

Abstract

Experience gained in a workplace characterised by decision-making and learning-by-doing is modelled via a process of signal accumulation under several different frameworks. We initially look at the probability of success based on uninterrupted signal accumulation, then consider the impact of rapid labour turnover under two alternative regimes. The first allows new workers to gain some of their predecessor's experience through Bayesian inference on reported earlier actions. The means of information transfer between workers is therefore similar to observational learning in herding or informational cascade models. The second regime considers all experience to be lost when a worker is replaced. We see that although with valuable experience the first regime appears a much better outcome for firms, transferring some knowledge to future workers carries with it the risk of excess inertia in decision-making.

Keywords: herding, private information, experience, labour turnover, excess inertia

JEL Classification: D82, D83, J63

Modelling Experience as Signal Accumulation

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1. INTRODUCTION

Quite often the replacement of an employee is a troubling period for a firm as his ‘lost experience’ has to be recouped. Furthermore, workers will often keep their methods to themselves which exacerbates this loss of experience. Nevertheless when a new employee replaces a worker, he can usually obtain some of the wisdom of his predecessor through examining his past record, his successes and failures and his most recent decisions. To give another story, there is often surprise expressed when new employees fail to break new ground and instead follow the practices of their predecessors, even when specifically hired because of their predecessor’s failings. A new chief executive may follow the actions of his predecessor for some time before initiating a U-turn in company policy. These stylized facts are shown in a new light in this paper. The worker is modelled as a decision-maker who gradually improves his ability to make good choices, and though a subsequent employee can get partial access to the information of his predecessor, the disappearance of workers’ accumulated past signals will be costly for any firm with rapid rates of labour turnover.²

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²The recruitment industry frequently attempts to measure the costs of rapid turnover, but tends to focus on the more measurable and easily understood elements. Glancing at the literature provided by modern recruitment firms provides a good idea of what are considered the main costs of labor-turnover. The “direct costs” emphasized in the literature include the costs of hiring and firing, interviewing, head-hunting and training. The more subtle “indirect costs” relate to the zero-productivity of an empty desk. Very exhaustive industry reports might also add losses due to joint productivity. Getting closer to the concerns of this paper, there is some awareness that training

Consider a learning model in which a sequence of signals taken from a distribution centered around the ‘correct action’ build up to provide a clear and improving indication of what a decision-maker should do. Depending upon the quality of the signal draws, the ability to get the decision right with a high degree of accuracy can be obtained often very quickly. This is our basic model of the building of experience. A worker will receive signals - proxies for reports, interactions, telephone conversations, and so on, which allow a worker to improve in his decision-making over time. These signals are, by their very nature, private. While a worker will announce his decision to the firm, it is often not necessary, very expensive in terms of time and money, and indeed very difficult, for him to reveal the precise rationale behind his decision. In fact it is in his interests to keep his signals private. So the firm sees a sequence of decisions, while the employee sees a sequence of signals gradually improving his odds of success. At some point however a new worker will replace the previous employee. It is clear that rather than simply imagining all the earlier decision-making of his predecessor as lost, it would be in the new worker’s interests to add to his basic knowledge about his new job, by attempting to infer the signals of his predecessor. The most obvious approach is to assume that a new employee will use Bayesian inference on the observed past actions of his predecessor to estimate the value of his signals and act on this.

There is a similarity between this and existing herding models in which past actions but not signals can be observed such as Banerjee (1992) and Bikchandani *et al* (1992). In such an *informational cascade* we might see agents trapped in a herd on a particular decision because of the observation that the decision was popular in the past. A few early choices can therefore create cascades of decision-making into a particular choice, though only because the very first decision-makers felt that was the best option. Understanding herding theory allows some immediate hypotheses about what is likely to emerge. Decisions will persist - so replacement workers will for a prolonged period of time follow the actions of their predecessors often beyond when the earlier worker would have opted for a new course of action. It will be in worker’s interests to capitalise on all of this by keeping their information as private as possible and so

will not be enough to produce a perfect replacement for the previous employee, and it will still take time.

extract rents on the divergence between the odds of success they can obtain and that of a new employee who has access to coarser information based on inference of actions rather than a long sequence of private signals. Finally, we might imagine that the herd externality based on this loss of information, even given fully rational Bayesian updating, will be greater for industries and firms where individuals are free to make decisions which are individually valuable enough to warrant the collection of a great deal of information. We might imagine the impact of a replacement on a law firm, or medical clinic to be considerable whereas the decision-making in a blue-collar factory is not likely to have the same effect. A lawyer knowing that his replacement will lead to the disappearance of a great deal of built-up experience will wish to keep as much of this knowledge to himself as possible to command higher rents from his employer after some time in the same job, while a firm will necessarily try to combat this by requiring more and more record-keeping to enable future employees to better infer the decisions of their predecessors.

The existing work which gets closest to this paper is Hirshleifer and Welch (2002) which attempts a more general application of herding theory to “the psychology of change” and mirrors the notion of excess-inertia in decision-making in this paper. Early applications in herding theory came first to the financial sector in Welch (1992), and more recent applications to marketing and other firm decisions are given in SgROI (2000) and SgROI (2001). Meyer (1991) shows the importance of odd and even sets of information, used in this paper, through an application to biased tournaments.

The paper continues in section 2 with a simple model of decision-making under which the worker is employed and examines the normal learning process by which experience is gained as a form of signal accumulation. Section 3 provides ways of measuring experience relative to two benchmarks. The first assumes that some of the information gathered by workers being replaced can be obtained by new workers via a process of Bayesian inference on existing records. The second benchmark instead considers the situation where there are no maintained records. In both cases experience is lost, though clearly the existence of records allows some to be recaptured. We can also measure the value of record-keeping using this approach. Section 4 examines the problem of excess inertia in decision-making which occurs when record-keeping allows some experience to be passed from worker to worker. Section 5 concludes.

2. THE MODEL

We begin with a benchmark model showing how decision-making improves over time within a firm when there is no turnover. Consider a binary state $V \in \{-1, 1\}$, where we consider $V = 1$ to represent the “good” state and $V = -1$ the “bad” state. There is a decision to be made, $A \in \{Y, N\}$. We denote the converse of an action by \bar{A} , so if $A = Y$, then $\bar{A} = N$. Actions may be changed over time, $t \in \mathbb{T}^{++}$, though the state is fixed at $t = 0$ by nature, before the first action can be taken. At this stage we partition \mathbb{T}^{++} into \mathbb{T}_{even}^{++} , the set of strictly positive even integers, and \mathbb{T}_{odd}^{++} the set of strictly odd integers. An agent receives a costless signal to assist in his decision every period, $x_t \in \{H, L\}$ which is informative in the sense that $\Pr[x_t = H \mid V = 1] = \Pr[x_t = L \mid V = -1] = p > 0.5$, and $\Pr[x_t = H \mid V = -1] = \Pr[x_t = L \mid V = 1] = 1 - p < 0.5$.

We use the terminology “ x_t suggests A_t ” meaning that the single signal $x_t = H$ is suggestive that the state is $V = 1$, therefore the action should be Y , so “ H suggests Y ” and similarly “ L suggests N ”. The history of signals is $H_t = \{x_1, \dots, x_t\}$. Information is defined as $I_t = \{H_{t-1}, x_t\}$. Each period the agent must make a decision and payoffs are:

$$u_t = \begin{cases} V & \text{if } A_t = Y \\ 0 & \text{if } A_t = N \end{cases}$$

Call a decision *correct* if it maximizes u_t , so if $V = 1$ then $A = Y$ is correct, if $V = -1$ then $A = N$ is correct.³ The objective of the agent is to maximize u_t which might represent a bonus gained once the correct value of the state is revealed to the employer. Cumulative payoffs are given as:

$$U_T = V \sum_{t=1}^T 1_{\{A_t=Y\}}$$

³For simplicity we suppress principle-agent concerns and consider a reduced form in which we might incorporate some form of performance related-pay. In particular we might imagine a final reference at the time an employee leaves his employment or a small percentage chance that shirking (failing to use all available information in this model) is discovered and rewarded with prompt dismissal. Putting all of this to one side will simply assume that though the value of the true state will only be revealed in the distant future to the firm, employees will act to get decisions correct.

Where $1_{\{A_t=Y\}}$ is the index function returning a value 1 if the condition $\{A_t = Y\}$ is satisfied, 0 otherwise. We assume the agent is risk neutral, so he cares only about maximizing the expected value of u_t by making as good a decision as he can at time t .

2.1. The Normal Learning Process. In this paper experience is gained through *learning-by-doing*. The worker's job is simply to match the value of the state (1 or -1) with the correct action (Y or N respectively). Each period a worker gains a signal and processes this to provide a suggested action. This action may be correct or not, but the basic premise of this paper is that as signals accumulate the odds of being correct will rise. The measure of experience is therefore equivalent to the rising probability of success as more signals are obtained.

Consider the agent's decision for the first few periods. At $t = 0$ nature chooses a state. At $t = 1$ the agent receives the signal x_1 , so $I_1 = \{x_1\}$ and acts upon it choosing an action A_1 according to the simple rule $x_1 = H \Rightarrow A_1 = Y$ & $x_1 = L \Rightarrow A_1 = N$. At period 2, the agent receives a second signal and his information set now includes two signals, $I_2 = \{x_1, x_2\}$. If his two signals agree he will clearly follow the suggested course of action. If they disagree he will be indifferent, and will happily follow either Y or N . More generally the decision which maximizes u_t by the agent at time $t \in \mathbb{T}_{odd}^{++}$, will be to follow the majority of signals in H_t .⁴ Now consider the binomial expansion of the set of signals which will produce a correct decision. By listing the combination of all possible majority signals of H in the state $V = 1$ we produce the *success rate* of getting the decision right at time t , which we denote as $\theta(p, t)$, or simply θ_t^{NLP} with p held fixed.

Definition 1. *The normal learning process (NLP) maximizes u_t for agent at time $t \in \mathbb{T}_{odd}^{++}$, by setting A_t to follow the majority of signals in H_t , and produces a strictly*

⁴The decision-making process is only considered for time indexed by $t \in \mathbb{T}_{odd}^{++}$, since the probability of success is identical for $t = k$ and $t = k + 1$, where $k \in \mathbb{T}_{odd}^{++}$, under the learning processes analysed below. This result is stated and proved more formally in Lemma 2 given in the appendix and greatly simplifies much of the analysis to come.

increasing monotonic success rate for $\theta(p, t)$, of:

$$\theta^{NLP}(p, t) = \sum_{k=0}^{\frac{t-1}{2}} \frac{t!}{(t-k)!k!} p^{t-k} (1-p)^k$$

Table 1 shows how quickly the agent's probability of getting the decision correct increases with an initial probability of success p . For $p = 0.9$ the worker achieves a success rate of 0.999 by $t = 9$, though he only achieves a success rate of 0.579 by $t = 99$ if $p = 0.51$.

t	$\theta^{NLP}(0.6, t)$	$\theta^{NLP}(0.8, t)$
1	0.600	0.800
3	0.648	0.896
5	0.683	0.942
7	0.710	0.967
9	0.733	0.980
11	0.753	0.988
13	0.771	0.993
15	0.787	0.996
17	0.801	0.997
19	0.814	0.998

Table 1: Learning Rates for Different Values of p and t .

3. MEASURING EXPERIENCE

In this section we examine a series of measures of experience by looking at the difference in success rates between the normal learning process given in the last section and some alternative situations in which experience is lost as a worker is replaced.

3.1. Success under Rapid Turnover. Consider what might happen when an experienced employee is replaced by a new less experienced employee. The new employee will not have access to his predecessors set of signals, but will easily enough be able to confirm the last action of his predecessor, since this is the action under which he finds the firm operating at his arrival. Alternatively he may even be able to ‘shadow’ his predecessor and observe his methodology first-hand just prior to replacing him.

Furthermore he knows the length of time his predecessor occupied the post, and has access to the same signal generating technology (the p remains the same, and is known to the new agent). The new employee can of course calculate his predecessor's probability of success based on the analysis above, and will slowly build up his own success probability by gaining new signals.

We model all of this by considering a firm which loses an employee at a certain date (t^*) and then replaces him with a series of new employees every two periods, so each worker gets two periods of employment. The new agent is granted an information set in his first period of employment of $I_\tau = \{\theta_{\tau-1}, x_\tau\}$, $\tau \in \mathbb{T}_{odd}^{++}$, so instead of a history of signals he receives only the coarser observation of the likelihood of success of his predecessor on replacement plus his own signal. In his second and final period of employment he has a further observation so $I_{\tau+1} = \{\theta_{\tau-1}, x_\tau, x_{\tau+1}\}$, $\tau + 1 \in \mathbb{T}_{even}^{++}$. His coarser information set will naturally reduce the speed at which his success rate rises. Following Lemma 2 given in the appendix we can disregard the stream of signals $\{x_\tau, \tau \in \mathbb{T}_{even}^{++}\}$, as not necessary for the updating process. We can model the improving chances of success using a simple *Bayesian learning process* as defined below.⁵

Definition 2. Let θ_t be the prior probability of success at t and $\theta_{t+2} = f(\theta_t, x_{t+1})$, $t \in \mathbb{T}_{odd}^{++}$ be the posterior. A Bayesian learning process (BLP) updates θ_t by the new signal x_{t+1} , which is correct with probability p , strictly increasing the odds of success to:

$$\theta_{t+2}^{BLP} = \frac{\theta_t p^2}{\theta_t p + (1 - \theta_t)(1 - p)} + \frac{\theta_t (1 - p)^2}{\theta_t (1 - p) + (1 - \theta_t)p}$$

While $\theta_{t+2}^{BLP} > \theta_t$, so success rates do improve, the improvement takes place more slowly than under the NLP. For comparison note the values of θ_t over time when we assume that the new agent takes over when $t = 5$; we will denote the point of replacement of the original worker as t^* .

⁵See footnote 4.

t	$\theta^{NLP}(2/3, t)$	$\theta^{BLP}(2/3, t)$
1	0.667	(0.667)
3	0.741	(0.741)
5	0.790	0.763
7	0.827	0.783
9	0.855	0.800
11	0.878	0.815
13	0.896	0.828
15	0.912	0.839
17	0.925	0.849
19	0.935	0.858

Table 2: A Comparison of the Speed of Learning with and without Turnover.

Note: At $t^* = 5$ replacement occurs in the $\theta^{BLP}(2/3, t)$ case, and rapid turnover begins.

In Table 2, θ_t^{NLP} is the probability of success under the NLP (where the worker is never replaced), whereas θ_t^{BLP} is the probability of success in a firm with rapid turnover (with the original worker replaced at t^*). Note that the initial probability of success in both cases is simply p , which we set as $2/3$.

3.2. The Informational Cost of Turnover. One obvious approach to measuring the informational cost of turnover is to take the difference in the odds of success under no turnover as opposed to under turnover.

Definition 3. *The informational value of experience (IVE) is defined as:*

$$IVE(p, t) = \theta^{NLP}(p, t) - \theta^{BLP}(p, t)$$

The cumulative difference between the success rates under the NLP and the BLP measures the overall cost of replacement, and is also simply defined.

Definition 4. *The cumulative informational value of experience (CIVE) defined within the range t^* (the time of replacement of the original worker) and T (the end date of*

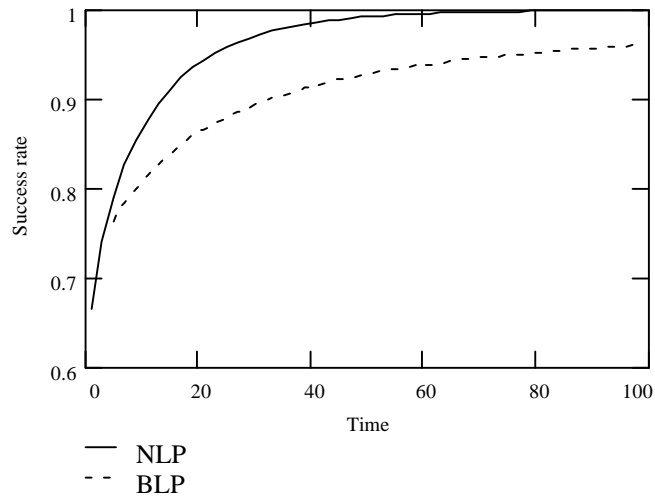


FIGURE 1. Figure 1: Success rates under the NLP and BLP over time.

the time horizon of interest) as:

$$CIVE(p, t^*, T) = \int_{s=t^*}^T [\theta^{NLP}(p, s) - \theta^{BLP}(p, s)] ds$$

Our final notion of the informational value of experience makes use of a few properties of $\theta^{NLP}(p, t)$ and $\theta^{BLP}(p, t)$.

Proposition 1. For $p > 0.5$, letting $T \rightarrow \infty$, we can find a bounded value of CIVE as:

$$CIVE(p, t^*, \infty) = \int_{s=t^*}^T [\theta^{NLP}(p, s) - \theta^{BLP}(p, s)] ds$$

Proof. See Appendix. ■

To give an idea of what this might look like Figure 1 considers the two success rates for $p = 2/3$, $t^* = 5$, and $T = 199$.

3.3. No Knowledge Transfer. We can ask the question: what if there is no transfer of knowledge between workers at all? So no records are kept and replacement workers know nothing about the signals or actions of their predecessors. In this case,

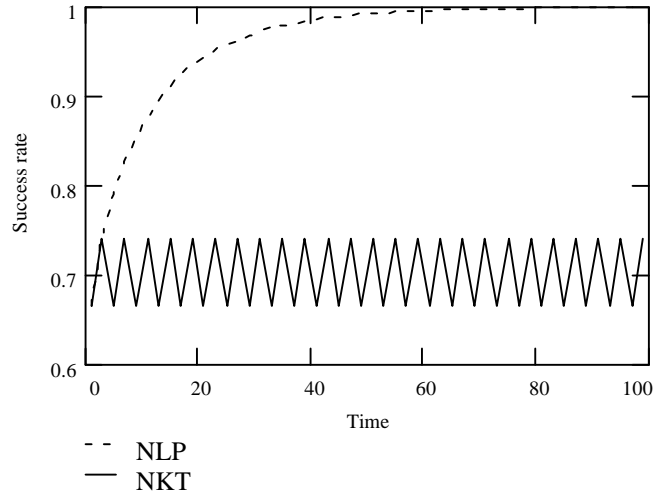


FIGURE 2. Figure 2: Success rates under NLP and NKT over time.

we have a brief period of learning by each worker followed by a full reset on replacement. To model this we simply consider the NLP returning values $\theta^{NLP}(p, 0)$ then $\theta^{NLP}(p, 1)$, and returning to $\theta^{NLP}(p, 0)$ in period 3 (with the replacement of the current worker), so we let each worker have two periods of employment. We call this saw-tooth pattern NKT for *no knowledge transfer*.

Definition 5. *No knowledge transfer (NKT) is a process which returns alternating values $\{\theta^{NLP}(p, 0), \theta^{NLP}(p, 1), \theta^{NLP}(p, 0), \theta^{NLP}(p, 1), \dots\}$ for T periods.*

The success rate under NKT is shown in Figure 2 for the case where $p = \frac{2}{3}$, compared with the standard normal learning process.

The simplicity of the situation with no knowledge transfer allows an equally simple derivation of the full value of experience.

Proposition 2. *The cumulative gain in success rates or ‘raw value of experience’ denoted $v(p, T)$ to period T with individual signal success probability p , of not replacing a worker when there is no prospect of any experience transfer between workers is*

$$v(p, T) = \int_{s=1}^T \theta^{NLP}(p, s) ds - \frac{T}{2} (\theta^{NLP}(p, 1) + p)$$

Proof. See Appendix. ■

While we might imagine various forms of ‘record keeping’ we will consider a record to be a full list of all past actions which is available to the current agent on demand. The list does not the signals of other agents (taken to be private information), though the current agent may be able to infer the past signals of others from the record.

Definition 6. A ‘record’ is a full list of actions $\{A_1, A_2, \dots, A_\tau\}$ given to the worker at time τ .

We can evaluate the value of keeping records of this type as simply the cumulative gain in success rates under BLP over and above the situation with no knowledge transfer, and is calculable in much the same way as $v(p, T)$ but with $\int_{s=1}^T \theta^{NLP}(p, s) ds$ replaced with $\int_{s=1}^T \theta^{BLP}(p, s) ds$.

Definition 7. The ‘value of record keeping’ under rapid turnover to period T , with individual signal success probability p , denoted $r(p, T)$ is

$$r(p, T) = \int_{s=1}^T \theta^{BLP}(p, s) ds - \frac{T}{2} (\theta^{NLP}(p, 1) + p)$$

So if a worker is replaced his experience is lost, unless a record remains which allows a future worker to infer some of his predecessor’s signals and therefore recover some of the lost experience.

Having developed a notion of the success rate of a worker in a decision-making occupation, how this might be affected by replacement, and various measures of experience lost, we can proceed to examine the impact on the firm of the experience gathering process of workers.

4. INERTIA IN DECISION-MAKING

Using the same model, we can characterize the problem of inertia in decision-making. We consider a particular problem: why is it that after an employee has been fired his replacement will often continue with the same policy? More specifically we aim to show that the original worker is more able to change actions than is his successor who has access to his predecessor’s last action and theorized success rate when there is record keeping so replacement workers use Bayesian learning. To give

a simple example, consider the following situation. The original worker receives the signals: $\{H, L, H, L\}$. His actions are: $\{Y, Y, Y, Y\}$. If he were to stay in the job and receive just one more L signal he would switch out of the series of Y actions and opt for an N . However a single L may well be insufficient for a new employee who has observed either a sequence of Y decisions by his predecessor, or who has calculated the success rate of his predecessor and knows this to greatly exceed the probability of his own L signal being correct (simply p). Intuitively it is clear then that the original worker's access to the full signal history grants him the ability to avoid what we will in this section call excess inertia.

4.1. Excess Inertia. For high values of t an incumbent worker's success rate of $\theta^{NLP}(p, t)$ will approach 1. Even for low t this will strictly exceed p . On replacement, at time $t = t^* > 1$, a new worker's signal will have an independent success probability of p , therefore we can say with certainty that the action taken in period $t^* - 1$, will dominate the action implied by signal x_{t^*} . This is given as Lemma 1 below and followed by the main result in this section.

Lemma 1. *Immediately after a new appointment at $t = t^* > 1$, a new worker will always follow the action of his direct predecessor, so $A_{t^*} = A_{t^*-1}$.*

Proof. See Appendix. ■

Proposition 3. *Excess Inertia. Consider two possible scenarios: (a) 'No Replacement'. An incumbent worker remains in his post at time $t = t^* > 1$, gaining a new signal x_{t^*} ; or (b) 'Replacement'. He is replaced by a new worker who is aware of both A_{t^*-1} and $\theta^{NLP}(p, t^* - 1)$, and who receives the same new signal x_{t^*} . The likelihood is higher that $A_{t^*} = \bar{A}_{t^*-1}$ under no replacement (a) than under replacement (b).*

Proof. See Appendix. ■

This proposition provides a clear message which is at odds with common folklore on incumbent workers: an incumbent worker is more likely to change policy than would be his replacement if there is record-keeping. Since the actions of the hypothetical incumbent provide our first best efficient benchmark, we know that we have a problem of *excess inertia* after the initial replacement. In particular this provides a counter-acting reason why we might not want record-keeping. Records raise the odds of

success by recovering lost experience, but as they list actions not signals they may provide a distorted view of the past and in particular encourage inertia in behaviour.

Hirshleifer and Welch (2002) provides a nice example from 1993. Louis Gerstner a former CEO of RJR Nabisco taking over as CEO of IBM after John Akers, was widely criticized for failing to change his predecessor's policies despite a long period of poor performance (Business Week, 7/26/93). The finding here provides two messages for firms. Firstly, a firm should not expect a rational replacement worker to forge ahead quickly with new policies when his predecessor held down the job for a non-trivial period of time and had a well-documented policy of action. In order to change policies the new employee needs time to build up sufficient signals which suggest a new course of action. Secondly, if a firm's shareholders have some belief that change is needed they should furnish the new employee with additional signals which suggest this is a good idea in order to help compensate for excess inertia, or simply keep the second employee isolated from the behaviour of his predecessor.

5. CONCLUSION

In industries characterized by learning-by-doing and decision-making, experience has various important affects. It generates an economic rent which an incumbent can partially obtain over and above the outside salary option. It makes labour-turnover expensive to the firm, and justifies record keeping when private information is important. It explains inertia in decision-making after the replacement of an employee.

What should firms do to take this cost into account? Largely this is done already. In decision-making industries where signals are hard to extract (the legal and medical sectors being obvious examples) economic rents are undoubtedly captured by experienced employees, and partnerships are the natural way for a private firm to be organized. In financial firms not organized in partnerships salaries may be even more inflated through experience. Some, notably hedging funds, are often small with owners acting as fund-managers and making the bulk of decisions within the firm, so once again the expensive bargaining process between worker and owner can be effectively ignored. The steady rise of salaries through promotion and even simply through a measure of duration with a firm is testimony to the value placed on experience and an overall concern about fast turnover. Labour hoarding in times of recession may also

be another sign. There is an entire macroeconomics literature on this which goes well beyond the current paper. Excess inertia is also clearly a problem, and in most cases the response is simply to wait until a new employee can build up enough experience to confidently overturn a past policy. One possible fact which might be explained via excess inertia is the often surprising decision to bring in a high-level employee from outside of the entire sector. The aim might be to insulate a new employee from too many preconceptions about the correct policy to pursue.

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APPENDIX

Proof of Proposition 1:

Proof. By definition, for $p > 0.5$, the processes $\theta^{NLP}(p, t)$ and $\theta^{BLP}(p, t)$ are monotonic strictly increasing functions. By definition as probabilities, the functions are both bounded above by 1. Consider any $\varepsilon > 0$ such that $\theta^{NLP}(p, t)$ or $\theta^{BLP}(p, t)$ asymptote to $1 - \varepsilon$. By choice of a sufficiently great value of t we can find $\theta^{NLP}(p, t) > 1 - \varepsilon$, and $\theta^{BLP}(p, t) > 1 - \varepsilon$. This clearly holds true for any value of $\varepsilon > 0$. Therefore we must set $\varepsilon = 0$, which establishes that $\lim_{t \rightarrow \infty} \{\theta^{NLP}(p, t)\} = 1$ and $\lim_{t \rightarrow \infty} \{\theta^{BLP}(p, t)\} = 1$. Further, by definition, $\theta^{NLP}(p, t^*) = \theta^{BLP}(p, t^*)$. Therefore $\lim_{t \rightarrow \infty} \{IVE(p, t)\} = 0$ and $IVE(p, t^*) = 0$. Now letting $T \rightarrow \infty$, by the definition of CIVE we have a bounded value as required. ■

Proof of Proposition 2:

Proof. Experience under no knowledge transfer returns alternate values of $\theta^{NLP}(p, 0)$, $\theta^{NLP}(p, 1)$, $\theta^{NLP}(p, 0)$, etc. Since $\theta^{NLP}(p, 0) = p$, the difference between these is simply $\theta^{NLP}(p, 1) - p$. To find the area under this process corresponding to cumulative experience we can use simple geometry. Take the triangular areas (given by the simple formula $\frac{1}{2}$ base \times height for a right-angled triangle) $\frac{1}{2} (\theta^{NLP}(p, 1) - p)$ plus the square area below p , which is simply $\theta^{NLP}(p, 0) = p$. Now add these and multiply by the length of the process T to give $\frac{T}{2} (\theta^{BLP}(p, 1) + p)$. The area under the NLP is simply $\int_{s=0}^T \theta^{NLP}(p, s) ds$. The value $\int_{s=1}^T \theta^{NLP}(p, s) ds - \frac{T}{2} (\theta^{BLP}(p, 1) + p)$, therefore returns the extra cumulative success rate gained by not replacing the worker. ■

Proof of Lemma 1:

Proof. For $t^* > 1$, we have that $\theta^{NLP}(p, t^* - 1) > p$. Since $\Pr[x_t = H \mid V = 1] = p$ and $\Pr[x_t = L \mid V = 0] = p$, by definition for $\forall t$, including $t = t^*$, then, ignoring the actions of the proceeding worker and following the advice of the signal x_{t^*} , results in an expected unconditional payoff of pV , whereas following the final action of the proceeding worker returns an expected unconditional payoff of $\theta^{NLP}(p, t^* - 1)V > pV$. Therefore setting $A_{t^*} = A_{t^*-1}$ returns a higher expected payoff than setting $A_{t^*} = \bar{A}_{t^*-1}$, regardless of the value of x_{t^*} . ■

Proof of Proposition 3:

Proof. From Lemma 1 we have $\Pr [A_{t^*} = \bar{A}_{t^*-1} \mid \text{scenario (b)}] = 0$. It serves to simply show $\Pr [A_{t^*} = \bar{A}_{t^*-1} \mid \text{scenario (a)}] > 0$. This is easily achieved by finding one or more examples of positive probability under which $A_{t^*} = \bar{A}_{t^*-1}$. As only one example is sufficient, consider the following event, $\eta = [\{x_t\}_{t=1}^{t^*} = \{H, L, H, L, L\} \ \& \ \text{coin flips in favor of } A_t = Y \text{ at times } t = 2 \text{ and } t = 4]$. The event η will produce $A_{t^*-1} = Y$, and $A_{t^*} = N \neq A_{t^*-1}$ and occurs with probability $\frac{1}{4}p^2(1-p)^3 > 0$ for $p > 0.5$ under $V = 1$ and probability $\frac{1}{4}p^3(1-p)^2 > 0$ for $p > 0.5$ under $V = 0$. Therefore $\Pr [A_{t^*} = \bar{A}_{t^*-1} \mid \text{scenario (a)}] > \Pr [A_{t^*} = \bar{A}_{t^*-1} \mid \text{scenario (b)}] = 0$. ■

Lemma 2. *It is weakly optimal to disregard signal x_τ , $\tau \in \mathbb{T}_{\text{even}}^{++}$ when deciding on action A_τ .*

Proof. Consider an agent at time $\tau \in \mathbb{T}_{\text{even}}^{++}$ with a history of signals $H_{\tau-1}$. Now we can partition the history of signals into two alternatives: (a) $\sum_{t=1}^{\tau-1} 1_{\{A_\tau=Y\}} > \sum_{t=1}^{\tau-1} 1_{\{A_\tau=N\}}$; and (b) $\sum_{t=1}^{\tau-1} 1_{\{A_\tau=Y\}} < \sum_{t=1}^{\tau-1} 1_{\{A_\tau=N\}}$.

First consider (a). We can further subdivide (a) into two alternatives: (i) $\sum_{t=1}^{\tau-1} 1_{\{A_\tau=Y\}} > \sum_{t=1}^{\tau-1} 1_{\{A_\tau=N\}} + 2$; and (ii) $\sum_{t=1}^{\tau-1} 1_{\{A_\tau=Y\}} = \sum_{t=1}^{\tau-1} 1_{\{A_\tau=N\}} + 1$. In case (i) the utility maximizing choice of action at time $t = \tau$ is $A_\tau = Y$, irrespective of x_τ . In case (ii) if $x_\tau = H$, then $E[u_\tau(A_\tau = Y) \mid I_\tau] = E[u_\tau(A_\tau = N) \mid I_\tau]$, so he might as well choose action $A_\tau = Y$. Action $A_\tau = Y$ is therefore weakly optimal regardless of x_τ .

Now consider (b). We can again further subdivide (b) into two alternatives: (i) $\sum_{t=1}^{\tau-1} 1_{\{A_\tau=Y\}} + 2 < \sum_{t=1}^{\tau-1} 1_{\{A_\tau=N\}}$; and $\sum_{t=1}^{\tau-1} 1_{\{A_\tau=Y\}} + 1 = \sum_{t=1}^{\tau-1} 1_{\{A_\tau=N\}}$. In case (i) the utility maximizing choice of action at time $t = \tau$ is $A_\tau = N$, irrespective of x_τ . In case (ii) if $x_\tau = H$, then $E[u_\tau(A_\tau = Y) \mid I_\tau] = E[u_\tau(A_\tau = N) \mid I_\tau]$, so he might as well choose action $A_\tau = N$. Action $A_\tau = N$ is therefore weakly optimal regardless of x_τ .

Note that $\sum_{t=1}^{\tau-1} 1_{\{A_\tau=Y\}} = \sum_{t=1}^{\tau-1} 1_{\{A_\tau=N\}}$ is ruled out as $\tau - 1 \in \mathbb{T}_{\text{odd}}^{++}$.

Therefore it is true for every case that the rule: choose the action implied by the history $H_{\tau-1}$, regardless of the signal x_τ , is weakly optimal. ■