for specimen B1/25 encompassed the full length of the shear span, starting at the support and extending past the load point as seen in Figure 3.20.

![Figure 3.20 - Specimen B1/25 after failure](image)

The crack angle through the web region was approximately 45°. However, in the flange the crack was very shallow, running at an angle of approximately 10°. One
would expect this shallow angle due to the compression forces in the flange. Equation 3-1 can be developed based on Mohr's circle.

\[ \tan 2\theta = \frac{2v}{f_{pc}} \]

One can see that if the longitudinal compression force, \( f_{pc} \), was increased with a constant shear stress, \( v \), the crack angle, \( \theta \), must reduce. Thus in the flange where compressive stresses due to flexure were present, the crack angle was reduced.

### 3.8.3 B2/45/H/20 – Strengthened Beam with 45° Holes

As a first attempt at developing a strap orientation that penetrated the compression flange sufficiently to provide significant shear enhancement, two 45° degree holes were drilled into the flange as indicated in Figure 3.21. Based on a preliminary analysis of the depth of the neutral axis along the length of the beam using Response-2000, a beam analysis program developed based on the Modified Compression Field Theory (Bentz and Collins 2000), this hole configuration should have penetrated at least a portion of the compressive region in the flange.

![Figure 3.21 - Hole Configuration of B2/45/H/20](image)

**3.8.3.1 Shear-Deflection Results**

Specimen B2/45/H/20 failed at a shear force of 95.5kN. This represented an improvement of 8.5% over the capacity of the control specimen. This result was
disappointing as previous work by Chan, using the same cross section, internal reinforcement diameter and CFRP straps (although the straps were supported in a different way), had yielded an increase in capacity of 48%. Also, in her tests the failure mode of the retrofitted beam was flexure. In this case the failure mode was still shear.

3.8.3.2 Failure Mode

The shear failure for specimen B2/45/H/20 was nearly identical to B1/25 as illustrated in Figure 3.22.

![Figure 3.22 - Specimen B2/45/H/20 after failure](image)

The only difference between B2/45/H/20 and B1/25 was that the shear cracks in the web started next to the CFRP strap adjacent to the support. After that the crack continued through the web at approximately 45-degrees and through the flange at a much shallower angle. The effectiveness of the first strap in changing the crack location was probably what generated the slight increase in capacity between B1/25 and B2/45/H/20. Since the straps in B2/45/H/20 did not fully penetrate the compression flange, the primary crack appears to have passed above the other two straps. This suggested that these straps were ineffective in terms of tying the secondary arches into the primary arch according to the Kani et al. analogy. De Lorenzis and Nanni (2001) were able to increase the capacity of their T-beam specimens by 41% without penetrating the flange. However, their flange to overall
depth ratio of 0.25 was significantly less than the 0.39 used in this study. Thus it was possible that the neutral axis in their beams fell below the flange, increasing the possibility that their FRP shear reinforcement would carry shear forces. Even so, when their NSM FRP bars did penetrate the flange, the capacity increase was significantly higher at 106%. Similar results were achieved by Melo et al. (2003) who used carbon fibre laminates to retrofit T-beams. Without penetrating the flange they obtained only a 17% increase in capacity over a control specimen whereas when the section was fully wrapped an increase of 84% was achieved. These results indicate that the depth of flange penetration plays a critical role in the level of shear enhancement.

The Kani et al. theory also suggested that the strap closest to the support would have been ineffective as it is within a distance $d$ from the support, whereas this strap seems to have been responsible for the slight capacity enhancement as it was the only strap to change the path of the shear crack.

The holes within the flange may also have served as weak points through which the crack could pass. This notion will be discussed in greater detail in Chapter 4.

### 3.8.4 B3/30/H/22 – Strengthened Beam with 30° Holes

Specimen B3/30/H/22 was designed to overcome the problem seen in B2/45/H/20 where the cracks followed a path that went above two of the three CFRP straps. A new hole orientation was used as seen in Figure 3.23. The advantage of this hole configuration was that the straps now encompassed a much greater area of the compressive flange. At the time of the design it was felt that the compressive stresses in the flange in the shear span were low enough that the stress could distribute around the holes without the concrete failing in compression.
3.8.4.1 Shear-Deflection Results

B3/30/H/22 attained a maximum shear force of 91kN. This represented an increase of only 3.4% over the control specimen. This result was surprising since this hole arrangement meant that the straps encompassed far more of the compression region than B2/45/H/20. One possible cause of this failure was the presence of the holes, which occupied approximately 35% of the compression flange. This was a significant reduction in compressive area and could have reduced the beam’s ability to transfer loads through the compression struts. The position of the holes, as high up in the compression flange as possible, may well have created unreasonably high local stress concentrations and disturbed the “flow” of the compressive stresses leading to a shear failure.

3.8.4.2 Failure Mode

The failure of B3/30/H/22 was quite different from the previous two specimens as seen in Figure 3.24.
Rather than the 45-degree crack forming in the web right next to the support, or next to the first strap, the failure was forced to a location between the middle and inner straps. This meant that the crack within the flange formed at a much steeper angle, approximately 23-degrees. The crack angle in the flange should only have changed (from B1/25) if there was a failure plane which required less energy to form as discussed by Loov (1998), suggesting the holes had created a new critical failure plane. Closer examination revealed that the crack in the flange followed the path of the holes. This meant that towards the edge of the flange the crack was not as steep as in the middle of the flange.

3.8.5 B3/30/Hb/27 – Unstrengthened Beam with 30° Holes

In order to ascertain whether the holes had a detrimental effect on the beam capacity, specimen B3/30/Hb/27 was tested. The straps were removed from the span of B3/30/H/22 that did not fail in shear. The end support was moved in so that the beam was loaded with a single point load but the maximum moment in the beam, as well as the shear, remained the same as in previous tests.

3.8.5.1 Shear-Deflection Results

The maximum shear force was approximately 82kN in B3/30/Hb/27. This represented a 10% reduction in capacity over specimen B3/30/H/22 and a 7% reduction compared
to B1/25. However, it was difficult to make an accurate comparison of these specimens for numerous reasons. First, specimen B3/30/Hb/27 was already damaged, and the existing crack pattern was the result of testing B3/30/H/22 with CFRP straps installed. It is unlikely, based on the crack pattern of the unretrofitted specimen, B1/25, that the crack pattern of B3/30/Hb/27 would have formed if the beam had been initially undamaged. Second, the concrete had had an extra 8 days to harden before specimen B3/30/Hb/27 was tested. The compressive capacity had increased by 20%. Depending on the test used, the tensile capacity had either decreased by 5% (modulus of rupture results) or increased by 13% (split cylinder results). The increase in compressive capacity was especially important since the area of the compressive flange had been reduced by the holes, so it is possible that this decrease in area was offset by the increase in compressive capacity when compared to B3/30/H/22. Finally, changing the test from two point loads to one may have affected the results. The beam response was stiffer, as seen in Figure 3.19, and rather than having a region of zero shear, there was a region where the shear essentially changed direction under the load point. Based on these three considerations, the two tests could not be compared accurately. However, the fact that there was a 10% reduction in shear capacity versus the retrofitted specimen even though the compressive capacity had increased by 20% tended to suggest that the holes played a role in the lack of capacity enhancement shown by specimen B3/30/H/22.

3.8.5.2 Failure Mode

The failure of B3/30/Hb/27 was different from that of the previous three beams. Rather than having a 45-degree crack that formed in the web and a shallower crack in the flange, the crack instead formed at a relatively constant angle as seen in Figure 3.25. The crack also seemed to run directly from the support to the holes closest to the load point. The only difference between this specimen and the control specimen was the holes, and yet the bilinear crack path of the control specimen was replaced by a crack running at a relatively constant angle. Once again this indicated that the holes played a significant role in the shear failure of these beams. However, since many of the shear cracks had formed during the testing of B3/30/H/22, it was impossible to say if this crack pattern was indicative of what would have happened to an undamaged specimen.
3.8.6 B4/30/G/25 – Strengthened Beam with 30° Grouted Holes

In an attempt to prevent the premature shear failure of B3/30/H/22, the holes in specimen B4/30/G/25 were filled with grout as shown in Figure 3.26.

3.8.6.1 Shear-Deflection Results

Specimen B4/30/G/25 failed at a shear force of 105kN. This represented a 19% increase over the control specimen. However, the failure was still in shear and as can be seen by Figure 3.19, there was no significant increase in ductility. The capacity also increased by 10% over the capacity of B2/45/H/20. This indicated that
encompassing more of the compressive flange with the strap has increased the shear capacity. Similarly it represented a 15% increase over specimen B3/30/H/22 illustrating that filling the holes with grout has also improved the shear capacity. It was difficult to say which one of these improvements had a greater impact on the shear capacity because B4/30/G/25 employed both improvements simultaneously. However, since B3/30/H/22 and B4/30/G/25 were the same except for the grouting, it can be concluded that the grouting seems to have had an impact.

3.8.6.2 Failure Mode

The flange of specimen B4/30/G/25 suffered significant damage as seen in Figure 3.27. Unlike specimen B2/45/H/20, the crack in the flange only runs across the first strap. The most significant web cracking has also formed between the middle and inner straps. Thus by increasing the penetration of the strap, the crack path has changed, increasing the capacity. This observation agrees well with work by Loov that suggests that steeper crack angles lead to higher shear capacities. However, there were also some unexpected horizontal cracks that formed next to the grouted holes. These cracks are not typical of a shear failure and they appeared to have formed as a result of the grout. If a stiffer intrusion were placed in the middle of a compressive region, one might expect a stress concentration to form in the area around the intrusion. This phenomenon is investigated further in Chapter 4.

Figure 3.27 - Specimen B4/30/G/25 after failure
3.8.7  B5/30/C/27 – Strengthened beam with groove cast in concrete

In order to determine the full effect of the material differences between the concrete and grout, B5/30/C/27 was cast. The CFRP straps sat directly in grooves cast into the concrete as illustrated in Figure 3.28 thus eliminating the grout.

![Figure 3.28 - Strap configuration of Specimen B5/30/C/27](image)

3.8.7.1 Shear-Deflection Results

Specimen B5/30/C/27 failed in shear at 110kN. This represented a 25% increase in shear capacity over the control specimen. More importantly, it represented a 5% increase in capacity over specimen B4/30/G/25. This suggested that while grout may have played a small role in the failure of B4/30/G/25, it did not account for the fact that the increase in shear capacity was not as significant as that observed by Chan. In fact, as per Figure 3.19, there was little improvement in maximum shear capacity or ductility regardless of whether the strap grooves were formed in grouted holes or cast directly into the concrete.

3.8.7.2 Failure Mode

Figure 3.29 shows the damage to B5/30/C/27 after failure. The crack pattern was quite similar to B4/30/G/25 including the formation of a horizontal crack running back from the load pad, although not at the same height. Figure 3.30 shows the area around
the load pad once the spalled concrete had been removed. There were several failure planes each falling away from the pad at approximately 45° creating a 3-D failure problem. For a standard shear failure, one would expect a bilinear failure plane running in the same direction as the longitudinal axis of the beam. These slopes around the loading pad seemed to be more indicative of a bearing failure. If the bearing stress were calculated as a standard load over area relationship, the bearing stress under the 100 by 100mm pads would be 11MPa at the failure load whereas the bearing capacity of the concrete was approximately 23MPa. However, since the bearing failure only occurred on one side of the loading pad, the stress distribution may not have been uniform under the pad. If, for example, one were to assume a linear stress distribution under the pad with the stress increasing from zero to a maximum, the maximum stress would be 22MPa, which was much closer to the bearing capacity. This would be an oversimplification of the problem as the presence of flexural, shear, and bearing stresses in this area of the flange makes the problem quite complex. Especially since the 3-D bearing stresses mean that a plane stress assumption is no longer valid. However, this calculation served to illustrate that with a redistribution of bearing stresses, bearing failure could be much more of a problem than the uniform stress distribution would have indicated. Thus the capacity of the beam would not be a reflection of the ultimate shear capacity, but a combination of its bearing, shear and flexural capacity. This bearing problem will be investigated further in Chapter 4. Since it was the goal of this testing program to determine the ultimate shear capacity of the beams obtainable through the use of an under-slab strap installation technique, bearing had to be eliminated as a cause of failure, which was the goal of the next specimen.
3.8.8 B6/30/C/44 – Strengthened beam with improved bearing capacity

In order to ensure that a bearing failure was prevented, two parameters were adjusted in specimen B6/30/C/44. First of all, the concrete strength was increased to approximately 40MPa. Also, wider bearing pads were used that spanned the entire width of the flange. The new pads loaded over an area of 140 by 250mm as opposed to the previous 100 by 100mm pads. The increase in bearing length from 100mm to 140mm was simply due to the fact that those were the dimensions of the available rollers. Chan also employed wider bearing pads so this change brought the testing setup in line with her work.
3.8.8.1 Load-Deflection Results

Although B6/30/C/44 still failed in shear, the ultimate load of 141kN represented a 59% increase in load carrying capacity over the control specimen. The failure was slightly more ductile with a plateau of approximately 1.5mm forming at the peak load as indicated by Figure 3.19. It could be argued that at least some of this shear enhancement was due to increasing the concrete strength. Chapter 4 and 5 will investigate numerically whether a lower strength concrete (25MPa) beam could still achieve a flexure failure when retrofitted with the CFRP straps.

3.8.8.2 Failure Mode

As can be seen in Figure 3.31, the crack path for specimen B6/30/C/44 was different from that in previous retrofitted specimens. Except for B2/45/H/20, the primary shear crack in the web of the retrofitted specimens occurred between the strap closest to the load point and the middle strap. This was due to the fact that weak points (holes and inadequate bearing pads) had allowed the crack in the flange to form at a much steeper angle than the compressive stresses would normally dictate in these specimens. Having removed these potential failure mechanisms, the crack path of specimen B6/30/C/44 was a more typical bilinear one with a shallower crack angle in the flange. This crack path suggested that the problems encountered in the previous beams had been overcome. Unlike Chan’s retrofitted specimen, this beam still failed in shear. However, an examination of the longitudinal stresses shown in Figure 3.32 indicates the longitudinal reinforcement was yielding, suggesting that a flexural failure was imminent.
3.8.9 B7/30/G/36 – Strengthened beam with grouted holes

Although specimen B6/30/C/44 showed promising results, the method of strap installation was not realistic. Specimen B7/30/G/36 was tested with the straps installed in the same way as specimen B4/30/G/25. However, due to a problem during the grouting process, the grout had to be drilled out of the holes and the holes regrouted. This added a further element of realism to the installation process as this meant that the 30° holes also had to be drilled out, whereas in previous tests these
holes had been formed using inserts cast into the concrete. Thus every operation that was performed in attaining this configuration could be repeated on-site. Based on an analysis of strap strains (see section 3.10) it was decided to tighten the strap spacing to 200mm. The middle strap was left in the same location and the other two straps were brought 50mm closer to it.

3.8.9.1 Load-Deflection Behaviour

Specimen B7/30/G/36 reached a maximum shear force of 135kN, which represented a 53% increase over the control specimen. This was also the first specimen to fail in flexure. As a result of this much more ductile failure mode, B7/30/G/36 obtained a maximum deflection of 29.1mm as seen in Figure 3.19, representing an approximate increase in deflection capacity of 93% over the control specimen. This ductile behaviour is desirable in structural design, as it gives some indication that failure is imminent before it actually occurs. Part of the reason that B7/30/G/36 failed in flexure, while B6/30/C/44 failed in shear at a higher load, was thought to be the concrete strength. As seen in Table 3.4, the concrete used in B6/30/C/44 had a compressive cube strength of 44MPa versus 36.1MPa for B7/30/G/36. This was enough to give B6/30/C/44 slightly more flexural capacity, which seems to have led to a shear failure. This notion will be discussed further in Chapter 6.

3.8.9.2 Failure Mode

Unlike previous specimens, the failure occurred due to crushing of the concrete in the constant moment region for B7/30/G/36 as illustrated in Figure 3.33. Although the shear cracks began to extend into the flange, they were not as significant as in previous tests.
3.9 Beam Stiffness

All seven beams that were tested in four-point bending (specimen B3/30/Hb/27 was much stiffer because of its shorter span) initially had approximately identical stiffness. However, the stiffness of the beams began to diverge at approximately 40kN as seen in Figure 3.19. This was to be expected since shear cracks began to appear at 40kN and the deflection was now the result of a flexural and a shear component. Specimen B1/25 saw the largest reduction in stiffness due to the lack of any additional shear reinforcement to help minimize crack openings and thus reduce deflections. The notion of beam stiffness and shear versus flexural deflection will be examined in greater detail in Chapter 6.

3.10 Transverse Strain

Section 3.8 looked at the overall performance of each beam, and how the failure mode of one specimen led to the design of the next specimen. In this section local strain results will be examined to differentiate the performance of each beam and help to explain the cause of failure. In order to simplify the discussion of the transverse strains, only those strains in the side of the beam where failure occurred will be examined. For every beam except B3/30/H/22 and B7/30/G/36, this was the east side. For specimen B3/30/H/22 this was the west side whilst specimen B7/30/G/36 failed in
flexure. However for the sake of comparison, strains in the east side of this beam will be examined.

Figure 3.34 indicates the position of the straps and shear links relative to each other and the load points for specimens B2/45/H/20 through B6/30/C/44. It also indicates where the outer, middle, and inner locations are for discussion purposes. Figure 3.35 shows the layout for B7/30/G/36.
3.10.1 Strap Strains

Although strain gauges were placed on both outside faces of the straps, only the gauges placed on the face of the strap that did not contain the weld will be used for discussion here. The strain gauges placed on the welds give a much stiffer response than those placed opposite the weld. To illustrate this, Figure 3.36 shows the strap load vs. strain results for 5 of the straps during prestressing of the straps for B6/30/C/44. Figure 3.36a shows the strap load vs. strain results for the gauges on the outside face of the strap opposite the weld. The average slope of the lines was 4684kN (which represents a modulus of elasticity of 122GPa, quite similar to that calculated by the rule of mixtures). Figure 3.36b presents the load vs. strain data for the gauges on the weld. The average stiffness was much higher at 6751kN (176GPa). The reason for this is believed to be due to the increased area of material at the weld, which results in reduced strains in this area. However, these strains and the corresponding modulus of elasticity do not reflect the strap behaviour away from the weld. As a result of this, the following analysis uses only the data from gauges on the outside face of the strap opposite the weld since it was deemed to be more representative of overall strap behaviour.

![Graph](image)

a) Load vs. strap strain opposite the weld
b) Load vs. strap strain on the weld

Figure 3.36 – Load vs. strap strain – B6/30/C/44

The strap load vs. strain data was acquired for specimens B3/30/H/22 through B7/30/G/36 during the prestressing operations. The average slope of the prestressing data trendlines for all six straps, as well as the associated stiffness, is given in Table 3.9 for each specimen as well as the mean, standard deviation and the % error. The results in Table 3.9 will be used in the next section to determine the load in each strap.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Slope (kN)</th>
<th>Stiffness (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B3/30/H/22</td>
<td>4867.06</td>
<td>126.746</td>
</tr>
<tr>
<td>B4/30/G/25</td>
<td>4526.33</td>
<td>117.873</td>
</tr>
<tr>
<td>B5/30/C/27</td>
<td>4782.19</td>
<td>124.536</td>
</tr>
<tr>
<td>B6/30/C/44</td>
<td>4683.64</td>
<td>121.970</td>
</tr>
<tr>
<td>B7/30/G/36</td>
<td>4429.98</td>
<td>115.364</td>
</tr>
<tr>
<td>Mean</td>
<td>4657.84</td>
<td>121.298</td>
</tr>
<tr>
<td>Standard Deviation, s</td>
<td>179.66</td>
<td>4.679</td>
</tr>
<tr>
<td>% error</td>
<td>3.86</td>
<td>3.86</td>
</tr>
</tbody>
</table>

Table 3.9 – Strap stiffness
3.10.1.1 Outer Strap Strains

Figure 3.37 gives the shear force vs. outer strap strain relationship for the six beams with CFRP straps. Since the amount of prestress in each strap varied, the strain due to prestress was subtracted from each data series to make the results more comparable. Examining Figure 3.38, which presents the shear force vs. strap strain including the strain due to prestress, it was observed that despite a variation in strain due to prestress of over 100%, the straps perform in a similar manner. This large variation in initial strain is attributed to three issues. First, the strain gauges were placed on the straps before they had been tightened resulting in initial non-zero strains as the straps straightened. Also, as the steel pad was pulled up and the straps tightened together during prestressing, unless the pad was pulled up perfectly vertically, one side of the strap would receive more strain than the other side once the strap had tightened against the load pad. This meant that though there was 15kN of force in the strap, the strains on either side of the strap could be out of balance. Finally, because of the limited number of steel shims available for holding the prestress, it was never possible to achieve a prestress of exactly 15kN, which was also reflected in the strain variation. Despite this, the straps all started to develop increased tensile loads just after the shear cracks appeared at approximately 40kN, which validates the use of strains with the prestress removed as discussed in this section.

The axes of Figure 3.37 have been made the same as that for the middle (Figure 3.39) and inner (Figure 3.40) straps so the relative effectiveness of each strap can be compared. The most effective outer strap in terms of strain at a given load appeared to be that on B2/45/H/20 (although once B2/45/H/20 failed the strap strains on B7/30/G/36 reached higher values at higher loads). This made sense since the shear crack in B2/45/H/20 bypassed the other two straps, forcing the outer strap to take more load. The strain at the ultimate load was only 0.0014, which translated to a load of 6.5kN which when combined with the prestressing force represented about 33% of the total strap capacity. The effectiveness of this strap was contrary to the report presented by Kani et al. that suggested straps within a distance $d$ of the support would be ineffective. Although the strains did not approach failure, they were not insignificant and there was a capacity enhancement, both of which disputed the work by Kani et al. The strain in the strap on B3/30/H/22 was significantly less at a given
load than for B2/45/H/20. Once again this was to be expected since the major shear crack for this specimen formed between the middle and inner straps. The strains for specimen B7/30/G/36 were about twice as high as those in B6/30/C/44. This behaviour was the result of moving the straps 50mm closer to the middle strap in specimen B7/30/G/36 and the lower concrete strength of B7/30/G/36. This placed the outer strap closer to the widest shear cracks (as observed during testing) and also meant the concrete contribution was reduced resulting in higher strains in the straps.

Figure 3.37 - Shear force vs. outer strap strain
3.10.1.2 Middle Strap Strains

The middle straps on each beam developed significantly higher strains than either the outer or inner straps as illustrated in Figure 3.39. The strains in the middle straps were approximately three times greater than in the other two straps. The exception to this was B2/45/H/20 where the outer and middle strap strains at the peak load were approximately equal. One might expect the strain in the middle strap to be even lower for this specimen as the shear crack passed over the top of the strap. The middle straps on specimens B4/30/G/25 and B5/30/C/27 actually saw the highest strains for a given load over most of the loading range. This suggests that the straps were effectively embedded into the compression zone. The increase in strap strain when compared with B6/30/C/44 and B7/30/G/36 is also a reflection of the lower concrete strength, and thus lower concrete shear contribution in specimens B4/30/G/25 and B5/30/C/27, which is balanced by a higher strap contribution. Though the strap strains in B3/30/H/22 are quite high at failure, these increased strains do not result in an increase in overall capacity, instead the strap seems to be taking load that is being shed from elsewhere. The straps on specimens B6/30/C/44 and B7/30/G/36 behaved in an almost identical fashion. The only difference was that the strap on B6/30/C/44 continued to strain due to the beam’s higher flexural capacity. This very similar strap
strain profile means that B7/30/G/36 may also have failed in shear had its concrete strength and corresponding flexural capacity been slightly higher.

![Graph](image)

**Figure 3.39 - Shear force vs. middle strap strain**

### 3.10.1.3 Inner Strap Strains

The shear force vs. inner strap strain is plotted in Figure 3.40 for the 6 retrofitted specimens. The strain for B2/45/H/20 never exceeded the prestressing strain, which was a result of the crack path forming above the strap. This result emphasized the need for greater penetration into the compressive flange. The inner strap on B3/30/H/22 actually began unloading before the failure load was reached. This correlated well with the observation that failure occurred through the holes in this specimen since if the shear crack had formed below the top of the strap, the strains should have increased until failure. The straps on specimens B4/30/G/25 and B5/30/C/27 continued to work effectively until failure, following a similar loading path to B7/30/G/36. This indicated that the failure of B4/30/G/25 and B5/30/C/27 did not occur in the area of the straps as they performed in a similar manner to a specimen that failed in flexure. This tied in well with the observed failure around the bearing pad, and suggested that if this failure could have been prevented, a higher capacity was possible as later tests proved. The strains in B6/30/C/44 are lower than those in
B7/30/G/36; the reasons for this are two-fold. First, the straps in B7/30/G/36 were moved closer to the middle strap so that they share more of the load. Second, the concrete strength, and thus the concrete contribution, was slightly lower for B7/30/G/36, resulting in higher strap strains. The fact that the inner strap forces increased also suggested that Kani et al.'s conclusion that transverse reinforcement within a region of length $d$ away from the load point is ineffective was incorrect.

![Graph showing Shear Force vs. Inner Strap Strain](image)

**Figure 3.40 - Shear force vs. inner strap strain**

**3.10.2 Shear Link Strains**

There were two gauges on each shear link, one on either side of the link. In most cases the results discussed in the following section were the average of those two gauges. However, in instances where one of the gauges was not working, or failed prematurely, the result from the gauge on only one side of the shear link was plotted.

**3.10.2.1 Outer Shear Link Strains**

Figure 3.41 illustrates that the shear links experience almost no strain until a shear force of between 30 and 50kN. In all cases, this load corresponds to the observed formation of shear cracks during the tests. The control specimen, B1/25, developed the highest strains at a given load (apart from B3/30Hb which was a special case).
This was anticipated, as B1/25 had no additional shear reinforcement to help carry the shear force. The performance of the links in B2/45/H/20 was almost identical to that of B1/25 up until 80kN, after which point the strains were slightly less. At 80kN, the outer straps in B2/45/H/20 began developing higher strains explaining why the strains in the link were lower. As mentioned, specimen B3/30/Hb/27 saw higher strains at a given load than any other specimen. This was initially due to the fact that the beam had been preloaded and so the shear cracks had already formed transferring more load to the links from the outset. In the later stages, the higher strains were due to the reduced concrete contribution because of the holes, causing the links to pick up additional load. The key thing to note was that each of the retrofitted specimens had lower link strains at a given load than the control specimen, indicating that the shear force was being shared between the straps and the links. In the case of B7/30/G/36, the strains in the links were approximately half of those in B1/25 at a load of 88kN. At this load the outer link in B1/25 was yielding resulting in a link force of approximately 32.7kN whereas at that same load the force in the outer link in B7/30/G/36 was approximately 12.4kN. This represented a significant reduction, and illustrates one way in which the straps add additional shear capacity by both adding additional transverse capacity and increasing the applied shear force required to yield the steel links.

![Shear force vs. outer shear link strain](image)

Figure 3.41 - Shear force vs. outer shear link strain
Interestingly, the strains in the outer links of B7/30/G/36 were significantly lower than those in B6/30/C/44. At the failure load of B7/30/G/36, the strain in the links was only 0.0020 versus 0.0049 for B6/30/C/44. On the other hand, the outer strap strains for B7/30/G/36 are higher than those for B6/30/C/44 as can be seen from Figure 3.37. This suggests that by tightening the strap spacing, thereby bringing the outer strap 50mm closer to the outer link, the load sharing between the strap and the link has been changed. Whilst it is not possible to draw definitive conclusions from this single result, it does suggest that the placement of the straps relative to the shear links and areas of significant shear cracking can affect the load sharing, which could result in overloading of the straps if care is not taken. Further research in this area is required to confirm this result.

3.10.2.2 Middle Shear Link Strain

The shear force vs. middle shear link strain plot in Figure 3.42 showed the same general trends that were observed for the outer links. Once again the retrofitted specimens have lower link strains than the two unreetrofitted specimens. The performance of B2/45/H/20 lies in between the two types of beams, which was a result of the shear crack forming farther along the beam due to the effectiveness of the first strap. In section 3.9, it was noted that the stiffness of the retrofitted specimens increased over the control specimen. Figure 3.42 shows that at a given shear force above 40kN the strains in the shear links were significantly lower in the retrofitted specimens. Lower strains generally correlate to smaller cracks. In turn smaller cracks result in a stiffer beam response as the rotation of the beam is decreased.
3.10.2.3 Inner Shear Link Strain

For completeness, the shear force vs. inner link strain plot is given in Figure 3.43. It should be noted that the strain axis scale is much smaller. It was virtually impossible to distinguish between individual beams when the results were plotted at the same scale as Figures 3.41 and 3.42, the strains in the inner links were just too small. These small strains were expected since the link was located under the load point in a region that did not see the full shear force. However, the mechanics in this ‘disturbed’ region are not well understood so predicting strains based on beam theory would be incorrect. The main observation to be taken from this plot was that the links did not yield, suggesting that shear retrofitting was not required in this area, nor was it provided.
3.11 Conclusions

Based on initial strap tests, it was decided that in terms of strap strength, any orientation that ensured the strap did not bend below a minimum radius of 12.5mm would provide adequate strap capacity. The material used to support the strap did not have a significant impact on the capacity as long as the strap cross section was kept flat, which is one reason why using nothing in the holes is ineffective. However, less stiff materials lead to excessive strap deformations without developing strap stresses, which could affect the capacity of the retrofitted beam.

The beam tests revealed that encompassing a significant portion of the compressive flange was key to developing the full capacity of the beam. Otherwise crack paths developed that avoided the straps. The presence of holes in the compression flange also had a significant effect on the shear capacity. The holes created a weak plane through which shear failure occurred at lower loads than expected. Though the straps can offer slight increases in capacity in some of these cases, the full capacity enhancement could not be achieved and thus large holes should be avoided. The grout’s affect on the overall capacity was minimal, making it a useful material for retrofitting beams. Bearing failure under the load pads also seemed to play a role in the failure of the beams, and will be examined further in Chapter 4. However, once
the affects of bearing were eliminated, it was discovered that an under-slab installation technique was viable. Tests using a practical installation technique showed a load carrying capacity enhancement of 58% and a 93% increase in deflection capacity over a control specimen. Strain gauge data showed that the straps helped to reduce the tensile force in the shear links by approximately 50%, contributing to the increased shear capacity.
Chapter 4
Finite Element Analysis

This chapter will examine the results of various analyses performed using the DIANA finite element analysis (FEA) package. A brief introduction to the program and the models used will be presented. Based on the experimental beam results, there is reason to believe that the presence of holes, and to a lesser extent materials with different levels of stiffness, in the flange will adversely affect the capacity of RC beams. In order to model these intrusions in the beams using 2-D FEA, a new concrete model is needed to account for their effects on the concrete capacity and stiffness. Several concrete specimens with and without intrusions were tested and then modelled using FEA. The results of these models are then employed to develop the new concrete model. DIANA is also used to predict the capacity of seven of the static beam tests. The effect of intrusions, strap penetration and bearing area is examined in greater detail. The software’s ability to predict strap strains is evaluated since an accurate prediction of these strains is crucial for design purposes. The FEA is then employed to redesign the retrofitting system for the lower strength concrete beams, to see if a flexural failure is possible for lower concrete strengths. Finally, conclusions are drawn about the use of FEA to design CFRP strap based shear retrofitting systems.

4.1 DIANA Analysis Software

In order to ensure that designers could apply the results of this study to predict beam capacities, an FEA package that is commercially available was used. Two packages were initially considered: VecTor2 and DIANA. VecTor2 is an FEA program developed by the University of Toronto (Wong and Vecchio 2002) and based on the MCFT. The demonstration version of the program was used to model the control specimen, B1/25, but gave inconsistent results. Instead it was decided to use the DIANA (2003) package because of its available concrete models, the fact that there were full versions of the program available within the structures group and because there was existing expertise at Cambridge in its use. Previous research by Kesse (2000 and 2003) successfully used DIANA to predict the load carrying capacity of retrofitted specimens with good accuracy. His research also provided tremendous
insights into the deficiencies of DIANA (as discussed in the following sections) allowing results produced by this program to be better understood. Al-Mahaidi et al. (2000) used DIANA to predict the capacity of T-beams to within 10% of the actual test results, also predicting the correct failure mode. Thus, based on its commercial availability as well as the success of previous studies, DIANA Release 8.1.2 was chosen as the FEA package for this research program.

4.1.1 Concrete Models

In order to predict the capacity of any RC structure properly, models are required for the various aspects of material behaviour. Whilst some of these models, such as elastic-perfectly plastic stress-strain behaviour for the steel reinforcement, are fairly straightforward, others are not. The behaviour of concrete requires careful examination since it is both non-linear and different in compression versus tension. The following sections outline the material models chosen, and why they were chosen.

4.1.1.1 Concrete Compression

Kesse (2000) looked at three possible criteria for failure of concrete in compression: von-Mises, Drucker Prager, and the Thorenfeldt total strain criteria. He calibrated his model against experiments performed by Kupfer et al. (1969) who tested a series of plain concrete specimens in stress states ranging from equal compression in both directions to equal tension in both directions. Kesse concluded that all three of these failure criteria predicted uniaxial stress conditions well. In terms of biaxial compression, the von-Mises criterion tends to underestimate the actual test results whereas Drucker Prager overestimates these results. The Thorenfeldt criterion actually gave the most satisfactory results in biaxial compression. However, Kesse found that this model was very inaccurate in the case of combined tension-compression. He instead recommended the use of the von-Mises criterion, which gave the best results in combined tension-compression, suggesting that the equal biaxial compression stress state does not occur frequently in beams, minimizing the impact of any deficiencies. In 2003, Kesse again recommended the von-Mises criterion, this time pointing out that fewer parameters were required for input into the FEA program.
with no loss of accuracy when compared to the Drucker Prager and Mohr-Coulomb approaches. Al-Mahaidi et al. do not indicate which criterion they used beyond stating that plane stress elements were employed. As such, based on the preceding discussion, a von-Mises failure criterion is implemented in this study.

Kesse used a stress-strain curve developed by Wang et al. (1978) to model the compression behaviour of the concrete with good results. In this study two compression curves were considered, a curve presented by Collins and Mitchell (1997) based on a curve developed by Thorenfeldt et al. (1987) and that given by Wang et al. The two curves were used to model the behaviour of the control specimen, B1/25, and it was found that the Wang et al. curve produced better results in terms of the overall beam stiffness. This curve was developed to fit the results of numerous compressive tests on cylinders carried out by Wang et al. The curve equation requires the user to solve for four unknown parameters to define the ascending portion of the curve and then four further parameters to solve for the descending branch. Fortunately these expressions can all be calculated using the compressive cylinder strength. Interestingly the Wang et al. curve gives a lower initial elastic modulus than more traditional curves such as that given in Collins and Mitchell, or those proposed by codes (ACI Committee 318 1995), as illustrated in Figure 4.1. Wang et al. suggest that this is a result of the nature of their testing scheme. In the current work, the lower concrete stiffness results in a better prediction of the beam stiffness as mentioned above, and so the Wang et al. model will be used in this study. Once the points on the Wang et al. stress-strain curve have been determined using a spreadsheet, they are entered into DIANA as a strain-hardening diagram with yield assumed to occur at $0.45f'_c$. 

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4.1.1.2 Cracking

Cracking is an important aspect of RC beam behaviour. It can be accounted for in FE models either discretely or using a smeared approach. Discrete cracks are modelled by specifying the precise location of the crack and providing physical separation by splitting the nodes at that location. They are difficult to model as they require knowledge of the crack locations and the ability to change the model geometry during the analysis. Rots (1991) suggests that these problems can be overcome by first predicting the location of the cracks using a smeared crack approach and then linking the nodes of the cracks together using interface elements. However, in instances where the distance between cracks is quite small and the crack patterns are quite complicated these models can be difficult to implement. The other possibility is to use a smeared crack approach. In this case, when the cracking stress is reached, the material properties of the element are changed to account for this cracking. The smeared approach does not require the user to predefine crack locations and can also account for multiple cracks affecting a single element. It is because of this more straightforward application that this approach will be used here, although potential problems will also be noted. In DIANA, the smeared crack model requires the user to specify the tension cut-off, tension softening and shear retention parameters.

Figure 4.1 – Concrete compressive curve comparison – 30MPa
Kesse (2000 and 2003) found that eight-noded elements performed poorly after cracking as they had a tendency to take on an 'hourglass' shape. This behaviour was also observed by Crisfield (1986) who noted that while the mid-side node of the eight-noded elements is normally restrained by the surrounding elements, if there is significant cracking this is not the case and erroneous displacement results can occur. Kesse recommends the use of four-noded elements to overcome these difficulties and so four-noded elements will be employed here.

### 4.1.1.3 Tension Cut-off

The tension cut-off parameter allows the program to decide when an element has cracked. DIANA offers two possibilities for modelling the tension cut-off: constant and linear. A constant tension cut-off uses the same tensile capacity for the concrete in the principal stress direction regardless of the stress being applied in the other direction. A linear tension cut-off, on the other hand, reduces the allowable compressive capacity with increasing tensile stress along the other principal stress direction, as illustrated in Figure 4.2. In this study the linear tension cut-off model was used, as it is a more realistic representation of concrete behaviour as demonstrated by the experimental work of Kupfer et al. (1969) as well as Vecchio and Collins (1986) among others.

![Figure 4.2 – Linear tension cut-off](image-url)
4.1.1.4 Tension Softening

Once a crack forms, the tensile capacity of the concrete does not immediately drop to zero. Instead there is a gradual reduction in tensile capacity, as determined experimentally by numerous researchers including Phillips and Binsheng (1993), which is referred to as tension softening. DIANA offers four main tension softening models: brittle (once the crack forms the tensile capacity immediately drops to zero), linear (the tensile capacity reduces linearly as illustrated in Figure 4.3), multi-linear, and non-linear. Kesse (2000) used the results of panel tests performed in uniaxial tension by Wollrab et al. (1996) to determine the most effective tension model. He concluded that a linear softening model was the most robust in terms of modelling post-cracking behaviour. Al-Mahaidi et al. also employed the linear tension softening model and so the same model will be used in the current study.

![Figure 4.3 – Linear tension softening model](image)

In DIANA, the linear tension softening curve is based on the tensile strength, $f_t$, and the ultimate tensile strain in the concrete, $\varepsilon_{ult}$. The ultimate tensile strain in the concrete is derived from either equation 4-1 if the element is reinforced or 4-2 if it is a plain concrete element. For the beam models equation 4-2 was used as it was felt that plain concrete behaviour would dominate, as the vast majority of elements did not contain reinforcement. Also, for the element size chosen (approximately 25mm) there is not a significant difference between the results of 4-1 and 4-2.

$$\varepsilon_{ult} = \frac{f_y}{E_s} \quad 4-1$$

where

- $f_y = \text{yield strength of the reinforcement (MPa)}$
- $E_s = \text{modulus of elasticity of the reinforcement (MPa)}$
\[ \varepsilon_{\text{ult}} = \frac{2G_f}{f_t h_b} \]

where \( G_f \) = fracture energy (N/mm)
\( f_t \) = tensile strength of the concrete (MPa)
\( h_b \) = estimated numerical crack bandwidth (mm)

The term \( G_f \) has been determined by Phillips and Binsheng from tensile tests on plain concrete specimens. They were able to relate \( G_f \) (in N/m) to the compressive strength of the concrete (in MPa) as given in Eq. 4-3 and the tensile strength (in MPa) in Eq. 4-4.

\[ G_f = 43.2 + 1.13f_{cu} \]  
\[ G_f = 30.5 + 6.64f_t^2 \]

The term \( h_b \) in Eq. 4-2 is a function of the element size in DIANA as seen in equation 4-5. As the element size increases, the ultimate strain decreases which accounts for the increasing influence of a crack on larger elements using the smeared crack approach.

\[ h_b = \sqrt{2A_{\text{element}}} \]

Kesse further concluded that work by Bazant and Planas (1998) was correct in assuming the element size should be three times the maximum aggregate size. However, in his 2003 work, Kesse used elements that were both two and three times the maximum aggregate size. Both of these element sizes functioned adequately although the smaller elements seemed to model performance slightly better but with a corresponding increase in analysis time. Based on these considerations, and the overall geometry of the T-beams used in this study, an element size of approximately 25mm (2.5 times the maximum aggregate size) was used.

**4.1.1.5 Shear Retention**

The ability of cracks to carry shear by aggregate interlock has, since the earliest days of FEA (Hand et al. 1973), been modelled by multiplying the elastic shear modulus, \( G \), by a reduction factor. In DIANA this is referred to as the shear retention factor, \( \beta \).
There are two supported shear retention factors in DIANA: full and constant. A full shear retention factor means that the cracks can carry the full shear even after opening. A partial shear retention factor reduces the shear modulus of an element by a set amount after cracking. Kesse (2003) adjusted the constant $\beta$ value until the ultimate load predicted by DIANA for his control specimen was approximately the same as the experimental ultimate load of his control specimen. He then set $\beta$ to this value, which he determined to be 0.2, in the FEA models of his remaining beams. A similar approach will be used in this study.

Table 4.1 gives a summary of the concrete models used.

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Four-noded quadrilateral plane stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression</td>
<td>Wang et al. (1978) stress-strain curve</td>
</tr>
<tr>
<td>Failure Criterion</td>
<td>von-Mises</td>
</tr>
<tr>
<td>Cracking</td>
<td>Smeared</td>
</tr>
<tr>
<td>Tension Cut-off</td>
<td>Linear</td>
</tr>
<tr>
<td>Tension Softening</td>
<td>Linear tension softening using equation 4-2 with the fracture energy term developed by Phillips and Binsheng (1993).</td>
</tr>
<tr>
<td>Shear</td>
<td>Constant shear retention factor, $\beta$</td>
</tr>
</tbody>
</table>

Table 4.1 – FEA concrete models

4.1.2 Steel Reinforcement

The steel reinforcement is specified as embedded reinforcement in DIANA. This means that the reinforcement increases the stiffness of the concrete, or ‘mother’, element to which it is attached. This also means that the reinforcement does not have any degrees of freedom of its own, but uses the displacement field of the mother element, which assumes perfect bond between the reinforcement and the surrounding concrete. For the specimens used in this study, this assumption was valid as the longitudinal reinforcement is hooked to develop full anchorage at the end points. There is still the potential for localized slip between the reinforcement and surrounding concrete, which will not be predicted by DIANA, but this should not affect the overall capacity. Similarly, closed stirrups are used to provide effective end anchorage. However, because smooth bars were used for the stirrups, it is possible that localized debonding could occur in the beam specimens. The average strain in the
bars should still be similar between the test specimens and the FEA model. The perfect bond assumption may not always be valid, in which case the use of contact and link elements may need to be considered for reinforcement in future models. DIANA also has the option of assuming no bond between the embedded reinforcement and the concrete elements.

The longitudinal reinforcement steel was modelled as elastic perfectly plastic using the elastic modulus, $E$, and yield stress, $f_y$, values given in Table 3.2. The transverse steel reinforcement was modelled using the strain-hardening curve given in Figure 4.4 and the $E$ value from Table 3.2.

![Figure 4.4 – Strain-hardening curve for transverse steel reinforcement](image)

4.1.3 CFRP Straps

Kesse (2000) employed the no-bond feature available in DIANA to model the CFRP straps as reinforcement elements. According to the DIANA (2003) manual, if the no bond feature is specified, the stiffness of the reinforcement does not affect the stiffness of the surrounding concrete element. Similarly, the stresses and strains in the concrete element do not affect the reinforcement. However, Kesse noted that the strain did change along the length of the strap element, despite not being bonded to
the beam. As such, Kesse (2003) employed truss elements connected only at the end points and then prestressed. These elements maintained a constant stress and strain along their length, which was more realistic. In the current research, two-noded truss elements are also employed. The results of the beam analysis showed no variation in strap stress or strain along the length of the element, and thus these elements were deemed to be suitable.

The CFRP was modelled as an elastic brittle material with an elastic modulus of 121000 MPa and a rupture stress of 1300 MPa.

4.1.4 Solution Algorithm

Several iteration schemes were used to model the control beam in this study to decide which one was the most effective. The constant stiffness method converged at every load step. However, the results were erroneous as the final predicted beam capacity was well above the actual control specimen capacity and the deflections tended to suggest a level of ductility that did not exist in the actual specimen. DIANA also produced an initial warning message in the output that suggested that this solution technique was not suitable for the analysis type. The regular Newton-Raphson method with arc-length control was also tried. Although this method seemed to produce more accurate results, the solutions occasionally diverged from the load-displacement curve at unexpected points. As well, convergence at every load stage was not always achieved and the out of balance force was sometimes quite significant. The DIANA manual suggested that the modified Newton-Raphson approach might still converge when the regular Newton-Raphson approach cannot. The modified Newton-Raphson approach calculates the stiffness at the beginning of a load step and employs that stiffness throughout the load step as opposed to the regular Newton-Raphson, which recalculates the stiffness for every iteration. The results produced by the modified Newton-Raphson were more stable than the regular Newton-Raphson solver and seemed to model the actual beam performance quite well. However, there was still a tendency for this approach not to converge at every load step. This was deemed acceptable if the out of balance force was relatively small (within the same order of magnitude as the load convergence tolerance of 0.001) and subsequent load stages did converge. Convergence in these analyses was based on force convergence, which
DIANA takes to be the ratio of the Euclidian norm of the out-of-balance force vector for iteration, \(i\), to the initial out-of-balance force as given in Equation 4-6.

\[
\frac{\sqrt{g_i^T g_i}}{\sqrt{g_o^T g_o}} \leq 0.001 = \text{the convergence tolerance} \tag{4-6}
\]

where

- \(g_i\) = out-of-balance force vector for iteration \(i\)
- \(g_i^T\) = transpose of out-of-balance force vector for iteration \(i\)
- \(g_o\) = out-of-balance force vector for initial iteration
- \(g_o^T\) = transpose of out-of-balance force vector for initial iteration

The load-displacement plot was also examined to ensure that this lack of convergence had not led to spurious results such as sudden changes in slope. According to Memon and Su (2004), the problem with the modified Newton-Raphson method is that it does not capture the full load-displacement behaviour, failing near the limit point (ultimate load). Indeed this was observed to be the case for more ductile specimens. As such, two solution techniques were employed. The modified Newton-Raphson was used because it was able to converge at almost every load stage and the out of balance force was not significant when convergence was not achieved. The regular Newton-Raphson approach with arc-length control was also employed to develop the full load-displacement curve and the results checked against the more reliable convergence of the modified Newton-Raphson method.

### 4.2 Intrusion Modelling

Specimens B2/45/H/20 and B3/30/H/22 had strap configurations that involved holes in the flange of the beam. These holes seemed to be the cause of a difference in capacity between B3/30/H/22 and beams with similar strap configurations, bearing pad layouts and concrete strengths (B4/30/G/25 and B5/30/C/27). However, there also seemed to be slight differences in capacity between B4/30/G/25 and B5/30/C/27 where the material used to fill the hole around the strap was the only variable.

Unfortunately, these hole layouts cannot be modelled in a two-dimensional beam FEA because the geometry of the holes varies along the z-axis. This is especially true for specimen B3/30/H/22 where the presence of both vertical and 30° holes made the geometry quite complex in the z-direction as illustrated in Figure 4.5. One possibility
would be to create a three-dimensional beam model, however the number of elements required to do this effectively, especially to capture the complex geometry in the area of the circular holes, made analysis times prohibitively long. Instead, it was decided to create a new concrete model that would account for the affects of the holes but still allow a two-dimensional beam model to be used.

![Diagram of hole layouts in B2/45/H/20 and B3/30/H/22](image)

**Figure 4.5** – Hole layouts in B2/45/H/20 and B3/30/H/22

In order to develop this new concrete model, some understanding of the behaviour of concrete with intrusions was required. As such, a series of compression tests were performed on specimens using plain concrete, concrete with holes, and concrete with holes filled with grout. The tests were modelled using DIANA to see if comparable results could be obtained. These models could then be used to develop a concrete curve to simulate the presence of holes for use with the two-dimensional beam analysis.

### 4.2.1 Specimens

As a first step towards developing an equivalent concrete model, a series of compression tests were conducted to gauge the impact of the intrusions. The compression test specimens are illustrated in Figure 4.6. Each specimen was a rectangular prism and had exterior dimensions of 100 by 100 by 300mm. These specimens were designed so that the stresses in the central section would not be influenced by the confinement provided by the loading plates. The concrete mix design was the same as that given in Table 3.3 and used for specimens B1/25 through B5/30/C/27. This mix design gave low concrete strengths resulting in less stiff...
behaviour in compression. If there were stress concentrations due to variations in material stiffness between the concrete and the grout, the use of a weaker concrete should accentuate this problem. The grout properties are given in Table 3.7.

The specimens were tested in an Avery Denison cube-testing machine at a load rate of 180kN/min.

### 4.2.2 Test Results

The results of the compression tests are given in Table 4.2. In each case two specimens were tested (except for the cubes where three specimens were tested). The table gives the maximum average load obtained by each specimen type, the standard deviation (S.D.) of the maximum load, the relative strength (expressed as the ratio of the maximum load for that specimen to the maximum load carried by the plain concrete specimen), stress over the gross area, and stress over the net area.

<table>
<thead>
<tr>
<th>Specimen Type</th>
<th>Max. Load (kN)</th>
<th>S.D. (kN)</th>
<th>Relative Strength</th>
<th>Gross Stress (MPa)</th>
<th>Net Stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain</td>
<td>177.6</td>
<td>3.5</td>
<td>1</td>
<td>17.8</td>
<td>17.8</td>
</tr>
<tr>
<td>Hole</td>
<td>150.9</td>
<td>2.4</td>
<td>0.85</td>
<td>15.1</td>
<td>20.1</td>
</tr>
<tr>
<td>Grouted Hole</td>
<td>192.9</td>
<td>4.1</td>
<td>1.09</td>
<td>19.3</td>
<td>19.3</td>
</tr>
<tr>
<td>Plain Cube</td>
<td>239.7</td>
<td>7.4</td>
<td>1.35</td>
<td>24.0</td>
<td>24.0</td>
</tr>
</tbody>
</table>

Table 4.2 – Results of intrusion specimen tests
The capacity of the specimen with the hole was lower than that of the plain concrete specimen. This was expected since there was a reduction in the area of concrete available to carry the load at the centre of the specimen. However, the net stress, the total load divided by the reduced area of concrete due to the hole, was actually higher than the gross stress of the plain specimen. This suggested that there was a localized effect that allowed the concrete in the area of the hole to absorb higher stresses without failing. The closed form solution for the stress distribution around a hole in an infinitely wide plate was first developed by Kirsch (1898) and is taken here from Timoshenko and Goodier (1970) who give the full derivation for the stress in any direction using polar coordinates. Equation 4-7 gives the stress in the direction of loading at any point along the cross section passing through the centre of the hole. Although equation 4-7 is derived for a hole in an infinitely wide plate, Timoshenko and Goodier state that as long as the total plate width is greater than four hole diameters, the error should be less than 6%. In this case the specimens were exactly four diameters wide so a reasonably accurate approximation of the stress state should be expected.

\[
\sigma_{y\hat{z}x} = \frac{\sigma_{\text{ave}}}{2} \left( 2 + \frac{r_{\text{hole}}^2}{r^2} + 3 \frac{r_{\text{hole}}^4}{r^4} \right)
\]

where

- \( \sigma_{y\hat{z}x} \) = stress in the \( y \) direction at \( x \) along the cross section (MPa)
- \( \sigma_{\text{ave}} \) = average stress applied to the specimen away from hole (MPa)
- \( r_{\text{hole}} \) = radius of the hole (mm)
- \( r \) = radial distance away from the centre of the hole (mm)

The stress distribution from equation 4-7 is plotted for one half of the specimen with a hole in Figure 4.7. The average stress is taken as 15.1MPa from Table 4.2
From Figure 4.7 it can be seen that the stress at the edge of the hole is 45.3MPa, or three times the average applied stress. This value is probably an overestimate for concrete as the closed form solution was developed for linear elastic materials. Thus, both the non-linear stress-strain profile of the concrete, as well as its lower tensile strength will not be accounted for. Nevertheless, it offers a conservatively high estimate on the possible levels of stress the concrete would reach locally, and suggests that care will have to be taken when modelling this. The stress changes significantly over a distance of about 10mm from the edge of the hole, indicating that in order to accurately capture this behaviour using FEA a fairly small mesh will be required.

The capacity of the specimen with the grouted hole was actually higher than the plain concrete specimen. Initially this seems counterintuitive as one might expect that the stiffer grout material will create stress concentrations in the surrounding concrete, causing premature failure. Or one might expect the specimen to achieve the same load as a plain concrete specimen since the majority of material is concrete. However, it is also possible that the stiffer grout attracts load towards the centre of the specimen. If this is the case, it is feasible that the concrete in the area of the grout is able to take this increased load as it benefits from confinement provided by the surrounding concrete. This would result in an overall increase in the load carrying capacity of the
specimen. The following section will examine whether the FEA reveals this to be the case.

4.2.3 Finite Element Modelling

The prism tests were modelled using FEA with the ultimate goal of developing a concrete model that would account for the effects of the intrusions, especially the holes. Unfortunately the Wang et al. model seems to give erroneous results for compressive strengths lower than 20MPa. It seems their curve fit of the post-peak behaviour was not designed to work for concrete strengths below 20MPa, and solving their equations results in a kink developing in this portion of the curve. As such, the FEA models were run using a concrete compressive strength of 20MPa, and the relative magnitude of results was compared. The tensile strength of the concrete was 2.7MPa, which was taken from tensile tests using similar concrete mixes. The grout compressive strength was 47MPa, which was determined from cylinder tests on the grout. The finite element meshes are illustrated in Figure 4.8. By using symmetry, only one quarter of each specimen needed to be modelled thus reducing analysis times. The choice of the mesh size was difficult since as mentioned earlier, Kesse found that elements on the order of 2 to 3 times the maximum aggregate size give the best results. Here the mesh had to be much smaller to capture both the geometry and stress changes involved. There is a trade-off between keeping the elements large enough to properly model material behaviour and small enough to measure localized changes. Examining Figure 4.7 it can be seen that major changes in slope can be reasonable captured if the curve is split into 5mm sections. As such, the average element size used was 5mm. To ensure that the end conditions were stiff enough to simulate the compressive testing machine, a 30mm thick steel load plate was modelled on top of the specimen and displacement loading was used.
a) Model of plain concrete specimen

b) Model of specimen with hole

c) Model of specimen with grouted hole

Figure 4.8 – Finite element meshes for intrusion models
4.2.4 Evaluation of FEA modelling

The results of the FEA are given in Table 4.3. The table gives the maximum average load obtained by each model, the relative strength (expressed as the ratio of the maximum load obtained by that model to the plain concrete model), stress over the gross section, and the stress over the net section for the three different specimen types.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Max. Load (kN)</th>
<th>Relative Strength</th>
<th>Gross Stress (MPa)</th>
<th>Net Stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain</td>
<td>194</td>
<td>1</td>
<td>19.4</td>
<td>19.4</td>
</tr>
<tr>
<td>Hole(^1)</td>
<td>125</td>
<td>0.64</td>
<td>12.5</td>
<td>16.7</td>
</tr>
<tr>
<td>Hole(^2)</td>
<td>137</td>
<td>0.71</td>
<td>13.7</td>
<td>18.3</td>
</tr>
<tr>
<td>Hole(^3)</td>
<td>148</td>
<td>0.76</td>
<td>14.8</td>
<td>19.7</td>
</tr>
<tr>
<td>Grouted Hole</td>
<td>192</td>
<td>0.99</td>
<td>19.2</td>
<td>19.2</td>
</tr>
</tbody>
</table>

\(^1\) using smeared cracking  
\(^2\) using a weak plane  
\(^3\) using a discrete crack and no hardening curve

Table 4.3 – FEA results for intrusion tests

The plain concrete specimen did not quite reach a stress of 20MPa, which was the uniaxial concrete strength specified. This was because the steel loading plate partially restrained the concrete due to a difference in the Poisson’s ratio of the two materials (0.3 for steel and 0.18 for concrete), which resulted in horizontal compressive stresses being developed in the concrete near the load plate. In 2-D, these horizontal compressive stresses do not offer the significant strength increases due to biaxial confinement when using the von-Mises model that they would in 3-D with triaxial confinement. In fact, in this case the presence of horizontal compressive stresses actually served to lower the overall strength of the specimen slightly since failure was due to the von-Mises criteria being exceeded in the area where the horizontal stresses were developed. Whilst expansion to a 3-D model might give better results in terms of confinement, the beam models were 2-D so it was decided that a 2-D analysis was more useful. Kesse also discovered this limitation of the von-Mises criterion but decided that the overall accuracy of the model outweighed its lack of accuracy in biaxial confinement, as this stress state does not generally govern beam behaviour.

The hole specimen model failed at a much lower load than expected. One cause for this lowered capacity seemed to be the use of the smeared cracking. As can be seen
from Figure 4.9, the cracking occurs over the full width of the specimen. This result does not correlate well with the experimental work of Sammis and Ashby (1986) who investigated loading brittle porous solids in compression. They tested a series of 10mm thick glass and polymethyl methacrylate (PMMA) plates with holes of 5, 10, and 20mm diameters and widths of between 50 to 100mm. Each plate was tested in uniaxial compression. In every case, a single crack developed parallel to the loading axis at the top and bottom of the hole. These cracks then grew significantly as the compressive stress was increased. How far they grew was dependant upon the ratio of the hole diameter to overall width with shorter cracks being developed as the ratio of the hole diameter to width decreased. Similar results have been achieved numerically by Tang et al. (2005) using a technique called material failure process analysis, which they suggest is extendable to materials such as concrete. When they modelled the work of Sammis and Ashby using this method, which is similar to FEA, they again discovered that a single crack developed at the top and bottom of the hole.

![Figure 4.9 - Cracking in hole specimen with smeared cracking](image)

In order to bring the FEA results in line with the literature, the smeared cracking was removed from the model and instead a weak plane running vertically above the hole was considered. The elements in this weak plane were given a tensile strength of 2.7MPa but no tension softening properties, while the remaining elements were modelled as having the same stress-strain behaviour in tension as in compression. This model resulted in an increased capacity for the specimen (137kN versus 125kN), however it was noted that the tensile stresses in the elements around the hole area exceeded the tensile strength of the concrete. In an attempt to remedy this, the crack was modelled discretely. This was done by releasing the x-restraint on the nodes up to
the same height as the crack predicted using the weak plane, which was 113mm above the hole as illustrated in Figure 4.10. At the same time, since fracture in the elements was no longer an issue due to the use of a discrete crack, the mesh element size was reduced to 2.5mm in order to better model the compression behaviour. The discrete crack resulted in much lower tensile stresses around the hole. However, there was no increase in capacity and the response was not as stiff as with the weak plane model. This lack of stiffness was due to two effects. First, by releasing the nodes, the crack is present from the first loading stage instead of gradually forming as would happen in reality. Second, by using a non-linear compressive stress-strain curve for the concrete, the effects of cracking are overestimated in the area of the crack. This is because the stress-strain curve already accounts for the effects of microcracking in its non-linear shape so when it is combined with a discrete crack the stiffness of the specimen is reduced. In an effort to overcome this, the model was run a final time, with the concrete modelled as a linear elastic material using the initial Young’s modulus given by Wang et al. The model specimen capacity increased to 148kN and the response was slightly stiffer as illustrated in Figure 4.11. The behaviour is still non-linear, despite using a linear elastic concrete compression model, due to the effects of the discrete crack. The real response is probably between these two discrete crack curves since microcracking would have an effect on the elements farther away from the crack. However, it would seem that using the linear elastic model gives a more realistic result in terms of compressive capacity, though it is still underestimated.

Figure 4.10 – Discrete crack model of specimen with hole
Although these models helped to explain the behaviour of the prism with the hole in it, they still did not accurately predict the overall capacity of the specimens. One possible explanation for this was the tendency of the von-Mises criterion to underestimate the effects of biaxial compression as noted earlier. In order to achieve the experimental capacity, the net section stresses have to be considerably higher than the uniaxial compressive strength. Failing to properly predict confinement increases due to biaxial stresses around the hole would result in an underestimate of these increased localized stresses and the overall specimen capacity. However, when the Drucker Prager failure criterion was used, which Kesse suggested gives higher results in biaxial compression, no increase in capacity was developed. It should be noted that even Sammis and Ashby stated that “[t]he behaviour of brittle porous solids in compression is very complicated.” So, whilst the result is puzzling, the goal of this FEA was to develop a concrete model to account for the effect of the holes. Thus, the conservative result obtained here can still be applied to the beam models (see section 4.3.5.2), as it will demonstrate whether a reduced compressive capacity and stiffness affected the beam’s performance.
Although these models helped to explain the behaviour of the prism with the hole in it, they still did not accurately predict the overall capacity of the specimens. One possible explanation for this was the tendency of the von-Mises criterion to underestimate the effects of biaxial compression as noted earlier. In order to achieve the experimental capacity, the net section stresses have to be considerably higher than the uniaxial compressive strength. Failing to properly predict confinement increases due to biaxial stresses around the hole would result in an underestimate of these increased localized stresses and the overall specimen capacity. However, when the Drucker Prager failure criterion was used, which Kesse suggested gives higher results in biaxial compression, no increase in capacity was developed. It should be noted that even Sammis and Ashby stated that "[t]he behaviour of brittle porous solids in compression is very complicated." So, whilst the result is puzzling, the goal of this FEA was to develop a concrete model to account for the effect of the holes. Thus, the conservative result obtained here can still be applied to the beam models (see section 4.3.5.2), as it will demonstrate whether a reduced compressive capacity and stiffness affected the beam's performance.

![Figure 4.11 - Stress-strain curves for hole models - FEA](chart)
The model with the grouted hole reached a maximum load of 191kN. This meant that the relative strength ratio was 0.99 as opposed to the 1.09 ratio that was achieved experimentally. As was suggested earlier, the stiffer grout attracted more load towards the centre of the specimen according to the model. Once again, since the von-Mises failure criterion does not properly account for the beneficial effects of confinement in 2-D, this load concentration caused failure to occur when the von-Mises failure criteria was exceeded in the concrete directly above the grout intrusion. If the model were capable of accounting for the localized confinement effects at the centre of the specimen provided by the surrounding concrete, the result might have been closer to the experimental result. Even so, it served to demonstrate that the grout has little effect on the capacity of the concrete and should, therefore, have little effect on the capacity of the beam. This conclusion has been verified by both the results of the beam tests (B6/30/C/44 which did not have grout intrusions and B9/30/G/42, a specimen discussed in Chapter 6 which had grout intrusions, had almost identical ultimate capacities) and the compressive tests in this section.

4.3 Beam Modelling
4.3.1 Finite Element Mesh

The mesh element size was kept as close to 25mm as possible for all specimens as previously discussed. Figure 4.12a shows the mesh for the control beam, B1/25, along with the loading area and constraints. Figure 4.12b shows the mesh for specimen B2/45/H/20, Figure 4.12c gives the mesh for B3/30/H/22 through B6/30/C/44 while Figure 4.12d presents the mesh for specimen B7/30/G/36. It should be noted that for specimens B1/25 through B5/30/C/27 the load was applied as a pressure over a 100mm length to reflect the smaller bearing pads used in those experiments. For specimens B6/30/C/44 and B7/30/G/36 the length over which pressure was applied was increased to 150mm. In order to allow for full rotation at the support, the beam was supported on a steel bearing pad, which was constrained against vertical translation at only one node. In between the beam and the bearing pad there was a transition material (denoted as plaster in Figure 4.12a) that had the same compressive properties as the concrete in the beam, but was not susceptible to cracking. This material was employed by Kesse (2000) as well to eliminate spurious cracking near the supports at higher loads.
Figure 4.12 – Beam mesh layout and boundary constraints
4.3.2 Concrete strength

In his research Kesse (2003) used the cylinder strength of the concrete, which he calculated as the cube strength multiplied by 0.8. In the current research the concrete cube strength is used. The reason for this decision is two-fold. Initially the beam models were run using the cylinder strength as calculated by Kesse in conjunction with the concrete compression curve developed by Wang et al. The model beam stiffness, especially in the linear elastic region, was lower than the actual beam stiffness when the cylinder strength was used. Employing the cube strength gave more realistic predictions of the stiffness. It is believed that the concrete in the flange, due to the presence of both of additional links as well as longitudinal reinforcement, is benefiting from confinement effects. This could explain why Kesse achieved better results using the cylinder strength, as his specimens were rectangular and may not have benefited from this additional confinement. Another advantage of the stiffer beam response is that it allows for the use of a lower shear retention factor, $\beta$. This is due to the fact that the shear modulus, which is calculated using the elastic modulus, is slightly higher for the stiffer concrete cube model. So to obtain the same effective shear modulus a lower shear retention factor is required. The importance of using a lower shear retention factor will be presented later in this chapter when strap strains are discussed.

4.3.3 Determining the Shear Retention Factor

A constant shear retention factor is employed here in order to calculate the amount of shear carried by the concrete. Choosing the correct value of $\beta$ is difficult since it will affect the ultimate capacity, the beam stiffness and the transverse reinforcement strains. The ultimate capacity is affected since higher $\beta$ factors will enhance the so-called concrete contribution to the overall shear capacity. The beam stiffness after cracking should initially be lower than the experimental results since the initial cracked shear modulus is underestimated using a constant shear retention factor. Finally, unless the concrete contribution is predicted accurately, the transverse reinforcement contribution will also be incorrect since the total capacity of the beam is the result of both.
In order to choose a $\beta$ factor, one must decide which of the three affected parameters is most important. In this case, it was decided that designers would be most concerned about the ultimate capacity with secondary consideration given to stiffness and transverse reinforcement strains. As such, the $\beta$ factor in this study was chosen based on which one gave the best prediction for the capacity of the control beam. However, the transverse reinforcement strains can be very important, especially for the CFRP, as will be discussed later. Five possible $\beta$ factors were tried. The shear force versus deflection results of each analysis performed for B1/25 are presented in Figure 4.13. For clarity, only the results obtained using the modified Newton-Raphson method are presented. However, these do not vary significantly from those achieved using the regular Newton-Raphson method with arc-length control since the failure mode in each case is brittle.

It can be seen that using a $\beta$ factor of 0.08 gives the best prediction of the failure load. Interestingly, using a $\beta$ factor of 0.2, which was used by Kesse (2003) to give the best results for his rectangular sections, overestimates the capacity of the control specimen by 20%. This is believed to be due to the fact that the flange of the T-beam remains
largely uncracked throughout the analysis, thus contributing significant shear capacity. To counteract this, the shear capacity of the cracked elements needs to be reduced to a greater extent than would be required for an ordinary rectangular section where the contribution of the uncracked portion of the beam to the shear capacity would be smaller. Al-Mahaidi et al. used a $\beta$ factor of 0.05 to obtain fairly accurate results, further suggesting that T-beams require lower $\beta$ factors. A $\beta$ factor of 0.08 will be employed for all the FEA models, unless otherwise stated.

4.3.3.1 Stiffness Predictions

The stiffness results demonstrate the expected behaviour, with higher $\beta$ factors producing stiffer responses. The initial uncracked stiffness predictions are quite accurate validating the use of the Wang et al. curve in conjunction with the cube strength. Once the beam cracks, the predicted stiffness drops due to the lower shear modulus. The FE peak displacement of 13.9mm is within 10% of the actual peak displacement and the FE capacity of 88.6kN is within 1% of the actual beam capacity. Although the beta factor of 0.08 was chosen based on the beam capacity prediction, the stiffness prediction is also fairly accurate despite the use of a constant shear retention factor.

4.3.3.2 Failure Mode

The failure for a shear retention factor of 0.08 appears to be due to a loss of stiffness in the concrete elements under the pressure loading. Ordinarily one might not expect failure directly under the loading. However, in this case failure in this region is entirely possible due to the use of smaller bearing pads as will be discussed later. This loss of stiffness does not immediately indicate the type of failure and so various failure modes must be checked.

The model is first checked for flexural failure. Examining the von-Mises stresses for each element reveals that none of the concrete elements have yet reached the failure criterion, although they are quite close to failure in the vicinity of the loading. A check of the longitudinal reinforcement stresses during the final load step indicates
that none of the bars have yielded. These two results tend to rule out the possibility of a flexural failure.

The second potential failure mode is shear. Two of the shear links have entered the non-linear portion of the stress-strain curve suggesting that a significant amount of shear force has been transferred to the links. The failure is also quite sudden with no evidence of a yield plateau regardless of the solver used. The crack pattern indicates shear cracks running from the support point into the flange just below the load point. Finally, by using a higher $\beta$ factor, an increase in capacity is possible as illustrated in Figure 4.13 indicating that an increase in the concrete shear contribution will increase the capacity. All of these considerations indicate that failure is driven by shear.

It is interesting to note that with full shear retention, a flexural failure is possible. Figure 4.13 illustrates that the curve using a full shear retention factor starts to plateau at approximately 120kN. An examination of the FEA longitudinal reinforcement stresses revealed that the bars had begun to yield, another strong indication of a flexural failure. This suggests that it might be possible to retrofit these specimens and achieve a ductile flexural failure.

Bearing in the region of the loading is also a possible cause of failure. However this will be discussed in greater detail in section 4.3.8.

4.3.3.3 Transverse Strains

The shear link strains for test specimen B1/25 are plotted against the FEA link strains in Figure 4.14. The overall behaviour of the transverse reinforcement seems to be predicted adequately. There is an initial period before the shear cracks form when the link strains are negative. According to the experimental results shear cracking, and the subsequent increase in link strains, first occurs at around 34kN whereas the FEA places this value closer to 37kN, which seems to be in reasonably good agreement.
The strain in the inner link is considerably overestimated by the FEA. A comparison of the crack pattern predicted by DIANA to the actual crack pattern, given in Figure 4.15, reveals that the extent of shear cracking under the load points is over predicted by FEA resulting in the excessive strains in the inner links. Where one would expect a single discrete crack forming at the load pad, as seen in the experiments, DIANA has smeared this crack throughout the region, causing an overestimate of the link strains.

The middle and outer link strains are predicted more accurately although the use of a constant shear retention factor means that they are not exact. The FEA suggests that the strains in the outer link will exceed those in the middle link. Although this result disagrees with the experimental data, it is difficult to know whether it is actually incorrect. The strain gauges were placed at the mid-height of the outer link, whereas the shear crack formed towards the bottom of the outer link in the specimen. Thus it is possible that the experimental results do not reflect the peak strain in the link and that the FE prediction, which was taken about 50mm below the mid-height of the link, is actually more accurate. It is more important to note that the similarity in strains indicates that the FEA steel contribution prediction is reasonable. However, since steel is a ductile material, small inaccuracies in the calculated strain are balanced by
load redistribution. This is not the case when considering the brittle behaviour of the CFRP straps in later specimens.

\[ \text{Crack strains} \quad > 0.0057 \quad < 0.0057 \]

![Diagram showing crack patterns](image)

a) DIANA prediction

![Actual crack patterns](image)

b) Actual

Figure 4.15 – B1/25 crack patterns

### 4.3.4 Specimen Predictions

A summary of the experimental failure patterns is given in Figure 4.16. The major crack path, bearing pad sizes and cross section for selected specimens is illustrated. Because the major crack paths were often three-dimensional when the smaller bearing pads were used, the major crack path has been shown at the centre and edge of the specimens.

The results of the FEA are summarized in Table 4.4. The experimental capacity (Exp.) and the FE predicted capacity (FE) are given as well their ratio, which gives an indication of the accuracy of the FEA. The deflection (Δ) results are also given. The experimental failure mode (Exp. F.M.) is compared to the predicted failure mode (FE F.M.). The two main failure modes are shear in the concrete (S. C.) or flexure.
<table>
<thead>
<tr>
<th>Specimen</th>
<th>Exp. (kN)</th>
<th>FE (kN)</th>
<th>FE / Exp.</th>
<th>Exp. Δ (mm)</th>
<th>FE Δ (mm)</th>
<th>Exp Δ / FE Δ</th>
<th>Exp. F. M.</th>
<th>FE F. M.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1/25</td>
<td>88.2</td>
<td>88.6</td>
<td>1.00</td>
<td>15.5</td>
<td>13.9</td>
<td>0.90</td>
<td>S. C.</td>
<td>S. C.</td>
</tr>
<tr>
<td>B2/45/H/20</td>
<td>95.4</td>
<td>100</td>
<td>1.05</td>
<td>15.6</td>
<td>15.1</td>
<td>0.97</td>
<td>S. C.</td>
<td>S. C.</td>
</tr>
<tr>
<td>B3/30/H/22</td>
<td>91.4</td>
<td>98.4</td>
<td>1.07</td>
<td>16.0</td>
<td>15.0</td>
<td>0.94</td>
<td>S. C.</td>
<td>S. C.</td>
</tr>
<tr>
<td>B4/30/G/25</td>
<td>105.2</td>
<td>106</td>
<td>1.01</td>
<td>16.9</td>
<td>15.9</td>
<td>0.94</td>
<td>S. C.</td>
<td>S. C.</td>
</tr>
<tr>
<td>B5/30/C/27</td>
<td>111.0</td>
<td>111</td>
<td>1.00</td>
<td>16.9</td>
<td>16.3</td>
<td>0.96</td>
<td>S. C.</td>
<td>S. C.</td>
</tr>
<tr>
<td>B6/30/C/44</td>
<td>140.9</td>
<td>143</td>
<td>1.01</td>
<td>22.1</td>
<td>29.3</td>
<td>1.33</td>
<td>S. C.</td>
<td>Flexure</td>
</tr>
<tr>
<td>B7/30/C/36</td>
<td>134.7</td>
<td>138</td>
<td>1.02</td>
<td>29.1</td>
<td>24.6</td>
<td>0.85</td>
<td>Flexure</td>
<td>Flexure</td>
</tr>
</tbody>
</table>

Table 4.4 – Comparison of actual and FEA predicted results

Figure 4.16 – Specimen major crack paths, bearing pads and cross sections
The FEA was very accurate in terms of predicting the ultimate capacity and also showed promising accuracy in terms of displacements. Specimens B2/45/H/20 through B5/30/C/27 failed in a manner similar to B1/25, with the elements under the pressure loading losing stiffness causing the analysis to diverge, which was deemed to be a shear failure. Specimens B6/30/C/44 and B7/30/G/36 appeared to fail in flexure with the longitudinal reinforcement reaching the yield strain followed by failure of the compression elements. Given the promising accuracy of the models, the results will now be used to explore various key factors affecting capacity that were identified during the experimental program, namely the role of intrusions in the flange (both holes and grout filled holes), strap penetration, bearing capacity and strap strains.

4.3.5 Role of the holes in the flange

Two approaches were used to gauge the impact of the holes in the flange for B2/45/H/20 and B3/30/H/22. They will be examined separately below.

4.3.5.1 B2/45/H/20

In the case of B2/45/H/20, since the holes were fairly low in the compression flange it was possible to model the holes into the beam discretely. This was done by removing an element from the mesh just above the strap, which created a void in the flange of the beam. This approach should be conservative as it assumes the hole passes through the entire width of the flange whereas in reality only part of the flange was affected, although the real hole was at an angle. The effect of the hole could then be gauged by comparing these results to those from a specimen without the holes.

There was actually no difference in load carrying capacity between the models with and without the holes as each specimen achieved a capacity of 99kN. However, the holes did have a significant effect on the crack pattern as can be seen in Figure 4.17. Figure 4.17a shows the crack pattern when holes are not modelled into the flange. In this case the cracks are mainly in the web except for a couple cracks that form above the straps. When holes are introduced, as illustrated in Figure 4.17b, there is much more extensive cracking in the flange. The cracks are widest adjacent to the middle
holes. This is in good agreement with the analysis on the prisms with holes that suggested the holes would act as crack propagators, with cracks forming parallel to the principal compressive stress direction.

The fact that there was no difference in specimen capacity suggests that while the holes increased the level of cracking, they did not reach high enough into the flange to affect the specimen capacity. In other words, a shear plane could not be formed that reached the top of the flange due to the holes alone.

4.3.5.2 B3/30/H/22

Because the holes in specimen B3/30/H/22 extended from the bottom of the flange to the top, it was not possible to model them discretely as was the case for B2/45/H/20. Instead the results of the FE analysis on the prism with a hole were used to develop a modified material model to account for the presence of holes in the flange. The holes have a two-fold effect on the compressive capacity of the beam, as they reduce both
the ultimate compressive capacity and the stiffness of the surrounding concrete. The FEA model of the prism with a hole and a discrete crack, as illustrated in Figure 4.10, was run using the concrete strength for this beam of 22.3MPa to develop such a concrete model for the beam. In order to create the stress-strain curve, one needs to determine the gauge length over which to measure the strain. As can be seen from Figure 4.18, the choice of gauge length has a dramatic impact on the stiffness. If the gauge length was taken as 150mm as shown in Figure 4.19, the response was much stiffer than if the gauge length was taken as 50mm. Similarly the response was even less stiff if the gauge length used was 25mm. The other issue was where to measure the strain. The FEA strains were higher when taken above the hole than they were when measured at the edge of the prism. Since the goal of this analysis was to determine whether the reduced compressive capacity and stiffness had an adverse effect on specimen capacity, the most conservative approach of using a 25mm gauge length directly above the hole was taken.

![Figure 4.18 - Concrete stress-strain curves for B3/30/H/22](image-url)
This new material was used in 100mm wide sections in the flange of the beam as illustrated in Figure 4.20. The width of the region was chosen based on the distance over which the holes had an impact on stiffness. As can be seen in Figure 4.18, the behaviour of the concrete using a gauge length that extended from 50mm beyond the centre of the hole to the end of the prism (denoted as ‘Hole Concrete – 50mm beyond hole’ in Figures 4.18 and 4.19) is actually stiffer than the plain concrete, so modelling this region using the weak material would be inappropriate. Thus the area affected by the hole was taken to be 50mm on either side of the hole and the concrete model with a 25mm gauge length was used. Another built-in conservative assumption was the area of the flange occupied by the holes. At the top of the flange where the compressive stresses will be highest, the area occupied by the holes was actually quite small. Whereas by using the results of the prism tests, it is assumed that 25% of the flange area will be occupied by holes. This assumption is unconservative towards the bottom of the flange where the holes occupy more than 25% of the flange, but the compressive stresses in this area are also considerably lower and so the top of the flange should still be critical.
Despite having significantly reduced the maximum capacity and stiffness of the concrete in the region of the holes, this specimen still achieved the same ultimate shear force as a specimen with regular concrete throughout the flange. This indicated that failure in this specimen was not affected by the reduced compressive capacity due to the holes.

The other potential problem created by the presence of holes is the formation of tensile cracks parallel to the compressive stress direction. Unlike for the prism test models, it was not possible to model discrete cracks into the flange by releasing nodes. This was because if compressive stresses had developed in the area of the cracks, there was the possibility that the released nodes could cross over each other creating a situation where two elements overlapped. In order to avoid this undesirable situation, the first three rows of elements in the weak region closest to the pressure loading, as illustrated in Figure 4.21, were given a small tensile strength of 0.2MPa with no tension softening. This meant that cracking developed in these elements at very low loads, which is the same behaviour developed when a hole is present. In order to isolate cracking as a problem, the compressive strength and stiffness of the elements in all the weak regions was made the same as the surrounding concrete.
The model developed extensive cracking in the flange, as can be seen in Figure 4.21, and failed at a load of 93.5kN which was 5.5kN less than the models with regular and reduced compressive capacity. This is still slightly higher than the experimental specimen capacity (91kN), however what is not accounted for is the area of the holes themselves. The model assumes that while cracks develop at an early load, there is still concrete present where the holes are to take the shear whereas this was not the case in the actual specimen. Thus, a further reduction in capacity would be expected if the area of the holes could be accounted for. Also, whilst the actual crack was observed to follow the path of the hole, in the model the cracking occupies several rows of elements. Using a 2-D analysis, it is impossible to represent the actual 3-D shear path. However, what the analysis does illustrate is the detrimental effect that cracks developed at low loads in the flange have on the overall capacity of the specimen. It seems the failure of specimen B3/30/H/22 was not caused by a reduction in compressive capacity, but instead by the development of cracks caused by the holes. Since the holes occupied the full height of the flange, these cracks were able to form a complete failure plane, unlike B2/45/H/20, resulting in a premature failure.

4.3.6 Grout intrusions in the flange

It was noted in Chapter 3 that there was a slight difference between the capacity of specimens B4/30/G/25 and B5/30/C/27. This difference was thought to be due to the use of a stiffer material, grout, to fill the holes. However, the predictions given in Table 4.5 for both specimens were quite accurate, even though the grout intrusions were not modelled into B4/30/G/25. The difference in capacity instead seems to be due to a slight variation in concrete strength (24.6MPa for B4/30/G/25 versus 26.7MPa for B5/30/C/27). This result correlates well with the results of the prism tests and FEA presented earlier in the chapter, which indicated that the grout would not reduce the concrete compressive capacity.

4.3.7 Strap Penetration

In the experimental work, it was noted that the cracks passed over the straps in specimen B2/45/H/20 and so B3/30/H/22 was developed to remedy this issue.
Unfortunately the presence of the holes meant that B3/30/H/22 actually had a lower capacity than B2/45/H/20. Using FEA, the beams could be rerun with no holes in the flange in order to explore the effects of strap penetration. To eliminate the concrete strength as a variable, both beams were also run with the same concrete model having a cube strength of 24.8MPa (the same as the control beam). This meant that the results in this analysis differ slightly from those given in Table 4.4 and so to avoid confusion the models will be referred to as the ‘lower penetration specimen’ and the ‘higher penetration specimen’. The only variable in this analysis is therefore the depth of penetration of the strap into the flange, which is either 22.5mm from the bottom of the flange for the lower penetration specimen or 90mm for the higher penetration specimen.

The ultimate capacity of the higher penetration specimen was 102.5kN while the lower penetration specimen reached 100kN. This indicated that the placement of the straps in the flange had a slight impact on capacity, however not as significant as was expected. Figure 4.22 shows the crack pattern for each specimen.

![Crack Patterns](image)

**a)** Lower penetration specimen – similar to B2/45/H/20

**b)** Higher penetration specimen – similar to B5/30/C/27

Figure 4.22 – Crack patterns for strap penetration models
In the lower penetration specimen, cracks formed above the middle straps. Unfortunately the smeared crack model does not seem ideal for modelling this type of behaviour. As one can see from the crack patterns, there is still extensive cracking in the web underneath the middle strap, which was not developed in the experimental specimen B2/45/H/20 where a single discrete crack formed above the strap. Similarly the smeared crack model suggests extensive cracking underneath the load pad, which was not present in either specimen. Instead a single discrete shear crack formed in the flange around the bearing pad. However, even using the smeared cracking approach it can be seen that when the straps do not penetrate high enough into the flange cracks can develop above the straps. These cracks could form a plane of weakness allowing the beam to fail in shear. The smeared cracking approach has also lead to the strap strains being poorly predicted, as will be discussed in section 4.3.9.

The difference in capacity between the two beams seems to be driven by the stiffness of the concrete. As seen in Figure 4.22, the elements under the loading have lost stiffness at 100kN in the lower penetration specimen as indicated by the dots. At the same load the elements in the higher penetration still have stiffness remaining, although this stiffness is lost at the next load stage. It would seem, based on the results of this analysis, that strap penetration alone had only a minor effect on beam capacity. However, the crack predictions differ from those obtained experimentally and cast some doubt on this result.

4.3.8 Bearing capacity

The model of the control beam, B1/25, was rerun with a larger loading area in order to determine whether bearing played a significant role in failure. The length of the bearing area was increased from 100mm to 150mm. The maximum capacity did increase from 88.6kN to 93.4kN; this represented just a 5% increase in load carrying capacity, whereas the bearing area was increased by 50%. Specimen B4/30/G/25 (a beam where there was a suspicion that bearing contributed to the failure) was also run with larger bearing pads and in this case no increase in capacity was noted. So, while bearing may have played a small role in the failure of the beam, it did not seem to be the key factor.
However, it is still possible that the size of the load pad played a role in the failure. In specimens B1/25 to B5/30/C/27, because the load pad was only as wide as the web and not the flange, shear cracks developed in the flange beside the load pad as can been seen in Figure 4.23. In the tests performed by Chan (2000) using load pads that extended across the full width of the flange, the shear cracks only reached the edge of the load pad. The loss of stiffness directly under the centre of the loading predicted by the FEA for B1/25 through B5/30/C/27 should not occur if a wider bearing pad is used, based on these experimental observations.

![Constant Moment Region Shear Span Cracking around Load Pad](image)

**Figure 4.23 – Shear cracks around load pad – B1/25**

Specimen B7/30/G/36 also initially failed due to a loss in stiffness in the elements under the pressure loading. While this failure was acceptable for the previous specimens due to the size of the bearing pad, the wider bearing pad should have prevented this type of failure in this case. In order to remedy this, the model for B7/30/G/36 was rerun with a different type of material used for the elements directly under the pressure loading. Only the compressive aspects of the material were input so the tension model was the same as the compression model. By modifying the concrete so that it was not susceptible to cracking in the region directly under the load pad, it was hoped that this would override the effects of smeared cracking elsewhere and provide a more realistic failure. Indeed when this approach was used, the predicted maximum displacement for B7/30/G/36 increased from 21.3mm with regular concrete elements under the loading area to 24.6mm with the compression only elements as illustrated in Figure 4.24.
Thus, while bearing does not appear to be an issue, the width of the load pad seems to have an affect on the nature of the shear failure. A wider pad seems to restrict the growth of shear cracks in the flange, which ultimately affects the capacity as illustrated by specimen B7/30/G/36.

![Graph](image)

**Figure 4.24 - Shear force versus mid-span deflection – B7/30/G/36**

This notion of restricting the development of the shear failure plane is also illustrated by Figure 4.16. For specimen B1/25, there was very little difference in the angle of the failure plane at the centre of the specimen versus that at the edge, which suggests that the size of the load pad had little effect on the ultimate capacity. The capacity of Chan’s control specimen, which had full width load pads, was only slightly higher than B1/25 (100kN versus 88kN for B1/25) despite Chan’s control specimen having a concrete cube strength of 50MPa (102% higher than B1/25). So in the case of the control specimen, because the cracks in the flange already followed such a shallow path, the size of the load pad appears to have had little effect on the shear capacity. For B3/30/H/20 and B4/30/G/25, one can see from Figure 4.16 that the difference in angle between the central shear plane and the edge shear plane is much greater. Both the work of Loov (1998) and equation 3-1 suggest that shear planes with lower angles result in lower shear capacities. The smaller pad allowed these weaker shear planes to form towards the edge of the specimens, resulting in an overall capacity reduction.
4.3.9 Strap strains

The strap strains in each specimen followed two distinct trends in behaviour. In the case of B2/45/H/20 they were overestimated and as such will be discussed in further detail below. In all other specimens the middle strap strains were underestimated. B6/30/C/44 was chosen to represent this group and will also be discussed further.

The shear force vs. strap strain behaviour for the FE model of B2/45/H/20 is plotted in Figure 4.25. The inner and middle strap strains have been predicted inaccurately. This is due in part to the use of a smeared crack model. In the actual specimen, the entire shear crack formed above these two straps. In the FEA, though the highest crack strains did form near the top of the straps, the crack strains were smeared across the web of the beam resulting in artificially high strap strains. These erroneous results are less likely to occur when the strap encompasses more of the beam section, eliminating the possibility of significant discrete cracks forming outside of the strap. The strain prediction for the outer strap, which encompasses most of the cracks and is therefore not likely to be affected by the difference between smeared and discrete cracking, is very accurate at the ultimate load. Thus, the FEA has not modelled strap behaviour as accurately as one would have hoped, but many of these problems might be overcome when the straps are moved higher into the flange.

![Figure 4.25 - FE comparison - Shear force vs. strap strain – B2/45/H/20](image-url)
Figure 4.26 gives the shear force versus strap strain behaviour for the FE model of specimen B6/30/C/44, a specimen where the straps were higher in the flange. In this case, whilst the outer and inner straps were predicted rather well, the middle strap strain was underestimated by 62%. Though the straps did not rupture during the actual test, an underestimate of this magnitude could easily result in a designer predicting a strap is adequate when in fact it will rupture.

The question becomes: why were the capacities of the beams predicted so accurately, if the strap strains are predicted so badly? The answer is believed to be due to the failure mode. In specimens B2/45/H/20 through B5/30/C/27 failure was controlled by a loss of stiffness under the loading. In these cases, as long as the stiffness of these elements remained critical, failure would occur there regardless of the strains in the straps. Failure in B6/30/C/44 and B7/30/G/36 was dominated by flexure. Thus as long as there was enough shear capacity to force a flexural failure, the distribution of stresses would not matter. To further investigate this point, the shear retention factor, $\beta$, was varied for specimen B6/30/C/44. The shear force versus middle strap strain results are presented in Figure 4.27 for shear retention factors of 0.03, 0.05, 0.08 as well as a variable shear retention factor. A variable shear retention factor adjusts the
shear stiffness of the concrete based on the crack strain, reducing the stiffness as

This type of model could be very effective at modelling the strap

resulting in an increase in strap strains. Unfortunately whilst DIANA has a variable

shear retention factor, it is no longer supported. However, an analysis was run on

B6/30/C/44 using the variable shear retention factor, which is still available in

DIANA despite not being supported. The variable shear retention factor, \( \beta \), is
calculated using equation 4-8.

\[
\beta = \frac{1}{1 + 4447 \varepsilon_{cr}}
\]

where \( \varepsilon_{cr} \) is the crack strain in an element

The variable shear retention factor actually gave the worst prediction with lower strap
strain results than those based on a shear retention factor of 0.08. This result was
somewhat odd since taking the average crack strains from the analysis at failure as
approximately 0.005, the variable shear retention factor should have been 0.04 using
equation 4-8. So one might have expected higher strains from the variable shear
retention factor than if a constant value of 0.08 was used. Unfortunately since the
shear stiffness of the concrete based on the crack strain, reducing the stiffness as cracks grow larger. This type of model could be very effective at modelling the strap behaviour as it would reduce the concrete contribution as the shear force increases, resulting in an increase in strap strains. Unfortunately whilst DIANA has a variable shear retention factor, it is no longer supported. However, an analysis was run on B6/30/C/44 using the variable shear retention factor, which is still available in DIANA despite not being supported. The variable shear retention factor, $\beta$, is calculated using equation 4-8.

$$\beta = \frac{1}{1 + 4447 \varepsilon_c}$$

where $\varepsilon_c$ is the crack strain in an element

The variable shear retention factor actually gave the worst prediction with lower strap strain results than those based on a shear retention factor of 0.08. This result was somewhat odd since taking the average crack strains from the analysis at failure as approximately 0.005, the variable shear retention factor should have been 0.04 using equation 4-8. So one might have expected higher strains from the variable shear retention factor than if a constant value of 0.08 was used. Unfortunately since the
variable shear retention factor is no longer supported, the DIANA manual does not give any indication as to why this is the case. The most accurate prediction in terms of middle strap strain at ultimate load was obtained using the lowest shear retention factor, 0.03. It should be noted that while yielding of the longitudinal reinforcement did occur using this value, there was almost no ductility in the specimen. This illustrates the huge amount of variability and uncertainty involved in the choice of the shear retention factor. For example, using a factor of 0.05, whilst entirely reasonable for the beams in the current study, would underestimate the capacity of the beams tested by Kesse (2003). The constant shear retention factor seems to be a poor model when using FEA approaches to predict the strap strains.

This fact was also discovered by Kesse (2003) who wrote a computer code that instead calculated the shear stiffness based on work by Yoshikawa et al. (1989). In this approach, the cracked stiffness matrix is a function of the normal and transverse stiffness along the crack and is non-linear so, as the crack opens, the shear stiffness of the concrete decreases. When Kesse employed this technique on simple models containing only a few elements, he found that the strap strain was also significantly more non-linear than when a constant shear retention factor was used. Unfortunately Kesse’s model was not always numerically stable and required large amounts of memory as significant quantities of data had to be stored at each load step to evaluate the crack stiffness at the next load step. It seems that a variable approach to shear stiffness is difficult to implement in a stable and reliable manner, which is also evidenced by the lack of such a factor in DIANA. It is recommended that the constant shear retention factor with smeared cracking models should not be used to attempt to design CFRP strap retrofitting systems. There is too much variability for a designer approaching a problem for the first time, and most likely lacking the significant amount of data available in the current study as well as to Kesse, to suggest that their strap design would be safe beyond a reasonable doubt.

4.4 Redesign of low strength concrete specimen retrofit

Specimens B4/30/G/25 and B5/30/C/27 both failed in shear experimentally before reaching their flexural capacity despite having no holes in the flange to cause such a reduction. In order to determine whether it is possible for low strength concrete beams
of this type to achieve a flexural failure when retrofitted, a redesigned low strength concrete beam was modelled. One issue that needs to be reconciled is the effect of the bearing pad. The elements under the pressure loading were replaced with elements that used the same stress-strain model for tension as for compression (the same technique that was used to remodel specimen B7/30/G/36). The length of the load pad was also increased to 150mm to further reduce any bearing issues. Finally, the strap spacing was reduced to 200mm as the experimental results of B7/30/G/36 indicated that this was a more efficient arrangement.

The model was then rerun with the same strap area and prestress to see if any further changes were required. Although the overall beam capacity increased from 106kN to 111kN, failure still occurred in the elements under the pressure load towards the constant moment region end. It seemed that the combined effects of the stress due to the pressure loading and the stress due to bending were causing these elements to exceed the von-Mises stresses and then fail. Since it was possible that the load pad provided localized confinement effects, the model was run again but with the compressive capacity of the elements under the loading increased by 20% to 30MPa. The choice of this level of increase was arbitrary; it was simply used to see if the capacity of the beam could be increased if the localized compressive strength under the loading were increased. When the model was rerun, there was no increase in capacity, the failure merely shifted from the elements directly under the loading, to the element next to the applied load in the constant moment region. Whilst it was possible to justify raising the compressive capacity of the elements directly under the applied loading, it is not as easy to justify raising the compressive capacity of the elements next to the loading since they do not benefit from confinement due to the load pad. As such, it seems that with the low strength concrete (approximately 25MPa), a flexural failure cannot be achieved using FEA in these specimens due to localized effects.

This localized failure highlights an important point about shear retrofitting in general: the need to check all possible failure modes. There is a temptation when designing a shear retrofitting system to check that there is enough shear and flexural capacity and then assume that this is an acceptable design. However, there is the possibility that having globally increased the capacity, the failure is then driven by a local issue, such
as the combined effects of bearing and flexure for example. Thus, the designer should perform all the relevant structural checks to ensure that the capacity is not affected by a factor other than flexure or shear failure.

4.5 Conclusions

The intrusion tests produced several interesting results. First of all, they indicated that while the presence of a hole reduces the compressive capacity of the specimen, this reduction is not equivalent to the area of the hole multiplied by the compressive strength of the plain concrete. Localized increases in compressive capacity near the hole allow the specimen to take more load before failing. The FEA did not properly estimate these increases in capacity. Using smeared cracking, it was also difficult to develop the single tensile crack running through the hole produced in the research of others. The cracking instead needed to be modelled discretely in order to achieve the observed behaviour. The presence of a grout intrusion actually increased the capacity of the experimental compressive specimens. Although the FEA did not reveal such an increase, it did show that the stiffer grout increased the stress in the concrete in the region of the grout. It is believed that this concrete actually benefits from the confinement of the surrounding concrete and thus has increased capacity. These confinement effects were not captured by the FEA resulting in a prediction for the grout intrusion specimens that was almost identical to the plain concrete specimens. However, based on this result, grout intrusions are not expected to affect the overall beam capacity significantly.

The goal of the intrusion modelling was to develop a concrete model to account for the presence of holes in the compression flange of B3/30/H/22. Because the beam analysis was two-dimensional, it was not possible to discretely model the holes in the flange. Instead, by modelling the results of the prism tests, a concrete model with reduced compressive capacity and stiffness was developed, which could then be used to see what role, if any, the holes played in the beam capacity. Although the FEA model of the prism with a hole in it gave conservative results when compared to the experiments, this was deemed to be acceptable since it would further highlight any effects the holes had on the capacity of the beam.
The constant shear retention factor of 0.08 was obtained from a calibration with the experimental results of the control beam, B1/25, and seems reasonable based on a similar study (Al-Mahaidi et al. 2000). Using this shear retention factor, the capacities of the other six static beam tests were predicted quite accurately.

The effect of the holes in specimens B2/45/H/20 and B3/30/H/22 seemed to be quite different. The holes were modelled discretely in the case of B2/45/H/20 and seemed to have no affect on beam capacity. In B3/30/H/22, because the holes occupied the full depth of the flange, their impact needed to be checked indirectly. Their effect on compression was investigated by using the model developed from the prism tests to reduce the compressive capacity in 100mm sections around each hole in the flange. Despite using this weak material, the overall capacity of the beam remained the same.

The influence of cracks that develop parallel to the compressive stress direction around holes was then investigated by reducing the tensile capacity of the concrete in the area of the holes, to simulate the presence of these cracks. The cracks were found to lower the capacity of the beam. It seems that holes in the flange affect the capacity of the beam by serving as crack propagators, creating a plane of weakness in shear. However, they only affect capacity if they reach high enough into the flange to create a weak plane that reaches the top surface.

The role of strap penetration was investigated by running two FE models that were identical except for the level of strap penetration. It was found that though the capacity was affected slightly, strap penetration did not seem to play a significant role in the failure. However, the crack patterns shed some doubt on this conclusion, as the discrete crack present in the flange of B2/45/H/20 was not predicted using the smeared crack model.

The difference in capacity between B4/30/G/25 and B5/30/C/27 was found to be due to slight variations in the concrete strength, and not the presence of grout intrusions as previously believed.

The role of bearing capacity was checked by increasing the size of the load pads for specimens B1/25 and B4/30/G/25. In each case, the increase in bearing area was found to have little impact on the overall capacity. However, the shape of the bearing
pads was still believed to play a role in failure. Because the pads were not the full width of the flange for specimens B1/25 through B5/30/C/27, shear cracks were allowed to form alongside the pad. This was reflected by the loss of stiffness in elements under the loading in the FEA. When attempts were made to ensure that failure did not occur under the pad but beside the pad as is typically seen experimentally, the deflection capacity of B7/30/G/36 increased. Thus, it seems the pad shape influenced the shear capacity of the beam, and not the bearing capacity as previously speculated.

The smeared crack model with constant shear retention factor was found to be fairly inaccurate in terms of predicting strap strains. This conclusion was supported by the work of Kesse (2003). Considerable underestimations of the strap strains at failure were possible if the shear retention factor is too high and underestimates of the beam capacity are possible if the factor is too low. This coupled with the fact that it is difficult to know what shear retention factor to use in the first place, means that the smeared crack model with constant shear retention is probably not a suitable modelling approach for this shear retrofitting system as strap rupturing cannot be predicted with certainty.

The FEA was then used to see if it was possible to achieve a flexural failure using the strap retrofitting system on the lower strength beams ($f_{cu} \approx 25$MPa). It was concluded that due to a localized stress concentration at the edge of the load pad in the constant moment region, these beams could not actually achieve a flexural failure according to the FEA. This highlights the need to check all possible failure modes in order to determine the critical one.
Chapter 5
Design Equation Development

If a retrofitting system is to be used by designers, there must be numerical models in place to allow them to accurately estimate the capacity enhancement offered by the system. To this end, in this chapter the shear models that were chosen in Chapter 2 will be presented in detail, modified to include the straps and used to estimate the capacity of beams from several different test series to check their accuracy. These series include the current one and the two T-beams tested by Chan (2000), which will demonstrate the accuracy of the T-beam predictions. Kesse’s (2003) specimens will be used to gauge how effectively each model accounts for strap spacing, stiffness, and prestress with rectangular beams. Finally, a series of deep beams that was tested by Stenger (2000) will also be examined. The level of prestress plays a much greater role for deep beams, as will be discussed. Once the most accurate model is chosen, material safety factors will be recommended and a design procedure will be outlined with a worked example.

5.1 Shear Models
5.1.1 Modified Compression Field Theory
5.1.1.1 Background

Vecchio and Collins (1986) developed the Modified Compression Field Theory (MCFT). The basic principle of this model is that any concrete structure can be subdivided into a series of elements. If one understands the behaviour of these elements, then the response of the entire structure can be predicted using either a sectional approach or FEA.

The theory was developed based on an experimental program consisting of 30 RC panels. These panels were designed to simulate the elements referred to above. In order to be used in the model, the panels needed to meet certain criteria. First, the panels had to be large enough so that the stresses and strains could be averaged over the element and local behaviour did not overshadow the global behaviour. The panels used were 890mm by 890mm by 70mm thick. Also, the reinforcement had to be uniform in both the x and y directions, so that an average response could be gauged.
The panels were tested in a membrane element tester that was capable of applying any of the possible in-plane stress states. The applied forces were monitored and the strains were measured at various places within the element.

The results of the panel tests allowed Vecchio and Collins to develop models for the average material properties of both steel and concrete. It should be noted that the average material properties are different from those that would be acquired from standard material tests. A cylinder test, for example, would give a different stress-strain curve for concrete than one based on the panel tests because of the complex interaction of stresses and materials involved in the panel tests. Concrete has a higher compressive capacity in biaxial compression than if one of the principal stresses is tensile, as is often the case in a beam. The stress in the reinforcement is a minimum between cracks and a maximum at the crack and thus the average stress-strain response of the reinforcement lies somewhere in between. The assumption is made that the strain is the same in the concrete and the reinforcement, which assumes no overall bond slip. For any given set of strains in the $x$ and $y$ coordinate system, the principal strains can be calculated using Mohr’s circle. The response of the reinforced concrete element is then the addition of the stresses from the average concrete and steel stress-strain curves corresponding to the principal strains. These average models are illustrated in Figure 5.1. However, it is not enough just to consider the average material models, as increased stresses in the reinforcement at the crack could cause failure. In this case, the stress in the reinforcement increases because it no longer benefits from the added capacity of the concrete in tension at a crack. However, the concrete still contributes some capacity due to aggregate interlock as given in Figure 5.1. The amount of capacity is dependant on the crack width, which is a function of the crack spacing and the principal tensile strain.
The MCFT can be used to predict the capacity of beams using a sectional approach (Vecchio and Collins 1988). The beam cross section is split into a series of horizontal layers. The longitudinal strain distribution and shear stress distribution are then assumed. The stress in each layer is calculated based on these assumed quantities and the material stress-strain relationships developed in the MCFT. The resultant forces in each layer are then calculated and the moment, shear, and axial force in the section can be determined. These results are then compared to the actual loading on the section and the process is iterated if necessary. Clearly such a lengthy process is ideally suited to a computer-based implementation and Professor E. Bentz at the University of Toronto has developed such a program, called Response-2000 (Bentz and Collins 2000). The program allows the user to define unusual cross sections, such as T-beams, and since it divides the beam into horizontal layers it should predict the response of the whole section including the flanges unlike design codes that only consider the web. As with the MCFT, Response-2000 also assumes that the bond between the concrete and the reinforcement is perfect. In order to incorporate transverse reinforcement, a smeared model is used, so the analysis assumes that proper reinforcement detailing has been employed.
5.1.1.2 Inclusion of the Straps

Response-2000 does not explicitly allow for the inclusion of prestressed transverse reinforcement. However, assumptions can be made to allow for the prestress by modeling the CFRP with a bilinear stress-strain curve that is initially very stiff. The second branch of the curve has the regular CFRP stiffness, thus creating a pseudo-prestress. The depth of strap penetration can also be modeled by altering the cover to the transverse reinforcement but as with all the models, specifying any holes in the cross section is quite difficult and was not done in the comparison study.

5.1.1.3 Advantages and Disadvantages

The MCFT has the advantage that it uses material stresses and strains to calculate the response of the section. Thus there is no need to assume strains in the transverse reinforcement, which is appealing for the straps where failure is possible if the strains are calculated incorrectly. There are two main drawbacks to the MCFT and Response-2000. First, the reinforcement is considered to be perfectly bonded to the concrete, which is not the case with the CFRP straps, and may lead to high localized strains in the straps and correspondingly high strap contributions. Second, because the transverse reinforcement is smeared in the analysis, predictions for specimens where a large strap spacing is used may be inaccurate.

5.1.2 Compressive Force Path

5.1.2.1 Background

The Compressive Force Path concept developed by Kotsovos (1988) disputes many of the assumptions on which other shear design models are founded. His conclusions are based on a series of beam tests where the main variables were the span-to-depth ratio, and the placement of the shear links. For two of the span-to-depth ratios used (3.3 and 4.4), he either placed no links in the shear span, links throughout the shear span, or links in only part of the shear span. For a shear span-to-depth ratio of 1.5, he carried out four tests where the location of the shear links was varied as illustrated in Figure 5.2. His results indicated that only providing shear links in part of the shear span was enough to significantly increase the overall shear capacity, in some cases up to the
same level as the specimens with shear reinforcement throughout the span. With links only in the zero shear region, Figure 5.2(d), an increase in capacity over the specimen with no links was also noted. Kotsovos argues that these results suggest that the sectional approach used by codes is incorrect.

Kotsovos explains these increases in capacity by suggesting that almost all the shear force in a concrete beam is carried through the compression zone of the beam as illustrated in Figure 5.3. Shear failures are the result of tensile stresses being introduced into this compressive zone. There are four main factors that contribute to these tensile stresses: changes in compressive path direction (developing tensile force $T_v$ in Figure 5.3), a varying intensity of compressive stress along the path (shown as $T_{\text{direction}}$ in Figure 5.3), the tips of inclined cracks entering the path (illustrated as $T_{\text{crack}}$ in Figure 5.3), and bond failure of the tensile reinforcement. He suggests that careful link placement reduces the possibility of these failure modes and results in the observed capacity increases. The amount of compressive stress carried by the path is often greater than the compressive strength of the concrete, which he indicates is possible because the concrete compressive strength is based on uniaxial compression tests. If the beneficial effects of triaxial compression are considered, the concrete has
sufficient capacity to carry the necessary compressive stress. His work directly disputes two of the main concepts assumed in current code-based shear design: the importance of aggregate interlock and the use of the truss analogy. Citing the significant width of observed cracks, he suggests that the amount of shear that could actually be carried across the crack faces is minimal. He submits that the truss analogy (Ritter 1899 and Morsch 1909) is implausible because many of his specimens did not have enough links to form the tension ties required for the truss, and yet developed full capacity.

Kotsovos et al. (1987) used the Compressive Force Path concept to predict the behaviour of T-beams. Their experimental program included three types of beams. Two types were 6.6m long and varied in cross section. Type I had a T-beam cross-section along the full tested span. Type II consisted of a rectangular cross section the same width as the flange for the first metre of the span and then a T-beam cross section for the rest of the span. Type III had a considerably shorter span of 3.2m. The T-beams had up to 200% the capacity of similarly loaded rectangular beams. Also, there was an increase in capacity of Type II beams over Type I beams. They justified this increase by suggesting that the extra beam width in the region close to the support allowed the compressive force path to transfer the shear from the flange, through the web and into the support without large increases in compressive stress. Furthermore, they observed that some of the cracks prior to failure measured up to 3mm in width. A width that previous research (Fenwick and Pauley 1968) would have suggested led to negligible aggregate interlock and an insufficient contribution to the shear capacity. Kotsovos et al. suggest that T-beams instead act as tied arches. They recommend the

![Compressive Force Path](image)

Figure 5.3 – Compressive force path
use of equation 5-1 as proposed by Bobrowski and Bardhan-Roy (1969) for the design of T-beams for shear. Equation 5-1 is based on earlier work by Whitney (1957) who developed an empirical formula to fit the data of various slab, frame and beam tests.

\[
M_{xx} = 0.875a_{xx}d \left( 0.342b + 0.3 \frac{M_{fi}}{d^2} \sqrt{\frac{z}{a_{xx}}} \right) \quad V_{xx} = 16.66 \quad 5-1
\]

where:
- \(M_{xx}\) = the moment corresponding to shear failure at point \(x\) (Nmm)
- \(M_{fi}\) = the flexural capacity (Nmm)
- \(a_{xx} = M_{xx} / V_{xx}\) (mm)
- \(M_{ax}\) = the applied bending moment at \(x\) (Nmm)
- \(V_{ax}\) = the applied shear force at \(x\) (N)
- \(z\) = flexural lever arm (mm)
- \(d\) = the effective depth (mm)
- \(\rho_s\) = tensile reinforcement ratio \(= A_s/(b_w d)\)
- \(A_s\) = area of longitudinal reinforcement (mm²)
- \(b_w\) = web width (mm)
- \(f_y\) = the yield strength of the tension steel (MPa)
- \(b\) = the effective width (mm)

The shear capacity of the specimen, \(V_{fi}\), can then be calculated using equation 5-2:

\[
V_{fi} = M_{xx} / a_{xx} \quad 5-2
\]

It should be noted that for the T-beams used in the current study, the width of the beam, \(b\), used in equation 5-1 is the same as the web width. Bobrowski and Bardhan-Roy only allow a greater width to be used if flange fillets are present. This assumption is made despite the inclusion of an appendix in their paper illustrating that the effective flange area for resisting shear is bounded by 45° lines extending from the intersection of the flange and the web into the flange as previously illustrated in Figure 2.8. Equation 5-1 is the same equation used in Kotsovos and Bobrowski’s (1993) procedure for the design of rectangular members, the only term that would create any increase in capacity over that of a rectangular section is the flexural capacity, \(M_{fi}\). Nowhere in Bobrowski and Bardhan-Roy’s paper do they mention the limits on the flange width that can be used for calculating the flexural capacity. Instead they limit the concrete compressive strength to 0.624 times the cube strength and the depth of the rectangular stress block to 0.5d.
Kotsovos and Bobrowski elaborated on this design model by allowing for the inclusion of stirrups if the flexural capacity predicted by equation 5-1 is less than the actual flexural capacity of the beam. In other words, if the load at which a shear failure occurs is less than the load at which a flexural failure would occur, stirrups may be added to ensure a flexural failure. According to their theory, the transverse reinforcement allows the compressive force path to change direction by balancing the unresolved force component in the vertical direction, as illustrated by $T_v$ in Figure 5.3. The amount of shear reinforcement required is a function of the span-to-depth ratio. They recommend a total shear link cross-sectional area to be placed over a length $d$ within the shear span as given by equation 5-3 in order to prevent a shear failure for the span-to-depth ratio used in the current study (3.33).

$$A'_{sv} = \left( V_{ax} - V_{fs} \right) / f_{ys} \quad 5-3$$

where $A'_{sv} =$ total stirrup area placed over a length $d$ (mm$^2$)

$f_{ys} =$ yield stress of the steel shear link (MPa)

5.1.2.2 Inclusion of the Straps

In order to account for the capacity of the transverse reinforcement that is actually provided, equation 5-3 can be rewritten as 5-4. Also included in this equation is a term to account for the capacity of the CFRP straps, which has to be added in to the Kotsovos model.

$$F_{trans} = \frac{A_{sv}f_{ys}d}{s_v} + \frac{A_{FRP}f_{FRP}d}{s_{FRP}} \quad 5-4$$

where $A_{sv} =$ area of steel shear link (mm$^2$)

$s_v =$ steel link spacing (mm)

$A_{FRP} =$ area of CFRP strap (mm$^2$)

$f_{FRP} =$ stress in CFRP strap (MPa)

$f_{FRP} = E_{FRP} (\varepsilon_{prestress} + \varepsilon_{crack})$

$E_{FRP} =$ elastic modulus of CFRP straps (MPa)

$\varepsilon_{prestress} =$ strain due to prestress

$\varepsilon_{crack} =$ strain due to crack opening

$s_{FRP} =$ FRP reinforcement spacing (mm)
Whilst the shear links are assumed to yield defining their contribution to $T_v$, Kotsovos offers little guidance in terms of strap stress. The force in the strap should be at least equal the prestressing force as that is applied before loading of the beam, but the additional force provided by strain in the straps due to crack openings, $\varepsilon_{\text{crack}}$, is harder to define. The Compressive Force Path theory assumes that little or no shear is carried by aggregate interlock, and so the width of the shear cracks is not relevant in determining the capacity. However, the width of the shear cracks is crucial for determining the strain, and thus the stress, in the straps. Since there is no guidance within the model in the forthcoming comparison two total strap stresses (including the prestress) will be assumed: 50% of the ultimate capacity and 100% of the ultimate capacity. Based on strap tests from Chapter 3, the ultimate capacity of each strap is assumed to be 1500MPa. It is also difficult to account for the effects of strap penetration into the flange so in the comparison section all straps were assumed to be fully embedded in the compression flange.

5.1.2.3 Advantages and Disadvantages

The Compressive Force Path theory assumes that all the applied shear force is transferred in the compressive region, an assumption that separates it from the other models considered. If the assumption is correct, it has the potential to be the most accurate model. The major difficulty presented by this model lies in calculating the contribution of the straps because crack widths are not essential to the shear force transfer mechanism, thus the additional stress due to the crack openings must be assumed.

5.1.3 Chen and Teng CFRP Strap Model

5.1.3.1 Background

Chen and Teng (2001) have developed a model for use with the CFRP straps employed in this experimental program. It is a slightly different approach from the other three models as it is only for the CFRP strap contribution, allowing the designer to use code models to calculate the concrete and steel contribution. The model is based on another developed by Chen and Teng (2003) to predict the shear capacity of
beams retrofitted with FRP sheets. They suggest that there is little difference between the CFRP straps, and FRP sheets that have debonded from the concrete beam but are fully wrapped around the beam and so fail due to rupture of the fibres. The main difference is that the straps are discrete elements whereas the sheets are continuous. According to the model, the prestressing effect of the straps on the concrete capacity can be neglected since once it stops confining the concrete, which they assume to be the case at failure, it no longer provides any additional concrete capacity. However, the strain capacity of the strap used up by prestressing cannot be ignored and to accommodate this they propose the strain distribution factor, $D_{fp}$, given in equation 5-5. The strain distribution factor is plotted against the normalized prestressing strain, $\bar{\varepsilon}_{fp,0}$, in Figure 5.4.

$$D_{fp} = \frac{1 + \bar{\varepsilon}_{fp,0}}{2}$$

where

$$\bar{\varepsilon}_{fp,0} = \frac{\varepsilon_{\text{prestress}}}{\varepsilon_{fp,\text{rup}}}$$

$\varepsilon_{fp,\text{rup}}$ = the rupture strain of the CFRP

![Figure 5.4 - Variation in stress distribution with prestressing strain](image)

Figure 5.4 – Variation in stress distribution with prestressing strain
The strain distribution factor assumes that the crack width is zero at the crack tip and a maximum width, which will cause the CFRP straps to rupture, at the other end of the crack. As can be seen from Figure 5.4, if there is no prestress in the straps, the strain distribution factor would reduce to 0.5, indicating that the average strain in the straps is half the maximum. If the straps were prestressed to their full tensile capacity, the strain distribution factor becomes 1, which suggests that the crack cannot open at all as any additional strain will cause failure of the straps. Chen and Teng (2001) suggest limits on the prestressing strain in order to prevent such a failure from occurring immediately after first cracking. The minimum level of prestressing strain, \( \varepsilon_{pp,0}^{\text{min}} \), is given by equation 5-6, while the maximum level, \( \varepsilon_{pp,0}^{\text{max}} \), is given by equation 5-7.

\[
\begin{align*}
\varepsilon_{pp,0}^{\text{min}} & = \varepsilon_{pp,\text{rup}} - \varepsilon_{\text{max}} \quad 5-6 \\
\varepsilon_{pp,0}^{\text{max}} & = \varepsilon_{pp,\text{rup}} - \varepsilon_{\text{min}} \quad 5-7
\end{align*}
\]

The minimum prestressing strain is defined as the rupture strain reduced by a maximum strain term, \( \varepsilon_{\text{max}} \). Chen and Teng propose that \( \varepsilon_{\text{max}} \) be taken as 0.012 based on their test data. The maximum prestressing strain is based on a minimum strain, \( \varepsilon_{\text{min}} \), required to develop aggregate interlock. Unfortunately, the authors were unable to give guidance about this minimum strain due to a lack of experimental evidence. As discussed in Chapter 2, other researchers have recommended a value 0.004 as the maximum strain to develop aggregate interlock. However, because the straps are unbonded this assumption may not be appropriate, as discussed later. Based on this problem, it would seem better to use the prestressing strain recommendations of Kesse, who suggested that prestressing values between 5 and 50% of the ultimate strap capacity, when combined with the correct strap spacing and stiffness, would lead to an adequate retrofit.

The strain distribution factor is then used to calculate the effective stress in the FRP at failure, \( f_{pp,\varepsilon} \), given in equation 5-8.

\[
f_{pp,\varepsilon} = D_{pp} f_{\text{pult}}
\]

where \( f_{\text{pult}} \) = the ultimate strength of the FRP (MPa)
The strain distribution factor assumes that the crack width is zero at the crack tip and a maximum width, which will cause the CFRP straps to rupture, at the other end of the crack. As can be seen from Figure 5.4, if there is no prestress in the straps, the strain distribution factor would reduce to 0.5, indicating that the average strain in the straps is half the maximum. If the straps were prestressed to their full tensile capacity, the strain distribution factor becomes 1, which suggests that the crack cannot open at all as any additional strain will cause failure of the straps. Chen and Teng (2001) suggest limits on the prestressing strain in order to prevent such a failure from occurring immediately after first cracking. The minimum level of prestressing strain, $\varepsilon_{\text{pp},0}^{\text{min}}$, is given by equation 5-6, while the maximum level, $\varepsilon_{\text{pp},0}^{\text{max}}$, is given by equation 5-7.

$$\varepsilon_{\text{pp},0}^{\text{min}} = \varepsilon_{\text{pp},\text{rup}} - \varepsilon_{\text{max}} \quad 5-6$$

$$\varepsilon_{\text{pp},0}^{\text{max}} = \varepsilon_{\text{pp},\text{rup}} - \varepsilon_{\text{min}} \quad 5-7$$

The minimum prestressing strain is defined as the rupture strain reduced by a maximum strain term, $\varepsilon_{\text{max}}$. Chen and Teng propose that $\varepsilon_{\text{max}}$ be taken as 0.012 based on their test data. The maximum prestressing strain is based on a minimum strain, $\varepsilon_{\text{min}}$, required to develop aggregate interlock. Unfortunately, the authors were unable to give guidance about this minimum strain due to a lack of experimental evidence. As discussed in Chapter 2, other researchers have recommended a value 0.004 as the maximum strain to develop aggregate interlock. However, because the straps are unbonded this assumption may not be appropriate, as discussed later. Based on this problem, it would seem better to use the prestressing strain recommendations of Kesse, who suggested that prestressing values between 5 and 50% of the ultimate strap capacity, when combined with the correct strap spacing and stiffness, would lead to an adequate retrofit.

The strain distribution factor is then used to calculate the effective stress in the FRP at failure, $f_{\text{frp,e}}$, given in equation 5-8.

$$f_{\text{pp.e}} = D_{\text{pp}} f_{\text{frp,ult}} \quad 5-8$$

where $f_{\text{frp,ult}}$ = the ultimate strength of the FRP (MPa)
Chen and Teng propose the use of a standard shear model adding the concrete ($V_c$), steel ($V_s$), and FRP ($V_{frp}$) terms together to give the total shear capacity ($V_{tot}$). The authors have taken a model derived for continuous FRP and modified it to give the FRP contribution for discrete straps in equation 5-9.

$$V_{frp} = 2f_{frp} t_{frp} w_{frp} \frac{0.9d(cot \theta + cot \alpha) \sin \alpha}{s_{frp}}$$

where

- $t_{frp}$ = thickness of the FRP (mm)
- $w_{frp}$ = width of the FRP (mm)
- $\alpha$ = angle of fibres with respect to the longitudinal axis

In equation 5-9, the crack is assumed to reach a height of 0.9 times the effective depth, $d$. The number of straps that cross the crack and contribute to the shear capacity is therefore dependent on the crack angle and the strap spacing as well as the strap orientation (although that has been 90° in all experiments to date). Because equation 5-9 was derived from models for continuous FRP, limits have to be placed on the strap spacing to account for the fact that the straps are discrete elements. Otherwise it would be possible to encounter a situation where no straps cross the crack. The limit suggested by Chen and Teng is 0.45$d$ to ensure that the predicted strap enhancement is achieved. This limit is possibly more stringent than required based on the work of Kesse (2003), who suggested a maximum strap spacing of $d$ would provide adequate capacity when the correct levels of stiffness and prestress were provided. However, Chen and Teng had limited data available to them when they developed their model and thus made a conservative estimate of the required spacing, which is possibly more appropriate for design. The effect of strap penetration could be accounted for in this model, by changing the crack height (from 0.9$d$) in equation 5-9. However, it was decided to use the model as presented by the authors.

5.1.3.2 Advantages and Disadvantages

The Chen and Teng approach has the advantage of being specifically for the CFRP straps. It also presents a shear model involving adding shear terms for the concrete, steel and FRP with a recommendation to use standard code equations for the concrete and steel terms, meaning that designers would be familiar with the approach. The
major disadvantage of the model is the assumption that the straps at the widest point of the crack rupture at failure. This assumption can give erroneous results, especially for deep beams, as will be discussed in the comparison section.

5.1.4 Shear Friction Approach

This approach is presented in great detail as it was found to be the most accurate of the models based on the comparison sections that follow. The level of detail may make the approach seem more complicated than it is, and so to aid the reader a design example is presented in section 5.8.

5.1.4.1 Background

Deniaud and Cheng (2001 and 2003) proposed an equation for the shear capacity of an FRP retrofitted specimen based on the shear friction approach developed by Loov (1998). Shear friction assumes that some of the total shear force can be carried along a discontinuity within a concrete member once slip along that discontinuity takes place. The amount of shear that can be carried depends on the roughness of the crack, the width of the crack, and the clamping action provided by external forces. Although commonly applied to interfaces between dissimilar materials and different casts of concrete, Loov considers that a shear crack also represents such a discontinuity.

Equation 5-10 gives the shear friction strength, $v_{ult}$, as proposed by Loov.

$$v_{ult} = k \sqrt{\sigma_c^*} \quad 5-10$$

The parameter $k$ was determined by Kumaraguru (1992) as 0.6 based on an analysis of push-off tests conducted by Walraven et al. (1987), Hofbeck et al. (1969) and Mattock and Hawkins (1972). Interestingly Walraven et al. and Mattock and Hawkins propose shear friction equations of their own that include a so-called ‘cohesion’ term, which Loov does not require. The value of $k$ is conservative for uncracked concrete and unconservative for cracked concrete. Loov argues that along a given shear crack line there is a combination of both cracked and uncracked concrete with the concrete in the compression zone being uncracked. Thus $k$ is an average of the two. $\sigma_c^*$
represents the average normal stress acting along the failure plane. This is a function of the clamping force provided by the shear reinforcement as well as the component normal to the crack of the force in the longitudinal reinforcement as the cracks slide. This term could also be used to introduce the force provided by the CFRP straps as an additional clamping force. Finally, $f'_c$ is the specified compression strength of the concrete cylinder. Figure 5.5 illustrates the forces involved in Loov’s model. The normal stress, $\sigma$, is equivalent to the resultant force, $R$, divided by the area of the shear plane allowing Equation 5-10 to be rewritten as Equation 5-11.

![Figure 5.5 - Forces acting along the shear plane](image)

$$S = k \sqrt{RAf'_c}$$

\[ S = (T - N)\cos \theta + \left( V_n - \sum T_v \right)\sin \theta \quad \text{from equilibrium (N)} \]

\[ R = (T - N)\sin \theta - \left( V_n - \sum T_v \right)\cos \theta \quad \text{from equilibrium (N)} \]

\[ A = \frac{b_v h}{\sin \theta} \quad \text{(mm$^2$)} \]

\[ T = A_d E_v \epsilon_v \leq A_d f'_v \quad \text{(N)} \]

$N$ = axial force applied to the beam (N)

$T_v$ = tensile resistance of stirrup (N)

$h$ = the depth of the beam (mm)

Solving equation 5-11 for $V_n$, the shear strength, gives a quadratic equation, the solution for which is given by equation 5-12.
\[ V_n = 0.5k^2 C \left[ \frac{T - N}{0.25k^2 C} + \cot^2 \theta \right]^{1/2} - \cot \theta \left[ 1 + \cot^2 \theta \right] - (T - N) \cot \theta + \sum T_v \]

where \( C = f_c' b_v h \) (N)

Loov presents the equation in its factored form for design but the safety factors have been removed from 5-12 so that the accuracy of the model can be determined. The main variables in this equation are the crack angle, \( \theta \), the force in longitudinal reinforcement, \( T \), the force in the shear links, \( T_v \), and the normal force, \( N \). The crack angle Loov suggests can be bounded between a crack stretching from the support to the load point, and a crack angle of 90°. The crack angle that produces the lowest value of shear capacity is assumed to be the one that causes failure. By analyzing various results of this equation, Loov determined that the failure plane would have the lowest possible angle in order to minimize the concrete contribution whilst also intersecting the fewest number of stirrups to minimize the reinforcement contribution. The implication is that these failure planes govern the ultimate capacity, despite the possibility that other crack patterns can form at lower loads and under different loading conditions. This assumption correlates with the work of Walraven et al. who concluded from push-off tests that load history has little effect on ultimate shear friction capacity. Three such failure planes that are critical for beams with stirrups are illustrated in Figure 5.6. Plane 1 starts at the bottom of one stirrup and goes to the top of the next thus having the minimum angle while crossing no stirrups. Plane 2 goes from the bottom of one stirrup, through the second stirrup and to the top of the third stirrup. Plane 2 has a much shallower angle, leading to a lower concrete contribution, but crosses a stirrup, increasing the reinforcement contribution. Plane 3 has a lower angle still, decreasing the concrete contribution, but crosses two stirrups, increasing the steel contribution. The optimum solution is the one that gives the minimum shear capacity, \( V_n \).
Concrete and Longitudinal Reinforcement Contribution

$T$, $T_r$, $N$ and $\theta$ are still unknowns for design. As a simplification, Loov assumed that the axial force, $N$, acting on the beam was zero (a reasonable assumption for most beams). He then plotted $V_n/C$ versus $s_r/d_s$ (equivalent to varying $\cot \theta$) using equation 5-12 for varying longitudinal reinforcement ratios ($T/C$) considering failure along plane 1 only as shown in Figure 5.7. It was discovered that the curve given by equation 5-13 closely fits the data. By employing equation 5-13, $T$ can be expressed in terms of the crack angle and is eliminated as a variable for design. In order to compensate for the fact that 5-13 gives an unconservative solution, Loov recommends the use of a value of $k$ equal to 0.5.

\[
\frac{V_n}{C} \leq 0.25k^2 \frac{d_s}{s_v} \tag{5-13}
\]

where $d_s = \text{the length of the stirrup (mm)}$
Transverse Reinforcement Contribution

The force in the stirrups, \( T_v \), is then integrated into equation 5-13 by considering the number of stirrups along the crack, \( n_v \), to have yielded producing equation 5-14. By assuming that the links yield, their contribution is only dependant on the number of stirrups crossed, which is a function of the crack angle, \( \theta \). Equation 5-13 has been multiplied by the compressive force, \( C \), and expanded. The term \( d_s/s_v \) in 5-13 has been replaced by \( \tan \theta \) to give it in the form used by Deniaud and Cheng. Thus, the only variable left is the crack angle, \( \theta \), which can be varied to give the critical minimum value of the shear force, \( V_n \).

\[
V_n = 0.25k^2 f_c b_w h \tan \theta + T_v n_v 
\]  
5-14

Maximum Shear Capacity

Loov indicates that the shear friction approach cannot be applied without limit. He proposes that the maximum possible shear capacity be given by equation 5-15 for concrete cylinder strengths of less than or equal to 28MPa and by equation 5-16 for cylinder strengths greater than 28MPa.
\[ V_n = 0.25 f'_c b_u h \quad \text{for} \quad f'_c \leq 28 \text{MPa} \]
\[ V_n = 7.0 b_u h \quad \text{for} \quad f'_c > 28 \text{MPa} \]

**Deniaud and Cheng FRP Contribution**

In order to account for the increased capacity of a T-beam section, equation 5-14 can be further modified based on the work of Tozser and Loov (1999). They assume the effective area shown in Figure 2.8 can be used. The model proposed for a T-beam accounts for the change in crack angle between the web and the flange by allowing for the use of two variable crack angles. Equation 5-17 is the retrofitted capacity of a RC T-beam as developed by Deniaud and Cheng (2003) for beams retrofitted with continuous FRP in a ‘U’ configuration on the web only. They propose that the value of \( k \) be taken as 0.5 for the web and 0.7 for the flange. This is based on the suggestion by Loov that a \( k \) of 0.6 is conservative for uncracked sections (the flange) and unconservative for cracked sections (the web).

\[
V_n = 0.25 f'_c \left( k_w^2 A_{cw} \tan \theta_w + k_f^2 A_{cf} \tan \theta_f \right) + T_u + T_{FRP} \tag{5-17}
\]

where
- \( k_f = 0.7 \)
- \( k_w = 0.5 \)
- \( A_{cf} \) = effective flange concrete area (mm²)
- \( A_{cw} = b_u (h - h_f) \) = web concrete area (mm²)
- \( h_f \) = flange depth (mm)
- \( \theta_f \) = shear plane angle in the flange
- \( \theta_w \) = shear plane angle in the web
- \( T_{FRP} \) = tensile capacity of the FRP (N)

The contribution of the FRP is then given by equation 5-18.

\[
T_{FRP} = d_{FRP} t_{FRP} E_{FRP} \varepsilon_{FRPave} R_L / \tan \theta_w \tag{5-18}
\]

where
- \( d_{FRP} \) = height of FRP sheets (mm)
- \( E_{FRPave} \) = average FRP strain
- \( R_L \) = remaining bonded length over initial bonded length ratio

Equation 5-18 was derived for continuous sheets of FRP bonded to the surface of the beam. The \( R_L \) term is used to account for FRP within the crack length that has failed due to debonding. Similarly, the \( \varepsilon_{FRPave} \) term was developed by considering debonding and the maximum strain at the crack. Because the CFRP straps are unbonded, they do
not experience stress concentrations at the cracks but rather the strain is uniform along the unsupported length of the strap and so these terms do not apply. As such, the FRP term in this equation requires reevaluation to take into account the unique properties of the straps.

5.1.4.2 Inclusion of the Straps

Because a crack could form at any distance along the beam, it is impossible to know where the straps and stirrups will intersect the crack. As such, a 'smears' approach will be used with both the straps and stirrups in this new model. Equation 5-17 is thus rewritten as 5-19. The Loov method requires the checking of concrete failure planes that pass between transverse reinforcement elements, as illustrated by plane 1 in Figure 5.6, so it will be necessary to check the transverse reinforcement spacing against failure of this plane in the final design procedure. The force in the smeared transverse reinforcement is given by equations 5-20 and 5-21 for the steel shear links and CFRP straps respectively. The required dimensional notation is given in Figure 5.8.

\[ V_c = 0.25 f_c^\prime k_w^2 A_{cw} \tan \theta_w + k_f^2 A_{cf} \tan \theta_f + F_c + F_{FRP} \]  \hspace{1cm} 5-19

\[ F_c = \rho_c b_w L_{web} \tan \theta_w + \rho_s b_w L_{flange} \tan \theta_f \] \hspace{1cm} 5-20

where

- \( \rho_c \) = steel transverse reinforcement ratio = \( A_s/(b_s s_v) \)
- \( L_{web} = (h - h_f - c_v)/\tan \theta_w \) (mm)
- \( c_v \) = cover to steel transverse reinforcement (mm)
- \( L_{flange} = (h_f - c_v)/\tan \theta_f \) (mm)

\[ F_{FRP} = F_{FRPweb} + F_{FRPflange} \] \hspace{1cm} 5-21

where

- \( F_{FRPweb} = \rho_{FRP} b_w L_{FRPweb} E_{FRP} (\varepsilon_{FRP}) \) (N)
- \( \rho_{FRP} \) = FRP reinforcement ratio = \( A_{FRP}/(b_o s_{FRP}) \)
- \( L_{FRPweb} = (h - h_f - c_{FRPb})/\tan \theta_w \) (mm)
- \( c_{FRPb} \) = bottom cover to straps (mm)
- \( F_{FRPflange} = \rho_{FRP} b_w L_{FRPflange} E_{FRP} (\varepsilon_{FRP}) \) (N)
- \( L_{FRPflange} = (h_f - c_{FRP})/\tan \theta_f \) (mm)
- \( c_{FRP} \) = top cover to straps (mm)
By including a top cover to the CFRP term \( c_{FRPT} \), this method is capable of calculating the effects of varying strap penetration.

**Assumed Strap Strain Model**

In order to estimate the strain in the FRP straps, \( \varepsilon_{FRP} \), the width of the crack they cross must be known. The crack width is a function of the force in the reinforcement across the crack. It thus becomes an iterative process to determine the crack width and strain in the reinforcement, unless an approximation of the required strain can be made. The UK Concrete Society (2000) has recommended a maximum allowable average strain in the FRP of 0.004 based on the work of Khalifa et al. (1998) who suggested this strain as a limit on developing aggregate interlock as discussed in Chapter 2. In order to make this method straightforward for designers to apply, this value could be used as \( \varepsilon_{FRP} \), the total strain in the CFRP strap due to both prestress and crack openings, in equation 5-21. However, there are several problems with this approach. First, this value of strain is actually lower than the prestressing strain used in many of the specimens tested by Kesse (2003) and others, without even considering the strain due to cracking. Second, this strain is independent of the depth of the section, which may be acceptable when the FRP is bonded and the strain is localized around the crack. However, for the straps, which are not bonded to the concrete, this strain could be
significantly higher than the strain developed due to crack openings in deeper sections. The approach does have the advantage of being fairly straightforward, as iterations are not required to determine the strain.

**Confined Crack Strap Strain Model**

A better approximation of the strap strain could be achieved if the shear crack widths were estimated. In order to estimate the maximum capacity provided by aggregate interlock, Vecchio and Collins (1986) give equation 5-22, which is based on the average shear crack width, \( w \). Equation 5-22 requires the maximum shear stress, \( v_{c,\max} \), whereas the Loov shear model uses the average shear stress, \( v_{ci} \), along the failure plane. In order to convert the maximum shear stress to the average shear stress, Vecchio and Collins give equation 5-23 which is an empirical equation based on the test results of Walraven (1981). The use of Walraven’s results suggests that there should be a strong correlation between the shear friction approach and these equations, as the coefficient, \( k \), was based on the results of Walraven *et al.* (1987) among others. The term, \( f_{ci} \), is the compressive stress normal to the cracks and can be calculated based on the force in the straps and shear links as given in 5-24. As a first guess, the compressive stress is calculated as the force in the straps due to prestress plus the force due to yielding of the shear links divided by the area effected by each transverse reinforcement element. The area effected by each strap is assumed to be the strap spacing, \( s_{FRP} \), multiplied by the web width, \( b_w \), and similarly for the shear links it is the shear link spacing, \( s_v \), multiplied by the web width. The component of this compressive stress perpendicular to the crack is then found to give the result seen in 5-24. Equation 5-25 is the concrete shear force term from equation 5-17, divided by the area of the concrete to give an average shear stress, \( v_{ci} \). Equation 5-23 can be solved for the maximum shear stress, as given in 5-26, using the quadratic equation. Finally, equation 5-22 can then be rewritten in terms of \( w \), the crack width, as given in equation 5-27.

\[
v_{ci,\max} = \frac{\sqrt{f'_c}}{0.31 + \frac{24w}{a_{agg} + 16} + 16}
\]

where
- \( v_{c,\max} \) = maximum shear stress (MPa)
- \( a_{agg} \) = maximum aggregate size (mm)
- \( w \) = crack width (mm)
\[
v_{\text{ci}} = 0.18v_{\text{ci,max}} + 1.64f_{\text{ci}} - 0.82\frac{f^2_{\text{ci}}}{v_{\text{ci,max}}}
\]

\[
f_{\text{ci}} = \frac{A_{\text{FRP}}E_{\text{FRP}}}{s_{\text{FRP}}b_w} \cos \theta + \frac{A_{\text{rr}}f_{\text{yy}}}{s_{\text{rr}}b_w} \cos \theta
\]

\[
v_{\text{ci}} = 0.25f_{\text{c}}k^2 \tan \theta
\]

where \( k = 0.5 \) for the web of a T-beam

\[ k = 2.1(f'_{\text{c}})^{0.4} \leq 0.5 \] for rectangular sections

\[ \theta = \theta_{\text{v}} \] for T-beams

\[
v_{\text{ci,max}} = \frac{-1.64f_{\text{ci}} + v_{\text{ci}} + \sqrt{(1.64f_{\text{ci}} - v_{\text{ci}})^2 - 4(0.18)(-0.82f^2_{\text{ci}})}}{2(0.18)}
\]

\[
w = \frac{\left(\sqrt{f_{\text{c}}} - 0.31\right)\left(a_{\text{agg}} + 16\right)}{24}
\]

By employing the Vecchio and Collins approach to crack widths, the treatment of the shear capacity across the cracks becomes quite similar to the MCFT. However, because this model does not consider behaviour away from the crack, the interaction between stress and strain in the \( x \) and \( y \) directions is not required, and the number of iterations needed to obtain a solution is reduced.

**Simple Crack Strap Strain Model**

The preceding procedure for determining the average crack width is complicated, especially since once the crack width has been established the additional strain in the CFRP strap created by the crack opening means an increased compressive force perpendicular to the crack. Thus an iterative approach is required to determine the strain in the CFRP straps, as will be discussed in the next section. A simplified approach for obtaining the crack width has been proposed by Collins and Mitchell (1997). This approach assumes the compressive stress acting perpendicular to the crack is negligible and thus the average shear stress is given by equation 5-28. By ignoring the compressive stress, the confining effects of the transverse reinforcement including the prestress in the straps are also ignored, which is conservative. However
iterations will not be required to obtain the crack width. Equation 5-28 can then be rearranged to give the crack width in equation 5-29.

\[
v_{ci} = \frac{0.18\sqrt{f_c'}}{0.3 + \frac{24w}{a_{agg} + 16}}
\]

5-28

\[
w = \frac{0.18\sqrt{f_c'} - 0.3}{v_{ci}}(a_{agg} + 16)
\]

5-29

### Calculating Strap Strains

Using equations 5-27 or 5-29, it is now possible to estimate the crack width for a given average shear stress in the concrete, \(v_{ci}\). In the case of a T-beam, it is suggested in the current work that cracking be ignored in the flange and only calculated for the web. Otherwise the increased average shear stress in the flange as calculated using equation 5-19, based on the fact that the flange is largely uncracked, results in an overestimated crack width and corresponding FRP contribution. Once the average crack width, \(w\), has been calculated for the web based on the shear stress, the average crack width in the direction of the strap, \(w_{crFRP}\), can be calculated from equation 5-30 using trigonometry.

\[
w_{crFRP} = \frac{w}{\cos \theta}
\]

5-30

The strain in the strap caused by the crack opening is thus the crack width in the direction of the strap divided by the height of the strap, \(h_{FRP}\). The total average strain in the strap, \(\varepsilon_{FRP}\), given in equation 5-31 is the strain due to cracking plus the strain due to prestress, \(\varepsilon_{prestress}\).

\[
\varepsilon_{FRP} = \varepsilon_{prestress} + \frac{w_{crFRP}}{h_{FRP}}
\]

5-31

This term can now be used in equation 5-21 to gain a better estimate of the CFRP strap contribution. For a rectangular section the designer should only consider the web terms. This model was programmed into a spreadsheet. For any given set of beam dimensions and material properties, the maximum shear capacity can be found by
varying the crack angles until a minimum value of equation 5-19 is achieved. If the Vecchio and Collins model is used to estimate the crack width, a set of crack angles must first be assumed. The value of 5-24 is calculated assuming that the strain in the straps is equal to the prestressing strain for the first iteration. The result of 5-24 is then used to get the crack width from 5-23 and 5-22. The crack width is then rotated into the strap direction using 5-30 and the strain in the strap is estimated using 5-31. The result of 5-31 is inserted into 5-21 and then finally the shear capacity for that crack angle can be calculated using 5-19. This process should be repeated for varying values of the crack angles until a minimum is reached. The strap strain from 5-31 that results in a minimum value of 5-19 is then used in equation 5-24 as the next guess for the strain in the straps and the whole process is iterated. It usually requires three iterations to come to a converged value of equation 5-19.

5.1.4.3 Advantages and Disadvantages

The shear friction approach has several advantages over other models. It allows for the use of a bi-linear crack path in T-beam sections, which is more realistic based on test results. It also has a term that considers the additional shear capacity provided by the flange. Finally, because it is based on the results of tests that consider the shear capacity as a function of the crack width, there is the potential within the model to better estimate the strap strain. There are two main drawbacks to the shear friction approach. One is that it was derived considering only shear equilibrium and so the moment capacity must be calculated using a separate model, in this case the MCFT is used through Response-2000. However, this is not unusual as most code approaches treat shear and flexural capacity independently. The other drawback is that it is an iterative approach. This is not a significant problem if the method is implemented into a spreadsheet.

5.2 T-beam Predictions

The models were first used to predict the capacity of the beams tested in the current study as well as the two beams presented in Chan (called ‘Control’ and ‘Retrofit’). Table 5.1 indicates how each component of the shear (i.e. concrete, steel, and straps) and flexural capacity was determined for the models as some do not calculate every
component of the capacity as mentioned previously. The table then gives the predictions of the four models (with the safety factors removed) for each specimen. Also given for each specimen is the ratio of the predicted capacity to the actual capacity and the mode of failure according to the model. The notation ‘S. S.’ refers to a shear failure where the straps ruptured whereas ‘S. C.’ refers to a shear failure without strap rupture. In the case of the Kotsovos model, three predictions are given. First a prediction using a flexural capacity calculated using only the width of the web and strap stresses equal to 100% of the ultimate strap capacity is presented. Then the flexural capacity prediction from Response-2000 (which is based on the MCFT and is denoted as such in Table 5.1) was used and two different assumed strap forces of 50% and 100% of the ultimate strap capacity are employed to give the second and third prediction. Because Chen and Teng do not specify a concrete or steel model, suggesting instead that appropriate code models are used, three different models for the concrete and steel capacity were employed: the Canadian code (CSA 1994), the British code (BS 8110 1997) and the shear friction model by Loov. Both models that employ the Loov shear friction approach (i.e. Chen and Teng, and the modified Deniaud and Cheng) do not give a flexural capacity prediction so the flexural capacity prediction from Response-2000 (denoted as MCFT) was used for both of these models to determine whether failure was due to shear or flexure. The final four columns of Table 5.1 give the average value of the ratios of predicted to actual capacity for each model (referred to as the ‘Mean’), and the standard deviation of those ratios (referred to as ‘S.D.’). The first set of Mean and S.D. columns includes all the beam specimens. The second set of columns does not include specimens B2/45/H/20, B3/30/H/22, B4/30/G/25, and B5/30/C/27, as indicated by an ‘**’ next to the Mean and S.D. headings. The ultimate capacities of these specimens, herein referred to as the ‘weak specimens’, are somewhat questionable as the combined effects of having holes in the flanges and smaller bearing pads may have had a detrimental effect, as discussed in Chapters 3 and 4. As mentioned earlier, the holes were not accounted for in the models.

The MCFT offers unexpectedly conservative estimates for the capacity of most beams. This is surprising since unlike most code approaches, it should take the shear capacity of the flange into account. One would expect that even if the straps were not modeled correctly at least the capacity of the unretrofitted T-beams would be
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Table 5.1 - Model comparisons for T-beams.
predicted accurately. Another problem with the MCFT and Response-2000 is that they consider the straps to be bonded to the concrete. This should result in higher capacity predictions, as the strain in each strap will be localized around the crack resulting in a higher strap force contribution, which makes the conservative predictions of the program harder to explain. However, the Mean is still fairly reasonable at between 0.87 and 0.89. Having consistently low predictions does not eliminate the MCFT as a possible model since a conservative approach is generally warranted when looking at brittle shear behaviour. The S.D. of the model is also quite consistent, although it is higher than some of the other models.

The Kotsovos model is extremely conservative in its predictions when only the web is used to calculate the flexural capacity. However, the predictions become unconservative when a more accurate model is used for the flexural capacity. This is because the assumed stresses in the straps lead to an overestimate of the capacity of most beams, which is a result of the lack of guidance about crack widths offered by this model. It is difficult to make definitive conclusions about the model accuracy because of the weak specimens. It is possible that, had they not failed due to other effects, the weak specimens would have attained the capacities predicted by Kotsovos. When the weak specimens are not included, the Kotsovos model has a reasonable Mean although the standard deviation is the highest of any model.

The Chen and Teng model was combined with the ‘simplified method’ of shear design from the Canadian code to produce consistent results when the weak specimens are not considered. Similarly the Chen and Teng approach works quite well with both the British code and the Loov model in terms of consistency, and is also very accurate when used with the Loov model. It should be noted that the overall accuracy of this model is partially due to the flexural predictions of the MCFT. Of course, this still means that the shear predictions were sufficiently accurate so as to suggest that flexural failure would govern.

The modified Deniaud and Cheng model with an assumed strap strain of 0.004 gives exactly the same results as the Chen and Teng combined with the Loov model if the weak specimens are not considered. This is to be expected since both control beams use only the results of the Loov model and flexure dominates for B6/30/C/44,
B7/30/G/36 and the ‘Retrofit’ specimen where the flexural predictions come from the MCFT. If the weak specimens are considered this model produces more accurate results than Chen and Teng. To some extent this accuracy is more due to luck than the quality of the model. The stress in the FRP was assumed to be 40% of the ultimate strap capacity in this model based on the recommendation of the UK Concrete Society. On the other hand Chen and Teng predicted the average stress in the straps to be 62.5% of the ultimate strap capacity based on a strain due to prestress of 0.0025 and their assumption that the maximum stress in the straps will equal the ultimate stress. In the case of the weak specimens, because failure was driven by factors other than the straps rupturing, the straps never reached this level of stress. Since the modified Deniaud and Cheng model employed a lower average strap stress, it was more accurate. This discrepancy highlights the need for a model that will accurately calculate the strap stresses and corresponding contribution to shear capacity.

Finally, the modified Deniaud and Cheng approach is used with both the Collins and Mitchell approach to crack width estimation (referred to as the ‘Simple Crack Model’) and the Vecchio and Collins approach (referred to as the ‘Confined Crack Model’) to estimate the beam capacities. The models predict the capacity of the T-beams well with an accuracy of approximately 0.98 when the weak specimens are not considered. Even when the weak specimens are included the Mean was still 1.00 with a standard deviation of 0.05 using the simple crack model. The confined crack model gave higher predictions. It is difficult to say which model is the most accurate, as they both overestimate the capacity of the weak specimens. However, the simple crack model produces more conservative predictions, which is appealing from a design point of view. If the confined-crack model is more accurate, the results indicate that the size of the bearing pads causes a 15% reduction in the beam capacities. Regardless of the crack model used, the shear predictions for the weak specimens are still below the flexural capacity of each beam (estimated as approximately 127kN for B4/30/G/25 and 128kN for B5/30/C/27 using Response-2000), indicating that these specimens would have failed in shear even with larger bearing pads. Calculating the amount of CFRP reinforcement required to force a flexural failure will be used as a design example in section 5.8. The crack-width based models seem to give accurate predictions of the T-beam capacities although the presence of the weak specimens makes drawing definitive conclusions difficult. Finally, the confined crack model may
produce unconservative results and also increases the number of iterations required to obtain a solution suggesting that the simple crack model may be more appropriate.

Based on the analysis of the T-beam sections, in terms of accuracy, the approaches based on the Loov model are the best. They also seem to be the most consistent. The Kotsovos approach seems to either under or overestimate the capacity significantly depending on the flexural model, which suggests it is a poor option. Whilst both the Chen and Teng, and modified Deniaud and Cheng models with fixed strap strain offered good results, they do not have the ability to accurately predict the strap strain for all specimens. The modified Deniaud and Cheng approach used in conjunction with crack width models seemed to be most accurate.

5.3 Rectangular Section Predictions

The predictions for Kesse’s (2003) tests are given in Table 5.2. The results of all four models are given with the Mean and the S.D. values as before. The specimen name is divided into sections: B#-#s-#l-#p. The first section, B#, is simply the specimen number. The second section, #s, gives the number of straps used while the third section, #l, gives the number of loops in each strap. Finally, #p refers to the amount of prestress in each strap. For specimens B7 and B8 there is also a ‘34d’ term included in the designation, which indicates that these specimens were loaded to 34kN and then unloaded before the straps were installed. The cross section of each of Kesse’s specimens is illustrated in Figure 5.9. The transverse reinforcement layouts are given in Figure 5.10. Table 5.3 gives the key parameters for each of Kesse’s specimens including the number of straps used on each specimen, the number of loops in each strap, the amount of prestress and the concrete cube strength, $f_{cu}$. 
<table>
<thead>
<tr>
<th>Models</th>
<th>No straps</th>
<th>Retrofitted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Capacity</td>
<td>Failure Mode</td>
<td></td>
</tr>
<tr>
<td>Chen &amp; Teng</td>
<td>CSA</td>
<td>Chen &amp; Teng</td>
</tr>
<tr>
<td>Chen &amp; Teng</td>
<td>BS</td>
<td>Chen &amp; Teng</td>
</tr>
<tr>
<td>Chen &amp; Teng</td>
<td>Loov</td>
<td>Chen &amp; Teng</td>
</tr>
<tr>
<td>Deniaud &amp; Cheng</td>
<td>UK C. Soc. Max. Allow</td>
<td>Simple Crack Model</td>
</tr>
<tr>
<td>Simple crack model</td>
<td>Veale/Vact</td>
<td>Simple Crack Model</td>
</tr>
<tr>
<td>Confined crack model</td>
<td>Veale/Vact</td>
<td>Confined Crack Model</td>
</tr>
</tbody>
</table>

Table 5.2 - Model comparisons for rectangular sections
Figure 5.9 – Cross section of Kesse’s specimens

Figure 5.10 – Transverse reinforcement layout of Kesse’s specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Number of Straps</th>
<th>Number of Loops</th>
<th>Prestress (% of ult. strap capacity)</th>
<th>$f_{cu}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2-ns-ns</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>62.3</td>
</tr>
<tr>
<td>B5-2s-10l-50p</td>
<td>2</td>
<td>10</td>
<td>50</td>
<td>49.7</td>
</tr>
<tr>
<td>B6-1s-5l-50p</td>
<td>1</td>
<td>5</td>
<td>50</td>
<td>47.9</td>
</tr>
<tr>
<td>B7-1s-5l-50p-34d</td>
<td>1</td>
<td>5</td>
<td>50</td>
<td>43.5</td>
</tr>
<tr>
<td>B8-2s-10l-50p-34d</td>
<td>2</td>
<td>10</td>
<td>50</td>
<td>44.0</td>
</tr>
<tr>
<td>B9-1s-10l-50p</td>
<td>1</td>
<td>10</td>
<td>50</td>
<td>41.0</td>
</tr>
<tr>
<td>B10-2s-5l-50p</td>
<td>2</td>
<td>5</td>
<td>50</td>
<td>43.5</td>
</tr>
<tr>
<td>B11-2s-10l-25p</td>
<td>2</td>
<td>10</td>
<td>25</td>
<td>45.9</td>
</tr>
<tr>
<td>B12-2s-10l-5p</td>
<td>2</td>
<td>10</td>
<td>5</td>
<td>45.9</td>
</tr>
</tbody>
</table>

Table 5.3 – Key parameters of Kesse’s specimens
The MCFT gives consistent results with a standard deviation of 0.07. The accuracy of the model is also good, with the predictions being on average 89% of the actual capacity. Its prediction of the control beam capacity is excellent at 97% of the actual, however, its prediction for the capacity of a beam with only one strap, B6-1s-5l-50p, is poor at 75% of the actual. This is probably due to the fact that Response-2000 smears the transverse reinforcement. Unless the crack occupies the full shear span, the full capacity of the strap will not be engaged as it would if the strap were modeled as a discrete element. However, since this beam failed due to strap rupturing, a conservative prediction is perhaps warranted as the failure mode is very brittle. If the strap spacing recommendations of Kesse are followed (in B6-1s-5l-50p the strap spacing is greater than $d$), smearing of the transverse reinforcement is less of an issue. Once again, the major concern with the use of Response-2000 is that the straps are considered to be bonded to the concrete. However, this does not seem to have resulted in an overestimate of the strap contribution and the program gives reasonable predictions.

The accuracy of the Kotsovos model for rectangular sections is good. Unfortunately its standard deviation is the worst of any model. As well, the model always predicts shear failures. The fact that a strap stress must be assumed for this model is the most probable cause of the poor standard deviation. In some cases the prestress in the strap and the assumed strap stress used in the Kotsovos model are the same so the model does not account for any increase in stress above this value when the beam is loaded. Similarly, the prestress in other specimens was quite low, so the assumed strap stress may be too high in these cases. Whilst the model never overestimates the specimen capacity, in order to be a viable option, a model should give consistent results.

The Chen and Teng model seems to be fairly consistent at modeling rectangular beams. The model is more accurate when employed with the Canadian and British codes than it was for the T-beams. This is because both codes only consider the web to be effective in carrying shear, which results in more conservative predictions for T-beams. When employed with the Loov model, the Chen and Teng approach is very accurate with the predictions averaging 98% of the actual results. However, there is a decrease in consistency compared to the T-beam predictions as several beam
capacities are overestimated. The most significant of these is the capacity of Kesse’s control specimen (B2-ns-ml), which is overestimated by 8%. This seems to be due to the concrete cube strength, 62.5MPa, used in this specimen. Because the shear friction capacity is based on the concrete compressive strength, higher strength concrete beams achieve higher shear capacities due to the concrete contribution. Loov and Peng (1998) discovered that the value of \( k \) used in the shear friction approach should be reduced for higher concrete strengths, proposing the relationship given in equation 5-32 and plotted in Figure 5.11.

\[
k = 2.1(f_c^{-0.4}) \leq 0.5
\]

This reduction was employed when predicting the capacity of Kesse’s control beam, suggesting that even this value of \( k \) is unconservative and may need to be reevaluated. However, most of the beams tested by Kesse used lower strength concrete (between 40 and 50MPa) and as a result the Loov approach gives better results. Kesse tested a second control specimen with a concrete cube strength of 48.7MPa, which reached a maximum load of 48kN before the test was halted. Unfortunately the test was not taken to failure, but it can be seen that for a 22% reduction in concrete cube strength there was only an 8%, or lower, reduction in shear capacity. As such, there is the potential for overestimates of the shear capacity at higher concrete strengths. The
quality of the predictions for specimens B1/25 and Control in the T-beam series suggests the method works well for unretrofitted beams with lower concrete strengths.

The modified Deniaud and Cheng model with fixed strap strain also appeared to be quite accurate. The only beam capacity that was overestimated was the control, which was due to the impact of concrete strength on the Loov model. Another promising aspect of this model is its ability to predict the failure mode. Specimen B12-2s-10l-5p is the only beam for which the failure mode is predicted incorrectly. This incorrect prediction highlights a potential problem with this method due to assuming the strain in the CFRP straps. In the case of B12-2s-10l-5p, the prestress in the straps is quite low (5% of the ultimate strap capacity) and so it is possible that the assumed strap strain of 40% of ultimate is inaccurate.

Using the modified Deniaud and Cheng approach with the crack width models resulted in good accuracy with a Mean of 0.94 for the simple crack model and 0.99 for the confined crack model. However, the consistency of the predictions was weak at 0.08 and 0.07. The reason for this lack of consistency was due to two specimens. First, the capacity of the control specimen, B2-ns-nl, is overestimated using the Loov model. The other overestimate was specimen B10-2s-5l-50p where the model predicted a ductile flexural failure. This overestimate seems to be the result of incorrectly predicting the strap strains since the specimen failed due to strap rupturing. Strap strain predictions will be dealt with in Chapter 6. The confined crack model predicts a flexural failure for specimen B12-2s-10l-50p when the experimental failure mode was shear. This is due to the conservative flexural capacity prediction of the MCFT, as the confined crack model actually accurately predicts a shear failure at 89.8kN (within 1% of the actual), although strap rupturing was still not predicted. However, the confined crack model gives slightly unconservative predictions for several specimens, which should be avoided when considering a brittle failure mode such as shear.

It seems that, as with the T-beams, being able to accurately estimate the strap strains results in the best predictions. The modified Deniaud and Cheng approach with the crack models is once again the most accurate model.
5.4 Deep Beam Predictions

Stenger (2000) tested a series of 1200mm deep RC beams. The beams were tested in constant shear by displacing one end of the beam with respect to the other. Beam ST2 was a control specimen. Beams ST1, ST4, and ST5 were wrapped with 25 loop CFRP straps prestressed to 56% of the ultimate strap capacity. Beam ST3 had the same area of CFRP straps as the other retrofitted beams but a prestress of only 4%. A summary of the key specimen parameters for Stenger’s tests can be found in Table 5.4. Figure 5.12 illustrates the cross section of Stenger’s specimens while Figure 5.13 gives the transverse reinforcement spacing. He discovered that if minimal prestress was used (ST3), there was almost no capacity enhancement due to the straps over the control beam (ST2). This is quite different from Kesse’s (2003) conclusion that prestress had very little effect on the overall capacity if the correct spacing and stiffness of straps was employed. This suggests that the effect of the straps on deep beams is quite different than for smaller sections. The chosen model will have to account for both these situations. The predictions for each model are given in Table 5.5.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Number of Strap Loops</th>
<th>Prestress (% of ultimate strap capacity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST1</td>
<td>25</td>
<td>56</td>
</tr>
<tr>
<td>ST2</td>
<td>0</td>
<td>0</td>
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<td>ST3</td>
<td>25</td>
<td>4</td>
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<tr>
<td>ST4</td>
<td>25</td>
<td>56</td>
</tr>
<tr>
<td>ST5</td>
<td>25</td>
<td>56</td>
</tr>
</tbody>
</table>

Table 5.4 – Key parameters of Stenger’s specimens
6 - Ø26mm longitudinal bars

6 - Ø8mm longitudinal bars (3 each face)

Figure 5.12 – Cross section of Stenger’s specimens

25 Loop CFRP Straps

Ø6mm Shear Links @ 375mm spacing

Specially Stiffened End Region

370 500 500 500

Test Region L = 2240

Specially Stiffened End Region

Figure 5.13 - Transverse reinforcement spacing for Stenger’s specimens
<table>
<thead>
<tr>
<th>Models</th>
<th>Concrete</th>
<th>Steel</th>
<th>Straps</th>
<th>Flexure</th>
<th>ST1</th>
<th>ST2</th>
<th>ST3</th>
<th>ST4</th>
<th>ST5</th>
<th>Mean</th>
<th>S.D.</th>
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<tr>
<td>Actual Capacity</td>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>Failure Mode</td>
<td></td>
<td></td>
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<tr>
<td>MCFT</td>
<td>MCFT</td>
<td>MCFT</td>
<td>MCFT</td>
<td>MCFT</td>
<td>577.9</td>
<td>366.3</td>
<td>570.1</td>
<td>580.8</td>
<td>584.5</td>
<td>0.91</td>
<td>0.16</td>
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<td>Veale/Vact</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Failure Mode</td>
<td>Kotsovos</td>
<td>Kotsovos</td>
<td>Assumed strap strain</td>
<td>Kotsovos</td>
<td>554.0</td>
<td>399.6</td>
<td>563.6</td>
<td>559.0</td>
<td>563.3</td>
<td>0.87</td>
<td>0.16</td>
</tr>
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<td>Kotsovos (50%)</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Veale/Vact</td>
<td>Kotsovos</td>
<td>Kotsovos</td>
<td>Assumed strap strain</td>
<td>Kotsovos</td>
<td>711.0</td>
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<td>720.5</td>
<td>715.9</td>
<td>720.2</td>
<td>1.11</td>
<td>0.24</td>
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<td>Chen and Teng (Can)</td>
<td>CSA</td>
<td>CSA</td>
<td>Chen and Teng</td>
<td>CSA</td>
<td>399.6</td>
<td>209.8</td>
<td>340.7</td>
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<td>404.2</td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>Failure Mode</td>
<td>BS</td>
<td>BS</td>
<td>Chen and Teng</td>
<td>BS</td>
<td>448.3</td>
<td>257.3</td>
<td>384.6</td>
<td>448.3</td>
<td>448.3</td>
<td>0.67</td>
<td>0.09</td>
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<td>Chen and Teng (Brit)</td>
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<td></td>
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<tr>
<td>Veale/Vact</td>
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<td></td>
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<tr>
<td>Failure Mode</td>
<td>Loov</td>
<td>Loov</td>
<td>Chen and Teng</td>
<td>MCFT</td>
<td>616.0</td>
<td>371.2</td>
<td>571.4</td>
<td>631.9</td>
<td>644.5</td>
<td>0.96</td>
<td>0.15</td>
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<td>Chen and Teng (Loov)</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Veale/Vact</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Failure Mode</td>
<td>Loov</td>
<td>Loov</td>
<td>UK C. Soc., Max. Allow. Strain</td>
<td>MCFT</td>
<td>528.5</td>
<td>371.2</td>
<td>551.5</td>
<td>540.2</td>
<td>550.8</td>
<td>0.87</td>
<td>0.16</td>
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<tr>
<td>Simple crack model</td>
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<tr>
<td>Veale/Vact</td>
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<td></td>
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</tr>
<tr>
<td>Failure Mode</td>
<td>Loov</td>
<td>Loov</td>
<td>Simple Crack Model</td>
<td>MCFT</td>
<td>591.3</td>
<td>371.2</td>
<td>429.2</td>
<td>604.2</td>
<td>616.0</td>
<td>0.88</td>
<td>0.06</td>
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<tr>
<td>Confinned crack model</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Veale/Vact</td>
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<td></td>
</tr>
<tr>
<td>Failure Mode</td>
<td>Loov</td>
<td>Loov</td>
<td>Confinned Crack Model</td>
<td>MCFT</td>
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<td>371.2</td>
<td>484.6</td>
<td>659.8</td>
<td>672.4</td>
<td>0.95</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 5.5 - Model comparisons for deep beams
The MCFT was very accurate in terms of predicting the capacity of these specimens but the consistency of the predictions was not as good. One reason for this lack of consistency is the overestimation of the capacity of specimen ST3. Despite being able to account for varying levels of strap prestress using the MCFT, the assumption that the straps are bonded to the concrete means that when a crack develops the stress in the strap increases significantly in that area and results in an overestimate of capacity. In reality, because the strap is unbonded, the opening of a crack does not have a significant effect on the strap strain for deep beams because the change in length due to the crack opening over the total strap length (1300mm in this case) is quite small.

The Kotsovos model gives quite consistent predictions for four out of the five specimens when a strap stress of 50% is used. Unfortunately the prediction for ST3 is 18% above the actual capacity, which is due to the assumed stress in the strap. While the assumption that the stress in the strap is 50% of the ultimate capacity works well with the specimens that had high prestress, it does not work well for the specimen with low prestress. Similar observations can be made when a strap stress of 100% is used, although in this case more of the specimen capacities are overestimated.

The Chen and Teng model did not prove to be as consistent as it had been for the previous sets of beams. Once again it is the beam with the 4% prestress that has had an adverse effect on the consistency of the model. This error is caused by Chen and Teng's assumption that one strap will always reach its ultimate capacity.

The modified Deniaud and Cheng model with assumed CFRP strain suffers from the same problems as the Kotsovos and Chen and Teng model.

The modified Deniaud and Cheng approach with crack width models works well with the deep beam sections giving Mean values of 0.88 and 0.95 for the simple and confined crack models respectively. It is also one of the more consistent models in terms of predictions. Perhaps most importantly, while all of the previous models overestimated the effect of a small amount of prestress, this model does not.
5.5 Model Selection

Based on the preceding comparison, it is apparent that the modified Deniaud and Cheng approach with crack models gives the most accurate predictions and thus is the best available approach for modeling the CFRP strap retrofitting system. The accuracy of this approach is illustrated in Figures 5.14 and 5.15 for the simple and confined crack models respectively. In order to fit the deep beam results on the same graph as the other specimens, their shear forces, $V$, have been divided by 10.

![Graph showing accuracy comparison between predicted and experimental shear forces for different types of beams](image)

**Figure 5.14 – Accuracy of modified Deniaud and Cheng approach with simple crack model**
The line $V_{\text{predicted}} = V_{\text{experimental}}$ represents perfect accuracy. Any points that fall below this line represent conservative predictions. Based on the proximity of the points to this line, both models seem quite accurate. Almost all the predictions using the simple crack model fall below this line. The predictions of the confined crack model tend to be scattered above and below this line, suggesting better overall accuracy, but less conservative predictions. As such, the simplified crack model seems to offer the right combination of accuracy and conservatism for design. However, the model is currently based on a limited database of test results. This database should be expanded to ensure that the model works for various $a/d$ ratios, transverse reinforcing schemes, and loading arrangements to ensure that it is robust enough for design.

5.6 Safety factors

In design, safety factors are required to account for variability in material strength to ensure that the predicted capacity of a structural element does not exceed the capacity of the actual element due to weaker materials being supplied on-site.
5.6.1 Concrete and steel

The material safety factors, $\gamma_m$, for the existing steel and concrete should be taken from the relevant design code. For example, BS 8110 recommends a safety factor, $\gamma_s$, of 1.05 for reinforcing steel and a safety factor, $\gamma_c$, of 1.5 for concrete in flexure.

5.6.2 CFRP straps

The design capacity of the CFRP straps also needs to be reduced to account for material variations. The results of tensile tests performed on the CFRP straps of specimens B7/30/G/36, B8/30/G/46 and B9/30/G/42 (specimens B8/30/G/46 and B9/30/G/42 will be discussed in greater detail in Chapter 6) are given in Table 5.6. The design strength was defined as the average strap capacity minus three standard deviations, in other words, the 99% confidence interval. If the average strap capacity is divided by the design strength, the resulting ratio should give a safety factor with only a 1% chance of exceedance. In this case, the average strap capacity was 58.1 kN whilst the average design strength was 53.1 kN giving a ratio of 1.09. This would yield a material safety factor for the straps, $\gamma_{FRP}$, of about 1.1. However, this factor is perhaps unconservative given the limited database of experimental work. Other researchers have suggested higher values such as Khalifa et al. (1998) who proposed a $\gamma_{FRP}$ of 1.43 and Triantafillou (1998) who recommended 1.15 for CFRP rupture. ISIS Canada (2001) employs an even more conservative value, using a $\gamma_{FRP}$ of 2 in their design examples. Given the number of tensile tests that have been performed on the CFRP straps over the course of this research, the material properties should be understood well enough that a factor of 2 would be unnecessarily conservative. However, given the brittle nature of the material, a $\gamma_{FRP}$ of 1.25 would seem to be both conservative and more in line with other research.

<table>
<thead>
<tr>
<th></th>
<th>‘Ideal’</th>
<th>B7/30/G/36</th>
<th>B8/30/G/46</th>
<th>B9/30/G/42</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>59.3</td>
<td>58.3</td>
<td>59.2</td>
<td>56.9</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>2.31</td>
<td>1.69</td>
<td>2.07</td>
<td>1.18</td>
</tr>
<tr>
<td>Design</td>
<td>52.4</td>
<td>53.2</td>
<td>52.9</td>
<td>53.4</td>
</tr>
<tr>
<td>% of Ideal</td>
<td>-</td>
<td>1.02</td>
<td>1.01</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Table 5.6 – Ultimate strap capacities
5.5.2 Application of Safety Factors

There are two possible ways to apply the material safety factors. One approach is to multiply the material strengths by the safety factors and then insert those values into equation 5-19. The safety factors could also be applied to each term of equation 5-19 once the minimum failure angle has been found. The difference in approaches does not result in significant differences in the predictions. Equation 5-19 can be rewritten as 5-33 for use as a design equation.

\[
V_f = \frac{0.25f'_c}{\gamma_c} \left( k_w^2 A_{ew} \tan \theta_w + k_t A_{tr} \tan \theta_f \right) + \frac{F_v}{\gamma_s} + \frac{F_{FRP}}{\gamma_{FRP}}
\]

5-33

Also, the maximum experimental FRP strap strain should be modified to give the allowable strap strain, \( \varepsilon_{allowable} \), in equation 5-34. The allowable strain has been evaluated based on the experimental results of this study, but may require reevaluation based on the specific CFRP straps used in an application.

\[
\varepsilon_{allowable} = \frac{\varepsilon_{experimental}}{\gamma_{FRP}} = \frac{0.0125}{1.25} = 0.01
\]

5-34

5.6 Design algorithm

1. Calculate the maximum factored applied shear force on the beam, \( V_{required} \). This value will most likely be the shear force to cause flexural failure, and should not exceed this force. Calculate the available shear resistance, \( V_{available} \), of the beam using equations 5-33 (without the FRP term) and 5-20. If \( V_{available} > V_{required} \) a retrofit is not required. If \( V_{available} < V_{required} \) proceed to step 2.

2. Decide on a strap spacing, \( s_{FRP} \), such that it is less than the effective depth, \( d \). Assume a strain due to prestress, \( \varepsilon_{prestress} \), in the straps. For the material used in this study, this value should not exceed 0.005, or 50% of the ultimate strain, so that there is available capacity to allow for strains due to the applied load, as well as redistribution due to concrete creep.

3. Estimate the required amount of CFRP straps by assuming the complete difference in shear capacity, \( V_{difference} = V_{required} - V_{available} \), is taken by one
strap and that the only strain in the strap is that due to the prestress, 
\( \varepsilon_{\text{prestress}} \). This approach is used because the crack angle and actual strap 
strain are not yet known. Equation 5-35 gives a conservative first of 
estimate of the FRP reinforcing ratio, \( \rho_{\text{FRPinit}} \).

\[
\rho_{\text{FRPinit}} = \frac{V_{\text{difference}}}{\varepsilon_{\text{prestress}}E_{\text{FRP}}S_{\text{FRP}}b_w}
\]  
5-35

4. Calculate the retrofitted capacity of the beam with \( \rho_{\text{FRPinit}} \) by using 
equations 5-33, 5-20 and 5-21. The strain in the FRP can be evaluated 
using equation 5-31. Since the predictions using the simple crack model 
were more conservative than those using the confined crack model, 
equations 5-28 and 5-29 are recommended for determining the crack 
width. The crack angles, \( \theta_v \) and \( \theta_b \), are varied until a minimum value of \( V_f \) 
is achieved.

5. Check to see if the retrofitted capacity, \( V_f \), exceeds the required capacity, 
\( V_{\text{required}} \). Given the conservative assumptions employed in equation 5-35 
this is quite likely and the designer can proceed to step 6. However, a more 
efficient design is almost certainly possible and an improved estimate of 
\( \rho_{\text{FRP}} \) can be calculated using equation 5-36. Then steps 4 and 5 can be 
repeated until the designer is satisfied with the efficiency of the design. As 
a final check, the FRP reinforcement ratio should be calculated using the 
actual area of FRP to be used. In practice these areas should probably be 
specified in 5 loop increments for ease of installation in the field.

\[
\rho_{\text{FRPn}} = \frac{V_{\text{required}} - \left(0.25f_c' \left(k_w^2 A_{\text{ct}} \tan \theta_u + k_f^2 A_{\text{cf}} \tan \theta_f \right) \right)}{F_{\text{FRPn-1}}} - F_{\text{in-1}}
\]  
5-36

6. Once the actual number of loops and spacing has been selected, failure of 
the concrete between transverse reinforcement elements should be checked 
by setting the crack angles, \( \theta_v \) and \( \theta_b \), to values whereby the total length of 
the crack is less than the transverse reinforcement spacing. The capacity is

172
calculated using only the concrete terms of 5-33. The maximum allowable shear strength should also be checked using either equation 5-15 or 5-16.

7. Once an appropriate spacing, prestress, and number of loops has been decided upon for the retrofit, the strap strains must be checked to ensure that both initial and long-term rupture is not an issue. This will be dealt with in greater detail in Chapter 6.

5.8 Design example

As suggested in section 5.2, according to the modified Deniaud and Cheng approach with crack width models, specimens B4/30/G/25 and B5/30/C/27 would still have failed in shear even if the size of the loading pad had not be an issue. In the following example, specimen B4/30/G/25 will be redesigned (including changing the strap spacing from 250mm to 200mm) using the modified Deniaud and Cheng approach with the simple crack model in order to achieve a flexural failure for this lower concrete strength.

Example

Given:

\[ f'_{ct} = 25 \text{MPa} \]
\[ k_f = 0.7 \]
\[ k_w = 0.5 \]
\[ h = 280 \text{mm} \]
\[ h_f = 110 \text{mm} \]
\[ h_{FRP} = 280 \text{mm (free height of strap)} \]
\[ b_w = 105 \text{mm} \]
\[ b_y = 250 \text{mm} \]
\[ c_v = 30 \text{mm} \]
\[ c_{FRP} = 0 \text{mm} \]
\[ c_{FRPT} = 20 \text{mm} \]
\[ a = 820 \text{mm} \]
\[ a_{agg} = 10 \text{mm} \]
\[ A_v = 56.5 \text{mm}^2 \]
\[ s_v = 250 \text{mm} \]

\[ \rho_v = \frac{A_v}{b_w s_v} = \frac{56.5}{105(250)} = 0.002154 \]

\[ f_{sy} = 570 \text{MPa} \]

Step 1: nb: since this is an attempt to predict the actual amount of retrofitting required to achieve flexural failure, safety factors will not be employed in this example.
\( V_{\text{required}} = 115 \text{kN} \) (moment capacity from Response-2000)

\[
V_{\text{available}} = 0.25 f' c \left( k_w^2 A_w \tan \theta_w + k_f^2 A_f \tan \theta_f \right) + \rho_v b_w \frac{(h-h_f-c_v)}{\tan \theta_w} f_{yv} + \rho_b b_w \frac{(h_f-c_v)}{\tan \theta_f} f_{yv}
\]

\[
A_w = b_v (h-h_f) = 105(280-110) = 17850 \text{mm}^2
\]

The calculation of the flange area, \( A_{cf} \), varies depending on the width of the flange. If the flange is sufficiently wide such that \((b_f-b_w)/2\) is greater than the flange depth, \( h_f \), as illustrated in Figure 5.16(a), then equation 5-37 may be used. If the flange is not that wide, as in Figure 5.16(b) and as was the case with the current specimens, then equation 5-38 must be used.

![Flange and Web Effective Areas](image)

(a) \( \frac{b_f - b_w}{2} > h_f \)

(b) \( \frac{b_f - b_w}{2} < h_f \)

Figure 5.16 – Effective concrete areas

\[
A_{cf} = b_w h_f + h_f^2 \quad \text{5-37}
\]

\[
A_{cf} = b_w h_f + h_f^2 - \left( \frac{b_w + h_f - b_f}{2} \right)^2 \quad \text{5-38}
\]

\[
A_{cf} = 105(110) + 110^2 - \left( \frac{105}{2} + 110 - \frac{250}{2} \right)^2 = 22244 \text{mm}^2
\]

By varying the crack angles, \( \theta_w \) and \( \theta_f \), the minimum value of \( V_{\text{available}} \) can be found.

In this case the minimum occurs when \( \theta_w = 42^\circ \) and \( \theta_f = 24^\circ \) resulting in:
\[ V_{\text{available}} = 0.25(20)(0.5^2 \times 17850 \times \tan 42^\circ + 0.7^2 \times 22244 \times \tan 24^\circ) + 0.002154(105) \frac{280 - 110 - 30}{\tan 42^\circ} (570) + 0.002154(105) \frac{110 - 30}{\tan 24^\circ} (570) = 87568 \text{N} \]

\[ V_{\text{available}} = 87.6 \text{kN} < V_{\text{required}} = 115 \text{kN} \]

\[ \therefore \text{shear retrofiting will be required.} \]

**Step 2:**

A strap spacing of 200mm will be used which is less than the effective depth, \(d = 226\text{mm} \). A prestress of 25\% will be assumed so as to allow for strains due to cracking as well as long term creep strain.

**Step 3:**

Estimate the required FRP reinforcing ratio using equation 5-35:

\[ \rho_{\text{FRPini}} = \frac{V_{\text{difference}}}{\epsilon_{\text{prestr}} E_{\text{FRP}} s_{\text{FRP}} b_w} = \frac{115000 - 87568}{0.0025 \times 121000 \times 200 \times 105} = 0.004318 \]

**Step 4:**

Calculate the capacity of the beam with \(\rho_{\text{FRPini}}\). By varying the crack angles, \(\theta_w\) and \(\theta_t\), the minimum value of \(V_r\) can be found. In this case the minimum occurs when \(\theta_w = 60^\circ\) and \(\theta_t = 33^\circ\) resulting in:

\[ V_{cl} = 0.25 f_c' k^2 \tan \theta_w = 0.25 \times 20 \times 0.5^2 \tan 60^\circ = 2.165 \text{MPa} \]

\[ W_{\text{crFRP}} = \frac{0.18 \times \sqrt{f_c'}}{24 \cos \theta_w} \left( \frac{v_{ci}}{V_{cr}} - 0.3 \right) (a_{agg} + 16) \]

\[ W_{\text{crFRP}} = \frac{0.18 \times \sqrt{20}}{24 \cos 60^\circ} \left( \frac{2.165}{0.13} - 0.3 \right) (10 + 16) = 0.156 \text{mm} \]
\[ V_n = 0.25 f'_c \left( k_w A_{cw} \tan \theta_w + k_f A_{cf} \tan \theta_f \right) + \rho_v b_w \frac{(h - h_f - c_v)}{\tan \theta_w} f_{yw} \]
\[ + \rho_v b_w \frac{h_f - c_v}{\tan \theta_f} f_{yw} + \rho_{FRP} b_w \frac{h - h_f}{\tan \theta} E_{FRP} \left( \varepsilon_{\text{prestress}} + \frac{w_{\text{FRP-web}}}{h_{\text{FRP}}} \right) \]
\[ + \rho_{FRP} b_w \frac{h_f - c_{FRP}}{\tan \theta_f} E_{FRP} \left( \varepsilon_{\text{prestress}} \right) \]
\[ V_n = 0.25(20) \left( 0.5^2 \times 17850 \times \tan 60^\circ + 0.7^2 \times 22244 \times \tan 33^\circ \right) \]
\[ + 0.002154 \times 105 \times \frac{280 - 110 - 30}{\tan 60^\circ} \times 570 \]
\[ + 0.002154 \times 105 \times \frac{110 - 30}{\tan 33^\circ} \times 570 \]
\[ + 0.004318 \times 105 \times \frac{280 - 110}{\tan 60^\circ} \times 121000 \times \left( \frac{0.0025 + 0.156}{280} \right) \]
\[ + 0.004318 \times 105 \times \frac{110 - 20}{\tan 33^\circ} \times 121000 \times 0.0025 \]
\[ V_n = 135805N \]

**Step 5:**

Since this \( \rho_{\text{FRP}^{\text{init}}} \) provides considerably more capacity than required (135kN >> 115kN), equation 5-36 is employed to iterate towards a more efficient solution:

**Iteration 2:**

\[ \rho_{\text{FRP}^{2}} = \rho_{\text{FRP}^{1}} \frac{V_{\text{required}} - \left( 0.25 f'_c \left( k_w A_{cw} \tan \theta_w + k_f A_{cf} \tan \theta_f \right) \right)}{F_{\text{FRP}^{1}} - F_{\text{v}}} \]
\[ \rho_{\text{FRP}^{2}} = 0.004318 \frac{115000 - 74037 - 26304}{35464} = 0.001785 \]

Crack angles for minimum solution: \( \theta_w = 54^\circ, \theta_f = 28^\circ \)

\[ v_{ci} = 1.720 \text{MPa} \]

\[ w_{cr,\text{FRP}} = 0.309 \text{mm} \]

\[ V_n = 111894N \]

**Iteration 3:**

\[ \rho_{\text{FRP}^{3}} = 0.002066 \]

Crack angles for minimum solution: \( \theta_w = 55^\circ, \theta_f = 29^\circ \)

\[ v_{ci} = 1.785 \text{MPa} \]

\[ w_{cr,\text{FRP}} = 0.285 \text{mm} \]

\[ V_n = 114971N \]
This solution is now accurate enough to select an actual strap area and make sure that it will work. In order to provide $\rho_{FRP}$ of 0.002066 at a spacing of 200mm, a strap area of 43.4mm$^2$ is required. This is just greater than the 38.4mm$^2$ provided by 10 loops, so the designer should go up to 15 loops thus providing 57.6mm$^2$. The actual $\rho_{FRP}$ of 0.002743 should now be checked:

$\rho_{FRP} = 0.002743$

Crack angles for minimum solution: $\theta_{v} = 57^\circ$, $\theta_{r} = 30^\circ$

$\nu_{ci} = 1.925$MPa

$w_{FRP,web} = 0.235$mm

$V_{n} = 121838$N > 115000N

.: from a strength point of view, 15 loops spaced at 200mm are adequate.

**Step 6:**

To ensure that failure cannot occur in the concrete between the transverse reinforcement elements, the capacity of the concrete must be checked using equation 5-33 without the transverse reinforcement terms. In this case because the distance between transverse reinforcement elements is relatively small, the two crack angles are set equal to one another.

$s_{trans} = 125$mm

$h - c_{v} - c_{FRP} = 280 - 30 - 20 = 230$mm

$\theta = \tan^{-1}\left(\frac{230}{125}\right) = 61^\circ$

$V_{n} = 0.25 f_{c}^{'\prime} k_{w} A_{cw} \tan \theta_{w} + k_{f}^f A_{cf} \tan \theta_{f} \right)$

$= 0.25(20)(0.5^2 \times 17850 \times \tan 61^\circ + 0.7^2 \times 22244 \times \tan 61^\circ) = 139kN > 115kN$

.: OK

In order to ensure that the capacity of the concrete is not exhausted by the retrofit, the maximum allowable capacity must also be checked using equation 5-15:

$V_{n} = 0.25 f_{c}^{'\prime} b_{w} h = 0.25 \times 20 \times 105 \times 280 = 147kN > 115kN$

.: OK

**Step 7:**

The strap strains now need to be calculated. This could be done by using the crack width from the simplified crack model and equation 5-31. However, the simplified
crack model provides a conservative estimate of the FRP contribution and thus a lower than actual strap strain. The confined crack model provides more realistic strap strains, as it is a more accurate model. However, as will be discussed in the next chapter, the strap strains increase with time under long-term loading and so a strap strain model capable of predicting these long-term increases will be required. Such a strap strain model is presented in Chapter 6.

Design Verification

A T-beam with approximately the same properties as those used in this example was tested as part of the PhD research program of Samir Hassan Dirar. The beam had a cube strength of 32MPa, a flange depth of 105mm and an overall depth of 270mm but was otherwise the same. Although the beam failed in shear, it failed at a load of 128kN. This result provides three important conclusions. First, that it is possible to provide a significant shear capacity enhancement (45% increase in capacity over the control specimen) to a low strength concrete beam using the under-slab strap installation technique. Second, that the proposed design method provides a conservative approach to estimating the capacity of retrofitted beams. Finally, though the flexural capacity prediction was conservatively low, the estimate of the shear capacity given the cube strength and dimensions used was 130kN, which was within 2% of the actual failure load of 128kN.

As was discussed in the previous chapter, the designer must check all possible failure modes including local effects to determine if increasing the shear capacity has made the capacity of another element of the beam critical. One such consideration that will be dealt in the next chapter is fatigue of materials.

5.8 Conclusions

The goal of this chapter was to pick a model for designing retrofitted beams that gives an accurate prediction of the capacity. By comparing the predictions of each potential model to a database of experimental results with several different strap and beam configurations it became apparent that the model needed to accurately estimate the strap strains. This was especially apparent for the deep beam sections tested by
Stenger where prestress played a significant role in the beam capacity. The modified Deniaud and Cheng approach with the simple crack model was chosen based on its accuracy. The one disadvantage of the model is that it is an iteration-based approach. However, only the crack angle needs to be varied and when implemented into a spreadsheet convergence can be obtained quite quickly.

A design algorithm was then proposed and checked using experimental results. The results given by the model seemed quite accurate and validated the proposed approach.
Chapter 6
Long-term Capacity Testing

This chapter examines two further experiments, a sustained load and a cyclic load beam test, that were undertaken to validate the long-term capacity of the under-slab installation technique and the CFRP straps. An important aspect of gaining acceptance from designers involves understanding and predicting the long-term behaviour of a structural system. These tests are a first step towards gaining this acceptance. This chapter examines the choice of testing parameters, as well as the set-ups of the sustained and cyclic load tests. The results of the tests are discussed. A method of determining beam deflections is then developed and the results are used to determine the initial and long-term strap strains. Finally, failure tests on both specimens are discussed with conclusions at the end of the chapter.

6.1 Design Philosophy

In order to provide comparability of results, the same cross section as specimen B7/30/G/36, illustrated in Figure 3.23, was employed for the long-term tests. The reinforcement properties are given in Table 3.2 while the concrete mix design is given in Table 3.4. The concrete properties for the long-term specimens are given in Table 6.1 where B8/30/G/46 is the sustained load test specimen and B9/30/G/42 is the cyclic load specimen. The table includes the compressive cube strength, \( f_{cu} \), the split cylinder strength, \( f_{ct} \), the modulus of rupture strength, \( f_{t2} \), and the age at first loading of the specimens. The average strap stiffness was taken as 121 GPa as determined in Chapter 3.

<table>
<thead>
<tr>
<th></th>
<th>B8/30/G/46 (Start of Long-term Test)</th>
<th>B8/30/G/46 (On day of Static Failure Test)</th>
<th>B9/30/G/42</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{cu} ) (MPa)</td>
<td>42.9</td>
<td>45.7</td>
<td>41.7</td>
</tr>
<tr>
<td>( f_{ct} ) (MPa)</td>
<td>2.70</td>
<td>3.30</td>
<td>3.30</td>
</tr>
<tr>
<td>( f_{t2} ) (MPa)</td>
<td>4.59</td>
<td>5.09</td>
<td>5.70</td>
</tr>
<tr>
<td>Age (days)</td>
<td>90</td>
<td>363</td>
<td>159</td>
</tr>
</tbody>
</table>

Table 6.1 – Concrete properties
Specimen B8/30/G/46 was loaded with a sustained load for 260 days then unloaded and finally tested to failure. Specimen B9/30/G/42 was exposed to a cycling load for 2.1 million cycles before being tested to failure.

The sustained shear force applied to B8/30/G/46 was 110kN, which was chosen for several reasons. First, it was between the capacity of the unretrofitted specimen (B1/25) of 88kN and the capacity of the retrofitted specimen (B7/30/G/36) that failed in flexure at 135kN. Work by Tilly (1979) suggests that the ratio of dead to live load applied to typical RC bridges is between 0.2 and 0.4. As a typical live load, the standard fatigue vehicle from BS 5400 (1980) was used. This vehicle has a weight of 80kN per axle, which when combined with a dead load calculated using Tilly’s ratio results in two point loads of 110kN each (unfactored dead plus live load). It should be noted that the standard fatigue vehicle loading would not normally be applied to a beam with these dimensions. However, this loading (approximately 80% of the beam’s retrofitted capacity) provided an extreme test of the sustained load capacity as well as corresponding to the point at which significant strap strains were developed during the static tests.

For specimen B9/30/G/42 the shear force was cycled between 70 and 110kN at a frequency of 2Hz as illustrated in Figure 6.1. This frequency was chosen because it was greater than the natural frequency of the beam, calculated as 1.33Hz, avoiding the possibility of resonance. A higher frequency was not used because it was felt that the beam would not have fully recovered from the last loading cycle before the next was applied, resulting in an artificially stiff beam response. Ideally the shear force should have been cycled between 30 and 110kN to represent the 80kN fatigue vehicle specified in BS 5400. However, it was not possible to design a testing rig with the available equipment to accommodate this load differential. The current loading was deemed adequate since the lower limit of the loading range, 70kN, was below the capacity of the control specimen (88kN) whilst the upper limit, 110kN, was above this value. Also, based on the strap strain readings from the static tests, this load range creates a significant difference between the minimum strap strain (0.00029 excluding prestress, or 2% of the ultimate strain for B7/30/G/36) and the maximum strap strain (0.00209 excluding prestress, or 17% of the ultimate strain for B7/30/G/36). Whilst
2.1 million load cycles represents only about 3% of the 73 million loading cycles a bridge may see in its lifetime (Heffernan et al. 2004), it should allow any trends to become evident. The magnitude of the loading is also quite high, varying between approximately 0.5 and 0.8 times the ultimate retrofitted capacity of the specimen. An actual structure would probably not be exposed to frequent loading of this magnitude, so as with the sustained load test, this test provides an indication of the long-term capacity of the retrofitting system under extreme conditions.

![Diagram of cyclic loading range](image)

**Figure 6.1 – Cyclic loading range**

### 6.2 Sustained Load Test Set-up

The set-up for the sustained load test is shown in Figure 6.2. As with the static tests, the beam was 3m long and had a clear span of 2.5m. The two point loads were applied 0.75m in from each support. In order to apply the 110kN of shear force (and 220kN of total load) two threaded rods were used. Each 30mm diameter rod was turned down in the middle and a full bridge strain gauge configuration was placed on the turned down area. The rods were then subjected to known loads in a tensile testing machine while the resistance across the full bridge was measured. The load versus resistance readings were used to develop a calibration factor for each rod for use in the data logger. The rods served as load cells allowing the total force being applied to the beam to be measured. The rods were then screwed into the threaded sockets in the strong floor.
A cross beam and a spreader beam transferred the load from the strong floor through the threaded rods into the test specimen as illustrated in Figure 6.2. Once these beams were in place, the load was applied by tightening nuts on the threaded rods against the cross beam. The disadvantage of this system was that as the beam crept, the tension in the rods reduced, which lowered the total force being applied to the beam. Thus the nuts had to be periodically tightened to ensure 110kN of shear force was maintained in the specimen. Although this situation was not ideal, the amount of shear force required made the use of dead weight for loading impractical.

Figure 6.3 illustrates the location of the instrumentation on specimen B8/30/G/46. The mid-span displacement of the beam was measured with a mechanical dial gauge. The strains on the internal longitudinal and transverse reinforcement bars were measured using 6mm strain gauges. Strain gauges were placed at the mid-span of the beam on one of the 20mm longitudinal bars as well as on one of the top 8mm bars. Gauges
were placed at mid-height on one side of each shear link in one shear span, and on the middle shear link in the other span. The strains in the CFRP straps were also measured using 6mm strain gauges at the mid-height of the outer layer on both sides of the strap. The applied loads and strains were recorded at 6-hour intervals.

![Instrumentation on B8/30/G/46](image)

**Figure 6.3 – Instrumentation on B8/30/G/46**

### 6.3 Cyclic Load Test Set-up

The cyclic load testing rig was a self-reacting frame shown in Figure 6.4. Each element of the frame was checked against the S-N curves of BS 7608 (1993) to ensure the frame would not fail during the 2.1 million loading cycles. The specimen was placed on top of a 280 ASB 100 (Asymmetric) beam, which served as the reaction beam. The reaction beam was then attached to two specially made channel sections that were bolted to the columns. Two channel sections were used as cross beams at the top of the columns and supported the dynamic jack making the system completely self-contained preventing fatigue failure of the strong floor. Each bolt was class 8.8 tightened to a torque of 475Nm in order to achieve a stress of 0.7 times the proof load in the bolts as dictated by BS 7608.
An Amsler testing machine capable of applying both a constant base load and a pulsating dynamic load was used with a single 250kN dynamic (500kN static) capacity jack. The load was spread from the jack to the specimen through an I-beam that had vertical stiffeners at the reaction points. Under the spreader beam horizontal sliders and rollers were used in an attempt to eliminate any horizontal restraint provided by the loading system. Only one of the specimen supports on the reaction beam restricted horizontal movement in a further attempt to eliminate horizontal restraint.

The applied load was measured using a 500kN load cell. Five LRDTs were used with one at each support, one at each load point, and one at the midpoint of the beam as illustrated in Figure 6.5. The same strain gauge configuration that was used for B8/30/G/46 was employed here except the strain gauges on each CFRP strap were placed only on the outside face opposite the weld. The results were monitored using a high-speed data acquisition system capable of scanning each data channel 100 times a second. The data acquisition program was designed to record two seconds of data every hour so that changes with time could be measured.
6.4 Sustained and Cyclic Load Test Results

6.4.1 Strap Strains

In order to determine the long-term behaviour of the prestressed CFRP straps in the absence of external load, the strap strains were measured on specimen B9/30/G/42 during a period of 76 days between prestressing and the commencement of cyclic loading. These results, plotted in Figure 6.6, provide a baseline CFRP strap performance. The prestressing force in the strap corresponded to approximately 15kN although the initial strap strains were quite variable for the reasons discussed in Chapter 3.

The strap strain versus time results for specimen B8/30/G/46 (subjected to a sustained load for a period of 260 days) are illustrated in Figure 6.7. Figure 6.8 gives the maximum CFRP strap strains versus number of cycles for specimen B9/30/G/42. Figure 6.9 presents the minimum and maximum CFRP strap strains versus number of cycles for selected straps on specimen B9/30/G/42.

Figure 6.5 – Instrumentation on B9/30/G/42
Figure 6.6 – Strap strain versus time – unloaded beam specimen

Figure 6.7 – Strap strain versus time – B8/30/G/46
The change in strain and additional tensile force for each strap is given in Table 6.2. The additional tensile force is calculated as the change in strain multiplied by the modulus of elasticity and the total strap area. Table 6.3 presents the same results for the 2.1 million cycles for specimen B9/30/G/42. Finally, Figure 6.10 offers a direct comparison of strap performance, giving the strap strain versus time relationship for selected straps on both specimens over the first 12 days of loading.
<table>
<thead>
<tr>
<th>Strap Location</th>
<th>Change in Strain</th>
<th>Additional Tension (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>South Outer</td>
<td>0.00068</td>
<td>3.11</td>
</tr>
<tr>
<td>North Outer</td>
<td>0.00072</td>
<td>3.30</td>
</tr>
<tr>
<td>South Middle</td>
<td>0.00129</td>
<td>5.92</td>
</tr>
<tr>
<td>North Middle</td>
<td>0.00123</td>
<td>5.60</td>
</tr>
<tr>
<td>South Inner</td>
<td>0.00039</td>
<td>1.76</td>
</tr>
<tr>
<td>North Inner</td>
<td>0.00062</td>
<td>2.85</td>
</tr>
</tbody>
</table>

Table 6.2 – Strap strain changes with time – sustained load (B8/30/G/46)

<table>
<thead>
<tr>
<th>Strap Location</th>
<th>Change in Strain</th>
<th>Additional Tension (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>West Outer</td>
<td>0.00054</td>
<td>2.78</td>
</tr>
<tr>
<td>East Outer</td>
<td>0.00048</td>
<td>2.49</td>
</tr>
<tr>
<td>West Middle</td>
<td>0.00099</td>
<td>5.03</td>
</tr>
<tr>
<td>East Middle</td>
<td>0.00162</td>
<td>8.29</td>
</tr>
<tr>
<td>West Inner</td>
<td>0.00025</td>
<td>1.26</td>
</tr>
<tr>
<td>East Inner</td>
<td>0.00071</td>
<td>3.61</td>
</tr>
</tbody>
</table>

Table 6.3 – Strap strain changes with cycling – cyclic load (B9/30/G/42)

Figure 6.10 – Strap strain versus time – B8/30/G/46 & B9/30/G/42

6.4.2 Beam Deflections

The beam mid-span deflection versus time results for the sustained load test, B8/30/G/46, are presented in Figure 6.11. The minimum and maximum mid-span deflection versus number of cycles are given for B9/30/G/42 in Figure 6.12.
6.5 Discussion of Sustained and Cyclic Load Test Results

6.5.1 Strap Strains

Figure 6.6 illustrates that the strap strains on the unloaded beam have decreased with time by about 5% on average for all the straps over 77 days. This reduction in strain is believed to be due to creep in the concrete under the prestressing force of the strap. At
the same time, there may also be further losses due to relaxation in the CFRP strap. Whilst the strain gauge readings will not indicate relaxation, experimental work by Saadatmanesh and Tannous (1999) on CFRP prestressing rods indicates that relaxation losses can be between 5 and 10% of the prestressing force over a 50-year period. Using their results, relaxation losses of about 7.5% could be expected for the straps over 76 days.

In contrast, under a sustained applied shear force of 110kN, the CFRP strap strains on B8/30/G/46 increased over time as illustrated in Figure 6.7. The maximum strap strains have increased by approximately 0.0013, or 75%, from an initial strain of 0.0018 if the strain due to prestress is removed. The true long-term strap strain increases would be, if anything, slightly larger if the strain losses due to concrete creep were removed. The magnitude of the increase is larger for straps where the initial strain (with or without prestress) after loading the beam is higher. This suggests that the strap strain increase could be a function of the magnitude of the initial strain.

It was observed in chapter 3 that the inner straps on specimen B7/30/G/36 seemed to be the least effective. In terms of long-term strains, they also see the lowest increases. The most effective straps, the middle ones, see the largest strain increases. Thus, focusing on the design of the most effective (in this case the middle) straps would seem prudent since their strains govern both the immediate and long-term behaviour.

If the strap strains were to eventually exceed the ultimate strain, the straps could fail, resulting in collapse of the structure. Thus, it is necessary to develop a method of calculating the long-term increase in strap strain to have reasonable confidence in the long-term capacity of the straps. The increase in strap strain could be connected to the beam deflection, as a comparison of Figures 6.7 and 6.11 reveals that the two relationships have a similar shape with time. Thus, an examination of the beam deflections will be undertaken in the next section before a method of calculating the long-term strap strains is developed.

The shape of the maximum strap strain versus number of cycles profile in Figure 6.8 is quite similar to that for the sustained load test in Figure 6.7. The major difference is the overall increase in the strap strains. For most straps this is lower than for the sustained load test as illustrated by the outer strap strains in Figure 6.10. The reason
for this is believed to be that the shorter duration of loading combined with the lower average load results in lower strains. This suggests that strap strains due to creep of the sustained load specimen might be critical. One exception to this was the middle strap in the east span of the cyclic specimen as seen in Figure 6.10. The middle strap strain in the west span of the cyclic specimen was less than the middle strap strain due to the sustained load over the first 12 days of testing. However, the middle strap strain in the east span of the cyclic specimen remained similar to the sustained load test for approximately the first four days, and then began to exceed the sustained load test. The first increase corresponds to a four-day period when the cyclic specimen was unloaded and realigned in the testing rig. This increase is possibly due to a widening of the crack caused by damage along the crack when the specimen was completely unloaded. At approximately 1.6 million cycles, this strap underwent a second sudden increase in strain. Although the increase looks significant, it actually only represents about 3% of the total strap capacity and translates to an increase in shear crack width of approximately 0.08mm. The rise in strain in this strap, as well as a smaller rise in the outer strap strain on the same side could be the result of the internal steel link between the straps losing stiffness. Unfortunately, the strain gauge on this link failed at 1.3 million cycles (which may in itself be an indication of increasing strains in this link) making it impossible to compare the two strain results. However, the maximum strain in the link, before gauge failure, of 0.0022 is close to the strain at which the stiffness of the link begins to change (the stress-strain curve for the links, given in Figure 3.13, does not have a well-defined yield plateau). If the stiffness of the link decreased, more load would be transferred to the CFRP straps since their stiffness remains constant. Although this represented a small change in strap strain, the fatigue performance of all elements of the beam must be considered, as will be discussed in later sections, since if the link were to eventually fail in fatigue the strap loading would increase dramatically.

Figure 6.9 illustrates that the difference between the minimum and maximum strain in the straps, and corresponding stress range, increases over the duration of the cyclic test. This is based on the assumption that the straps are linear elastic, and that the stress can be related to the strain through the modulus of elasticity, which is assumed to remain constant with time. The middle straps have the largest strain range, which starts at approximately 0.00079 and corresponds to a stress range of 96MPa. The
strain range increases by about 22% to a value of approximately 0.00096 after 2.1 million cycles, corresponding to a stress range of 116MPa. However, an increase in strain of 0.0002 (from 0.008 to 0.01) translates into a crack opening of only approximately 0.05mm, which suggests that the beam stiffness over this loading range is not changing significantly. An increase in the strain, and the corresponding stress, range is problematic since fatigue performance is generally a function of the stress range. If the straps, as well as the steel links, are exposed to an increasing stress range, this could shorten their fatigue life. Although the strain range did increase over the duration of loading, the straps appear to have the required fatigue capacity. It also appears that the increases in strain range are becoming smaller as the number of cycles increase. This suggests that the strain range might eventually stabilize at a constant value. However, even under the maximum shear force the strains in the straps are only about 50% of the ultimate strap strain capacity, so future work with higher average strains and with higher strain ranges carried out over a greater number of cycles should be performed to confirm this result.

6.5.2 Deflections

Specimen B8/30/G/46 was loaded for approximately 260 days. This loading resulted in a total deflection of 24.3mm. The increase in deflection above the initial deflection of 15.4mm represented a factor of 1.58 as illustrated in Figure 6.11.

One can see from Figure 6.12 that the mid-span deflection of the cyclic load specimen also increases over time. The initial deflection of 12.5mm for the cyclic load specimen was lower than the initial deflection of 15.4mm for the sustained load specimen despite similar load levels and material properties. This was probably due to the loading method of tightening bolts on threaded rods used for the sustained load specimen, where it took several hours to reach the sustained load. The sustained load specimen most likely underwent significant creep during this initial loading, which has not been accounted for. The overall deflection increase factor for the cyclic load test of 1.21 is not as high as the 1.58 noted for the sustained load test. This can be partially explained by the duration of cyclic loading, as the load was applied over a much shorter time frame for the cyclic test. However, over the first 12 days of the sustained load test, the same duration as the cyclic test, the deflection had increased
by a factor of 1.29. Thus even over an equivalent time period, the sustained load test deflections were still greater. The average load applied to the specimen in the cyclic load test is lower, at 90kN, than was used in the long-term test. This results in lower concrete stresses and reduced creep. The combined effects of shorter duration and lower average load led to the lower deflections of the cyclic specimen.

Interestingly there is little change in the difference between the maximum and minimum deflections over time for the cyclic load specimen. This indicates that during the 2.1 million loading cycles the beam stiffness over this load range is remaining relatively constant since a drop in stiffness would result in a larger change in the maximum deflection versus the minimum deflection. At the same time the overall beam stiffness is decreasing as both the maximum and minimum deflections are increasing with time. This indicates that the change in stiffness, and overall deflection, is a function of the sustained base load, and not the additional cyclic load. The change in CFRP strap strain over time may be less of an issue for retrofitted structures where the sustained load will remain at the same level as before, as this load seems to have the largest impact on strap strains. If the sustained load remains the same and the retrofit is instead to enhance the maximum load capacity, the increase in long-term deflection of the beam should be minimal, especially in a structure that has had many years to creep. With little increase in deflection, the strap strain increases with time may also be minimal if there is a correlation between deflections and strap strains as discussed in the following sections. However, as was noted earlier, there was a slight increase in the cyclic strain range for the straps, despite there being no change in beam stiffness over the cyclic loading range. Further research is necessary to develop a better understanding of the relationship between sustained load and load range on strap stresses.

There is a slight drop in the overall deflection of B9/30/G/42 at approximately 920000 cycles and then an increase thereafter. This corresponds to a period of 4 days when the beam had to be completely unloaded and realigned in the rig. The initial decrease in deflection is believed to be due to creep recovery while the beam was unloaded.

As noted previously, the long-term strap strain and deflection relationships may be connected and allow the designer to calculate the strap strain based on the long-term
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As noted previously, the long-term strap strain and deflection relationships may be connected and allow the designer to calculate the strap strain based on the long-term
deflection. However, a viable method of calculating deflections must first be
developed before any such relationship could be considered. As such, the available
methods for calculating deflections need to be examined.

6.5.2.1 Deflections due to Flexure

According to most approaches, the deflection of RC beams increases over time due
to the creep and shrinkage of concrete (ACI Committee 435 2003). Creep in the concrete
causes increasing strains in the longitudinal reinforcement, which translates to an
overall increase in the beam deflection. There exist approaches (ACI Committee 435
2003) that are capable of estimating flexural deflections to a high degree of accuracy.
Unfortunately they require knowledge of the materials and environmental conditions
that typically would not be available for an existing structure. Instead it would seem
more practical to use a simplified method such as the ACI factor approach (ACI
Committee 318 1995), given in equation 6-1, despite the lack of accuracy of such an
approach (Espion and Halleux 1990). Adjustment factors could then be used to
account for the inaccuracies in the method.

\[
\delta_i = \delta_o \left(1 + \frac{\xi}{1 + 50 \rho_c} \right) \quad 6-1
\]

where

- \( \delta_i \) = the deflection at time \( t \) (mm)
- \( \delta_o \) = the initial flexural deflection at time \( t = 0 \) (mm)
- \( \xi \) = a factor to account for the duration of loading
- \( \rho_c = A_c / b_d d \) = the compressive reinforcement ratio
- \( A_c \) = the area of compressive reinforcement (mm\(^2\))

The values for \( \xi \) are 1 at three months, 1.2 at six months, 1.4 at one year and 2 at five
years.

The initial flexural deflection, \( \delta_o \), is calculated using the elastic modulus of the
concrete, \( E_c \), tensile strength, \( f_t \), and effective second moment of area, \( I_{\text{eff}} \), given by
ACI Committee 318. The formulas are reproduced below in equations 6-2, 6-3, and 6-4 for ease of reference.
\[ E_c = 4700 \sqrt{f'_c} \text{ (MPa)} \]

where \( f'_c \) = compressive cylinder strength of the concrete (MPa)

\[ f_t = 0.639 \sqrt{f'_c} \text{ (MPa)} \]

\[ I_{\text{eff}} = \left( \frac{M_{\text{cr}}}{M_{\text{av}}} \right)^4 I_g + \left( 1 - \left( \frac{M_{\text{cr}}}{M_{\text{av}}} \right)^4 \right) I_{\text{cr}} \]

where \( M_{\text{cr}} = f_t I_g / y \) = the cracking moment (Nmm)
\( I_g \) = the gross second moment of area (mm\(^4\))
\( I_{\text{cr}} \) = the cracked second moment of area of the transformed section (mm\(^4\))
\( y \) = the distance from the centroid to the tension face (mm)

Since this method requires the concrete cylinder strength, the concrete cube strengths were multiplied by 0.8, which is a conversion based on data from Neville (1988).

Using the ACI procedure on specimen B8/30/G/46, an initial deflection of 10.2mm was calculated. This value was approximately 34% less than the observed deflection of 15.4mm. Using the \( \xi \) value for six months of 1.2, the long-term deflection is calculated as 18.7mm, which was still 22% lower than the observed deflection at 6 months of 23.9mm. There appears to be some element of the deflection which the ACI approach is not accounting for properly. Figure 6.13 shows a comparison of the actual deflected shape of specimen B6/30/C/44 against the deflected shape predicted by the ACI formula at loads of 40kN and 110kN. Specimen B6/30/C/44 (discussed in greater detail in Chapter 3) was used here because the concrete strength was similar to that of B8/30/G/46, but LRDT readings were taken at 250mm intervals along the beam length during testing, allowing for a more comprehensive plot of the deflected shape.

At a load of 40kN, which was just above the load at which shear cracks were observed to form, the ACI prediction is 8% less than the actual deflection, which is more accurate than at higher loads. However, even at this lower load a difference can be seen in the shape of the deflection diagrams. In the shear span, the ACI prediction seems to be more curved than the actual shape. This fact is especially evident at the load points (0.75 and 1.75m along the span) where a definite change in slope can be seen in the actual beam deflection whereas the ACI prediction is much smoother. This type of linear deflection profile in the outer spans seems to be indicative of shear deflections. At a load of 110kN the maximum ACI prediction is 26% below the actual
deflection. The amount of net deflection in the region of constant moment is approximately the same for both the actual and the ACI prediction. Thus, the ACI underestimates the deflection by 3.6mm, but all of that is in the shear spans of the member. These irregularities point to a need to estimate the shear deflections.

![Diagram showing deflection comparison between actual and ACI predictions](image)

**Figure 6.13 – Actual vs. ACI predicted deflected shape – B6/30/C/44**

### 6.5.2.2 Deflections due to Shear

Interestingly, the deflection calculations in most design codes are based on flexural effects. However, if the deflections of B8/30/G/46 and B9/30/G/42 were due exclusively to flexural effects, one would not anticipate any increase in strap strain over time, as all the long-term strain redistribution would be longitudinal. If anything, a slight decrease in strap strain due to creep in the concrete might be expected as illustrated in Figure 6.6. Thus an increase in strap strain further suggests that there is a shear component to the deflection. This result has been verified by Nie and Cai (2000) who calculated deflections due to shear of between 13 and 35% of the total long-term deflection using a method developed by Neville et al. (1983).

Neville et al. indicate that once inclined shear cracks form, deflections due to shear develop. They suggest a method first proposed by Dilger (1967) to calculate the shear deformations. In this approach, the shear rotation, $\chi$, is the combination of elongation
of the shear reinforcement, $\Delta_{sv}$, with contraction of the concrete compressive strut, $\Delta_{cv}$, as illustrated in Figure 6.14.

Thus, the shear rotation, $\gamma$, is given by equation 6-5 when small angle theory is applied.

$$\gamma = \frac{\Delta_{cv}}{\sin \theta} + \frac{\Delta_{sv}}{\sin \alpha} \frac{z}{z (\cot \alpha + \cot \theta)}$$

where

- $\Delta_{cv} =$ displacement of concrete compressive strut (mm)
- $\Delta_{sv} =$ elongation of transverse reinforcement (mm)
- $z = d - x/3$

$x =$ depth from the top of the beam to the neutral axis of the linear elastic transformed section (mm)

The deformation, $\Delta$, in each case can be calculated as the strain in the member multiplied by the length of the member. Thus equation 6-5 can be rewritten as 6-6.
\[
\gamma = \frac{-\varepsilon_v}{(\cot \alpha + \cot \theta) \sin^2 \theta} + \frac{\varepsilon_v}{(\cot \alpha + \cot \theta) \sin^2 \alpha}
\]

where \( \varepsilon_v \) = strain in the concrete compression strut (negative)

\[
\varepsilon_v = \Delta_v \times \frac{\sin \theta}{z}
\]

\[
\varepsilon_{sv} = \text{strain in the shear reinforcement}
\]

\[
\varepsilon_{sv} = \Delta_{sv} \times \frac{\sin \alpha}{z}
\]

The strain in each element can then be calculated using the element stress and assuming linear elastic behaviour. The element force divided by the element area gives the element stress. Using the variable angle truss analogy, the force in the compression strut is given by equation 6-7 and in the transverse reinforcement by equation 6-8. The area of each member is calculated based on its inclination and in the case of the transverse reinforcement using the transverse reinforcement ratio. It is worth noting that Neville et al. consider the transverse reinforcement to be smeared, and so proper detailing of transverse reinforcement should be followed. Equation 6-9 gives the area for the concrete strut and 6-10 gives the area of the tension member.

\[
C_v = \frac{-V}{\sin \theta} \quad 6-7
\]

\[
T_{links} = \frac{V}{\sin \alpha} \quad 6-8
\]

where \( V = \) the applied shear force (N)

\[
A_{cv} = b_w z (\cot \alpha + \cot \theta) \sin \theta \quad 6-9
\]

\[
A_{links} = \rho_v b_w z (\cot \alpha + \cot \theta) \sin \alpha \quad 6-10
\]

By dividing 6-7 by 6-9 and 6-8 by 6-10 to get the stress in each element and then dividing by the appropriate elastic modulus to get the strains, equation 6-6 can be written in terms of the shear force, \( V \), as in equation 6-11.

\[
\gamma = -\frac{V}{E_c b_w z (\cot \alpha + \cot \theta) \sin^4 \theta} + \frac{V}{E_v b_w z \rho_v (\cot \alpha + \cot \theta) \sin^4 \alpha}
\]

Neville et al. recommend an empirical modification to 6-11 to account for the fact that this method will overestimate the strain in the stirrups when compared to experimental results. A modification factor, \( k_v \), is proposed to lower the strains in the
transverse reinforcement and bring them in line with experimental results. Neville et al. also adjust the concrete term to allow for the calculation of long-term shear displacements. In this case, the concrete term is multiplied by a creep adjustment term as given in equation 6-12.

\[
\gamma = \frac{V(1 + \phi)}{E_v b_w z (\cot \alpha + \cot \theta)^2 \sin^4 \theta} + \frac{V k_c}{E_v b_w z \rho_v (\cot \alpha + \cot \theta)^2 \sin^4 \theta}
\]

where
- \( \phi \) = creep coefficient
- \( k_c = 1 - \frac{V_c}{V} \)
- \( V_c \) = concrete shear contribution (N)

Neville et al. refer to the denominators of 6-12 as the shear stiffness, \( K \). This allows 6-12 to be rewritten in the simplified form of equation 6-13.

\[
\gamma = \frac{V(1 + \phi)}{K_{conc} \left( 1 + \frac{k_c}{K_{link}} \right)}
\]

Up until this point, the approach has been concerned only with shear rotations. In order to obtain the shear deflection, \( \delta_{\text{shear}} \), the general formula given by Neville et al. is used as given in equation 6-14.

\[
\delta_{\text{shear}} = \int_0^l \frac{V_{ul} V_{ul}}{K(x)} \, dx
\]

where
- \( \delta_{\text{shear}} \) = deflection due to shear (mm)
- \( V_{ul} \) = shear force due to a unit load applied at the point where the deflection is required (N)
- \( \frac{1}{K(x)} = \frac{(1 + \phi)}{K_{conc}(x)} + \frac{k_c}{K_{link}(x)} \)

Equation 6-14 can be simplified for beams with two equal point loads applied symmetrically, as in this study, to equation 6-15.

\[
\delta_{\text{shear}} = \gamma a
\]

where \( a \) = the shear span (mm)
6.5.2.3 Further Modifications to the Shear Deflection Model

Since the Neville et al. model was not derived for use with beams retrofitted with CFRP straps several modifications to the model were required. The derivation of the shear rotation, $\gamma$, for the T-beams used in the current study is illustrated in Figure 6.15.

![Diagram of beam section showing shear rotation and strain](image)

---

**Figure 6.15 – Derivation of the shear rotation for retrofitted beams**

An adjustment has been made to the original method of calculating the strain in the shear reinforcement to account for the contribution of the CFRP straps. The transverse reinforcement ratio is taken as the sum of the shear link ratio, $\rho_s$, and the CFRP strap ratio, $\rho_{FRP}$, as given in 6-16. The modulus of elasticity for the transverse reinforcement, $E_{trans}$, is then calculated using a weighted average between the CFRP

---
and steel moduli based on the reinforcement ratios as given in equation 6-17.

Equation 6-12 can then be rewritten as 6-18 with \( \alpha = 90^\circ \).

\[
\rho_{\text{trans}} = \rho_v + \rho_{\text{FRP}} \quad 6-16
\]

\[
E_{\text{trans}} = E_v + \frac{E_{\text{FRP}}}{E_v} \quad 6-17
\]

\[
\gamma = \frac{V(1 + \phi)}{E_v b_w z (\cot \theta)^2 \sin^4 \theta + V \rho_{\text{trans}} b_w z \rho_{\text{trans}} (\cot \theta)^2} \quad 6-18
\]

In the method laid out by Neville et al., no limit is given on the possible strains in the shear links. This is problematic since if the shear links yield, the CFRP straps then take the full component of the shear force in the transverse reinforcement resulting in increased strap strains. As such, it is recommended that the shear rotation calculation should be done in two parts. First, the shear rotation up to the shear force at which the links yield is calculated. This force can be determined using equation 6-19.

\[
V_y = \frac{E_{\text{trans}} b_w z \rho_{\text{trans}} (\cot \theta) \epsilon_{\text{yield}}}{k_v} \quad 6-19
\]

where \( \epsilon_{\text{yield}} \) = the yield strain of the steel transverse reinforcement.

The shear rotation between the shear force at which the steel links yield and the shear force at which the shear rotation is required, \( V_{\text{required}} \), is then calculated using equation 6-20.

\[
\gamma_{\text{yield}} = \frac{V'}{K_{\text{conc}}} + \frac{k_v V'}{K_{\text{FRP}}} \quad 6-20
\]

where \( V' = V_{\text{required}} - V_y \)

\( K_{\text{FRP}} = E_{\text{FRP}} b_w z \rho_{\text{FRP}} (\cot \theta)^2 \)

The total shear rotation, \( \gamma_{\text{tot}} \), required to calculate the deflection is then given by equation 6-21.

\[
\gamma_{\text{tot}} = \gamma + \gamma_{\text{yield}} \quad 6-21
\]
6.5.2.4 Determination of $k_v$ and $\theta$

The term $k_v$ is an important element of this method, as it will determine how much of the shear stiffness is contributed by the transverse reinforcement. As the transverse reinforcement area is much smaller than the concrete compressive strut area, the transverse reinforcement stiffness is an order of magnitude smaller and so results in the largest shear rotations. Neville et al. recommend an empirical formulation for calculating the concrete shear contribution based on the concrete compressive strength, longitudinal reinforcement ratio, beam geometry, and applied forces at the section under investigation. However, given the importance of the $k_v$ term, it stands to reason that the best possible estimate of the concrete and total shear capacities would provide the most accurate displacement predictions. The previous chapter looked at several of the current shear models and determined that a modified version of the Deniaud and Cheng model that accounted for the crack widths gave the best predictions. Based on that accuracy, it seems reasonable to employ this model here. Thus, $k_v$ in the following comparison was determined using the terms of equation 5-19 with the simple crack model. It should be noted that by using this approach, the prestress in the straps is accounted for in the $k_v$ term as the prestress was used in the calculation of the FRP contribution to $V$.

The angle of the compressive strut, which is the same as the crack angle in the model, must also be determined. Neville et al. recommend the use of a 45° angle for simplicity. However, since the modified Deniaud and Cheng approach will be used to determine $k_v$, it seems reasonable to employ the angles developed by this model as well. The Neville et al. model is derived based on a single angle, $\theta$, whereas the modified Deniaud and Cheng model produces angles for the web, $\theta_w$, and the flange, $\theta_f$. An average angle, $\theta_{ave}$, was used to estimate the overall shear deflection. This angle is a weighted average of the angle in the web and the flange as given in equation 6-22.

$$\theta_{ave} = \theta_w \left( \frac{h - h_f}{h} \right) + \theta_f \left( \frac{h_f}{h} \right)$$ 6-22
By determining $k_v$ and $\theta$ using the modified Deniaud and Cheng approach, one assumes that the failure state is critical, both in terms of load sharing and the crack angle, for deflections at all load levels. This assumption is incorrect as the load sharing between the concrete and the transverse reinforcement changes with increasing load. Assuming behaviour at failure should be conservative as the maximum transverse reinforcement contribution is developed at this load. Since the strain in the transverse reinforcement has the most significant impact on shear deflections, shear deflections at lower loads should be conservatively overestimated.

6.5.3 Deflection Predictions

In order to validate the proposed deflection procedure, the deflection results of several beam tests will be predicted. First the seven beams discussed in Chapter 3 (B1/25 through B7/30/G/36) will be examined. Then a beam tested by Chan (2000) will be investigated as well as a series of T-beams tested by Czaderski (1998) and Czaderski and Motavalli (2004). An attempt will also be made at predicting the results of tests by Kesse (2003).

6.5.3.1 B1/25 through B7/30/G/36

For the specimens in the current series, the calculated values for $k_v$ were in the range of 0.45 to 0.5. The average crack angle values ranged between 35° and 45°. The shear links were assumed to yield at a strain of 0.0027. Table 6.4 presents the results of the deflection analysis on specimens B1/25 through B7/30/G/36. The table gives the load at which the deflection is calculated, the experimental deflection (Exp Defl), the predicted flexural deflection (Pred Flex Defl) using the ACI approach, the ratio of the predicted flexural deflection to the experimental deflection (Pred Flex Defl / Exp Defl), the predicted shear deflection (Pred Shear Defl) using the modified Neville et al. method, the predicted flexural plus shear deflection (Pred Tot Defl), and the ratio of the predicted total deflection to the experimental deflection (Pred Tot Defl / Exp Defl) for each specimen.
The analysis shows that the flexural deflection prediction consistently underestimates the beam deflection, the average prediction being 73% of the actual. The combined flexure and shear deflection predictions are more accurate with an average prediction that is identical to the actual. Interestingly the standard deviation for the combined flexure and shear method is higher, but this is mainly due to specimen B1/25. For B1/25, though the maximum load exceeded the shear force at which the links yielded, the links were not assumed to yield. This was because there were no CFRP straps on that beam so if the links yielded, the transverse reinforcement stiffness would have been zero above the yield load, resulting in infinite displacements. In fact the actual transverse reinforcement does not have a well-defined yield plateau and so there would still be some transverse reinforcement stiffness in the actual beam. In this case, full shear link stiffness was assumed resulting in smaller displacements and a poor prediction at higher loads. If this result is removed, the standard deviation of the model improves and the overall accuracy continues to suggest that this is an excellent approach for estimating deflections.
6.5.3.2 Other T-beam Specimens

To further check the accuracy of this approach, the model was used to predict the results of tests conducted by Chan, Czaderski and Motavalli and Czaderski. Both Czaderski, and Czaderski and Motavalli looked at T-beams with L-shaped CFRP plates bonded to the side of the specimen. The tests also involved a variety of cross section dimensions and span-to-depth ratios, which should test the versatility of the proposed model. The cross section and testing layout for Czaderski and Motavalli’s specimen (S6) are illustrated in Figure 6.16 and for Czaderski’s specimens (T1, T2, and T3) in Figure 6.17. In the case of Chan’s specimen (Retrofit) the dimensions are the same as the beams tested in this study. The adjustment factor, $k_v$, and crack angle for Retrofit were estimated using the modified Deniaud and Cheng model. For specimen (S6) the adjustment factor, $k_v$, was set at 0.5 and the crack angle, $\theta$, was set at 45° based on values for other beams. For the Czaderski specimens (T1, T2, and T3) the adjustment factor was determined based on the concrete and total shear forces given by the author ($k_v = 0.18$ for T1, $0.23$ for T2 and $0.28$ for T3) and the crack angle was set to 45°. The results of this analysis are given in Table 6.5.

![Diagram of Czaderski and Motavalli’s (2004) specimen (S6)](image-url)

Figure 6.16 – Czaderski and Motavalli’s (2004) specimen (S6)
L-shaped CFRP plates ($A_{\text{FRP}} = 60\text{mm}^2$ @ 300mm spacing)  
(Shear links Ø6mm @ 400mm spacing)

(a) Specimen T1

L-shaped CFRP plates ($A_{\text{FRP}} = 54\text{mm}^2$ @ 300mm spacing)  
(Shear links Ø6mm @ 400mm spacing)

(b) Specimen T2

L-shaped CFRP plates ($A_{\text{FRP}} = 60\text{mm}^2$ @ 300mm spacing)  
(Shear links Ø6mm @ 400mm spacing)

(c) Specimen T3

Figure 6.17 – Czaderski’s (1998) specimens
<table>
<thead>
<tr>
<th>Specimen</th>
<th>Load (kN)</th>
<th>Act Defl (mm)</th>
<th>Flex Defl (mm)</th>
<th>Flex Defl / Act Defl</th>
<th>Shear Defl (mm)</th>
<th>Tot Defl (mm)</th>
<th>Tot Defl / Act Defl</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retrofit</td>
<td>100 (67%)</td>
<td>11.5</td>
<td>9.5</td>
<td>0.83</td>
<td>2.4</td>
<td>11.9</td>
<td>1.03</td>
</tr>
<tr>
<td>S6</td>
<td>225 (59%)</td>
<td>9.6</td>
<td>6.7</td>
<td>0.7</td>
<td>2.8</td>
<td>9.5</td>
<td>0.99</td>
</tr>
<tr>
<td>T1</td>
<td>250 (77%)</td>
<td>29.0</td>
<td>27.0</td>
<td>0.93</td>
<td>2.7</td>
<td>29.7</td>
<td>1.02</td>
</tr>
<tr>
<td>T2</td>
<td>400 (77%)</td>
<td>11.3</td>
<td>7.0</td>
<td>0.62</td>
<td>4.5</td>
<td>11.5</td>
<td>1.02</td>
</tr>
<tr>
<td>T3</td>
<td>200 (79%)</td>
<td>29.8</td>
<td>26.7</td>
<td>0.90</td>
<td>4.1</td>
<td>30.8</td>
<td>1.03</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>0.80</td>
<td></td>
<td></td>
<td>1.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.13</td>
<td></td>
<td></td>
<td></td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.5 – Other T-beam specimens’ deflection predictions

Once again the predictions using the combined method are more accurate than if only flexure is considered. In this case there is quite a high standard deviation on the flexural predictions because of the variety of specimens analysed. T1 and T3 had 6m total spans with shear spans of approximately 1.6m whereas specimen T2 was 3m long with shear spans of 1.25m. Shear deflections played a much greater role in T2 because it was a much shorter beam. The quality of predictions is especially good when the variety of specimens is considered.

6.5.3.3 Kesse’s Specimens

The method was used to predict the deflections of Kesse’s (2003) specimens with poor results given in Table 6.6. The average accuracy is just 75% with a standard deviation of 25%. Part of this poor performance may be due to the fact that the shear deflection method was designed for use with T-beams, where the crack angle is generally more stable. A parametric study on the Neville et al. model showed that a 5° change in crack resulted in a 58% increase in shear deflection when modelling Kesse’s specimens. Much of this increase can be attributed to the increase in angle causing the steel links to yield at lower loads and increasing the strap strains. Even a slight variation in crack angle can change the prediction significantly. Also, Kesse’s specimens tended to form shear cracks at higher shear forces (as a percentage of the ultimate) than was the case for the T-beams. When the transverse reinforcement starts
to take load will have significant effect on the displacement predictions. If the
deflections were calculated at lower loads, the shear deflection should be considerably
overestimated. This overestimate of shear deflection would actually improve the
accuracy of the overall deflection prediction at lower loads for most specimens, which
tends to suggest that the flexural deflection is underestimated. Finally, because
Kesse’s specimens were cantilevers, it is difficult to say whether rotation at the
support has been accounted for properly. It seems that whilst excellent results can be
achieved with T-beams, another approach may be required for estimating the shear
displacements in rectilinear beam sections.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Load (kN)</th>
<th>Act Defl (mm)</th>
<th>Flex Defl (mm)</th>
<th>Flex Defl / Act Defl</th>
<th>Shear Defl (mm)</th>
<th>Tot Defl (mm)</th>
<th>Tot Defl / Act Defl</th>
</tr>
</thead>
<tbody>
<tr>
<td>B5-2s-10l-50p</td>
<td>68</td>
<td>8.7</td>
<td>2.4</td>
<td>0.28</td>
<td>3.0</td>
<td>5.4</td>
<td>0.62</td>
</tr>
<tr>
<td>B6-1s-5l-50p</td>
<td>57</td>
<td>5.4</td>
<td>2.0</td>
<td>0.37</td>
<td>4.4</td>
<td>6.4</td>
<td>1.19</td>
</tr>
<tr>
<td>B9-1s-10l-50p</td>
<td>61</td>
<td>6.5</td>
<td>2.2</td>
<td>0.34</td>
<td>3.5</td>
<td>5.7</td>
<td>0.88</td>
</tr>
<tr>
<td>B10-2s-5l-50p</td>
<td>56</td>
<td>6.2</td>
<td>2.0</td>
<td>0.32</td>
<td>2.3</td>
<td>4.3</td>
<td>0.69</td>
</tr>
<tr>
<td>B11-2s-10l-25p</td>
<td>68</td>
<td>7.6</td>
<td>2.5</td>
<td>0.33</td>
<td>1.8</td>
<td>4.3</td>
<td>0.57</td>
</tr>
<tr>
<td>B12-2s-10l-5p</td>
<td>62</td>
<td>7.3</td>
<td>2.2</td>
<td>0.30</td>
<td>1.4</td>
<td>3.6</td>
<td>0.49</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.32</td>
<td></td>
<td>0.74</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.03</td>
<td></td>
<td>0.26</td>
</tr>
</tbody>
</table>

\(^1\) all deflections evaluated at a load of approximately 70\% of ultimate

Table 6.6 – Deflection predictions for Kesse’s specimens

### 6.5.3.4 Long-term Deflections

Neville *et al.* also proposed a method for calculating the long-term shear deflections.
They suggested that the long-term change in transverse reinforcement strain was
negligible and that the increase in shear deflection was the product of creep in the
concrete. As can be seen from Figure 6.7 and 6.8, this assumption is incorrect for the
current specimens, where the maximum strap strains have increased with time by
approximately 75\%. Increases in strain in the middle steel links were also observed.
An increase in transverse reinforcement strains would seem logical if shear
deformations are significant. Without an increase in the applied shear force, the only other way for the shear strains to increase (and thus the corresponding shear displacements) would be for the stiffness of the concrete to decrease, a fact that is acknowledged in the Neville et al. approach by the inclusion of a creep coefficient, \( \phi \). A decrease in the concrete’s stiffness would suggest a corresponding increase in the transverse reinforcement stresses and strains under the same load due to their constant stiffness, a fact that is not accounted for in their approach. As such, a different model is considered for calculating the long-term deflections due to shear. The initial flexural and shear deflections are calculated using the methods given. The long-term deflections, \( \delta_l \), are then calculated by multiplying both of these initial values by the ACI factor of equation 6-1.

The ACI factor approach was intended to be used with flexural deflections and so its applicability to shear deflections is uncertain. However, since the factors were derived based on curve fitting of numerous experimental results, it is likely that shear deflections have been built into the method; assuming beams with smaller span-to-depth ratios were included in the database. To determine if this approach is valid, the long-term deflection predictions using the ACI factor approach are checked against the results of specimens B8/30/G/46 and B9/30/G/42. The results for B8/30/G/46 under a sustained shear force of 110kN are given in Table 6.7.

<table>
<thead>
<tr>
<th>Time (months)</th>
<th>Exp Defl (mm)</th>
<th>Pred Flex Defl (mm)</th>
<th>Pred Flex Defl / Exp Defl</th>
<th>Pred Shear Defl (mm)</th>
<th>Pred Tot Defl (mm)</th>
<th>Pred Tot Defl / Exp Defl</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15.4</td>
<td>10.2</td>
<td>0.66</td>
<td>2.5</td>
<td>12.7</td>
<td>0.82</td>
</tr>
<tr>
<td>3</td>
<td>23.0</td>
<td>17.3</td>
<td>0.75</td>
<td>4.3</td>
<td>21.6</td>
<td>0.94</td>
</tr>
<tr>
<td>6</td>
<td>23.9</td>
<td>18.7</td>
<td>0.78</td>
<td>4.7</td>
<td>23.4</td>
<td>0.98</td>
</tr>
</tbody>
</table>

**Table 6.7 – Long-term deflection predictions – B8/30/G/46**

Initially the total deflection prediction does not seem as accurate as for previous specimens. This is believed to be due to creep during the initial loading of the specimen as discussed earlier. The long-term predictions seem quite accurate. Also, the results indicate that the ratio of flexural to shear deflection remains relatively constant over time. Otherwise one would expect the long-term predictions to be less accurate than initial predictions (Table 6.4) as both the shear and flexural deflections...
are multiplied by the ACI factor to calculate the long-term deflection. This notion of a constant ratio between the shear and flexural deflection as well as the relationship between the transverse strains and the shear deflection will be used to develop the long-term strap strain model later.

In order to model the long-term deflection of B9/30/G/42, a $\xi$ value for a duration of loading equal to 12 days was developed. To do this, the values of $\xi$ were plotted versus duration of loading in months on a logarithmic scale. A line was then fitted through the points and used to extrapolate the 12-day factor. This factor, $\xi = 0.3$, was used in equation 6-1 to calculate the long-term deflection increase multiplier of 1.21 and the long-term deflections of B9/30/G/42. Since the experimental results suggested that only the minimum shear force of 70kN created the deflection increase, only this deflection was multiplied by the long-term deflection factor. To get the long-term deflection at 110kN, the difference between the initial deflection at 70kN and 110kN is added to the long-term deflection at 70kN. These results are presented in Table 6.8.

<table>
<thead>
<tr>
<th>Load (kN) / Time (days)</th>
<th>Exp Defl (mm)</th>
<th>Pred Flex Defl (mm)</th>
<th>Pred Flex Defl / Exp Defl</th>
<th>Pred Shear Defl (mm)</th>
<th>Pred Tot Defl (mm)</th>
<th>Pred Tot Defl / Exp Defl</th>
</tr>
</thead>
<tbody>
<tr>
<td>70 / 0</td>
<td>7.4</td>
<td>6.5</td>
<td>0.88</td>
<td>1.5</td>
<td>8.0</td>
<td>1.08</td>
</tr>
<tr>
<td>110 / 0</td>
<td>12.3</td>
<td>10.2</td>
<td>0.83</td>
<td>2.6</td>
<td>12.8</td>
<td>1.04</td>
</tr>
<tr>
<td>70 / 12</td>
<td>11.3</td>
<td>7.9</td>
<td>0.70</td>
<td>1.8</td>
<td>9.7</td>
<td>0.86</td>
</tr>
<tr>
<td>110 / 12</td>
<td>14.9</td>
<td>11.6</td>
<td>0.78</td>
<td>2.9</td>
<td>14.5</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 6.8 – Long-term deflection predictions – B9/30/G/42

It is interesting to note that the difference between the initial experimental deflections at 70 and 110kN of 4.9mm is greater than the constant 3.6mm difference noted throughout the cyclic loading experiment. This is because upon initial loading the beam has not yet reached the maximum load of 110kN and so the steel links have not yet undergone any permanent displacements due to yielding. The modified Neville et al. model suggests the links will yield at approximately 100kN. Once the beam has been up to the maximum load, the minimum deflection will increase because of irrecoverable deformations due to link yielding. Failure to account for this yielding is the reason the minimum deflection at 70kN and 12 days is underestimated.
6.5.4 Prediction of CFRP Strap Strains

The Neville et al. approach for estimating shear deflections has proven to be quite accurate, both for initial and long-term deflections. As such, this approach will be used to estimate the initial and long-term strap strains. This is a logical extension of this approach, as the transverse reinforcement strains were required to calculate the deflections. Although the strap strains could be estimated using the crack widths calculated in Chapter 5, the Neville et al. approach is potentially better as it offers a method of calculating the long-term strap strains.

6.5.4.1 Initial Strap Strains

T-beams

In order to calculate the strap strains, the values for \(k_v\) and \(\theta\) can be obtained from the modified Deniaud and Cheng model as before with one modification. Rather than considering the total \(k_v\) and the average crack angle, \(\theta_{ave}\), only the web terms will be used. Whilst using the average approach worked well for calculating the total shear deflection, the strap strains are critical where there is extensive cracking in the web as has been demonstrated experimentally. Thus it is appropriate to calculate the strains in this critical region.

First, the transverse reinforcement strain at the point when the shear links yield is assumed to be the same as the yield strain of the steel links, \(\varepsilon_{yv}\). The strain in the CFRP straps after yielding of the steel links, \(\varepsilon_{transy}\), can be calculated based on the Neville et al. model and is given by equation 6-23. The transverse strains, \(\varepsilon_{yv}\) and \(\varepsilon_{transy}\), are calculated over a depth \(z\) so to convert them into strap strains the transverse strains need to be multiplied by the ratio of the depth \(z\) to the strap height, \(h_{FRP}\). The strap strain, \(\varepsilon_{FRP req}\), at the required load, \(V_{required}\), is thus given by equation 6-24. Generally this equation would be used with the maximum beam capacity to determine whether strap rupturing was an issue but it can also be used to determine initial strains at another load level in order to calculate the long-term strains.
\[ \varepsilon_{\text{transy}} = \frac{k_{\text{web}} (V_{\text{required}} - V_{\text{yweb}})}{E_{\text{FRP}} b_w z \rho_{\text{FRP}} (\cot \theta_w)} \]

where

\[ k_{\text{web}} = 1 - \frac{V_{\text{yweb}}}{V_{\text{totweb}}} \]

\[ V_{\text{yweb}} = \frac{E_{\text{trans}} b_w z \rho_{\text{trans}} (\cot \theta_{\text{web}}) \varepsilon_{y}}{k_{\text{web}}} \]

\[ \varepsilon_{\text{FRP, required}} = \left( \varepsilon_{y} + \varepsilon_{\text{transy}} \right) \frac{z}{h_{\text{FRP}}} \]

In order to check the validity of this approach, the predicted strap strains at the ultimate load for specimens B3/30/H/22 through B7/30/G/36 are compared to the experimental results (including \( \varepsilon_{\text{prestress}} \)) in Table 6.9. Because the initial experimental prestressing strain was actually quite variable for the reasons discussed in Chapter 3, this value was subtracted from the experimental strain values and replaced with a value of 0.0025 to provide a better comparison.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Ultimate Load (kN)</th>
<th>Actual Strap Strain</th>
<th>Predicted Strap Strain</th>
<th>Predicted / Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>B3/30/H/22</td>
<td>95 (actual)</td>
<td>0.006384</td>
<td>0.006940</td>
<td>1.09</td>
</tr>
<tr>
<td>B4/30/G/25</td>
<td>105 (actual)</td>
<td>0.006087</td>
<td>0.007472</td>
<td>1.23</td>
</tr>
<tr>
<td>B5/30/C/27</td>
<td>111 (actual)</td>
<td>0.006692</td>
<td>0.007702</td>
<td>1.15</td>
</tr>
<tr>
<td>B6/30/C/44</td>
<td>136 (predicted)</td>
<td>0.006614</td>
<td>0.006981</td>
<td>1.06</td>
</tr>
<tr>
<td>B7/30/G/36</td>
<td>132 (predicted)</td>
<td>0.006055</td>
<td>0.006694</td>
<td>1.11</td>
</tr>
</tbody>
</table>

Table 6.9 – Evaluation of strap strain predictions

The model generally overestimates the strap strains, which is conservative and acceptable for a design approach. Part of the reason for the high predictions is believed to be the assumption that the link steel has a well-defined yield strain. In reality though the links begin to lose stiffness at the assumed yield strain, they still have some stiffness and would take load. These results may prove more accurate for beams with steel links that are elastic-perfectly plastic.

**Rectangular Sections**

The strain model was then used to predict the strap strains for Kesse’s specimens. The results of this analysis are given in Table 6.10. The table gives the load at which the strap strains were evaluated, the actual strap strains including \( \varepsilon_{\text{prestress}} \), the predicted strap strains including \( \varepsilon_{\text{prestress}} \), and the ratio of predicted to actual. Also given is the
predicted crack angle, $\theta$, from the modified Deniaud and Cheng model and the crack angle that is required in the strap strain model to give the actual strap strain.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Load (kN)</th>
<th>Actual Strap $\varepsilon$</th>
<th>Predicted Strap $\varepsilon$</th>
<th>Predicted / Actual</th>
<th>Predicted $\theta$</th>
<th>Required $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B5-2s-10l-50p</td>
<td>88(^1)</td>
<td>0.0059</td>
<td>0.0040</td>
<td>0.68</td>
<td>43°</td>
<td>51°</td>
</tr>
<tr>
<td>B6-1s-5l-50p</td>
<td>71(^1)</td>
<td>0.0061</td>
<td>0.0077</td>
<td>1.26</td>
<td>28°</td>
<td>26°</td>
</tr>
<tr>
<td>B9-1s-10l-50p</td>
<td>76(^1)</td>
<td>0.0058</td>
<td>0.0065</td>
<td>1.12</td>
<td>35°</td>
<td>33°</td>
</tr>
<tr>
<td>B10-2s-5l-50p</td>
<td>78(^2)</td>
<td>0.0108</td>
<td>0.0060</td>
<td>0.56</td>
<td>37°</td>
<td>50°</td>
</tr>
<tr>
<td>B11-2s-10l-25p</td>
<td>87(^1)</td>
<td>0.0050</td>
<td>0.0031</td>
<td>0.62</td>
<td>39°</td>
<td>49°</td>
</tr>
<tr>
<td>B12-2s-10l-5p</td>
<td>80(^1)</td>
<td>0.0043</td>
<td>0.0020</td>
<td>0.46</td>
<td>36°</td>
<td>51°</td>
</tr>
</tbody>
</table>

\(^1\) predicted failure load  
\(^2\) actual failure load

Table 6.10 – Strap strain results for Kesse’s specimens

The results indicate that the model has a tendency to overestimate the strap strains for beams with only one strap and underestimate them for beams with two straps. Underestimating the strap strain is a problem if, as was the case for specimen B10-2s-5l-50p discussed in Chapter 5, failure is caused by the straps rupturing. Not accounting for rupturing can result in an overestimation of the beam capacity. It is interesting to note that for all the beams with two straps, the correct strap strains are predicted if the crack angle is assumed to be 50°. For Kesse’s beam depth of 280mm, this angle translates to a horizontal distance along the beam of 235mm. This is the same distance as the strap spacing, which could suggest that the critical strain occurs when the area of only one strap is considered. However, it may also indicate that steeper cracks are critical when considering rectangular sections. For the beams with only one strap the predictions are more accurate. More data is required to validate these observations in order to be able to use the model with full confidence for rectangular sections.
6.5.4.2 Long-term Strap Strains

T-beams

As discussed earlier, the CFRP strap strains increase over time under a sustained load. It seems that the ACI factor is quite accurate in terms of predicting the long-term deflection. The long-term deflection predictions also indicated that the ratio of shear to flexural deflection remains constant over time. Since the strap strains are closely related to the shear deflection, it is suggested that the long-term strap strain is a function of the shear component of the long-term deflection.

A relationship between the initial strap strain and the long-term increase is proposed in equation 6-25.

\[
\varepsilon_{FRPLT} = \varepsilon_{FRPreqired} \left( \frac{\varepsilon}{1 + 50 \rho_c} \right) \left( \frac{\delta_{\text{shear}} - \delta_{\text{shearyield}}}{\delta_{\text{shear}} + \delta_{\text{shearyield}} + \delta_{\text{yield}} - \delta_{\text{yield}}} \right) + \varepsilon_{FRPreqired}
\]

6-25

In this equation, the initial strain in the strap is multiplied by two terms to get the change in strap strain with time and then added to the initial strain. The \( A \) term is the ACI modification factor which is used to account for the effects of creep. However, the strap strain should be unaffected by creep due to longitudinal effects, and thus the \( B \) term gives the ratio of shear deflection to total deflection and is applied to isolate the creep effects due to shear. A modification to equation 6-25 is required if the steel links have begun to yield at the required load. This is because the \( B \) term in equation 6-25 uses a combination of the contribution of the steel links and the straps, however if the steel links yield, only the straps will be able to take further load redistribution due to creep. Equation 6-25 is rewritten as 6-26 for beams where the steel links are assumed to have yielded at the required load, such as those in the current study.

\[
\varepsilon_{FRPLT} = \varepsilon_{FRPreqired} \left( \frac{\varepsilon}{1 + 50 \rho_c} \right) \left( \frac{\delta_{\text{shear}} - \delta_{\text{shearyield}}}{\delta_{\text{shear}} - \delta_{\text{shearyield}} + \delta_{\text{yield}} - \delta_{\text{yield}}} \right) + \varepsilon_{FRPreqired}
\]

6-26

where \( \delta_{\text{shearyield}} = \text{shear deflection when steel links yield (mm)} \)

\( \delta_{\text{yield}} = \text{flexural deflection when shear links yield (mm)} \)
In order to determine whether this method provides a reasonable estimate of the long-term strap strains, the actual strap strains (including $\varepsilon_{\text{prestress}} = 0.0025$) at 110kN are compared with the calculated strains at 110kN for specimens B8/30/G/46 and B9/30/G/42 in Table 6.11. Figure 6.18 shows a semi-log plot of the outer and middle strap strains for B8/30/G/46 with time. Also plotted are the predicted long-term middle strap strain using the ACI factor and equation 6-26 as well the results of equation 6-26 using the initial experimental outer strain.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Time</th>
<th>Actual Strap Strain</th>
<th>Predicted Strap Strain</th>
<th>Predicted / Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>B8/30/G/46</td>
<td>0</td>
<td>0.004265</td>
<td>0.004228</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>3 months</td>
<td>0.005335</td>
<td>0.005314</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>6 months</td>
<td>0.005476</td>
<td>0.005532</td>
<td>1.01</td>
</tr>
<tr>
<td>B9/30/G/42</td>
<td>0</td>
<td>0.004033</td>
<td>0.004366</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>12 days</td>
<td>0.005656</td>
<td>0.004700</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>12 days*</td>
<td>0.005656</td>
<td>0.005096</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Table 6.11 – Long-term strap strain predictions

![Figure 6.18 - Strap strains with time comparison – B8/30/G/46](image)

The predictions for the sustained load specimen, B8/30/G/46, are quite accurate. This result seems to justify the use of both the Neville et al. method and the ACI factor to account for creep. Figure 6.18 indicates that just using the ACI factor to account for creep is too conservative, which is to be expected, as this factor should account for the effects of creep on both flexure and shear. It is worthwhile to note that both the
experimental strap strains and the ACI factor prediction plot linearly on the semi-log graph indicating that the use of the logarithmic ACI factor terms is appropriate for calculating the long-term strap strains. The predictions become much more accurate when the deflection ratio is also employed. This makes sense since the Neville et al. approach suggests there is a direct correlation between shear deflection and transverse strains and so if one can isolate the creep due to shear one should be able to predict the strap strains with time. The ‘predicted’ outer strap strain was obtained by entering the initial experimental outer strap strain as $\varepsilon_{\text{FRP, required}}$ in equation 6-26, which means the outer strain value gets multiplied by the same factors as the predicted middle strain. What the result indicates is that for lower initial strap strains, equation 6-26 predicts lower long-term strap strains as evidenced by the shallower slope of the line (which also matches the slope of the experimental results). This correlates well with the observation that the magnitude of the experimental long-term strain increases was dependant on the magnitude of the initial strap strain and further validates the use of equation 6-26.

While the initial strap strain prediction for the cyclic load test is both accurate and conservative, the strain prediction after 2.1 million cycles (12 days) is unconservative. Part of this may be due to the link next to this strap that seemed to lose stiffness resulting in a sudden increase in the strap strain. When loss of stiffness in the link is accounted for by lowering the value of $V_{\text{jwet}}$ to 70kN (the base cycling load) the prediction, denoted with an *, improves although it is still unconservative. The ACI factor may also have affected this prediction. Since the 12-day factor had to be extrapolated based on the available ACI factors, it is possible that the value is incorrect. The deflection prediction using this factor was also low suggesting that with a more accurate ACI factor, the predicted strain may also have been more accurate. Whilst this method shows initial promise, the database of results needs to be expanded to ensure that the method is accurate and conservative. However, it does suggest that there is a correlation between the increased strap strains and long-term shear deflections.
Rectangular Sections

At present there are no long-term strap strain results for retrofitted rectangular sections, making the development of a model purely speculative.

Despite the emphasis placed on long-term CFRP strap strains, it is worth noting the following: most structures where this retrofitting technique would be employed will have already undergone the majority of creep, so significant increases in strap strain should not be an issue. The sustained load test also considered the most severe loading case of a live load applied over a long period of time. Most structures are unlikely to see the full live load applied for significant periods of time and so increases in deflection and strap strains should be lower. Finally, since the CFRP strap technique is not limited by the constraints of other FRP retrofitting techniques (bond, anchorage, and localized stress concentrations), the number of strap loops can be increased to reduce the strain required to obtain the same level of prestress so that the long-term strain can be accommodated.

6.6 Other Considerations
6.6.1 Steel Reinforcement Fatigue

The fatigue capacity of a retrofitted beam is limited by the fatigue capacity of its component parts. As such, the designer must not only consider the fatigue capacity of the retrofitting material (the CFRP straps) but also the reinforcement and concrete in the existing beam.

During the cyclic test, the minimum stress level in the middle steel links was approximately 250MPa whilst the maximum was 370MPa as illustrated in Figure 6.19. This led to a stress range of 120MPa, which was below the 280MPa that design codes (BS5400 1996) deem to be acceptable for this type of reinforcement. Tilly (1979) suggested that the fatigue properties of the reinforcement were dependant upon both the stress range and the average mean stress. For the case of the transverse reinforcement, the average mean stress is 310MPa, which is higher than the values tested by Tilly. He observed that increasing the average stress from 159MPa to 275MPa resulted in a 40MPa reduction in the stress range that caused failure at $10^6$ cycles. The stress range to cause failure at an average stress of 275MPa was found to
be approximately 210MPa. The data given by Tilly seems to exhibit fairly linear behaviour (although this requires confirmation) so by extrapolation another 40MPa reduction in stress range will occur between an average stress of 275MPa and 391MPa, making the stress range to cause failure 170MPa at an average stress of 391MPa. A fatigue failure in the transverse reinforcement should not be critical as the observed stress range of 120MPa is less than this extrapolated value.

![Figure 6.19 - Middle link stresses versus number of cycles – B9/30/G/42](image)

Unfortunately the strain gauges on the longitudinal reinforcement gave erroneous results during the cyclic test due to strain gauge debonding. Results from previous static tests indicate that the minimum stress in the longitudinal reinforcement is approximately 310MPa with a maximum of approximately 475MPa at the cyclic test load levels (giving an average stress of 393MPa). This leads to a stress range of 165MPa, which is within the code limit of 220MPa for this bar diameter. However, when one considers the extrapolation of Tilly’s work given above, an average stress of 393MPa results in a stress range of 170MPa to cause failure at $10^6$ load cycles, which is close to the actual stress range of 165MPa. This suggests the possibility of a fatigue failure in the longitudinal reinforcement. If the full intended loading range of 30 to 110kN had been used, then the stress would have exceeded these limits. A method of predicting fatigue failure in the tensile reinforcement has been presented by Heffernan et al. (2004). Their method indicates that the stresses in the reinforcement...
need to be increased by a factor of 1.2 to allow for stress concentrations at cracks and by a further factor of 1.05 to account for tensile strain increases due to concrete softening. This would further reduce the potential stress range in the longitudinal reinforcement by increasing the average stress. Thus, any designer hoping to employ this shear retrofitting system should give careful consideration to fatigue of the existing reinforcement as well as the straps.

6.6.2 Concrete Fatigue

Although not specifically investigated in this experiment, Czaderski and Motavalli (2004) also noted the importance of the concrete capacity. In their experiments on T-beams retrofitted with CFRP L-shaped plates, they loaded a specimen between 39 and 59% of the ultimate specimen capacity for five million cycles. They noted that the concrete compression strain had increased significantly during the course of their test. They recommend that the designer consider carefully whether there is enough remaining concrete capacity in the structure to be retrofitted. Interestingly, although concrete capacity did not seem to be an issue for the cyclic specimen, B9/30/G/42, it did affect the overall capacity of the sustained load specimen, B8/30/G/46, when tested to failure (see section 6.8.2.1).

6.7 Unloading of the Sustained Load Specimen

In order to test specimen B8/30/G/46 to failure, it was first unloaded so that it could be moved from the long-term test rig into the static test rig. The mid-span deflection versus time for the unloading of the beam is given in Figure 6.20. The mid-span deflection after unloading was 6.2mm, which meant a net change of 18.1mm from the maximum displacement under sustained load. Thus the 15.4mm of initial displacement was recovered as well as 2.7mm of creep displacement. The mid-span deflection further decreased over the next 10 days until it plateaued at 5.3mm. These results compare well with those of other researchers (Neville et al. 1983) including the duration of creep recovery, which is fairly short in comparison to the duration of creep, which is essentially infinite. The creep deflection recovery of 3.6mm represents approximately 40% of the total creep deflection.
As illustrated in Figure 6.21, several vertical cracks developed in the flange of the beam after unloading. The cracks were widest in the constant moment region of the beam where they developed at a spacing of approximately 180 mm. Several cracks also developed in the shear spans although the width of these cracks was not as significant. These cracks are believed to be due to a combination of shrinkage strains, and stress redistribution within the flange due to concrete creep. If the cracks were due to shrinkage alone, one would expect them to occur at an even spacing with relatively constant widths along the full length of the beam. However, since the cracking appeared to be concentrated in the region of maximum compressive stress, it was felt that creep also played a role. It is well known that compressive reinforcement reduces the amount of creep strain in concrete, hence its inclusion in the denominator of equation 6-1. As the concrete loses stiffness due to creep, the stiffer compressive reinforcement attracts stress from the concrete. Once the beam is unloaded, the concrete strain does not return to zero but a portion of the strain remains irrecoverable as indicated here by the beam deflection remaining non-zero. Since the strains in the reinforcement and surrounding concrete must remain the same to satisfy beam theory, once the concrete reaches its zero stress state, there is still compressive stress in the reinforcement since the strain is non-zero (and equal to the creep strain in the concrete). As a result, the surrounding concrete goes into tension in order to reach equilibrium, and cracks form in the flange where the tensile capacity of the concrete is
exceeded. From a practical point of view, these cracks could cause damage to the road surface or floor finishes being supported on the beam. The designer should think carefully about this when bracing the structure during repairs.

Figure 6.21 – Crack pattern – B8/30/G/46 unloaded

6.8 Static Test to Failure of the Sustained and Cyclic Load Specimens

To determine whether long-term loading has had any affect on the ultimate capacity of the retrofitted beams, B8/30/G/46 and B9/30/G/42 were tested to failure. The cyclic testing rig was used for the static load tests. The only difference was that a new Amsler pump capable of taking advantage of the full 500kN capacity of the jack was hooked into the system. Specimen B8/30/G/46 was loaded to failure. Specimen B9/30/G/42 was loaded to a total load of 220kN and then unloaded to 140kN to see if there were any significant changes in beam performance at the cyclic minimum and maximum loads, and then loaded to failure. These two static failure tests will be compared with the results of B6/30/C/44 and B7/30/G/36, specimens with similar concrete strengths but that were undamaged prior to being tested to failure. By comparing these results one can gauge what effect the long-term loading had on the ultimate capacity.

6.8.1 Failure mode

Specimens B8/30/G/46 and B9/30/G/42 both failed due to crushing of the concrete in the constant moment region. Specimen B6/30/C/44 failed in shear after reaching the
ultimate load and undergoing additional deformation. Examination of B9/30/G/42 after failure revealed that there were shear cracks in the flange extending to the top surface adjacent to the load pad. There was no evidence of a potential shear failure in specimen B7/30/G/36, which had a slightly lower concrete strength (36 versus 42MPa). The shear cracks in specimen B8/30/G/46 whilst extending into the flange did not extend to the top surface. Although B8/30/G/46 had the highest cube strength on the day of testing (45.7MPa), its flexural capacity was actually lower than B7/30/G/36, which was due to concrete creep effects from the 260 days of sustained loading. Heffernan et al. note that as the concrete softens over time, the depth to the neutral axis of the beam increases, reducing the lever arm between the concrete and steel resultant forces. To support a constant moment, the stresses in the concrete must increase to offset this reduction in lever arm over time. In the failure test of specimen B8/30/G/46, this reduced lever arm caused by the sustained loading test meant that higher concrete stresses developed for a given applied moment. Thus the compressive capacity of the concrete was reached at a lower applied load and resulted in a lower flexural capacity. In the current test series, beams with lower flexural capacities did not fail in shear while beams with higher flexural capacities seemed more susceptible to this type of failure. This suggests that variations in concrete strength and duration of loading could have an impact on the failure mode if the flexural and shear capacities are similar. It is rather difficult to draw definitive conclusions about this behaviour as B6/30/C/44 demonstrated ductility before failing in shear and although specimen B9/30/G/42 developed extensive shear cracks, it failed due to concrete crushing. From a materials point of view this behaviour makes sense. Shear cracks form when the tensile strength of the concrete is exceeded. The tensile strength of concrete is usually defined as being proportional to the square root of the compressive strength. Thus as the compressive strength increases, the tensile capacity does not increase by the same magnitude, suggesting that there is a point where the concrete will have enough compressive capacity for flexure but will be unable to support the shear stresses. Further experiments using higher strength concrete beams will be required in order to ascertain whether the current specimens were on the verge of a shear failure due to the capacity of the concrete.
6.8.2 Shear-Deflection Results

Figure 6.22 gives the shear force versus deflection response for specimen B6/30/C/44, the sustained load specimen, B8/30/G/46, and the cyclic load specimen, B9/30/G/42.

![Shear force versus mid-span deflection](image)

Figure 6.22 – Shear force versus mid-span deflection – B6/30/C/44, B8/30/G/46 & B9/30/G/42

6.8.2.1 B8/30/G/46 – Sustained Load Specimen

Specimen B8/30/G/46 attained a maximum shear force of 131.2kN. This capacity is approximately 6% lower than the other two specimens, despite the concrete cube strength for B8/30/G/46 being the highest at the time of testing. This lowered moment capacity is the result of concrete softening as suggested earlier.

At the maximum load the performance of B8/30/G/46 is quite ductile with 5.3mm of deflection at the maximum load before crushing of the concrete in the constant moment region led to failure. However, its performance is quite different to the other specimens. The initial stiffness of the beam, up to approximately 40kN, is much lower. This is believed to be a result of the cracks in the compression region (which formed when the sustained load was removed) closing during which time the compression reinforcement takes the majority of the load. After this initial loading
period, the stiffness of the beam increases by about 50% but still remains lower than the other two specimens, which is due to the reduced modulus of elasticity of the concrete caused by creep. The reduced stiffness of the beam, coupled with the initial irrecoverable creep deflection of 5.3mm, means that the beam reaches its maximum load at a deflection approximately 7.5mm greater than the other two beams.

6.8.2.2 B9/30/G/42 – Cyclic Load Specimen

B9/30/G/42 attained a maximum shear force of 139.2kN. The performance of the beam was quite ductile supporting the maximum load for approximately 4 mm of deflection before crushing of the concrete. While B9/30/G/42 started out with an initial deflection offset from the cyclic testing of 2.5mm, at the ultimate load the deflections of B6/30/C/44 and B9/30/G/42 are virtually identical. Czaderski and Motavalli (2004) also noted that there was very little difference between the ultimate deflection of a cyclically loaded beam and its statically loaded counterpart. However, as discussed above, creep in the concrete had a significant impact on the ultimate behaviour of the sustained load specimen. The reason that this is not the case for the cyclic beam is thought to be due to the lower load level and the shorter duration of loading. As well, once a load of 70kN was reached (the base load for the cyclic test which appeared to cause the majority of creep) the stiffness of the concrete should be quite similar to B6/30/C/44 since the creep effects would be minimal above that load. The combination of these factors results in the ultimate performance of the cyclic specimen being very similar to a static test on a virgin specimen. If the cyclic test had been run over a longer time period, the effects of creep might have had more impact on the ultimate performance, as they did for the sustained load specimen.

The overall behaviour of B9/30/G/42 is quite promising as it suggests that cyclic loading has had no effect on the ultimate capacity of the specimen, or of the retrofitting system. Also, despite having grouted holes, B9/30/G/42 has achieved the same capacity as B6/30/C/44, a beam with comparable concrete strength but without grouted holes. This validates the conclusion drawn earlier that grouting the holes does not have a significant affect on the beam capacity.
6.8.3 Straps

The shear force versus middle strap strain relationships for specimens B6/30/C/44 through B9/30/G/42 are given in Figure 6.23. The strap strains for the two long-term test specimens start from non-zero values even when the strain due to prestressing has been subtracted due to the presence of residual strains from earlier loading. The initial strain in the strap on the cyclic load specimen is actually higher than the comparable strap on the sustained load specimen. One might expect that having undergone more creep the residual strains in the straps on the sustained load specimen might be higher. However, the main cause of strains in the straps is the width of the shear crack. Since the sustained load beam was only loaded and unloaded once, the shear cracks were able to close with relative ease, although there is some residual strain. The cracks in the cyclic specimen on the other hand were exposed to 2.1 million cycles as well as several complete unloading cycles. This has resulted in greater damage to the material along the crack and higher residual strains in the strap. It should be noted that while these higher strap strains most likely indicate higher residual deflections due to shear, the sustained load specimen still has a larger overall residual deflection due to flexural creep effects caused by the longer duration of loading.

![Figure 6.23 – Shear force vs. middle strap strains – B6/30/C/44 through B9/30/G/42](image-url)
The slopes of the shear force versus strap strain curves also demonstrate this notion of greater damage along the cracks in the cyclic specimen. After an initial period of increased strains in the middle strap during the failure test of the sustained load specimen, B8/30/G/46, which corresponds to the overall lack of stiffness in the specimen, the rate of strain change decreases dramatically. B9/30/G/42 was loaded statically up to the maximum load several times before the cyclic loading was started. The middle strap strain results for the second static loading of B9/30/G/42 have been plotted (labelled as ‘B9/30/G/42 initial’ in Figure 6.23). The rate of change for B8/30/G/46 becomes quite similar to the initial loading results for the middle strap on B9/30/G/42. This is to be expected since that was only the second time the shear cracks had been opened in both specimens. However, the strap strains in specimen B9/30/G/42 after 2.1 million cycles increase at a greater rate as load is applied. This suggests that the straps carry more shear force over this load range than they did when the specimen was first loaded indicating that the load has been redistributed over time.

While the strap strains for B9/30/G/42 correspond quite closely to the strains from the static tests (B6/30/C/44 and B7/30/G/36) at the ultimate load, the strains for B8/30/G/46 do not. This is believed to be due to the increased concrete capacity at the time of testing of B8/30/G/46 (approximately 46MPa). Whilst concrete creep affects the flexural capacity, Walraven et al. (1987) suggest, based on a series of push-off tests conducted under both sustained and cyclic load, that load history has no effect on ultimate shear capacity. As such a higher concrete contribution at the ultimate load would result in lower strap strains. However, this result is also slightly misleading because the beam was fully unloaded prior to testing, allowing for creep recovery. As can be seen from Figure 6.23, the strain in the middle strap on B8/30/G/46 at the ultimate load does not actually exceed the maximum strain attained during the sustained load test. The failure strain instead passes through the strain range from the sustained load test approximately 60% of the way along this range. This correlates quite well with the observation that the specimen recovered 40% of its total creep deflection as it seems that the strap has recovered 40% of the strain due to creep at this load level. This validates the conclusion that there is a correlation between specimen deflections and strap strains.
An important point to note is that each strap has additional capacity remaining at failure. This suggests that even with lower steel yield strengths or concrete capacities, this particular strap configuration could prevent a shear failure and allow a ductile flexural failure to occur.

As a final check of the strap capacities, the straps on both long-term capacity specimens were tested to failure. The results of these tests are presented in Table 5.6 and indicate that the straps still had adequate capacity comparable to straps on previously undamaged specimens.

### 6.9 Conclusions

The results of the long-term tests are quite promising in terms of validating the performance of the CFRP strap retrofitting system. The sustained load specimen supported a load equal to 80% of its retrofitted capacity for 260 days. The cyclic load specimen withstood 2.1 million cycles of loading varying between 50 to 80% of the retrofitted capacity. In both cases the straps withstood the full loading range. Since the under-slab installation technique was also employed in both cases, these tests also validated the use of this system for long-term loading. However, it was noted that the strap strains increase over time as well as with the number of loading cycles. If the strap strains were to increase until they reached the rupture strain of the CFRP, the straps could fail, possibly resulting in failure of the structure. Thus a method of predicting the initial and long-term strap strains needed to be developed.

It was observed that the strap strain behaviour seemed to have much the same shape as the deflection behaviour for both long-term specimens. Deflection modelling was examined with the goal of developing a strap strain model. Most design codes suggest that RC beam deflections are due primarily to flexural effects. However, when beams in the current test series were modelled using flexure-based deflection models, the models underestimated the actual deflections. This underestimation was believed to be due to the failure to account for shear deflections. When a model developed by Neville et al. (1983) was implemented to calculate the shear deflections, the resulting combined flexure and shear predictions were much more accurate. The Neville et al. method also gives the transverse reinforcement strains at any given load.
Unfortunately the long-term deflection model given by Neville et al. does not account for increases in long-term transverse reinforcement strains. As such, the model was modified further to a combined flexure and shear deflection approach to calculate initial deflections and the ACI factor approach to calculate long-term deflections. This method gave accurate results when checked against the long-term beam deflections in this study.

A strap strain model was then developed using the shear deflection model. Initial strap strains were calculated quite accurately although slightly conservatively. The long-term strap strain model also showed promise. However, the database of beam tests with straps needs to be expanded before full confidence in the model can be achieved, especially for rectangular sections. The fatigue capacity of other elements of the beam needs to be considered carefully, as load redistribution from these elements could still cause failure of the straps unless they are accounted for properly.

Both specimens failed in a ductile manner due to concrete crushing in the constant moment region when tested statically to failure. The sustained load specimen, B8/30/G/46 failed at a lower load than would have been predicted using the concrete cube strength. This was a consequence of concrete creep that occurred during the sustained load test affecting the flexural capacity. The cyclic load specimen having not been exposed to the same duration or magnitude of loading as the sustained load specimen failed at the same shear force as B6/30/C/44, a beam of similar concrete strength that had not been damaged before being tested to failure. The straps provided satisfactory long-term performance as in both cases the beams withstood the loading and failed in a ductile manner.

After the beams had failed, failure tests on the straps indicated that they still had similar capacities to previously unloaded straps, which again suggested that their long-term performance is adequate.
Chapter 7
Conclusions and Recommendations

7.1 Conclusions

The initial goal of this research program was to develop a method of installing a CFRP strap shear retrofitting system, which had been used successfully in other studies, from underneath the beam without requiring access to the top surface. If successful, this system would eliminate the need for metal support pads on the beam surface as well as reducing or eliminating delay times due to these repairs.

The experimental portion of this research consisted of three main programs: strap configuration tests, static beam tests with different strap configurations, and long-term capacity tests. The strap configuration tests to develop the under-slab installation technique resulted in three conclusions about optimising the strap capacity. First, when the straps are passed through holes in the concrete, they should not press against the sharp edges of the holes. If the straps come in contact with sharp edges, the straps tear which causes significant reductions in strap capacity. Second, the strap cross section should be kept as flat as possible. If the cross section takes on a curved profile, each fibre in the strap is loaded at a different level and failure occurs slowly as each fibre ruptures independently resulting in a reduced capacity. Finally, the strap must be supported on a material that is stiff enough to ensure that the force in the strap can be transferred to the beam without experiencing significant deformations. If the support material is not stiff enough, the strap simply bears into the support material without developing significant strains.

The static RC T-beam tests identified three significant issues that had to be overcome in order to develop an effective under-slab installation technique using drilled holes in the flange. First, the strap needs to extend far enough into the compression flange to properly tie the beam together. If the straps did not extend high enough in the compression flange, shear cracks developed above the straps and they did not take significant load. This result validated the conclusions of Kani et al. (1979) who suggested that the transverse reinforcement has to tie the entire depth of the beam together in order to be effective. This problem was overcome using a hole pattern that increased the penetration of the strap. The presence of holes in the compression flange
also played a role. A specimen with holes occupying 35% of the total flange area offered little capacity enhancement over the control beam, despite the straps encompassing almost the full beam depth. This problem was dealt with by grouting the holes. Finally, the area of the loading pad seemed to have an effect. Specimens where the pad only occupied the web width failed in shear, whereas when the pad occupied the full flange width, flexural failures were possible. Once these issues were overcome, a ‘realistic’ specimen was tested. The holes were drilled into the flange of this specimen and then filled with grout except for a groove for the strap, just as they would be on site. This retrofitted specimen failed in flexure with a considerable amount of ductility. Thus, the most important conclusion of these tests is that a significant capacity enhancement is possible using the CFRP straps with an underslab installation technique.

A commercially available FEA package called DIANA was used to better understand the experimental results. The role of intrusions (holes and grout filled holes) was examined by modelling a series of concrete prism tests. The results suggested that a single discrete crack developed in the specimens with holes, running parallel to the direction of principal compressive stress. The maximum capacity and stiffness of a prism with a hole was predicted to be lower than an equivalent plain concrete specimen. This result was extended to the beam models by creating a concrete curve with reduced stiffness and capacity. This new concrete curve was used to model the concrete in the vicinity of the holes in the 2-D FE beam analysis. The results indicated that the reduced stiffness and compressive capacity did not have a significant impact on the overall beam capacity. Instead it seemed that the tensile cracks that formed parallel to the direction of compressive stresses in the region around the holes created a weak plane in the concrete that caused a premature shear failure. Holes that were lower in the flange had less of an impact because the compressive stresses were lower in this region and the weak plane did not reach the top of the beam. The FE also indicated that strap penetration into the flange played a minor role in the specimen capacity with shear cracks forming above straps that did not go far enough into the flange. The size of the loading pad seemed to be an issue not because of bearing capacity, but because of the formation of shear cracks. If the loading pads did not occupy the full width of the flange, shear cracks formed at shallower angles next to the pads, resulting in reduced beam capacity. Finally, whilst the use of a smeared
crack model with a constant shear retention factor worked well in terms of predicting the capacities of the test specimens, this was because failure was never strap critical. The smeared crack model with constant shear retention factor is not recommended for design, as the strap strain predictions are not accurate, and sometimes very unconservative.

A design procedure to determine the retrofitted capacity for use in codes was then developed. A model was selected from those available in the literature based on how well it predicted the results of several series of beam tests with varying cross sections. The model was based on a shear friction approach developed by Loov (1998) and Deniaud and Cheng (2001 and 2003) which was modified to better account for the effects of prestress and strains due to opening cracks. With these modifications in place this model proved to give accurate predictions for all the tests, including the deep beam tests of Stenger (2000) where estimating the strap strain correctly proved to be crucial. A design procedure was then outlined with appropriate safety factors and limits on strap strain. This procedure was used to prove numerically that lower strength concrete specimens of the type tested in this study can be retrofitted to achieve a flexural failure, a result which had not been confirmed experimentally until this point. A similar specimen was tested, which failed in shear but further validated the model because the shear prediction was within 2% of the actual capacity. The specimen failed at a load approximately 45% higher than the control specimen proving that the straps, and the under-slab installation technique, can provide significant capacity enhancement to low strength concrete beams.

The long-term capacity tests were undertaken to ensure that the straps, which up until this point had only been tested statically in conjunction with beams, would not fail after being loaded for an extended period of time. The straps were installed using the under-slab installation technique developed in this program. Although only two long-term capacity tests were undertaken, the loading used was quite severe. The sustained load specimen was exposed to a load of 80% of its retrofitted capacity for 260 days. The cyclic load specimen was loaded between 50 and 80% of its retrofitted capacity for 2.1 million cycles. In both cases the straps remained intact throughout the duration of the tests validating both the long-term use of the straps and the under-slab installation technique. However, the strap strains did increase over time. If the strap
strains were to eventually reach the rupture strain, the straps would fail which could result in structural collapse. A model was thus developed based on the shear deflections to predict the strap strains both initially and in the long-term. The model was quite accurate when used in conjunction with T-beams. The results were less accurate for rectangular sections although more data is required to fully validate the model.

7.2 Future Work

Although the long-term capacity tests produced promising results in terms of both proving the long-term potential of the straps and highlighting key issues, the database of results in this area needs to be expanded. Further experiments need to investigate the effect different $a/d$ ratios and varying cross sections have on the strap performance. Long-term strap strain results for rectangular sections need to be produced so that the strap strain model can be checked and improved. An expanded database of results is also required to validate the modified Deniaud and Cheng model for different cross sections, $a/d$ ratios and loading patterns. Also, experimental work on other durability parameters such as temperature variation and exposure to hostile environments would further increase the confidence of designers.

Work is required towards making the strap system a more commercially saleable product. This includes developing a simplified welding apparatus that is both durable and easy to use in the field. The prestressing operation should also be made less complicated. One such possibility is to develop a pressurized grout pad where the pressure applied to the grout can be used to determine the prestressing force. The prestress is then held in the strap by the hardened grout. Once this system has been made user friendly, both the design and construction communities will be more willing to consider the CFRP strap option.
Chapter 8
References

ACI Committee 318, 1995, Building Code Requirements for Reinforced Concrete (318-95) and Commentary (318-95R), American Concrete Institute, Farmington Hills, MI, 369pp.

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