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ON GREEN GROWTH WITH SUSTAINABLE CAPITAL

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On Green Growth with Sustainable Capital*

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1. Introduction

In the UK, the flagship of growth Industrial Strategy is to boost green growth through the promotion of cost effective low carbon technologies. While the industrial strategy lays out the goals of clean growth, it is less clear about the trade-offs facing the economy in meeting this target. The challenge emanates from a long standing theoretical and policy debate in resource and environment economics on whether growth is possible without exhausting natural resources. Since natural resources (hereafter natural or green capital) are part of the capital stock, we define sustainable growth in terms of the aggregate capital stock. Broadly, aggregate capital is the sum total of natural and man-made capital. According to Heal (2017), an economy is sustainable if the value of aggregate capital stock is nondecreasing.

The proponents of strong sustainability (e.g, Daly, 1997; Ayres, 2007), however, take the stand that the stock of natural capital must be non-decreasing which disallows substitution between natural and man-made capital. Solow (1974) and Nordhaus and Tobin (1972) take a weak sustainability view that some degree of substitution is possible between these two types of capital.[1] The crux of the debate boils down to whether natural and man-made capital are substitutable and a socially acceptable sustainable low carbon growth is achievable. If so, what policy instruments could accomplish this task?

This paper addresses the question using the lens of a simple endogenous growth model. The endogeneity of growth is crucial for understanding sustainability of

[1] For a recent survey on the sustainability issues of growth, see Cerkez (2018).

growth. An exogenous growth model would not be helpful because the sustainable growth depends on some unexplained exogenous engine of growth which is the central focus of this paper. Growth economists are deeply divided on the remaining issue whether there is enough empirical validity of the endogenous growth models. The standard tests of the endogenous growth model involve testing the convergence hypothesis and finding evidence in favour of broad based reproducible capital which mitigates the diminishing returns to capital. Barro (1991), Barro and Sala-i-Martin (2004), Mankiw, Romer and Weil (1992) and Young (1995) find some evidence of convergence. However, their convergence results are criticized by Bernard and Durlauf (1995), Quah (1996), Philips and Sul (2003) among others. A prototype AK endogenous growth model of Rebelo (1991) uses a broad based capital as a vehicle of growth. This model is criticized by Jones (1995) but defended strongly by McGrattan (1998).[2] We develop a variant of this AK endogenous growth model where the broad based capital is composed of man-made and natural capital.

In our model, sustainable growth implies a low-carbon balanced growth. Man-made carbon intensive capital is augmented by private investment. The private sector, while determining its optimal accumulation of man-made capital, does not internalize the damage it inflicts on the natural or green capital base due to emissions. A benevolent government designs a Pigovian type tax-subsidy and a public investment programme to correct for this externality. Doing so, the government seeks a Pareto optimal mix of man-made and green capital. The underlying production technology is kept general to allow for different degrees of substitution between

[2] For an excellent survey of the empirics of endogenosus growth models, see Capoloupo (2009).

green and man-made capital. The strong sustainability approach to growth arises as a special case in our setting when the production function has zero substitutability between green and man-made capital.

The punch-line of our analysis is that when the source of emissions is production, there is a trade-off between environmental policy and growth. This trade-off arises due to the fact that a carbon tax distorts the resource allocation. This adverse effect on growth is fundamentally due to the absence of a pollution abatement technology. We then present a scenario where an emissions abatement technology is in place. In this scenario, a combination of carbon tax, public investments in abatement and green capital replenishment could restore the Pareto optimal proportion of man-made to green capital. Greater efficiency in pollution abatement boosts the long run growth and lowers the depreciation of green capital and lowers the carbon tax. A pollution abatement technology also presents a pathway to resilience to a climate shock.

Our results are consistent with the current envrironmental policy of net-zero carbon emissions which aims to lower emissions while recognizing that zero emissions is not possible.[3] In our model, green depreciation can be effectively eliminated by an optimal carbon tax and a carbon abatement technology. The cost of such cabon tax is the distortion inflicted on the private sector which can be considerably lowered by making abatement technology more efficient. After netting out this cost, a net-zero carbon emissions is still possible.

[3] The UK is the first major economy that has committed itself to a legally binding net-zero carbon emission target by 2050.

Finally, we extend our model to characterize the effect of consumption based emissions. In this version, it is consumption not production that contributes to the green house effect. In a progress report to the UK parliament, the Committee on Climate Change (CCC) reports that consumption based emissions are more pervasive in UK than emissions resulting from production. An example is food wastage. The UK consumption emissionss were rising before the financial crisis but then fell between 2007 and 2009. Since then it fell by only 3% while production emissions declined by 15% (CCC, June 2018, pp. 32).

We ask the question: what are the consequences of consumption based emissions on sustainable growth? We show that growth is unaltered by these emissions because they have no effect on the static efficiency condition for resource allocation. Since consumption is the source of emission, the optimal carbon tax is zero in this settting. A corrective consumption tax is needed to finance public investment programme in order to replenish the green capital destroyed by emissions. Although such consumption based emissions have no adverse effect on growth, they negatively impact societal welfare due to lost consumption from consumption tax. If the emissions are due to consumption waste, a consumption tax can correct for this by financing a public investment programme in green capital. Our model based simulation suggests that the consumption equivalent of welfare loss is higher in an economy with higher consumption based emissions.

The remainder of the paper is organized as follows. Section 2 describes the background and briefly surveys the related literature. Section 3 sets up a social planning problem which characterizes the socially optimal sustainable growth with optimal public and private investments in green and man-made capital. Section 4 develops a model of a decentralized economy with a benevolent government in order to determine the optimal carbon tax, subsidy and public investment which could replicate the allocation of the social planning optimum. The strong sustainability view is shown as a special case of our baseline model where there is strict complementarity between man made and natural capital. Section 5 reports the simulation results of our baseline model. Section 6 extends the model to include public investment in pollution abatement. Section 7 analyzes the case of consumption based emissions. Section 8 concludes.

2. Background

In the UK and other countries, the moves to decarbonise the economies are driven by the desire to avoid the economic, social and ecological consequences of climate change and environmental degradation. These efforts are supported by increasing scientific and economic reasoning. The scientific case for action is based on the wide-reaching consequences of climate change and the urgency to act in order to avoid the tipping point beyond which the impacts become irreversible (Lenton et al., 2019). The Intergovernmental Panel on Climate Change (IPCC) has warned that large scale discontinuity in the climate system is likely even at a temperature threshold of 1.5 degree C. Various adverse effects of climatic change can manifest which include river and coastal flooding and steep rise in energy cooling.

Moreover, the economic case for decarbonisation suggests that the cost of acting to decarbonise sooner is lower than the damage from inaction which is increasing with time (Stern, 2007). At the macroeconomic level the decarbonisation effort is closely linked to the issue of economic growth and sustainability. More precisely, our central interest is addressng the following question: how can decarbonisation be achieved while maintaining a sustained growth with a socially desrable proportion of green to man-made capital?

According to the European Commission, green or natural capital is defined as environment friendly replenishable resources. Examples of replenishable natural capital are reforestation, use of solar energy, and improving air and water quality. According to the UK Office for National Statistics (ONS), natural capital includes resources such as mineral reserves, energy reserves, net greenhouse gas sequestration, outdoor recreation, agricultural land and timber, and water abstracted for public water supply. In comparison, man-made capital consists of assets such as machinery and urban land (EC, 2017). ONS (2019) estimates the value of the UK's main ecoservices in 2016 at about £958 bill.

In principle, green economic growth can be achieved by substituting polluting man-made capital with non-polluting green capital and investment to increase the stock of natural capital (EFTEC, 2015). The substitution needs to be facilitated by developing new technologies. However, the existing energy and clean technologies are not sufficient for achieving high levels of decarbonisation of the economy. Deep decarbonisation requires substantial investments as well as development of new technologies, something which requires investment and time.

According to the UK Committee for Climate Change (CCC, 2018) the total annual capital investment in the UK economy ranged from 15% to 24% of GDP over

1990-2017; our scenarios imply an extra green investment of around 1% in 2050. The CCC (2018) scenarios estimate the investment requirement of achieving net-zero to be comparable to the 2008 estimates for 80% emissions reduction relative to 1990 levels. The lower new estimates reflect a reduction in the cost of renewable and green technologies through innovation.

Connections to literature

Our paper relates to a wave of literature on the effect of environmental tax on economic growth. Forster (1973) analyzes optimal capital accumulation in the presence of pollution. His framework was subsequently extended by Gruver (1976), Luptacik and Schubert (1982), and Siebert (1987). Gradus and Smulders (1993) do a comprehensive analysis of the environmental policy in terms of pollution abatement.[4] Using a learning by doing technology and pollution distaste in the utility function, Michel and Rotillon (1995) argue that capital should be mostly taxed to combat the pollution distaste. A feature of their model is that a social optimum that internalizes pollution distaste might lead to a zero long run growth unless there is strong consumption compensation for pollution distaste. Gars and Olovsson (2019) document that countries using fossil fuel instead of biofuel embark on a higher growth path and develop an endogenous growth model that explains this. In many of these papers, a common theme is that there is a trade-off between environmental protection and growth.

[4] Using a two good general equilibrium model, Hollady et al. (2018) examine the effect of environmental regulation on the emissions leakage in the presence trade frictions They analyze the effect of an emissions tax but abstract from capital accumulation, growth and production based externality from emission which is our primary focus in this paper.

Our model is closer in spirit to the Dynamic Integrated Model of Climate and the Economy (DICE) of Nordhaus (2018) who uses Cass-Koopmans growth models with forward looking agents to analyze the economic effects of climate change. In the DICE model, many aspects of the environment are mapped into temperature as a single state variable. Such a mapping is motivated by natural science modules where fossil fuel emissions lead to higher temperature due to greenhouse effect. In the spirit of the DICE model, we characterize the production of a single composite final good with labour, fossil intensive man-made capital and natural capital. The natural capital depreciates due to green house effects of carbon intensive man-made capital.

The technology of final goods production is similar to Gars and Olovsson (2019). We have two kinds of capital, man-made (fossil fuel intensive) and green capital (biofuel intensive) in our production function. The novelty of our setting is that we let the stock of green capital erode due to carbon emissions from man-made capital to model the effects of climate shock on the aggregate economy. Although man-made capital erodes the green capital base, there is inherent complementarity between these two types of capital in the production process. This complementarity gives rise to a socially optimal positive sustainable growth. We demonstrate this by setting up a social planning problem which lays out the Pareto optimal ratio of man-made to green capital where man-made capital can damage the green capital base. We then describe a market economy where the private sector fails to internalize the adverse effect of its investment in man-made capital on green capital. A corrective tax-subsidy and green public investment programme are then designed which could replicate the socially optimal green growth rate.

Our market economy model replicates the efficient allocation using the tax-subsidy mechansim as an environmental policy instrument. Alternatively, one can introduce pollution permits as an environmental policy instrument where a fixed number of pollution permits are auctioned off by the government to pollutant firms. Invoking Coase theorem, one can hope to achieve efficient allocations. We do not take this avenue because of the limitations of this approach due to the free rider problem pointed out by Chari and Jones (2000).

3. Sustainability of growth as a social planning problem

The economy produces the final output (Y_t) with broad based capital (K_t) and a unit raw labour with a linear technology as in Rebelo (1991):

$$Y_t = AK_t \tag{1}$$

where A is a constant total factor productivity (TFP) term. The aggregate capital is composed of man-made (K_t^p) and green capital (K_t^g) based on the following constant elasticity of substitution (CES) aggregation:

$$K_t = \left[(1-\nu)K_t^{p^{\varphi}} + \nu K_t^{g^{\varphi}}\right]^{1/\varphi} \tag{2}$$

with $0 < \nu < 1$,and $\varphi = (\sigma - 1)/\sigma$ where σ is the elasticity of substitution. Note that since σ is positive by construction $-\infty < \varphi < 1$.[5]

The man-made capital evolves according to the linear depreciation rule:

$$K_{t+1}^{p} = (1 - \delta_p) K_t^p + I_t^p \tag{3}$$

where I_t^p is the level of private investment in man-made capital and δ_p is its rate of depreciation.

A benevolent social planner invests a fraction of final output, i_{yt}^g to replenish green capital by planting trees among other means.[6] The law of motion of the green capital stock is given by:[7]

$$K_{t+1}^{g} = (1 - \delta_{gt}) K_t^g + i_{yt}^g Y_t \tag{4}$$

The depreciation rate of green capital (δ_{gt}) is proportional to the ratio of private to green capital. More man-made capital relative to green capital causes erosion of

[5] Our production fnction is similar in spirit to Gars and Olovsson (2019). In their model, the production of final goods requires the use of biofuel and fossil fuel which are produced capital stock different varieties. In our setting, we abstract from varieties and focus on a production function involving green (biofuel intensive) capital and man-made (fossil intensive) capital.

[6] We represent the investment in man-made capital in level but green investment in rate. This distinction is crucial to justify a carbon tax rate in a decentralized economy.

[7] Here, we are assuming that there is no adjustment cost for changing man-made and green capital. Aggregate output can be converted to man-made and green capital with equal ease. While this is a stretch from the reality, it is also true that differential adjustment costs for changing these two types of capital may not have any long run growth consequences. One can add differential adjustment costs of man-made and green capital which is zero in the steady state. Thus the balanced growth rate will be unaltered by the presence of such adjustment cost. Since the central interest of this paper is on sustainable growth, we make this simplifying assumption.

green capital (in the form of deforestation and climate change). In other words:

$$\delta_{gt} = \omega_t \frac{K_t^p}{K_t^g} \tag{5}$$

A few clarifications about the green depreciation rate, δ_{gt} are in order. The term ω_t represents net erosion of green capital per unit of man-made capital due to the carbon emissions of the latter capital. This erosion is caused by the technology of investment, but it can be managed by pollution abatement technology to which we turn later. In principle, green capital can regenerate and the net erosion could be negative. In our model, ω_t is the single state variable as the temperature is in the DICE model of Nordhaus (2018). For our baseline model, we assume that ω_t is time invariant meaning $\omega_t = \overline{\omega}$ for all t and is exogenous. Hereafter, we call $\overline{\omega}$ the rate of green erosion. The social planner takes the emission technology (5) and the net erosion rate as given and designs a Pareto optimal ratio of man-made to green capital and a path of public investment in green capital.

Plugging (5) into (4), the law of motion of green capital reduces to:

$$K_{t+1}^g = K_t^g - \overline{\omega} K_t^p + i_{yt}^g Y_t \tag{6}$$

The social planner determines a socially desirable sustainable green growth that maximizes the welfare of a representative infinitely lived agent. Noting that C_t is the consumption of the agent at date t and β is a constant discount factor, formally the optimization problem is written as:

$$Max \quad \sum_{t=0}^{\infty} \beta^t \ln C_t \tag{7}$$

s.t.

$$C_t + I_t^p \leq (1 - i_{yt}^g) Y_t \tag{8}$$

and (1), (2), (3), (6), (8) and also the inequality constraint $i_{yt}^g \leq 1$. We do not impose any non-negativity constraint on either i_{yt}^g and I_t^p because we allow for disinvestment in both types of capital.

Assuming an interior solution, the planner chooses the time paths of man-made and green capital to equate the marginal product of man-made with the marginal product of green capital, net of depreciation rates of both types of capital.[8] In other words, the following static efficiency condition must hold:

$$\Theta\left(\frac{K_t^g}{K_t^p}\right) = \Psi\left(\frac{K_t^g}{K_t^p}\right) + \overline{\omega} + \delta_p \tag{9}$$

where

$$\Theta\left(\frac{K_t^g}{K_t^p}\right) = A\frac{\partial K_t}{\partial K_t^p} = A(1-\nu)\left[(1-\nu) + \nu\left(\frac{K_t^g}{K_t^p}\right)^{\varphi}\right]^{\frac{1-\varphi}{\varphi}} \tag{10}$$

and

$$\Psi\left(\frac{K_t^g}{K_t^p}\right) = A\frac{\partial K_t}{\partial K_t^g} = A\nu\left[\nu + (1-\nu)\left(\frac{K_t^g}{K_t^p}\right)^{-\varphi}\right]^{\frac{1-\varphi}{\varphi}} \tag{11}$$

[8]The derivation is available in the appendix. We assume an interior solution for the social planning problem assuming the green investment rate i_{yt}^g does not hit the upper bound. For plausible parameter values, we find that this is a reasonable assumption which keeps the growth self sustained.

We have the following proposition.

Proposition 1. *Based on the static efficiency condition (9), a unique ratio of green to man-made capital,* $\frac{K_t^g}{K_t^p}$ *exists.*

Proof. It follows from the fact that $\Theta(0) = A(1-\nu)^{1/\varphi}$, $\Theta'\left(\frac{K_t^g}{K_t^p}\right) > 0$ and $\Psi(0) = \infty$, $\Psi'\left(\frac{K_t^g}{K_t^p}\right) < 0$. Thus, there exists a unique crossing point in the positive quadrant between $\Theta\left(\frac{K_t^g}{K_t^p}\right)$ and $\Psi\left(\frac{K_t^g}{K_t^p}\right)+\varpi+\delta_p$ schedules. Figure 1 demonstrates the existence of a unique K_t^g/K_t^p ■

Next, note that since there is no non-negativity restriction on both types of investment, there is no transitional dynamics in this environment. Regardless of the initial stocks of both types of capital, the following balanced growth rate (γ) is attained immediately.

$$1+\gamma = \beta\left[1+\Psi\left(\frac{K_t^g}{K_t^p}\right)\right] \tag{12}$$

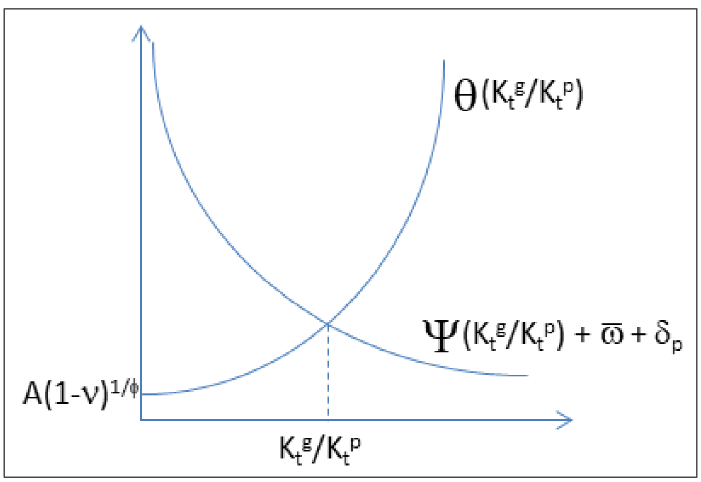

Figure 1: Existence of K_t^g/K_t^p

Using the implicit function theorem, and exploiting the fact that $\Theta'\left(\frac{K_t^g}{K_t^p}\right) > 0$

and $\Psi'\left(\frac{K_t^g}{K_t^p}\right) < 0$, it is straightforward to verify that

$$\frac{\partial(K_t^g/K_t^p)}{\partial\overline{\omega}} = \frac{1}{\left[\Theta'\left(\frac{K_t^g}{K_t^p}\right) - \Psi'\left(\frac{K_t^g}{K_t^p}\right)\right]} > 0$$

The efficiency condition dictates that a shift to a technology that causes greater erosion of green capital (higher $\overline{\omega}$) requires more stringent quantity control of man-made capital by either divesting in man-made capital or investing in green capital. Either of these two actions or a combination of them boosts the ratio K_t^g/K_t^p. The social planner mandates a higher ratio of green to man-made capital when the environmental damage is higher. This can also be easily checked from Figure 1. Higher $\overline{\omega}$ makes the $\Psi(.) + \overline{\omega} + \delta_p$ shift out resulting a higher equilibrium K_t^g/K_t^p.

The balanced growth rate (γ) must satisfy the following conditions:

$$1 + \gamma = \beta\left[1 + \Psi\left(\frac{K_t^g}{K_t^p}\right)\right] \tag{13}$$

Since $\Psi'(.) < 0$, the implication is that a higher green erosion rate ($\overline{\omega}$) unambiguously lowers the balanced growth rate via a rise in K_t^g/K_t^p. Therefore, growth is highest with zero erosion.

Using (6), the steady state investment ratio in green capital is given by:

$$i_y^g = \frac{\gamma + \overline{\omega}(K_t^p/K_t^g)}{A\left[(1-\nu)(K_t^p/K_t^g)^{-\varphi} + \nu\right]^{1/\varphi}} \tag{14}$$

Higher erosion ($\overline{\omega}$) lowers growth (γ) as well as the socially optimal ratio of man-made to green capital (K_t^p/K_t^g). The effect on the fraction of final output invested to replenish green capital, i_y^g is nonlinear. It depends on the erosion rate ($\overline{\omega}$) and the resulting substitution of man-made by green capital. If this substitution is strong, the efficient investment in green capital could fall due to a decline in $\overline{\omega}(K_t^p/K_t^g)$.

4. A decentralized economy with carbon tax

We now describe how a government can replicate the social planning allocation described in the preceding section by a corrective tax-subsidy scheme in a decentralized economy. The private sector consists of firms and households. Competitive firms produce final goods using the production function (2). Households own the man-made capital, accumulate it and rent it at a competitive price (r_t) every period to the firms for final goods production. Households supply one unit of labour for the production of final goods at a competitive wage (w_t). While producing final goods, the private sector does not internalize the damage caused to green capital based on (5). The government imposes a carbon tax (τ_t) on the rental income of firms in a Pigovian fashion to correct for the externality and uses the tax proceeds to finance green investments and transfers (T_t) to households. The government budget constraint is:

$$\tau_t r_t K_t^p = i_{yt}^g Y_t + T_t \tag{15}$$

where the public investment ratio $\{i_{yt}^g\}$ satisfies (6).

The household takes the stock of green capital $\{K_{gt}\}$ as well as the sequences $\{\tau_t\}$, $\{T_t\}$, $\{w_t\}$ and $\{r_t\}$ as parametrically given, and maximizes (7) subject to the following flow budget constraints and the private investment technology (3):[9]

$$C_t + I_t^p = w_t + (1 - \tau_t) r_t K_t^p + T_t \tag{16}$$

The Euler equation facing the household is:

$$\frac{C_{t+1}}{C_t} = \beta \left[(1 - \tau_{t+1}) r_{t+1} + 1 - \delta_p \right] \tag{17}$$

The zero profit condition dictates that the competitive rental price of capital equals the marginal product of private capital which means

$$r_{t+1} = \Theta \left(\frac{K_{t+1}^g}{K_{t+1}^p} \right) \tag{18}$$

4.1. Optimal carbon tax

The government designs the time path of the carbon tax such that the private marginal benefit of investing in man-made capital exactly balances the social marginal benefit given by the social planner's Euler equation (13). The optimal carbon tax is:

$$\tau_t = \frac{\overline{\omega}}{\Theta \left(\frac{K_t^g}{K_t^p} \right)} \tag{19}$$

Plugging the efficient time path of K_t^g / K_t^p from the social planning problem, one can generate the time path of the carbon tax, τ_t.

[9] Since the household takes K_t^g as given, it faces a constant returns to scale technology involving K_t^p and inelastic labour which is normalized at unity. .

4.2. Strong sustainability

Proponents of strong sustainability take the stand that natural and man made capital are not substitutable. This arises as a special case when there is zero elasticity of substitution ($\varphi \longrightarrow -\infty$) between these two types of of capital. In this case the production function (1) takes the Leontief form:

$$Y_t = A \ \min\left[K_t^p, K_t^g\right] \tag{20}$$

The efficient ratio of green to man-made capital is unity. Based on (5), the green depreciation rate along a balanced growth path is given by:

$$\delta_{gt} = \overline{\omega} \tag{21}$$

Since strict complementarity disallows any substitution between two types of capital, higher emissions rate ($\overline{\omega}$) causes irreversible damage to green capital base.

The balanced growth rate is given by:

$$1 + \gamma = \beta \left(1 + \frac{A - \overline{\omega} - \delta_p}{2}\right) \tag{22}$$

Higher erosion rate unambiguously lowers the long run growth rate as in the previous scenario because of the destruction caused by man-made capital. The optimal investment rate in green capital (14) is:

$$i_y^g = \frac{\beta\{1 + 0.5(A - \delta_p)\} - 1 + \overline{\omega}(1 - 0.5\beta)}{A} \tag{23}$$

Higher $\overline{\omega}$ unambiguously raises the socially optimal public investment in green capital. The government has to engage in a public investment programme to replenish green capital, permanently damaged by man-made capital.[10]

Such an investment programme must be financed by a carbon tax. The optimal carbon tax in the case of a fixed coefficient production function (20) is given by:

$$\tau = 0.5 + A^{-1}(1 - 0.5\delta_p) + 0.5A^{-1}\overline{\omega} \tag{24}$$

which rises unambiguously with respect to $\overline{\omega}$.

5. Simulation

We perform model simulations of our baseline production based emission model to assess the effects of green capital erosion on the aggregate economy. In order to carry out any quantitative exercise, we take a stand on setting the long run growth target for the UK economy. We set a baseline target growth rate for the UK economy of 2% at a zero emission rate. This target is in line with the long term annual average growth rate of UK real GDP over the period 1947-2018 from the St. Louis Federal Reserve database (FRED) which is found to be 2.47%. One may debate whether this is a reasonable target given that the UK economy, in recent years, has slowed down (1.47% in 2019). Since there are no reliable GDP growth rate forecasts for the UK, we take 2% as a reasonable growth target.

Regarding the choice of the value of the discount factor β opinions considerably

[10] See the appendix for a proof of (23).

differ. Prescott (1986) sets β equal to 0.96 for calibrating the US economy to annual data which means a 4% steady state real interest rate. This estimate is used in many calibration exercises of macro models. Given the assumption of a logarithmic utility function which is also widely used in quantitative macroeconomics literature following Prescott (1996), a 2% growth rate together with β equal to 0.96 implies a social discount rate of 6%.[11] This social discount rate is too high in the context of climate change involving the future generation's welfare. Green Book (2018) suggests that the social discount rate is 3.5% based on a 2% growth rate and an implicit assumption of a logarithmic utility function. On the other hand, the Stern report (2007) takes a radical stand that the social discount rate is around 0.05%. We fix β equal to 0.98 which means that the social discount rate is 4%. We also perform a sensitivity analysis in order to check how our quantitative analysis differs when β is changed in this neighborhood.

Following Prescott (1996), the depreciation rate of man-made capital, δ_p, is fixed at 0.1 which implies a 2.5% quarterly depreciation rate used in several studies. With all these parameter values, the TFP parameter, A, needs to be fixed at 0.127. The elasticity and share parameters are fixed at $\varphi = 0.5$ and $\nu = 0.5$ respectively. With these values, we obtain a long run annual growth target of 2% for the UK economy at $\overline{\omega} = 0$. At this target zero emissions, the ratio of consumption to GDP and man

[11] For a mature economy on a balanced growth path, the so called accounting rate of interest is equal to the consumption rate of interest. The standard rule in social cost benefit literature is that along the balanced growth path, the social discount rate (ρ) is equal to growth rate (g) times the intertemporal elasticity of substitution in consumption (say, σ) plus the impatience rate $(1 - \beta)$. See Bell (2003, Ch 10) for a discussion of this and other rules for ρ. Given our $g = 0.02$ and $\sigma = 1$ due to our logarithmic utility assumption, it implies that $\rho = 0.04$.

made investment to GDP are found to be 42.7% and 19.2% respectively while the green investment rate is 38.7%.

Figure 2 plots the effect of green erosion ($\overline{\omega}$) on the aggregate economy. Starting from a zero erosion, a higher $\overline{\omega}$ can be thought of as a climate shock. In response to such a shock, the carbon tax rate rises sharply from zero to a rate which induces firms to substitute man-made capital for green capital. Public investment rate in green capital required to replenish green capital rises while the private investment rate falls. The consumption rate of the current generation rises which reflects a substitution effect of carbon tax encouraging the household to consume more and invest less in man-made capital. The green depreciation rises because the rise in the ratio of green to man-made capital is not enough to lower the depreciation of green capital. However, the ratio of green depreciation remains close to zero. The lower growth reflects the distortionary effect of a higher carbon tax highlighting the classic environmental policy and growth trade-off. Although the consumption rate is higher, the negative growth effect depresses the societal welfare.[12]

[12] The welfare function is specified later.

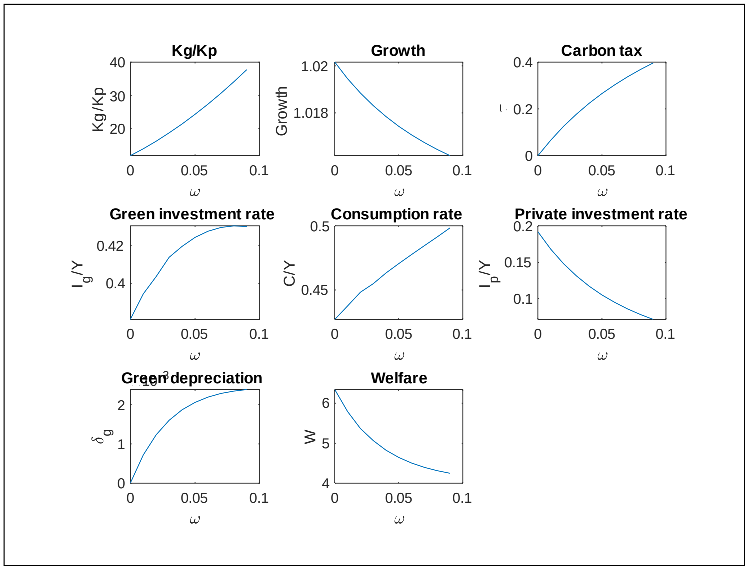

Figure 2: Effect of a climate shock on the aggregate economy

Anticipating disparate opinions about the choice of the social discount rate, we perform a sensitivity analysis of the key variables by changing the social discount rate. Changing the discount factor β from 0.98 to 0.995 is equivalent to changing the social discount rate from 4% to 2.5% given the same balanced growth rate of 2%. Table 1 presents the results of this sensitivity analysis. A lower social discount rate raises the target growth rate from 2 to 3.58%. This increase in green investment takes place at the expense of a drastic reduction in man-made investment rate and lower societal consumption rate. This reduction of man-made investment happens since at a zero emission rate green capital does not depreciate while man-made capital does. If society is more forward looking, people are better off investing more in green capial with zero depreciation.

Table 1: Sensitivity of zero emission targets with respect to the social discount rate

β	0.98	0.985	0.99	0.995
γ	2.02%	2.54%	3.06%	3.58%
i_g/y	38.10%	47.9%	57.78%	67.63%
i_p/y	19.2%	20.03%	21.36%	10.68%
c/y	42.7%	32.03%	20.86%	21.69%

6. Overcoming adverse effect of emissions on growth: Decarbonisation

The punch-line of our baseline production based emissions model is that there is a critical trade-off between environmental policy and long-run growth unless there is an effort to abate the emissions by lowering ω_t. This requires public investment in emissions abatement. In this section, we extend our model to explore such possibility. Suppose in addition, to green investment (i^g_{yt}), a fraction of GDP (i^ω_{yt}) is spent on emissions abatement. Formally, we introduce an emissions abatement technology as follows:

$$\omega_t = \varpi - \varkappa(\varpi) i^\omega_{yt} \tag{25}$$

If there is no public investment in emission abatement, emission is simply ϖ. The higher the investment in emissions abatement, the lower the emissions via the abatement technology (25). The effectiveness of the emissions abatement is captured by the parameter $\varkappa$ which is an increasing function of the exogenous emissions ϖ. We call $\varkappa(\varpi)$ an intervention function for combating climate shock. A higher green house effect (higher ϖ) can be combated by a more efficient abatement technology (e.g. efficient carbon capturing) which means a higher $\varkappa$.[13] The exact functional form for $\varkappa(\varpi)$ depends on the time to intervene and proactiveness of the pollution agency in response to a climate shock. In the following section, we give an illustration of a specific intervention pattern.

The social planning problem (7) now changes to:

$$Max \quad \sum_{t=0}^{\infty} \beta^t \ln C_t \tag{26}$$

s.t.

$$C_t + I_t^p \leq (1 - i_{yt}^g - i_{yt}^\omega) Y_t \tag{27}$$

and (1), (2), (3), (6), (8) and $i_{yt}^g + i_{yt}^\omega < 1$.

The new first order condition for abatement investment (i_{yt}^ω) equates the marginal benefit of abatement investment to the marginal abatement cost in terms of foregone national output. In other words,

$$\varkappa(\varpi) K_t^p = A K_t \tag{28}$$

[13]There are various ways of abating pollution. The Global Commission on the Economy and Climate in their technical report suggests several pathways for this which include: (i) more compact urban form with greater use of public transport, (ii) improving agricultural productivity, (iii) removal of fossil fuel subsidies, (iv) transition from coal, (iv) phasing out short lived climate pollutants such as black carbon, methane, HFCs, (v) emissions from oil and gas, (v) reduced food wastage called waste resource action programme (WRAP). See
https://newclimateeconomy.report/workingpapers/wp-content/uploads/sites/5/2016/04/NCE-technical-note-emission-reduction-potential_final.pdf

which immediately pins down the Pareto optimal ratio of green to man-made capital as follows:

$$\frac{K_t^g}{K_t^p} = \left[\frac{(\varkappa(\varpi)/A)^{\phi} - 1 + \nu}{\nu}\right]^{1/\phi} \tag{29}$$

Notice that the ratio of green to man-made capital is constant and it holds in both short run and long run equilibrium. Higher abatement efficiency ($\varkappa$) unambiguously raises the ratio of green to man-made capital.

The static efficiency condition (9) is modified after including abatement investment as follows:

$$\Theta\left(\frac{K_t^g}{K_t^p}\right) = \Psi\left(\frac{K_t^g}{K_t^p}\right) + (\omega_t + \delta_p)/(1 - i_{yt}^{\omega}) \tag{30}$$

Plugging (29) into the modified static efficiency condition (30), the optimal abatement investment is:

$$i_{yt}^{\omega} = \frac{\Psi\left(\frac{K_t^g}{K_t^p}\right) - \Theta\left(\frac{K_t^g}{K_t^p}\right) + \delta_p + \bar{\omega}}{\varkappa + \Psi\left(\frac{K_t^g}{K_t^p}\right) - \Theta\left(\frac{K_t^g}{K_t^p}\right)} \tag{31}$$

The balanced growth equation (13) now nets out the abatement investment. It is given by:

$$1 + \gamma = \beta\left[1 + (1 - i_{yt}^{\omega})\Psi\left(\frac{K_t^g}{K_t^p}\right)\right] \tag{32}$$

The steady state green investment ratio (14) changes to:

$$i_y^g = \frac{\gamma + (\varpi\ - \varkappa i_{yt}^{\omega})(K_t^p/K_t^g)}{A\left[(1-\nu)(K_t^p/K_t^g)^{-\varphi} + \nu\right]^{1/\varphi}}$$

Finally, note that since the private investors do not internalize the investment in green capital and emissions abatement, the Pigovian tax has to be adjusted in order to pay for both types of investment. The optimal carbon tax is:

$$\tau_t = \frac{\bar{\omega}}{\Theta\left(\frac{K_t^g}{K_t^p}\right)} + i_{yt}^{\omega}[1 - \varkappa/\Theta(K_t^g/K_t^p)] \tag{33}$$

The appendix presents an outline of the key equations of this model. For a linear technology ($\varphi = 1$), the model admits the following closed form solutions for the optimal abatement investment rate, growth rate and the depreciation rate are:

$$i_{yt}^{\omega} = \frac{A(2\nu - 1) + \delta_p + \overline{\omega}}{A(2\nu - 1) + \delta_p + \varkappa} \tag{34}$$

$$1 + \gamma = \beta[1 + (1 - i_{yt}^{\omega})A\nu] \tag{35}$$

$$\delta_{gt} = (\bar{\omega} - \varkappa i_{yt}^{\omega})[\nu/\{\varkappa A^{-1} - 1 + \nu\}] \tag{36}$$

The optimal carbon tax rate (33) reduces to:

$$\tau_t = \frac{\overline{\omega}}{A(1 - \nu)} + i_{yt}^{\omega}.(1 - \varkappa/A(1 - \nu)) \tag{37}$$

6.1. Combating a climate shock with technological intervention

Our model provides a pathway to technological intervention to deal with a large climate shock. Consider an abatement technology with a specific intervention function which combats emission with a four-year time lag. A climate shock hits the economy in the form of a green house effect and this effect progressively rises. This is modelled by raising $\overline{\omega}$ from unity to 1.05 over a period of five years. An intervention takes place in the form an efficient abatement technology after five years from the onset of the green house effect which is modelled by an upward shift of $\varkappa$. In the next eleven years, another technological discovery means a further upward shift of $\varkappa$. After then $\varkappa$ progressively rises. We fix the other parameter values at $A = 0.4$, $\nu = 0.5$, $\delta_p = 0.01$. Figure 3 illustrates the effects of this intervention. Growth rate initially falls due to this climate shock but as soon as the technology is in place, it starts rising. Abatement investment initially rises at the expense of a lower green investment. As soon as a more efficient abatement technology is in place, abatement investment falls due to lower cost of such abatement which is offset by a rise in green investment. The green capital base expands reflecting a higher ratio of green to man made capital. The carbon tax initially rises and then it falls due to a lower cost of abatement. Green depreciation first rises, then falls and eventually turns negative which means green capital regenerates.

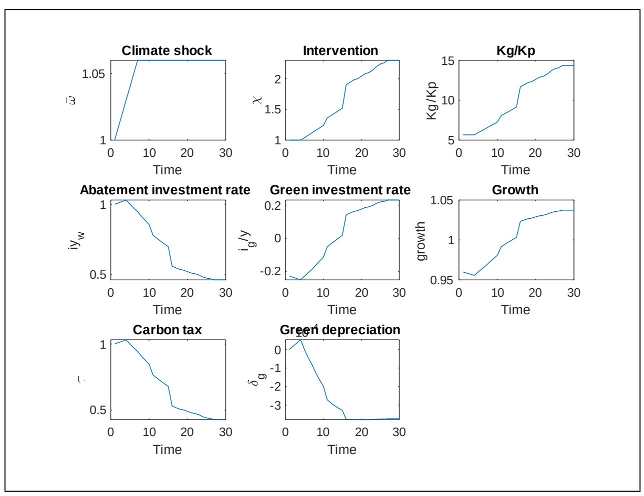

Figure 3: Effect of climate change in the presence of an abatement technology

7. Emissions from Consumption

Until now we have exclusively focused on scenarios where green capital erosion results purely from production process by carbon intensive man-made capital. We now turn to a scenario where consumption is the main cause of greenhouse effect.

The damage function due to consumption based emission is formulated as follows:

$$\delta_{gt} = \omega_t^c \left[\frac{C_t}{K_t^g} \right] \tag{38}$$

The term ω_t^c now represents erosion of green capital due to consumption. Assume that $\omega_t^c = \overline{\omega}^c$. The same principle can be used to derive a balanced growth path. Sustainable growth is still possible in this environment.

The appendix shows that the static efficiency condition is:

$$\Theta \left(\frac{K_t^g}{K_t^p} \right) - \delta_p = \Psi \left(\frac{K_t^g}{K_t^p} \right) \tag{39}$$

which means that regardless of the initial condition, the economy approaches K_t^g / K_t^p that solves the above static efficiency condition. The allocation of green and man made capital is Pareto optimal because it is independent of the consumption emission factor $\overline{\omega}^c$. The balanced growth rate is unaffected by consumption based emission. The optimal carbon tax (τ_t^c) is thus zero.

Although Pigovian carbon tax is zero, a corrective tax-subsidy is still needed to compensate for the loss of natural capital due to consumption based emission. We have the following proposition.

Proposition 2. *If the emission is consumption based, the optimal corrective tax* *policy is to impose a flat rate consumption tax (τ_t^c) on the household to finance green public investment given by:*

$$\tau_t^c = \frac{\gamma + \overline{\omega}^c (C_t/K_t^p)(K_t^p/K_t^g)}{A\left[(1-\nu)(K_t^p/K_t^g)^{\varphi} + \nu\right]^{1/\varphi}}$$

with $\frac{\partial \tau_t^c}{\partial \overline{\omega}^c} > 0$.

Proof. Appendix. ■

Since the source of green house effect is consumption based, households are taxed on consumption. The tax revenue is used for public investment in green capital to replenish the stock of green capital eroded by consumption based emission. Such a tax has welfare consequence because it hurts household's consumption but it has no distortionary effect on capital accumulation.

7.1. Comparing welfare losses from production and consumption based emissions

How does two economies, (i) with production based emissions and (ii) consumption based economy compare in terms of welfare loss? To obtain a consumption equivalent of such welfare loss, we first compute the welfare along the balanced growth path given the stocks of two types of capital at date t.

$$W(C_t/Y_t,\ \gamma) = \frac{\ln(C_t/Y_t)}{(1-\beta)} + \frac{\beta \ln(1+\gamma)}{(1-\beta)^2} \tag{40}$$

Since the stocks of two types of capital, K_t^p and K_t^g are given at date t, it means at the start of date t, the final output Y_t is also given. Since there is no transitional dynamics in this model, the balanced growth rate is immediately achieved. Thus the welfare at date t can be conveniently broken down in the consumption/GDP ratio

(C_t/Y_t) at date t and the balanced growth rate γ. The consumption equivalent of the welfare loss due to carbon emission can be computed by equating the welfare for zero emission state with positive emission state and computing the compensating variation (Δ_c) in consumption/GDP ratio. Denoting the zero emission and positive emission states by a zero and one suffixes, the compensating variation Δ_c is characterized by:

$$W_0 = W_1((C_t/Y_t)^1 + \Delta_c \ , \gamma^1) \tag{41}$$

It is straightforward to verify that Δ_c can be explicitly solved as:

$$\Delta_c = e^{(1-\beta)W^0 - \frac{\beta \ln(1+\gamma)}{1-\beta}} - (C_t/Y_t)^1 \tag{42}$$

Table 2 reports the consumption equivalent of welfare loss (Δ_c) as shown in 41) in two environments, (i) when emissions are production based and (ii) when the emissions are consumption based. At a low level of emissions (below $\omega = 0.03$), the consumption loss is slightly higher in case of production based emissions. If the emissions cross a threshold level 0.02, consumption based emissions hurt welfare more because of a steeper consumption tax on the households to finance green investment. Notice that this loss is computed after a corrective tax-subsidy policy is already in place. This is a deadweight loss due to the erosion of green capital due to emissions.

Table 2: Percent Consumption loss for production and consumption based emissions

ω	Baseline	Consumption Emissions
0.01	1.13	0.99
0.02	2.05	1.96
0.03	2.74	2.91
0.04	3.33	3.84
0.05	3.80	4.76

8. Conclusion

This paper extends conventional endogenous growth models to demonstrate the trade-offs facing the policy maker to balance sustainable growth with a clean environment policy. Since the private sector does not internalize the damage to the environment by carbon emissions, the policy maker imposes a corrective carbon tax on the private sector. Using alternative models, we show that higher carbon tax can nearly eliminate the depreciation of green capital caused by emissions if the production technology allows sufficient substitution of man-made capital by green capital. However, the distortionary effect of this tax lowers long run growth. To have a sustainable clean growth and to meet the UK Industrial Policy goal, efforts should be made to develop carbon free technologies. The existing technologies are not sufficient to achieve the ambitious policy goals.

To demonstrate the role of a carbon free technologies, we extend our model environment to include public investment in emissions abatement. Our model shows that with a highly efficient pollution abatement technology, the adverse growth effect of environmental control can be mitigated or even reversed if the abatement technology is efficient and proactive to climate shock. The carbon tax could be also lowered. The policy lesson is that the adverse effect of carbon tax on growth can be reversed by emissions abatement technology in the form of carbon capture solutions such as forestation, carbon capture and storage. In addition, this alternative technology should be supplemented by more green investment. A carbon tax can help the transition to this new technology.

We finally extend our model to depict a scenario of consumption based emissions. If the green house effect arises primarily due to food wastage, it has no effect on the allocative efficiency of the economy. As a result, the optimal carbon tax should be zero. However, a positive consumption tax is warranted to finance the public investment in natural capital to replenish the lost green capital. Such a consumption tax entails a deadweight loss to the society. Welfare comparison suggests that the loss from both production and consumption based emissions are non-trivial.

Our key results are likely to be robust even if we add a pollution distaste function in the preference as Michelle and Rotillon (1995). A useful future extension of our model is to consider adverse health effect of emissions as in Gradus and Smulders (1993). Such an extension would strengthen the case for a steeper Pigovian carbon tax. However, the effect on growth caused by the carbon tax is likely to be ambiguous. While the distortionary effects of carbon tax would lower the long run growth, a positive effect on health may promote growth via human capital.

A. Appendix

The present value Lagrangian is given by:

$$L^p = \sum_{t=0}^{\infty} \beta^t \ln C_t + \sum_{t=0}^{\infty} \lambda_t \left[(1-i^g_{yt})AK_t + (1-\delta_p)K^p_t - C_t - K^p_{t+1}\right] \tag{A.1}$$
$$+ \sum_{t=0}^{\infty} \mu_t \left[K^g_t + i^g_{yt}AK_t - \omega K^p_t - K^g_{t+1}\right]$$

where $\{\lambda_t\}$ and $\{\mu_t\}$ are the lagrange multipliers. The first order conditions are:

$$C_t : \ \beta^t / C_t \ - \lambda_t = 0 \tag{A.2}$$

$$K^p_{t+1} : \ \ - \lambda_t + \lambda_{t+1} \left\{(1-i^g_{yt+1})A\frac{\partial K_{t+1}}{\partial K^p_{t+1}} + 1 - \delta_p\right\} - \mu_{t+1}\omega + \mu_{t+1}Ai^g_{yt+1}\frac{\partial K_{t+1}}{\partial K^p_{t+1}} = 0 \tag{A.3}$$

$$K^g_{t+1} : \ \lambda_{t+1}(1-i^g_{yt+1})A\frac{\partial K_{t+1}}{\partial K^g_{t+1}} - \mu_t + \mu_{t+1}\left\{1 + Ai^g_{yt+1}\frac{\partial K_{t+1}}{\partial K^g_{t+1}}\right\} = 0 \tag{A.4}$$

$$i^g_{yt} : \ \ - \lambda_t + \mu_t = 0 \tag{A.5}$$

Eq (A.5) is the foundation of the crucial static efficiency condition that equates the marginal distortion from the tax rate to the marginal benefit of the tax to finance green capital. Plugging (A.5) into (A.3) and using (A.2), we get:

$$\frac{C_{t+1}}{C_t} = \beta\left[A\frac{\partial K_{t+1}}{\partial K^p_{t+1}} + 1 - \delta_p - \bar{\omega}\right] \tag{A.6}$$

Likewise, plugging (A.5) into (A.4) and using (A.2), we get:

$$\frac{C_{t+1}}{C_t} = \beta\left[A\frac{\partial K_{t+1}}{\partial K^g_{t+1}} + 1\right] \tag{A.7}$$

Equating (A.6) to (A.7), one obtains the static efficiency condition (9).

To get the optimal carbon tax formula (19), equate the right hand sides of (A.6) and (A.7).

A.1. Case of strict complementarity

Since the production function in (20) is Leontief type, the efficient ratio K_t^P/K_t^g is pinned down by the technology and is equal to unity. Equation (4) reduces to:

$$1+\gamma = 1-\omega + i_y^g A \tag{A.8}$$

To get the optimal green investment ratio i_y^g, we need to recast the social planning problem and derive the balanced growth rate from the social planner's perspective. The social planner now no longer chooses the ratio of green to made made capital because it is pinned down by the technology at a fixed proportion ($K_t^P/K_t^g = 1$). Setting $K_t^g = K_t^p$, the economy wide resource constraint can be reduced to:

$$C_t + 2K_{t+1}^p - (2-\delta_p-\omega)K_t^p = AK_t^p$$

The present value lagrangian can be written as:

$$L^p = \sum_{t=0}^{\infty} \beta^t \ln C_t + \sum_{t=0}^{\infty} \lambda_t^{'} \left[(2+A-\omega-\delta_p)AK_t^p - C_t - 2K_{t+1}^p\right] \tag{A.9}$$

where $\{\lambda_t^{'}\}$ is the sequence of Lagrange multipliers associated with the flow resource constraints.

The first order conditions are:

$$C_t: \ \beta^t/C_t \ - \lambda_t^{'} = 0 \tag{A.10}$$

$$K_{t+1}^p: \quad -2\lambda_t^{'} + \lambda_{t+1}^{'}(2 + A - \omega - \delta_p) = 0 \tag{A.11}$$

Now it is straightforward by using (A.10) and (A.11) that the balanced growth rate is given by:

$$1 + \gamma = \beta \left(1 + \frac{A - \omega - \delta_p}{2}\right) \tag{A.12}$$

Using (A.8) and (A.12), the optimal investment ratio in green capital given by:

$$i_y^g = \frac{\beta\{1 + 0.5(A - \delta_p)\} - 1 + \omega(1 - .5\beta)}{A} \tag{A.13}$$

To get the optimal carbon tax, we need to use the household's Euler equation (19) which reduces to:

$$\frac{C_{t+1}}{C_t} = \beta\left[(1 - \tau_{t+1})A + 1 - \delta_p\right] \tag{A.14}$$

Along the balanced growth path (A.14) reduces to:

$$1 + \gamma = \beta\left[(1 - \tau)A + 1 - \delta_p\right] \tag{A.15}$$

Equating (A.12) with (A.15), we get:

$$\tau = 0.5 + A^{-1}(1 - 0.5\delta_p) + 0.5A^{-1}\omega \tag{A.16}$$

A.2. Model with pollution abatement

The present value Lagrangian is given by:

$$L^p = \sum_{t=0}^{\infty} \beta^t \ln C_t + \sum_{t=0}^{\infty} \overline{\lambda}_t \left[(1 - i_{yt}^g - i_{yt}^\omega) A K_t + (1 - \delta_p) K_t^p - C_t - K_{t+1}^p \right] \tag{A.17}$$
$$+ \sum_{t=0}^{\infty} \overline{\mu}_t \left[K_t^g + i_{yt}^g A K_t - \omega(i_{yt}^\omega) K_t^p - K_{t+1}^g \right]$$

where $\{\overline{\lambda}_t\}$ and $\{\overline{\mu}_t\}$ are the lagrange multipliers. The first order conditions are:

$$C_t : \;\; \beta^t / C_t \;\; - \overline{\lambda}_t = 0 \tag{A.18}$$

$$K_{t+1}^p : -\overline{\lambda}_t + \overline{\lambda}_{t+1} \left\{ (1 - i_{yt+1}^g - i_{yt+1}^\omega) A \frac{\partial K_{t+1}}{\partial K_{t+1}^p} + 1 - \delta_p \right\} - \overline{\mu}_{t+1} \left\{ \omega(i_{yt+1}^\omega) + A i_{yt+1}^g \frac{\partial K_{t+1}}{\partial K_{t+1}^p} \right\} = 0 \tag{A.19}$$

$$K_{t+1}^g : \overline{\lambda}_{t+1} (1 - i_{yt+1}^g - i_{yt+1}^\omega) A \frac{\partial K_{t+1}}{\partial K_{t+1}^g} - \overline{\mu}_t + \overline{\mu}_{t+1} \left\{ 1 + A i_{yt+1}^g \frac{\partial K_{t+1}}{\partial K_{t+1}^g} \right\} = 0 \tag{A.20}$$

$$i_{yt}^g : \;\; - \overline{\lambda}_t + \overline{\mu}_t = 0 \tag{A.21}$$

$$i_{yt}^\omega : \;\; - \overline{\mu}_t \omega'(i_{yt}^\omega) K_t^p - \overline{\lambda}_t A K_t = 0 \tag{A.22}$$

Use (25), (A.21) and (A.22) to verify (29). Use (A.19) and (A.21) to get the balanced growth equation (32).

A.3. Model of consumption based emissions

The present value Lagrangian is given by:

$$L^p = \sum_{t=0}^{\infty} \beta^t \ln C_t + \sum_{t=0}^{\infty} \lambda_t \left[(1 - i_{yt}^g) A K_t + (1 - \delta_p) K_t^p - C_t - K_{t+1}^p \right] \tag{A.23}$$
$$+ \sum_{t=0}^{\infty} \mu_t \left[K_t^g + i_{yt}^g A K_t - \overline{\varpi}^c C_t - K_{t+1}^g \right]$$

where $\{\lambda_t\}$ and $\{\mu_t\}$ are the lagrange multipliers. The first order conditions are:

$$C_t : \ \beta^t / C_t \ - \lambda_t - \overline{\varpi}^c \mu_t = 0 \tag{A.24}$$

$$K_{t+1}^p : \ \ - \lambda_t + \lambda_{t+1} \left\{ (1 - i_{yt+1}^g) A \frac{\partial K_{t+1}}{\partial K_{t+1}^p} + 1 - \delta_p \right\} + \mu_{t+1} A i_{yt+1}^g \frac{\partial K_{t+1}}{\partial K_{t+1}^p} = 0 \tag{A.25}$$

$$K_{t+1}^g : \ \lambda_{t+1} (1 - i_{yt+1}^g) A \frac{\partial K_{t+1}}{\partial K_{t+1}^g} - \mu_t + \mu_{t+1} \left\{ 1 + A i_{yt+1}^g \frac{\partial K_{t+1}}{\partial K_{t+1}^g} \right\} = 0 \tag{A.26}$$

$$i_{yt}^g : \ \ - \lambda_t + \mu_t = 0 \tag{A.27}$$

Using (A.24) and (A.26), one gets, $\lambda_t = \beta^t / (1 + \overline{\varpi}^c)$ which upon substitution in (A.25) and (A.26), yields

$$\frac{C_{t+1}}{C_t} = \beta \left[A \frac{\partial K_{t+1}}{\partial K_{t+1}^p} + 1 - \delta_p \right] \tag{A.28}$$

$$\frac{C_{t+1}}{C_t} = \beta \left[A \frac{\partial K_{t+1}}{\partial K_{t+1}^g} + 1 \right] \tag{A.29}$$

Use of (A.28) and (A.29) yields the static efficiency condition.

A.4. Proof of Proposition 2

Plug in (38) into (4) to get:

$$K_{t+1}^g = K_t^g - \varpi^c C_t + i_{yt}^g Y_t$$

which can be rewritten after imposing balanced growth condition as:

$$1+\gamma = 1 - \varpi^c (C_t/K_t^p)(K_t^p/K_t^g) + i_{yt}^g (Y_t/K_t^g)$$

Plugging the production function (1) and (2), one obtains

$$i_{yt}^g = \frac{\gamma + \overline{\omega}^c (C_t/K_t^p)(K_t^p/K_t^g)}{A\left[(1-\nu)(K_t^p/K_t^g)^{\varphi} + \nu\right]^{1/\varphi}} \tag{A.30}$$

Rewrite the economy-wide resource constraint: $C_t + I_t^p = (1 - i_{yt}^g)Y_t$ as:

$$(1+\overline{\omega}^c)(C_t/K_t^p) + \gamma(1 + K_t^g/K_t^p) + \delta_p = A\left[(1-\nu) + \nu(K_t^g/K_t^p)^{\varphi}\right]^{1/\varphi}$$

Since K_t^g/K_t^p and the balanced growth rate (γ) are independent of the growth rate, verify that $\frac{\partial \ln(C_t/K_t^p)}{\partial \ln \overline{\omega}^c} = \frac{-\overline{\omega}^c}{1+\overline{\omega}^c}$. In other words, the absolute value of the elasticity of C_t/K_t^p with respect to $\overline{\omega}^c$ is less than unity. This means $\overline{\omega}^c(C_t/K_t^p)$ in eq (A.30) is increasing in $\overline{\omega}^c$. Since K_t^p/K_t^g is invariant to $\overline{\omega}^c$, $\frac{\partial i_{yt}^g}{\partial \overline{\omega}^c} > 0$.

Since the carbon tax is zero, it follows from the government budget constraint (15) that $i_{yt}^g Y_t + T_t = 0$ which means that $-T_t/Y_t = i_{yt}^g$. Use of the household's budget constraint (16) immediately reveals that this is equivalent to a flat rate consumption tax rate (τ_t^c) equal to i_{yt}^g.

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