

When and How Should Firms Differentiate?

Quality and Advertising Decisions in a Duopoly

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One of the hallmarks of competitive interaction is the desire to differentiate from rivals. In this paper, we examine under what conditions firms will elect to differentiate through product quality vs. advertising intensity. Consumers purchase from the set of products they are informed about through advertising, and choose the alternative that maximizes their utility. In the main model analyzed, firms select product quality in a first stage, advertising level in a second stage, and price in the last stage. The probability a consumer is informed of a firm's product depends on the level of its advertising expenditure. We find that when advertising is not cost-effective both firms choose a light ad spending. This allows them to minimally differentiate in qualities without concern of intense price competition, as each firm expects to have a segment of 'captive' consumers who are only informed of its product. When advertising is moderately cost-effective, one firm shifts to expending heavily on advertising. However, the rival prefers to differentiate by advertising lightly, while choosing the same maximal quality level. This strategy softens price competition by inducing the heavy-advertiser to price high more often to capitalize on its large captive segment. When advertising is very cost-effective, both firms advertise heavily and prefer to differentiate in qualities. Three extensions are examined. In the first, we assume upfront fixed costs of choosing quality. While our results generalize in some parts of the parameter space, we are also able to sustain an equilibrium with both quality and advertising differentiation. In the second, we allow for continuous advertising levels and discrete product qualities. We find that varying the cost-effectiveness of advertising in this set up produces directionally similar results as those in the main model. In the third, advertising and pricing decisions are simultaneous rather than sequential. Our main-model results qualitatively hold in this setting, although mixed-strategies in advertising levels can arise in equilibrium. Taken together, our work shows that letting market awareness be determined endogenously suggests far less product differentiation than previously suspected and reveals regions where advertising actions are differentiated.

(Product Quality, Advertising, Differentiation, Competition)

Introduction

In most markets, firms must figure out how to contend with competitors. The presence of rivals means that consumers have multiple offerings to choose from, and hence demand for each player is by no means guaranteed. As they seek to navigate this challenge, firms typically consider ways to set themselves apart from the competition in order to avoid a “race to the bottom” in prices that would erode any and all profits. A common approach to softening such detrimental competitive intensity is through product positioning. In particular, vertical product differentiation may help avoid the dreaded “head to head” battle with rivals and allow each firm to carve out its own demand in the marketplace. Several examples of such a strategy come to mind. In the hybrid-electric vehicle market between 2003-2013, for instance, Toyota’s Prius had several performance advantages over other hybrids, such as the Honda Civic hybrid (e.g., in miles per gallon, battery life, and handling; see Reynolds 2006). And, as assessed by Consumer Reports, a host of categories such as vacuum cleaners, oven ranges, laptops and fitness trackers, exhibit a substantial degree of quality variation between high- and low-end brands (see Table 1 top panel).

However, causal empiricism suggests that there are also industry contexts in which firms’ products are quite similar in quality and yet they are able to achieve positive profits. Cordless drills intended for “tougher jobs” are one such instance where several brands, such as Makita, DeWalt and Milwaukee, offer products with similar specs and overall quality; along with categories such as lawn mowers, snow blowers, washers, and dryers that also seem to exhibit a relatively small degree of quality dispersion across models (see Table 1 bottom panel). Thus, it is likely the case that these firms are withstanding competitive pressures by “differentiating” through other strategic choices, despite having the option of selecting dissimilar qualities.

[Insert Table 1 about here]

One prominent action, aside from product, that firms have at their disposal is advertising, which is intended to help firms communicate their offerings. Yet as with product, here too one observes a range of behaviors. Specifically, some markets are characterized by firms selecting similar levels of advertising, whereas other markets exhibit asymmetric levels across firms. For example, home improvement retailers Home Depot and Lowe’s spent similar amounts on advertising in 2019, while major wireless carriers (AT&T, Verizon, T-Mobile) spent different amounts.¹

¹See Advertising Age, “Leading National Advertisers 2020 Fact Pack,” June, 2020. A survey of CMOs also

Complicating matters, there is sometimes an interaction between product quality and ad levels, with several studies finding that higher quality firms advertise more extensively than lower quality rivals (e.g., Archibald et al. 1983, Tellis and Fornell 1988), while in other cases no such systematic empirical relationship was found (e.g., Kash and Miller 2009, Caves and Greene 1996).

It is not intuitively obvious why such heterogeneity in firm behavior exists and how to reconcile the variety of strategies along the key decision variables of product quality and advertising, as well as the implications for pricing – three of the so-called marketing “4Ps”. The objective of this paper is to shed light on these issues by addressing the following research questions.

- When should we expect firms to differentiate in product qualities and when in advertising levels, assuming both strategies are available to them?
- Would firms ever choose to minimally differentiate in quality as well as in advertising? What pricing approach is needed to enable this outcome?
- If quality entails a fixed upfront cost, or if advertising is chosen from a continuous set or simultaneously with prices, how does that impact firms’ incentives to differentiate?

To address these questions, we develop a duopoly model where consumers are heterogeneous with respect to their valuation for quality. Yet in order for consumers to consider the purchase of a product, they must first be aware of its existence and informed about its characteristics through advertising.² In the main setup analyzed, firms choose product quality in a first stage, select advertising levels in a second stage, and set prices in the last stage.

On a given purchase occasion, consumers evaluate the various products they are informed about and choose the one that delivers maximum utility. Consequently, the return on advertising for a firm will critically depend on how its offering compares to the other alternatives in the marketplace and on how aggressively those products are advertised. In this context, the decisions of what quality product to offer and then how heavily to promote it through advertising become intertwined, and further depend on the resulting pricing the firms will pursue. This structure allows us to examine the research questions laid out above; thereby shedding light on which strategic lever(s) a firm should pull in an effort to effect differentiation from a rival.

points to heterogeneity in spending when one looks at the percent of revenue budgeted to marketing (ranging from 2.4% to 18.9%; The CMO Survey and Deloitte Digital, 2018, www.cmosurvey.org).

²In the U.S. alone, companies spent close to \$240 billion in 2019 to advertise their offerings to consumers (Advertising Age, “Leading National Advertisers FactPack 2020,” June, 2020).

Our analysis focuses on advertising commonly used to inform consumers about products (for empirical support on advertising’s informative role see, e.g., Akerberg 2001, Caves and Greene 1996, Bagwell 2007). In the main model firms choose between two levels of advertising reach, which correspond to the probability a consumer becomes informed about the firm’s product, and each level is associated with a cost. Specifically, a heavy ad spend results in a relatively large fraction of consumers receiving the firm’s message and considering its product, while a light ad spend results in a relatively small fraction of consumers receiving the message. We find that three equilibria can emerge depending on the relative costs and reach of advertising – in particular, on how the extra spend a firm has to incur when shifting from the light to heavy ad level compares to the bump in reach it achieves from this shift. When advertising is not cost-effective (i.e., the heavy ad level entails a large extra cost and results in a small gain in reach), firms are minimally differentiated: they choose the same product quality, the same advertising level, and the same pricing strategy. Price competition is softened because firms’ advertising choices create informational disparities in the market by limiting the number of consumers who will consider both products. Hence the firms earn healthy profits while selecting the same high quality position. We further show that, in a part of this equilibrium region, by choosing to co-locate in quality a firm “prevents” its rival from shifting to the greater ad reach level, even though a monopolist would already find it profitable to do so in order to gain from greater consumer awareness.

When advertising is moderately cost-effective (i.e., the heavy ad level entails a small extra cost and results in an intermediate gain in reach), asymmetric advertising choices with no product differentiation occurs. Specifically, one firm has an incentive to choose a high-quality product and advertise heavily. Interestingly, the rival still finds it optimal to select the same high quality level, but in order to soften price competition it must advertise lightly (otherwise, if both advertise heavily and are undifferentiated in quality then price competition intensifies and profits will be negatively impacted). In doing so, the light advertiser concedes a large segment of consumers to the heavily advertising firm, yet benefits from the higher average equilibrium prices that ensue. Finally, when advertising is very cost-effective (i.e., the heavy ad level entails a small extra cost and results in a large gain in reach), quality differentiation with no advertising differentiation is the equilibrium outcome. In this case, a firm cannot afford to advertise lightly because it will then be relinquishing a very large segment of consumers who will only be informed of its rival’s product. Instead, both firms advertise heavily with one firm choosing maximal quality and a high

price, while the rival differentiates with a lower quality product and a cheaper price.

Thus, our findings suggest that firms tend to utilize at most one of the actions at their disposal to soften competition – (i) advertising differentiation: selecting the same product strategy yet using distinct advertising levels to endogenously segment the market in terms of awareness (i.e., consumer informedness), or (ii) product differentiation: choosing dissimilar product positions, thereby leveraging customer heterogeneity in willingness to pay for quality, while both electing to advertise heavily. In these instances, pricing strategies are also distinct. In particular, in case (i) the heavy advertiser on average selects higher prices than the light advertiser, and in case (ii) the high-quality firm prices higher than its low-quality rival. Minimal differentiation in product quality and advertising is feasible as well, as long as the advertising levels chosen are light; in which case the resulting prices are in similar mixed strategies. We further characterize equilibrium profits and uncover a non-monotonic pattern: a firm’s profits can increase and then decrease as a function of advertising reach when it advertises lightly. In other words, a firm can be better or worse off if the advertising medium it uses is able to impact a greater fraction of consumers.

We also analyze three modeling extensions. First, because in some R&D contexts greater product improvements can entail more substantial outlays, we explore the sensitivity of our findings to incorporating a fixed upfront cost of choosing quality. We find that when firms are expected to be differentiated in their advertising strategies, they may also differentiate in product qualities. This is because a firm advertising lightly facing a rival advertising heavily will not be able to recoup the costs associated with maximal quality if these costs are too high; and hence it would select a lower quality level than its rival. Second, we relax the assumption of discrete ad levels and allow advertising choices to be continuous, with an associated convex cost function. We find that, directionally, the results from this setup correspond to those from the main model. Specifically, equilibria where firms are undifferentiated in discrete qualities can be sustained and, in that case, as advertising becomes more cost effective the firms tend to exhibit greater differentiation in their advertising choices. In addition, when advertising is very cost effective, equilibria where firms differentiate in discrete qualities can be sustained, while advertising choices become similar. Third, we examine an alternative setup whereby advertising and pricing are chosen simultaneously rather than sequentially. For some advertisers, particularly those using digital platforms extensively, the ability to bid for ad placements in near real time suggests that such a model formulation may be more appropriate. We are still able to characterize regions where firms co-locate

in the quality space and then “suppress” intense price competition by selecting light advertising levels. Interestingly, when advertising is moderately cost-effective we find that firms will revert to mixed advertising strategies, while still co-locating in quality.

Collectively, the results uncover when and how firms strategically use product quality and advertising to effectively compete: in some cases differentiating in their advertising approach while electing similar qualities, and in some instances differentiating through quality divergence while electing similar advertising levels. There are also scenarios where both actions are similar across firms. The range of patterns that can arise in equilibrium may thus help explain the variation observed in practice with respect to these decision variables and the seemingly inconsistent empirical research findings on the link between product quality and advertising intensities.

The rest of the paper is organized as follows. The next section relates our work to the relevant literature and summarizes our contribution. This is followed by a description of the main model setup and its analysis. The ensuing section presents three model extensions. The paper concludes by offering managerial and empirical implications, discussing model limitations, and outlining future research opportunities. All proofs are presented in the Web Appendix.

Related Literature

Our work is primarily related to two streams of literature, the first pertaining to quality differentiation and the second to the relationship between advertising and product quality.

Within the stream of literature on vertical product differentiation, the widely known model of Shaked and Sutton (1982) examines price competition between firms that first choose product quality. Since consumers are assumed to be fully informed about all products in their model, in equilibrium firms choose different qualities to reduce price competition. Moorthy (1988) relaxes Shaked and Sutton’s zero production cost assumption by introducing a quadratic cost function for quality. While this can result in an equilibrium where the firm choosing the lower quality is better off, it is still the case that firms always differentiate in qualities. Choi and Shin (1992) establish conditions such that the low quality firm always chooses a quality level that is a fixed proportion of the high quality firm’s choice. Choudhary et al. (2005) allow personalized pricing, which can intensify competition to the detriment of the high quality firm, and Jing (2006) identifies the conditions on the cost structure under which producing the low-quality good can be more profitable. These studies all show that quality differentiation is a robust equilibrium outcome.

However, in reality similar quality products are often observed in the marketplace. Rhee (1996) cites evidence for this and offers an explanation that incorporates consumer heterogeneity along unobservable attributes into the vertical differentiation model. If consumers are sufficiently heterogeneous on these extra dimensions, in equilibrium firms offer products that are identical on the observed quality dimension yet differentiated on the unobserved dimensions. Such work is linked to the broader research on multi-dimensional product positioning (e.g., Ansari et al. 1988, Ecomomides 1989, Vandebosch and Weinberg 1995, Lauga and Ofek 2011). This literature, which assumes that all products and dimensions are known to consumers, shows that when marginal production costs are relatively low the robust outcome is for firms to maximally differentiate on only one dimension and minimally differentiate on the other dimension (the so called, Max-Min equilibrium). In our model, there is only a single product dimension (“quality”) and advertising as a strategic action introduces different considerations than a second product dimension. In particular, the advertising level chosen impacts which products are in consumers’ consideration set. We characterize when this structure results in minimal differentiation in quality coupled with minimal or maximal differentiation in advertising, as well as when it leads to partial differentiation in quality and minimal differentiation in advertising.

Notably, the papers in this stream analyze firms’ quality choices under various price and cost assumptions, but presume that all consumers consider all products. Hence, these works ignore the fact that in many markets consumers are ex-ante uninformed about the various offerings, with advertising serving as an additional lever firms can use to differentiate and soften competition.

The second literature stream related to our work studies the connection between advertising choices and product positions. It is theoretically well-established in economics and marketing that advertising levels can signal product quality (Milgrom and Roberts 1986); though scant empirical evidence exists to conclusively support this idea (Bagwell 2007, and references therein). Such works ignore advertising’s role in informing consumers about which products exist (Nelson 1974, Butters 1977) and thus its effect on market size. Grossman and Shapiro (1984) model advertising as a continuous variable in a Salop (1979) horizontal set up with fixed product locations, whereby receiving ads increases the probability a consumer finds a product that better matches their tastes (i.e., advertising is informative). They find that as the ad technology becomes less efficient, in the sense that it becomes more costly to increase advertising reach, two effects on profits arise: a direct negative effect due to the higher costs associated with advertising and an indirect positive

strategic effect due the reduced level of price competition (since firms are induced to choose lower advertising intensities in equilibrium). In our main model, advertising levels are discrete and firms endogenously select product qualities in the first stage. Consequently, quality choices impact firms' advertising selections – a driving force that is absent in Grossman and Shapiro (1984) and an aspect we later show holds up even if advertising is continuous (but qualities are discrete). At the same time, anticipating the implications of advertising is shown to affect firms' quality decisions. We examine how the cost vs. reach tradeoff associated with choosing the heavy rather than light ad level affects equilibrium outcomes; and also look at the impact on profits.

Iyer et al. (2005) investigate targeted advertising when consumers have horizontal tastes and find that firms advertise more often to consumers with a strong preference for their product, as a way to soften price competition and eliminate wasted advertising. Other papers (e.g., Chen et al. 2001, Zhang and Katona 2012) show that imperfect targetability can reduce competition. While some of these forces will be relevant in our context, much of the literature studying targeting aspects of advertising treats qualities as given. By contrast, we show that endogenizing both product quality and advertising critically affects equilibrium outcomes.³

Our paper also bears on the debate of whether higher quality products should be associated with higher levels of advertising and prices, with the empirical literature finding mixed evidence for such a relationship (Erdem et al. 2008, Kash and Miller 2009, Caves and Greene 1996, Archibald et al. 1983, Tellis and Fornell 1988, Song et al. 2016). Our analysis can help reconcile these seemingly inconsistent observations by delineating conditions for when a positive relationship should hold between these actions and when it should not in equilibrium.

Lastly, we mention research on quantity commitment – as the notion that firms restrict output to soften price competition resembles some intuitions we present with respect to advertising choices. In particular, work by Nasser and Turcic (2016) finds that firms' quantity commitments depend on the degree of exogenous horizontal differentiation. Our approach differs as we focus on vertical preferences with multiple decision variables (quality and advertising). Moreover, advertising in our model is stochastic and creates informational disparities among consumers. The combination of these factors yields divergent implications. For example, we find that firms can

³In Zhu and Dukes (2017), firms decide which product attribute(s) to make prominent (e.g., through advertising) when consumers have limited attention and horizontal tastes. They show that firms may make the same attribute prominent, which can impact quality choices. Our setup is different: we assume only one attribute and a firm's ad level affects the fraction of consumers informed of its product, independently of its rival's advertising (in the Conclusion section we discuss ad spillover effects, which is a characteristic of the model in Zhu and Dukes 2017).

minimally differentiate in qualities and select asymmetric advertising levels, while in Nasser and Turcic (2016) a high degree of product differentiation is needed to sustain asymmetric quantity commitments. We also characterize an outcome in which both firms advertise heavily; an analogous result where neither firm commits to quantity does not exist in Nasser and Turcic (2016).

In summary, our contribution lies in extending the first stream of literature by exploring the strategic role of advertising as another action, in addition to product quality, that firms can take to withstand competitive pressures. Relative to the second stream, we endogenize product qualities and analyze how this decision is impacted by foreseeing the need to advertise to inform consumers and by the ensuing pricing equilibrium. In a sense, we combine the two sources of differentiation appearing in these literature streams into a single model, and characterize when and how firms will use each of the two actions (quality and advertising) to differentiate; thereby uncovering important interactions between these variables.

Main Model Setup

We consider two competing firms that seek to sell a product in a given market. We index the firms by the numbers 1 and 2 or the letters i and j , always assuming that $i \neq j$. If firms offer different quality products, we denote the firm offering the lower quality product by 1 and the firm offering the higher quality product by 2. We assume that every consumer purchases at most one unit. We further assume that consumers are heterogeneous with respect to their valuation of quality, denoted by ϑ . The parameter ϑ is uniformly distributed in the interval $[0, 1]$. A consumer with parameter ϑ gains utility $\vartheta s - p$ from a product with quality s priced at p , and purchases the product for which his/her utility is greater (provided net utility is non-negative). However, consumers can only purchase products they are informed about (Keller and Kotler 2011). Advertising allows a firm to communicate the existence and characteristics of its product, thereby affecting the likelihood that it enters a consumer's consideration set. Our setup is thus consistent with the informative view of advertising (e.g., Tirole 1988, Bagwell 2007) and consumer behavior research (e.g., Mitra and Lynch 1995).⁴

⁴Trivially, an individual cannot consider buying a product that she is not aware of. Yet the consideration set can be narrower than the awareness set as some alternatives may be excluded before carefully comparing them (Hauser and Wernerfelt 1990). Because informative advertising can impact the salience of products in memory, our model applies even if we assume consumers are aware of all products. All we require for our results to go through is that advertising increases the likelihood a product enters the consumer's consideration set. This is also consistent with the repeat-business effect of advertising (Nelson 1974).

Firm Actions: Firms choose qualities, advertising levels, and prices. Qualities, s_i , are non-negative and have an upper bound \bar{s} ($0 < \bar{s}$), i.e., each firm chooses its quality from the set $s_i \in [0, \bar{s}]$. Without loss of generality, we assume $s_1 \leq s_2$. Advertising levels a_i and a_j reflect the probabilities that each firm's ad reaches (or is attended to by) a particular consumer and are independent. For simplicity, we assume two discrete levels of advertising: light (a_L) and heavy (a_H), with costs $0 < c_L < c_H$, respectively. The advertising levels satisfy $0 < a_L < a_H \leq 1$, that is, a firm can either choose to heavily advertise at a higher cost, thereby achieving a relatively broad advertising reach, or to lightly advertise at a lower cost, thereby achieving a relatively narrow reach. We use the notation $\Delta c \equiv c_H - c_L$ for the advertising cost differential, and $\Delta a \equiv a_H - a_L$ for the advertising reach differential between the two ad levels.⁵ Prices are chosen from the set $p_i \in [0, \infty)$. In the Model Extensions section we present results from an alternative setup where advertising is continuous and qualities are discrete, and in the Conclusion section we discuss other possible advertising settings (e.g., spillover effects and persuasive and targeted advertising).

Timing of Moves: The timing of the game is as follows. First, firms choose their qualities. Second, firms make their advertising decisions by selecting whether to advertise lightly or heavily. Third, firms set prices. Finally, consumers make purchase decisions. This timing reflects the notion that the quality position choice tends to be a long-term decision, whereas prices can be easily altered. The time-scope of advertising decisions is somewhere in between. It is further common for advertising allocation decisions to come after product positioning has been determined but before prices are set (this timing assumption is consistent with several prior works, e.g., McAfee 1994, Roy 2000). In practice, while prices can be adjusted on a weekly, daily or even hourly basis, advertising budgets and media planning are typically set on a quarterly or annual basis. Such a sequence is also consistent with papers that assume consumers can costlessly obtain information on price after observing an ad (see, e.g., Mayzlin and Shin 2011). That said, given that there are contexts where advertising intensity can be changed relatively quickly (e.g., on digital platforms), in the Model Extensions section we discuss an alternative timeline where advertising and prices are chosen simultaneously in the second stage.

Costs, profits, and Equilibrium: We assume no fixed entry costs, hence both firms participate in the market. Furthermore, we assume that variable production costs are constant and normalize

⁵To avoid trivial corner solutions we assume Δa is not too small. See the Web Appendix for the formal condition.

them to zero.⁶ The implications of fixed costs associated with quality are examined in the Model Extensions section. Advertising costs c_i are as described above. Firms' profits are, therefore, simply their revenues (price \times quantity sold) minus advertising expenditures: $\Pi_i = p_i D_i - c_i$.

We solve for the sub-game perfect equilibria of the game. In the main model analysis, we focus on pure strategies for quality and advertising decisions, yet allow mixed strategies for prices (reflecting the fact that in several markets firms tend to run occasional price promotions).

Main Model Analysis

Solving the game backwards, we first need to characterize the demand for each firm conditional on product, advertising, and pricing decisions. We note that once firms have advertised their products, consumers may not all have the same consideration set. Some consumers will be informed of both products, and they constitute what we call the *competitive segment*. We also refer to these consumers as *comparison shoppers* since they are able to compare both products (on quality and price) before making their purchase decision. The size of the competitive segment is endogenous and can be as small as a_L^2 when both firms advertise lightly or as large as a_H^2 when both firms advertise heavily. In addition to the competitive segment, a firm might have what we call a *captive segment*, composed of consumers who only consider its product and not the competitor's. Captive segment sizes are also endogenous and are summarized in Table 2. As can be seen from the table, the size of a firm's captive segment depends on its own and its rival's advertising decision. For completeness, we note that there also exists a segment of consumers who are unaware of any offerings (of expected size $(1 - a_i)(1 - a_j)$).

[Insert Table 2 about here]

A firm generally has the option of focusing on its captive segment and not competing for comparison shoppers (except in the extreme case of $a_H = 1$ and facing a rival advertising heavily). When doing so, the firm would act as a monopolist over its captive segment. Thus, as a benchmark, we first solve the monopolist case.

⁶Normalizing production costs to zero is done for several reasons. First, it allows focusing on the strategic incentives to differentiate in qualities when advertising is another option to differentiate. Including production costs would complicate this analysis without providing much added insights, as our results should hold over a certain range. Second, this assumption corresponds to Shaked and Sutton (1982), allowing us to compare our results to theirs. Third, the extant literature has examined the implications of including quality-dependent production costs, with the general finding that as these costs rise firms tend to choose lower qualities (e.g., Jing 2006). Fourth, for a growing number of goods (e.g., digital products) variable costs are insignificant regardless of quality.

A consumer who receives an ad for the monopolist's product buys it if and only if $\vartheta s_m - p_m \geq 0$. That is, the monopolist's demand consists of consumers who are informed about the product and value quality sufficiently, satisfying the following condition: $\vartheta \geq p_m/s_m$. Hence, the monopolist's demand is: $D_m = (1 - p_m/s_m)a_m$, where $a_m \in \{a_L, a_H\}$. Given a_m and s_m , the monopolist chooses the price $p_m^* = s_m/2$ to maximize its revenue: $p_m(1 - p_m/s_m)a_m$. Its profit becomes: $\Pi_m = s_m a_m/4 - c_m$, which is increasing in s_m regardless of the advertising level. The monopolist thus always sets $s_m^* = \bar{s}$ in the first stage, and chooses $a_m = a_H$ in the second stage if and only if $(\bar{s}a_L/4 - c_L) < (\bar{s}a_H/4 - c_H)$ or $\Delta c < \frac{\bar{s}\Delta a}{4} \equiv \Delta c_m$. Specifically, the monopolist advertises heavily rather than lightly if the extra revenue that results from shifting to the wider advertising reach, which is a function of Δa , is greater than the added cost Δc . In analyzing the duopoly case, therefore, if a firm advertises lightly when the extra cost of switching to the heavy ad level is less than Δc_m , then we are assured that this is a result of the strategic interaction between the firms.

Let us now turn to the duopoly case. We seek to understand under what conditions a firm will elect to use product quality vs. advertising level to soften competition with its rival; also allowing for the possibility of minimal differentiation in these decision variables. We start by analyzing the profits in the pricing subgame conditional on the advertising levels chosen, assuming that firms have elected to co-locate on quality in the first stage. Subsequently, we analyze the endogenous choices of advertising and quality. We introduce the following notation: let (L, L) , (L, H) , (H, L) and (H, H) denote the possible advertising strategies, whereby the first argument is the advertising level chosen by Firm 1 and the second is the level chosen by Firm 2.

Lemma 1 *Under no product differentiation ($s_1 = s_2 = s$), the profits of Firm i as a function of its advertising level (a_i) and its competitor's advertising level (a_j) are:*

- *If Firm i advertises heavily while Firm j advertises lightly ($a_i = H, a_j = L$),*

$$\pi_i = a_H(1 - a_L)\frac{s}{4} - c_H.$$
- *If Firm i advertises lightly ($a_i = L, a_j = L$ or H), $\pi_i = a_L(1 - a_L)\frac{s}{4} - c_L.$*
- *If both firms advertise heavily ($a_i = H, a_j = H$), $\pi_i = a_H(1 - a_H)\frac{s}{4} - c_H.$*

To understand the intuition for the various profit levels in Lemma 1, first note that the pricing equilibrium when firms co-locate in qualities has to be in mixed strategies. If Firm i were to choose

a given price with probability one, two scenarios could unfold: (i) when Firm i 's price is high, its rival is prompted to undercut it slightly to sell to all the comparison shoppers; but then Firm i would want to deviate and undercut its rival's price, (ii) when Firm i 's price is low, its rival would concede the comparison shoppers and focus on its own captive segment. But then Firm i would have an incentive to deviate and sell at a higher price (just below Firm j 's price). In either case, Firm i cannot choose a single price in equilibrium and hence plays a mixed pricing strategy. For the same reasons, Firm j 's best response is to also play a mixed pricing strategy.

Now consider the scenario where Firm i advertises heavily but its rival advertises lightly (H, L). The size of Firm i 's captive segment is, per Table 2, $a_H(1 - a_L)$ and is relatively large, such that it is profitable for Firm i to sometimes focus exclusively on that segment to extract monopolist rents, i.e., its support has a mass point at price $\frac{s}{2}$. Indeed, as shown in the proof, Firm i sets this price with probability $(1 - \frac{a_L}{a_H})$. As Firm i has to be indifferent between all the prices it mixes over, profits are equal to the monopoly rents over its captive segment, i.e., $\pi_i = a_H(1 - a_L)\frac{s}{4} - c_H$.

Next, consider the scenario where Firm i advertises lightly ($a_i = L$). In this case, Firm i 's captive segment is not large enough to sustain a mass point at the monopoly price, i.e., the cumulative distribution function (CDF) is continuous. When the rival also chooses a light ad level ($a_j = L$), both firms have a continuous CDF. Given that each firm has to be indifferent among all the prices played, including at the monopoly price where demand is equal to the captive segment of size $a_L(1 - a_L)$, we get that profits will be $\pi_i = a_L(1 - a_L)\frac{s}{4} - c_L$. When the rival chooses a heavy ad level ($a_j = H$) instead, consistent with the explanation above, it plays the monopoly price of $\frac{s}{2}$ with a positive probability of $(1 - \frac{a_L}{a_H})$. By undercutting this price infinitesimally, Firm i will serve its own captive segment of size $a_L(1 - a_H)$, as well as capture the entire competitive segment of size $a_L a_H$ with the probability $(1 - \frac{a_L}{a_H})$. Hence, given the requirement of being indifferent among all prices played, Firm i 's profits will be $\pi_i = a_L(1 - a_H)\frac{s}{4} + a_L a_H \frac{s}{4}(1 - \frac{a_L}{a_H}) - c_L = a_L(1 - a_L)\frac{s}{4} - c_L$.⁷

The last scenario to discuss is when both firms advertise heavily (H, H). In this case, a substantial proportion of consumers are informed of both products. Hence, the captive segments are relatively small in size and more intense price competition is triggered. Consequently, firms select lower prices and do so more often than in the other two cases. As before, and as long as

⁷The details of all the distribution functions from which mixed-strategy equilibrium prices are drawn are provided in the Web Appendix as part of the proof of Lemma 1. We further show there that the property whereby the profits of a firm advertising lightly are independent of its rival's advertising level can be generalized to the case of asymmetric advertising competencies (i.e., firms have different heavy advertising reach).

$a_H < 1$, the CDFs are continuous and each firm has to be indifferent between all the prices it mixes over, including the monopoly price and serving only the captive segment of size $a_H(1 - a_H)$, which leads to profits of $a_H(1 - a_H)\frac{s}{4} - c_H$. Notably, if $a_H = 1$, i.e., a heavy ad spend informs the entire market, then prices are driven to zero as all consumers are aware of both products that are assumed to be non-differentiated; profits are negative because the firms still incur the cost of advertising heavily.

One conclusion worth highlighting from Lemma 1 is that when a firm advertises lightly it earns the same profit of $a_L(1 - a_L)\frac{s}{4} - c_L$, regardless of the advertising choice of its rival. This property implies that Firm 1 can always consider a deviation to co-locate its product with the higher-quality Firm 2 (resulting in no quality differentiation) and choose to advertise lightly. Since a profitable deviation should not exist in equilibrium, Firm 1's profit has to be at least as high as the profit from this potential deviation. In other words, $\pi_1^* \geq a_L(1 - a_L)\frac{s}{4} - c_L$.

Another conclusion arising from Lemma 1 is that firms with undifferentiated qualities can still sustain positive revenues. Notably, by selecting a light as opposed to a heavy advertising level, a firm strategically lets its rival have a larger captive segment, which softens the ensuing price competition because fewer consumers will be informed of both products.

With these insights in hand, we can discuss the implications of Lemma 1 for the best response advertising choices when product qualities are undifferentiated. If the rival advertises heavily, by matching this ad level Firm i creates a large competitive segment that, as explained above, triggers intense price competition and negatively impacts profits; yet both firms still incur the cost of advertising heavily. Thus, Firm i can be better off advertising lightly if it expects its rival to advertise heavily. However, if its rival advertises lightly, Firm i needs to compare the profits from advertising heavily and broadening the set of consumers aware of its product vs. advertising lightly and limiting the overlapping number of consumers aware of both products (i.e., shrinking the size of the competitive segment). Lemma 1 reveals that when $a_L(1 - a_L)\frac{s}{4} - c_L \leq a_H(1 - a_L)\frac{s}{4} - c_H$, or $\Delta c \leq \Delta a(1 - a_L)\frac{s}{4}$, advertising heavily is more profitable. In other words, when the ad-reach differential Δa and the quality level s are high enough compared to the cost differential Δc , Firm i tends to prefer advertising heavily when its rival advertises lightly. Another way to frame this condition is that when advertising is *cost-effective*, defined as when the shift from the light to the heavy ad level entails a sufficiently small extra cost relative to the change in advertising reach, the firm will have a strong incentive to opt for H when its rival selects L .

We now turn to characterizing the equilibrium of the entire game by solving for the subgame perfect ad levels chosen in the second stage and the quality positions chosen in the first stage. Formal details of the cutoff values in each region of the proposition are given in the Web Appendix.

Proposition 1 *The following differentiation regions arise in equilibrium:*

1. *When advertising is not cost-effective (Δc is high relative to Δa), both firms choose maximal quality coupled with advertising lightly: minimal product and advertising differentiation.*
2. *When advertising is moderately cost-effective (Δc is low enough and Δa is intermediate), both firms choose maximal quality with one firm advertising heavily while the other advertising lightly: only advertising differentiation.*
3. *When advertising is very cost-effective (Δc is very low and Δa is sufficiently high), firms choose distinct quality levels with both advertising heavily: only product differentiation.*

The proposition uncovers an interesting general finding: firms never elect to concurrently differentiate both in product qualities and in advertising levels. It is optimal to either only differentiate in advertising levels or in product qualities, or not to differentiate in these decision variables. Graphically, Figure 1 illustrates the type of differentiation (in product, advertising, or none), along with pricing strategies and the consumer information structure, as a function of how cost-effective it is to shift from the light to heavy advertising level.

[Insert Figure 1 about here]

We now discuss the intuition behind the results in Proposition 1, starting from the not cost-effective case, i.e., part 1 of the proposition (the region marked ‘None’ in Figure 1). When selecting maximal product quality in this region, firms anticipate that a light ad spend by both will lead to informational differentiation among consumers via the emergence of sizable captive segments. Light advertising levels can thus act to mitigate price competition by, in essence, creating segments that possess distinct information sets. The mixed pricing strategies played result in each firm primarily catering to its endogenous captive segment and earning monopoly rents from these consumers. Alternatively, conditional on Firm 2 selecting the highest quality level and advertising lightly, Firm 1 could elect to differentiate in quality. This strategy is less

profitable because it entails selling a product of lower quality at a lower price and not realizing the full amount of profits from the captive segment. Conversely, Firm 2 might like to advertise heavily. But by selecting minimal product differentiation in the first stage, and given that advertising heavily is relatively costly in this region, Firm 1 prevents this from occurring by making it an unprofitable move; inducing Firm 2 to advertise lightly. Thus, throughout this region where advertising is not cost-effective, the firms' actions are minimally differentiated – they select the exact same quality level, the same advertising level, and play the same mixed pricing strategy.

Recall that when the advertising cost differential is greater than $\Delta c_m = \frac{\bar{s}\Delta a}{4}$ a monopolist would advertise lightly, and when it is lower than Δc_m it would advertise heavily. Naturally, if a monopolist does not want to invest in advertising heavily, neither would firms in a duopoly. Notably, Proposition 1 contains a sub-region where both firms select a light ad spend even where a monopolist would already shift to advertising heavily (the region to the right of the dashed line and to the left of the first bold line in Figure 1). This happens because the minimal product differentiation strategy embraced in the first stage lowers the benefits of advertising heavily, and discourages each firm from switching to this level even though a monopolist would do so.

As advertising becomes more cost-effective (Proposition 1 part 2, the region marked 'Advertising' in Figure 1), one firm has an incentive to advertise heavily; thereby generating broad awareness for its offering. If Firm 1 expects its rival to select the top quality ($s_2 = \bar{s}$) followed by heavy advertising, what are its optimal product and advertising strategies? Firm 1 can either co-locate on quality and advertise lightly (L, H) (it would never advertise heavily when co-locating in this region, as doing so would lead to intense price competition and lower profits, i.e., $\pi_1(H, H) < \pi_1(L, H)$; as discussed in connection with Lemma 1), or it can differentiate its offering by choosing lower quality followed by a heavy ad spend (H, H). Firm 1's optimal strategy depends on the implications of its decisions for the pricing stage. If Firm 1 co-locates in quality ($s_1 = s_2 = \bar{s}$) and advertises lightly, its rival Firm 2 that advertises heavily will have a large captive segment and can earn monopoly profits on this segment. Since Firm 2 will indeed charge as much as its monopoly price quite often, and since prices are strategic complements, Firm 1's average mixed-strategy price will be higher than what it would be under product differentiation and advertising heavily. Thus, when advertising is moderately cost-effective, quality co-location coupled with a light ad spend generates higher profits than product differentiation followed by a heavy ad spend. Said differently, by opting for no quality differentiation with its rival in the

first stage, Firm 1 in effect commits to advertising lightly, since this level of market awareness for its product generates more profits. For Firm 2, conditional on Firm 1 co-locating and expected to choose the light ad level, the best response is to choose a heavy ad level, given that the size of the resulting captive segment will allow it to price higher on average. It is worth noting that this asymmetric equilibrium in advertising strategies holds despite the firms being identical at the outset: they go on to choose the same quality path but a different advertising path.

The last case to analyze is when advertising is very cost-effective (Proposition 1 part 3; the region marked ‘Product’ in Figure 1). If Firm 2 advertises heavily, a very substantial portion of the market is informed of its product (as a_H is very large in this region). Firm 1 again compares two strategies: product co-location with its rival followed by light advertising (L, H) vs. quality differentiation ($s_1 < s_2 = \bar{s}$) followed by a heavy advertising level (H, H). Given Firm 2’s very broad reach, choosing the light advertising level results in a small captive segment for Firm 1 and in relatively modest profits, despite having high quality. By contrast, if Firm 1 chooses a lower quality and also advertises heavily (H, H), the market becomes more “competitive” and the majority of consumers are informed about both products. Hence, it would have to price cheaply enough to generate positive profits. Given the demand disadvantage when advertising lightly in this case, and if the extra cost of shifting to advertising heavily is sufficiently small (Δc very low), Firm 1 can earn greater profits by following the latter approach: differentiating in product quality and matching the heavy advertising level of its rival.

We further highlight that the equilibrium product quality s_1^* is a decreasing function of a_H (see the Web Appendix). This makes sense: the greater the overlap in reach between the two rivals the larger the size of the competitive segment, which prompts Firm 1 to want to further separate itself at the product stage from Firm 2 (that chooses $s_2^* = \bar{s}$). In the special case of $a_H = 1$, i.e., advertising heavily covers the entire market, we have $s_1^* = \frac{4}{7}\bar{s}$. This corresponds to the solution in Shaked and Sutton (1982), but with full awareness in our model an endogenous outcome of the game (and note that for any $a_H < 1$ the degree of product differentiation is smaller, i.e., $s_1^* > \frac{4}{7}\bar{s}$).

Clearly, firms’ profits are negatively impacted by greater advertising costs in the respective equilibrium regions. We now discuss how profits change as advertising achieves greater reach. The next corollary shows that the relationship is not always monotonic, as one might have conjectured.

Corollary 1 *Holding constant advertising costs c_L and c_H ,*

- *When a firm advertises lightly, its profits vary non-monotonically, first increasing and then decreasing, as its advertising reach expands (a_L increases).*
- *When a firm advertises heavily, its profits will either increase or decrease as its advertising reach expands (a_H increases), depending on its rival's product and advertising choices.*

The parameter a_L represents the reach of communication vehicles when a firm opts for the light advertising level. One might have expected that as these vehicles improve, a firm should earn greater profits. The corollary indicates that this is not always true. The intuition is as follows. When advertising is not cost-effective, from Proposition 1 we know that firms do not differentiate their actions and play (L, L) . Firms' profits mirror a monopolist's rents from the captive segment ($\pi^* = a_L(1 - a_L)\frac{\bar{s}}{4} - c_L$). As a_L initially increases, each firm's captive segment grows and is greater than the size of the competitive segment; hence both benefit from higher advertising reach. However, as a_L further increases (beyond $\frac{1}{2}$) this pattern reverses - the competitive segment becomes larger than the captive segments; hence both firms see their profits decline.

When advertising is moderately cost-effective, firms play (L, H) . The heavy advertiser sees its profits decrease in a_L yet increase in a_H . This is because its captive segment shrinks in the former and expands in the latter ad level. The light advertiser, however, may again see an inverted-U pattern in a_L : its captive segment increases in a_L , which positively affects profits and is initially the dominating effect, but as a_L increases beyond a certain level the heavy advertiser lowers its average price to compete, which negatively impacts the light advertiser's profits.

Lastly, when advertising is very cost effective both firms advertise heavily (H, H) . In this case, there is an interesting interaction between advertising reach (a_H) and product differentiation. Specifically, as a_H increases competition intensifies and Firm 1 is induced to select a lower quality (as $\frac{\partial s_1^*}{\partial a_H} < 0$) and set a lower price. Hence, its profits decline in advertising reach a_H . Conversely, the rival choosing maximal quality benefits from the greater product differentiation and sees its profits rise. Corollary 1 thus suggests that greater advertising reach can be a double-edged sword, having the potential to positively or negatively impact profits. Several of these effects of advertising can be seen as consistent with those reported in Grossman and Shapiro (1984).

To conclude the analysis in this section, we note that two ex-ante identical firms choose to differentiate in either advertising or product quality or not at all. When firms are minimally differentiated in qualities they avoid intense price competition by choosing a reduced level of

consumer awareness (with at least one firm electing to advertise lightly). This leads to the endogenous creation of segments whereby not all consumers are informed about both products and some consumers are captive. This, in turn, softens price competition and allows the firms to derive positive profits. When firms differentiate, they either choose different advertising levels or different qualities coupled with different pricing. The findings reveal that by endogenizing all three decisions (quality, advertising, and price) we can characterize a much richer set of outcomes.

The specific equilibrium predicted was shown to depend in large part on the degree of advertising cost-effectiveness – how costly it would be to shift from a light to a heavy advertising level relative to the gain in reach. In practice, several measures show considerable variance across markets in terms of how cost-effective advertising is at informing consumers. For instance, advertising reach per ad placement on some digital platforms tends to differ by category, e.g., social media ads for apparel are noticed and clicked over twice more often than ads for financial services and insurance (see Irvine 2020). Similarly, traditional media exhibit variance in the costs of reaching a given number of prospects (Thomas et al. 2000). Such disparities might help explain why some categories exhibit minimal differentiation across firms in the quality of products offered, while others exhibit considerable differentiation in advertising levels or product qualities. Furthermore, the results may help explain why some studies fail to find a significant positive correlation between quality and advertising (Kash and Miller 2009, Caves and Greene 1996). Specifically, if advertising plays an informative role as modeled here: when one firm advertises heavily while its rival advertises lightly both may offer similar quality products (a scenario common when national brands compete with virtually identical private labels, Kane 2014); and if they choose different qualities they are both expected to choose a heavy advertising level. Such behaviors, when aggregated, would in fact yield minimal observed correlation between these decision variables.

Model Extensions

We now present findings from three model extensions: a) incorporating upfront fixed costs of selecting quality, b) allowing continuous advertising choices, and c) letting advertising and pricing decisions be made simultaneously. The analysis suggests that most of the results from the main model are robust (at least directionally) to these alternative setups, yet several new or nuanced findings do emerge. Proofs for these extensions are provided in the Web Appendix.

Incurring Upfront Quality Costs

To concentrate on firms' strategic incentives to choose quality when having to take into account the subsequent need to engage in advertising, we assumed in the main model that there were no costs associated with the quality decision. This further allowed comparison to prior literature. Notwithstanding, in practice, one might expect that higher quality products are increasingly more difficult to develop and thus entail a greater upfront fixed cost. To understand how incorporating this possibility would affect firms' equilibrium strategies we extended our model, focusing the analysis on scenarios where including such a cost could most impact the findings.

Assume that offering a product entails an upfront fixed cost that is increasing and convex in quality. Specifically, let this cost take the form $\frac{1}{2}ks_i^2$, where k reflects the sensitivity of the cost to the quality level selected. All other aspects of this extension are as in the main model setup. We wish to examine whether firms will still co-locate on a high quality level in the presence of this cost. In particular, one can imagine that if the advertising subgame is asymmetric, the firm choosing the light ad level may find it difficult to justify a hefty upfront cost of selecting maximal quality ($s = \bar{s}$). This is because its profits are limited by the fact that relatively few consumers are informed about its product vis-à-vis the rival's. To explore this situation, we concentrate on the case whereby advertising is moderately cost effective and, to fix ideas, set $a_L = 0.5$ and $a_H = 1$. With these assumptions, our analysis reveals the following outcome.

Proposition 2 *When advertising is moderately cost effective (Δc is low enough and $\Delta a = 0.5$), there exist \underline{k} and \bar{k} such that the following differentiation strategies arise in equilibrium:*

- *For any $0 \leq k < \underline{k}$, both firms choose maximal quality with one firm advertising heavily while the other advertising lightly: only advertising differentiation.*
- *For any $\underline{k} < k < \bar{k}$, firms choose distinct quality levels with one firm advertising heavily while the other advertising lightly: product and advertising differentiation.*

Thus, Proposition 2 reveals that when upfront quality costs are low ($0 < k < \underline{k}$) our findings from the main model fully carry over: if advertising heavily is not too costly and yields an intermediate bump in reach, a strategy of matching the rival's maximal quality is sustained. This holds because, despite the upfront cost of quality incurred, the subsequent differentiation in advertising strategies provides sufficient returns. However, when this upfront cost rises beyond

a certain threshold, one firm must lower its quality level as the subsequent profits under (L, H) would not cover the expenses associated with maximal quality for the firm advertising lightly. Consequently, here we observe differentiation in both product and advertising. The Web Appendix provides further discussion of this extension (e.g., what happens when $\bar{k} < k$).

Continuous Advertising and Discrete Quality Levels

In the main setup, quality and price were modeled as continuous decision variables (consistent with prior literature), whereas advertising levels were discrete. This allowed keeping the analysis tractable, while still capturing relevant intuitions. The fact that firms often decide on the scale of their ad campaigns, e.g., an aggressive vs. a small-scale effort, conforms to our “heavy” vs. “light” designations. Yet in reality, the exact budget allocated to advertising can be more granular.

To understand whether our findings would hold up qualitatively if advertising were a continuous variable, we analyzed the following setup.⁸ Let firms choose advertising from a continuous set $a_i \in [0, 1]$, where again a_i is the probability a consumer receives the firm’s ad. Further assume that the cost of advertising is $A(a_i) = \frac{r}{2}a_i^2$, which is strictly convex ($r > 0$). Hence, the parameter r can be thought of as capturing the cost effectiveness of advertising, such that a smaller r implies a more cost-effective advertising technology. Firms choose qualities from a discrete set $s_i \in \{s_L, s_H\}$ at costs K_i , with $0 < s_L < s_H$ and $0 < K_L < K_H$. All other aspects of the setup are as in the main model. Our primary objective is to show that there can exist equilibria in which firms either co-locate or differentiate in quality, and that the second-stage advertising choices correspond to those from the main model as a function of the cost-effectiveness of advertising.

Proposition 3 *There exist \underline{r} and \bar{r} such that:*

- *For $r > \bar{r}$, it is possible to sustain an equilibrium where firms select the same maximal quality levels ($s_1 = s_2 = s_H$), i.e., no product differentiation. In any such equilibrium, the advertising choices satisfy $\frac{\partial(a_2 - a_1)}{\partial r} = \frac{\partial \Delta a}{\partial r} < 0$, i.e., as advertising becomes less (more) cost effective firms will be less (more) differentiated in advertising.*
- *For $r < \underline{r}$, it is possible to sustain an equilibrium where firms select distinct quality levels ($s_1 = s_L, s_2 = s_H$), i.e., product differentiation. In any such equilibrium, the advertising choices satisfy $\frac{\partial(a_2 - a_1)}{\partial r} = \frac{\partial \Delta a}{\partial r} \geq 0$, i.e., as advertising becomes more (less) cost effective firms will be weakly less (more) differentiated in advertising.*

⁸We thank the Associate Editor for proposing this alternative modeling set up and analysis approach.

It is easy to see that these findings are consistent (at least directionally) with those from the main model. In particular, Proposition 1 revealed that when advertising was not cost effective firms co-located in qualities and selected the same ad levels, yet when advertising became moderately cost effective firms differentiated in advertising. In the alternative setup, this corresponds to the region where r is large and indeed we can characterize equilibria with firms choosing the same quality levels. Importantly, the advertising stage subgame in any such quality co-location equilibrium exhibits the following property: if advertising becomes more cost effective (r decreases, though still satisfies $r > \bar{r}$), firms' advertising choices will be more differentiated (since $\frac{\partial \Delta a}{\partial r} < 0$).

Proposition 1 also revealed that when advertising is very cost effective, firms opt for distinct quality levels with no advertising differentiation. Here too, when r is small enough we can sustain equilibria where firms select different quality levels. And as advertising becomes more cost effective (r decreases) the firms will tend to be less differentiated in advertising (as now $\frac{\partial \Delta a}{\partial r} \geq 0$).⁹

Simultaneous Advertising and Pricing Decisions

In the main model, we assumed that advertising and pricing decisions were made sequentially. As such, a firm observed the ad level of the rival prior to determining its pricing strategy. While this timing makes sense when media buys need to be executed well in advance or ad budgets are set only periodically, there can be contexts or media (e.g., digital platforms) where advertising decisions are made more flexibly. Thus, we seek to understand how an alternative timeline, in which advertising and pricing decisions are made simultaneously, affects the findings. For ease of exposition, we solve for the case of $a_H = 1$, which does not qualitatively affect the results. All other aspects of the setup are as in the main model. We start by analyzing the advertising-pricing subgame, assuming that firms have elected to co-locate in quality in the first stage.

Lemma 2 *Under no product differentiation ($s_1 = s_2 = s > 0$), the equilibrium of the advertising-pricing subgame is as follows:*

- *When advertising is not cost effective (Δc is high relative to Δa), both firms advertise lightly ($a_i = L, a_j = L$) and $\pi_i = a_L(1 - a_L)\frac{s}{4} - c_L$.*
- *When advertising is sufficiently cost effective (Δc is low relative to Δa), both firms mix between light and heavy advertising levels and $\pi_i = \frac{a_L c_H - c_L}{1 - a_L}$.*

⁹Specifically, the inequality will initially be strict (i.e., advertising differentiation will decrease as r decreases, or $\frac{\partial \Delta a}{\partial r} > 0$). Yet as $r \rightarrow 0$, at some point both firms will be induced to select $a_i = 1$, and we will have $\frac{\partial \Delta a}{\partial r} = 0$. See the Web Appendix for further details, including relevant values of K_i to sustain quality differentiation.

Lemma 2 reveals that, if firms co-locate in quality, the advertising-pricing subgame entails one of two outcomes: when advertising is not cost-effective both firms advertise lightly, yet when advertising is sufficiently cost-effective both mix between the heavy and light levels. The intuition for the former scenario is similar to that in the main model: shifting to the heavy ad level is costly and since the products are co-located the ability to reap rewards from this shift are limited. The latter scenario reveals a somewhat different eventuality compared to the main model, stemming from the simultaneity of advertising and pricing decisions. Specifically, if advertising is cost-effective, neither firm can select the heavy level deterministically. To understand why, assume trying to sustain an equilibrium with one firm choosing the light ad level and the rival selecting the heavy ad level.¹⁰ If the light advertiser deviates to the heavy ad level, it can improve its profits since the deviation is not observable and it benefits from the greater awareness and from undercutting its rival's price. We now turn to characterizing the equilibrium of the entire game.

Proposition 4 *The following differentiation regions arise in equilibrium:*

1. *When advertising is not cost-effective (Δc is high relative to Δa), both firms choose maximal quality coupled with advertising lightly: minimal product and advertising differentiation.*
2. *When advertising is moderately cost-effective (Δc is low enough and Δa is intermediate), both firms choose maximal quality followed by mixing between advertising lightly and heavily: no product differentiation, advertising differentiation occurs with a positive probability.*
3. *When advertising is very cost-effective (Δc is very low and Δa is sufficiently high), firms choose distinct quality levels and both advertise heavily: only product differentiation.*

Proposition 4 reveals that equilibrium outcomes in the simultaneous setting bear close similarity to those in the main model (per Proposition 1). In particular, when advertising is not cost-effective firms elect minimal differentiation in their actions. As advertising becomes more cost-effective, firms find it beneficial to continue selecting maximal quality. In doing so, each firm prevents its rival from selecting the heavy ad level with certainty. The equilibrium that emerges is in mixed advertising strategies, per Lemma 2, implying that firms play (L, H) or (H, L) with a positive probability. Thus, while in the intermediate region for Δc vs. Δa we can't fully sustain

¹⁰Clearly, under no product differentiation, a subgame equilibrium with both firms selecting the heavy ad level deterministically cannot be sustained.

advertising differentiation - such differentiation has some positive likelihood of occurring. Lastly, when advertising is very cost effective neither firm can afford not to generate broad advertising reach. Foreseeing this, firms realize they must differentiate in qualities in the first stage.

Conclusion

Summary and Managerial Implications

In this paper, our goal has been to characterize the type of differentiation strategies firms are expected to pursue to best withstand competitive pressures, with a particular emphasis on the interaction between endogenous quality and advertising choices.

In general, we find that the equilibrium strategies depend on how cost-effective advertising is, i.e., how expensive it is to shift from a light to a heavy advertising level relative to the increase in customer reach arising from such a shift. When advertising is not cost-effective, firms minimally differentiate on all fronts (quality, advertising and pricing). Even though product qualities are the same, price competition is not intense due to the endogenous presence of captive segments (in a sense, the market is “differentiated” in the informedness of consumers). When advertising is moderately cost-effective, firms still minimally differentiate in product qualities but switch to advertising differentiation: one firm elects to advertise heavily while the other lightly. The light advertiser is at a disadvantage given its smaller captive segment and hence mainly competes for comparison shoppers (who are aware of both products) by occasionally undercutting its rival’s price. This undercutting does not, however, need to be aggressive because the heavy advertiser has a relatively large captive segment and tries to extract monopoly rents from these customers much of the time; thus keeping overall prices higher on average. Committing in the first stage to being undifferentiated in qualities, coupled with one firm strategically choosing a light ad level, is what enables this equilibrium outcome. Finally, there is a scenario where both firms choose the heavy advertising spend and, in turn, the only way to avoid intense price competition is to select dissimilar qualities. This product differentiation result, which corresponds to the classic finding in the literature, holds only when advertising is very cost-effective. Otherwise, firms prefer to create sizable captive customer segments by at least one of them advertising lightly. Minimal product differentiation serves a strategic purpose in achieving this outcome.

These findings have several important managerial and empirical implications as they bear on three of the so-called “4Ps” of marketing— product, promotion and pricing— in a single framework.

Foremost, they suggest that the conventional wisdom whereby a firm should seek a high degree of product differentiation from its rival to avoid intense price competition is qualified, and depends on whether there is a role for informative advertising that generates awareness of the offerings. Furthermore, the type of differentiation strategy depends on how cost-effective advertising is. Many managerial treatments of advertising classify media based on their impact on consumers and their cost (see, e.g., Thomas et al. 2000, Gotter 2018). Our results thus tell managers how the characteristics of communication vehicles pertaining to their market should affect the quality of products to offer and the level of advertising to select vis-à-vis rivals. The findings also bear on the pricing strategies that should be implemented. When product differentiation is minimal, firms should engage in “flexible” pricing, i.e., a mixed strategy where they price promote often; and if they advertise at different levels, the heavy advertiser should stick to the higher list price more frequently than its rival. Under product differentiation, however, firms should set distinct yet stable prices. From an empirical standpoint, a testable implication of our theory is that as the cost-effectiveness of advertising rises we should observe less variance or fluctuations in pricing.

Our analysis also suggests that when setting innovation strategy, which is often aimed at improving quality (e.g., performance on key attributes), it is crucial for firms to consider how the new products will be advertised. In their seminal work surveying managers on appropriating returns from R&D, Levin et al. (1987) found that marketing and sales were cited as the primary vehicle. This suggests that managers need to understand the strategic implications of communication actions on the products they elect to develop, compared to rivals, and that differentiation is achievable through various means. For instance, learning that a rival is developing an equally advanced product may not spell disaster, if the firms can refrain from both advertising heavily.

Limitations and Future Research

Although our study encompasses several important aspects of the quality-advertising-pricing set of decisions in a competitive context, and we have justified many of our assumptions as well as examined three alternative modeling setups (upfront quality costs, continuous advertising choices, and simultaneous advertising-pricing decisions), we acknowledge several limitations.

First, advertising in our model is informative. Yet one can imagine situations where advertising plays a persuasive role by influencing product quality perceptions. In an extension presented in the Web Appendix, we show how our modeling framework can incorporate this form of ad-

vertising. Our main finding is that firms may differentiate less in objective qualities, yet the divergence between their persuasive advertising levels results in the same degree of perceived quality differentiation as when consumers fully know objective qualities.

Second, advertising in our model was indiscriminate or “blanket”: each firm communicated to the entire market with the likelihood of its ad reaching any consumer affected by the level chosen. However, firms are sometimes able to target their ads to specific segments. In the Web Appendix, we solve an extension that explores this possibility. Our findings suggest that minimal differentiation in quality can work in this case, provided firms are able to “stay out of each other’s turf” by differentiating their advertising strategies, with each firm focusing on a distinct segment. We further show that a firm need not lower its price so as to serve all the consumers who have been targeted with ads; it may wish to ignore consumers with moderate willingness to pay.

Third, we assumed that advertising reach levels were the same across firms for given costs. In the Web Appendix, we show why relaxing this assumption does not alter our main conclusions.

Fourth, we assumed that consumers become informed about products solely through advertising. In reality, consumers have other ways to learn about offerings, e.g., through word of mouth or search, and may decide to do so after seeing an ad (as in Mayzlin and Shin 2011). Assuming that consumers have some positive probability of being informed about the firm’s product even without receiving an ad for it would not change our model findings. In this case, we could interpret advertising in our model as increasing this base probability.¹¹ Our results would also hold if consumer search costs are sufficiently high. However, if these costs are moderate, uninformed consumers may have an incentive to search, which could impact our findings. We leave for future research the investigation of how consumer search affects endogenous quality positions and advertising levels. The model could also be extended to incorporate spillover effects, that is, when consumers receive an ad from one firm there is some likelihood that they will become aware of the other firm’s product. This could capture a “category expansion” effect for advertising beyond the “brand specific” effect modeled in this paper. In that case, the competitive segment would be larger than in our analysis, which could create a disincentive to advertise. The existence conditions for the different equilibria would likely change, but our key results should remain valid.

Fifth, although we extended the model to incorporate fixed quality costs, we employed stylized

¹¹We could easily re-formulate our model so that consumers have a base probability a_L of being informed through non-advertising mechanisms. If a firm decides to advertise (vs. not advertise), this probability increases to a_H .

assumptions on marginal production costs. This allowed focusing on the strategic forces that drive quality and advertising differentiation. The assumption is also reasonable in categories where variable costs are either negligible, e.g., digital goods or pharmaceutical drugs, or only weakly related to quality positions (e.g., some beverages and clothing). Future research could examine the implications of including variable costs that are a function of quality.

Lastly, we modeled heterogeneity in willingness to pay for quality in order to study how vertical differentiation is impacted by the need to inform consumers about products. In some settings, consumers may exhibit heterogeneity in horizontal tastes. It is possible that firms' choice of product positioning along a horizontal continuum is impacted by the need to engage in informative advertising. It would be interesting to see if an equilibrium can be sustained where firms co-locate in the horizontal product space as in the vertical setup. The interaction between endogenous product location and advertising in a horizontal setting can be fruitful ground for future research.

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Table 1: Quality and Price Dispersion for Select Categories (per Consumer Reports)

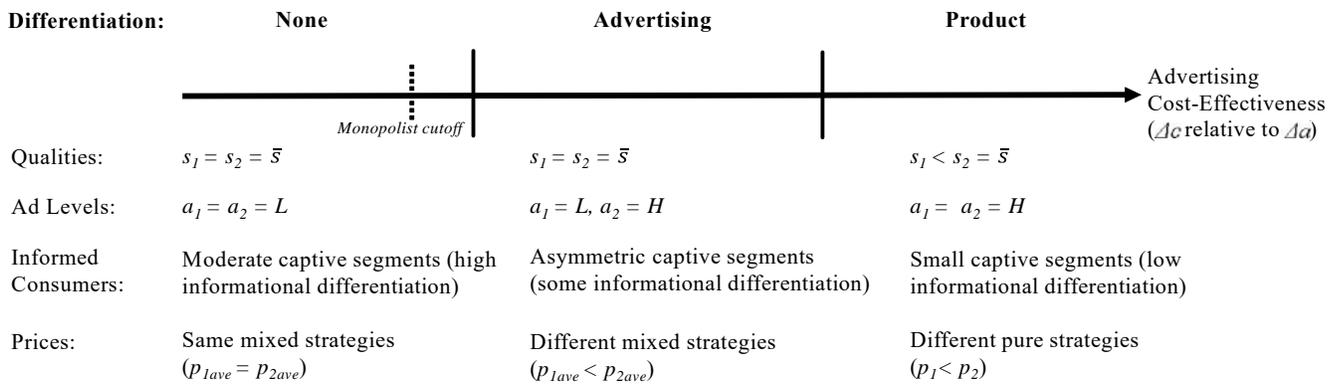
	Product	Average Quality	Standard Dev. Quality	Average Price	Standard Dev. Price
Large Quality Dispersion	Computers, Laptop: 10- to 11-inch	55.4	13.3	\$385	290.3
	Ranges: gas and dual-fuel, single oven (30")	58.9	12.6	\$1,437	788.7
	Computers, Laptop: 11- to 15-inch laptops	68.8	11.6	\$843	318.7
	Ranges: smoothtop, double oven (30")	72.8	10.7	\$1,658	428.6
	Ranges: pro-style gas and dual-fuel (30")	56.2	10.3	\$4,178	1077.9
	Dishwashers	70.4	9.7	\$854	407.3
	Fitness Trackers: built-in data readout	72.6	9.6	\$153	56.3
	Vacuum Cleaners: bagless upright	51.9	9.3	\$215	112.8
Small Quality Dispersion	Snow Blowers: two-stage gas	87.1	3.8	\$1,235	266.7
	Generators: large stationary	90.75	3.3	\$3,950	1618.5
	Dryers: gas	78	2.7	\$1,127	227.9
	Dryers: electric	78.1	2.6	\$1,044	241.8
	Washers: front loaders	82.7	1.5	\$1,134	287.4
	Cordless Drills: tougher job/driver	82.7	1.5	\$280	0
	Lawn Mowers: gas self-propel, single-speed	21.2	1.2	\$298	36.6
	Lawn Mowers: battery-powered	19.5	0.7	\$425	47.2

Note: Quality rated on a 0-100 scale. Data compiled from Consumer Reports Buying Guides (2014, 2017)

Table 2: Expected Size of Captive Segments as a Function of Advertising Choices

Ad Level	Heavy ($a_j = a_H$)	Light ($a_j = a_L$)
Heavy ($a_i = a_H$)	Both firms: $a_H(1 - a_H)$	Firm i : $a_H(1 - a_L)$ Firm j : $a_L(1 - a_H)$
Light ($a_i = a_L$)	Firm i : $a_L(1 - a_H)$ Firm j : $a_H(1 - a_L)$	Both firms: $a_L(1 - a_L)$

Figure 1: Equilibrium Strategies and Differentiation as Advertising Cost-Effectiveness Varies



Note: p_{iave} is firm i 's average price in a mixed strategy equilibrium.