Theoretical and Experimental Investigation of Driver Noncooperative-Game Steering Control Behavior

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*Abstract*—This paper investigates two noncooperative-game strategies which may be used to represent a human driver’s steering control behavior in response to vehicle automated steering intervention. The first strategy, namely the Nash strategy is derived based on the assumption that a Nash equilibrium is reached in a noncooperative game of vehicle path-following control involving a driver and a vehicle automated steering controller. The second one, namely the Stackelberg strategy is derived based on the assumption that a Stackelberg equilibrium is reached in a similar context. A simulation study is performed to study the differences between the two proposed noncooperative- game strategies. An experiment using a fixed-base driving simulator is carried out to measure six test drivers’ steering behavior in response to vehicle automated steering intervention. The Nash strategy is then fitted to measured driver steering wheel angles following a model identification procedure. Control weight parameters involved in the Nash strategy are identified. It is found that the proposed Nash strategy with the identified control weights is capable of representing the trend of measured driver steering behavior and vehicle lateral responses. It is also found that the proposed Nash strategy is superior to the classic driver steering control strategy which has widely been used for modeling driver steering control over the past. A discussion on improving automated steering control using the gained knowledge of driver noncooperative-game steering control behavior was made.

*Index Terms*—Driver, vehicle, noncooperative game, steering control strategy, experiment, model identification.

[[1]](#footnote-1)

# INTRODUCTION

T

HE introduction of automated driving systems to modern vehicles brought marked improvement to driving safety. Meanwhile, this raised interesting questions of the influence of vehicle automation on drivers’ control behavior. Assessment of the performance of automated driving systems currently relies heavily on experimental approaches using test drivers. The drivers’ subjective feelings on the performance of automated driving systems are interpreted into engineering terms to advise design changes. A consequence is that the development of automated driving systems is time-consuming and expensive. Besides, new development in technology may expand vehicle performance envelope into a region where previous knowledge of the mapping from test driver’s subjective feelings to vehicle design changes becomes inapplicable [1]. Towards this end, mathematical models capable of representing human drivers’ control behavior in response to the intervention of automated driving systems in are in demand. Such models are believed useful in supporting the development of automated driving systems in low-cost computer-aided design phases. However, an absence of thorough theoretical understanding of what control strategy a human driver may adopt to interact with automated driving systems is presently limiting the application of mathematical driver models in technology development.

Elsewhere in engineering, economics, and neuroscience, game theory has been widely used to model situations in which two or more individuals make decisions that influence one another’s welfare [2]. As a descendant of game theory, dynamic game theory specializes itself in dealing with circumstances where players make decisions repeatedly over time [3]. Dynamic games can be classified into noncooperative games and cooperative games, according to their mode of play [4]. In a noncooperative game, each player focuses on pursuing his/her individual interest while in a cooperative game, players arrive at a binding agreement of interest [5]. The strategy that a game player adopts has predominant influence on his/her welfare. A strategy of a player is essentially a mapping of the states of the game to the player’s action [6]. The strategies of all players constitute a strategy set. An equilibrium is such a strategy set that every player has the belief that his/her strategy has reached an optimum in terms of maximizing his/her welfare. In other words, no player is willing to change his/her strategy unilaterally [7]. Nash and Stackelberg are two typical equilibria observed in noncooperative games. A Nash equilibrium may emerge when each player chooses to develop his/her strategy by taking the others’ behavior into account, and all players take actions simultaneously. A Stackelberg equilibrium may emerge in games of the leader-follower form, where the leader player derives strategy by considering all the followers’ strategies while all the followers stick to their own optimal responses [8].

In a dynamic game, players may have different knowledge about the states of the game. The term “information pattern” was proposed by Başar and Olsder [3] to describe a player’s knowledge of the states of a game. Players who have open-loop information pattern are defined to know only the initial states of a game. In contrast, players who have closed-loop information pattern can get access to intermediate states of a game. More detailed classification of the closed-loop information pattern, e.g. in terms of players’ measurability and memorization of game states are explained in [3].

A linear quadratic (LQ) game is a special case of a dynamic game. In a LQ game, the dynamic evolution of the game system is describable by a linear differential equation, and the players’ cost functions are describable by affine-quadratic terms [4]. Since a LQ game is analytically tractable and numerically solvable, it has been extensively used for modelling and solving dynamic game problems. Of particular note here is the work done by Braun *et al.* [9] who constructed a LQ game model to predict human players’ hand movements in a two-person rope- pulling game. High degree of agreement was achieved between model-predicted and experiment-measured human players’ arm movements. This implies that human sensorimotor interaction may be modeled and represented in a dynamic game context.

The interaction between a human driver and an automated vehicle may also be understand from a dynamic game point of view: the driver and the automated driving controller are game players that make decisions and take actions in a repeated way, and their decisions are continuously influence by each other’s actions. Inspired by the work done by Braun *et al.* [9] and enlightened by the dynamic-game nature of driver-automation interaction, the authors of the present paper proposed a set of dynamic-game schemes in [10], aiming at modeling and representing human drivers’ steering interaction with an Active Front Steering (AFS) system [11] in a path-following scenario, where the AFS system is capable of making decision and applying steering control independently of the human driver. One scheme was derived by assuming that a Pareto equilibrium is reached when the driver and the AFS controller are playing a cooperative game in vehicle steering control. Such a scheme was investigated via simulation analysis in a recent publication by the authors [12]. Another two schemes were derived based on the assumption that the driver interacts with the AFS in a noncooperative game context. Specifically, one involves driver adopting a steering control strategy that has a Nash equilibrium property while the other involves a Stackelberg equilibrium. It was not covered in [10] how the driver noncooperative-game steering control strategies can be validated via experiments.

The assumption that human drivers adopt noncooperative- game strategies to interact with vehicle automation was implied in some research on vehicle dynamics control. Tamaddoni *et al.* [13] reported a vehicle yaw stability control algorithm based on the assumption that the driver’s steering control and the vehicle controller’s yaw moment control can reach a Nash equilibrium. Dextreit and Kolmanovsky [14] reported a controller for energy management in hybrid vehicles based on the assumption that the driver’s speed control and the controller’s speed control can reach a Stackelberg equilibrium. Flad *et. al* [15] developed a steering torque assistance controller that seeks to reach a Nash equilibrium with the driver’s steering torque. Most Recently, Ji *et al.* [16] [17] developed a noncooperative-game framework for designing driver-automation shared control, where the vehicle automation in [16] was implemented via an automated angle-overlay steering system, and later in [17] a torque- overlay system. In all these studies, the effectiveness of the proposed vehicle controllers was tested via simulation and/or experiments. However, so far it has been an open question whether drivers in reality interact with vehicle automation by using some degree of noncooperative-game control strategy.

The present paper attempts to field the question through investigating whether human drivers may interact with an AFS system by adopting noncooperative-game steering control strategies. To achieve this goal, an extension to the authors’ previous work [10] from theoretic perspective is first carried out. This involves conducting a numerical study to explore the difference between a driver’s noncooperative Nash and his/her noncooperative Stackelberg steering control strategies. Another extension to [10] from experimental perspective is then carried out. This involves fitting the formula of driver noncooperative Nash strategy to realistic driver steering control behavior measured using a fix-base driving simulator. On this basis, a discussion on the improvement of vehicle automated steering according to the observed driver steering behavior is made.

The contribution of the paper may be summarized as follows. First, the significant difference in driver steering behavior between adopting Nash and Stackelberg strategies are revealed. Second, the validity of the proposed Nash steering strategy for representing human drivers’ realistic steering interaction with vehicle automated steering control is demonstrated. Third, detailed design improvement to existing automated steering control is suggested based on the gained knowledge of driver steering behavior in response to automated steering control. It is noteworthy that besides game theory, reinforcement learning is a powerful approach to dealing with noncooperative and distributed decision-making problems, especially when only partial information is available to some of the decision-making entities, e.g. [18] [19]. A key reason that the game-theoretic approach is adopted in the present work is that it allows for a synthesis of linear quadratic representation of a mechanistic driver model and that of a dynamic game system. In such a circumstance, the mechanism of a human driver’s steering control can be explicitly described using model parameters, including driver prediction horizon, control horizon, control weights on target path tracking error, game-system convex iteration steps etc. On determining the values of these model parameters, quantitation of an individual driver’s driving skill and style can be conducted, such as the work reported in [20] and [21]. In view of this, it is expected that penetrating insight into the underlying mechanism of driver-automation interaction may be achieved by using a game-theoretic driver modeling approach. On this basis, optimization and customization of present or future automated driving interface, e.g. shared control [22], and parallel driving [23] becomes a possibility.

The remainder of the paper is organized as follows. Section II delineates the theoretic schemes for studying noncooperative steering interaction between a human driver and a vehicle AFS controller. Section III formulates the derivation of a driver’s noncooperative Nash and Stackelberg steering strategies. A simulation study is carried out to explore their differences. Section IV describes an experiment for measuring human driver’s steering behavior in response to AFS control in a path-following scenario. Section V presents the outcomes of fitting the driver Nash steering control strategy to the measured data and illustrates the superiority of the Nash strategy over a classic steering strategy in representing driver steering behavior Section VI suggests possible design improvement of vehicle automated steering control by using the gained knowledge of human drivers’ noncooperative-game steering control behavior. Section VII draws conclusions and suggests future work.

# Driver-AFS Noncooperative Game Theoretic Steering Control Schemes



Fig. 1. Driver-AFS Nash steering control scheme

The two driver-AFS noncooperative game theoretic steering control schemes to be investigated are illustrated respectively in Fig. 1 and Fig. 2. The scheme shown in Fig. 1 is the driver- AFS Nash steering control scheme, in which the driver adopts a steering control strategy having the Nash equilibrium property. The one shown in Fig. 2 is the driver-AFS Stackelberg steering control scheme, in which the driver adopts a steering control strategy having the Stackelberg equilibrium property.



Fig. 2. Driver-AFS Stackelberg steering control scheme

## Driver-AFS Nash steering control scheme

In the driver-AFS Nash steering control scheme shown in Fig. 1, the AFS controller determines its steering angle  at time step *k* using vehicle state , AFS target path , and driver steering angle . The AFS control law is obtained by minimizing cost function  at time step *k*. Such a cost function concerns penalizing the difference between vehicle state  and AFS target . Both driver steering angle  and AFS angle  are involved because vehicle state  is subject to both of them. As a result, the AFS control law can be conceptually expressed as:

 (1)

Here  denotes the rule that maps ,  and  to the AFS steering angle . The vehicle state  is a vector composed of vehicle dynamic responses such as lateral velocity , lateral displacement  and yaw rate . They can be measured using low-cost sensors or estimated using practical methods, e.g. those reviewed in [24]. The AFS target path  can be determined independently of the driver’s intention by using various onboard sensing hardware and software. The driver steering angle  can be measured using low-cost sensors that have been equipped in most modern vehicles. In (1),  is taken into account by the AFS system so that the AFS can compensate for undesirable driver steering behavior, which is possibly due to human error. The idea of compensating driver action in vehicle control was reported in several studies, e.g. by Anderson *et al.* [25] and Li *et al.* [26].

The human driver, on the other hand, controls the vehicle to follow his or her own target path . Since the AFS system concerned in this paper is capable of planning its own target path  independently of the driver, there is the possibility that the driver’s planned target path  can differ from the AFS controller’s planned target path . This may in turn cause the driver and the AFS controller to steer the vehicle in different directions, e.g. the driver decides to take a left lane change while the AFS decides to take a right lane change. The driver has the chance to sense vehicle dynamics using his or her visual, vestibular and somatosensory systems [27] and identify the characteristics of the AFS control. On this basis, the driver may form an ‘internal model’ describing his/her understanding of the dynamics of the whole automated vehicle system [1]. The internal model hypothesis is widely adopted in neuroscience studies. Driver steering control models based on the internal model hypothesis have been validated in several studies, e.g. [28] and [29]. Given that the driver may develop an internal model of the automated vehicle system, it is possible that the driver adopts a steering control strategy that compensates for the effect of AFS steering angle . Such a steering control strategy can be derived by minimizing a cost function  at time step *k*. This cost function evaluates the deviation of vehicle dynamic responses represented by state  from the driver’s target path , as explained in [1]. The cost function involves driver steering angle  and AFS control angle  since both affect vehicle state . By minimizing the cost function, the driver may derive a steering angle  that satisfies . It is noted that in this condition, the AFS steering angle  follows (1). This implies that the driver cannot further reduce his/her cost function if the driver applies any other steering angle than . Such a feature is known as the Nash equilibrium property [3]. Hence, the strategy that maps ,  and  to driver steering angle  is the driver’s Nash steering control strategy. It can be expressed conceptually as:

 (2)

where  denotes the mapping rule.

## Driver-AFS Stackelberg steering control scheme

In the driver-AFS noncooperative Stackelberg steering control scheme shown in Fig. 2, the AFS controller still adopts control law (1). The driver is assumed to interact with the AFS controller in a different manner from that in the Nash scheme. Specifically, the driver in the Stackelberg scheme is assumed to derive his or her steering control strategy by compensating for the entire AFS steering control law (1), rather than just the AFS steering angle . Such an assumption may be accepted in situation where the driver has already built up a fairly accurate internal model of AFS control law (1). This may be achieved through repeated training. It should be remarked here that in the Nash scheme the driver is assumed to have derived an internal model of the dynamics of the vehicle, subject to both driver steering angle  and AFS angle . In contrast, in the Stackelberg scheme the driver is assumed to have developed an internal model incorporating both vehicle dynamics and AFS control law (1). Mathematically, the interaction between driver and AFS can be modeled as a leader-follower game, as defined in [8]. Specifically, the AFS serves as the follower by sticking to its control law (1) in response to any driver steering angle, while the driver acts as the leader by knowing the AFS control law (1) and takes it into account in deriving the driver’s own steering control strategy. The driver’s cost function can be thus described as , where the AFS control law (1) is explicitly involved. Accordingly, the driver may arrive at such a steering angle  that enables the cost function above to reach its minimum. Such a  therefore bears the Stackelberg equilibrium property [3]. The strategy mapping ,  and  to driver steering angle  is therefore the driver’s Stackelberg steering control strategy. It can be expressed conceptually as:

 (3)

where  denotes the mapping rule.

# Derivation of Driver Nash and Stackelberg Steering Control Strategies

In this section, derivation of a driver’s Nash and Stackelberg steering control strategies is presented. A simulation study is then carried out to explore their differences.

## Vehicle Dynamics Model

The dynamics of a vehicle is represented using the linear time-invariant “bicycle” model, as described in [10]:

 (4)

 is vehicle state vector at time step *k*. It comprise vehicle lateral velocity , yaw rate , lateral displacement , lateral displacement integral , and yaw angle , i.e. .  is the state matrix,  and  are input vectors associated with driver steering angle  and AFS steering angle .  is the output matrix relating  to output vector .  characterizes the vehicle’s position and orientation. It is defined as .

## AFS Control Law

The noncooperative Model Predictive Control (MPC) approach is used to derive the analytical expressions of the AFS control law in this subsection. It will be used to derive the driver’s Nash and Stackelberg steering control strategies later.

Noncooperative MPC was proposed by Rawlings and Mayne [30] as a combination of noncooperative game theory and distributed MPC method. The Distributed MPC method was developed first as a solution to industrial process control of large-scale systems. The core idea of distributed MPC involves decomposing a large-scale system into smaller subsystems, and deploying local controllers to control the subsystems. Hence, the communication between the local controllers has a major impact on the control performance. The noncooperative MPC approach is characterized by designing the communication between local controllers based on noncooperative game theory: each local controller views the others’ control actions as known disturbances and tries to compensate for their effects.

Rawlings and Mayne [30] explained that the communication can be alternatively designed based on cooperative game theory, leading to the cooperative MPC approach. The communication can also be removed to simplify the control problem, leading to the decentralized MPC approach. However, neither of them will be touched in this paper.

Continuing to follow Rawlings and Mayne [30], three steps are followed to derive the AFS control law through using the noncooperative MPC approach. First, the AFS controller’s prediction equation is established by iterating vehicle dynamics equation (4) for *N*2 steps ahead. *N*2 thus denotes the AFS controller’s prediction horizon. In other words, future vehicle dynamic responses are predicted based on vehicle model (4). During the process, both the AFS and the driver steering angles are kept unchanged over the prediction horizon *N*2. As a result, the AFS controller’s prediction equation can be obtained:

 (5)

where

, , , and.

 denotes future vehicle dynamic responses predicted by the AFS controller at time *k*.  consists of vectors from  up to .  is the vehicle output vector at time *k*, as per (4).  for  denotes future vehicle output vector at time *k* + *i*, which is predicted by iterating (4) for *i* steps ahead of time *k*. As the prediction is model based, the accuracy of the prediction is largely influenced by the fidelity the prediction model (4), as explained in [31]. For general vehicle predictive control problems, the prediction model in use may become less accurate as the vehicle’s operation condition changes, e.g. change of speed or road surface. Various methods have been developed to adapt the prediction model to such uncertainties, as those discussed in [32]. As the focus of the present paper is the modelling of driver steering control behavior, the vehicle is set to operate in a linear region that can be completely described by the time-invariant “bicycle” model (4). This point will be re-emphasized later in the numerical simulation and the driving simulator experiment. Over the iteration, future AFS angle  and future driver steering angle  are involved. As mentioned above, a solution technique is adopted involving setting future AFS angles  () unchanged from , i.e. , and so for . In other words, the control horizon is one time step. Such a setting was used in some vehicle control systems to reduce computational burden, e.g. in [33]. This setting also conforms to the steering control of human drivers who were found less likely to change steering angle action over his or her prediction horizon [29]. ,  and  involved in (5) are matrices of appropriate dimensions.

Second, the AFS controller at time *k* is assigned to apply steering angle  to minimize the difference between its predicted vehicle output  and its target path . Such a control objective can be formulated as to minimize the following cost function  at time *k*:

 (6)

where  denotes the AFS controller’s path-following error.  is a diagonal matrix composed of weights ,  and . These three weights penalize the AFS controller’s path-following errors associated with ,  and , respectively. In (6) no future AFS angle  is penalized but just the AFS angle  at time *k*. This is due to the setting  introduced above.

Finally, the AFS cost function (6) is minimized subject to its prediction equation (5). This results in the analytical expression of the AFS control law (7):

 (7)

where ,  and  are gain arrays, which are functions of , , , , ,  and . (7) is the analytical expression of its conceptual counterpart (1). It should be noted that since there is only  rather any future AFS angle  involved in cost function (6), minimization of (6) straight gives (7); the “receding horizon” strategy is not needed.

It can be seen from (7) that the AFS angle  depends on vehicle state , AFS target path , but also driver steering angle . Rawlings and Mayne [30] observed that the dependency of AFS angle  on driver steering angle  is the key feature of noncooperative MPC. Generally, in noncooperative MPC problems, a local controller’s optimal action depends not only on the state of the system and its target, but also the other controllers’ decisions. In contrast, the optimal control action of a conventional MPC controller is independent of the decision of other controllers. Increasing weights ,  and  allows the vehicle to follow the AFS controller’s target path  more firmly, at an expense of increased AFS steering angle . Details on the determination of the AFS control weights will be provided in Section IV.

## Driver Nash Steering Control Strategy

In this subsection, the driver Nash steering control strategy is derived using the noncooperative MPC approach [30].

First, the driver’s prediction equation is built up by iterating vehicle dynamics equation (4) for *N*1 steps ahead, where *N*1 is the driver’s prediction horizon. During this process, the driver’s steering angle  may vary for the first *M*1 steps and then hold constant up to *N*1. *M*1 therefore denotes the driver’s control horizon which must satisfy . As a result of the iteration, the driver’s prediction equation can be written as

 (8)

where

, ,

, , and

.

It is worth noting that in (8) the AFS steering angle  is kept unchanged throughout the driver prediction horizon *N*1 so as to conform to the setting made for (5) in the previous subsection.

Second, the driver’s cost function  at time step *k* is defined as to minimize the driver’s path-following error   over his or her steering angle vector :

 (9)

where  is a diagonal matrix composed of weights ,  and . They are used by the driver to penalize path-following errors associated with ,  and , respectively.

Following Maciejowski’s solution technique [34], the driver steering angle vector  that minimizes cost function (9) is the least-squares solution to the following equation:

 (10)

where  satisfies . Maciejowski [34] explained that such a least-squares problem can be solved using the QR algorithm (invoked in MATLAB using the backslash operator “\”). This leads to the solution to (10) for :

 (11)

where  is a matrix resulting from the QR algorithm:



 is a function of , , , , ,  and .

The driver steering angle vector  determined by (11) ensures that the driver’s cost function (9) reaches minimum in response to a given AFS steering angle .  contains a series of driver steering angles from  to , however, the driver can only apply one specific steering angle to vehicle at time *k*. To this end, a solution technique is adopted, involving setting driver control horizon *M*1 to unity in driver prediction equation (8), i.e. setting . Such a setting was adopted for two reasons. First, this  setting was found in [29] effective of representing the steering behaviour of a variety of drivers. Second, the authors attempt to do a comparative study between the driver Nash steering control strategy to be developed later and a “classic” driver steering strategy to be introduced in Section V, in order to investigate which strategy is more appropriate in representing drivers’ steering behavior. The classic strategy already incorporates the  setting. To allow a fair comparison, the driver Nash steering control strategy to be developed should involve this same setting. On adopting , the driver’s prediction equation (8) becomes

 (12)

where

, , and .

And the driver’s cost function reduces from (9) to:

 (13)

Minimizing (13) subject to (12) then givens

 (14)

where ,  and  are gain arrays, which are functions of , , , , ,  and . (14) is a steering control strategy that enables the driver to minimize cost function (13) in response to a given AFS angle .

Continuing to follow Rawlings and Mayne [30], a convex iteration approach is adopted to facilitate the development of the driver’s Nash-equilibrium-based steering control strategy. This starts with introducing the following iterative equation:

 (14a)

where *p* denotes the step of convex iteration, and *w*1 the convex iteration weight that must satisfy . It can be seen that (14a) combines the driver steering angle  from (14) and the current iterate  to determine the next iterate . (14) can be substituted into (14a) to yield:



(14b)

In view that the AFS steering angle  appearing in (14b) is determined via AFS control law (7), the expression of (7) can be adapted to the convex iteration scenario by incorporating the current iterate . This gives:

 (14c)

Substituting (14c) into (14b) then results in:



(14d)

According to Rawlings and Mayne [30], the convergence of the convex iteration described in (14d) is governed by the scalar term . If it locates inside the unit circle, (14d) will converge to the following equation as step .



(14e)

In (14e) AFS angle  vanishes. (14e) allows the driver to minimize cost function (13) for any AFS angle. In other words, the driver cannot further reduce the value of (13) if he or she adopts any strategy other than (14e). Such a property is known as Nash equilibrium [3]. (14e) is thereby the driver’s Nash steering control strategy. It should be remarked that the convex iteration weight *w*1 does not appear in (14e), however, it does influence the existence of a Nash equilibrium. Rawlings and Mayne [30] explained that the Nash equilibrium given in the form of (14e) exists as a unique solution only if the scalar term  shown in (14d) is inside the unit circle. This will be checked in the simulation and experiment analysis later, where specified driver and AFS parameters are employed.

To simply the expression, (14e) is rewritten as:

 (15)

where  is  used in (14e), and gain arrays ,  and  represent corresponding terms in (14e). It is noteworthy that (14) can result in the same driver steering angle as (15) does, given that the AFS control law (7) is in use. Hence (14) is equivalent to (15) in terms of calculating the driver Nash-equilibrium-sensed steering angle. (14) is the analytical expression of its conceptual counterpart (2).

## Driver Stackelberg Steering Control Strategy

In the Stackelberg case, the driver is assumed to develop his or her prediction equation in a different way from the Nash case. Specifically, the driver substitutes the AFS control law (7) into his or her prediction equation (12) employed in the Nash case to generate a more sophisticated prediction equation (16):

 (16)

where,  and. This reflects the key difference between the Nash and the Stackelberg schemes. As explained in Section II, the driver in the Stackelberg scheme decides to derive his or her steering strategy by compensating for the entire AFS control law.

The driver’s cost function in the Stackelberg case is still (13), which suggests that the driver still focus on minimizing his or her own path-following error . Again by following Maciejowski [34], the driver steering angle  minimizing cost function (13) is the least-squares solution to:

 (17)

Solving (17) for  then gives:

TABLE I

Driver Model Parameters Used for Simulation Study

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Symbol | Quantity | Value | | |
| Case A | Case B | Case C |
|  | lateral displacement  error weight | 2.0 10-4 | 4.0 10-4 | 2.0 10-1 |
|  | lateral displacement  integral error weight | 4.8 10-7 | 9.6 10-7 | 4.8 10-7 |
|  | yaw error weight | 1.2 10-2 | 2.4 10-2 | 1.2 10-2 |
| *N*1 | prediction horizon (step) | 200 | 200 | 150 |

 (18)

where  can be calculated again by using the QR algorithm:



It can be seen that (18) is a steering strategy minimizing driver cost function (13) subject to prediction equation (16). As (16) implies the driver’s consideration of AFS control law in his/her prediction, (18) is thereby a strategy that bears the Stackelberg equilibrium property in a leader-follower game [8], where the AFS acts as the follower who sticks to its control law (7) while the driver acts as the leader who keeps in mind what control law the AFS sticks to. (18) can be then written as:

 (19)

where ,  and  respectively represent ,  and  appearing in (18). (19) is the analytical expression of the driver’s Stackelberg equilibrium strategy. Its conceptual counterpart is (3).

It is noted that the derivation of driver Stackelberg steering control strategy (19) does not involve the convex iteration that was adopted in deriving the driver Nash strategy (15). Instead, (19) is derived following a procedure very similar to that for a conventional MPC problem described in [34]. This is because the driver’s prediction equation (16) in the Stackelberg case does not involve the AFS steering angle  but its target path  instead, which can be viewed as a known disturbance at each time step *k*. Maciejowski [34] explained that the solution in the form of (19) exists as a unique optimal solution since the Hessian, i.e. the second derivative of cost function (13) with respect to driver angle , is positive-definite. This condition is guaranteed by the diagonal weight matrix  that is composed of non-negative weights ,  and .

It can be seen that the driver’s Nash steering control strategy (15) and the driver’s Stackelberg strategy (19) represent different steering behaviors. The Nash strategy (15) suggests that the driver determines his or her steering angle  by taking the AFS angle  into account while the Stackelberg strategy (19) implies that the driver takes the AFS control law as a whole into account. To illustrate the differences between the two types of driver steering control strategies, a simulation study is carried out in the next subsection.

## Simulation Study

A driving scenario where the AFS controller and the driver hold different target paths is designed for simulation study. In this scenario, the AFS controller is assumed to have detected an obstacle ahead and decided to perform a lane change maneuver to avoid crash. The driver, on the other hand, is assumed to aim at keeping the vehicle travelling straight ahead. Such a conflict may occur when there is an error in AFS target path planning, or when the driver thinks his or her target path is more morally acceptable even it may eventually lead to a crash [35]. The vehicle parameters used for this study are set the same as those used in [10]. The vehicle longitudinal speed *U* is kept to 20 m/s to suit the linear “bicycle” model assumption. The sample time *T*s is set 0.01 s. The AFS controller’s prediction horizon *N*2 is set to 200 steps, i.e. 2.0 s. This gives a preview distance of 40 m, which is achievable using onboard cameras. The AFS control weights are set ,  and . These weights were tested to be able to produce good path- following performance while offering drivers the opportunity to override AFS control. The values of the weights will be used in the driving simulator experiment to be described in the next section, where explanations on how they were determined will be provided. In this subsection, the influence of driver Nash and Stackelberg steering control strategies on steering angles and vehicle motion are studied. Specifically, the influence of driver control weights ,  and , and that of driver prediction horizon *N*1 will be illustrated through studying three cases of parameters, i.e. Cases A, B and C as shown in Table I.



(a)



(b)



(c)

Fig. 3. Simulation results of vehicle lateral displacement and driver and AFS steering angles. Subplots (a), (b) and (c) respectively show results obtained under Cases A, B and C (as per Table I). Dashed: driver adopting Nash steering control strategy; dotted: driver adopting Stackelberg steering control strategy.

Prior to the numerical simulation, stability of the closed-loop driver-AFS Nash scheme (Fig. 1) and that of the Stackelberg scheme (Fig. 2) were analyzed using the procedure suggested by Rawlings and Mayne [30]. Under the Nash scheme, AFS control law (7) and driver Nash strategy (15) were injected into vehicle model (4) to generate the following closed-loop system:

 (20)

where

,

, and

.

The stability of (20) is therefore determined by the eigenvalues of . It was found that for all the three cases concerned, i.e. Cases A, B and C, the eigenvalues of  were located inside the unit circle of the complex plane. This indicates that (20) is asymptotically stable in all these three cases. By following a similar procedure, the close-loop system for the Stackelberg scheme can be obtained, which has the same structure as (20). Analysis of the eigenvalues indicates the asymptotical stability of the closed-loop Stackelberg scheme in the three cases.

The existence and uniqueness of the Nash equilibrium point represented by the driver Nash steering control strategy (15) are analyzed based on the term  shown in (14d). It was found that for all the three cases concerned, the absolute value of this term is always smaller than 1 over the prescribed range of *w*1, i.e. . This indicates that the driver Nash strategy (15) exists as a unique solution for all the three cases.

TABLE III

AFS Lane Change Target Path Profiles Used for Experiment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *L*1 (m) | *L*2 (m) | *L* (m) | *D* (m) |
| Path 1 | 160 | 30 | 400 | 2.2 |
| Path 2 | 210 | 25 | 2.5 |
| Path 3 | 137 | 13 | 1.9 |
| Path 4 | 175 | 25 | - 2.2 |
| Path 5 | 152 | 18 | - 2.1 |
| Path 6 | 185 | 35 | - 2.4 |

TABLE II

Test Subject Details

|  |  |  |  |
| --- | --- | --- | --- |
| Subject Number | Gender | Age | Driving Experience (years) |
| 1 | Female | 26 | 3 |
| 2 | Male | 34 | 12 |
| 3 | Male | 27 | 4 |
| 4 | Male | 25 | 5 |
| 5 | Female | 26 | 1 |
| 6 | Male | 24 | 4 |

Following the analysis made above, Cases A, B and C were simulated. Fig. 3 (a) illustrates the results from Case A. The left subplot shows the driver’s straight-line target path (square marker), the AFS controller’s lane-change target path (cross marker), and the resultant vehicle lateral displacement (dashed line for driver Nash strategy; dotted line for driver Stackelberg strategy). The right subplot shows driver and AFS steering angles (dashed line for Nash; dotted for Stackelberg). In Case A, driver Nash strategy (15) results in larger steering angles than driver Stackelberg strategy (19). Meanwhile, the adoption of Nash strategy allows the driver to follow his or her straight-line target path more closely than that of Stackelberg strategy.

Fig. 3 (b) shows the results from Case B, where the values of driver control weights are doubled from Case A. This allows the vehicle to follow the driver’s straight-line target path more closely compared to that shown in Case A. However, there is no significant change in driver and AFS angles from Case A.

Fig. 3 (c) shows the results from Case C. The driver control weights are kept the same as those used in Case A, while the driver prediction horizon is reduced to 150 steps. In this case, the driver’s path-following performance declines significantly compared to that in Case A. It also can be seen that under the Stackelberg scheme, driver and AFS steering angles decrease significantly compared to that shown in Case A.

Based on the observations above, a conclusive remark can be drawn: increase of driver control weights ,  and , or increase of driver prediction horizon *N*1 results in changes in driver and AFS steering angle profiles, meanwhile allows the driver to be able to follow his or her target path more closely.

# Driving Simulator Experiment

In this section, the experiment using a driving simulator for measuring human drivers’ steering behavior in response to AFS control law (7) is described.

## Driving Simulator and Test Drivers

A fixed-base driving simulator developed in the Department of Engineering at the University of Cambridge using Matlab Real-Time Workshop was used. A driver can apply control to the simulator vehicle model via steering wheel, accelerator pedal and brake pedal. A variety of studies on driver behavior has been conducted using this driving simulator, e.g. [36]. Six test drivers with an average driving experience of 4.8 years were employed. They were marked respectively as Test Drivers 1 to 6. Their details are provided in Table II.

## A FS Lane Change Maneuver for Driver Steering Behavior Measurement

In the experiment, an AFS system adopting control law (7) was programed into the driving simulator. The simulator was set to run the linear time-invariant “bicycle” vehicle model (4) with a sample time *T*s = 0.01 s at a constant longitudinal speed *U* = 20 m/s. The vehicle parameters used for the experiment are the same as those used for the numerical simulation in Section III. The AFS was then set to perform lane change maneuvers automatically at some point in the experiment. The lane change maneuvers mimic the collision avoidance actions that an AFS system may take following detecting an obstacle in front of the vehicle. Each test driver will be asked to apply steering action to keep the vehicle moving straight ahead. As a result, conflict over steering control will occur between driver and AFS.

### AFS Target Paths

Six AFS lane change target paths of different profiles were used for the experiment, as shown in Table III. Three of them involve leftward lane change and the others involve rightward lane change. The dimensions were designed carefully to ensure that these lane change maneuvers could represent realistic crash avoidance path while maintain the vehicle model (4) to operate in a regime that would give linear behavior of a real vehicle.

### AFS Controller Parameters

Parameters of the AFS controller used in the experiment are the same as those used in the simulation study in Section III: the prediction horizon *N*2 is set 200 steps (2.0 s) and the control weights are set ,  and . The reason for setting *N*2 = 200 was explained in Section III.  was set 0 because in this experiment the key task of the AFS controller is to control the vehicle to follow its lane change target paths, suggesting that the role of yaw angle error weight  is less significant than that of lateral deviation error weight  or the integral weight .

The values of  and  were determined in a trial prior to the experiment. It is noteworthy that the trial was conducted using the same driving simulator but only for determining  and  for the AFS controller. In contrast, the experiment was carried out mainly for measuring driver steering behavior in response to AFS automatic lane change, where the AFS controller was set to use the determined  and, and to follow the target paths presented in Table III. In other words, the trial is of a completely separate activity from the experiment. We would like to kindly remind our readers not to confuse the experiment with the trial. Details of the trial can be found in [37]. Below a summary of it is provided.



Fig. 4. Scheme of experiment track including AFS lane change maneuvers and disturbance events.



Fig. 5. Experiment results of vehicle lateral displacement and driver and AFS steering angles for Test Drivers 2, 3, 5 and 6 along AFS lane change Path 4. Solid: Test Driver 2, dashed: Test Driver 3, dotted: Test Driver 5, dash-dot: Test Driver 6.



Fig. 6. Experiment results of vehicle lateral displacement and driver and AFS steering angles for Test Drivers 1 and 4 along AFS lane change Path 4. Solid: Test Driver 1, dotted: Test Driver 4.

In the trial, the following three considerations were taken for determining  and :

1. *AFS controller’s path-following performance*.  and  shall be adequately large to allow the AFS controller to control the vehicle to follow its lane change target paths desirably firmly.
2. *Drivers’ ability to override AFS control.*  and  shall be not too large so that human drivers have the ability to override the AFS control.
3. *Drivers’ steering wheel angle magnitude when overriding AFS lane change control.* When a human driver decides to override the AFS lane change control, the steering wheel angle the driver applies shall not exceed 150 degrees so as to avoid the driver getting his or her arms crossed over the steering wheel.

Eight test drivers were employed in the trail to determine  and  for the AFS controller. The driving scenario used in the trial involves conflicting steering control between driver and AFS. Specifically, the AFS controller was programmed to control the vehicle to follow a specific lane change target path defined by ISO 3888-1, while test drivers were requested to override the AFS control and keep the vehicle moving straight ahead.  and  were determined by seeking a compromise between the first two considerations listed above while taking the third one as a hard constraint. The outcome of the trial was  and . It was illustrated in [37] that the AFS control law (7) with such weights is capable of giving good path-following performance and neutralizing the effect of disturbing steering control produced by drivers.

## Experiment Track and Results

Following the trail for determining AFS control weights, the experiment was started. The six test drivers described in Table II were invited to drive the simulator vehicle and interact with the programmed AFS controller that carried out automatic lane change follow control law (7) to track its six target paths shown in Table III. Specifically, each test driver was asked to try his or her best to apply steering control to keep the vehicle travelling straight. As a result, conflict over steering control occurred between driver and AFS. Such conflict is likely to happen when there is an error in the AFS system, or when drivers thinks their target paths are morally more acceptable [35]. Test drivers’ steering angles in response to AFS control were measured by the driving simulator. It should be noted that in the experiment, test drivers were not informed the existence of AFS control but simply reminded that there might be some disturbance to their driving. They were neither given by the driving simulator any additional stimulus that may excite them to change their driving behavior. Once the AFS starts a lane change, the driver would only feel a change in steering wheel torque feedback, as what they would experience exactly in driving a real vehicle on road, due to the development of tire lateral force. In other words, test drivers were not intentionally guided to reach any equilibrium status. Instead, they were left by themselves to develop their own control in response to AFS intervention. Such a design bears the same nature as that employed by Braun *et al.* [9], who found that their human test subjects’ sensorimotor interaction in a rope-pulling game can be reproduced using the Nash equilibrium solution of a dynamic noncooperative game.

It can be imagined that if the AFS automatic lane change maneuvers were frequently imposed to a test driver over a short period, the driver may memorize the characteristics of the AFS control. As such memorization may change a driver’s steering behavior, it should be avoided. Therefore, several disturbance events were designed to impair a test driver’s memorization of the AFS automatic lane change control. The disturbance events are implemented by applying to the simulator vehicle (i) a step angle overlaid at its front wheels, (ii) an impulse angle at its front wheels, (iii) a step lateral force at its center of gravity, and (iv) an impulse lateral force at its center for gravity. These events were parameterized to excite vehicle lateral responses in different strength. The disturbance events and the AFS lane change maneuvers were triggered one by one in a random order along a straight experiment track, as shown in Fig. 4.

In the experiment, all the six test drivers’ steering angles and the vehicle’s lateral displacement were measured. Figs. 5 and 6 illustrate the measurements obtained under AFS lane change Path 4, as per Table III. The right subplot in Fig. 5 shows the steering angles applied by Test Drivers 2 (solid), 3 (dashed), 5 (dotted) and 6 (dash-dot), and corresponding AFS angles. The left subplot in Fig. 5 shows the profile of AFS lane change Path 4 (cross marker), that of driver straight-line target path (square marker), and the vehicle lateral displacements resulting from the interaction between test drivers and the AFS controller. Fig. 6 shows the results from Drivers 1 (solid) and 4 (dotted) in the same pattern. It can be seen that Drivers 2, 3, 5 and 6 (shown in Fig. 5) were inclined to apply larger steering angles to suppress AFS automatic lane change control, compared to Drivers 1 and 4 (show in Fig. 6). As a result, they did better at bringing the vehicle back to the driver’s straight line target paths, compared to Drivers 1 and 4. In contrast, Drivers 1 and 4 tended to expand less steering effort to oppose AFS control, which caused the vehicle to continuously deviate from the driver’s straight line target path. The difference in driver steering behavior between what was captured in Figs. 5 and 6 was not peculiar under AFS lane change Path 4, but universal among all the six AFS lane change paths. More details on test drivers’ steering angles were provided in [37]. With some prior knowledge of the driver model identification results (to be presented in Section V), the classification of the six test drivers’ steering behavior into two categories (according to Figs. 5 and 6) was found to agree with the distribution of the driver model identification errors. More details and discussions will be provided in Section V.



Fig. 7. Block diagram of identification procedure

It should be noted that the AFS system is a time-critical system. In real vehicle applications, the real-time performance of the AFS system was guaranteed through both software and hardware optimization, such as that described in [11]. For the experiment concerned in this paper, the AFS system was programmed into the driving simulator, and its real-time performance was guaranteed by using Matlab Real-Time Workshop. No violation of real-time execution was reported by the driving simulator throughout the experiment. This suggests that the AFS system used in this research maintained real-time performance successfully. This in turn allows test drivers to have sufficiently realistic feeling of driving in terms of interacting with the AFS system. It is also noted that all the six test drivers’ steering behavior were measured in a condition that the vehicle and AFS parameters were fixed. The authors acknowledged that a human driver’s steering behavior may change when these parameters change. However, since the authors’ aim is to investigate whether the proposed driver noncooperative-game steering control strategies is valid for representing measured driver steering behavior, the authors decided to fix vehicle and AFS parameters so as to minimize the complexity of the problem. Robustness of the proposed driver model to varying vehicle and AFS parameters is a topic of significance and will be examined in detail in future work.

# Driver Model Identification

To investigate the validity of the proposed noncooperative- game steering control strategies for representing real driver steering behavior, a model identification task was carried out. This involves looking for a set of driver model parameters that allows a closest fit of the driver noncooperative steering control strategies to measured driver steering behavior.

In this section only the driver Nash steering control strategy is fitted to measured driver steering behavior while the driver Stackelberg strategy is not. This is because it would be more practical to assume that a driver is able to somehow recognize the AFS controller’s steering action, as required in the Nash strategy, rather than that the driver knows the AFS control law, as required in the Stackelberg strategy. However, one shall bear in mind that once a driver is imparted the AFS control law, by means of repeated training, it may become more acceptable to assume that a driver can adopt the Stackelberg strategy.

## Identification Procedure

Identification of the driver Nash steering control strategy was conducted using the “indirect” method described by Ljung [38]. The procedure is depicted in Fig. 7.

The “Experiment” block in Fig. 7 describes what a test driver did in the experiment. The driver drove the simulator vehicle model (4) to follow his/her straight-line target path. Meanwhile, the AFS controller used AFS control law (7) to control the vehicle to follow its lane-change target path, e.g. Path 1 as per Table III.  is the driver’s steering angle measured at time step *n* under AFS lane-change Path 1, where *n* ranges from 1 to 2000. Here this 2000 is determined based on AFS lane change path length *L* = 400 m, vehicle longitudinal speed *U* = 20 m/s, and sample time *T*s = 0.01 s.

The “Simulation” block in Fig. 7 shows how the driver-AFS Nash steering control scheme was implemented in a numerical simulation environment. The driver Nash steering strategy (14), which is mathematically equivalent to (15), as explained early in the paper was set to work with vehicle model (4) and AFS control law (7).  denotes the steering angle resulting from driver Nash strategy (14). As explained in Section III,  is primarily determined by driver prediction horizon *N*1 and driver control weights ,  and . In order to improve the efficiency of identification, *N*1 was fixed at 200 steps, i.e. 2.0 s. This value was reported by both Keen [20] and Odhams [36] to give the best representation of human drivers’ preview manner. As a result,  is dominated by driver control weights ,  and .

A test driver’s identification error under AFS lane change Path 1 is denoted by , which can be calculated using (21):

 (21)

 is the sum of squared difference between measured and simulated driver steering angle over the entire length of AFS Path 1, i.e. 400 m. The test driver’s identification errors under AFS Paths 2 to 6 are calculated following the same way. On this basis, the test driver’s averaged identification error  over all the six AFS lane change target paths is calculated as:



Fig. 8. Test Driver 4’s simulation results from Nash strategy with identified weight set  (denoted using suffix “Sim”), and experiment results (denoted using suffix “Exp”) along AFS lane change Path 5.



Fig. 9. Test Driver 2’s simulation results from Nash strategy with identified weight set  (denoted using suffix “Sim”), and experiment results (denoted using suffix “Exp”) along AFS lane change Path 5.

 (22)

The Matlab function “fminsearch” was then used to search for a set of  that minimizes  iteratively, as shown in Fig. 7. Such a set allows a closest fit of the simulated driver steering angles to the experiment measurement.

The computation time spent for running the identification program shown in Fig. 7 using Matlab on a computer with a 2.4 GHz CPU and an 8 GB RAM was recorded. It took 508.56 seconds on average to identify a  set for a test driver. This involves around 410 steps of iteration, which suggests that it took around 1.24 seconds to simulate the driver-AFS Nash scheme shown in Fig. 1 under a simulation time span of 20 seconds.

## Identification Outcomes

The identification procedure described above implies that the driver adopts the Nash strategy to interact with the AFS controller. However, in the experiment, the drivers were never guided or implied to do so, as explained in the previous section. Identification outcomes were analyzed to examine how well the driver Nash strategy fits the measured driver steering angles. To this end, the identified set  was put into the driver Nash steering control strategy (14) to generate simulated driver steering angles. Fig. 8 illustrates Test Driver 4’s steering angles generated in this way (“Driver Sim” in the right subplot; dashed) under AFS lane change Path 5, and the resultant vehicle lateral displacement (“Veh Sim” in the left; dotted). In the same figure, the test driver’s steering angles measured from the experiment (“Driver Exp”; solid), and the measured vehicle lateral displacement (“Driver Exp”; solid) are overlaid. It can be found that the simulated driver steering angles are capable of representing the trend of the measured data, despite that the simulated driver steering angles are too smooth to capture the oscillation existing in measured data. Such oscillation may be due to some high-order neuromuscular-steering dynamics that were not modeled in the driver Nash strategy. Fig. 9 illustrates Driver 2’s identification outcome following the same pattern. It can be seen that the trend of the simulated driver steering angles agree with that of the measured data. However, the simulated angles are not good enough to capture the oscillation existing in the measured data. With regard to the identification outcomes of the other four test drivers, readers are advised to see [37].

 

Fig. 10. Driver-AFS classic steering control scheme Fig. 13. Identification errors of all six test drivers



Fig. 11. Test Driver 4’s simulation results from “classic” strategy with identified weight set  (denoted using suffix “Sim”), and experiment results (denoted using suffix “Exp”) along AFS lane change Path 5. This figure is presented as a comparator to Fig. 8.



Fig. 12. Test Driver 2’s simulation results from “classic” strategy with identified weight set  (denoted using suffix “Sim”) and experiment results (denoted using suffix “Exp”) along AFS lane change Path 5. This figure is presented as a comparator to Fig. 9.

Based on all the six test drivers’ identification outcomes, a conclusive remark is made: the driver Nash steering control strategy (14) is capable of representing the trend of each of all the six test drivers’ measured steering angles and that of the corresponding vehicle lateral response. However, (14) is less capable of capturing the oscillation existing in the test drivers’ measured steering behavior.

## Comparative Study between Driver Nash and Classic Steering Control Strategies

The subsection above shows the outcomes of fitting driver Nash steering control strategy (14) to measured data. In order to better understand the superiority of (14) in representing human drivers’ steering behavior, a comparator to (14) was introduced. This comparator was named the driver’s classic steering control strategy. The term “classic” is used because the strategy was proposed first by MacAdam in 1981 [28], and was extensively validated in a number of separate studies, where the vehicle was controlled by a driver, without the presence of an automated system able to steer the vehicle independently of the driver.

Following the introduction of the driver classic strategy, an alternative driver-AFS steering control scheme which involves the driver adopting this classic strategy to interact with the AFS was established, as shown in Fig. 10. Such a scheme can be viewed as a modification to the driver-AFS Nash scheme (Fig. 1) by removing the driver’s accounting for the impact of the AFS control. Consequently, the driver’s prediction equation in the classic scheme can be written by reducing from (12) to:

 (23)

The driver classic steering strategy can be thereby expressed as:

 (24)

Compared to the driver Nash strategy (14), the classic strategy (24) excludes the driver’s compensation for AFS angle .

By replacing the driver Nash steering control strategy (14) in Fig. 7 with the classic strategy (24), an alternative weight set  can be identified for each of the six test drivers. The symbol “^” is used to distinguish from the weight set identified using the Nash strategy (14). The  set was then injected into (24) to generate simulated driver steering angles and vehicle lateral responses. Fig. 11 shows Test Driver 4’s simulated results using his  under AFS lane change Path 5, and his measured data from the experiment. Fig. 12 shows Test Driver 2’s simulated and measured data. It can be seen that Driver 4’s simulated steering angles given by the classic strategy (shown in Fig. 11) are very close to those given by the Nash strategy (Fig. 8). In other words, both strategies are able to represent the trend of measured driver steering behavior. But this is not the case for Test Driver 2. Driver 2’s simulated steering angles given by the classic strategy (Fig. 12) fails to capture the trend of his experimental measurement. This led to a continuing offset of 0.3 m between simulated and measured vehicle lateral displacement, as shown in Fig. 12. Such an offset did not appear when the Nash strategy was used (Fig. 9). This implies that the Nash strategy (14) is superior to the classic strategy (22) in representing Test Driver 2’s steering behavior.

The observations made above are reflected in Fig. 13 which shows all six test driver’s identification errors  obtained from the Nash strategy (14) (circle marker) and the classic strategy (24) (triangle marker). It can be seen that Driver 4’s  from the Nash strategy (14) is very close to that from the classic strategy (24). Hence the two steering control strategies gave nearly identical simulated results in Driver 4’s case. In contrast, Driver 2’s  from the Nash strategy (14) is much smaller than that from the classic strategy (24). Therefore, the Nash strategy is superior to the classic strategy in Driver 2’s case. It can be concluded from Fig. 13 that for Test Drivers 2, 3, 5 and 6, the Nash strategy (14) is superior to the classic one (24) in representing their measured steering behavior. However, for Test Drivers 1 and 4, the two strategies are of similar effect for representing measured driver steering behavior. This coincides with the note made early in Section III that Drivers 2, 3, 5 and 6 did better at bringing the vehicle back to the driver’s straight- line target paths, compared to Drivers 1 and 4.

# Discussion

In this section, a discussion on a possible improvement of vehicle AFS control based on gained knowledge of test drivers’ steering behavior was provided.

Early in Section IV, the weights of the AFS controller were determined as ,  and . As  penalizes vehicle lateral deviation from AFS target path and  penalizes the integral of the deviation, the AFS control law (7) has a nature of a PI controller that regulates vehicle lateral deviation. Vehicle path-following controllers following similar principle have been reported and demonstrated in many publications. Here the authors would like to point out that the introduction of  in AFS path-following control, that is, the introduction of integral control would cause the driver to have to put in excessive effort to override the AFS control. In the experiment results shown in this paper, this was reflected as that the test drivers had to continuously increase their steering angles to suppress AFS control, as illustrated in Figs. 5 and 6.

Regarding the development of automated driving technology, human drivers’ reaction to the intervention of automation must always be considered. For liability and safety reasons, drivers are expected to retain their control of the vehicle [39]. This means that drivers should be offered the chance to override the automated driving technology. In view of this, the test drivers’ continuous increase of steering angles shown in Figs. 5 and 6 is not at all desirable. Specifically, if the AFS controller for some reason decides to keep on conducting lane change, the driver will have to keep on increasing steering angles to override the AFS. When the steering wheel reaches its limit position, the driver will have no more chance to override AFS. To this end, some improvement shall be made to the AFS control weights so as to avoid the driver’s continuous increase of steering angle.

A preliminary improvement is reducing the value of  to zero, that is, to fully remove the penalization of the integral of vehicle lateral deviation. On this basis, a further improvement involving raising  from  to  can be made. As a result, the improved AFS weight set finally contains  and . The effectiveness of this improved weight set shall be carefully inspected. To this end, three steps of inspection were carried out one after another.

First, it is useful to inspect the path-following performance of the AFS control law (7) with the improved weight set when the driver is assumed to give null steering control. In view of this, the path-following performance of the AFS control using the improved weight set is compared to that using the original weight set in simulation. Vehicle parameters and simulation time step were all set the same as those used in Section III. Simulation results in terms of AFS steering angles and vehicle lateral displacement are presented in Fig. 14. It can be seen that there is subtle difference between the results from the improved weight set (dotted) and those from the original weight set (solid). It can be seen that both weight sets give decent vehicle path-following performance with driver steering in absence.



Fig. 14. Performance of AFS path-following control when driver’s steering control is in absence. Solid: AFS control with original weight set: ,  and ; dotted: AFS control with improved weight set:  and .



Fig. 15. Test Driver 4’s simulation results from Nash strategy (denoted using suffix “Sim”) and experiment results (denoted using suffix “Exp”) along AFS lane change Path 5, with improved AFS control weight set  and  being in use.



Fig. 16. Test Driver 2’s simulation results from Nash strategy (denoted using suffix “Sim”) and experiment results (denoted using suffix “Exp”) along AFS lane change Path 5, with improved AFS control weight set  and  being in use.

The next step is to inspect whether a driver gains the ability to override the improved AFS control, and whether the driver can fulfill this without the need for increasing steering angle in a continuous manner. To this end, the AFS control law (7) with the improved weight set was programed into the driving simulator, and all the six test drivers shown in Table II were invited to drive the simulator vehicle and interact with the AFS controller again. Test drivers’ steering angles and vehicle dynamic responses were recorded. Figs. 15 and 16 respectively illustrate Test Driver 4’s and Test Driver 2’s experiment results obtained under AFS lane change Path 5 (solid). Simulated results generated using driver Nash strategy (14) with identified set  were then overlaid (dotted). It can be seen that in response to the AFS control law with the improved weight set, test drivers were still capable of overriding the AFS control and keeping the vehicle travelling along the driver’s straight-line target path. Particularly, test drivers’ steering angles at first increased and then maintained a steady-state value of approximately 90 degrees (see solid “Driver Exp” in Figs. 15 and 16). This is distinct from the test drivers’ steering reaction to the original AFS weight set shown in Figs. 8 and 9, where the drivers continuously increased their steering angles. Moreover, the driver steering angles simulated by the Nash strategy (14) were capable of capturing the steady-state steering angles measured in the experiment (dotted “Driver Sim”). This demonstrates the validity of the driver Nash strategy (14) for representing human drivers’ steering interaction with AFS.

In a nutshell, the outcomes of the three steps’ inspection demonstrate that the improved AFS control weight set can help a driver to override AFS control in an easier way, without sacrificing the performance of AFS path-following control.

# Conclusion

This paper concerned theoretical modeling and experimental validation of noncooperative-game-theoretic driver steering control strategies. Two strategies, namely the Nash strategy and the Stackelberg strategy were derived based on the equilibrium solutions of a noncooperative game, where a human driver and an Active Front Steering (AFS) controller hold different target paths and they compete for vehicle path-following control. The driver Nash steering control strategy was fitted to measured human driver steering behavior, and the driver control weights involved in the Nash strategy were identified using a system identification procedure. The conclusions below were reached:

1. *The Nash and Stackelberg steering control strategies represent different driver steering behaviors.* The Nash strategy suggests that the driver determines his or her own steering angle by taking the AFS angle into account while the Stackelberg strategy suggests that the driver goes one step further by taking the AFS control law into account.
2. *The Nash strategy in theory results in more aggressive driver steering control.* The steering angles given by the Nash strategy was found larger than those given by the Stackelberg strategy in a simulation study.
3. *The Nash strategy is capable of representing measured driver steering angles and vehicle lateral responses.* Following adopting the driver control weights identified from measured human driver behavior, the Nash strategy was found capable of representing the trend of measured driver steering angles and vehicle lateral displacement for all the six test drivers used in the experiment.
4. *The Nash strategy is superior to the classic strategy in representing human driver steering interaction with AFS.* Compared to the driver Nash steering control strategy, the driver classic strategy was found less capable of capturing the trend of some test drivers’ measured steering behavior and resultant vehicle lateral responses. This suggests that the driver classic strategy in some case may be less suitable for modeling human drivers’ steering behavior in response to vehicle automation intervention; the Nash strategy could be a more suitable choice.

In the next step, neuromuscular dynamics of drivers’ arms will be modeled and incorporated into the driver Nash steering strategy for analysis. It is expected that the inclusion of driver arm neuromuscular dynamics can help the driver model capture the oscillation found in measured human driver steering angles.

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