



## RESEARCH ARTICLE

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**Imaging the Earth Using Time-Domain Surface-Wave Measurements: Evaluation and Correction of the Finite-Frequency Phase Shift**Yihe Xu<sup>1,2</sup> , Sergei Lebedev<sup>1,2</sup>, and Thomas Meier<sup>3</sup><sup>1</sup>Department of Earth Sciences, Bullard Laboratories, University of Cambridge, Cambridge, UK, <sup>2</sup>Geophysics Section, School of Cosmic Physics, Dublin Institute for Advanced Studies, Dublin, Ireland, <sup>3</sup>Institute of Geophysics, Christian-Albrechts University Kiel, Kiel, Germany

## Key Points:

- We identify a source of errors that can cause 0.2%–0.8% error in surface-wave phase-velocity measurements
- The error applies to all the time-domain methods that use bandpass-filtered waveforms
- We derive an explicit formula to evaluate the error and provide a numerical method to compute and remove it

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**Abstract** Surface waves propagating from earthquakes, active sources or within the ambient noise wavefield are widely used to image Earth structure at various scales, from centimeters to hundreds of kilometers. The accuracy of surface-wave, phase-velocity measurements is essential for the accuracy of the Earth models they constrain. Here, we identify a finite-frequency phase shift in the phase travel time that causes systematic errors in time-domain, phase-velocity measurements. The phase shift arises from the approximation of monochromatic surface waves with narrow-band filtered surface waves. We derive an explicit formula of the finite-frequency phase shift and present a numerical method for its evaluation and for the correction of the measurements. Applications to high-frequency and long-period examples show that the phase shift is typically around  $\pi/60$ – $\pi/16$  for the common settings of ambient-noise imaging studies, which translates to 0.2%–0.8% phase-velocity measurement errors. The finite-frequency phase shift depends on the (a) second derivative of the wavenumber with respect to frequency; (b) width of the narrow-band filter; (c) epicentral or interstation distance; and (d) center frequency of the filter. In conversion to phase velocity, the last two factors cancel out. Frequency-domain methods for phase-velocity measurements have the advantage of not producing the finite-frequency phase shift. Both time- and frequency-domain measurements, however, can be impacted by a break-down of the far-field approximation (near-field phase shift), which our calculations also show. Our method offers an effective means of improving the accuracy of the widely used time-domain, phase-velocity measurements via the evaluation of and corrections for the finite-frequency phase shift.

**Plain Language Summary** Surface waves sample the “surface layer” of the Earth with the thickness of the layer depending on the wave's wavelength. Combining surface waves of different wavelengths, or different frequencies, can provide a detailed image of the Earth's structure over a large depth range. Measuring the velocity at which surface waves travel is a crucial step in this process. In this study, we identify a source of errors in the measurements that has not received much notice so far. We first derive a formula for the possible bias and then develop a numerical method to evaluate and correct it. Applications of the numerical method reveal the effects of the bias in two common scenarios, in the surface-wave studies of the shallow (kilometers deep) and deep (hundreds of kilometers deep) structure of the Earth. Our results provide a means to improve the accuracy of the surface wave measurements and, thus, the accuracy of the surface-wave imaging of the Earth.

**1. Introduction**

Surface waves sample the Earth's interior at scales from centimeters to hundreds of kilometers, with varying resolution depending on their frequency. Intermediate- and long-period (>5 s) surface waves generated by earthquakes provide essential constraints on the shear-wave velocity structure and anisotropy of the crust (e.g., Agius & Lebedev, 2014, 2017; Bourjot & Romanowicz, 1992; Levshin & Ratnikova, 1984; U. Meier et al., 2007; Polat et al., 2012; Press & Ewing, 1955), lithosphere and asthenosphere (e.g., Bonadio et al., 2021; Deschamps et al., 2008; Ekström & Dziewonski, 1998; El-Sharkawy et al., 2020; Masters et al., 1996; T. Meier et al., 2004; Pasyanos et al., 2014; Shapiro & Ritzwoller, 2002; Zhang et al., 2009), and crustal and upper-mantle interfaces (e.g., Bartzsch et al., 2011; Beghein et al., 2019; Lebedev et al., 2013). High-frequency (>5 Hz) surface waves generated by active sources have been used to constrain upper-crustal and near-surface structure (e.g., Mi et al., 2020; Pan et al., 2019; Socco et al., 2010; Wathelet et al., 2004; Xia et al., 1999). Ultrasonic (>20 kHz)

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surface waves image the top few centimeters beneath the surface, providing information about hidden cracks in concrete and weathering in historical sites (e.g., Aggelis et al., 2010; Bodet et al., 2005; T. Meier et al., 2017).

During the last two decades, ambient noise interferometry (cross-correlation of noise recorded at two receivers yielding estimates of the Green's functions) has been widely adopted as an additional approach for surface-wave imaging. It provides abundant surface wave observations and fills the frequency gap between earthquakes and active sources. It also facilitates the imaging of regions where earthquakes and active sources cannot provide sufficient illumination (e.g., Li et al., 2016; Mordret et al., 2013; Nicolson et al., 2014; Zhan et al., 2014). Ambient noise surface wave tomography has been successfully applied to imaging subsurface structure and anisotropy from the crustal and upper mantle scale (e.g., Moschetti et al., 2007; Pawlak et al., 2012; Sabra et al., 2005; Shapiro et al., 2005; Yao et al., 2006), to the scale of a basin, a volcano, or a fault region (e.g., Brenguier et al., 2007; Delorey & Vidale, 2011; Inzunza et al., 2019; Martins et al., 2019; Mordret et al., 2019; Roux et al., 2011), and to a mineral-deposit scale (e.g., Bellefleur et al., 2015; Hollis et al., 2019; Mordret et al., 2013; Xu et al., 2021). The method is also applied in other fields, such as cryoseismology (e.g., Lindner et al., 2019; Preiswerk & Walter, 2018).

Accurate group or phase velocity measurements are the foundation of most surface-wave studies at any scale. The measurement methods fall into two categories, with the measurements performed either in the time domain or in the frequency domain. Time-domain methods measure the arrival time or the instantaneous phase of surface waves in the time domain and then convert them to the velocity. One representative method is the frequency-time analysis (Bensen et al., 2007; Dziewonski et al., 1969; Dziewonski et al., 1972; Levshin et al., 1972; Ritzwoller & Levshin, 1998; Yao et al., 2006). Using a set of narrow-band filters, the original, broadband surface waves are turned into a set of narrow-band surface-wave signals. One can then measure the phase travel time or instantaneous phase on the narrow-band seismograms to obtain phase velocities, or measure the group travel time on the envelopes to obtain group velocities. Similar measurements can also be obtained using wavelet transforms (e.g., Kulesh et al., 2008). Using multiple stations can solve the ambiguity in the phase velocity measurement (e.g., Martins et al., 2019; Xia et al., 1999), and even determine the detailed direction of surface wave propagation (e.g., Kolínský et al., 2019).

The time-domain methods for the determination of the phase velocity implicitly assume that the narrow-band signals represent the monochromatic surface waves at the center frequencies of the filters. The approximation can result in systematic errors in the measurements. By contrast, frequency-domain methods measure the phase of the spectra using the Fourier transform (Bonadio et al., 2018, 2021; Fry et al., 2010; Kästle et al., 2016; T. Meier et al., 2004; Molinari et al., 2015; Verbeke et al., 2012; Zhang et al., 2007). The frequency-domain methods do not rely on the assumption of monochromaticity of narrow-band surface waves. However, they require effective procedures for identifying the correct  $2\pi$  branch of the phase of the surface wave (e.g., Bonadio et al., 2018, 2021; Soomro et al., 2016).

Both the time- and frequency-domain methods, originally developed for earthquake seismograms, have been applied in ambient noise studies with minor modifications. The difference is that the initial phase of virtual sources is zero (Bensen et al., 2007; Bonadio, 2019; Kästle et al., 2016; Yao et al., 2006), as the cross-correlation in noise interferometry cancels out the original initial phase from noise sources. Moreover, the zero initial phase leads to an alternative frequency-domain method as well (Aki, 1957; Ekström, 2014; Ekström et al., 2009). The spectrum of the noise cross-correlation functions (NCFs) is found to resemble a Bessel function with the argument of  $\omega\Delta/c(\omega)$ , where  $\omega$  is angular frequency,  $\Delta$  is interstation distance, and  $c$  is the phase velocity (Aki, 1957; Ekström et al., 2009). Phase velocity can be obtained by fitting the Bessel function to the real part of the spectrum of NCFs.

In this paper, we focus on the accuracy of phase velocity measurements using the time-domain methods, most popular due to their simplicity. Successful measurements of phase velocity are usually more accurate than those of group velocity (e.g., Shen et al., 2016) and, also, can be related to Earth structure and anisotropy more unambiguously (Dahlen & Zhou, 2006). Phase-velocity measurements, however, are more complicated than group-velocity ones, even after the unknown initial phase is resolved.

Unlike with the group velocity, converting measured phase or phase travel time to correct phase velocity does not follow a simple relationship of  $t = \Delta/c$ , where  $t$  and  $c$  are phase travel time and phase velocity, respectively. Instead, misalignments have been long observed between the monochromatic surface wave and its corresponding

Fourier component,  $\cos(\omega(t - \Delta/c))$  (Brune et al., 1961; Toksöz & Anderson, 1966; Tromp & Dahlen, 1992; Z. Wang et al., 1993). The misalignments are generally referred to as phase shifts, which must be accounted for during the conversion; otherwise, the resulting phase velocity is biased.

We refer to phase shifts as phase advances or delays (Aki & Richards, 2002; Brune et al., 1961), where a phase advance refers to an event arriving earlier than expected and a phase delay refers to a late arrival. The representation is unambiguous regardless of the sign of frequency and the convention of the Fourier transforms. The value of a phase shift is defined as the amount of deviation of the measured phase from a predicted phase that usually from theoretical approximations. A phase advance, if not corrected, will result in a higher phase velocity, and vice versa. Phase advances and delays correspond to decreases and increases of the absolute magnitude of the phase in frequency domain, respectively.

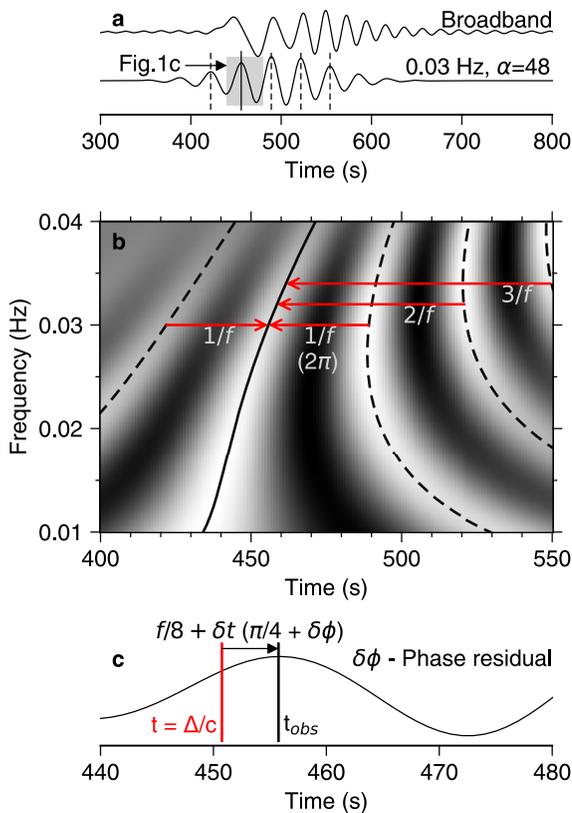
A number of types of the phase shifts have been identified. Quantifying and taking them into account is essential for the accuracy of the measurements and for the accuracy of the Earth-structure models computed using the measurements. A phase delay of  $\pi/4$  is commonly used in noise cross-correlation studies (e.g., Yao et al., 2006). The phase delay, which is physically generated during interference of surface waves from noise sources from different directions (e.g., Kästle et al., 2016), is referred as (virtual) source phase shift hereafter as it is analogous to the  $\pi/4$  phase delay in surface waves generated by earthquakes. The  $\pi/4$  is a good approximation in the far field (several wavelengths away) of the (virtual) source. In the near field, the amount of the phase shift deviates from  $\pi/4$ . Consequently, most noise cross-correlation studies discard short-distance measurements, for example, less than 1–3 wavelengths (Bensen et al., 2007; Luo et al., 2015; Shapiro et al., 2005). The phase delay turns to a phase advance of  $\pi/4$  when using NCFs (Kästle et al., 2016) instead of empirical Green's functions that equals the opposite of the first derivative of the NCFs with respect to time. For earthquake sources, the frequency-dependent initial phase at the source can also have an effect on the group or phase velocity measurements (Levshin et al., 1999).

Another type of phase shift is the caustic phase shift. It is generated when surface waves pass a caustic where rays of surface waves cross (e.g., Tromp & Dahlen, 1992; Z. Wang et al., 1993). Surface waves stack constructively at a caustic and change their phase. The caustic phase shift is a  $\pi/2$  phase advance in the far field of the caustic. Each passage of a caustic causes a  $\pi/2$  phase advance. One example of the caustic phase shift is the polar phase shift (Brune et al., 1961), which occurs when surface waves travel across the antipode of the source and the polar area (the source). The corresponding caustics are exactly at the source and its antipode in a spherically symmetric Earth model and can deviate from the antipode for  $10^\circ$ – $20^\circ$  in a heterogeneous Earth model (Z. Wang et al., 1993). The source phase shift can be regarded as a “semi-caustic” as well. Its difference from a caustic is that rays converge and diverge at a caustic but only diverge at the source (Brune et al., 1961).

In the vicinity (near field) of a caustic or semi-caustic, the phase advance deviates from its far-field value (Kästle et al., 2016; Pollitz, 2001; Schwab & Kausel, 1976; Tromp & Dahlen, 1993; Wielandt, 1980). The exact amount of the phase shift can be either estimated analytically, by adding higher-order terms of Taylor expansions into the analysis (Herrmann, 1973; Wielandt, 1980), or computed numerically (Schwab & Kausel, 1976). In this paper, we use near-field phase shifts to refer to the difference between a (semi-)caustic phase shift and its far-field approximation.

In the time-domain methods for phase-velocity measurement, approximating monochromatic surface waves by narrowband ones causes additional types of phase shifts due to the interference between neighboring frequencies. The origins of such errors and corresponding solutions have been discussed in the context of group-velocity measurements (Dziewonski et al., 1972; Herrmann, 1973; Levshin et al., 1992; Shapiro & Singh, 1999). The two origins identified are rapid changes in the group-velocity dispersion curve and variations in the amplitude spectra. The proposed solutions for group velocity changes normally involve an iterative procedure that reduces most of the changes in group velocity by subtracting the group velocity measured in the original seismogram (Bensen et al., 2007; Dziewonski et al., 1972; Ritzwoller & Levshin, 1998). Solutions for non-constant amplitude spectra involve replacing the center frequency by an instantaneous frequency (Ritzwoller & Levshin, 1998) or a centroid frequency (Shapiro & Singh, 1999).

The narrow-band approximation also has an effect on phase-velocity measurements. Herrmann (1973) showed an additional phase term in the mathematical representation of Gaussian filtered surface waves, but he did not discuss its further implication. Wielandt and Schenk (1983) approached the problem from the perspective of



**Figure 1.** Time-domain phase velocity measurement and the finite-frequency phase shift. (a) Broadband and narrow-bandpass-filtered surface waves. The broadband surface wave is a synthetic seismogram generated using the velocity model ak135 (Kennett et al., 1995). For the narrow-band surface wave, a Gaussian filter with the center frequency of 0.03 Hz and  $\alpha = 48$  is used. The arrival times of the peaks are marked by vertical lines, with the peak corresponding to the correct phase travel time marked by a solid line and other peaks by dashed lines. (b) Frequency-time representation of the broadband surface wave. Each row of the image corresponds to a narrow-band filtered surface wave. Connected peaks form ridges, with the “correct” ridge marked by a solid line and other ridge by dashed lines. The separation between these ridges are  $n/f$ , or  $2n\pi$  in phase. (c) The finite-frequency phase shift. The measured phase travel time is  $f/8 + \delta t$  s later than the predicted phase travel time of  $\Delta/c$ , equivalent to a  $\pi/4 + \delta\phi$  phase delay, where  $\delta\phi$  is the finite-frequency phase shift.

the narrowband surface waves, or “ridges” in the frequency-time representation (Figure 1b). The 0th ridge is often referred as the “correct” ridge (e.g., Y. Wang et al., 2017; Yao et al., 2006), though other ridges can also generate correct phase velocity measurements if the time shift  $n/f_c$  is corrected (Bonadio et al., 2018; T. Meier et al., 2004; Soomro et al., 2016; Xu et al., 2021).

The finite-frequency phase shift,  $\delta\phi$ , corresponds to the time offset  $\delta t$  ( $= \delta\phi/2\pi f_c$ ) between the peaks of the narrowband and monochromatic surface waves (Figure 1c),

$$t_n = \frac{\Delta}{c} + \frac{1}{8f_c} + \frac{n}{f_c} + \frac{\phi_s}{\omega} + \delta t, \quad n = 0, \pm 1, \pm 2, \dots \quad (2)$$

Unlike the “systematic error” due to the variations in amplitude spectra, discussed by Shapiro and Singh (1999), the phase shift is non-zero even for constant amplitude spectra. It needs to be corrected along with the  $\pi/4$  phase delay to get an accurate measurement of phase velocities,

time-domain windowing, implemented with a formula similar to ours but with the center of the time windows being a variable, which created some additional complexities. They also suggested that the error is strong enough to be noticed only at periods longer than beyond 200 s. We note that this is true when the epicentral or interstation distance is thousands of kilometers. For small distances (e.g., 4 km), we show in this paper that the error is observable at periods longer than 0.5 s. To our knowledge, there are no recent systematic reviews on the phase shifts due to the narrow-band approximation and their effects on the measurements and the resulting Earth models, especially in the context of noise cross-correlations.

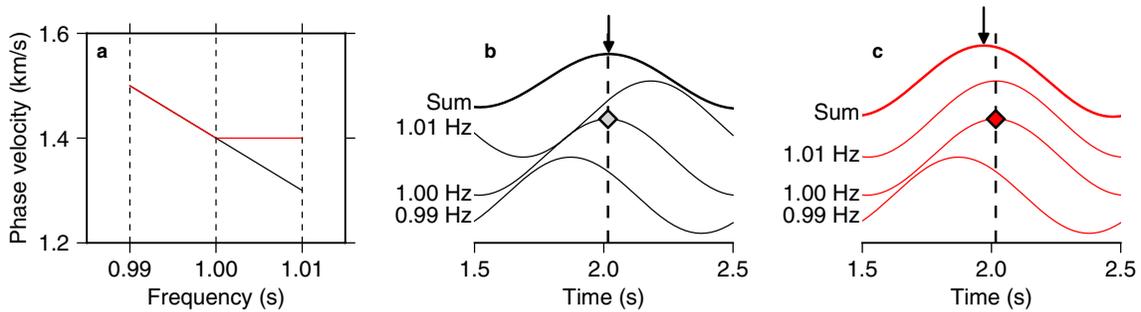
The purpose of this paper is to present a systematic study of—and practical ways to correct for—the phase shift caused by approximating monochromatic surface waves by narrow-band surface waves. We refer to the phase shift as the finite-frequency phase shift, as it originates from a replacement of an infinitesimally narrow frequency band by a finite-width band. We first derive an explicit expression of the phase shift, under certain assumptions, and then present a numerical method to accurately evaluate the phase shift. We also demonstrate the effect of the finite-frequency phase shift on the phase velocity measurements and offer recipes for the correction of the phase shift in order to obtain accurate phase-velocity measurements. The results apply to measurements with time-domain methods on surface waves from earthquakes, active sources, and ambient seismic noise.

## 2. Finite-Frequency Phase Shift

Surface waves at different frequencies travel at different speeds. To measure phase velocities at different frequencies, a comb of narrow-bandpass filters is often used to isolate individual frequencies, such as in the multiple filter technique (Dziewonski et al., 1969). The filtered surface wave resembles a cosine function with a period of  $1/f_c$ , where  $f_c$  is the center frequency of the filters (Figure 1a). Assuming the narrowband surface waves share the same periodicity with the monochromatic surface waves, the arrival time of the peaks can be written explicitly,

$$t_n = \frac{\Delta}{c} + \frac{1}{8f_c} + \frac{n}{f_c} + \frac{\phi_s}{\omega_c}, \quad n = 0, \pm 1, \pm 2, \dots, \quad (1)$$

where  $\Delta$  is the distance,  $c$  the phase velocity at the frequency  $f_c$ ,  $\omega_c = 2\pi f_c$  the angular frequency, and  $\phi_s$  is the initial phase of the source (Dahlen & Tromp, 1998; Yao et al., 2006). The  $1/(8f_c)$  is the manifestation of the  $\pi/4$  phase delay caused by the source term. We refer to  $n$  as the order of peaks in



**Figure 2.** Finite-frequency phase shift caused by using a bundle of frequencies to estimate the center frequency. (a) Phase velocity curves used in the example. The black line denotes the curve that is linear, whereas the red line denotes a curve with a curvature (the kink at 1 Hz). (b) Monochromatic components at 0.99, 1.00, and 1.01 Hz and their superposition. The linear curve is used. The diamond and the dashed line mark the arrival time of the peak of the 1-Hz component. The arrow points to the arrival time of the peak of the superposition. (c) Same as (b) but using the curve with the kink.

$$c = \frac{\Delta}{t - (1/8f_c) - (n/f_c) - (\phi_s/\omega_c) - \delta t}, \quad (3)$$

or

$$c = \frac{\omega_c \Delta}{\omega_c t - (\pi/4) - 2n\pi - \phi_s - \delta \phi}. \quad (4)$$

Figure 2 illustrates the origin of the finite-frequency phase shift. Monochromatic surface waves are generated at three equal-spaced frequencies, 0.99, 1.00, and 1.01 Hz. Superposition of the three produces a narrow-band surface wave (Figures 2b and 2c). The three monochromatic surface waves have the same amplitude, so as to avoid the influence of the spectral amplitude variation. When the phase velocity dispersion curve is linear, the peak in the narrow-band surface wave arrives at the same time as the peak of the center-frequency (1 Hz) surface wave. By contrast, when the dispersion curve is not linear, the two peaks have a slight misalignment. The misalignment is the finite-frequency phase shift. If not accounted for, the misalignment will result in a biased phase velocity measurement.

### 3. Theoretical Derivation

We can derive an explicit expression for the finite-frequency phase shift assuming the far field approximation and narrow frequency bands. By far field, we mean large  $|\omega|\Delta/c = 2\pi\Delta/\lambda$ , that is, a large number of wavelengths fitting between the source and station, or between two stations in the case of noise interferometry and two-station methods. Starting from the spectrum of surface waves, we integrate the spectrum over frequency in order to obtain the seismogram of surface waves, to which we refer as broadband surface waves. Then, we apply Gaussian filters to obtain the explicit expression for the narrow-band surface waves, from which, finally, we obtain the explicit expression for the finite-frequency phase shift.

#### 3.1. Stationary Phase Approximation

The stationary phase approximation method is used repeatedly in the following derivation in order to approximate integrals. We briefly summarize the points that are relevant to our derivation. For more details, we refer the reader to Aki and Richards (2002). The method is used to estimate the integral of the form

$$\int_a^b F(\omega) \exp(f(\omega)) d\omega, \quad (5)$$

where  $F(\omega)$  and  $f(\omega)$  are complex functions and  $\omega$  is a dummy variable that does not necessarily mean frequency. Both  $a$  and  $b$  can be infinity. This type of integral is dominated by several discrete points where  $f'(\omega) = 0$ . They are termed stationary phase points or saddle points after the shape of the integrand surface near these points in the

complex domain of  $\omega$ . If the  $|f''(\omega)|$  values at the saddle points are sufficiently large, the integral can be estimated as a summation of the contribution of the saddle points,

$$\sum_{\omega_0 \in \text{saddle points}} F(\omega_0) \exp(f(\omega_0)) \sqrt{\frac{2\pi}{|f''(\omega_0)|}} \times \begin{cases} \exp\left(i\frac{\pi - \phi}{2}\right), & \phi \in (0, \pi] \\ \exp\left(i\frac{-\pi - \phi}{2}\right), & \phi \in (-\pi, 0) \end{cases}, \quad (6)$$

where  $\omega_0$  are saddle points within the interval  $[a, b]$  and  $\phi = \arg f''(\omega_0)$ . When a saddle point is at the end of the interval, its contribution should be halved.

### 3.2. Broadband Time-Domain Waveforms

Surface waves can be written as a superposition of monochromatic surface waves,

$$x(t) = \frac{1}{\pi} \Re \int_0^{+\infty} A(\omega) \exp\left(-i\frac{\omega\Delta}{c(\omega)} + i\varphi(\Delta)\right) \exp(i\omega t) d\omega, \quad (7)$$

where  $\Re$  is the real part operator that arises from the reduction of the interval  $(-\infty, +\infty)$  to  $[0, +\infty)$  using the Hermitian symmetry of the spectrum,  $A(\omega)$  is the amplitude spectrum, and  $\varphi(\Delta)$  represents the overall phase shift regarding propagation of monochromatic surface waves, comprising the initial phase from the source mechanism, the source phase shift and the caustic phase shift when passing a polar or antipodal region. The Fourier transform we used is

$$\begin{aligned} F(\omega) &= \int_{-\infty}^{+\infty} f(t) \exp(-i\omega t) dt, \\ f(t) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) \exp(i\omega t) d\omega, \end{aligned} \quad (8)$$

although the conclusion is independent of the convention of Fourier transform. For any given distance, Equation 7 can be evaluated using the stationary phase approximation when  $k''(\omega_0)\Delta$  is sufficiently large,

$$x(t) \approx \sqrt{\frac{2}{\pi\Delta|k''(\omega_0)|}} \times \begin{cases} \cos\left(\omega_0 t - k(\omega_0)\Delta + \varphi - \frac{\pi}{4}\right), & k''(\omega_0) > 0 \\ \cos\left(\omega_0 t - k(\omega_0)\Delta + \varphi + \frac{\pi}{4}\right), & k''(\omega_0) < 0 \end{cases}, \quad (9)$$

where  $k(\omega) = \omega/c(\omega)$  is the wavenumber and  $\omega_0$  is the saddle point which satisfies  $k'(\omega_0) = t/\Delta$ . It shows that the classical phase deviation of  $\pm\pi/4$  in broadband surface waves comparing to its corresponding monochromatic component (Aki & Richards, 2002).

### 3.3. Narrowband Time-Domain Waveforms

#### 3.3.1. Far-Field Approximation

Bandpass filtering can change the instantaneous phase of surface waves. Applying a Gaussian filter to the broadband surface wave (Equation 7), we have

$$x(t) = \frac{1}{\pi} \Re \int_0^{+\infty} A(\omega) \exp\left(-i\frac{\omega\Delta}{c(\omega)} + i\varphi(\Delta)\right) \exp\left(-\alpha\frac{(\omega - \omega_c)^2}{\omega_c^2}\right) \exp(i\omega t) d\omega, \quad (10)$$

where  $\omega_c$  is the center frequency of the filter and  $\alpha (>0)$  is a parameter that controls the width of the filter. Using the stationary phase approximation, we can obtain the explicit expression for the time-domain waveform of the filtered surface wave,

$$x(t) \approx \sqrt{\frac{2}{\pi |ik''\Delta + \frac{2\alpha}{\omega_c^2}|}} A(\omega_0) \exp\left(-\alpha \frac{(\omega_0 - \omega_c)^2}{\omega_c^2}\right) \times \begin{cases} \cos\left(\omega_0 t - k\Delta + \varphi(\Delta) + \frac{-\pi - \beta}{2}\right), & k''(\omega_0) > 0 \\ \cos\left(\omega_0 t - k\Delta + \varphi(\Delta) + \frac{\pi - \beta}{2}\right), & k''(\omega_0) < 0 \end{cases} \quad (11)$$

where the stationary phase point  $\omega_0$  is determined by

$$-ik'(\omega_0)\Delta + it - 2\alpha \frac{\omega_0 - \omega_c}{\omega_c^2} = 0, \quad (12)$$

and

$$\beta = \arg f''(\omega) = \arg\left(-\frac{2\alpha}{\omega_c^2} - ik''(\omega_0)\Delta\right). \quad (13)$$

When using a wide-band Gaussian filter ( $\alpha \rightarrow 0$ ),  $\beta$  in Equation 11 approaches  $-\pi/2$  for  $k''(\omega_0) > 0$  and  $\pi/2$  for  $k''(\omega_0) < 0$ . Thus, Equation 11 degenerates to its broadband counterpart, Equation 9. When using a narrow-band filter ( $\alpha \rightarrow \infty$ ),  $\beta \rightarrow -\pi$  ( $k''(\omega_0) > 0$ ) and  $\pi$  ( $k''(\omega_0) < 0$ ), the cosine term in Equation 11 in both cases become  $\cos(\omega_0 t - k\Delta + \varphi(\Delta))$ , which has the same phase as the monochromatic surface waves in Equation 7. This is why narrow-bandpass filtered surface waves can be used to measure phase velocity at the center frequencies of the filters. However,  $\alpha$  cannot be  $\infty$  when making real measurements. It is crucial to know how much is the error for using a finite  $\alpha$ . As the saddle points move away from the real axis into the complex domain for  $0 < \alpha < \infty$ , it is not trivial to evaluate the phase shift using the stationary phase approximation, so we turn to another approximation under the narrow-band assumption.

### 3.3.2. Far-Field and Narrow-Band Approximation

To obtain an explicit expression for the finite-frequency phase shift, we re-evaluate the integral in Equation 10 using Taylor expansions at the center frequency  $\omega_c$ . In addition to the far-field approximation, we limit ourselves to narrow-band filters. In this case, let

$$F(\omega) = \frac{A(\omega)}{\pi},$$

$$f(\omega) = \omega t - k\Delta + \varphi(\Delta), \quad (14)$$

we can expand  $F(\omega)$  and  $\exp(if(\omega))$  at  $\omega_c$  to their second-order terms,

$$F(\omega) \approx F(\omega_c) + F'(\omega_c)(\omega - \omega_c) + \frac{1}{2}F''(\omega_c)(\omega - \omega_c)^2$$

$$\exp(if(\omega)) \approx \exp(if(\omega_c)) \left[1 + if'(\omega_c)(\omega - \omega_c) + \frac{1}{2}(if''(\omega_c) - f'^2(\omega_c))(\omega - \omega_c)^2\right]. \quad (15)$$

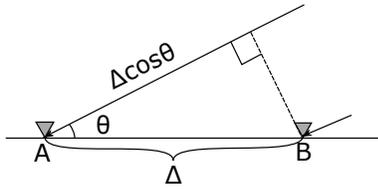
Substituting Equation 15 into Equation 10, the integral can be organized as a series of integrals  $\int (\omega - \omega_c)^n \exp(-\alpha(\omega - \omega_c)^2) d\omega$ , which can be evaluated analytically. Summing the results yields the expression for the time-domain waveform of narrowband surface waves,

$$x(t) \approx \frac{A(\omega_c)B}{\pi} \omega_c \sqrt{\frac{\pi}{\alpha}} \cos(i(\omega_c t - k\Delta + \varphi(\Delta) - \delta\phi)), \quad (16)$$

where  $B$  and  $\delta\phi$  are the amplitude and phase, respectively, of a complex number given by

$$B \exp(i\delta\phi) = F(\omega_c) + \frac{\omega_c^2}{2\alpha} \left[-\frac{1}{2}F(\omega_c)f'(\omega_c)^2 + \frac{1}{2}F''(\omega_c)\right] - i\frac{\omega_c^2}{2\alpha} \left[\frac{1}{2}F(\omega_c)f''(\omega_c) + F'(\omega_c)f'(\omega_c)\right]. \quad (17)$$

The  $\delta\phi$  is the finite-frequency phase shift we are looking for. We intentionally use  $-\delta\phi$  instead of  $\delta\phi$  in Equation 16, so that  $\delta\phi > 0$  corresponds to phase delay and  $\delta\phi < 0$  corresponds to phase advance.



**Figure 3.** Incoming plane wave at a pair of stations. The plane wave arrives at the stations at an angle  $\theta$ . The distance between stations A and B is  $\Delta$ . The plane wave needs to travel farther by  $\Delta \cos \theta$  to reach station A compared to station B. The corresponding time delay is  $\Delta \cos \theta / c$  and phase delay is  $\omega \Delta \cos \theta / c$ .

Assuming a constant amplitude spectrum,  $F'(\omega_c)$  and  $F''(\omega_c)$  are zero. Therefore, we can evaluate the finite-frequency phase shift as

$$\delta \phi = \arg \left\{ \frac{4\alpha}{\omega_c^2} - (t - k'(\omega_c) \Delta)^2 + ik''(\omega_c) \Delta \right\}. \quad (18)$$

For narrow-band filters when  $\alpha$  is sufficiently large, the first term dominates the real part. We can therefore simplify the mathematical representation of  $\delta \phi$  to

$$\delta \phi = \arctan \frac{k''(\omega_c) \Delta}{4\alpha / \omega_c^2} \approx \frac{\omega_c^2 k''(\omega_c) \Delta}{4\alpha}. \quad (19)$$

The formula in Equation 19 reveals four factors that affect the finite-frequency phase shift, namely, (a) the center frequency of the Gaussian filter  $\omega_c$ , (b) the second derivative of the wavenumber with respect to frequency  $k''(\omega_c)$ , (c) the epicentral distance  $\Delta$ , and (d) the width of the Gaussian filter  $\alpha$ . Since  $\omega_c$ ,  $\Delta$ , and  $\alpha$  are all positive, the sign of the phase shift is solely determined by  $k''(\omega_c)$ . Physically,  $k''(\omega_c) > 0$  means a positive curvature (with respect to frequency) in the phase dispersion curve or a positive slope (with respect to period) in the group dispersion curve.

### 3.4. Finite-Frequency Phase Shift in Noise Cross-Correlation Functions

Here we show that the finite-frequency phase shift in the surface waves in the NCFs shares exactly the same formula with ballistic surface waves. First, we compute the spectrum of NCFs by summing the cross-correlation of incoming plane waves over all the azimuths. Assuming that we have two stations, station A and station B, separated by the distance  $\Delta$ , and an incoming plane wave that arrives at the angle  $\theta$  (measured counter-clockwise from the east; Figure 3), the cross-correlation spectrum of the plane wave at the station A with one at station B is  $\exp(i\omega \Delta \cos \theta / c)$ , where  $\omega$  is the angular frequency and  $c$  is the phase velocity at this frequency. Assuming a perfect illumination, when the amplitude of the incoming plane waves is constant over azimuth, the spectrum of the noise cross-correlation can be written as

$$C(\omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp \left( i\omega \frac{\Delta \cos \theta}{c(\omega)} \right) d\theta = J_0 \left( \frac{\omega \Delta}{c(\omega)} \right), \quad (20)$$

where  $J_0(x)$  is the zeroth-order of Bessel functions of the first kind. Evaluating the integral using the stationary phase approximation yields the far-field approximation of the spectrum of the noise cross-correlation,

$$C(\omega) \approx \sqrt{\frac{2}{\pi \frac{|\omega| \Delta}{c(\omega)}}} \cos \left( \frac{|\omega| \Delta}{c(\omega)} - \frac{\pi}{4} \right). \quad (21)$$

where far field implies a large  $\omega \Delta / c = 2\pi \Delta / \lambda$  and  $\lambda$  is the wavelength. Alternatively, the approximation can be obtained using asymptotic forms of Bessel functions (Gradshteyn & Ryzhik, 2014). The contributions of the causal branch and the acausal branch can also be separated in the far-field approximation (Table 1).

Applying inverse Fourier transform to the far-field approximation of the spectrum of the causal branch (Table 1) yields the expression of broadband surface waves in the causal branch of NCFs,

$$\begin{aligned} C_+(t) &\approx \frac{1}{2\pi^2} \Re \int_0^\infty \sqrt{\frac{2\pi}{k\Delta}} \exp(i\omega t - ik\Delta) \exp\left(i\frac{\pi}{4}\right) d\omega \\ &\approx \frac{1}{\pi \Delta} \frac{1}{\sqrt{|k(\omega_0)| |k''(\omega_0)|}} \times \begin{cases} \cos(\omega_0 t - k(\omega_0) \Delta), & k''(\omega_0) > 0 \\ \cos\left(\omega_0 t - k(\omega_0) \Delta + \frac{\pi}{2}\right), & k''(\omega_0) < 0 \end{cases} \end{aligned} \quad (22)$$

**Table 1**  
Spectra of the Causal and Acausal Branches of the Noise Cross-Correlation Under the Far-Field Approximation

Branch	$\omega > 0$	$\omega < 0$
Causal	$\exp\left(-i\left(\frac{\omega \Delta}{c(\omega)} - \frac{\pi}{4}\right)\right)$	$\exp\left(-i\left(\frac{\omega \Delta}{c(\omega)} + \frac{\pi}{4}\right)\right)$
Acausal	$\exp\left(i\left(\frac{\omega \Delta}{c(\omega)} - \frac{\pi}{4}\right)\right)$	$\exp\left(i\left(\frac{\omega \Delta}{c(\omega)} + \frac{\pi}{4}\right)\right)$

Note. The common factor  $\sqrt{2c(\omega)/\pi|\omega|\Delta}$  is omitted. The expressions correspond to the convention of Fourier transforms indicated by Equation 8. Using other conventions will result in changes of signs or an additional constant factor.

where  $\omega_0$  is the saddle point which satisfies  $k'(\omega_0) = t/\Delta$ . We can also obtain the expression for the surface waves in the acausal branch,

$$C_-(t) \approx \frac{1}{\pi\Delta} \frac{1}{\sqrt{|k(\omega_0)||k''(\omega_0)|}} \times \begin{cases} \cos(\omega_0 t + k(\omega_0)\Delta), & k''(\omega_0) > 0 \\ \cos\left(\omega_0 t + k(\omega_0)\Delta - \frac{\pi}{2}\right), & k''(\omega_0) < 0 \end{cases} \quad (23)$$

where  $k'(\omega_0) = -t/\Delta$  ( $t < 0$ ). The two branches form an even function of  $t$ .

Unlike the ballistic surface waves, surface waves in NCFs have a known  $\pi/4$  advance instead of an unknown phase shift of  $\phi(\Delta)$ . However, the  $\pi/4$  phase advance can be canceled out by the  $\pi/4$  phase delay that arises in the frequency integration when  $k''(\omega_0) > 0$ , making the instantaneous phase of the surface wave at a certain time  $t$  is in-phase (no phase shift) with the corresponding monochromatic surface wave  $\cos(\omega_0 t - \omega_0 \Delta/c(\omega_0))$ . To illustrate the phenomenon, we generate a broadband surface-wave synthetic using the exact spectrum in Equation 20 and ak135 (Kennett et al., 1995) and then compare it with the monochromatic surface wave (Figure 4). The alignment is easier to examine at the peaks, so we chose the frequency 0.0355 Hz that is the saddle point corresponding to the peak at  $t = 901.27$  s. A good consistency is observed between the monochromatic and broadband surface waves at 901.27 s, confirming their alignment in terms of phase. Furthermore, the instantaneous frequency at 901.27 s is also 0.0355 Hz, which can be verified by computing the first derivative of the instantaneous phase  $\phi(t) = \omega_0 t - k(\omega_0)\Delta$ , which is

$$\omega(t) = \frac{\partial\phi(t)}{\partial t} = \omega_0 + \frac{\partial\omega_0}{\partial t} (t - k'(\omega_0)\Delta) = \omega_0(t) \quad (24)$$

in the causal branch. The second term is zero because the saddle point  $\omega_0$  follows the relationship  $k'(\omega_0) = t/\Delta$ .

The narrowband time-domain waveforms of NCFs can be computed using the same approach as in Section 3.3,

$$C(t) \approx \frac{1}{2\pi^2} \Re \int_0^\infty \sqrt{\frac{2\pi}{k\Delta}} \exp(i\omega t - ik\Delta) \exp\left(i\frac{\pi}{4}\right) \exp\left(-\alpha \frac{(\omega_0 - \omega_c)^2}{\omega_c^2}\right) d\omega \quad (25)$$

$$\approx \frac{B}{2\pi^2} \omega_c \sqrt{\frac{\pi}{\alpha}} \Re \left\{ \exp\left(i\left(\omega t - k\Delta + \frac{\pi}{4} - \delta\phi\right)\right) \right\},$$

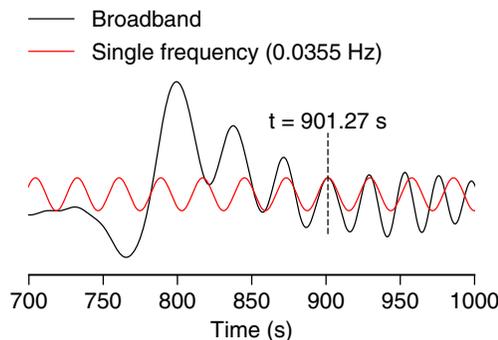
where  $B$  and  $\delta\phi$  share exactly the same definition as ballistic surface waves, with a slightly different definition of  $F(\omega)$  and  $f(\omega)$ ,

$$F(\omega) = \frac{1}{2\pi^2} \sqrt{\frac{2\pi}{k\Delta}},$$

$$f(\omega) = \omega t - k\Delta + \frac{\pi}{4}. \quad (26)$$

Despite that difference in  $F(\omega)$  and  $f(\omega)$ , surface waves from noise interferometry share the identical formula for the finite-frequency phase shift with surface waves from earthquakes and active sources,

$$\delta\phi \approx \frac{\omega_c^2 k''(\omega_c) \Delta}{4\alpha}. \quad (27)$$



**Figure 4.** Broadband and monochromatic surface waves. The broadband surface wave synthetic is computed using a trapezoid integration from 2 to 50 mHz, with an interval of 0.01 mHz. There is no misalignment between the peak at  $t = 901.27$  s and its corresponding single frequency component, as the phase delay of  $\pi/4$  that arises from the frequency integration is canceled by the phase advance of  $\pi/4$  that arises from the integration over azimuth.

### 3.5. Finite-Frequency Phase Shift in Two-Station Methods

Two-station methods are commonly used to measure phase velocities when the initial phase of the source is unknown (e.g., Brilliant & Ewing, 1954; Kolínský et al., 2019; T. Meier et al., 2004). It avoids the initial phase by measuring the travel time difference between two stations that are in line with the source. The difference can be captured by cross-correlating the surface waves on the two stations, whose spectrum has the following form,

$$C(\omega) = \exp\left(-i \frac{\omega \Delta}{c(\omega)}\right). \quad (28)$$

The narrowband surface waves can be obtained following the same procedure as in the ballistic and noise cross-correlation cases,

$$C(t) \approx \frac{1}{\pi} \Re \int_0^\infty \exp(i\omega t - ik\Delta) \exp\left(-\alpha \frac{(\omega_0 - \omega_c)^2}{\omega_c^2}\right) d\omega \approx \frac{B}{\pi} \omega_c \sqrt{\frac{\pi}{\alpha}} \Re \{ \exp(i(\omega t - k\Delta - \delta\phi)) \}. \quad (29)$$

Again, the finite-frequency phase shift is,

$$\delta\phi \approx \frac{\omega_c^2 k''(\omega_c) \Delta}{4\alpha}. \quad (30)$$

#### 4. Numerical Computation of the Finite-Frequency Phase Shift

To evaluate the finite-frequency phase shift accurately without the far-field and narrow-band assumptions, we now compute it numerically. The results can be used to correct the errors in phase-velocity measurements caused by this specific phase shift.

We compute the phase shift by comparing the measured phase travel times, obtained from surface-wave synthetics, to their far-field prediction, given by  $t_0 = \Delta/c(f_c) - 1/(8f_c)$ . The computation only needs a phase dispersion curve,  $c = c(f)$ . As for the “measured” phase travel time, we first generate a synthetic surface wave by superposing monochromatic surface waves. Here, we use the exact expression of the causal branch of monochromatic noise cross-correlation functions,

$$\sum_{i=0}^{N-1} w_i \Re \left[ \left( H_0^{(2)} \left( \frac{\omega_i \Delta}{c(f_i)} \right) \right) \exp(i\omega_i t) \right] df, \quad f_i = f_0 + idf, \quad (31)$$

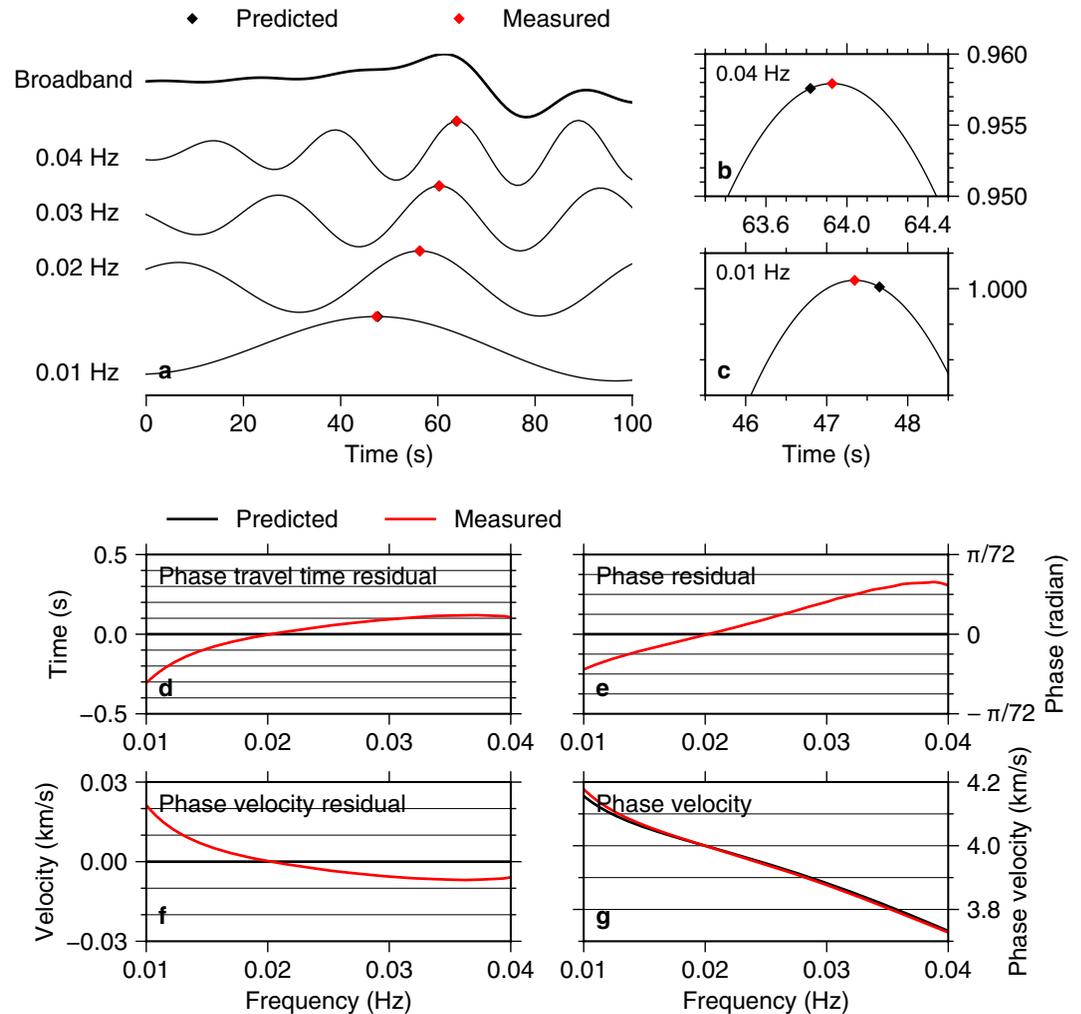
where  $H_0^{(2)}(x)$  is the zeroth-order Hankel functions of the second kind,  $\omega_i = 2\pi f_i$ , and  $w_i$  is the weighting coefficient determined by the integration method used. The results apply to ballistic surface waves as well. For two-station methods, one should replace the Hankel functions by cosine functions, but they share the same far-field patterns. We use the trapezoidal integration, with  $w_i = 0.5$  when  $i = 0, N - 1$  and  $w_i = 1$  when  $i = 1, \dots, N - 2$ . Next, we measure the phase travel time using the method of Xu et al. (2021), who used a frequency-dependent  $\alpha = 2\pi f \gamma^2$ , with  $\gamma$  being an empirical parameter (Soomro et al., 2016). Finally, we compute the difference  $\delta t$  between the measured and predicted phase travel times and multiply it by  $2\pi f_i$  to get the finite-frequency phase shift  $\delta\phi$ . The process should be repeated for each distance. For dense arrays, we can accelerate the computation by interpolating a phase shift table pre-computed at a set of distance. The phase velocity measurement method should be the same as the one used to obtain the real measurements so as to avoid introducing errors due to the difference in the measurement methods.

In order to correct the finite-frequency phase shift in a real phase-velocity dataset, we need a phase dispersion curve to compute the finite-frequency phase shift. A good start is the average dispersion curve or a reference model inverted from the dataset. We recommend the reference-model approach as the dispersion curve it generates is free of unphysical bumps. Once the phase shift  $\delta\phi$  is computed, one can correct the phase velocity by

$$\frac{1}{c} = \frac{1}{\tilde{c}} - \frac{\delta\phi}{\omega\Delta}, \quad (32)$$

where  $\tilde{c}$  and  $c$  are the phase velocity before and after the correction.

Figure 5 illustrates the computation of the finite-frequency phase shift for a station pair separated by 250 km. We used a phase dispersion curve calculated using ak135 to generate the broadband surface-wave synthetic (Herrmann, 2013; Figure 5a). We integrated from 2 to 50 mHz with an interval of 0.01 mHz. The phase travel time is measured on the narrow-band-passed waveforms. The resulting phase shift is mostly positive (phase delay; Figures 5b and 5e), except for low frequencies (Figures 5c and 5e). We found that this low-frequency anomaly is related to the near-field phase shift, which we discuss further in Section 5.1. The phase shift causes noticeable perturbations in the corresponding “measured” phase velocity curve (up to 0.02 km/s; Figures 5f and 5g).

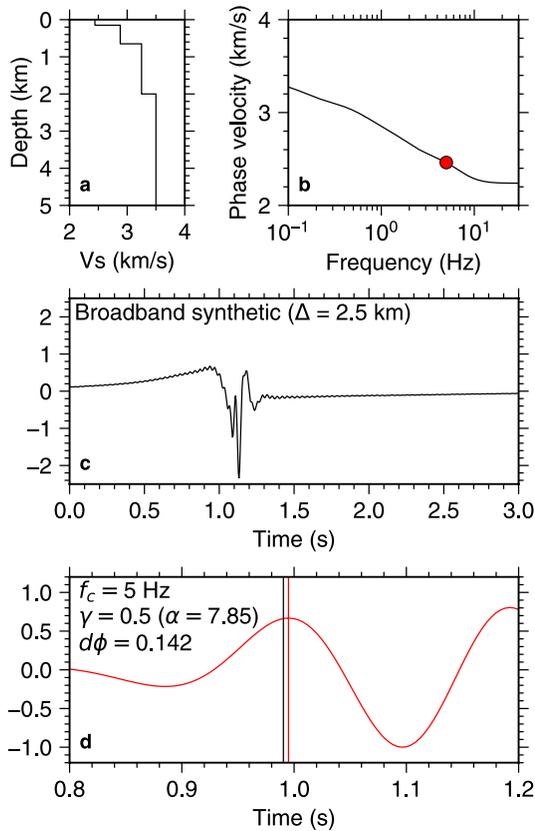


**Figure 5.** Numerical computation of the finite-frequency phase shift. (a) Broadband synthetic and narrow-band filtered synthetics. We use  $\gamma = 16$  to determine the width of the Gaussian filters, the same as Soomro et al. (2016) used for station pairs with similar distance ranges. Red and black diamonds denote the measured and predicted phase travel time, respectively. (b) Zoom-in around the measured phase travel time of the filtered synthetic around 0.04 Hz. (c) Same as (b), but for 0.01 Hz. (d) Phase travel time residual, the difference between the measured (red) and predicted (black) phase travel time. (e) Corresponding phase residual. (f) Corresponding phase velocity residual. (g) Measured and true phase velocity dispersion curve.

#### 4.1. High-Frequency Example (1–25 Hz)

We now compute the phase shift in a high-frequency scenario, relevant for the surface wave imaging of the shallow crust. We use this to demonstrate how to use the numerical method and to illustrate the factors that control the finite-frequency phase shift.

We use a model of the Dublin Basin, simplified from one obtained previously using receiver functions (Licciardi & Piana Agostinetti, 2017). The model has four layers and a half space below them (Figure 6a; Table 2). The P-wave velocity and density are converted from the S velocity using the formulas in Brocher (2005). After computing the Rayleigh-wave phase-velocity dispersion curve, we evaluate the phase shift for interstation distances from 0.5 to 4 km, at a 0.1 km step (Figure 6b). For each distance, we compute the synthetics using Equation 31 and a frequency range of 0.1–30 Hz, with an interval of 0.01 Hz (Figure 6c). We select 51 frequency points logarithmically distributed between 0.1 and 30 Hz as the center frequencies of the Gaussian filters. Figure 6d shows an example of the finite-frequency phase shift measured for the Gaussian filter with a center frequency of 5 Hz, which is 0.142 radians.



**Figure 6.** Example of the evaluation of the finite-frequency phase shift using a simplified model of the Dublin Basin. (a) The velocity model (only the top 5 km portion of the shear-wave velocity model is shown). (b) Rayleigh-wave phase-velocity dispersion curve. The red dot denotes the frequency (5 Hz) used in panel (d). (c) Broadband synthetic of the noise cross-correlation function at the interstation distance of 2.5 km. (d) Filtered waveform. The Gaussian filter is centered at 5 Hz, with  $\gamma = 0.5$ , or  $\alpha = 7.85$ . The finite-frequency phase shift is 0.142 radians. We used a smaller  $\gamma$  for a clearer illustration of the phase shift.

**Table 2**  
*Velocity Model of the Dublin Basin Simplified After Licciardi and Piana Agostinetti (2017)*

Layer no.	Layer thickness (km)	Vp (km/s)	Vs (km/s)	Density (g/cm <sup>3</sup> )
1	0.15	4.17	2.44	2.42
2	0.50	4.85	2.88	2.51
3	1.35	5.49	3.25	2.62
4	8.00	5.96	3.50	2.71
5	$\infty$	6.34	3.70	2.79

We use  $\gamma = 1$  to compute the finite-frequency phase shift over the distance range of 0.5–4 km and the frequency range of 1–25 Hz (Figure 7; a slightly smaller  $\gamma = 0.5$  is used, for clarity, in Figure 6c). The amplitude of the phase shift ranges from  $-0.015$  to  $0.026\pi$ , much smaller than the  $\pi/4$  phase shift caused by the source. The prominent features are the two high-amplitude anomalies around 5 and 1.4 Hz. These are the frequencies of the two peaks in the second derivative of the wavenumber,  $k''(\omega)$ , which confirms the effect predicted by the Equation 27. The effect of the distance is clear as well. The amplitudes of the two anomalies decrease with decreasing distance, from  $0.026\pi$  at 4 km to  $0.0019\pi$  at 0.6 km. Additionally, the phase shift increases linearly with frequency due to a combined effect of  $\omega^2$  and the frequency-dependent  $\alpha$ . It makes the 5-Hz anomaly stronger than the 1.4-Hz anomaly despite a smaller  $k''(\omega)$  at 5 Hz.

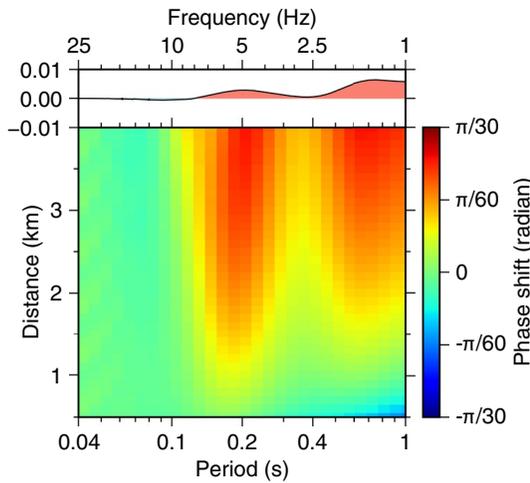
The corresponding phase-velocity perturbation ranges from  $-0.79\%$  to  $2.3\%$  and shows a pattern of variations with period and distance different from that for the phase shift (Figure 8). First, the sign of the phase-velocity perturbation is opposite to the phase shift. That is because a positive phase shift causes a delay in travel time, translating to a lower phase velocity. Second, the change of amplitude with distance disappears and the change with frequency is proportional to  $k''(\omega)$ . The reason is that the effects of distance and frequency cancel out when the phase shift is converted to phase velocity (Equation 3). Therefore, the variation of the phase-velocity perturbation only depends on  $k''(\omega)$ . It is noteworthy that the cancellation of the frequency dependence is because we used a frequency-dependent  $\alpha (=2\pi f\gamma^2)$ . If a constant  $\alpha$  is used in Gaussian filters, the frequency dependence remains. The positive anomaly observed in the lower-right corner of Figure 8 corresponds to low frequency and short distance or, equivalently, the near field (small  $\Delta/\lambda$ ). The distinctive feature is a result of the near-field effects rather than the finite-frequency phase shift, as we discuss further in Section 5.1.

The finite-frequency phase shift is larger if a smaller  $\gamma$  is used, when the width of filter widens to include more frequencies (Figure 9). The phase shifts range from  $-\pi/128$  to  $7\pi/128$  for  $\gamma = 0.5$ , with a group of filters centered at 0.05, 0.1, 0.2, and 0.5 s. The phase shifts decrease when  $\gamma$  increases and converge to a small (non-zero) value determined by the near-field effects. However, using a large  $\gamma$  in real-data applications may reduce the signal-to-noise ratio in the filtered surface waves, hence increasing errors in phase velocity measurements. Therefore, the problem of the finite-frequency phase shift cannot be solved by simply adopting a large  $\gamma$ .

#### 4.2. Long-Period Example

The phase shift can be observed in long-period measurements as well (Wielandt & Schenk, 1983). In this example, we use the phase-velocity dispersion curve computed for the global reference model ak135. The phase shifts computed here illustrate the errors that can be incurred in long-period measurements performed with time-domain methods, which are widely used for crustal and lithospheric imaging. We also discuss a technique—ridge jumping—that can improve the quality of measurements, especially at shorter periods.

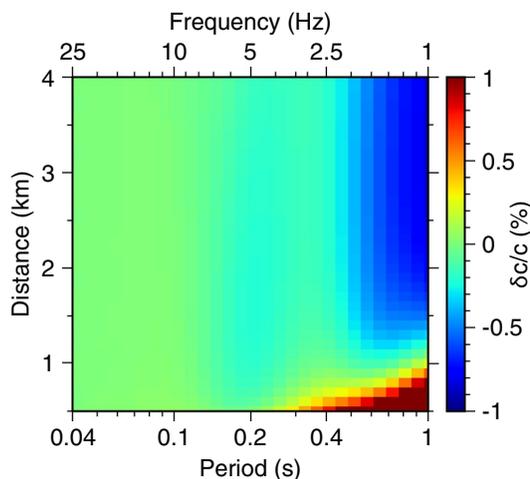
We evaluated the finite-frequency phase shift within the distance range of 100–3000 km (every 100 km) and the period range of 25–100 s. The broadband synthetic was computed by integrating over 2–50 mHz with an interval of 0.01 mHz, and the width of the Gaussian filters was set to  $\gamma = 16$



**Figure 7.** Phase shift (bottom) and the second derivative of  $k(\omega)$  (top). (Top) The red shaded region marks the positive  $k''(\omega)$ . (Bottom) The positive phase shift indicates phase delay; the negative phase shift—phase advance.

used in the surface wave studies. It can be large, however, at (a) long distance, (b) high frequencies, (c) with relatively wide-band filters, or (d) rapid changes with frequency in the group velocity curve. In terms of phase velocity perturbations, their dependences on the distance and frequency cancel out, so that only the bandwidth of the filters and the rate of group-velocity change—or, equivalently,  $k''(\omega)$ —have a significant effect. We can tune the bandwidth of the filters, but  $k''(\omega)$  is determined by the Earth's structure.

We also observed another type of phase shift, in addition to the finite-frequency phase shift. This phase shift dominates in the near field and is discussed below. We also show effects of inaccurate dispersion curves and non-flat amplitude spectra on the phase shift, and the effect of the finite-frequency phase shift on the inverted velocity models. Finally, we discuss whether the finite-frequency phase shift exists in the frequency-domain methods.



**Figure 8.** Phase velocity perturbation corresponding to the phase shift in Figure 7. The perturbation is with respect to the phase-velocity dispersion curve that is used to generate the synthetics.

(Soomro et al., 2016). We adopted the ridge jumping technique (Bonadio et al., 2018; T. Meier et al., 2004; Soomro et al., 2016; Xu et al., 2021), selecting high-amplitude ridges when scanning from low to high frequency, rather than staying on one ridge for the entire frequency band (Figure 10a). For the stability of the results, we only jump to the adjacent ridges and do so only when their amplitude is 1.5 times higher than that of the current ridge (Xu et al., 2021). The ridge jumping may produce discontinuities in the observed phase shift (Figure 10b), but it can reduce the phase shift at higher frequencies ( $>0.03$  Hz).

The phase shift for the model ak135 ranges from  $-\pi/44$  to  $\pi/16$ , if we use multiple ridges (Figure 10d). This is  $\sim 18$  times smaller than the values obtained when using the single, 0th ridge ( $-\pi/2$  to  $\pi/40$ ; Figure 10c). The corresponding velocity perturbation is up to 0.2% at interstation or source-station distances above 300 km (Figure 11). Below 300 km, the near-field effect becomes dominant and increases the perturbation to 0.9%.

## 5. Discussion

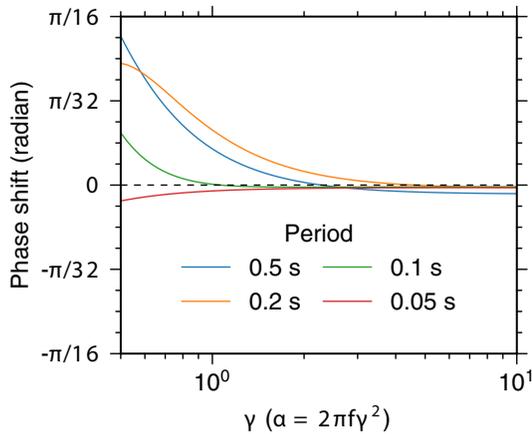
Using the calculations above, we found that the finite-frequency phase shift is relatively small in the range of distance, frequency, and  $\gamma$  that are commonly used in the surface wave studies. It can be large, however, at (a) long distance, (b) high frequencies, (c) with relatively wide-band filters, or (d) rapid changes with frequency in the group velocity curve. In terms of phase velocity perturbations, their dependences on the distance and frequency cancel out, so that only the bandwidth of the filters and the rate of group-velocity change—or, equivalently,  $k''(\omega)$ —have a significant effect. We can tune the bandwidth of the filters, but  $k''(\omega)$  is determined by the Earth's structure.

### 5.1. The Near-Field Phase Shift

The phase shift computed using the Dublin Basin model is negative at 1 Hz around the distance of 0.5 km, in contrast to the positive phase shift at the same frequency but longer distances ( $\Delta > 1$  km). Furthermore, it is seemingly inconsistent with the positive  $k''(\omega)$  at 1 Hz. The inconsistency is caused by a different type of phase shift, which we refer to as the near-field phase shift. The near-field phase shift arises from the far-field approximation, assumed when Bessel functions are replaced by cosine functions to represent monochromatic surface waves.

The phase shift computed by the numerical method described in Section 4 is the phase difference between the monochromatic cosine functions and narrowband Bessel functions. It comprises both the finite-frequency phase shift, caused by the difference between monochromatic Bessel functions and narrowband Bessel functions, and the near-field phase shift, caused by the difference between monochromatic cosine functions and monochromatic Bessel functions.

We separated the two types of phase shift in Figure 12. In the isolated finite-frequency phase shift, the negative anomaly no longer exists in the near-field corner, which is more consistent with the prediction by Equation 27



**Figure 9.** Phase shift as a function of  $\gamma$ . We compute the example using an interstation distance of 2.5 km and Gaussian filters centered at 2 Hz, 5 Hz, 10 Hz, and 20 Hz.

(Figure 12b). By contrast, the near-field phase shift is consistently negative and only visible in the near field (Figure 12c), which can be predicted by

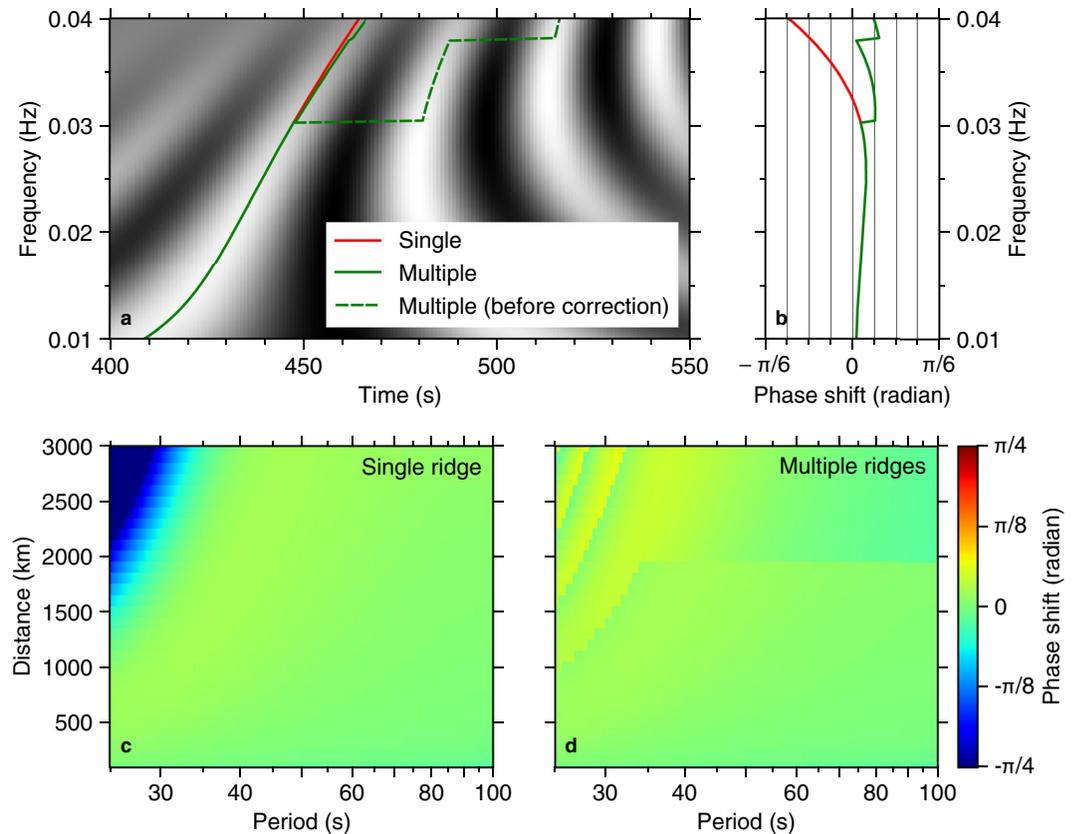
$$\delta\phi_{near} = -\arg H_0^{(2)}\left(\frac{\omega\Delta}{c}\right) - \left(\frac{\omega\Delta}{c} - \frac{\pi}{4}\right). \quad (33)$$

The phase of  $H_0^{(2)}\left(\frac{\omega\Delta}{c}\right)$  should be unwrapped to obtain a continuous phase, which approaches  $-\omega\Delta/c + \pi/4$  at high frequencies, decreasing  $\delta\phi_{near}$  to zero.

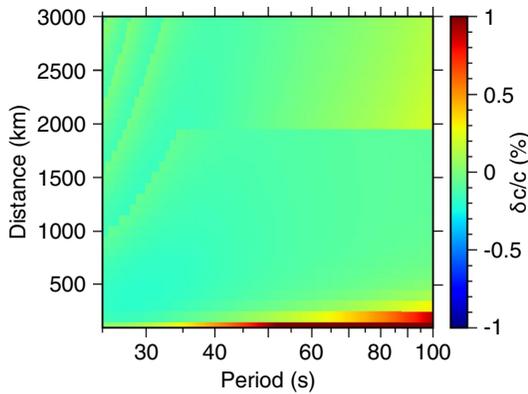
### 5.2. Effect of Inaccurate Dispersion Curves

In practice, measured dispersion curves are inaccurate because of the finite-frequency phase shift and random errors. Strong random errors in the measured dispersion curve can greatly reduce the accuracy in the computation of finite-frequency phase shifts, which relies on the second derivative of the dispersion curve. The problem can be resolved by inverting the noisy curve to a velocity model. Because of the smooth sensitivity kernel of surface waves, the synthetic dispersion curve computed from a velocity model is smooth. The smooth dispersion curve can then be used to compute the phase shifts.

Figure 13 shows such an example using the Dublin Basin model. The measured dispersion curve, despite being slightly slower than the true dispersion curve (Figure 13a), yields a good prediction of the finite-frequency



**Figure 10.** Phase shift in the measurements made on synthetic traces computed for ak135. (a) Frequency-time representation of the synthetic surface wave at 1750 km. Red and green solid lines denote the phase travel time measured using single ridge and multiple ridges, respectively. The green dashed line marks the ridge picked by the ridge jumping method. (b) Phase shift computed using the single-ridge and multiple-ridge phase travel time at the distance of 1750 km. (c) Phase shift computed using single ridges at the distance range of 100–3000 km. (d) Same as (c), but using the ridge jumping method.



**Figure 11.** Phase velocity perturbations corresponding to the phase shifts shown in Figure 10.

dispersion curve (Figure 13d), comparing to the result in Figure 7. Adding 1% of noise produces a wrong estimation (Figure 13e), but after a dispersion inversion, the inverted dispersion curve gives a reasonable good estimation (Figure 13f). We fit the noisy dispersion using a least-squares inversion, with a layered model and a relatively small damping parameter, 0.4, to penalize the roughness of the model. The thickness of the layers follows a logarithmic distribution, with the thinnest layer (5 m) at the surface and the thickest layer (250 m) at the bottom of the model (3 km depth).

### 5.3. Non-Constant Spectral Amplitude

Variations in spectral amplitude can bias the phase velocity measurement as well, in a similar way as in the group velocity measurements (Shapiro & Singh, 1999). Measured phase travel time will be biased toward the frequencies with higher amplitude. The effect can be demonstrated using the simplest example in Figure 2. We compare the phase travel time of the summation of the three monochromatic surface waves, at 0.99, 1, and 1.01 Hz, to the monochromatic surface wave at 1 Hz. The phase velocity at the three frequencies are 2.856, 2.853, and 2.850 km/s, respectively (Figure 14a).

We change both the amplitude and the curvature of the phase dispersion curve in this experiment. Amplitude variations of the three frequencies follow  $A \propto f^n$ , where  $A$  is amplitude,  $f$  the frequency and  $n$  a constant number. Variations in curvature are represented by second derivative of phase velocity with respect to the frequency  $f$ .

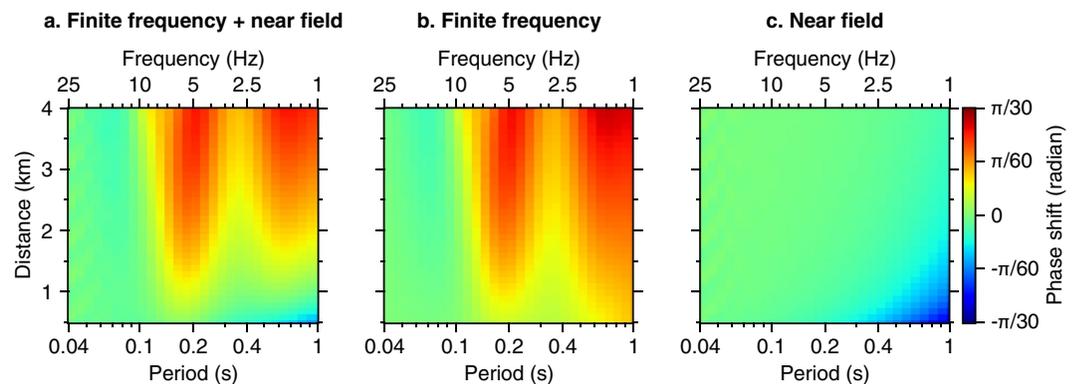
The result shows that amplitude decay leads to a smaller phase travel time, as the faster 0.99 Hz has a higher weight. Bias caused by variations in phase velocity and spectral amplitude can superpose either constructively or destructively.

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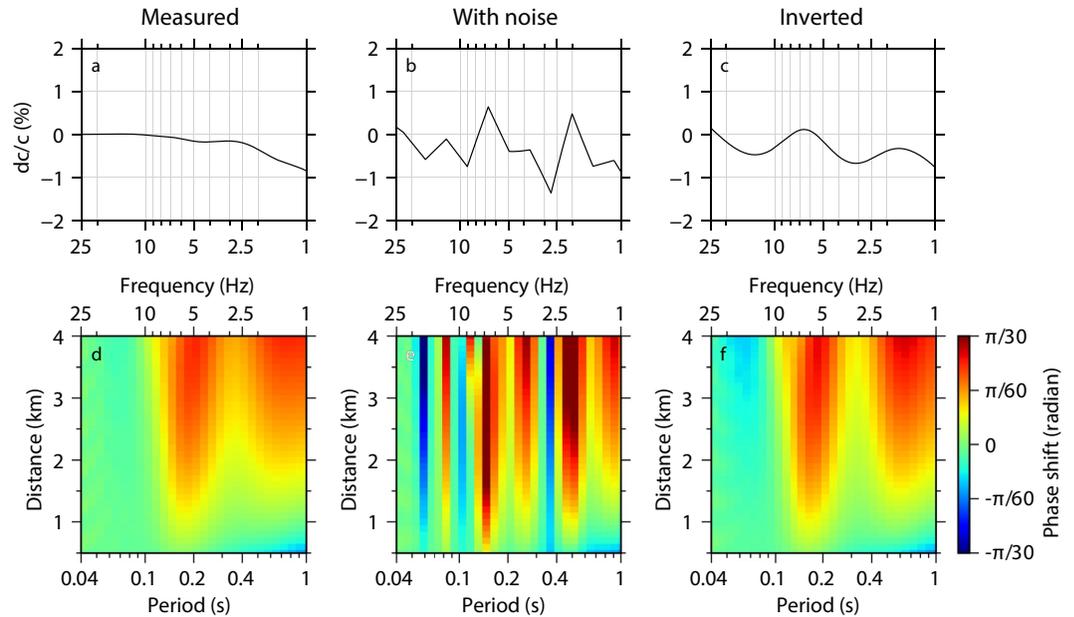
### 5.4. Effect on Velocity Models

Finite-frequency phase shift can cause noticeable changes in the inverted velocity models, once other errors in the measurements become small enough. Here we demonstrate the difference in velocity models in both the high-frequency and long-period examples.

Two dispersion curves are inverted for each example, with exactly the same inversion settings, for example, initial models and damping parameters. The first dispersion curve is computed directly from the reference velocity model, specifically, the Dublin Basin model for the high-frequency example and ak135 for the long-period example. The second dispersion curve is measured from synthetic surface waves generated using the first dispersion



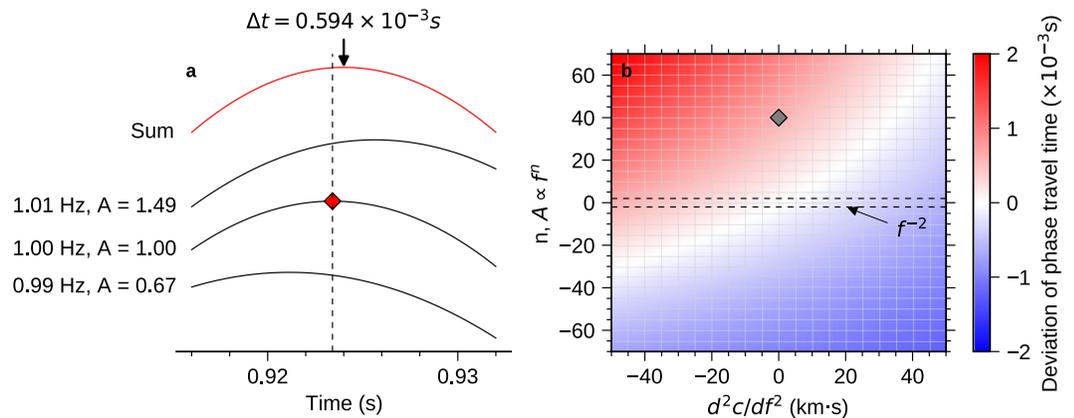
**Figure 12.** The origins of the finite-frequency and near-field phase shifts. The frames show the phase differences (a) between narrow-band Bessel functions and monochromatic cosine functions; (b) between narrow-band and monochromatic Bessel functions; and (c) between monochromatic Bessel functions and monochromatic cosine functions. The phase of monochromatic cosine functions is  $\omega/\Delta - \pi/4$ . The phase of narrow-band ones is measured using synthetics.



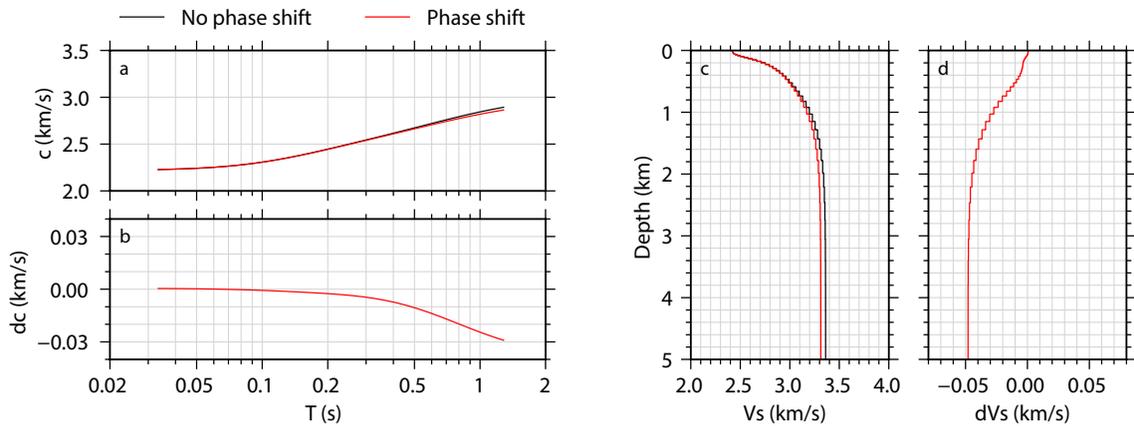
**Figure 13.** Phase shift computed by inaccurate dispersion curves. (a) Relative difference between the measured dispersion curve and the synthetic dispersion curve. (b) Measured dispersion curve with random noise of 1%. (c) Best-fit dispersion curve to the one in (b) using 1-D dispersion inversion. (d) Phase shifts computed using the measured dispersion curve in panel (a). (e) Phase shifts computed using the noisy dispersion curve in panel (b). (f) Phase shifts computed using the inverted dispersion curve in panel (c).

curve. The two curves have different velocities because of the finite-frequency phase shift and near-field phase shift. We cut the dispersion curves at 1.43 and 200 s, respectively, to remove influence of the near-field phase shift.

The seismic velocity models yielded by inversion of the dispersion data show consistent difference (Figures 15 and 16). In the high-frequency example, the shear-wave velocity in the two models has a difference of 0.045–0.48 km/s below 2 km depth. In the long-period example, a similar velocity difference, 0.045 km/s, is observed, below 300 km depth. They translate to 1.4% and 0.95% in terms of relative velocity changes, respectively. Interestingly, errors in shear wave velocities in the two examples are in the same order of magnitude, capped at 0.05 km/s, despite the huge difference in the frequency ranges. Errors in phase velocities, capped at 0.03 km/s



**Figure 14.** Deviation of phase travel time for non-uniform spectral amplitude. (a) Linear dispersion curve but with non-uniform amplitude (labeled on the left end of each line). (b) A map of deviation of phase travel time with respect to the decay rate of the amplitude and the second derivative of  $c(f)$ . The decay rate of  $f^{-2}$  corresponds the roll-off rate of displacement spectrum of regular earthquakes. The gray diamond at  $n = 40$  corresponds to the example in panel (a).



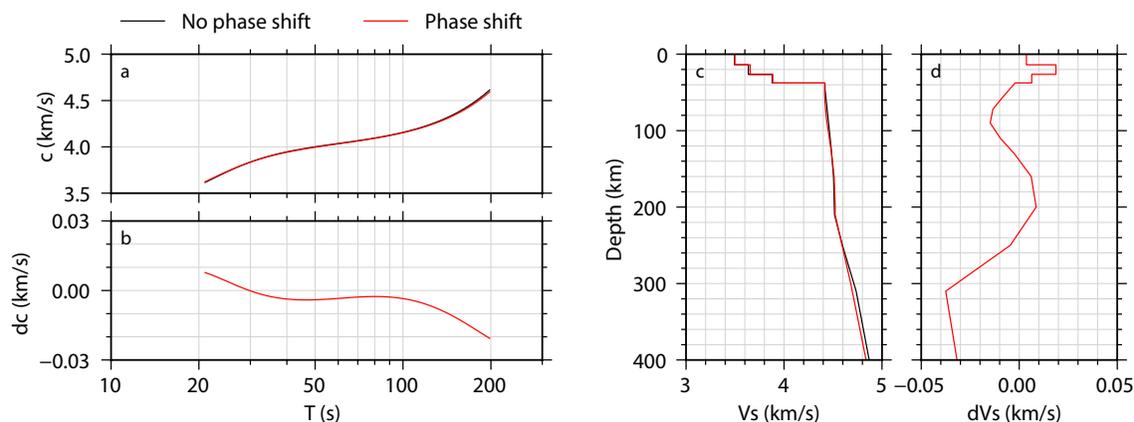
**Figure 15.** Comparison of models with and without the finite frequency phase shift in a high-frequency scenario. (a) Dispersion curves with and without finite-frequency phase shift. (b) Difference between the two dispersion curves. (c) 1-D shear wave velocity models inverted from the two dispersion curves in panel (a). (d) Difference between the two models.

(high-frequency) and 0.02 km/s (long-period), are also at the same level. It may suggest that the effect of the finite-frequency phase shift on the velocity models does not scale up with frequency and distance, which is consistent with Figures 8 and 11 that show phase velocity perturbations is independent of frequency and distance, except  $k''(\omega)$ .

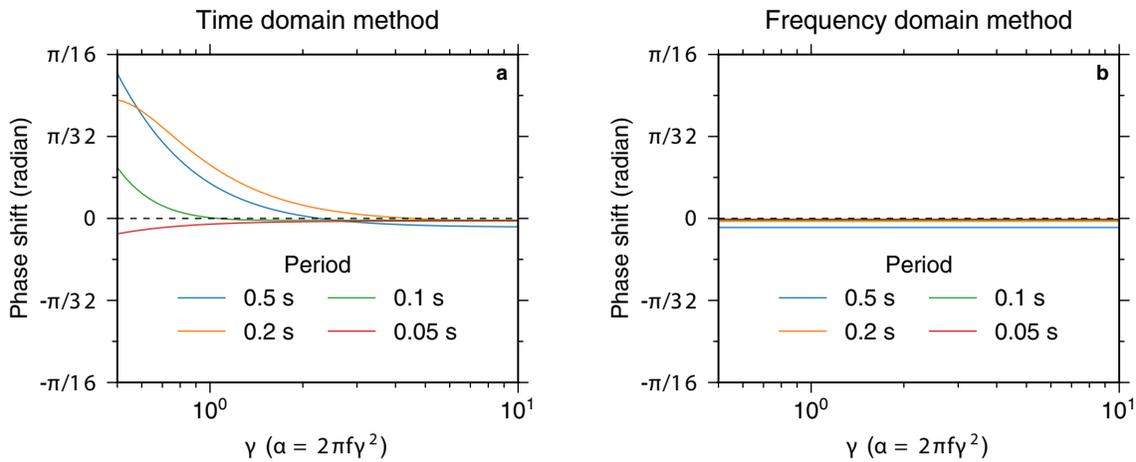
### 5.5. Time-Domain Method Versus Frequency-Domain Method

The finite-frequency phase shift only applies to measurement methods that use narrow-band waveforms to measure the phase travel time in the time domain. Frequency-domain methods (e.g., phase unwrapping method; T. Meier et al., 2004; Sato, 1955; Soomro et al., 2016) do not have such phase shift. The difference between the two methods is that the phase unwrapping method measures the phase of the spectrum at the center frequency, so that its results are monochromatic measurements, instead of narrow-band measurements. The measured phase is then unwrapped, converted to the phase travel time, and compared to the predicted phase travel time as in the time domain method.

Figure 17 shows a comparison of the phase shift associated with the time domain method and the phase unwrapping method. We can see that the phase shift associated with the frequency domain method is at constant values for different  $\gamma$ . These constant values are not zero, which reflect the near-field phase shift.



**Figure 16.** Comparison of models with and without the finite frequency phase shift in a long-period scenario. Same as Figure 15, but with dispersion curves at long periods.



**Figure 17.** Phase shift as a function of  $\gamma$  for (a) the time-domain method and (b) the frequency-domain method.

The frequency-domain method is an attractive alternative to time-domain ones as it is free of the finite-frequency phase shift. The time-domain methods, however, remain very popular due to their simplicity. We note that the near-field phase shift is present regardless of the approach chosen.

## 6. Conclusions

The finite-frequency phase shift arises in the phase velocity measurements obtained using the time-domain methods due to the approximation of the monochromatic surface waves by narrow-band surface waves. It applies to both ambient noise studies and those using ballistic surface waves from earthquakes or active sources. The phase shift biases the measured phase travel time and, hence, the measured phase velocity and the resulting velocity models. The finite-frequency phase shift can be up to  $\pi/4$  when using a very broadband filter and down to nearly zero when using a very narrowband filter. The exact amount depends on four factors: (a) the second derivative of the wavenumber  $k$  with respect to  $\omega$ ; (b) the width of the filter; (c) the distance; and (d) the center frequency of the Gaussian filter. For common settings in surface wave studies, the phase shift is about  $\pi/60$  to  $\pi/16$ , much smaller than the polar phase shift (phase advance of  $\pi/2$ ) and source phase shift (phase delay of  $\pi/4$ ), identified previously.

The finite-frequency phase shift can cause non-negligible phase-velocity perturbations of around 0.2% in long-period (25–100 s) studies and around 0.8% in high-frequency (1–25 Hz) studies. The exact amount can vary depending on the filter parameters and local Earth structure. If the acceptable level of errors in the phase velocity measurements is comparable to or lower than the finite-frequency phase shift, the shift should be corrected.

The finite-frequency phase shift, and its corresponding perturbation in phase velocities, can be corrected once having an estimation of the phase shift. A dispersion curve, computed from the 1-D velocity model inverted from an average dispersion curve or from the entire dataset of dispersion curves, can be used to compute a good estimation of the phase shift. It can be done by comparing the measured phase travel time in surface wave synthetics generated by the dispersion curve to the far-field prediction of the phase travel time. The approach can also correct the near-field phase shift when using Bessel functions instead of cosine functions in generating synthetics.

Because the finite-frequency phase shift results from the monochromatic approximation, it does not exist in the frequency-domain methods that do not rely on the approximation. In this sense, switching to frequency-domain methods is an alternative approach for increasing the accuracy of phase-velocity measurements.

## Data Availability Statement

The velocity models used in the paper and codes for computing the finite-frequency phase shift and reproducing some of the figures in this paper are publicly available (Xu, 2021; <http://doi.org/10.5281/zenodo.5718901>).

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