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Network-constrained Models of Liberalized Electricity Markets: the Devil is in the Details

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Network-constrained models of liberalized electricity markets: The devil is in the details[0]

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Numerical models for electricity markets are frequently used to inform and support decisions. How robust are the results? Three research groups used the same, realistic data set for generators, demand and transmission network as input for their numerical models. The results coincide when predicting competitive market results. In the strategic case in which large generators can exercise market power, the predicted prices differed significantly. The results are highly sensitive to assumptions about market design, timing of the market and assumptions about constraints on the rationality of generators. Given the same assumptions the results coincide. We provide a checklist for users to understand the implications of different modelling assumptions.

1. INTRODUCTION

Due to advances in mathematical modeling capabilities, numerical models of strategic behavior in power networks are gaining increasing attention. Such models are being used to support policy

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decisions on market design, merger analysis and other regulatory tasks (*e.g.*, Nordic competition authorities (2003)).

Several research groups have been developing mathematical models for calculating static equilibria among oligopolistic generators. This paper is result of two workshops in which three of these models were compared. We wanted to understand the representation and implications of different assumptions. These assumptions include the role of fringe generators, the timing of the energy and transmission markets, different levels of bounded rationality of strategic generators, and transmission and energy contract coverage by generators. In models that assume perfectly competitive behavior these assumptions are not critical, therefore all models should (and indeed do) predict the same equilibrium prices in the competitive mode.

In strategic models, the main issue to be considered is the causal relationships between different components of the market. For example, do strategic generators assume that their output decisions directly influence the output of fringe generators? The models show that such additional responsiveness can reduce average market prices. However, the inclusion of power network constraints complicates the results in sometimes surprising ways. For instance, different zonal price levels imply changes of allocation of transmission capacity in the network such that net imports into some nodes can be reduced, inducing local price increases even when average prices fall.

Another issue is the sequence of energy and transmission markets. In the highly meshed European electricity network two designs for the allocation of scarce transmission capacity are currently debated. In the separated transmission and energy markets design, transmission capacity is first allocated in an auction, and then local energy spot markets clear. In contrast, in the integrated energy and transmission market (also called nodal/zonal pricing, market splitting, and market coupling), the system operator accepts energy bids at all locations and clears the energy markets using available transmission capacity. The integrated market design implies that transmission capacity is allocated in reaction to the energy bids of strategic generators, which can increase effective demand elasticity (Neuhoff 2003), assuming that generators correctly anticipate how their actions will affect transmission prices. This additional responsiveness reduces the exercise of market power and in the example of this paper results in price reductions in all zones, confirming previous theoretical results (Ehrenmann et al., 2003).

A third strategic modeling issue concerns bounded rationality. One could assume that strategic generators simplify the world when deciding on their optimal output decision (Barquin and Vazquez, 2003). This bounded rationality could imply, for instance, that a generator always assumes that the last period's transmission constraints will remain binding and that no other transmission constraints will limit the solution when calculating the equilibrium Nash output decisions of his fellow strategic generators and himself. If the anticipated constraints coincide with the realised constraints then believes of the agents are consistent and the algorithm converges towards a local Nash equilibrium. In our calculations, the approach resulted in similar prices to the full rationality model of an integrated market design, although the solution procedure often resulted in oscillations.

However, the mathematical model for an integrated market is inherently non-convex and difficult to solve. The structure of that model is an "equilibrium problem with equilibrium constraints" (EPEC), and such models may have no pure strategy equilibria or several equilibria (Hobbs and Helman, 2004). One way to avoid the related difficulties is to use a Bertrand assumption on transmission prices, which simplifies the market model (Metzler *et al.*, 2003). The Bertrand assumption can be interpreted in the framework of bounded rationality: strategic generators do not anticipate that they will influence transmission prices. Under this assumption their output decision is less affected by transmission constraints and therefore they continue to bid more competitively. In our data set with a highly meshed network, the approach resulted in lower prices.

The purpose of this paper is to document the differences in the results of these various model approaches, and to relate those differences to the assumptions they make. In Section 2, we give a literature review, followed by the description of the data set in Section 3. In Section 4, a basic model of strategic generators in networks is introduced. We then expand the model to represent different cost functions (Section 5), and more complex interactions such as generators anticipating their impact on fringe generators (Section 6), and generators anticipating the reaction of the system operator (Section 7). Section 8 discusses alternatives representations of the system operator to facilitate numerical tractability. Additional features that can increase model realism, such as long-term contracts, are presented in Section 9. A set of conclusions in Section 10 closes the paper, including the results of a poll of modelers and model users on desirable future directions in model development and comparisons. Appendix 1 summarizes each of the models considered.

2. LITERATURE REVIEW

Because transmission constraints can isolate markets and enhance market power, a number of models of strategic interaction on networks have been developed (see reviews by Daxhalet and Smeers, 2001; Day et.al., 2002; Ventosa et al., 2003). Most models of generator competition take a general approach of defining a market equilibrium as a set of prices, generation amounts, transmission flows, and consumption that satisfy each market participant's first-order conditions for maximization of their net benefits while clearing the market. If a market solution exists that satisfies this set of conditions, it will have the property that no participant will want to alter their decision unilaterally (as in a Nash equilibrium). Although it is recognized that no modeling approach can precisely predict prices in oligopolistic markets, there appears to be agreement that equilibrium models are valuable for gaining insights on modes of behavior and relative differences in efficiency, prices, and other outcomes of different market structures and designs (Smeers, 1997).[1]

Equilibrium market models differ in many ways, including the market mechanisms modeled, the type of game assumed, fidelity to the physics of power transmission, and computational methods. Regarding market clearing mechanisms, most studies of generation markets have implicitly or explicitly assumed a single buyer or "pool"-type centralized bidding process supervised by an Independent System Operator (ISO) (*e.g.*, Cardell et al., 1997). This process results in a set of publicly disclosed market clearing prices. Other studies model bilateral trading with or without

[1] There are, of course, other ways to project the extent and impacts of market power, including empirical studies based on past behavior in the market of interest or similar markets (Borenstein et al., 2002), experiments with live subjects (e.g., Smith, 2000), and simulations using artificial automata (e.g., Bower and Bunn, 2000). The approaches have complementary advantages. The main advantages of equilibrium models are the ability to prove general results, verifiability and replicability, ease of computation, and their grounding in accepted economic and game theoretic concepts.

the presence of traders/arbitragers (Metzler et al., 2003; Smeers and Wei, 1999). Some studies assume that that transmission services and energy markets are cleared simultaneously or are well arbitraged, while others assume a sequential process. The practical differences between these formulations are one focus of the comparisons we report in this paper.

Turning to the type of game represented, most models assume some type of Nash game. Cournot models appear to be most popular, and are the emphasis of this paper. Several variants exists to represent network-constraints in Cournot models. Some assume that generators are price-takers (Bertrand) relative to the cost of transmission (Blake, 2003; Metzler et al., 2003; see Section 8, *infra*), while more sophisticated (but more difficult to solve) models represent generators as being Stackelberg leaders with respect to transmission pricing and allocation decisions by transmission system operators (TSOs) (Borenstein *et al.*, 2000; Cardell *et al.*, 1997; see the two-stage model of Section 7, *infra*). The quantity strategies in the Cournot models are usually the amounts generated by existing power plants. However, a few multistage dynamic Cournot models have been formulated which represent capacity investments, although transmission constraints are disregarded (Chuang et al., 2001; Murphy and Smeers, 2002).

Nash games in other types of strategies have also been modeled in transmission constrained markets, including games in prices (Hobbs and Schuler, 1985) and games in supply functions, in which each firm submits a schedule of the quantities it is willing to deliver under different prices. As an example of the latter, system operators in Denmark and California have applied models in which each company decides a fixed amount or percentage by which it will mark up all its marginal costs when constructing bids (Kristoffersen et.al. (2003); London Economics, 2003). As further examples, Hobbs *et al.* (2000) and Weber and Overbye (1999) have represented bidding games among competitive generators who are also Stackelberg leaders with respect to TSO decisions about transmission. Research is also being done on other symmetric games among generators, such as conjectural variations (Garcia et al., 2002), conjectured rival supply functions (Day et al., 2002), and “supergames” in which collusive solutions are bounded by incentive compatibility constraints (Harrington et al., 2003). In addition, there are a few models of asymmetric games — in particular, Stackelberg games — in which larger generators act as Stackelberg leaders with respect to a set of smaller generators who are either Cournot players or price-takers (e.g., Chen et al., 2003). One of the models compared in this paper is of the latter type, in which the competitive fringe is modeled as a Stackelberg follower.

Early research in power markets usually disregarded transmission constraints or considered only Kirchhoff's current law, disregarding the voltage law that forces power to flow in parallel paths. However, because the voltage law results in tighter constraints in flows and can yield surprising pricing results, more recent transmission-constrained models have included both of Kirchhoff's laws.[2] This is usually accomplished using the linearized "DC" load flow model (Schweppe et al., 1988), in which constant "power transmission distribution factors" $PTDF_{ijk}$ describe how many MW of flow occur on a particular line k in response to an assumed injection of 1 MW at node (or "bus") i and a matching withdrawal of MW at bus j. The DC model's linearity allows use of the principle of superposition, which greatly simplifies load flow calculations for market models relative to the complete nonlinear AC load flow model. The DC model disregards reactive power flows and voltage constraints, and usually excludes calculations of resistance losses. A very few oligopoly models have either included nonlinear resistance losses in the DC model (Chen et al., 2003) or a full AC representation (Bai et al., 1997).

Finally, turning to solution methodology, a variety of approaches have been used to solve equilibrium models that are sufficiently complex that closed form solutions are not possible. One basic approach is to discretize the strategy space, and then either examine all possible combinations for possible Nash equilibria (Bai et al., 1997), or heuristically search for an equilibrium (Kristoffersen et.al. (2003)[3], ; London Economics, 2003). The former is possible only with very small models and limited numbers of strategies. Discretization of the decision space has been used for models for which it would be impossible to consider continuous strategy spaces, for example AC load flow-based models or network-constrained supply function equilibrium models.

[2] As an example of the counterintuitive effects of Kirchhoff's Voltage Law, an addition of a transmission line can lower the transmission capacity of a system (Wu *et al.*, 1996); an optimal strategy of Cournot generator can be to increase output in order to congest lines and keep out competition (Cardell *et al.*, 1997); and decreased market concentration can increase prices by worsening transmission congestion at critical locations (Hobbs *et al.*, 2003).
[3] More detailed description on http://www.eltra.dk/composite-15381.htm.

The alternative is to retain the continuous strategy space. If "nice" first-order conditions can be defined for each market player's optimization problem, then an equilibrium can often be found by simultaneously solving *n* equilibrium conditions (including each player's KKT conditions and market clearing) for *n* variables representing the decisions and market quantities and prices.[4] Equilibria for models with continuous strategy spaces can also be sought using diagonalization (Cardell *et al.*, 1997; Ehrenmann and Neuhoff, 2003; Hobbs *et al.*, 2001; Hobbs and Schuler, 1985; Weber and Overbye, 1999). In each iteration of the diagonalization algorithm (a type of Gauss-Seidel algorithm), one agent chooses the optimal value of its strategic variables taking the decisions made by other firms in previous iterations as being fixed. Once no firm wants to change its decisions, a Nash equilibrium has been found.

In the case of power market models representing both of Kirchhoff's laws and therefore an endogenous allocation of link based transmission capacity, most are Cournot-based (e.g., Metzler *et al.*, 2003; Stoft, 1999; Wei and Smeers 1999). However, one paper uses conjectured supply functions (Day et al., 2002).

In terms of the above classification, the models compared in this paper are Nash-Cournot equilibrium models of generator competition on linearized DC networks, in which continuous strategy spaces are considered and solutions are obtained either by diagonalization or simultaneous solutions of equilibrium conditions. A variety of market-clearing mechanisms are considered. The models considered here and their distinguishing features are summarized in Figure 1 and described in more detail in Appendix 1.

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[4] In general, the complete set of KKT and market clearing conditions defines a *mixed complementarity problem* (MCP). The general form of a MCP problem is as follows: find vectors $\boldsymbol{x}, \boldsymbol{y}$ that satisfy the conditions $\boldsymbol{x} \geq 0$, $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) \leq 0$, $\boldsymbol{x}^T\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}) = 0$, and $\boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}) = 0$, where $\boldsymbol{f}$ and $\boldsymbol{g}$ are vector valued functions. There should be exactly as many conditions as variables. In another type of model (the mathematical program with equilibrium constraints, or MPEC), the KKT conditions for one player are embedded as constraints in another player's optimization model (as in a Stackelberg model in which a leader's model correctly anticipates the response of the Stackelberg followers, as represented by their KKTs).

	ECN,	Madrid,	Cambridge
Cournot, SF	both	both	Cournot
2 Stage Implement	no	yes	yes
T reaction to Gen	no	E,fixed C	both
T contracts	both	no	no
Fringe	expect.	Realised	both
Cost function	p.l. /step	Cont./p.l.	p.l.

Figure 1 Summary of Different Models

- Cambridge I: Cournot generators assume that all of their output is sold at the location generated without anticipating that the TSO will alter the amount of imported or exported electricity. As an option, behavior of the fringe generators can be chosen as non-responsive (1 stage game) or responsive (2 stage game) to the generators output choices.
- Cambridge II (Ehrenmann and Neuhoff 2003): Cournot generators correctly anticipate how the TSO will arbitrage price differences by changing the amounts of power it buys and sells at different nodes. Cambridge II is a 2-stage model in which the optimality conditions of the TSO are included as constraints in the generators problems. The fringe generators output quantities are modeled as being responsive to the strategic generators output decision.
- Madrid (Barquin and Vazquez, 2003): Cournot producers anticipate how their output decisions will affect transmission prices under the assumption that the binding transmission constraints will not change. An iterative procedure is proposed in order to obtain a set of binding transmission constraints consistent both with generators and TSO actions.
- COMPETES (Hobbs *et al.*, 2003): Cournot generators are Bertrand (price-taking) with respect to the price of transmission services, which is an even greater (but computationally convenient) simplification of the full two-stage model (see Section 8, *infra*). Competes furthermore offers the option to use linear or stepwise cost functions for generators. Other versions of COMPETES represent each generating firm’s conjectures regarding changes in rivals’ outputs and transmission prices in response to the firm’s decisions.

We restricted ourselves to models with a limited number of free parameters. This allowed us to focus on the impact of different model specifications and already provided for a large number of scenarios to compute, compare and analyse. For this reason we did not run COMPETES in the mode of linear conjectured supply functions but only as Cournot model. Otherwise we would have had to either assume the slope (typical) or the intercept exogenously to obtain unique solutions.

3. Test System

A realistic case study was created for the purposes of this model comparison.[5] Figure 1 shows the representation of the electricity grid that is used for the current modelling purpose. This representation contains fifteen nodes and is derived from a more detailed representation of the network that was already used in COMPETES. Demand for electricity and production capacity is allocated to seven of these nodes, the other nodes are used as intermediate nodes of the linearized DC network. In the Netherlands demand and production is allocated to three nodes (Zwol, Krim, Maas), in Belgium demand and production is split among two nodes (Merc, Gram) and in France and Germany all demand and production is allocated to one node (respectively F and D), the other nodes in these two countries are intermediate.

As shown in Figure 1 by the brown lines the nodes are interconnected by 28 different flowgates. In the COMPETES and Cambridge formulation these flowgates correspond to the transmission constraints. All flowgates are characterised by an upper limit in MWe, and by power transmission distribution functions (PTDF's) characterising the increase of the amount of energy that passes through each flowgate when electricity is transmitted from a specific node to the reference node 'D' in Germany.

In the operation of an electricity network, the operator should take into account the possibility of a line outage in order to avoid possible interruptions of supply. For this purpose TSO's have developed the so-called (n-1) principle as a security criterion to ensure a reasonable level of network reliability. It means that a network must be able to cope with a sudden outage of one of

[5] For a detailed listing of the problem data, please refer to www.electricitymarkets.info

the network components (such as a circuit failure) without interrupting the supply of electricity. In a study performed by IAEW and Consentec[6] two relevant line outages contingencies are identified that may limit the flows within the Benelux, France and Germany.

When calculating the PTDF's and the upper limit of the flowgates, the critical contingency - an outage of one of the two "Selfkant" tie line circuits between the nodes MAAS and ROMM - is considered.[7] Therefore instead of two circuits only one circuit is available on this flow-gate. The capacity of the flow gate is reduced and the increased reactance changes the PTDF's relative to a non-security constrained world.

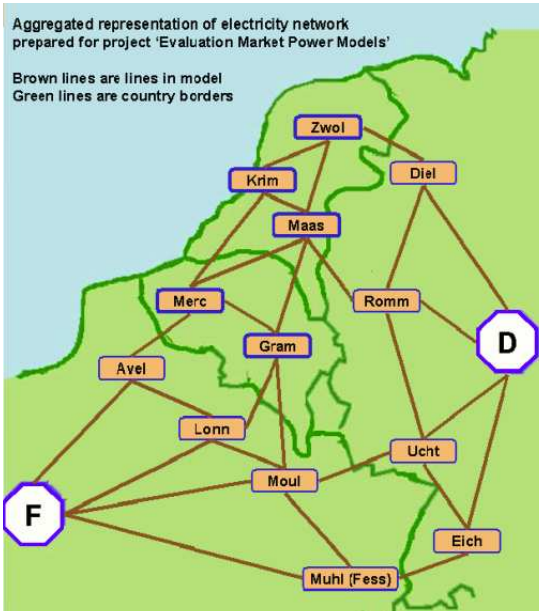

Figure 2 Grid used in the model comparison

Generators and their capacity

Eight firms are considered as strategic generators with production in one or several countries. These strategic generators are shown in Table 1 in the countries where they are active. Production units in Belgium and Netherlands are divided between the nodes in each country

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[6] H.J. Haubrich, C. Zimmer, K. von Sengbusch, F. Li, W. Fritz, S. Kopp, W. Schulz, F. Musgens, and M. Peek, "Analysis of Electricity Network Capacities and Identification of Congestion," Inst. Power Systems and Power Economics, Aachen University of Technology, www.iaew.rwth-aachen.de/publikationen/EC_congestion_final_report_appendix.pdf, 2001.

[7] Two potential critical contingencies are considered in the Aachen report (see previous footnote). For the model comparison project only one was taken into account. In the more detailed dataset of ECN both are considered.

according to their location. In Germany and France all units are allocated to the national nodes *D* and *F*. The remaining generation plants, not owned by one of the strategic players, are assumed to bid competitively.

Table 1 Strategic players per country

Countries	Strategic companies
Germany	EOn, ENBW, RWE, Vattenfall, EDF,
Belgium	Electrabel
France	EdF, ENBW
Netherlands	Essent, Nuon, E.ON, Electrabel

Variable costs of the generation units are represented in two separate ways: first, using a two-part linear cost curve and second, using a four-step cost function. The detailed data on generation capacity per generator and per node and their corresponding variable cost will be made available on the project website.

Demand data

To create ten different demand scenarios the empirical load data for the summer and winter super peak have been used as the first two scenarios. Then additional four summer and four winter scenarios were produced by reducing the peak output with a scenario and location specific random factor. The corresponding demands are assumed to be requested at a price of 30 Euro/MWh and a linear demand slope is imposed which corresponded to a demand elasticity of 0.1 at 30 Euro/MWh. Figure 3 shows the load assumed in the different scenarios.

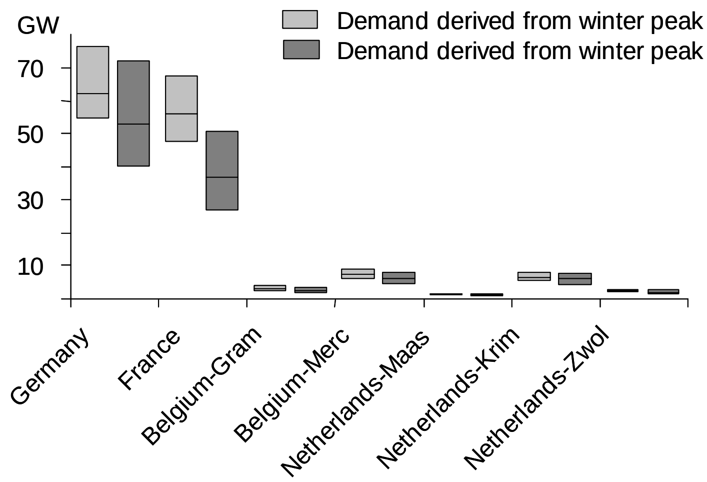

Figure 3 Demand levels in MW at the nodes for five summer and five winter scenarios

All participating models calculated the competitive case with this data set and obtained the same equilibria for all scenarios. This serves as the benchmark for the strategic cases.

4. Non-responsive Allocation of Transmission Capacity

The basic model consists of strategic generators deciding on their outputs assuming that their decisions will not affect the output decisions of their competitors (Nash-Cournot) or the fringe generators; the allocation of transmission capacity by the TSO to traders; or the amounts of power moved by traders from one location to another. Generators only sell at the location of the electricity production, consistent with a "Pool" market design. Traders (or the TSO itself) move power from one location to another to arbitrage price differences. A key implication of these assumptions is that each producer believes that any change in output by any of its generation units must be absorbed by local demand; this results in a relatively inelastic residual demand curve at each node (and, as we shall see, higher prices than under alternative assumptions). The TSO allocates transmission rights to traders as suggested by Chao and Peck (1996) such that the allocation would maximise social welfare if the bids submitted by all participants were competitive. This standard assumption for the TSO will be retained throughout the paper.

Under these assumption, an equilibrium is achieved if each generator chooses their profit maximising outputs for all their plants, the TSO achieves the welfare maximising allocation of transmission rights (naively assuming bids for transmission are competitive), and the competitive fringe produces until its marginal cost equals the locational price or it faces capacity constraints. This basic model has the nice feature that with convex costs it becomes a convex problem for each player. Players have a compact set of output choices; therefore existence of a solution is guaranteed (Harker and Pang 1990).

Two approaches can be applied to find an equilibrium. First, the stationarity (first-order) conditions of all agents are compiled in one mixed complementarity problem (MCP) and solved by algorithms such as PATH (Dirkse and Ferris, 1995). Second, using a diagonalisation algorithm, the optimisation problems for each of the strategic generators, the system operator and each of the fringe generators are solved sequentially.

COMPETES used the MCP-formulation and Cambridge I calculated the equilibrium using a diagonalisation approach. For all scenarios both models calculated the same equilibria (Figure 3).

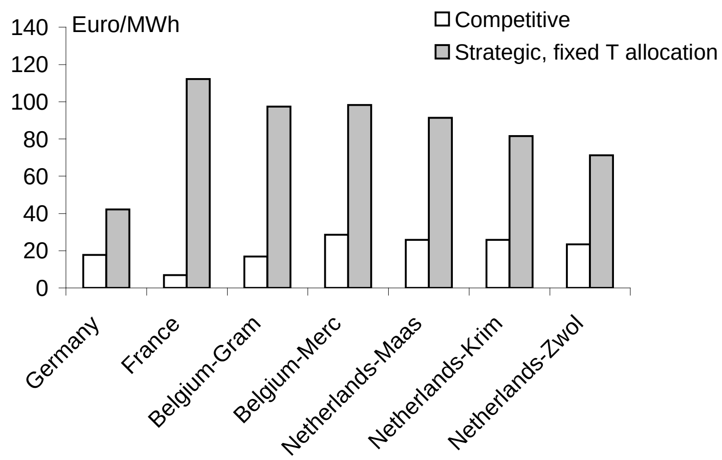

Figure 4 Prices in competitive solution and strategic solution with non-responsive transmission allocation and fringe generation (averaged over 5 summer scenarios).

Figure 4 gives the equilibrium prices calculated both by COMPETES and Cambridge for all nodes averaged over five summer scenarios.[8] The calculated strategic prices exceed the competitive equilibrium prices by a large margin. These prices are high because of the low price elasticity for each location's demand curve, and because each generator's perceived effective demand only includes the local demand, and not the effect of export opportunities or competitive imports.

5. Step vs. linear cost function

The previous calculations used continuous, linearised costs functions with two steps as illustrated in Figure 5.

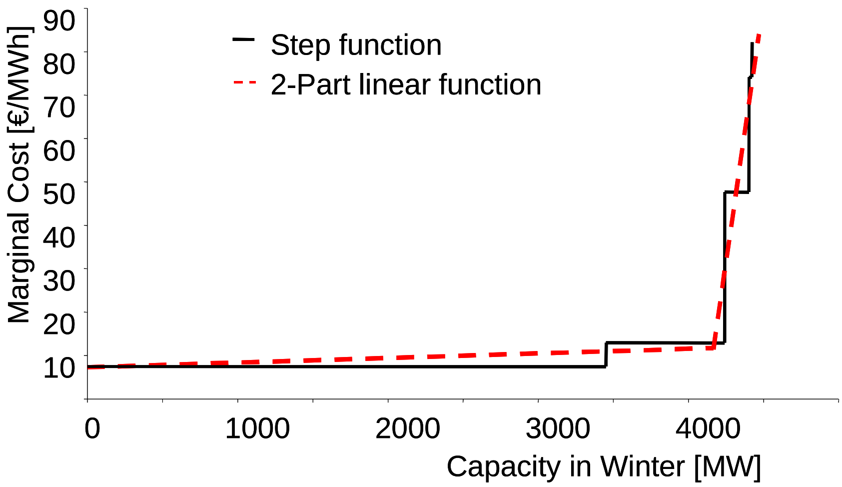

Figure 5 Marginal cost curves for Electrabel generators at node Gram.

To see whether the linearisation impacts the results, each model was also run with step cost functions having up to four steps. The average price increased by 1 Euro/MWh with a standard deviation of 1.6 Euro/MWh. The impact can be considered small relative to the price levels.[9]

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[8] The complete results of all models are available at www.econ.cam.ac.uk/electricity.

[9] The same calculations were performed for responsive transmission allocation with Bertrand assumption, discussed in Section 8. The step wise cost function increased the average price by 0.7 Euro/MWh with a standard deviation of 2 Euro/MWh.

6. Strategic generator anticipates fringe reaction

The basic one-stage model of Section 4 is built on the assumption that each strategic generator does not anticipate that the fringe generators will change their output in response to their strategic output decisions. This can be justified as a bounded rationality in the form of generators paying limited attention to the detailed market structure. However, typical spot market arrangements allow fringe generators to submit several bids to the spot market, such that they are called to produce if the day ahead price exceeds their marginal costs. If generators understand this mechanism, then they will anticipate the change of fringe supply when calculating the profits they could obtain with different output choices. In real power markets, demand actually has very little short-run elasticity, so that the fringe (including perhaps imports) provides most of the elasticity in the effective demand curves faced by generators (Bushnell, 2003).

One approach to representing the reaction of the fringe is to approximate its effect by increasing the elasticity of the demand functions (*ibid.*). Another is to explicitly represent the reaction of fringe generators to the output of strategic generators in a two-stage (Stackelberg) model. In the second stage, the fringe generators (Stackelberg followers) chose their outputs such that their marginal costs equal the local spot price. Then in the first stage, the strategic generators (Stackelberg leaders) chose output to maximise their profits and the TSO decides on net-transmission to maximises welfare. Both decisions are made in anticipation of the reaction of demand and fringe generation in the second stage.

The nice property of the basic one-stage model that its solution exists and is usually unique is lost because capacity constraints or kinks in cost functions of fringe generators in the second stage result in non-concavities of the profit functions in the first stage. The Cambridge I model nevertheless succeeded in calculating the equilibrium output choices. The algorithms can only ensure local stationarity of a solution.[10] It is difficult to prove that the local solution is global because of the large strategy space. Furthermore, local uniqueness of the equilibria is no longer guaranteed, and Cambridge did find multiple solutions for some of the scenarios. The differences

[10] For a maximization problem we call a point (B-) stationary for a function $f:R^n \rightarrow R$ if its directional derivative $f'(x^*,d)\leq 0$ for all feasible directions d.

between the solutions were however small relative to differences between the results for different market designs, as discussed in more detail in Ehrenmann and Neuhoff (2003).

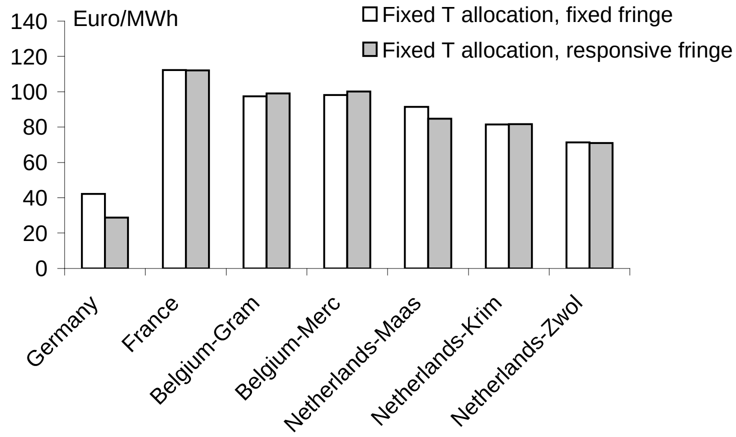

Figure 6 Effect of allowing generators to assume that fringe output reacts to their output decision (averaged over five summer scenarios)

Figure 6 shows for the average of the summer scenarios that the additional demand elasticity provided by fringe generators induces generators to increase their outputs. The result is lower prices at most nodes. The effect is particularly strong in Germany because of large the amount of fringe generation. Also, Netherlands-Maas has more fringe generation than other Low Country nodes, which causes a greater price decrease at that location than elsewhere in Belgium and the Netherlands. Due to the different prices, the equilibrium allocation of transmission capacity by the TSO also differs, with smaller net imports at some nodes. If the reduction of net-imports exceeds output increases by generators, as in Belgium-Merc, then in a few scenarios local equilibrium prices are higher.

7. Responsive Allocation of Transmission Capacity

In the previous models, strategic generators assume that net-imports by traders (or the TSO) to each of the nodes do not change as result of their output decision. However, in some market designs, TSOs allocate transmission capacity dependent on the bids submitted by generators, e.g., in PJM or Nordpool. If generators submit less energy to one node, then the higher price at the node makes energy imports into that node more valuable. As a result, more of the scarce transmission capacity will be used to transmit energy to that node, effectively increasing the net-demand elasticity generators face at any one node (Neuhoff, 2003).

In Cambridge II, the strategic generators not only anticipate the reaction of fringe generators, but also the reaction of the TSO to their output decisions. This is represented by using the first-order conditions of the TSO and the fringe generators as constraints in the optimisation problems of strategic generators (Ehrenmann and Neuhoff, 2003). The resulting equilibrium problem among the generators is a non-convex EPEC (see Section 2). As mentioned in Section 2, some previous Cournot models have represented this Stackelberg game between generators and TSO in a similar manner (Borenstein *et al.*, 2000; Cardell *et al.*, 1997).

Cambridge II uses a diagonalisation approach to solve the mathematical problem with equilibrium constraints (MPEC) for each player sequentially. Unlike before, local equilibria were only found in 6 of the 10 scenarios, which is not surprising since their existence cannot be guaranteed for nonconvex EPECs. The solutions that we found were not unique, and we suspect they were part of a continuous set of equilibria.

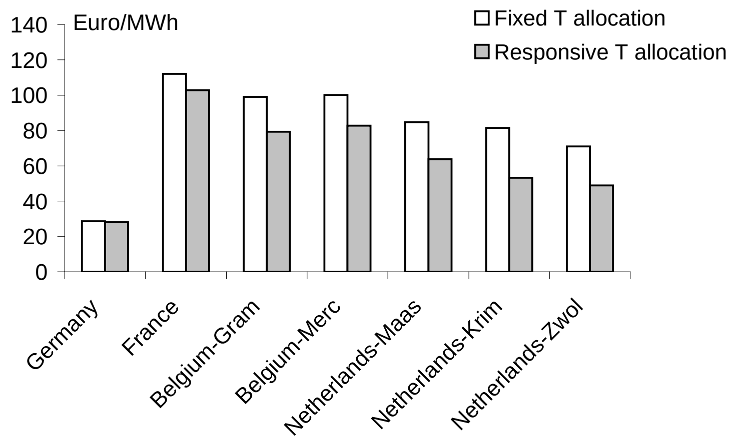

Figure 7 Generators anticipate TSO transmission allocation decisions as response to their output decision. (averaged over five summer scenarios)

Figure 7 shows that increased demand elasticity provided by flexible allocation of import capacity increases competitiveness and therefore results in significantly lower prices in all locations except Germany. The basic reason is that producers now recognize that they can compete not only for load at their own locations, but also elsewhere, subject to transmission limitations; furthermore, they are also competing with generators located throughout the system. However, the prices are still well above competive levels (cf. Figs. 3 and 5).

8. Alternative approaches to model responsive transmission allocation

Solving two-stage models is an active field of research in part because they suffer from severe difficulties. First, solution are not even generically locally unique and it is not clear if any of them is 'better' than the others. Second, one can easily construct a network which has no pure strategy equilibrium solutions (*e.g.*, Berry *et al.*, 1999). Madrid and COMPETES use alternative ways to model the integrated market design thereby avoiding the difficulties of solving a two-stage model.

The Madrid model makes the assumption that generators believe that a fixed set of transmission lines is constrained. This gives a closed-form expression for net demand at all nodes, which can be substituted into the profit function of generators. Then the generators' optimisation problems can be solved in a single stage. The Madrid group complemented the approach with an iteration algorithm to search for a set of consistent beliefs, which can oscillate between different sets of binding constraints. Unfortunately, such an oscillation means that a full equilibrium has not been found, although it does not rule out the existence of such an equilibrium. Figure 8 illustrates such an oscillation for the summer peak scenario; fortunately, the upper and lower prices reached during one of these oscillations are typically close to an equilibrium calculated with the full two-stage approach (Cambridge II).

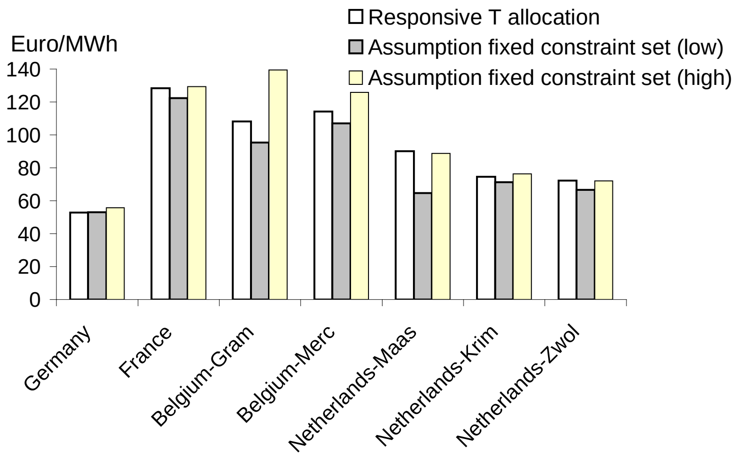

Figure 8 Price range of model with generators choosing output based on fixed set of transmission constraints (at example of scenario summer 1)

A second approach is used in the version of COMPETES used here, and is based upon Metzler et al. (2003). This version uses the Bertrand assumption for transmission rights: generators assume that the TSO sets prices for moving power from one location to another and offers, in effect, an unlimited amount of rights at this price. Generators therefore assume transmission prices are exogenously given in their optimisation problem and are not influenced by their output decision. This can be interpreted as bounded rationality on the part of strategic generators; such an assumption is more likely to be credible if the transmission system is a grid rather than radial in nature, and if the generation market is not too concentrated. In equilibrium, the TSO will set prices such that transmission constraints are satisfied. To represent an integrated market with the Bertrand assumption, the transmission operator or traders have to determine their transmission flows in reaction to the output decisions of strategic generators. This would still require a two-stage implementation. However, Metzler et al. (2003) show that if generators are allowed to sell their output at several locations using virtual transmission rights, then the two-stage problem with Betrand assumption can be implemented as a mixed complementarity problem.[11]

[11] Virtual transmission rights are sold simultaneously with the energy market. They do not share the typical property of transmission rights of being sold in a contracting stage preceding the energy market. This additional contracting stage usually creates pre-commitment and therefore changes generators' incentives to exercise market power in the spot market (Joskow and Tirole 2000). Virtual transmission rights do not provide additional contracting incentives and do not influence the exercise of market power. The easiest proof of this is that under nodal pricing the first order conditions for strategic generators are identical with and without virtual transmission contracts.

The profit function of a generator producing at node r and selling quantity s_i at node i consists of the sales revenue minus costs for transmission contracts (which under nodal pricing equal the price difference between the selling node i and the production node r) and production costs:

The results of the integrated market with Bertrand assumption are compared to the results of the integrated market with perfect rationality. Figure 9 shows that for our set-up the Betrand assumption results in lower prices. The two applied models furthermore differ, because in Cambridge implementation fringe generation reacts to the output changes of generators. As discussed in section 6, this leads to slightly lower prices calculated by Cambridge II than would have been obtained based on the assumption that the fringe is non-price responsive.

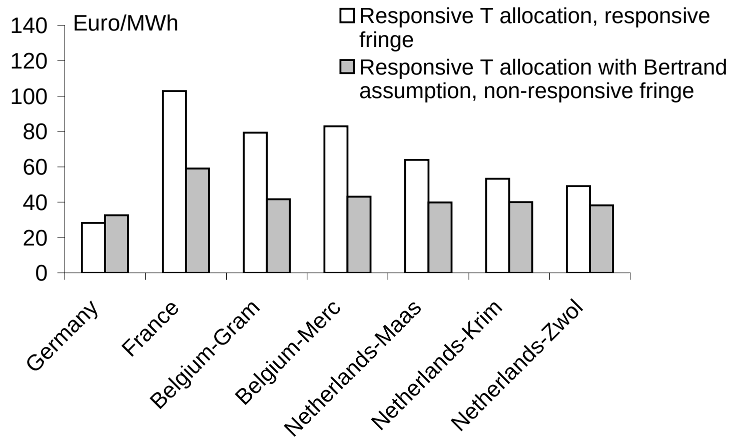

Figure 9 Comparison of full two-stage model with COMPETES (generators sell at several nodes and assume that they can obtain unlimited transmission rights at constant *p*)

9. Long term contracts

The previous graphs showed prices above the levels that are actually observed in those markets (with the exception of the competitive and, possibly, COMPETES results of Figure 4 and Figure 9 respectively). As a modeler, one is tempted to calibrate the model to match the observed prices. One frequently used approach is to increase demand elasticities thereby reducing the extent to which players in Cournot models exercise market power. However, there are other economic

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$$\pi = \sum_i s_i p_i - \sum_i s_i (p_i - p_r) - C(\sum_i s_i).$$

The optimal sales quantity at any one node is given by the first order condition with respect to s_k:

$$\frac{\partial \pi}{\partial s_k} = p_k + \sum_i s_i \frac{\partial p_i}{\partial s_k} - p_k + p_r - \sum_i s_i \left(\frac{\partial p_i}{\partial s_k} - \frac{\partial p_r}{\partial s_k} \right) - C'(\sum_i s_i) = p_r + \frac{\partial p_r}{\partial s_k} \sum_i s_i - C'(\sum_i s_i).$$

This corresponds to the first order condition of the generator only selling at his home node.

The construction of virtual transmission contracts that do not distort the exercise of market power ensures that the COMPETES model can represent an integrated energy and transmission market based on the Bertrand assumption as a one-stage model. This representation as a one-stage model facilitates the identification of a solution. Generic local-uniqueness and existence of a solution is guaranteed under very mild assumptions (Metzler et al., 2003).

reasons why actual prices are not at the modeled Cournot level. One reason frequently suggested (e.g., Newbery 1998) is that generators sign long-term contracts for their electricity output. If generators only sell a fraction of their output in the spot market, then they benefit less from pushing up prices by withholding output. At the same time they incur the same costs of forgone revenue on the withheld output, and so are less inclined to withhold output. The gray bars in

Figure 10 represent the results when EDF is modeled in Cambridge II as selling 20 GW forward in France, while Electrabel in Belgium contracts 2 GW each at the nodes Gram and Merc. This reduces prices at all three nodes relative to the situation with only nodal pricing. The higher aggregate production also decreases prices in the Netherlands.

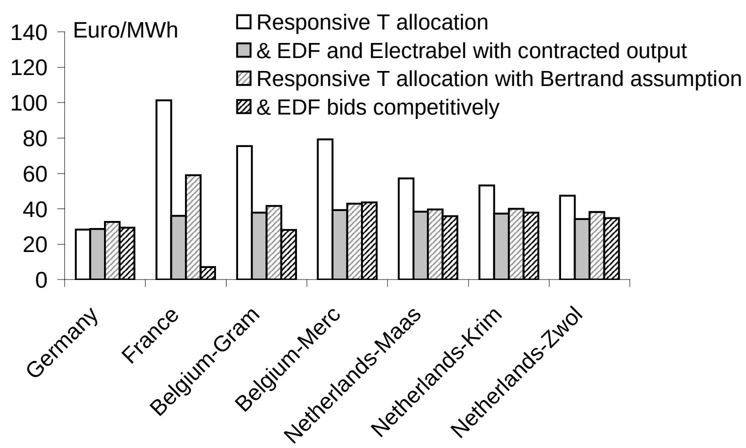

Figure 10 Changes in Cambridge II and COMPETES assumptions that yield prices closer to observed levels (averaged over five summer scenarios)

An alternative approach is to assume that the threat of regulatory intervention prevents monopolists to exercise market power. This can be represented by assuming that EDF bids competitively. The dark striped bars in Figure 10 show the resulting prices, and illustrates that they are lower than the two-stage reference case (Cambridge II).

Transmission contracts are a further group of instruments that can reduce the market power of generators. Gilbert et al. (2003) show that allocating transmission contracts in uniform price auctions always mitigates the market power of generation companies. The representation of the separate allocation of such contracts before the energy market in numerical models is an outstanding research issue.

Clever choice of reasonable assumptions can be used to replicate any observed price; this is a generic problem with over-parameterized models. For this reason, we based the model comparison project on models which did not include assumptions about forward contracting.

10 Closing comments

This paper has shown how structural and market design assumptions can affect the solutions of transmission-constrained electricity market models. The results of models developed by three different research groups were compared and analyzed. In the competitive mode all models predicted the same price, confirming that market design is of less relevance if strategic behavior by generation companies is ignored. We then used a simple, one step implementation; COMPETES and Cambridge consistently predicted the same price increase. COMPETES can use different representation of marginal cost curves. The results for piecewise linear and step-functions were surprisingly similar across the cost functions. Models can represent fringe generators as responsive to the bidding decisions of strategic generators or as only anticipating the equilibrium decision of strategic generators. Cambridge implemented both options and showed that at nodes with a significant contribution of fringe generators the responsiveness reduces the market power of strategic generators and lowers prices. The previous simulations assumed that transmission capacity is allocated at the same stage with bids in to the energy spot market.

Many market designs allow the system operator to determine the optimal use of the transmission network after receiving energy bids. All models were used to represent this more demanding modeling task. The Cambridge group uses a two-stage implementation to replicate the complete causality. Solutions are not guaranteed and not necessarily locally unique, but for all analyzed scenarios at least one solution could be identified. The Madrid group assumes that generators have a shared expectation on which transmission lines will congested. However, their subsequent output decisions resulted in congestion patterns that differ from their expectation. In an iterative calculation generators always base their expectation on the current congestion pattern and the model predicts oscillating congestion patterns. It is reassuring that the Madrid and Cambridge model predicted similar price regions. COMPETES avoids the numerical difficulties of the two stage implementation by introducing a Bertrand assumption: Generators take transmission prices as given and do not assume that they will not change in reaction to their output decisions. The model provides the additional feature that generators can sell at other locations. This allows the representation of a responsive transmission allocation with Bertrand assumption. Simulated prices are significantly lower than in the other two implementations. This shows that the Bertrand assumption makes generators bid more competitively if we assume that they cannot anticipate their impact on transmission prices.

These models and their results were discussed at a workshop held in October 2003 in Berlin by eleven experts in power market modeling and regulation, who considered their implications for future model development and comparisons. The experts included four staff from French, Belgian, and Dutch regulatory agencies and seven researchers representing a Danish system operator, universities from Spain, the UK, and the US, and the Netherlands Energy Research Center (ECN). During the workshop, the experts were surveyed regarding needs and priorities for future model developments. The four questions addressed concerned policy questions that need to be addressed by power market equilibrium models, shortcomings of present models, prerequisites for models to be useful in public forums, and desirable enhancements. A Nominal Group procedure was used to generate ideas (Van de Ven and Gustafson, 1975). This was done in four steps: (1) ideas were written down silently and anonymously, followed by (2) a group discussion and (3) anonymous vote, and (4) concluding with a group review of the results. The votes consisted of a 1-2-3 ranking by each person of the most preferred or important responses to each question. Table 2 groups those responses that received at least one first place vote, and tallies the points received by each category of responses. First place votes received three points, while second and third place votes were assigned two and one points each, respectively.

The table shows that the most important questions that models should be applied to concern the effect of market power, how it can be mitigated, and generation and transmission investment

(Question 1, Table 2). Thus, the focus of the models of this paper on strategic behavior is well placed; however, the models have a short-run perspective, and do not represent the dynamics of investment, which are of concern to policy makers. Therefore, the workshop participants would like to see explicit representations of long-run decisions, including allocation of forward contracts, in the models (Question 4, Table 2). Although the absence of long-run decisions in present models is of concern to the workshop participants, the table reveals that an even greater concern is the robustness, assumptions, transparency, and possible misuse of present models (Questions 2 and 3, Table 2). A goal of this paper has been to partially address the latter concern by clarifying the assumptions of different models and assessing the impact of those assumptions on the results. This comparison can help make models more useful in regulation by illustrating what can—and cannot—be learned from them. It is interesting to note that a requirement to match observed market behavior is ranked very low by the participants; insight rather than specific predictions are what is desired.

References

X. Bai, S.M. Shahidehpour, V.C. Ramesh, and E. Yu, "Transmission Analysis by Nash Game Method," *IEEE Trans. Power Sys., 12*(3), 1997, 1046-1052.

J. Barquin, and M. Vazquez, "Cournot Equilibrium Computation on Electricity Networks", 2nd International Workshop Liberalization and Modernization of Power Systems: Congestion Management Problems, August 11-14, 2003, Irkutsk, Russia.

C.A. Berry, B.F. Hobbs, W.A. Meroney, R.P. O'Neill, and W.R. Stewart, Jr., "Analyzing Strategic Bidding Behavior in Transmission Networks," *Util. Policy*, 8(3), 1999, 139-158.

M.S. Blake, "Game Theory Models in PLEXOS", Drayton Analytics, Adelaide, Australia (2003), www.plexos.info/kb/part_03/KB0307007.htm .

S. Borenstein, J. Bushnell, and S. Stoft, "The Competitive Effects of Transmission Capacity in a Deregulated Electricity Industry," *Rand J. Econ.*, 31(2), 2000, 294-325.

S. Borenstein, J. Bushnell, and F.A. Wolak, "Measuring Market Inefficiencies in Claifornia's Restructured Wholesale Electricity Market," *Amer. Econ. Rev.*, 92(5), 2002, 1376-1405.

J. Bower and D. Bunn, "Model Based Comparisons of Pool and Bilateral Markets for Electricity," *Energy J., 21*(3), 2000, 1-29.

J. Bushnell, "Looking for Trouble: Competition Policy in the U.S. Electricity Industry," CSEM Working Paper 109, University of California Energy Institute, Berkeley, April 2003, www.ucei.berkeley.edu/PDF/csemwp109.pdf .

J. Cardell, C.C. Hitt, and W.W. Hogan, "Market Power and Strategic Interaction in Electricity Networks," *Resource and Energy Econ*, *19*(1-2), 1997, 109-137.

H. P. Chao and S. Peck, "A Market Mechanism for Electric Power Transmission," *Journal of Regulatory Economics*, 10(1), 1996, 25-60.

Y.-S. Chen, B.F. Hobbs, S. Leyffer, and T. Munson, "Solution of Large-Scale Leader-Follower Market Equilibrium Problems: Electric Power and NO_x Allowances Markets," Working Paper, Johns Hopkins University, Dec. 2003.

A.S. Chuang, F. Wu, and P. Varaiya, "A Game-Theoretic Model for Generation Expansion Planning: Problem Formulation and Numerical Comparisons," *IEEE Trans. Power Sys.*, 16(4), 2001, 885-891.

O. Daxhelet and Y. Smeers, "Variational Inequality Models of Restructured Electric Systems," in M.C. Ferris, O.L. Mangasarian, and J.-S. Pang, *Applications and Algorithms of Complementarity*, Kluwer, 2001.

C.J. Day, B.F. Hobbs, and J.-S. Pang, "Oligopolistic Competition in Power Networks: A Conjectured Supply Function Approach," *IEEE Trans. Power Systems*, 17(3), 597-607, Aug. 2002.

S.P. Dirkse and M.C. Ferris, "The PATH Solver: A Non-Monotone Stabilization Scheme for Mixed Complementarity Problems," *Optim. Methods and Software*, *5*, 1995, 123-156.

A. Ehrenmann, 'Manifolds of multi-leader cournot equilibria' Operations Research Letters 32(2), 2004, 121-125.

A. Garcia-Alcalde, M. Ventosa, M. Rivier, A. Ramos, and G. Relano, "Fitting Electricity Market Models: A Conjectural Variations Approach," *Proceedings, PSCC 2002*, in process.

R. Gilbert, K. Neuhoff, D. Newbery, "*Allocating Transmission to Mitigate Market Power in Electricity Networks*," CMI working paper 07

J.E. Harrington, B.F. Hobbs, J.S. Pang, A. Liu, and G. Roch, "Collusive Game Solutions via Optimization," in P. Sauer, ed., *Proceedings, NSF EPNES Workshop, Orlando, Florida*, October 2003.et al.

P.T. Harker, J.S. Pang, Finite-dimensional variational inequalities and nonlinear complementarity problems: a survey of theory, algorithms and applications, Math. Programming 48 (1990) 161-220

B.F. Hobbs and U. Helman, "Complementarity-Based Equilibrium Modeling for Electric Power Markets", in D. Bunn, ed., *Modeling Prices in Competitive Electricity Markets*, J. Wiley, in press (2004).

B.F. Hobbs, C. Metzler, and J.-S. Pang, "Calculating Equilibria in Imperfectly Competitive Power Markets: An MPEC Approach," *IEEE Trans. Power Sys., 15*(2), 2000, 638-645.

B.F. Hobbs, F.A.M. Rijkers, and A.D. Wals, "Modeling Strategic Generator Behavior with Conjectured Transmission Price Responses in a Mixed Transmission Pricing System I: Formulation and II. Application," *IEEE Trans. Power Systems*, in press (2003).

B.F. Hobbs and R.E. Schuler, "Assessment of the Deregulation of Electric Power Generation Using Network Models of Imperfect Spatial Competition," *Papers Reg. Sci. Asso., 57*, 1985, 75-89.

X. Hu and D. Ralph, "A Bilevel Game Theory Model in Restructured Electricity Markets with Locational Marginal Prices", mimeo, Judge Institute, Cambridge, 2003

P. Joskow and J. Tirole, Transmission rights and market power on electric power networks. RAND Journal of Economics 31(3), 450-487, 2000.

B. Kristoffersen, B. Donslund, P.B. Eriksen, "Impacts of Large-Scale Wind Power on the Power Market." Proceedings from fourth International Workshop on Large-Scale Integration of Wind Power. October 20-21, 2003, Billund, Denmark. Royal Institute of Technology, Stockholm.

London Economics, "Economic evaluation of the Path 15 and Path 26 transmission expansion projects in California", Prepared for the California ISO, Cambridge, MA, 2003, www.londoneconomics.com/pdfs/ca_paprt_executive%20summary.pdf.

C. Metzler, B.F. Hobbs, and J.S. Pang, "Nash-Cournot Equilibria in Power Markets on a Linearized DC Network with Arbitrage: Formulations and Properties," *Networks & Spatial Theory*, 3(2), 123-150, 2003.

F.H. Murphy and Y. Smeers, "Generation capacity expansion in imperfectly competitive restructured electricity markets," CORE Discussion Paper 2002/69, Center for Operations Research and Econometrics, Universite' Catholique de Louvain, Louvain-la-Neuve, Belgium, 2002.

K. Neuhoff, “Combining Energy and Transmission Markets Mitigates Market Power”, CMI Working Paper 17, 2003

D. M. Newbery, “Competition, contracts, and entry in the electricity spot market,” *Rand Journal of Economics 29(4), 1998, 726-749*

Nordic Competition Authorities, "A Powerful Competition Policy", Appendix 1, Report from the Nordic competition authorities, No.1/2003. Copenhagen, Oslo, Stockholm, 20 June 2003.

F.C. Schweppe, M.C. Caramanis, R.E. Tabors, and R.E. Bohn, *Spot Pricing of Electricity*, Kluwer, Norwell, MA, 1988.

Y. Smeers, “Computable Equilibrium Models and the Restructuring of the European Electricity and Gas Markets,” *Energy J., 18*(4), 1997, 1-31.

S. Stoft, "Using Game Theory to Study Market Power in Simple Networks," in H. Singh, ed., *Game Theory Tutorial*, *IEEE Winter Power Meeting*, IEEE, Parsippany, NJ, Jan. 1999.

J.D. Weber and T.J. Overbye, “A Two-Level Optimization Problem for Analysis of Market Bidding Strategies,” Paper WE099, IEEE Power Engineering Meeting, Edmonton, July 1999.

J.-Y. Wei and Y. Smeers, “Spatial Oligopolistic Electricity Models with Cournot Generators and Regulated Transmission Prices,” *Oper. Res., 47*(1), 1999, 102-112.

Wu Felix, Pravin Varaiya, Pablo T. Spiller and Shmuel Oren, “Folk Theorems on Transmission Access: Proofs and Counterexamples” *Journal of Regulatory Economics* 1996, 10:5-23.

Appendix 1. Description of Modelling Approaches

COMPETES (Hobbs et al., 2003)

This model is a generalization of previous complementarity-based models for simulation of Cournot and Conjectured Supply Function (CSF) models of competition among electricity generators on a transmission network. Generating firms compete to sell power at the local price in each market (each node of the network, most generally), and they pay a TSO for transmission services from their generators to the point of sale. COMPETES is a static equilibrium model in which power producers either play a Cournot game at each location in the network (as in Metzler et al., 2003) or a more sophisticated CSF game in which producers anticipate that rival supply will respond in a locally linear manner to perturbations in energy prices (as in Day et al., 2002). The latter case can be viewed as a generalization of the Cournot assumption of no rival response to price changes, and is related to the notion of "conjectural variations." By parametrically changing the slope of the conjectured rival supply functions, different degrees of competitive intensity can be modeled, ranging from pure (Bertrand) competition (very large response by rivals to price increases) to Cournot competition (zero response). Positively sloped CSFs represent degrees of intensity between these cases.

Regarding transmission, the model assumes that generators are Bertrand price takers with respect to the transmission services market, which is represented as being constrained by a linearized DC load flow. The price charged to generators consists only of congestion-based fees needed to clear the market for transmission services. COMPETES has implemented the option to generalizes this transmission representation in two major respects. First, an individual generator can conjecture that transmission prices will change if it alters its demand for energy services, as opposed to the Bertrand assumption. Thus, a large firm (such as Electrabel) upstream of a transmission constraint might recognize that if it decreases the amount of transmission services it requests, the price for services will decline. Just as for the CSF model, the degree of price response is an exogenous parameter.

The second generalization concerning the transmission system allows for inefficient pricing mechanisms, reflective of actual institutions. These inefficient mechanisms include: lack of netting (no credit for congestion relief by counterflows), export taxes, and path-based transmission pricing (disregarding of parallel flows). These features, nor the conjectured supply and transmission price response, are not considered in the model comparison of this paper.

The model is formulated as a linear mixed complementarity problem (MCP). The steps involved in constructing and solving a linear MCP for a power market are as follows:

1. Define a linear or quadratic optimization problem for each market participant (generating firm, TSO, consumer, purveyor of emissions allowances, arbitrager/trader, etc.), in which each participant maximizes its profit or other objective subject to internal constraints, prices, and conjectures about how those prices are affected by its decisions.
2. Derive the first-order conditions for optimization of each problem (generally, Karush-Kuhn-Tucker conditions).
3. Define market clearing conditions.
4. Collect the first order and market clearing conditions into a single complementarity problem, which should be "square" (as many conditions as variables).
5. Implement the conditions using a matrix generation language for optimisation problems (AIMMS, in the case of COMPETES), and then use PATH or another numerical complementarity solver to obtain a solution to the problem.

Formulation as a LMCP makes possible the solution of large-scale models using efficient complementarity solvers (Dirkse and Ferris, 1995). COMPETES and other MCP-based models have been formulated and solved for very large systems with thousands or tens of thousands of variables. For instance, such models have been used to assess the relative competitiveness of different regions within the North American Eastern Interconnection (Hobbs and Helman, 2004). The MCP formulation also facilitates the establishment of existence and uniqueness properties.

Further description of an application of COMPETES to a more detailed network of northwest Europe is documented in Hobbs et al. (2003). There, the effects of transmission pricing policies, generation market structure, and expansion of transmission capacity are evaluated assuming competitive and oligopolistic market conditions.

Cambridge II Model — Integrated Energy and Transmission Markets (Ehrenmann and Neuhoff, 2003)

The static bilevel equilibrium model is motivated by Cardell et al. (1997) and allows the study of market power and transmission pricing issues in a deregulated electricity market under an independent system operator regime. Generators submit quantity bids to an independent TSO. Following the Nash equilibrium assumption each generator assumes that the bids of his competitors stay fixed. The TSO clears the market by deciding about dispatching generation and consumption based on submitted quantity bids by generators and demand curves of generators and marginal cost curves of competitive generators. The TSO dispatches such that social welfare would be maximised, were the bid curves based on locational marginal costs.

Each strategic generator's problem is an equilibrium problem with equilibrium constraints (EPEC); the overall problem is therefore an equilibrium problem with equilibrium constraints. The steps involved in construct and solving such an EPEC for a power market are as follows:

1. Define linear or quadratic optimization problem for the TSO in which the TSO maximizes social wellfare with respect to internal constraints (network constraints and Kirchhoff's Laws).
2. Derive the first order conditions for the optimization problem of the ISO and express it as constraints in the optimisation problem of the strategic generators.
3. Define the quadratic optimization problems for the generating firms and consumers in which the participants optimize profit with respect to internal constraints and the optimality conditions of the TSO.
4. Implement the conditions using a modeling language (GAMS).
5. Use an MPEC solver to solve the generators optimization problems sequentially (diagonalisation).
6. If the diagonalisation procedure converges, check for optimality for all generators. (existence of solution).

Nonconvexities created by transmission constraints can prevent the uniqueness or existence of a solution (Weber and Overbye, 1999) and the fact that all the generators share the same complementarity constraints can imply that a continuum of solutions exists. We would like to assess to what extend either issue causes a problem in the modeling of real(istic) networks.

Analytical models show that the ability of generators to exercise market power depends on the market design. The two basic design options are separate transmission auctions with subsequent energy spot markets or combined energy and transmission markets (nodal pricing, market splitting). Neuhoff (2003) shows that the second approach is preferable, but could not provide analytic solutions for situations with generation companies owning generation assets at several locations. More detailed descriptions neregarding the economic implications and different implementations will be available in Ehrneman/Neuhoff 2003 (forthcoming).

Cambridge I Model - Separate markets for energy and transmission (Ehrenmann and Neuhoff, 2003)

The Cambridge II model effectively implements the combined energy and transmission markets, therefore a second model was developed to represent the separate markets. It is assumes that not the system operator would arbitrage the markets but a competitive group of traders. These traders buy transmission rights from in an auction for transmission rights and then arbitrage the energy spot markets. The difference to Cambridge II is that traders have to submit their quantity bids to the energy spot markets at the same time as the strategic generators submit bids. This is realised by moving the system operator from the second stage - in which his scheduling decisions are contingent on the output choices of generators - to the first stage.

Madrid (Barquin and Vazquez 2003)

While the previous models were constructed to allow for implementation using standard optimisation algorithms, the Madrid model instead evolved from models with a more detailed representation of generation structure. To allow for the use of any differentiable cost function the model requires a consistent belief among generators and TSO about which transmission constraints will be binding.

The Madrid model iterates two different submodels. The first submodel simulates the behavior of the TSO when receiving a set of quantity offers from the different generators. The TSO is assumed to maximize the apparent social welfare implied by offers and bids taking into account network constraints. Demand is assumed to behave non-strategically. This submodel's main output is a matrix that relates the price sensitivity in every bus to power injections in each bus. Mathematically, a solution of a linear system is all that is required. The second submodel computes the oligopolistic market equilibrium when every generator conjectures the same price sensitivities, that is, input data for this submodel. Generators behave as Cournot oligopolists. Even though this equilibrium problem is not *per se* an optimization problem, it is shown that it can be transformed to one. Consequently, standard techniques which are able to deal with large systems can be applied. If quadratic cost functions (linear marginal cost functions) are used, then the optimization problem is a quadratic program with linear constraints. The output of this submodel is the quantity offer submitted by each generator that it is required by first submodel. Full consistency requires that the generators' conjectures used in the second submodel to have the same values as sensitivities obtained from market clearing performed by the TSO, as simulated by the first submodel. Therefore, there is a need to iterate between the submodels.

If the model converges, a pure strategy Nash-Cournot equilibrium has been computed. However, there are circumstances when no equilibrium exists. In this case, the algorithm oscillates between two or more states, defined by a different sets of constrained on or off interconnections. The algorithm stops when detecting these oscillations.

Table 1. Summary of Responses to Questionnaire on Model Uses and Capabilities (Numbers in brackets represent importance attributed to issue by participants)

Q1: What sort of questions would you like answers for?

A. Market Power Impacts & Mitigation (30)
- Give insights about impacts of market structure and design on market power, collusion, strategic gaming
- What are alternatives for mitigating market power?
- What is the impact of market transparency (e.g., information on binding transmission constraints in real time) on the exercise of market power? Is it different for radial and meshed systems?

B. Generation Investment (16)
- When should we invest in new production capacity?
- How do market uncertainties & market power interact to affect capacity addition decisions? Will there be enough capacity?
- How does market power mitigation affect incentives for capacity additions?
- How effective is long-term contracting in encouraging entry and controlling market power?

C. Effects of Transmission Investment on Market (12)

D. What criteria to use to choose the right market equilibrium in case of multiple equilibria/learning (2)

Q2: What are your concerns about present market equilibrium models?

A. Assumptions, Robustness, Calibration (15)
- Robustness of simulations – sensitivity analyses with respect to assumptions made
- Too many behavioral assumptions are made when modelling strategies
- What difference does different behavioral assumptions (type of game) make? Is it really, e.g., Cournot? What approach is most appropriate to actual markets? How are long-term considerations (like entry deterrence) relevant to bidding behavior?

B. Appropriate Use (13)
- People/policy makers take the exact numerical results too seriously
- Numbers/outcomes take on life of their own, unless model inputs/outputs and restrictions/possibilities are well explained

C. Transparency of assumptions and algorithms (12)

D. Inclusion of Market Features (10)
- Absence of capacity expansion capabilities
- Interface with other (possibility non-electrical) agents' concerns
- They don't account for liquidity problems
- Use of market-based methods when there is no market and when one player is better informed than others

E. Data: The availability and correctness of data, as producers dislike revealing good, realistic data, adding errors to the assumptions and results of the model (8)

F. Ability to anticipate the consequences of market design decisions such as choice of capacity allocation method, coupling of geographic markets (3)

Table 1, Continued ….

Q3: What would we require to use models to defend regulatory decisions?

A. Appropriate Use and Cautions (17)
- Understanding of what models can and cannot tell us
- Scientific "agreement" or best practice—no technical discussion in court
- Caution with numerical results – could ruin trust

B. Robustness and testing of results (15)

C. Transparency about model assumptions and limitations (13)

D. Not an appropriate use (11)
- (Our agency) does not intend to use models to defend its decisions. It rather wants to use them to prepare its decisions. Therefore, models should be simple but realistic and include a "parameterisation" of most possible solutions: (a) it is better to admit model limitations; and (b) we are interested in qualitative behavior, not absolute levels.

E. Realistic Inputs/Outputs, Match Observed Behavior (1)
- Strong empirical evidence to support decisions made

F. Inclusion of Certain Market Features (1)
- Integration of gas/electricity markets

Q4: What capabilities do we need next to answer the questions identified above?

A. Long-run decisions: capacity investments, forward contracts (22)
- Endogenous long-run decisions
- Represent different instruments for capacity markets
- Consider initial position (initial degree of forward contracting) in modeling day-ahead / contracting decisions
- Include entry deterrence and other long-term considerations in bidding behavior

B. Interactions among Multiple Markets (emissions trading, gas/electricity markets, ancillary services) (8)

C. Risk (7)
- Uncertainty modeling (demand, fuel prices, contingencies, …)
- What affects liquidity; how does liquidity affect market power?

D. Transmission Issues (6)
- Define quantities of transmission capacity allocated for spot markets and longer term
- Model the role of the system operator as a strategic agent
- In-depth evaluation of "netting" of interconnector flows (as opposed to "no netting": constraining transactions in each direction separately)

E. Rules (6)
- Effect of bidding rules and market power mitigation on equilibria
- Model flexibility: it should be easy to include new rules and markets features

F. Market Dynamics (multisettlement markets, dynamic market power models) (6)