

Supplementary Information.

**Field-dependent specific heat of the canonical underdoped cuprate superconductor
 $\text{YBa}_2\text{Cu}_4\text{O}_8$**

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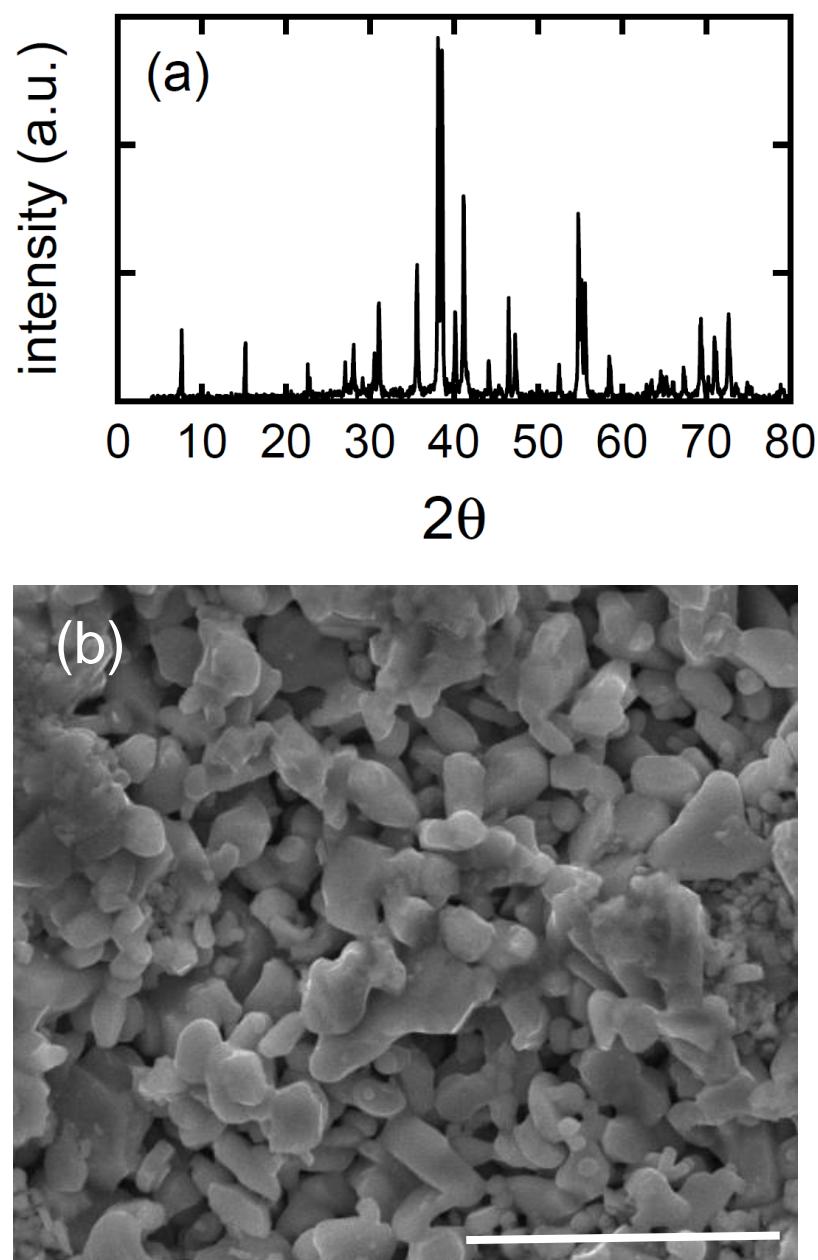


Figure S1 (a) typical x-ray diffraction pattern for the $\text{YBa}_2\text{Cu}_4\text{O}_8$ polycrystalline samples showing essentially single-phase composition (Co-K α radiation). (b) SEM micrograph of the surface of a sample pellet. The white bar indicates 10 μm scale.

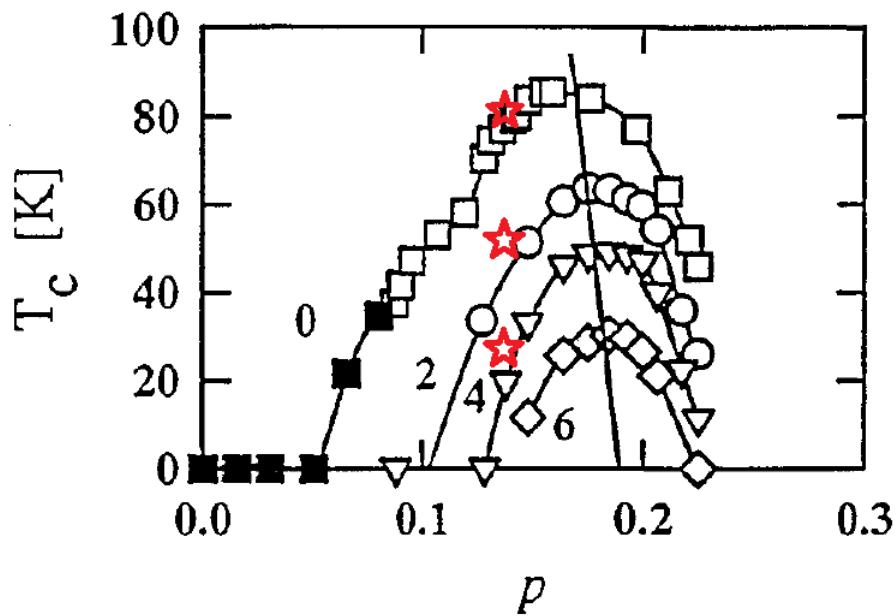


Figure S2. Black symbols: T_c versus hole concentration for $\text{Y}_{0.8}\text{Ca}_{0.2}\text{Ba}_2\text{Cu}_3\text{O}_{7-\delta}$ for 0, 2, 4 and 6% planar Zn concentration¹. Red stars: T_c values for $\text{YBa}_2\text{Cu}_4\text{O}_8$ for 0, 2 and 4 % planar Zn concentration (this work).

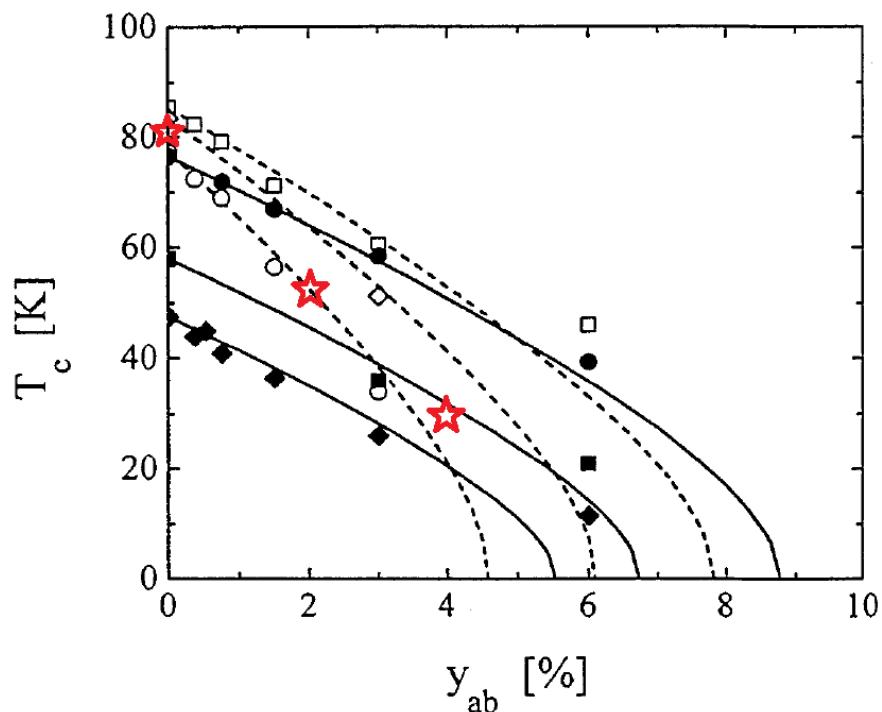


Figure S3. Black symbols: T_c versus planar Zn concentration¹ for $\text{Y}_{0.8}\text{Ca}_{0.2}\text{Ba}_2\text{Cu}_3\text{O}_{7-\delta}$. Solid curves and symbols: overdoped. Dashed curves and open symbols: underdoped. Red stars: T_c versus planar Zn concentration for $\text{YBa}_2\text{Cu}_4\text{O}_8$ (this work).

Comparison of ^{89}Y Knight shift with electronic entropy

The ^{89}Y Knight shift, $^{89}\text{K}_s$, for $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ is reported by Alloul *et al.*² To convert to entropy units we must first convert to the spin susceptibility, χ_s . This was done by Alloul by comparing the temperature dependence of $^{89}\text{K}_s$ with that of the bulk magnetic susceptibility, χ_m . We use this relationship. The comparison of the T -dependent components is robust, however, each of $^{89}\text{K}_s$ and χ_m has an additive constant that must be identified if a comparison of absolute values is to be undertaken. For $^{89}\text{K}_s$ this additive constant is the chemical shift, $^{89}\sigma$, which is evaluated by Alloul as ranging from -200 ppm for $x = 0.41$ to -370 ppm for $x = 1$. In contrast Takigawa *et al.*³ evaluate $^{89}\sigma$ as -152 ± 10 ppm independent of x . Our analysis below is consistent with this value, independent of x . This is the value that we also used⁴ for $^{89}\sigma$ in $\text{YBa}_2\text{Cu}_4\text{O}_8$. For χ_m , the additive constant, χ_0 , comprises a diamagnetic term and a van Vleck term ($\chi_0 = \chi_{\text{dia}} + \chi_{\text{vv}}$) estimated by Alloul as $\chi_{\text{dia}} = -2.65 \times 10^{-7}$ emu/g and $\chi_{\text{vv}} = 1.95 \times 10^{-7}$ emu/g, i.e. $\chi_0 = -0.7 \times 10^{-7}$ emu/g.

In view of the uncertainty of these T -independent parts we simply convert $^{89}\text{K}_s$ to χ_m using Alloul's Fig. 4 and multiply by a_w , as plotted in Fig. S4, for comparison with S/T for $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$. The $a_w\chi_m$ values for each x value were then displaced vertically (by an additive constant) to coincide with the entropy data. The first thing to note is that the T -variation of the susceptibility and entropy data for each specific value of x are in excellent agreement. Now if we take the value of this additive constant and work back to the chemical shift σ_0 we obtain values that vary quite narrowly between -130 and -150 ppm, very consistent with Takigawa³. This baseline uncertainty of ± 10 ppm corresponds to ± 0.04 mJ/g.at.K² in Fig. S4 and is rather small.

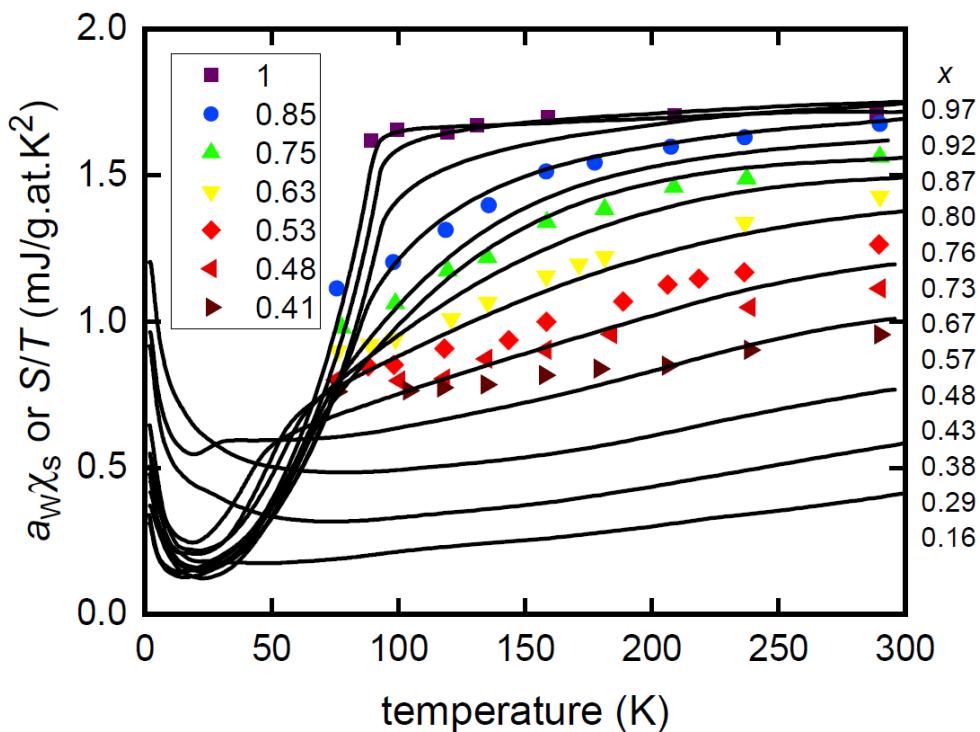


Figure S4. Data points: spin susceptibility for $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ from the ^{89}Y Knight shift (reported by Alloul²) multiplied by the Wilson ratio in order to express in entropy units. x values are annotated. Solid curves: electronic entropy divided by T as reported by Loram *et al.*^{5,6}

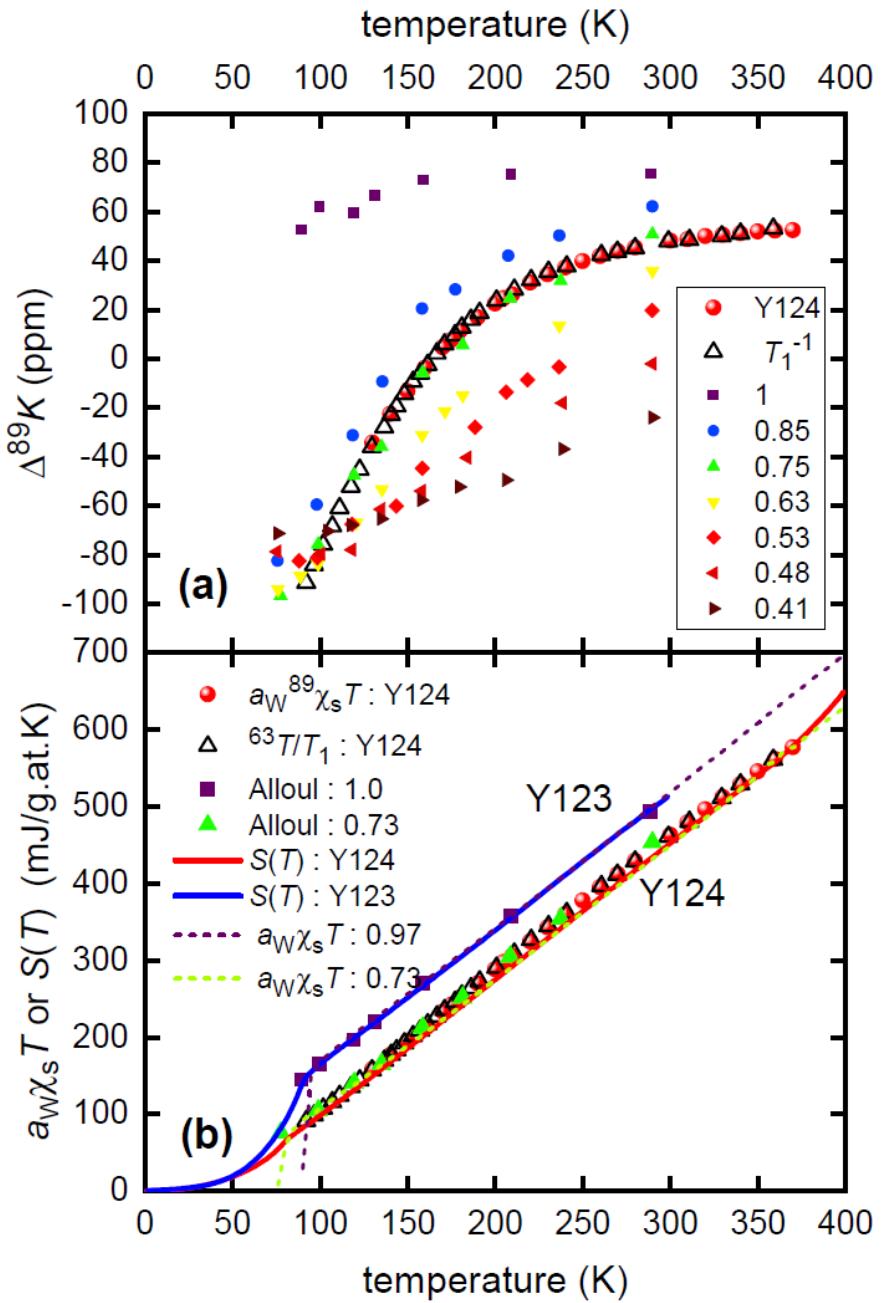


Figure S5. A reproduction of Fig. 7 but with bulk susceptibility data, $a_W \chi_s T$, (green dashed curve) overlaid on top of the entropy data, $S(T)$, (red solid curve). In Fig. 7 the susceptibility data was hidden by the entropy data. Here it is evident that the two agree closely over the entire temperature range.

References:

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3. Takigawa, M., Hults, W. L. & Smith, J. L. *Phys. Rev. Lett.* **71**, 2650-2653 (1993).

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