SPACE-CONSTRAINED AUTONOMOUS REVERSING OF ARTICULATED VEHICLES

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This dissertation is submitted to the University of Cambridge for the degree of

Doctor of Philosophy
Declaration

I hereby declare that this dissertation is the result of my own work and includes nothing, which is the outcome of work done in collaboration except as declared in the preface and Acknowledgements and specified in the text. It is not substantially the same as any work that I have submitted, or, is being concurrently submitted for a degree or diploma or other qualification at the University of Cambridge or any other University or similar institution except as declared in the preface and Acknowledgements and specified in the text. I further state that no substantial part of my dissertation has already been submitted, or, is being concurrently submitted for any such degree, diploma or other qualification at the University of Cambridge or any other University or similar institution except as declared in the preface and Acknowledgements and specified in the text. It does not exceed the prescribed word limit for the Engineering Degree Committee.

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Xuanzuo Liu
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Abstract

This dissertation presents several modern control methods for autonomous reversing of long combination vehicles (LCVs). These approaches not only significantly improve the performance of previous autonomous reversing systems, but also address a major gap in the reversing control literature for LCVs. The methods were validated by implementing them at full-scale on experimental articulated vehicles owned by the ‘Cambridge Vehicle Dynamics Consortium’ (CVDC). Experimental results were in very good agreement with simulation results.

Previous path following control methods for autonomous reversing of LCVs have focussed on minimising the tracking error between the rear-end of the combination and a desired path, irrespective of the motion of the rest of the vehicle. A significant disadvantage of these strategies is that the other parts of the vehicle, particularly the tractor unit, can experience large excursions from the reference path, thus making their implementation infeasible for manoeuvres in spatially-limited areas, such as warehouse yards and normal roads. The ‘Minimum Swept Path Control’ (MSPC) method was devised to reduce this problem by relaxing the requirement for very accurate path following, while minimising the maximum excursion of the vehicle. This strategy weights both the path-following error at the rear end of the vehicle and the swept path of the front end of the vehicle. MSPC enables the swept path to be reduced by about 50%, compared with path following control, which gives this method more realistic applications.

The ‘Lane-bounded Reversing Control’ (LBRC) method requires a vehicle to satisfy the reversing objectives while constraining the motion to be within a specified ‘lane’. The LBRC controller is ‘intelligent’, which means it can pursue an optimum route without tracking a desired path generated by a path planner and can proactively avoid future potential clashes. Hence, this controller enables the autonomous reversing system to perform a precise, minimum-cost and collision-free manoeuvre to a specified terminal position by planning ahead
and making optimal decisions. The controller performance was evaluated in numerous realistic scenarios, both simulated and in field tests at full-scale.

The analysis of LBRC provides a solid foundation for the development of more advanced control methods. Adaptive Lane-bounded Reversing Control (ALBRC) and Adaptive Bi-directional Control (ABC) systems were designed to improve the LBRC method.

The tuning of the LBRC controller was based on empirical experience and there are many weights to be tuned in the controller configuration. To offset this drawback, the ALBRC algorithm was developed by attaching ‘virtual bumpers’ to the vehicle system states, and allowing the controller weights to adapt to lane boundaries and obstacles. The ALBRC method simplifies the original tuning process significantly.

The ALBRC controller performs well in most cases. However, a solution is not guaranteed if the preview and control horizons are not tuned properly or a vehicle reverses from an arbitrary position. Hence, the ABC algorithm incorporates a so-called ‘cusp technology’, which allows a vehicle to move forward and backward to realign its position and orientation between attempts at the reversing manoeuvre. In this case, the preview and control horizons do not need to change in different scenarios. The ABC method can significantly reduce computational time compared with using a long preview horizon.
Supervisor: Professor David Cebon

Advisor: Doctor David Cole

Examiners: Professor Jan Maciejowski
           Professor Timothy Gordon
To Baoqi, Binxia, and Cheng
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Table of Contents

DECLARATION.................................................................................................................................. II

ABSTRACT ........................................................................................................................................ III

ACKNOWLEDGEMENTS ............................................................................................................. VII

PUBLICATIONS ............................................................................................................................ XIII

LIST OF FIGURES ......................................................................................................................... XIV

LIST OF TABLES ............................................................................................................................ XX

NOMENCLATURE ......................................................................................................................... XXI

CHAPTER 1 INTRODUCTION ......................................................................................................... 1

1.1 BACKGROUND ............................................................................................................................... 1
1.2 LITERATURE REVIEW .................................................................................................................. 3
1.3 ‘PRE-PLANNED’ REVERSING METHODS .................................................................................. 4
   1.3.1 Linear controllers .................................................................................................................. 4
   1.3.2 Non-linear controllers ......................................................................................................... 7
1.4 ‘ACTIVE-PLANNED’ REVERSING METHODS ........................................................................ 9
   1.4.1 Advanced control methods ................................................................................................ 9
   1.4.2 Machine learning methods ............................................................................................... 10
1.5 ANALOGOUS RESEARCH ....................................................................................................... 12
   1.5.1 Forward driving .................................................................................................................. 12
1.6 CONCLUSIONS .......................................................................................................................... 13
1.7 SCOPE OF THE THESIS ............................................................................................................. 14
   1.7.1 Research questions ............................................................................................................. 14
   1.7.2 Outline of Thesis .............................................................................................................. 15
1.8 TABLES ......................................................................................................................................... 17
1.9 FIGURES ...................................................................................................................................... 18

CHAPTER 2 VEHICLE MODELLING .......................................................................................... 19

2.1 VEHICLE DYNAMICS MODEL ................................................................................................. 19
2.2 LINEARISATION ........................................................................................................................ 22
2.3 VEHICLE PARAMETERS .......................................................................................................... 25
2.4 VERIFICATION OF LINEAR MODEL ....................................................................................... 25
2.5 TYRE MODEL ............................................................................................................................. 26
2.6 CONCLUSIONS .......................................................................................................................... 27
2.7 TABLES ......................................................................................................................................... 29
5.3.2 Limits of manoeuvrability ................................................................. 129
5.3.3 Sharp 90-degree bend with tight boundaries .................................. 130
5.3.4 Misaligned narrow gate ................................................................. 130
5.3.5 Mountain road .................................................................................. 131
5.4 B-DOUBLE CASE .................................................................................. 131
5.4.1 Lane change manoeuvre ................................................................. 132
5.4.2 Misaligned narrow gate ................................................................. 132
5.4.3 Sharp 90-degree bend ....................................................................... 133
5.5 B-TRIPLE CASE ................................................................................... 133
5.5.1 Lane change manoeuvre ................................................................. 133
5.6 STABILITY AND FEASIBILITY OF THE GENERAL N-TRAILER VEHICLE ....................................................... 134
5.7 CONCLUSIONS .................................................................................. 137
5.8 TABLES ............................................................................................... 139
5.9 FIGURES ............................................................................................. 143

CHAPTER 6 IMPLEMENTATION OF LANE-BOUNDED REVERSING CONTROL (LBRC)
................................................................................................................................................ 159

6.1 VEHICLE TESTING FRAMEWORK ........................................................ 159
6.1.1 System architecture ........................................................................ 159
6.1.2 Estimations of the articulation angle rates ....................................... 160
6.1.3 Estimations of the lateral and yaw velocities of the tractor ............... 161
6.2 LBRC SUBSYSTEM .............................................................................. 162
6.2.1 Solution scheme ............................................................................... 162
6.2.2 Reduced computation scheme ......................................................... 163
6.3 ASYNCHRONOUS SIMULATION MODEL ........................................... 164
6.3.1 Simulation results ........................................................................... 165
6.4 FIELD TESTING RESULTS ................................................................. 166
6.4.1 Field tests using the tractor-semitrailer vehicle ............................... 166
6.4.2 Field tests using the B-double vehicle ............................................ 168
6.5 CONCLUSIONS .................................................................................. 169
6.6 TABLES ............................................................................................... 172
6.7 FIGURES ............................................................................................. 174

CHAPTER 7 ADVANCED CONTROL STRATEGIES - EXPLORATORY RESEARCH ....186

7.1 ADAPTIVE LANE-BOUNDED REVERSING CONTROL (ALBRC) METHOD ...................................................... 186
7.1.1 Simulation analysis .......................................................................... 189
7.2 ADAPTIVE BI-DIRECTIONAL CONTROL (ABC) .............................................. 191
CHAPTER 8 CONCLUSIONS AND FUTURE WORK ................................................................. 207

8.1 SUMMARY OF MAIN CONCLUSIONS ........................................................................... 207
  8.1.1 Literature review (Chapter 1) ................................................................................ 207
  8.1.2 Vehicle modelling (Chapter 2) .............................................................................. 208
  8.1.3 Minimum Swept-Path Control (Chapters 3 and 4) ................................................. 208
  8.1.4 Lane-Bounded Reversing Control (Chapters 5 and 6) ........................................... 209
  8.1.5 Advanced control methods – exploratory research (Chapter 7) .......................... 211

8.2 RECOMMENDATIONS FOR FUTURE WORK ............................................................. 212
  8.2.1 Improvements in current research ........................................................................... 212
  8.2.2 Dynamic obstacles .................................................................................................. 212
  8.2.3 Implementation of the developed advanced control methods ................................ 212
  8.2.4 Reinforcement learning methods ............................................................................ 212
  8.2.5 Integration with vehicle perception systems ............................................................ 213

CHAPTER 9 REFERENCES ....................................................................................................... 214

APPENDIX A: SOLVER FOR A QUADRATIC PROGRAMMING PROBLEM ......................... 228
Publications

The following peer-reviewed publications have resulted from this work [1]–[4]:


- X. Liu and D. Cebon, “Minimum Swept Path Control for Autonomous Reversing of Long Combination Vehicles,” in the 15th International Symposium on Heavy Vehicle Transportation Technology, HVTT15, Rotterdam, the Netherlands, 2018


Three more academic papers about the ‘Lane-Bounded Reversing Control’ (LBRC) and advanced control methods are in preparation to be submitted for publication in peer-reviewed journals and conferences.
List of figures

FIGURE 1.1 TYPES OF COUPLING: (A) A-TYPE COUPLING; (B) B-TYPE COUPLING ........................................ 18
FIGURE 1.2 ‘ON-AXLE HITCHING’ AND ‘OFF-AXLE HITCHING’ .......................................................... 18
FIGURE 2.1 VEHICLE DIAGRAM SHOWING DIMENSIONS OF THE VEHICLE UNITS ................................ 33
FIGURE 2.2 VEHICLE VELOCITY AND FORCE ANALYSIS ...................................................................... 33
FIGURE 2.3 1° STEER ANGLE DISTURBANCE ADDED BETWEEN 4.995 S AND 5.005 S ...................... 34
FIGURE 2.4 1° STEER ANGLE DISTURBANCE ADDED FROM 5 S ONWARDS ..................................... 34
FIGURE 2.5 UNIT IMPULSE RESPONSE AT THE SPEED OF -1 M/S BETWEEN 4.9 S AND 5.2 S FOR THE 1° STEER ANGLE DISTURBANCE .............................................................................................. 35
FIGURE 2.6 STEP RESPONSE AT THE SPEED OF 1 M/S BETWEEN 0 S AND 60 S FOR THE 1° STEER ANGLE DISTURBANCE .............................................................................................................. 35
FIGURE 2.7 20° STEER ANGLE DISTURBANCE ADDED BETWEEN 4.995 S AND 5.005 S ................ 36
FIGURE 2.8 20° STEER ANGLE DISTURBANCE ADDED FROM 5 S ONWARDS .............................. 36
FIGURE 2.9 IMPULSE RESPONSE AT THE SPEED OF -1 M/S BETWEEN 4.9 S AND 5.2 S FOR THE 20° STEER ANGLE DISTURBANCE .............................................................................................................. 37
FIGURE 2.10 STEP RESPONSE AT THE SPEED OF 1 M/S BETWEEN 0 S AND 60 S FOR THE 20° STEER ANGLE DISTURBANCE .............................................................................................................. 37
FIGURE 2.11 STEADY-STATE VALUES FOR DIFFERENT STEER ANGLE DISTURBANCES .................. 38
FIGURE 2.12 EIGENVALUES OF THE CLOSED-LOOP SYSTEM MATRICES OF THE B-DOUBLE COMBINATION .................................................................................................................................................. 39
FIGURE 2.13 A NORMALISED NON-LINEAR TYRE MODEL ................................................................. 40
FIGURE 2.14 COMPARISON BETWEEN THE NON-LINEAR AND LINEARISED TYRE MODELS IN THE CASE OF A STRAIGHT LINE ............................................................................................................ 40
FIGURE 3.1 COMPARISON BETWEEN PATH FOLLOWING CONTROL AND MINIMUM SWEPT PATH CONTROL ......................................................................................................................................................... 59
FIGURE 3.2 RELATIONSHIP BETWEEN THE FRONT AND REAR AXLE TRACKING ERRORS ........... 60
FIGURE 3.3 MSPC STRATEGY. INTERMEDIATE TRAILER UNITS BETWEEN THE TRACTOR AND THE REAR SEMITRAILER ARE REPRESENTED BY ARTICULATION ANGLE, $\gamma$ ........................................... 60
FIGURE 3.4 CALCULATION OF THE EQUILIBRIUM ARTICULATION ANGLE OF THE REAR TRAILER ...... 61
FIGURE 3.5 THE LANE CHANGE MANOEUVRE ...................................................................................... 61
FIGURE 3.6 A SAMPLE TIME HISTORY OF LATERAL OFFSETS DURING THE LANE CHANGE MANOEUVRE, WHEN $W_f = 1, W_r = 1$ AND $L_{pd} = 7$ M ................................................................................................................. 62
FIGURE 3.7 TUNING RESULTS IN THE CASE OF THE TRACTOR-SEMITRAILER VEHICLE PERFORMING THE LANE CHANGE MANOEUVRE ................................................................................................................. 63
Figure 3.8 Relationship between the optimised preview distance and the total length of vehicles in the case of the tractor-semi trailer vehicle performing the lane change manoeuvre, when given $W_{fa} = 1$ and $W_{ra} = 1$................................. 64

Figure 3.9 The roundabout manoeuvre........................................................................................................... 64

Figure 3.10 Tuning results for the tractor-semi trailer vehicle performing the roundabout manoeuvre ........................................................................................................... 65

Figure 3.11 Tuning results in the case of the B-double performing the lane change manoeuvre ........................................................................................................................................ 65

Figure 3.12 Tuning results in the case of the B-double performing the roundabout manoeuvre ........................................................................................................................................ 66

Figure 3.13 Contours of the lateral offsets for the tractor-semi trailer vehicle .................. 68

Figure 3.14 Conflict diagram of $|y_{fa}|_{max}$ versus $|y_{ra}|_{max}$ for fixed $W_{fa}$ for the tractor- semi trailer vehicle .................................................................................................................. 69

Figure 3.15 Weight selection criteria for the tractor-semi trailer vehicle............................... 69

Figure 3.16 Optimal working weights plotted on the contours of the lateral offsets for the tractor-semi trailer vehicle ........................................................................................................... 70

Figure 3.17 Conflict diagram of $|y_{fa}|_{max}$ versus $|y_{ra}|_{max}$ for fixed $W_{fa}$ for the B-double vehicle .................................................................................................................. 71

Figure 3.18 Weight selection criteria for the B-double ............................................................... 71

Figure 3.19 Simulation results of the tractor-semi trailer vehicle performing the lane change manoeuvre ........................................................................................................... 72

Figure 3.20 Simulation results of the tractor-semi trailer vehicle performing the roundabout manoeuvre ........................................................................................................... 73

Figure 3.21 Simulation results of the B-double performing the lane change manoeuvre ........................................................................................................... 74

Figure 3.22 Simulation results of the B-double performing the roundabout manoeuvre ........................................................................................................... 75

Figure 4.1 Test vehicles .................................................................................................................. 90

Figure 4.2 Schematic of the on-board equipment layout ................................................................ 91

Figure 4.3 Architecture of the autonomous reversing system .................................................. 92

Figure 4.4 RT3022 dual antenna inertial and GPS navigation system ..................................... 93

Figure 4.5 The OXTS base station and a radio aerial on the rooftop of the Mondeo on the testing field .................................................................................................................. 94

Figure 4.6 The experimental configuration for calibrating the articulation angle sensor .................................................................................................................. 94
Figure 4.7 The string potentiometer mounted on the underside of the tractor and attached to the steering radius arm to measure the actual steer angle.

Figure 4.8 Hardware installation in the tractor cabin.

Figure 4.9 Components of the steering robot and the cable connections.

Figure 4.10 The Autobox with the DS4302 CAN interface board.

Figure 4.11 The front axle calibration.

Figure 4.12 The relationship between the measured left and right front wheel angles and the hand wheel angle.

Figure 4.13 Illustration of the Ackermann steering geometry.

Figure 4.14 The effective ‘single-track’ steer angle calculated from the measured left and right front wheel angles respectively.

Figure 4.15 The relationship between the hand wheel angle, the effective ‘single-track-average’ steer angle, and the sensor voltage of the string potentiometer.

Figure 4.16 The articulation angle sensor calibration method. The laser was glued to the sensor arm perpendicularly that passed through the centre of the kingpin.

Figure 4.17 The relationship between the articulation angle and the sensor voltage for the articulation angle sensor.

Figure 4.18 The sinusoidal signal tracking test for the AB Dynamics SR30 steering robot.

Figure 4.19 The local coordinate frame set up for the testing. A closed shape testing was carried out in the local grid using the INS.

Figure 4.20 Schematic of the MSPC subsystem.

Figure 4.21 Schematic of the steer angle tracking subsystem.

Figure 4.22 Measured steering angle with and without the PID controller.

Figure 4.23 Comparison between the simulated and experimental results during the lane change manoeuvre for the tractor-semi trailer vehicle.

Figure 4.24 Comparison between the simulated and experimental results during the roundabout manoeuvre for the tractor-semi trailer vehicle.

Figure 4.25 Comparison between the simulated and experimental results during the lane change manoeuvre for the B-double.

Figure 4.26 Comparison between the simulated and experimental results during the roundabout manoeuvre for the B-double.

Figure 5.1 Notation.

Figure 5.2 Transformation from the global to local coordinates. As an example, the jth trailer is shown for illustration.

Figure 5.3 Control strategy flow chart.
FIGURE 5.4 SIMULATION RESULTS OF THE TRACTOR-SEMITRAILER VEHICLE DURING THE LANE CHANGE MANOEUVRE................................................................................................................................. 147
FIGURE 5.5 LIMITS OF MANOEUVRABILITY......................................................................................................................... 149
FIGURE 5.6 SIMULATION RESULTS OF THE TRACTOR-SEMITRAILER VEHICLE PERFORMING A 90-DEGREE TURN MANOEUVRE........................................................................................................ 150
FIGURE 5.7 SIMULATION RESULTS OF THE TRACTOR-SEMITRAILER VEHICLE PERFORMING A MIXED MANOEUVRE........................................................................................................................................ 151
FIGURE 5.8 SIMULATION RESULTS OF THE TRACTOR-SEMITRAILER VEHICLE PERFORMING A U-SHAPED MANOEUVRE WITH THE PARKED ‘LIMOUSINE’ (‘OBSTACLE’)... ................................................................. 152
FIGURE 5.9 SIMULATION RESULTS OF THE TRACTOR-SEMITRAILER VEHICLE PERFORMING A U-SHAPED MANOEUVRE WITH THE PARKED ‘LORRY’ (‘OBSTACLE’).......................................................................................... 153
FIGURE 5.10 SIMULATION RESULTS OF THE B-DOUBLE VEHICLE PERFORMING VARIOUS MANOEUVRES ................................................................................................................................................ 154
FIGURE 5.11 SIMULATION RESULTS OF THE B-TRIPLE VEHICLE PERFORMING A LANE CHANGE MANOEUVRE......................................................................................................................................... 156
FIGURE 5.12 MOTION PATHS FOR A TRACTOR ARTICULATED WITH 7 B-LINK TRAILERS PERFORMING A LANE CHANGE MANOEUVRE............................................................................................................ 157
FIGURE 5.13 THE RELATIONSHIP BETWEEN THE RELATIVE COMPUTATIONAL EFFICIENCY INDEX AND THE NUMBER OF TRAILERS, FOR FIXED SAMPLE TIME, PREVIEW HORIZON, AND CONTROL HORIZON................................................................................................................................. 157
FIGURE 5.14 THE RELATIONSHIP BETWEEN THE RELATIVE COMPUTATIONAL EFFICIENCY INDEX AND THE PREVIEW HORIZON FOR A TRACTOR ARTICULATED WITH 7 B-LINK TRAILERS AND FIXED SAMPLE TIME AND CONTROL HORIZON................................................................................................................................. 158
FIGURE 5.15 THE EMPIRICAL RELATIONSHIP BETWEEN THE AVERAGE ACTUAL COMPUTATIONAL TIME PER SIMULATION STEP FOR A FEASIBLE SOLUTION OF EACH COMBINATION VEHICLE AND THE NUMBER OF TRAILERS, WITH THE PREVIEW DISTANCE SET TO 1.7 TIMES THE VEHICLE LENGTH ......................................................................................................................................... 158
FIGURE 6.1 ARCHITECTURE OF THE LBRC SYSTEM .......................................................................................................................... 174
FIGURE 6.2 UNFILTERED AND FILTERED ESTIMATED ARTICULATION ANGLE RATES .............................................................. 175
FIGURE 6.3 SCHEMATIC OF THE LBRC SUBSYSTEM .......................................................................................................................... 176
FIGURE 6.4 SCHEMATIC OF THE ASYNCHRONOUS SIMULATION MODEL ............................................................................................ 176

FIGURE 6.8 THE EXPERIMENTAL CONFIGURATION FOR THE CONSTRAINED LANE CHANGE MANOEUVRE ON THE RUNWAY AT BOURN AIRFIELD .................................................................................................................................................... 178

FIGURE 6.9 THE EXPERIMENTAL CONFIGURATION FOR THE CONSTRAINED NARROW GATE MANOEUVRE ON THE RUNWAY AT BOURN AIRFIELD .................................................................................................................................................... 179

FIGURE 6.10 THE EXPERIMENTAL CONFIGURATION FOR THE CONSTRAINED 90-DEGREE SHARP TURN MANOEUVRE ON THE RUNWAY AT BOURN AIRFIELD .................................................................................................................................................... 179

FIGURE 6.11 COMPARISON BETWEEN THE EXPERIMENTAL AND SIMULATED RESULTS OF THE TRACTOR-SEMITRAILER VEHICLE PERFORMING THE CONSTRAINED LANE CHANGE MANOEUVRE .................................................................................................................................................... 180

FIGURE 6.12 COMPARISON BETWEEN THE EXPERIMENTAL AND SIMULATED RESULTS OF THE TRACTOR-SEMITRAILER VEHICLE PERFORMING THE CONSTRAINED NARROW GATE MANOEUVRE .................................................................................................................................................... 181

FIGURE 6.13 COMPARISON BETWEEN THE EXPERIMENTAL AND SIMULATED RESULTS OF THE TRACTOR-SEMITRAILER VEHICLE PERFORMING THE CONSTRAINED 90-DEGREE SHARP TURN MANOEUVRE .................................................................................................................................................... 182

FIGURE 6.14 COMPARISON BETWEEN THE EXPERIMENTAL AND SIMULATED RESULTS OF THE B-DOUBLE PERFORMING THE CONSTRAINED LANE CHANGE MANOEUVRE .................................................................................................................................................... 183

FIGURE 6.15 COMPARISON BETWEEN THE EXPERIMENTAL AND SIMULATED RESULTS OF THE B-DOUBLE PERFORMING THE CONSTRAINED NARROW GATE MANOEUVRE .................................................................................................................................................... 184

FIGURE 6.16 COMPARISON BETWEEN THE EXPERIMENTAL AND SIMULATED RESULTS OF THE B-DOUBLE PERFORMING THE CONSTRAINED 90-DEGREE SHARP TURN MANOEUVRE .................................................................................................................................................... 185

FIGURE 7.1 THE NON-REVERSING AREAS USING THE LBRC METHOD .................................................................................................................................................... 198

FIGURE 7.2 THE NOTATIONS USED FOR A VEHICLE UNIT .................................................................................................................................................... 198

FIGURE 7.3 ILLUSTRATION FOR SITUATIONS WHEN A CORNER POINT MOVES CLOSER TO A BOUNDARY THAN THE OTHER THREE .................................................................................................................................................... 199

FIGURE 7.4 SIMULATION RESULTS OF THE TRACTOR-SEMITRAILER VEHICLE PERFORMING THE LANE CHANGE MANOEUVRE .................................................................................................................................................... 200

FIGURE 7.5 SIMULATION RESULTS OF THE TRACTOR-SEMITRAILER VEHICLE PERFORMING A MIXED MANOEUVRE .................................................................................................................................................... 201

FIGURE 7.6 SIMULATION RESULTS OF THE TRACTOR-SEMITRAILER VEHICLE PERFORMING A 90-DEGREE TURN MANOEUVRE .................................................................................................................................................... 202

FIGURE 7.7 SIMULATION RESULTS OF THE TRACTOR-SEMITRAILER VEHICLE WHILE MOVING ON A MOUNTAIN ROAD WITH A PARKED ‘LIMOUSINE’ .................................................................................................................................................... 203
FIGURE 7.8 SIMULATION RESULTS OF THE TRACTOR-SEMITRAILER MOVING ON A MOUNTAIN ROAD WITH A PARKED ‘LORRY’ ..................................................................................................................... 204

FIGURE 7.9 A SCENARIO SIMULATING THAT THE TRACTOR-SEMITRAILER VEHICLE STARTS OFF FROM A POSITION VERY CLOSE TO THE UPPER BOUNDARY ............................................................................. 205

FIGURE 7.10 A SCENARIO SIMULATING THAT THE TRACTOR-SEMITRAILER VEHICLE STARTS OFF FROM A POSITION VERY CLOSE TO THE LOADING BAY .......................................................................................... 206

This dissertation has 105 figures.
List of tables

TABLE 1.1 TYPICAL LCVs .................................................................................................................... 17
TABLE 2.1 TRACTOR UNIT PARAMETERS .......................................................................................... 29
TABLE 2.2 B-LINK TRAILER PARAMETERS ...................................................................................... 30
TABLE 2.3 SEMITRAILER PARAMETERS .......................................................................................... 31
TABLE 2.4 DETAILS OF THE SIMULATED VEHICLE COMBINATIONS ............................................. 31
TABLE 2.5 NON-LINEAR TYRE MODEL PARAMETERS ...................................................................... 32
TABLE 2.6 CALCULATED LINEAR AXLE LATERAL STIFFNESS COEFFICIENTS IN THE CASE OF A
    STRAIGHT LINE ............................................................................................................................. 32
TABLE 3.1 THE MAXIMUM NATURAL FREQUENCY AND SAMPLE TIME OF THE DISCRETE-TIME MODELS
    AT THE SPEED OF -1 M/S ................................................................................................................ 58
TABLE 3.2 THE OPTIMISED PREVIEW DISTANCES FOR THE TRACTOR-SEMITRAILER AND B-DOUBLE
    VEHICLES ...................................................................................................................................... 58
TABLE 3.3 THE OPTIMAL WEIGHTS AND GAINS FOR THE TRACTOR-SEMITRAILER AND B-DOUBLE
    VEHICLES ...................................................................................................................................... 58
TABLE 4.1 GAINS FOR THE PID COMPENSATOR ............................................................................... 89
TABLE 5.1 SIMULATION PARAMETERS FOR THE TRACTOR-SEMITRAILER VEHICLE .................. 139
TABLE 5.2 SIMULATION PARAMETERS FOR THE B-DOUBLE VEHICLE .......................................... 140
TABLE 5.3 RECOMMENDED DEFAULT VALUES FOR THE SIMULATION PARAMETERS IN THE CASE OF
    TRACTOR-SEMITRAILER AND B-DOUBLE VEHICLES ............................................................... 141
TABLE 5.4 SIMULATION PARAMETERS FOR THE B-TRIPLE VEHICLE ........................................... 142
TABLE 6.1 THE SAMPLING FREQUENCY OF THE REAL-TIME APPLICATIONS ................................. 172
TABLE 6.2 PARAMETERS FOR THE SECOND ORDER BUTTERWORTH-TYPE IIR FILTER ................. 172
TABLE 6.3 THE CONTROLLER PARAMETERS OF THE REAL-TIME APPLICATIONS .......................... 173
TABLE 7.1 SIMULATION PARAMETERS FOR THE TRACTOR-SEMITRAILER VEHICLE .................. 197
This dissertation contains 19 tables.
## Nomenclature

<table>
<thead>
<tr>
<th>Abbreviations</th>
<th>Full Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>Adaptive Bi-directional Control</td>
</tr>
<tr>
<td>ADALINE</td>
<td>Adaptive Linear Neurons</td>
</tr>
<tr>
<td>ADAS</td>
<td>Advanced Driver Assistance System</td>
</tr>
<tr>
<td>ALBRC</td>
<td>Adaptive Lane-Bounded Reversing Control</td>
</tr>
<tr>
<td>CAN</td>
<td>Controller Area Network</td>
</tr>
<tr>
<td>C.o.M.</td>
<td>Centre of Mass</td>
</tr>
<tr>
<td>CVDC</td>
<td>Cambridge Vehicle Dynamics Consortium</td>
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<tr>
<td>DDPG</td>
<td>Deep Deterministic Policy Gradient</td>
</tr>
<tr>
<td>DQN</td>
<td>Deep Q Network</td>
</tr>
<tr>
<td>EKF</td>
<td>Extended Kalman Filter</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite Impulse Response</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>HGV</td>
<td>Heavy Goods Vehicle</td>
</tr>
<tr>
<td>IIR</td>
<td>Infinite Impulse Response</td>
</tr>
<tr>
<td>IMU</td>
<td>Inertial Measurement Unit</td>
</tr>
<tr>
<td>INS</td>
<td>Inertial Navigation System</td>
</tr>
<tr>
<td>IT</td>
<td>Information Technology</td>
</tr>
<tr>
<td>LBRC</td>
<td>Lane-Bounded Reversing Control</td>
</tr>
<tr>
<td>LCV</td>
<td>Long Combination Vehicle</td>
</tr>
<tr>
<td>LHV</td>
<td>Longer and Heavier Goods Vehicle</td>
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<td>LKC</td>
<td>Lane Keeping Control</td>
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<td>LQR</td>
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<td>MLD</td>
<td>Mixed Logical Dynamics</td>
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<td>MPC</td>
<td>Model Predictive Control</td>
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<td>MSPC</td>
<td>Minimum Swept Path Control</td>
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<tr>
<td>NMPC</td>
<td>Non-linear Model Predictive Control</td>
</tr>
<tr>
<td>ODE</td>
<td>Ordinary Differential Equation</td>
</tr>
<tr>
<td>OxTS</td>
<td>Oxford Technical Solution</td>
</tr>
<tr>
<td>PCI</td>
<td>Peripheral Component Interconnect</td>
</tr>
<tr>
<td>PFC</td>
<td>Path Following Control</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
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<tr>
<td>PI</td>
<td>Proportional-Integral</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-Integral-Derivative</td>
</tr>
<tr>
<td>PPO</td>
<td>Proximal Policy Optimisation</td>
</tr>
<tr>
<td>ReLU</td>
<td>Rectified Linear Unit</td>
</tr>
<tr>
<td>RL</td>
<td>Reinforcement Learning</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Squared</td>
</tr>
<tr>
<td>RTK</td>
<td>Real Time Kinematic</td>
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<tr>
<td>SAC</td>
<td>Soft Actor Critic</td>
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<td>SFC</td>
<td>State Feedback Control</td>
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<tr>
<td>SL</td>
<td>Supervised Learning</td>
</tr>
<tr>
<td>TD</td>
<td>Temporal Difference</td>
</tr>
<tr>
<td>VFO</td>
<td>Vector-Field-Orientation</td>
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<tr>
<td>V.S.E.</td>
<td>Vehicle Systems Engineering</td>
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<table>
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<tr>
<th>Matrices</th>
<th>Full Descriptions</th>
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<td>$\mathbf{0}$</td>
<td>Zero matrix</td>
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<tr>
<td>$\mathbf{b}$</td>
<td>Constraint matrix at sample instant</td>
</tr>
<tr>
<td>$\mathbf{q}$</td>
<td>Weighting matrix at sample instant</td>
</tr>
<tr>
<td>$\mathbf{r}$</td>
<td>Weighting matrix at sample instant</td>
</tr>
<tr>
<td>$\mathbf{\rho}$</td>
<td>Transformation matrix of articulation angles</td>
</tr>
<tr>
<td>$\mathbf{A}$</td>
<td>System matrix in a linear state space model</td>
</tr>
<tr>
<td>$\mathbf{B}$</td>
<td>Input matrix in a linear state space model</td>
</tr>
<tr>
<td>$\mathbf{C}$</td>
<td>Output matrix in a linear state space model</td>
</tr>
<tr>
<td>$\mathbf{D}$</td>
<td>Transformation matrix of state vector at current sample instant</td>
</tr>
<tr>
<td>$\mathbf{E}$</td>
<td>Transformation matrix of input at sample instant</td>
</tr>
<tr>
<td>$\mathbf{F}$</td>
<td>Defined matrix</td>
</tr>
<tr>
<td>$\mathbf{G}$</td>
<td>Hessian matrix</td>
</tr>
<tr>
<td>$\mathbf{H}$</td>
<td>Hessian matrix</td>
</tr>
<tr>
<td>$\mathbf{I}$</td>
<td>Identity matrix</td>
</tr>
<tr>
<td>$\mathbf{K}$</td>
<td>Gain matrix or prediction matrix</td>
</tr>
<tr>
<td>$\mathbf{K}_{N_p}$</td>
<td>Terminal control gains</td>
</tr>
<tr>
<td>$\mathbf{L}$</td>
<td>Transformation matrix in element-wise inequalities</td>
</tr>
</tbody>
</table>
Symbols | Full Descriptions | Units
---|---|---
\(a\) | Distance from C.o.M. to front axle of tractor unit or front hitch point of trailer units | m
\(b\) | Distance from C.o.M. to rear axle of tractor unit or first rear axle of B-link trailers or second rear axle of semitrailers | m
\(c\) | Distance from rear hitch point or rear end to rear axle of tractor unit or first rear axle of B-link trailers or second rear axle of semitrailers | m
\(d\) | Distance | m
\(d\) | Constant distance vector | |
\(e\) | Axle spacing | m
\(e\) | Error vector | m
\(f_D\) | Ordinary differential equation function for dynamic vehicle model | |
\(f_L\) | Defined vector | |
\( f_t \) Generic tyre force function

\( f_0 \) Front overhang \( m \)

\( f_s \) Sample rate \( Hz \)

\( g \) Lagrangian dual function

\( k \) Sample instant

\( \text{inf} \) Infimum

\( l \) Vehicle length or moving distance \( m \)

\( l_{\text{eff}} \) Equivalent wheelbase \( m \)

\( ln \) Natural logarithm

\( m \) Vehicle mass \( kg \)

\( n \) Number of trailers

\( n_a \) Number of axles

\( p \) Boundary constraints

\( r \) Ratio of controller weights

\( ro \) Rear overhang \( m \)

\( u \) Longitudinal velocity or sensor drift \( m/s \)

\( v \) Lateral velocity or sensor voltage \( m/s \)

\( t_a \) Averaged actual elapsed running time \( s \)

\( t_s \) Simulation time \( s \)

\( w \) System natural frequency \( Hz \)

\( wd \) Vehicle width \( m \)

\( x \) Longitudinal position of vehicle unit in a fixed global coordinate system \( m \)

\( x_L \) Extended state vector of LBRC model

\( y \) Lateral position of vehicle unit in a fixed global coordinate system \( m \)

\( y \) Constraint vector of vehicle corner positions

\( y_{fa} \) Lateral offset of the front axle of the tractor to a reference path \( m \)

\( y_{lat} \) Swept path lateral width \( m \)

\( y_{ra} \) Lateral offset of the rear axle of the last trailer unit to a reference path \( m \)

\( z_L \) Defined state vector of LBRC model
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{p1}$</td>
<td>Tyre cornering coefficient</td>
<td>1/rad</td>
</tr>
<tr>
<td>$C_{p2}$</td>
<td>Tyre curvature coefficient</td>
<td>1/Nrad</td>
</tr>
<tr>
<td>$C^n$</td>
<td>Continuous curvature to the $n^{th}$ derivative</td>
<td></td>
</tr>
<tr>
<td>$D_{jp}$</td>
<td>Preview distance for the $(j + 1)^{th}$ vehicle unit</td>
<td>m</td>
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<tr>
<td>$F_y$</td>
<td>Generic lateral tyre force</td>
<td>N</td>
</tr>
<tr>
<td>$F^n_y$</td>
<td>Non-linear lateral tyre force</td>
<td>N</td>
</tr>
<tr>
<td>$F_f$</td>
<td>Lateral tyre force of the front axle</td>
<td>N</td>
</tr>
<tr>
<td>$F_r$</td>
<td>Lateral tyre force of the rear axle</td>
<td>N</td>
</tr>
<tr>
<td>$F_z$</td>
<td>Vertical load at one tyre</td>
<td>N</td>
</tr>
<tr>
<td>$l_c$</td>
<td>Relative computational efficiency index</td>
<td></td>
</tr>
<tr>
<td>$l_z$</td>
<td>Yaw moment of inertia of vehicle unit about the C.o.M.</td>
<td>kg·m$^2$</td>
</tr>
<tr>
<td>$J$</td>
<td>Cost function</td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>Controller gain</td>
<td></td>
</tr>
<tr>
<td>$K_y$</td>
<td>Linear lateral tyre stiffness coefficient or local gradient of a non-linear tyre model</td>
<td>N/rad</td>
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<tr>
<td>$L_{pd}$</td>
<td>Preview distance</td>
<td>m</td>
</tr>
<tr>
<td>$N$</td>
<td>Preview or control horizon</td>
<td></td>
</tr>
<tr>
<td>$N_l$</td>
<td>Number of the vehicle moving towards lower boundary in a forward direction</td>
<td></td>
</tr>
<tr>
<td>$N_u$</td>
<td>Number of the vehicle moving towards upper boundary in a forward direction</td>
<td></td>
</tr>
<tr>
<td>$O$</td>
<td>Objective or cost function</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of curvature</td>
<td>m</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Sample time</td>
<td>s</td>
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<tr>
<td>$V$</td>
<td>Control Lyapunov function</td>
<td></td>
</tr>
<tr>
<td>$V_f$</td>
<td>Terminal cost</td>
<td></td>
</tr>
<tr>
<td>$W$</td>
<td>Controller weights</td>
<td></td>
</tr>
<tr>
<td>$X$</td>
<td>Longitudinal coupling force of the rear hitch point</td>
<td>N</td>
</tr>
<tr>
<td>$Y$</td>
<td>Lateral coupling force of the rear hitch point</td>
<td>N</td>
</tr>
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<td>$Y$</td>
<td>Transformed constraint vector of vehicle corner positions</td>
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<tr>
<td>$Z_D$</td>
<td>State vector of dynamic vehicle model</td>
<td></td>
</tr>
<tr>
<td>$Z_M$</td>
<td>Extended state vector of MSPC model</td>
<td></td>
</tr>
</tbody>
</table>
\( \alpha \)  
Generic side slip angle \( \text{rad} \)

\( \alpha_f \)  
Side slip angle of the front axle \( \text{rad} \)

\( \alpha_r \)  
Side slip angle of the rear axle \( \text{rad} \)

\( \delta \)  
Tractor unit front axle steer angle \( \text{rad} \)

\( \varepsilon \)  
Allowable minimum distance \( \text{m} \)

\( \zeta \)  
Damping ratio

\( \theta \)  
Heading angle of vehicle unit in a fixed global coordinate system \( \text{rad} \)

\( \lambda \)  
Eigenvalue of system matrix

\( \lambda \)  
Lagrange multiplier vector

\( \mu \)  
Friction Coefficient

\( \Gamma \)  
Articulation angle \( \text{rad} \)

\( \Gamma \)  
Constraint vector of articulation angles

\( \Delta \)  
Small deviation from equilibrium position

\( \theta \)  
Front wheel angle \( \text{Rad} \)

\( \Omega \)  
Yaw velocity \( \text{Rad/s} \)

**Superscripts**

\( . \)  
First derivative with respect to time

\( .. \)  
Second derivative with respect to time

\( \sim \)  
Estimated value

\( \sim \)  
Surrogate symbol

\( \textit{cl} \)  
Corresponding to closed-loop system

\( \textit{g} \)  
Global coordinate frame

\( \textit{l} \)  
Local coordinate frame

\( \textit{n} \)  
Corresponding to non-linearity

\( \textit{MSPC} \)  
Corresponding to MSPC method

\( \textit{PFC} \)  
Corresponding to PFC method

\( \tau \)  
Transpose

\( * \)  
Optimum value
<table>
<thead>
<tr>
<th>Subscripts</th>
<th>Full Descriptions</th>
</tr>
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<tr>
<td>0</td>
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<td>a</td>
<td>Corresponding to articulation angle or actual steer angle</td>
</tr>
<tr>
<td>cl</td>
<td>Corresponding to control horizon</td>
</tr>
<tr>
<td>cl2</td>
<td>Corresponding to closed-loop system</td>
</tr>
<tr>
<td>d</td>
<td>Demanded value</td>
</tr>
<tr>
<td>e</td>
<td>Equilibrium value</td>
</tr>
<tr>
<td>eff</td>
<td>Equivalent wheelbase calculation</td>
</tr>
<tr>
<td>f</td>
<td>Corresponding to front wheel/tyre</td>
</tr>
<tr>
<td>fa</td>
<td>Corresponding to front axle</td>
</tr>
<tr>
<td>h</td>
<td>Corresponding to hand wheel</td>
</tr>
<tr>
<td>i</td>
<td>Corresponding to (i^{th}) trailer unit or inner front wheel</td>
</tr>
<tr>
<td>imu</td>
<td>Corresponding to IMU measurements</td>
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<tr>
<td>j1</td>
<td>Corresponding to front-left corner point of ((j+1)^{th}) vehicle unit</td>
</tr>
<tr>
<td>j1l</td>
<td>Corresponding to lower constraint of front-left corner point of ((j+1)^{th}) vehicle unit</td>
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<td>Corresponding to upper constraint of front-left corner point of ((j+1)^{th}) vehicle unit</td>
</tr>
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<td>Corresponding to front-right corner point of ((j+1)^{th}) vehicle unit</td>
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<td>Corresponding to upper constraint of front-right corner point of ((j+1)^{th}) vehicle unit</td>
</tr>
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<td>j3</td>
<td>Corresponding to rear-right corner point of ((j+1)^{th}) vehicle unit</td>
</tr>
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<td>j3l</td>
<td>Corresponding to lower constraint of rear-right corner point of ((j+1)^{th}) vehicle unit</td>
</tr>
<tr>
<td>j3u</td>
<td>Corresponding to upper constraint of rear-right corner point of ((j+1)^{th}) vehicle unit</td>
</tr>
<tr>
<td>j4</td>
<td>Corresponding to rear-left corner point of ((j+1)^{th}) vehicle unit</td>
</tr>
<tr>
<td>j4l</td>
<td>Corresponding to lower constraint of rear-left corner point of ((j+1)^{th}) vehicle unit</td>
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<td>Symbol</td>
<td>Description</td>
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<td>--------</td>
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<td>Corresponding to upper constraint of rear-left corner point of $(j + 1)^{th}$ vehicle unit</td>
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<td>$jc$</td>
<td>Corresponding to C.o.M. of $(j + 1)^{th}$ vehicle unit</td>
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<td>Corresponding to lower constraint of C.o.M. of $(j + 1)^{th}$ vehicle unit</td>
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<tr>
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<td>Corresponding to upper constraint of C.o.M. of $(j + 1)^{th}$ vehicle unit</td>
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<td>Corresponding to front end of $(j + 1)^{th}$ vehicle unit</td>
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<tr>
<td>$jfl$</td>
<td>Corresponding to lower constraint of front end of $(j + 1)^{th}$ vehicle unit</td>
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<td>$jfu$</td>
<td>Corresponding to upper constraint of front end of $(j + 1)^{th}$ vehicle unit</td>
</tr>
<tr>
<td>$jr$</td>
<td>Corresponding to rear end of $(j + 1)^{th}$ vehicle unit</td>
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<td>$jrl$</td>
<td>Corresponding to lower constraint of rear end of $(j + 1)^{th}$ vehicle unit</td>
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<td>Corresponding to upper constraint of rear end of $(j + 1)^{th}$ vehicle unit</td>
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<tr>
<td>$l$</td>
<td>Corresponding to lower constraints</td>
</tr>
<tr>
<td>$p$</td>
<td>Corresponding to preview horizon</td>
</tr>
<tr>
<td>$r$</td>
<td>Corresponding to rear wheel/tyre or right wheel</td>
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<td>Corresponding to rear axle</td>
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<td>$s$</td>
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<td>$th$</td>
<td>Threshold</td>
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<td>Corresponding to lateral tyre</td>
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<td>$ya$</td>
<td>Corresponding to front axle offset</td>
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<td>Corresponding to rear axle offset</td>
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<tr>
<td>$CT$</td>
<td>Continuous time model</td>
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<td>$D$</td>
<td>Derivative gain</td>
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<td>Corresponding to dynamic vehicle system</td>
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<td>$DT$</td>
<td>Discrete time model</td>
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<td>Integral gain</td>
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$L$  
Corresponding to LBRC control system

$M$  
Corresponding to MSPC control system

$H$  
Corresponding to rear hitch point

$p$  
Proportional gain

$pp$  
Preview point

$\delta$  
Corresponding to steer angle

$\gamma$  
Corresponding to articulation angle

**Transfer Functions**  
**Full Descriptions**

$H(z)$  
Discrete-time parallel PID compensator or discrete-time Butterworth filter
Chapter 1 Introduction

1.1 Background

With the outbreak of the coronavirus pandemic and the subsequent global blanket lockdowns, the logistics industry has witnessed the worst contraction since 2000 [5]–[7], but it is still the backbone of the global economy. Despite sluggish global economic growth over the past decade, it is anticipated that road freight volumes will continue to increase in the foreseeable future, compared with other transport modes like rail, air or maritime, especially on the European, American, and Australian continents [8], [9]. Even though there was a moderate decline in the road freight portion of EU-28 inland goods transport from 89% in 2009 to 74.9% in 2015, the share was still nearly triple that of the combination of rail and inland waterways (18.4% and 6.7% respectively in 2015) [9], [10]. The Logistics Report 2019 [11] pointed out that in the U.K., the road freight industry experienced positive year-on-year growth and still remained optimistic for the future. In spite of Brexit, the mass of goods lifted increased by 4% and 19% for the domestic and international road freight activity respectively from April 2018 to March 2019 [12], which was partly driven by the increasing consumer demand for online retail [11], [13].

In recent years, there has been increasing pressure to reduce carbon footprints and fuel consumption in the road freight industry. Long Combination Vehicles (LCVs) with multiple articulation points provide an important route to improving fuel efficiency [14], [15]. Woodrooffe et al. [16] concluded that a full-laden heavier LCV could produce lower emission and fuel consumption rates per tonne of payload, offering 18% to 32% decrease in fuel consumption in comparison with conventional articulated vehicles [16]. This was confirmed by Knight et al. [17] who performed extensive study of the applicability of LCVs in the U.K.

LCVs are combinations of Heavy Goods Vehicles (HGVs), such as Tractor-semitrailer, B-double, B-triple, A-double and Nordic Combination shown in Table 1.1 [18]–[20]. The tractor-semitrailer combination is a tractor connected to a semitrailer unit by an articulation joint. It is the most common type of HGV used worldwide. As seen in Figure 1.1 (a) and (b), the ‘B’ and ‘A’ in the configuration descriptions refer to two different types of couplings: a B-type coupling connects two trailers through a fifth wheel; whereas an A-type coupling uses a dolly with a drawbar [21]. The ‘Nordic combination’, has a rigid truck towing a dolly and semitrailer. The B-double and Nordic combinations have two articulation points, and the A-double and B-triple combinations have three articulation points.
In this study, a tractor-semitrailer combination was implemented in the initial simulations to test the proposed reversing controllers. The control methods were then applied to a tractor with multiple trailers.

Reversing LCVs into small parking bays or interchanging trailers are common tasks for drivers. However, unlike forward driving, reversing of LCVs is inherently unstable, with non-holonomic characteristics and jack-knifing effects. It is beyond the ability of most lorry drivers. Moving obstacles or stationary obstacles such as other lorries, cars and workers in the vicinity of the LCVs exacerbate the difficulty. Experienced professional drivers capable of performing those tasks are rare and highly sought after, as described in [22], [23]. A study, “Longer and/or Longer and Heavier Goods Vehicles (LHVs) - a Study of the Likely Effects if Permitted in the UK: Final Report”, [17] showed that the skill required to reverse LCVs is one of the barriers to their adoption in the U.K. Hence, it is important to design assistive controllers for drivers reversing multiply-articulated vehicle combinations.

Autonomous driving is an extensively researched field in the automotive and information technology (IT) industries. Companies from traditional vehicle manufacturers to IT start-ups have been racing to build self-driving cars and striving for a big share in the promising future consumer market. On the roadmap towards the era of driverless vehicles, there have been numerous Advanced Driver Assistance Systems (ADASs) developed and employed for cars [24]–[31], including adaptive cruise control systems, automated emergency braking systems, vehicle collision avoidance systems, lane keeping and path following control systems, and automated car parking systems, but very few technologies are commercially applied to LCVs, especially for the reversing assistance systems.

A few companies committed to manufacturing autonomous trucks and have launched prototypes to showcase modern technologies, but these models have not gone into production, because the technologies still need to mature and be fully tested [32], [33]. Some ADAS applications, e.g. automated parking systems for a single truck, address control system design and included vehicle testing [34]. However, the research projects concerned with ‘park assist’ for lorries lack experimental implementation on real full-scale vehicles [35]–[38].

The objective of previous work on reversing systems for LCVs, such as Path Following Control (PFC) developed by Rimmer [19], [39]–[41], was to reduce the path-tracking errors between the rearmost axle of the LCVs and a specified (pre-planned) path. The PFC approach can cause large excursions of the other vehicle units from the target path, especially the tractor unit, thus
increasing the overall swept path width. This means that path following controllers cannot be implemented in confined spaces such as narrow road lanes, because the maximum deviation of the vehicle from the nominal path can exceed the available lane width or manoeuvring space. Moreover, because the manoeuvre has to be pre-planned, it is not possible to avoid unanticipated obstacles using the PFC method.

Consequently, the aim of this project is to investigate more advanced control strategies to improve reversing controller performance by minimising the overall swept path and actively adapting to constrained boundaries. The methods devised in this study can enable autonomous reversing systems to perform accurate, minimum swept path, and collision-free manoeuvre to a pre-defined target terminal position. The LCVs can travel within confined spaces and automatically avoid obstacles. Hence, these control strategies are more versatile and usable for practical applications.

1.2 Literature review

There are two main control methods devised in this project: the ‘Minimum Swept Path Control’ (MSPC) method is aimed at minimising the overall swept path while a vehicle is tracking a pre-specified path in reverse. The ‘Lane-Bounded Reversing Control’ (LBRC) method allows a vehicle to actively avoid obstacles and look for an optimum route to a target position. The MSPC method is a new concept in the control literature, but the idea was inspired by path-tracking methods. Hence, this literature review is divided into two sections presented in the subsequent sections: ‘pre-planned’ reversing methods and ‘actively-planned’ reversing methods. A distinction between the two categories is if the controller follows a pre-defined target path by a path planner (‘pre-planned’ reversing methods) or whether it pursues an optimised path by itself (‘actively-planned’ reversing methods). The MSPC and classic path-tracking methods belong to the ‘pre-planned’ reversing methods, and the LBRC method and Machine Learning (ML) techniques are within the domain of the ‘actively-planned’ reversing methods. Optimal control theories and Reinforcement Learning (RL) methods have a large intersection and plenty of similarities, as both methods can be used to solve optimal sequential decision-making problems [42]–[44].

Two types of connections between vehicle units defined in the literature are illustrated in Figure 1.2: ‘on-axle hitching’ occurs when the articulation point is coincident with the axle centre; ‘off-axle hitching’ occurs when the articulation point is in front of or behind the axle centre.
1.3 ‘Pre-planned’ reversing methods

The pre-planned reversing methods can be categorised as either linear or non-linear controllers. In this study, the inversion of non-linear control laws [45] for path-following problems is included in linear controller development, and training a neural network to act as a non-linear path-tracking controller is considered under non-linear controller development.

1.3.1 Linear controllers

In 1985, Kageyama and Saito [46] investigated the use of optimal control theory to reverse articulated vehicles automatically, and showed that the rearmost unit reaches control limits very easily, making reversing difficult for a human driver. They also showed the controllability of reversing multiply articulated vehicles is geometrically determined for straight-line and lane change manoeuvres.

Laumond [47]–[49] mathematically proved the controllability for a general nonholonomic multibody system with on-axle hitching and illustrated the standard steps for the differential geometrical control theory based on a kinematic model. Altafini [50] extended Laumond’s proof by analysing a general n-trailer system with off-axle hitching. He showed that there are more singularities in the case of off-axle hitching than the on-axle hitching.

In 1991, Sampei et al. [51] developed a velocity-dependent linear state feedback controller for reversing a trailer-like mobile robot with on-axle hitching. The steer angle was adapted to the speed of the model trailer by choosing an appropriate time-scale function of the robot velocity. The control strategy was based on a kinematic model and the controller weights were hand-tuned.

Stergiopoulos et al. [52]–[55] presented a time-varying state feedback controller that used a kingpin sliding mechanism to control the position and velocity of the joint, so as to avoid jackknifing for multiple trailers. While Werner et al. [53] used a linear quadratic regulator (LQR) approach for a kinematic model of a tractor towing with an actively steered drawbar on a straight line, DeSantis [54] combined the state feedback and proportional-integral-derivative (PID) control methods to stabilise a tractor-trailer robot following straight lines and circular arcs. However, these methods were all based on simple models and some lacked experimental validation.

Rimmer [19], [39]–[41] developed a linear path-tracking controller based on the state feedback control law and a vehicle dynamics model, with a ‘look-ahead’ distance from the rear trailer.
The controller was tuned by the LQR method and tested on full-scale tractor-semitrailer and
B-double vehicles. The experimental results showed very small tracking errors. Rimmer
showed through simulation that the method could be used to reverse a B-train with up to 6 hitch
points.

A very similar method as Rimmer was adopted by Evestedt [56]. However, Evestedt [56] only
considered a kinematic vehicle model and articulation angles in the state space. The controller
was only tested on a small LEGO platform.

Pradalier and Usher [57]–[59] used a proportional-integral (PI) controller as an inner loop to
stabilise the hitch angle between a ride-on mower and a small trailer. A path-tracking outer
loop was constructed by applying the control law presented in [60], which defined a desired
hitch angle based on the compensations of heading errors, lateral errors and curvature errors.
It was found that the performance of the linear controller was independent of the system non-
linearity in terms of the path-tracking accuracy. However, there were many gains to be tuned,
and the tuning technique was complicated and relied on an empirical process. It could not be
applied to multiple trailers easily. The PI method was also used in [61] to develop a hitch angle
controller to stabilise reverse motion.

A set-point feedback control, which used the axle mid-point of the last trailer as a guidance
point, was proposed in [37] for vehicles with arbitrary numbers of trailers. The strategy was
based on a kinematic model with on-axle hitching, characterised by the use of the velocity
propagation via the mid-point of those rigid axles of passive trailers (which was known as the
guidance point). Through the calculated position error of the guidance point, a Vector-Field-
Orientation (VFO) stabiliser consisting of the orienting control of the angular velocity and the
pushing control of the longitudinal velocity could converge those errors to zero and then
manoeuvre the vehicles on the desired path. A significant disadvantage is that the control law
only applied to vehicles with on-axle hitching.

A hybrid control scheme was outlined in [35] to tackle the problem of jack-knifing found in
reversing truck-trailer combinations. To stabilise the system, a linear quadratic optimisation
method based on Jacobian linearisation was designed to calculate the first switching surface,
which determined if the controller enabled the vehicle to reverse stably. The second switching
surface determined forward driving instead of backward to realign the relative angles, when
jack-knifing and instability were imminent. Experiments were carried out on a miniaturized
vehicle rather than a full-scale LCV, so the practical results still remained uncertain.
Stahn et al. [62], [63] developed a laser scanner-based navigation system and a multi-phase tracking controller for a tractor-semitrailer vehicle to achieve collision-free manoeuvring to a destination. The researchers tested the system on a truck and a full trailer. Depending on the distance to the target, the motion controller decided whether to use either a single backward motion, ‘reversal points’ or ‘virtual tractor’. The obstacle avoidance was achieved by generating a navigable and collision-free path rather than the motion controller. Nevertheless, it was found to be difficult to apply the control strategy to multiply-articulated vehicles because of limitations of the path planner.

In terms of the above concept of ‘virtual tractor’, this method makes the rear trailer reverse like a single tractor to guide the other passive trailers. The ‘virtual tractor’ method adopts the algorithm of backward set-point propagation based on a kinematic model of vehicle geometry. Similar methods were employed in [64]–[71], and the main difference between them was the hitching type. In [64]–[66], the testing was on a robot with on-axle hitching, whereas in [67]–[70], the approach was implemented on a mobile robot with on- and off-axle hitching. [65] lacked experimental evaluation and [71] considered sliding parameter estimation in its kinematics model.

The time scale transformation and exact linearisation [72]–[74] can be used to convert a non-linear control system to a linear system, and then a linear controller is designed for the linearised control system. Two linear path-tracking controllers were developed using two different non-linear feedback control laws for an exactly linearised single trailer system in [36], [75]. The two controllers corresponded to straight lines and circular arcs respectively, so a switching strategy was needed to deal with a path consisting of both straight lines and circular arcs. In this case, the vehicle was not able to exactly track complicated paths, e.g. lane change manoeuvres with continuously changing curvature. Simulation results for an 8-shaped path consisting of two adjacent circles were presented for the tractor-semitrailer vehicle. The same method was also applied to a B-double vehicle [76] and a wheel loader [77], but the vehicle could only follow a straight line in reverse.

Bolzern et al. [78] complemented the exact linearisation technique and applied the method to articulated vehicles with off-axle hitching, whose kinematic model was not exactly feedback linearisable. A ‘ghost’ tractor with on-axle hitching was deliberately designed to exhibit a very similar behaviour as the original tractor under the steady-state compatibility condition. Therefore, the exact linearisation method became applicable. Any deviations between the
‘ghost’ and original tractors were viewed as perturbations. Simulations for two and three trailers validated the transformation method. Compared to the state feedback linearisation, an input-output linearisation controller based only on the linearisation of the offsets dynamics, proposed by Bolzern later [79], underperformed for the same paths. The main disadvantage was that the stability of zero dynamics of choosing a guide point on the rear trailer was only guaranteed by the positive off-axe distances. The input-output linearisation technique was also used in [80], [81].

1.3.2 Non-linear controllers

Non-linear control methods, based on the Lyapunov function and robustness design techniques, have often been employed to stabilise articulated vehicles in reverse [38], [61], [82]–[87]. Some of these were effective only in the case of kinematics or on-axle hitching; some were difficult to implement in practice. For instance, in [82], different control laws had to be applied for different reversing trajectories. In [38], [83], the Lyapunov and backstepping methods had to be applied together for multiple trailers to eliminate oscillation. The Lyapunov technique was also used in [61], [84] to develop angle controllers. The backstepping method was implemented in a similar way [85] for a wheeled mobile robot for motion about circular arcs. A high-gain feedback control system was devised for a tricycle robot articulated with a trailer with off-axe hitching, and the Lyapunov method was used to show the stability [86], but the vehicle dynamics were neglected and an only backward rectilinear manoeuvre was performed. Kim et al. [87] also constructed two Lyapunov functions to achieve globally asymptotic stability and implemented the controller on a model trailer. Experimental results showed the vehicle response was stable and it was able to converge towards target paths.

In [88]–[90], a curvature-based control approach based on a differential Lyapunov function was created for multiple trailers with off-axe hitching. The method could be used for both forward and backward path tracking. An active speed limiter and an extended Kalman filter (EKF) sideslip estimator were designed to improve the accuracy. However, the speed controller was not able to entirely compensate for the substantial lag caused by the system inertia.

A linear Model Predictive Control (MPC) method was employed for autonomous reversing tractor-trailer systems along reference paths by Wu and Huang [91]. The researchers developed an MPC controller based on a kinematics model of a single trailer system and fed forwards the equilibrium value of the steer angle to reduce the tracking errors. The simulation results showed
the lateral error was smaller than that under an LQR method for a ‘Dubins path’\(^1\) [92] consisting of straight lines and circular arcs.

Beglini et al. [93] applied an intrinsically stable MPC (IS-MPC) method [94], [95], which was proposed for humanoid gait generation, to the reversing problem of a tractor trailer. The main idea of the IS-MPC method was to divide an unstable system into stable and unstable components and construct an initialisation function as an MPC stability constraint to converge the unstable part. The lateral and yaw velocities of the tractor were taken as the control inputs, and two controllers were used simultaneously: a state feedback controller was devised as the main controller to minimise the position errors between a random point on the tractor and a reference path. The IS-MPC controller was used to correct the tracking errors as a compensator. A stable state trajectory was generated by the feedback controller forwards tracking the reference path from a past sample instant up to the current time (the so-called ‘retrograde initialisation’), and the corresponding control inputs were used to divide the vehicle kinematics system and find the stability constraint. The control method was implemented on a radio-controlled toy tractor-trailer model for rectilinear and circular manoeuvres, and the controller gave good performance.

Non-linear MPC (NMPC) methods were used for vehicle stability during path-tracking manoeuvres in [96], [97]. A differential braking controller using an NMPC method for a tractor-semi-trailer combination was devised to actuate braking actions on either the tractor or the trailer, thus maintaining the yaw stability of the vehicle [96]. This paper also stated that the differential braking controller along with either a hitch angle tracking or a yaw rate tracking controller could improve the vehicle’s handling and avoid jack-knifing. The NMPC methods were employed on articulated agricultural vehicles in [98]–[101]. Considering the partial observability of the system states and system delays, Backman et al. [98]–[100] developed an EKF to obtain accurate state estimates for an NMPC controller. A variable prediction horizon depending on the computational capacity was also proposed. The controller was tested on a standard tractor and towed trailer and performed well. Wang et al. [101] developed an adaptive min-max MPC controller by bounding external disturbance in a cost function and characterising the disturbance as a ‘virtual spring’, in order to alleviate the computational burden of a standard min-max MPC controller.

\(^1\) A ‘Dubins path’ is the shortest curve between two points with curvature constraints and fixed initial and terminal tangents.
A hybrid control framework integrating the NMPC and the mixed logical dynamics (MLD) methods was constructed in [102], [103]. The reversing problem was transformed and solved using a ‘multi-parametric mixed-integer quadratic programming’ (mp-MIQP) technique. Sinusoidal paths were simulated for a tractor-trailer vehicle, which showed asymptotic stability.

A non-linear control structure consisting of an inner-loop and an outer-loop controller in a cascade was built to achieve asymptotic stability for reference paths with constant or smoothly varying curvature in [104]. This significantly extended a previous control law used in robotics [105]. However, the prerequisites for the control law were ‘sign-homogeneous hitching’ (i.e. the hitch points had to be on the same side of unicycle robotic vehicles) and ‘segment platooning’ (i.e. all vehicle units move in the same direction). The two controllers were implemented on a three-trailer laboratory-scale robotic vehicle.

Hejsae et al. [106] argued that controllers might fail if articulation angle rates and steer angle rate were not considered in the control design. An angle-rate-bounded manoeuvrability and safety condition was proposed and a ‘look-ahead’ distance was also employed. On top of these, a three-layer neural genetic network with Rectified Linear Units (ReLUs) was trained to follow a reference path and reduce lateral and heading errors. Only a simulation of a sharp turn was run.

1.4 ‘Actively-planned’ reversing methods
There are only a few techniques in the literature to achieve optimised routes to a terminal position, without a pre-planned reference path. They can be divided into two categories: advanced control theory and machine learning methods. However, there is a huge gap in the literature of control theory for the autonomous reversing of multiply-articulated vehicles, because only fuzzy control methods have been used to enable a reversing system to pursue an optimised route rather than follow a collision-free path generated by a path planner. These fuzzy control methods are discussed next.

1.4.1 Advanced control methods
Hierarchical fuzzy controllers can be used to make decisions based on a qualitative transformation from prior experience and knowledge about the non-linear vehicle system [107]–[110]. For instance, [107] presented an expert reversing system using a fuzzy control system and a forward-chaining inference scheme. The controller was able to reverse a toy truck trailer into a loading bay by taking different actions forwards and backwards according to a pre-defined ‘linguistic state and action table’. The prior knowledge and know-how were
acquired to create the look-up table, and a large number of fuzzy rules had to be made correspondingly. The system only worked for one trailer performing a specific task. It is not possible to apply the theory to more complicated tasks or to multiple trailers, which human drivers are not capable of operating and for which the prior knowledge does not exist.

1.4.2 Machine learning methods

In 1989, use of neural networks to learn how to reverse a tractor with a single trailer from arbitrary initial positions was proposed by Nguyen and Widrow [111], [112]. First, a three-layer neural network including a 7-variable input layer with hidden ‘Adaptive Linear Neurons’ (i.e. ADALINE units) was trained to learn a vehicle kinematics model and behave like an emulator of the tractor-trailer system. Then, another three-layer perceptron\(^2\) including a 6-node input layer were trained to be a non-linear controller using the trained neural-net emulator. The weights of the controller were adjusted via a back-propagation method using the final state error vector as a metric. The training proceeded until the tractor and trailer hit something and stopped. The simulation results showed that after the training process, the emulator was able to represent the non-linearity of the vehicle system, because of the universal function approximation of neural networks. The trained controller could reverse the tractor and trailer from any initial position, even from a jack-knifing position by driving forwards and backwards. It was also seen that early control decisions had significant impact on the reversing manoeuvre, and even though some early moves may not move towards the terminal position, they adjusted the vehicle position for the ultimate success. The authors stated that many trials were needed to train the neural networks.

Specifically, focused on the truck reversing task, Jenkins et al. [113] improved Nguyen and Widrow’s networks by decomposing the problem into constituent subtasks, e.g. the relationship between the tractor heading angle, the trailer heading angle, and the wheel angle, thereby significantly reducing the number of neurons required in the controller. The ordinary differential equations (ODEs) of the vehicle kinematics model were used instead of the neural-network-based emulator, as the authors argued that the emulator was not able to provide accurate measurements, thus causing instability in the network controller. The performance of the controller was as good as the large neural networks. However, Jenkins’ method is not a generalised neural paradigm for a complex system because a specific analysis is required to decouple the system.

\(^2\) A class of a feedforward artificial neural network (ANN).
Koza [114], [115] applied a ‘tree-based’ genetic algorithm to a control problem of reversing a tractor-trailer vehicle without giving a mathematically exact solution. When a terminal domain and fitness measures for evaluation of each tree node were specified, only a few basic functions\(^3\) were needed to solve the problem. A new offspring was produced by randomly selecting and recombining the existing parental tree nodes. The highest fitness value of a tree node among a generation was chosen as the best-of-generation individual. The algorithm proceeded from generation to generation until terminal criteria were satisfied. An advantage of the genetic programming is that it is highly scalable, since no complex mathematic model is required. However, the stability and feasibility of the method is not guaranteed, and the computation is very expensive.

A combination of the genetic algorithm and neural networks called ‘Neuro-genetic’ was used in [116]. Trained neural nets were used as intermediate agents to replace the standard tree nodes. The trajectory length was also considered in the fitness measure to avoid unnecessary detours. The impact of the number of inputs on control performance was investigated for single starting point learning. However, for multi-point learning ensemble, the fitness measure had to be fine-tuned and the computation increased significantly. The ‘hyperparameters’ of the neural networks still depended on empirical experience. Ho et al. [117] simplified the Jenkins’ algorithm [113] by replacing the back-propagation process with a genetic algorithm. A similar method was also employed and a comparison between the neuro-controllers evolved by the genetic programming and a LQR controller was made in [118]. Kinjo et al. concluded that the neuro-genetic algorithm outperformed the LQR method.

For the Supervised Learning (SL) and genetic programming methods, a mismatch exists between the learned action distribution and the realistic optimal action distribution [119]. Therefore, for arbitrary starting and terminal positions, the neural networks may not generalise well and substantial manual tuning is required.

Reinforcement Learning (RL) approaches have been employed for multi-stage decision-making problems in recent years. A tabular Q-learning algorithm based on a dynamic state space discretisation method was developed by Vollbrecht [120] for wall avoidance and the docking task for a tractor with a single trailer. This method adopted a hierarchical architecture for the state representation. The main disadvantage was the discrete state and action space, because it became very large when a discrete space was used to replicate continuous space.

\(^3\) Including arithmetic operations, mathematical functions, conditional logic operations, et cetera.
Gatti et al. [121] used a classical temporal difference method called $TD(\lambda)$ and a neural network to reverse a truck-trailer vehicle. The neural network was built as the value function approximator to deal with continuous state space, but only three discrete actions (i.e. the steer angle is either -1 rad, 0, or 1 rad) were chosen. A reward function was constructed to train the agent (‘controller’) to avoid jack-knifing, exceedance of specified domain boundaries, and a large heading angle of the trailer. The training process was also confined to simple assumptions about the initial vehicle position and heading. To cope with the continuity in both the state and action space, McDowell [122] applied the deep deterministic policy gradient (DDPG) method to the truck reversing problem and compared the performance with an LQR controller. A path planner was used to generate numerous paths to provide error states for training the neural networks, but the feasible paths were visually checked. Unlike previous methods, varied terminal positions were considered.

Common issues with the RL algorithms are convergence and manual tuning for the reward function. In general, there are more hyperparameters to be tuned than the SL methods. Moreover, many different configurations of LCVs are used in practice, and the learning algorithms need to be trained specifically for each one, requiring large training data sets.

In conclusion, it is not straightforward to apply the ML methods including the SL and RL algorithms to a general case.

1.5 Analogous research
1.5.1 Forward driving

Driver path-following, Lane Keeping Control (LKC), and ‘potential field’ approaches have often been implemented on cars for forward driving [123]–[127]. It was thought that aspects of the above methods may be relevant to the autonomous reversing of LCVs in this study.

Several control methods have been used to assist drivers to follow a specified path. Odhams [128] developed a preview point controller to feed back the lateral error between the desired path and a ‘look-ahead’ distance. Cole et al. [127] compared different types of LQR controllers with an MPC controller and discussed the influences of the preview and control horizon. Some optimal preview control methods were also employed in [129]–[131] and ML techniques can be found in [132]–[137].

A linear quadratic state feedback controller using the integral of the lateral offset error was proposed in [123] for improving the performance of lane keeping controllers on a curved road.
and reducing the oscillation in the yaw rate. An LQR method was used to tune the controller gains. The controller was tested on a car. It was shown that a shorter ‘look-ahead’ distance can reduce the stability of closed loop system. Therefore, it suggested using a longer preview distance. An MPC method can be integrated with this LKC controller for real-time motion control [138].

Netto et al. [124] showed that feeding-back lateral offsets at the preview distance can improve the performance of LKC controllers. For a particular configuration, [139] stated that there always exists a sufficiently large ‘look-ahead’ distance to guarantee the stability and robustness of the closed-loop system.

A simple ‘look-ahead’ control scheme for LKC was designed in [140] where the Lyapunov method was used to prove the vehicle stability even with highly saturated tyres (on the point of sliding). Coupled with a longitudinal controller and based on path position and wheel slip, the LKC algorithm was used to create an autonomous race car.

In [141]–[143], an adaptive lane keeping controller, which did not use lateral velocity measurements, was designed for cars driving forwards. However, too many variables needed to be estimated adaptively, which made the controller very complicated. The controller required 24 integral actions, which could not be applied in practice. In order to solve these problems, an improved version was proposed in [144]. The new controller had good robustness and reduced high frequency oscillations in the input of steer angle. Nevertheless, the approach has not been tested on real cars.

A ‘potential field’ controller was developed by Rossetter [145] to constrain the vehicle lateral motion within bounded lanes, under time-varying perturbations. ‘Virtual bumpers’ were attached to the environment to avoid a scaling issue and virtual control force related to hazards was applied to the vehicle. The controller gave good performance on a modified steer-by-wire car. Similar approaches could be found in [146]–[149].

1.6 Conclusions

1. There are mainly two types of controllers proposed in the literature for accurately approaching a terminal position in reverse: one is to follow a collision-free path generated by a path planner; the other is to solely pursue an optimised route and avoid obstacles actively. Most research has focused on the first category.
2. The majority of path-following controllers are linear, either through Jacobian linearization or non-linear inversion techniques. For smooth paths, the performance of the linear controllers is comparable with the non-linear controllers. The disadvantages of non-linear controllers are the implementation and design issues, but they have a competitive edge for complex manoeuvres.

3. There has been an increasing interest in ‘active-planning’ reversing methods, particularly induced by the enthusiasm for machine learning techniques. However, only fuzzy control methods have been employed in the control literature. Very few reinforcement learning methods have been applied to the reversing of LCVs.

4. An advantage of using modern control methods is that the controllers can be analytically proven to be globally asymptotically stable; on the other hand, neural networks can learn complex non-linear functions.

5. Most vehicle models used in reversing studies have been based on vehicle kinematics, typically with on-axle hitching. These models neglect the wheel slip that is fundamental to multiple axle-groups on trailers.

6. Most of the methods lacked experimental validation on full-scale tractor and trailers.

1.7 Scope of the thesis

Previous path following control methods caused large swept path, particularly for the tractor unit. This has practical limitations because it prevents vehicles from being able to reverse in some situations, due to hard boundaries including the edges of the road or parked vehicles, et cetera.

The literature review has highlighted a number of areas requiring further research. These are posed below as research questions to be addressed by this thesis.

1.7.1 Research questions

1. How can the performance of the previous path following methods (i.e. pre-planned reversing methods) be improved to reduce the swept path, while maintaining an appropriate path-tracking accuracy? What are the key factors associated with the swept path reduction? How can the optimal criteria be formally evaluated?

2. How can an LCV be controlled actively to avoid obstacles and pursue an optimised route? What are realistic scenarios that can be used for performance evaluation in simulation and in
practice on a full-scale vehicle? What are the stability and feasibility conditions of a general n-trailer vehicle?

3. How can the tuning problem be alleviated for an actively-planned reversing controller? How can a vehicle reverse from any arbitrary initial condition including a jack-knifing position?

1.7.2 Outline of Thesis

This thesis tackles the reversing problem using a combination of theoretical development, simulation and practical testing at full-scale on an experimental test vehicle.

A non-linear vehicle dynamics model is derived in Chapter 2 and linearised for controller development in the following chapters. The differences between the linearised and non-linear models are quantified.

A ‘pre-planned’ reversing controller, ‘Minimum Swept Path Control’ (MSPC), is designed in Chapter 3 to reduce the overall swept path of an LCV while reversing along a pre-defined path. Optimal performance criteria are defined and a systematic approach to controller tuning is described. The results are compared with a path-following controller for various manoeuvres.

The practical implementation of the MSPC reversing controller is investigated on full-size tractor-semitrailer and B-double vehicles in Chapter 4. The real-time controller performance of the experimental system is compared with the simulation results from Chapter 3.

An ‘actively-planned’ reversing controller, ‘Lane-Bounded Reversing Control’ (LBRC), is investigated in Chapter 5. This controller enables an LCV to perform a space-confined, collision-free, and precise manoeuvre to a specified terminal position. The controller performance and manoeuvrability are assessed for various realistic scenarios. The stability and feasibility conditions for using LBRC for a general n-trailer vehicle is discussed and key factors associated with stability and computational time are addressed.

Chapter 6 investigates the practical implementation of LBRC on the experimental tractor-semitrailer and B-double vehicles. It identifies some important practical issues (e.g. time lags, noisy analogue signals, and computational complexity versus accuracy) and discusses numerical approaches to alleviating the issues. The real-time experimental controller performance is compared with the simulation results from Chapter 5.

Further improvements to the actively-planned reversing controller are discussed in Chapter 7. These are based on the foundation of the controller developed in Chapter 5.
### 1.8 Tables

*Table 1.1 Typical LCVs*

<table>
<thead>
<tr>
<th>Type</th>
<th>Typical Length / m</th>
<th>Typical Payload / tonnes</th>
<th>Countries Used</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Tractor-Semitrailer</td>
<td>14.63-16.5</td>
<td>24-32.3</td>
<td>Most countries</td>
<td>[18]–[20]</td>
</tr>
<tr>
<td>Combination</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II. A-double Combination</td>
<td>22.1-39.1</td>
<td>23.6-42.8</td>
<td>Australia, Canada, Mexico, USA</td>
<td></td>
</tr>
<tr>
<td>III. B-double Combination</td>
<td>20.4-26</td>
<td>23.5-44.5</td>
<td>Australia, Canada, Argentina, Spain, Germany, USA, et al.</td>
<td>[18]–[20]</td>
</tr>
<tr>
<td>IV. B-triple Combination</td>
<td>33.3</td>
<td>60</td>
<td>Australia</td>
<td></td>
</tr>
<tr>
<td>V. Nordic Combination</td>
<td>25.2</td>
<td>40</td>
<td>Denmark, Netherlands</td>
<td></td>
</tr>
</tbody>
</table>
1.9 Figures

Figure 1.1 Types of coupling: (a) A-type coupling; (b) B-type coupling

Figure 1.2 'On-axle hitching' and 'off-axle hitching'
Chapter 2 Vehicle modelling

This chapter mainly discusses the development of a non-linear vehicle dynamics model and its linearised equivalent to represent articulated vehicles. The vehicle parameters, equivalent wheelbase calculation, tyre model, and some other dynamics related topics are described.

2.1 Vehicle dynamics model

The vehicle model is shown in Figure 2.1. Many trailers require 2 or 3 rear axles under lorry loading regulations [150]. Multiple rear axles can cause tyre lateral scrubbing in tight corners. Based on an arbitrary combination of Heavy Goods Vehicles (HGVs), a single-track ‘bicycle’ vehicle dynamics model [151], [152] was extended to include an arbitrary number of trailer units with any number of axles. The model assumes one equivalent axle at the rear of the tractor and trailer units for simplicity, and a constant longitudinal speed of the tractor, \( u_0 \), as shown in Figure 2.2. For the controller development in the subsequent chapters, the model is extended to have multiple axles in each axle group of the trailer units. In reality, the hitch point would be above, or slightly in front of the ‘effective drive axle’ [153]. It is shown here as being rearward of the ‘effective drive axle’, for convenience of presentation. Some extra assumptions are listed as below:

i. Load transfer effects are neglected;
ii. The system is free of disturbances, perturbations, and noise;
iii. The steering assembly has perfect Ackermann geometry;
iv. Saturation and rate limits of steering angle are neglected;
v. Tyre realigning moments, longitudinal and spin creep are neglected.

Variables relating to the tractor unit have subscript ‘0’ and those related to the trailers have subscripts starting from ‘1’ to ‘n’, where ‘n’ is the total number of the trailers.

Considering the independent variables of the dynamic system, the state vector, \( Z_D \), and the corresponding first derivative, \( \dot{Z}_D \), can be defined as follows, including \((2n + 2)\) elements:

\[
Z_D = \begin{bmatrix} v_0 & \Omega_0 & \Gamma_1' & \cdots & \Gamma_i' & \cdots & \Gamma_n' & \hat{r}_1 & \cdots & \hat{r}_n \end{bmatrix}^T,
\]

\[
\dot{Z}_D = \begin{bmatrix} \dot{v}_0 & \dot{\Omega}_0 & \dot{\Gamma}_1' & \cdots & \dot{\Gamma}_i' & \cdots & \dot{\Gamma}_n' & \ddot{r}_1 & \cdots & \ddot{r}_n \end{bmatrix}^T,
\]

where \( Z_D \in R^{(2n+2) \times 1} \) and \( \dot{Z}_D \in R^{(2n+2) \times 1} \). As shown in Figure 2.2,

- \( v_0 \): the lateral velocity of the centre of mass (C.o.M.) of the tractor;
- $\Omega_0$: the yaw velocity of the C.o.M. of the tractor;
- $\Gamma_i$: the articulation angle between the $i^{th}$ and $(i+1)^{th}$ vehicle units;
- $\dot{\Gamma}_i$: the articulation angle rate between the $i^{th}$ and $(i+1)^{th}$ vehicle units.

From Figure 2.2, the yaw rate, longitudinal and lateral velocities of the $i^{th}$ trailer’s C.o.M., $\Omega_i$, $u_i$ and $v_i$, are written as follows:
\[
\Omega_i = \Omega_{i-1} + \dot{\Gamma}_i, \tag{2-3}
\]
\[
u_i = u_{i-1} \cos(\Gamma_i) - ((b_{i-1} + c_{i-1})\Omega_{i-1} - v_{i-1}) \sin(\Gamma_i), \tag{2-4}
\]
\[
u_i = -u_{i-1} \sin(\Gamma_i) - ((b_{i-1} + c_{i-1})\Omega_{i-1} - v_{i-1}) \cos(\Gamma_i) - a_i \Omega_i, \tag{2-5}
\]
where $i = 1, 2, 3, \ldots, n$.

- $b_{i-1}$: the distance between the rear axle and the C.o.M. of the $i^{th}$ vehicle unit;
- $c_{i-1}$: the distance between the rear axle and the hitch of the $i^{th}$ vehicle unit;
- $a_i$: the distance between the front axle and the C.o.M. of the $i^{th}$ trailer;
- $u_{i-1}$: the longitudinal velocity at the C.o.M. of the $i^{th}$ vehicle unit;
- $v_{i-1}$: the lateral velocity at the C.o.M. of the $i^{th}$ vehicle unit;
- $\Omega_{i-1}$: the yaw velocity at the C.o.M. of the $i^{th}$ vehicle unit.

Differentiating equations (2-3) to (2-5) gives the equations of first derivatives of $u_i$, $v_i$ and $\Omega_i$:
\[
\dot{\Omega}_i = \dot{\Omega}_{i-1} + \ddot{\Gamma}_i, \tag{2-6}
\]
\[
\dot{u}_i = \dot{u}_{i-1} \cos(\Gamma_i) - \left((b_{i-1} + c_{i-1})\dot{\Omega}_{i-1} - \dot{v}_{i-1}\right) \sin(\Gamma_i) - \dot{\Gamma}_i \left(u_{i-1} \sin(\Gamma_i) + ((b_{i-1} + c_{i-1})\Omega_{i-1} - v_{i-1}) \cos(\Gamma_i)\right), \tag{2-7}
\]
\[
\dot{v}_i = -\dot{u}_{i-1} \sin(\Gamma_i) - \left((b_{i-1} + c_{i-1})\dot{\Omega}_{i-1} - \dot{v}_{i-1}\right) \cos(\Gamma_i) - a_i \Omega_i - \dot{\Gamma}_i \left(u_{i-1} \cos(\Gamma_i) - ((b_{i-1} + c_{i-1})\Omega_{i-1} - v_{i-1}) \sin(\Gamma_i)\right), \tag{2-8}
\]
From Figure 2.2, the following force and moment equations are written:
\[
m_0 \dot{v}_0 + u_0 \Omega_0 = F_{f_0} \cos(\delta) + F_{r_0} + Y_{H_0}, \tag{2-9}
\]
\[
I_{z_0} \dot{\Omega}_0 = F_{f_0} \cos(\delta) a_0 - F_{r_0} b_0 - Y_{H_0} (b_0 + c_0), \tag{2-10}
\]
\[
I_{z_i} \dot{\Omega}_i = -Y_{H_i} (a_i + b_i + c_i) + m_i \dot{v}_i + u_i \Omega_i) a_i - F_{r_i} (a_i + b_i), \tag{2-11}
\]
\[ Y_{H_{i-1}} = \left( Y_{H_i} + F_{r_i} - m_i(\dot{v}_i + u_i\omega_i) \right) \cos(I_i) + \left( X_{H_i} - m_i(\dot{u}_i - v_i\omega_i) \right) \sin(I_i), \quad (2-12) \]
\[ X_{H_{i-1}} = -\left( Y_{H_i} + F_{r_i} - m_i(\dot{v}_i + u_i\omega_i) \right) \sin(I_i) + \left( X_{H_i} - m_i(\dot{u}_i - v_i\omega_i) \right) \cos(I_i), \quad (2-13) \]

where \( i = 1, 2, 3, \ldots, n. \)

- \( a_0 \): the distance between the front axle and the C.o.M. of the tractor;
- \( b_0 \): the distance between the C.o.M. and the rear axle of the tractor;
- \( c_0 \): the distance between the rear axle and the hitch of the tractor;
- \( b_i \): the distance between the C.o.M. and the rear axle of the \( i^{th} \) trailer;
- \( c_i \): the distance between the rear axle and the hitch of the \( i^{th} \) trailer;
- \( m_0 \): the mass of the tractor;
- \( I_{x_0} \): the yaw moment of inertia of the tractor;
- \( I_{x_i} \): the yaw moment of inertia of the \( i^{th} \) trailer;
- \( X_{H_{i-1}} \): the longitudinal coupling force of the hitch point between the \( i^{th} \) and the \((i + 1)^{th}\) vehicle units;
- \( Y_{H_{i-1}} \): the lateral coupling force of the hitch point between the \( i^{th} \) and the \((i + 1)^{th}\) vehicle units;
- \( X_{H_i} \): the longitudinal coupling force of the hitch point between the \( i^{th} \) and the \((i + 1)^{th}\) trailers;
- \( Y_{H_i} \): the lateral coupling force of the hitch point between the \( i^{th} \) and the \((i + 1)^{th}\) trailers;
- \( F_{r_0} \): the lateral tyre force of the front axle of the tractor;
- \( F_{r_i} \): the lateral tyre force of the rear axle of the \( i^{th} \) trailer.

The sign of slip angles depends on the motion direction of the tractor. With the steering angle, \( \delta \), the slip angles are defined as follows:

\[ \alpha_{f_0} = \text{sign}(u_0) \ast (\tan^{-1}\left(\frac{v_0 + a_0\omega_0}{u_0}\right) - \delta), \quad (2-14) \]
\[ \alpha_{r_0} = \text{sign}(u_0) \ast \tan^{-1}\left(\frac{v_0 - b_0\omega_0}{u_0}\right), \quad (2-15) \]
\[
\alpha_{r_i} = \text{sign}(u_0) \ast \tan^{-1}\left(\frac{v_{i-b_i}}{u_i}\right),
\]

where \(i = 1, 2, 3, \ldots, n\).

- \(\text{sign}(u_0)\): the sign of the longitudinal velocity of the tractor;
- \(\alpha_f\): the slip angle of the front axle of the tractor;
- \(\alpha_r\): the slip angle of the rear axle of the tractor;
- \(\alpha_{r_i}\): the slip angle of the rear axle of the \(i^{th}\) trailer.

According to the correlation between slip angles and lateral tyre forces, a general equation can be defined, regardless of which tyre model is used.

\[
F_y = f_t(\alpha),
\]

where \(\alpha\) and \(F_y\) represent the side slip angle and the corresponding lateral tyre force, and \(f_t\) is a tyre force function, depending on the tyre model used.

Combining the above equations results in a differential equation in terms of state vectors, \(Z_D\) and \(\dot{Z}_D\), which can be fed into an ODE solver.

\[
\dot{Z}_D = f_D(\delta, Z_D),
\]

where \(f_D\) symbolises the general non-linear function after rearranging equations (2-3) to (2-16).

The following relationships between the vehicle C.o.M. positions, heading, and yaw velocities can be used to estimate the heading and positions of each unit.

\[
\dot{\theta}_j = \Omega_j,
\]

\[
\dot{x}_j = u_j \cos(\theta_j) - v_j \sin(\theta_j),
\]

\[
\dot{y}_j = u_j \sin(\theta_j) + v_j \cos(\theta_j),
\]

where \(\theta_j\), \(x_j\), and \(y_j\) represent the heading and location of the C.o.M. of each vehicle unit in a fixed global coordinate system \((j = 0, 1, 2, \ldots, n)\).

### 2.2 Linearisation

Unlike linear systems that have analytic solutions, the non-linear ODEs outlined in Section 2.2 only have numeric solutions. They can be used to simulate the behaviour of articulated vehicles,
during turning when the articulation angles are large. It is difficult and expensive to tune the
dynamic system and analyse its properties using the non-linear equations, but a linearised
model is able to represent the non-linear dynamics model by capturing some important
characteristics [154]. Linearisation is a necessary step for stability and state space analyses and
controller design [155], [156].

An effective approach is to calculate small deviations from a given equilibrium state, thereby
generating the linearised model through Jacobian linearisation [157]. The assumptions made
in Section 2.2 still hold, and some supplementary assumptions need to be made:

vi. Deviations from any equilibrium point are small;

vii. When a non-linear tyre model is employed, the tyre properties are assumed to be locally
linear within the range of deviations and the corresponding lateral stiffness coefficients
are determined by calculating the gradient of the tyre curve.

The state vector of the non-linear dynamics model, $\mathbf{Z}_D$, and its derivative, $\dot{\mathbf{Z}}_D$, can be linearised
as small deviations around any equilibrium point, as follows:

$$
\Delta \mathbf{Z}_D = [\Delta v_0 \Delta \Omega_0 \Delta \Gamma_1 \cdots \Delta \Gamma_i \cdots \Delta \Gamma_n \Delta \dot{\Gamma}_1 \cdots \Delta \dot{\Gamma}_i \cdots \Delta \dot{\Gamma}_n]^T, \quad (2-22)
$$

$$
\Delta \dot{\mathbf{Z}}_D = [\Delta \dot{v}_0 \Delta \dot{\Omega}_0 \Delta \dot{\Gamma}_1 \cdots \Delta \dot{\Gamma}_i \cdots \Delta \dot{\Gamma}_n \Delta \ddot{\Gamma}_1 \cdots \Delta \ddot{\Gamma}_i \cdots \Delta \ddot{\Gamma}_n]^T, \quad (2-23)
$$

where $\Delta \mathbf{Z}_D \in \mathbf{R}^{(2n+2)\times 1}$ and $\Delta \dot{\mathbf{Z}}_D \in \mathbf{R}^{(2n+2)\times 1}$. $\Delta$ represents a small deviation from any
equilibrium point, e.g. $\Delta v_0 = v_0 - v_{0e}$ ($v_{0e}$ is the tractor’s lateral velocity at any equilibrium
point). $i = 1, 2, 3, \cdots, n$.

As the longitudinal speed of the tractor, $u_0$, is constant, $u_0 = u_{0e}$ and $\Delta u_0 = 0$.

Linearising equations (2-3) – (2-8) around an equilibrium point is carried out as follows. The
subscript ‘e’ in the following linearisation equations stands for the equilibrium state calculated
by assuming that the derivatives are zero.

$$
\Delta \Omega_i = \Delta \Omega_{i-1} + \Delta \dot{\Gamma}_i \quad (2-24)
$$

$$
\Delta u_i = \Delta u_{i-1} \cos(I_{ie}) - \left( (b_{i-1} + c_{i-1}) \Delta \Omega_{i-1} - \Delta v_{i-1} \right) \sin(I_{ie}) - \Delta \dot{\Gamma}_i \left( u_{(i-1)e} \sin(I_{ie}) + \right. \\
\left. \left( (b_{i-1} + c_{i-1}) \Omega_{(i-1)e} - v_{(i-1)e} \right) \cos(I_{ie}) \right) \quad (2-25)
$$

$$
\Delta v_i = -\Delta u_{i-1} \sin(I_{ie}) - \left( (b_{i-1} + c_{i-1}) \Delta \Omega_{i-1} - \Delta v_{i-1} \right) \cos(I_{ie}) - a_i \Delta \Omega_i - \\
\Delta \dot{\Gamma}_i \left( u_{(i-1)e} \cos(I_{ie}) - \left( (b_{i-1} + c_{i-1}) \Omega_{(i-1)e} - v_{(i-1)e} \right) \sin(I_{ie}) \right) \quad (2-26)
$$
\[ \Delta \dot{\theta}_i = \Delta \dot{\theta}_{i-1} + \Delta \dot{f}_i \]  
\[ \Delta \dot{u}_i = \Delta \dot{u}_{i-1} \cos(\Gamma_{ie}) - \left( (b_{i-1} + c_{i-1}) \Delta \dot{\theta}_{i-1} - \Delta \dot{v}_{i-1} \right) \sin(\Gamma_{ie}) - \Delta \dot{f}_i \left( u_{(i-1)e} \sin(\Gamma_{ie}) + \left( b_{i-1} + c_{i-1} \right) \Omega_{(i-1)e} - v_{(i-1)e} \right) \cos(\Gamma_{ie}) \]  
\[ \Delta \dot{v}_i = -\Delta \dot{u}_{i-1} \sin(\Gamma_{ie}) - \left( (b_{i-1} + c_{i-1}) \Delta \dot{\theta}_{i-1} - \Delta \dot{v}_{i-1} \right) \cos(\Gamma_{ie}) - a_i \Delta \dot{\theta}_i - \Delta \dot{f}_i \left( u_{(i-1)e} \cos(\Gamma_{ie}) - \left( b_{i-1} + c_{i-1} \right) \Omega_{(i-1)e} - v_{(i-1)e} \right) \sin(\Gamma_{ie}) \]  

Linearising equations (2-14) – (2-16) results in:

\[ \Delta \alpha_{f_0} = \text{sign}(u_0) \left( \frac{1}{1 + (\frac{v_{0e} + a_0 \Delta \theta_0}{u_0})^2} \frac{\Delta v_0 + a_0 \Delta \theta_0}{u_0} - \Delta \delta \right) \]  
\[ \Delta \alpha_{r_0} = \text{sign}(u_0) \left( \frac{1}{1 + (\frac{v_{0e} - b_0 \Delta \theta_0}{u_0})^2} \frac{\Delta v_0 - b_0 \Delta \theta_0}{u_0} \right) \]  
\[ \Delta \alpha_{r_i} = \text{sign}(u_0) \left( \frac{1}{1 + (\frac{v_{ie} - b_i \Delta \theta_i}{u_{ie}})^2} \left( \frac{\Delta v_i - b_i \Delta \theta_i}{u_{ie}} - \Delta u_i \frac{v_{ie} - b_i \Delta \theta_i}{u_{ie}} \right) \right) \]  

Linearising equation (2-17) gives:

\[ \Delta F_y = K_y \Delta \alpha, \]  

where \( K_y \) symbolises the linear lateral tyre stiffness coefficient or the local gradient of the non-linear tyre model, and \( \Delta \alpha \) is a general term standing for \( \Delta \alpha_{f_0} \) and \( \Delta \alpha_{r_j} \) \( (j = 0, 1, 2, \cdots, n) \).

Linearising equations (2-9) – (2-13) generates:

\[ m_0 (\Delta \dot{v}_0 + u_0 \Delta \theta_0) = \Delta F_{f_0} \cos(\delta_e) - F_{f_{0e}} \sin(\delta_e) \Delta \delta + \Delta F_{r_0} + \Delta Y_{H_0} \]  
\[ l_{z_0} \Delta \dot{\theta}_0 = \Delta F_{f_0} \cos(\delta_e) a_0 - F_{f_{0e}} \sin(\delta_e) a_0 \Delta \delta - b_0 \Delta F_{r_0} - (b_0 + c_0) \Delta Y_{H_0} \]  
\[ l_{z_i} \Delta \dot{\theta}_i = -\Delta Y_{H_i} (a_i + b_i + c_i) + m_i (\Delta \dot{v}_i + \Delta u_i \Omega_{ie} + u_{ie} \Delta \Omega_i) a_i - \Delta F_{r_i} (a_i + b_i) \]  
\[ \Delta Y_{H_{i-1}} = \left( \Delta Y_{H_i} + \Delta F_{r_i} - m_i (\Delta \dot{v}_i + \Delta u_i \Omega_{ie} + u_{ie} \Delta \Omega_i) \right) \cos(\Gamma_{ie}) + \left( \Delta X_{H_i} - m_i (\Delta \dot{u}_i - \Delta v_i \Omega_{ie} - v_{ie} \Delta \Omega_i) \right) \sin(\Gamma_{ie}) - \left( Y_{H_{ie}} + F_{r_{ie}} - m_i u_{ie} \Omega_{ie} \right) \sin(\Gamma_{ie}) \Delta \Gamma_i + \left( X_{H_{ie}} + m_i v_{ie} \Omega_{ie} \right) \cos(\Gamma_{ie}) \Delta \Gamma_i \]  

24
\[
\Delta X_{H_{i-1}} = -\left(\Delta Y_{H_i} + \Delta F_{r_i} - m_i(\Delta \dot{v}_i + \Delta u_i \Omega_{ie} + u_{ie} \Delta \Omega_i)\right) \sin(\Gamma_{ie}) + \left(\Delta X_{H_i} - m_i(\Delta \dot{v}_i - \\
\Delta v_i \Omega_{ie} - v_{ie} \Delta \Omega_i)\right) \cos(\Gamma_{ie}) - \left(Y_{H_ie} + F_{r_ie} - m_i u_{ie} \Omega_{ie}\right) \cos(\Gamma_{ie}) \Delta \Gamma_1 + \left(X_{H_ie} + \\
m_i v_{ie} \Omega_{ie}\right) \sin(\Gamma_{ie}) \Delta \Gamma_1
\] (2-38)

Substituting equations (2-24) – (2-33) and (2-37) – (2-38) into (2-34) – (2-36) gives a linear state space model in terms of the linearised state vector, \(\Delta Z_D\), system matrix, \([A_D]\), input matrix, \([B_D]\), and control input, \(\Delta \delta\).

\[
\Delta \dot{Z}_D = [A_D] \Delta Z_D + [B_D] \Delta \delta,
\] (2-39)

where \([A_D] \in \mathbb{R}^{(2n+2) \times (2n+2)}\) and \([B_D] \in \mathbb{R}^{(2n+2) \times 1}\).

2.3 Vehicle parameters

Parameters for each vehicle unit are shown in Tables 2.1 – 2.3. The geometry was measured from test vehicles owned by CVDC. Some inertias, mass and C.o.M. locations and were provided by Volvo Trucks. Otherwise, they were estimated based on previous research [19], [158]–[161]. All test vehicles are unladen. The vehicle combinations simulated in this thesis are outlined in Table 2.4.

Winkler extended Pacejka’s work [162]–[165] and simplified the analysis of handling characteristics of vehicle systems with multiple axles. The ‘equivalent wheelbase’ was calculated using the Winkler’s method [153] to represent axle groups of trailer units.

2.4 Verification of linear model

The non-linear model was verified previously by simulating a tractor-semitrailer vehicle moving forwards and backwards and comparing the responses to known linear models in [166]. To verify the correctness of the linearisation, a comparison between the non-linear and linearised models was made. If deviations from the equilibrium states are small, there should be no significant difference between the two approaches. ‘Impulse’ and step function perturbations were used for the tractor-semitrailer vehicle. The vehicle units were assumed to run along a straight line at a constant longitudinal speed. A unit disturbance, \(1\), is added to the model input, steer angle, from 4.995 s to 5.005 s, creating an ‘impulse’ function perturbation as shown in Figure 2.3, and from 5 s onwards, creating a step function perturbation as shown in Figure 2.4. The subsequent responses of the model states are correspondingly shown in Figures 2.5 and 2.6.
Figures 2.7 and 2.8 show when a bigger disturbance, $20^\circ$, is reapplied to both systems, the consequent responses are plotted in Figures 2.9 and 2.10. The solid lines stand for the non-linear model, and the dashed lines represent the linearised model. As seen in the above figures, solid lines exactly overlap the dashed lines when the steer angle disturbance is $1^\circ$, whereas solid and dashed lines are noticeably different when the steer angle disturbance is $20^\circ$. This means there are no discernible changes for small disturbances (e.g. $1^\circ$) but some visible changes for larger ones (e.g. $20^\circ$). For the step function disturbances, as the perturbations increase in Figure 2.11, the differences in the steady state between two models become obvious. Hence, it is substantiated that the linearisation is correct.

The system matrix, $[A_p]$, has $(2n + 2)$ eigenvalues, but $n$ of them, which are equivalent to the number of trailers, are in the right half plane of the complex plane because of the non-holonomic vehicle characteristics. Take a typical tractor and twin trailer combination as an example. The eigenvalues of the system matrix, $[A_p]$, in equation (2-39) are plotted in Figure 2.12 (a), along with a magnified view of the positive eigenvalues in Figure 2.12 (b). The blue circles represent the stable eigenvalues in the left half plane, whereas the red cross marks show the unstable eigenvalues in the right half plane. Therefore, the open-loop system is unstable.

### 2.5 Tyre model

In cases where tyre non-linearity is important, a non-linear tyre model was used to account for the saturation of side forces and the correlation between vertical loads and lateral stiffness [167]. The brush tyre model [165], [167], [168] modified by Jujnovich [159] was used in this project because: (i) it accounts for large slip angles and variations in normal load when necessary; (ii) it has simple physical parameters for which information was available; (iii) its use in this application was verified previously in [19].

Considering a slip angle, $\alpha$, the non-linear side force, $F_y^n$, can then be calculated via the following equations:

\[
\frac{F_y^n}{\mu F_z} = \begin{cases} 
\frac{-\bar{c}}{\mu} \alpha + \frac{\bar{c}^2}{3\mu^2} |\alpha| \alpha - \frac{\bar{c}^3}{27\mu^3} \alpha^3 & |\alpha| < \frac{2\mu}{\bar{c}} \\
-sgn(\alpha) & otherwise
\end{cases}
\]  

\[\bar{C} = C_{p1} + C_{p2} F_z\]  

where

- $C_{p1}$: the cornering coefficient of the non-linear tyre model;
• $C_{p2}$: the curvature coefficient of the non-linear tyre model;
• $F_z$: the vertical load at one tyre;
• $\mu$: the friction coefficient between the road and the tyre.

The tyre parameters used for tractor and semitrailer vehicles in this project are shown in Table 2.5. A static analysis method was used to calculate vertical tyre loads, assuming that the tyres on each axle carried equal vertical loads.

When slip angles are not so big that the influence of side forces on vehicle lateral offsets becomes very significant, a linear tyre model [169] was used in conjunction with the dynamic bicycle model. The linear lateral stiffness coefficients, $K_y$, are given by:

$$K_y = -\tilde{C}F_z, \quad (2-42)$$

The calculated $K_y$ for all tyres around a straight line on the specific tractor-semitrailer vehicle are shown in Table 2.6.

Equations (2.40) and (2.41) are plotted in normalised form in Figure 2.13. For the non-linear model, the ratio between the lateral force, $F_y^n$, and the tyre friction force, $\mu F_z$, can only be within the range from -1 to 1, for any slip angle, $\alpha$. The linear model (equation (2.42)) is superimposed on the non-linear curves in Figure 2.14. The two curves agree at small slip angles.

The use of linear and non-linear tyre models gives small differences in lateral offsets. Hence, to satisfy the stabilisation and convergence criteria, a linear tyre model was firstly developed to design controllers, and then a non-linear model was used to confirm the performance of the controller for larger slip angles.

2.6 Conclusions

1. An extended dynamics model based on the standard two-axle bicycle model was established for a tractor articulated with arbitrary number of trailers having any number of axles.

2. Vehicle parameters for specific vehicle combinations were defined. The Winkler’s method was used to model an ‘equivalent axle’ to replace multiple-axle groups of trailers.

3. A linearised model was built to capture the significant characteristics of the non-linear vehicle dynamics model for the state space analysis and the development of controllers. The linearised model matched the non-linear model for small slip angles.

4. The open-loop system for reversing is unstable.
5. A modified brush tyre model was used to improve the vehicle dynamics model for larger slip angles.
## 2.7 Tables

*Table 2.1 Tractor unit parameters*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from the front axle to the centre of mass</td>
<td>$a$</td>
<td>1.416</td>
<td>m</td>
</tr>
<tr>
<td>Distance from the rear axle to the centre of mass</td>
<td>$b$</td>
<td>2.684</td>
<td>m</td>
</tr>
<tr>
<td>Distance from the rear axle to hitch point</td>
<td>$c$</td>
<td>-0.96</td>
<td>m</td>
</tr>
<tr>
<td>Front overhang</td>
<td>$fo$</td>
<td>1.4</td>
<td>m</td>
</tr>
<tr>
<td>Rear overhang</td>
<td>$ro$</td>
<td>1.25</td>
<td>m</td>
</tr>
<tr>
<td>Vehicle width</td>
<td>$wd$</td>
<td>2.5</td>
<td>m</td>
</tr>
<tr>
<td>Number of axles</td>
<td>$n_a$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Equivalent wheelbase</td>
<td>$l_{eff}$</td>
<td>4.1</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>$m$</td>
<td>7,020</td>
<td>kg</td>
</tr>
<tr>
<td>Yaw moment of inertia</td>
<td>$I_z$</td>
<td>24,275</td>
<td>kg·m²</td>
</tr>
</tbody>
</table>
Table 2.2 B-link trailer parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from the front hitch point to the centre of mass</td>
<td>(a)</td>
<td>4.527</td>
<td>m</td>
</tr>
<tr>
<td>Distance from the first rear axle to the centre of mass</td>
<td>(b)</td>
<td>4.773</td>
<td>m</td>
</tr>
<tr>
<td>Distance from the first rear axle to the rear hitch point</td>
<td>(c)</td>
<td>0.75</td>
<td>m</td>
</tr>
<tr>
<td>Front overhang</td>
<td>(f_0)</td>
<td>1.61</td>
<td>m</td>
</tr>
<tr>
<td>Rear overhang</td>
<td>(r_0)</td>
<td>1.393</td>
<td>m</td>
</tr>
<tr>
<td>Vehicle width</td>
<td>(w_d)</td>
<td>2.5</td>
<td>m</td>
</tr>
<tr>
<td>Number of axles</td>
<td>(n_a)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Axle spacing</td>
<td>(e)</td>
<td>1.45</td>
<td>m</td>
</tr>
<tr>
<td>Equivalent wheelbase</td>
<td>(l_{eff})</td>
<td>10.077</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>(m)</td>
<td>17,205</td>
<td>kg</td>
</tr>
<tr>
<td>Yaw moment of inertia</td>
<td>(l_z)</td>
<td>258,603</td>
<td>kg·m²</td>
</tr>
</tbody>
</table>
Table 2.3 Semitrailer parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from the front hitch point to the centre of mass</td>
<td>$a$</td>
<td>5.66</td>
<td>m</td>
</tr>
<tr>
<td>Distance from the second rear axle to the centre of mass</td>
<td>$b$</td>
<td>2.34</td>
<td>m</td>
</tr>
<tr>
<td>Distance from the second rear axle to the rear-end</td>
<td>$c$</td>
<td>3.5</td>
<td>m</td>
</tr>
<tr>
<td>Front overhang</td>
<td>$f_o$</td>
<td>1.43</td>
<td>m</td>
</tr>
<tr>
<td>Rear overhang</td>
<td>$r_o$</td>
<td>3.33</td>
<td>m</td>
</tr>
<tr>
<td>Vehicle width</td>
<td>$w_d$</td>
<td>2.5</td>
<td>m</td>
</tr>
<tr>
<td>Number of axles</td>
<td>$n_a$</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Axle spacing</td>
<td>$e$</td>
<td>1.43</td>
<td>m</td>
</tr>
<tr>
<td>Equivalent wheelbase</td>
<td>$l_{eff}$</td>
<td>8.17</td>
<td>m</td>
</tr>
<tr>
<td>Mass</td>
<td>$m$</td>
<td>33,022</td>
<td>kg</td>
</tr>
<tr>
<td>Yaw moment of inertia</td>
<td>$l_z$</td>
<td>165,080</td>
<td>kg·m²</td>
</tr>
</tbody>
</table>

Table 2.4 Details of the simulated vehicle combinations

<table>
<thead>
<tr>
<th>Vehicle combination</th>
<th>Vehicle unit 1</th>
<th>Vehicle unit 2</th>
<th>Vehicle unit 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tractor-semitrailer</td>
<td>tractor</td>
<td>semitrailer</td>
<td></td>
</tr>
<tr>
<td>B-double</td>
<td>tractor</td>
<td>B-link trailer</td>
<td>semitrailer</td>
</tr>
</tbody>
</table>
### Table 2.5 Non-linear tyre model parameters [19]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction Coefficient</td>
<td>( \mu )</td>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>Tractor Tyre Cornering Coefficient</td>
<td>( C_p_1 )</td>
<td>8.78</td>
<td>1/rad</td>
</tr>
<tr>
<td>Tractor Tyre Curvature Coefficient</td>
<td>( C_p_2 )</td>
<td>-4.95e-5</td>
<td>1/Nrad</td>
</tr>
<tr>
<td>Trailer Tyre Cornering Coefficient</td>
<td>( C_p_1 )</td>
<td>6.28</td>
<td>1/rad</td>
</tr>
<tr>
<td>Trailer Tyre Curvature Coefficient</td>
<td>( C_p_2 )</td>
<td>-3.40e-6</td>
<td>1/Nrad</td>
</tr>
</tbody>
</table>

### Table 2.6 Calculated Linear Axle Lateral Stiffness Coefficients in the case of a straight line

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tractor Front Axle Lateral Stiffness Coefficient</td>
<td>( K_y )</td>
<td>-3.683e5</td>
<td>N/rad</td>
</tr>
<tr>
<td>Tractor Rear Axle Lateral Stiffness Coefficient</td>
<td>( K_y )</td>
<td>-3.338e5</td>
<td>N/rad</td>
</tr>
<tr>
<td>Semitrailer Rear Axle Lateral Stiffness Coefficient</td>
<td>( K_y )</td>
<td>-4.119e5</td>
<td>N/rad</td>
</tr>
</tbody>
</table>
2.8 Figures

Figure 2.1 Vehicle diagram showing dimensions of the vehicle units

Figure 2.2 Vehicle velocity and force analysis
Figure 2.3 1° steer angle disturbance added between 4.995 s and 5.005 s

Figure 2.4 1° steer angle disturbance added from 5 s onwards
Figure 2.5 Unit impulse response at the speed of -1 m/s between 4.9 s and 5.2 s for the 1° steer angle disturbance

Figure 2.6 Step response at the speed of 1 m/s between 0 s and 60 s for the 1° steer angle disturbance
Figure 2.7 $20^\circ$ steer angle disturbance added between 4.995 s and 5.005 s

Figure 2.8 $20^\circ$ steer angle disturbance added from 5 s onwards
Figure 2.9 Impulse response at the speed of -1 m/s between 4.9 s and 5.2 s for the 20° steer angle disturbance

Figure 2.10 Step response at the speed of 1 m/s between 0 s and 60 s for the 20° steer angle disturbance
Figure 2.11 Steady-state values for different steer angle disturbances
(a) Eigenvalues of the closed-loop system matrix of the B-double combination

(b) A magnified view of the positive eigenvalues of the closed-loop system matrix of the B-double combination.

Figure 2.12 Eigenvalues of the closed-loop system matrix of the B-double combination
Figure 2.13 A normalised non-linear tyre model

Figure 2.14 Comparison between the non-linear and linearised tyre models in the case of a straight line
Chapter 3 Minimum Swept Path Control (MSPC)

To date, automatic reversing control strategies for HGVs have aimed to ensure that the rear-end of the vehicles exactly follows a specified path, irrespective of the motion of the rest of the combination [19], [39]–[41]. The disadvantage of these approaches is that the other parts of the vehicle, particularly the tractor unit, can experience large excursions from the nominal path, which can render those strategies impractical. For instance, the path-following control (PFC) method proposed by Rimmer [19], [39]–[41] for a common tractor-semitrailer combination minimises the rear error but causes very large swept path, as shown in Figure 3.1 (a). Consequently, the aim of this chapter is to address this problem by investigating ‘Minimum Swept Path Control’ (MSPC) to minimise the overall swept path, by relaxing the constraint on the path-tracking error of the rear axle of the last trailer and reducing the maximum lateral offset of the front axle of the tractor unit, as presented in Figure 3.1 (b).

Figure 3.2 illustrates a relationship between the front and rear axle tracking errors. The PFC method stays at the top-left zone, corresponding to a small rear error but a large front error. As the rear error keeps increasing, the vehicle units start to drift away from reference paths and ultimately the system becomes unstable. The aim of the MSPC method is to achieve the best compromise between the front and rear axle tracking errors, so as to minimise the swept path of the vehicle.

3.1 Controller design

The MSPC algorithm aims to reverse vehicle units by reducing the maximum lateral offset of all parts of the vehicle while generating an acceptable tracking error. In this sense, a cost function is developed to penalise the lateral offsets of the front axle of the tractor, \( y_{fa} \), and the rear axle of the rear trailer, \( y_{ra} \), by applying two weights, \( W_{fa} \) and \( W_{ra} \), to the squared lateral offsets. See Figure 3.3 for definitions of \( y_{fa} \) and \( y_{ra} \). The discrete-time cost function at sample \( k \), \( J_M \), is defined as follows:

\[
J_M = \sum_{k=1}^{\infty} \left( W_{ra} y_{ra}(k)^2 + W_{fa} y_{fa}(k)^2 + W_\delta \delta(k)^2 \right),
\]

where \( W_\delta \) is the weight to the steer angle, \( \delta(k) \).

To implement this cost function, two independent variables, \( y_{ra} \) and \( y_{fa} \), are added to the original continuous-time system state vector, \( Z_D \), without losing full rank of the system’s controllability and observability matrices [170]. Consequently, the extended state vector, \( Z_M \), and its derivative, \( Z_M' \), are defined as follows:
\( Z_M = [Z_D^T \gamma_{fa} \gamma_{ra}]^T, \) \hspace{1cm} (3-2)

where \( Z_M \in \mathbb{R}^{(2n+4) \times 1}. \)

\( \dot{Z}_M = [\dot{Z}_D^T \dot{\gamma}_{fa} \dot{\gamma}_{ra}]^T, \) \hspace{1cm} (3-3)

where \( \dot{Z}_M \in \mathbb{R}^{(2n+4) \times 1}. \)

MSPC is a linear controller that combines State Feedback Control (SFC) [131] with an optimised preview distance [128], [171]. The main strategy is to control the articulation angles between the tractor and trailer units, the lateral offset of the rear axle, and the heading angle of the rear trailer by feeding back those states into the system, while implementing an optimised preview distance to predict future vehicle steady state and feed these forwards to minimise state errors. The control input of steer angle at sample \( k \) is a function of the equilibrium steer angle, the rear lateral offset, the heading angle error, and the articulation angle errors, given by the following equation:

\[
\delta_d(k) = \delta_e(k) + K_{yra} \gamma_{ra}(k) + K_{\theta ra} \left( \theta_p(k) - \theta_{ra}(k) \right) + \sum_{i=1}^{n} K_{ri} \left( I_{ie}(k) - \Gamma_{i}(k) \right),
\]

where \( i = 1, 2, 3, \ldots, n. \) The variables shown in Figure 3.3, and the controller gains are defined as follows:

- \( \delta_d \): the demanded steer angle control input;
- \( \delta_e \): the equilibrium steer angle;
- \( \Gamma_{ie} \): the equilibrium articulation angle between the \( i^{th} \) and \( (i + 1)^{th} \) vehicle units;
- \( \delta_e \) and \( \Gamma_{ie} \) are calculated from the future equilibrium state (when the rear axle, \( P_{ra} \), reaches the current position of the preview point, \( P_{pp} \), as illustrated in Figure 3.4);
- \( \theta_p \): the heading of the reference path;
- \( \theta_{ra} \): the heading of the rear trailer;
- \( \Gamma_{i} \): the actual articulation angle between the \( i^{th} \) and \( (i + 1)^{th} \) vehicle units;
- \( K_{yra} \): the corresponding control gain for the rear lateral offset error;
- \( K_{\theta ra} \): the corresponding control gain for the heading angle error;
- \( K_{ri} \): the corresponding control gain for the articulation angle error.
As the vehicle units reverse at a low speed and the desired path is defined to have continuous curvature and curvature derivatives, small slip angles are assumed. The vehicle geometry is used to calculate the equilibrium articulation angle of the rear trailer, $\Gamma_{ne}$, as follows:

$$R_{pp} \sin(\Gamma_{ne}) = c_{n-1} + l_{eff} \cos(\Gamma_{ne}),$$  

(3-5)

where, as seen in Figure 3.4:

- $R_{pp}$: the radius of curvature of the preview point on the path, $P_{pp}$;
- $c_{n-1}$: the distance from the ‘equilibrium axle’ to the rear hitch point of the penultimate trailer unit;
- $l_{eff}$: the distance from the front hitch point to the ‘equilibrium axle’ of the rear trailer.

The equilibrium (steady state) configurations of the vehicle are calculated by setting all derivatives, $\dot{Z}_M$, in equation (3-3) to zero. These configurations are the same as for the vehicle driving forwards along the path at the same speed. In these configurations, the equilibrium articulation angle of the rear trailer, $\Gamma_{ne}$, is used to calculate the equilibrium steer angle, $\delta_e$, and the other articulation angles, $\Gamma_{je}$, where $j = 1, 2, \cdots, n - 1$.

Apart from the controller gains, the preview distance, $L_{pd}$, is also a very important factor affecting the lateral offsets. Use of a relatively long preview distance can significantly reduce the maximum lateral offset. The tuning of the preview distance will be discussed in the subsequent section.

Linear Quadratic Regulator (LQR) theory is used to tune state feedback controllers so that a pre-defined quadratic cost function is minimised over an infinite time horizon, as shown in equation (3-1) [172].

The linearised vehicle dynamics model represented by equation (2-39) is adopted to build the MSPC system. The system state vector, $Z_M$, and its derivative, $\dot{Z}_M$, are linearised as small deviations around any equilibrium point, as follows:

$$\Delta Z_M = [\Delta Z_D^T \Delta y_{fa} \Delta y_{ra}]^T,$$  

(3-6)

where $\Delta Z_M \in \mathbb{R}^{(2n+4)\times 1}$.

$$\Delta Z_M = [\Delta Z_D^T \Delta \dot{y}_{fa} \Delta \dot{y}_{ra}]^T,$$  

(3-7)

where $\Delta \dot{Z}_M \in \mathbb{R}^{(2n+4)\times 1}$. 

43
Following the approach of Rimmer [19], linear analysis for a straight-line case is sufficient to
tune the controller by capturing the closed-loop characteristics of the system. This implies that
all the equilibrium state variables and path-heading angles are zero. Then, the front and the rear
axle lateral offsets can be determined from the heading and the longitudinal and lateral speeds
of the vehicle units as follows:

\[
y_{fa} = \int_{0}^{t_{s}} (u_{0}(t) \sin(\theta_{0}(t)) + v_{0}(t) \cos(\theta_{0}(t))) \, dt + a_{0} \sin(\theta_{0}(t_{s})), \tag{3-8}
\]

\[
y_{ra} = \int_{0}^{t_{s}} (u_{n}(t) \sin(\theta_{n}(t)) + v_{n}(t) \cos(\theta_{n}(t))) \, dt - (l_{eff_{n}} - a_{n}) \sin(\theta_{n}(t_{s})), \tag{3-9}
\]

where \(t_{s}\) is the simulation time.

The differential equations for the front and the rear axle lateral offset deviations from the
straight-line equilibrium state are generated by differentiating and linearising equations (3-8)
and (3-9) as follows:

\[
\Delta \dot{y}_{fa} = \Delta v_{0} + u_{0e} \Delta \theta_{0} + a_{0} \Delta \Omega_{0}. \tag{3-10}
\]

\[
\Delta \dot{y}_{ra} = \Delta v_{n} + u_{ne} \Delta \theta_{n} - (l_{eff_{n}} - a_{n}) \Delta \Omega_{n} \tag{3-11}
\]

Considering the correlation between the axle lateral offsets and the vehicle heading angles in
the straight-line case, the vehicle position and heading deviations are given as follows:

\[
\Delta y_{fa} = \Delta y_{ra} + \sum_{j=0}^{n} l_{j} \Delta \theta_{j} + l_{eff_{n}} \Delta \theta_{n}, \tag{3-12}
\]

where \(l_{j} = a_{j} + b_{j} + c_{j}\). See Figure 2.1 for definitions of \(a_{j}, b_{j}, \) and \(c_{j}\).

\[
\Delta \theta_{i} = \Delta \theta_{0} + \sum_{j=1}^{i} \Delta \Gamma_{j}, \tag{3-13}
\]

where \(i = 1, 2, 3, \ldots, n\).

Combining (3-12) and (3-13) gives the relationship between the heading angle of the tractor,
the lateral offsets, and the articulation angles:

\[
\Delta \theta_{0} = \frac{1}{\sum_{j=0}^{n-1} l_{j} + l_{eff_{n}}} \Delta y_{fa} - \frac{1}{\sum_{j=0}^{n-1} l_{j} + l_{eff_{n}}} \Delta y_{ra} - \sum_{i=1}^{n} \frac{\sum_{j=0}^{n-1} l_{j} + l_{eff_{n}}} {\sum_{j=0}^{n-1} l_{j + l_{eff_{n}}}} \Delta \Gamma_{i} \tag{3-14}
\]

Substituting equations (3-13) and (3-14) into (3-10) and (3-11) generates two linear equations
between \(\Delta \dot{y}_{fa}, \Delta \dot{y}_{ra}, \) and \(\Delta Z_{M}\). Adding these equations to the original linearised vehicle
dynamics model, equation (2-39), gives a continuous-time state space model in terms of \(\Delta Z_{M}\),
system matrix, \([A_{M_{CT}}]\), input matrix, \([B_{M_{CT}}]\), and control input, \(\Delta \delta\).
\[ \Delta \dot{Z}_M = [A_{M_{CT}}] \Delta Z_M + [B_{M_{CT}}] \Delta \delta, \]  
(3-15)

where \([A_{M_{CT}}] \in R^{(2n+4) \times (2n+4)}\) and \([B_{M_{CT}}] \in R^{(2n+4) \times 1}\).

The natural frequencies of the system are calculated by the following equation.

\[ w_i = \frac{\|\lambda_i\|_2}{2\pi}, \]  
(3-16)

where \(i = 1, 2, \cdots, 2n + 4\).

- \(w_i\): a system natural frequency in Hertz;
- \(\|\lambda_i\|_2\): an eigenvalue of the system matrix, \([A_{M_{CT}}]\), measured in 2-norm.

According to the Nyquist-Shannon sampling theorem [173]–[176], the sample rate, \(f_s\), should be selected as follows.

\[ f_s > 2 \times \max(w_i) \]  
(3-17)

Discretising the continuous-time model at a frequency of \(f_s\) and assuming zero-order hold over the sample time, \(T_s\), give the following discrete-time state space model at sample \(k\).

\[ T_s = \frac{1}{f_s}, \]  
(3-18)

\[ \Delta Z_M(k+1) = [A_{M_{DT}}] \Delta Z_M(k) + [B_{M_{DT}}] \Delta \delta(k), \]  
(3-19)

where \([A_{M_{DT}}]\) and \([B_{M_{DT}}]\) denote the system and input matrices of the discrete-time model. \([A_{M_{DT}}] \in R^{(2n+4) \times (2n+4)}\) and \([B_{M_{DT}}] \in R^{(2n+4) \times 1}\).

For the linearised discrete-time system, the cost function in equation (3-1) can be written in the following quadratic form:

\[ J_M = \sum_{k=1}^{\infty} (\Delta Z_M(k)^T [Q] \Delta Z_M(k) + \Delta \delta(k)^T [R] \Delta \delta(k)), \]  
(3-20)

where \([Q]\) and \([R]\) are the weighting matrices. \([Q] \geq 0\) and \([R] > 0\) can penalise the state errors and the control input, respectively. In this work, \([R] = 1\) was chosen. \([Q]\) is decomposed into the following quadratic form:

\[ [Q] = [C]^T [C], \]  
(3-21)
where \([C] = \begin{bmatrix} [0_{2 \times (2n+2)}] \begin{bmatrix} \sqrt{W_{fa}} & 0 \\ 0 & \sqrt{W_{ra}} \end{bmatrix} \end{bmatrix} \) is a \(2 \times (2n+2)\) zero matrix. \([C] \in \mathbb{R}^{2 \times (2n+4)}\), which is a sparse matrix.

Then the cost function in equation (3-20) is rearranged by substituting equation (3-21) into (3-20).

\[
J_M = \sum_{k=1}^{\infty} \left( ([C] \Delta Z_M(k))^T ([C] \Delta Z_M(k)) + \Delta \delta(k) \Delta \delta(k)^T [R] \Delta \delta(k) \right),
\]

where \([C] \Delta Z_M(k) = \begin{bmatrix} \sqrt{W_{fa}} \Delta y_{fa}(k) \\ \sqrt{W_{ra}} \Delta y_{ra}(k) \end{bmatrix}\).

Before the original control input equation (3-4) is linearised in terms of the system state vector, \(\Delta Z_M, \Delta Z_M\) has to be converted into a new state vector, \(\Delta Z_{eq}\), which includes the heading angle of the rear trailer. \(\Delta Z_{eq}\) is defined as follows:

\[
\Delta Z_{eq} = [ \Delta Z'_D \Delta \theta_n \Delta y_{ra} ]^T,
\]

where \(\Delta Z_{eq} \in \mathbb{R}^{(2n+4) \times 1}\).

The state transformation from \(\Delta Z_M\) to \(\Delta Z_{eq}\) connects the cost function and the control input function. The two functions can eventually be represented by the same system state vector, \(\Delta Z_M\) through a transformation process.

A coordinate transformation matrix, \([T]\), is defined to transform \(\Delta Z_M\) to \(\Delta Z_{eq}\), as follows:

\[
\Delta Z_{eq}(k) = [T] \Delta Z_M(k)
\]

The linearised control input equation is now written in terms of \(\Delta Z_{eq}\) as follows:

\[
\Delta \delta(k) = -[K] \Delta Z_{eq}(k),
\]

where \([K]\) is the gain matrix and defined by:

\[
[K] = \begin{bmatrix} 0 & 0 & K_{r_1} & \cdots & K_{r_n} & 0 & \cdots & 0 & K_{\theta ra} & -K_{y ra} \end{bmatrix}
\]

According to equations (3-13) and (3-14), the relationship between the axle lateral offsets and the heading angle of the rear trailer is written as:

\[
\Delta \theta_n = \frac{1}{\sum_{j=0}^{n-1} l_{j+k} \cdot \text{eff}_n} \Delta y_{fa} - \frac{1}{\sum_{j=0}^{n-1} l_{j+k} \cdot \text{eff}_n} \Delta y_{ra} + \sum_{i=1}^{n} \frac{\sum_{j=0}^{i-1} l_{j+k} \cdot \text{eff}_n}{\sum_{j=0}^{i} l_{j+k} \cdot \text{eff}_n} \Delta \Gamma_i
\]

46
Consequently, \([T]\) can be defined as:

\[
[T] = \begin{bmatrix}
I_{2n+2} & 0 & 0 \\
S_{1,2n+2} & \frac{1}{\sum_{j=0}^{n-1} l_j + l_{eff_n}} & -\frac{1}{\sum_{j=0}^{n-1} l_j + l_{eff_n}} \\
0_{1,2n+2} & 0 & 1
\end{bmatrix}, \quad (3-28)
\]

where

\[
S_{1,2n+2} = \begin{bmatrix}
0 & 0 & \sum_{j=0}^{n-1} l_j & \ldots & \sum_{j=0}^{n-1} l_j \\
\sum_{j=0}^{n-1} l_j & \sum_{j=0}^{n-1} l_j & \ldots & \sum_{j=0}^{n-1} l_j & 0 & \ldots & 0
\end{bmatrix}, \quad (3-29)
\]

and \([S_{1,2n+2}]\) is a \(1 \times (2n + 2)\) matrix. \([I_{2n+2}]\) denotes a \((2n + 2) \times (2n + 2)\) identity matrix, and \([0_{1,2n+2}]\) stands for a \(1 \times (2n + 2)\) zero matrix. \([T] \in \mathbb{R}^{(2n+4) \times (2n+4)}\).

The transformation matrix, \([T]\), which converts the lateral offset of the front axle of the tractor, \(\Delta y_{fa}\), into the heading of the rear trailer, \(\Delta \theta_{ra}\), is purely based on the vehicle geometry. This is a similarity transformation \([177], [178]\).

After substituting equation (3-24) into (3-25), the linearised control input equation is as follows:

\[
\Delta \delta(k) = -[K][\Delta Z_{eq}(k)] = -[K][T][\Delta Z_M(k)] \quad (3-30)
\]

To determine the controller gains in (3-30), the quadratic cost function in equation (3-20) can be minimise, subject to (3-19), by forming the following discrete-time algebraic Riccati equation \([179], [180]\):

\[
[A_{MDT}]^T[S][A_{MDT}] - [S] - [A_{MDT}]^T[S][B_{MDT}] \left([B_{MDT}]^T[S][B_{MDT}] + [R]\right)^{-1}[B_{MDT}]^T[S][A_{MDT}] + [Q] = 0,
\]

where \([S]\) is the symmetric positive semi-definite solution of the associated Riccati equation.

It is necessary for the existence of a solution \([S]\) that the pair \([(Q), [A_{MDT}]\)] be observable and \([(A_{MDT}), [B_{MDT}]\)] be controllable \([170]\).

The LQR optimal gains can be derived from \([S]\) as follows:

\[
[K_{LQR}] = \left([B_{MDT}]^T[S][B_{MDT}] + [R]\right)^{-1}[B_{MDT}]^T[S][A_{MDT}]
\]

Combining equations (3-30) and (3-32) gives the full state feedback gain matrix, \([K_{LQR}]\).
\[ [K_{LQR}] = [K_{M_{LQR}}][T]^{-1} \]  

(3-33)

As shown in equation (3-30), \([K]\) is a partial state feedback gain matrix, yet \([K_{LQR}]\) has feedback gains for all the state variables. At a low constant longitudinal speed of the tractor unit, the gains for the articulation angles, the rear lateral offset, and the rear heading angle are much larger than the gains for the lateral velocity, the yaw velocity, and the articulation angle rates. The \(gain \times state\) products for the lateral velocity, the yaw velocity, and the articulation angle rates are negligible. It can be shown by plotting the poles of the closed loop system that the partial feedback gain matrix barely changes the eigenvalues of closed-loop matrix, thereby not affecting the controller’s stability. Consequently, \([K]\) is extracted from \([K_{LQR}]\) by setting the gains for the lateral and yaw velocities of the tractor and all articulation angle rates to zero.

Replacing \(\Delta \delta(k)\) in equation (3-19) with (3-30) generates the following closed-loop matrix, \([A_{cl}] \in \mathbb{R}^{(2n+4) \times (2n+4)}:\)

\[ [A_{cl}] = [A_{M_{DT}}] - [B_{M_{DT}}][K][T] \]

(3-34)

The characteristics of the closed-loop response can be determined by using the eigenvalues of \([A_{cl}]\). If all of the eigenvalues lie inside the unit circle, the system will be asymptotically stable [181], [182]. The damping ratio, \(\zeta_i\), corresponding to a stable eigenvalue is defined as follows.

\[ \zeta_i = |\cos(\angle \ln(\lambda_i^{cl}))|, \]

(3-35)

where \(\lambda_i^{cl}\) is an eigenvalue of the closed-loop system matrix, \([A_{cl}] (i = 1, 2, \ldots, 2n + 4)\). ‘\(\ln\)’ means the natural logarithm. The number of eigenvalues is the same as the rank of \([A_{cl}]\), which also equals the number of system states. The eigenvalue with the lowest damping ratio dominates the closed-loop response.

### 3.2 Simulation analysis

For some manoeuvres, e.g. lane changes, the lateral offsets are nearly equally distributed at both sides of a desired path. For other manoeuvres, e.g. roundabouts, the maximum excursion on one side exceeds that on the other side significantly. The sum of two maximum deviations on both sides of the desired path can be used to decide whether or not vehicle units are able to complete the manoeuvre. It is a key parameter of the overall swept path to determine how much space is needed. Hence, the distance from one side peak to the other, \(|y_{lat}|_{max}\) defined below,
is calculated as an underlying indicator for the tuning methods described in the subsequent sections.

\[ |y_{lat\text{max}} = \max(|y_{fa}, y_{ra}, 0|) + \min(|y_{fa}, y_{ra}, 0|) \]  

(3-36)

The criteria for the tuning method are to keep a balance between \( |y_{fa}\text{max} \) and \( |y_{ra}\text{max} \), and ensure that the maximum width of the swept path, \( |y_{lat}^{\text{MSPC}}\text{max} \), using the MSPC method does not exceed the swept path, \( |y_{lat}^{\text{PFC}}\text{max} \), using the baseline PFC method. Hence, the distance from one side peak to the other using the PFC method is also an upper bound for the maximum absolute value of the lateral offsets using the MSPC method, as represented in equation (3-37).

\[ \max \left( |y_{fa}\text{max}^{\text{MSPC}}, |y_{ra}\text{max}^{\text{MSPC}} \right) \leq |y_{lat}\text{max}^{\text{MSPC}} \leq |y_{lat}\text{max}^{\text{PFC}} \]  

(3-37)

The preview distance, \( L_{pd} \), can change the front and rear lateral offsets significantly. The optimised preview distance depends on the details of desired paths because of the above criteria, regardless of the controller weights. This means that for each target path, varying the preview distance exhibits an underlying trend. On the other hand, for each given preview distance, adjusting the weights is believed to give comparable performance. For instance, an increase in \( W_{fa} \) penalises the rear offsets more when \( W_{ra} \) remains unchanged. Therefore, the preview distance is tuned and determined before optimising the controller weights.

The desired path used in the initial simulations to tune the controller gains is shown in Figure 3.5. It is used to simulate a 20 m lane change manoeuvre of the vehicle units in reverse. This is an extreme manoeuvre with continuously changing curvature to test the performance of the proposed controller. Tuning the controller gains for other paths is expected to give similar tuning effect. This means tuning for the lane change manoeuvre implies the controller will also be effective on other manoeuvres.

The longitudinal velocity of the tractor is assumed to be -1 m/s, where the minus sign indicates moving backwards. The maximum natural frequency and selected sample time of both the tractor-semitrailer and B-double combinations are shown in Table 3.1.

### 3.2.1 Optimum preview distance analysis

For the manoeuvre in Figure 3.5 and the simulated tractor-semitrailer configuration, the preview distance, \( L_{pd} \), was varied to find the optimal value. The ratio of the controller weights, \( r \), is defined as follows:
\[ r = \frac{W_{fa}}{W_{ra}} \]  \hspace{2cm} (3-38)

A sample time history showing \( y_{fa} \) and \( y_{ra} \) during the lane change manoeuvre is presented in Figure 3.6. It is also seen that \( |y_{lat|_{\text{max}}^{MSPC}} \) is from one side peak to the other. Figure 3.7 (a) shows variations of the maximum axle lateral offsets of the front and rear axles \( (|y_{fa}|_{\text{max}} \) and \( |y_{ra}|_{\text{max}} \), as a function of the preview distance, \( L_{pd} \), for different combinations of \( W_{fa} \) and \( W_{ra} \). The ratio, \( r \), spans a wide range from 0.1 to 10. A sharp minimum, corresponding to the optimised preview distance, can be seen for different combinations of the controller weights.

In this case, the optimised preview distance is approximately 6 m. The optimised preview distance is independent of the controller weights. When \( L_{pd} \) is very small, the MSPC controller tends towards the PFC controller, giving very large front offsets and very small rear offsets. Increasing \( L_{pd} \) reduces \( |y_{fa}|_{\text{max}} \) at the expense of \( |y_{ra}|_{\text{max}} \) until the large rear offsets have a negative impact on the front offsets. This leads to the simultaneous increases in both \( |y_{fa}|_{\text{max}} \) and \( |y_{ra}|_{\text{max}} \), when \( L_{pd} \) increases beyond the optimised preview distance.

The maximum width of the swept path, \( |y_{lat|_{\text{max}}^{MSPC}} \), is plotted against the preview distance in Figure 3.7 (b). This graph is used to check whether the optimised preview distance violates the tuning criteria. As seen in Figure 3.7 (b), all solid lines follow the same pattern. At the pre-selected optimised preview distance, \( |y_{lat|_{\text{max}}^{MSPC}} \) is the lowest compared to that at the other preview distances for all values of \( W_{fa} \) and \( W_{ra} \). \( |y_{lat|_{\text{max}}^{MSPC}} \) is always less than \( |y_{lat|_{\text{max}}^{PFC}} \) using the baseline PFC method (the dashed line in Figure 3.7 (b)).

The optimised preview distance is plotted as a function of the total length in Figure 3.8. The ratio between the optimised preview distance and the combination length is approximately constant, independent of the controller weights, \( W_{fa} \) and \( W_{ra} \). This relationship is expected to change slightly for different manoeuvres.

Unlike the lane change manoeuvre, which has nearly equal excursions on both sides, the roundabout manoeuvre depicted in Figure 3.9 has a dominant side, as there is a long steady-state part in the middle, and the tractor stays on one side of the centre line for most of the time and only moves to the other side during the period of transition. Figure 3.10 (a) shows the impact of variations of \( L_{pd} \) on the \( |y_{fa}|_{\text{max}} \) and \( |y_{ra}|_{\text{max}} \) for \( W_{fa} = 1 \) and \( W_{ra} = 0.1 \). As depicted in Figure 3.10 (a), to keep a balance between \( |y_{fa}|_{\text{max}} \) and \( |y_{ra}|_{\text{max}} \), the optimised preview distance is around 6 m, the same range as the lane change manoeuvre.
However, the presumed range violates the tuning criteria, as seen in Figure 3.10 (b), where \( |y_{lat}^{MSPC}| \) is plotted as a function of \( L_{pd} \) against the baseline \( |y_{lat}^{PFC}| \). When \( L_{pd} \) is in the neighbourhood of 6 m, \( |y_{lat}^{MSPC}| \) is larger than \( |y_{lat}^{PFC}| \). The minimum is found when \( L_{pd} \) is around 2 m. This is because after a threshold, the rear offset grows rapidly, and the increase in the rear axle error cannot be compensated by the reduction in the front axle error. Therefore, the overall reduction in the swept path deteriorates by using a relatively large preview distance.

It is found that the tractor excursions dominate both sides, and the smaller of the front offset peaks on both sides, given in equation (3-39), is the threshold. Hence, according to the Liebig’s law of the minimum [183], [184], on the side with less overshoot, where the smaller peak of the front offsets exists, the path-tracking errors are constrained from increasing. The reduction in the front axle errors is also bounded.

\[
y_{th} = \left| \min(\max(y_{fa}), \min(y_{fa}), 0) \right|,
\]

(3-39)

where

- \( y_{fa} \): the front axle lateral offset on the both sides. It comes with the positive or negative sign;
- \( \min(\max(y_{fa}), \min(y_{fa}), 0) \): zero or the front offset peak in the direction of the negative ‘Y axis’;
- \( y_{th} \): the threshold and upper bound for the rear axle lateral offset on the less overshoooting side, where the relatively smaller front offset peak exists.

Both relationships are investigated and plotted for the B-double (twin trailer) combination in the case of the lane change and roundabout manoeuvres, as shown in Figures 3.11 and 3.12. As the total length of the B-double vehicle is increased, the optimised preview distance is increased.

The precise optimised preview distances for the tractor-semitrailer and B-double vehicles in the case of the lane change and roundabout manoeuvres are provided in Table 3.2.

### 3.2.2 Gain tuning approach

After optimising the preview distance, the controller gains need to be tuned for the tractor-semitrailer and B-double vehicles in the case of the lane change manoeuvre depicted in Figure 3.5.
For the tractor-semitrailer combination, contour diagrams of the maximum axle lateral offsets, $|y_{fa}|_{max}$ and $|y_{ra}|_{max}$, with the optimised preview distance, $L_{pd}$, set to 5.84 m, are plotted in Figure 3.13 to show the relationship with weights, $W_{fa}$ and $W_{ra}$.

Two opposite trends for reducing $|y_{fa}|_{max}$ and $|y_{ra}|_{max}$ are seen in Figure 3.13. $|y_{fa}|_{max}$ is minimum in the bottom right of Figure 3.13 (a) (high $W_{fa}$ and low $W_{ra}$). $|y_{ra}|_{max}$ is minimum in the top left of Figure 3.13 (b) (low $W_{fa}$ and high $W_{ra}$). When $W_{ra}$ is fixed, increasing $W_{fa}$ reduces $|y_{fa}|_{max}$ but increases $|y_{ra}|_{max}$. When $W_{fa}$ is fixed, increasing $W_{ra}$ has an opposite effect on $|y_{fa}|_{max}$ and $|y_{ra}|_{max}$. As illustrated in Figure 3.13, when the ratio, $r = \frac{W_{fa}}{W_{ra}}$ in equation (3-38), is appropriate, there is an approximate linear relationship between $W_{fa}$ and $W_{ra}$ for the contour lines (e.g. the line with $|y_{fa}|_{max} = 0.69$ m in Figure 3.13 (a) and the line with $|y_{ra}|_{max} = 0.27$ m in Figure 3.13 (b)). When $W_{ra}$ is very low and therefore $r$ is large, varying $W_{fa}$ has an insignificant impact on $|y_{fa}|_{max}$ and $|y_{ra}|_{max}$ (e.g. the line with $|y_{fa}|_{max} = 0.63$ m in Figure 3.13 (a) and the line with $|y_{ra}|_{max} = 0.33$ m in Figure 3.13 (b)). The two lines nearly level off regardless of the changes in $W_{fa}$ once $r$ becomes large.). When $W_{ra}$ is very large and therefore $r$ is very small, the slope of the contour lines becomes very steep, which means a very high $W_{fa}$ is necessary to keep $|y_{fa}|_{max}$ and $|y_{ra}|_{max}$ constant (e.g. the line with $|y_{fa}|_{max} = 0.87$ m in Figure 3.13 (a) and the line with $|y_{ra}|_{max} = 0.18$ m in Figure 3.13 (b)). In these ranges, the minimum damping ratio, $\zeta_i$, is near to 1, so all eigenvalues lie on the negative real axis and the motion is well damped.

As there are two controller weights, keeping one weight constant and varying the other is a way to investigate the relationship between the weights and the maximum axle excursions. A reasonable range for both weights is from 0 to 10, but both cannot be zero at the same time, because $[Q]$ must be a positive semi-definite matrix and $([A_{M,x}],[Q])$ must be observable [172].

A conflict diagram plotting the maximum excursion of the front axle in the lane change manoeuvre against the maximum excursion of the rear axle is drawn in Figure 3.14. In Figure 3.14, each solid line represents varying $W_{ra}$ with a fixed $W_{fa}$. It is seen that increasing one of the axle weights penalises the other axle’s response. For example, increasing $W_{fa}$ reduces $|y_{fa}|_{max}$, but increases $|y_{ra}|_{max}$. Likewise, decreasing $W_{ra}$ gives the same effect, making the
front axle excursions smaller and the rear axle’s larger. The diagram also shows contours of the ratio, \( r = \frac{W_{fa}}{W_{ra}} \), (dashed lines). Along each of these contours, although the values of weights vary, their ratio, \( r \), is a constant. Varying \( W_{fa} \) with a fixed \( W_{ra} \) will generate a similar conflict diagram.

One way to decide on the best weights would be to strike a balance between \( |y_{ra}|_{\text{max}} \) and \( |y_{fa}|_{\text{max}} \). The line of \( |y_{fa}|_{\text{max}} = |y_{ra}|_{\text{max}} \) is drawn on the same conflict diagram but with adjusted axis scales including zero, in Figure 3.15. It is observed that no matter which weighting set is selected, \( |y_{fa}|_{\text{max}} \) is always greater than \( |y_{ra}|_{\text{max}} \). Considering the shortest distance perpendicular to the line in Figure 3.15 gives the set, \( W_{fa} = 1 \) and \( W_{ra} = 0.1 \), i.e. point ‘A’. This implies \( |y_{fa}|_{\text{max}} \approx 0.60 \text{ m} \) and \( |y_{ra}|_{\text{max}} \approx 0.35 \text{ m} \). The least error norm \((\|\text{error}\|_2^2 = |y_{fa}|_{\text{max}}^2 + |y_{ra}|_{\text{max}}^2)\) is also considered as a possible criterion and is shown in Figure 3.15. The same set, \( W_{fa} = 1 \) and \( W_{ra} = 0.1 \), is found, which shows both criteria gave the same solution. For these weights, \( \zeta \) is approximately 1.

The optimal weights, \( W_{fa} = 1 \) and \( W_{ra} = 0.1 \), are plotted on the contours of \( |y_{fa}|_{\text{max}} \) and \( |y_{ra}|_{\text{max}} \) in Figure 3.16, indicating the possible ranges of \( |y_{fa}|_{\text{max}} \) and \( |y_{ra}|_{\text{max}} \).

In the B-double case, a similar conflict diagram is plotted in Figure 3.17, along with the contours of \( r \). The same trends as the tractor-semi trailer case are found. The same weight selection criteria applied for the B-double vehicle give the optimal weights, \( W_{fa} = 1 \) and \( W_{ra} = 0.1 \), i.e. point ‘B’ in Figure 3.18, implying that \( |y_{fa}|_{\text{max}} \approx 1.45 \text{ m} \) and \( |y_{ra}|_{\text{max}} \approx 1.23 \text{ m} \).

The optimum weights and corresponding gains for the tractor-semi trailer and B-double vehicles are shown in Table 3.3.

3.2.3 Comparison with the baseline PFC method

3.2.3.1 Tractor-semi trailer case

The vehicle motion paths in reverse for the lane change manoeuvre are sketched in Figure 3.19 (a). The tractor-semi trailer vehicle reverses from right to left. The blue lines stand for the tractor unit and the red lines denote the semitrailer.
The simulation results for the reversing manoeuvre, comparing the paths of the tractor-semitrailer vehicle for the proposed MSPC with $W_{fa} = 1$, $W_{ra} = 0.1$ and $L_{pd} = 5.84$ (the tuned static optimised preview distance), and the corresponding optimal pure PFC with $W_{ra} = 5$ and the dynamic preview distance, are plotted in Figure 3.19 (b). The axle lateral offsets are compared in Figure 3.19 (c). Both controllers can follow the desired path, but there are visible differences in both axles’ excursions. From Figure 3.19 (c), it is seen that with the MSPC, the front axle’s maximum lateral offset (solid curve) is much less than that of the PFC (dashed curve) by more than 45%. However, the rear axle has a comparatively larger excursion of 0.35 m. As the maximum excursions for the lane change manoeuvre are nearly equal at both sides, the distance from one side peak to the other is also reduced by more than 45%. This means the vehicle units take up 45% less space when they perform the lane change manoeuvre, compared to the PFC method.

For the roundabout manoeuvre depicted in Figure 3.9, the same optimal gains as the lane change manoeuvre but the different optimised preview distance ($L_{pd} = 2$) are used for the simulation studies. The vehicle motion paths for the reversing manoeuvre are drawn in Figure 3.20 (a). The track performance comparison between the PFC and MSPC methods is made in Figure 3.20 (b). It is seen that for the roundabout manoeuvre, the tractor stays outside of the reference path all the time except during the transition period, during which the tractor moves below the straight line to readjust the semitrailer’s position. This is the main reason for the deviations for the roundabout being so unequal.

The lateral offsets are plotted against the axle travelling distances in Figure 3.20 (c). It is seen that the tractor’s excursions dominate both sides, but the maximum deviation in the direction of the negative ‘Y axis’ is much smaller than that in the opposite direction. As explained in the section of ‘Optimum preview distance analysis’, in this case, the peak front offset in the direction of the negative ‘Y axis’ is the threshold, $y_{th}$, which constrains the reduction of the path-tracking errors. Otherwise, if the rear offsets are relaxed too much by the MSPC method, exceeding the tractor extreme excursion on the less overshooting side, the decrease in $|y_{fa}|_{max}$ cannot offset the increase in $|y_{ra}|_{max}$. Hence, the $|y_{lat}|_{max}^{MSPC}$ will become far larger, beyond the baseline $|y_{lat}|_{max}^{PFC}$, as seen above in Figure 3.10 (b). Due to this constraint, the overall swept path for the PFC method (dashed curves) can only be reduced by 10% by employing the MSPC method (solid curves).
3.2.3.2 B-double case

The MSPC algorithm was used for the B-double vehicle with the same tuned weights ($W_{fa} = 1$ and $W_{ra} = 0.1$) but different optimised preview distances. The vehicle motion for the lane change manoeuvre is drawn in Figure 3.21 (a). The lines denoting the tractor and semitrailer units remain the same, and the additional lines, the black lines, stand for the ‘B-link’ trailer, which joins the other two vehicle units. They travel backwards from right to left.

Figure 3.21 (b) shows the tracking performance of the front and rear axles for the PFC and MSPC methods. Noticeable differences in both axles’ excursions are found. As seen in Figure 3.21 (c), the maximum absolute value of the front axle lateral offsets is reduced from 2.87 m for the PFC method to 1.45 m for the MSPC method. The maximum width of the swept path is reduced by approximately 57%, as the maximum excursions for both axles span equally at both sides. $|y_{fa}|_{max}$ and $|y_{ra}|_{max}$ are balanced at the same level. The conflict diagram for the B-double vehicle in Figure 3.18 also shows that the point ‘B’ is very close to the line of $|y_{fa}|_{max} = |y_{ra}|_{max}$, which meets one of the tuning criteria. In this case, the relaxation of the path-tracking errors is fully exploited, and therefore, the overall reduction is almost maximised (about 57%), compared to the tractor-semitrailer case (about 45%). The B-double combination can reverse through much narrower lanes when controlled by MSPC than by the baseline PFC method.

Figure 3.22 (a) illustrates the B-double vehicle performing the roundabout manoeuvre. The tractor unit travels further away from the reference path, compared to the case of the tractor-semitrailer vehicle, due to the increase in the total length of the combination. It is seen in Figure 3.22 (b) that around the steady-state part, there is no visible difference between the PFC and MSPC trajectories. However, the reversing tracks tend to part from each other in the transition phases. The semitrailer moves inwards to pull the tractor unit closer to the target path.

The lateral offsets throughout the roundabout simulation are plotted against the rear axle’s travelling distances in Figure 3.22 (c). The reduction in the maximum excursion is about 33% at the expense of the path-tracking errors, which is much larger than the tractor-semitrailer case (about 10%). The reason for the improvement is that the tractor’s excursions dominate both sides, and the relatively smaller front offset peak, $y_{th}$, which is in the direction of the negative ‘Y axis’ in the B-double case, is much larger than the tractor-semitrailer case, enabling the controller to further reduce the swept path. The maximum width of the swept path is reduced from 17.83 m for the PFC method to 12.60 m for the MSPC method.
3.3 Conclusions

1. A Minimum Swept Path Control (MSPC) strategy was devised to improve the performance of autonomous reversing of articulated vehicles. This approach guarantees the accuracy of following a desired path in reverse, and reduces the maximum excursion of the vehicle units significantly.

2. Two additional states, $y_{fa}$ and $y_{ra}$, were added to the original vehicle dynamics model built in Chapter 2, and two corresponding weights, $W_{fa}$ and $W_{ra}$, were placed on them in a cost function for minimising the excursions of the vehicle from a target path in reverse motion. The modified model was linearised and discretised. Then, a discrete-time state space model was built for the controller development and tuning.

3. The preview distance was optimised before the weight selection. It is subject to the distribution of axle deviations along target paths, regardless of the controller weights. The tractor excursions dominate both sides of the desired path. According to the Liebig’s law of the minimum, the peak on the less overshooting side (i.e. the relatively smaller front offset peak, $y_{fh}$) restricts the overall reduction in the swept path for the MSPC method. When the rear offset becomes larger than this threshold, the increase in the rear errors cannot be offset by the reduction in the front errors. This is the main reason why the reduction in the swept path is bounded for the manoeuvres with unequally distributed excursions, e.g. roundabout manoeuvres.

4. An empirical relationship between the lateral offsets of both axles and the corresponding weights in the control cost function was found and used to select the optimal controller weights. Linear Quadratic Regulator (LQR) theory was used to tune the controller gains by solving the discrete time Riccati equation.

5. The MSPC controller significantly reduces the swept path for the manoeuvres with equally distributed excursions, e.g. lane change manoeuvres.

6. The overall swept path reduction is more substantial for long combination vehicles, e.g. the B-double vehicle, as employing the PFC method causes much larger maximum excursions. For the lane change manoeuvre, the reduction of MSPC compared to PFC is about 45% for the tractor-semitrailer vehicle and about 57% for the B-double vehicle; for the roundabout manoeuvre, the reduction is about 12% for the tractor-semitrailer vehicle and about 33% for the B-double vehicle. The trade-off is also satisfactory in both cases.
7. The controller is capable of good performance for different reference paths, e.g. lane change manoeuvres with constantly changing curvature, and roundabout manoeuvres with the steady-state phase (i.e. constant curvature in the middle).
### 3.4 Tables

**Table 3.1 The maximum natural frequency and sample time of the discrete-time models at the speed of -1 m/s**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Vehicle combinations</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tractor-semitrailer</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B-double</td>
<td></td>
</tr>
<tr>
<td>Maximum natural frequency, $w_{max}$</td>
<td>28.89</td>
<td>23.37 Hz</td>
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<tr>
<td>Maximum sample time</td>
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<td>0.021 s</td>
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<tr>
<td>Selected sample time for the simulations, $T_s$</td>
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<td>4.28e-4 s</td>
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</table>

**Table 3.2 The optimised preview distances for the tractor-semitrailer and B-double vehicles**

<table>
<thead>
<tr>
<th>Optimised preview distance, $L_{pd}$</th>
<th>Vehicle combinations</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tractor-semitrailer</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B-double</td>
<td></td>
</tr>
<tr>
<td>$L_{pd}$ for the lane change manoeuvre</td>
<td>5.84</td>
<td>8.84 m</td>
</tr>
<tr>
<td>$L_{pd}$ for the roundabout manoeuvre</td>
<td>2</td>
<td>5.84 m</td>
</tr>
</tbody>
</table>

**Table 3.3 The optimal weights and gains for the tractor-semitrailer and B-double vehicles**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Vehicle combinations</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tractor-semitrailer</td>
<td>B-double</td>
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<tr>
<td>$W_{fa}$</td>
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<td>1</td>
</tr>
<tr>
<td>$W_{ra}$</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$K_{f1}$</td>
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<tr>
<td>$K_{f2}$</td>
<td>-</td>
<td>-36.23</td>
</tr>
<tr>
<td>$K_{dera}$</td>
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</tr>
<tr>
<td>$K_{yra}$</td>
<td>1.05</td>
<td>-1.05</td>
</tr>
</tbody>
</table>
3.5 Figures

(a) Path Following Control (PFC) using a common tractor-semitrailer combination as an example. The dashed lines represent the future positions of vehicle units.

(b) Minimum Swept Path Control (MSPC) using the same tractor-semitrailer combination presented in (a). The dashed lines represent the future positions of vehicle units.

Figure 3.1 Comparison between Path Following Control and Minimum Swept Path Control
Figure 3.2 Relationship between the front and rear axle tracking errors

Figure 3.3 MSPC Strategy. Intermediate trailer units between the tractor and the rear semitrailer are represented by articulation angle, $\Gamma_i$
Figure 3.4 Calculation of the equilibrium articulation angle of the rear trailer

Figure 3.5 The lane change manoeuvre
Figure 3.6 A sample time history of lateral offsets during the lane change manoeuvre, when $W_{fa} = 1$, $W_{ra} = 1$ and $L_{pd} = 7$ m
(a) The optimised preview distance selection for the tractor-semitrailer vehicle performing the lane change manoeuvre

(b) Relationship between the maximum width of the swept path and the preview distance in the case of the tractor-semitrailer vehicle performing the lane change manoeuvre

Figure 3.7 Tuning results in the case of the tractor-semitrailer vehicle performing the lane change manoeuvre
Figure 3.8 Relationship between the optimised preview distance and the total length of vehicles in the case of the tractor-semitrailer vehicle performing the lane change manoeuvre, when given $W_{fr} = 1$ and $W_{ra} = 1$

Figure 3.9 The roundabout manoeuvre
(a) Relationship between $|y_f|_{\text{max}}$ and $|y_r|_{\text{max}}$ in the case of the tractor-semitrailer vehicle performing the roundabout manoeuvre

(b) Relationship between the maximum width of the swept path and the preview distance in the case of the tractor-semitrailer vehicle performing the roundabout manoeuvre

Figure 3.10 Tuning results for the tractor-semitrailer vehicle performing the roundabout manoeuvre
(a) Relationship between $|\gamma_{fa}|_{\text{max}}$ and $|\gamma_{ra}|_{\text{max}}$ in the case of the B-double vehicle performing the lane change manoeuvre

(b) Relationship between the maximum width of the swept path and the preview distance in the case of the B-double vehicle performing the lane change manoeuvre

Figure 3.11 Tuning results in the case of the B-double vehicle performing the lane change manoeuvre
(a) Relationship between $|\gamma_{fa}|_{\text{max}}$ and $|\gamma_{ra}|_{\text{max}}$ in the case of the B-double vehicle performing the roundabout manoeuvre

(b) Relationship between the maximum width of the swept path and the preview distance in the case of the B-double vehicle performing the roundabout manoeuvre

Figure 3.12 Tuning results in the case of the B-double vehicle performing the roundabout manoeuvre
(a) Contours of $|y_f\alpha|_{\max}$ for the tractor-semitrailer vehicle

(b) Contours of $|y_r\alpha|_{\max}$ for the tractor-semitrailer vehicle

Figure 3.13 Contours of the lateral offsets for the tractor-semitrailer vehicle
Figure 3.14 Conflict diagram of $|y_{fa}|_{max}$ versus $|y_{ra}|_{max}$ for fixed $W_{fa}$ for the tractor-semitrailer vehicle

Figure 3.15 Weight selection criteria for the tractor-semitrailer vehicle
(a) Optimal working weights ($W_{fa} = 1$ and $W_{ra} = 0.1$, e.g. the red cross point) plotted on the contours of $|y_{ra}|_{\text{max}}$ for the tractor-semitrailer vehicle.

(b) Optimal working weights ($W_{fa} = 1$ and $W_{ra} = 0.1$, e.g. the red cross point) plotted on the contours of $|y_{ra}|_{\text{max}}$ for the tractor-semitrailer vehicle.

Figure 3.16 Optimal working weights plotted on the contours of the lateral offsets for the tractor-semitrailer vehicle.
Figure 3.17 Conflict diagram of $|y_{fa}|_{max}$ versus $|y_{ra}|_{max}$ for fixed $W_{fa}$ for the B-double vehicle

Figure 3.18 Weight selection criteria for the B-double vehicle
(a) The motion paths of the tractor-semitrailer vehicle performing the lane change manoeuvre

(b) Tracking performance comparison between the PFC and MSPC methods for the tractor-semitrailer vehicle performing the lane change manoeuvre

(c) Lateral offset comparison between the PFC and MSPC methods for the tractor-semitrailer vehicle performing the lane change manoeuvre

Figure 3.19 Simulation results of the tractor-semitrailer vehicle performing the lane change manoeuvre
(a) The motion paths of the tractor-semitrailer vehicle performing the roundabout manoeuvre

(b) Tracking performance comparison between the PFC and MSPC methods for the tractor-semitrailer vehicle performing the roundabout manoeuvre

(c) Lateral offset comparison between the PFC and MSPC methods for the tractor-semitrailer vehicle performing the roundabout manoeuvre

Figure 3.20 Simulation results of the tractor-semitrailer vehicle performing the roundabout manoeuvre
(a) The motion paths of the B-double vehicle performing the lane change manoeuvre

(b) Tracking performance comparison between the PFC and MSPC methods for the B-double vehicle performing the lane change manoeuvre

(c) Lateral offset comparison between the PFC and MSPC methods for the B-double vehicle performing the lane change manoeuvre

Figure 3.21 Simulation results of the B-double vehicle performing the lane change manoeuvre
(a) The motion paths of the B-double vehicle performing the roundabout manoeuvre

(b) Tracking performance comparison between the PFC and MSPC methods for the B-double vehicle performing the roundabout manoeuvre.

(c) Lateral offset comparison between the PFC and MSPC methods for the B-double vehicle performing the roundabout manoeuvre.

Figure 3.22 Simulation results of the B-double vehicle performing the roundabout manoeuvre
Chapter 4 Implementation of Minimum Swept Path Control (MSPC)

This chapter mainly discusses the experimental evaluation of MSPC for autonomous reversing of articulated vehicles and illustrates the vehicle testing framework. The controller for the MSPC method was implemented on the full-size tractor-semitrailer and B-double (twin trailer) combinations owned by CVDC. An inner-loop compensator using the PID method was developed and tuned to track the desired hand wheel steer angle generated by the main controller. Two reversing manoeuvres used in the simulations; the lane change with continuously changing curvature (Figure 3.5), and the roundabout with constant curvature in the middle (Figure 3.9), were performed to test the performance of the controller under transient and steady-state conditions. The field tests took place at an airfield in Bourn, Cambridgeshire, and the results of the testing are discussed in the subsequent section. The vehicle testing was also filmed.

4.1 Vehicle testing framework

4.1.1 Vehicle configuration

The test vehicles are shown in Figure 4.1, and the vehicle parameters are shown in Tables 2.1 – 2.3. The first rear axle of the tractor and the B-link trailer were lifted to match the vehicle dynamics model in simulation. The actively steered system on the rear trailer was switched off so the rear axles of the trailers were locked in the straight-ahead position. Both trailers were unladen.

4.1.2 System architecture

The equipment layout is presented in Figure 4.2, and the overall system architecture is shown in Figure 4.3. Figure 4.2 illustrates the approximate location of equipment mounted on each vehicle unit and the physical connections. Figure 4.3 shows how main signals flows. The integrated control system consisted of two subsystems: the MSPC subsystem and the steer angle tracking subsystem.

An Oxford Technical Solutions (OxTS) RT3022 dual antenna inertial and GPS navigation system [185] was mounted at the rear of the semitrailer to determine the speed, position, and heading of the semitrailer and communicate these signals to the global controller in the tractor cabin via a Controller Area Network (CAN) bus. Figure 4.4 (a) shows the measurement origin of the inertial measurement unit (IMU) was accurately mounted on top of the ‘equivalent rear axle’ of the semitrailer, which corresponded to the cross point of the two orthogonal lines.
shown in Figure 4.4 (b). The two magnetic antennas were located on the centre of steel plates, mounted on the rooftop of the semitrailer, as shown in Figure 4.4 (c). The antennas were mounted in the same orientation and at least 0.3 m from the edges of the steel plates. The mounting bracket and the steel plates were designed to secure the IMU and antennas and also to determine their position and orientation precisely, as seen in Figures 4.4 (c) and 4.4 (d). The IMU was connected to a laptop via an Ethernet cable. A warm up process, which is thoroughly explained in Section 4.1.6, was used to offset some slight misalignments caused by the hardware installation.

The GPS base station [186] shown in Figure 4.5 was used on the testing field to improve the measurement accuracy by providing Real Time Kinematic (RTK) differential corrections via radio modem aerials. One aerial was placed on the rooftop of the semitrailer, as seen in Figure 4.4 (c), and the other one was placed on the rooftop of a stationary car, as seen in Figure 4.5. The car battery was used to power the base station.

The Vehicle Systems Engineering (V.S.E.) articulation angle sensors [187], [188] were installed on the kingpins of the trailers to measure the articulation angles between the vehicle units. The calibration arrangements are shown in Figure 4.6. The analogue articulation angle signals were converted to 16-bit digital form, through a low pass filter and an analogue-digital converter [189]. These signals were then fed into the MSPC subsystem via the CAN bus.

Figure 4.7 shows that the string potentiometer [190] attached to the front left steering radius arm on the tractor unit. The analogue signal was low-pass filtered, digitised, and fed into the steer angle tracking subsystem via the CAN bus. The steering string pot was calibrated to measure the effective ‘single-track-average’ steer angle.

An Anthony Best Dynamics (AB Dynamics) SR30 steering robot [191] replaced the original steering wheel through a steering column adapter [192] and was connected to its ‘Omni’ controller, presented in Figure 4.8. The robot was powered by its own battery pack. Figure 4.9 illustrates the main components of the steering robot, e.g. the joystick with the start button and activation switch. The steering wheel could be manually steered when the robot was not in use. The ‘Omni’ controller was connected to the laptop via a USB connection and to the global controller via the CAN bus.

The tractor speed was obtained from CAN messages read from its FMS port.
The real time system was run on an xPC for the implementation on the semitrailer and on a dSpace AutoBox [193] for the B-double implementation. The xPC, shown in Figure 4.8, was a dual-core target PC without operating systems and had two Softing AC2-PCI dual CAN bus cards [194] in the PCI slots. The AutoBox had a DS1007 dual-core board (2.0 GHz) [195], [196] and a DS4302 CAN interface board [197], as seen in Figure 4.10. Both the xPC and Autobox were connected to the laptop via an Ethernet cable.

4.1.3 Steering system calibration

The relationship between the hand wheel angle, the effective ‘single-track-average’ steer angle, and the sensor voltage of the string potentiometer was measured. This relationship was non-linear due to steering system imperfections (e.g. backlash).

Figure 4.11 (a) depicts the calibration method for this relationship. The steering robot was initiated to perform a ‘lock to lock’ manoeuvre and then set the zero position of the steering wheel and front wheels. The black rectangle represents a front wheel at zero position, and the dashed rectangle shows the rotated front wheel. Aluminium sheets were set up, at a known distance perpendicular to the front wheel at zero position, as seen in Figure 4.11 (b). The vertical distance between the aluminium sheets and the centre of the front wheel, \(d_f\), was measured. Two lasers [198], [199] on the heavy duty mounting clamps [200] were mounted on a magnetic base, which was attached to the vertical line passing through the centre of the steel wheel trim. One laser was aligned to point to the aluminium sheets horizontally, and the other one was pointed to the origin of the local coordinate frames vertically, as shown in Figure 4.11 (c). As the front wheel rotated, the horizontal and vertical coordinates, \(d_m\) and \(l_m\), and the moving distance on the aluminium sheets, \(l_f\), were measured in synchronisation. The two lasers were set up very close to each other and the distance between them was negligible compared to the other measurements. A positive front wheel angle, \(\theta\), was assumed for an anti-clockwise rotation and a negative \(\theta\) for a clockwise rotation. \(\theta\), as illustrated in Figure 4.11 (a), was calculated as follows:

\[
\theta = -\arctan \left( \frac{l_f + l_m}{d_f + d_m} \right),
\]

where \(l_f\), \(l_m\), \(d_f\) and \(d_m\) were assumed positive.

\(\theta\) for an anti-clockwise rotation is calculated as follows:
\[ \theta = \arctan \left( \frac{l_f - l_m}{d_f - d_m} \right), \]  

(4-2)

The calibration was simultaneously carried out for both front wheels from the zero position, to the ‘lock to lock’, and then back to the zero position. Throughout the whole process, the angle sensor on the steering robot was used to monitor the hand wheel angle.

The measured left and right front wheel angles, \( \theta_l \) and \( \theta_r \), were plotted against the hand wheel angle, as depicted in Figure 4.12. It is seen that with the hand wheel angle increasing, there was an obvious difference between \( \theta_l \) (the circular marks) and \( \theta_r \) (the cross marks). It shows that there was good Ackermann steering geometry [201] for the tractor, which satisfied the assumption of the vehicle dynamics model in Chapter 2. As seen in Figure 4.12, some hysteresis occurred when the front wheels rotated back.

As illustrated in Figure 4.13, the effective ‘single-track’ steer angle was calculated from the outer and inner front wheel angles, \( \theta_o \) and \( \theta_i \), respectively, as follows.

\[ \delta_o = \arctan \left( \frac{l_{eff \; o}}{\tan(\theta_o) \; \frac{wd}{2}} \right), \]  

(4-3)

\[ \delta_i = \arctan \left( \frac{l_{eff \; i}}{\tan(\theta_i) \; \frac{wd}{2}} \right), \]  

(4-4)

where the subscripts, ‘o’ and ‘i’, stand for the outer and inner front wheels.

- \( \delta_o \): the effective ‘single-track’ steer angle calculated from the outer front wheel;
- \( \delta_i \): the effective ‘single-track’ steer angle calculated from the inner front wheel;
- \( l_{eff} \): the tractor wheelbase;
- \( wd \): the tractor width.

Figure 4.14 shows the effective ‘single-track’ steer angles calculated from the measured left and right wheel angles, \( \delta_i | \theta_i \) and \( \delta_r | \theta_r \), were in good agreement. The effective ‘single-track-average’ steer angle, \( \delta \), given by the following equation, was the only input to the control system.

\[ \delta = \frac{\delta_i + \delta_r}{2} \]  

(4-5)

The relationship between the effective ‘single-track-average’ steer angle, \( \delta \), and the voltage of the string potentiometer, \( v_s \), was investigated and can be seen in Figure 4.15 (a). Linear and
polynomial regression methods [202] were used to fit the data points. The linear fit was found to be sufficiently accurate. Consequently, the following linear relationship was used.

\[ \delta = 0.16059 \times (v_s - u_s), \]  

(4-6)

where \( v_s \) is the sensor voltage and \( u_s \) is the sensor bias.

The string potentiometer was zeroed at the start of each test to eliminate the small drift in the bias.

The demanded steer angle calculated from the MSPC subsystem was converted to a hand wheel angle, which was the input to the steering robot. The hand wheel angle was plotted as a function of the effective ‘single-track-average’ steer angle, \( \delta \), in Figure 4.15 (b). A cubic fit was found to be more accurate than a linear fit, so a cubic was adopted for use.

\[ \theta_h = 832.09 \times \delta^3 + 14.58 \times \delta^2 + 1350.9 \times \delta, \]  

(4-7)

where \( \theta_h \) was the hand wheel angle. The intercept was neglected for sensor zeroing.

Finally, \( \theta_h \) was converted to a CAN message through a low pass filter with a cut off frequency of 33 Hz.

4.1.4 Articulation angle calibration

The articulation angle sensors were calibrated to convert the sensor output voltage to accurate angles. When the tractor and trailers were connected together, the kingpin of the trailing vehicle unit was placed inside the fifth wheel of the preceding vehicle unit, so that the arm of the articulation angle sensor fitted snugly in the ‘wedge’ opening of the 5\textsuperscript{th} wheel as depicted in Figure 4.16. By this arrangement, the body of rotary pot was fixed to the tractor and the arm of the sensor, which was attached to the shaft of the pot through a hole drilled along the centreline of the kingpin, rotated with the trailer: thereby transducing the articulation angle. The solid lines in Figure 4.16 indicate the initial position and the dotted lines the rotated position. For instance, when the \( i \)\textsuperscript{th} vehicle unit made a clockwise turn, there was a positive articulation angle, \( \Gamma_i \), between the \( i \)\textsuperscript{th} and \((i + 1)\)\textsuperscript{th} vehicle units.

According to the sensor specification [203], there was a linear relationship between the sensor voltage and the articulation angle for the common operating range. The sensor output voltage in middle position, corresponding to a zero articulation angle, was \( 2.5 \pm 0.05 \, V \), when the input voltage was 5 \( V \).
A laser was glued to the sensor arm perpendicularly that passed through the centre of the kingpin, as seen in Figure 4.6. Then, the middle position of the sensor was determined by identifying the sensor output voltage. Wooden sheets were set up in parallel to the sensor arm so that the laser pointed to the wood sheets perpendicularly. The vertical distance between the centre of the kingpin and the wooden sheets, $d_i$, and the moving distance on the tape measure, $l_i$, were both measured. Assuming a clockwise turn, the articulation angle, $\Gamma_i$, was calculated as follows.

$$\Gamma_i = \arctan\left( \frac{l_i}{d_i} \right), \quad (4-8)$$

For an anti-clockwise turn, a negative sign was added in front of the formula.

The relationship between the articulation angle and the sensor output voltage is plotted in Figure 4.17 for the sensors on the semitrailer and the B-link trailer. Linear regression was used to curve-fit the data.

For the semitrailer, the calibration gave the following equation.

$$\Gamma = -0.14303 \times (v_a - u_a), \quad (4-9)$$

where $v_a$ was the sensor voltage and $u_a$ was the bias.

The output voltage was measured and updated before each test to zero the articulation angle sensor.

Likewise, for the B-link trailer, the equation was given as follows.

$$\Gamma = -0.14299 \times (v_a - u_a) \quad (4-10)$$

The slopes of the two fitted lines were nearly identical within 0.03%, which validates that the calibration was conducted with good accuracy.

For the tractor-semitrailer combination, the analogue angle was low-pass filtered with a Butterworth filter, at a cut off frequency of 32 Hz. For the B-double vehicle, the articulation angles were filtered at the frequency of 25 Hz and 32 Hz, respectively.

4.1.5 Steering robot setup

The steering robot was used to follow the hand wheel angle demand calculated by the steer angle tracking subsystem, as shown in Figure 4.3. The steering robot was configured to follow a demand signal, via a CAN bus connection, at a corner frequency of 25 Hz. A sinusoidal signal
was used to test the response of the steering robot. Figure 4.18 shows that the robot accurately followed a demanded signal at approximately 0.16 Hz. The steering robot is able to react at up to 6.5 Hz when the torque is smaller than 7 Nm.

The ‘Omni’ controller for the steering robot was reset at the beginning of each day of testing. After warming-up of the navigation system, the steering robot was used to perform a ‘lock to lock’ manoeuvre and then set the zero position of the hand wheel by driving in a straight line.

After setup, the start button on the top of joystick, shown in Figure 4.9, was pressed to commence each test session. The activation switch (‘dead-man’s handle’) had to be held to activate the test. Once the activation switch was released, the test was terminated.

4.1.6 Inertial navigation system setup

The RT3022 inertial navigation system (INS) mainly consisted of three accelerometers, three gyros, and a GPS receiver, generating high precision measurements of motion in real time. With the aid of dual antennas and the base station, the GPS could achieve a positioning accuracy of 20 cm RMS via differential corrections.

As the antenna separation distance was not estimated by the RT3022 [185], the position and heading accuracy were very sensitive to the accuracy of the separation distance measurement. In this work, the primary and secondary antennas were separated by 2.024 m (larger separation distance gives better accuracy).

The RT3022 was configured using the supplied software, NAVconfig. Then, the INS was dynamically initiated by driving in a straight line above the threshold speed (5 m/s by default). A warm up, involving figure of eight manoeuvres, straight line driving at constant speed and acceleration or braking, was completed for at least 15 minutes. This allowed the Kalman filter in the RT3022 to correct any errors and improve the system accuracy. The original settings in the configuration were improved during the warm up and could be saved for the next initialisation, reducing the warm up time for later tests.

A local coordinate frame was set up for each testing day using NAVdisplay [204]. A right-handed set with the Z-axis up was used for the local coordinates. The INS generated the displacement from the origin and the yaw angle in the local coordinate grid. The yaw angle was defined as the angle between the vehicle heading and the configured ‘X axis’ in the clockwise direction. The range of the yaw angle was $(-180°, 180°]$. Then, the yaw angle needed to be transformed to conform to the definition of the heading of the target paths. An
appropriate global position (latitude and longitude) and heading were selected to be the origin of the local map. Then, the ‘X axis’ was set by driving a minimum of 100 m along a desired direction. In this work, the desired ‘X axis’ of the local grid was set the same as that of the testing paths. The zero accuracy was tested by driving the vehicle in a closed, approximately rectangular shape, returning to the starting location. Figure 4.19 shows that a local coordinate grid was set up at the starting point and the trajectory was continuous without any jumps. The measured accuracy of closure of the manoeuvre was about 2 cm. All testing manoeuvres were recorded in this local coordinate frame.

The related signals, e.g. local positions and local yaw angle, were extracted from the RT3022’s CAN bus output interface.

4.1.7 Global controllers

The code of global control algorithms, written in C++, Python, and MATLAB scripts in this work were ultimately converted to the block diagram environment, Simulink, which is useful for the real time implementation and rapid prototyping. Standalone C code for embedding in the global controller was automatically generated using the in-built Simulink Coder tool.

For the xPC (used for tractor-semitrailer testing), an executable file, which was linked to the object code of the C source code, was created using Microsoft Visual Studio 2017, and the real time application was then downloaded to the target PC. The Simulink CAN blocksets along with the Softing CAN blocks were used to pack and unpack the CAN messages. Some real time commands in the MATLAB scripts were used to run the model, change the parameters at run time, and save the data on the host laptop.

For the dSpace Autobox (used for the B-double testing), the GNU C compiler was used to generate the executable file and the application was downloaded to the DS1007 processor. ‘fno-fast-math’ was selected for the compilation. The dSpace RTI CAN blockset was added to the Simulink models. The DS4302 CAN interface board can support 4 CAN buses at most. The board was tested in the hardware in the loop to differentiate and number the CAN buses. The real time testing was selected and enabled. dSpace provided an interactive user interface tool called ‘ControlDesk’ for handling the real time applications. It was used to monitor, modify, and save the real time parameters by simply dragging variables to its layout.
For the implementation of the MSPC method, the applications for the tractor-semitrailer and B-double vehicles ran at the frequency of 100 Hz, which met the sampling criterion (equation (3-17)) and was compatible with other equipment.

4.2 MSPC subsystem

The signal flow diagram in Figure 4.20 shows the structure of the real-time MSPC subsystem. The demanded steer angle, $\delta_d$, was calculated according to equation (3-4).

After the local coordinate frame was set up to match the coordinate frame of the target path, the real-time local position and heading of the ‘effective rear axle’ of the semitrailer along with the measured articulation angles were used to calculate the position and heading of the front axle of the tractor using the vehicle geometry. Then, the axle offsets from the reference path were calculated, and the rear offset along with the real time articulation and rear heading angles were fed into the controller. The equilibrium states for each manoeuvre were pre-computed. Hash tables storing the equilibrium states and the path details were created to provide a very quick look-up for interpolation. The state errors were calculated and then multiplied by the corresponding gains.

4.3 Steering angle tracking subsystem

The steer angle tracking subsystem enabled the actual steer angle, $\delta_a$, to follow the demanded steer angle, $\delta_d$, in order to improve the reversing performance. The system structure was a combination of feedforward and PID control, as shown in Figure 4.21.

The transfer function, $H(z)$, for the discrete-time parallel PID compensator was as follows:

$$H(z) = K_p + K_i \frac{T_s}{z-1} + K_D \frac{c_D z}{1 + c_D z T_s}, \quad (4-11)$$

where $T_s$ was the real-time processor’s sample time. In this work, $T_s = 0.01 \, \text{s}$. $K_p$, $K_i$ and $K_D$ were the proportional, integral and derivative gains, respectively. $C_D$ was the coefficient for the derivative filter.

The output of PID compensator was saturated using a model of the physical limitations of the tractor steering system and rate limited by the tractor speed to prevent the dry steering, which is the act of turning the steering wheel of the tractor while the tractor is stationary. Then it was converted to the hand wheel angle based on the measured relationship between the hand wheel angle and the effective ‘single-track-average’ steer angle given in equation (4-7). The demanded hand wheel angle was sent to the steering robot.
A saturated ramp signal for the steer angle demand, $\delta_d$, was used to tune the PID controller experimentally. The steering robot was set to follow the external signal. Firstly, the gains, $K_p$, $K_i$, and $K_d$, were set to zero, and the raw steady-state offset between the input signal and the measured steer angle, $\delta_a$, was measured and plotted in Figure 4.22. Then with $K_i$ and $K_d$ held at zero, $K_p$ was increased gradually to reduce the steady-state offset until there was insignificant effect on the steady-state error. After tuning $K_p$, $K_d$ was held constant and $K_i$ was increased to eliminate the remaining steady-state error. Lastly, $K_D$ was increased slightly to improve the settling time and stability. The measured steer angle using the tuned PID gains is superimposed in Figure 4.22.

The gains for the PID compensator are shown in Table 4.1.

### 4.4 Field testing results

The tractor and trailers were aligned on the ‘X axis’ with small yaw angle and position offset. The controllers were able to correct the vehicle position and heading automatically for small perturbations (e.g. the initial yaw angle and position offset), enabling the vehicle to perform its manoeuvres.

The desired paths, i.e. the lane change and roundabout manoeuvres, were depicted previously in Figures 3.5 and 3.9 for the tractor-semitrailer and B-double vehicles.

The tractor speed was controlled manually at approximately -1 m/s, where the negative sign means reversing.

#### 4.4.1 Field tests on the tractor-semitrailer configuration

Figure 4.23 (a) shows that the target path and test vehicle paths using the MSPC method and the baseline PFC method, during the lane change manoeuvre for the tractor-semitrailer vehicle. The tractor and semitrailer travelled backwards from right to left. With the aid of the tracking subsystem, the tractor’s front wheel was actuated to follow the demanded steer angle calculated by the main MSPC controller. On straight lines, the controller continued adjusting the steer angle to keep the vehicle travelling straight. Both the PFC and MSPC methods enabled the rear trailer to follow the desired path and ultimately converge to the terminal position. There is little visible difference in the simulation and experimental MSPC results (solid and dashed lines). It is seen from the uneven straight-line movements that the controllers were under continuous external disturbances (e.g. due to the uneven pavement surface).
The steer angle tracking performance during the lane change manoeuvre is shown in Figure 4.23 (b). The solid blue line is the measured steer angle, $\delta_a$, and the dashed red line is the demanded steer angle, $\delta_d$. The measured steer angle, $\delta_a$, was in good agreement with the MSPC control output, $\delta_d$.

To understand the difference between the MSPC method and the baseline PFC method, Figure 4.23 (c) shows the lateral offsets from the nominal path in the two approaches. The overshooting of the measured lateral offsets was smaller than that of the simulation results, and the shape of the measured and simulated curves were almost the same. The experimental performance was better than the simulation, with slightly smaller lateral offsets. For example, in the MSPC case, the overall swept path width was reduced by over 50% compared to the baseline (PFC case).

For the roundabout manoeuvre, the comparison of the reversing tracks between the PFC and the simulated and measured MSPC results is shown in Figure 4.24 (a). The rear axle was able to track the reference path using both methods. The simulated MSPC rear axle (the dashed red line) was overlapped by the measured MSPC rear axle (the solid green line). However, the measured MSPC front axle (the solid magenta line) was closer to the target path than the simulated MSPC front axle (the dashed blue line).

The axle lateral offsets are plotted in Figure 4.24 (b) to examine the difference. During the transition phases, the measured MSPC rear offset overshot more than the simulated MSPC rear offset. The measured MSPC front offset was much lower than the simulated MSPC front offset during the steady-state period. The overall swept path width was further reduced by approximately 10%, compared to the simulation results.

4.4.2 Field tests on the B-double configuration

The controller was also implemented on the B-double vehicle to validate the MSPC method for LCVs and used the same target paths for testing. For the longer vehicle, there was a significant increase in the lateral offsets, particularly for the front axle.

The reversing tracks for both methods are compared in Figure 4.25 (a). When the vehicle units converged towards the terminal straight line, there was noticeable difference between the measured MSPC lateral offsets (the solid lines) and the simulated lateral offsets (the dashed lines). Both the solid magenta and green lines had smaller overshoots, compared to the dashed blue and red lines.
Figure 4.25 (b) shows a comparison of the lateral offsets during the maneuver. The experimental results showed excellent agreement with the simulation results. It is also seen that at the last transition phase, both the measured front and rear offsets were slightly smaller than the simulated offsets. Compared to the baseline PFC method, the MSPC was able to reduce the overall swept path by more than 50% during the field tests.

The roundabout maneuver was also performed with the B-double vehicle, and the test vehicle paths are superimposed in Figure 4.26 (a). (The test vehicle moved in the same direction from right to left.) It is seen that the agreement between the simulation and experimental results was good and the measured lateral offsets were smaller than the simulated results. During the steady-state phase, the semitrailer followed the desired path in a slightly smaller radius circle and achieved an equilibrium, pulling the other parts of the vehicle closer to the reference, thereby reducing the overall swept path.

The axle lateral offsets from the target path for the experimental evaluation are compared to the simulation results in Figure 4.26 (b). The PFC front offsets dominated both sides of the desired path. The measured MSPC tracking errors of the rear trailer were acceptable during the maneuver, not exceeding the negative maximum excursion of the PFC front offsets. The simulated and measured MSPC offsets agreed well with each other, but the experimental results had smaller overshoots than predicted. Particularly during the steady-state period, the measured rear offsets (the solid green line) were smaller than the simulated rear offsets (the dashed red line), and due to the reduction in the rear axle, the measured front offsets (the solid magenta line) were also smaller than the simulated front offsets (the dashed blue line). It is likely that the reason for this is the actual equilibrium values were larger than the pre-computed equilibrium values from the model. This implies there may be a way to reduce the overall swept path further by changing the equilibrium values for maneuvers with constant curvature, e.g. roundabouts.

The overall swept path using the MSPC method was reduced by about 40% experimentally, compared to the baseline PFC method, exceeding the expectations from the simulation.

4.5 Conclusions
1. The MSPC controller developed in Chapter 3 was implemented on two different vehicle configurations: a tractor-semitrailer and a B-double, using the CVDC’s own vehicles, and tested at Bourn airfield.
2. Various hardware, e.g. the AB Dynamics SR30 steering robot, the V.S.E. articulation angle sensors, the UniMeasure string potentiometer, and the OxTS RT3022, were mounted on the test vehicle units and connected via Ethernet and USB cables, and CAN buses.

3. The string potentiometer and articulation angle sensors were calibrated. The relationship between the sensor voltage of the string potentiometer, the effective ‘single-track-average’ steer angle, and the hand wheel angle was investigated. The relationship was non-linear due to the backlash. The results show that the front axle had good Ackermann geometry.

4. A steering tracking subsystem was developed using the PID control method to follow the steer angle demand calculated by the main MSPC controller. The inner loop compensator was tuned experimentally.

5. For both the lane change and roundabout manoeuvres performed by the tractor-semitrailer and B-double vehicles, the simulated and experimental results were in good agreement. The experimental results show that the controller exhibited good real-time performance, reducing the overall swept path width by more than 40%, compared to the previous path following method. By contrast with the simulated responses, the experimental responses had smaller overshoots. Hence, the real time performance slightly exceeded the simulation.
### 4.6 Tables

*Table 4.1 Gains for the PID compensator*

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<th>Parameter</th>
<th>Value</th>
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</tr>
<tr>
<td>$C_D$</td>
<td>100</td>
</tr>
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4.7 Figures

(a) Tractor-semitrailer test vehicle. Distances shown between the front axle and the hitch point for the tractor, and hitch to second rear axle for the semitrailer

(b) B-double test vehicle. Distances shown between the front axle and the hitch point for the tractor, hitch to hitch point for the B-link trailer, and hitch to second rear axle for the semitrailer

Figure 4.1 Test vehicles
Figure 4.2 Schematic of the on-board equipment layout
Figure 4.3 Architecture of the autonomous reversing system
(a) A photograph of the RT3022 IMU mounted on top of the ‘equivalent rear axle’ of the semitrailer and the cable connections

(b) Design of the mounting bracket for the RT3022 IMU with the cross point of the two orthogonal lines showing the origin of measurement

(c) A photograph of the position of the dual antennas and radio aerial on the roof of the semitrailer

(d) Design of the steel plate for the dual antennas. The cross point of the two orthogonal lines was the antenna origin

Figure 4.4 RT3022 dual antenna inertial and GPS navigation system
Figure 4.5 The OxTS base station and a radio aerial on the rooftop of the stationary Mondeo on the testing field

Figure 4.6 The experimental configuration for calibrating the articulation angle sensor
Figure 4.7 The string potentiometer mounted on the underside of the tractor and attached to the steering radius arm to measure the actual steer angle

Figure 4.8 Hardware installation in the tractor cabin
Figure 4.9 Components of the steering robot and the cable connections

Figure 4.10 The Autobox with the DS4302 CAN interface board
(a) The front axle calibration diagram, using lasers to project spots onto a ‘wall’

(b) The experimental configuration for calibrating the front axle

(c) The local coordinate frame set up for the front axle calibration

Figure 4.11 The front axle calibration
Figure 4.12 The relationship between the measured left and right front wheel angles and the hand wheel angle.

Figure 4.13 Illustration of the Ackermann steering geometry.
Figure 4.14 The effective ‘single-track’ steer angle calculated from the measured left and right front wheel angles respectively.
(a) The relationship between the effective ‘single-track-average’ steer angle and the sensor voltage of the string potentiometer. The equations for the linear and cubic fitting methods are displayed.

(b) The relationship between the hand wheel angle and the effective ‘single-track-average’ steer angle. The equations for the linear and cubic fitting methods are displayed.

Figure 4.15 The relationship between the hand wheel angle, the effective ‘single-track-average’ steer angle, and the sensor voltage of the string potentiometer.
Figure 4.16 The articulation angle sensor calibration method. The laser was glued to the sensor arm perpendicularly that passed through the centre of the kingpin.
(a) The relationship between the articulation angle and the sensor voltage for the articulation angle sensor installed on the semitrailer. The equation for the linear regression method is displayed.

\[ y = -0.14303x + 0.021527 \]

(b) The relationship between the articulation angle and the sensor voltage for the articulation angle sensor installed on the B-link trailer. The equation for the linear regression method is displayed.

\[ y = -0.14299x + 0.0046529 \]

Figure 4.17 The relationship between the articulation angle and the sensor voltage for the articulation angle sensor.
Figure 4.18 The sinusoidal signal tracking test for the AB Dynamics SR30 steering robot

Figure 4.19 The local coordinate frame set up for the testing. A closed shape testing was carried out in the local grid using the INS
Figure 4.20 Schematic of the MSPC subsystem

Figure 4.21 Schematic of the steer angle tracking subsystem

Figure 4.22 Measured steering angle with and without the PID controller
(a) The position of the tractor-semitrailer vehicle during the lane change manoeuvre

(b) The demanded and measured steer angle during the lane change manoeuvre

(c) Comparison of the lateral offsets during the lane change manoeuvre

Figure 4.23 Comparison between the simulated and experimental results during the lane change manoeuvre for the tractor-semitrailer vehicle
(a) The position of the tractor-semitrailer vehicle during the roundabout manoeuvre

(b) Comparison of the lateral offsets during the roundabout manoeuvre for the tractor-semitrailer vehicle

Figure 4.24 Comparison between the simulated and experimental results during the roundabout manoeuvre for the tractor-semitrailer vehicle
(a) The position of the B-double vehicle during the lane change manoeuvre

(b) Comparison of the lateral offsets during the lane change manoeuvre for the B-double vehicle

Figure 4.25 Comparison between the simulated and experimental results during the lane change manoeuvre for the B-double vehicle
(a) The position of the B-double vehicle during the roundabout manoeuvre

(b) Comparison of the lateral offsets during the roundabout manoeuvre for the B-double vehicle

Figure 4.26 Comparison between the simulated and experimental results during the roundabout manoeuvre for the B-double vehicle
Chapter 5 Lane-Bounded Reversing Control (LBRC)

Lane-Bounded Reversing Control is developed in this chapter to enable autonomous reversing systems to perform precise, space-confined, and collision-free manoeuvres to target terminals. Examples include passing through a narrow gate, getting around a parked vehicle, or reversing into a loading bay. The controller is non-linear, based on the Model Predictive Control (MPC) theory [127], [205]–[207] with constraints. The LBRC controller plans ahead and makes optimal decisions to avoid future collisions and reach the ultimate target position and orientation. In order to satisfy the small angle approximation for linearisation, each vehicle unit is treated in a local vehicle frame.

As shown in Figure 5.1 (a), the local longitudinal (‘X Axis’) and lateral (‘Y Axis’) axes are fixed on the middle of the front end of the tractor and the rear end of the rear trailer, and for the rest of vehicle units, the local axes are fixed on the C.o.M.. Necessary constraints are imposed to make the vehicle stay within the bounded lanes.

The two main forms of notation used in this chapter are defined as follows:

1. The main format for motion variables is $v_{jm}^s$.
   (i) ‘$v$’ is a variable, representing any lower-case scalar, e.g. ‘$y$’ and ‘$\theta$’ seen in Figure 5.1 (b). Note that ‘$y$’ refers to the lateral offset, which is the lateral distance to the fixed local or global longitudinal axis (‘X Axis’); ‘$\theta$’ denotes the heading angle, which is the angle that the vehicle unit is pointing compared to the fixed local or global longitudinal axis (‘X Axis’).
   (ii) Superscript, ‘$s$’, stands for an element of \{l, g\}, and ‘$l$’ and ‘$g$’ are short for the local and global coordinate frame, respectively.
   (iii) The first subscript, ‘$j$’, represents the $(j + 1)^{th}$ vehicle unit, and $j \in [0, n]$. ‘0’ and ‘$n$’ means the tractor and the rear trailer, respectively.
   (iv) The second subscript, ‘$m$’, is an element of \{f, c, r, 1, 2, 3, 4\}. It describes a position on the vehicle unit, as illustrated in Figure 5.1 (b), ‘$f$’ and ‘$r$’ refer to the middle point of the front and rear end of the vehicle unit, ‘$c$’ corresponds to the C.o.M., and the integers, ‘1’, ‘2’, ‘3’ and ‘4’, denote the front-left, front-right, rear-right and rear-left corner point of the vehicle units respectively, which are numbered clockwise.

2. The second form of the notations, $v_{jmc}^s$, defines the corresponding constraints of $v_{jm}^s$ (i.e. the distances to the boundaries), as shown in Figure 5.1 (c).
(i) The additional subscript, ‘c’, is an abbreviation for ‘constraint’, and \( c \in \{ u, l \} \). ‘u’ and ‘l’ here represent the upper and lower constraints derived from predefined upper and lower boundaries.

(ii) Examples: \( y_{0f}^l \) is the lateral offset of the middle point of the front end of the tractor in the local vehicle frame, and \( y_{0fu}^l \) is the upper constraint of \( y_{0f}^l \) in the same coordinate system; \( y_{nr}^g \) is the lateral offset of the middle point of the rear end of the rear trailer in the global coordinate frame, and \( y_{nrl}^g \) is the lower constraint of \( y_{nr}^g \).

(iii) Note that pairs of constraints at the same end of each vehicle unit are equal in the local coordinate frame, e.g. \( y_{01u}^l = y_{02u}^l \), et cetera.

5.1 Controller design

Additional variables are added to the original system state vector, \( \mathbf{Z}_D \in \mathbb{R}^{(2n+2) \times 1} \), and its derivative, \( \dot{\mathbf{Z}}_D \), in equations (2-1) and (2-2) to create the new control system state vector, \( \mathbf{x}_L \), and its derivative, \( \dot{\mathbf{x}}_L \), defined as follows:

\[
\mathbf{x}_L = [\mathbf{Z}_D^T \ y_{0f}^l \ \theta_{0f}^l \ y_{jc}^l \ \theta_{jc}^l \ y_{nr}^l \ \theta_{nr}^l]^T, \quad (5-1)
\]

\[
\dot{\mathbf{x}}_L = [\dot{\mathbf{Z}}_D^T \ y_{0f}^l \ \dot{\theta}_{0f}^l \ y_{jc}^l \ \dot{\theta}_{jc}^l \ y_{nr}^l \ \dot{\theta}_{nr}^l]^T, \quad (5-2)
\]

where \( \mathbf{x}_L \in \mathbb{R}^{(4n+4) \times 1} \), \( j \in (1, n) \).

- \( y_{0f}^l \): the lateral offset of the front-end point of the tractor in its local coordinate frame;
- \( \theta_{0f}^l \): the heading angle of the front-end point of the tractor in its local coordinate frame;
- \( y_{jc}^l \): the lateral offset of the C.o.M. of the \( j^{th} \) trailer in its local coordinate frame;
- \( \theta_{jc}^l \): the heading angle of the C.o.M. of the \( j^{th} \) trailer in its local coordinate frame;
- \( y_{nr}^l \): the lateral offset of the rear-end point of the rear trailer in its local coordinate frame;
- \( \theta_{nr}^l \): the heading angle of the rear-end point of the rear trailer in its local coordinate frame;
- \( \dot{y}_{0f}^l, \dot{\theta}_{0f}^l, \dot{y}_{jc}^l, \dot{\theta}_{jc}^l, \dot{y}_{nr}^l, \) and \( \dot{\theta}_{nr}^l \) are their first derivatives correspondingly.

Then the new dynamic system is linearised about the longitudinal axis in each vehicle frame, in which all equilibrium states are zero. The small angle approximation is applied.

\[
\Delta \mathbf{x}_L = [\Delta \mathbf{Z}_D^T \ \Delta y_{0f}^l \ \Delta \theta_{0f}^l \ \Delta y_{jc}^l \ \Delta \theta_{jc}^l \ \Delta y_{nr}^l \ \Delta \theta_{nr}^l]^T, \quad (5-3)
\]
\[ \Delta \dot{x}_L = [\Delta \dot{Z}_D^T \Delta \dot{y}_0^l \Delta \dot{\theta}_{0f}^l \cdots \Delta \dot{y}_{jc}^l \Delta \dot{\theta}_{jc}^l \cdots \Delta \dot{y}_{nr}^l \Delta \dot{\theta}_{nr}^l]^T, \]  
(5-4)

\[ \Delta \dot{y}_{0f}^l = \Delta v_0 + \Delta u_0 \Delta \theta_{0f}^l + (a_0 + f_{o0})\Delta \Omega_0, \]  
(5-5)

\[ \Delta \dot{\theta}_{0f} = \Delta \Omega_0, \]  
(5-6)

\[ \Delta \dot{y}_{jc}^l = \Delta v_j + \Delta u_j \Delta \theta_{jc}^l, \]  
(5-7)

\[ \Delta \dot{\theta}_{jc}^l = \Delta \Omega_j, \]  
(5-8)

\[ \Delta \dot{y}_{nr}^l = \Delta v_n + \Delta u_n \Delta \theta_{nr}^l + (l_{effn} - a_n + r_{on})\Delta \Omega_n, \]  
(5-9)

\[ \Delta \dot{\theta}_{nr}^l = \Delta \Omega_n, \]  
(5-10)

where

- \( \Delta \): a small deviation from any equilibrium point;
- \( f_{o0} \): the distance between the front end and the front axle of the tractor;
- \( r_{on} \): the distance between the rear end and the ‘equivalent rear axle’ of the rear trailer.

The linearised continuous-time system is built as a function of \( \Delta x_L \) and \( \Delta \delta \).

\[ \Delta \dot{x}_L = [A_{Lct}] \Delta x_L + [B_{Lct}] \Delta \delta \]  
(5-11)

where \([A_{Lct}] \in \mathbb{R}^{(4n+4) \times (4n+4)}\) and \([B_{Lct}] \in \mathbb{R}^{(4n+4) \times 1}\) are the system matrix and the input matrix, respectively, of the continuous-time model.

Then the linear system is discretised with sample time, \( T_s \), and zero-order hold is assumed over \( T_s \). The discrete-time system model at sample \( k \) is shown below.

\[ \Delta x_L(k + 1) = [A_{Ldt}] \Delta x_L(k) + [B_{Ldt}] \Delta \delta(k) \]  
(5-12)

where \([A_{Ldt}] \in \mathbb{R}^{(4n+4) \times (4n+4)}\) and \([B_{Ldt}] \in \mathbb{R}^{(4n+4) \times 1}\) are the system matrix and the input matrix, respectively, of the discrete-time model. \( \Delta \delta(k) \) and \( \Delta x_L(k) \) are the discrete-time linearised steer angle and system state at sample \( k \). The discrete-time system state is defined as follows:

\[ \Delta x_L(k) = [\Delta \dot{Z}_D^T(k) \Delta y_{0f}^l(k) \Delta \theta_{0f}^l(k) \cdots \Delta y_{jc}^l(k) \Delta \theta_{jc}^l(k) \cdots \Delta y_{nr}^l(k) \Delta \theta_{nr}^l(k)]^T \]  
(5-13)

The discrete-time state vector, \( \Delta x_L(k) \), is transformed to \( \Delta z_L(k) \), defined as follows.

\[ \Delta z_L(k) = [\Delta y_{0f}^l(k) \Delta y_{0f}^l(k) \cdots \Delta y_{jc}^l(k) \Delta y_{jc}^l(k) \cdots \Delta y_{nr}^l(k) \Delta y_{nr}^l(k)]^T, \]  
(5-14)
where \( \Delta z_L(k) \in \mathbb{R}^{(2n+2) \times 1} \).

The relationship between \( \Delta z_L(k) \) and \( \Delta x_L(k) \) is established as follows:

\[
\Delta z_L(k) = [C] \Delta x_L(k) \quad (5-15)
\]

where the transformation matrix, \([C]\), is a \((2n+2) \times (2n+2)\) zero matrix. The transformation matrix, \([C] \in \mathbb{R}^{(2n+2) \times (4n+4)},\) is a sparse matrix.

The preview horizon, \( N_p \), is defined as the number of future sample intervals incorporated into the MPC optimisation, and the control horizon, \( N_c \), is the number of control input samples to be optimised. The control input varies only up to the control horizon, \( N_c \). After \( N_c \) steps, up to the preview horizon, \( N_p \), the control input stays unchanged. The state update in the preview horizon by iterating equation (5-12) for \( N_p \) steps from any sample instant \( k \) is shown below.

\[
\begin{bmatrix}
\Delta x_L(k+1) \\
\Delta x_L(k+2) \\
\vdots \\
\Delta x_L(k+N_c) \\
\vdots \\
\Delta x_L(k+N_p)
\end{bmatrix}
= 
\begin{bmatrix}
A_{LDT} \\
A_{LDT}^2 \\
\vdots \\
A_{LDT}^{N_c} \\
\vdots \\
A_{LDT}^{N_p}
\end{bmatrix} \Delta x_L(k) +
\begin{bmatrix}
B_{LDT} \\
A_{LDT} B_{LDT} \\
\vdots \\
A_{LDT}^{N_c-1} B_{LDT} \\
\vdots \\
A_{LDT}^{N_p-1} B_{LDT}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta (k) \\
\Delta \delta (k+1) \\
\vdots \\
\Delta \delta (k + N_c - 1)
\end{bmatrix} +
\begin{bmatrix}
\Delta \delta (k) \\
\Delta \delta (k+1) \\
\vdots \\
\Delta \delta (k + N_c - 1)
\end{bmatrix}
\begin{bmatrix}
\sum_{j=0}^{N_p-N_c} A_L^{j} B_{LDT}
\end{bmatrix}
\quad (5-16)
\]

Equation (5-16) can be simplified as follows:

\[
\Delta X_L(k) = [K_L] \Delta x_L(k) + [\Gamma_L] \Delta U_L(k) \quad (5-17)
\]
where

\[ \Delta X_L(k) = \begin{bmatrix} \Delta x_L(k+1) \\ \Delta x_L(k+2) \\ \vdots \\ \Delta x_L(k+N_c) \\ \vdots \\ \Delta x_L(k+N_p) \end{bmatrix}; \]

- \[ [K_L] = \begin{bmatrix} A_L^1 DT \\ A_L^2 DT \\ \vdots \\ A_L^{N_c} DT \\ \vdots \\ A_L^{N_p} DT \end{bmatrix}; \]

- \[ [\Gamma_L] = \begin{bmatrix} B_L DT \\ A_L^1 DT B_L DT \quad B_L DT \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ A_L^{N_c-1} DT B_L DT \quad \ldots \quad A_L^1 DT B_L DT \quad \ldots \quad A_L^{N_p-1} DT B_L DT \quad \ldots \quad \sum_{j=0}^{N_p-N_c} \quad A_L^j DT B_L DT \end{bmatrix}; \]

- \[ \Delta U_L(k) = \begin{bmatrix} \Delta \delta(k) \\ \Delta \delta(k+1) \\ \vdots \\ \Delta \delta(k+N_c-1) \end{bmatrix}. \]

\[ [K_L] \in \mathbb{R}^{(4n+4)N_p \times (4n+4)} \text{ and } [\Gamma_L] \in \mathbb{R}^{(4n+4)N_p \times N_c} \] are the prediction matrices of the system model.

The cost function for the MPC problem is defined as follows:

\[
\min_{\Delta U_L} V_L(\Delta z_L, \Delta U_L) = \sum_{i=0}^{N_p} \| \Delta z_L(k+i) - p_L(k+i) \|^2_{q_{k+i}} + \sum_{i=0}^{N_c-1} \| \Delta \delta(k+i) \|^2_{r_{k+i}}, \quad (5-18)
\]

where

- \[ p_L(k+i) \in \mathbb{R}^{(2n+2)\times 1} \]: the boundary matrix containing upper and lower constraints of each element of \( \Delta z_L(k+i) \) in the corresponding vehicle frame, as defined in equation (5-19) and shown in Figure 5.1 (a). The upper and lower constraints at sample instant \( k \) are the points of lateral intersection between the vehicle unit and the pre-defined boundaries. The elements of \( p_L(k+i) \) are sampled from the intersection point
onwards at a step size of $u_j T_s$ up to the preview horizon, where $u_j$ is the longitudinal speed of the $(j + 1)^{th}$ vehicle unit and $j \in [0, n]$;

- $q_{k+i} \in \mathbb{R}^{(2n+2) \times (2n+2)}$: the weighting matrix for the deviation offsets, defined in equation (5-20);
- $r_{k+i}$: the weight on the steer angle, defined in equation (5-21).

\[
p_L(k + i) = \begin{bmatrix}
y_{0f}^l(k + i) \\
y_{0fl}^l(k + i) \\
\vdots \\
y_{jcu}^l(k + i) \\
y_{jcl}^l(k + i) \\
\vdots \\
y_{nru}^l(k + i) \\
y_{nrl}^l(k + i)
\end{bmatrix},
\]

where $j \in [1, n]$.

- $y_{0fu}^l(k + i)$: the local upper constraint of $\Delta y_{0f}^l(k + i)$;
- $y_{0fl}^l(k + i)$: the local lower constraint of $\Delta y_{0f}^l(k + i)$;
- $y_{jcu}^l(k + i)$: the local upper constraint of $\Delta y_{jcu}^l(k + i)$;
- $y_{jcl}^l(k + i)$: the local lower constraint of $\Delta y_{jcl}^l(k + i)$;
- $y_{nru}^l(k + i)$: the local upper constraint of $\Delta y_{nru}^l(k + i)$;
- $y_{nrl}^l(k + i)$: the local lower constraint of $\Delta y_{nrl}^l(k + i)$.

\[
q_{k+i} = \text{diag} \left( w_{0fu}(k + i), w_{0fl}(k + i), \ldots, w_{jcu}(k + i), w_{jcl}(k + i), \ldots, w_{nru}(k + i), w_{nrl}(k + i) \right),
\]

where ‘diag’ means that $q_{k+i}$ is a diagonal matrix. $w_{0fu}(k + i), w_{0fl}(k + i), w_{jcu}(k + i), w_{jcl}(k + i), w_{nru}(k + i)$ and $w_{nrl}(k + i)$ are the weights on each corresponding deviation offset.

\[
r_{k+i} = w_\delta(k + i),
\]

where $w_\delta(k + i)$ is the weight on the steer angle.

Then, equation (5-18) can be decomposed as follows:
\[
\min_{\Delta U_L} \left( \Delta Z_L, \Delta U_L \right) = \|\Delta Z_L(k) - p_L(k)\|^2_{q_k} + \sum_{i=1}^{N_p} \|\Delta Z_L(k + i) - p_L(k + i)\|^2_{q_{k+i}} + \\
\|\Delta Z_L(k + N_p) - p_L(k + N_p)\|^2_{q_{k+N_p}} + \sum_{i=0}^{N_e-1} \|\Delta \delta(k + i)\|^2_{r_{k+i}},
\] (5-22)

where \(q_k\) and \(q_{k+N_p}\) are the initial and terminal weighting matrices of the deviation offsets.

\(\Delta Z_L(k), P_L(k), Q_L(k)\) and \(R_L(k)\) at any sample instant \(k\) are defined as:

\[
\Delta Z_L(k) = \begin{bmatrix}
\Delta z_L(k + 1) \\
\vdots \\
\Delta z_L(k + i) \\
\vdots \\
\Delta z_L(k + N_p)
\end{bmatrix}
\] (5-23)

\[
P_L(k) = \begin{bmatrix}
p_L(k + 1) \\
\vdots \\
p_L(k + i) \\
\vdots \\
p_L(k + N_p)
\end{bmatrix}
\] (5-24)

\[
Q_L(k) = \begin{bmatrix}
q_{k+1} \\
\vdots \\
q_{k+i} \\
\vdots \\
q_{k+N_p}
\end{bmatrix}
\] (5-25)

\[
R_L(k) = \begin{bmatrix}
r_k \\
\vdots \\
r_{k+i} \\
\vdots \\
r_{k+N_e-1}
\end{bmatrix}
\] (5-26)

Equation (5-22) can be further simplified as:

\[
\min_{\Delta U_L} V_L = \|\Delta Z_L(k) - p_L(k)\|^2_{q_k} + \|\Delta Z_L - P_L\|^2_{Q_L} + \|\Delta U_L\|^2_{R_L},
\] (5-27)

where \(\Delta Z_L, P_L, \Delta U_L, Q_L\) and \(R_L\) symbolise \(\Delta Z_L(k), P_L(k), \Delta U_L(k), Q_L(k)\) and \(R_L(k)\) respectively for simplicity. \(\Delta Z_L \in \mathbb{R}^{(2n+2)N_p \times 1}, P_L \in \mathbb{R}^{(2n+2)N_p \times 1}, \Delta U_L \in \mathbb{R}^{N_c \times 1}, Q_L \in \mathbb{R}^{(2n+2)N_p \times (2n+2)N_p}\) and \(R_L \in \mathbb{R}^{N_c \times N_c}\).

As \(\|\Delta Z_L(k) - p_L(k)\|^2_{q_k}\) is only determined by initial conditions, the cost function’s optimisation is totally dependent on \(\|\Delta Z_L - P_L\|^2_{Q_L} + \|\Delta U_L\|^2_{R_L}\). Hence, the minimisation of \(V_L\) is equivalent to the minimisation of the cost function, \(O_L\), defined as follows:
\[
\min_{\Delta U_L} O_L = \|\Delta Z_L - P_L\|_{Q_L}^2 + \|\Delta U_L\|_{R_L}^2
\] (5-28)

According to equations (5-15) to (5-17), \(\Delta Z_L(k)\) can be represented by \(\Delta x_L(k)\) and \(\Delta U_L(k)\).

\[
\Delta Z_L(k) = \begin{bmatrix}
CA_{LDT} \\
CA_{LDT}^2 \\
\vdots \\
CA_{LDT}^{N_c} \\
\vdots \\
CA_{LDT}^{N_p}
\end{bmatrix} \Delta x_L(k) + \\
\begin{bmatrix}
CB_{LDT} \\
CA_{LDT} B_{LDT} \\
\vdots \\
CA_{LDT}^{N_c - 1} B_{LDT} \\
\vdots \\
CA_{LDT}^{N_p - 1} B_{LDT}
\end{bmatrix} \begin{bmatrix}
\Delta \delta(k) \\
\Delta \delta(k + 1) \\
\vdots \\
\Delta \delta(k + N_c - 1)
\end{bmatrix}
\] (5-29)

Then, equation (5-29) can be simplified as follows:

\[
\Delta Z_L(k) = [\Psi_L] \Delta x_L(k) + [\Theta_L] \Delta U_L(k),
\] (5-30)

where \([\Psi_L] \in R^{(2n+2)N_p \times (4n+4)}\) and \([\Theta_L] \in R^{(2n+2)N_p \times N_c}\).

- \([\Psi_L] = \begin{bmatrix}
CA_{LDT} \\
CA_{LDT}^2 \\
\vdots \\
CA_{LDT}^{N_c} \\
\vdots \\
CA_{LDT}^{N_p}
\end{bmatrix}\)

- \([\Theta_L] = \begin{bmatrix}
CB_{LDT} \\
CA_{LDT} B_{LDT} \\
\vdots \\
CA_{LDT}^{N_c - 1} B_{LDT} \\
\vdots \\
CA_{LDT}^{N_p - 1} B_{LDT}
\end{bmatrix} \begin{bmatrix}
\Delta \delta(k) \\
\Delta \delta(k + 1) \\
\vdots \\
\Delta \delta(k + N_c - 1)
\end{bmatrix}
\]

Substituting equation (5-30) into equation (5-28) and introducing a new variable, \(e_L(k) = [\Psi_L] \Delta x_L(k) - P_L(k)\), give the following optimisation problem.

\[
\min_{\Delta U_L} O_L = \frac{1}{2} \Delta U_L^T [2(\Theta_L^T Q_L \Theta_L + R_L)] \Delta U_L + (2\Theta_L^T Q_L e_L)^T \Delta U_L + e_L^T Q_L e_L.
\] (5-31)
where $\mathbf{e}_L$ represents $\mathbf{e}_L(k)$.

Equation (5-31) can be simplified further by defining $\mathbf{G}_L = 2(\mathbf{\Theta}_L^T \mathbf{Q}_L \mathbf{\Theta}_L + \mathbf{R}_L)$ and $\mathbf{F}_L = 2\mathbf{\Theta}_L^T \mathbf{Q}_L$, where $\mathbf{G}_L \in \mathbb{R}^{N_c \times N_c}$ and $\mathbf{F}_L \in \mathbb{R}^{N_c \times (2n+2)N_p}$:

$$\min_{\Delta \mathbf{U}_L} O_L = \frac{1}{2} \Delta \mathbf{U}_L^T \mathbf{G}_L \Delta \mathbf{U}_L + (\mathbf{F}_L \mathbf{e}_L)^T \Delta \mathbf{U}_L + \mathbf{e}_L^T \mathbf{Q}_L \mathbf{e}_L$$  \hspace{1cm} (5-32)

$\mathbf{e}_L^T \mathbf{Q}_L \mathbf{e}_L$ is a constant term in equation (5-32), so the optimisation of $O_L$ is only dependent on the first two terms. Therefore, the cost function is further simplified as follows:

$$\min_{\Delta \mathbf{U}_L} O_L(\Delta \mathbf{U}_L) = \frac{1}{2} \Delta \mathbf{U}_L^T \mathbf{G}_L \Delta \mathbf{U}_L + \mathbf{f}_L^T \Delta \mathbf{U}_L,$$  \hspace{1cm} (5-33)

where $\mathbf{f}_L = \mathbf{F}_L \mathbf{e}_L$, $\mathbf{f}_L \in \mathbb{R}^{N_c \times 1}$.

As the vehicle units travel within bounded lanes and have physical limitations, some constraints need to be imposed:

$$\begin{aligned}
\delta_l(k + i) &\leq \delta(k + i) \leq \delta_u(k + i) & i = 0, 1, \ldots, N_p - 1 \\
\dot{\delta}_l(k + i) &\leq \dot{\delta}(k + i) \leq \dot{\delta}_u(k + i) & i = 0, 1, \ldots, N_p - 1 \\
\Gamma_{ji}(k + i) &\leq \Gamma_j(k + i) \leq \Gamma_{ju}(k + i) & i = 0, 1, \ldots, N_p \\
y^i_{j1}(k + i) &\leq y^i_{j1}(k + i) \leq y^i_{j1u}(k + i) & i = 0, 1, \ldots, N_p \\
y^i_{j2}(k + i) &\leq y^i_{j2}(k + i) \leq y^i_{j2u}(k + i) & i = 0, 1, \ldots, N_p \\
y^i_{j3}(k + i) &\leq y^i_{j3}(k + i) \leq y^i_{j3u}(k + i) & i = 0, 1, \ldots, N_p \\
y^i_{j4}(k + i) &\leq y^i_{j4}(k + i) \leq y^i_{j4u}(k + i) & i = 0, 1, \ldots, N_p 
\end{aligned}$$  \hspace{1cm} (5-34)

where

- $\delta_u(k + i)$: the upper limit of the steer angle, $\delta(k + i)$;
- $\delta_l(k + i)$: the lower limit of the steer angle, $\delta(k + i)$;
- $\dot{\delta}_u(k + i)$: the upper limit of the steer angle rate, $\dot{\delta}(k + i)$;
- $\dot{\delta}_l(k + i)$: the lower limit of the steer angle rate, $\dot{\delta}(k + i)$;
- $\Gamma_{ju}(k + i)$: the upper limit of the $j^{th}$ articulation angle, $\Gamma_j(k + i)$, where $j \in [1, n]$;
- $\Gamma_{ji}(k + i)$: the lower limit of the $j^{th}$ articulation angle, $\Gamma_j(k + i)$, where $j \in [1, n]$;
- $y^j_{j1u}(k + i)$: the upper limit of the local lateral offset of the front-left corner point of the $(j + 1)^{th}$ vehicle unit, $y^j_{j1}(k + i)$, where $j \in [0, n]$;
- $y^j_{j1l}(k + i)$: the lower limit of the local lateral offset of the front-left corner point of the $(j + 1)^{th}$ vehicle unit, $y^j_{j1}(k + i)$, where $j \in [0, n]$;
• $y^l_{2u}(k+i)$: the upper limit of the local lateral offset of the front-right corner point of the $(j+1)^{th}$ vehicle unit, $y^l_{2}(k+i)$, where $j \in [0,n]$;
• $y^l_{2l}(k+i)$: the lower limit of the local lateral offset of the front-right corner point of the $(j+1)^{th}$ vehicle unit, $y^l_{2}(k+i)$, where $j \in [0,n]$;
• $y^u_{3u}(k+i)$: the upper limit of the local lateral offset of the rear-right corner point of the $(j+1)^{th}$ vehicle unit, $y^u_{3}(k+i)$, where $j \in [0,n]$;
• $y^u_{3l}(k+i)$: the lower limit of the local lateral offset of the rear-right corner point of the $(j+1)^{th}$ vehicle unit, $y^u_{3}(k+i)$, where $j \in [0,n]$;
• $y^l_{4u}(k+i)$: the upper limit of the local lateral offset of the rear-left corner point of the $(j+1)^{th}$ vehicle unit, $y^l_{4}(k+i)$, where $j \in [0,n]$;
• $y^l_{4l}(k+i)$: the lower limit of the local lateral offset of the rear-left corner point of the $(j+1)^{th}$ vehicle unit, $y^l_{4}(k+i)$, where $j \in [0,n]$.

The steer angle and its rate of change are discretised with a small sampling time, $\Delta t$, and the forward Euler method is used. Their relationship can be written in a linear form:

$$\delta(k+i-1) + \Delta t \dot{\delta}(k+i) \leq \delta(k+i) \leq \delta(k+i-1) + \Delta t \dot{\delta}(k+i),$$  \hspace{1cm} (5-35)

where $\delta(k+i-1)$ is a preceding steer angle at sample instant $(k+i-1)$.

Combining inequality (5-35) and the constraint for the steer angle in inequalities (5-34) can give a surrogate inequality for the steer angle.

$$\dot{\delta}(k+i) \leq \delta(k+i) \leq \dot{\delta}_u(k+i),$$  \hspace{1cm} (5-36)

where

- $\dot{\delta}_u(k+i) = \min (\delta_u(k+i), \delta(k+i-1) + \Delta t \dot{\delta}_u(k+i));$
- $\dot{\delta}(k+i) = \max (\delta_l(k+i), \delta(k+i-1) + \Delta t \dot{\delta}_l(k+i)).$

This transformation not only increases the computation speed, but also makes the controller implementation more realistic. This is because in real-time applications, accurate measurement of the rate of change of steer angle requires inertial systems, which can be avoided by the transformation.

The angle and position constraints in inequalities (5-34) can be rearranged as element-wise inequalities as follows:
where

- \( \mathbf{\rho}(k + i) = \begin{bmatrix} \mathbf{0}_{n,2}, [\mathbf{I}_n], [\mathbf{0}_{n,3n+2}] \end{bmatrix} \in \mathbb{R}^{n \times (4n+4)} \) : a transformation matrix for articulation angles. \([\mathbf{0}_{n,2}] \) and \([\mathbf{0}_{n,3n+2}] \) denote \( n \times 2 \) and \( n \times (3n + 2) \) zero matrices, and \([\mathbf{I}_n]\) is an \( n \times n \) identity matrix;
- \( \mathbf{\Gamma}_l(k + i) \in \mathbb{R}^{n \times 1} \): the lower constraint matrix for the articulation angles containing the lower limit of each angle as described in inequalities (5-34);
- \( \mathbf{\Gamma}_u(k + i) \in \mathbb{R}^{n \times 1} \): the upper constraint matrix for the articulation angles containing the upper limit of each angle as described in inequalities (5-34);
- \( \mathbf{S}_f(k + i) \in \mathbb{R}^{4 \times (4n+4)} \): a position transformation matrix for the system state vector. This is used to transform \( \Delta y_{0f}(k + i), \Delta y_{jc}(k + i), \) and \( \Delta y_{nu}(k + i) \) in the system.
state vector, $\Delta \mathbf{x}_L(k + i)$, into the four vehicle corner positions in the local coordinate frame of each vehicle unit, as shown in Figure 5.1 (c), $j \in [0, n]$;

- $Y_{jl}(k + i)$: a transformed lower constraint matrix of the vehicle corner positions and $j \in [0, n]$;
- $Y_{ju}(k + i)$: a transformed upper constraint matrix of the vehicle corner positions and $j \in [0, n]$.

In terms of each vehicle frame, the constraints of the corner points are given as follows:

$$y_{jl}(k + i) \leq S_j(k + i) \Delta \mathbf{x}_L(k + i) + d_j(k + i) \leq y_{ju}(k + i), \quad (5-41)$$

$$y_{jl}(k + i) = \begin{bmatrix} y_{j1l}(k + i) \\ y_{j2l}(k + i) \\ y_{j3l}(k + i) \\ y_{j4l}(k + i) \end{bmatrix}, \quad (5-42)$$

$$y_{ju}(k + i) = \begin{bmatrix} y_{j1u}(k + i) \\ y_{j2u}(k + i) \\ y_{j3u}(k + i) \\ y_{j4u}(k + i) \end{bmatrix}, \quad (5-43)$$

$$d_j(k + i) = \begin{bmatrix} w_{d_j}/2 \\ -w_{d_j}/2 \\ -w_{d_j}/2 \\ w_{d_j}/2 \end{bmatrix}, \quad (5-44)$$

where

- $y_{jl}(k + i)$: the lower constraint matrix containing the local lower limits of the corner points of the $(j + 1)^{th}$ vehicle unit, where $j \in [0, n]$;
- $y_{ju}(k + i)$: the upper constraint matrix containing the local upper limits of the corner points of the $(j + 1)^{th}$ vehicle unit, where $j \in [0, n]$;
- As shown in Figure 5.1 (c), the two front corner points have the same points of the lateral intersection between the vehicle unit and the two boundaries. This means they have the same lower and upper limits, i.e. $y_{j1u}(k + i) = y_{j2u}(k + i)$ and $y_{j1l}(k + i) = y_{j2l}(k + i)$, where $j \in [0, n]$;
Likewise, the two rear corner points have the same lower and upper limits, so 
\[ y_{j3u}(k+i) = y'_{j4u}(k+i) \text{ and } y_{j3u}(k+i) = y'_{j4u}(k+i), \]
where \( j \in [0, n] \);

- \( w_{d_j} \): the vehicle width of the \((j+1)^{th}\) vehicle unit, where \( j \in [0, n] \);
- \( d_j(k+i) \): a \( 4 \times 1 \) constant matrix, denoting the lateral distances of the four corner points from the longitudinal axis in the vehicle’s local ego coordinate frame.

Combining inequality (5-41), and equations (5-42) to (5-43) gives a simplified form, which is merged in the element-wise inequalities (5-37).

\[ \begin{bmatrix} -S_j(k+i) \\ S_j(k+i) \end{bmatrix} \Delta x_L(k+i) \leq \begin{bmatrix} -y_{j\nu}(k+i) + d_j(k+i) \\ y_{j\nu}(k+i) - d_j(k+i) \end{bmatrix} \]  

(5-45)

Compared to the element-wise inequalities (5-37), the relationship between \( Y_{j\nu}(k+i) \) and \( y_{j\nu}(k+i) \) is found:

\[ Y_{j\nu}(k+i) = y_{j\nu}(k+i) - d_j(k+i), \]  

(5-46)

Likewise, it gives a similar equation for \( Y_{j\mu}(k+i) \) and \( y_{j\mu}(k+i) \).

\[ Y_{j\mu}(k+i) = y_{j\mu}(k+i) - d_j(k+i), \]  

(5-47)

where \( Y_{j\nu}(k+i) \in \mathbb{R}^{4 \times 1} \) and \( Y_{j\mu}(k+i) \in \mathbb{R}^{4 \times 1} \).

The element-wise inequalities (5-37) can be rewritten for the preview horizon as follows:

\[ \begin{cases} M(k+i) \Delta x_L(k+i) + E(k+i)\Delta \delta(k+i) \leq b(k+i) \quad i = 0, 1, \ldots, N_p - 1 \\ M(k+N_p) \Delta x_L(k+N_p) \leq b(k+N_p) \end{cases}, \]  

(5-48)

\[ M(k+i) = \begin{bmatrix} 0 \\ 0 \\ -\rho(k+i) \\ \rho(k+i) \\ -S_0(k+i) \\ S_0(k+i) \\ \vdots \\ -S_j(k+i) \\ S_j(k+i) \\ \vdots \\ -S_n(k+i) \\ S_n(k+i) \end{bmatrix} \]  

(5-49)
\[ E(k + i) = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \]  

\[ b(k + i) = \begin{bmatrix} -\delta_i(k + i) \\ \delta_u(k + i) \\ -\Gamma_l(k + i) \\ \Gamma_u(k + i) \\ -Y_{0l}(k + i) \\ Y_{0u}(k + i) \\ \vdots \\ -Y_{jl}(k + i) \\ Y_{ju}(k + i) \\ \vdots \\ -Y_{nt}(k + i) \\ Y_{nu}(k + i) \end{bmatrix} \]  

\[ M(k + N_p) = \begin{bmatrix} -\rho(k + N_p) \\ \rho(k + N_p) \\ -S_0(k + N_p) \\ S_0(k + N_p) \\ \vdots \\ -S_f(k + N_p) \\ S_f(k + N_p) \\ \vdots \\ -S_n(k + N_p) \\ S_n(k + N_p) \end{bmatrix} \]
\[
b(k + N_p) = \begin{bmatrix}
-G_1(k + N_p) \\
G_n(k + N_p) \\
-Y_{0l}(k + N_p) \\
Y_{0u}(k + N_p) \\
\vdots \\
-Y_{jl}(k + N_p) \\
Y_{ju}(k + N_p) \\
\vdots \\
-Y_{nl}(k + N_p) \\
Y_{nu}(k + N_p)
\end{bmatrix}
\]

(5-53)

where \( M(k + i) \in \mathbb{R}^{(10n+10) \times (4n+4)} \), \( E(k + i) \in \mathbb{R}^{(10n+10) \times 1} \), \( b(k + i) \in \mathbb{R}^{(10n+10) \times 1} \), \( M(k + N_p) \in \mathbb{R}^{(10n+8) \times (4n+4)} \), and \( b(k + N_p) \in \mathbb{R}^{(10n+8) \times 1} \).

Iterating the element-wise inequalities (5-48) in the preview horizon generates a constraint inequality as follows:

\[
\begin{bmatrix}
M(k) \\
0 \\
\vdots \\
0
\end{bmatrix} \Delta x_L(k) + \begin{bmatrix}
0 & \cdots & 0 \\
M(k + 1) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & M(k + N_p)
\end{bmatrix} \begin{bmatrix}
\Delta x_L(k + 1) \\
\Delta x_L(k + i) \\
\vdots \\
\Delta x_L(k + N_p)
\end{bmatrix}
\]

\[
\begin{bmatrix}
E(k) \\
\vdots \\
0 \\
0 \\
\vdots \\
0 \\
0 \\
0
\end{bmatrix} \Delta \delta(k) \leq \begin{bmatrix}
b(k) \\
b(k + 1) \\
\vdots \\
b(k + N_c - 1) \\
b(k + N_c - 1) \\
\vdots \\
b(k + N_p)
\end{bmatrix}
\]

(5-54)

The element-wise inequalities (5-54) is simplified as follows:

\[
[D_L] \Delta x_L(k) + [\Delta_L] \Delta X_L(k) + [\varepsilon_L] \Delta U_L(k) \leq [\beta_L],
\]

(5-55)

where

- \([D_L] = \begin{bmatrix}
M(k) \\
0 \\
\vdots \\
0
\end{bmatrix} \in \mathbb{R}^{(N_p+1)(10n+10)-2 \times (4n+4)} \),

- \([\Delta_L] = \begin{bmatrix}
0 & \cdots & 0 \\
M(k + 1) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & M(k + N_p)
\end{bmatrix} \in \mathbb{R}^{((N_p+1)(10n+10)-2) \times (4n+4)N_p} \),
Replacing $\Delta X_L(k)$ in the element-wise inequalities (5-55) with equation (5-17) gives the following element-wise inequalities:

$$[[\Pi_L]]\Delta U_L(k) \leq [\beta_L] + [L_L]\Delta x_L(k), \quad (5-56)$$

where

- $[[\Pi_L]] = [\epsilon_L] + [\Delta L][T_L]$ and $[[\Pi_L]] \in R^{(N_p+1)(10n+10)-2 \times N_c}$,
- $[L_L] = -[D_L] - [\Delta L][K_L]$ and $[L_L] \in R^{((N_p+1)(10n+10)-2) \times (4n+4)}$.

At any sample instant $k$, the aim of this control method is to find the optimal value of the control input sequence, $\Delta U_L^*$, which minimizes value of the cost function, $O_L$, while satisfying the constraints in the element-wise inequalities (5-56):

$$\min_{\Delta U_L} O_L = \frac{1}{2} \Delta U_L(k)^T G_L \Delta U_L(k) + f_L^T \Delta U_L(k)$$

such that $[[\Pi_L]]\Delta U_L(k) \leq [\beta_L] + [L_L]\Delta x_L(k)$

This is a quadratic programming problem with linear constraints and the only independent variable matrix is $\Delta U_L(k)$. The optimal control input at any sample $k$ is the first element of $\Delta U_L(k)$.

Note that the path boundary matrix, $p_L(k+i)$, and the vehicle corner constraint matrices, $y_{H}(k+i)$ and $y_{ju}(k+i)$, are transformed from the world coordinates to the vehicle local frame using the following equation.

$$\begin{bmatrix} x^l \\ y^l \end{bmatrix} = \begin{bmatrix} x^\theta - x^\theta_p \\ y^\theta - y^\theta_p \end{bmatrix} \begin{bmatrix} \cos(\theta^\theta_p) & \sin(\theta^\theta_p) \\ -\sin(\theta^\theta_p) & \cos(\theta^\theta_p) \end{bmatrix}$$

(5-57)
where, as shown in Figure 5.2,

- $x^l$ and $y^l$: the coordinates of the path boundaries in the vehicle local frame;
- $x^g$ and $y^g$: the global coordinates of the path boundaries;
- $x_o^g$ and $y_o^g$: the origin position of the vehicle local frame in the world coordinate system;
- $\theta_o^g$: the heading of the vehicle in the global coordinate frame.

### 5.1.1 Solution method

There are two different types of matrices in the constrained cost function. One type is constant, which only depends on the discrete-time state-space model and the constraint transformation matrices. Examples are $[G_L]$, $[\Pi_L]$, and $[L_L]$. The other type of matrix is variable, determined by the system state vector, $\Delta x_L(k)$, the estimated future boundary paths, $[P_L(k)]$, and the estimated constraints, $b(k + i)$. Examples are: $[f_L(k)]$ and $[\beta_L(k)]$.

$[f_L(k)]$ defined below, can be calculated from both $\Delta x_L(k)$ and $[P_L(k)]$ after the discrete-time state-space model is built.

$$
[f_L(k)] = [F_L]([\Psi_L] \Delta x_L(k) - [P_L(k)])
$$

(5-58)

Figure 5.3 illustrates a flow chart of the control strategy. When the vehicle moves to the next position along the pre-defined boundary paths, the system state vector, $\Delta x_L(k)$, is calculated at sample instant $k$. Then, the future position of each vehicle unit is estimated by sampling the boundaries from the current vehicle position onwards, thus generating the boundary path matrix, $[P_L(k)]$. Hence, $[f_L(k)]$ is calculated using equation (5-58). After the exceedance of the constraints is checked, the constraints of the vehicle corner points up to the preview horizon are estimated and transformed from the global coordinates to each vehicle local frame, thus giving the constraint matrix, $[\beta_L(k)]$.

Lagrangian duality theory [208]–[210] can be used to find the minimum of the linearly constrained quadratic programming problem. The primal cost function can be transformed to a dual function using a non-negative Lagrange multiplier vector, $[\lambda]$. The Lagrangian function at sample instant $k$ is constructed as:

$$
L(\Delta U_L, \lambda) = \frac{1}{2} \Delta U_L^T G_L \Delta U_L + f_L^T \Delta U_L + \lambda^T (\Pi_L \Delta U_L - Y_L),
$$

(5-59)

where

- $[Y_L(k)] = [\beta_L] + [L_L] \Delta x_L(k)$ and $[Y_L(k)] \in R^{(N_p+1)(10n+10) - 2 \times 1}$;
• \( \Delta U_L \) represents \( \Delta U_L(k) \) for notational convenience;
• \([\lambda] \in R^{(N_p+1)(10n+10)-2 \times 1}\).

\([G_L]\) is a symmetric positive definite Hessian matrix.

The Lagrangian dual function, \( g(\lambda) \), is defined as the infimum of \( L(\Delta U_L, \lambda) \) by solving equation (5-61).

\[
g(\lambda) = \inf_{\Delta U_L} L(\Delta U_L, \lambda),
\]

where ‘inf’ stands for the infimum.

\( g(\lambda) \) is found from \( \Delta U_L \) by first solving the equation:

\[
\nabla_{\Delta U_L} L(\Delta U_L, \lambda) = 0
\]

\( (5-61) \)

\[
\Delta U_L^* = -[G_L]^{-1}(f_L + \Pi_L^T \lambda),
\]

where \( \Delta U_L^* \) is the solution of equation (5-61).

Substituting equation (5-62) into equation (5-59) gives the Lagrangian dual problem, defined as:

\[
\max_{\lambda \geq 0} \ g(\lambda) = -\frac{1}{2} \left(f_L + \Pi_L^T \lambda\right)^T [G_L]^{-1} \left(f_L + \Pi_L^T \lambda\right) - \lambda^T Y_L
\]

\( (5-63) \)

The Lagrangian dual problem is also a convex quadratic programming problem. The duality gap, defined as the difference between the optimal values of the primal and dual problems, can be used to confirm optimality. The primal-dual solver using Lagrangian duality theory and numeric iterative methods was initially written by Hartley [211] and then modified by the author of this thesis to solve this specific cost function (see Appendix A). The matrices, \([G_L]\), \([f_L(k)]\), \([\Pi_L]\), \([L_L]\), and \([\beta_L(k)]\) are the parameters of the solver, and the outputs are the optimal matrix, \( \Delta U_L(k) \) and the duality gap.

### 5.2 Simulation analysis

The following sections investigate simulations of tractor-semitrailer and B-double vehicles for several scenarios. The test vehicle parameters and geometry are shown in Tables 2.1 – 2.3 and Figure 2.1. In the simulations, the longitudinal speed of the tractor, \( u_0 \), is a constant, -1 m/s, where the negative sign represents reversing. The remaining simulation parameters for different manoeuvres, i.e. the sample time, \( T_s \), the preview and control horizons, \( N_p \) and \( N_c \),
and the weights, are shown in Tables 5.1 and 5.2 for the tractor-semitrailer and B-double vehicles respectively. These controller parameters were hand-tuned for the extreme manoeuvres described later in this section. The default values in Table 5.3, recommended by the author, are suitable for the most general scenarios. The controller needs an appropriate preview distance to plan ahead and optimise the motion paths. The preview distance for the \((j + 1)^{th}\) vehicle unit, \(D_{jp}\), is defined as:

\[
D_{jp} = u_j N_p T_s,
\]

where \(j \in [0, n]\).

As \(T_s\) decreases, the controller’s ability to reject unknown disturbance usually improves and then levels off. For a fixed preview distance, \(D_{jp}\), if \(T_s\) is small, \(N_p\) would be large, resulting in an expensive computation. If \(T_s\) is large, the controller is likely to be unstable. For a long one-step preview distance \((u_j T_s)\), some obstacles may be overlooked (due to under-sampling), and the controller may be unable to detect the obstacles. As the open-loop system is unstable and the closed-loop response time is unknown, the sample time has to be selected based on trial and error to find a trade-off between the controller performance and the computational effort [212].

In this study, \(T_s = 1 \text{s}\) was selected empirically based on the simulation trials for the tractor-semitrailer vehicle and \(T_s = 2 \text{s}\) for the B-double vehicle. The recommended default values were used at the beginning of the tuning process and adapted in the extreme cases (e.g. the case of the mountain road with a parked lorry in Section 5.3.5). If the vehicle exceeded the boundaries, the simulation stopped and the values of \(N_p, N_c\) and controller weights were re-tuned. The simulations were run using a discrete solver at a fixed fundamental sample rate of 100 Hz. In this study, the processor used for the simulations was Intel (R) Core (TM) i7-7700 CPU running at 3.60 GHz and the installed RAM was 32 G.

In terms of the physical limitations of the test vehicles used in this study, the constraints for the steer angle and articulation angles were constants, e.g. \(\delta \in [-45^\circ, 45^\circ]\), \(\dot{\delta} \in [-18^\circ/\text{s}, 18^\circ/\text{s}]\), and \(\gamma_j \in [-90^\circ, 90^\circ]\), where \(j \in [1, n]\).

### 5.3 Tractor-semitrailer case

Several different scenarios were simulated to test the controller’s performance. These included:

(i) a lane change manoeuvre with obstacles on each corner;
(ii) passing by a parked car or a
lorry on a mountain road; (iii) a lane change into a narrow loading bay; (iv) and a 90-degree corner preceding a narrow gate. The simulation parameters for each manoeuvre are displayed in Table 5.1. The magenta rectangle stands for the tractor, and the black rectangle represents the semitrailer. The vehicle travels from right to left in reverse.

5.3.1 Lane change manoeuvre

A tube-shaped path with upper and lower boundaries is depicted in Figure 5.4 (a). The lane width and vehicle width are about 6 and 2.55 m, respectively. There are two 7-metre-length, 1.5-metre-width obstacles (i.e. ‘Obstacle 1’ and ‘Obstacle 2’) located on each corner. These are included in the definition of the path contours.

A typical nominal path does not need to be defined using the LBRC method. Due to the hard boundaries and obstacles, the reversing spaces are significantly confined.

As shown in Figure 5.4 (b), the controller enables the tractor-semitrailer vehicle to get around the two obstacles on the corner and converge to the terminal without any collisions. Before passing by the obstacles, the vehicle starts to deviate from the middle line of the boundaries and move towards to the opposite side of the obstacle location, as illustrated in Figures 5.4 (c) and 5.4 (d). During the obstacle avoidance, the controller also needs to predict the future positions, estimate the distance to the boundary, and prevent the vehicle units from hitting the boundary. After passing by the obstacles, the vehicle recovers from the intended deviations caused by the obstacle avoidance and resumes travelling along the middle line within the boundaries, towards the terminal.

As shown in Figure 5.4 (e), in terms of the LBRC method, the trajectories of the mid-points of the front end of the tractor (the solid blue line) and the rear end of the semitrailer (the solid red line) deviate significantly from the nominal path, and the trajectory of the front end has two obstacle-shaped dents, compared to those using the PFC and MSPC methods. To see the difference clearly, the axes are not equally scaled. The tractor excursions using the PFC (the dotted blue line) and MSPC (the dashed blue line) methods are much larger than those using the LBRC method. The tractor hits both obstacles using the PFC method, and crashes around ‘Obstacle 2’ using the MSPC method. The advantage of MSPC method over the PFC method is to minimise the overall swept path, but both methods still have the same purpose of following a target path and do not proactively consider the path boundaries. The MSPC method can be used to avoid some obstacles by the large reduction in the overall swept path, but its controller is not ‘aware’ of doing this. Unlike the PFC and MSPC approaches, using the LBRC method
allows the vehicle to avoid the obstacles actively during the manoeuvre. The PFC and MSPC controllers are not capable of predicting potential collisions and taking effective actions in advance to prevent them from happening.

5.3.2 Limits of manoeuvrability

A series of simulations of lane change manoeuvres were run to test the controller’s limits of manoeuvrability as shown in Figures 5.5 (a) – 5.5 (f). The differences between the simulations shown in the figures are the distance between the starting and terminal positions and whether or not there is a transition phase (see Figures 5.5 (a) – 5.5 (d)) in the boundaries connecting to the terminal. The desired tube-shaped boundaries are drawn to the same scale and the lane width remains the same for comparison. The starting distance was shortened progressively until the LBRC controller failed to accomplish the manoeuvre without exceeding the boundaries. The graphs for the starting distance between 0 and 60 m are omitted for simplicity. The length of the tractor-semitrailer vehicle is approximately 17 m. The transition phase in Figures 5.5 (a) – 5.5 (c) has two affine lines with 20 m horizontal span. Hence, the definition of the boundaries has $C^0$ continuity with cusps. The length of the transition phase in Figure 5.5 (d) has been halved because of the reduced length of the manoeuvre. The lane change curves are connected to the loading bay directly without any transition phase in Figures 5.5 (e) and 5.5 (f). When the distance from the vehicle start point to the loading bay is 20 m or less, there is no feasible path.

As shown in Figures 5.5 (b) and 5.5 (c), the controller can handle the discontinuous curvature of the boundaries and allows the vehicle units to make good use of the available space to reverse into the terminal. Consequently, minimal effort is needed to define the path parameters and hence plan the path of the vehicle. This is very different to the PFC method [19], which requires paths with continuous curvature derivatives up to the number of trailers. When the terminal emerges in the preview horizon of the controller, the heading of the semitrailer is adjusted to be directed towards the terminal so that the whole combination can exactly pass the narrow mouth of the loading bay. The controller is able to plan feasible paths to pass through the confined spaces to the terminal and estimate all future vehicle positions and headings during the entire manoeuvre at the very beginning of the movements, as shown in Figures 5.5 (d) – 5.5 (f), although it needs large computing power. For instance, for the first simulation step, it costed approximately 1.15 s for a fixed step size of 0.01 s, and the average time of the following simulation steps was about 0.24 s after some data were cached in the first simulation step. The large computational time was caused by the long preview distance. As seen in Figure 5.5 (d), the tractor takes the semitrailer very close to the edge of the first corner so as to utilise the
available space to realign its position for entry to the loading bay. Then it manages to put the
tail of the semitrailer through the mouth. The controller’s performance is very good even in the
extremely confined spaces as depicted in Figure 5.5 (f). It is also shown that the continuous
boundary curvature is not required for the controller because the constraints on the steer angle
guarantee a smooth path.

5.3.3 Sharp 90-degree bend with tight boundaries

A tight 90-degree turn with 4.6-metre-width is shown in Figure 5.6 (a). This manoeuvre was
aimed to test the controller’s performance for large changes in heading angles and large
curvatures. Compared to lane change manoeuvres, the tractor experiences much larger
excursions from the nominal path for sharp turns. It is difficult to keep the vehicle units within
the narrow boundaries using the PFC and MSPC methods.

As seen in Figure 5.6 (a), the tail of semitrailer starts to move towards the inner boundary at
the beginning, very close to the corner. Then it moves along the direction of a tangent line to
the inner circle, which could leave the maximal spaces for the movements of tractor unit. The
tractor almost touches the outer boundary so that the semitrailer passes by the corner. The
tractor then returns to the nominal path.

Compared to the PFC and MSPC methods in Figure 5.6 (b), the LBRC method enables the
vehicle units to adapt to the width of boundaries. The rear-end middle point of the LBRC
vehicle (the solid red line) is inside the nominal path rather than along the path during the
manoeuvre, and after the corner, it converges to the straight line again. The middle point of the
front end of the PFC (the dotted blue line) and MSPC (the dashed blue line) vehicles are much
further from the nominal path than the LBRC (the solid blue line). The corner points of both
PFC and MSPC vehicles also exceed the outer boundary during the sharp turn. This means the
LBRC balances the excursions of the tractor and semitrailer so that both units reverse within
the boundaries.

5.3.4 Misaligned narrow gate

A mixed manoeuvre, which consists of a 90-degree bend connecting to a parking lot through a
3-metre-width gate, is shown in Figure 5.7 (a). The middle line of each single part is misaligned,
which requires the LBRC to respond promptly during transitions and the vehicle units to pass
through the gate with a small offset in heading angle. The vehicle starts off with initial position
offsets, i.e. a lateral offset from the middle line and a small heading offset. The robustness of
the controller is tested by the disturbance.
The vehicle gets back to the middle line quickly from the initial position, as shown in Figure 5.7 (b). Just after the tractor emerges from the corner, the gate appears in the preview horizon of the controller, so the tractor starts to change its direction and push the tail of the semitrailer towards the middle line of the gate. Given the misalignment of the gate and the terminal position, passing through the gate with a small heading angle is an optimal behaviour, because the vehicle units have already adjusted their internal offset while passing through the gate. This means it takes less spaces and time to converge to the terminal path after the gate. Figure 5.7 (c) shows a magnified view of the motion of each unit through the gate. The vehicle units make good use of the limited space, while continuously making small changes to their orientations.

5.3.5 Mountain road

A U-shaped mountain road with a 6-metre-length ‘limousine’ parked on the left side was simulated, as shown in Figure 5.8 (a). The vehicle reverses past the car, starting at the top right of the figure. As the length of the parked car is approximately one third of the total length of the tractor-semitrailer combination, while the combination passes by the car, only a small part of the road is blocked by the car. As seen in Figure 5.8 (b), the parked car acts as a short-period perturbation because the overall trajectories still remain around the nominal line and only the part around the car is affected. Figure 5.8 (c) shows that the semitrailer passes very close to the parked car, and when the tractor moves within the unobstructed lane between the car and the inner boundary on the right side, it starts to turn left to push the semitrailer back to the nominal line.

In Figure 5.9 (a), the length of the obstacle is extended to 16 m, which is approximately the length of the tractor-semitrailer combination: simulating a lorry parked at the same side of the mountain road. Unlike the case of the parked ‘limousine’, there is a short steady state in the unobstructed lane between the parked lorry and the inner boundary on the right side, as seen in the magnified view in Figure 5.9 (b). After the semitrailer enters the unobstructed lane, it continues to move along the middle line. The vehicle units in Figure 5.9 (c) pass further away from the obstacle than the paths in Figure 5.8 (b).

The controller is able to adapt to the different lengths of obstacles by changing the vehicle behaviour.

5.4 B-double case

The theory in section 5.1 can be used to reverse long combination vehicles with multiple articulation points. It is applied to a doubly-articulated (‘B-double’) vehicle in this section.
Three typical manoeuvres were simulated to test the controller’s performance for the B-double combination. As the total length of the whole combination is approximately 26 m, a relatively longer preview distance is required for motion planning. However, a long preview distance slows the computation and simulation speeds down. An effective way to solve the problem is to increase the sample time, $T_s$, appropriately. The simulation parameters for each manoeuvre are displayed in Table 5.2. The magenta, green and black rectangles denote the tractor, B-link trailer and semitrailer, respectively.

5.4.1 Lane change manoeuvre

Figure 5.10 (a) shows a manoeuvre consisting of tube-like boundaries with an 8-metre-length obstacle at each corner of a 20-metre lane change, connected to a 4-metre-width loading bay. There is no transition phase between the boundaries of the lane change path and the loading bay. The two obstacles block the vehicle movements so that it cannot follow the middle line. The controller has no difficulty stabilising the reversing motion of the vehicle. When the obstacles occur within the preview horizon of the controller, the controller recognises their positions and plans ahead to optimise the steer angle trajectory. By doing this, the semitrailer is pushed closer to the opposite side of the obstacles while passing by the obstacles. After avoiding the second obstacle, the tractor turns left to aim the tail of the semitrailer towards the loading bay. Then, the vehicle units move towards the loading bay smoothly, and eventually stop at the terminal position.

5.4.2 Misaligned narrow gate

In Figure 5.10 (b), the position of the B-double combination is misaligned with the terminal. The boundaries imitate a rural scenario consisting of road curbs, walls and a 3-metre-width gate access into a farm yard. The curvatures of the boundaries are discontinuous. A wall partly blocks the initial position of the vehicle. There is a strict requirement for the vehicle orientations to pass through the narrow gate.

When the semitrailer approaches around ‘$x = -10$ m’ prior to the wall, the controller detects the wall in the preview horizon and makes the tractor turn left to push the second trailer and semitrailer away from the wall. After the tractor avoids hitting the wall, the controller allows the vehicle units to be realigned to pass through the gate with a small heading offset so as to drive directly towards the terminal position with minimal corrections.
5.4.3 Sharp 90-degree bend

A tight 90-degree turn with a 10-metre-length obstacle around the corner is shown in Figure 5.10 (c). The 4 m loading bay emerges in the end of this manoeuvre as the terminal. For this manoeuvre, the tractor experiences the largest excursions compared to the two trailers. The trajectory of the front overhang of the tractor is drawn in Figure 5.10 (d) for cases with and without the obstacle. There is an obvious difference in the front-end trajectories in the two cases. The path with the obstacle (the solid blue line) is much closer to the inner boundary than the path without the obstacle (the dashed blue line). This is because the obstacle obstructs the original vehicle optimal route and thus changes the controller’s optimal line.

5.5 B-triple case

The LBRC theory is applied to a triply-articulated (‘B-triple’) vehicle in this section to investigate the controller stability subject to the constrained conditions, e.g. obstacles, cusps, discontinuous curvature, and a loading bay. It is not guaranteed that the recommended controller parameters, e.g. the sample time, the preview and control horizons, and the controller weights, work in every scenario. This means that the controller parameters may need to be hand-tuned again to adapt to a new scenario. As the number of trailers increases, the preview horizon needs to increase, which will increase the computational burden significantly. There are many parameters that can be tuned in the controller, so a systematic approach is essential. In this section, for the B-triple vehicle, a complex scenario is simulated with re-tuned controller parameters. When the number of trailers is beyond 3, tuning the controller for complex scenarios becomes much harder. Hence, a generic simpler scenario will be used to discuss the controller stability of the general n-trailer vehicle in section 5.6.

The simulation parameters are shown in Table 5.4.

5.5.1 Lane change manoeuvre

A 20 m lane change with two 20 m long and 6 m wide obstacles at each corner, connected to a 4 m wide loading bay, is shown in Figure 5.11 (a). The pre-defined tube boundaries have $C^0$ continuity and there is no transition phase between the lane change and loading bay. The solid magenta, green, cyan and black rectangles represent the tractor, the two B-link trailers and the semitrailer respectively. The vehicle successfully avoids the two large obstacles and stop at the terminal position in the loading bay.
Figures 5.11 (b) and 5.11 (c) show a magnified view of motion of each vehicle unit around the obstacles (‘Obstacle 1’ and ‘Obstacle 2’). Before it passes the obstacles, the tractor has already adjusted the vehicle position to the middle line between the obstacle and the boundary so that the trailers move along the middle line smoothly and keep an appropriate distance away from both the obstacle and the boundary. When the tractor moves close to the obstacles, it starts to change its direction so as to push the trailers back to the nominal path, because the obstacles disappear in the preview horizon of the rear trailer. Hence, only the tractor experiences large excursions in the vicinity of the obstacles and the trailers resume their motion along the original route after getting past the obstacles.

A comparison between the cases with and without the obstacles is shown in the magnified view in Figures 5.11 (d) and 5.11 (e). The path of the front-end of the tractor unit (point ‘f’ in Figure 5.1(b)) with the obstacles (the solid blue line) is much further away from the obstacles, compared to the path without the obstacles (the dashed blue line). It is seen that there is a short steady-state phase around the obstacles because of their relatively long length (20 m). The front-end point of the tractor moves along the original nominal path before and after the obstacles, and along the middle line between the obstacle and boundary when the tractor travels within the confined space. The obstacles have a significant impact on the controller’s decision making.

The controller is robust and stable, even in the case of B-triple vehicle performing the constrained lane change manoeuvre.

5.6 Stability and feasibility of the general n-trailer vehicle

As discussed in section 5.5, a generic simpler scenario is used to investigate the stability of the general n-trailer vehicle. The manoeuvre has a lane-change with \( C^0 \) continuous boundaries, terminating in a loading bay, with no obstacles (See Figure 5.12).

Unlike the LQR-tuned methods (e.g. PFC and MSPC), the LBRC strategy has a finite horizon causing the open-loop prediction to diverge from the closed-loop system. Following [213]–[216], to eliminate the divergence and ensure stability, \( q_{k+i} \), \( r_{k+i} \), and \( q_{k+N_p} \) must be positive definite such that the terminal cost, \( V_f \) shown in (5-22), is a control Lyapunov function, \( V \):

\[
V_f = \| \Delta z_L(k + N_p) - p_L(k + N_p) \|^2_{q_{k+N_p}}
\]  

(5-65)

with a stabilising terminal control law defined in equation:
\[ \Delta \delta(k + N_p) = \begin{bmatrix} K_{N_p} \end{bmatrix} \Delta x_L(k) \]  

(5-66)

Here \( \begin{bmatrix} K_{N_p} \end{bmatrix} \) is the terminal control gains for the system state vector.

The terminal control law can be derived from the discrete-time LQR method.

The discrete-time Lyapunov function can be constructed as:

\[ V = (A_{LDT} + B_{LDT} K_{N_p})^T Q_{N_p} (A_{LDT} + B_{LDT} K_{N_p}) - Q_{N_p} + C^T q_{k+N} + K_{N_p}^T r_{k+i} K_{N_p} \]  

(5-67)

where \( Q_{N_p} \) is calculated by the discrete-time Lyapunov equation, \( V = 0 \).

The relationship \( Q_{N_p} \) and \( q_{k+N} \) is defined as:

\[ Q_{N_p} = C^T q_{k+N} C \]  

(5-68)

Hence, the calculated \( q_{k+N} \) from the terminal control law can be used to guarantee the Lyapunov stability [217]–[219] for the LBRC method.

For recursive feasibility [206], [213], [214], theoretically the terminal transformation and constraint matrices, \( M(k + N_p) \) and \( b(k + N_p) \) should be calculated such that \( M(k + N_p) \Delta x_L(k + N_p) \leq b(k + N_p) \) is constraint admissible and invariant for the closed-loop system defined below.

\[ \Delta x_L(k + 1) = \left( A_{LDT} + B_{LDT} K_{N_p} \right) \Delta x_L(k) \]  

(5-69)

The invariant and constraint admissible conditions are met by inequalities (5-70) and (5-71), respectively, for the closed-loop system [213], [214].

\[ M(k + N_p) \left( A_{LDT} + B_{LDT} K_{N_p} \right) \Delta x_L(k + N_p) \leq b(k + N_p) \]  

(5-70)

\[ (M(k + i) + E(k + i) K_{N_p}) \Delta x_L(k + N_p) \leq b(k + i) \]  

(5-71)

Hence, the terminal constraint satisfying the above inequalities can be used to guarantee a solution of the LBRC method.

However, in practice when solving the constrained reversing problem, for different definitions of constrained boundaries, it is not possible to guarantee that inequalities (5-70) and (5-71) are satisfied at every sample instant before the simulations are completed. This is because \( b(k + i) \)
derived from the pre-defined boundaries is likely to vary all the time and moreover is not known beyond the preview horizon.

Without the constructed terminal cost function and terminal constraint set, the best strategy is for $N_p$ to be sufficiently large to reduce the chance of running into infeasibility. This approach was found to work well in all of the cases examined in this study. However, the extremely expensive computation becomes a hurdle for determining a sufficiently long horizon for the general n-trailer vehicle. This is because tuning the controller parameters is based on empirical simulation results, but the large computational time thwarts the attempt to pursue a feasible solution.

A tractor articulated with 7 B-link trailers is stably controlled within the $C^0$ continuous boundaries using the LBRC method, as shown in Figure 5.12. In the case of more than 7 B-link trailers, the computation becomes intractable. This compares with Rimmer [19], who found that it was only possible to stabilise a B-train with up to 6 trailers using the PFC controller.

Two factors influencing the computing speed are investigated here: one is the number of trailers, $n$; the other one is the length of preview horizon, $N_p$. Although the two factors are correlated, varying one factor and fixing the other controller parameters can be used to investigate the relationship between the computing speed and the factors.

The fixed simulation time step used in this study is, $t_s = 0.01 \text{ s}$. The relative computational efficiency index, $I_c$, is defined as:

$$I_c = \frac{t_a}{t_s}$$

where $t_a$ denotes the averaged actual elapsed running time for the first 10 simulation steps. $I_c$ is plotted against the number of trailers, $n$, in Figure 5.13. The vertical axis represents how many times the real-time computational time exceeds the fixed time step. In this case, the sample time, preview horizon, and control horizon are set to 2 s, 30 and 10, respectively; the controller weights are all 1. When the number of trailers increases to 10, the simulation slows down more than 150 times, regardless of the expected effect of increasing the preview and control horizons. The increase in computation speed is essentially linear, because the number of trailers and the length of preview horizon are naively assumed to be uncorrelated.

The relationship between $I_c$ and $N_p$ is investigated in Figure 5.14 by varying the preview horizon for a tractor articulated with 7 B-link trailers. The sample time, the control horizon and
the weights remain the same. Figure 5.14 shows that when the preview horizon is increased by 10, the computation power increases by approximately 22.2 times, which is worse than the increase in the number of trailers. For instance, completing the lane change manoeuvre defined in Figure 5.12 generally needs about 27,000 simulation steps. In the case of the 7 trailers with $N_p = 60$, it takes about 13 hours to complete one simulation.

The empirical relationship between the preview distance and the vehicle length is that the best preview distance is approximately 1.7 times the vehicle length. The average actual computation time, $t_a$, for a feasible solution of different combination vehicles, with varying preview horizons, is plotted in Figure 5.15. This shows how much time is exhausted for a single iteration of the fixed 0.01 s step size. As the number of trailers increases, the actual computational time increases dramatically.

When the two parameters, $N_p$ and $n$, are both varied independently, Figures 5.13 and 5.14 show the relationship between the computational time and changing only one factor results in a linear relationship. This is because the number of rows and columns of matrices depends on both $N_p$ and $n$. When only one factor changes, the relationship is linear; when the two factors are modified together for a feasible solution, the relationship is non-linear, as shown in Figure 5.15.

Hence, from the perspective of real-time applications, a simplified LBRC algorithm will be presented in Chapter 6, to reduce the computational burden significantly. However, the simplified algorithm is still significantly more complex than the LQR methods and the running time is considerably larger than the vehicle response time. Therefore, a time lag is considered in Chapter 6 for the controller implementation.

5.7 Conclusions

1. The LBRC is a non-linear method, devised to enable autonomous reversing systems to perform precise, space-confined, and collision-free manoeuvres to target terminals. The theory applies to vehicles with multiple trailers.

2. The cost function is a linearly constrained quadratic programming problem and its minimum can be found by the modified primal-dual solver.

3. The effectiveness of the control strategy was demonstrated using simulations of tractor-semi-trailer, B-double, and B-triple vehicles reversing along a range of difficult manoeuvres.
past obstacles, around sharp corners, and through narrow gates to a terminal position in a loading bay.

4. The controller is robust to the details of boundary contours and initial perturbations. The tube boundaries do not require continuous curvature and can be discontinuous in gradient.

5. Theoretically, the terminal cost function and constraints can be constructed to guarantee the Lyapunov stability and recursive feasibility for the LBRC method. However, in practice, it is not possible to guarantee the recursive feasibility in advance for different constrained manoeuvres, because the constraints beyond the preview horizon are not known until they have entered the preview horizon. Hence, the preview horizon, $N_p$, has to be sufficiently large to reduce the chance of running into infeasibility.

6. A tractor articulated with 7 B-link trailers can be stabilised using the LBRC method. This is an improvement on the PFC method.

7. To find the stability limit of the general n-trailer vehicle experimentally is constrained by the available computing power.

8. The relationships between the computing speed, the number of trailers, and the preview horizon were investigated by varying one factor and keeping the other parameters constant. Increasing the preview horizon by 10 (i.e. the increase in preview distance is approximately 10 m) has a more significant impact on the computing speed than an extra trailer (i.e. the vehicle wheelbase is approximately 10 m in this study). However, the number of trailers is correlated with the preview horizon. When the number of trailers increases, the preview horizon increases correspondingly, so the computation time increases approximately parabolically with the number of trailers.

9. When taking the computational time into account, it is not possible to implement the controller using the complete LBRC theory. Hence, a simplified LBRC algorithm will be discussed in Chapter 6 to reduce the computational burden and time lag significantly.

10. The main drawback of LBRC method is the need to tune the controller parameters manually, although the default values can be used in most cases. An ‘Adaptive Lane-Bounded Reversing Control (ALBRC)’ method is described in Chapter 7 to offset this disadvantage.
## 5.8 Tables

*Table 5.1 Simulation parameters for the tractor-semi trailer vehicle*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sample time, $T_s$</th>
<th>Preview horizon, $N_p$</th>
<th>Control horizon, $N_c$</th>
<th>Weight on the upper constraint of the front end of the tractor, $w_{0fu}$</th>
<th>Weight on the lower constraint of the front end of the tractor, $w_{0fl}$</th>
<th>Weight on the upper constraint of the rear end of the semitrailer, $w_{nrBu}$</th>
<th>Weight on the lower constraint of the rear end of the semitrailer, $w_{nrBl}$</th>
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139
Table 5.2 Simulation parameters for the B-double vehicle

<table>
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<th>Misaligned narrow gate</th>
<th>Sharp 90-degree bend</th>
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Table 5.3 Recommended default values for the simulation parameters in the case of tractor-semitrailer and B-double vehicles

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<th>Vehicle combination</th>
<th>Tractor-semitrailer</th>
<th>B-double</th>
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<td>Control horizon, $N_c$</td>
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5.9 Figures

(a) Path boundaries in each vehicle local frame

(b) Notation for vehicle positions
(c) Notation for vehicle corner boundary constraints

Figure 5.1 Notation

Figure 5.2 Transformation from the global to local coordinates. As an example, the $j^{th}$ trailer is shown for illustration
Figure 5.3 Control strategy flow chart
(a) A lane change manoeuvre with two obstacles at each corner for the tractor-semitrailer vehicle

(b) Motion paths for the lane change manoeuvre with two obstacles at each corner for the tractor-semitrailer vehicle
Figure 5.4 Simulation results of the tractor-semitrailer vehicle during the lane change manoeuvre

(c) Motion paths around the first obstacle during the lane change manoeuvre for the tractor-semitrailer vehicle

(d) Motion paths around the second obstacle during the lane change manoeuvre for the tractor-semitrailer vehicle

(e) Trajectory comparison between the LBRC, MSPC and PFC control strategies during the lane change manoeuvre for the tractor-semitrailer vehicle
(a) A lane change manoeuvre with a loading bay as the terminal and a 20 m long transition phase connected to the loading bay. The tractor-semitrailer vehicle starts moving at the origin of the global frame.

(b) The tractor-semitrailer vehicle starts moving at $x = -60 \text{ m}$ in the global frame.

(c) The tractor-semitrailer vehicle starts moving at $x = -70 \text{ m}$ in the global frame.
(d) The lane change manoeuvre with a 10 m long transition phase connected to the loading bay. The tractor-semitrailer vehicle starts moving at \( x = -80 \, \text{m} \) in the global frame

(e) The lane change manoeuvre without the transition phase connected to the loading bay. The tractor-semitrailer vehicle starts moving at \( x = -90 \, \text{m} \) in the global frame

(f) The tractor-semitrailer vehicle starts moving at \( x = -95 \, \text{m} \) in the global frame

Figure 5.5 Limits of manoeuvrability
(a) A 90-degree turn manoeuvre with tight boundaries for the tractor-semitrailer vehicle

(b) Trajectory comparison between the LBRC, MSPC and PFC control strategies during the 90-degree turn manoeuvre for the tractor-semitrailer vehicle

Figure 5.6 Simulation results of the tractor-semitrailer vehicle performing a 90-degree turn manoeuvre
(a) A mixed manoeuvre for the tractor-semitrailer vehicle

(b) Motion paths during the mixed manoeuvre for the tractor-semitrailer vehicle

(c) Motion paths around the gate during the mixed manoeuvre for the tractor-semitrailer vehicle

Figure 5.7 Simulation results of the tractor-semitrailer vehicle performing a mixed manoeuvre
Figure 5.8 Simulation results of the tractor-semitrailer vehicle performing a U-shaped manoeuvre with the parked ‘limousine’ (‘Obstacle’) on a mountain road.

(a) A U-shaped manoeuvre for the tractor-semitrailer vehicle and a parked ‘limousine’ (‘Obstacle’) on a mountain road.

(b) Motion paths during the U-shaped manoeuvre with the parked ‘limousine’ (‘Obstacle’) for the tractor-semitrailer vehicle.

(c) Motion paths around the parked ‘limousine’ (‘Obstacle’) during the U-shaped manoeuvre for the tractor-semitrailer vehicle.

Figure 5.8 Simulation results of the tractor-semitrailer vehicle performing a U-shaped manoeuvre with the parked ‘limousine’ (‘Obstacle’).
Figure 5.9 Simulation results of the tractor-semitrailer vehicle performing a U-shaped manoeuvre with the parked ‘lorry’ (‘Obstacle’) for the tractor-semitrailer vehicle

(a) The U-shaped manoeuvre for the tractor-semitrailer vehicle and a parked ‘lorry’ (‘Obstacle’) on the mountain road

(b) Motion paths around the parked ‘lorry’ (‘Obstacle’) during the U-shaped manoeuvre for the tractor-semitrailer vehicle

(c) Motion paths during the U-shaped manoeuvre with the parked ‘lorry’ (‘Obstacle’) for the tractor-semitrailer vehicle
(a) Motion paths for the B-double vehicle performing a lane change manoeuvre with two obstacles at each corner

(b) Motion paths for the B-double vehicle passing through a narrow gate

(c) A 90-degree turn manoeuvre with an obstacle at the corner for the B-double vehicle

(d) Comparison of the front-end trajectory with and without the obstacle

Figure 5.10 Simulation results of the B-double vehicle performing various manoeuvres
(a) Motion paths for the B-triple vehicle performing a lane change manoeuvre with two obstacles at each corner

(b) Motion paths around the first obstacle (‘Obstacle 1’)

(c) Motion paths around the second obstacle (‘Obstacle 2’)

(d) Comparison of the trajectory of the front-end of the tractor, with and without the first obstacle ('Obstacle 1')

(e) Comparison of the trajectory of the front-end of the tractor, with and without the second obstacle ('Obstacle 2')

Figure 5.11 Simulation results of the B-triple vehicle performing a lane change manoeuvre
Figure 5.12 Motion paths for a tractor articulated with 7 B-link trailers performing a lane change manoeuvre

Figure 5.13 The relationship between the relative computational efficiency index and the number of trailers, for fixed sample time, preview horizon, and control horizon
Figure 5.14 The relationship between the relative computational efficiency index and the preview horizon for a tractor articulated with 7 B-link trailers and fixed sample time and control horizon.

Figure 5.15 The empirical relationship between the average actual computational time per simulation step for a feasible solution of each combination vehicle and the number of trailers, with the preview distance set to 1.7 times the vehicle length.
Chapter 6 Implementation of Lane-Bounded Reversing Control (LBRC)

This chapter presents the implementation of the LBRC method on the CVDC-owned, full-scale tractor-semitrailer and B-double vehicles, as shown in Figure 4.1. As the constrained objective function (equation (5-32)) is a function of the discrete-time system state, $\Delta x_L(k)$, the initial vehicle testing framework discussed in Chapter 4 was modified to measure and estimate all state variables, e.g. the lateral and yaw velocities of the tractor, and the articulation angle rates. Section 6.1 describes the major modifications and the estimation methods. The core algorithm is elaborated in Section 6.2.

Since the LBRC is a non-linear control method including an optimiser, the computation is expensive and time-consuming. There is a long-time lag between the vehicle response and the global controller processing. This was taken into account in the original simulation model described in Chapter 5. The sampling frequency of the real-time applications is shown in Table 6.1. Three common scenarios were devised to test the performance of the controller in planning and decision-making. Two different versions of the simulation model were used: synchronous (without time delays) and asynchronous (with time delays). The differences between these models is discussed in Section 6.3.

The experimental tests were performed at Bourn airfield, Cambridgeshire. The reversing manoeuvres were set out for the experiment using traffic cones as obstacles. Precise and collision-free manoeuvres were achieved by employing the LBRC method. The experimental and simulation results are compared in Section 6.4. The vehicle testing was recorded on videos that are available on YouTube. The video links are as follows:


6.1 Vehicle testing framework

6.1.1 System architecture

The overall system architecture is shown in Figure 6.1. By comparison with the MSPC system (See Figure 4.3), it is seen that the core component, i.e. the controller subsystem, was replaced and most of the remaining parts were unchanged. Therefore, the main hardware, including the AB Dynamics S30 steering robot, the V.S.E. articulation angle sensors, the UniMeasure string potentiometer, and the OxTS INS and base station, remained in use, as explained in Chapter 4. The calibration functions for the analogue sensors and the steering system remained the same.
As shown in Equation (5-13), the system states of the LBRC state space model consist of the lateral and yaw velocities of the tractor, $v_0$ and $\Omega_0$, the articulation angles, $\Gamma_i$, the articulation angle rates, $\dot{\Gamma}_i$, and the local lateral offsets derived from the position and heading of each vehicle unit. The articulation angles, and the velocities, position and heading of the semitrailer were directly measured using the hardware. Because there was only one precise IMU (i.e. the RT3022 that was installed above the ‘effective rear axle’ at the end of the rear trailer), the articulation angle rates, and the velocities, position and heading of the other vehicle units had to be estimated, using the articulation angle measurements. Hence, a ‘Vehicle Motion Estimator’ component was developed and added to the system architecture. The outputs of this subsystem were the discrete-time system state, $\Delta x_L(k)$, along with the measured and estimated position and heading of each vehicle unit.

The position and heading of the other vehicle units were estimated using the vehicle geometry, the measured articulation angles, and the measured position and heading of the rear trailer. The coordinates of each vehicle corner point were estimated at each time step to check the exceedance of the lane boundaries.

### 6.1.2 Estimations of the articulation angle rates

The articulation angle rate was estimated by numerically differentiating the articulation angle sensor readings at each sampling interval.

$$\hat{\Gamma}_i(k) = \frac{\tilde{\Gamma}_i(k) - \tilde{\Gamma}_i(k-1)}{T_s}, \quad (6-1)$$

where $k \in (0, +\infty]$ and the unit is rad/s.

- $\hat{\Gamma}_i(k)$: the estimated articulation angle rate between the $i^{th}$ and $(i + 1)^{th}$ vehicle units at the current sample instant $k$. The initial condition, $\hat{\Gamma}_i(0)$, was 0, as the vehicle was stationary at the very start of each test session;
- $\tilde{\Gamma}_i(k)$: the measured articulation angle between the $i^{th}$ and $(i + 1)^{th}$ vehicle units at the current sample instant $k$;
- $\tilde{\Gamma}_i(k - 1)$: the measured articulation angle between the $i^{th}$ and $(i + 1)^{th}$ vehicle units at the preceding sample instant $k - 1$. The initial condition, $\Gamma_i(0)$, was 0, as the vehicle units were aligned on a straight line and the sensor drift was offset at the very start of each test session;
- $T_s$: the sample time of the real-time application.
The estimated articulation angle rate, $\hat{\Gamma}_i(k)$ as per equation (6-1), was very noisy. This is because the noise of the articulation angle signals was amplified and the signal-to-noise ratio was degraded via the numerical differentiation [220]–[222]. A Butterworth-type infinite impulse response (IIR) filter [223] was used to smooth the signal spectrum and attenuate the high frequency noise. (The IIR filter responded faster and required less memory space in the real-time applications, compared to alternative finite impulse response (FIR) filters.) The transfer function of the second-order, low-pass, discrete-time Butterworth filter is given as follows.

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 (z^{-1})^2}{a_0 + a_1 z^{-1} + a_2 (z^{-1})^2},$$  \hspace{1cm} (6-2)

where $z^{-1}$ is the delay operator.

The discrete filter was tuned using the testing data for the lane change and roundabout manoeuvres, as shown in Figure 6.2. The solid blue shows the estimated articulation angle rate via differencing and the solid red line represents the filtered version. The introduced time lag was about 0.017s, much smaller than the updated sampling time of the global controller. The parameters of the filter are shown in Table 6.2.

### 6.1.3 Estimations of the lateral and yaw velocities of the tractor

After the articulation angle rate was estimated, the velocities of the C.o.M. of each vehicle unit could be calculated. The longitudinal, lateral, and yaw velocities of the ‘effective rear axle’ of the semitrailer (the last vehicle unit) were measured by the IMU. Then, the estimated velocities of the C.o.M. of the semitrailer were determined as follows.

$$\hat{\Omega}_n(k) = \hat{\Omega}_{imu}(k), \hspace{1cm} (6-3)$$

$$\hat{v}_n(k) = \hat{\tilde{v}}_{imu}(k) + (l_{effn} - a_n)\hat{\Omega}_{imu}(k), \hspace{1cm} (6-4)$$

$$\hat{u}_n(k) = \hat{\tilde{u}}_{imu}(k), \hspace{1cm} (6-5)$$

where

- $\Omega_{imu}(k)$: the measured yaw velocity of the ‘effective rear axle’ of the semitrailer at sample $k$;
- $\hat{\Omega}_n(k)$: the estimated yaw velocity of the C.o.M. of the semitrailer at sample $k$;
- $\hat{\tilde{v}}_{imu}(k)$: the measured lateral velocity of the ‘effective rear axle’ of the semitrailer at sample $k$;
• $\hat{v}_n(k)$: the estimated lateral velocity of the C.o.M. of the semitrailer at sample $k$;
• $\hat{u}_{lma}(k)$: the measured longitudinal velocity of the ‘effective rear axle’ of the semitrailer at sample $k$;
• $\hat{u}_n(k)$: the estimated longitudinal velocity of the C.o.M. of the semitrailer at sample $k$.

The lateral and yaw velocities of the C.o.M. of the other vehicle units could be estimated from $\hat{\Omega}_n(k), \hat{v}_n(k)$ and $\hat{u}_n(k)$, given the following equations. (See geometry in Figure 2.2)

\[
\hat{\Omega}_{i-1}(k) = \hat{\Omega}_i(k) - \hat{\Gamma}_i(k), \\
\hat{u}_{i-1}(k) = \hat{u}_i(k) \cos \left( \hat{\Gamma}_i(k) \right) - \left( a_i \hat{\Omega}_i(k) + \hat{v}_i(k) \right) \sin \left( \hat{\Gamma}_i(k) \right), \\
\hat{v}_{i-1}(k) = \hat{u}_i(k) \sin \left( \hat{\Gamma}_i(k) \right) + \left( a_i \hat{\Omega}_i(k) + \hat{v}_i(k) \right) \cos \left( \hat{\Gamma}_i(k) \right) + (b_{i-1} + c_{i-1})\hat{\Omega}_{i-1}(k)
\]

where $i = n, n - 1, \ldots, 0$.

$\hat{v}_0(k)$ and $\hat{\Omega}_0(k)$ were the estimations of the lateral and yaw velocities of the tractor, included in the outputs of the ‘Vehicle Motion Estimator’, as indicated in Figure 6.1.

### 6.2 LBRC subsystem

#### 6.2.1 Solution scheme

Figure 6.1 shows how the measured and estimated signals, e.g. $\hat{x}_L(k)$, were fed to the LBRC subsystem, in which the core control algorithm was implemented. The LBRC subsystem consists of three main components, ‘Boundary Path Predictor’, ‘Constraint Checker and Predictor’, and ‘Essential Matrices Hash Table’, as shown in Figure 6.3.

In the ‘Boundary Path Predictor’ block, the current and future upper and lower limits in the preview horizon were extracted from the pre-defined maps and then transformed to each vehicle’s local coordinate frame, thus generating the boundary matrix, $[P_L(k)]$. The boundary exceedances for all vehicle corner points were checked first in the ‘Constraint Checker and Predictor’ block. If any corner point was very close to the boundary or about to hit the boundary, the stop function would be actuated. The local constraints of steer angle, articulation angles, and vehicle corner points were estimated in the preview horizon, giving the constraint matrix, $[\beta_L(k)]$. The hash table stored the necessary matrices, e.g. $[\Psi_L], [F_L], [K_L], [P_L]$, and $[H_L]$, which were computed offline to reduce the real-time computational burden. The Hessian matrix, $[H_L]$, was calculated using equation (6-9) to eliminate the numerical error of the matrix, $[G_L]$. This is because the quadratic form in the object function (equation (5-32)) had to be symmetric.
\[ \mathbf{G}_L \] was therefore replaced by \[ \mathbf{H}_L \] in the object function. These essential matrices are only determined by the discrete-time state space model, the preview and control horizons, the controller weights, and the fixed constraints.

\[ \mathbf{H}_L = \frac{[\mathbf{G}_L] + [\mathbf{G}_L]^T}{2} \]  

(6-9)

After the matrix substitution, the constrained objective function became:

\[
\min_{\Delta \mathbf{U}_L} O_L = \frac{1}{2} \Delta \mathbf{U}_L(k)^T \mathbf{H}_L \Delta \mathbf{U}_L(k) + \mathbf{f}_L(k)^T \Delta \mathbf{U}_L(k) 
\]

(6-10)

subject to \[ [\mathbf{\Pi}_L] \Delta \mathbf{U}_L(k) \leq [\mathbf{X}_L(k)]. \]

where \[ \mathbf{f}_L(k) = \mathbf{F}_L(k)([\mathbf{\Psi}_L] \Delta \mathbf{x}_L(k) - \mathbf{P}_L(k)) \] and \[ \mathbf{X}_L(k) = [\mathbf{\beta}_L(k)] + [\mathbf{L}_L] \Delta \mathbf{x}_L(k). \]

The matrices that depend on the current system state, e.g. \[ [\mathbf{f}_L(k)] \] and \[ [\mathbf{X}_L(k)] \], were calculated at run time in the ‘Matrix Calculation’ block.

The dense ‘Primal-Dual Solver’ using the numerical iterative solution method was used to minimise the quadratic form of cost function subject to inequalities, which was the same form as equation (6-10). All coefficient matrices in the object function (i.e. \[ [\mathbf{H}_L] \] and \[ [\mathbf{f}_L(k)] \]) and constraint matrices (i.e. \[ [\mathbf{\Pi}_L] \] and \[ [\mathbf{X}_L(k)] \]) were sent to the quadratic programming solver to compute the optimised trajectory, \[ [\Delta \mathbf{U}_L(k)] \]. The first entry of this matrix was the demanded steer angle, \( \delta_d(k) \).

**6.2.2 Reduced computation scheme**

To reduce the real-time computational burden and make the B-double implementation realistic, the control algorithm described in Chapter 5 was simplified by removing the states of the B-link trailer from the system state, \( \Delta \mathbf{x}_L(k) \). This meant that the future position and heading estimation, and constraint checking in the preview horizon for the intermediate trailer were not calculated. Consequently, the sizes of all matrices, particularly the constraint matrix, \( [\mathbf{\beta}_L(k)] \), were much smaller than in the original simulation model. This saved considerable computation time. In most cases, the tractor experienced larger excursions than the trailer units, and the trajectory of the corner points of the B-link trailer was basically covered by the rear corner points of the tractor and front corner points of the semitrailer. Hence, there was no significant difference between the original and simplified algorithms, but the simplified approach reduced computational requirements dramatically. For example, the computational time for the same settings was decreased by approximately 5 times, from about 0.4 s per iteration for the original
algorithm to about 0.08 s per iteration for the simplified algorithm. Even though the original algorithm was simplified, the sampling rate of the real-time B-double application was still at the low frequency of 12.5 Hz, which was just about one eighth of the sampling frequency of the vehicle sensors. This limits the maximum speed at which reversing manoeuvres can be performed.

For much longer combination vehicles with more than 2 articulation points, e.g. a tractor articulated with 7 trailers, the full computation would be extremely expensive and the same reduced computation scheme could be used. The simplified scheme is a feasible alternative for the longer vehicles, because the tractor experiences much larger excursions than the other trailers in most cases.

6.3 Asynchronous simulation model

Due to the complexity of the LBRC algorithm, there was a long lag between the computation time of the global controller and the vehicle response time. Even though the real-time LBRC algorithm was a simplified version, the global controller of the LBRC method ran at a much lower frequency than the controller for the MSPC method. Moreover, for the real-time applications, some signals, i.e. the articulation angle rates, and the lateral and yaw velocities of the tractor, were estimated and filtered rather than measured. Therefore, the simulation model built in Chapter 5 had to be modified to take these implementation details into account.

As shown in Figure 6.4, the blocks of ‘Vehicle Dynamics Model’ and ‘Global Controller’ were separated, and sampling rate transition blocks were added in between. The modified simulation model ran at multiple sample rates simultaneously. The sampling frequency of the vehicle dynamics system was set at 100 Hz, according to equations (3-17) and (3-18). An ‘INS’ model was built inside to measure the position, heading, and velocities of the IMU on top of the ‘equivalent rear axle’ of the semitrailer. The outputs of the ‘Vehicle Dynamics Model’ were mainly the measured articulation angles and the INS measurements, as per as the real-time applications. These outputs were resampled from a high rate (100 Hz) to lower rates of 33.3 Hz for the tractor-semitrailer vehicle and 12.5 Hz for the B-double vehicle shown in Table 6.1. The optimal steer angle calculated using the LBRC algorithm was the only output of the ‘Global Controller’. It was then fed back to the ‘Vehicle Dynamics Model’ block using a low-to-high rate transition block.
Inside the subsystem of the ‘Global Controller’, the articulation angle rates were estimated by differencing and smoothing using a second-order Butterworth-type IIR filter. The filter parameters were set the same as those used in the experimental evaluation.

6.3.1 Simulation results

Three commonly used scenarios were devised to simulate the realistic and practical applications:

1) the first scenario involved a lane change manoeuvre with two car-like obstacles on each corner, connected to a 4 m wide loading bay, as depicted in Figure 6.5;
2) the second scenario shown in Figure 6.6 was designed such that the vehicle was partially blocked by a wall at the very beginning, aiming at a narrow gate towards a farm-yard. Unlike the lane change manoeuvre with continuously changing curvature, the narrow gate scenario was only made up of straight lines and thus had a $C^0$ continuity;
3) the third scenario was a sharp 90-degree turn with a trailer-like obstacle on the corner, connecting to a 4-metre-width loading bay, as shown in Figure 6.7.

The simulations for the three scenarios were run using both the synchronous and asynchronous models. To demonstrate the impact of the rate transition and the use of digital filters, both models adopted the simplified control algorithm. The synchronous model was fundamentally a simplified version of the original simulation model described in Chapter 5. The asynchronous model was based on the synchronous model with the multiple sample rates, and the estimations of the system variables being further taken into account. The longitudinal speed of the tractor was -1 m/s, where the negative sign represents reversing. The recommended default values for the preview horizon, control horizon, and controller weights were adopted for both models, as presented in Table 6.3.

To understand the difference, the trajectory of the ‘equivalent rear axle’ of the semitrailer is compared in Figures 6.5 – 6.7. The dashed red line represents the asynchronous model and the solid cyan line denotes the synchronous model. In the three scenarios, the controller managed to enable the vehicle to avoid the potential collisions along the path and ultimately reverse into the loading bay. It shows that the simplified algorithm worked well in most cases. There were minor differences in the trajectories between the synchronous and asynchronous models for the lane change and narrow gate manoeuvres. The dashed red line was closer to the boundaries compared to the solid cyan line, because the vehicle took action for obstacle avoidance earlier.
in the synchronous model than in the time-lagged model. Eventually, the two lines converged towards the same straight line.

The vehicle heading during the lane change and narrow gate scenarios did not change significantly. However, there was a large change in the vehicle heading during the 90-degree turn. In this case, the delayed actions had a marked impact on the performance. Due to the time lag, the rear axle in the asynchronous model drifted towards the outer boundary and had larger excursions, compared to the synchronous model. This is seen from the visible difference between the dashed and solid lines in Figure 6.7. After the sharp turn, the offset rear axle moved back to the nominal path in the section before entering the loading bay.

6.4 Field testing results

The three scenarios designed for the simulation study were set up using traffic road cones on the runway at Bourn airfield, Cambridgeshire, as presented in Figures 6.8 – 6.10, respectively. The surface of the runway is poorly maintained and very rough, with numerous bumps, joints and potholes. A group of cones was used to imitate the obstacles on the pre-defined maps, and two rows of cones were placed at a 4 m separation to represent the loading bay. For the narrow gate scenario (Figure 6.8), in addition to using a cluster of cones to represent the wall, two cones were placed before the loading bay as the 3 m wide gate. The photographs of the experimental configuration for the narrow gate and 90-degree turn scenarios are shown in Figures 6.9 and 6.10. Two cameras were used to film each of these two manoeuvres from different directions. The exact target manoeuvres were used in the controller and the cones were placed on the road surface as closely as possible to the positions defined by the target manoeuvres.

The tractor speed was manually controlled by a driver to reverse at a constant longitudinal speed of approximately 1 m/s. The parameters used in the real-time controller were those shown in Table 6.3. These were the same values as those used in the asynchronous simulation model. Measured initial perturbations, e.g. the initial position and heading offsets, were also added to the modified simulation model, when the experimental and simulated results were compared.

6.4.1 Field tests using the tractor-semitrailer vehicle

The measured position of the tractor-semitrailer combination during the lane change manoeuvre is plotted in Figure 6.11 (a). The dashed magenta rectangles represent the position of the tractor whilst the dashed black block shows the position of the semitrailer. The vehicle
successfully avoided the two car-like obstacles and reversed into the standard loading bay. The tractor kept changing its direction to adjust the position of semitrailer and just avoided hitting the first obstacle (i.e. ‘Obstacle 1’). The front left corner point of the tractor passed very close to the first obstacle. Due to the slow response of the global controller, there was a significant delay in the demanded steer angle, so the vehicle overshot its target position slightly. The controller was also continuously disturbed by sensor noise and measurement inaccuracies, e.g. the initial position and heading inaccuracies from the INS measurements. This is believed to be the reason why the semitrailer drifted towards the upper boundary. When the semitrailer was close to the upper boundary, the global controller promptly calculated its optimal steer angle based on the new constraints, enabling the tractor to change the direction of semitrailer to get away from the upper boundary. Once the loading bay appeared in the preview horizon, the tail of the semitrailer started to aim at the location of the loading bay. Because of the time lag and the direction of motion, the semitrailer tended to sink to the bottom of the boundary tube before moving towards the loading bay. Therefore, the vehicle moved just past the lower boundary of the loading bay and then towards the middle line. The trajectory of the ‘equivalent rear axle’ of the semitrailer between the experimental and simulation results is compared in Figure 6.11 (b). The measured rear axle trajectory (the solid blue line) was almost the same as the simulated one (the dashed red line), except that the measured rear axle was slightly closer to the upper boundary than the simulated rear axle after the semitrailer moved past the first obstacle.

The experimental motion of the tractor-semitrailer vehicle passing through the narrow gate towards the loading bay is shown in Figure 6.12 (a). Initially, the tractor-semitrailer vehicle was partially obstructed by the wall. When the wall was detected by the controller, the tractor started to move towards the semitrailer to increase the articulation angle, thus enabling the semitrailer to get around the wall by changing its direction of motion. The gate was very narrow so that the vehicle was only able to pass through it with a small heading angle. Therefore, the controller had to align the tractor and semitrailer before entering the gate. To understand the difference between the real-time and the simulated performance, the reversing trajectory of the rear axle is plotted in Figure 6.12 (b). The trajectory of the measured rear axle (the solid blue line) overlaps that of the simulated rear axle (the dashed red line). The only visible difference was that the measured position of the real axle moved upwards slightly, compared to the simulation result. This was likely because of the external noise, e.g. the sensor noise or the uneven pavement surface.
Figure 6.13 (a) shows the measured vehicle position during the 90-degree turn scenario. The stability of the controller was tested under initial perturbations, e.g. a large initial position offset and a small heading offset. The tractor and semitrailer managed to quickly get back to the middle line of the boundaries and avoid hitting the trailer-like obstacle (i.e. ‘Obstacle 1’) on the corner. This shows that the controller was robust to the external disturbances. Due to the time lag, the vehicle moved close to the upper boundary and after the sharp bend, it started to reverse towards the loading bay. As shown in Figure 6.13 (b), the trajectories of the real axle in the experiment (the solid blue line) and simulation (the dashed blue line) were in good agreement. During the sharp bend, the vehicle units slightly drifted outwards.

6.4.2 Field tests using the B-double vehicle

The second car-like obstacle (i.e. ‘Obstacle 2’) used in the lane change manoeuvre performed by the tractor-semitrailer vehicle was replaced with a larger obstacle, whose length was similar to the tractor-semitrailer combination. The motion of the B-double combination during the revised lane change manoeuvre is plotted in Figure 6.14 (a). The magenta rectangle represents the tractor, the green one is the B-link trailer, and the black rectangle is the semitrailer. The performance of the B-double vehicle was slightly worse than that of the tractor-semitrailer vehicle, because its sampling frequency was even lower than the sampling frequency used in the tractor-semitrailer case and the time delay was longer. After the vehicle passed around the first obstacle on the corner (i.e. ‘Obstacle 1’), it drifted close to the upper boundary. The two main reasons for this were likely as follows: (i) when the time lag became very long, the vehicle was not able to react promptly to the updated constraints; (ii) due to the use of the longer obstacle on the other corner (i.e. ‘Obstacle 2’), the vehicle tended to push itself towards the upper boundary. When the loading bay emerged within the preview horizon, the tractor moved nearly halfway past the second obstacle and then turned anti-clockwise to adjust the tail of the semitrailer. The vehicle reversed into the loading bay just past the lower constraint and was ultimately parked slightly above the centreline of the loading bay. The experimental trajectory of the ‘equivalent rear axle’ of the semitrailer is compared to the simulated trajectory in Figure 6.14 (b). There was no significant difference between the two trajectories. Along the ‘Obstacle2’, the measured rear axle position (the solid blue line) was slightly offset toward the upper boundary, compared to the position of the simulated rear axle (the dashed red line).

Figure 6.15 (a) shows the measured vehicle position during the narrow gate manoeuvre for the B-double combination. The straight wall partially obstructed the motion path of the vehicle at the very beginning. As a result, the tractor moved clockwise to enable the semitrailer to get
past the wall. The distance from the wall to the narrow gate was not long, compared to the length of the double trailer combination and the preview horizon. This meant that when the semitrailer got just around the wall, the narrow gate had already come into the preview horizon. Therefore, the tractor changed its direction again while passing by the wall. This is thought to be the reason why the tractor was very close to the wall, as seen in Figure 6.15 (a). The double trailers were realigned nearly straight before passing through the narrow gate, but the tractor had to continue wobbling slightly to maintain the straight state of the trailers due to the long length of the combination and the uneven road surface. The front wheels of the tractor were continuously disturbed by the roughness of the road. Even while the tractor was passing through the gate, it still moved slightly to adjust the trailers’ direction in the yard. The comparison of the path of the ‘equivalent rear axle’ between the experimental and simulated results is made in Figure 6.15 (b). The only significant difference is that the measured trajectory of the rear axle was upwards very slightly.

Compared to the 90-degree turn manoeuvre performed by the tractor-semi trailer vehicle, a much larger obstacle was set to occupy the corner (i.e. ‘Obstacle 1’) in the B-double case, as shown in Figure 6.16 (a). To test the controller stability and robustness, the vehicle was started off with position and heading offsets and managed to get back to the normal track. The controller was able to keep the tractor and twin trailers moving slightly outwards from the middle line between the obstacle and the lower boundary. Then, the vehicle drifted slightly due to the time lag, just past the upper boundary, and converged into the loading bay. To make a comparison between the experimental and simulated results, the trajectory of the rear axle is plotted in Figure 6.16 (b). The initial external disturbances were considered in the asynchronous simulation model. The trajectory of the measured rear axle (the solid blue line) agreed with that of the simulated rear axle (the dashed red line) very well.

6.5 Conclusions

1. The testing framework of the MSPC system described in Chapter 4 was modified to implement the LBRC method. A subsystem called ‘Vehicle Motion Estimator’ was built into the system architecture to estimate the vehicle position, heading and kinematic parameters.

2. The articulation angle rates were estimated via differencing the articulation angle measurement, and then filtered by a tuned second-order Butterworth-type IIR filter.

3. The lateral and yaw velocities of the C.o.M. of the tractor were calculated using the sensor measurements and the estimated articulation angle rates.
4. The vehicle position and heading were estimated using the vehicle geometry, the estimated system state, and the IMU measurements.

5. The LBRC method was implemented in a real-time controller. For each iteration, the ‘Boundary Path Predictor’ was to generate the boundary matrix, \([P_L(k)]\); the ‘Constraint Checker and Predictor’ was to check the boundary exceedance of each vehicle corner point and give the constraint matrix, \([\beta_L(k)]\). The solver took the matrices, i.e. \([H_L], [f_L(k)], [\Pi_L],\) and \([\chi_L(k)]\), as inputs and computed the optimal matrix, \(\Delta U_L(k)\).

6. To make the implementation realistic and reduce the computational burden, except that the matrices dependent on the system state (e.g. \([f_L(k)]\) and \([\chi_L(k)]\)) had to be determined at run time, the other essential matrices (e.g. \([H_L]\) and \([\Pi_L]\)) were all pre-calculated and stored at compile time. The original algorithm described in Chapter 5 was also simplified by removing the state variables of the B-link trailer from the system state. This reduced controller still had adequate performance. An asynchronous model was built to take the time delay, the estimation methods and the algorithm simplification into account.

7. Three test scenarios were designed and used to test the controller performance in planning and decision-making: (i) a lane change manoeuvre with obstacles on each corner, connected to a loading bay; (ii) an obstructive wall and misaligned narrow gate ahead of a loading bay; (iii) a sharp 90-degree bend with an obstacle on the corner, connecting to a loading bay.

8. The simulated vehicle managed to avoid the obstacles and approach the target, but drifted during the test manoeuvres, mainly because of the time lag. There was almost no visible difference between the synchronous and asynchronous models for the small heading-changing manoeuvres, e.g. the lane change and narrow gate manoeuvres. However, a significant difference was observed for the large heading-changing manoeuvres, e.g. the 90-degree sharp turn. The time lag was a very crucial factor in the controller performance.

9. The LBRC method was experimentally evaluated using two different vehicle configurations, i.e. the tractor-semitrailer and the B-double vehicles. The scenarios designed in the simulation study were set up using the traffic road cones on the airfield runway. The experiments were also filmed and demonstrated.

10. For the three manoeuvres performed by the tractor-semitrailer and B-double combinations, the experimental and simulated results were in good agreement. The experimental results
showed that the controller exhibited good real-time performance, enabling the vehicle to avoid the obstacles, pass through the narrow gate, and reverse into the loading bay.

11. To improve the real-time performance, the INS, e.g. RT3022, could be installed on each vehicle unit to measure the vehicle state accurately instead of the numerical estimations, and a much faster PC could be used to reduce or eliminate the time delay.

12. The controller was robust to the details of pre-defined maps, even without a $C^0$ continuity. The stability of the controller was tested under the initial perturbations, e.g. the initial position and heading offsets, during the 90-degree sharp turn manoeuvre.
6.6 Tables

Table 6.1 The sampling frequency of the real-time applications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Tractor-semitrailer</th>
<th>B-double</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling frequency of the global controller, $f_c$</td>
<td>33.3 Hz</td>
<td>12.5 Hz</td>
</tr>
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</table>

Table 6.2 Parameters for the second order Butterworth-type IIR filter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
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</tr>
<tr>
<td>$a_1$</td>
<td>-1.5610</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.6414</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.0201</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.0402</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.0201</td>
</tr>
<tr>
<td>Parameter</td>
<td>Tractor-semitrailer</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>Sample time, $T_s$</td>
<td>1 s</td>
</tr>
<tr>
<td>Preview horizon, $N_p$</td>
<td>20</td>
</tr>
<tr>
<td>Control horizon, $N_c$</td>
<td>10</td>
</tr>
<tr>
<td>Weight on the upper constraint of front end</td>
<td>1</td>
</tr>
<tr>
<td>of tractor, $w_{foub}$</td>
<td></td>
</tr>
<tr>
<td>Weight on the lower constraint of front end</td>
<td>1</td>
</tr>
<tr>
<td>of tractor, $w_{rolb}$</td>
<td></td>
</tr>
<tr>
<td>Weight on the upper constraint of rear end</td>
<td>1</td>
</tr>
<tr>
<td>of semitrailer, $w_{roub}$</td>
<td></td>
</tr>
<tr>
<td>Weight on the lower constraint of rear end</td>
<td>1</td>
</tr>
<tr>
<td>of semitrailer, $w_{rolb}$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 6.1 Architecture of the LBRC system.
(a) Unfiltered and filtered estimated articulation angle rates derived from measured articulation angles, for the tractor-semitrailer vehicle during a lane change manoeuvre

(b) Unfiltered and filtered estimated articulation angle rates for the tractor-semitrailer vehicle during a roundabout manoeuvre

Figure 6.2 Unfiltered and filtered estimated articulation angle rates
Figure 6.3 Schematic of the LBRC subsystem

Figure 6.4 Schematic of the asynchronous simulation model
Figure 6.5 The trajectory comparison of the ‘equivalent rear axle’ of the semitrailer between the asynchronous and synchronous models in the case of the tractor-semitrailer vehicle performing the constrained lane change manoeuvre.

Figure 6.6 The trajectory comparison of the ‘equivalent rear axle’ of the semitrailer between the asynchronous and synchronous models in the case of the tractor-semitrailer vehicle performing the constrained narrow gate manoeuvre.
Figure 6.7 The trajectory comparison of the ‘equivalent rear axle’ of the semitrailer between the asynchronous and synchronous models in the case of the tractor-semitrailer vehicle performing the constrained 90-degree sharp turn manoeuvre.

Figure 6.8 The experimental configuration for the constrained lane change manoeuvre on the runway at Bourn airfield.
Figure 6.9 The experimental configuration for the constrained narrow gate manoeuvre on the runway at Bourn airfield

Figure 6.10 The experimental configuration for the constrained 90-degree sharp turn manoeuvre on the runway at Bourn airfield
(a) The motion paths of the tractor-semitrailer vehicle during the experiment of the constrained lane change manoeuvre

(b) The trajectory comparison of the ‘equivalent rear axle’ of the semitrailer between the experimental and simulated results in the case of the tractor-semitrailer vehicle performing the constrained lane change manoeuvre

Figure 6.11 Comparison between the experimental and simulated results of the tractor-semitrailer vehicle performing the constrained lane change manoeuvre
(a) The motion paths of the tractor-semitrailer vehicle during the experiment of the constrained narrow gate manoeuvre

(b) The trajectory comparison of the ‘equivalent rear axle’ of the semitrailer between the experimental and simulated results in the case of the tractor-semitrailer vehicle performing the constrained narrow gate manoeuvre

Figure 6.12 Comparison between the experimental and simulated results of the tractor-semitrailer vehicle performing the constrained narrow gate manoeuvre
(a) The motion paths of the tractor-semi-trailer vehicle during the experiment of the constrained 90-degree sharp turn manoeuvre

(b) The trajectory comparison of the ‘equivalent rear axle’ of the semi-trailer between the experimental and simulated results in the case of the tractor-semi-trailer vehicle performing the constrained 90-degree sharp turn manoeuvre

Figure 6.13 Comparison between the experimental and simulated results of the tractor-semi-trailer vehicle performing the constrained 90-degree sharp turn manoeuvre
(a) The motion paths of the B-double vehicle during the experiment of the constrained lane change manoeuvre

(b) The trajectory comparison of the ‘equivalent rear axle’ of the semitrailer between the experimental and simulated results in the case of the B-double vehicle performing the constrained lane change manoeuvre

Figure 6.14 Comparison between the experimental and simulated results of the B-double vehicle performing the constrained lane change manoeuvre
(a) The motion paths of the B-double vehicle during the experiment of the constrained narrow gate manoeuvre

(b) The trajectory comparison of the ‘equivalent rear axle’ of the semitrailer between the experimental and simulated results in the case of the B-double vehicle performing the constrained narrow gate manoeuvre

Figure 6.15 Comparison between the experimental and simulated results of the B-double vehicle performing the constrained narrow gate manoeuvre
(a) The motion paths of the B-double vehicle during the experiment of the constrained 90-degree sharp turn manoeuvre

(b) The trajectory comparison of the ‘equivalent rear axle’ of the semitrailer between the experimental and simulated results in the case of the B-double vehicle performing the constrained 90-degree sharp turn manoeuvre

Figure 6.16 Comparison between the experimental and simulated results of the B-double vehicle performing the constrained 90-degree sharp turn manoeuvre
Chapter 7 Advanced control strategies - exploratory research

The theory of the LBRC method described in Chapter 5 lays a foundation for the development of more advanced control strategies. Both the experimental and simulated results show that the LBRC controller excels in planning ahead, making optimal decisions and obstacle avoidance. However, one drawback of the controller is that there are many parameters that must be manually tuned and these change under different scenarios. Even the simplified algorithm discussed in Chapter 6 requires 6 parameters to be tuned, including 4 controller weights and the preview and control horizons. The controller tuning is heuristic. To overcome this drawback, a new technique was developed: ‘Adaptive Lane-Bounded Reversing Control’ (ALBRC) method.

Even when the controller is properly tuned, extreme cases exist where significant computing power is needed to run the algorithm due to the large preview horizon required. If a small preview horizon is chosen to reduce the computational burden, a feasible solution may not exist. Moreover, an arbitrary starting position and orientation may also lead to infeasibility. For example, when an obstacle is just behind the rear trailer; or when a vehicle is initially very close to a lane boundary, as shown in Figure 7.1. A vehicle is not able to pass through the narrow gate from any starting position within the areas bounded by the red dashed lines using the LBRC method. To overcome the problems of large computational times and infeasible solutions, an ‘Adaptive Bi-directional Control’ (ABC) method is proposed in this chapter. This method makes the implementation more realistic by using a small preview horizon. The scenario illustrated in Figure 7.1 is used to demonstrate the capability of the ABC controller in Section 7.2.1.

Both the ALBRC and ABC methods can be integrated with the fast implementation algorithm described in Chapter 6 and applied to control multiple trailers.

Simulations of these controllers were performed for the tractor-semitrailer combination and various manoeuvres. No experimental testing has been undertaken for the controllers described in this chapter.

7.1 Adaptive Lane-Bounded Reversing Control (ALBRC) Method

In Chapter 5, the controller weights of the LBRC method were used to penalise the distances to path boundaries and obstacles. The relationship between the controller weights and the lateral distances can be conceptualised as ‘virtual bumpers’ attached to the front, rear end and
sides of the vehicle. Alternatively, the vehicle can be thought to move within an ‘imaginary force field’ constrained by the lane boundaries, and the force can be adjusted by varying the controller weights. Increasing the controller weight of one side of the vehicle is like pulling the vehicle towards that side, so the controller weights are set as a function of the lateral distances.

The front and rear end of the tractor and rearmost trailer are included in the system state vector. When the corresponding controller weights change, it is analogous to adding virtual lateral forces to the front of the tractor unit and the rear of the last trailer to pull them away from the boundaries. For the intermediate vehicle units, it is analogous to adding virtual forces to the C.o.M..

To develop a fast and feasible algorithm, the approach presented in Section 6.2 is used as the foundation. Hence, the discrete-time system state vector, \( \mathbf{x}_L(k) \), is simplified as:

\[
\mathbf{x}_L(k) = [\mathbf{Z}_D^T(k) \ y_{df}^l(k) \ \theta_{df}(k) \ y_{nr}^l(k) \ \theta_{nr}^l(k)]^T
\]  

Therefore, the number of the controller weights is reduced to 4. \( w_{df}(k), w_{df}(k), w_{nr}(k) \) and \( w_{nr}(k) \) are the controller weights placed on the corresponding lateral offsets from the upper and lower boundaries, i.e. \( y_{df}^l(k), y_{df}^l(k), y_{nr}^l(k), \) and \( y_{nr}^l(k) \). The upper and lower limits in the local vehicle coordinate frame are simply the lateral offsets from the upper and lower boundaries.

The quantity, \( r_{jmc}(k) \), is defined to quantify the lateral offsets from lane boundaries, as illustrated in Figure 7.2.

\[
r_{jmc}(k) = \frac{y_{jmc}^l(k)}{\min(y_{jmu}^l(k), y_{jml}^l(k))},
\]  

where the subscript notation is clarified as follows:

- \( j \): an element of \( \{0, n\} \), where 0 and \( n \) denote the tractor and the semitrailer at the rear, respectively;
- \( m \): an element of \( \{f, r\} \), where \( f \) and \( r \) refer to the front end of the tractor and the rear end of the semitrailer, respectively;
- \( c \): an element of \( \{u, l\} \), where \( u \) or \( l \) represent the lateral offset from the upper or lower boundary.
The reason behind the choice of a ‘min’ function is the weight placed on the steer angle, \( w_\delta(k) \), was set as a constant (i.e. 1) in this study. The minimum of \( r_{jmc}(k) \) is 1. This means the penalty for a large lateral offset is greater than the penalty for a larger steer angle (See equation (5-18)).

For any vehicle unit, the first step is to identify which corner point is closest to the boundaries or obstacles. Then, the vehicle position and orientation are adjusted by varying the controller weights to exert the virtual force and torque.

Take the tractor as an example for illustration. As seen in Figure 7.3 (a), the lateral distance from the middle front-end point (i.e. Point ‘f’) to the upper boundary, \( y_{0fu}(k) \), is smaller than the other three. This means the distance between Point ‘f’ and the lower boundary is larger than the distance between Point ‘f’ and the upper boundary. In this case, \( r_{0fu}(k) = 1 < r_{0fl}(k) \). In order to make Point ‘f’ gradually move away from the upper boundary, let \( w_{0fu}(k) = r_{0f}(k) \) and \( w_{0f}(k) = r_{0f}(k) \). This means that \( w_{0fu}(k) < w_{0fl}(k) \). Consequently, \( y_{0fl}(k) \) will be penalised more than \( y_{0fu}(k) \) in the subsequent steps. This is analogous to exerting a virtual force at Point ‘f’ towards the lower boundary. This pushes Point ‘f’ away from the upper boundary so that \( y_{0fu}(k) \) increases until it is no longer the smallest lateral offset. As the virtual force can only be added at Point ‘f’ to adjust the tractor’s position and orientation, it is slightly different when the middle point of the rear end (i.e. Point ‘r’) is the closest to the boundaries. For instance, Figure 7.3 (b) shows that the lateral distance from Point ‘r’ to the upper boundary, \( y_{0ru}(k) \), is smaller than the other three. In this case, \( r_{0r}(k) = 1 < r_{0rl}(k) \). Let \( w_{0fu}(k) = r_{0r}(k) \) and \( w_{0fr}(k) = r_{0r}(k) \), which results in \( w_{0fu}(k) > w_{0fr}(k) \). This means more control weights are placed on \( y_{0fu}(k) \) than \( y_{0fr}(k) \). This is analogous to adding a virtual force to Point ‘f’ pointing to the upper boundary. Hence, the rear-left corner point (i.e. Point ‘3’) gradually moves away from the upper boundary.

Varying the controller weights adjusts the position and orientation of the semitrailer in a similar way; the only difference is that the virtual force can only be added at Point ‘r’ rather than Point ‘f’.

188
The adaptive learning algorithm is summarised in pseudo code as below:

1. at sample instant \( k \), calculate the lateral distances of the tractor and semitrailer, i.e. \( y_{0fu}^i(k) \), \( y_{ofl}^i(k) \), \( y_{0r}^l(k) \), \( y_{orl}^l(k) \), \( y_{nfu}^l(k) \), \( y_{nfl}^l(k) \), \( y_{nr}^u(k) \) and \( y_{nrl}^l(k) \);
2. calculate the corresponding deviation ratios according to equation (7-2), i.e. \( r_{0fu}^l(k) \), \( r_{ofl}^l(k) \), \( r_{0r}^l(k) \), \( r_{orl}^l(k) \), \( r_{nfa}^l(k) \), \( r_{nfl}^l(k) \), \( r_{nru}^l(k) \) and \( r_{nrl}^l(k) \);
3. find the shortest distance and assign the ratios to the front end of the tractor and the rear end of the semitrailer, respectively.

For the tractor:
if \( \min \left( y_{0fu}^l(k), y_{ofl}^l(k) \right) \leq \min \left( y_{bru}^l(k), y_{orl}^l(k) \right) \)
\[
\begin{align*}
    w_{0fu}(k) &= r_{0fu}(k) \\
    w_{of}(k) &= r_{ofl}(k)
\end{align*}
\]
else
\[
\begin{align*}
    w_{0fu}(k) &= r_{orl}(k) \\
    w_{of}(k) &= r_{nr}(k)
\end{align*}
\]
For the semitrailer:
if \( \min \left( y_{nru}^l(k), y_{nrl}^l(k) \right) \leq \min \left( y_{nfa}^l(k), y_{nfl}^l(k) \right) \)
\[
\begin{align*}
    w_{nru}(k) &= r_{nru}(k) \\
    w_{nr}(k) &= r_{nrl}(k)
\end{align*}
\]
else
\[
\begin{align*}
    w_{nru}(k) &= r_{nfa}(k) \\
    w_{nr}(k) &= r_{nfl}(k)
\end{align*}
\]

7.1.1 Simulation analysis
To make a comparison with the LBRC method, the ALBRC controller was tested in the scenarios described in Chapter 5, which are as follows:

1) a lane change manoeuvre with a misaligned starting position, followed by a loading bay;
2) passing through a misaligned narrow gate in front of a farmyard and the vehicle begins from a position closer to the lower boundary;
3) a 90-degree sharp bend;
4) a 6 m long ‘limousine’ parked on a mountain road;
5) a similar-sized ‘lorry’ parked on a mountain road.
The controller parameters are shown in Table 7.1. As the controller weights adjust themselves to the different scenarios, only the preview and control horizons need to be tuned based on a trial and error method (See Section 5.2). The dashed magenta rectangle denotes the tractor and the dashed black rectangle is the semitrailer. The longitudinal speed of the tractor was set to -1 m/s (negative value indicates reversing).

As shown in Figure 7.4 (a), the ALBRC method enables the reversing system to perform the lane change manoeuvre and reverse into the loading bay from a starting position very close to the upper boundary. Figure 7.4 (b) shows the controller weights vary continuously during the lane change manoeuvre. The vehicle is closer to the upper boundary than the lower boundary initially, so \( w_{0fl} \) and \( w_{nrl} \) are larger than \( w_{0fu} \) and \( w_{nru} \). This means the virtual force gradually pulls the vehicle towards the lower boundary. However, the virtual torque also causes the tractor to rotate clockwise, pushing both the rear-left corner point of the tractor and the front-left corner point of the semitrailer close to the upper boundary. As a result of this, \( w_{0fu} \) and \( w_{nru} \) become larger than \( w_{0f} \) and \( w_{nrl} \). Before the tractor arrives at the origin of the coordinates, \( w_{0fu} \) and \( w_{0fl} \) are rapidly changing under the influence of the virtual ‘force’, as seen in Figure 7.4 (b). When the tractor and the semitrailer move around the middle line between the upper and lower boundaries, there are no significant changes in controller weights.

The vehicle motion when passing through a narrow gate (3 m wide) is depicted in Figure 7.5 (a). The magnified view of the same manoeuvre in Figure 7.5 (b) shows that the vehicle just passes through the gate with a small heading angle. As the gate is misaligned with the farmyard, the vehicle starts adjusting itself while passing through the gate until it stops at the middle line. When the tractor moves towards the upper boundary in order to get the semitrailer past the first 90-degree bend, there is a spike in \( w_{0fl} \), as shown in Figure 7.5 (c). With large \( w_{nrl} \), the tractor manages to get around the sharp narrow bend.

Figure 7.6 (a) shows the tractor-semitrailer combination performing a sharp corner turn. As discussed above, when the front-left corner point of the tractor is close to the outer boundary, \( w_{0fl} \) grows rapidly, as seen in Figure 7.6 (b). From the figure it is evident that \( w_{nru} \) is larger than \( w_{nrl} \) throughout the manoeuvre. This is because the front-left corner point of the semitrailer is also close to the outer boundary and the weights can only be placed on the rear end of the semitrailer. By increasing \( w_{nru} \), the front-left corner point of the semitrailer moves away from the outer boundary.
The scenario in Figure 7.7 (a) includes a ‘limousine’ obstacle (approximately 6 m long and 2.5 m wide) parked on the left side of a mountain road. The changes in the controller weights throughout the manoeuvre are shown in Figure 7.7 (b). When the obstacle emerges in the preview horizon (after a travel distance of approximately 40 m), \( w_{0ft} \) begins increasing so the tractor starts to move slightly rightwards to push the tail of the semitrailer away from the obstacle. Then, just after the tail of the semitrailer passes the obstacle, the tractor adjusts the position of the semitrailer to resume moving around the middle line of the outer and inner boundaries. There is an obviously similar shift in the curve shape of the controller weights, shown in Figure 7.7 (b). The obstacle is considered to be a small disturbance because the size of the obstacle is much smaller than the length of the tractor-semitrailer vehicle.

Next, the ‘limousine’ in Figure 7.7 (a) is replaced by an approximately 16 m long and 2.5 m wide ‘lorry’ parked on the same road. As shown in Figure 7.8 (a), the ALBRC controller enables the vehicle to just avoid hitting the corner of the obstacle. From the magnified view in Figure 7.8 (b), the difference from the ‘limousine’ scenario is a short period during which the vehicle moves parallel to ‘lorry’. This causes \( w_{0ft} \) and \( w_{nru} \) to gradually increase after 100 m of travel distance where the obstacle emerges, and then become larger than those in the ‘limousine’ scenario, as shown in Figure 7.8 (c). This is because when the tractor and the semitrailer enter the very small gap between the obstacle (i.e. ‘limousine’) and the inner boundary, they tend towards the obstacle at the start. Compared to Figure 7.7 (b), the duration of \( w_{0ft} \) and \( w_{nru} \) decreasing to realign the vehicle is longer (i.e. from around 100 m to 150 m), which is affected by the obstacle length. That is why the tractor and semitrailer move around the middle line of the gap for a while after the readjustment, as seen in Figure 7.8 (b).

Compared to the mountain road scenarios depicted in Figures 5.8 (b) and 5.9 (c), where the LBRC method was applied, using the ALBRC method allows the vehicle to follow the middle line of the two boundaries rather than tend towards one side. This balances the spaces on both sides of the vehicle.

### 7.2 Adaptive Bi-directional Control (ABC)

The preview and control horizons still need to be tuned using the ALBRC method for different scenarios. However, both the LBRC or the ALBRC methods may not result in a feasible solution under some conditions. For instance, there might be insufficient space to reverse at the start. Moreover, even when a feasible solution exists, substantial computing power is required, i.e. the simulations of the limits of manoeuvrability in Section 5.3.2. This makes it difficult to
implement the algorithms. Hence, the ‘ABC’ algorithm was developed on the basis of the ALBRC method to allow the vehicle to realign its position and orientation by moving forwards and backwards. This resembles the so-called ‘cusp’ approach, which allows a vehicle to realign its position when encountering a non-drivable ‘cusp’ point during path following. Using a small preview horizon also enables a significant reduction in the computational burden.

The underlying framework of both the ALBRC and the ABC methods is the LBRC. The LBRC theory discussed in Chapter 5 can be used for both forward and backward driving. This provides a solid foundation for the ABC algorithm.

Some major differences between the ABC method and the LBRC and ALBRC methods are:

(i) a vehicle is considered to be parallel to the terminal position, i.e. inside a loading bay when the heading angle of the vehicle is within a tolerable range of the terminal heading angle. In this case, the vehicle can adjust itself by driving forwards with equal controller weights to rebalance the lateral distances to the boundaries;

(ii) the maximum forward driving distance set to approximately 1.5-times the vehicle length. The vehicle will stop driving forwards when it reaches the maximum distance or is likely to hit an obstacle in front;

(iii) the number of transitions between forward and backward driving is limited. Moreover, the controller weights are a polynomial function of the number of transitions. With the number going up, the controller weights are increased correspondingly. This means a greater penalty is incurred each time in order to reduce the number of transitions;

(iv) the ratios, \( r_{mc}(k) \), are tracked each time the vehicle fails to reverse because of potential collisions. The ratios can indicate how far the vehicle is away from the boundaries and provide a guide to the direction for the subsequent forward motion;

(v) for reversing, the tail of the rear trailer should be directed at the terminal position. Therefore, when a collision is about to occur, it is necessary to track which rear corner point is likely to hit the boundaries. This can provide additional information for the subsequent forward motion.
The main strategy is summarised in pseudo code as follows:

If the semitrailer is parallel to the terminal position

\[ w_{0fu} = w_{0ft} = w_{nru} = w_{nrl} = 1 \]

if \( u_0 > 0 \)

- the vehicle moves forwards and then stops either at the maximum forward driving distance or when a collision is likely to happen
elseif \( u_0 < 0 \)

- reversing using the above controller weights
else

if \( u_0 < 0 \)

\[ r_u(k) = \max \left( r_{0fu}(k), r_{0ru}(k), r_{nfu}(k), r_{nru}(k) \right) \]

\[ r_l(k) = \max \left( r_{0ft}(k), r_{0rl}(k), r_{nft}(k), r_{nrl}(k) \right) \]

if \( y_{nru}(k) \leq \varepsilon \) and \( y_{nrl}(k) > \varepsilon \)

\[ w_{0f} = w_{nru} = \min(r_{nru}(k), r_{nrl}(k)) \]

\[ w_{0f} = w_{nrl} = \max(r_{nru}(k), r_{nrl}(k)) * \frac{3}{\sqrt{N_u}} \]
elseif \( y_{nrl}(k) \leq \varepsilon \) and \( y_{nru}(k) > \varepsilon \)

\[ w_{0fu} = w_{nru} = \max(r_{nru}(k), r_{nrl}(k)) * \frac{3}{\sqrt{N_l}} \]

\[ w_{0f} = w_{nrl} = \min(r_{nru}(k), r_{nrl}(k)) \]
elseif \( y_{nru}(k) > \varepsilon \) and \( y_{nrl}(k) > \varepsilon \)

\[ w_{0fu} = w_{0ft} = w_{nru} = w_{nrl} = 1 \]
else

\[ w_{0fu} = w_{nru} = r_{nru}(k) \]

\[ w_{0ft} = w_{nrl} = r_{nrl}(k) \]
elseif \( u_0 > 0 \)

if \( r_u(k) > r_l(k) \)

\[ w_{0fu} = w_{0ft} = w_{nrl} = 1 \]

\[ w_{nru} = 3 * \frac{3}{\sqrt{N_u}} \]

- the vehicle moves forwards and then stops either at the maximum forward driving distance or when a collision is likely to happen
elseif \( r_u(k) < r_l(k) \)

\[ w_{0fu} = w_{0ft} = w_{nru} = 1 \]

\[ w_{nrl} = 3 * \frac{3}{\sqrt{N_l}} \]

- the vehicle moves forwards and then stops either at the maximum forward driving distance or when a collision is likely to happen
else

\[ w_{0fu} = w_{0ft} = w_{nru} = w_{nrl} = 1 \]

- the vehicle moves forwards and then stops either at the maximum forward driving distance or when a collision is likely to happen

where

- \( \varepsilon \): the allowable minimum distance away from the boundaries;
- \( N_u \) and \( N_l \): the number of the vehicle moving towards the upper and lower boundary in a forward direction, respectively. \( N_u \) and \( N_l \) start from 1.
7.2.1 Simulation analysis

7.2.1.1 Manoeuvres

The preview and control horizons are fixed at 30 and 10, respectively. Two scenarios are used to test the ABC controller:

1) the tractor and semitrailer begin from a position very close to the upper boundary, as seen in Figure 7.9 (a). There is not enough space for the vehicle to reverse, and so no feasible solution exists when using the LBRC or the ALBRC methods;

2) the shortest lane change manoeuvre from Section 5.3.2 was used (See Figure 5.5 (f)), so that the distance between the starting and terminal position was only 25 m. This is small compared to the total length of the combination (i.e. approximately 17 m). There is no transition phase between the lane change manoeuvre and the loading bay. In Section 5.3.2, it was found necessary to use the LBRC method with the preview horizon set at 60 to negotiate this manoeuvre. This was very computationally expensive. The reason for adopting the same lane change manoeuvre is to compare the performance of the ABC and LBRC controllers.

The colours representing the tractor and semitrailer in the following figures are the same as the previous case. The longitudinal speed of the tractor is also set to -1 m/s.

7.2.1.2 Results

Figures 7.9 (a) and 7.9 (b) show the vehicle motion from the starting position during the first transition to the first cusp. Initially, the controller intends to reverse the vehicle towards the narrow gate, but it ends up with the rear-left corner point of the tractor very close to the upper boundary, causing the vehicle to stop. In this case, \( r_u(k) < \eta(k) \), which means the space between the vehicle and the lower boundary is larger than the space between the vehicle and the upper boundary. According to the algorithm, the tractor begins moving forwards to the right, then straightening near to the centreline, with \( w_{0fu} = w_{0fl} = w_{nru} = 1 \) and \( w_{nrl} = 3 \). After driving about a 1.5-times vehicle length distance, the tractor stops around the middle line of the upper and lower boundaries. Then, the vehicle starts reversing with the continuously varying controller weights until the front-left corner point of the tractor gets very close to the upper boundary again. The tractor moves forwards to the right again to realign the vehicle position and ends up with the position of the solid rectangles, as shown in Figure 7.9 (c). Figure 7.9 (d) shows that the vehicle eventually manages to reverse through the narrow gate from the position close to the lower boundary.
If the preview horizon is increased, the number of cusps is believed to be decreased for this manoeuvre (i.e. the vehicle would be able to navigate to the end of the manoeuvre after the first cusp) and on the other hand, the computation time is believed to be increased. The trade-off between the number of cusps and the preview horizon is worth investigating further, which is recommended in Future Work (See Section 8.2).

The vehicle motion during the lane change manoeuvre is depicted in Figure 7.10. It shows the ABC controller enables the vehicle to reverse into the loading bay from a nearby starting position by moving forwards and backwards with adaptive controller weights and a much smaller preview horizon, compared to the LBRC controller. The controller makes good use of the confined spaces and guarantees that there are no collisions with the boundaries.

7.3 Conclusions

1. The ALBRC method was developed to simplify the tuning approach to the controller weights of the LBRC method. Only the preview and control horizons need to be tuned for different scenarios using the ALBRC method. The method for tuning the preview and control horizons used for the ALBRC method is the same as that used for the LBRC method.

2. The ALBRC demonstrated good performance in the following five scenarios: a lane change manoeuvre, a 90-degree sharp bend, passing through a narrow gate, and two partially obstructive mountain roads. The simulation results were compared to those using the LBRC method, and it was shown that the vehicle tends towards the middle line between the boundaries using the ALBRC method. This is superior to the LBRC method.

3. The ABC controller was developed by incorporating the basic theory of the LBRC method, a ‘cusp’ technology, and the adaptive weight tuning of the ALBRC method. Using the ABC method allows the vehicle to change direction to gain more space for reversing. This can enable the vehicle to negotiate paths that would otherwise be infeasible due to the initial conditions or shape of the path. The computational burden is significantly improved by using shorter preview horizon.

4. Two difficult scenarios for the tractor-semitrailer combinations were simulated to demonstrate the ability of the ABC controller. The scenarios include: (i) passing through a narrow gate with a very short distance from the vehicle starting position to the boundary, and (ii) a very tight lane change manoeuvre. In the first case, no feasible solutions exist using either the LBRC or the ALBRC methods. In the second case, the solution using LBRC method
required a very large preview horizon (i.e. $N_p = 60$). The ABC controller managed to perform both manoeuvres with a small preview horizon (i.e. $N_p = 30$).
### 7.4 Tables

**Table 7.1 Simulation parameters for the tractor-semitrailer vehicle**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lane change manoeuvre</th>
<th>Sharp 90-degree bend</th>
<th>Misaligned narrow gate</th>
<th>Mountain road (parked limousine)</th>
<th>Mountain road (parked lorry)</th>
</tr>
</thead>
<tbody>
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<td>Sample time, $T_s$</td>
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<td>1 s</td>
<td>1 s</td>
<td>1 s</td>
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<tr>
<td>Preview horizon, $N_p$</td>
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<td>30</td>
</tr>
<tr>
<td>Control horizon, $N_c$</td>
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<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
7.5 Figures

**Figure 7.1** The non-reversing areas using the LBRC method

**Figure 7.2** The notations used for a vehicle unit
(a) The situation when the front-left corner point (i.e. Point ‘1’) is the closest to a boundary

(b) The situation when the rear-left corner point (i.e. Point ‘3’) is the closest to the boundary

Figure 7.3 Illustration for situations when a corner point moves closer to a boundary than the other three
(a) A lane change manoeuvre for the tractor-semitrailer vehicle

(b) The controller weights during the lane change manoeuvre using the ALBRC method

Figure 7.4 Simulation results of the tractor-semitrailer vehicle performing the lane change manoeuvre
(a) The vehicle motion of the tractor-semitrailer vehicle passing through a misaligned gate

(b) A magnified view of the tractor-semitrailer vehicle passing through the narrow gate

(c) The controller weights during the mixed manoeuvre using the ALBRC method

Figure 7.5 Simulation results of the tractor-semitrailer vehicle performing a mixed manoeuvre
(a) The vehicle motion of the tractor-semitrailer vehicle during a 90-degree bend

(b) The controller weights in the scenario of getting around the 90-degree bend using the ALBRC method

Figure 7.6 Simulation results of the tractor-semitrailer vehicle performing a 90-degree turn manoeuvre
(a) The vehicle motion of the tractor-semi-trailer passing by a parked ‘limousine’ on a mountain road

(b) The controller weights in the scenario of passing by a parked ‘limousine’ on the mountain road using the ALBRC method

Figure 7.7 Simulation results of the tractor-semi-trailer vehicle while moving on a mountain road with a parked ‘limousine’
(a) The vehicle motion of the tractor-semitrailer passing by a parked ‘lorry’ on the mountain road

(b) A magnified view of the tractor-semitrailer vehicle passing by the ‘lorry’ on the mountain road

(c) The controller weights during the manoeuvre using the ALBRC method

Figure 7.8 Simulation results of the tractor-semitrailer moving on a mountain road with a parked ‘lorry’
(a) A scenario simulating that the tractor-semitrailer vehicle starts off from a position very close to the upper boundary

(b) The first transition from backward to forward driving using the ABC method

(c) The second transition from backward to forward driving using the ABC method

(d) The tractor-semitrailer vehicle manages to pass through the narrow gate from the terminal position of the second transition using the ABC method

Figure 7.9 A scenario simulating that the tractor-semitrailer vehicle starts off from a position very close to the upper boundary
(a) A scenario simulating that the tractor-semitrailer vehicle starts off from a position very close to the loading bay

(b) The sketched vehicle motion of the tractor-semitrailer vehicle during the confined lane change manoeuvre using the ABC method

Figure 7.10 A scenario simulating that the tractor-semitrailer vehicle starts off from a position very close to the loading bay
Chapter 8 Conclusions and Future Work

8.1 Summary of main conclusions

Reversing an articulated vehicle is a very common operation in practice. The open-loop motion is unstable and has to be stabilised by drivers. It is almost impossible for human drivers to achieve this when the vehicle has more than one articulation joint. The control methods proposed in this study can be used to assist drivers with daily stressful tasks, e.g. reversing into small parking lots or loading bays, and interchanging swap bodies.

8.1.1 Literature review (Chapter 1)

Chapter 1 summarises the outlook for the road freight sector against the backdrop of the coronavirus pandemic and climate change, and the advantages of using Long Combination Vehicles (LCVs). Different types of LCVs including the tractor-semitrailer, B-double and A-double combinations, were discussed. Two types of hitching, i.e. ‘on-axle hitching’ and ‘off-axle hitching’ were explained.

With the recent advances in autonomous driving technology, numerous Advanced Driver Assistance Systems (ADAS) have been developed and implemented. However, vehicle manufacturers have been more focused on developing autonomy for passenger and light duty vehicles rather than LCVs. Hence self-driving technology for LCVs is still in an early stage (i.e. L2 level [224], [225]).

Reversing LCVs into constrained parking bays are common tasks for drivers. However, unlike forward driving, reversing of LCVs is unstable, with non-holonomic characteristics. Moving obstacles such as cars, workers or trollies in the vicinity of the LCVs exacerbate the difficulty. Hence, it is important to design autonomous systems for drivers reversing truck-trailer combinations.

A literature review was conducted on two different types of controllers for reversing LCVs in Chapter 1: the first is defined as a ‘pre-planned’ controller, because the controller has to follow a collision-free path generated by a path planner and minimise the tracking errors to guarantee feasibility; the second is an ‘actively-planned’ controller, which is able to pursue an optimum route and proactively avoid obstacles without the benefit of a pre-determined path. The ‘pre-planned’ reversing controllers are categorised into linear and non-linear controllers, and the ‘actively-planned’ reversing methods include advanced control methods and machine learning methods. Very few ‘actively-planned’ reversing methods have been used for reversing of LCVs.
The majority of the research work has neglected the dynamic characteristics of the vehicle and employed kinematic vehicle models. There are also many limitations on the application of these methods to multiply-articulated vehicles. For example, some methods can only be applied to articulated vehicles with ‘on-axle hitching’. Moreover, very few have been implemented and tested on full-scale LCVs.

Hence, the research aim of this thesis is to address the above issues and fill gaps in the literature of the ‘actively-planned’ reversing methods for a general n-trailer system.

8.1.2 Vehicle modelling (Chapter 2)

An extended dynamics model was developed for a general n-trailer system with an arbitrary number of axles, based on the standard two-axle, single-track model. A modified brush model for truck tyres was used to improve the fidelity of the non-linear vehicle model.

The vehicle dynamics model was linearised using the Jacobian linearisation method for state space analysis and the development of the controllers. The open-loop system is unstable for reversing because of the non-holonomic characteristics, and the number of unstable poles is equal to the number of trailers. Winkler’s method was used to specify an ‘equivalent’ wheelbase to replace multiple-axle groups of trailers for modelling path tracking.

The linearised dynamics model was compared with the original non-linear model using impulse and step function perturbations in steer angle. The two models agreed well for small disturbances.

The vehicle parameters were defined through measurements performed on the real-world testing vehicles used in this study.

8.1.3 Minimum Swept-Path Control (Chapters 3 and 4)

A new ‘pre-planned’ control method called ‘Minimum Swept-Path Control’ (MSPC) was devised to improve the path-following performance for realistic applications, following a pre-determined path. The main aim of the MSPC method was to reduce the overall swept path while guaranteeing the accuracy of path tracking during manoeuvres, enabling reversing on narrower lanes than previous path-following approaches.

The vehicle dynamics model was modified to calculate the lateral offsets of the front axle of the tractor and the rear axle of the rear trailer in the state space. Both of the weighted displacements were incorporated into the controller cost function instead of only the tracking error of the rear end of the vehicle in the previous PFC method.
The relationships between the lateral offsets of both axles, the preview distance and the controller weights were investigated and then used in the tuning process. Linear Quadratic Regulator (LQR) theory was used to tune the controller gains by solving the discrete-time Riccati equation.

Simulations of lane change and roundabout manoeuvres were run for the tractor-semitrailer and B-double vehicles. The simulation results showed that the MSPC controller significantly reduced the overall swept path and was robust to the details of the target paths.

Two different full-scale test vehicles, a tractor-semitrailer and a B-double, were used to test the performance of the MSPC controller at Bourn airfield. Various hardware and sensors (AB Dynamics SR30 steering robot, V.S.E. articulation angle sensors, UniMeasure string potentiometer, and OxTS RT3022 inertial and GPS navigation system) were mounted on the test vehicle units. The front steer angle sensor and the articulation angle sensors were calibrated carefully, to measure relationships between the sensor voltage and the corresponding angles. A dual antenna inertial and GPS navigation system with a GPS base station was used to measure vehicle motion accurately.

A steering tracking subsystem was also developed using a proportional-integral-derivative (PID) control method to follow the steer angle demand calculated by the main MSPC controller. The inner loop compensator was tuned experimentally.

The simulation and experimental results were found to agree well for both the lane change and roundabout manoeuvres performed by the tractor-semitrailer and B-double vehicles. The experimental results show that the MSPC controller exhibited good real-time performance, reducing the overall swept path width by more than 40%, compared to the previous path following methods. The experimental responses had slightly less overshoot than the simulation results. Hence, the real-time performance exceeded the simulation.

8.1.4 Lane-Bounded Reversing Control (Chapters 5 and 6)

An ‘actively-planned’ reversing method called ‘Lane-Bounded Reversing Control’ (LBRC) was developed to enable autonomous reversing of multiply-articulated vehicles in space-constrained and collision-free manoeuvres to target terminals, without a pre-calculated reference path. It was proved mathematically that the Lyapunov stability can be guaranteed by constructing the terminal cost function or by using a sufficiently large preview horizon. The
optimal steer angle was obtained by solving a linearly constrained quadratic programming problem, using a modified primal-dual solver.

In order to demonstrate the effectiveness of the LBRC method, a number of difficult manoeuvres in confined spaces were simulated for the tractor-semitrailer, B-double, and also B-triple configurations. The manoeuvres included combinations of navigating obstacles, traversing sharp bends, and passing through narrow gates to a terminal position in a loading bay. The boundaries of these manoeuvres are not required to have continuous position, gradient or curvature. The simulation results show that the LBRC controller exhibited good performance in these scenarios and was robust to the details of boundary contours and initial disturbance. A numerical tractor articulated with 7 B-link trailers was stabilised using the LBRC method (exceeding the previous maximum of 6 trailers).

The relationship between the average computational time per iteration for a feasible solution, the length of the preview horizon, and the number of trailers was measured. As the number of trailers and the length of the preview horizon increase, the computational time increases quadratically.

The LBRC controller requires significant computational resources. To implement the controller on the test vehicles, a rapid algorithm was devised by removing the states of the intermediate trailers. The original and fast implementation algorithms both performed similarly for the testing manoeuvres. An asynchronous model was built using the rapid algorithm, taking the system time lags and numerical estimation methods into consideration. The comparison between the synchronous and asynchronous models shows that the time delay is a crucial factor in the controller performance.

The testing framework of the MSPC system was adopted and modified for the implementation of the LBRC controller. A subsystem called ‘Vehicle Motion Estimator’ was established in the system architecture to estimate the vehicle position, heading, and kinematic parameters. The articulation angle rates were estimated via numerically differencing the real-time articulation angle measurements, and then filtered by a tuned second-order Butterworth infinite impulsive response (IIR) filter. The lateral and yaw velocities of the Centre of Mass (C.o.M.) of the tractor were calculated based on the inertial measurements of the rear trailer, the vehicle geometry, and the estimated articulation angle rates. To speed up the real-time calculation, some matrices were pre-computed and stored in the cache.
Three scenarios were used to test the controller’s planning and decision-making performance on the tractor-semitrailer and B-double combination. These included: (i) a lane change manoeuvre with obstacles on each corner, connected to a loading bay; (ii) negotiating an obstructive wall and passing through a misaligned narrow gate ahead of a loading bay; (iii) a sharp 90-degree bend with an obstacle on the corner, connected to a loading bay. For these manoeuvres, the experimental and simulated results were found to be in good agreement. The experimental results showed that the controller exhibited good real-time performance, enabling the test vehicle to negotiate all of the manoeuvres satisfactorily.

8.1.5 Advanced control methods – exploratory research (Chapter 7)
Two advanced control methods were developed to compensate the disadvantages of using the LBRC method, which are computational time and complex tuning requirements.

The ‘Adaptive Lane-Bounded Reversing Control’ (ALBRC) method addresses the heuristic tuning problem caused by the LBRC method. The controller weights adapt to different manoeuvres and only the preview and control horizons require tuning. Five scenarios used in the development of the LBRC controller were adopted to provide a comparison between the ALBRC and LBRC controllers. The simulation results showed that the ALBRC controller performs well and tends to make the vehicle travel closer to the middle line of the boundaries than the LBRC.

The ‘Adaptive Bi-directional Control’ (ABC) method was devised to tackle situations where there is no feasible solution, or to overcome the substantial computing power required to generate a solution with long preview horizons. The ABC method integrates the theory of the LBRC method and the adaptive tuning method with the ‘cusp’ technology, which allows the vehicle to travel forwards and straighten-up when there is no feasible solution in reverse. The ABC controller with a small preview horizon enables a vehicle to move forwards and backwards to realign the vehicle position and maximises the use of confined spaces for reversing, while significantly reducing computation time.
8.2 Recommendations for future work

8.2.1 Improvements in current research

To improve the real-time performance of the LBRC controller, accurate inertial sensors are required on each vehicle unit to directly measure the vehicle velocities instead of estimation methods. Kalman filters can also be used to estimate the vehicle motion or the articulation angle rates to improve the estimation accuracy.

8.2.2 Dynamic obstacles

Dynamic obstacles can be simulated in the maps to test the response time of the LBRC and find the relationship between the preview horizon, controller weights, and the response time. Estimation methods, e.g. Kalman filters, could be used to estimate the expected trajectory of the vehicle and the moving obstacles. The LBRC method will need to be modified to avoid dynamic obstacles.

8.2.3 Implementation of the developed advanced control methods

The code for both the ALBRC and ABC methods requires work to be optimised and both methods should have a fast-real-time algorithm with very small running time. Then, the computational time between the ALBRC, ABC and LBRC solutions should be compared. The ALBRC and ABC controllers need to be tested first using hardware-in-the-loop and eventually on full-scale test vehicles. For the ABC method, the relationship between the number of cusps and the preview horizon should be investigated, in order to make the controller exhibit optimal performance.

8.2.4 Reinforcement learning methods

As discussed in the literature review, reinforcement learning methods are suitable for multi-stage decision-making problems. There have been many breakthroughs in the field of reinforcement learning, but only very few methods have been applied to reversing of LCVs. Methods like ‘Deep Deterministic Policy Gradient’ (DDPG) [226], ‘Proximal Policy Optimisation’ (PPO) [227], ‘Soft Actor Critic’ (SAC) [228], or even a ‘Deep Q-Network’ (DQN) [229]–[232] method could be used to solve the reversing problem. The number of trailers could be used as an input in the training and so the methods could be applied to a general n-trailer system.
8.2.5 Integration with vehicle perception systems

The developed controllers could also be integrated with vehicle perception systems using computer vision methods. Using lidars, radars, and cameras to perceive the vehicle surroundings and generate suitable boundaries for the controllers according to obstacles measured in real time. The integration of the two systems is highly recommended because this can make the autonomous reversing system complete. First, the perception system would be used to monitor static and dynamic obstacles and update the boundary map. Then, the controllers could react, based on the boundary map. Such a system could be combined with mapping information about a manoeuvring area, communicated to the vehicle by a central control tower in a depot.
Chapter 9 References


[126] V. Cerone, M. Milanese, and D. Regruto, “Combined Automatic Lane-Keeping and Driver’s Steering Through a 2-DOF Control Strategy,” IEEE Transactions on Control


[211] E. Hartley, Dense primal-dual solver [Source code]. Sep. 06, 2013. The function is compatible with Embedded MATLAB.


Appendix A: Solver for a quadratic programming problem

Various methods, e.g. interior-point-convex [233], active-set [234], and trust-region-reflective [235], can be used to solve quadratic objective functions with linear constraints. The built-in MATLAB function, i.e. ‘quadprog’ [236] with the interior-point-convex algorithm, was used for the simulations in this thesis. This is because the Hessian matrix, \( \mathbf{G}_L \), in the cost function (see equation (5-33)) is dense and positive definite so that it is a convex quadratic programming problem. The interior-point solver is based on a primal-dual algorithm with a Mehrotra predictor-corrector [233].

However, the ‘quadprog’ function did not support code generation in MATLAB 2017a, which could not be used in xPC and AutoBox for real-time applications. Instead, an Embedded MATLAB function using the primal dual algorithm for the convex linearly constrained quadratic programming problem was coded by Hartley [211]. This function adopted almost the same algorithm as the interior-point solver of the ‘quadprog’ function, and was previously verified and tested by Jia [237] for his vehicle tests. It showed good real-time performance. Therefore, the dense primal dual solver was selected in this project.

To increase the computation speed, the major modification made by the author was removing all calculations related to equality constraints in the source code, because the cost function (see equation (5-33)) does not have equality constraints. Then the modified function was incorporated in the xPC and AutoBox models for the real-time applications in this thesis.