Quantum Gases in Optical Boxes

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Quantum atomic and molecular gases are flexible systems for studies of fundamental many-body physics. They have traditionally been produced in harmonic electromagnetic traps and thus had inhomogeneous density, but the recent advances in light shaping for optical trapping of neutral particles have led to the development of homogeneous samples created in optical box traps. Box trapping simplifies the interpretation of experimental results, provides more direct connections with theory, and in some cases allows qualitatively new, hitherto impossible experiments. It has now been achieved for both Bose and Fermi atomic gases in various dimensionalities, and also for gases of heteronuclear molecules, and has allowed breakthroughs in the studies of both equilibrium and non-equilibrium phenomena such as superfluidity, turbulence, and the dynamics of phase transitions. Here we review progress in this emerging field.

Since the earliest days of ultracold atomic gases, the success in using them to study many-body physics [1–3] has owed a lot to the possibility to trap the atoms in versatile potentials, including low-dimensional traps [4], double wells [5], and optical lattices [6–8]. The electromagnetic trapping potentials are often also dynamically tuneable, which has allowed experiments ranging from studies of elementary excitations [9–11] to reversible crossing of phase transitions [12, 13].

More recent advances in the shaping of optical potentials have opened many new possibilities. One major development is the increasingly popular use of the uniform (flat-bottom) optical-box traps [14–19], as opposed to the traditionally used (optical or magnetic) harmonic ones. Box traps have allowed scientific breakthroughs in a wide range of areas, including studies of superfluidity, turbulence, and the dynamics of phase transitions. For example, the gas homogeneity has been beneficial for measurements of density-dependent quantities, such as the speeds of different types of sound in various superfluids [20–27], the quantum depletion in a condensed Bose gas [28], or the pairing gap in a Fermi gas [29, 30]. Qualitatively new observations have also been made, such as recurrences in closed quantum systems [31], the unexpected observation of the quantum Joule-Thomson effect [32], or the discovery of a novel type of breathers [33].

Here we briefly review the development of box traps and the scientific successes in this new and growing field.

Before starting, we also draw the reader's attention to contemporary reviews on two related emerging fields - (i) the creation of 'atomtronics' circuits for coherent matter waves, such as ring traps that support persistent currents [34], and (ii) the trapping of individual atoms or molecules in arrays of optical tweezers [35]. All three fields take advantage of the advances in light shaping, and there are also scientific connections; as a prominent example, the dynamics of phase transitions in a homogeneous system have been studied in ring traps [36], 2D and 3D box traps [15, 37], and a 1D tweezers-array [38].

MAKING BOX TRAPS

In this section we briefly introduce the key ideas and tools involved in the making of optical boxes. The basic concept behind most box traps is to use sculpted repulsive (blue-detuned) laser beams to construct the box walls that confine the particles (see Fig. 1a). Three-dimensional (3D) box traps are most commonly cylindrical and are made using one hollow tube beam and two sheet end-cap beams. To make a homogeneous 3D potential, one also levitates the particles against gravity, which for atoms is usually done using a static magnetic field gradient [14], while polar molecules can be levitated using a static electric field gradient [19]. To make a low-dimensional box one freezes out the particle motion along some direction(s) using very tight confinement, which can be harmonic; this dimensionality reduction is analogous to the making of a low-dimensional harmonic trap [4]. One could also make red-detuned (attractive) box traps, and also cancel gravity using a light field of linearly varying intensity [39], but this is technically more demanding because of the need to sculpt high intensity light such that the variations in the optical potential are smaller than all the relevant energy scales in the gas.

The development of optical boxes was greatly aided by two complementary types of programmable spatial light modulators (SLMs) - the liquid-crystal SLMs that modulate the phase of laser beams and the Digital Micromirror Devices (DMDs) that modulate their amplitude (see Fig. 1b). A liquid-crystal SLM is a rectangular array of $\sim 10^6$ pixel elements (each $\sim 10\ \mu$m in size) with individually controllable indices of refraction; using it to imprint a spatially-modulated phase delay on a laser beam, one controls the intensity pattern in the vicinity of the conjugate (Fourier) plane. Similarly, a DMD is a rectangular array of $\sim 10^6$ mirrors (each $\sim 10\ \mu$m in size) that

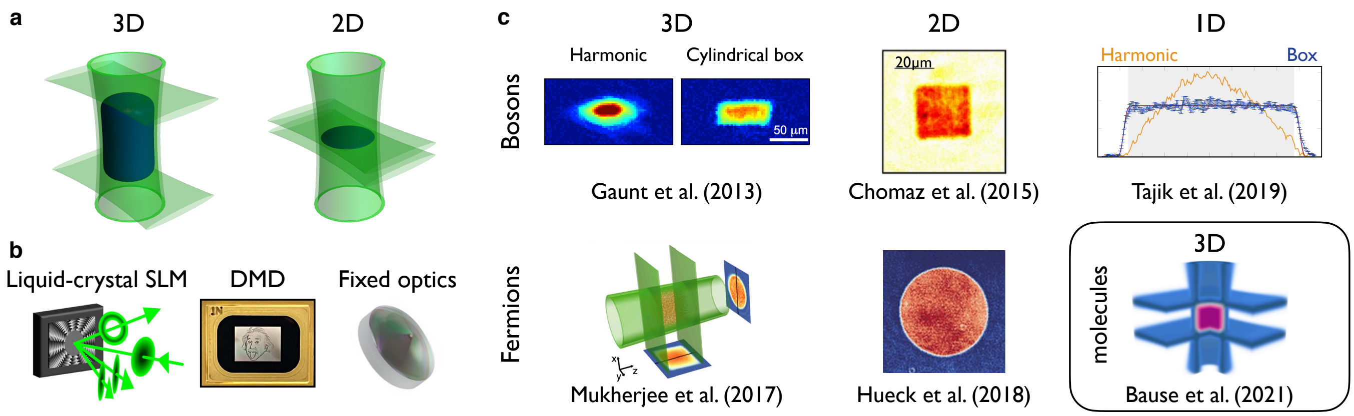


FIG. 1. **Optical box traps.** (a) The concept. The walls of the box are made using repulsive laser beams (green) and the particles (blue) are confined in the dark region between them. Here, a cylindrical 3D box is sculpted using one hollow-tube beam and two end-cap beams. A 2D box is made by freezing out the particle motion along one direction (here vertical) using very tight confinement; to create a 1D box one freezes out the motion along two directions. (b) The box tools. Programmable spatial light modulators (SLMs) - the phase-controlling liquid-crystal SLM and the amplitude-controlling Digital Micromirror Device (DMD) - offer great flexibility for sculpting of laser beams; in the cartoon, starting with a single Gaussian beam, a single liquid-crystal SLM is used to create all three beams for a cylindrical 3D box [14]. However, box traps can also be made using specialised fixed optics, *e.g.* an axicon to create a tube beam. (c) Box-trap gallery. Optical boxes have been realised for Bose and Fermi atoms in various dimensionalities, as well as for (fermionic) heteronuclear molecules in 3D [14–19].

can be individually turned 'on' or 'off' (by changing their tilt angle) to spatially modulate the amplitude of a beam; an arbitrary intensity pattern can then be imaged onto the cloud [40].

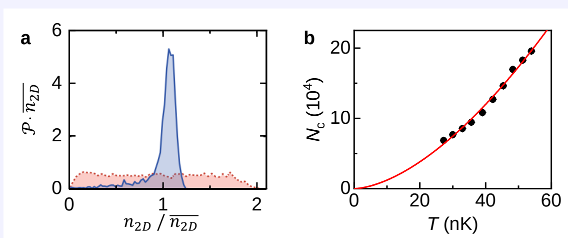
Liquid-crystal SLMs are convenient for creating multiple beams needed for a box trap using a single device [14], and are generally more power-efficient than DMDs. On the other hand, DMDs are more convenient for making arbitrarily-shaped boxes, such as squares in 2D or cubes in 3D, and much better for creating dynamical potentials. Due to their (currently) sub-kHz refresh rate, the liquid-crystal SLMs cannot be reprogrammed during an experimental run without the particles escaping while the phase pattern is being updated and the trap temporarily turned off. Meanwhile, the ~ 10 kHz refresh rate of DMDs is sufficiently high for the trapping pattern to be dynamically changed without the ultracold particles moving significantly during the updates [41].

The versatility of SLMs has been essential for experimental exploration [42], but box walls can also be made using non-tuneable tools, such as axicons [17, 18, 43] and custom-manufactured masks [15], which can be more cost-effective and power-efficient. Yet another option is to use 'painted', time-averaged potentials created by fast spatial scanning of laser beams [44]. For a comprehensive review of recent advances in light shaping for atom trapping, including comparisons of different methods, see Ref. [41].

Using the various light-sculpting methods, boxes of various shapes and dimensionalities have been created for both atomic and molecular gases [14–19], as illustrated in Fig. 1c; see also Refs. [45–48] for earlier examples of confining laser-cooled atoms with blue-detuned beams, and Refs. [49, 50] for early prototypes of 1D box traps.

As a final point in this section, optical box traps are not perfect. The sharpness of their walls is limited by the op-

Box 1: Characterising box traps



(a) One simple measure of the gas homogeneity is the distribution of the real-space densities. Here we show the distribution of column densities, $n_{2\mathrm{D}}$, extracted from *in situ* images of 3D clouds [17]. In a box trap (blue) the probability distribution $\mathscr{P}(n_{2\mathrm{D}})$ is narrow, strongly peaked near the average value $\overline{n_{2\mathrm{D}}}$. This means, for example, that most atoms experience essentially the same mean-field potential. For comparison, the corresponding distribution in a non-degenerate harmonically trapped gas (red) is very broad; the expected distribution is uniform between 0 and $2\overline{n_{2\mathrm{D}}}$.
(b) A complementary characteristic of box traps is the single-particle density of states, which is seen, for example, in the dependence of the critical atom number for Bose-Einstein condensation, N_c, on the temperature T [32]. In a 3D harmonic trap $N_\mathrm{c} \propto T^3$, while in a perfect 3D box $N_\mathrm{c} \propto T^{3/2}$. Here the experimental data is captured by $N_\mathrm{c} \propto T^\alpha$ with $\alpha = 1.65$ (red line). A common way [14] to characterise (imperfect) box traps is to model them by an isotropic power-law potential $V(\mathbf{r}) \propto r^p$, with $p \gg 1$. Then $\alpha = 3/2 + 3/p$, and for a Fermi gas one similarly gets that the Fermi energy is [17] $E_\mathrm{F} \propto N^{1/\alpha}$. In current experiments $p > 10$ is readily achieved, and there are indications that values of p up to ~ 100 are feasible [18].

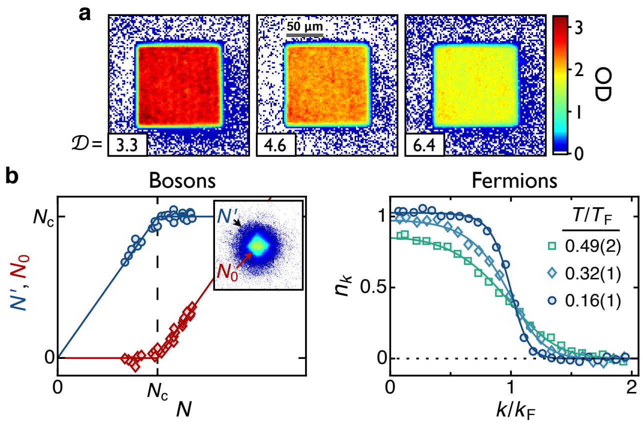


FIG. 2. **Quantum statistics in homogeneous gases.** (a) *In situ* absorption images of homogeneous quantum-degenerate 3D gases with different phase space densities $\mathcal{D} > 1$. The gas with lowest density (lowest optical density OD) is the coldest and actually has the highest $\mathcal{D}$. However, in the images one cannot see this, or even whether the gases are fermionic or bosonic (here they are fermionic ^{6}Li gases). (b) In contrast, the effects of quantum statistics and degeneracy are striking in momentum space. For a Bose gas, the statistical nature of the BEC phase transition, driven by the saturation of the excited states, is revealed more clearly than in the corresponding harmonic-trap experiments [52]. Here the plot shows the total number of atoms in the thermal cloud (N') and the condensate (N_0), as the number of atoms in the cloud (N) is varied at constant temperature [32]; the inset shows an image of a partially condensed gas after release from the box and free expansion. For a Fermi gas below the Fermi temperature T_F, the effects of the Pauli exclusion principle are clearly observed - the occupation of individual momentum states are limited to $n_k \leq 1$, and the Fermi surface forms at the Fermi wavevector k_F (adapted from [17]).

tical wavelength of $\sim 1\ \mu$m, which is not negligible compared to the typical box dimensions of $10 - 100\ \mu$m. Moreover, additional fields used for levitation, the creation of low-dimensional gases [15, 16, 51], or tuning of interactions, can lead to further imperfections. In Box 1, we discuss two complementary methods used to quantify the uniformity of box-trapped gases. The currently achieved uniformity has been good enough for the success stories we discuss in the next section, but how close to perfect a box needs to be ultimately depends on the specific scientific problem (see Outlook).

SUCCESS STORIES

Here we outline the scientific advances afforded by homogeneous atomic gases, which also illustrate the general types of problems for which the box traps are advantageous.

Quantum statistics. We start with experiments on purely quantum-statistical phenomena (Fig. 2). In harmonic traps the real and momentum space are coupled such that, for example, one can clearly observe real-space effects of Bose-Einstein condensation (BEC) [53] and Fermi pressure [54]. In a box trap, the signatures of quantum statistics are harder to see in real space (see Fig. 2a), but in momentum space they are revealed more cleanly than in harmonic-trap experiments (Fig. 2b).

For bosons, one clearly observes the statistical nature of the BEC transition, which is driven by the saturation of the total occupation of all excited (momentum) states [32]. As the total atom number in the gas is increased at a fixed temperature, the number of atoms in the thermal cloud saturates at the critical value for condensation, and all the extra atoms accumulate in the condensate; in harmonic traps this textbook effect is obscured by a combination of geometric and mean-field interaction effects [52].

For fermions, one instead observes the occupation-number saturation at the level of individual momentum states, as prescribed by the Pauli exclusion principle. As the temperature (normalised to the Fermi temperature T_F) is reduced, the occupation of all states remains bound to $n_k \leq 1$ and the Fermi surface forms [17] at the Fermi wavevector k_F.

Experiments on the thermodynamics of nearly-ideal box-trapped gases also lead to the unexpected observation of the Joule-Thomson effect that arises solely from quantum correlations [32]. While the ideal classical gas does not change temperature under isoenthalpic rarefaction, the ideal Bose gas cools and the ideal Fermi gas is expected to heat [55].

Equilibrium properties of interacting systems. We now move on to the experiments on the many-body physics of interacting gases. We start with the broad class of spectroscopic and transport measurements in which one weakly perturbs a system in order to extract information on its equilibrium properties (Fig. 3).

A lot of attention has been given to long-wavelength sound waves [20–27]; in Fig. 3a(i) we show examples of sound propagation in 3D Fermi [22] and 2D Bose [21] gases. The key quantities studied in such experiments are the sound speed and the sound attenuation (or equivalently diffusivity). Both of these quantities are density dependent, and the crucial advantage of box traps for interpreting the measurements is that they are constant in space.

In Fig. 3a(ii) we illustrate two scientific highlights of the experiments on sound in homogeneous superfluids. In both 3D and 2D unitary Fermi gases the quantum limit of sound diffusivity, set by $\hbar/m$ (where m is the atom mass), was demonstrated [22, 26]; this universal limit should also be relevant for other strongly interacting Fermi systems such as neutron stars. In a weakly interacting 2D Bose gas, first and second sound [58, 59] in a Berezinskii–Kosterlitz–Thouless (BKT) superfluid [60, 61] were observed for the first time, and the measurements of the two sound speeds revealed the universal superfluid-density jump at the BKT transition [62]. Related experiments on superfluidity have investigated the Josephson effect [63] and the critical velocity [64] in a 2D Fermi gas, while a more complex trapping geometry allowed studies of a Bose-gas superfluid flow through a constriction between two reservoirs [65].

In Fig. 3b we illustrate the benefits of gas homogeneity for spectroscopic measurements that globally probe the sys-

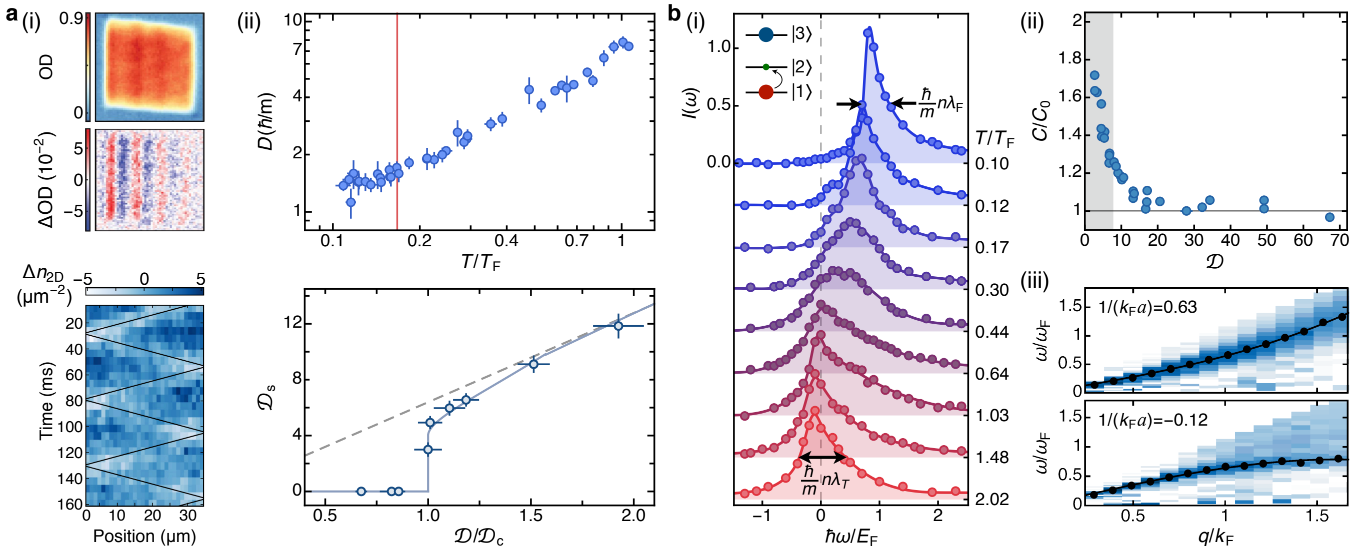


FIG. 3. **Sound and spectroscopy measurements on box-trapped gases** (a) Sound: (i) Low-energy sound modes can be probed by perturbing the gas with an external potential and observing the evolution of the resulting density modulations in time and space. Images show examples of measurements in a 3D Fermi gas [22] (top) and a 2D Bose gas [21] (bottom). (ii) Top: the sound diffusivity D (seen in the attenuation of the wave) in the low-temperature unitary Fermi gas approaches the universal quantum limit, $D \approx \hbar/m$; the red line indicates T_c, the critical temperature for superfluidity (adapted from [22]). Bottom: the superfluid phase-space density $\mathcal{D}_s$ in a 2D Bose gas, which was deduced from the measured speeds of first and second sound, undergoes a universal jump from 0 to 4 at the BKT phase transition; here $\mathcal{D}$ is the total phase-space density and $\mathcal{D}_c$ its critical value [25]. (b) Spectroscopy: (i) Particle-ejection spectroscopy on the unitary 3D Fermi gas [56]. Rabi rf spectroscopy was performed on the whole cloud and the differences induced by small changes in T/T_F are observable only due to the lack of inhomogeneous broadening. Such measurements probe short-range correlations and have also revealed non-Fermi-liquid behaviour in the normal unitary gas, at $T > T_c$. (ii) Ramsey rf spectroscopy on a 2D Bose gas was used to determine the two-body contact (C) as a function of $\mathcal{D}$; the shaded region indicates the non-superfluid phase (adapted from [57]). (iii) Bragg spectroscopy was used to measure the excitation spectrum in a strongly interacting 3D Fermi gas, here shown on the BEC (top) and the BCS (bottom) side of the BEC-BCS crossover [29].

tem [28, 29, 56, 57, 66–69]; for experiments on the extraction of homogeneous-gas properties by local probing of harmonically trapped gases see [70–73].

The Rabi radio-frequency (rf) spectra shown in Fig. 3b(i) measure the energy cost of removing a particle from a spin−1/2 3D Fermi gas at different reduced temperatures T/T_F [56]. Measurements were performed on the whole sample and the spectra taken at closely-spaced T/T_F values are clearly distinct only thanks to the lack of inhomogeneous broadening; in a harmonic trap global measurements would mix signals for a wide range of $T/T_F(\mathbf{r}) \propto n(\mathbf{r})^{-2/3}$, where $n(\mathbf{r})$ is the local density. Rabi rf spectroscopy of 3D Fermi gases [56, 68] has, for example, provided an observation of non-Fermi-liquid behaviour in a normal strongly interacting gas [56], while Ramsey rf spectroscopy of 2D Bose gases has provided measurements of short-range correlations across the BKT transition (Fig. 3b(ii)) and an observation of magnetic-dipole interactions [57, 69].

The lack of inhomogeneous broadening is similarly beneficial for Bragg spectroscopy [74, 75]. Fig. 3b(iii) shows measurements of the excitation spectrum of a strongly interacting 3D Fermi gas, which were used to extract the concavity of the dispersion relation and the density-dependent pairing gap in the BEC-BCS crossover [29]; see also Ref. [30] for similar measurements on 2D Fermi gases. Bragg-spectroscopy experiments on condensed homogeneous 3D Bose gases have provided an observation of Heisenberg-limited long-range coherence [66], the confirmation of Bogoliubov's theory of quantum depletion [28], and an observation of the breakdown of Bogoliubov's theory of the excitation spectrum for sufficiently strong interactions [67].

Non-equilibrium phenomena. In another large class of experiments, homogeneous interacting gases have been driven or quenched far from equilibrium (Fig. 4).

One paradigmatic topic in non-equilibrium physics is turbulence in strongly driven systems. Depending on the system and the excitation protocol, turbulent dynamics can be dominated by either waves or vortices, and the advent of box traps has led to new insights in both cases (Fig. 4a).

Wave turbulence is theoretically described in terms of dynamics that are local in momentum space [79], so it is advantageous to experimentally study it in momentum space, for which box traps (with their plane-wave eigenstates) provide the natural setting. In 'shaken' 3D box-trapped Bose gases the power-law momentum distribution characteristic of a direct turbulent cascade was observed [20], and the elusive particle and energy fluxes through the cascade have also been measured [76].

Vortex dynamics have been studied in turbulent (quasi-)2D box-trapped Bose gases. Such dynamics can arise due to var-

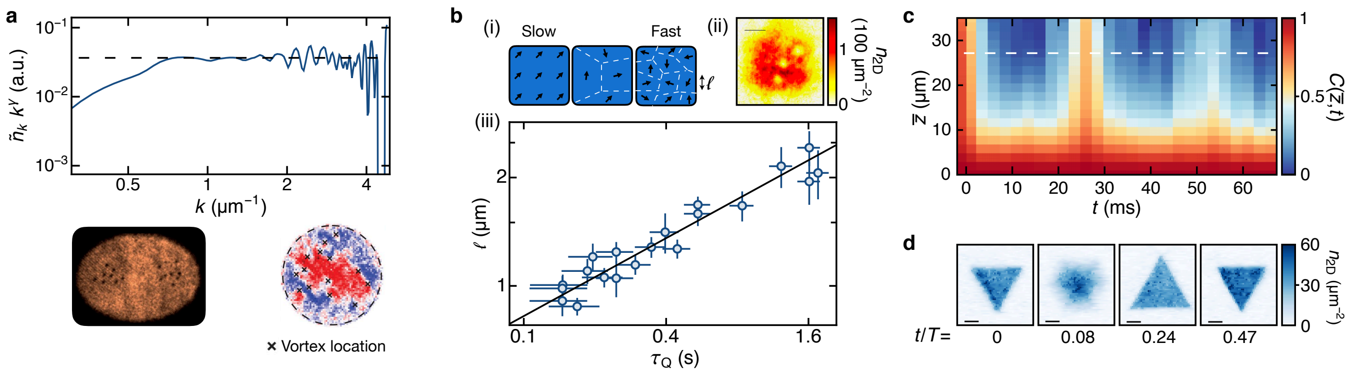


FIG. 4. **Non-equilibrium phenomena** (a) Wave and vortex turbulence. Top: power-law momentum distribution, $\tilde{n}_k \propto k^{-\gamma}$, in a 3D Bose gas driven on a large lengthscale reveals a wave-turbulence cascade towards smaller lengthscales [20, 76]. Bottom: experiments on vortex turbulence in quasi-2D Bose gases showed large-scale vortex clustering, which was observed by probing the density distribution [77] (left) and the superfluid-velocity field [78] (right). (b) Critical dynamics. (i) Critical slowing down results in nonadiabatic crossing of the BEC transition and formation of domains with different spontaneously chosen condensate phases (indicated by the arrows). (ii) Topological defects form at the domain boundaries; here the image shows vortices spontaneously generated in a quench-cooled gas [15]. (iii) The power-law dependence of the average domain size, ℓ, on the quench time, τ_Q, is in agreement with the Kibble-Zurek theory [37]. (c) Recurrences of phase correlations were observed in a 1D Bose gas [31]. (d) A novel breather was seen in a 2D Bose gas; for particular initial density distributions, such as a uniform triangle prepared in a box trap, the cloud evolving in a harmonic potential periodically returns to its initial state [33].

ious reasons, including vortex interactions and density gradients; eliminating the latter allowed clean observations of vortex clustering corresponding to negative temperatures [77, 78] (see also [80, 81]), as predicted by Onsager in 1949 [82]. In another recent experiment, on box-trapped 2D Fermi gases, the interplay of accelerated vortices and waves has also been observed [83].

Another major topic for which homogeneous gases have distinct advantages is the critical behaviour near second-order phase transitions, where the range of correlations in the gas diverges (see Fig. 4b and Box 2). This is fundamentally homogeneous-system physics and it is hard to study it in inhomogeneous systems, specifically because due to the divergence of the correlation length, the local-density approximation (LDA) breaks down. In a non-equilibrium context, dynamic crossing of such a transition results in causally disconnected domains that display different choices of the symmetry-breaking order parameter. The Kibble-Zurek (KZ) theory [84, 85] that describes these dynamics was originally developed specifically for homogeneous systems and its key assumption is that when it comes to the choice of the order parameter all parts of the system have 'equal voting rights' [86, 87]. Box-trap experiments have provided quantitative tests of the KZ predictions for how the domain size [37] and the resulting density of defects in the ordered state [15] depend on the quench rate (see also [36, 38, 88] for ring-trap and optical-tweezers experiments).

A number of further non-equilibrium experiments have been made possible by different properties of homogeneous gases. The form of the excitation spectrum of a weakly interacting 1D Bose gas allowed the observation of recurrences in a closed quantum system [31, 94] (Fig. 4c). A momentum-space study of an energy-quenched far-from-equilibrium 3D Bose gas revealed bidirectional universal scaling dynamics [95]. Finally, in a 3D Bose gas quenched to unitarity, the fact that all parts of a non-equilibrium homogeneous cloud evolve in the same way allowed the observation of universal loss and prethermalization dynamics [96, 97].

Other box-enabled experiments. Finally, box-trapping and related technologies have also facilitated many experiments that are less directly related to the physics of homogeneous gases. One example of this is the discovery of a novel breather in a 2D Bose gas [33]; the breather shown in Fig. 4d is observed in a harmonic potential, but the initial state had to be prepared in a box trap [33]. Another similar example is the deterministic preparation of a Townes soliton in Ref. [98] (see also Refs. [99, 100] for other observations of Townes solitons in box traps); this 2D soliton is an inhomogeneous ground state of the system, but its deterministic preparation was based on starting with a homogeneous gas and imprinting arbitrary density profiles using a DMD [101]. A different example of a practical advantage of box traps is the observation of the transition from an atomic to a molecular condensate [102]; in this case the creation of a (quasi-)equilibrium condensed gas of unstable molecules was facilitated by the use of a 2D box trap to minimise losses and heating. Further examples of box-enabled experiments include observation of the weak collapse of a condensate with attractive interactions [103] and the studies of the modulation instabilities that lead to emission of matter-wave jets, pattern formation and quasi-particle pair-production [104–107].

OUTLOOK

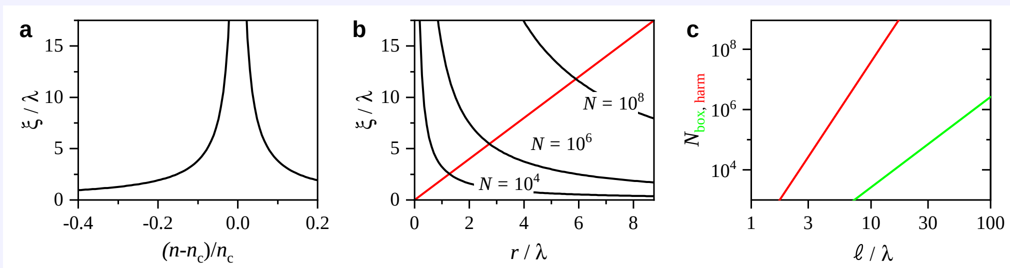
The scientific exploitation of box-trapped quantum gases is still in its infancy. Here we highlight some exciting possibili-

Box 2: Critical phenomena in harmonic and box traps

Box traps are particularly advantageous for studies of phenomena associated with long-range correlations, such as those emerging near second-order phase transitions. Here we illustrate this advantage with a simple ideal-gas calculation, by directly comparing the range of correlations that can be observed in harmonic and box traps near the BEC critical point.

In a homogeneous system, near the critical point the correlation length ξ diverges as illustrated in panel **a**. At a fixed temperature, $\xi/\lambda = A(|n-n_c|/n_c)^{-\nu}$, where λ is the thermal wavelength, n_c is the critical density, ν the critical exponent, and A is a non-universal prefactor; for the ideal-gas BEC transition [89] $n_c = 2.612/\lambda^3$, $\nu = 1$, and $A = 1/2.612$.

For a harmonically trapped gas with spatially uniform T, one can evaluate a spatially varying $\xi(r)$ within the local density approximation (LDA), *i.e.* assuming that ξ at each r is the same as in a homogeneous system with density $n(r)$. However, this approach breaks down if $n(r=0)$ approaches n_c, because the deduced ξ becomes larger than the lengthscale over which n and hence ξ itself vary significantly. One can still, at the cost of reducing the experimental signal, focus on the central part of the cloud and assume that n is constant within some non-infinitesimal volume (as in Refs. [70, 71, 73, 90]). In reality n and ξ vary within this region, but one can still directly probe correlations on a lengthscale ℓ if $\xi(r) > \ell$ for all $r < \ell/2$; this approach was used in Ref. [91].



Setting $n(r=0) = n_c$, assuming an isotropic potential $(1/2)m\omega^2 r^2$, and expanding the ideal-gas distribution [1] near $r = 0$ gives $\xi(r)/\lambda \approx k_B T/(\hbar\omega) \times [2\pi r/\lambda]^{-1}$. Noting that $k_B T/(\hbar\omega) = (N/1.202)^{1/3}$, where N is the total number of atoms in the trap, in panel **b** we plot ξ/λ versus r/λ for different N (black lines). The intersects of these curves with the line $\xi = 2r$ (red) then give the achievable $\ell = 2r$ for a given N, irrespective of the choices of ω and corresponding T. Conversely, the atom number needed to directly observe correlations on a lengthscale ℓ in a harmonic trap is

$$N_{\text{harm}} = 1.202\,\pi^3 (\ell/\lambda)^6\,, \tag{1}$$

shown by the red line in panel **c**. With a typical $N \sim 10^6$, one can reach only $\ell \sim 5\lambda$.

On the other hand, working with a box trap, one just needs a box of size ℓ and the corresponding atom number

$$N_{\text{box}} = n_c \ell^3 \approx 2.612 \times (\ell/\lambda)^3\,, \tag{2}$$

shown by the green line in panel **c**. In this case the same $N \sim 10^6$ is sufficient for measurements up to $\ell \sim 70\lambda$.

For an interacting gas, for which [91–93] $\nu \approx 0.67$, one gets similar conclusions. In this case $N_{\text{box}} \propto (\ell/\lambda)^3$ is essentially the same, with just the prefactor ($\propto n_c/\lambda^3$) changing slightly, and one can estimate N_{harm} in various ways: still assuming ideal-gas $n(r)$ gives $N_{\text{harm}} \propto (\ell/\lambda)^{3+3/\nu} = (\ell/\lambda)^{7.5}$, while approximating $n(r)$ as a Gaussian gives $N_{\text{harm}} \propto (\ell/\lambda)^{3+3/(2\nu)} = (\ell/\lambda)^{5.25}$. In either case one concludes that with the same atom-number resources one can directly observe much-longer-range correlations in a box trap.

ties for the future, and also some open technical challenges.

The successful studies of phase-transition dynamics could be extended to the infinite-order BKT transition [108–112] and the bubble-nucleation dynamics associated with first-order transitions, including some believed to be relevant for the physics of the early universe [113–116]. Another general area where box traps could offer great advantages is topological physics [117, 118]; sharp boundaries could allow real-space studies of edge states [119], which have so far been observed in cold-atom systems exploiting synthetic dimensions associated with internal (spin) degrees of freedom [120–122]. It has also been predicted [123] that the supersolid phases of gases with strong dipolar interactions [124–129] should be qualitatively different in a box trap.

Further possibilities are offered by combining box traps and other trapping methods. For example, combining box traps and optical lattices has already facilitated the observation of a low-entropy homogeneous state with long-range antiferromagnetic correlations [130] and studies of competing magnetic orders in the bilayer Hubbard model [131]. Further possibilities are suggested by the hybrid trap of Ref. [17], which is box-like along two directions and harmonic along the third. In this case the harmonic direction provides tuning of the local chemical potential, while probing the system along a perpendicular direction retains many of the advantages of box traps, at least as long as the local density approximation (LDA) is valid. Such a setup could be used to study interfaces between different phases of matter, and could also facilitate searches for exotic states that are expected to occur only in narrow regions of bulk phase diagrams; an important example of such a still-sought-for phase is the Fulde-Ferrell-Larkin-Ovchinikov (FFLO) superfluid [132–134].

While the range of scientific possibilities is broad and exciting, we can already anticipate that some of them will also require further technological developments.

The first issue is that many interesting experiments are likely to require increasingly larger and closer to perfect box traps. This is particularly true for studies of critical phenomena (see Box 2), and more generally long-range correlations. As an illustration, a paradigmatic problem for which the current technology is still marginal is that of the critical temperature for Bose–Einstein condensation in an interacting homogeneous gas [135–140]. Critical fluctuations in a repulsively-interacting gas are predicted to raise T_c above the ideal-gas value. However, in a harmonic trap one observes the opposite [141–145] - the beyond-mean-field correlation shift of T_c is diminished because only a small fraction of the cloud is critical at T_c, and it is overpowered by a geometric mean-field (MF) effect [146] that reduces T_c. For a general power-law trap, $V(r) \propto r^p$ (see Box 1), with increasing p the beyond-MF shift should be more pronounced and the MF one should diminish. Based on an LDA estimate, we find that for the currently typical $p \sim 10$ the two effects are still comparable, and that one needs $p \gtrsim 100$ to cleanly observe the beyond-MF correlation shift of T_c. We expect other fundamental correlation-physics problems to similarly create a moving target for the tolerable box imperfections.

The second issue is that many exciting possibilities rely on specific features of different atomic species and their mixtures, but the methods for the levitation of gases in 3D box traps are generally species specific. The simplest magnetic levitation [14] strictly speaking works only for single-component gases, but can work well enough for mixtures of species that have similar ratios of mass and magnetic moment [17]. For two spin states of the same isotope, rapid swapping of the spins of individual particles can be used to levitate them simultaneously even if the two magnetic moments are significantly different [147]. Optical levitation can also extend the possibilities further, to multiple species with similar ratios of mass and polarizability [39]. However, creating arbitrary homogeneous mixtures of different chemical elements, different isotopes, or even just different spin states of the same isotope, remains an open challenge.

These two issues are in fact related, since even in single-species experiments the limitations for making the box traps larger and closer to uniform (with larger p values) are often related to the need to levitate particles with additional fields. An exciting possibility for the future would therefore be to send box-trap setups to space and perform many-body experiments in a microgravity environment [148–150].

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Acknowledgements. We thank C. Eigen for help in the preparation of the figures and critical reading of the manuscript. We also thank R. Lopes and S. Nascimbene for comments on the manuscript, and M. Zwierlein, P. Patel, B. Mukherjee, J. Beugnon, J. Dalibard, R. Saint-Jalm, H. Biss, T. Lompe, and H. Moritz for sharing their data. This work was supported by the EPSRC (grant nos EP/N011759/1, EP/P009565/1, and EP/T019913/1), ERC (QBox), QuantERA (NAQUAS, EPSRC grant no. EP/R043396/1), NSF CAREER (grant no. 1945324), and DARPA (grant no. 00010372). N. N. acknowledges support from the David and Lucile Packard Foundation, and the Alfred P. Sloan Foundation. R.P.S. acknowledges support from the Royal Society. Z.H. acknowledges support from the Royal Society Wolfson Fellowship.

Competing interests. The authors declare no competing interests.

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