Corrigendum to: On Translating Between Logics

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The proof in Dewar (2018) that classical and intuitionistic logic are not inter-translatable is defective. The proof claims that up to logical equivalence, there are only three non-trivial one-place schemata in intuitionistic logic ($\lambda\phi.\phi$, $\lambda\phi.\neg\phi$ and $\lambda\phi.\neg\neg\phi$). This is false, for two reasons. First, we could include arbitrary further propositional constants in a one-place schema: for instance, the schema $\lambda\phi.(\phi \vee P)$ is still a one-place schema. Second, even if we restrict our attention to one-place schemata that do not include any propositional constants, there are still infinitely many (logically inequivalent) schemata: this follows from the result of Nishimura (1960) that for any propositional constant, there are infinitely many intuitionistically inequivalent formulae containing only that propositional constant. However, the result still stands, as the following proof demonstrates.

Proof. For contradiction, suppose that $\mathcal{C}$ and $\mathcal{I}$ are intertranslatable*, via translations* $\tau : \mathcal{C} \to \mathcal{I}$ and $\sigma : \mathcal{I} \to \mathcal{C}$ induced by schematic interpretations (respectively) $\mathrm{T}_\bullet$ and $\Sigma_\bullet$. Nishimura (1960) demonstrates that any formula of one propositional constant A is equivalent (intuitionistically) to some 'basic formula' whose only connectives are $\neg$, $\to$, and $\vee$. Let us suppose that in the schema $\Sigma\alpha$ there occur the propositional constants

$B_1, \ldots, B_m$; that in the schema $\Sigma_\neg$ there occur the propositional constants $C_1, \ldots, C_n$; that in the schema $\Sigma_\rightarrow$ there occur the propositional constants $D_1, \ldots, D_p$; and that in the schema $\Sigma_\vee$ there occur the propositional constants $E_1, \ldots, E_q$.

It follows that for any intuitionistic formula whose only propositional constant is A, the formula $\sigma(A)$ may contain at most the propositional constants A, $B_1, \ldots, B_m$, $C_1, \ldots, C_n$, $D_1, \ldots, D_p$, $E_1, \ldots, E_q$. But in classical logic, there are (up to logical equivalence) only finitely many such formulae. Since (up to intuitionistic equivalence) there are infinitely many formulae whose only propositional constant is A, there must exist formulae ϕ, ψ whose only propositional constant is A which are intuitionistically inequivalent and yet for which $\sigma(\phi)$ and $\sigma(\psi)$ are classically equivalent. But then, since translations* are required to preserve consequence, $\tau(\sigma(\phi))$ and $\tau(\sigma(\psi))$ must be intuitionistically equivalent. If τ and σ really do witness the intertranslatability* of $\mathcal{C}$ and $\mathcal{I}$, then it follows that ϕ is intuitionistically equivalent to ψ; so we have a contradiction, and the result follows. $\square$

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References

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