

Scope for Cost Minimization in Public Debt Management: the case of the UK

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Abstract

This paper provides a framework for an empirical analysis of the scope for cost minimization in public debt management. It assumes that a debt manager aims at minimizing the expected cost of government's debt portfolio for a given level of short term interest rate and subject to a number of risk and market impact constraints. The analysis is applied to the UK government debt over the period April 1985 to March 2000, by simulating "real time" interest costs of alternative portfolios constructed using monthly forecasts of return spreads based on recursive modelling (RM) procedure recently developed by Pesaran and Timmermann (1995, 2000), which limits the extent of data snooping. Statistically significant evidence of predictability of return spreads are provided before the introduction of reforms of the UK debt management system in 1995, although there seems to be little evidence of predictability once the post reform sample is included. Nevertheless, there appears to have been some scope for a *small* reduction in interest costs over the 1985-2000 period even if portfolio shares and their monthly changes are constrained to lie within historically observed upper and lower bounds in order to minimize the market impact effects of such changes.

Keywords: Public debt management, cost minimization, recursive modelling, data snooping
JEL Classifications: E17, E44, G12, H63.

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1 Introduction

One of the important issues in public debt management is the possibility of reducing interest costs through changes in the maturity structure of the debt without distorting financial markets and endangering the effectiveness and efficiency of monetary policy. The cost minimization objective has been at the heart of many debates on public debt management worldwide. Its importance has been stressed repeatedly by the UK debt management authorities. For example, the 1990 HM Treasury's *Financial Statement and Budget Report* contains the following statement regarding debt management:

- “(a) [debt management] must support and complement monetary policy in pursuit of the Government’s objectives for inflation;
- (b) subject to (a), it should operate in a way which avoids distorting financial markets;
- (c) subject to (a) and (b), it should be conducted at least cost and risk.” HM Treasury (1990, p. 23)

More recently, Angela Knight, Economic Secretary to the Treasury, declares that:

“[M]y job is to minimise [interest] cost, by making sure that we manage our debt in an efficient and effective way.” HM Treasury (1995, p. 1)

The possibility of reducing interest costs by manipulating the average maturity of the debt portfolio has also received particular attention in the US over recent years. In the early 1990s, the Clinton Administration argued that this approach would significantly lower interest costs (see Congressional Budget Office, 1993, p. 70). As a result, in May 1993 the US Treasury cut back its sales of long maturity bonds and switched approximately \$55 billion into shorter maturities.¹ Both Campbell (1995) and Hall and Sargent (1997) note that the early 1990s’ average maturity reduction was aimed at exploiting historical patterns in interest costs and that these would not necessarily be repeated in the future. Nevertheless, US debt management authorities continue to regard shortening the average maturity as a cost-effective strategy. For example, in a press release which introduced the US Treasury’s debt buybacks program, the Treasury Secretary, Lawrence Summers (2000), claims that:

“...by paying off debt that has substantial remaining maturity, buybacks enable us to prevent what would otherwise be a potentially costly and unjustified increase in the average maturity of our debt ...”²

The cost minimization objective is also an important consideration in the World Bank-IMF “Guidelines for Public Debt Management” (2001, p. 9):

¹According to Hall and Sargent (1997, p16) this approach was supported by Alan Blinder. In 1992, he argued that the US Treasury should “...painlessly pare billions from its interest bill by refinancing the government’s existing debt with bonds that mature more quickly”.

²The World Bank-IMF Guidelines (2001, p19) list “excessive reliance” on short-term debt as a common pitfall of debt management but do not imply that the US strategy was excessively reliant. For many (poorer) countries, short maturity debt might result in very volatile interest costs. See the discussion of debt management objectives below.

“The main objective of public debt management is to ensure that the government’s financing needs and its payment obligations are met at the lowest possible cost over the medium to long run, consistent with a prudent degree of risk.”

Arguments for cost minimization have also been advanced in the academic literature, for example, by Tobin (1971), who further emphasizes the importance of debt management in the macro economic stabilization process through its possible effects on long-term interest rates. A related reference is Modigliani and Sutch (1966). A comprehensive review of the debt management literature is provided by Missale (1999).

In this paper, we examine the scope for cost minimization in public debt management using a methodology that limits the impact of data snooping. We consider a debt manager who aims to minimize the expected cost of the government’s bond portfolio using publicly available information and is subject to a number of constraints aimed at limiting the potential market impacts of changes in the maturity structure of the debt. We shall also take the short term interest rate as given, thus avoiding possible conflicts in monetary policy objectives as reflected in the short term rate and the cost minimization objective. To limit the effects of “data snooping” on our analysis we shall use the recursive modelling approach advanced in Pesaran and Timmermann (1995, 2000) for the analysis of stock market predictability.

This approach explicitly acknowledges the biases associated with *ex post* specification searches and advocates the use of *ex ante* model selection techniques where forecasting equations are selected recursively from a given set of base regressors fixed at the start of the analysis. The recursive analysis will be conducted for four of the main model selection criteria frequently used in the literature, namely Theil’s (1958) \bar{R}^2 , Akaike’s (1973) information criterion (AIC), Schwarz’s (1978) Bayesian criterion (SBC) and Hannan and Quinn’s (1979) criterion (HQC). In this way we avoid possible biases due to *ex post* searches across model selection criteria which tend to augment the data snooping problem in practice.

In our particular application of the recursive modelling approach, we focus on forecasting equations for return spreads, defined as the differences in holding period returns on bonds of different maturities. We shall consider three maturity bands, namely “short” (less than 7 years), “medium” (7 – 14 years) and “long” (more than 14 years), and consider two return spreads, namely the returns on the medium and long term bonds relative to the return on the short term bond. Forecast equations are selected recursively for both of these two return spreads simultaneously, as compared to the single excess return regressions usually considered in the finance literature. These forecasts exploit statistical patterns that exist between the return spreads and a variety of publicly observed macroeconomic variables. A base set of forecasting variables is established and, at the start of every time period (month), the best fitting regression is selected for each return spread by searching across all possible models. Model choice is based upon a pre-defined criterion and selection utilizes only information in the public domain.

To our knowledge, this is the first application of the recursive modelling approach which examines the time-varying relationships between return spreads and business cycle indicators. Using UK data, we find statistical and economic evidence of predictability using a common set of macroeconomic factors. This finding is robust to the choice of the model selection criterion. Taking the SBC as an example, we predict correctly the maturity band with the minimum holding period return for 51%

of the months in our evaluation period from April 1985 to March 2000. We find that the predictive power of business cycle variables for return spreads is time dependent. Again using the SBC, the variables selected most often include lagged changes in the UK Treasury Bill rate, the yield spread between short and long term US government debt, and the growth rate of industrial production.

The recursive forecasts obtained for different model selection criteria are then used in the simulations of interest rate costs under alternative restrictions on the debt portfolio. For plausible values of the parameters that describe restrictions on portfolio shares we find the scope for interest cost reductions relative to the UK's historical experience to be rather small. For example, after we impose what we consider to be realistic constraints on the share of the debt that can be re-allocated across different maturities at the start of each month we find that only modest cost savings (compared to the UK historical experience) can be achieved, namely of the order of £150 million per year on a portfolio of £99 billion. This finding is reasonably robust across different model selection criteria.

The rest of this paper is organized as follows. The next section considers a framework for cost minimization as a debt management objective. In section 3, recursive modelling and UK bond portfolio simulations are discussed. Some general conclusions are drawn in the final section.

2 A Framework for Expected Cost Minimization

In practice, debt managers are concerned with a number of issues which make unconstrained cost minimization undesirable. First, debt management policy can affect the other aspects of government macroeconomic policy, namely monetary and fiscal policies. The literature on monetary credibility emphasizes the impacts of debt management on the incentive to inflate.³ The literature on tax smoothing shows that debt instruments can be used to smooth tax distortions.⁴

Second, debt portfolios that are heavily concentrated in particular maturities might be exposed to the risk of interest rate volatility. Cost minimizing portfolios may result in interest costs that vary greatly with the business cycle.

Third, (even if macroeconomic policies are separable) large switches in the debt portfolio may affect interest rates. The theory of preferred habitats predicts that interest rates are affected by the supply of bonds. There are two potential effects: the immediate response by prices to the supply change and (assuming risk averse investors) the impact on any risk premium.⁵ In the former case, the debt manager faces a difficulty familiar to any non-atomistic agent that conducts trades in an asset market—the market will move against the agent.

In our counterfactual experiments, we adopt a pragmatic approach to these many practical debt management concerns. Clearly, for shares that are very close to the debt manager's actual portfolio at all times, these issues are of minor importance. On the other hand, as the above arguments make clear, cost minimizing portfolios that deviate radically from actual portfolios are unlikely to be preferred. But the critical deviation at which these many concerns dominate the (short-term) cost consideration is unknown.

Many of the above considerations can be formalized using a constrained optimization approach.

³See, for example, Fischer (1983), Calvo and Guidotti (1992) and Favero, Missale and Primiceri (1999).

⁴See, for example, Barro (1979) and Bohn (1988, 1990).

⁵See Taylor (1992), Egginton and Hall (1994) and Hess (1999) on supply effects in UK bond markets.

Consider a government debt portfolio composed of $N + 1$ bonds of different maturities, indexed in the order of their maturities by $i = 0, 1, \dots, N$. The total holding cost of the portfolio (excluding transaction and administrative costs) in period t is given by

$$C_t = \sum_{i=0}^N B_{it} R_{it}, \quad (1)$$

where B_{it} is the market value of the debt of maturity type i at time t , and R_{it} is the associated holding period cost (return) per unit of debt of type i .

Suppose that the debt manager is concerned with the expected cost of managing a given (pre-specified) amount of government debt,

$$B_t = \sum_{i=0}^N B_{it}, \quad (2)$$

with a given short term rate of interest, R_{0t} , set by monetary authorities, whilst at the same time taking account of the likely adverse impacts that an active debt management policy might have on the term structure of interest rates.

To isolate the effect of absolute changes in B_t from changes in the debt portfolio we use (2) and write (1) as

$$c_t = \sum_{i=0}^N w_{it} R_{it},$$

where $w_{it} = B_{it}/B_t$, and $c_t = C_t/B_t$. Since $\sum_{i=0}^N w_{it} = 1$, c_t can also be written as

$$c_t = R_{0t} + \sum_{i=1}^N w_{it} r_{it},$$

where

$$r_{it} = R_{it} - R_{0t}. \quad (3)$$

Since R_{0t} (set by monetary authorities) is assumed given, the part of c_t that can be minimized is given by

$$\rho_t = \sum_{i=1}^N w_{it} r_{it} = \mathbf{w}'_t \mathbf{r}_t, \quad (4)$$

where $\mathbf{w}_t = (w_{1t}, w_{2t}, \dots, w_{Nt})'$ and $\mathbf{r}_t = (r_{1t}, r_{2t}, \dots, r_{Nt})'$. In deriving the portfolio weights, \mathbf{w}_t , we assume that the debt management authorities solve the following optimization problem:

$$\min_{\mathbf{w}_t} \left[\mathbf{w}'_t E(\mathbf{r}_t \mid \mathbf{I}_{t-1}) + \frac{\lambda}{2} \mathbf{w}'_t \text{Var}(\mathbf{r}_t \mid \mathbf{I}_{t-1}) \mathbf{w}_t \right], \quad (5)$$

subject to the following constraints:

$$0 \leq w_{it} \leq 1, \quad i = 0, 1, \dots, N, \quad w_{0t} = 1 - \sum_{i=1}^N w_{it} \geq 0, \quad (6)$$

$$\sum_{i=1}^N w_{it} \leq 1, \quad (7)$$

$$w_{it} \in [\underline{w}_i, \bar{w}_i], \quad i = 0, 1, 2, \dots, N, \quad (8)$$

and

$$(1 - \alpha)w_{i,t-1} \leq w_{it} \leq (1 + \alpha)w_{i,t-1}, \quad i = 0, 1, 2, \dots, N, \quad (9)$$

where \mathbf{I}_{t-1} is the publicly available information set, $E(\mathbf{r}_t | \mathbf{I}_{t-1})$ is the $N \times 1$ conditional mean return spread vector, $Var(\mathbf{r}_t | \mathbf{I}_{t-1})$ is the associated conditional variance matrix, assumed as given, and α is a small positive constant. This formulation is a mirror image of the familiar efficient portfolio problem in finance. The risk parameter λ captures the relative weight attached by the debt manager to the interest cost volatility, and has a similar status as the risk aversion coefficient.⁶ The first constraint, (6), reflects the fact that debt agencies do not take part in short selling of assets. (7) is the standard adding up restriction. The last two constraints are intended to limit the market impacts of changes in the maturity structure of the debt. Constraints (8) and (9) restrict the the proportion of debt in each maturity to lie within certain pre-specified bounds. (8) imposes universal bounds, motivated by institutional considerations. For example, for continuity of markets in certain maturity bands it might be desirable not to allow their share to fall below certain minimum (historical) values. The rationale behind (9) is altogether different and aims at limiting market impacts of marginal re-allocations across maturities and applies equally to purchases and sales of bonds of a particular maturity.

These constraints apply to new issues and retirements as well as to maturity changes carried out by the debt manager in the pursuit of lower interest costs. One can think of the debt manager as making two types of intervention in each period. The first consists of moving existing debt between maturities motivated by interest costs. The second type of intervention is related to satisfying the government's budget constraint. That is, issuing debt when the government runs a deficit and buying back debt when it runs a surplus. The parameter α represents a threshold beyond which cross maturity trades would be regarded as market distorting. Plausible choices for \underline{w}_i , \bar{w}_i and α , will be discussed below.

In the UK application that follows, for each of our candidate debt management strategies, we calculate the (time averaged) interest costs from our simulated portfolios in each month (weighted by share of market value) and compare them with those resulting from the UK's actual portfolio. The difference captures the scope for reducing interest costs under alternative forecast strategies and constraints.

⁶In principle, the loss function could encompass all of the many aspects of debt management. The case studies included with the World Bank-IMF Guidelines (2001) reveal that in practice debt managers' objectives are quite parsimonious. The World Bank-IMF's own interpretation of debt management objectives appears to be representative.

3 Recursive Modelling of the Term Premia and UK Bond Portfolio Simulations

Within our framework, the evaluation of alternative debt management strategies requires forecasts of the return spreads and their volatilities. It is also important that, as far as possible, such forecasts are not subject to the data snooping problem. To generate the point forecasts of the return spreads we utilize an adaptation of the recursive modelling approach developed in Pesaran and Timmermann (1995, 2000). The analysis assumes that the expected holding period returns (and return spreads) vary over the business cycle and can be (partly) forecast using a number of pre-specified business cycle indicators.⁷ Although, it is widely acknowledged that, in principle, return spreads are predictable using macroeconomic indicators, there is no consensus how this could be achieved *a priori*. Given the possibility of technological change and switches in policy regimes, it is very unlikely that a given forecasting model could be applicable at all times.⁸

We model the behavior of the UK debt management authorities as searching recursively at the start of each month for the “best” forecasting equations.⁹ Given the data available to us we shall consider UK government debt aggregated by maturity into three broad maturity bands, namely less than 7 years, between 7 to 15 years and more than 15 years.¹⁰ We denote these as “short”, “medium” and “long” dated maturity bonds. The observations are monthly and cover the period April 1980 to March 2000.¹¹ The period April 1980 to March 1985 is used as the training sample, and all evaluations (forecasts and portfolio simulations) are based on an expanding sample from April 1985 to March 2000 (inclusive).

Table 1 shows the sample means and standard deviations of the holding period returns for the three maturities over various sub-periods. Sample statistics for the UK’s actual portfolio shares (by market value) appear in Table 2. A time plot of these portfolio shares is displayed in Figure 1. Three characteristics of the portfolio shares are particularly worth emphasizing. First, the portfolio is often concentrated heavily in the short maturity band but it is never “all short”. Second, for

⁷The idea that bond returns vary over the course of business cycle has a long standing in the literature and can be traced back at least as far as Lutz (1940).

⁸In the case of the UK, a review of the post-1980 issues of The Bank of England Quarterly Bulletin confirms that the UK debt management authorities were aware of these statistical relationships. (The Bank of England and HM Treasury were jointly responsible for debt management prior to the establishment of the UK Debt Management Office.) We note the first reference to each macroeconomic variable used in our UK application in the Data Appendix. Neither the Report of the Debt Management Review (HM Treasury, 1995) nor the relevant issues of The Bank of England Quarterly Bulletin indicate that the UK authorities used a particular model for these underlying relationships.

⁹Pre-1998, the Bank of England was responsible for UK debt management, acting together with, and as an agent for HM Treasury. In April 1998, the responsibility was transferred from the Bank of England to the UK Debt Management Office (an executive agency of HM Treasury). This shift in responsibility was part of the new Labour Government’s attempt to separate responsibility for monetary and fiscal policies. For simplicity, we refer to the various government organizations involved as the “debt management authorities”.

¹⁰Index-linked bonds were excluded since their markets were rather thin in the early part of our sample. Partly-paid and floating rate bonds were also omitted, as were bonds with negligible amounts outstanding (designated “rumps”). Unfortunately, we were not provided with data aggregated by duration or by alternative maturity bands.

¹¹The (end-month) bond data were provided by the UK Debt Management Office. The Data Appendix contains further details.

much of the sample, the share of longs is very low but always non-zero. Third, the amount by which the shares change in each month is fairly small and stable. Over the period March 1985 to February 2000, the average (absolute value) change in the (market value) share of the debt in the short maturity band is 2.0% per month. For the medium and long maturity bands these figures are a little higher at 2.5% and 3.7%.¹² The limited nature of the changes observed in the maturity structure presumably reflect (expected) costs of market impacts and disruptions associated with large movements in the debt portfolio. Although it is clear from Table 1 that, in general, short bonds have the lowest holding period returns, this is not the case in all sub-periods. Therefore, a strategy that had aimed at reducing the average debt maturity would not have always resulted in lower interest costs.

In view of the data available to us our UK application focuses on the following two return spreads

$$r_{mt} = R_{mt} - R_{st} \quad \text{and} \quad r_{lt} = R_{lt} - R_{st}, \quad (10)$$

where R_{st} , R_{mt} and R_{lt} denote the holding period returns over the period $t - 1$ to t for short, medium and long maturity bonds, respectively. Under recursive modelling, the vector of return spreads, $\mathbf{r}_t = (r_{mt}, r_{lt})'$, $t = t_0 + 1, t_0 + 2, \dots, T$ is modelled at each point in time in terms of a base set of regressors contained in the publicly available information set, \mathbf{I}_{t-1} .¹³ The recursive one-step ahead conditional forecasts of the two return spreads, r_{mt} and r_{lt} , for $t = t_0 + 1, t_0 + 2, \dots, T$ are used in the cost minimization exercise. Recursive modelling extends recursive estimation, a frequently used technique, by allowing the prediction model to vary over time. At each point in time a forecasting model is chosen from a set of available models spanned by an *a priori* chosen base set, and this process is repeated for all the time periods, $t_0 + 1, t_0 + 2, \dots, T$. Unlike recursive estimation, which simply updates the parameters of a given model, recursive modelling admits the possibility of model change and provides an *automated* search procedure. This automated approach reduces considerably, but does not eliminate, the “data mining” or “data snooping” problems highlighted, for example, by Sullivan, Timmermann, and White (1999).

We consider 2^k *a priori* linear models constructed from a base set of k indicators.¹⁴ For our UK application, a detailed review of the macroeconomic indicators discussed in the various post-1980 issues of The Bank of England Quarterly Bulletin suggests the following macroeconomic variables:¹⁵

$$\mathbf{Z}_{1,t-1} = \{\Delta TB_{t-1}, USSP_{t-1}, \Delta FT_{t-1}, \Delta ED_{t-1}, \Delta EE_{t-1}, \Delta POIL_{t-1}, \Delta M0_{t-2}, \Delta RPIX_{t-2}, \Delta IP_{t-3}\}, \quad (11)$$

where ΔTB_{t-1} and $USSP_{t-1}$ denote the change in the UK Treasury Bill rate and the yield spread between long and short maturity US government bonds, respectively. The variables ΔFT_{t-1} , ΔED_{t-1} ,

¹²Over the same sample period the maximum (absolute value) of these changes are 9.5%, 12.4% and 20.2% for short, medium and long bonds respectively.

¹³The initial observations $t = 1, 2, \dots, t_0$ are used to start the recursive process. The period $t = 1$ to $t = t_0$ is often referred to as the training period. To some extent, the choice of t_0 is arbitrary, but should exceed the total number of variables in the base set by some suitable multiple, often taken to be 2 or 3 times the variables in the base set.

¹⁴Non-linear terms can be included in the base set, so long as they can be constructed *a priori*. The GAP variable introduced below provides an example. But extension of the recursive modelling strategy to non-linear specifications involving unknown parameters increases the computational requirements significantly.

¹⁵The Data Appendix identifies the sources of the data and the dates where each of the macroeconomic indicators were first mentioned.

ΔEE_{t-1} and $\Delta POIL_{t-1}$ represent the growth rates of the Financial Times Index, the Sterling-US Dollar exchange rate, the Sterling-Euro exchange rate and the spot price of oil, respectively. The variables $\Delta M0_{t-2}$, $\Delta RPIX_{t-2}$, and ΔIP_{t-3} represent the growth rates of the monetary base, the Retail Price Index (excluding mortgage interest payments) and Industrial Production, respectively. For this last set of variables, we adopted the City analysts' convention of using 12-month rates of change to limit the impact of data revisions. The macroeconomic indicators have different release dates into the public domain. Hence, each variable enters the model with a lag that reflects the availability of the most recent observation.

In addition to these macroeconomic variables, the debt manager could also exploit regressors that are functions of past bond prices. As possible examples of such regressors for prediction of medium-short return spread, r_{mt} , we shall consider the following :

$$\mathbf{Z}_{2,t-1} = \{r_{m,t-1}, (p_{m,t-1} - p_{s,t-1}), (p_{l,t-1} - p_{s,t-1}), \Delta GAP_{s,t-1}\}, \quad (12)$$

where $(p_{m,t-1} - p_{s,t-1})$ and $(p_{l,t-1} - p_{s,t-1})$ are the differences between the (log) prices of medium and short, and long and short bonds, respectively. These variables exploit the possibly cointegrating nature of the bond prices of different maturities and their introduction is intended to capture the error correcting effects of such variables on the return spreads. The variable $\Delta GAP_{s,t-1}$ denotes the lagged first difference of the gap between the (log) price of short maturity bonds and the maximum (log) price of short bonds over the sample to date. That is, $GAP_{st} = \text{Max}(p_{s1}, p_{s2}, \dots, p_{st}) - p_{st}$. This term captures non-linear effects similar to those in Beaudry and Koop (1993) and their extensions in Pesaran and Potter (1997). Notice, however, that we are including changes of the GAP variable in our forecasting equations rather than the GAP variable itself originally considered by Beaudry and Koop. The GAP variable tends to be highly persistent and its inclusion in the return spread equations could yield statistically spurious results.

Similarly, for the long-short return spread, r_{lt} , we add the following variables to the base set, $\mathbf{Z}_{1,t-1}$:

$$\mathbf{Z}_{3,t-1} = \{r_{l,t-1}, (p_{m,t-1} - p_{s,t-1}), (p_{l,t-1} - p_{s,t-1}), \Delta GAP_{s,t-1}\}, \quad (13)$$

which replaces $r_{m,t-1}$ by $r_{l,t-1}$ in $\mathbf{Z}_{2,t-1}$.

Accordingly, the base set for predicting r_{mt} is given by

$$\begin{aligned} \mathbf{Z}_{m,t-1} &= \mathbf{Z}_{1,t-1} \cup \mathbf{Z}_{2,t-1} \\ &= \left\{ \begin{array}{l} r_{m,t-1}, (p_{m,t-1} - p_{s,t-1}), (p_{l,t-1} - p_{s,t-1}), \Delta GAP_{s,t-1}, \Delta TB_{t-1}, USSP_{t-1}, \\ \Delta FT_{t-1}, \Delta ED_{t-1}, \Delta EM_{t-1}, \Delta POIL_{t-1}, \Delta M0_{t-2}, \Delta RPIX_{t-2}, \Delta IP_{t-3}, \end{array} \right\} \end{aligned} \quad (14)$$

and the base set for predicting r_{lt} by¹⁶

$$\begin{aligned} \mathbf{Z}_{l,t-1} &= \mathbf{Z}_{1,t-1} \cup \mathbf{Z}_{3,t-1} \\ &= \left\{ \begin{array}{l} r_{l,t-1}, (p_{m,t-1} - p_{s,t-1}), (p_{l,t-1} - p_{s,t-1}), \Delta GAP_{s,t-1}, \Delta TB_{t-1}, USSP_{t-1}, \\ \Delta FT_{t-1}, \Delta ED_{t-1}, \Delta EM_{t-1}, \Delta POIL_{t-1}, \Delta M0_{t-2}, \Delta RPIX_{t-2}, \Delta IP_{t-3}. \end{array} \right\} \end{aligned} \quad (15)$$

¹⁶The exclusion of $r_{l,t-1}$ from $\mathbf{Z}_{m,t-1}$ and $r_{m,t-1}$ from $\mathbf{Z}_{l,t-1}$ is intended to avoid multicollinearity and reduce computational costs.

Our first step towards forecasting the two return spreads involves the selection of a preferred model among the many possible models that are implied by the different subsets of \mathbf{Z}_{mt} and \mathbf{Z}_{lt} . Let M_{imt} refer to model i at time t for r_{mt} :

$$M_{imt} : r_{mt} = \alpha_{im} + \boldsymbol{\gamma}'_{im} \mathbf{z}_{im,t-1} + u_{imt}, \quad (16)$$

for $i = 1, 2, 3, \dots, 2^{k_l}$, $t = 1, 2, \dots, T$, where $\mathbf{z}_{im,t-1}$ is a subset of the variables in $\mathbf{Z}_{m,t-1}$, u_{imt} is a disturbance term, and $k_m = 13$, the number of predictor variables in the bases set. Each of the $2^{13} = 8192$ models is identified by a $k_m \times 1$ vector of binary codes, \mathbf{e}_{im} , with a unity for an included variable and zero for an excluded variable. All forecasting equations contain an intercept term, and the unknown parameters, α_{im} and $\boldsymbol{\gamma}_{im}$, are estimated for each model by Ordinary Least Squares.

Similarly, models M_{ilt} , $i = 1, 2, 3, \dots, 2^{k_l}$ are estimated for r_{lt} :

$$M_{ilt} : r_{lt} = \alpha_{il} + \boldsymbol{\gamma}'_{il} \mathbf{z}_{il,t-1} + u_{ilt}, \quad (17)$$

where $\mathbf{z}_{i,l,t-1} \in \mathbf{Z}_{l,t-1}$. Each model M_{ilt} is identified by a $k_l \times 1$ vector of binary code, \mathbf{e}_{il} , with $k_l = 13$.

At the start of each month, we estimate 2×8192 models (8192 models for r_{mt} and 8192 models for r_{lt}). We choose the optimal models, M_{*mt} and M_{*lt} for each period $t = t_0 + 1, t_0 + 2, \dots, T$, using one of a number of standard model selection criteria, namely Theil's (1958) \bar{R}^2 , Akaike's (1973) information criterion (AIC), Schwarz's (1978) Bayesian criterion (SBC) and Hannan and Quinn's (1979) criterion (HQC). Each of these criteria offers the researcher a different degree of the trade-off between parsimony and fit. Throughout, the two base sets of regressors, $\mathbf{Z}_{m,t-1}$ and $\mathbf{Z}_{l,t-1}$, are kept unchanged. Although, under recursive modelling the selected subsets of these regressors will of course be time dependent and could differ across different model selection criteria.¹⁷

For each model selection criteria the recursive procedure identifies the optimal subset of forecasting variables, \mathbf{z}_{*mt} and \mathbf{z}_{*lt} , to be used at the start of month $t + 1$ to forecast $r_{m,t+1}$ and $r_{l,t+1}$, respectively. The one-step ahead point forecasts of $r_{m,t+1}$ and $r_{l,t+1}$ will be denoted by $\hat{r}_{m,t+1}$ and $\hat{r}_{l,t+1}$, respectively, although it will be understood that \mathbf{z}_{*mt} and \mathbf{z}_{*lt} and the forecasts implied by them are specific to the choice of the model selection criteria employed.

3.1 Forecast performance

Recall that our forecasts are the result of a recursive, two-step procedure for each time period. In the first stage, we select an optimal model (for each selection criterion). In the second stage, we use that model to generate one-step ahead forecasts of return spreads. Using a training sample from April 1980 to March 1985, we generate monthly forecasts based on recursively selected models over the evaluation period, April 1985 to March 2000.

Figure 2 shows the standard errors of the regression equations selected recursively using the SBC model selection criteria over our evaluation period.¹⁸ We shall refer to these as recursive

¹⁷In practice it is also possible to allow the base set of predictor variables to evolve over time in a pre-specified manner as an automated response to major regime changes. For a discussion see Pesaran and Timmermann (2000) where oil price changes are introduced in the base set in response to the quadrupling of oil prices in early 1970s.

¹⁸All of the figures presented in this section pertain to the SBC. This is merely to save space and does not reflect a preference for this criteria. Similar results are obtained for the other criteria, and are available upon request.

model (RM) standard errors, to distinguish them from the familiar recursive standard errors where the estimates are updated using a *fixed* regression equation. These estimates show a slight downward trend over the sample, except in four periods: the first occurs in mid-1986 which coincide with the Big Bang (see Bank of England Quarterly Bulletin (March 1986, p. 71-73)), but prior to the equity market crash of October 1987. The second occurs around 1990, reflecting perhaps the increased uncertainties surrounding the UK's position in the European Exchange Rate Mechanism (ERM). The third occurs in 1994 and maybe a by-product of increased uncertainty that may have preceded the reforms to UK debt management in 1995. Finally, late 1998 and early 1999 sees another rise in the RM standard errors, in particular for the long-short spread, r_{lt} . The RM standard errors for the medium-short spreads, r_{mt} , are systematically lower than those of the long-short return spreads. This result is robust to the choice of the model selection criteria and clearly shows the systematically higher uncertainty that surrounds the forecasts of r_{lt} as compared to that of r_{mt} .

Evaluation of forecasts can be carried out from many different perspectives, and in general depends on the loss function being used.¹⁹ The most usual criterion is the root mean squared forecast errors, but since our primary purpose is to investigate the extent to which the forecasts are likely to be useful in the development of cost-minimizing strategies, more relevant statistical criteria are market timing statistics used extensively in the empirical finance literature. In the case of our application we shall consider the market timing characteristics of the forecasts of the two return-spreads taken individually as well as jointly, for all the four model selection criteria.

We present market timing statistics over two sample periods, April 1985 to March 1995 and April 1985 to March 2000. As has already been alluded to, beginning in 1995 a number of reforms to the debt management were initiated in the UK. In the 1995 Debt Management Review, the UK authorities concluded that active management contributed to bond market volatility (HM Treasury (1995)). As a consequence, the authorities committed to publishing projections for debt stocks and flows, including auction calendars (see, for example, HM Treasury (2000)). A second reform was the transference of responsibility for debt management policy from the Bank of England to the UK Debt Management Office (DMO), an executive agent of HM Treasury, in 1998, as an attempt to separate control of monetary and debt management policies. These developments could have contributed to the increased difficulty in predicting return spreads from 1998 onwards, as seen in Figure 2, particularly in the case of the long-short spread. Consistent with a decline in predictability following these reforms, we find stronger evidence of predictability for the shorter sample that ends in March 1995 rather than for the full sample that covers the post reform period.

The market timing statistics are based on the proportion of times that the direction of the return spreads are predicted correctly. This information is summarized in Table 3. Using the SBC, the sign of $r_{m,t+1}$ ($r_{l,t+1}$) is correctly forecast in 61.7% (60.0%) of the months over the period April 1985 to March 1995. The corresponding figures for the \bar{R}^2 , the AIC, and the HQC are 59.2% (61.7%), 55.8% (65.0%), and 58.3% (65.8%), respectively. When we extend the sample to March 2000 these figures fall by between one and six percentage points. This is consistent with the view that there has been a fall in the predictability of UK bond spreads following the debt management reforms of the mid 1990s.

We interpret these statistics as a preliminary indication that our forecasts contain useful infor-

¹⁹A review of alternative approaches to forecast evaluation can be found in Pesaran and Skouras (2002).

mation for market timing at least in the period preceding the reforms to UK debt management operations. To formally test this hypothesis, we use Pesaran and Timmermann’s (1992) non-parametric test of market timing (PT Test). This tests the null hypothesis:

$$H_0 : \sum_{i=1}^2 (\pi_{ii} - \pi_{i0}\pi_{0i}) = 0 \quad (18)$$

where π_{11} (π_{22}) represents the probability that both the forecast of a particular return spread and its realized value are negative (positive), π_{10} (π_{20}) represents the probability that the forecast is negative (positive) and π_{01} (π_{02}) represents the probability that the realized value is negative (positive). Under the null hypothesis that the forecasts and realized values are independently distributed the PT test statistic is asymptotically distributed as $N(0, 1)$. Note that a forecasting model (such as a random walk model) that always predicts that a particular return spread is positive (or negative) will result in a PT statistic which is identically equal to zero.²⁰ Table 3 shows that H_0 is rejected for both of our return spreads for all the four model selection criteria at the 10% significance level or better over the April 1985 to March 1995 period and with one exception ($r_{m,t+1}$ and AIC) over the full sample. These results also confirm that the RM forecasts outperform the random walk model of return spreads when the objective of the exercise is to predict the signs of the $r_{m,t+1}$ and $r_{l,t+1}$.

Since the cost minimization problem is concerned with the reallocation of debt across three maturity bands, we also need to consider the relative forecast accuracy of the $r_{m,t+1}$ and $r_{l,t+1}$ as well as their expected signs. Given the point forecasts, $\hat{r}_{m,t+1}$ and $\hat{r}_{l,t+1}$ we can derive a “predicted ordering” for the three holding period returns.²¹ With three maturity bands there are six possible predicted orderings and corresponding orderings of the actual data; which we refer to as “realized orderings”. This provides us with a 6×6 matrix of predicted and realized orderings. Alternative tests of predictability can now be based on the individual elements of this matrix or on linear combination(s) of some of its elements. Pesaran and Timmermann (1994) consider such tests and argue in favour of focussing on the diagonal elements of this matrix and suggest testing the null hypothesis that

$$H_0 : \sum_{i=1}^6 (\pi_{ii} - \pi_{i0}\pi_{0i}) = 0,$$

where π_{ii} denotes the joint probability that the ordering in category i is correctly predicted and $\pi_{i0}\pi_{0i}$ is the probability of this joint event on the assumption that processes generating the predicted and realized orderings are independently distributed. π_{i0} is the marginal probability of the realized ordering in category i , and π_{0i} is the associated marginal probability of the predicted ordering. This is a direct generalization of the PT test and is referred to as the Generalized Henriksson-Merton (GHM) test in recognition of the seminal contribution of Henriksson and Merton (1981) to the development of market timing statistics.

The GHM statistics computed for the return spreads and their forecasts for all the four model selection criteria are presented in the last row of Table 3. Once again the statistics are presented

²⁰For example, consider a model that produces $\hat{r}_{m,t+1} > 0$ for all t . Here $\pi_{11} = \pi_{01}$ and $\pi_{10} = 1$ ensures the first term of the summation is zero and $\pi_{22} = \pi_{20} = 0$ ensures the second term is also zero.

²¹For example, if $\hat{r}_{l,t+1} > 0$ and $\hat{r}_{m,t+1} < 0$, then $\hat{R}_{l,t+1} > \hat{R}_{s,t+1} > \hat{R}_{m,t+1}$.

for the sample period April 1985 to March 1995 and the full sample that ends in March 2000. The GHM test results are in line with the PT tests applied to the individual return spread forecasts and as before do not seem to be unduly sensitive to the choice of model selection criteria. The null hypothesis of no market timing involving both return spreads is rejected at the 10% level or less for all the four model selection criteria in the case of the sample ending in March 1995, but not if we consider the full sample that includes the post reform period. Again, this is consistent with the view that the return spreads have become less predictable following the reforms to UK debt management operations. This does not, of course, necessarily mean that there exists no other forecasting model that could perform better than the RM forecasts over the April 1995 to March 2000 period. Rather our results suggest that identification of such a forecasting model without the benefit of hindsight (or without data snooping) could be difficult.

Having established that the forecasts are useful as indicators of market timing, it is also interesting to investigate the frequency with which different regressors in the base set, $\mathbf{Z}_{m,t-1}$ and $\mathbf{Z}_{l,t-1}$ defined by (14) and (15), have contributed to the forecasts. The inclusion frequency of the various regressors are summarized in Table 4. Recall that, in theory, the recursively selected models need not include any of the variables in the base sets. In practice, we find that a number of variables are always included in the forecasting models, regardless of the selection criterion used. All model selection procedures select models with fewer regressors than the 14 in the base set (13 plus the constant). Consistent with having the most lenient penalty for adding a regressor, the \overline{R}^2 criteria picks the most regressors, typically between seven and ten, while, on average, the SBC picks the fewest, usually three to six. The variables $r_{m,t-1}$, $r_{l,t-1}$ and ΔFT_{t-1} are rarely selected. The most commonly selected regressors are the relative price variables, $(p_{m,t-1} - p_{s,t-1})$ and $(p_{l,t-1} - p_{s,t-1})$, the non-linear variable, $\Delta GAP_{s,t-1}$, the change in the UK Treasury Bill rate, ΔTB_{t-1} , the US term spread, $USSP_{t-1}$ and the change in industrial production, ΔIP_{t-3} .

Before discussing the simulated debt management strategies, it is worth dwelling on the robustness of our finding so far that return spreads have been forecastable pre 1995. It is well known that predictability studies are sometimes sensitive to both the method of model selection and the “real-time” remeasurements of macroeconomic data.

For example, Aiolfi and Favero (2002) argue that there is stronger evidence of predictability in US stock returns if forecasts are generated by “thick modelling”. For each model selection criterion this would involve ranking all the forecasting models and selecting the “best” forecast as an average of the forecasts from the top $x\%$ of the models under consideration. In practice, the use of this procedure is further complicated since in addition to the choice of the model selection one must also decide on the percentage of the top models to be used in the averaging (pooling) procedure. However, since our primary aim here is to investigate the robustness of our results to the idea of thick modelling we did not concern ourselves with an optimal choice of $x\%$. Rather we experimented with a large number of alternative values of $x\%$ in the range 0.1% to 50% , but found no evidence of improved predictability.²²

In a study of the impact of real-time macroeconomic data remeasurements, Egginton, Pick and Vahey (2002) report an improvement in the forecast performance of models for UK inflation. In the case of our application only two of the variables in the base set could be subject to the remeasure-

²²In fact, these results suggest slightly weaker predictability, and are available on request.

ment problem, namely Industrial Production and M0. Faced with multiple measurements of these variables over time a further choice (and hence possible data snooping) ensues. Should one use the latest measurements of these variables available in “real time”, or the historical measurements which might not be available at the time forecasts are formed? Even if we confine ourselves to the measurements that are available in real time there are many possible combination of data vintages that could be used. Instead of getting into these issues we decided to check to see if our results held up to the exclusion of variables subject to the remeasurement problem. To this end we computed RM forecasts using a smaller set of regressors with $\Delta M0_{t-2}$ and ΔIP_{t-3} (the only variables subject to the remeasurement problem) excluded from the base set. The PT and GHM test results applied to these forecasts were only marginally worse than the ones obtained when $\Delta M0_{t-2}$ and ΔIP_{t-3} were included in the analysis. See the last panel of Table 3. In the first instance this might seem odd considering that both of these regressors (particularly ΔIP_{t-3}) had been frequently included in the forecasting equations (see Table 4). But when dealing with a set of closely related regressors (as we do), exclusion of some need not necessarily have an adverse effects on the forecasts if there are other regressors that could fill the gap.

3.2 Simulated portfolios

In this section we use the forecasts generated from the recursive modelling procedure to carry out a number of debt management simulation exercises over the evaluation period, April 1985 to March 2000. We conduct our portfolio simulations over the whole sample rather than just the pre-reform sample in order to minimize the possibility of data-snooping. While it might be the case that predictability fell following the reforms of the mid-1990s, and with it the opportunities to lower costs, this may not have been apparent to the debt manager at the time.

For each exercise we compute the portfolio weights, w_{it} , $i = m, l$ over the evaluation period by solving the minimization problem given by equations (5) to (9).²³

This optimization problem depends on the risk parameter, λ , the values of α , \underline{w}_i , and \overline{w}_i (the parameters of the constraints), the forecasts of the return spreads, $\hat{r}_{m,t+1}$ and $\hat{r}_{l,t+1}$, and $\Omega_{t|t-1} = Var(\mathbf{r}_t | \mathbf{I}_{t-1})$, the 2×2 return spread conditional volatility matrix. Recursive estimates of $\Omega_{t|t-1}$ are computed using the RiskMetric estimator given by

$$\hat{\Omega}_{t|t-1} = \left(\frac{1 - \gamma}{1 - \gamma^n} \right) \sum_{j=1}^n \gamma^{j-1} \mathbf{e}_{t-j} \mathbf{e}'_{t-j},$$

where \mathbf{e}_t is the 2×1 vector of recursive forecast errors, γ is a decay coefficient and n is the size of the observation window. This estimator is used extensively in the professional finance literature and has a number of desirable properties and is particularly suited to the recursive modelling strategy.²⁴ It can be readily extended to a larger number of return spreads. It is very simple to compute recursively, and yields a positive definite covariance estimator when n is sufficiently large relative to the dimension of \mathbf{e}_t . For most financial assets γ is taken to be in the range 0.94 – 0.96, and n

²³The numerical solution to the constrained optimization problem was carried out using the subroutine FMINCON in Matlab. The technical details are available on request.

²⁴The RiskMetric estimator has been popularized in Finance by J.P. Morgan and Goldman Sachs.

is chosen to be around 5 years (or 60) in the case of monthly observations. A more complicated alternative would be to fit multi-variate GARCH models to the recursive forecast errors, with the GARCH model itself being estimated recursively. Firstly, it is not clear that the outcome would be superior to the RiskMetric approach. Secondly, given the known convergence problems that surrounds the estimation of the multi-variate GARCH models its use in recursive computations could be problematic. Finally, the multi-variate GARCH estimates are likely to break down in the case where N , the number of return spreads, is relatively large while the RiskMetric estimator could still be operational.²⁵ In our applications we follow the literature and set $n = 60$ and $\gamma = 0.95$.²⁶

The portfolio simulation results are summarized in Tables 5 and 6, with the former giving the results for the risk neutral case ($\lambda = 0$), and the latter for the risk averse case with λ set to its “consensus” value of 2.0.²⁷ All the simulations are subject to the constraint (9) where the proportion of debt in each maturity band is constrained not to exceed $1 \pm \alpha$ of its realized value in the preceding month, with $\alpha = 0.10, 0.05, 0.03$ and 0.02 .

Panel (a) of these tables refer to the case where the portfolio shares are constrained to lie within their historically observed minimum and maximum values summarized in Table 2. Whilst the simulation results reported in panel (b) refer to the case where no additional restrictions are imposed on the portfolio shares apart from requiring them to lie in the $[0, 1]$ range and satisfying the adding up restriction, (7).

The extent to which interest costs can be minimized critically depends on the constraints that are imposed on the portfolio shares. The α constraints seem to be less binding than the max-min constraints. Without the latter constraints all the simulation results suggest important cost reductions could have been achieved by making use of the RM forecasts. This result is robust to the choice of the model selection criteria and the particular value of α in the range $[0.02 - 0.10]$. The situation is less clear cut when the additional max-min constraints are also imposed. In the presence of both types of constraints little or no cost savings would have been achieved if RM forecasts were based on \bar{R}^2 or the *AIC* criteria, whilst some modest cost savings would have followed if the forecasts had been based on SBC or HQC. Amongst various model selection criteria, the HQC implies the highest cost saving of around £0.15 billion per year at 1990 prices (evaluated at the bond portfolio market value of £99 billion in February 1990). However, the amount of switching involved can be substantial: when $\alpha = 0.10$ the average monthly debt switches are 3.6%, 4.2% and 5.8% for the short, medium and long maturity bands respectively. Note that these figures are higher than those we quoted earlier for the actual switches in the DMA’s portfolio shares. In addition, for the long maturity band the restriction on the amount of the debt that can be switched is binding in 43% of periods, for the medium and short maturity bands these figures are 29% and 9% respectively. Such frequent and large shifts in the portfolio might not have been compatible with the other aims of debt management, such as the preservation of reasonably liquid markets in all the three maturity bands.

Amongst the various values of α in the range 0.02 to 0.10, the ones at the lower end seem more realistic and generate more realistic average monthly changes in debt shares. For example, for

²⁵For a comparison of the RiskMetric estimator with a number of alternative multi-variate GARCH specifications see Engle (2000) who also proposes a new two-stage estimator.

²⁶Results for other values of n and γ are similar and available on request.

²⁷We also experimented with other values of λ in the range of 0 to 3 and obtained very similar results.

$\alpha = 0.03$ the average monthly changes in debt shares are 1.5%, 1.6% and 2.3% for the three maturity bands.²⁸ For this choice of α , the HQC and the SBC simulations again generate the lowest interest costs, namely 10.66% and 10.68% on average per annum, respectively. Again, the figure for the HQC represents the highest saving of just under £0.15 billion (1990) per year. Figure 3 illustrates that the potential for cost savings is small throughout our simulation period. The recursive average interest costs from our simulation (using the SBC) are almost indistinguishable from the recursive average interest rates from the UK’s actual debt portfolio.²⁹ Figure 4 shows the time path of the portfolio shares using the SBC for model selection. The portfolio share for the short maturity is in the region of 53% (its historical high) by the end of the sample period.³⁰ Indeed as Figure 4 shows the share of the debt in the short maturity band is at or very near the upper constraint for most of the simulation periods. Beginning in April 1986 we see a steady rise in the proportion of the debt in the short maturity band until early 1988 when this upper constraint begins to bind.

For comparison, Table 5 panel (b) also presents average annualized interest rates from simulated portfolios with no constraints on the level of the shares (except for adding-up and non-negativity). In this case, the interest costs resulting from the models selected by the \bar{R}^2 criterion, the AIC, the HQC, and the SBC are all lower than those resulting from the UK’s actual portfolio. At annualized rates, for a value of $\alpha = 0.03$ these annualized average interest rates range from 9.84% (SBC) to 10.42%, (\bar{R}^2). The former figure implies a cost saving of around £0.97 billion per year at 1990 prices (evaluated at the bond portfolio market value of £99 billion in February 1990). Figure 3 illustrates that these cost savings start to show up from 1993 onwards. However, Figure 4 also shows that the proportion of the debt in the short maturity for the SBC strategy would have risen to 95% by the mid 1990s. Therefore, one interpretation is that while aware of the cost saving opportunities the UK authorities did not to take advantage of them as a portfolio with 95% of the debt in short maturity bonds would have been in conflict with other debt management objectives.

In Table 6 we present results from simulations for which the loss function involves a penalty for the variation in interest costs, with $\lambda = 2$.³¹ These results are almost identical to those presented in table 5. For our UK application, ignoring interest cost volatility in the loss function (but not the constraints) has little impact on the scope for cost minimization.

4 Summary and Conclusions

In this paper we consider the scope for cost minimization in public debt management. We model a debt manager who aims to minimize the expected cost of the government’s bond portfolio for a given level of short term interest rate (set by the monetary authorities) and subject to a number of risk and market impact constraints.

In an application to the UK, we evaluate a variety of debt management strategies using the recursive modelling (RM) procedure recently developed by Pesaran and Timmermann (1995, 2000)

²⁸Switch auctions were introduced in the UK in 1996. These are roughly the same size as contemporary conventional auctions, which rarely exceed £4 billion at current prices.

²⁹Figures for other criteria are similar and available on request.

³⁰Figures for other model selection criteria are similar and available on request.

³¹We also experimented with other non-zero values for λ . The results are similar to those reported in table 6. Results of debt management simulations using these alternative values are available on request.

to generate monthly forecasts of return spreads over the period April 1985 to March 2000. The RM procedure limits the extent of data snooping and allows us to generate candidate “real-time” debt portfolio strategies. We provide statistically significant evidence of predictability of return spreads before the introduction of reforms of the UK debt management system in 1995, although there seems to be little evidence of predictability once the post reform sample is included. Nevertheless, our results show that there would have been scope for a small reduction in interest costs over the 1985-2000 period even if the portfolio shares and their monthly changes were constrained to lie close to those which were historically observed. Further cost savings would have followed if the portfolio shares were allowed to deviate from their historical highs and lows, although we would be then faced with the difficult question of how to trade off the cost minimization objective with the other aims of debt management, particularly the extent to which it would be prudent to concentrate 80% to 90% of government debt in short maturity bonds.

Our results are reasonably robust to the choice of model selection criteria (we consider four such criteria), the use of “thick modelling” where average forecasts of top models rather the forecast from the single “best” model is considered, and the exclusion of macroeconomic indicators that are subject to the remeasurement problems (in our application confined to industrial production and money supply indicators).

The next stage in the present analysis would be to check the robustness of our results to further changes in the base set of the regressors used in the forecast analysis, the choice of the estimation sample (observation window) for forecasting (currently we are using an expanding window), and alternative approaches to modelling explicitly the market impact of an active debt management policy aimed at reducing (expected) interest costs.

Finally, it would be interesting to see if similar results are obtained if the debt simulation approach of this paper is applied to the government debt portfolios of other OECD countries.

Data Appendix

We use data from four sources. The UK Debt Management Office (DMO), the Office for National Statistics databank (ONS), Citibase and the Federal Reserve Economic Database (FRED). Researchers interested in acquiring bond data should contact the DMO, or alternatively can be downloaded from

<http://www.econ.cam.ac.uk/dae/research/debt/>

The ONS and FRED databanks are available online at

<http://www.data-archive.ac.uk/online/ons/>

and

<http://www.stls.frb.org/fred/index.html>

R_s	: Monthly Holding Period Return, Short Bonds. Source DMO.
R_m	: Monthly Holding Period Return, Medium Bonds. Source DMO.
R_l	: Monthly Holding Period Return, Long Bonds. Source DMO.
r_m	: Medium to Short Return Spread, $R_m - R_s$.
r_l	: Long to Short Return Spread, $R_l - R_s$.
p_s	: ln (Price), Short Bonds, End Month, Clean. Source DMO.
p_m	: ln (Price), Medium Bond, End Month, Clean. Source DMO.
p_l	: ln (Price), Long Bond, End Month, Clean. Source DMO.
B_s	: Quantity, Short Bonds, End Month. Source DMO.
B_m	: Quantity, Medium Bonds, End Month. Source DMO.
B_l	: Quantity, Long Bonds, End Month. Source DMO.
ΔTB	: Change in 3 Month Treasury Bill rate. Source: ONS, AJRP.
$USSP$: Yield Spread between U.S. Long-term Government Bonds and 3 Month Treasury Bills. Calculated as $(1 + LTGOVTBD/100)^{1/12} - (1 + TB3MA/100)^{1/12}$. Source: FRED.
ΔFT	: Percentage Change in the Financial Times All Share Index. Calculated as $\ln(FT_t/FT_{t-1})$. Source: ONS, AJMA.
ΔED	: Percentage Change in the Sterling-Dollar Exchange Rate. Calculated as $\ln(ED_t/ED_{t-1})$. Source: ONS, AJFA
ΔEE	: Percentage Change in the Sterling-Euro (Deutchmark pre Euro) Exchange Rate Calculated as $\ln(EE_t/EE_{t-1})$. Source: ONS, THAP.
$\Delta POIL$: Percentage Change in the Spot Price of Oil. Calculated as $\ln(POIL_t/POIL_{t-1})$. Source: Citibase, MEEFPP.
$\Delta M0$: Year on Year Percentage Change in the Monetary Base. Calculated as $EUAF/100$. Source: ONS.
$\Delta RPIX$: Year on Year Percentage Change in the Retail Price Index (excluding mortgage interest). Calculated as $CDKQ/100$. Source: ONS.
ΔIP	: Year on Year Percentage Change in the Industrial Production Index. Calculated as $\ln(IP_t/IP_{t-12})$. Source: ONS, DVZI.

Notes:

1) There are three other variables in the base sets. These are $(p_m - p_s)$, $(p_l - p_s)$ and ΔGAP_s . The first is the difference between the natural logs of the prices of medium maturity and short maturity bonds. The second is the difference between the natural logs of the prices of long maturity and short maturity bonds. The third is constructed as follows:

$$\Delta GAP_{st} = GAP_{st} - GAP_{s,t-1}$$

where

$$GAP_{st} = \max(p_{s1}, p_{s2}, \dots, p_{st}) - p_{st}$$

and p_s is the natural log of the price of short maturity bonds.

2) Bond data

The holding period return data were obtained from UK Debt Management Office. The individual bond data are aggregated (with equal weights) into maturity bands: short (≤ 7 years), medium (between 7 and < 15 years) and long (≥ 15 years). The monthly data used in this study are the averages of underlying daily observations.

The daily holding period return on an individual bond is defined as the first difference of (the log of) the closing price, adjusted to reflect “ex-dividend” effects using the 3-month London Interbank Offered Rate (*LIB3*). Namely

$$R_{dt} = \ln \left(\frac{P_{dt} + A_{dt}}{P_{d,t-1}} \right),$$

where P_{dt} denotes the daily close of business dirty price at time t and A_{dt} denotes the adjustment for ex-dividend periods.

3) Macroeconomic indicators

Our macroeconomic indicators were selected based on a review of *The Bank of England Quarterly Bulletins* from 1980 to 1985. In the listed issues, the following indicators were mentioned in the *Financial Review* section of the Bulletins:

Measure	Indicator mentioned	Date of the Bulletin
<i>TB</i>	Treasury Bill rate*	March 1980
<i>USSP</i>	US rates	June 1980
<i>FT</i>	Equity prices*	March 1980
<i>ED</i>	Sterling-US dollar exchange rate	September 1980
<i>EM</i>	Sterling- Mark exchange rates	September 1980
<i>POIL</i>	Oil prices	March 1985
<i>M0</i>	Monetary base	March 1980
<i>RPIX</i>	Inflation	June 1980
<i>IP</i>	Industrial production*	September 1980.

Indicators denoted by * were discussed generally in the Bulletins, but not specifically in connection to the bond markets. References to variables after September 1981 are from the “Gilt-edged” subsection of the “Operation of monetary policy”.

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Table 1: Means and Standard Deviations of Holding Period Returns & Spreads

	Holding Period Returns			Spreads	
	R_s	R_m	R_l	r_m	r_l
April 1980 - March 1985	14.45 (1.45)	16.66 (2.96)	17.40 (3.56)	2.21 (1.85)	2.94 (2.57)
April 1985 - March 1990	9.10 (1.26)	9.63 (2.56)	7.75 (2.95)	0.53 (1.45)	-1.35 (1.90)
April 1990 - March 1995	11.65 (1.13)	14.26 (2.36)	14.74 (2.85)	2.61 (1.43)	3.08 (2.10)
April 1995 - March 2000	7.67 (0.65)	11.41 (1.72)	14.98 (2.36)	3.75 (1.19)	7.31 (1.92)
April 1980 - March 2000	10.69 (1.17)	12.96 (2.44)	13.66 (2.96)	2.27 (1.49)	2.97 (2.14)
April 1985 - March 2000	9.46 (1.05)	11.75 (2.24)	12.44 (2.73)	2.29 (1.36)	2.98 (1.98)

The means of both the holding period returns and the spreads have been converted into annual rates. The figures in parenthesis are standard deviations of monthly rates.

Table 2: Bond Shares by Market Values April 1985 - March 2000

	Short (0 – 7 years)	Medium (7 – 15 years)	Long (15+ years)
Mean	0.4576	0.3742	0.1682
Standard Deviation	0.0332	0.0456	0.0402
Minimum	0.3871	0.2541	0.0881
Maximum	0.5334	0.4686	0.2621

These figures refer to monthly debt shares that are measured by market value outstanding.

Table 3: Predictive Performance

	April 1985 - March 1995				April 1985 - March 2000			
	\bar{R}^2	AIC	HQC	SBC	\bar{R}^2	AIC	HQC	SBC
(a) Proportion of return spreads whose signs are correctly predicted								
$r_{m,t+1}$	0.592	0.558	0.583	0.617	0.567	0.544	0.556	0.561
$r_{l,t+1}$	0.617	0.650	0.658	0.600	0.556	0.589	0.594	0.578
(b) Proportion of periods where the minimum (maximum) of the holding period returns are correctly predicted								
Minimum	0.500	0.533	0.567	0.533	0.433	0.461	0.478	0.506
Maximum	0.417	0.417	0.450	0.442	0.367	0.367	0.389	0.378
(c) Market timing statistics								
PT Statistic: $r_{m,t+1}$	2.221	1.598	1.883	3.196	1.732	1.022	1.478	1.724
PT Statistic: $r_{l,t+1}$	2.745	3.733	3.994	2.707	1.593	2.405	2.627	1.972
GHM Statistic:	1.802	1.568	2.233	3.261	0.052	0.461	0.499	1.177
(d) Market timing statistics when $\Delta M0_{t-2}$ and ΔIP_{t-3} are excluded from base set								
PT-Statistic: $r_{m,t+1}$	2.159	2.175	2.286	1.759	1.524	1.037	1.323	1.021
PT-Statistic: $r_{l,t+1}$	2.074	3.022	3.474	2.707	1.0726	1.779	2.095	1.972
GHM Statistic:	1.610	2.315	2.262	1.817	0.339	0.904	0.901	0.431

The proportion of correct signs refers to the proportion of the forecast evaluation period (April 1985 - March 2000) that the sign of the prediction of a particular holding period return spread is the same as the realized sign, PT Statistic refers to the Pesaran and Timmerman (1992) non-parametric test of predictive performance and GHM statistic refers to the generalized Henriksson-Merton test of market timing proposed by Pesaran and Timmerman (1994). The PT and GHM statistics are distributed asymptotically $N(0, 1)$ under the null hypothesis. Therefore, for a one-sided test the 10% (5%) critical value is 1.28 (1.65).

Table 4: Factor Inclusion Rates April 1985 - March 2000

	Equation for r_{mt}				Equation for r_{lt}			
	\bar{R}^2	AIC	HQC	SBC	\bar{R}^2	AIC	HQC	SBC
ΔTB_{t-1}	1	0.983	0.978	0.939	1	1	1	0.889
$USSP_{t-1}$	1	0.894	0.539	0.600	1	1	1	0.900
ΔFT_{t-1}	0	0	0	0	0.050	0	0	0
ΔED_{t-1}	0.122	0.044	0.039	0.011	0.528	0.228	0.067	0.011
ΔEE_{t-1}	0.833	0.261	0	0	0.678	0.178	0	0
$\Delta POIL_{t-1}$	0.800	0.628	0.294	0.078	0.667	0.383	0.083	0.011
$\Delta M0_{t-2}$	0.933	0.656	0.361	0.067	0.583	0.061	0	0
$\Delta RPIX_{t-2}$	0.261	0.061	0.022	0.011	0.294	0.167	0	0.006
ΔIP_{t-3}	1	1	0.983	0.556	1	0.994	0.583	0.222
$r_{m,t-1}$	0.022	0	0	0
$r_{l,t-1}$	0.528	0.017	0	0
$p_{m,t-1} - p_{s,t-1}$	0.961	1	0.983	0.761	0.767	0.778	0.783	0.444
$p_{l,t-1} - p_{s,t-1}$	0.150	0.094	0.044	0.072	0.355	0.300	0.222	0.456
$\Delta GAP_{s,t-1}$	1	0.983	0.978	0.939	1	1	1	0.889

These figures refer to the proportion of the months for which we generate forecasts of the holding period returns spreads (1985m4-2000m3) that each factor is included in a particular forecasting model. The forecasting models are time dependent and are chosen from the set of all possible models spanned from the base set, using model selection criteria applied to a sample that begins in 1980M4 and ends with the month before that being forecast. We do not encounter any ties.

Table 5: Annualized Mean Interest Rate (%) and Standard Deviation of Monthly Interest Rates from Debt Management Simulations for $\lambda = 0$: April 1985 - March 2000

UK Actual			10.81 (1.74)	
	\overline{R}^2	AIC	HQC	SBC
(a) Weights restricted to lie between sample minimum and maximum				
$\alpha = 0.10$	10.88 (1.79)	10.88 (1.76)	10.66 (1.74)	10.75 (1.72)
$\alpha = 0.05$	10.88 (1.79)	10.93 (1.76)	10.68 (1.72)	10.72 (1.67)
$\alpha = 0.03$	10.81 (1.77)	10.84 (1.75)	10.66 (1.70)	10.68 (1.66)
$\alpha = 0.02$	10.77 (1.76)	10.77 (1.74)	10.66 (1.70)	10.68 (1.66)
(b) Weights restricted to lie in $[0, 1]$				
$\alpha = 0.10$	10.12 (1.91)	9.62 (1.66)	9.51 (1.53)	9.64 (1.27)
$\alpha = 0.05$	10.31 (1.79)	9.85 (1.59)	9.76 (1.52)	9.71 (1.35)
$\alpha = 0.03$	10.42 (1.73)	10.07 (1.60)	9.97 (1.54)	9.84 (1.41)
$\alpha = 0.02$	10.50 (1.71)	10.24 (1.62)	10.16 (1.58)	10.00 (1.47)

Standard deviations are in parentheses. α is the maximum proportion by which the share of the market value of the debt in each maturity band is allowed to change in any one month.

Table 6: Annualized Mean Interest Rate (%) and Standard Deviation of Monthly Interest Rates from Debt Management Simulations for $\lambda = 2$: April 1985 - March 2000

UK Actual			10.81 (1.74)	
	\overline{R}^2	AIC	HQC	SBC
(a) Weights restricted to lie between sample minimum and maximum				
$\alpha = 0.10$	10.84 (1.78)	10.88 (1.76)	10.66 (1.74)	10.72 (1.72)
$\alpha = 0.05$	10.85 (1.79)	10.91 (1.76)	10.67 (1.71)	10.71 (1.67)
$\alpha = 0.03$	10.76 (1.76)	10.84 (1.74)	10.65 (1.70)	10.82 (1.69)
$\alpha = 0.02$	10.75 (1.75)	10.76 (1.73)	10.64 (1.70)	10.67 (1.66)
(b) Weights restricted to lie in $[0, 1]$				
$\alpha = 0.10$	10.00 (1.87)	9.58 (1.66)	9.54 (1.47)	9.64 (1.27)
$\alpha = 0.05$	10.21 (1.75)	9.84 (1.58)	9.75 (1.50)	9.71 (1.34)
$\alpha = 0.03$	10.31 (1.69)	10.04 (1.57)	9.94 (1.53)	9.84 (1.41)
$\alpha = 0.02$	10.45 (1.69)	10.22 (1.61)	10.13 (1.57)	9.99 (1.46)

See the notes to Table 5.

Figure 1: Actual UK Bond Shares by Market Value

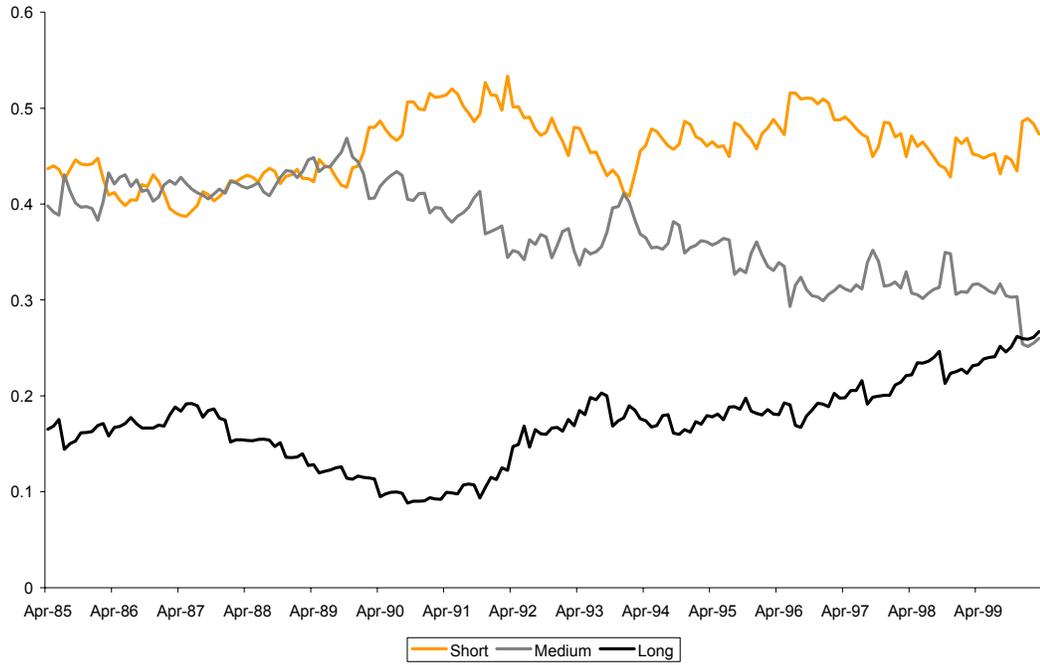


Figure 2: Recursive Standard Errors from Models Selected using SBC

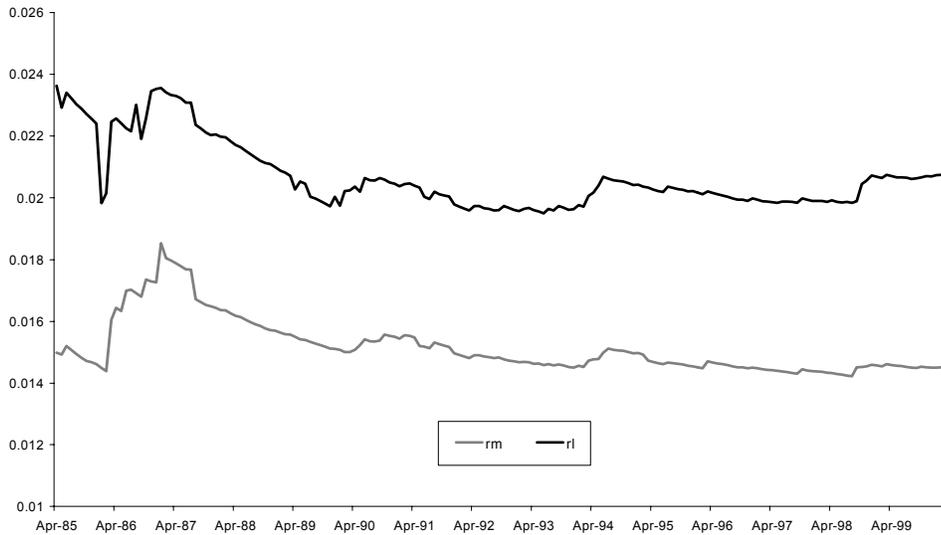


Figure 3: Recursive Average Interest Costs, Simulated (SBC) and UK Actual Portfolios



Figure 4: Proportion of Debt in Short Maturity, Simulated (SBC) and UK Actual Portfolios

