

TYPE IA SUPERNOVAE ARE EXCELLENT STANDARD CANDLES IN THE NEAR-INFRARED

ARTURO AVELINO <sup>1</sup>, ANDREW S. FRIEDMAN <sup>2</sup>, KAISEY S. MANDEL <sup>3,4</sup>, DAVID O. JONES <sup>5</sup>, PETER J. CHALLIS,<sup>1</sup> AND  
ROBERT P. KIRSHNER <sup>1,6</sup>

<sup>1</sup>*Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138*

<sup>2</sup>*University of California, San Diego, La Jolla, California 92093, USA*

<sup>3</sup>*Institute of Astronomy and Kavli Institute for Cosmology, Madingley Road, Cambridge, CB3 0HA, UK*

<sup>4</sup>*Statistical Laboratory, DPMMS, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WB, UK*

<sup>5</sup>*University of California, Santa Cruz, Santa Cruz, California 95064, USA*

<sup>6</sup>*Gordon and Betty Moore Foundation, 1661 Page Mill Road, Palo Alto, CA 94304*

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ABSTRACT

We analyze a set of 89 Type Ia supernovae (SN Ia) that have both optical and near-infrared (NIR) photometry to derive distances and construct low redshift ( $z \leq 0.04$ ) Hubble diagrams. We construct mean light curve (LC) templates using a hierarchical Bayesian model. We explore both Gaussian process (GP) and template methods for fitting the LCs and estimating distances, while including peculiar velocity and photometric uncertainties. For the 56 SN Ia with both optical and NIR observations near maximum light, the GP method yields a NIR-only Hubble-diagram with a RMS of  $0.117 \pm 0.014$  mag when referenced to the NIR maxima. For each NIR band, a comparable GP method RMS is obtained when referencing to NIR-max or  $B$ -max. Using NIR LC templates referenced to  $B$ -max yields a larger RMS value of  $0.138 \pm 0.014$  mag. Fitting the corresponding optical data using standard LC fitters that use LC shape and color corrections yields larger RMS values of  $0.179 \pm 0.018$  mag with SALT2 and  $0.174 \pm 0.021$  mag with SNooPy. Applying our GP method to subsets of SN Ia NIR LCs at NIR maximum light, even without corrections for LC shape, color, or host-galaxy dust reddening, provides smaller RMS in the inferred distances, at the  $\sim 2.3$ - $4.1\sigma$  level, than standard optical methods that do correct for those effects. Our ongoing RAISIN program on the Hubble Space Telescope will exploit this promising infrared approach to limit systematic errors when measuring the expansion history of the universe to constrain dark energy.

*Keywords:* distance scale – supernovae: cosmology, general, infrared observations, optical observations, photometry

1. INTRODUCTION

The increasing sample of high quality, low-redshift (low- $z$ ), near-infrared (NIR) light curves (LCs) of Type Ia supernovae (SN Ia) provides an opportunity to further investigate their utility as cosmological standard candles. Optical samples of SN Ia are large enough now that systematic uncertainties are major limitation to ac-

curate cosmological constraints. Infrared observations of SN Ia can help in that essential way because supernovae are more nearly standard candles in the NIR and the effects of dust are diminished. This paper explores ways to use NIR observations of SN Ia to measure distances. This investigation is for a low- $z$  sample, but we are working to extend this technique to cosmologically-interesting distances with the Hubble Space Telescope (HST).

Before NIR photometry became practical for large samples of SN Ia, photometry and spectroscopy of SN Ia at optical wavelengths enabled the unexpected 1998 dis-

covery of cosmic acceleration (Riess et al. 1998; Schmidt et al. 1998; Perlmutter et al. 1999). Since then, a suite of independent cosmological methods has confirmed the SN Ia results (see Frieman et al. 2008; Weinberg et al. 2013 for reviews). The prevailing view is that the mechanism behind cosmic acceleration is some form of dark energy. The constraints on cosmological parameters from the SN Ia Pantheon sample (Scolnic et al. 2018) combined with the Planck 2015/2018 Cosmic Microwave Background data (Planck Collaboration et al. 2016b, 2018), as well as Baryon Acoustic Oscillations (Alam et al. 2017) and local Hubble constant measurements (Riess et al. 2016, 2018c,b,a) are consistent with this view. Among the major cosmological techniques, SN Ia provide precise measurements of extragalactic distances and the most direct evidence for cosmic acceleration (see Goobar & Leibundgut 2011; Kirshner 2013; Goobar 2015; Davis & Parkinson 2016; Riess et al. 2018c for reviews).

Optical SN Ia LCs are known to be excellent *standardizable* candles that exploit correlations between intrinsic luminosity and LC shape and color (Phillips 1993; Phillips et al. 1999; Hamuy et al. 1996; Riess et al. 1996, 1998; Perlmutter et al. 1997; Goldhaber et al. 2001; Tonry et al. 2003; Wang et al. 2003; Prieto et al. 2006; Jha et al. 2006, 2007; Astier et al. 2006; Takahashi et al. 2008; Conley et al. 2008; Mandel et al. 2009; Guy et al. 2005, 2007, 2010; Mandel et al. 2011, 2017). Recent work has demonstrated that SN Ia in the NIR are more nearly *standard* candles, even before correction for LC shape or host galaxy dust reddening (e.g. Krisciunas et al. 2004a; Wood-Vasey et al. 2008; Mandel et al. 2009; Krisciunas et al. 2009; Friedman 2012; Kattner et al. 2012). NIR LCs are  $\sim 5$ – $11$  times less sensitive to dust extinction than optical  $B$ -band data (Cardelli et al. 1989). When constructing SN Ia Hubble diagrams using NIR data, the distance errors produced by extinction are small: ignoring dust would be fatal for optical studies, but nearly not as serious for NIR studies like Wood-Vasey et al. 2008 or the present work. An improved approach would use optical and infrared data simultaneously to determine the extinction (Mandel et al. 2011).

Optical-only samples yield typical Hubble diagram intrinsic scatter of  $\sigma_{\text{int}} \sim 0.12$  mag and a RMS of 0.141 mag after applying light-curve shape, host-galaxy dust, and host-galaxy mass corrections, assuming a peculiar-velocity uncertainty of  $250 \text{ km s}^{-1}$  (e.g. Scolnic et al. 2018). For simplicity, we adopt a conservative peculiar-velocity uncertainty for the host galaxies in our sample of  $150 \text{ km s}^{-1}$ . If the typical redshifts in the sample were large enough, this would be of no consequence, but for

our nearby sample, the inferred intrinsic scatter of the supernova luminosities depends on the value we choose. As a result, though we have confidence when comparing the RMS and intrinsic scatter for various subsamples containing the same SN with both optical and infrared data, the real value of the scatter should be determined from observations that are securely in the Hubble flow beyond  $10,000 \text{ km s}^{-1}$ .

When including a peculiar-velocity uncertainty of  $150 \text{ km s}^{-1}$ , our best method yields intrinsic scatters as small as  $\sim 0.03$ – $0.11$  mag, depending on the NIR filter subset, and a RMS of  $\sim 0.087 \pm 0.013$  mag for the best NIR  $YJH$ -band subset, confirming and strengthening previous results for NIR methods (Meikle 2000; Krisciunas et al. 2004a, 2005, 2007; Folatelli et al. 2010; Burns et al. 2011; Wood-Vasey et al. 2008; Mandel et al. 2009, 2011; Kattner et al. 2012; Dhawan et al. 2015). Assuming a larger peculiar-velocity uncertainty, such as  $250 \text{ km s}^{-1}$ , makes our estimated intrinsic scatter even *smaller*. In addition, our best NIR method using any of the  $YJHK_s$  bands yields an RMS of only  $0.117 \pm 0.014$  mag, compared to  $0.179 \pm 0.018$  mag and  $0.174 \pm 0.021$  mag for SALT2 and SNooPy fits to optical  $BVR$  data for the same 56 SN Ia, respectively. While using LC shape, color, and host galaxy dust corrections would likely lead to improvements, the simpler approaches in this paper are still remarkably effective.

Overall, a substantial body of evidence indicates that rest-frame LCs of SN Ia in NIR are both better standard candles than at optical wavelengths and less sensitive to the confounding effects of dust. When NIR data are combined with  $UBVRI$  photometry, this yields accurate and precise distance estimates (Krisciunas et al. 2004b, 2007; Wood-Vasey et al. 2008; Folatelli et al. 2010; Burns et al. 2011; Friedman 2012; Phillips 2012; Kattner et al. 2012; Burns et al. 2014; Mandel et al. 2009, 2011, 2014, 2017).

This is significant for supernova cosmology because, along with photometric-calibration uncertainties (Scolnic et al. 2015; Foley et al. 2018), uncertain dust extinction estimates and the intrinsic variability of SN Ia colors present challenging and important systematic problems for dark energy measurements (Wang et al. 2006; Jha et al. 2007; Wood-Vasey et al. 2007; Hicken et al. 2009a; Kessler et al. 2009; Guy et al. 2007, 2010; Chotard et al. 2011; Conley et al. 2007, 2011; Komatsu et al. 2011; Campbell et al. 2013; Rest et al. 2013; Scolnic et al. 2013; Narayan 2013; Betoule et al. 2014; Rest et al. 2014; Mosher et al. 2014; Scolnic et al. 2014b,a, 2015; Narayan et al. 2016; Scolnic et al. 2017; Mandel et al. 2017; Foley et al. 2018; Scolnic et al. 2018; Brout et al. 2018a; Kessler et al. 2018). Combining optical and NIR

LCs promises to reduce these systematic distance uncertainties (Folatelli et al. 2010; Burns et al. 2011; Kattner et al. 2012; Mandel et al. 2011, 2014).

This work is organized as follows. In §2, we review previous results with SN Ia in NIR, detail our analysis selection criteria, and discuss host galaxy redshifts. In §3, we outline our Gaussian process (GP) procedure to fit LCs and our hierarchical Bayesian model to construct mean  $YJHK_s$  LC templates. In §4, we use these templates and GP fits to individual LCs to construct Hubble diagrams in each NIR band, as well as a combined  $YJHK_s$  NIR Hubble diagram. We compare this to optical  $BVR$  Hubble diagrams for the very same set of 56 supernovae that use the SALT2 and SNooPy LC fitters. We end with §5 by documenting how, even without correcting for LC shape or dust, SN Ia in the NIR using our GP fits at NIR maximum are better standard candles than optical SN Ia observations corrected for these effects. Mathematical details of the Gaussian process, the hierarchical Bayesian model, and the method for determining the intrinsic scatter are presented in the Appendices.

## 2. SN Ia IN NIR AS STANDARD CANDLES

Pioneering studies by Meikle (2000) and Krisciunas et al. (2004a) demonstrated that SN Ia have smaller luminosity variation in the NIR  $JHK_s$  bands than in the optical  $BV$  bands at the time of  $B$ -band maximum light ( $t_{Bmax}$ ). Krisciunas et al. (2004a) found that optical LC shape and intrinsic NIR luminosity were uncorrelated in a sample of 16 SN Ia, while measuring a NIR absolute magnitude scatter of  $\sigma_J = 0.14$ ,  $\sigma_H = 0.18$ , and  $\sigma_{K_s} = 0.12$  mag. Following this, Wood-Vasey et al. (2008) used a homogeneously-observed sample of 18 spectroscopically-normal SN Ia in the  $JHK_s$  bands, with intrinsic root-mean-square (RMS) absolute magnitudes of 0.15 mag in the  $H$ -band, *without applying any reddening or LC shape corrections*. By combining these 18 objects with 23 SN Ia from the literature, the sample in Wood-Vasey et al. (2008) yielded an  $H$ -band RMS of 0.16 mag, strengthening the evidence that normal SN Ia are excellent NIR standard candles. In the present work, we show that SN Ia in NIR yield a narrow distribution of  $YJHK_s$  peak magnitudes with RMS Hubble diagram scatter as small as  $0.087 \pm 0.013$  mag for the combined  $YJH$  bands and as large as  $0.179 \pm 0.029$  mag for the  $K_s$  band, consistent with previous results.

Following Wood-Vasey et al. 2008, Mandel et al. 2009 developed a new hierarchical Bayesian model (BAYESN) and a template model to account for  $J$ -band LC shape variation to the existing SN Ia in NIR sample, finding a marginal scatter in the peak absolute magnitudes of

0.17, 0.11, and 0.19 mag, in  $JHK_s$ , respectively, while finding that  $J$ -band LC shape does correlate with NIR intrinsic luminosity. Subsequent work by Folatelli et al. 2010 applied a different LC shape correction method, but found scatters of 0.12–0.16 mag in  $YJHK_s$ , consistent with the results of Mandel et al. (2009).

Additional work by Kattner et al. (2012) found an absolute magnitude scatter of 0.12, 0.12, and 0.09 mag in  $YJH$ , respectively, by analyzing a subset of 13 well-sampled normal NIR SN Ia LCs with relatively little host galaxy dust extinction. Kattner et al. 2012 also showed evidence for a correlation between the  $JH$ -band absolute magnitudes at  $t_{Bmax}$  and,  $\Delta m_{15}(B)$ , the light-curve decline rate parameter in  $B$ -band after 15 days of  $t_{Bmax}$  (Phillips 1993), with no evidence for strong correlation in the  $Y$ -band. This is also consistent with the results of Mandel et al. 2009, who found that  $J$ -band LC shape and luminosity are correlated.

Using a small data set of 12 SN Ia  $JH$ -band LCs, each with only 3-5 data points, Barone-Nugent et al. 2012, 2013 find a scatter of 0.116 mag and 0.085 mag in the  $J$  and  $H$ -bands, respectively. In the first data release of the SweetSpot survey, Weyant et al. 2014 present a similarly small sample of 13 low- $z$  SN Ia, each with 1-3 LC points, finding an  $H$ -band scatter of 0.164 mag. This was followed by a second SweetSpot data release, which included a total of 33 SN Ia with 168  $JHK_s$  observations in the redshift range  $0.02 < z < 0.09$ , well into the smooth Hubble flow, but which did not yet include NIR Hubble diagrams (Weyant et al. 2018).

By analyzing 45 NIR LCs with data near NIR-maximum, Stanishev et al. 2018 find an intrinsic Hubble diagram scatter of  $\sim 0.10$  mag, after accounting for potential new correlations between light curve shape, color excess, and  $J - H$  color at NIR-max. Stanishev et al. 2018 also present single-epoch  $JH$  photometry for 16 new SN Ia with  $z > 0.037$ . The Carnegie Supernova Project (CSP) final data release (CSP-I; Krisciunas et al. 2017), was recently analyzed in Burns et al. (2018), which found peculiar velocity corrected Hubble diagram dispersions of  $\sim 0.08 - 0.15$  mag, depending on the subset of the 120 SN Ia they considered. Additional CSP-II photometric data, to be published in 2019, was recently described in Phillips et al. 2019. Hsiao et al. (2019) present an overview of the NIR SN Ia spectroscopy obtained by the CSP and the Center for Astrophysics (CfA) Supernova Group.

While the current sample of optical SN Ia LCs exceeds 1000 (Scolnic et al. 2018), and will be increased by orders of magnitude by ongoing and future surveys including the Dark Energy Survey (DES; DES Collaboration et al. 2018a,b; Brout et al. 2018b; D’Andrea et al. 2018), the

Zwicky Transient Facility (ZTF; [Smith et al. 2014](#)), and the Large Synoptic Survey Telescope (LSST; [Ivezic et al. 2008](#); [Zhan & Tyson 2017](#)), the number of normal SN Ia with published NIR LCs is still less than 250. Nevertheless, the NIR sample has the potential to improve systematics compared to optical-only SN Ia cosmology samples, which are already systematics limited ([Scolnic et al. 2018](#)).

Overall, the growing sample of photometric data suggests that NIR observations of SN Ia present a promising path to standardize SN Ia for distance estimates ([Dhawan et al. 2015](#); [Shariff et al. 2016](#); [Burns et al. 2018](#); [Stanishev et al. 2018](#)), Hubble constant estimates ([Cartier et al. 2014](#); [Efstathiou 2014](#); [Riess et al. 2016](#); [Cardona et al. 2017](#); [Dhawan et al. 2018](#); [Burns et al. 2018](#)), and eventually, cosmological parameter estimates, when the nearby and high- $z$  samples are combined as in the HST RAISIN program (**RAISIN: Tracers of cosmic expansion with SN Ia in the IR**, PI. R. Kirshner, HST GO-13046, GO-14216).

### 2.1. Nearby SN Ia in NIR Sample and Data Cuts

This work analyzes a suitable subset including 89 objects from the current sample of low-redshift photometric data for SN Ia NIR  $YJHK_s$ -band LCs including data releases 1 and 2 from the Carnegie Supernova Project ([Schweizer et al. 2008](#); [Contreras et al. 2010](#); [Stritzinger et al. 2010, 2011](#); [Taddia et al. 2012](#)), now superseded by CSP data release 3 ([Krisciunas et al. 2017](#)), the CfA ([Wood-Vasey et al. 2008](#); [Friedman 2012](#); [Friedman et al. 2015](#)), and other groups (e.g. [Krisciunas et al. 2000, 2004b,c, 2005, 2007](#)). We limit our analysis to spectroscopically normal SN Ia from Table 3 of [Friedman et al. 2015](#), plus the definitive version of the CSP-I DR3 sample of low- $z$  SN Ia ([Krisciunas et al. 2017](#)), and other groups. Additional CSP-II photometric data, to be published in 2019, was recently described in [Phillips et al. 2019](#) and will be analyzed in future work.

We determine  $\Delta m_{15}(B)$  and  $E(B - V)_{\text{host}}$  with the SNooPy LC fitting package ([Burns et al. 2011](#)). In the process of applying  $K$ -corrections to transform the data to the rest-frame, we use SNooPy to transform all observer-frame  $JHK$ -band data to the natural system of the PAIRITEL/2MASS telescope used by the CfA ([Friedman et al. 2015](#)), with the observed  $Y$ -band data remaining in the natural system of the CSP Swope telescope, since only the CSP data included  $Y$ -band observations ([Krisciunas et al. 2017](#)).

We apply the following data cuts to analyze a subset of 89 SN Ia with NIR data. Table 1 shows how the initial sample of 177 SN Ia decreases after applying the

different cuts, and Table 2 lists the general properties of the remaining 89 SN Ia.

- Optical light curve shape parameter  $0.8 < \Delta m_{15}(B) < 1.6$ , to consider *normal* SN Ia only ([Hicken et al. 2009b](#)). Objects must have accompanying  $B$ -band optical data to measure  $\Delta m_{15}(B)$ .
- Host galaxy reddening:  $-0.15 < E(B - V)_{\text{host}} < 0.4$ . This cut is inspired by the standard SALT2 cut in color,  $-0.3 < c < 0.3$ , in optical-only analysis ([Betoule et al. 2014](#); [Scolnic et al. 2018](#)) but with a less stringent cut considering that SN Ia in the NIR are less sensitive to dust.
- One advantage of the relative NIR insensitivity to dust reddening is that it also allows us to set a large threshold for Milky Way color excess:  $E(B - V)_{\text{MW}} < 1$  mag, to exclude highly reddened SN Ia. All 177 SN Ia in the sample passed this cut. SN2006lf with  $E(B - V)_{\text{MW}} = 0.8135$  mag has the largest color excess in the initial sample.
- Redshift range:  $z < 0.04$ . The maximum redshift cut limits the effects of Malmquist bias. Section 2.2 describes corrections to deal with SN Ia at  $z < 0.01$ , that suffer from peculiar velocity bias.
- Duplicates: For a given supernova observed by multiple surveys, we use the CSP data ([Krisciunas et al. 2017](#)), which typically has smaller photometric uncertainties than the CfA PAIRITEL data ([Friedman et al. 2015](#)).
- We include only spectroscopically normal SN Ia as identified by the Supernova Identification Code (SNID) [Blondin & Tonry \(2007\)](#).
- At least 3 photometric points in a given band for each SN Ia LC. A large fraction of the NIR data from [Barone-Nugent et al. \(2012\)](#), [Stanishev et al. \(2018\)](#), and the SweetSPOT survey with WIYN ([Weyant et al. 2014, 2018](#)) did not meet this criterion, so we chose not to analyze these data in this work.

### 2.2. Host Galaxy Redshifts

Heliocentric galaxy recession velocities and CMB frame redshifts are shown in Tables 2 and 3. We obtained heliocentric host galaxy recession velocities using the NASA/IPAC Extragalactic Database (NED), using

**Table 1.** Data cuts

| Cuts                                   | # SN Ia after cuts |
|--|--------------------|
| Initial sample                         | 177                |
| $0.8 < \Delta m_{15} < 1.6$            | 138                |
| $-0.15 < E(B - V)_{\text{host}} < 0.4$ | 122                |
| $E(B - V)_{\text{MW}} < 1$             | 122                |
| $z_{\text{CMB}} < 0.04$                | 111                |
| Remove duplicates                      | 100                |
| Normal spectrum                        | 95                 |
| $\geq 3$ LC points                     | 89                 |

Reduction of the initial sample based on data cuts

measurements with the smallest reported uncertainty.<sup>1</sup> If the host galaxy was anonymous or had no reported NED redshift, we used redshifts reported in the literature. When no uncertainties are available, we assume a recession velocity uncertainty of 50 km/s.

To further correct the CMB frame redshifts for local velocity flows and to estimate uncertainties, we used the model of Carrick et al. 2015.<sup>2</sup> Such corrections are most important for SN Ia with  $z < 0.01$  ( $v < 3000$  km/s), but we also use them for SN Ia further into the Hubble flow.

In special cases, we did not use the Carrick et al. 2015 flow model and instead used independent information for individual objects. For several SN Ia that have  $v_{\text{helio}} > 3000$  km/s, but are members of known galaxy clusters, to avoid large peculiar velocities from the cluster velocity dispersion, following Dhawan et al. 2018, we used the mean recession velocity of the cluster based on the cluster redshift from NED to estimate the CMB frame recession velocity for the SN Ia host galaxy. For SN 2008hs in Abell 347, we used  $v_{\text{CMB}} = 5655 \pm 13$  km/s. For SN 2010ai in the Coma cluster, we used  $v_{\text{CMB}} = 7166 \pm 54$  km/s (Pimblet et al. 2014).

To further avoid peculiar velocity systematics for SN Ia with  $v_{\text{helio}} < 3000$  km/s, where available, we also used redshift-independent distance information from Cepheid variable stars, surface brightness fluctuations (SBF), or the Tully-Fisher method (TF) to estimate an effective CMB frame redshift (see Tables 3-4 for references).

<sup>1</sup> However, even if earlier SDSS data releases report a smaller redshift error, we use the SDSS DR13 (2016) reported heliocentric redshift from NED where available (Albaret et al. 2017; [http://www.sdss.org/dr13/data\\_access/bulk/](http://www.sdss.org/dr13/data_access/bulk/)).

<sup>2</sup> [http://cosmicflows.iap.fr/table\\_query.html](http://cosmicflows.iap.fr/table_query.html). We used defaults of  $\Omega_M = 0.3$  (implicitly  $\Omega_\Lambda = 0.7$  for a flat universe),  $H_0 = 73.24$  km s<sup>-1</sup>Mpc<sup>-1</sup> (Riess et al. 2016),  $\beta = 0.43$ , and bulk flows of  $(V_x, V_y, V_z) = (89, -131, 17)$  km/s (Carrick et al. 2015).

Of the 19 SN Ia with Cepheid distances  $\mu_{\text{Cepheid}}$  and uncertainties  $\sigma_{\mu_{\text{Cepheid}}}$  in the HST SHOES program (Table 5 of Riess et al. 2016), 7 with NIR data are included in our Table 3 (SN 2001el, SN 2003du, SN 2005cf, SN 2007af, SN 2007sr, and SN 2011by). One other SN Ia (SN 1998bu) also has Cepheid distance from the HST Key Project (Table 4 of Freedman et al. 2001).<sup>3</sup> Lastly, 2 more SN Ia with NIR data (SN 2002dj, SN 2003hv) had redshift-independent host galaxy distance information from TF and/or SBF (Courtois & Tully 2012; Tully et al. 2013, 2016).

For all of these methods, we convert the reported distance modulus  $\mu'$  on a given Hubble scale  $H'_0$  to the Hubble scale of  $H_0 = 73.24$  km s<sup>-1</sup>Mpc<sup>-1</sup> as measured by Riess et al. 2016 and use this value of  $H_0$  throughout the rest of the paper. More specifically, for Hubble constants in units of km s<sup>-1</sup>Mpc<sup>-1</sup>, the distance modulus  $\mu_{\text{eff}}$  on our fiducial Hubble scale  $H_0$  is given by

$$\mu_{\text{eff}} = 5 \log_{10} \left( \frac{H'_0}{H_0} \right) + \mu' \quad (1)$$

See Table 4.

For these objects, we convert the redshift independent distance modulus  $\mu_{\text{eff}}$  to an effective CMB frame recession velocity with Hubble's law in the linear regime:

$$v_{\text{eff}} = c z_{\text{eff}} = H_0 d_L(\mu_{\text{eff}}) = H_0 \times 1\text{Mpc} \times 10^{\frac{\mu_{\text{eff}} - 25}{5}} \quad (2)$$

with an uncertainty given by<sup>4</sup>

$$\sigma_{v_{\text{eff}}} = c \sigma_{z_{\text{eff}}} = c z_{\text{eff}} \left( \frac{\ln 10}{5} \sigma_{\mu_{\text{eff}}} \right) \quad (3)$$

For SN Ia with Cepheid distances, we assume that the only contribution to the recession velocity uncertainty comes from Eq. 3 and therefore adopt a peculiar velocity uncertainty of 0 km/s for these objects.

For objects without Cepheid or other redshift-independent distances, we assume a peculiar velocity uncertainty of  $\sigma_{\text{pec}} = 150$  km/s, following Radburn-Smith et al. 2004.<sup>5</sup> As shown in Section 5, the value of  $\sigma_{\text{pec}} = 150$  km/s yields a more conservative determination of the Hubble diagram intrinsic scatter compared with larger values of  $\sigma_{\text{pec}}$  that tend to produce a misleadingly small value. However, statistics like the RMS, which we also use to compare various methods, are relatively insensitive to the assumed value of  $\sigma_{\text{pec}}$ .

<sup>3</sup> We use the metallicity corrected values  $\mu_Z$  and  $\sigma_Z$  from Table 4 of Freedman et al. 2001.

<sup>4</sup> We do not propagate the uncertainty on  $H_0$  in Eq. 2 because we have fixed the Hubble scale for this work.

<sup>5</sup> Estimates in the literature range from  $\sigma_{\text{pec}} = 150 - 400$  km/s:  $\sigma_{\text{pec}} = 150$  km/s (Radburn-Smith et al. 2004), 300 km/s (Davis et al. 2011),  $\sigma_{\text{pec}} = 360$  km/s (Kessler et al. 2009), or  $\sigma_{\text{pec}} = 400$  km/s (Wood-Vasey et al. 2007).

Table 2. SN Ia Light Curve Parameters

| SN name  | $z^a$ <sub>(helio)</sub> | $z^b$ <sub>CMB</sub> | $\sigma^c$ <sub>pec</sub><br>(mag) | LC Data<br>Source <sup>d</sup> | $t^e$ <sub>Bmax</sub><br>(MJD days) | $\Delta m_{15}(B)^f$<br>(mag) | $E(B-V)^g$ <sub>host</sub><br>(mag) | $E(B-V)^h$ <sub>MW</sub><br>(mag) |       |       |        |       |       |        |
|----------|--------------------------|----------------------|------------------------------------|--------------------------------|-------------------------------------|-------------------------------|-------------------------------------|-----------------------------------|-------|-------|--------|-------|-------|--------|
| SN1998bu | 0.0030                   | 0.000003             | 0.0025                             | 0.00023                        | 0.475                               | CfA                           | 50953.11                            | 0.08                              | 1.076 | 0.012 | 0.351  | 0.006 | 0.022 | 0.0002 |
| SN1999ee | 0.0114                   | 0.000010             | 0.0112                             | 0.00050                        | 0.137                               | CSP                           | 51469.61                            | 0.04                              | 0.802 | 0.007 | 0.384  | 0.004 | 0.017 | 0.0001 |
| SN1999ek | 0.0176                   | 0.000007             | 0.0178                             | 0.00050                        | 0.886                               | K04c                          | 51482.60                            | 0.19                              | 1.113 | 0.031 | 0.277  | 0.014 | 0.479 | 0.0187 |
| SN2000bh | 0.0229                   | 0.000027             | 0.0240                             | 0.00050                        | 0.064                               | CSP                           | 51636.16                            | 0.25                              | 1.055 | 0.019 | 0.065  | 0.012 | 0.047 | 0.0064 |
| SN2000ca | 0.0236                   | 0.000200             | 0.0239                             | 0.00050                        | 0.064                               | CSP                           | 51666.25                            | 0.18                              | 0.917 | 0.019 | -0.033 | 0.010 | 0.057 | 0.0025 |
| SN2000E  | 0.0047                   | 0.000003             | 0.0056                             | 0.00050                        | 0.273                               | V03                           | 51577.20                            | 0.13                              | 1.041 | 0.027 | 0.217  | 0.011 | 0.319 | 0.0086 |
| SN2001ba | 0.0296                   | 0.000033             | 0.0302                             | 0.00050                        | 0.051                               | CSP                           | 52034.47                            | 0.17                              | 0.997 | 0.020 | -0.072 | 0.009 | 0.054 | 0.0017 |
| SN2001bt | 0.0146                   | 0.000033             | 0.0142                             | 0.00050                        | 0.108                               | K04c                          | 52064.69                            | 0.07                              | 1.199 | 0.009 | 0.216  | 0.008 | 0.056 | 0.0007 |
| SN2001cn | 0.0152                   | 0.000127             | 0.0154                             | 0.00050                        | 0.100                               | K04c                          | 52071.93                            | 0.19                              | 1.044 | 0.012 | 0.176  | 0.008 | 0.051 | 0.0008 |
| SN2001cz | 0.0155                   | 0.000027             | 0.0171                             | 0.00050                        | 0.090                               | K04c                          | 52104.10                            | 0.10                              | 0.956 | 0.014 | 0.136  | 0.008 | 0.079 | 0.0005 |
| SN2001el | 0.0039                   | 0.000007             | 0.0045                             | 0.00014                        | 0.000                               | K03                           | 52182.38                            | 0.10                              | 1.080 | 0.019 | 0.277  | 0.010 | 0.012 | 0.0003 |
| SN2002dj | 0.0094                   | 0.000003             | 0.0083                             | 0.00152                        | 0.421                               | P08                           | 52451.04                            | 0.14                              | 1.111 | 0.019 | 0.093  | 0.013 | 0.082 | 0.0009 |
| SN2003du | 0.0064                   | 0.000013             | 0.0094                             | 0.00035                        | 0.000                               | St07                          | 52765.01                            | 0.09                              | 1.010 | 0.015 | -0.033 | 0.010 | 0.008 | 0.0008 |
| SN2003hv | 0.0056                   | 0.000037             | 0.0049                             | 0.00034                        | 0.267                               | L09                           | 52891.49                            | 0.11                              | 1.501 | 0.006 | -0.092 | 0.007 | 0.013 | 0.0008 |
| SN2004ef | 0.0310                   | 0.000017             | 0.0301                             | 0.00050                        | 0.051                               | CSP                           | 53264.90                            | 0.05                              | 1.422 | 0.011 | 0.116  | 0.006 | 0.046 | 0.0013 |
| SN2004eo | 0.0156                   | 0.000003             | 0.0152                             | 0.00050                        | 0.101                               | CSP                           | 53278.90                            | 0.04                              | 1.318 | 0.006 | 0.077  | 0.005 | 0.093 | 0.0010 |
| SN2004ey | 0.0158                   | 0.000003             | 0.0154                             | 0.00050                        | 0.100                               | CSP                           | 53304.81                            | 0.04                              | 1.025 | 0.011 | 0.006  | 0.004 | 0.130 | 0.0139 |
| SN2004gs | 0.0274                   | 0.000007             | 0.0287                             | 0.00050                        | 0.054                               | CSP                           | 53356.75                            | 0.05                              | 1.546 | 0.006 | 0.189  | 0.006 | 0.026 | 0.0006 |
| SN2004S  | 0.0093                   | 0.000003             | 0.0107                             | 0.00050                        | 0.143                               | K07                           | 53040.00                            | 0.29                              | 1.052 | 0.021 | 0.112  | 0.014 | 0.086 | 0.0014 |
| SN2005bo | 0.0135                   | 0.000027             | 0.0144                             | 0.00050                        | 0.107                               | CfA                           | 53479.67                            | 0.15                              | 1.310 | 0.020 | 0.272  | 0.007 | 0.044 | 0.0006 |
| SN2005ef | 0.0064                   | 0.000017             | 0.0069                             | 0.00036                        | 0.000                               | CfA                           | 53534.3                             | 0.06                              | 1.072 | 0.023 | 0.088  | 0.010 | 0.084 | 0.0013 |
| SN2005el | 0.0149                   | 0.000017             | 0.0153                             | 0.00050                        | 0.101                               | CSP                           | 53647.42                            | 0.04                              | 1.370 | 0.006 | -0.102 | 0.005 | 0.098 | 0.0004 |
| SN2005iq | 0.0340                   | 0.000123             | 0.0336                             | 0.00050                        | 0.046                               | CSP                           | 53688.14                            | 0.06                              | 1.280 | 0.012 | -0.049 | 0.006 | 0.018 | 0.0007 |
| SN2005kc | 0.0151                   | 0.000003             | 0.0149                             | 0.00050                        | 0.106                               | CSP                           | 53698.31                            | 0.08                              | 1.112 | 0.023 | 0.350  | 0.012 | 0.114 | 0.0023 |
| SN2005ki | 0.0195                   | 0.000010             | 0.0203                             | 0.00050                        | 0.076                               | CSP                           | 53706.01                            | 0.04                              | 1.365 | 0.004 | -0.065 | 0.004 | 0.027 | 0.0009 |
| SN2005lu | 0.0320                   | 0.000037             | 0.0317                             | 0.00050                        | 0.048                               | CSP                           | 53712.08                            | 0.23                              | 0.834 | 0.008 | 0.324  | 0.011 | 0.022 | 0.0009 |
| SN2005na | 0.0165                   | 0.000003             | 0.0162                             | 0.00050                        | 0.107                               | CSP                           | 53739.37                            | 0.30                              | 1.027 | 0.014 | -0.050 | 0.012 | 0.068 | 0.0025 |
| SN2005ac | 0.0231                   | 0.000010             | 0.0237                             | 0.00050                        | 0.065                               | CfA                           | 53781.35                            | 0.10                              | 1.189 | 0.008 | 0.066  | 0.010 | 0.014 | 0.0006 |
| SN2006ax | 0.0167                   | 0.000020             | 0.0180                             | 0.00050                        | 0.085                               | CSP                           | 53827.78                            | 0.04                              | 1.058 | 0.012 | -0.009 | 0.005 | 0.041 | 0.0019 |
| SN2006bh | 0.0108                   | 0.000013             | 0.0107                             | 0.00050                        | 0.143                               | CSP                           | 53834.14                            | 0.06                              | 1.408 | 0.007 | -0.043 | 0.004 | 0.023 | 0.0004 |
| SN2006bt | 0.0321                   | 0.000003             | 0.0307                             | 0.00050                        | 0.090                               | CSP                           | 54049.69                            | 0.23                              | 1.037 | 0.034 | 0.023  | 0.012 | 0.013 | 0.0007 |
| SN2006cp | 0.0223                   | 0.000003             | 0.0223                             | 0.00050                        | 0.069                               | CfA                           | 53897.45                            | 0.15                              | 1.023 | 0.046 | 0.134  | 0.022 | 0.022 | 0.0011 |
| SN2006D  | 0.0085                   | 0.000017             | 0.0090                             | 0.00050                        | 0.171                               | CfA                           | 53757.84                            | 0.08                              | 1.460 | 0.013 | 0.062  | 0.009 | 0.039 | 0.0004 |
| SN2006ej | 0.0204                   | 0.000003             | 0.0205                             | 0.00050                        | 0.084                               | CfA                           | 53879.24                            | 0.15                              | 1.394 | 0.014 | 0.016  | 0.011 | 0.008 | 0.0005 |
| SN2006ff | 0.0200                   | 0.000010             | 0.0211                             | 0.00050                        | 0.079                               | CSP                           | 54041.56                            | 0.05                              | 1.058 | 0.008 | 0.007  | 0.006 | 0.010 | 0.0020 |
| SN2006if | 0.0132                   | 0.000017             | 0.0121                             | 0.00050                        | 0.127                               | CfA                           | 54045.56                            | 0.06                              | 1.406 | 0.010 | -0.054 | 0.010 | 0.814 | 0.0503 |
| SN2006N  | 0.0143                   | 0.000083             | 0.0145                             | 0.00050                        | 0.106                               | CfA                           | 53761.48                            | 0.15                              | 1.457 | 0.013 | -0.030 | 0.007 | 0.083 | 0.0010 |
| SN2007A  | 0.0176                   | 0.000007             | 0.0172                             | 0.00050                        | 0.107                               | CSP                           | 54113.67                            | 0.13                              | 1.037 | 0.034 | 0.022  | 0.014 | 0.016 | 0.0007 |
| SN2007af | 0.0055                   | 0.000013             | 0.0056                             | 0.00018                        | 0.000                               | CSP                           | 54174.97                            | 0.04                              | 1.116 | 0.010 | 0.183  | 0.005 | 0.034 | 0.0008 |
| SN2007ai | 0.0317                   | 0.000137             | 0.0327                             | 0.00050                        | 0.047                               | CSP                           | 54174.03                            | 0.26                              | 0.844 | 0.021 | 0.339  | 0.013 | 0.286 | 0.0035 |
| SN2007bc | 0.0208                   | 0.000007             | 0.0211                             | 0.00050                        | 0.073                               | CSP                           | 54200.82                            | 0.09                              | 1.282 | 0.012 | 0.039  | 0.006 | 0.019 | 0.0006 |
| SN2007bd | 0.0304                   | 0.000100             | 0.0311                             | 0.00050                        | 0.049                               | CSP                           | 54207.43                            | 0.06                              | 1.270 | 0.012 | -0.018 | 0.010 | 0.029 | 0.0009 |
| SN2007ca | 0.0141                   | 0.000010             | 0.0145                             | 0.00050                        | 0.106                               | CSP                           | 54228.20                            | 0.14                              | 1.037 | 0.024 | 0.376  | 0.012 | 0.057 | 0.0016 |
| SN2007co | 0.0270                   | 0.000110             | 0.0274                             | 0.00050                        | 0.056                               | CfA                           | 54264.91                            | 0.23                              | 1.040 | 0.040 | 0.208  | 0.017 | 0.096 | 0.0037 |
| SN2007cq | 0.0260                   | 0.000080             | 0.0252                             | 0.00050                        | 0.061                               | CfA                           | 54280.90                            | 0.10                              | 1.062 | 0.021 | 0.051  | 0.011 | 0.092 | 0.0020 |
| SN2007jg | 0.0371                   | 0.000013             | 0.0380                             | 0.00050                        | 0.040                               | CSP                           | 54366.64                            | 0.25                              | 1.088 | 0.034 | 0.150  | 0.017 | 0.090 | 0.0020 |
| SN2007le | 0.0067                   | 0.000003             | 0.0065                             | 0.00050                        | 0.237                               | CSP                           | 54399.85                            | 0.07                              | 1.027 | 0.016 | 0.379  | 0.008 | 0.029 | 0.0003 |
| SN2007ge | 0.0240                   | 0.000050             | 0.0236                             | 0.00050                        | 0.065                               | CfA                           | 54429.59                            | 0.10                              | 0.988 | 0.023 | 0.069  | 0.014 | 0.033 | 0.0008 |
| SN2007sr | 0.0055                   | 0.000030             | 0.0044                             | 0.00025                        | 0.000                               | CSP                           | 54449.73                            | 0.19                              | 1.084 | 0.015 | 0.173  | 0.009 | 0.040 | 0.0010 |
| SN2007st | 0.0212                   | 0.000030             | 0.0211                             | 0.00050                        | 0.073                               | CSP                           | 54455.09                            | 0.32                              | 1.486 | 0.019 | 0.101  | 0.018 | 0.014 | 0.0004 |
| SN2008af | 0.0134                   | 0.000007             | 0.0130                             | 0.00050                        | 0.045                               | CSP                           | 54499.69                            | 0.43                              | 1.178 | 0.010 | -0.028 | 0.023 | 0.029 | 0.0012 |
| SN2008ar | 0.0262                   | 0.000010             | 0.0290                             | 0.00050                        | 0.053                               | CSP                           | 54535.22                            | 0.07                              | 1.032 | 0.014 | 0.081  | 0.008 | 0.031 | 0.0011 |
| SN2008bc | 0.0151                   | 0.000120             | 0.0156                             | 0.00050                        | 0.098                               | CSP                           | 54550.41                            | 0.08                              | 1.015 | 0.019 | 0.003  | 0.008 | 0.225 | 0.0042 |
| SN2008bf | 0.0235                   | 0.000003             | 0.0234                             | 0.00050                        | 0.061                               | CSP                           | 54585.91                            | 0.06                              | 1.013 | 0.015 | -0.013 | 0.016 | 0.018 | 0.0007 |
| SN2008C  | 0.0166                   | 0.000013             | 0.0175                             | 0.00050                        | 0.088                               | CSP                           | 54466.00                            | 0.23                              | 1.075 | 0.019 | 0.239  | 0.010 | 0.072 | 0.0023 |
| SN2008fl | 0.0199                   | 0.000103             | 0.0199                             | 0.00050                        | 0.077                               | CSP                           | 54721.85                            | 0.13                              | 1.328 | 0.006 | 0.080  | 0.005 | 0.157 | 0.0058 |
| SN2008fr | 0.0390                   | 0.002001             | 0.0384                             | 0.00050                        | 0.040                               | CSP                           | 54733.93                            | 0.26                              | 0.920 | 0.014 | -0.002 | 0.011 | 0.040 | 0.0012 |
| SN2008fw | 0.0085                   | 0.000017             | 0.0086                             | 0.00050                        | 0.178                               | CSP                           | 54739.29                            | 0.15                              | 1.844 | 0.009 | 0.112  | 0.008 | 0.050 | 0.0030 |
| SN2008gb | 0.0370                   | 0.000167             | 0.0381                             | 0.00050                        | 0.040                               | CfA                           | 54748.22                            | 0.34                              | 1.183 | 0.014 | 0.080  | 0.018 | 0.171 | 0.0035 |
| SN2008gg | 0.0320                   | 0.000023             | 0.0311                             | 0.00050                        | 0.049                               | CSP                           | 54750.93                            | 0.34                              | 1.036 | 0.028 | 0.155  | 0.013 | 0.019 | 0.0010 |
| SN2008gf | 0.0340                   | 0.000003             | 0.0340                             | 0.00050                        | 0.046                               | CSP                           | 54768.70                            | 0.09                              | 1.319 | 0.010 | 0.030  | 0.006 | 0.024 | 0.0008 |
| SN2008gp | 0.0330                   | 0.000070             | 0.0335                             | 0.00050                        | 0.046                               | CSP                           | 54779.62                            | 0.04                              | 1.047 | 0.008 | -0.018 | 0.004 | 0.104 | 0.0051 |
| SN2008hj | 0.0379                   | 0.000130             | 0.0372                             | 0.00050                        | 0.041                               | CSP                           | 54802.26                            | 0.12                              | 1.055 | 0.027 | 0.038  | 0.012 | 0.030 | 0.0008 |
| SN2008hm | 0.0197                   | 0.000077             | 0.0210                             | 0.00050                        | 0.073                               | CfA                           | 54804.74                            | 0.21                              | 0.993 | 0.025 | 0.182  | 0.014 | 0.380 | 0.0085 |
| SN2008hs | 0.0174                   | 0.000070             | 0.0189                             | 0.00050                        | 0.058                               | CfA                           | 54812.94                            | 0.14                              | 1.531 | 0.015 | 0.129  | 0.024 | 0.050 | 0.0003 |
| SN2008hv | 0.0126                   | 0.000007             | 0.0140                             | 0.00050                        | 0.110                               | CSP                           | 54817.65                            | 0.04                              | 1.328 | 0.006 | -0.065 | 0.006 | 0.028 | 0.0008 |
| SN2008ia | 0.0219                   | 0.000097             | 0.0225                             | 0.00050                        | 0.068                               | CSP                           | 54813.67                            | 0.09                              | 1.340 | 0.009 | 0.003  | 0.007 | 0.195 | 0.0050 |
| SN2009aa | 0.0273                   | 0.000047             | 0.0280                             | 0.00050                        | 0.054                               | CSP                           | 54878.81                            | 0.04                              | 1.172 | 0.008 | 0.020  | 0.005 | 0.029 | 0.0009 |
| SN2009ab | 0.0143                   | 0.000020             | 0.0103                             | 0.00050                        | 0.149                               | CSP                           | 54883.89                            | 0.08                              | 1.088 | 0.016 | 0.050  | 0.010 | 0.184 | 0.0028 |
| SN2009ad | 0.0284                   | 0.000003             | 0.0287                             | 0.00050                        | 0.054                               | CSP                           | 54886.91                            | 0.07                              | 0.949 | 0.013 | 0.020  | 0.007 | 0.095 | 0.0011 |
| SN2009ag | 0.0086                   | 0.000007             | 0.0102                             | 0.00050                        | 0.151                               | CSP                           | 54890.23                            | 0.16                              | 1.088 | 0.019 | 0.343  | 0.009 | 0.218 | 0.0012 |
| SN2009af | 0.0221                   | 0.000080             | 0.0224                             | 0.00050                        | 0.066                               | CfA                           | 54897.14                            | 0.26                              | 0.876 | 0.019 | 0.084  | 0.012 | 0.022 | 0.0003 |
| SN2009an | 0.0092                   | 0.000007             | 0.0107                             | 0.00050                        | 0.144                               | CfA                           | 54898.56                            | 0.09                              | 1.327 | 0.010 | 0.063  | 0.010 | 0.016 | 0.0003 |
| SN2009bv | 0.0366                   | 0.000017             | 0.0385                             | 0.00050                        | 0.040                               | CfA                           | 54927.07                            | 0.20                              | 0.948 | 0.033 | -0.026 | 0.019 | 0.008 | 0.0008 |
| SN2009cz | 0.02                     |                      |                                    |                                |                                     |                               |                                     |                                   |       |       |        |       |       |        |

Table 3. SN Ia Recession Velocities

| SN name <sup>a</sup> | RA (deg)                    | DEC (deg)                   | Host Galaxy                     | $v_{\text{helio}}$ <sup>d</sup> | $v_{\text{CMB}}$ <sup>e</sup> | $v_{\text{CMB,flow}}$ <sup>f</sup> | Ref(s) <sup>g</sup> | Code <sup>h</sup> |
|----------------------|-----------------------------|-----------------------------|---------------------------------|---------------------------------|-------------------------------|------------------------------------|---------------------|-------------------|
|                      | $\alpha(2000)$ <sup>b</sup> | $\delta(2000)$ <sup>b</sup> | (or cluster) <sup>c</sup>       | ( $\text{km s}^{-1}$ )          | ( $\text{km s}^{-1}$ )        | ( $\text{km s}^{-1}$ )             |                     |                   |
| SN1998bu             | 161.69167                   | -11.83528                   | NGC 3268                        | 888 ± 3                         | 757 ± 70                      | 232 ± 150                          | NED:F01             | Cepheid           |
| SN1999ee             | 334.04167                   | -36.8444                    | IC 5179                         | 3419 ± 3                        | 3160 ± 3                      | 3368 ± 150                         | NED:C15             | Flow              |
| SN1999ek             | 84.13167                    | 16.63833                    | UGC 03329                       | 5266 ± 2                        | 5292 ± 2                      | 5340 ± 150                         | NED:C15             | Flow              |
| SN2000E              | 309.30750                   | 66.09722                    | NGC 6951                        | 1424 ± 1                        | 1267 ± 1                      | 1685 ± 150                         | NED:C15             | Flow              |
| SN2000bh             | 185.31292                   | -21.99889                   | ESO 573-G 014                   | 6854 ± 8                        | 7196 ± 8                      | 7188 ± 150                         | NED:C15             | Flow              |
| SN2000ca             | 203.84583                   | -34.16028                   | NGC 383-G 032                   | 7080 ± 60                       | 7351 ± 62                     | 7167 ± 150                         | NED:C15             | Flow              |
| SN2001ba             | 174.50750                   | -32.33083                   | MCG -05-28-001                  | 8861 ± 10                       | 9193 ± 10                     | 9060 ± 150                         | NED:C15             | Flow              |
| SN2001bt             | 288.44500                   | -59.28972                   | IC 4830                         | 4388 ± 10                       | 4331 ± 10                     | 4260 ± 150                         | NED:C15             | Flow              |
| SN2001cn             | 281.57417                   | -65.76167                   | IC 4758                         | 4543 ± 38                       | 4523 ± 38                     | 4626 ± 150                         | NED:C15             | Flow              |
| SN2001cz             | 191.87583                   | -39.58000                   | NGC 4679                        | 4933 ± 8                        | 4930 ± 8                      | 5124 ± 150                         | NED:C15             | Flow              |
| SN2001el             | 56.12750                    | -44.63972                   | NGC 1448                        | 1168 ± 2                        | 1340 ± 42                     | 1568 ± 150                         | NED:R16             | Cepheid           |
| SN2002dj             | 198.25125                   | -19.51917                   | NGC 5018                        | 2816 ± 1                        | 2479 ± 457                    | 2023 ± 150                         | NED:Co12            | SBF/TF            |
| SN2003du             | 218.64917                   | -59.33444                   | UGC 9391                        | 1914 ± 4                        | 2809 ± 105                    | 3165 ± 150                         | NED:R16             | Cepheid           |
| SN2003hv             | 46.03875                    | -26.08556                   | NGC 1201                        | 1686 ± 11                       | 1470 ± 101                    | 1723 ± 150                         | NED:Tu13            | SBF/TF            |
| SN2004S              | 101.43125                   | -31.23111                   | MCG -05-16-021                  | 2788 ± 1                        | 2937 ± 1                      | 3213 ± 150                         | NED:C15             | Flow              |
| SN2004ef             | 340.54175                   | 19.99456                    | UGC 12158                       | 9289 ± 5                        | 8931 ± 5                      | 9015 ± 150                         | NED:C15             | Flow              |
| SN2004eo             | 308.22579                   | 9.92853                     | NGC 6928                        | 4664 ± 1                        | 4298 ± 6                      | 4560 ± 150                         | NED:C15             | Flow              |
| SN2004ey             | 327.28254                   | 0.44422                     | UGC 11816                       | 4749 ± 1                        | 4405 ± 1                      | 4617 ± 150                         | NED:C15             | Flow              |
| SN2004gs             | 129.59658                   | 17.62772                    | MCG +03-22-020                  | 8214 ± 2                        | 8475 ± 2                      | 8590 ± 150                         | NED:C15             | Flow              |
| SN2005fo             | 192.42096                   | -11.09647                   | NGC 4708                        | 4166 ± 8                        | 4503 ± 9                      | 4314 ± 150                         | NED:C15             | Flow              |
| SN2005ef             | 230.38417                   | -7.41306                    | MCG -01-59-003                  | 1929 ± 5                        | 2077 ± 109                    | 12034 ± 150                        | NED:R16             | Cepheid           |
| SN2005el             | 77.95300                    | 5.19428                     | NGC 1819                        | 4470 ± 5                        | 4466 ± 5                      | 4574 ± 150                         | NED:C15             | Flow              |
| SN2005iq             | 359.63542                   | -18.70917                   | ESO 538-G 013                   | 10206 ± 37                      | 9880 ± 36                     | 10058 ± 150                        | NED:C15             | Flow              |
| SN2005kc             | 338.53058                   | 5.56842                     | NGC 7311                        | 4524 ± 1                        | 4159 ± 1                      | 4343 ± 150                         | NED:C15             | Flow              |
| SN2005ki             | 161.11758                   | 9.29233                     | NGC 3332                        | 5833 ± 3                        | 6185 ± 3                      | 6580 ± 150                         | NED:C15             | Flow              |
| SN2005lu             | 39.01546                    | -17.26389                   | ESO 545-G038                    | 9596 ± 11                       | 9389 ± 11                     | 9515 ± 150                         | NED:C15             | Flow              |
| SN2005na             | 105.40458                   | 14.13325                    | UGC 3634                        | 7891 ± 25                       | 8045 ± 25                     | 8162 ± 150                         | NED:C15             | Flow              |
| SN2006D              | 193.14122                   | -9.77522                    | MCG -01-33-034                  | 2556 ± 5                        | 2891 ± 6                      | 2691 ± 150                         | NED:C15             | Flow              |
| SN2006N              | 92.13000                    | 64.72361                    | CGCG 308-009                    | 4260 ± 25                       | 4325 ± 25                     | 4354 ± 150                         | NED:C15             | Flow              |
| SN2006ac             | 190.43708                   | 35.08528                    | NGC 4619                        | 6923 ± 3                        | 7175 ± 3                      | 7113 ± 150                         | NED:C15             | Flow              |
| SN2006ax             | 171.01442                   | -12.29144                   | NGC 3663                        | 5014 ± 6                        | 5382 ± 6                      | 5386 ± 150                         | NED:C15             | Flow              |
| SN2006bh             | 340.06708                   | -66.48508                   | NGC 7529                        | 3252 ± 4                        | 3148 ± 4                      | 3222 ± 150                         | NED:C15             | Flow              |
| SN2006bt             | 239.12721                   | 12.07211                    | CGCG 108-013                    | 9632 ± 8                        | 9278 ± 8                      | 9211 ± 150                         | NED:R17             | Cepheid           |
| SN2006cp             | 184.81208                   | 22.42722                    | UGC 7357                        | 6682 ± 1                        | 6990 ± 1                      | 6673 ± 150                         | NED:C15             | Flow              |
| SN2006ej             | 9.74904                     | -9.01572                    | NGC 191A                        | 6110 ± 2                        | 5780 ± 2                      | 6152 ± 150                         | NED:C15             | Flow              |
| SN2006kf             | 55.46033                    | 8.15694                     | UGC 2829                        | 6007 ± 3                        | 5861 ± 3                      | 5821 ± 150                         | NED:C15             | Flow              |
| SN2006lf             | 69.62962                    | 4.02361                     | NGC 3882                        | 6348 ± 3                        | 6382 ± 3                      | 6348 ± 150                         | NED:C15             | Flow              |
| SN2007A              | 6.31942                     | 12.88681                    | NGC 105                         | 5290 ± 26                       | 4940 ± 24                     | 5162 ± 150                         | NED:C15             | Flow              |
| SN2007af             | 215.58763                   | -0.39378                    | NGC 5584                        | 1638 ± 4                        | 1667 ± 53                     | 1418 ± 150                         | NED:R16             | Cepheid           |
| SN2007ai             | 243.22392                   | -21.63019                   | MCG-04-38-094                   | 5492 ± 11                       | 5915 ± 41                     | 9815 ± 150                         | NED:C15             | Flow              |
| SN2007as             | 141.90004                   | -80.18756                   | ESO 018-G 018                   | 5268 ± 138                      | 5368 ± 141                    | 5503 ± 150                         | NED:C15             | Flow              |
| SN2007bc             | 169.81071                   | 20.80903                    | UGC 6332                        | 6221 ± 2                        | 6548 ± 2                      | 6333 ± 150                         | NED:C15             | Flow              |
| SN2007bd             | 127.88867                   | -1.19944                    | UGC 4455                        | 9126 ± 30                       | 9408 ± 31                     | 9318 ± 150                         | NED:C15             | Flow              |
| SN2007ca             | 202.77421                   | -03.72183                   | MCG -02-34-061                  | 4217 ± 3                        | 4520 ± 3                      | 4339 ± 150                         | NED:C15             | Flow              |
| SN2007co             | 275.76500                   | 29.89722                    | MCG +05-43-016                  | 8083 ± 33                       | 7963 ± 33                     | 8229 ± 150                         | NED:C15             | Flow              |
| SN2007cq             | 333.66835                   | 5.08028                     | 2MASX J22144070+0504435         | 7307 ± 24                       | 7448 ± 23                     | 7564 ± 150                         | Ch13:C15            | Flow              |
| SN2007jg             | 52.46175                    | 0.05683                     | SDSS J032950.83+000316.0        | 11113 ± 4                       | 10955 ± 4                     | 11379 ± 150                        | NED:C15             | Flow              |
| SN2007le             | 354.70171                   | -6.52258                    | NGC 7721                        | 2017 ± 1                        | 1660 ± 1                      | 1939 ± 150                         | NED:C15             | Flow              |
| SN2007qe             | 358.55417                   | 27.40917                    | NSF J235412.09+272432.3         | 7183 ± 15                       | 6842 ± 14                     | 7067 ± 150                         | Ch13:C15            | Flow              |
| SN2007sr             | 180.47000                   | -18.97269                   | NGC 4038                        | 1641 ± 9                        | 1327 ± 75                     | 611 ± 150                          | NED:R16             | Cepheid           |
| SN2007st             | 27.17696                    | -48.64939                   | NGC 692                         | 6350 ± 9                        | 6195 ± 9                      | 6330 ± 150                         | NED:C15             | Flow              |
| SN2008C              | 104.29804                   | 20.43714                    | UGC 3611                        | 4983 ± 4                        | 5121 ± 4                      | 5260 ± 150                         | NED:C15             | Flow              |
| SN2008af             | 224.86875                   | 16.65333                    | UGC 9640                        | 10020 ± 2                       | 10199 ± 2                     | 10195 ± 150                        | NED:C15             | Flow              |
| SN2008ar             | 186.15800                   | 10.83817                    | IC 3284                         | 7846 ± 3                        | 8180 ± 3                      | 8680 ± 150                         | NED:C15             | Flow              |
| SN2008bc             | 144.63012                   | -63.97378                   | KK 1524                         | 4525 ± 36                       | 4711 ± 37                     | 4677 ± 150                         | NED:C15             | Flow              |
| SN2008bf             | 181.01208                   | 17.04517                    | ambiguous                       | 7045 ± 50                       | 7365 ± 52                     | 7608 ± 150                         | K17:C15             | Flow              |
| SN2008fl             | 294.18683                   | -37.55125                   | NGC 6805                        | 5960 ± 31                       | 5815 ± 30                     | 5980 ± 150                         | NED:C15             | Flow              |
| SN2008fr             | 17.95475                    | 14.64083                    | SDSS J011149.19-143826.5        | 11692 ± 600                     | 11373 ± 584                   | 11503 ± 150                        | NED:C15             | Flow              |
| SN2008fw             | 157.25321                   | -16.66544                   | NGC 3251                        | 4280 ± 25                       | 4278 ± 25                     | 4354 ± 150                         | NED:C15             | Flow              |
| SN2008gb             | 44.48922                    | 46.86383                    | UGC 4227                        | 11092 ± 50                      | 10921 ± 49                    | 11428 ± 150                        | NED:C15             | Flow              |
| SN2008gg             | 21.34600                    | -18.17244                   | NGC 539                         | 9598 ± 7                        | 9321 ± 7                      | 9330 ± 150                         | NED:C15             | Flow              |
| SN2008gl             | 20.22842                    | 4.80531                     | UGC 881                         | 10198 ± 35                      | 9885 ± 34                     | 9954 ± 150                         | NED:C15             | Flow              |
| SN2008hp             | 50.75304                    | 19.96189                    | MCG +00-9-74                    | 1392 ± 21                       | 1011 ± 21                     | 1010 ± 150                         | NED:C15             | Flow              |
| SN2008hj             | 1.00796                     | -11.16875                   | MCG -02-01-014                  | 11360 ± 39                      | 11018 ± 38                    | 11140 ± 150                        | NED:C15             | Flow              |
| SN2008hm             | 51.79542                    | 46.94444                    | 2MFGC 02845                     | 5895 ± 23                       | 5752 ± 22                     | 6282 ± 150                         | NED:C15             | Flow              |
| SN2008hs             | 36.37333                    | 41.84306                    | NGC 0910 (Abell 347)            | 5207 ± 21                       | 5655 ± 13                     | 6186 ± 150                         | NED:Dh18            | Cluster           |
| SN2008hw             | 136.891925                  | 3.39225                     | NGC 2765                        | 3775 ± 2                        | 4097 ± 2                      | 4185 ± 150                         | NED:C15             | Flow              |
| SN2008ix             | 132.64646                   | -61.27994                   | ESO 125-G 006                   | 6578 ± 29                       | 6761 ± 30                     | 6754 ± 150                         | NED:C15             | Flow              |
| SN2009D              | 58.59512                    | -19.18172                   | MCG -03-10-052                  | 7497 ± 10                       | 7397 ± 10                     | 7275 ± 150                         | NED:C15             | Flow              |
| SN2009Y              | 220.59938                   | -17.24678                   | NGC 5728                        | 2793 ± 8                        | 3019 ± 9                      | 2827 ± 150                         | NED:C15             | Flow              |
| SN2009aa             | 170.92817                   | -22.27069                   | ESO 570-G 020                   | 3342 ± 14                       | 8543 ± 15                     | 8597 ± 150                         | NED:C15             | Flow              |
| SN2009ab             | 64.15162                    | 2.76417                     | UGC 2998                        | 3349 ± 6                        | 3260 ± 6                      | 3090 ± 150                         | NED:C15             | Flow              |
| SN2009ad             | 75.88908                    | 6.65992                     | UGC 3236                        | 8514 ± 1                        | 8496 ± 1                      | 8602 ± 150                         | NED:C15             | Flow              |
| SN2009ag             | 107.92004                   | -26.68508                   | ESO 492-2                       | 2590 ± 2                        | 2774 ± 2                      | 3056 ± 150                         | NED:C15             | Flow              |
| SN2009af             | 162.84196                   | 8.57853                     | NGC 3425                        | 6627 ± 24                       | 6982 ± 25                     | 7007 ± 150                         | NED:C15             | Flow              |
| SN2009an             | 185.69750                   | 65.85111                    | NGC 4332                        | 2764 ± 2                        | 2867 ± 2                      | 3207 ± 150                         | NED:C15             | Flow              |
| SN2009bv             | 196.83542                   | 35.78444                    | MCG +06-29-039                  | 10966 ± 5                       | 11201 ± 5                     | 11539 ± 150                        | NED:C15             | Flow              |
| SN2009cz             | 138.75008                   | 29.73531                    | NGC 2789                        | 6344 ± 3                        | 6601 ± 3                      | 6548 ± 150                         | NED:C15             | Flow              |
| SN2009kk             | 97.43588                    | -3.26444                    | 2MFGC 03182                     | 834 ± 4                         | 3729 ± 44                     | 3653 ± 150                         | NED:C15             | Flow              |
| SN2009kq             | 129.06292                   | 28.06722                    | MCG +05-21-001                  | 3507 ± 6                        | 3739 ± 6                      | 3766 ± 150                         | NED:C15             | Flow              |
| SN2010ai             | 194.85000                   | 27.99639                    | SDSS J125925.04+275948.2 (Coma) | 5507 ± 37                       | 7166 ± 54                     | 7298 ± 150                         | NED:P14             | Cluster           |
| SN2010dw             | 230.66792                   | -5.92111                    | 2MASX J15224062-0555214         | 11428 ± 45                      | 11600 ± 46                    | 11759 ± 150                        | NED:C15             | Flow              |
| SN2010hw             | 151.31250                   | 27.57817                    | UGC 6470                        | 6918 ± 2                        | 6614 ± 2                      | 6911 ± 150                         | NED:C15             | Flow              |
| SN2010kg             | 70.03500                    | 7.35000                     | NGC 1633                        | 4986 ± 2                        | 4931 ± 2                      | 5128 ± 150                         | NED:C15             | Flow              |
| SN2011B              | 133.95208                   | 78.21750                    | NGC 2655                        | 1400 ± 1                        | 1419 ± 1                      | 1670 ± 150                         | NED:C15             | Flow              |
| SN2011ao             | 178.46250                   | 33.36278                    | IC 2973                         | 3210 ± 1                        | 3487 ± 1                      | 3592 ± 150                         | NED:C15             | Flow              |
| SN2011bv             | 178.94000                   | 55.26411                    | NGC 3972                        | 834 ± 1                         | 1529 ± 61                     | 1796 ± 150                         | NED:R16             | Cepheid           |
| SN2011df             | 291.89000                   | 54.38639                    | NGC 6801                        | 4361 ± 6                        | 4205 ± 6                      | 4500 ± 150                         | NED:C15             | Flow              |
| SNF20080514-002      | 202.30625                   | 11.26889                    | UGC 8472                        | 6577 ± 3                        | 6866 ± 3                      | 6480 ± 150                         | NED:C15             | Flow              |

NOTE—

<sup>a</sup> SN Ia name. All SN Ia in this work are spectroscopically normal (see §2.1).<sup>b</sup> Epoch J2000 equatorial coordinates in decimal degrees.<sup>c</sup> Host galaxy or cluster from NASA/IPAC Extragalactic Database (NED) or the literature. See Ref(s) column.<sup>d</sup> Heliocentric recession velocity from NED with smallest reported uncertainty (SDSS DR13 values are used even if earlier SDSS reported smaller uncertainties). When no uncertainty is reported we assume an error of 50 km/s.<sup>e</sup> CMB frame recession velocity  $v_{\text{CMB}}$  using  $v_{\text{helio}}$ , RA, DEC, and CMB dipole from (Planck Collaboration et al. 2016a).<sup>f</sup>  $v_{\text{CMB,flow}}$  takes  $v_{\text{CMB}}$ , RA, DEC as input and further corrects to the CMB frame via the local velocity flow model of Carrick et al. (2015) (hereafter C15), with assumed uncertainty of 150 km/s (see §2.2).<sup>g</sup> The first reference listed is for  $v_{\text{helio}}$  from NED or the literature. The second reference is for the effective  $v_{\text{CMB}}$  derived using either the C15 local flow model or independent distance information for nearby galaxies with  $v_{\text{helio}} \lesssim 3000$  km/s, special cases where host galaxy identification from NED is ambiguous, or some clusters which may have  $v > 3000$  km/s (see §2.2 and Table 4). Reference codes: C15: Carrick et al. 2015; Ch11: Childress et al. 2011; Ch13: Childress et al. 2013; Co12: Courtois & Tully 2012; Dh18: Dhawan et al. 2018; F01: Freedman et al. 2001; F15: Friedman et al. 2015; H12: Hicken et al. 2012; K17: Krisciunas et al. 2017; MO00: Mould et al. 2000; P14: Pimblet et al. 2014; Pr07: Prieto et al. 2007; R14: Rest et al. 2014; R16: Riess et al. 2016; Tu13: Tully et al. 2013; Tu16: Tully et al. 2016.<sup>h</sup> If Code=Flow, we use  $v_{\text{CMB,flow}}$  from C15 in our Hubble diagram. If Code  $\neq$  Flow, we use  $v_{\text{CMB}}$ . Other codes include Cepheid: HST Cepheid distances from SHOES Riess et al. 2016 or HST Key Project Freedman et al. 2001; Cluster: Mean redshift of galaxy cluster from NED (e.g. Virgo, Coma, Abell\*); SBF/TF: Surface Brightness Fluctuations (SBF) or Tully-Fisher relation (TF) (e.g. Courtois & Tully 2012; Tully et al. 2013, 2016).

**Table 4.** SN Ia With Redshift-Independent Distance Moduli

| SN name  | Host Galaxy<br>(or cluster) | $\mu'$<br>(mag) <sup>a</sup> | $H'_0$<br>(km s <sup>-1</sup> Mpc <sup>-1</sup> ) | $\mu_{\text{eff}}$<br>(mag) <sup>b</sup> | Ref. <sup>c</sup> | Code <sup>d</sup> |
|----------|-----------------------------|------------------------------|---|--|-------------------|-------------------|
| SN1998bu | NGC 3368                    | 30.110 ± 0.200               | 72.00 ± 8.00                                      | 30.073 ± 0.200                           | F01               | Cepheid           |
| SN2001el | NGC 1448                    | 31.311 ± 0.045               | 73.24 ± 1.74                                      | 31.311 ± 0.045                           | R16               | Cepheid           |
| SN2002dj | NGC 5018                    | 32.570 ± 0.400               | 75.90 ± 3.80                                      | 32.647 ± 0.400                           | Co12              | SBF/TF            |
| SN2003du | UGC 9391                    | 32.919 ± 0.063               | 73.24 ± 1.74                                      | 32.919 ± 0.063                           | R16               | Cepheid           |
| SN2003hv | NGC 1201                    | 31.120 ± 0.250               | 74.40 ± 3.00                                      | 31.154 ± 0.250                           | Tu13              | SBF/TF            |
| SN2005cf | MCG -01-39-003              | 32.263 ± 0.102               | 73.24 ± 1.74                                      | 32.263 ± 0.102                           | R16               | Cepheid           |
| SN2007af | NGC 5584                    | 31.786 ± 0.046               | 73.24 ± 1.74                                      | 31.786 ± 0.046                           | R16               | Cepheid           |
| SN2007sr | NGC 4038                    | 31.290 ± 0.112               | 73.24 ± 1.74                                      | 31.290 ± 0.112                           | R16               | Cepheid           |
| SN2011by | NGC 3972                    | 31.587 ± 0.070               | 73.24 ± 1.74                                      | 31.587 ± 0.070                           | R16               | Cepheid           |

NOTE—

<sup>a</sup> Reported distance modulus  $\mu'$  on Hubble scale  $H'_0$ .<sup>b</sup> This is converted to a distance modulus  $\mu_{\text{eff}}$  on the Hubble scale of  $H_0 = 73.24 \text{ km s}^{-1} \text{ Mpc}^{-1}$  via Eq. (1). For SN Ia with Cepheid distances from [Riess et al. 2016](#), since  $H'_0 = H_0$  and  $\mu' = \mu_{\text{eff}}$ , effective distance moduli  $\mu_{\text{eff}}$  are already on the Hubble scale used for this paper. We compute the effective CMB frame recession velocity  $v_{\text{eff}}$  in Table 3 via Eqs. 2-3 using  $\mu'$  and  $H'_0$  (or equivalently  $\mu_{\text{eff}}$  and  $H_0$ ). This is then used to construct an effective redshift or recession velocity for use in Hubble diagrams.<sup>c</sup> Reference codes: Co12: [Courtois & Tully 2012](#); F01: [Freedman et al. 2001](#); R16: [Riess et al. 2016](#); Tu13: [Tully et al. 2013](#); Tu16: [Tully et al. 2016](#)<sup>d</sup> Same as in Table 3.

### 3. NIR LC TEMPLATES

We determine the normalized mean  $YJHK_s$  LC templates, as shown in Figure 1 and Table 5, using the SN Ia listed in Table 2 and shown in Figure 2 as follows. In each band, we convert the photometry from the observer-frame apparent magnitude to the rest-frame absolute magnitude. We further apply  $K$ -corrections to the rest-frame and correct for Milky Way dust extinction. These steps are detailed in §3.1. We then use a *Gaussian process* method, as described in §3.2, to fit the LC in each NIR band. Finally, in §3.3, using a hierarchical Bayesian model we average all the LCs in a given NIR band to determine the normalized mean LC template, its uncertainty, and the population standard deviation.

#### 3.1. Rest-Frame Absolute Magnitudes

For a given supernova  $s$  observed through filter  $O$ , we convert the apparent magnitude  $m_s$  datum observed at the modified Julian day (MJD)  $t_{\text{MJD}}$  to the absolute magnitude  $M_s$  at rest-frame phase  $t$ , via

$$M_s(t) = m_s(t) - \mu_{\Lambda\text{CDM}}(z_s) - K_{OQ}^s - A_O^s, \quad (4)$$

where  $z_s$  is the spectroscopic redshift of the supernova  $s$  with respect to the CMB, including any local flow models (see Table 2). The phase  $t \equiv (t_{\text{MJD}} - t_{B_{\text{max},s}})/(1 + z_{\text{helio},s})$  is the rest-frame observation time, corrected for cosmological time dilation,  $z_{\text{helio},s}$  is the heliocentric redshift, and  $t_{B_{\text{max}}}$  is the time of  $B$ -band maximum light. The term  $K_{OQ}$  is the  $K$ -correction from the observed band  $O$  to the rest-frame band  $Q$ , and  $A_O$  is the Milky Way foreground extinction defined as  $A_O = R_O E(B - V)_{\text{MW}}$ , where  $R_O$  is the total-to-selective extinction ratio in filter  $O$  and  $E(B - V)_{\text{MW}}$  is the Milky Way color excess. We use the Schlafly & Finkbeiner (2011) dust reddening map for  $E(B - V)_{\text{MW}}$ , and the CCM+O (O'Donnell 1994) reddening laws to determine  $R_O$  for the NIR and optical bands respectively. We assume a  $V$ -band total-to-selective extinction ratio for the Milky Way of  $R_V = 3.1$ .

We determine  $t_{B_{\text{max}}}$  and compute the  $K$ -correction  $K_{OQ}^s$  terms using a module in the SNooPy LC package (Burns et al. 2011), which uses the normal SN Ia spectroscopic template of Hsiao et al. 2007 that is “mangled” to match the actual colors derived from the data. Within SNooPy, the observer frame LCs are transformed into the natural systems of the CfA PAIRITEL/2MASS telescope for the  $JHK$ -bands and of the CSP Swope telescope for the  $Y$ -band, before being  $K$ -corrected to the corresponding rest-frame bandpass.

The theoretical distance modulus is defined as

$$\mu_{\Lambda\text{CDM}}(z_s) = 5 \log_{10} \left[ \frac{d_L(z_s)}{1 \text{Mpc}} \right] + 25 \quad (5)$$

We assume the luminosity distance  $d_L(z)$  for a spatially flat  $\Lambda$ CDM Universe, ignoring radiation, is approximately given by

$$d_L(z) = \left( \frac{c}{H_0} \right) (1 + z_{\text{helio}}) \int_0^z \frac{dz'}{E(z')} \quad (6)$$

where  $E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$  and  $c$  is the speed of light.<sup>6</sup> We assume fiducial values for the matter and energy density fractions of  $\Omega_m = 0.28$  and  $\Omega_\Lambda = 0.72$  and a Hubble constant of  $H_0 = 73.24 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Riess et al. 2016).

Every value of  $M_s$  has an error variance

$$\sigma_M^2 = \sigma_m^2 + \sigma_{\mu_{\text{pec}}}^2 + \sigma_A^2 + \sigma_{K_{\text{corr}}}^2, \quad (7)$$

where  $\sigma_m$  is the measurement uncertainty of the apparent magnitude  $m_s$ ,  $\sigma_{\mu_{\text{pec}}}$  is the uncertainty in the distance modulus  $\mu_{\Lambda\text{CDM}}(z_s)$  due to the peculiar velocity and redshift uncertainties, given as

$$\sigma_{\mu_{\text{pec},s}}^2(z_s) = \left( \frac{5}{z_s \ln(10)} \right)^2 \left[ \left( \frac{\sigma_{\text{pec}}}{c} \right)^2 + \sigma_{z,s}^2 \right]. \quad (8)$$

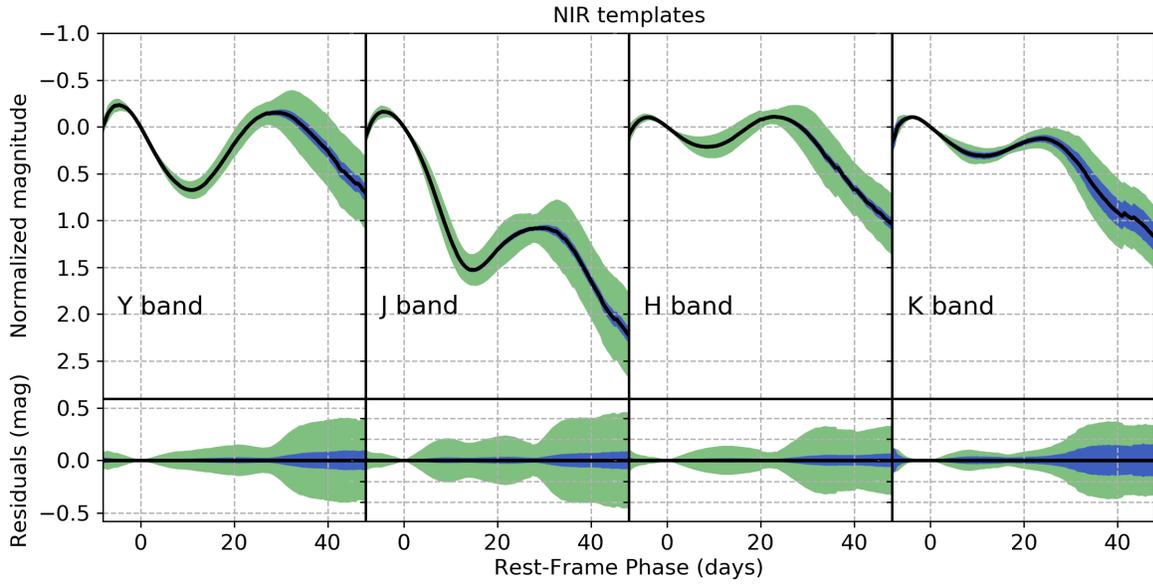
For the SN in Table 4 with independent distance estimates, we use those corresponding distance modulus uncertainties. The term  $\sigma_A$  in Eq. (7) is the uncertainty in the Milky Way dust extinction  $A_O$  computed as,  $\sigma_A = R_O \sigma_{\text{EBV}}$ , where  $\sigma_{\text{EBV}}$  is the uncertainty in the Milky Way color excess  $E(B - V)_{\text{MW}}$ , and  $\sigma_{K_{\text{corr}}}$  is the  $K$ -correction uncertainty estimated using Monte Carlo simulations of the full optical and NIR dataset  $\{m_s\}$  for a given SN. In this Monte Carlo approach, for each photometric datum at a given MJD time and band,  $m_s(t_{\text{MJD}})$ , we simulate a realization of this datum by drawing a random value from a Gaussian distribution with mean and standard deviation equal to the measured values  $m_s$  and  $\sigma_m$ . For each simulated full optical+NIR dataset, we compute the  $K$ -corrections and then determine the mean and standard deviation of the distribution of the  $K$ -correction values for each photometric datum at a given MJD time and band. We use this standard deviation as an estimation of the uncertainty of the  $K$ -correction value for that datum.

<sup>6</sup> As noted by Calcino & Davis 2017, using the CMB frame redshift  $z$  instead of the heliocentric redshift  $z_{\text{helio}}$  in Eq. 6 causes only a small error, which has a negligible effect on our analysis, especially for the low redshift sample analyzed in this work.

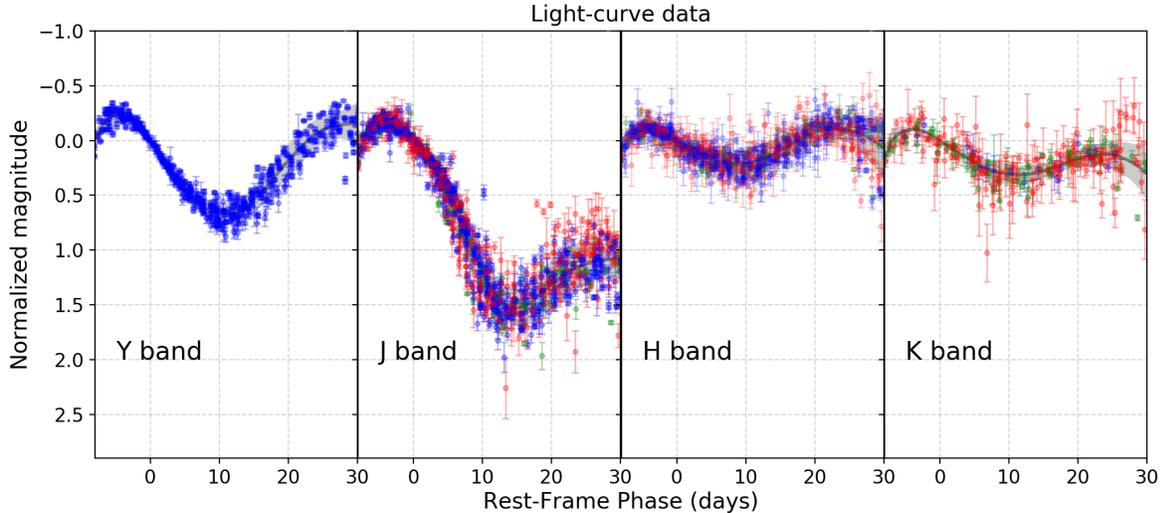
**Table 5.** Normalized  $YJHK_s$  LC Templates

| $t^*$<br>(days) | $\theta^{(Y)}$<br>(mag) | $\sigma_{\theta}^{(Y)}$<br>(mag) | $\theta^{(J)}$<br>(mag) | $\sigma_{\theta}^{(J)}$<br>(mag) | $\theta^{(H)}$<br>(mag) | $\sigma_{\theta}^{(H)}$<br>(mag) | $\theta^{(K)}$<br>(mag) | $\sigma_{\theta}^{(K)}$<br>(mag) |
|-----------------|-------------------------|----------------------------------|-------------------------|----------------------------------|-------------------------|----------------------------------|-------------------------|----------------------------------|
| -10             | 0.303 ± 0.067           | 0.168                            | 0.428 ± 0.037           | 0.136                            | 0.367 ± 0.047           | 0.184                            | 0.439 ± 0.290           | 0.178                            |
| -9              | 0.152 ± 0.034           | 0.108                            | 0.261 ± 0.025           | 0.106                            | 0.213 ± 0.032           | 0.136                            | 0.289 ± 0.335           | 0.194                            |
| -8              | -0.007 ± 0.026          | 0.092                            | 0.092 ± 0.020           | 0.094                            | 0.075 ± 0.023           | 0.104                            | 0.150 ± 0.125           | 0.112                            |
| -7              | -0.135 ± 0.023          | 0.092                            | -0.038 ± 0.015          | 0.076                            | -0.010 ± 0.016          | 0.070                            | 0.001 ± 0.058           | 0.106                            |
| -6              | -0.204 ± 0.018          | 0.078                            | -0.115 ± 0.013          | 0.070                            | -0.065 ± 0.012          | 0.054                            | -0.058 ± 0.034          | 0.066                            |
| -5              | -0.228 ± 0.016          | 0.072                            | -0.153 ± 0.012          | 0.070                            | -0.093 ± 0.010          | 0.046                            | -0.088 ± 0.021          | 0.034                            |
| -4              | -0.224 ± 0.012          | 0.056                            | -0.159 ± 0.010          | 0.060                            | -0.102 ± 0.007          | 0.034                            | -0.103 ± 0.016          | 0.020                            |
| -3              | -0.200 ± 0.008          | 0.042                            | -0.148 ± 0.008          | 0.050                            | -0.092 ± 0.006          | 0.032                            | -0.096 ± 0.011          | 0.014                            |
| -2              | -0.151 ± 0.005          | 0.028                            | -0.116 ± 0.005          | 0.038                            | -0.069 ± 0.004          | 0.024                            | -0.073 ± 0.008          | 0.010                            |
| -1              | -0.082 ± 0.003          | 0.014                            | -0.065 ± 0.003          | 0.022                            | -0.037 ± 0.002          | 0.014                            | -0.040 ± 0.004          | 0.006                            |
| 0               | 0.000 ± 0.000           | 0.000                            | 0.000 ± 0.000           | 0.000                            | 0.000 ± 0.000           | 0.000                            | 0.000 ± 0.000           | 0.000                            |
| 1               | 0.090 ± 0.002           | 0.012                            | 0.075 ± 0.003           | 0.024                            | 0.039 ± 0.002           | 0.016                            | 0.041 ± 0.005           | 0.014                            |
| 2               | 0.184 ± 0.005           | 0.024                            | 0.162 ± 0.007           | 0.050                            | 0.079 ± 0.005           | 0.032                            | 0.083 ± 0.011           | 0.030                            |
| 3               | 0.276 ± 0.007           | 0.036                            | 0.259 ± 0.010           | 0.080                            | 0.115 ± 0.007           | 0.048                            | 0.125 ± 0.017           | 0.046                            |
| 4               | 0.363 ± 0.009           | 0.050                            | 0.369 ± 0.014           | 0.108                            | 0.147 ± 0.009           | 0.064                            | 0.163 ± 0.022           | 0.062                            |
| 5               | 0.441 ± 0.011           | 0.060                            | 0.492 ± 0.018           | 0.140                            | 0.173 ± 0.012           | 0.078                            | 0.200 ± 0.026           | 0.074                            |
| 6               | 0.510 ± 0.013           | 0.072                            | 0.628 ± 0.022           | 0.168                            | 0.194 ± 0.013           | 0.092                            | 0.233 ± 0.030           | 0.086                            |
| 7               | 0.568 ± 0.015           | 0.080                            | 0.774 ± 0.025           | 0.188                            | 0.207 ± 0.015           | 0.104                            | 0.262 ± 0.034           | 0.096                            |
| 8               | 0.612 ± 0.016           | 0.086                            | 0.921 ± 0.026           | 0.202                            | 0.214 ± 0.017           | 0.114                            | 0.284 ± 0.037           | 0.100                            |
| 9               | 0.650 ± 0.016           | 0.088                            | 1.066 ± 0.028           | 0.210                            | 0.214 ± 0.018           | 0.122                            | 0.298 ± 0.037           | 0.100                            |
| 10              | 0.670 ± 0.017           | 0.092                            | 1.199 ± 0.027           | 0.208                            | 0.208 ± 0.018           | 0.128                            | 0.306 ± 0.038           | 0.094                            |
| 11              | 0.676 ± 0.017           | 0.096                            | 1.316 ± 0.027           | 0.200                            | 0.198 ± 0.019           | 0.134                            | 0.309 ± 0.035           | 0.086                            |
| 12              | 0.666 ± 0.019           | 0.104                            | 1.419 ± 0.026           | 0.186                            | 0.179 ± 0.020           | 0.138                            | 0.309 ± 0.035           | 0.080                            |
| 13              | 0.640 ± 0.020           | 0.108                            | 1.487 ± 0.024           | 0.170                            | 0.156 ± 0.020           | 0.138                            | 0.305 ± 0.034           | 0.078                            |
| 14              | 0.600 ± 0.022           | 0.116                            | 1.525 ± 0.024           | 0.166                            | 0.127 ± 0.021           | 0.138                            | 0.295 ± 0.033           | 0.074                            |
| 15              | 0.540 ± 0.023           | 0.118                            | 1.529 ± 0.024           | 0.170                            | 0.089 ± 0.019           | 0.134                            | 0.278 ± 0.032           | 0.068                            |
| 16              | 0.477 ± 0.024           | 0.126                            | 1.513 ± 0.026           | 0.182                            | 0.053 ± 0.019           | 0.128                            | 0.257 ± 0.028           | 0.058                            |
| 17              | 0.409 ± 0.025           | 0.132                            | 1.478 ± 0.028           | 0.195                            | 0.017 ± 0.019           | 0.122                            | 0.234 ± 0.027           | 0.056                            |
| 18              | 0.336 ± 0.026           | 0.138                            | 1.429 ± 0.029           | 0.208                            | -0.018 ± 0.018          | 0.116                            | 0.211 ± 0.028           | 0.064                            |
| 19              | 0.261 ± 0.026           | 0.142                            | 1.373 ± 0.030           | 0.216                            | -0.049 ± 0.017          | 0.108                            | 0.188 ± 0.030           | 0.074                            |
| 20              | 0.183 ± 0.027           | 0.144                            | 1.312 ± 0.032           | 0.226                            | -0.079 ± 0.016          | 0.100                            | 0.169 ± 0.031           | 0.082                            |
| 21              | 0.114 ± 0.029           | 0.150                            | 1.263 ± 0.033           | 0.232                            | -0.091 ± 0.014          | 0.080                            | 0.149 ± 0.033           | 0.088                            |
| 22              | 0.054 ± 0.028           | 0.148                            | 1.221 ± 0.034           | 0.236                            | -0.102 ± 0.014          | 0.076                            | 0.138 ± 0.034           | 0.094                            |
| 23              | -0.007 ± 0.027          | 0.142                            | 1.183 ± 0.032           | 0.228                            | -0.108 ± 0.014          | 0.082                            | 0.130 ± 0.036           | 0.100                            |
| 24              | -0.056 ± 0.026          | 0.134                            | 1.151 ± 0.032           | 0.218                            | -0.102 ± 0.016          | 0.094                            | 0.126 ± 0.039           | 0.110                            |
| 25              | -0.095 ± 0.025          | 0.126                            | 1.124 ± 0.029           | 0.198                            | -0.092 ± 0.019          | 0.120                            | 0.132 ± 0.043           | 0.122                            |
| 26              | -0.123 ± 0.024          | 0.120                            | 1.106 ± 0.026           | 0.180                            | -0.075 ± 0.023          | 0.152                            | 0.147 ± 0.047           | 0.134                            |
| 27              | -0.144 ± 0.024          | 0.124                            | 1.091 ± 0.025           | 0.170                            | -0.050 ± 0.027          | 0.182                            | 0.172 ± 0.054           | 0.156                            |
| 28              | -0.147 ± 0.026          | 0.134                            | 1.085 ± 0.024           | 0.154                            | -0.018 ± 0.031          | 0.214                            | 0.205 ± 0.060           | 0.174                            |
| 29              | -0.151 ± 0.031          | 0.162                            | 1.081 ± 0.028           | 0.178                            | 0.021 ± 0.036           | 0.244                            | 0.248 ± 0.068           | 0.200                            |
| 30              | -0.149 ± 0.038          | 0.196                            | 1.082 ± 0.034           | 0.224                            | 0.067 ± 0.041           | 0.272                            | 0.302 ± 0.084           | 0.238                            |
| 31              | -0.137 ± 0.046          | 0.238                            | 1.090 ± 0.040           | 0.274                            | 0.124 ± 0.043           | 0.294                            | 0.356 ± 0.095           | 0.262                            |
| 32              | -0.115 ± 0.053          | 0.274                            | 1.112 ± 0.048           | 0.322                            | 0.183 ± 0.046           | 0.312                            | 0.414 ± 0.111           | 0.306                            |
| 33              | -0.083 ± 0.058          | 0.304                            | 1.134 ± 0.054           | 0.354                            | 0.248 ± 0.049           | 0.316                            | 0.484 ± 0.118           | 0.322                            |
| 34              | -0.038 ± 0.065          | 0.332                            | 1.186 ± 0.059           | 0.382                            | 0.324 ± 0.052           | 0.326                            | 0.566 ± 0.133           | 0.342                            |
| 35              | 0.007 ± 0.070           | 0.340                            | 1.232 ± 0.063           | 0.396                            | 0.375 ± 0.052           | 0.320                            | 0.636 ± 0.138           | 0.350                            |
| 36              | 0.054 ± 0.075           | 0.356                            | 1.305 ± 0.065           | 0.408                            | 0.438 ± 0.053           | 0.320                            | 0.699 ± 0.144           | 0.358                            |
| 37              | 0.105 ± 0.078           | 0.364                            | 1.378 ± 0.065           | 0.396                            | 0.472 ± 0.050           | 0.298                            | 0.762 ± 0.146           | 0.362                            |
| 38              | 0.156 ± 0.077           | 0.370                            | 1.464 ± 0.068           | 0.408                            | 0.545 ± 0.050           | 0.294                            | 0.816 ± 0.148           | 0.372                            |
| 39              | 0.208 ± 0.077           | 0.372                            | 1.555 ± 0.069           | 0.408                            | 0.605 ± 0.050           | 0.294                            | 0.868 ± 0.141           | 0.368                            |
| 40              | 0.256 ± 0.082           | 0.384                            | 1.647 ± 0.068           | 0.402                            | 0.672 ± 0.050           | 0.292                            | 0.912 ± 0.148           | 0.370                            |
| 41              | 0.325 ± 0.084           | 0.388                            | 1.736 ± 0.069           | 0.406                            | 0.703 ± 0.049           | 0.274                            | 0.955 ± 0.142           | 0.356                            |
| 42              | 0.390 ± 0.086           | 0.396                            | 1.820 ± 0.071           | 0.418                            | 0.759 ± 0.050           | 0.282                            | 0.947 ± 0.140           | 0.334                            |
| 43              | 0.469 ± 0.090           | 0.404                            | 1.922 ± 0.076           | 0.436                            | 0.818 ± 0.053           | 0.294                            | 0.981 ± 0.133           | 0.332                            |
| 44              | 0.507 ± 0.094           | 0.404                            | 1.992 ± 0.077           | 0.440                            | 0.852 ± 0.055           | 0.302                            | 0.987 ± 0.149           | 0.335                            |
| 45              | 0.557 ± 0.091           | 0.400                            | 2.055 ± 0.081           | 0.454                            | 0.908 ± 0.060           | 0.312                            | 1.028 ± 0.149           | 0.338                            |

NOTE— Mean NIR LC templates in the  $YJHK_s$ -bands using the hierarchical Bayesian model and Gaussian process method described in §3. These are referenced to the time of  $B$ -band maximum light, such that  $t^* = 0$  at  $t_{B\max}$ . See Fig. 1.



**Figure 1.** Upper and lower panels show the normalized mean  $YJHK_s$  templates and residual plots, respectively. By construction, we normalize the templates so that they have magnitude zero at  $t^* = 0$ , with reference to the time of  $B$ -band maximum light. The numerical values of these templates are tabulated with 1-day sampling in Table 5. The black curves show the normalized mean magnitude  $\theta(t^*)$  vs. rest-frame phase  $t^*$ , while the green and blue bands correspond to the population standard deviation,  $\sigma_\theta(t^*)$ , and the uncertainty in  $\theta$ , respectively, determined using the hierarchical Bayesian model and Gaussian process method described in §3. We use 28, 67, 68 and 25 SN Ia that we can determine  $\mu^L$  as described in §3.2 to build the  $Y, J, H$  and  $K_s$  templates respectively.



**Figure 2.** Normalized SN Ia LC data vs. rest-frame phase  $t^*$  for the  $YJHK_s$  bands. Points are color coded by NIR photometric data source, including the CfA (red; Wood-Vasey et al. 2008; Friedman et al. 2015), the CSP (blue; Krisciunas et al. 2017), and other data from the literature (green; see Table 2 and references therein). Note that only the CSP used a Y-band filter. The black lines and gray bands in the background correspond to the NIR mean LC templates and population standard deviation shown in Fig. 1. The data for each supernova  $s$  was normalized by subtracting the estimated apparent magnitude  $\hat{m}_{0,s}$ , obtained from the template fit, from the apparent magnitude LC. The template fit in the plot is limited to the rest-frame phase range  $(-8, 30)$  days, since this was the phase range of data that minimized the scatter in the Hubble Diagram residuals using the template method discussed in §3-4. Data beyond  $> 30$  days was used to help fit the mean LC template in Fig. 1, but the increased scatter at those phases limited the utility of that phase range for distance estimates.

### 3.2. LC Fitting: Gaussian process

The Gaussian process technique is a non-parametric Bayesian method that we use to fit the NIR LCs for each SN Ia in Table 2. A Gaussian process defines a prior over functions. Realizations from a GP, evaluated on a discrete set of times, are random vectors drawn from a joint multivariate Gaussian distribution,  $\mathcal{N}$ , of dimension equal to the number of components in the vector. Given a dataset, the GP formalism allows us to coherently determine the posterior mean function that fits the dataset along with its posterior covariance. The GP methodology is especially helpful in accounting for missing data (in our cases, phases with no observations), and when the data are correlated as in the case of the SN Ia LCs. Rasmussen & Williams (2006) provide an introduction to GPs for machine learning.

The following description applies to a LC of a single supernova in a given NIR band. We model the absolute magnitude  $M$  at phase  $t$  as a noisy measurement of the latent (true) absolute magnitude  $\mathcal{M}$  at that phase, given by  $M(t) = \mathcal{M}(t) + \epsilon$ , where  $\epsilon \sim \mathcal{N}(0, \sigma_M^2)$ . In vector notation we express the collection of absolute magnitude data of a given LC as  $\mathbf{M} \equiv [M(t_1), M(t_2), \dots, M(t_n)]^\top$ , measured at phases  $\mathbf{t} \equiv [t_1, t_2, \dots, t_n]^\top$ , where  $n$  is the number of data in the LC, and  $\top$  means the transpose.

Using GP, we estimate the posterior mean,  $\boldsymbol{\mu}^{\text{post}}$ , and the posterior covariance,  $\boldsymbol{\Sigma}^{\text{post}}$ , of the latent absolute magnitudes  $\mathcal{M}^* \equiv [\mathcal{M}(t_1^*), \mathcal{M}(t_2^*), \dots, \mathcal{M}(t_{n^*}^*)]^\top$  on a regular grid of phases  $\mathbf{t}^* \equiv [t_1^*, t_2^*, \dots, t_{n^*}^*]^\top$ , where  $n^*$  is the number of times in the grid determined from a sequence of phases between  $t_{\min,s}$  and  $t_{\max,s}$  in steps of 0.5 days, where  $t_{\min,s}$  and  $t_{\max,s}$  are the minimum and maximum phases in  $\mathbf{t}^*$ . Thus the number of times in the regular grid is  $n^* = (t_{\max,s} - t_{\min,s})/0.5$ . In Appendix A, we provide the mathematical details to determine  $\boldsymbol{\mu}^{\text{post}}$  and  $\boldsymbol{\Sigma}^{\text{post}}$ .

#### 3.2.1. Normalization of the GP light curves

Our goal with the GP fitting is to determine the *shape* of the LC to be used later in Section 3.3 to construct NIR templates to fit the data and estimate distance moduli. So once we determine the posterior mean and covariance of the latent absolute magnitude LC for a given supernova  $s$  using GP, we *normalize* the LCs to extract the information about their shape regardless of their absolute magnitudes. The normalized LC  $L(t)$  is the function, over phase, of the difference in magnitudes relative to the peak phase, so that  $L(t_{B_{\max}}) = 0$ .

To estimate the distance moduli, we choose to use the phase of  $B$ -band maximum light,  $t_{B_{\max}}$ , as the reference time to derive the distances. In Section 4.2, we also implement the estimation of distance moduli using the

time of NIR-band maximum light instead of  $t_{B\max}$  as the reference time.

We define the vector  $\mathbf{L}$ , corresponding to the normalized LC derived from the latent absolute magnitude LC  $\mathcal{M}^*$ , evaluated on the phase grid  $\mathbf{t}^*$ , as

$$\mathbf{L} \equiv \mathcal{M}^* - \mathcal{M}_0 \mathbf{1} \quad (9)$$

where  $\mathcal{M}_0$  is the latent absolute magnitude at  $t_{B\max}$  and  $\mathbf{1}$  is a vector of dimension  $n^*$  with all its elements equal to one. Since this is a linear transformation of  $\mathcal{M}^*$  into  $\mathbf{L}$ , and  $\mathcal{M}^*$  is Gaussian, therefore  $\mathbf{L}$  is also Gaussian and described completely by its mean  $\boldsymbol{\mu}^L$  and covariance  $\boldsymbol{\Sigma}^L$ . See Appendix A.1 for details. In the next section we use  $(\boldsymbol{\mu}^L, \boldsymbol{\Sigma}^L)$  to construct the NIR light curve templates.

### 3.3. Hierarchical Bayesian Model for the Normalized Magnitudes

In this section, we describe how we construct NIR LC templates for the  $Y$ ,  $J$ ,  $H$ , and  $K_s$  bands that correspond to the mean *shape* of SN Ia LCs in each of these bands. To do so, we combine the normalized LCs described by  $(\boldsymbol{\mu}^L, \boldsymbol{\Sigma}^L)$ , from all the supernovae at a given phase  $t^*$  using a hierarchical Bayesian model to determine the mean normalized magnitude. Then we repeat the procedure described below over all the phases in  $\mathbf{t}^*$  to construct the final NIR LC templates.

First, we assume the normalized magnitude at phase  $t^*$ ,  $\mu_s^L$ , for the supernova  $s$  is drawn from a Gaussian distribution with true value  $\eta_s$  and standard deviation  $\sigma_{\eta,s}$ ,

$$\mu_s^L \sim \mathcal{N}(\eta_s, \sigma_{\eta,s}^2) \quad (10)$$

where the value of  $\sigma_{\eta,s}^2$  is given by the  $(t_s^*, t_s^*)$  element in the diagonal of the covariance matrix  $\boldsymbol{\Sigma}_s^L$  [see Eq. (A11)]. Next, we assume that the set of values  $\{\eta_s\}$  from all the  $N_T^*$  supernovae at phase  $t^*$ , are independent draws from a Gaussian population distribution with population mean  $\theta$  and variance  $\sigma_\theta^2$ ,

$$p(\{\eta_s\}|\theta, \sigma_\theta^2) = \prod_{s=1}^{N_T^*} \mathcal{N}(\eta_s|\theta, \sigma_\theta^2) \quad (11)$$

In Appendix B, we write the expression for the joint posterior distribution of the hierarchical model and describe additional decompositions in order to make the computations more tractable to determine the posterior inference<sup>7</sup> of  $(\{\eta_s\}, \theta, \sigma_\theta)$  given the data  $\{(\mu_s^L, \sigma_{\eta,s})\}$  at phase  $t^*$ .

<sup>7</sup> We use the *median* of the posterior probability distribution as the best estimated value.

We repeat the above procedure for all phases in the range  $t^* = (-10, 45)$  days, every 0.5 days, to determine  $(\theta, \sigma_\theta)$  for all  $t^*$  in this range. Figure 1 shows the  $YJHK_s$  templates constructed with this procedure and Table 5 reports the numerical values of the templates. The posterior estimates of the population mean and variance of the normalized LC,  $(\theta, \sigma_\theta^2)$ , and the uncertainty in the determination of  $\theta$ , are shown in Figure 1 as black curves, green bands, and blue bands, respectively.

## 4. HUBBLE DIAGRAM

We implement two different methods to derive the distance modulus for each supernova from the NIR LCs. We call them the *template method* and the *Gaussian-process method* (GP). The GP method requires data near the NIR maximum for all NIR bands being used, while the template method works for arbitrarily sampled data, even if the LC is sparse near maximum. For this reason, we have more objects in the template method Hubble diagrams. We describe these methods in more detail in the following sections.

Any of these NIR-only approaches approximately treat the information in each of the  $YJHK_s$  bands as independent. However, this simple approach does not take maximal advantage of the cross-band correlations between each of the NIR and optical bands, as is done using a more sophisticated hierarchical Bayesian model (e.g. BAYESN: Mandel et al. 2009, 2011, 2014). Nor does this approach use the fact that there is only one true distance to the supernova.

To alleviate this problem, we also derive the distance modulus for each supernova from the *combined* distance moduli in each NIR band. However, instead of computing a simple average distance modulus from the individual distance moduli, we instead estimate the covariance matrix of the  $YJHK_s$  distance moduli (and submatrices of it) and then derive the *weighted* average distance modulus. The advantage of this procedure is that it takes into account the correlations among the magnitudes in the NIR bands and then derives more realistic mean distance moduli and their uncertainties. More details are in Section 4.3.

For our NIR-only Hubble diagrams, only NIR LCs are used to directly construct distance moduli. However, auxiliary optical data is used to estimate  $t_{B\max}$ ,  $\Delta m_{15}(B)$ , and mangled K-corrections, and is employed in the input data selection cuts described in §2.1.

We make publicly available the scripts written to perform the analysis presented in this work at: <https://github.com/ArturoAvelino/SNIRfit>.

#### 4.1. Distance Modulus: Template method

To determine the photometric distance modulus  $\mu_s$  of the supernova  $s$  in a given NIR band, we use the normalized mean template,  $\theta$ , computed in Section 3, to determine the apparent magnitude at phase zero,  $m_{0,s} \equiv m_s(t=0)$ , by fitting the template to the sometimes sparse photometric LC data  $\{m_s(t)\}$ . We define the difference

$$\Delta m_s(t) \equiv m_s(t) - \theta(t) - m_{0,s} \quad (12)$$

where  $m_s(t)$  and  $\theta(t)$  are the apparent magnitude and the magnitude of the normalized template at phase  $t$ , respectively. We can express this difference for all the  $n$  phases in a given LC as the vector,

$$\Delta \mathbf{m}_s \equiv \begin{pmatrix} \Delta m_s(t_1) \\ \Delta m_s(t_2) \\ \vdots \\ \Delta m_s(t_n) \end{pmatrix}. \quad (13)$$

Then, to determine  $m_{0,s}$  we minimize the negative of the log likelihood function  $\mathcal{L}(m_{0,s})$  defined as

$$-2 \ln \mathcal{L}(m_{0,s}) = \Delta \mathbf{m}_s^\top \cdot \mathbf{C}^{-1} \cdot \Delta \mathbf{m}_s + \text{constant}, \quad (14)$$

where  $\mathbf{C}$  is the  $n$ -dimensional covariance matrix where the  $(t_i, t_j)$  component is given by:

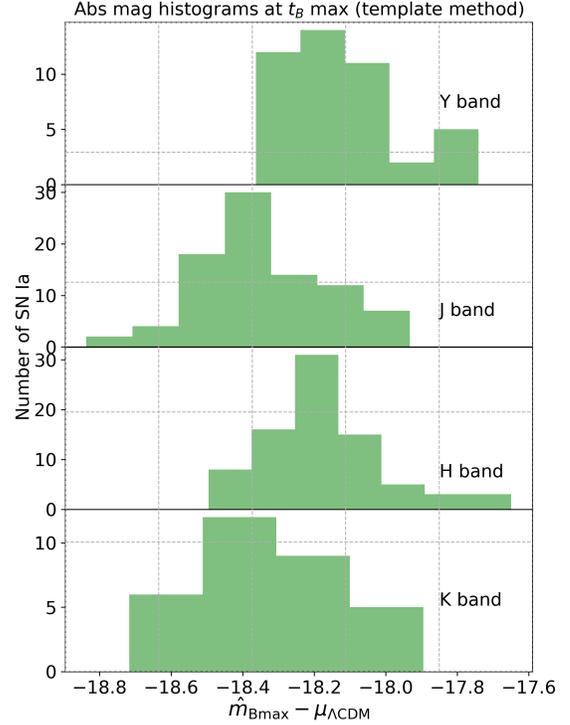
$$C_{ij} \equiv \text{Cov}(\Delta m_s(t_i), \Delta m_s(t_j)) \quad (15)$$

$$= \sigma_\theta(t_i) \sigma_\theta(t_j) \exp \left[ -\frac{(t_i - t_j)^2}{2l^2} \right] + \hat{\sigma}_{m,s}^2(t_i) \delta_{ij}, \quad (16)$$

where  $\sigma_\theta(t)$  is the population standard deviation of the sample distribution of magnitudes at phase  $t$ , determined from Eq. (B14) during the training process used to construct the mean LC template,  $\hat{\sigma}_{m,s}^2(t_i)$  is the photometric error of the datum  $m_s(t_i)$ , and  $l$  is the hyperparameter of GP kernel determined from Eq. (A9) and with values shown in Table 16.

From Eq. (14), we can calculate an analytic expression for the maximum likelihood estimator (MLE) of the apparent magnitude at  $B$ -band maximum light,  $\hat{m}_{0,s}$ , given by:

$$\hat{m}_{0,s} = \left[ \sum_{i,j}^n (C^{-1})_{ij} \right]^{-1} \times \sum_i^n \left[ \left( m_s(t_i) - \theta(t_i) \right) \sum_j^n (C^{-1})_{ij} \right], \quad (17)$$



**Figure 3.** Histograms of the absolute magnitudes at phase zero ( $t^* = t_{B\max}$ ), defined as  $M_{0,s} \equiv \hat{m}_{0,s} - \mu_{\Lambda\text{CDM}}(z_s)$  for the SN Ia sample using the template method. The sample mean, standard deviation, and the number of supernovae in each histogram are shown in Table 6.

with the MLE of the uncertainty of  $\hat{m}_{0,s}$  given as

$$\hat{\sigma}_{\text{fit},s} = \left[ \sum_{i,j}^n (C^{-1})_{ij} \right]^{-1/2}, \quad (18)$$

which corresponds to the fitting error of the light curve. This error incorporates the photometric measurement error and the sparsity of the actual data points.

Now, from the distribution of absolute magnitudes at phase zero estimated as  $M_{0,s} \equiv \hat{m}_{0,s} - \mu_{\Lambda\text{CDM}}(z_s)$  (see Fig. 3), we compute the sample mean absolute magnitude,  $\langle M_0 \rangle$ , and the sample standard deviation of the distribution, obtaining the values reported in Table 6. The sample standard deviation describes the total scatter of the absolute magnitude estimates. Below, we decompose this into the contributions from peculiar velocity-distance errors, measurement/fitting errors, and intrinsic dispersion.

**Table 6.** Mean  $YJK_s$  absolute magnitudes at  $t_{B\max}$  or  $t_{\text{NIR max}}$ .

| Band                               | $N_{\text{SN}}$ | $\langle M \rangle$<br>(mag) | Std. deviation<br>(mag) |
|------------------------------------|-----------------|------------------------------|-------------------------|
| Template method                    |                 |                              |                         |
| $Y$                                | 44              | -18.12                       | 0.15                    |
| $J$                                | 87              | -18.34                       | 0.17                    |
| $H$                                | 81              | -18.18                       | 0.17                    |
| $K_s$                              | 32              | -18.35                       | 0.21                    |
| Gaussian-process method at NIR max |                 |                              |                         |
| $Y$                                | 29              | -18.39                       | 0.11                    |
| $J$                                | 52              | -18.52                       | 0.14                    |
| $H$                                | 44              | -18.30                       | 0.11                    |
| $K_s$                              | 14              | -18.37                       | 0.18                    |
| Gaussian-process method at $B$ max |                 |                              |                         |
| $Y$                                | 29              | -18.16                       | 0.12                    |
| $J$                                | 52              | -18.34                       | 0.15                    |
| $H$                                | 44              | -18.19                       | 0.12                    |
| $K_s$                              | 14              | -18.28                       | 0.17                    |

NOTE— We use the sample mean values of the absolute magnitudes,  $\langle M \rangle$ , in each band to determine the distance modulus in the template and GP methods using Eqs. (19) and (23) respectively. For the template method  $\langle M \rangle \equiv \langle M_0 \rangle$  and for the GP method  $\langle M \rangle \equiv \langle M_{\text{NIRmax}} \rangle$ ,  $\langle M_{B\max} \rangle$ . Figs. 3, 4 and 5 show the histograms of  $M_{0,s}$  and  $M_{\text{NIRmax},s}$ , respectively. In this table, we also present the sample standard deviation of the absolute magnitude sample distribution just as a reference, we do not use those values in any part of the computations.

Finally, we estimate the photometric distance modulus for supernova  $s$  in a given NIR band as

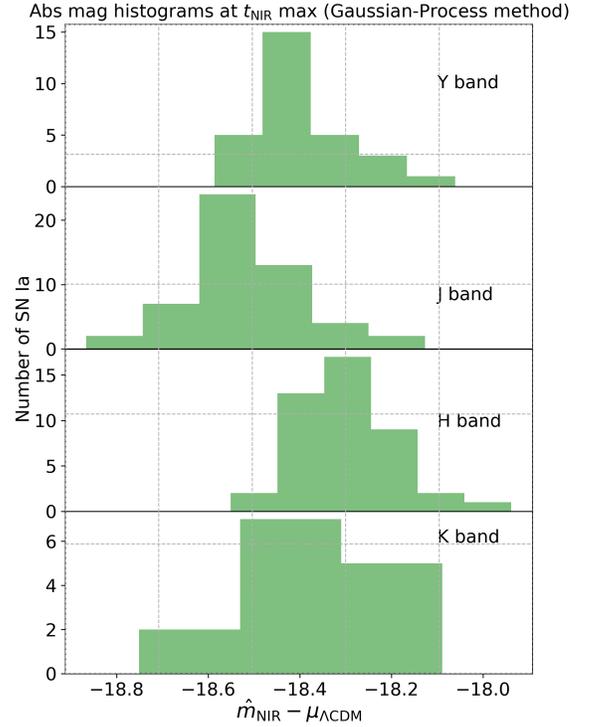
$$\hat{\mu}_s = \hat{m}_{0,s} - \langle M_0 \rangle. \quad (19)$$

The uncertainty on  $\hat{\mu}_s$  is composed of two sources of errors: the fitting uncertainty  $\hat{\sigma}_{\text{fit},s}$  estimated in Eq. (18) for each individual supernova, and the *intrinsic scatter*,  $\sigma_{\text{int}}$ , which primarily comes from the intrinsic variation of SN Ia absolute magnitudes and is estimated by fitting an entire sample of SN Ia on the Hubble diagram (see Appendix C for more details). So the variance of the photometric distance modulus is given as

$$\hat{\sigma}_{\mu,s}^2 = \hat{\sigma}_{\text{fit},s}^2 + \hat{\sigma}_{\text{int}}^2. \quad (20)$$

The Hubble residual for supernova  $s$  is defined as

$$\Delta\mu_s \equiv \hat{\mu}_s - \mu_{\Lambda\text{CDM}}(z_s). \quad (21)$$

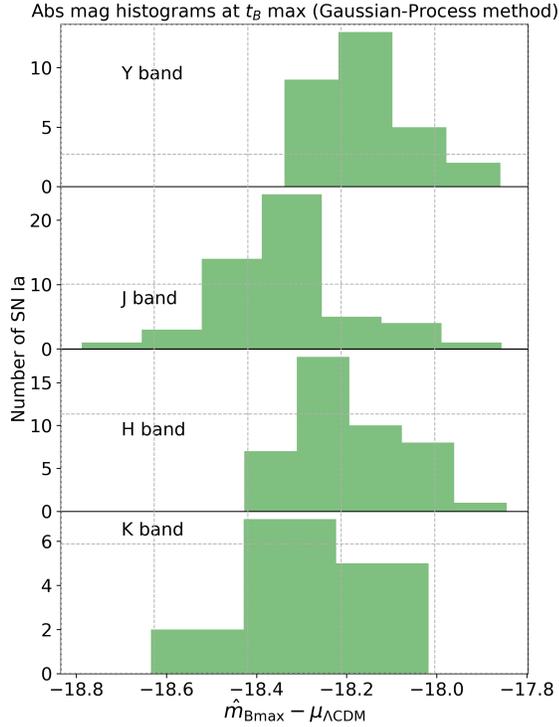

**Figure 4.** Histograms of the absolute magnitudes at phase =  $\text{NIR}_{\max}$ , defined as  $M_{\text{NIRmax},s} \equiv \hat{m}_{\text{NIRmax},s} - \mu_{\Lambda\text{CDM}}(z_s)$  for the SN Ia sample in the GP method at NIR max. The sample mean, standard deviation, and the number of supernovae in each histogram are shown in Table 6.

The uncertainty on  $\mu_{\Lambda\text{CDM}}(z_s)$  is given by Eq. (8). The variance on the Hubble residual for supernova  $s$ ,  $\sigma_{\Delta,s}^2$ , comes from the propagation of uncertainties on  $\hat{\mu}_s$  and  $\mu_{\Lambda\text{CDM}}(z_s)$ , it is,

$$\sigma_{\Delta,s}^2 = \hat{\sigma}_{\text{fit},s}^2 + \hat{\sigma}_{\text{int}}^2 + \sigma_{\mu_{\text{pec},s}}^2. \quad (22)$$

In addition to  $\sigma_{\text{int}}$ , to quantify the dispersion in the Hubble residuals, we also compute both the RMS and the inverse-variance weighted root-mean-square (wRMS, see Appendix C). The RMS and wRMS are measures of the total scatter in the Hubble diagram. The wRMS is relatively insensitive to the assumed value of the peculiar velocity uncertainty, and the formula for the RMS does not depend on the assumed value of  $\sigma_{\text{pec}}$  at all and is therefore more straightforward to compare with other works.

For the template method, the values of  $\hat{\mu}_s$ ,  $\sigma_{\text{int}}$  and wRMS in the Hubble diagram residual for a given NIR band depend on the phase range of the NIR LC template used to determine the distance modulus. We found that



**Figure 5.** Histograms of the absolute magnitudes at phase  $= B_{\max}$ , defined as  $M_{B_{\max},s} \equiv \hat{m}_{B_{\max},s} - \mu_{\Lambda\text{CDM}}(z_s)$  for the SN Ia sample in the GP method at  $B$  max. The sample mean, standard deviation, and the number of supernovae in each histogram are shown in Table 6.

phase range of  $t^* = (-8, 30)$  days in each of the  $YJHK_s$  bands minimized the scatter in the Hubble residual, as measured by  $\sigma_{\text{int}}$  or  $\text{wRMS}$ .

Table 12 reports the distance moduli  $\hat{\mu}_s$  and their fitting uncertainty  $\hat{\sigma}_{\text{fit},s}$  we obtain with this procedure for each supernova in each band, and Fig. 12 shows the Hubble diagram and residuals.

#### 4.2. Distance Modulus: Gaussian-process Method

The nearby low- $z$  NIR sample now contains a sufficient number of SN Ia well-sampled around maximum light in the  $YJHK_s$ -bands, that we can explore referencing various distance estimation approaches to the times of these NIR maxima, rather than  $B$ -max, for which there has long been sufficiently well sampled optical photometry.

An alternative approach that we implement to derive distance moduli is by determining the apparent magnitude at the time of NIR maximum light,  $t_{\text{NIRmax}}$ , and  $B$  maximum light,  $t_{B\text{max}}$ , using the GP technique to interpolate the LC data. The method follows the same pro-

cedure as the one described in Section 3.2, but instead of GP fitting the absolute magnitude LCs,  $\{M_s(t)\}$ , we directly GP fit the *apparent* magnitude LCs,  $\{m_s(t)\}$ . By doing this, we do not include  $\sigma_{\mu_{\text{pec}}}$  in the error budget for each  $m_s(t)$  because we do not subtract  $\mu_{\Lambda\text{CDM}}(z)$ .

To determine the posterior mean of the apparent magnitude LC,  $\{\bar{m}_s(t^*)\}$ , and the posterior covariance of a GP fit to  $\{m_s(t)\}$  we use the Eqs. (A7) and (A8) where we set  $\sigma_{\mu_{\text{pec},s}}^2 = 0$ . For each LC, we use the average of the apparent magnitude data as the GP prior mean, and use the same values for the hyperparameters of the GP kernel shown in Table 16, given that the shape and dispersion of the apparent magnitude LC data is very similar to the absolute magnitude LCs we fitted with GP in Section 3.2 for each supernova. We verified that the GP fits to the LCs are insensitive to these choices.

We only consider LCs that have data either around  $t_{\text{NIRmax}}$  or  $t_{B\text{max}}$  so that we can determine the GP fit at those reference phases. By construction,  $t_{B\text{max}}$  corresponds to a rest-frame phase  $t^* = 0$  days. To determine the NIR maximum using the GP method, we require LCs to have at least one data point observed at a phase  $t_{\text{NIRmax}} < -2.5$  days. This requirement excludes 33 of the 89 SN Ia used in the template-method sample, leaving us with only 56 SN Ia that are suitable for the GP method.

We also limit the search for the maximum to the phase range  $-8.5 < t_{\text{NIRmax}} < -2.5$  days to remove cases where maximum of the GP posterior mean happens *after*  $t_{B\text{max}}$ , which we found can be artifacts of the GP fit when there are too few data points before  $t_{B\text{max}}$ . For the rest of this section we denote the subscripts “NIRmax” and “Bmax” simply as “max”.

From each set  $\{\bar{m}_s(t^*)\}$ , we estimate,  $\hat{m}_{\text{max},s}$ , the GP interpolated apparent magnitude at  $t_{\text{max},s}$ . Then we estimate the distance modulus as

$$\hat{\mu}_s = \hat{m}_{\text{max},s} - \langle M_{\text{max}} \rangle \quad (23)$$

where  $\langle M_{\text{max}} \rangle$  is the mean absolute magnitude at  $t_{\text{max},s}$  from all the supernovae in a given NIR band (see Fig. 4), with  $M_{\text{max},s} \equiv \hat{m}_{\text{max},s} - \mu_{\Lambda\text{CDM}}(z_s)$ . The uncertainty on the photometric distance modulus  $\hat{\mu}_s$  in this case is  $\hat{\sigma}_{\text{fit},s}$ , which is equal to the uncertainty in the apparent magnitude at  $t_{\text{max},s}$  inferred from the GP fit to the LC.

Figure 13 shows Hubble diagrams constructed from the distance moduli inferred from the GP method for each of the  $YJHK_s$  bands, with numerical values reported in Table 13.

#### 4.3. Distance modulus from the combined NIR bands

From the estimated distance moduli ( $\hat{\mu}_s^Y, \hat{\mu}_s^J, \hat{\mu}_s^H, \hat{\mu}_s^K$ ) for a given supernova  $s$  determined from each NIR band

using either of the three methods described above, we estimate the weighted average of the distance modulus  $\mu_s$  from each method. First we define the vector of residuals

$$\delta\boldsymbol{\mu}_s \equiv \begin{pmatrix} \hat{\mu}_s^Y - \mu_s \\ \hat{\mu}_s^J - \mu_s \\ \hat{\mu}_s^H - \mu_s \\ \hat{\mu}_s^K - \mu_s \end{pmatrix}. \quad (24)$$

where  $\hat{\mu}_s^Y, \hat{\mu}_s^J, \hat{\mu}_s^H, \hat{\mu}_s^K$  are determined by either Eqs. (19) or (23), for the template or GP methods, respectively. Then, to estimate  $\mu_s$ , we minimize the negative of the likelihood function  $\mathcal{L}(\mu_s)$  defined as

$$-2 \ln \mathcal{L}(\mu_s) = \delta\boldsymbol{\mu}_s^\top \cdot C_\mu^{-1} \cdot \delta\boldsymbol{\mu}_s + \text{constant}, \quad (25)$$

where  $C_\mu$  is the sample covariance matrix computed from the Hubble residuals (see Eq. (21))  $\{\Delta\mu_s^Y, \Delta\mu_s^J, \Delta\mu_s^H, \Delta\mu_s^K\}$ , the collection of distance-modulus residuals from all SN Ia with observations in the four  $YJHK_s$  bands. For supernovae with observations in only three, two, or one bands, we construct the respective covariance matrices based on those supernova subsamples, and the vector defined in Eq. (24) becomes three, two, or one dimensional, respectively. In Appendix D, we provide numerical values of the covariance matrix  $C_\mu$  for these different subcases.

We derive an analytic expression for the minimization of Eq. (25) with respect to  $\mu_s$  and obtain the maximum likelihood estimate for the combined distance modulus given by,

$$\hat{\mu}_s = \sum_b w_b \hat{\mu}_s^b \quad (26)$$

where  $\hat{\mu}_s^b \in \{\hat{\mu}_s^Y, \hat{\mu}_s^J, \hat{\mu}_s^H, \hat{\mu}_s^K\}$  (the index  $b$  stands for band), and

$$w_b = \left[ \sum_{b'} (C^{-1})_{bb'} \right] \times \left[ \sum_{b', b''} (C^{-1})_{b'b''} \right]^{-1}. \quad (27)$$

Now, assuming that the uncertainties in the distance modulus estimated from each individual  $YJHK_s$  band,  $\hat{\sigma}_{\text{fit},s,Y}, \hat{\sigma}_{\text{fit},s,J}, \hat{\sigma}_{\text{fit},s,H}, \hat{\sigma}_{\text{fit},s,K}$ , are independent between bands  $b$  and also independent of the intrinsic scatter  $\sigma_{\text{int}}$ , then we can propagate the uncertainty in the combined distance modulus due to the fitting only as:

$$\hat{\sigma}_{\text{fit},s} = \sqrt{\sum_b w_b^2 \hat{\sigma}_{s,b}^2} \quad (28)$$

where  $\hat{\sigma}_{s,b} \equiv \hat{\sigma}_{\text{fit},s,Y}, \hat{\sigma}_{\text{fit},s,J}, \hat{\sigma}_{\text{fit},s,H}, \hat{\sigma}_{\text{fit},s,K}$ .

The last column in Tables 12-14 show the combined distance moduli we obtain with this procedure for the template and GP methods respectively. The reported uncertainties correspond to  $\hat{\sigma}_{\text{fit},s}$  in all cases.

#### 4.4. Distance modulus from optical bands

We wish to assess how well the SN Ia observed in NIR bands perform as standard candles, specifically when using  $t_{\text{NIRmax}}$  as opposed to  $t_{B\text{max}}$ , as the time reference to estimate their distance. To do so, we determine the distance moduli using only *optical BVR*-bands LCs for exactly the same 56 supernovae in the ‘‘any  $YJHK_s$ ’’ Hubble diagram set that was used for the GP method (see left panel in Fig. 7 and the SN listed in Table 13). Then we can compare the intrinsic scatter and RMS or wRMS in the Hubble-diagram residuals between the optical-only and NIR-only Hubble diagrams. A smaller intrinsic scatter, wRMS, or RMS, including the uncertainties, would indicate evidence that SN Ia are better standard candles using that data and Hubble diagram construction method.

##### 4.4.1. SALT2 distance modulus

We use the optical photometric data compiled in the public SNANA (Kessler et al. 2009) database<sup>8</sup> but replace the CMB redshift values in the SNANA photometric files with the  $z_{\text{CMB}}$  values in Table 2. Using the latest SALT2 model (SALT2.JLA-B14) (Guy et al. 2007) already trained on the JLA sample (Betoule et al. 2014), we fit the optical data and determine the SALT2 light-curve fit parameters for each supernova. For the CSP data, we added an additional 0.01 mag in quadrature to the photometric errors to have a more conservative uncertainties on those values when fitting the data in SALT2. We use the SALT2 outputs including the apparent magnitude  $m_B$  at  $B$ -band maximum light, the stretch parameter  $x_1$ , and the color term  $c$ , as well as their correlations.

We convert the SALT2-fit parameters to distance moduli for each supernova using the Tripp formula (Tripp 1998),

$$\mu_s = m_{B,s} - M_B + \alpha x_{1,s} - \beta c_s, \quad (29)$$

where  $M_B$  is the expected absolute magnitude at  $B$ -band maximum light for a SN Ia with  $x_1 = 0, c = 0$ , while  $\alpha$  and  $\beta$  are coefficients parametrizing correlations between luminosity and stretch or luminosity and color, respectively.

For the global parameters  $M, \alpha, \beta$  we use the values reported by Scolnic et al. (2018);  $\alpha = 0.147, \beta = 3.00$ , and assume the fiducial values of  $H_0 = 73.24 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $M_B = -19.36 \text{ mag}$ . We then adjust the latter to  $M_B = -19.44 \text{ mag}$  so that the weighted-average Hubble residual is zero.

<sup>8</sup> <http://snana.uchicago.edu>. Version Oct 18, 2017.

The standard deviation of the measurement error  $\sigma_{\text{fit},s}$  from the SALT2 fitting comes from propagating the uncertainties on Eq. (29), including their correlations. Interestingly we found that for the supernovae with high Milky Way color excess  $E(B - V) > 0.2$  the uncertainty on  $m_{B,s}$  is larger than the propagated uncertainty on the SALT2 distance modulus,  $\mu_s$ , derived from optical bands. This evidence further emphasizes how SN Ia are more negatively affected by dust when deriving distances using optical data, as compared to NIR observations.

The variance of the photometric distance modulus is given by

$$\sigma_{\mu,s}^2 = \sigma_{\text{fit},s}^2 + \sigma_{\text{int}}^2. \quad (30)$$

Using SALT2 in this way, we obtain an intrinsic scatter in the Hubble residuals of  $\sigma_{\text{int}} = 0.133 \pm 0.022$ , an inverse-variance weighted RMS of  $\text{wRMS} = 0.174 \pm 0.020$  mag, and a simple RMS =  $0.179 \pm 0.018$  mag. The third column of Table 15 and the left panel of Fig. 10 show the distance moduli derived from the SALT2 fits, along with the Hubble diagram and residuals, respectively. The uncertainties shown in Table 15 and Fig. 10 are the values of  $\sigma_{\text{fit},s}$ .

Note that we are not applying the usual SALT2 cuts to this subsample of SN because we are interested in comparing the scatter in the Hubble residuals using exactly the same 56 SN Ia used in the “any  $YJHK_s$ ” Hubble diagram for the GP method. We find that when applying the SALT2 cut on color,  $-0.3 < c < 0.3$ , there is only 1 SN Ia in the subsample that does not pass this cut. All SN Ia in the sample pass these SALT2 cuts:  $-3 < x_1 < 3$ , uncertainty in  $x_1 < 1$ , and uncertainty in  $t_{B\text{max}} < 2$  days. However, 21 SN Ia fail to pass the SALT2 cut requiring that the probability that the data are represented by the model, given the  $\chi^2$  per degree of freedom of the fit, is larger than 0.001 (a.k.a,  $\text{FIT-PROB} > 0.001$ ). However, a low fit probability does not necessarily indicate a poor SN Ia light curve fit and may instead be an indication that the photometric uncertainties or the model uncertainties are unrealistically small. We visually inspected the light curve fits of these 21 SN Ia, finding that they are reasonably well-fit by the model and can therefore be used to yield accurate distance measurements.

#### 4.4.2. SNooPy distance modulus

As a second cross check of the scatter in the optical-only Hubble diagram, we also fit the  $BVR$ -bands LCs using the SNooPy LC fitting package’s `EBV_model` (Burns et al. 2011), where every observed apparent magnitude  $m_s$  in band  $O \equiv B, V, R$  is compared to the

model

$$m_s(t) = \mu_s + T_Q(t, \Delta m_{15,s}) + M_Q(\Delta m_{15,s}) + R_O E(B - V)_{\text{MW},s} + R_Q E(B - V)_{\text{host},s} + K_{OQ}^s(z_s, t, E(B - V)_{\text{MW},s}, E(B - V)_{\text{host},s}), \quad (31)$$

where  $T_Q(t, \Delta m_{15,s})$  is a light-curve template for the rest-frame band  $Q$  that depend on  $t$  and  $\Delta m_{15,s}$ , and  $M_{Q,s}(\Delta m_{15,s})$  is the absolute magnitude band  $Q$ . In this model, the free parameters that SNooPy estimates (along with their uncertainties) are  $\mu_s$ ,  $\Delta m_{15,s}$ ,  $E(B - V)_{\text{host},s}$  and  $t_{B\text{max}}$ . We consider the estimated uncertainty on  $\mu_s$  output by SNooPy as the  $\sigma_{\text{fit},s}$  in our analysis. We refer the reader to Burns et al. (2011) for details on how SNooPy estimates the uncertainty on  $\mu_s$ .

We obtain an intrinsic scatter in the Hubble residuals of  $\sigma_{\text{int}} = 0.128 \pm 0.018$ , a  $\text{wRMS} = 0.159 \pm 0.019$  mag, and a  $\text{RMS} = 0.174 \pm 0.021$  mag. The fourth column of Table 15 and right panel of Fig. 10 show the distance moduli derived from the SNooPy fits, along with the Hubble diagram and residuals, respectively.

## 5. DISCUSSION

Tables 7 and 8 summarize the scatter in the Hubble residuals measured with the either the intrinsic scatter  $\sigma_{\text{int}}$ , the  $\text{wRMS}$ , or the  $\text{RMS}$ . We compute these both for our fiducial peculiar velocity uncertainty of  $\sigma_{\text{pec}} = 150$  km/s as well as the value  $\sigma_{\text{pec}} = 250$  km/s used in Scolnic et al. (2018).

While the formula for  $\text{RMS}$  in Eq. C23 does not depend on the assumed value of  $\sigma_{\text{pec}}$  (see Appendix C), the value of  $\sigma_{\text{int}}$  is quite sensitive to the assumed value of  $\sigma_{\text{pec}}$ . In particular, larger assumed values of  $\sigma_{\text{pec}}$  yield smaller inferred values of  $\sigma_{\text{int}}$  (see columns 4 and 5 of Tables 7 and 8). The assumption of  $\sigma_{\text{pec}} = 150$  km/s in this work therefore yields a more conservative estimate of  $\sigma_{\text{int}}$  compared with larger values of  $\sigma_{\text{pec}}$  because, in the latter case, most of the scatter in the Hubble residuals can be explained as arising solely from peculiar velocities. For instance, the Hubble residuals using only  $H$ -band LCs from the GP (NIR max) method produce an intrinsic scatter of zero when assuming  $\sigma_{\text{pec}} = 250$  km/s.

We found that  $\text{wRMS}$  is less sensitive than  $\sigma_{\text{int}}$  to the assumed value of  $\sigma_{\text{pec}}$ , producing differences of  $\sim 0.001$  mag between  $\sigma_{\text{pec}} = 150$  and  $\sigma_{\text{pec}} = 250$  km/s.

Of the three NIR methods used to derive distance moduli, the GP method at NIR max yields smaller  $\text{RMS}$ ,  $\text{wRMS}$ , and intrinsic scatter in the Hubble residuals than the template and GP methods at  $B$  max methods applied to the same 56 SN Ia with data from any of the  $YJHK_s$  bands. When we combine the GP dis-

**Table 7.** Hubble Residual Intrinsic Scatter,  $\sigma_{\text{int}}$ , and RMS.

| Band                        | Method             | $N_{\text{SN}}$ | $\sigma_{\text{int}}$ [mag]         | $\sigma_{\text{int}}$ [mag]         | wRMS [mag]                          | RMS [mag]         |
|-----------------------------|--------------------|-----------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------|
|                             |                    |                 | ( $\sigma_{\text{pec}} = 150$ km/s) | ( $\sigma_{\text{pec}} = 250$ km/s) | ( $\sigma_{\text{pec}} = 150$ km/s) |                   |
| Optical <i>BVR</i>          | SALT2              | 56              | $0.133 \pm 0.022$                   | $0.107 \pm 0.025$                   | $0.174 \pm 0.020$                   | $0.179 \pm 0.018$ |
| Optical <i>BVR</i>          | SNooPy             | 56              | $0.128 \pm 0.018$                   | $0.111 \pm 0.020$                   | $0.159 \pm 0.019$                   | $0.174 \pm 0.021$ |
| any <i>YJHK<sub>s</sub></i> | Template           | 56              | $0.112 \pm 0.016$                   | $0.096 \pm 0.019$                   | $0.140 \pm 0.016$                   | $0.138 \pm 0.014$ |
| any <i>YJHK<sub>s</sub></i> | GP (NIR max)       | 56              | $0.047 \pm 0.018$                   | $0.000 \pm 0.000^*$                 | $0.100 \pm 0.013$                   | $0.117 \pm 0.014$ |
| any <i>YJHK<sub>s</sub></i> | GP ( <i>B</i> max) | 56              | $0.066 \pm 0.016$                   | $0.044 \pm 0.023$                   | $0.106 \pm 0.010$                   | $0.115 \pm 0.011$ |

NOTE— We compare the Hubble residual scatter for the optical and NIR bands using exactly the same 56 supernovae for several methods. We compute the intrinsic scatter,  $\sigma_{\text{int}}$  (see Appendix C), the inverse-variance weighted root-mean square (wRMS), and the simple RMS, using two standard LC fitters: SALT2 (Guy et al. 2007) and SNooPy (Burns et al. 2011) to fit the optical *BVR*-band LC data, as well as the three NIR methods we implement in this work: NIR LC templates at *B*-max [Template], and GP regression at NIR-max [GP (NIR max) or *B*-band maximum [GP (*B* max)]]. We are limited to 56 SN Ia because these are all the supernovae that we can fit using the GP method. Columns 4 and 5 show  $\sigma_{\text{int}}$ , assuming  $\sigma_{\text{pec}} = 150$  and 250 km/s respectively. The estimated intrinsic scatter  $\sigma_{\text{int}}$  decreases as the assumed peculiar velocity uncertainty  $\sigma_{\text{pec}}$  increases from commonly assumed values of 150 km/s (Radburn-Smith et al. 2004) to 250 km/s (Scolnic et al. 2018), making  $\sigma_{\text{int}}$  somewhat model dependent. By contrast, the wRMS value only changes by thousandths of a magnitude for  $\sigma_{\text{pec}}$  in the same range. Column 6 shows the wRMS assuming  $\sigma_{\text{pec}} = 150$  km/s. Column 7 shows the simple RMS, which makes no assumptions about error weighting and does not depend on  $\sigma_{\text{pec}}$ . Both optical methods apply LC shape and dust corrections but still yield a larger scatter than the NIR methods quantified with any of  $\sigma_{\text{int}}$ , wRMS or RMS (see also Table 8). Figs. 7-10 show Hubble diagrams and residuals for this subsample.

\* For  $\sigma_{\text{pec}} = 250$  km/s, the estimated value of  $\sigma_{\text{int}}$  is consistent with 0. See the paragraph below Eq. (B.7) in Blondin et al. (2011) for the explicit approximation we use to estimate the uncertainty, which breaks down at  $\sigma_{\text{int}} = 0$ .

tance moduli for these same SN Ia referenced to the NIR maxima, we find an  $\text{RMS} = 0.117 \pm 0.014$ ,  $\text{wRMS} = 0.100 \pm 0.013$ , and intrinsic scatter of  $\sigma_{\text{int}} = 0.047 \pm 0.018$  mag. Using the GP method instead referenced to *B*-max for the same SN Ia yields  $\text{RMS} = 0.115 \pm 0.011$ ,  $\text{wRMS} = 0.106 \pm 0.010$ , and  $\sigma_{\text{int}} = 0.066 \pm 0.016$  mag. The NIR maxima thus yield comparable dispersion in the Hubble residuals than *B*-max for each individual NIR band subset with the GP method (see Table 11).

By comparison, when using the NIR template method referenced to *B*-max for these same SN Ia, we find a larger value of  $\text{RMS} = 0.138 \pm 0.014$ ,  $\text{wRMS} = 0.140 \pm 0.016$ , and  $\sigma_{\text{int}} = 0.112 \pm 0.016$  mag.

When we create the Hubble diagram using optical-only LCs of the same 56 supernovae, we find  $\text{RMS} = 0.179 \pm 0.018$ ,  $\text{wRMS} = 0.174 \pm 0.020$ , and  $\sigma_{\text{int}} = 0.133 \pm 0.022$  mag when using SALT2, and  $\text{RMS} = 0.174 \pm 0.021$ ,  $\text{wRMS} = 0.159 \pm 0.019$ , and  $\sigma_{\text{int}} = 0.128 \pm 0.018$  mag with SNooPy.

It is also of interest to compare the results for different observed subsamples. First, note that only the CSP observed with a *Y*-band filter, so it is not possible to compare these observations to other *Y*-band samples. Similarly, there are relatively few CSP *K*-band observations compared to the CfA and Others in the literature, so such *K*-band comparisons are not particularly useful. However, there are sufficient *J* and

*H* band observations to perform useful comparisons by subsample, which we present in Table 9.

As seen in Table 9, for each Hubble diagram method (Template, NIR max, B max), the CSP *JH* data yield smaller values than the data from the CfA or Others in the literature for the various measures of dispersion, including intrinsic scatter, wRMS, and RMS. In most cases the CfA sample yields smaller dispersions than the Others sample. The uncertainties on the dispersions are smaller for the CSP versus CfA samples because the photometric uncertainties for the CSP data are typically at least 2-3 times smaller at the brightest light curve point than the CfA data (Friedman et al. 2015). In addition, the smaller number of CfA vs. CSP light curves that passed our data analysis cuts leads to a larger inferred uncertainty on the dispersions for the CfA sample. However, for all of our Hubble diagram creation methods listed in Table 9, for each of the *J* and *H* bands, the dispersions are still consistent with one another to within 1-2 $\sigma$  for the samples from the CSP, CfA, and Others. This yields evidence that, for each method, estimates of the dispersion from each data sample are consistent with being drawn from the same intrinsic population of SN Ia.

Overall, as shown in Table 10, depending on the NIR *YJHK<sub>s</sub>* subset, the NIR-only GP method yields a RMS in the Hubble residuals that is as much as  $\sim 2.3$ -4.1 $\sigma$  smaller than the SALT2 and SNooPy fits using optical-

**Table 8.** Hubble Residual Intrinsic Scatter,  $\sigma_{\text{int}}$ , and RMS.

| Band                        | Method             | $N_{\text{SN}}$ | $\sigma_{\text{int}}$ [mag]         | $\sigma_{\text{int}}$ [mag]         | wRMS [mag]                          | RMS [mag]         |
|-----------------------------|--------------------|-----------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------|
|                             |                    |                 | ( $\sigma_{\text{pec}} = 150$ km/s) | ( $\sigma_{\text{pec}} = 250$ km/s) | ( $\sigma_{\text{pec}} = 150$ km/s) |                   |
| <i>Y</i>                    | Template           | 44              | $0.105 \pm 0.018$                   | $0.093 \pm 0.021$                   | $0.139 \pm 0.013$                   | $0.152 \pm 0.016$ |
| <i>Y</i>                    | GP (NIR max)       | 29              | $0.066 \pm 0.020$                   | $0.037 \pm 0.032$                   | $0.102 \pm 0.015$                   | $0.111 \pm 0.018$ |
| <i>Y</i>                    | GP ( <i>B</i> max) | 29              | $0.080 \pm 0.019$                   | $0.062 \pm 0.024$                   | $0.110 \pm 0.013$                   | $0.118 \pm 0.017$ |
| <i>J</i>                    | Template           | 87              | $0.136 \pm 0.016$                   | $0.122 \pm 0.018$                   | $0.170 \pm 0.013$                   | $0.175 \pm 0.013$ |
| <i>J</i>                    | GP (NIR max)       | 52              | $0.107 \pm 0.017$                   | $0.090 \pm 0.021$                   | $0.136 \pm 0.017$                   | $0.139 \pm 0.016$ |
| <i>J</i>                    | GP ( <i>B</i> max) | 52              | $0.124 \pm 0.019$                   | $0.110 \pm 0.021$                   | $0.153 \pm 0.020$                   | $0.151 \pm 0.021$ |
| <i>H</i>                    | Template           | 81              | $0.126 \pm 0.016$                   | $0.112 \pm 0.018$                   | $0.162 \pm 0.015$                   | $0.166 \pm 0.016$ |
| <i>H</i>                    | GP (NIR max)       | 44              | $0.032 \pm 0.027$                   | $0.000 \pm 0.000^*$                 | $0.095 \pm 0.010$                   | $0.114 \pm 0.015$ |
| <i>H</i>                    | GP ( <i>B</i> max) | 44              | $0.063 \pm 0.020$                   | $0.037 \pm 0.033$                   | $0.111 \pm 0.011$                   | $0.120 \pm 0.013$ |
| <i>K</i>                    | Template           | 32              | $0.175 \pm 0.032$                   | $0.163 \pm 0.035$                   | $0.211 \pm 0.023$                   | $0.207 \pm 0.020$ |
| <i>K</i>                    | GP (NIR max)       | 14              | $0.093 \pm 0.054$                   | $0.077 \pm 0.060$                   | $0.163 \pm 0.033$                   | $0.179 \pm 0.029$ |
| <i>K</i>                    | GP ( <i>B</i> max) | 14              | $0.094 \pm 0.054$                   | $0.059 \pm 0.069$                   | $0.162 \pm 0.035$                   | $0.170 \pm 0.027$ |
| any <i>YJHK<sub>s</sub></i> | Template           | 89              | $0.123 \pm 0.014$                   | $0.107 \pm 0.016$                   | $0.154 \pm 0.013$                   | $0.161 \pm 0.013$ |
| <i>JH</i>                   | Template           | 81              | $0.127 \pm 0.015$                   | $0.112 \pm 0.017$                   | $0.158 \pm 0.015$                   | $0.164 \pm 0.015$ |
| <i>JH</i>                   | GP (NIR max)       | 42              | $0.039 \pm 0.024$                   | $0.000 \pm 0.000^*$                 | $0.096 \pm 0.011$                   | $0.114 \pm 0.016$ |
| <i>JH</i>                   | GP ( <i>B</i> max) | 42              | $0.069 \pm 0.019$                   | $0.046 \pm 0.028$                   | $0.112 \pm 0.014$                   | $0.118 \pm 0.015$ |
| <i>YJH</i>                  | Template           | 40              | $0.093 \pm 0.018$                   | $0.080 \pm 0.022$                   | $0.121 \pm 0.013$                   | $0.137 \pm 0.018$ |
| <i>YJH</i>                  | GP (NIR max)       | 21              | $0.044 \pm 0.028$                   | $0.000 \pm 0.000^*$                 | $0.088 \pm 0.014$                   | $0.087 \pm 0.013$ |
| <i>YJH</i>                  | GP ( <i>B</i> max) | 21              | $0.068 \pm 0.023$                   | $0.056 \pm 0.031$                   | $0.097 \pm 0.014$                   | $0.098 \pm 0.014$ |

NOTE— Scatter in the Hubble residuals for different NIR band subsets, quantified by the intrinsic scatter,  $\sigma_{\text{int}}$ , the wRMS, and the simple RMS, using three methods: NIR LC templates at *B*-max [Template], and GP regression at NIR-max [GP (NIR max)] or *B*-band maximum [GP (*B* max)]. Column 3 shows the number of SN Ia in each Hubble diagram. Also see Table 7. Columns 4 and 5 show  $\sigma_{\text{int}}$ , assuming  $\sigma_{\text{pec}} = 150$  and 250 km/s respectively. Note that by increasing the value of  $\sigma_{\text{pec}}$ , the  $\sigma_{\text{int}}$  decreases even to zero in some cases with uncertainty denoted by  $\infty$ . For the GP method, we use exactly the same supernovae at *B*-max or NIR-max. For all NIR band subsets, the GP (NIR max) method produces the smallest scatter, followed by the GP (*B*-max) method, while the template method always yields the largest scatter and wRMS. Figs. 6-8 and 12-14 show Hubble diagrams and residuals for most of the NIR subsets listed in this table.

\* For  $\sigma_{\text{pec}} = 250$  km/s, the estimated value of  $\sigma_{\text{int}}$  in these cases is consistent with zero. See the paragraph below Eq. (B.7) in Blondin et al. (2011) for the explicit approximation we use to estimate the uncertainty, which breaks down at  $\sigma_{\text{int}} = 0$ .

only *BVR* data. Furthermore, our “any *YJHK<sub>s</sub>*” set of 56 SN Ia yields a RMS for our GP method at NIR max that is  $0.057 \pm 0.025$  mag smaller than SNooPy and  $0.062 \pm 0.023$  mag smaller than SALT2 applied to the corresponding *BVR* data, again at the  $\sim 2.7\sigma$  level. We interpret the smaller intrinsic scatter as additional evidence, at the  $\sim 2.5$ - $3.1\sigma$  level, that NIR SN Ia LCs at NIR maximum, without LC shape or dust corrections, are already *better* standard candles than optical-only SN Ia LCs referenced to *B*-max that apply such corrections. In addition, it is possible that NIR data or a combination of NIR and optical could yield even smaller intrinsic scatter if employing a method that applies LC shape and dust corrections, for example, using a hierarchical Bayesian approach like BAYESN (Mandel et al. 2009, 2011).

In Table 10, we note that the uncertainty on the difference in the dispersion estimates between any two methods has been computed conservatively. The uncertainty of the dispersion of each individual method has been

computed independently, and then the uncertainty in the difference is found by adding in quadrature, assuming the independence of the samples and therefore the individual uncertainties. However, this ignores the fact that the supernovae in our optical sample are exactly the same ones as those in our NIR sample. Therefore, the actual peculiar velocity-distance errors must be the same in each sample (and not just the variance of these errors). Because of this common component of scatter, the dispersion estimate for the optical Hubble diagram is (positively) correlated with that for the NIR Hubble diagram in each comparison. The effect of this positive correlation is to reduce the variance in the differences in dispersion. Using our estimates of  $\sigma_{\text{int}}$ ,  $\sigma_{\text{fit},s}$  and  $\sigma_{\mu_{\text{pec},s}}$  for the sample and each method, we have run simulations to account for this correlation and quantify this effect. For example, we find that the uncertainty in  $\Delta\text{RMS}$  for “SNooPy - any *YJHK<sub>s</sub>*” is  $\sim 30\%$  smaller than naive uncertainty assuming independent samples, resulting in a significance greater than  $3\sigma$ .

**Table 9.** Hubble Residual Intrinsic Scatter,  $\sigma_{\text{int}}$ , and RMS.

| Band     | Subsample | Method             | $N_{\text{SN}}$ | $\sigma_{\text{int}}$ [mag]<br>( $\sigma_{\text{pec}} = 150$ km/s) | wRMS [mag]<br>( $\sigma_{\text{pec}} = 150$ km/s) | RMS [mag]         |
|----------|-----------|--------------------|-----------------|--|---|-------------------|
| <i>J</i> | CfA       | Template           | 29              | $0.142 \pm 0.029$  | $0.179 \pm 0.023$                                 | $0.179 \pm 0.022$ |
| <i>J</i> | CSP       | Template           | 48              | $0.129 \pm 0.020$  | $0.160 \pm 0.017$                                 | $0.166 \pm 0.019$ |
| <i>J</i> | Others    | Template           | 10              | $0.159 \pm 0.055$  | $0.196 \pm 0.046$                                 | $0.201 \pm 0.039$ |
| <i>J</i> | CfA       | GP (NIR max)       | 15              | $0.125 \pm 0.037$  | $0.158 \pm 0.040$                                 | $0.149 \pm 0.032$ |
| <i>J</i> | CSP       | GP (NIR max)       | 31              | $0.094 \pm 0.019$  | $0.118 \pm 0.015$                                 | $0.113 \pm 0.014$ |
| <i>J</i> | Others    | GP (NIR max)       | 6               | $0.156 \pm 0.071$  | $0.195 \pm 0.056$                                 | $0.217 \pm 0.053$ |
| <i>J</i> | CfA       | GP ( <i>B</i> max) | 15              | $0.146 \pm 0.044$  | $0.182 \pm 0.045$                                 | $0.172 \pm 0.043$ |
| <i>J</i> | CSP       | GP ( <i>B</i> max) | 31              | $0.110 \pm 0.020$  | $0.129 \pm 0.015$                                 | $0.119 \pm 0.013$ |
| <i>J</i> | Others    | GP ( <i>B</i> max) | 6               | $0.188 \pm 0.078$  | $0.221 \pm 0.092$                                 | $0.226 \pm 0.076$ |
| <i>H</i> | CfA       | Template           | 27              | $0.173 \pm 0.033$  | $0.205 \pm 0.029$                                 | $0.200 \pm 0.029$ |
| <i>H</i> | CSP       | Template           | 44              | $0.102 \pm 0.018$  | $0.135 \pm 0.017$                                 | $0.140 \pm 0.019$ |
| <i>H</i> | Others    | Template           | 10              | $0.094 \pm 0.053$  | $0.147 \pm 0.032$                                 | $0.170 \pm 0.036$ |
| <i>H</i> | CfA       | GP (NIR max)       | 13              | $0.000 \pm 0.000^*$  | $0.107 \pm 0.019$                                 | $0.116 \pm 0.021$ |
| <i>H</i> | CSP       | GP (NIR max)       | 25              | $0.041 \pm 0.027$  | $0.088 \pm 0.013$                                 | $0.085 \pm 0.012$ |
| <i>H</i> | Others    | GP (NIR max)       | 6               | $0.000 \pm 0.000^*$  | $0.106 \pm 0.035$                                 | $0.189 \pm 0.051$ |
| <i>H</i> | CfA       | GP ( <i>B</i> max) | 13              | $0.000 \pm 0.000^*$  | $0.106 \pm 0.021$                                 | $0.108 \pm 0.018$ |
| <i>H</i> | CSP       | GP ( <i>B</i> max) | 25              | $0.071 \pm 0.023$  | $0.107 \pm 0.013$                                 | $0.101 \pm 0.012$ |
| <i>H</i> | Others    | GP ( <i>B</i> max) | 6               | $0.052 \pm 0.119$  | $0.135 \pm 0.042$                                 | $0.195 \pm 0.042$ |

NOTE— Dispersion in the *J* and *H*-band Hubble residuals by subsample (See caption of Fig. 8). In most cases, the CSP *JH* data yield smaller values of the various measures of dispersion (intrinsic scatter, wRMS, RMS) than the data from the CfA or Others subsamples, while the CfA subsample yields smaller dispersions than the Others subsample. The uncertainties on the dispersions are smaller for the CSP versus CfA samples because the photometric uncertainties for the CSP data are typically at least 2-3 times smaller at the brightest light curve point than the CfA data (Friedman et al. 2015). However, for each Hubble diagram method in Table 9 (Template, NIR max, *B* max), for both the *J* and *H* bands, the dispersions are still consistent with one another to within 1-2 $\sigma$  for the CSP, CfA, and Others subsamples. For the *Y* and *K<sub>s</sub>* bands, all or most of the data is from the CSP and CfA respectively, so it is either impossible (*Y*-band) or not useful (*K*-band) to perform similar comparisons for these subsamples.

For the Hubble diagrams created using just one of the *YJHK<sub>s</sub>* bands, when using the GP method at NIR max, the *Y* band has the smallest scatter with a RMS of  $0.111 \pm 0.018$  mag. When using the template method, the *Y* band has also the smallest scatter with RMS =  $0.152 \pm 0.016$ .

For every individual band and subset of NIR bands shown in Table 11, the GP method yields *smaller* intrinsic scatter when referencing to NIR max instead of *B* max, by mean amounts of up to  $\sim 0.03$  mag for the same SN Ia at up to the  $\sim 1.0\sigma$  level. While not as statistically significant as the NIR vs. optical comparison in Table 10, we note that the NIR maxima yield smaller intrinsic scatter  $\sigma_{\text{int}}$  and wRMS than *B* max for *all subsets* of the NIR data except for *K<sub>s</sub>*.<sup>9</sup> While NIR data at NIR max are *better* standard candles in comparison to optical data, they are also *at least as good or better*

than when referenced to *B*-max. Therefore, future analyses should consider using  $t_{\text{NIRmax}}$  as the reference time instead of the traditional  $t_{B_{\text{max},s}}$ .

As an additional comparison between NIR and optical Hubble residuals, in Fig. 11, we plot the histograms (dashed lines) with their Gaussian approximation (left panel), and the cumulative distribution function (right panel) for Hubble residuals using the same 56 SN Ia used for the “any *YJHK<sub>s</sub>*” GP method at NIR max (lower left panel on Fig. 7), SALT2 (lower left panel of Fig. 10), and SNooPy (lower right panel of Fig. 10). The Gaussian approximations of the histograms in the left panel of Fig. 11 show that the Hubble residuals are more narrowly distributed for the NIR data (solid red curve) compared to both optical methods (solid green and blue), while in the right panel of Fig. 11 the cumulative distribution function curve for the NIR Hubble residuals is steeper than for either optical curve. Both approaches suggest that the Hubble residual scatter is smaller in the NIR compared to the optical. A larger sample of SN Ia in the NIR would strengthen the evidence for this conclusion.

<sup>9</sup> The only exception we tested is the *K<sub>s</sub>*-band, which has only 14 SN Ia LCs, where we find an essentially equivalent wRMS  $\sim 0.163$  mag when referenced to either NIR max or *B*-max.

**Table 10.** Optical - NIR intrinsic scatter

| Optical <i>BVR</i> Method - NIR band(s) | $\Delta\sigma_{\text{int}}$<br>(mag) | $n\text{-}\sigma$ | $\Delta\text{wRMS}$<br>(mag) | $n\text{-}\sigma$ | $\Delta\text{RMS}$<br>(mag) | $n\text{-}\sigma$ |
|---|--------------------------------------|-------------------|------------------------------|-------------------|-----------------------------|-------------------|
| SALT2 - <i>Y</i>                        | $0.067 \pm 0.029$                    | 2.3               | $0.073 \pm 0.025$            | 2.9               | $0.068 \pm 0.026$           | 2.6               |
| SNooPy - <i>Y</i>                       | $0.062 \pm 0.027$                    | 2.3               | $0.057 \pm 0.024$            | 2.4               | $0.063 \pm 0.028$           | 2.2               |
| SALT2 - <i>J</i>                        | $0.027 \pm 0.028$                    | 1.0               | $0.038 \pm 0.026$            | 1.5               | $0.040 \pm 0.024$           | 1.6               |
| SNooPy - <i>J</i>                       | $0.021 \pm 0.025$                    | 0.8               | $0.023 \pm 0.025$            | 0.9               | $0.035 \pm 0.027$           | 1.3               |
| SALT2 - <i>H</i>                        | $0.101 \pm 0.035$                    | 2.9               | $0.079 \pm 0.022$            | 3.6               | $0.065 \pm 0.023$           | 2.8               |
| SNooPy - <i>H</i>                       | $0.095 \pm 0.033$                    | 2.9               | $0.063 \pm 0.021$            | 3.0               | $0.060 \pm 0.026$           | 2.3               |
| SALT2 - <i>K<sub>s</sub></i>            | $0.040 \pm 0.058$                    | 0.7               | $0.011 \pm 0.039$            | 0.3               | $0.000 \pm 0.035$           | 0.0               |
| SNooPy - <i>K<sub>s</sub></i>           | $0.034 \pm 0.057$                    | 0.6               | $-0.005 \pm 0.038$           | -0.1              | $-0.005 \pm 0.036$          | -0.1              |
| SALT2 - any <i>YJHK<sub>s</sub></i>     | $0.086 \pm 0.028$                    | 3.0               | $0.074 \pm 0.024$            | 3.2               | $0.062 \pm 0.023$           | 2.7               |
| SNooPy - any <i>YJHK<sub>s</sub></i>    | $0.080 \pm 0.026$                    | 3.1               | $0.059 \pm 0.023$            | 2.6               | $0.057 \pm 0.025$           | 2.3               |
| SALT2 - <i>JH</i>                       | $0.095 \pm 0.032$                    | 2.9               | $0.078 \pm 0.023$            | 3.5               | $0.065 \pm 0.024$           | 2.7               |
| SNooPy - <i>JH</i>                      | $0.089 \pm 0.030$                    | 2.9               | $0.062 \pm 0.022$            | 2.8               | $0.060 \pm 0.026$           | 2.3               |
| SALT2 - <i>YJH</i>                      | $0.089 \pm 0.036$                    | 2.5               | $0.086 \pm 0.024$            | 3.5               | $0.092 \pm 0.022$           | 4.1               |
| SNooPy - <i>YJH</i>                     | $0.084 \pm 0.034$                    | 2.5               | $0.070 \pm 0.024$            | 3.0               | $0.087 \pm 0.025$           | 3.5               |

NOTE— We show  $\Delta\sigma_{\text{int}}$ ,  $\Delta\text{wRMS}$ , and  $\Delta\text{RMS}$ , where the first is defined as the difference in Hubble residuals intrinsic scatter between the optical *BVR* data, fit using SALT2 or SNooPy, and the indicated subset of NIR data using the Gaussian process method at NIR max. The quantities  $\Delta\text{wRMS}$  and  $\Delta\text{RMS}$  are defined in a similar way to  $\Delta\sigma_{\text{int}}$  but using wRMS and RMS instead of the intrinsic scatter, respectively. The uncertainties are given by the quadrature sum of the  $\sigma_{\text{int}}$ , wRMS, or RMS, uncertainties from columns 4, 6, or 7, respectively of Tables 7 and 8 for  $\sigma_{\text{pec}} = 150$  km/s. Columns 3, 5, and 6, show  $n\text{-}\sigma$  defined as the number  $n$  of standard deviations  $\sigma$  by which the NIR data yields *smaller*  $\sigma_{\text{int}}$ , wRMS, or RMS, than the optical data using these methods, respectively. Excluding the *K<sub>s</sub>*-band on its own, where our LC compilation contains much less data than the *YJH* bands, in general, NIR data subsets yield smaller RMS than the optical data at the  $\sim 1.3\text{-}4.1\sigma$  level. In the best case, the *JH*, *YJH*, and *YJHK<sub>s</sub>*-bands perform  $\sim 2.3\text{-}4.1\sigma$  better than either SALT2 or SNooPy fits to the *BVR* data in terms of the RMS, while in the worst case, *J*-band, still performs  $1.3\sigma$  better than optical data. For simplicity, the stated uncertainties on the difference in dispersion estimates between any two methods ignores the fact that the actual peculiar velocity-distance errors are exactly the same between the optical and NIR samples, since they contain exactly the same SN. The effect of accounting for this correlation is to decrease the uncertainty of the difference, and increase the significance (§5).

## 6. CONCLUSIONS

This work bolsters and confirms a growing body of evidence that SN Ia in NIR are excellent standard candles in the *YJHK<sub>s</sub>* bands in comparison to the optical *BVR* bands. Depending on the NIR data subset, our GP method performs  $2.3\text{-}2.7\sigma$  better in RMS than either the SALT2 or SNooPy LC fitters for the same 56 SN Ia using *BVR* data and applying LC shape and color corrections. Using a suitable subset of the existing low-redshift sample including 89 spectroscopically normal SN Ia with NIR data, *YJHK<sub>s</sub>* photometry alone already provides a simple means to estimate accurate and precise host galaxy distances in each band, without the LC shape or host galaxy dust reddening corrections required for optical data.

In this work, we employed a hierarchical Bayesian model, combined with a Gaussian process LC fitter, to construct new mean NIR LC templates. We then used these templates, along with Milky Way dust corrections, NIR *K*-corrections, and the measured spectroscopic red-

shifts (corrected for local velocity flows), and redshift independent distance information (e.g. Cepheids) for special cases, to estimate host galaxy distances and uncertainties and construct Hubble diagrams in each of the individual *YJHK<sub>s</sub>* bands. When considering NIR-only methods, our GP method referenced to the time of NIR maximum yields slight smaller Hubble diagram intrinsic scatter and error weighted RMS than when referenced to *B* max and significantly smaller intrinsic scatter compared to the template method.

Our approach is intermediate in complexity between earlier analyses by our group by Wood-Vasey et al. 2008 and the BAYESN approach detailed in Mandel et al. 2009, 2011. The BAYESN methodology presents a coherent, principled, hierarchical Bayesian model that takes into account the full correlation structure between all the input optical and NIR bandpasses, both in color and phase, in order to determine the posterior distributions for distance moduli  $\mu$ , host galaxy dust estimates  $A_V$ , and separate  $R_V$  values for each supernova. Nev-

**Table 11.** GP Method intrinsic scatter for  $B$  max vs. NIR max

| NIR band(s)  | $\Delta\sigma_{\text{int}}$<br>(mag) | $n\text{-}\sigma$ | $\Delta\text{wRMS}$<br>(mag) | $n\text{-}\sigma$ | $\Delta\text{RMS}$<br>(mag) | $n\text{-}\sigma$ |
|--------------|--------------------------------------|-------------------|------------------------------|-------------------|-----------------------------|-------------------|
| $Y$          | $0.014 \pm 0.028$                    | 0.49              | $0.009 \pm 0.020$            | 0.42              | $0.007 \pm 0.025$           | 0.26              |
| $J$          | $0.018 \pm 0.025$                    | 0.70              | $0.017 \pm 0.026$            | 0.65              | $0.012 \pm 0.026$           | 0.46              |
| $H$          | $0.031 \pm 0.034$                    | 0.92              | $0.016 \pm 0.014$            | 1.12              | $0.006 \pm 0.020$           | 0.32              |
| $K_s$        | $0.001 \pm 0.076$                    | 0.01              | $-0.001 \pm 0.048$           | -0.03             | $-0.009 \pm 0.040$          | -0.23             |
| any $YJHK_s$ | $0.019 \pm 0.024$                    | 0.77              | $0.006 \pm 0.016$            | 0.38              | $-0.002 \pm 0.018$          | -0.10             |
| $JH$         | $0.030 \pm 0.031$                    | 0.99              | $0.016 \pm 0.018$            | 0.89              | $0.004 \pm 0.021$           | 0.17              |
| $YJH$        | $0.024 \pm 0.037$                    | 0.66              | $0.008 \pm 0.020$            | 0.41              | $0.011 \pm 0.019$           | 0.58              |

NOTE— Similar to Table 10, we show  $\Delta\sigma_{\text{int}}$ ,  $\Delta\text{wRMS}$ , and  $\Delta\text{RMS}$ , defined here as the difference in Hubble residuals scatter between the Gaussian process method referenced to  $B$  max or NIR max. As in Table 10, the uncertainties are given by the quadrature sum of the  $\sigma_{\text{int}}$  or wRMS uncertainties from columns 4 or 6 of Tables 7 and 8 for  $\sigma_{\text{pec}} = 150$  km/s. Columns 3, 5, and 7, show  $n\text{-}\sigma$  defined as the number  $n$  of standard deviations  $\sigma$  by which the NIR data referenced to NIR max yields *smaller* intrinsic scatter, wRMS, or RMS, than when referenced to  $B$ -max, respectively. For every individual band and subset of NIR bands, the GP method yields smaller estimated intrinsic scatter when referencing to NIR max instead of  $B$  max, where the largest difference is  $n\text{-}\sigma = 0.99\sigma$  for  $JH$  band. This trend is also observed when comparing the wRMS values, again, excluding,  $K_s$  band, where our sample lacks enough data to draw meaningful conclusions.

ertheless, BAYESN is considerably more complex to implement than the simpler analysis methods in this work, which perform quite well for our sample of NIR data.

Compared to optical LCs, NIR SN Ia LCs have a narrow luminosity distribution, and are less sensitive to host galaxy dust extinction. This could help to limit systematic galaxy distance errors that arise from the degeneracy between the intrinsic supernova colors and reddening of light by dust, that affects optical-only SN Ia cosmology (Krisciunas et al. 2004a; Wood-Vasey et al. 2008; Folatelli et al. 2010; Burns et al. 2011; Burns et al. 2014; Kattner et al. 2012; Mandel et al. 2009, 2011, 2017; Scolnic et al. 2014b, 2017). Studies combining NIR and optical SN Ia photometry have already shown that the addition of NIR data is an extremely promising way to break the degeneracy between intrinsic color and dust reddening, allowing distance estimates to become increasingly insensitive to the assumptions behind individual LC fitting models (Mandel et al. 2011, 2014).

We have recently begun to augment the existing low- $z$  SN Ia in NIR sample from the CfA, CSP, and other groups using the Hubble Space Telescope RAISIN program in Cycles 20 and 23 (Kirshner 2012; Foley et al. 2013a,b; Kirshner & The RAISIN TEAM 2014). In RAISIN1, we observed 23 SN Ia at  $z \sim 0.35$  in the rest-frame NIR with WFC3/IR, followed by observations of 24 additional SN Ia at  $z \sim 0.5$  for RAISIN2. Each of these HST NIR observations was accompanied by well-sampled ground based optical photometry from Pan-STARRS (PS1; Rest et al. 2014; Jones et al. 2018; Scolnic et al. 2018) and the Dark Energy Survey (DES; Dark Energy Survey Collaboration et al. 2016; DES Col-

laboration et al. 2018c; Brout et al. 2018b). Analysis of the RAISIN data will be presented in future work.

The evidence from this work further emphasizes the promise of NIR wavelength observations not only for the ongoing HST RAISIN project, but also for future space studies of cosmic acceleration and dark energy (Gehrels 2010; Beaulieu et al. 2010; Astier et al. 2011; Hounsell et al. 2017; Riess et al. 2018c). Upcoming missions that could exploit nearby NIR data as a low- $z$  anchor include the Large Synoptic Survey Telescope (LSST; Ivezić et al. 2008), the NASA Wide-Field Infrared Survey Telescope (WFIRST-AFTA; Gehrels 2010; Spergel et al. 2015), the European Space Agency’s EUCLID mission (Beaulieu et al. 2010; Wallner et al. 2017), as well as the NASA James Webb Space Telescope (JWST; Clampin 2011; Greenhouse 2016).

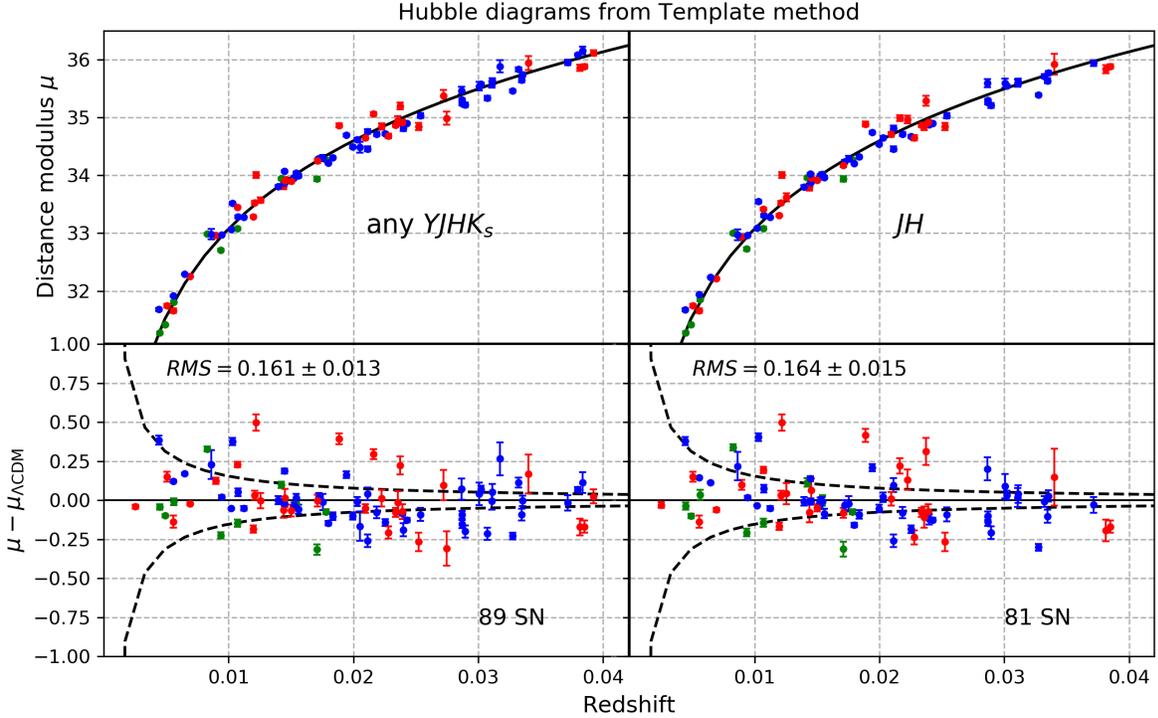
NIR photometry can also augment our knowledge of the spectral energy distribution of SN Ia, for example the Type Ia parametrized SALT2 model, which is currently poorly constrained at infrared wavelengths (Pierel et al. 2018b,a). This will dovetail nicely with the NIR capabilities of JWST and WFIRST and be useful for future SN Ia surveys.

Methods such as BAYESN (Mandel et al. 2009, 2011), SNooPy, and SALT2ext (Pierel et al. 2018b,a) that use empirical LC fitters and provide host galaxy distance estimates using both optical and NIR data can be extended to obtain cosmological inferences and dark energy constraints using both low- $z$  and high- $z$  samples. Combining the growing low-redshift SN Ia in NIR samples from the CfA, CSP, and other samples in the literature with higher redshift optical and NIR data sets will

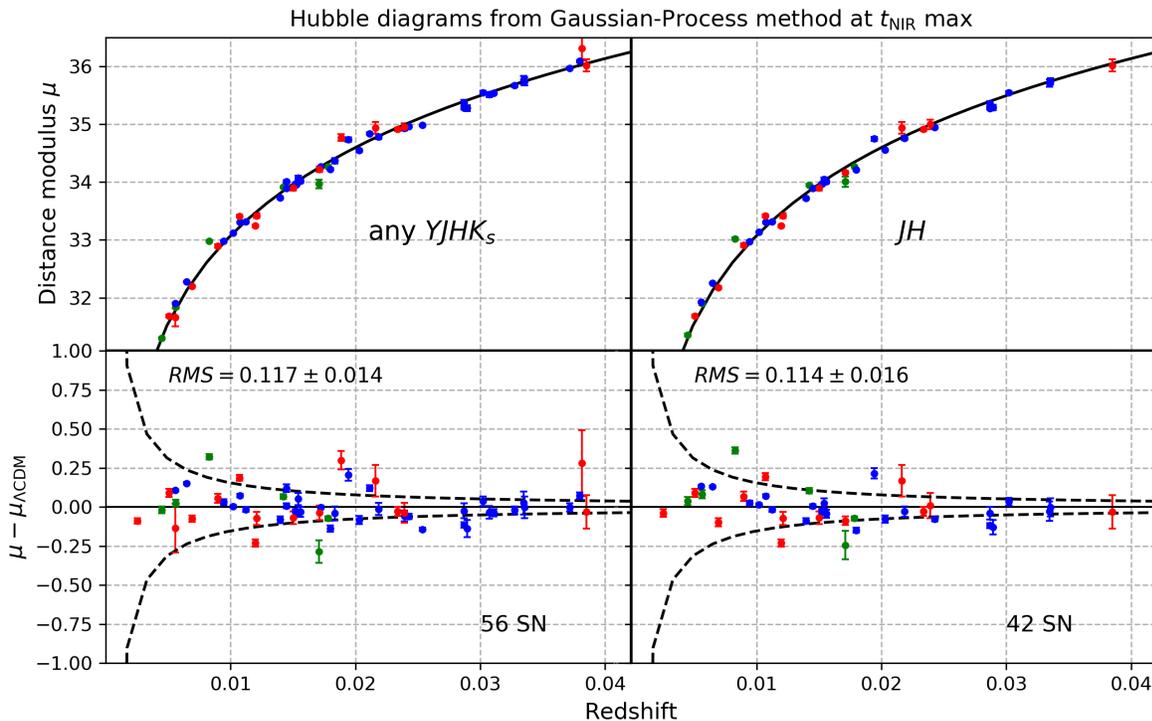
**Table 12.** SN Ia  $YJHK_s$  Distance Moduli from Template method

| SN name         | Source | $\hat{\mu}^Y$<br>(mag) | $\hat{\mu}^J$<br>(mag) | $\hat{\mu}^H$<br>(mag) | $\hat{\mu}^K$<br>(mag) | $\hat{\mu}$<br>(mag) |
|-----------------|--------|------------------------|------------------------|------------------------|------------------------|----------------------|
| SN1998bu        | CfA    | ...                    | 30.11 ± 0.03           | 30.02 ± 0.02           | 29.97 ± 0.01           | 30.03 ± 0.013        |
| SN1999ee        | CSP    | ...                    | 33.30 ± 0.02           | 33.27 ± 0.02           | ...                    | 33.28 ± 0.016        |
| SN1999ek        | Others | ...                    | 34.27 ± 0.02           | 34.27 ± 0.02           | ...                    | 34.27 ± 0.016        |
| SN2000bh        | CSP    | 34.83 ± 0.07           | 35.00 ± 0.06           | 34.83 ± 0.04           | ...                    | 34.81 ± 0.041        |
| SN2000ca        | CSP    | ...                    | 34.87 ± 0.02           | 34.92 ± 0.02           | ...                    | 34.91 ± 0.016        |
| SN2000E         | Others | ...                    | 31.72 ± 0.04           | 31.91 ± 0.04           | 31.78 ± 0.04           | 31.81 ± 0.022        |
| SN2001ba        | CSP    | ...                    | 35.56 ± 0.04           | 35.54 ± 0.04           | 35.66 ± 0.04           | 35.58 ± 0.023        |
| SN2001bt        | Others | ...                    | 33.94 ± 0.04           | 33.97 ± 0.03           | 33.92 ± 0.03           | 33.94 ± 0.021        |
| SN2001cn        | Others | ...                    | 34.06 ± 0.06           | 34.00 ± 0.06           | 33.95 ± 0.09           | 34.00 ± 0.039        |
| SN2001cz        | Others | ...                    | 33.88 ± 0.04           | 33.96 ± 0.06           | 33.96 ± 0.07           | 33.94 ± 0.034        |
| SN2001el        | Others | ...                    | 31.32 ± 0.03           | 31.27 ± 0.03           | 31.24 ± 0.03           | 31.28 ± 0.017        |
| SN2002dj        | Others | ...                    | 32.99 ± 0.03           | 33.00 ± 0.03           | 32.97 ± 0.03           | 32.99 ± 0.015        |
| SN2003du        | Others | ...                    | 32.64 ± 0.04           | 32.76 ± 0.03           | 32.72 ± 0.03           | 32.71 ± 0.020        |
| SN2003hy        | Others | ...                    | 31.38 ± 0.02           | 31.43 ± 0.02           | 31.45 ± 0.02           | 31.42 ± 0.011        |
| SN2004ef        | CSP    | 35.54 ± 0.07           | 35.66 ± 0.12           | 35.56 ± 0.10           | ...                    | 35.54 ± 0.073        |
| SN2004eo        | CSP    | 33.91 ± 0.01           | 33.95 ± 0.01           | 33.98 ± 0.02           | ...                    | 33.95 ± 0.013        |
| SN2004ey        | CSP    | 34.01 ± 0.02           | 33.95 ± 0.03           | 34.04 ± 0.04           | ...                    | 34.04 ± 0.026        |
| SN2004gs        | CSP    | 35.36 ± 0.07           | 35.65 ± 0.10           | 35.57 ± 0.09           | ...                    | 35.47 ± 0.068        |
| SN2004S         | Others | ...                    | 33.05 ± 0.04           | 33.10 ± 0.03           | 33.10 ± 0.05           | 33.08 ± 0.023        |
| SN2005bo        | CfA    | ...                    | 33.77 ± 0.11           | 33.81 ± 0.08           | 33.87 ± 0.06           | 33.81 ± 0.050        |
| SN2005cf        | CfA    | ...                    | 32.22 ± 0.02           | 32.21 ± 0.02           | 32.34 ± 0.01           | 32.25 ± 0.009        |
| SN2005el        | CSP    | 33.95 ± 0.02           | 34.02 ± 0.03           | 33.99 ± 0.03           | ...                    | 33.97 ± 0.020        |
| SN2005iq        | CSP    | 35.69 ± 0.02           | 35.76 ± 0.03           | 35.78 ± 0.05           | ...                    | 35.74 ± 0.033        |
| SN2005kc        | CSP    | 33.86 ± 0.02           | 33.88 ± 0.03           | 33.88 ± 0.02           | ...                    | 33.87 ± 0.019        |
| SN2005kf        | CSP    | 34.58 ± 0.02           | 34.64 ± 0.03           | 34.65 ± 0.04           | ...                    | 34.62 ± 0.029        |
| SN2005lu        | CSP    | 35.89 ± 0.11           | ...                    | ...                    | ...                    | 35.89 ± 0.106        |
| SN2005na        | CfA    | ...                    | 35.31 ± 0.13           | ...                    | 35.48 ± 0.15           | 35.38 ± 0.097        |
| SN2006ac        | CfA    | ...                    | 35.20 ± 0.08           | 35.32 ± 0.12           | 35.03 ± 0.09           | 35.20 ± 0.059        |
| SN2006ax        | CSP    | 34.18 ± 0.01           | 34.15 ± 0.02           | 34.23 ± 0.02           | ...                    | 34.22 ± 0.014        |
| SN2006bh        | CSP    | 33.27 ± 0.03           | 33.32 ± 0.04           | 33.31 ± 0.03           | ...                    | 33.29 ± 0.025        |
| SN2006bt        | CSP    | ...                    | 35.33 ± 0.04           | ...                    | ...                    | 35.33 ± 0.039        |
| SN2006cp        | CfA    | ...                    | 35.12 ± 0.11           | 34.90 ± 0.08           | 34.47 ± 0.10           | 34.85 ± 0.054        |
| SN2006D         | CfA    | ...                    | 32.93 ± 0.03           | 32.94 ± 0.04           | 33.03 ± 0.04           | 32.97 ± 0.021        |
| SN2006ej        | CSP    | 34.42 ± 0.14           | 34.60 ± 0.09           | ...                    | ...                    | 34.49 ± 0.094        |
| SN2006kf        | CSP    | 34.65 ± 0.02           | 34.75 ± 0.02           | 34.74 ± 0.03           | ...                    | 34.70 ± 0.022        |
| SN2006lf        | CfA    | ...                    | 33.48 ± 0.03           | 33.54 ± 0.04           | ...                    | 33.53 ± 0.030        |
| SN2006N         | CfA    | ...                    | 34.05 ± 0.11           | 33.92 ± 0.09           | 33.74 ± 0.09           | 33.91 ± 0.058        |
| SN2007A         | CSP    | 34.30 ± 0.02           | 34.18 ± 0.05           | 34.26 ± 0.04           | ...                    | 34.29 ± 0.029        |
| SN2007af        | CSP    | 31.91 ± 0.01           | 31.96 ± 0.01           | 31.93 ± 0.01           | ...                    | 31.92 ± 0.009        |
| SN2007ai        | CSP    | 35.57 ± 0.02           | 35.42 ± 0.03           | 35.38 ± 0.03           | ...                    | 35.46 ± 0.022        |
| SN2007as        | CSP    | 34.26 ± 0.02           | 34.27 ± 0.02           | 34.34 ± 0.04           | ...                    | 34.31 ± 0.025        |
| SN2007bc        | CSP    | 34.68 ± 0.03           | 34.80 ± 0.04           | 34.82 ± 0.06           | ...                    | 34.76 ± 0.042        |
| SN2007bd        | CSP    | 35.54 ± 0.03           | 35.58 ± 0.04           | 35.60 ± 0.08           | ...                    | 35.57 ± 0.057        |
| SN2007ca        | CSP    | 34.17 ± 0.01           | 34.07 ± 0.01           | 34.01 ± 0.02           | ...                    | 34.07 ± 0.016        |
| SN2007co        | CfA    | ...                    | ...                    | ...                    | 34.99 ± 0.11           | 34.99 ± 0.110        |
| SN2007cq        | CfA    | ...                    | 34.87 ± 0.03           | 34.84 ± 0.08           | ...                    | 34.85 ± 0.060        |
| SN2007jg        | CSP    | 36.05 ± 0.02           | 36.16 ± 0.02           | ...                    | ...                    | 36.09 ± 0.014        |
| SN2007le        | CSP    | 32.36 ± 0.01           | 32.24 ± 0.01           | 32.24 ± 0.01           | ...                    | 32.29 ± 0.007        |
| SN2007qe        | CfA    | ...                    | 34.70 ± 0.17           | 34.91 ± 0.07           | 35.26 ± 0.15           | 34.95 ± 0.075        |
| SN2007st        | CSP    | 31.68 ± 0.05           | 31.67 ± 0.06           | 31.68 ± 0.03           | ...                    | 31.68 ± 0.029        |
| SN2007st        | CSP    | ...                    | 34.22 ± 0.09           | 34.55 ± 0.04           | ...                    | 34.46 ± 0.041        |
| SN2008af        | CfA    | ...                    | 35.98 ± 0.19           | 35.90 ± 0.24           | 35.96 ± 0.18           | 35.94 ± 0.124        |
| SN2008ar        | CSP    | 35.30 ± 0.02           | 35.30 ± 0.04           | 35.17 ± 0.06           | ...                    | 35.22 ± 0.038        |
| SN2008bc        | CSP    | 34.07 ± 0.02           | 34.02 ± 0.04           | 33.95 ± 0.04           | ...                    | 34.00 ± 0.029        |
| SN2008bf        | CSP    | 34.96 ± 0.01           | 34.95 ± 0.01           | 35.07 ± 0.06           | ...                    | 35.03 ± 0.042        |
| SN2008C         | CSP    | 34.27 ± 0.08           | 34.24 ± 0.09           | 34.31 ± 0.06           | ...                    | 34.30 ± 0.053        |
| SN2008ff        | CSP    | 34.42 ± 0.03           | 34.52 ± 0.05           | 34.55 ± 0.03           | ...                    | 34.49 ± 0.024        |
| SN2008fr        | CSP    | 36.11 ± 0.06           | 36.23 ± 0.14           | ...                    | ...                    | 36.16 ± 0.067        |
| SN2008fw        | CSP    | 33.07 ± 0.11           | 33.06 ± 0.14           | 32.94 ± 0.12           | ...                    | 32.98 ± 0.091        |
| SN2008gb        | CfA    | ...                    | 35.98 ± 0.08           | 35.78 ± 0.09           | 35.83 ± 0.11           | 35.86 ± 0.053        |
| SN2008gg        | CSP    | 35.63 ± 0.05           | 35.60 ± 0.10           | 35.63 ± 0.07           | ...                    | 35.63 ± 0.051        |
| SN2008gl        | CSP    | 35.97 ± 0.02           | 35.70 ± 0.03           | 35.72 ± 0.05           | ...                    | 35.83 ± 0.033        |
| SN2008gp        | CSP    | 35.55 ± 0.02           | 35.50 ± 0.03           | 35.69 ± 0.06           | ...                    | 35.65 ± 0.038        |
| SN2008hj        | CSP    | 36.03 ± 0.05           | 36.02 ± 0.07           | 35.91 ± 0.07           | ...                    | 35.95 ± 0.049        |
| SN2008hm        | CfA    | ...                    | 34.59 ± 0.02           | 34.76 ± 0.06           | 34.57 ± 0.04           | 34.65 ± 0.027        |
| SN2008hs        | CfA    | ...                    | 34.86 ± 0.06           | 34.90 ± 0.06           | 34.82 ± 0.07           | 34.86 ± 0.035        |
| SN2008hv        | CSP    | 33.80 ± 0.02           | 33.78 ± 0.02           | 33.81 ± 0.04           | ...                    | 33.81 ± 0.026        |
| SN2008ia        | CSP    | 34.80 ± 0.02           | 34.72 ± 0.03           | 34.66 ± 0.03           | ...                    | 34.72 ± 0.022        |
| SN2009aa        | CSP    | 35.23 ± 0.03           | 35.27 ± 0.04           | 35.25 ± 0.03           | ...                    | 35.24 ± 0.026        |
| SN2009ab        | CSP    | 33.46 ± 0.02           | 33.51 ± 0.03           | 33.56 ± 0.03           | ...                    | 33.52 ± 0.023        |
| SN2009ad        | CSP    | 35.24 ± 0.01           | 35.21 ± 0.02           | 35.33 ± 0.04           | ...                    | 35.30 ± 0.025        |
| SN2009ag        | CSP    | 33.06 ± 0.00           | 33.11 ± 0.01           | 33.08 ± 0.01           | ...                    | 33.07 ± 0.005        |
| SN2009al        | CfA    | ...                    | 34.92 ± 0.05           | 34.84 ± 0.03           | ...                    | 34.87 ± 0.028        |
| SN2009an        | CfA    | ...                    | 33.46 ± 0.03           | 33.40 ± 0.03           | 33.51 ± 0.04           | 33.45 ± 0.017        |
| SN2009bv        | CfA    | ...                    | 36.03 ± 0.05           | 35.82 ± 0.05           | ...                    | 35.88 ± 0.040        |
| SN2009cz        | CSP    | 34.69 ± 0.05           | 34.68 ± 0.06           | 34.73 ± 0.04           | ...                    | 34.71 ± 0.037        |
| SN2009D         | CSP    | 34.90 ± 0.02           | 34.90 ± 0.01           | 34.90 ± 0.02           | ...                    | 34.90 ± 0.014        |
| SN2009kk        | CfA    | ...                    | 33.92 ± 0.05           | 34.04 ± 0.07           | ...                    | 34.01 ± 0.051        |
| SN2009kq        | CfA    | ...                    | 33.53 ± 0.09           | 33.65 ± 0.09           | 33.52 ± 0.06           | 33.57 ± 0.048        |
| SN2009Y         | CSP    | 32.97 ± 0.01           | 32.96 ± 0.02           | 32.97 ± 0.01           | ...                    | 32.97 ± 0.011        |
| SN2010ai        | CfA    | ...                    | 35.04 ± 0.03           | 34.87 ± 0.06           | ...                    | 34.92 ± 0.047        |
| SN2010dw        | CfA    | ...                    | 36.12 ± 0.04           | ...                    | ...                    | 36.12 ± 0.045        |
| SN2010iw        | CfA    | ...                    | 34.70 ± 0.04           | 34.63 ± 0.06           | 34.73 ± 0.10           | 34.68 ± 0.039        |
| SN2010kg        | CfA    | ...                    | 34.24 ± 0.04           | 34.14 ± 0.04           | 34.40 ± 0.11           | 34.25 ± 0.037        |
| SN2011ao        | CfA    | ...                    | 33.35 ± 0.03           | 33.29 ± 0.03           | 33.22 ± 0.06           | 33.29 ± 0.023        |
| SN2011B         | CfA    | ...                    | 31.62 ± 0.07           | 31.68 ± 0.05           | ...                    | 31.66 ± 0.038        |
| SN2011by        | CfA    | ...                    | 31.76 ± 0.06           | 31.74 ± 0.04           | ...                    | 31.75 ± 0.032        |
| SN2011df        | CfA    | ...                    | 33.97 ± 0.01           | 33.90 ± 0.03           | 33.83 ± 0.12           | 33.90 ± 0.037        |
| SNF20080514-002 | CfA    | ...                    | 35.03 ± 0.04           | 34.97 ± 0.07           | 35.23 ± 0.05           | 35.07 ± 0.032        |

NOTE— Distance moduli and their fitting uncertainties  $\hat{\sigma}_{\text{fit},s}$ , estimated from the different NIR bands, either alone (see columns 3-6) or combined (column 7), using the template method. Corresponding Hubble diagrams are shown in Figs. 6 and 12.



**Figure 6.**  $YJHK_s$  Hubble diagrams (top row) and residuals (bottom row) using the template method on the 89 supernovae that passed our cuts. The error bars plotted for each supernova correspond to the fitting uncertainties  $\hat{\sigma}_{\text{fit},s}$ . The left panel corresponds to the case when we determine a single distance modulus by combining any of the available 1, 2, 3, or 4  $YJHK_s$  distance moduli for a given SN Ia. The right panel shows the case when we require only SN Ia with  $J$  and  $H$ -band data, which allows us to include the majority of data from the CfA and CSP samples. Points are color coded by NIR photometric data source, including the CfA (red; Wood-Vasey et al. 2008; Friedman et al. 2015), the CSP (blue; Krisciunas et al. 2017), and other data from the literature (green; see Table 2 and references therein). Note that only the CSP used a  $Y$ -band filter. Table 8 summarizes the intrinsic scatter in the Hubble diagrams, while Table 12 reports the numerical values of the distance moduli from this figure.



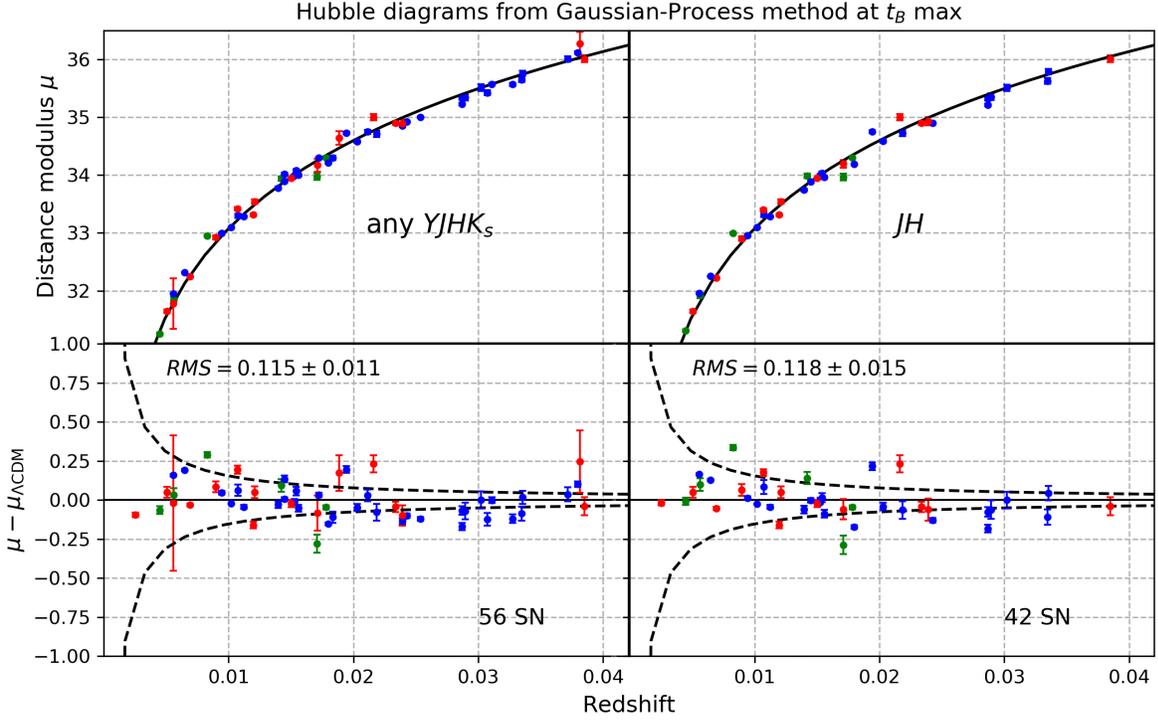
**Figure 7.** Similar to Fig. 6, but for  $YJHK_s$  Hubble diagrams (top row) and residuals (bottom row) using the GP method at NIR max. Again, Tables 7 and 8 summarize the intrinsic scatter in the Hubble diagrams, while Table 13 lists numerical values of the distance moduli from this figure.

continue to lay the foundation for ongoing and future, ground and space-based, supernova cosmology experiments, which seek to further test whether dark energy is best described by Einstein’s cosmological constant  $\Lambda$  or some other physical mechanism that varies on cosmic timescales.

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**Figure 8.** Similar to Fig. 7, but for  $YJHK_s$  Hubble diagrams (top row) and residuals (bottom row) using the Gaussian-process method at  $B$  max. Again, Tables 7 and 8 summarizes the intrinsic scatter in the Hubble diagram while Table 14 shows numerical values of the distance moduli from this figure.

## APPENDIX

### A. GAUSSIAN PROCESS REGRESSION

Given the dataset of observations in an absolute magnitude NIR LC,  $(\mathbf{M}, \mathbf{t})$  for a given supernova, we want to use this information to estimate the latent absolute magnitudes  $\mathcal{M}^*$  at a grid of phases  $\mathbf{t}^*$  described in Section 3.2. To do so, we define a Gaussian process with these data and variables.

To model the covariance  $\text{Cov}[\mathcal{M}^*, \mathcal{M}^{*\top}]$  we choose the squared exponential GP kernel that is defined as

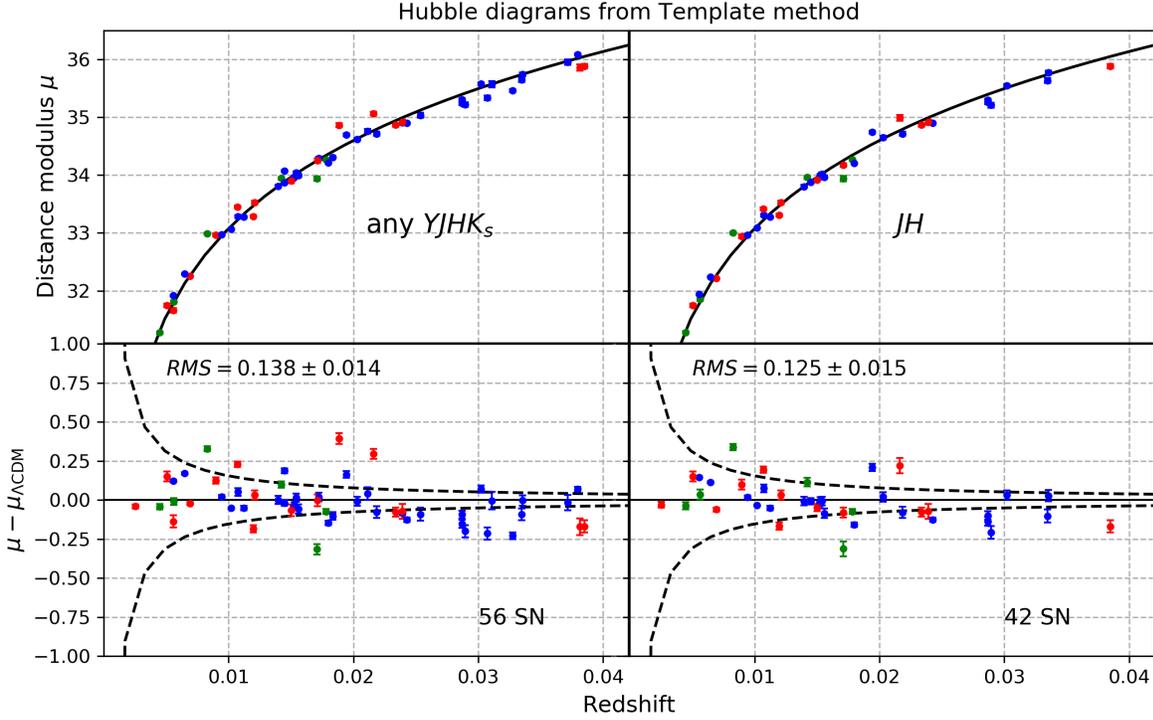
$$k(t_i, t_j) = \sigma_K^2 \exp \left[ -\frac{(t_i - t_j)^2}{2l^2} \right], \quad (\text{A1})$$

where  $\sigma_K$  and  $l$  are the GP kernel hyperparameters that we explain how to compute at the end of this section. We choose the GP kernel of Eq. (A1) because it is simple, produces smooth curves, and has the general properties we need to model the observed shapes of the NIR LCs: for two phases very close to each other,  $t_i \sim t_j$ , their covariance is close to 1, and for distant phases,  $t_i \ll t_j$ , then  $k(t_i, t_j) \sim 0$ , such that they are almost uncorrelated.

We also take into account the uncertainty associated with each datum  $M(t_i)$  in the variance  $\sigma_M^2 = \sigma_m^2 + \sigma_A^2 + \sigma_{K\text{corr}}^2 + \sigma_{\mu_{\text{pec}}}^2$  [see Eq. (7) for details], by defining the total covariance between two data points as

$$\text{Cov}[M_i, M_j] = k(t_i, t_j) + \delta_{ij}(\sigma_m^2 + \sigma_{K\text{corr}}^2) + \sigma_{\mu_{\text{pec}}}^2 + \sigma_A^2 \quad (\text{A2})$$

where  $\delta_{ij}$  is the Kronecker delta function, and we assume that the measurement and  $K$ -correction errors are independent between two different  $M_i$  and  $M_j$ , but that both the peculiar velocity-distance error and the Milky Way extinction error are not independent at different times because they are the same over the whole LC in a single filter for a given



**Figure 9.** Similar to Fig. 6 but applying the template method to exactly same 56 supernovae shown in Fig. 7 and 8. Again, Table 7 summarizes the intrinsic scatter in the Hubble diagram while Table 12 shows numerical values of the distance moduli from this figure.

supernova. In matrix notation we can write Eq. (A2) for all the data  $\mathbf{M}$  in a LC as

$$\text{Cov}[\mathbf{M}, \mathbf{M}^\top] = \mathbf{K}(\mathbf{t}, \mathbf{t}) + \mathbf{W} + (\sigma_{\mu_{\text{pec}}}^2 + \sigma_A^2) \mathbf{1} \cdot \mathbf{1}^\top, \quad (\text{A3})$$

where  $\mathbf{K}(\mathbf{t}, \mathbf{t})$  is a square matrix with elements given by Eq. (A1),  $\mathbf{W}$  is a diagonal matrix of dimension  $n \times n$  with elements given by

$$W_{ij} = \delta_{ij} (\sigma_m^2 + \sigma_{\text{Kcorr}}^2), \quad (\text{A4})$$

and  $\mathbf{1}$  is a vector of ones, so that the term  $(\sigma_{\mu_{\text{pec}}}^2 + \sigma_A^2) \mathbf{1} \cdot \mathbf{1}^\top$ , is a square matrix of dimension  $n \times n$  with elements all equal to  $(\sigma_{\mu_{\text{pec}}}^2 + \sigma_A^2)$ .

Following the standard GP formalism (e.g., Chapter 2 of Rasmussen & Williams (2006)), we first write the joint distribution of the observed absolute magnitudes,  $\mathbf{M}$ , and latent absolute magnitudes,  $\mathcal{M}^*$ , with a constant prior mean as

$$\begin{bmatrix} \mathbf{M} \\ \mathcal{M}^* \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} a\mathbf{1} \\ a\mathbf{1}^* \end{bmatrix}, \begin{bmatrix} \mathbf{K}(\mathbf{t}, \mathbf{t}) + \mathbf{W} + (\sigma_{\mu_{\text{pec}}}^2 + \sigma_A^2) \mathbf{1} \cdot \mathbf{1}^\top & \mathbf{K}(\mathbf{t}, \mathbf{t}^*) \\ \mathbf{K}(\mathbf{t}^*, \mathbf{t}) & \mathbf{K}(\mathbf{t}^*, \mathbf{t}^*) \end{bmatrix} \right) \quad (\text{A5})$$

where  $\mathbf{1}$  and  $\mathbf{1}^*$  are vectors of ones and of dimensions  $n$  and  $n^*$  respectively, and  $a$  is a scalar that we assign the value of  $-17.5$ ,  $-17$ ,  $-18$  and  $-18$  mag for the  $Y$ ,  $J$ ,  $H$  and  $K_s$  bands, respectively. We assume these values of  $a$  just for computational convenience in the GP fitting, and verified that the final templates are insensitive to these choices over a wide range of values for  $a$ . The matrices  $\mathbf{K}(\mathbf{t}, \mathbf{t})$ ,  $\mathbf{K}(\mathbf{t}^*, \mathbf{t})$ ,  $\mathbf{K}(\mathbf{t}, \mathbf{t}^*)$ , and  $\mathbf{K}(\mathbf{t}^*, \mathbf{t}^*)$ , are of dimensions  $n \times n$ ,  $n^* \times n$ ,  $n \times n^*$  and  $n^* \times n^*$  respectively, with elements defined by Eq. (A1).

The conditional distribution of  $\mathcal{M}^*$  given  $\mathbf{t}, \mathbf{t}^*$  and  $\mathbf{M}$ , can be written as

$$\mathcal{M}^* | \mathbf{t}, \mathbf{t}^*, \mathbf{M} \sim \mathcal{N}(\boldsymbol{\mu}^{\text{post}}, \boldsymbol{\Sigma}^{\text{post}}) \quad (\text{A6})$$



**Table 13.** SN Ia  $YJHK_s$  Distance Moduli from Gaussian-process Method at NIR max

| SN name         | Source | $\hat{\mu}^Y$<br>(mag) | $\hat{\mu}^J$<br>(mag) | $\hat{\mu}^H$<br>(mag) | $\hat{\mu}^K$<br>(mag) | $\hat{\mu}$<br>(mag) |
|-----------------|--------|------------------------|------------------------|------------------------|------------------------|----------------------|
| SN1998bu        | CfA    | ...                    | 30.09 ± 0.03           | 30.03 ± 0.03           | 29.87 ± 0.02           | 29.99 ± 0.018        |
| SN1999ee        | CSP    | ...                    | 33.30 ± 0.02           | 33.32 ± 0.02           | ...                    | 33.32 ± 0.016        |
| SN1999ek        | Others | ...                    | 34.24 ± 0.02           | 34.28 ± 0.02           | ...                    | 34.27 ± 0.014        |
| SN2000ca        | CSP    | ...                    | 34.94 ± 0.03           | ...                    | ...                    | 34.94 ± 0.034        |
| SN2000E         | Others | ...                    | 31.72 ± 0.02           | 31.93 ± 0.03           | 31.71 ± 0.04           | 31.84 ± 0.022        |
| SN2001ba        | CSP    | ...                    | 35.55 ± 0.02           | 35.55 ± 0.03           | 35.56 ± 0.08           | 35.55 ± 0.031        |
| SN2001bt        | Others | ...                    | 33.91 ± 0.03           | 33.96 ± 0.02           | 33.83 ± 0.03           | 33.91 ± 0.014        |
| SN2001cz        | Others | ...                    | 33.90 ± 0.05           | 34.03 ± 0.11           | 33.87 ± 0.11           | 33.97 ± 0.072        |
| SN2001el        | Others | ...                    | 31.45 ± 0.02           | 31.34 ± 0.03           | 31.17 ± 0.04           | 31.30 ± 0.022        |
| SN2002dj        | Others | ...                    | 33.01 ± 0.03           | 33.03 ± 0.02           | 32.89 ± 0.03           | 32.98 ± 0.017        |
| SN2004eo        | CSP    | 33.97 ± 0.01           | 33.97 ± 0.01           | 33.97 ± 0.01           | ...                    | 33.97 ± 0.011        |
| SN2004ey        | CSP    | 34.03 ± 0.01           | 33.96 ± 0.01           | 34.06 ± 0.04           | ...                    | 34.08 ± 0.038        |
| SN2005cf        | CfA    | ...                    | 32.21 ± 0.06           | 32.17 ± 0.03           | 32.25 ± 0.03           | 32.20 ± 0.020        |
| SN2005el        | CSP    | 33.88 ± 0.01           | 33.95 ± 0.01           | 33.97 ± 0.01           | ...                    | 33.96 ± 0.013        |
| SN2005iq        | CSP    | 35.76 ± 0.03           | 35.75 ± 0.04           | 35.74 ± 0.07           | ...                    | 35.74 ± 0.069        |
| SN2005kc        | CSP    | 33.92 ± 0.01           | 33.90 ± 0.01           | 33.89 ± 0.02           | ...                    | 33.90 ± 0.015        |
| SN2005ki        | CSP    | 34.58 ± 0.01           | 34.16 ± 0.01           | 34.55 ± 0.03           | ...                    | 34.55 ± 0.027        |
| SN2006ax        | CSP    | 34.17 ± 0.01           | 34.16 ± 0.01           | 34.22 ± 0.02           | ...                    | 34.22 ± 0.019        |
| SN2006bh        | CSP    | 33.31 ± 0.01           | 33.30 ± 0.01           | 33.31 ± 0.01           | ...                    | 33.31 ± 0.012        |
| SN2006bt        | CSP    | ...                    | 35.51 ± 0.04           | ...                    | ...                    | 35.51 ± 0.041        |
| SN2006D         | CfA    | ...                    | 32.85 ± 0.02           | 32.92 ± 0.04           | 32.86 ± 0.06           | 32.90 ± 0.029        |
| SN2006kf        | CSP    | 34.73 ± 0.01           | 34.77 ± 0.01           | 34.75 ± 0.04           | ...                    | 34.74 ± 0.039        |
| SN2006lf        | CfA    | ...                    | 33.40 ± 0.03           | 33.42 ± 0.05           | ...                    | 33.42 ± 0.043        |
| SN2007A         | CSP    | 34.26 ± 0.01           | 34.17 ± 0.02           | ...                    | ...                    | 34.27 ± 0.012        |
| SN2007af        | CSP    | 31.92 ± 0.01           | 32.00 ± 0.00           | 31.92 ± 0.01           | ...                    | 31.90 ± 0.007        |
| SN2007ai        | CSP    | 35.67 ± 0.02           | ...                    | ...                    | ...                    | 35.67 ± 0.025        |
| SN2007as        | CSP    | ...                    | ...                    | 34.37 ± 0.04           | ...                    | 34.37 ± 0.044        |
| SN2007bc        | CSP    | ...                    | 34.84 ± 0.02           | ...                    | ...                    | 34.84 ± 0.019        |
| SN2007bd        | CSP    | 35.54 ± 0.02           | 35.56 ± 0.04           | ...                    | ...                    | 35.54 ± 0.023        |
| SN2007ca        | CSP    | ...                    | ...                    | 34.01 ± 0.03           | ...                    | 34.01 ± 0.025        |
| SN2007jg        | CSP    | 36.09 ± 0.02           | 36.11 ± 0.02           | ...                    | ...                    | 36.09 ± 0.021        |
| SN2007le        | CSP    | 32.33 ± 0.01           | 32.23 ± 0.01           | 32.26 ± 0.01           | ...                    | 32.28 ± 0.006        |
| SN2008ar        | CSP    | 35.33 ± 0.01           | 35.34 ± 0.03           | 35.28 ± 0.05           | ...                    | 35.28 ± 0.054        |
| SN2008bc        | CSP    | 34.02 ± 0.01           | 33.98 ± 0.02           | 34.01 ± 0.03           | ...                    | 34.02 ± 0.031        |
| SN2008bf        | CSP    | 34.98 ± 0.01           | 34.87 ± 0.01           | ...                    | ...                    | 34.98 ± 0.010        |
| SN2008gb        | CfA    | ...                    | ...                    | ...                    | 36.31 ± 0.21           | 36.31 ± 0.211        |
| SN2008gp        | CSP    | 35.63 ± 0.03           | 35.53 ± 0.05           | 35.75 ± 0.07           | ...                    | 35.76 ± 0.074        |
| SN2008hj        | CSP    | 35.98 ± 0.02           | 36.08 ± 0.04           | ...                    | ...                    | 35.97 ± 0.026        |
| SN2008hs        | CfA    | ...                    | 34.86 ± 0.05           | ...                    | 34.67 ± 0.11           | 34.77 ± 0.059        |
| SN2008hv        | CSP    | 33.77 ± 0.00           | 33.72 ± 0.01           | 33.72 ± 0.02           | ...                    | 33.73 ± 0.022        |
| SN2009aa        | CSP    | 35.31 ± 0.01           | 35.28 ± 0.01           | 35.28 ± 0.02           | ...                    | 35.28 ± 0.018        |
| SN2009ad        | CSP    | 35.27 ± 0.01           | 35.27 ± 0.02           | 35.37 ± 0.05           | ...                    | 35.37 ± 0.048        |
| SN2009ag        | CSP    | 33.20 ± 0.01           | 33.20 ± 0.01           | 33.12 ± 0.01           | ...                    | 33.12 ± 0.009        |
| SN2009al        | CfA    | ...                    | 35.07 ± 0.02           | 34.88 ± 0.03           | ...                    | 34.91 ± 0.029        |
| SN2009an        | CfA    | ...                    | 33.40 ± 0.02           | 33.42 ± 0.03           | 33.39 ± 0.03           | 33.41 ± 0.019        |
| SN2009bv        | CfA    | ...                    | 36.07 ± 0.04           | 36.01 ± 0.13           | ...                    | 36.02 ± 0.108        |
| SN2009cz        | CSP    | 34.77 ± 0.01           | 34.72 ± 0.02           | 34.77 ± 0.04           | ...                    | 34.78 ± 0.038        |
| SN2009D         | CSP    | 34.98 ± 0.01           | 34.91 ± 0.01           | 34.95 ± 0.02           | ...                    | 34.97 ± 0.018        |
| SN2009Y         | CSP    | 33.02 ± 0.01           | 32.98 ± 0.01           | 32.97 ± 0.02           | ...                    | 32.98 ± 0.018        |
| SN2010ai        | CfA    | ...                    | 35.05 ± 0.05           | 34.99 ± 0.10           | 34.85 ± 0.09           | 34.95 ± 0.063        |
| SN2010kg        | CfA    | ...                    | 34.22 ± 0.03           | 34.16 ± 0.03           | 34.33 ± 0.14           | 34.22 ± 0.047        |
| SN2011ao        | CfA    | ...                    | 33.33 ± 0.05           | 33.23 ± 0.03           | ...                    | 33.24 ± 0.023        |
| SN2011B         | CfA    | ...                    | 31.66 ± 0.16           | ...                    | ...                    | 31.66 ± 0.156        |
| SN2011by        | CfA    | ...                    | 31.71 ± 0.05           | 31.68 ± 0.03           | ...                    | 31.69 ± 0.026        |
| SN2011df        | CfA    | ...                    | 33.94 ± 0.03           | 33.89 ± 0.05           | ...                    | 33.90 ± 0.041        |
| SNf20080514-002 | CfA    | ...                    | 35.03 ± 0.14           | 34.92 ± 0.12           | ...                    | 34.94 ± 0.101        |

NOTE— Distance moduli and their fitting uncertainties  $\hat{\sigma}_{\text{fit},s}$ , estimated from the different NIR bands, either alone (see columns 3-6) or combined (column 7) using the Gaussian-process method at NIR max. The Hubble diagrams from these data are shown in Figs. 7 and 13.

where the posterior mean  $\boldsymbol{\mu}^{\text{post}}$  and posterior covariance  $\boldsymbol{\Sigma}^{\text{post}}$  are given as

$$\boldsymbol{\mu}^{\text{post}} \equiv \mathbb{E}[\mathcal{M}^* | \mathbf{t}, \mathbf{t}^*, \mathbf{M}] = \mathbf{a}\mathbf{1} + \mathbf{K}(\mathbf{t}^*, \mathbf{t}) \left[ \mathbf{K}(\mathbf{t}, \mathbf{t}) + \mathbf{W} + (\sigma_{\mu_{\text{pec}}}^2 + \sigma_A^2) \mathbf{1} \cdot \mathbf{1}^\top \right]^{-1} (\mathbf{M} - \mathbf{a}\mathbf{1}) \quad (\text{A7})$$

$$\boldsymbol{\Sigma}^{\text{post}} \equiv \text{Cov} \left[ \mathcal{M}^*, \mathcal{M}^{*\top} | \mathbf{t}, \mathbf{t}^*, \mathbf{M} \right] = \mathbf{K}(\mathbf{t}^*, \mathbf{t}^*) - \mathbf{K}(\mathbf{t}^*, \mathbf{t}) \left[ \mathbf{K}(\mathbf{t}, \mathbf{t}) + \mathbf{W} + (\sigma_{\mu_{\text{pec}}}^2 + \sigma_A^2) \mathbf{1} \cdot \mathbf{1}^\top \right]^{-1} \mathbf{K}(\mathbf{t}, \mathbf{t}^*) \quad (\text{A8})$$

The final values we obtain from the GP regression are the vector  $\boldsymbol{\mu}^{\text{post}}$  and the matrix  $\boldsymbol{\Sigma}^{\text{post}}$ , that we estimate using Eqs. (A7) and (A8) respectively.

The coefficients  $\sigma_K$  and  $l$  in Eq. (A1) are called the *hyperparameters* of the GP kernel that we determine by assuming that the LCs for all the SN in a given NIR band are independent of each other, and that the GP hyperparameters describe the *population* of the SN LCs in a given band rather than each individual LC. With these assumptions, we can write the global marginal likelihood distribution

$$p(\{\mathbf{M}\}_s | \{\mathbf{t}\}_s, \sigma_K, l) = \prod_{s=1}^{N_T} \mathcal{N}(\mathbf{M}_s | \mathbf{a}\mathbf{1}_s, \mathbf{K}_s(\mathbf{t}_s, \mathbf{t}_s) + \mathbf{W}_s(\mathbf{t}_s, \mathbf{t}_s) + (\sigma_{\mu_{\text{pec},s}}^2 + \sigma_{A,s}^2) \mathbf{1}_s \cdot \mathbf{1}_s^\top), \quad (\text{A9})$$

where the subindex  $s$  refers to quantities for supernova  $s$ ,  $N_T$  is the number of SN Ia used to construct the normalized LC template in a given NIR band, and “ $\{\}_s$ ” means the collection of values from all the  $N_T$  SN Ia. To compute the

**Table 14.** SN Ia  $YJHK_s$  Distance Moduli from Gaussian-process Method at  $B$  max

| SN name         | Source | $\hat{\mu}^Y$<br>(mag) | $\hat{\mu}^J$<br>(mag) | $\hat{\mu}^H$<br>(mag) | $\hat{\mu}^K$<br>(mag) | $\hat{\mu}$<br>(mag) |
|-----------------|--------|------------------------|------------------------|------------------------|------------------------|----------------------|
| SN1998bu        | CfA    | ...                    | 30.11 ± 0.02           | 30.04 ± 0.02           | 29.90 ± 0.02           | 29.98 ± 0.014        |
| SN1999ee        | CSP    | ...                    | 33.31 ± 0.02           | 33.28 ± 0.02           | ...                    | 33.29 ± 0.013        |
| SN1999ek        | Others | ...                    | 34.26 ± 0.02           | 34.31 ± 0.02           | ...                    | 34.30 ± 0.013        |
| SN2000ca        | CSP    | ...                    | 34.86 ± 0.02           | ...                    | ...                    | 34.86 ± 0.019        |
| SN2000E         | Others | ...                    | 31.77 ± 0.04           | 31.96 ± 0.05           | 31.68 ± 0.06           | 31.85 ± 0.041        |
| SN2001ba        | CSP    | ...                    | 35.43 ± 0.09           | 35.54 ± 0.07           | 35.47 ± 0.09           | 35.51 ± 0.055        |
| SN2001bt        | Others | ...                    | 33.86 ± 0.06           | 34.02 ± 0.05           | 33.81 ± 0.06           | 33.94 ± 0.039        |
| SN2001cz        | Others | ...                    | 33.81 ± 0.05           | 34.01 ± 0.07           | 33.90 ± 0.08           | 33.97 ± 0.058        |
| SN2001el        | Others | ...                    | 31.36 ± 0.02           | 31.30 ± 0.03           | 31.20 ± 0.04           | 31.25 ± 0.023        |
| SN2002dj        | Others | ...                    | 32.97 ± 0.02           | 33.00 ± 0.02           | 32.87 ± 0.03           | 32.95 ± 0.018        |
| SN2004eo        | CSP    | 33.96 ± 0.01           | 33.94 ± 0.01           | 33.99 ± 0.01           | ...                    | 33.98 ± 0.009        |
| SN2004ey        | CSP    | 34.06 ± 0.02           | 33.92 ± 0.02           | 34.07 ± 0.04           | ...                    | 34.08 ± 0.027        |
| SN2005cf        | CfA    | ...                    | 32.23 ± 0.01           | 32.22 ± 0.01           | 32.28 ± 0.01           | 32.24 ± 0.009        |
| SN2005el        | CSP    | 33.96 ± 0.01           | 33.97 ± 0.02           | 34.01 ± 0.02           | ...                    | 33.99 ± 0.017        |
| SN2005jq        | CSP    | 35.70 ± 0.02           | 35.76 ± 0.03           | 35.80 ± 0.06           | ...                    | 35.76 ± 0.042        |
| SN2005kc        | CSP    | 33.90 ± 0.01           | 33.88 ± 0.02           | 33.89 ± 0.02           | ...                    | 33.90 ± 0.013        |
| SN2005ki        | CSP    | 34.59 ± 0.02           | 34.61 ± 0.03           | 34.58 ± 0.03           | ...                    | 34.58 ± 0.022        |
| SN2006ax        | CSP    | 34.20 ± 0.01           | 34.13 ± 0.01           | 34.21 ± 0.01           | ...                    | 34.21 ± 0.010        |
| SN2006bh        | CSP    | 33.28 ± 0.03           | 33.34 ± 0.09           | 33.31 ± 0.05           | ...                    | 33.30 ± 0.037        |
| SN2006bt        | CSP    | ...                    | 35.43 ± 0.04           | ...                    | ...                    | 35.43 ± 0.041        |
| SN2006D         | CfA    | ...                    | 32.89 ± 0.03           | 32.91 ± 0.05           | 32.94 ± 0.05           | 32.92 ± 0.035        |
| SN2006kf        | CSP    | 34.68 ± 0.01           | 34.73 ± 0.01           | 34.76 ± 0.03           | ...                    | 34.73 ± 0.022        |
| SN2006lf        | CfA    | ...                    | 33.45 ± 0.03           | 33.57 ± 0.05           | ...                    | 33.54 ± 0.038        |
| SN2007A         | CSP    | 34.34 ± 0.01           | 34.19 ± 0.04           | ...                    | ...                    | 34.30 ± 0.015        |
| SN2007af        | CSP    | 31.95 ± 0.01           | 31.96 ± 0.01           | 31.96 ± 0.01           | ...                    | 31.95 ± 0.007        |
| SN2007ai        | CSP    | 35.57 ± 0.03           | ...                    | ...                    | ...                    | 35.57 ± 0.027        |
| SN2007as        | CSP    | ...                    | ...                    | 34.30 ± 0.04           | ...                    | 34.30 ± 0.036        |
| SN2007bc        | CSP    | ...                    | 34.75 ± 0.04           | ...                    | ...                    | 34.75 ± 0.037        |
| SN2007bd        | CSP    | 35.56 ± 0.02           | 35.60 ± 0.04           | ...                    | ...                    | 35.57 ± 0.020        |
| SN2007ca        | CSP    | ...                    | ...                    | 34.02 ± 0.02           | ...                    | 34.02 ± 0.021        |
| SN2007jg        | CSP    | 36.08 ± 0.02           | 36.24 ± 0.03           | ...                    | ...                    | 36.12 ± 0.018        |
| SN2007le        | CSP    | 32.41 ± 0.01           | 32.24 ± 0.01           | 32.26 ± 0.01           | ...                    | 32.32 ± 0.005        |
| SN2008ar        | CSP    | 35.37 ± 0.03           | 35.41 ± 0.05           | 35.34 ± 0.08           | ...                    | 35.35 ± 0.053        |
| SN2008bc        | CSP    | 34.14 ± 0.02           | 34.08 ± 0.03           | 33.93 ± 0.03           | ...                    | 34.00 ± 0.023        |
| SN2008bf        | CSP    | 35.01 ± 0.02           | 34.98 ± 0.02           | ...                    | ...                    | 35.00 ± 0.012        |
| SN2008gb        | CfA    | ...                    | ...                    | ...                    | 36.28 ± 0.20           | 36.28 ± 0.200        |
| SN2008gp        | CSP    | 35.59 ± 0.01           | 35.50 ± 0.03           | 35.67 ± 0.06           | ...                    | 35.65 ± 0.044        |
| SN2008hj        | CSP    | 35.99 ± 0.05           | 36.05 ± 0.10           | ...                    | ...                    | 36.01 ± 0.044        |
| SN2008hs        | CfA    | ...                    | 34.96 ± 0.09           | ...                    | 34.60 ± 0.13           | 34.64 ± 0.115        |
| SN2008hv        | CSP    | 33.88 ± 0.01           | 33.83 ± 0.02           | 33.72 ± 0.03           | ...                    | 33.78 ± 0.022        |
| SN2009aa        | CSP    | 35.22 ± 0.02           | 35.18 ± 0.02           | 35.22 ± 0.03           | ...                    | 35.23 ± 0.023        |
| SN2009ad        | CSP    | 35.28 ± 0.01           | 35.24 ± 0.02           | 35.35 ± 0.04           | ...                    | 35.33 ± 0.027        |
| SN2009ag        | CSP    | 33.11 ± 0.00           | 33.12 ± 0.00           | 33.09 ± 0.00           | ...                    | 33.10 ± 0.003        |
| SN2009al        | CfA    | ...                    | 35.02 ± 0.04           | 34.86 ± 0.04           | ...                    | 34.90 ± 0.033        |
| SN2009an        | CfA    | ...                    | 33.48 ± 0.02           | 33.37 ± 0.03           | 33.49 ± 0.05           | 33.42 ± 0.026        |
| SN2009bv        | CfA    | ...                    | 35.94 ± 0.08           | 36.03 ± 0.07           | ...                    | 36.01 ± 0.058        |
| SN2009cz        | CSP    | 34.72 ± 0.08           | 34.77 ± 0.12           | 34.72 ± 0.06           | ...                    | 34.71 ± 0.054        |
| SN2009D         | CSP    | 34.96 ± 0.02           | 34.90 ± 0.01           | 34.90 ± 0.02           | ...                    | 34.92 ± 0.015        |
| SN2009Y         | CSP    | 33.01 ± 0.01           | 32.91 ± 0.02           | 32.98 ± 0.01           | ...                    | 32.99 ± 0.011        |
| SN2010ai        | CfA    | ...                    | 35.02 ± 0.03           | 34.91 ± 0.09           | 34.89 ± 0.08           | 34.89 ± 0.065        |
| SN2010kg        | CfA    | ...                    | 34.42 ± 0.08           | 34.13 ± 0.08           | 34.26 ± 0.25           | 34.17 ± 0.113        |
| SN2011ao        | CfA    | ...                    | 33.35 ± 0.03           | 33.30 ± 0.03           | ...                    | 33.31 ± 0.022        |
| SN2011B         | CfA    | ...                    | 31.78 ± 0.43           | ...                    | ...                    | 31.78 ± 0.434        |
| SN2011by        | CfA    | ...                    | 31.67 ± 0.06           | 31.64 ± 0.04           | ...                    | 31.65 ± 0.034        |
| SN2011df        | CfA    | ...                    | 33.99 ± 0.01           | 33.93 ± 0.03           | ...                    | 33.94 ± 0.022        |
| SNf20080514-002 | CfA    | ...                    | 35.03 ± 0.04           | 34.99 ± 0.07           | ...                    | 35.00 ± 0.054        |

NOTE— Same as Table 13 but using the Gaussian-process method at  $B$  max. The Hubble diagrams from these data are shown in Figs. 8 and 14.

MLE values for  $(\sigma_K, l)$ , we minimize the negative of the logarithm of Eq. (A9), obtaining the values shown in Table 16.

### A.1. Normalization of the GP light curves

In Section 3.2.1, we explained that we are primarily interested in the *shape* of the light curves. For this reason, after determining the posterior light curve described by  $(\boldsymbol{\mu}^{\text{post}}, \boldsymbol{\Sigma}^{\text{post}})$ , we *normalize* the LC using  $t_{B\text{max}}$  as the reference time where the light curve will have a value of zero.

First, for computational convenience, we rewrite the linear transformation of Eq. (9) as the matrix operation

$$\mathbf{L} = \mathbf{A}\boldsymbol{\mathcal{M}}^* \quad (\text{A10})$$

where  $\mathbf{A}$  is a  $n^* \times n^*$  square matrix defined as  $\mathbf{A} \equiv \mathbf{I} - \mathbf{V}_k$ , where  $\mathbf{I}$  is the identity matrix, and  $\mathbf{V}_k$  is a matrix containing only 1s in the  $k$ th column and zeros everywhere else, assuming that the  $k$ th element of  $\mathbf{t}^*$  correspond to phase  $t_k^* = t_{B\text{max}}$ .

We compute the mean of the normalized LC as,  $\boldsymbol{\mu}^L = \mathbb{E}[\mathbf{L}|\mathcal{D}] = \mathbf{A} \mathbb{E}[\boldsymbol{\mathcal{M}}^*|\mathcal{D}] = \mathbf{A}\boldsymbol{\mu}^{\text{post}}$ , where  $\mathcal{D} \equiv (\mathbf{t}, \mathbf{t}^*, \mathbf{M})$  is the conditional data in Eq (A6). And the covariance is given by

$$\boldsymbol{\Sigma}^L = \mathbf{A}\boldsymbol{\Sigma}^{\text{post}}\mathbf{A}^\top. \quad (\text{A11})$$

From these expressions at  $t_k^* = t_{B\text{max}}$ , the posterior mean and variance of the normalized LCs are both identically zero:

$$\mathbb{E}[L_k|\mathcal{D}] = 0, \quad \text{Var}[L_k, L_k|\mathcal{D}] = 0, \quad (\text{A12})$$

**Table 15.** SN Ia distance moduli from the optical *BVR* bands

| SN name         | Source | SALT2<br>(mag) | SNooPy<br>(mag) |
|-----------------|--------|----------------|-----------------|
| SN1998bu        | CfA    | 30.214 ± 0.052 | 30.562 ± 0.009  |
| SN2005cf        | CfA    | 32.327 ± 0.056 | 32.355 ± 0.017  |
| SN2006D         | CfA    | 32.923 ± 0.060 | 32.877 ± 0.015  |
| SN2006lf        | CfA    | 33.153 ± 0.196 | 33.254 ± 0.013  |
| SN2008gb        | CfA    | 36.062 ± 0.079 | 35.920 ± 0.042  |
| SN2008hs        | CfA    | 34.721 ± 0.072 | 34.649 ± 0.036  |
| SN2009al        | CfA    | 34.916 ± 0.064 | 35.065 ± 0.045  |
| SN2009an        | CfA    | 33.236 ± 0.059 | 33.225 ± 0.013  |
| SN2009bv        | CfA    | 36.292 ± 0.064 | 36.204 ± 0.036  |
| SN2010ai        | CfA    | 35.051 ± 0.062 | 34.895 ± 0.029  |
| SN2010kg        | CfA    | 33.814 ± 0.062 | 33.999 ± 0.018  |
| SN2011ao        | CfA    | 33.130 ± 0.057 | 33.359 ± 0.056  |
| SN2011B         | CfA    | 31.585 ± 0.055 | 31.678 ± 0.014  |
| SN2011by        | CfA    | 31.898 ± 0.052 | 31.953 ± 0.012  |
| SN2011df        | CfA    | 33.909 ± 0.060 | 33.999 ± 0.022  |
| SN1999ee        | CSP    | 33.138 ± 0.048 | 33.454 ± 0.010  |
| SN2000ca        | CSP    | 34.933 ± 0.058 | 34.910 ± 0.014  |
| SN2001ba        | CSP    | 35.607 ± 0.100 | 35.609 ± 0.020  |
| SN2004eo        | CSP    | 33.884 ± 0.054 | 33.823 ± 0.017  |
| SN2004ey        | CSP    | 34.096 ± 0.057 | 33.991 ± 0.006  |
| SN2005el        | CSP    | 34.070 ± 0.057 | 33.853 ± 0.015  |
| SN2005iq        | CSP    | 35.903 ± 0.057 | 35.771 ± 0.011  |
| SN2005kc        | CSP    | 33.936 ± 0.057 | 34.038 ± 0.013  |
| SN2005ki        | CSP    | 34.589 ± 0.057 | 34.479 ± 0.007  |
| SN2006ax        | CSP    | 34.329 ± 0.053 | 34.284 ± 0.010  |
| SN2006bh        | CSP    | 33.292 ± 0.055 | 33.178 ± 0.011  |
| SN2006bt        | CSP    | 35.672 ± 0.052 | 35.429 ± 0.060  |
| SN2006kf        | CSP    | 34.686 ± 0.077 | 34.602 ± 0.017  |
| SN2007A         | CSP    | 34.447 ± 0.057 | 34.436 ± 0.025  |
| SN2007af        | CSP    | 31.993 ± 0.053 | 32.041 ± 0.015  |
| SN2007ai        | CSP    | 35.695 ± 0.081 | 35.742 ± 0.023  |
| SN2007as        | CSP    | 34.848 ± 0.049 | 34.273 ± 0.015  |
| SN2007bc        | CSP    | 34.588 ± 0.057 | 34.645 ± 0.020  |
| SN2007bd        | CSP    | 35.554 ± 0.059 | 35.467 ± 0.027  |
| SN2007ca        | CSP    | 34.343 ± 0.053 | 34.462 ± 0.012  |
| SN2007jg        | CSP    | 36.189 ± 0.062 | 36.014 ± 0.039  |
| SN2007le        | CSP    | 32.117 ± 0.053 | 32.330 ± 0.014  |
| SN2008ar        | CSP    | 35.274 ± 0.057 | 35.279 ± 0.010  |
| SN2008bc        | CSP    | 34.074 ± 0.047 | 33.990 ± 0.010  |
| SN2008bf        | CSP    | 35.083 ± 0.054 | 34.979 ± 0.010  |
| SN2008gp        | CSP    | 35.659 ± 0.055 | 35.584 ± 0.012  |
| SN2008hj        | CSP    | 35.943 ± 0.056 | 35.905 ± 0.013  |
| SN2008hv        | CSP    | 33.715 ± 0.054 | 33.691 ± 0.011  |
| SN2009aa        | CSP    | 35.318 ± 0.056 | 35.284 ± 0.007  |
| SN2009ad        | CSP    | 35.328 ± 0.056 | 35.289 ± 0.011  |
| SN2009ag        | CSP    | 33.219 ± 0.064 | 33.261 ± 0.025  |
| SN2009cz        | CSP    | 34.797 ± 0.057 | 34.803 ± 0.017  |
| SN2009D         | CSP    | 34.965 ± 0.054 | 34.930 ± 0.015  |
| SN2009Y         | CSP    | 32.700 ± 0.053 | 32.650 ± 0.034  |
| SN1999ek        | Others | 34.380 ± 0.092 | 34.517 ± 0.017  |
| SN2000E         | Others | 31.655 ± 0.060 | 31.756 ± 0.028  |
| SN2001bt        | Others | 33.691 ± 0.057 | 33.836 ± 0.016  |
| SN2001cz        | Others | 33.964 ± 0.056 | 34.026 ± 0.011  |
| SN2001el        | Others | 31.457 ± 0.055 | 31.479 ± 0.017  |
| SN2002dj        | Others | 32.810 ± 0.056 | 32.899 ± 0.014  |
| SNf20080514-002 | Others | 35.102 ± 0.061 | 34.933 ± 0.017  |

NOTE— Distance moduli estimated by fitting the optical *BVR* bands for the same 56 supernovae listed in Table 13 using the SALT2 and SNooPy fitters. The Hubble diagrams for these 2 cases are shown in Fig. 10.

**Table 16.** Values of the GP hyperparameters

| band                 | $l$  | $\sigma_K$ |
|----------------------|------|------------|
| <i>Y</i>             | 7.90 | 0.70       |
| <i>J</i>             | 7.02 | 0.95       |
| <i>H</i>             | 9.81 | 0.75       |
| <i>K<sub>s</sub></i> | 8.19 | 0.55       |

which is required for self-consistency with the definition of the normalized LC.

## B. HIERARCHICAL BAYESIAN MODEL

Using Bayes' theorem, applying the product rule for probability, and assuming conditional independence of the means of the normalized LCs,  $\mu_s^L$ 's, with respect to the population mean and variance  $(\theta, \sigma_\theta^2)$ , we can write the joint posterior distribution in our hierarchical model as

$$p(\{\eta_s\}, \theta, \sigma_\theta | \{\mu_s^L, \sigma_{\eta,s}\}) \propto p(\theta, \sigma_\theta) \times p(\{\eta_s\} | \theta, \sigma_\theta) \times p(\{\mu_s^L\} | \{\eta_s\}, \{\sigma_{\eta,s}\}). \quad (\text{B13})$$

Inserting Eqs. (10) and (11) into Eq. (B13), we obtain,

$$p(\{\eta_s\}, \theta, \sigma_\theta | \{\mu_s^L, \sigma_{\eta,s}\}) \propto p(\theta, \sigma_\theta) \times \prod_{s=1}^{N_T^*} \mathcal{N}(\eta_s | \theta, \sigma_\theta^2) \times \prod_{s=1}^{N_T^*} \mathcal{N}(\mu_s^L | \eta_s, \sigma_{\eta,s}^2). \quad (\text{B14})$$

where  $N_T^*$  is the number of supernovae for which we have determined the best fitting function at phase  $t^*$ . Note that since each LC has a *different* number of photometric data points over *different* phase ranges, this implies that  $N_T^*$  is different for each phase  $t^*$ .

For computation convenience, following Gelman et al. (2014), we decompose the joint posterior distribution using the product rule as

$$p(\{\eta_s\}, \theta, \sigma_\theta | \{\mu_s^L, \sigma_{\eta,s}\}) \propto p(\{\eta_s\} | \theta, \sigma_\theta, \{\mu_s^L, \sigma_{\eta,s}\}) \times p(\theta | \sigma_\theta, \{\mu_s^L, \sigma_{\eta,s}\}) \times p(\sigma_\theta | \{\mu_s^L, \sigma_{\eta,s}\}), \quad (\text{B15})$$

where the first factor to the right of the proportionality sign of Eq. (B15) can be written for the supernova  $s$  as

$$p(\eta_s | \theta, \sigma_\theta, \mu_s^L, \sigma_{\eta,s}) = \mathcal{N}(\eta_s | \rho_s, R_s), \quad (\text{B16})$$

where

$$\rho_s \equiv \frac{\mu_s^L / \sigma_{\eta,s}^2 + \theta / \sigma_\theta^2}{1 / \sigma_{\eta,s}^2 + 1 / \sigma_\theta^2}, \quad (\text{B17})$$

and

$$R_s \equiv \frac{1}{1 / \sigma_{\eta,s}^2 + 1 / \sigma_\theta^2}. \quad (\text{B18})$$

The middle factor to the right of the proportionality sign of Eq. (B15) can be written as

$$p(\theta | \sigma_\theta, \{\mu_s^L, \sigma_{\eta,s}\}) = \mathcal{N}(\theta | \hat{\theta}, R), \quad (\text{B19})$$

where

$$\hat{\theta} \equiv \frac{\sum_{s=1}^{N_T^*} \mu_s^L (\sigma_{\eta,s}^2 + \sigma_\theta^2)^{-1}}{\sum_{s=1}^{N_{SN}} (\sigma_{\eta,s}^2 + \sigma_\theta^2)^{-1}}, \quad (\text{B20})$$

and

$$R^{-1} \equiv \sum_{s=1}^{N_T^*} \frac{1}{\sigma_{\eta,s}^2 + \sigma_\theta^2}. \quad (\text{B21})$$

Finally, the last term to the right of the proportionality sign can be written as

$$p(\sigma_\theta | \{\mu_s^L, \sigma_{\eta,s}\}) \propto R^{1/2} \prod_{s=1}^{N_T^*} (\sigma_{\eta,s}^2 + \sigma_\theta^2)^{-1/2} \exp\left(\frac{-(\mu_s^L - \hat{\theta})^2}{2(\sigma_{\eta,s}^2 + \sigma_\theta^2)}\right), \quad (\text{B22})$$

where we are assuming a uniform prior distribution  $p(\sigma_\theta) \propto 1$ .

We use Eq. (B15) combined with Eqs. (B16)-(B22) to simultaneously determine the posterior best estimates of  $(\{\eta_s\}, \theta, \sigma_\theta)$  at phase  $t^*$ , given the data  $\{\mu_s^L, \sigma_{\eta,s}\}$ , following the computational procedure described in Appendix C.3, subsection ‘‘Marginal and conditional simulation for the normal model’’, of Gelman et al. (2014). We use the R code presented there to build our R code to make the computations described in this work.

### C. RMS, WEIGHTED RMS, AND THE INTRINSIC SCATTER

We use the RMS to quantify the scatter in the Hubble residuals because it is simple and straightforward to compute and compare with the Hubble residuals reported by other authors. The definition we use is

$$\text{RMS} = \sqrt{N_{\text{SN}}^{-1} \left( \sum_{s=1}^{N_{\text{SN}}} \Delta\mu_s^2 \right)}, \quad (\text{C23})$$

where  $N_{\text{SN}}$  is the total number of SN Ia in the Hubble diagram. We compute the uncertainty on RMS using bootstrap resampling.

To weight the root mean square (RMS) by the uncertainties in each SN distance modulus estimate in each NIR band, we compute the inverse-variance weighted root mean square (wRMS) of the residuals as

$$\text{wRMS} = \sqrt{\left( \sum_{s=1}^{N_{\text{SN}}} w_s \right)^{-1} \sum_{s=1}^{N_{\text{SN}}} w_s \Delta\mu_s^2}, \quad (\text{C24})$$

where  $w_s \equiv 1/(\hat{\sigma}_{\text{fit},s}^2 + \hat{\sigma}_{\text{int}}^2 + \sigma_{\mu_{\text{pec},s}}^2)$  and  $\Delta\mu_s$  is defined in Eq. (21). We also compute the uncertainty on wRMS using bootstrap resampling.

We determine the *intrinsic scatter*,  $\sigma_{\text{int}}$ , in the Hubble residual following the procedure described in Eqs. (B.6)-(B.7) in Appendix B of Blondin et al. (2011). This dispersion tries to quantify the scatter due to intrinsic differences in the NIR SN Ia absolute magnitudes only and *not* due to the peculiar-velocity uncertainty of each SN. The intrinsic scatter corresponds to the remaining dispersion observed in the Hubble-diagram residuals *after* accounting for the uncertainty in distance modulus due to the peculiar-velocity uncertainty,  $\sigma_{\mu_{\text{pec},s}}^2$ , and the photometric errors  $\{\hat{\sigma}_{\text{fit},s}\}$ . When comparing our notation to Eqs. (B.6)-(B.7) of Blondin et al. (2011), note that where we use  $\sigma_{\text{fit},s}$ ,  $\sigma_{\text{int}}$  and  $\sigma_{\mu_{\text{pec},s}}$ , Blondin et al. (2011) instead uses the notation  $\sigma_{m,s}$ ,  $\sigma_{\text{pred}}$ , and  $\sigma_{\text{pec},s}$ , respectively.

### D. COVARIANCE MATRIX $C_\mu$ OF HUBBLE RESIDUALS

In this section we provide the numerical values for different cases of the covariance matrix  $C_\mu$ .

For the template method, we find the following values of the sample covariance matrix  $C_\mu$  for the  $YJH$  bands:

$$C_\mu = \begin{pmatrix} 0.0227 & 0.0192 & 0.0167 \\ 0.0192 & 0.0246 & 0.0201 \\ 0.0167 & 0.0201 & 0.0211 \end{pmatrix}, \quad (\text{D25})$$

and for the  $JHK_s$  bands:

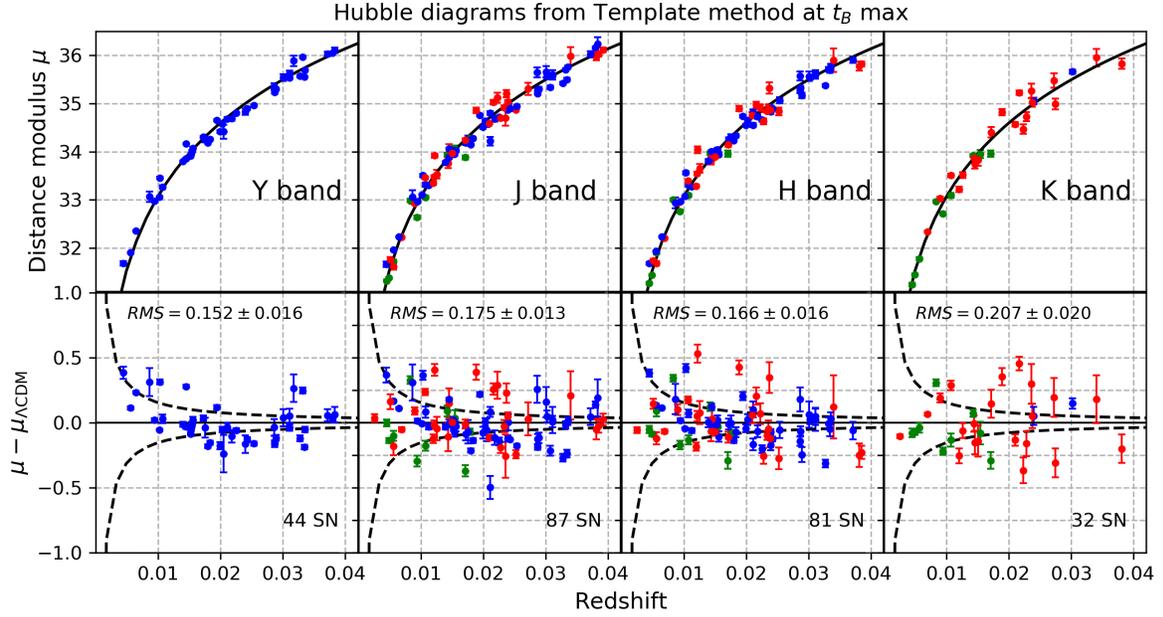
$$C_\mu = \begin{pmatrix} 0.0356 & 0.0276 & 0.0202 \\ 0.0276 & 0.0317 & 0.0237 \\ 0.0202 & 0.0237 & 0.0426 \end{pmatrix}. \quad (\text{D26})$$

For the GP method, we find the following values for the sample covariance matrix for the  $YJH$  bands:

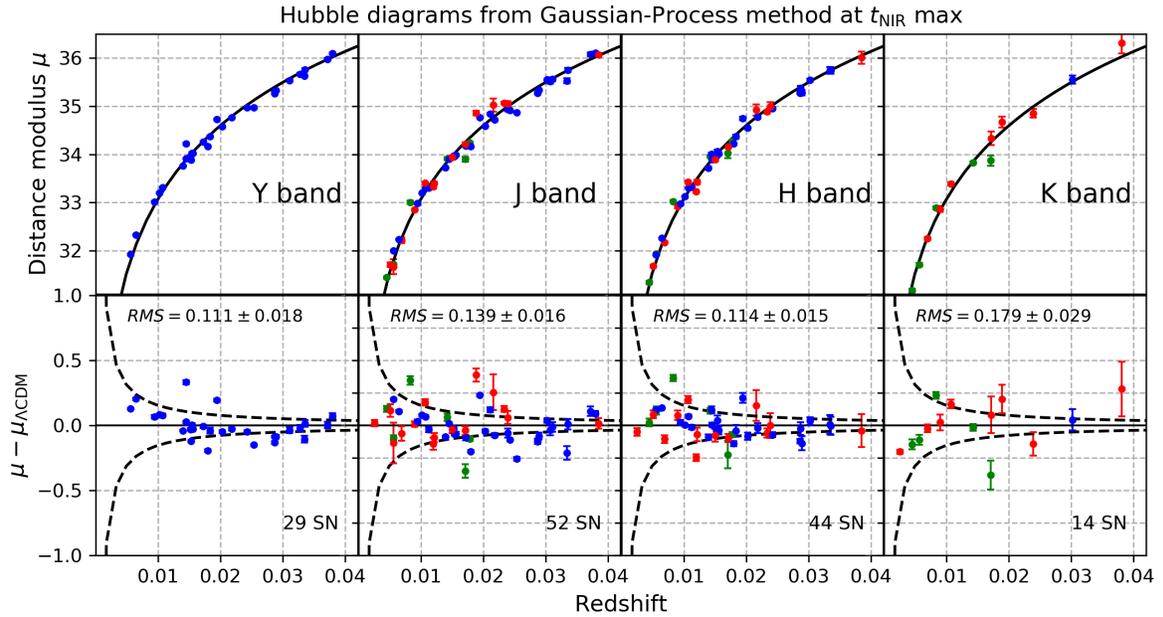
$$C_\mu = \begin{pmatrix} 0.0109 & 0.0110 & 0.0080 \\ 0.0110 & 0.0133 & 0.0084 \\ 0.0080 & 0.0084 & 0.0080 \end{pmatrix}, \quad (\text{D27})$$

and for the  $JHK_s$  bands:

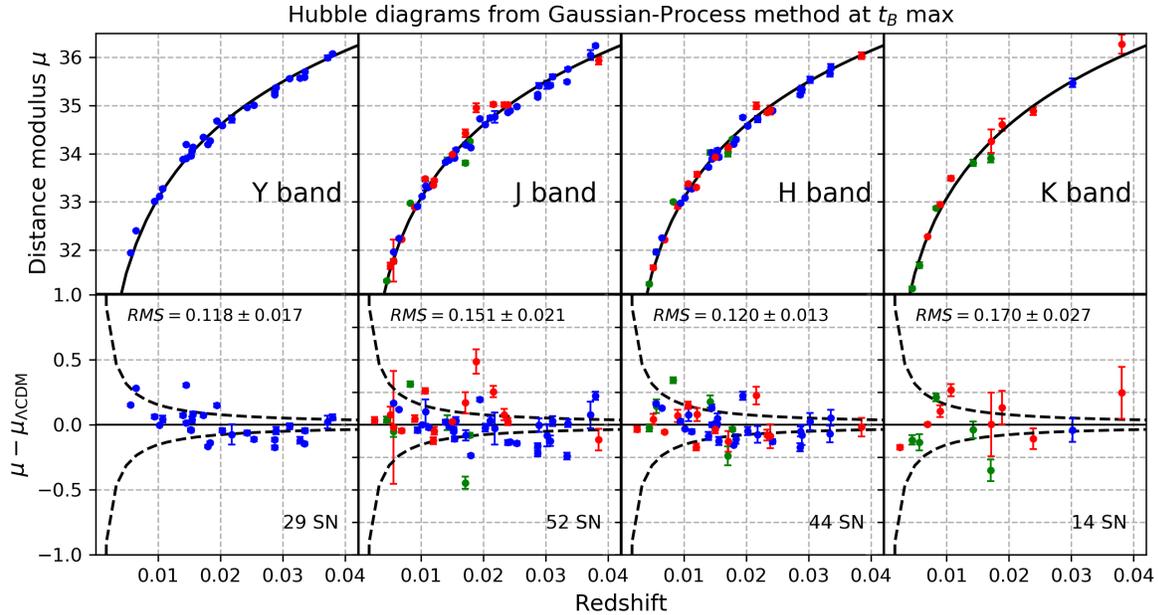
$$C_\mu = \begin{pmatrix} 0.0279 & 0.0217 & 0.0213 \\ 0.0217 & 0.0238 & 0.0192 \\ 0.0213 & 0.0192 & 0.0283 \end{pmatrix}. \quad (\text{D28})$$



**Figure 12.** Individual  $YJHK_s$  Hubble diagrams (top row) and residuals (bottom row) using the template method. Points are color coded by NIR photometric data source, including the CfA (red; Wood-Vasey et al. 2008; Friedman et al. 2015), the CSP (blue; Krisciunas et al. 2017), and other data from the literature (green; see Table 2). Note that only the CSP used a  $Y$ -band filter. In Table 12, we report the numerical values of the distance moduli shown in this figure. Table 8 shows the intrinsic scatter in the Hubble diagram.



**Figure 13.** Individual  $YJHK_s$  Hubble diagrams (top row) and residuals (bottom row) using the Gaussian-process method at NIR max. See the caption of Fig. 12. In Table 13, we report the numerical values of the distance moduli shown in this figure.



**Figure 14.** Individual  $YJHK_s$  Hubble diagrams (top row) and residuals (bottom row) using the Gaussian-process method at  $B$  max. See the caption of Fig. 12. In Table 14, we report the numerical values of the distance moduli shown in this figure.

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