Topics in Macro Finance

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This thesis is dedicated to my family; my lovely wife, beautiful son, beloved parents, brothers and my uncle (Inaam-ul-Haque) and aunt (Feroza Razzaq). I would like to take this opportunity to thank them deeply for supporting me through this long and often lonely journey. I would also like to express my deep appreciation to Prof. S.E. Satchell for his sustained guidance and strong support over the years.

DECLARATION

This dissertation is the result of my own work and therefore does not include any work which is the outcome of a collaborative effort, except that which is clearly declared below and specified in the text. It is not substantially the same as any that I have submitted, or is being concurrently submitted for a degree or diploma or other qualification at the University of Cambridge or any other university or similar institution, except as declared and specified in the declaration. Furthermore, it does not exceed the prescribed word limit for the relevant Degree Committee (total word count: 54,373).

In terms of full disclosure, what follows details the exact authorship profile of the various chapters that have been submitted as part of this thesis:

Chapter 1: Authored by Salman Ahmed under Prof. S.E. Satchell's guidance.

Chapter 2: Lead authored by Salman Ahmed. Co-authors include Prof. S.E Satchell, Prof. John Knight and Dr. Nandini Srivastva.

Chapter 3: Authored by Salman Ahmed. Co-authors include Mirko Cardinale. An abridged version of the paper was published in the *Journal of Asset Management* (2005).[1]

Chapter 4: Authored by Salman Ahmed under Prof. S.E. Satchell's guidance.

– Salman Ahmed

[1] See "Does inflation matter for equity returns?", *Journal of Asset Management*, Vol 6, No.4. December 2005, pp 259-273. 17 citations have been recorded as of 30th June 2016.

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INTRODUCTION

The Great Recession of 2008-2009 has forced both researchers and practitioners to re-assess the dynamics of risky-asset returns, volatility and investor behaviour under uncertainty. The impact of events such as the sharp fall in risky asset prices, and near collapse of the Western banking system triggered by the Lehman bankruptcy in September 2008, as well as rising unemployment in a number of advanced and emerging economies still continues to reverberate both in market-investor behaviour, as well as the general thrust of macro policy-making.

Before these events, it was common practice for both researchers and practitioners to use what can be considered as relatively small sample periods in calibrating return and risk drivers of risky assets such as equities. However, a sharp fall in the trend global economic growth and an increase in policy activism, particularly concerning monetary policy, witnessed after the 2008-2009 Great Recession is clearly starting to influence thinking around the dynamics of risky asset returns, volatility and investor behaviour.[2] In addition, direct central bank interventions in government and corporate bond markets as well as an increase in the incidence of broad-based negative nominal interest rates in a number of key government bond markets reflect the expectations of a sustained period of low economic growth/inflation[3] going forward.[4] This situation is unprecedented by historical standards.

Within academic research on the topic, the work of Shiller (2006) has crucially drawn attention to the importance of using very long-sample data-sets in assessing the nature of risky asset dynamics, especially when it comes to developing a firmer handle on the shape and form of the underlying drivers.[5] For instance, despite extensive research on the properties of asset market return volatility over the last 30 years, a study of the relationship between macro volatility and financial asset returns variability remains relatively unexplored. As Engel and Rangel (2008) note, within recent years the main focus in volatility research remains the construction of numerous time series models, while events such as the Great Recession and Eurozone debt crisis clearly show–at least intuitively–how gyrations in macro state variables can manifest themselves in risky asset return and volatility. As economic or financial crises tend to occur relatively infrequently, there is a value in using long-term data-sets to capture a variety of regimes that can improve and better understand model calibration. Indeed, usage of long-term data sample also helps in weakening the simultaneity bias, which may arise

[2] World economic growth averaged at 4.2% p.a. over the 1997/2007 period, compared to 3.2% p.a. over the 2008/16 period, based on IMF World Economic Outlook data and projections.
[3] IMF World Economic Outlook – April 2016 (Global Growth: Too Slow for too Long).
[4] See for example "Global Negative Yielding Bond Pile Nears $10 trillion", *Bloomberg News*, 6 July 2016.
[5] "Irrational Exuberance" – May 2006

when using contemporaneous variables (an issue encountered in Chapter 1: "Macro Drivers of Conditional Equity Volatility").

For both researchers and practitioners, a long data-set driven assessment is not only important from a purely empirical perspective, but is also useful when it comes to choosing the most appropriate risk model or volatility forecasting method. This is especially true in an asymmetrically dependent world.

In addition, when it comes to investor behaviour under uncertainty, the rise of behavioural economics first popularised by Kahneman and Tversky (1972) has, in recent years, shown an increased emphasis on the flaws in the classical expected utility theory framework. This framework has reigned for several decades as the dominant normative and descriptive model of decision-making under uncertainty. According to Machina (1982), this is mainly due to the simplicity and normative appeal of its axioms, the familiarity of the ideas it employs and the elegance of its characterizations of various types of behaviour in terms of the of properties of the utility function it uses. However, it is now generally agreed that the theory does not provide an appropriate description of behaviour under uncertainty as a substantial body of evidence shows that decision-makers systematically violate its basic tenets (for instance, see Hey (1997) for a discussion on the major alternative theories of decision making under uncertainty). Indeed, one of the main weaknesses of the expected utility framework is the existence of heterogeneous investor types–both individuals and institutional–with different investment objectives, preferences and information signals and the related implications on asset market price in an equilibrium setting. Although, heterogeneity does not directly contradict expected utility theory (EUT), EUT does has problems when dealing with practitioner models. This distinction is even more important in the post-Great Recession era, where the tightening of regulations such as Basel III, Volcker rule and Dodd-Frank is driving an even stronger wedge between the objective functions of regulated and non-regulated investors.

OBJECTIVE AND SCOPE OF THESIS

In terms of the specific topics covered, the research documented in this thesis sets out to further our understanding of risky asset returns and volatility and investor behaviour under uncertainty from an investor, policy maker and more generally a practitioner's perspective.

In three of the four chapters, the macro drivers of both risky asset returns (the first moment) and volatility (the second moment) are studied and analysed in detail across different geographies and various time periods. The use of both long sample sets and relevant sub-sample periods allows for a more in-depth assessment of the nature and form of these drivers as well as their influence on risky asset return and volatility dynamics. The earliest data used in this research starts from the 18th century.

In the first chapter, entitled “Macro Drivers of Conditional Equity Market Volatility”, the focus is on the analysis of macro state variables, which are shown to have a strong influence on the behaviour of equity return volatility (extension of the work done by Chen et al (1986), who concentrate on the importance of economic variables that a link with the behaviour of equity returns). Also, as Engel and Rangel (2008) note, the main thrust of volatility research in recent years has been the construction of numerous time series models such as GARCH, stochastic volatility and numerous others. Despite clear intuitive links, the relationship between macro-state factors, such as inflation, business cycle and interest rates, and equity returns volatility has not been studied with the same degree of attention.

Specifically, this chapter extends the work of Schwert (1989a and 1989b) by adopting a detailed empirical framework to study the precise nature of the empirical connection between macro state variables and the variability of equity returns for four of the largest advanced economies in the world: the US, Japan, the UK and Germany. Using a long-term historical data-set, this chapter shows that broad transitions in conditional equity returns volatility can be directly linked to the conditional volatility of key macro factors. This applies even after taking into account the lagged equity return volatility, which are known at time t and captured using the GARCH formulation. Moreover, the study finds that the behaviour of conditional equity returns volatility, when assessed against the variability in macro environment, displayed very different characteristics during the Great Recession (2008/9) and Great Depression periods (the 1920/30s) respectively. The sharp increase in conditional equity volatility during the 2008/9 period was more or less in line with estimates derived from the macro-based model. On the other hand, conditional equity volatility significantly overshot the relevant macro-based estimates during the Great Depression period. This empirical result appears to strengthen the stance adopted by policy interventionists such as the former Federal Reserve Chairman Ben Bernanke and Bank of England governor Sir Mervyn King, who have both argued that the unprecedented easing of monetary policy undertaken by key central banks played a crucial role in stabilising the economic situation in advanced countries in the aftermath of the Lehman bankruptcy in September 2008.

Moreover, the study also provides a fair-value assessment framework for implied equity market volatility (as measured by the VIX and VDAX indices) which is based on the variability of macro state factors. This framework generates important implications for both long- and short-term investors and policy-makers, especially given the sharp disconnect between equity returns and macro volatility that was visible during the pre-Great Recession years. Indeed, this disconnect was evident in all four countries studied. It appears that factors such as an increased leverage in the household and financial sector, historically low global real rates and deregulation, which incentivized excess lending, may also have caused this disconnect to appear before the onset of the Great Recession[6]. In terms of various econometric issues encountered in setting-up the estimation framework, usage of long-term data set (which includes a number of years when the size of the equity market relative to the economy was quite small compared to current levels) helps to weaken the "endogeneity" bias, which may arise from using contemporaneous data. In addition, care has been taken to address the regressed regressors issue, using findings from Pagan (1984).

Chapter two assesses the relative forecasting of GARCH, Stochastic Volatility (SV) and EGARCH models to forecast volatility, in a world where the true model can be depicted by an EGARCH(1,2) formulation. Applied economists and practitioners are often uncertain as to which of the common volatility models is better to use, especially in the context of forecasting. Overall, given the central role of volatility calibration in option pricing/trading and risk management systems, assessing the forecasting quality of various volatility models forms a weighty area of research. Studies such as Poon & Granger (2003), provide a summary of ninety-three research papers which focus on the forecasting performance of various volatility models. The authors report that conclusions based on the comparison exercises carried out in the different studies depend on the nature of the asset class studied together with the exact forecasting evaluation metric(s) employed. All in all, as Poon et al (2003) note, given the complexity of the issues involved and the importance of the volatility measure, volatility forecasting continues to be a subject area that attracts rigorous research focus. In terms of the choice of the candidate set of models used in the chapter, studies such as Hansen et al (2005) show that the threshold for replacing GARCH(1,1) formulation as the widely deployed volatility forecasting model remains high as shown by its widespread usage in practitioner models such as the MSCI BARRA Global Equity Model and Bloomberg factor model in PORT.

To avoid problems of data dependence, in chapter two, we assume that we know the true model and use artificially generated data to assess the candidate models' forecasting abilities. This has the advantage of making volatility known from the point of view of the simulator. We may therefore avoid using variations of realised volatility which are difficult to calculate in cases where the data are generated by processes with discontinuous moves and other irregularities.

[6] For instance, see "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles and Financial Crisis, 1870-2008", Schularick & Taylor, *American Economic Review* (2012), Vol 10, pp 1029-1061.

Specifically, we assume that the true model is EGARCH(1,2) based on persuasive empirical work by Pagan and Schwert (1990). This also takes into account the importance of asymmetric dependence in financial data. We further extend their analysis through an up-to-date dataset. Their analysis was based on the US equity market. We also apply it to the US 10 year bond returns.

The difficulty with any simulation exercise such as this one is that, through a clever choice of the true model, we can tilt the simulation to favour our preferred method. We would argue that we have fixed the true model to be different from both alternative models, which the econometrician assumes are GARCH(1,1) and SV(1,1) and include EGARCH (1,2) in the candidate set as a reference. Both the true model and the SV(1,1) model are log volatility models which may confer an advantage to SV. The SV possesses two sources of noise whilst the GARCH has only one, which may also favour SV. However EGARCH(1,2) has only one noise, so it is entirely possible that this could help GARCH. Neither assumed model has the more complex asymmetric lag structure of the EGARCH(1,2).

The detailed study carried out, which is also augmented by careful analytical analysis, confirms the superiority of the SV model under the normal distribution assumption using a variety of forecasting performance assessment metrics (including, the Diebold-Mariano test) and model parameter values. However, using t-distributed shocks, the quality of forecasting performance varies and appear to be dependent on the value of β , which relates to the behaviour of the given volatility model when β is close to 1. Overall, the study shows that simple estimators which ignore asymmetric dependence in volatility will forecast satisfactorily depending on the particular circumstances related to the actual distribution of the error process.

Turning to chapter three, the relationship between equity returns and inflation dynamics is explored using long-term historical data for the US, the UK, Germany and Japan. The basic theoretical concept in this area of research is commonly attributed to Fisher (1930), who hypothesised that nominal financial returns reflects full information concerning the possible future values of inflation. This effect is known as the "Fisher effect" and is widely accepted. Specifically, the Fisher hypothesis states that "expected nominal risky asset returns move one for one with expected inflation, such that expected real returns are independent of expected inflation".

Although, this theoretical framework could in principle hold independently of the holding period, previous studies have reported different results depending on whether a shorter or longer time horizon was considered. Therefore, this chapter employs a two-step sequential empirical hypothesis testing process to explore the relationship between equity returns and inflation from the point of view of a pension fund investor, or indeed any investor which has long-term liabilities linked to consumer price inflation. Indeed, from a pension fund's perspective, if over-long horizons equity returns do adjust to inflation, then even with a lag, the short term dynamics become almost irrelevant unless there is a clear mismatch between the liability maturity and the length of the equity/inflation adjustment cycle. On the other hand, if the above hypothesis fails to hold, there is a strong case for understanding the short-term dynamics of the equity-inflation relationship and to look for the possible existence of durable patterns. These patterns could then be used empirically for forecasting and can be potentially exploited by pension funds using tactical overlay strategies.

As noted above, unlike many previous studies on this topic, such as Boudoukh & Richardson (1993) and Lothian & McCarthy (2001), both the long- and short-term dimensions of the connection between inflation and equity returns are studied coupled with the role played by economic growth dynamics. In terms of the top-level results, mixed support was found for the hypothesis of a stable long-term equilibrium relationship between inflation and equity returns, while the short-term analysis showed evidence of asymmetric behaviour of equity markets during various inflationary environments across the different countries studied. Indeed, on this point, the study also expands on the work of Hess & Lee (1999), who examine the relationship between equity returns and inflation while conditioning on the source of the inflation shock. More recently, studies such as Ciner (2014) and Austin et al (2015) provide additional insights in this area of research.

Overall, the key implication of these results is that short-term dynamics cannot be completely ignored in the belief that the stock market will generate enough nominal returns to offset inflation in the long-term. This implication is also backed by mixed results found using cointergation analysis (which checks for long-term one-for-one relationship between equity and consumer goods prices) and inter-country differences uncovered using VECM estimations.

Finally, in chapter four the study carried out theoretically illustrates how both heterogeneous expectations and the quality of information related to different type of investors with different investment objectives affect risky asset price equilibrium using a representative agent driven wealth maximisation framework. It is important to note that a number of simplifying assumptions have been made in this chapter in order to focus on the role played by the difference in the structure of the utility maximisation function in shaping investor behaviour under uncertainty.

Within the traditional asset pricing framework, arbitrageurs soak up the demand shocks, thus ensuring that asset prices remain at their “fundamental price". Theoretical work by Delong et al (1990) and Shleifer and Vishny (1997) have shown how perfect arbitrage can break down, thereby allowing demand shifts to affect risky asset prices.

Specifically, herding as a form of connected behaviour takes place when investors copy and follow other investors' decisions while superseding their own private information and beliefs, see Devenow and Welch (1996) and Avery and Zemsky (1998). The drivers of herding can emanate from different sources depending on investor types (these may be individual or institutional) and their respective objective functions.

In this chapter, the phenomenon of herding is also explored by focusing on the use of benchmarks, which is a predominant practice amongst institutional investors such as pension funds and insurance companies. This can be captured by using a multi-attribute utility function. In addition, the role of commonality of information signals received by different types of investors is also studied.

An important behavioural explanation behind the wide-spread use of peer group benchmarks or market-capitalization based reference points is referred to as "regret risk" (see Shefrin (1999)). Representatives of institutional investors such as pension funds may experience "regret" if they use an asset allocation policy which is different from others and thus opens up the possibility of extreme deviation from the established norm. Shefrin (2000) argues that in the real world investors are partly driven by their emotions and these emotions are reflected in the use of benchmarks.

More specifically, regret theory specifies a two-attribute utility function where the investor faces a trade-off between two attributes, both impacting perceived utility under a choice-based framework (see Loomes & Sugden (1982) for more details). Here, the payoff from an investor's decision is compared to a hypothetical alternative choice, whereby, *ex ante,* if realised wealth is lower or higher than the outcome of the alternative choice–i.e. hypothetical wealth is generated by a benchmark portfolio–then the investor may experience "regret" or "jubilation".

Turning to studies focussed on multi-attribute utility functions, it becomes clear that while multivariate generalisations of risk aversion have been extensively developed (see Karni (1979); Pratt (1988); Gollier and Pratt (1996)); studies such as those of Li and Ziemba (1989), Finkelshtain & Chalfant (1993) and Grant & Satchell (2015) have also developed models of portfolio choice using multi-variate utility functions.

Specifically, chapter four builds on the work of Wagner (2002), who explored portfolio selection under a pure benchmark-based setting. In this chapter, we use a multi-attribute utility framework for certain type of investors, such as for instance, pension funds and insurance companies, while expanding the investor-type universe to include individuals that are modeled as pursuing absolute wealth maximization. In addition, the analytical findings of this chapter also augment the work done by Kapur and Timmermann (2005), who analyzed the implications of using relative performance contracts, when it comes to delegated investment management, on equity risk premium pricing and herding behavior in an equilibrium setting.

CONTRIBUTIONS OF THE THESIS

This thesis seeks to contribute to the knowledge on macro drivers of both equity return and volatility, by deploying long sample data-sets for a number of key economies. Events such as the Great Recession of 2008-2009 and the Eurozone debt crisis of 2011/12 have shown the deep interlinkages between financial sector stability, macro state variables and asset market dynamics. These interconnections not only manifest themselves by shaping the nature of mean equity market returns but also have a quantifiable influence on equity return volatility dynamics, properties of which vary over different episodes of significant equity market dislocations (Chapter 1).

In addition, our contribution to the literature involves showing a mixed support for the Fisher hypothesis, which postulates that equities are a long-term hedge for inflation using a bi-variate setting. The study also identifies the importance of incorporating short-term dynamics in studying equity return behaviour during different inflationary regimes and extends the work of a number of previous studies (Chapter 3).

From an applied economics/practitioner's perspective, the thesis (Chapter 2) further contributes to the literature by showing the relative strength of Stochastic Volatility model formulation, when compared to the widely used GARCH framework for forecasting volatility in a world of asymmetric dependency (the EGARCH model is also included in the candidate set). The empirical exercises carried out are further supported by detailed statistical work which shed further light on the statistical properties of the various models studied. Indeed, using a range of forecast performance metrics, the superiority of the SV model's forecasting ability under a normal distribution assumption is confirmed especially when the sample set is very large. In the case of t-distribution, on the other hand, the supremacy of the SV model appears to be reliant on the value of β parameter.

Finally, the thesis (Chapter 4) seeks to build on the classical expected utility framework, when used in a representative agent wealth maximisation setting. This is achieved by incorporating heterogeneous agents with different objective functions, preferences and information signals. Indeed, the formulation shown in this thesis can be used to better understand the analytical drivers of "herding" or commonality in investment positions, an empirical observation which is often regarded as one of the key reasons behind the deterioration in secondary market liquidity witnessed in the period after the Great Recession (for instance, see IMF - Global Financial Stability Report, April 2015).

STRUCTURE OF THE THESIS

For the sake of completeness, the thesis is divided into the following chapters. Chapter one examines the macro drivers of conditional equity returns volatility in four of the largest developed economies in the world. Chapter two studies the relative abilities of GARCH, Stochastic Volatility (SV) and EGARCH models to forecast risky asset volatility in an asymmetrically dependent world. Chapter three carries out an in-depth large sample set based empirical exercise to understand the importance of inflation in shaping equity market returns in a number of key advanced economies within a Fisher framework. Finally, chapter four analytically examines the behaviour of heterogeneous investors and related implications on risky asset market price equilibrium.

CHAPTER 1 - MACRO DRIVERS OF CONDITIONAL EQUITY RETURNS VOLATILITY

Against the backdrop of increased levels of both risk and uncertainty surrounding future macro outcomes in a number of major economies, this chapter explores the empirical relationship between variability in key macro state variables and the volatility of equity returns. This empirical connection is studied using a long-term historical data-set (including data from what is now referred to as the Great Recession period) for the four largest developed economies in the world: the US, Japan, the UK and Germany.

The empirical results presented in this chapter show that broad transitions in conditional equity returns volatility can be directly linked with the conditional volatility of key macro variables. Not only does this relationship hold over different sample periods, but it is also visible across the four countries studied. In addition, this chapter also explores the macro drivers of implied volatility as measured by the VIX and VDAX indices.

More specifically, the purpose of this chapter is to empirically study the drivers of realized and implied equity market volatility (especially, for the interest of regulators who are focused on financial stability as a policy goal) using data from various countries and different sample periods. It is important to note that the study does not seek to improve the forecasting ability of commonly used time-series based models (such as GARCH/ARCH) but instead focuses on exploring the linkages between the variability of macro-state variables and equity return volatility – which is very much in the spirit of Chen, Roll and Ross (1986), who concentrate on identifying key economic state variables that exhibit influence on the behavior of mean equity returns.

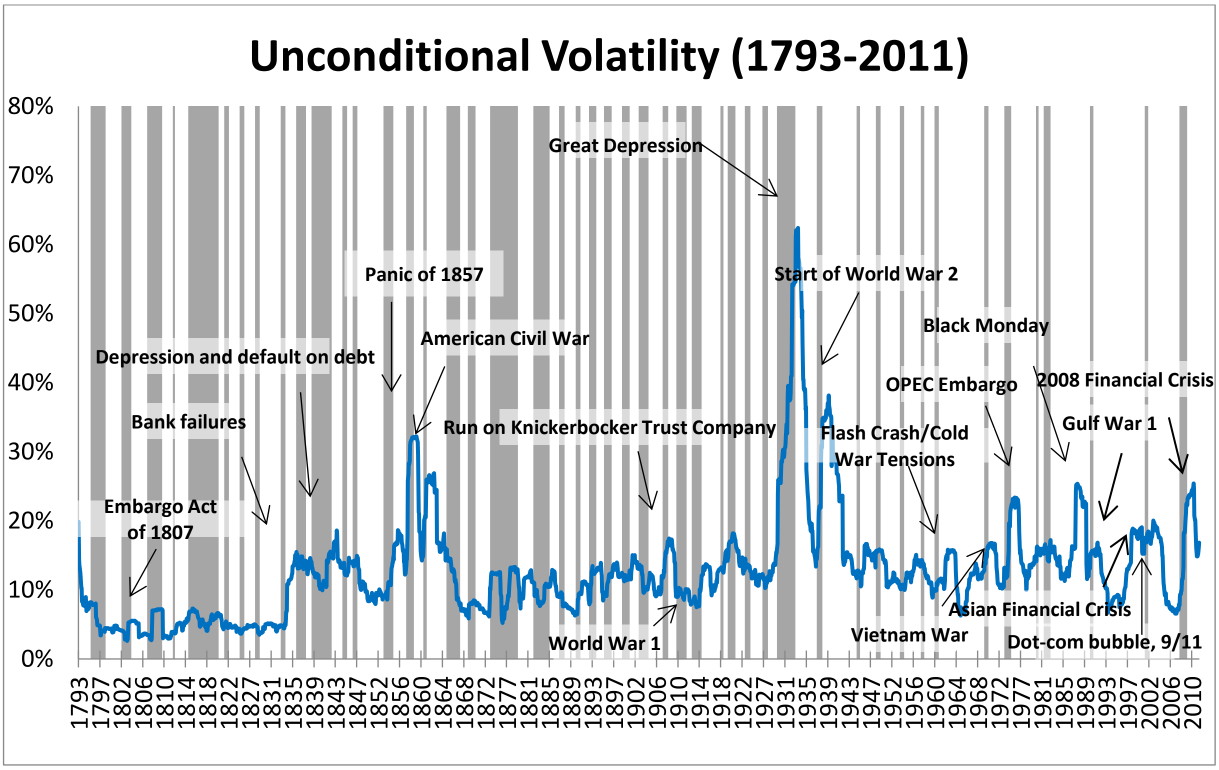
Overall, this chapter contributes to the existing literature by highlighting the important role played by macro volatility in shaping equity returns volatility and studies the differences/similarities among various episodes of significant equity market upheaval (such as the Great Depression and Great Recession years) once variability in key macro state variables has been taken into account.

1.1 INTRODUCTION

Despite extensive research that focuses on the properties of asset market return volatility over the last 30 years, the study of the relationship between macro volatility and financial asset returns variability remains relatively unexplored. As Engel and Rangel (2008) note, the main focus in volatility research in recent years has been the construction of numerous time series models (such as GARCH, stochastic volatility, etc.). However, and despite clearly intuitive links, the relationship between macro-state factors such as inflation, business cycle and interest rates, and equity return volatility has not been studied with the same degree of thoroughness.

Figure 1.1 plots the annualized volatility of US equity returns from 1793, as captured by the two-year rolling standard deviation of monthly equity returns. It reveals that the sharp increases in volatility of equity returns occurring from time-to-time can be linked to identifiable events. These events have important implications for macro outcomes as well, specifically economic growth and inflation.

Figure 1.1 Annualized Unconditional Volatility of US Equity Returns



Source: See Appendix 1.1 for details

In particular, over the last 100 years the average realised volatility of equity returns during recession periods has been 18% p.a., compared to 15.4% p.a. during non-recession periods, according to the National Bureau of Economic Research's recession markings.[7] The two clearest examples of this linkage between the macro environment and equity returns volatility are the 1920s Great Depression and the recent Global Recession period. Both periods witnessed a rise in equity volatility alongside sharp deterioration in economic fundamentals.

Broadly speaking, the onset of the now dubbed Great Recession in 2008, and the ensuing weak global economic recovery that started in 2009, and which then worsened on account of sovereign debt issues

[7] The difference between the two means is significant at 1% level of significance.

in the Eurozone's periphery, has increased the role of macro outcomes in shaping asset prices. This occurs at the level of both business cycle and inflation dynamics.

Looking back to late-2008, accommodative policy actions taken across the world, both on the fiscal and monetary fronts, played a key role in stemming systemic and financial contagion risks emanating in the aftermath of the Lehman Brothers bankruptcy. These policy interventions, which in some cases were genuinely unprecedented, not only helped determine macro outcomes (both actual and forward expectations), but were a major driver of change in asset prices such as equities, fixed income, FX and commodities observed during that period.

Moving forward, as the global economic recovery tapered off in 2010, key central banks once again started moving various monetary policy levers using unconventional tools in an effort to offset the slowdown in economic growth. Specifically, the Federal Reserve, the Bank of Japan, the Bank of England and the European Central Bank embarked on additional rounds of monetary policy easing in the shape of quantitative and credit easing measures, given near-zero short-term interest rates.

Indeed, in order to counter the weak post-recession recovery, the interventionist policy stance adopted by key central banks has added an important layer of sustained policy uncertainty and risk in the macro environment, a situation that has not been witnessed since the 1920s and 1930s.

In addition to this heightened central bank activism, the ongoing sovereign debt crisis in Southern European countries and the inability of EU authorities to credibly contain it has also introduced significant political, regulatory and therefore economic risk and uncertainty into the global macro environment during 2010/11. Indeed, since late-2009, when sovereign debt issues faced by Italy, Spain, Ireland, Portugal and Greece came to the fore, the situation has been exacerbated by the continued reluctance of core European countries (especially Germany) to provide these countries with unconditional support. This reluctance to provide a comprehensive solution to the crisis has led to a sharp rise in the debt servicing costs of these countries, which has in turn led to concerns around the stability of the entire global financial system, given strong linkages between the struggling sovereigns and the European financial sector.

Looking ahead *ex ante,* given the volatile expected policy path, both fiscal and monetary, followed by a number of advanced economies, as well as the heightened risk of a debt crisis in the European Union, the volatility of future macro outcomes (namely growth and inflation) has also risen. This assertion is further strengthened by the findings of Reinhart and Rogoff (2010) who, using 200 years of data for a number of countries, document a strong negative relationship between economic growth and public debt levels during periods when debt/GDP ratio is above the 90% threshold level. Recent data from the US, the UK, Japan and a number of European countries have public debt/GDP ratios that are higher than, or very close to, this threshold (for example, see IMF data on public debt statistics)[8].

Since Knight (1921) presented his seminal work in *Risk, Uncertainty and Profits*, the academic literature in this field has sought to distinguish between the concepts of risk and uncertainty. Knight made this distinction between risk—which he identifies as unknown outcomes whose probability of occurrence can be measured or at least modelled, and uncertainty—in which uncertain developments are very difficult to even articulate. This subtle distinction is an important one when modelling and studying the variability of both asset and macro-state factors. Moreover, given the extreme financial stresses, which at one time threatened to take down the entire global financial system, experienced in the aftermath of the Lehman bankruptcy, and again over different periods since late-2009 when the Greek debt issue came to attention, global asset markets have had to contend with both 'risk' and 'uncertainty'.

In this chapter, the focus of the empirical exercise is on the conditional volatility, as opposed to the unconditional volatility, of both equity returns and key macro-state variables. By construction, the conditional volatility framework is designed to detect periods of intense and concentrated volatility periods, which may be reflecting concerns about both uncertainty (i.e. regime change) and risk (i.e. different states of the world). This is in contrast with periods of high unconditional volatility, which may only be reflecting incidence of 'risk' rather than existence of 'uncertainty'.

To summarize, in this chapter, linkages between conditional volatility of equity returns and macro-state variability is empirically established using long-term historical data that includes the recent Great Recession period. In addition, the behaviour of market-implied equity volatility (as measured by the VIX and VDAX indices) [9] is connected to volatility/state of key macro variables.

Section 1.2 discusses the academic literature in this area of research. Section 1.3 discusses the theoretical underpinnings of the approach taken and shares the current thinking on the importance of various macro variables used in the empirical exercise carried out. In addition, both details of the estimation approach and results of the modelling exercise are laid-out. Then, the relationship between implied volatility and macro-state variability is presented in section 1.4. Finally, section 1.5 discusses the main conclusions of the study.

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[8] http://www.imf.org/en/Publications/WP/Issues/2016/12/31/A-Historical-Public-Debt-Database-24332

[9] VIX index is a weighted blend of prices for a range options on the S&P 500 index. Similarly, VDAX index is based on the DAX index and calculated by measuring square root of the implied variance across all options of a given time to expiration.

1.2 LITERATURE REVIEW

The study of linkages between macro factors and equity returns was first formalized in the work of Chen et al (1986), when they included a number of macro factors (such as industrial production, inflation, government bond yields etc) as economic state variables in their enhanced version of a multi-factor asset pricing model for equities. Indeed, several of these variables were found to be significant in explaining expected stock returns; most notably, industrial production, term structure of the yield curve and changes in inflation expectations. Their main conclusion was that stock returns are indeed exposed to economic variables and that they are priced in accordance with their exposures. This identification was achieved using CAPM-style linear multi-factor estimation techniques.

However, linkages between macro variables and equity return volatility were studied more than a decade earlier by Officer (1973). Using the 1897-1969 data-set, Officer empirically related the very high volatility of US equity returns during the 1930s to the variability of leverage and the volatility of industrial production. He showed that the variability of macro factors played an important role in explaining the sharp rise in stock market volatility during the Great Depression and its subsequent decline in the decades ahead. Using these empirical results, Officer cast doubt on the explanatory power of commonly assumed factors, such as the increased number of stocks in the broad index after the 1930s, thus creating more diversification, and the formation of Securities Exchange Commission (SEC) via the Securities Act in 1933. These factors were credited with the steady decline in stock market volatility after the Great Depression. Indeed, Officer's findings were backed by studies conducted by Black (1976) and Christie (1982), which showed how financial leverage can positively impact equity return volatility.

Schwert (1989a, 1989b) has contributed significantly to this area of research by directly relating changes in US equity market volatility to real macro volatility, in order to explain its time-varying nature. His work showed that both economic and equity return volatility were much higher in the 1929-1939 Great Depression, compared to the overall 1857-1987 sample period. Indeed, Schwert (1989a) also provided empirical evidence to show that many economic series are more volatile during periods of economic contraction and this applies to financial asset returns' volatility as well. Schwert ascribed this finding to the increase in operational leverage, which occurs during recessions. Similar to Officer's finding, Schwert (1989b) also found that financial leverage affects stock volatility; however, he showed that this effect only explains a small proportion of the changes in stock volatility seen over time. In addition, Schwert (1989b) found that, in terms of the direction of predictability, financial asset volatility helped predict future macro volatility. This finding supported the assertion by Fama (1990) that equity markets are forward-looking.

Moreover, Engle et al (2008) introduced the Spline-GARCH model in an attempt to link the high frequency financial data (specifically, equity returns) with low frequency macro data. They showed empirical evidence to support the positive effect of the long term volatility of macroeconomic fundamentals (such as GDP, interest rates and inflation) on the volatility of equity returns. They also found inflation rate volatility was relevant, but in this case the result was sensitive to the country set used in the research.

An alternative strand of research in this area focuses on the impact of economic news announcements on financial asset returns (both mean and volatility). In many cases, authors such as Almeida et al (1998), Anderson and Bollerslev (1998a), Andersen, Bollerslev, Diebold and Vega (2003), Balduzzi, Elton and Green (2001) and Fleming et al (1999) have used high frequency financial asset prices data (mainly government bond yields and foreign exchange rates) to study the impact of new economic news on financial asset price behaviour. More recent studies in this area include the work of Bauwens et al (2005), Conrad et al (2008) and Hanousek (2008). All the above reported research broadly documents a measurable impact of macro news announcements (mostly US) on the behaviour of various asset markets during relevant observation windows.

In equity markets, high frequency studies such as Pearce et al (1984) document the effect of unexpected economic news announcements on stock price movements using daily returns data. Using the 1977-1982 data sample, the authors found that unanticipated monetary policy announcements exerted a significant effect on equity prices and, with some degree of persistence, beyond the announcement day.

More recently, Flannery et al (2002) used 17 macro announcement data series from 1980 to 1996 to identify three nominal variables (CPI, PPI and a monetary aggregate) and three real variables (employment report, balance of trade and housing starts) as strong candidates for equity risk factors. Interestingly, using GARCH modelling methodology, the authors found that the real variables also exerted significant positive influence on conditional volatility of daily US equity returns. In addition, a study by Lahaye et al (2007) also analyzed and assessed the impact of macroeconomic announcements on the observed discontinuities (i.e. outsized moves in prices) in many assets including stock market index futures and highlighted the importance of US labor market data (especially, payrolls) in exerting a heavy influence on stock and bond future markets. Furthermore, Lee and Mykand (2006) also examined the relationship between macro announcements and jumps on individual equities and the S&P500 index returns with three months of high frequency data and found a strong role of macro announcements in explaining jumps in equity index return data. Finally, Beine et al (2007) studied the link between central bank interventions and jumps and found that interventions can cause rare but large discontinuities.

In the literature, study of the impact of US macro announcements hasn't been just limited to US asset markets only. For instance, Ruhl et al (2014) analyzed the effect of US macroeconomic announcements on European stock returns, return volatility and bid-ask spreads using intra-day data and found that certain announcements are important for European equity market and the direction of news is important.

1.3a MACRO DRIVERS OF EQUITY RETURNS VOLATILITY – THEORETICAL UNDERPINNINGS

Following Shiller (1981), it is useful to think about equity price, P_t^* as the discounted present value of actual future cash flows and P_t is its expectation based on information at time t:

$$\boldsymbol{P_t = E_t P_t^*} \qquad (1.1)$$

$$\boldsymbol{P_t^*} = \left(\sum_{i=0}^{\infty} \frac{\boldsymbol{C_{t+i}}}{[\boldsymbol{1+D_{t+i}}]^i} \right) \qquad (1.2)$$

where $\boldsymbol{C_{t+i}}$ and $1/[\boldsymbol{1} + \boldsymbol{D_{t+i}}]$ are the actual cash flows (earnings) and discount rate respectively, at time t+i. $\boldsymbol{E_t}$ is the standard conditional expectations operator, which denotes the equity holder's information set at time t.

At the top level, the present value of the equity depends on the expectation of both current and future earnings and the discount rate, which are in turn affected by the state of the business cycle and other relevant macro-state variables.

Theoretically, if the discount rate is assumed to be constant, then the variance of stock prices will be directly influenced by variance of earnings/cash flows: further assuming that cash flows are independent,

$$\textbf{i.e. var}(\boldsymbol{P_t}) = \left(\sum_{i=0}^{\infty} \boldsymbol{\sigma_{t+i}^2} \right) \qquad (1.3)$$

$$\boldsymbol{where\ \sigma_i^2 = var(C_i)}/(1+D_{t+i})^{2i} \qquad (1.4)$$ [10]

However, if the discount rate is allowed to vary as well, then the conditional variance of equity price is proportional to variance of both cash flows and the discount rate and is also a function of their co-variance.[11]

[10] Assuming variance of earnings is independent of time t and its growth rate is a stationary process (see Shiller (1981) for more details).

[11] See http://www.stat.cmu.edu/~hseltman/files/ratio.pdf for Taylor rule approximation of VAR ($\frac{X}{Y}$) that depends on both the variance and covariance of the two random variables X and Y.

Indeed, if macroeconomic variables provide information about the variability of expected cash flows and the discount factor, then under the set-up shown above, the volatility of key macro-state variables can certainly extern an influence on the volatility of equity returns as well.

1.3b MACRO DRIVERS OF EQUITY MARKET VOLATILITY – KEY MACRO-STATE VARIABLES

Guided by existing literature discussed in detail below, table 1.1 lists the four key variables which have been used in this chapter to capture transitions in the state of the macro economy, namely the state of the real business cycle, government bond market behavior and the general price level:

Table 1.1 Macro-state Variable Set

K_t	k_t
Consumer Price index	**π**
Industrial production index	**ip**
Unemployment Rate	**ue**
10yr govt bond yields	**y**

* k_t denotes the log difference of the relevant K_t series. i.e. (ln $(\frac{K_t}{K_{t-1}})$)

Inflation (CPI): When the inflation of goods' prices is volatile, the volatility of nominal asset returns should reflect inflation volatility. Moreover, in some instances, changes in the level and volatility of inflation may also be symptomatic of policy-induced changes to the business cycle, with the variability in inflation occurring due to a sudden shift in inflation expectations (for example, Germany during the Weimar years). In other instances, however, a shift in inflation volatility may reflect the incidence of supply-side shocks hitting the economy, such as, for instance, the rise in inflation volatility witnessed during the 1970s OPEC crisis.

Industrial Production and Unemployment: Since equity prices reflect the claims on future earnings of corporations, it is plausible that the volatility of economic activity is a major determinant of stock return volatility. Here, the variability of real industrial production and the unemployment rate are the state variables, which can be used to capture the volatility of the real business cycle.

Government Bond Yields: Government bond yields capture the interplay between policy (both fiscal and monetary) and the evolution of real business cycle, coupled with the pricing of any sovereign credit risk. Clearly, the volatility in the pricing of these three sub-components has major implications for equity return volatility via their influence on expectations of future cash flows and the discount factor.

The conventional wisdom when it comes to the linkages between inflation and financial stability (and risky asset market volatility being an important symptom of financial instability as it takes hold, as we saw recently during the Great Recession years) is succinctly summarised by Bordo et al (2000), who observed “that a monetary regime that produces aggregate price stability will, as a by-product, tend to promote stability of financial system”.

As noted by Borio et al (2002), from a broader point of view, few would disagree with the above statement, particularly the idea that volatility in the inflation rate can harm the stability of the financial system and by extension generate an increase in volatility in equity markets as a side-effect. Intuitively, an unexpected decline in inflation increases the real value of outstanding debt, making defaults more likely. On the other side, periods of declining inflation, particularly if they are linked with tight monetary or fiscal policies, are more likely to see pressure in the financial system than are periods with stable inflation.

Similarly, the vulnerability of the financial system and associated asset market volatility witnessed over the horizon of a couple of years tends to rise when inflation is higher than expected, particularly if macroeconomic policies need to be tightened significantly to reduce inflation. Furthermore, high inflation, even if it is relatively stable, can pose a threat to financial stability, particularly if it encourages leveraged asset acquisitions and the misallocation of resources. There is some empirical work to support these ideas. For example, Hardy and Pazarbasioglu (1999) find that an increase in inflation, followed by a sharp reduction, significantly increases the probability of a financial crisis, while Demirguc-Kunt and Detragiache (1997) reported that countries with high levels of current inflation are more likely to experience a financial crisis, which also manifests itself in high asset market volatility. In addition, Bordo et al (2000) argue that episodes of financial distress in the United States in the 18th and 19th centuries generally took place in a disinflationary environment following several years of high inflation.

Indeed, importance of inflation as a key macro variable in driving monetary policy has only risen in recent decades with the advent of *inflation targeting* as a more transparent monetary policy framework since the early 1990s (with New Zealand’s central bank as the first to adopt it in 1990 closely followed by Bank of Canada in 1991), that is now followed by a number of major and emerging countries across the world including the Eurozone, the UK, Japan and the US (the Federal Reserve runs a dual mandate of unemployment and inflation targeting).

However, despite being an important state variable, inflation may not in itself be the cause of a financial crisis (that manifests itself as a sharp fall in risky asset prices and the unwinding of financial imbalances built up in previous periods which drive episodes of high asset market volatility). The best example over recent times of financial imbalances building up in a low/stable inflation environment is the experience of Japan (a country we study in this chapter) in late 1980s as sharp rises in asset prices (equities and commercial property in particular) happened alongside near-zero inflation. The experience of Asian countries during mid-to late-90s produced similar dynamics with most countries experiencing a generalized gentle downtrend in inflation going into 1997.

On the business cycle side and its links with equity market volatility, the existing literature focuses on the “balance sheet view” [Bernanke et al (1995), Bernanke et al (1998)] which postulates that nominal and real shocks to the economy can be amplified by the “financial accelerator” effect. More specifically, this means that a fall in a firm’s net worth resulting from an initial shock (which shows up as a sharp change in unemployment rate/industrial production growth – variables we use to capture business cycle dynamics in this study) increases agency costs by worsening the potential conflicts between borrowers and lenders. This leads subsequently to higher external financing premiums, which in turn magnify the fluctuations in borrowing, spending, investment and consequently, asset market price variations.

With unemployment rate and industrial production (IP) being key business cycle variables, their link to aggregate consumption can be articulated through the lens of standard demand theory (i.e. as recession takes place, labor income falls generating a negative influence on consumption). This connection between the production/consumption side of the macro economy can then be theoretically motivated through the consumption-based asset pricing model explored in detail by Campbell et al (1999) and Campbell (2003) – a framework, which in addition to studying the relation between stock returns and consumption growth also analytically links the standard deviation of consumption growth to the standard deviation of expected asset returns. However, more directly, a production-based asset pricing model developed by Cochrane (1991) which ties stock returns to investment returns is also a good example of understanding the theoretical underpinnings behind the choice of using the unemployment rate and IP as business cycle variables, when it comes to understanding the drivers of equity return volatility.

Finally, from a more empirical perspective, work done by Fama and French (1989) shows that the variation in expected bond and stock returns is related to business conditions and their study shows evidence of a strong degree of co-movement between bond and equity returns.

1.4 EMPIRICAL ESTIMATION APPROACH

As noted in the introduction, the main advantage of using the conditional volatility specification is that it directly accounts for volatility clustering and other information embedded in the time series of equity returns volatility. Therefore, the inclusion of macro-state volatility factors allows the model to estimate the influence of these variables on the variability of equity returns, once "pure" time-series based properties of equity return volatility have been taken into account. However, before discussing in detail the two-step sequential approach adopted in this paper, it is useful to go over other relevant approaches which have been introduced to model volatility in a multi-variate setting, which have a focus on improving volatility forecasting.

The factor GARCH model introduced by Engle (1987) and the latent factor ARCH model of Diebold and Nerlove (1989), which were further explained by Sentana (1998), are plausible candidates for multivariate volatility parameterization. However, as discussed above, the aim of our study is to understand the macro drivers of equity returns volatility rather than improve the forecasting ability of GARCH/ARCH type models. That said, it is worth noting that conventional information criteria (IC) are not appropriate for choosing among GARCH models since the variance parameters and variance equations fit will not affect values of the IC as the information matrix is block diagonal (for instance, see Brooks and Burke (2003) for more details).

In statistical terms, it is possible to argue that the precise joint distribution of equity returns and the relevant macro factors is very complex and the approach used is a robust procedure. Finally, the two-step approach used is consistent with the main objective of this study, which aims to identify the drivers of equity returns volatility (after taking into account well-documented time varying properties of equity return volatility) rather than improving overall forecasting ability.

However, it is important to acknowledge that the estimation procedure outlined below in (1.6) may be subject to simultaneity bias as it is difficult to argue that the right-hand side variables listed in table 1.1 are pure exogenous variables with respect to the dynamics of equity returns.

In order to gauge the importance of equity market relative to the size of the economy and its evolution over time, Exhibit 1.a, below plots the ratio of market-capitalization of domestic companies as % of GDP for the US, UK, Germany and Japan from 1975. At the start of the sample period, this ratio in various countries varied between 10% to 40% and exhibits a similar pattern in subsequent years (with the exception of Japan). For the US, Germany and the UK, the 1990s show a sharp rise in the size of the equity market especially relative to that of the economy (which is consistent with the sharp run-up in prices leading into the dot com bubble, though Germany continues to lag on the basis of a significantly lower ratio compared to the other three countries)). On the other hand, in the case for Japan, the sharp increase in the size is visible during the 1980s and the subsequent decline is also visible as the bubble burst during the late 1980s/early 90s. Based on these numbers, there is little doubt that the importance of equity markets (with respect to the economy) has been quite strong especially since the early-90s (Germany is a clear exception here, where the latest reading shows a ratio of around 49.5%).

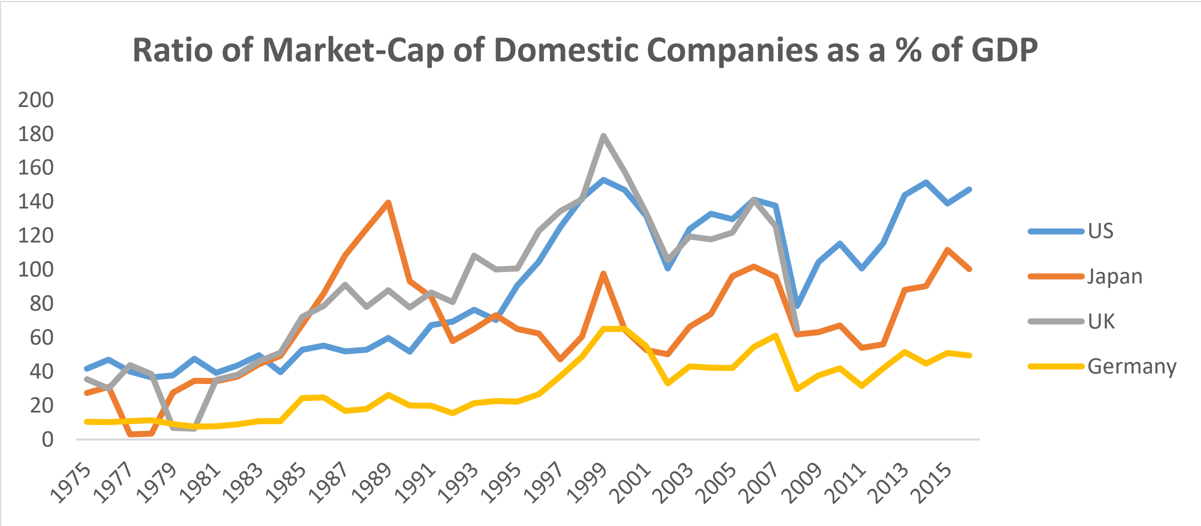
A useful procedure to weaken the biases induced by possible endogeneity of right-hand side macro variables is to study a much longer sample period over which the size of the equity market relative to the economy was significantly smaller, which then allows for the exogeneity assumption to hold.

Using the case of the US (due to data availability and also given that the current ratio is highest here amongst the four countries considered), we have studied the size of equity market relative to the economy before 1975 by using Shiller data (*Irrational Exuberance,* Princeton University Press, 2000). Specifically, using the ratio of the size of equity market to consumption expenditures (indexed to 100 from 1889) as a proxy (this indexed ratio tracks the actual ratio of market cap of domestic companies to GDP with a correlation of 98% over the 1975-2009 period[12]), we approximate the ratio shown for the US in Exhibit 1.b back to 1889. Indeed, the average ratio over the 1889-2009 period was estimated to be 43% with approximately half of the period showing a ratio around 20% compared to around 140% based on 2015 data.

Overall, we have followed the approach adopted by Chen et al (1986) who took equity market returns as endogeneous relative to other variables, as similar to ours (in spirit), their aim was to model equity returns as functions of macro variables and non-equity asset returns. Indeed, using a longer data-sample (given the changing importance of equity markets for the economy when assessed on the basis of relative size) has helped us to weaken this bias, when estimating equations which include contemporaneous variables.

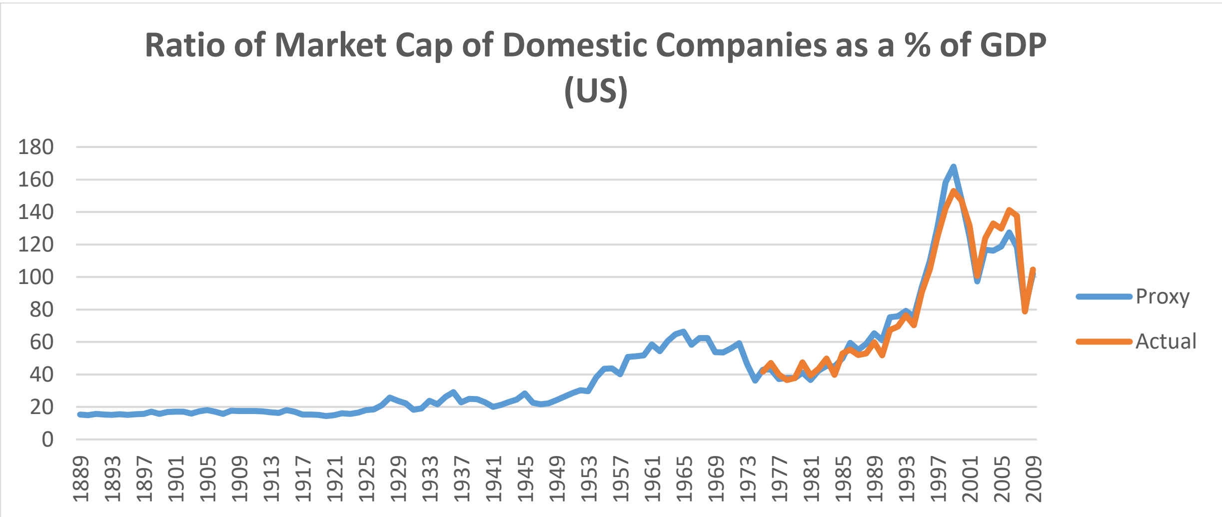
[12] Note Shiller data on real consumption is available till 2009 only.

Exhibit 1.a



Source: World Bank

Exhibit 1.b



Source: World Bank, Shiller Data

In term of the specifics of the approach, we follow a two-step sequential procedure, where the conditional volatility of a particular time series $\boldsymbol{k_t}$ is estimated using the GARCH modelling technique (see Bollerslev (1986) for more details). Specifically, the GARCH (p, q) model of a particular time series variable $\boldsymbol{k_t}$ (where p is the order of the GARCH terms σ^2, q is the order of the ARCH terms ε_t^2 and w_t is the i.i.d. residual term) is given by:

$$\sigma_{t,k}^2 = \alpha_{0,k} + \sum_{i=1}^{q} \alpha_i \, \epsilon_{t-i,k}^2 + \sum_{i=1}^{p} \beta_i \, \sigma_{t-i,k}^2 + w_{t,k} \qquad (1.5)$$

Using the GARCH (p,q) modelling approach, the underlying conditional volatility of individual macro and equity return series (denoted by $h_{t,i}$)[13] is estimated[14]. The optimal lag length (q and p) is decided using the Akaike information criterion. Indeed, the appropriate model specification, based on this criterion is found to be GARCH (1,1) for the various macro and equity return variables.

Additional model specification testing showed that a t-distributed error-term generated slightly better log-likelihood statistics for the equity returns variable compared to the Gaussian distribution assumption specification. Therefore, $h_{t,s}$[15] is assumed to follow a t-distribution error-process, during the estimation process.

Once the individual $h_{t,k}$ series for the five variables mentioned above have been derived from the relevant GARCH(1,1) models, the following ordinary least squares (OLS) regression is run[16] (z_t denotes the i.i.d. residual term):

$$\ln(h_{t,s}) = \alpha + \beta_1 \ln(h_{t,\pi}) + \beta_2 \ln(h_{t,ip}) + \beta_3 \ln(h_{t,ue}) + \beta_4 \ln(h_{t,y}) + z_t \quad (1.6)$$

Overall, equation (1.6) directly tests for a possible relationship between conditional volatility of equity returns and conditional variability of various macro-state variables. Here, it is also relevant to address the "generated regressors" issue which may arise due to the two-step procedure outlined above. Based on the work done by Pagan (1984), in instances when squared residuals or predictors are used as left-hand side variables, then resultant disturbances will exhibit autocorrelation and appropriate treatment would be needed to correct for heteroscedasticity and autocorrelation. Through-out the study, we have used a Newey-West estimator of the covariance matrix in order to take into account this issue.

The following sub-sections present the empirical results derived from estimating equation (1.6) for the four countries in question.

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[13] Where i denotes either the individual macro factors shown in table 1.1 or equity returns.

[14] Note that all $\boldsymbol{k_t}$series shown in table 1.1 are the individual inputs for the relevant univariate GARCH(p,q) model, from which the individual conditional volatilities shown in (1.6) are estimated. For 10 yr bond yields, $\boldsymbol{k_t}$ denotes log-difference of yield.

[15] Where $h_{t,s}$is conditional volatility of equity returns.

[16] OLS regressions were run on both log and non-log transformations, with the former producing better goodness of fit estimates. The tables show the results from the regressions employing the log transformations of the individual $h_{t,k}$series.

Empirical Results – US

Empirical results derived from estimating equation (1.6) for the US, are shown in table 1.2. Given different starting points for the underlying macro and equity return variables, the earliest common sample period for the five variables is May 1929 onwards. Estimation results over this sample period (May 1929 to August 2011)[17] provide convincing evidence of a statistically significant relationship between conditional volatility of macro factors and that of equity returns. Individually, with the exception of the unemployment rate volatility factor, the beta coefficients of the remaining three macro factors come out positive and strongly significant with p-values of less than 1%.[18] Indeed, the positive sign of the beta coefficients is in line with the theoretical prediction which postulate's that equity returns volatility tends to be positively influenced by macro-state volatility.

Furthermore, estimation results are found to be robust to both model specification and sample period adjustments, as dropping the unemployment factor from the model and taking the starting point back to 1920 (to include the pre-Great Depression period) yields similar results, with all three macro conditional volatility factors displaying positive links with conditional volatility of equity returns.

In addition, diagnostic testing on the error terms using Augmented Dickey-Fuller tests (ADF)[19] indicate that the residual series of the various regressions shown in table 1.2 are stationary (i.e. the null hypothesis of the presence of a unit root is rejected at 1% level of significance) – which is consistent with the observation that volatility tends to be mean reverting (for example, see Engle and Patton (2001) who outline some stylized facts about equity market volatility).

Moreover, table 1.2 also shows results for two single variable regressions[20] for which the data is available from 1791 and 1875 respectively. In both cases, the beta coefficient comes out as positive and statistically significant with p-values of at least 1.5%.

Broadly, the bond volatility factor appears to have the most consistent link with equity market volatility across time, with β_4 coming out as positive and significant over the various sample periods studied (using data since 1791). The ip volatility factor appears to have been an important determinant of equity market volatility during the 1920s and the 1990s. However, over the post-war period (1948-

[17] See table 1.2 for details.

[18] Given the incidence of autocorrelation in error-terms, Newey-West consistent covariance matrix is used to calculate p-values.

[19] Optimal lag length for the test is chosen on the basis of Akaike information criterion. A trend version of ADF test was also used that yielded the same conclusion (i.e. null hypothesis of the presence of a unit root was rejected).

[20] The two single variable regressions are ln($h_{t,s}$) regressed against ln($h_{t,y}$) and ln($h_{t,s}$) regressed against ln($h_{t,\pi}$) respectively.

1990) the link appears to have weakened, with inflation and bond volatility factors showing dominance over pure business cycle type variables.

Turning to the ue volatility factor, the explanatory power of this business cycle indicator appears to be a more recent phenomenon, with a high coefficient of +0.50 appearing during the 1990-2011 period. However, in the sample period including the Great Depression years (1929-1939), the ip volatility factor comes out as the more important one (in terms of statistical significance).

Table 1.2 Empirical Results (US)

Period	Π	ip	ue	y	t-bill	Adj R-Sq
Sep 1791 - Aug 2011	-	-	-	0.38**	-	13%
Oct 1875 - Aug 2011	0.10*	-	-	-	-	3%
Feb 1920 - Aug 2011	0.17**	0.21**	-	0.24**	-	33%
May 1929 - Aug 2011	0.22**	0.27**	-0.07	0.17**	-	39%
Feb 1920 - Dec 1947	0.15	0.25**	-	0.38*	-	33%
Jan 1948 - Aug 2011	0.12*	0.00	0.04	0.25**	-	17%
Jan 1965 - Dec 1980	0.23**	0.10	0.26*	0.21*	-	51%
Jan 1990 - Aug 2011	-0.05	0.17*	0.50**	0.37**	-	35%
Jan 2000 - Aug 2011	-0.47**	0.19 *	0.56**	0.64**	-	59%
Feb 1920 - Aug 2011	-	-	-	-	0.33**	36%
Feb 1920 - Aug 2011	0.12**	0.09**	-	-	0.27**	44%
May 1929 - Aug 2011	0.15**	0.15**	-0.04	-	0.22**	47%
Feb 1920 - Dec 1947	0.20**	0.19**	-	-	0.36**	69%
Jan 1948 - Aug 2011	0.11*	-0.03	-0.05	-	0.15**	13%
Jan 1965 - Dec 1980	0.19**	0.02	0.24	-	0.32**	54%
Jan 1990 - Aug 2011	-0.05	0.15	0.38*	-	0.17**	30%
Jan 2000 - Aug 2011	-0.61**	0.21*	0.20*		0.33**	54%

**, * indicate beta coefficients that are significant at 1% and 5% level of confidence respectively.

Estimation results also indicate significant variation in the overall explanatory power of macro volatility factors, as captured by the adjusted R-square statistic, over the various sample periods studied. For instance, since 2000 the conditional volatility of macro factors appears to explain a much higher proportion of changes in conditional equity return volatility compared to the long-term historical average. However, this is also the sample-period most vulnerable to endogeneity bias discussed above. That said, as we discuss in more detail below, this period still showed very similar features (both in terms of sign of coefficients and behavior of residuals) when compared to the much longer sample-based estimation especially during the Great Recession years.

In addition, and similar to the 2000-2011 period, the period including the Great Depression years (1929-1939) also shows relatively higher adjusted R-squared, indicating a stronger relationship between macro and equity market volatility during periods of major economic upheaval.

Focusing on the recent 2000-2011 sample period, all four macro conditional volatility factors are found to be significant with p-values close to zero. However, contrary to theoretical assertions, the inflation volatility factor appears to have a negative sign, while the other three macro factors retain the theoretically correct positive relationship with equity returns variability. Indeed, this is the only sub-sample period over the last 100 years, during which the inflation volatility factor shows a statistically significant negative sign.[21]

Here, a credible policy shift towards inflation targeting by key central banks, which arguably led to a lower and more stable inflation rate compared to the past, especially during the 2000-2008 period[22] may help explain this odd empirical result.[23] Specifically, any increase in inflation volatility against a backdrop of low level of inflation may indicate falling risks of deflation, therefore resulting in the counter-intuitive negative sign for the inflation volatility factor observed during this sample period.

The flip side of this unusual empirical observation (i.e. the negative inflation volatility beta during the 2000-2011 sample period) is apparent during the 1965-1980 period, when the inflation volatility factor showed a statistically significant and above-average positive beta (+0.26 vs +0.17 over the 1920-2011 sample period).[24] It is during this period that the two oil price shocks played a big role in shaping both inflationary dynamics and asset returns,[25] and the model appears to be correctly picking up the positive relationship between inflation and equity returns volatility prevalent during this time. Overall, inflation volatility factor has been an important determinant of equity returns volatility, but it appears to have lost its explanatory power over the last two decades as the level of inflation has come down sharply.

In addition to the four macro variables discussed above, table 1.2 also documents estimation results using the conditional volatility of 90-day t-bills yields as an additional explanatory factor. However, given the very high correlation between the 90-day t-bills yield volatility and the bond yield volatility

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[21] Using the 5% level of significance threshold.

[22] Based on data since 1929, conditional volatility of π during this period was found to be in the bottom decile.

[23] Mervyn King (Governor of Bank of England) characterised this phenomenon as "NICE" in his first speech as governor in the early-2000s. According to Mr. King, NICE stood for a **n**on-**i**nflationary **c**onsistently **e**xpansionary regime. See Bernanke et al (1997) for a detailed discussion on inflation targeting by central banks in advanced economies.

[24] The t-test of differences rejects the null of zero difference at 5% level of confidence.

[25] Ahmed et al (2005) document the relationship between inflation and equity returns using a long-term historical data set. The authors document a strong negative relationship between high inflation and mean equity returns in a number of advanced countries.

factors,[26] there was strong evidence of multi-collinearity, which supported the decision to drop 10-year government bond volatility factor from the revised model. Overall, the estimation exercise yielded broadly similar results, with the 90-day t-bills volatility factor also showing a consistently positive link with equity market volatility, over the various sample periods studied.

Overshoot of Conditional Equity Volatility

Using empirical results derived from estimating equation (1.6), figure 1.2 below plots the actual against fitted values of conditional volatility of US equity returns for the 1929-2011 sample period. Despite the close linkages between equity and macro-based conditional volatility captured by the regression model, empirical results clearly show overshooting of conditional equity returns volatility witnessed during the Great Depression years (when assessed on the basis of both size and sign of the residuals). Specifically, the actual conditional volatility of equity returns during this period was significantly higher than what can be explained by a macro–factor-based model, with the residuals derived from the regression registering more than 3.5 standard deviations above average. With regard to conditional equity returns volatility, this empirical result extends the findings of Officer (1973) and Schwert (1989b) both of whom also document the unusually high level of unconditional US equity returns volatility during the Great Depression years.

Here, as Schwert (1989b) notes, Robert Merton's characterization of the Great Depression as an example of the so-called Peso Problem makes sense, as there was a legitimate uncertainty about whether the economic system would survive the turmoil. This is a viewpoint that the spot conditional volatility of key economic fundamentals, which are used as explanatory variables in the model, is unable to capture.

On the other hand, if we move forward to the recent Great Recession period, estimation results (using both the 1929-2011 and 2000-2011 sample period) show little evidence of a similar overshoot, despite its similarity to the Great Depression period with regards to the potential incidence of the Peso Problem. Here, as figure 1.2 shows, the sharp rise in conditional volatility of equity returns witnessed during the 2008-9 period was more or less in line with the estimates derived from the macro-based model, thus leading to near-zero residuals. Moreover, as figure 1.3 shows, this observation appears to be independent of the sample period tested, as estimating the model (shown in equation (1.6)) over the Jan 1990-Aug 2011 period yielded similar results.

This lack of overshooting in the conditional volatility of US equity returns (confirmed by the various estimations based on different sample periods) is interesting given the sharp rise in global financial

[26] For instance, over the 2000-2011 sample period, the correlation between the 90 day t-bill volatility and bond volatility is more than 90%.

stresses, which took hold in the aftermath of the Lehman bankruptcy. As many financial economists now note, the unprecedented nature of global policy interventions which took place in late-2008 and early-2009 went a long way to reducing the likelihood of a total collapse of the global financial system. Indeed, the credible policy backstops deployed, which entailed the part nationalization of the banking sector in many advanced economies, reversed the negative implications of the Peso Problem, as confidence in the global financial system returned with governments in a number of advanced economies taking over the role of the lender of last resort. As such, the rapid, convincing and credible response of policy makers in reducing the likelihood of a complete collapse of the financial system can explain the lack of excess volatility observed during this period.

Moreover, this absence of overly high equity returns volatility during the Great Recession period, when judged against macro-driven estimates, appears to strengthen the case of policy interventionists, such as then current Chairman of US Federal Reserve, Ben Bernanke,[27] and Bank of England Governor, Sir Mervyn King. It is plausible that without such policy actions and backstops, sustained uncertainty about a regime shift would have added to the fundamental uncertainty reflected in spot volatility of macroeconomic data, thus leading to much higher equity returns volatility than what was observed during the Great Recession period.

Overall, in-depth analysis of the Great Depression years documented in studies including that of Bernanke (2000), highlight the important role played by the contraction in money supply and severe institutional weakness that contributed to a sharp deterioration in economic activity seen during the 1929-1939 period. Indeed, empirical results shown in this study confirm the presence of a Peso Problem-type phenomenon, which potentially led to overly high conditional equity returns volatility, in evidence during the Depression years. However, as noted above, the significantly more active response of policy makers during the more recent Great Recession period appears to have manifested itself in the form of a more "normal" rise in conditional equity returns volatility. This is despite the extreme, and in some cases unprecedented, nature of financial stresses witnessed in the immediate aftermath of the Lehman bankruptcy.[28]

[27] For example, see B. S. Bernanke (2000), "Essays on the Great Depression".

[28] See IMF (2009) for a summary and evolution of financial and inter-bank market stress indicators in the aftermath of the Lehman bankruptcy.

Figure 1.2 US - Actual vs. Fitted Conditional Equity Volatility

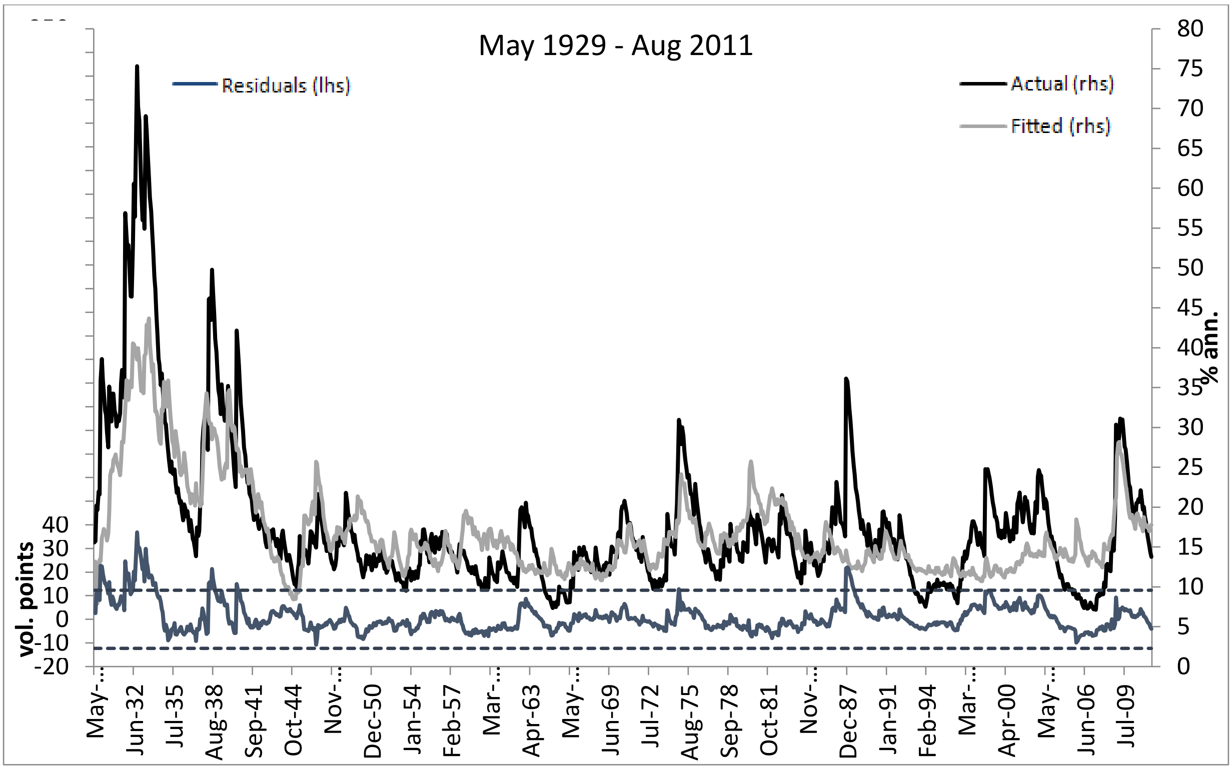
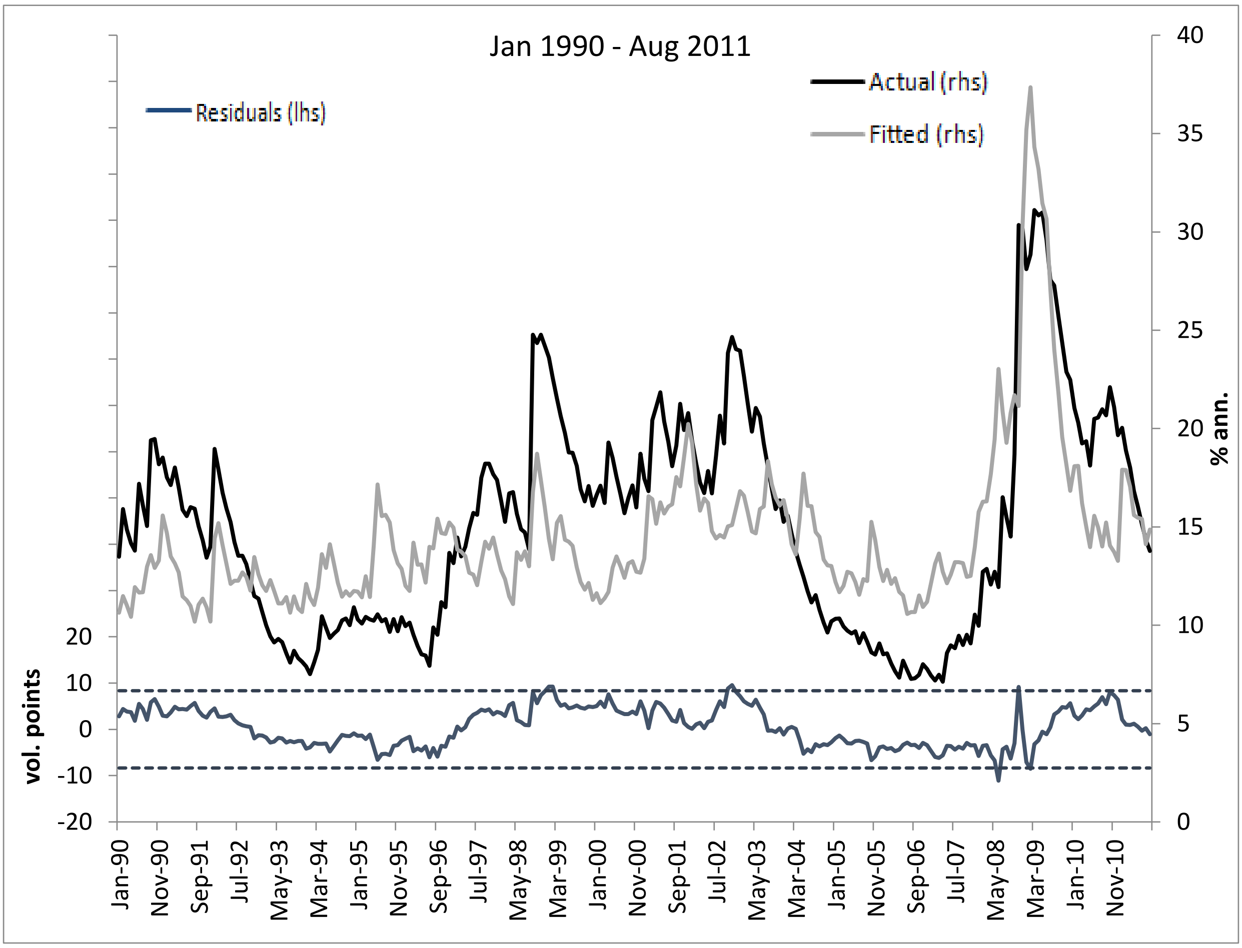


Figure 1.3 US – Actual vs. Fitted Conditional Equity Volatility



Undershoot of Conditional Equity Volatility (Pre-Great Recession)

Focusing on the period leading up to the Great Recession, it is interesting to note the negative gap between actual and estimated conditional equity volatility that was systematically present during the 2004-2007 period. Indeed, using the 1929-2011 sample period, the residual was found to be around two standard deviations below average during this period, while results for the 1990-2011 sample were only marginally less extreme and the result was consistent with the longer sample estimation in terms of its diagnosis. Of course, this systematic under-pricing of equity volatility during this period occurred at a time of rising imbalances, as reflected by a sharp increase in leverage in both the household and the financial sector. In addition, an abundance of liquidity and a lax lending environment also helped inflate the housing bubble, the collapse of which generated the significant economic upheaval of the following years.[29] Overall, the very low level of conditional equity volatility compared to macro-based estimates appears to have been another sign of mispricing of macro risk in equity markets.

Non-Macro-Induced Volatility Episodes

Looking back at the historical evolution of stock market variability, it is clear that the underlying source of volatility can differ across episodes. The October 1987 crash is an interesting case in point, with many studies, such as that of Wigmore (1998), showing no obvious links between the sharp rise in equity market volatility and the underlying macro environment during that period. Estimation results presented in this paper reconfirm this finding in conditional equity volatility[30] as well.

Another high volatility episode which displays similar features to the 1987 market crash is the 1998 LTCM crisis period. This episode also involved a sharp rise in actual conditional volatility without an accompanying macro-based move, and resulted in residuals greater than +2 standard deviations. Here, studies such as Wilson (2007) that use proxies of liquidity risk, such as US swap spreads, do a better job of explaining the volatility surge when compared to the pure macro-based framework used in this paper.

[29] For more details on the factors that contributed to the collapse of the US housing bubble, see IMF "Global Financial Stability Report", April 2008.

[30] Here, the extreme nature of the positive spread between actual and fitted conditional equity volatility recorded during this period was found to be robust to different sample sizes tested. Figure 1.2 shows the results for the 1929-2011 sample period.

Conditional Volatility of US equity Returns and Recessions

The previous sub-section analysed the relationship between various measures of macro volatility and the conditional variability of equity returns. Of course, there is reason to believe that equity return volatility is also connected to the general health of the economy. Here, during periods of economic contraction, the presence of operating leverage can amplify the impact of a fall in demand, as company profits fall faster than revenues in sectors with high fixed costs.

Specifically, table 1.3 shows the beta estimates of the relationship between the conditional volatility of equity returns and the level of economic activity, by running the regression shown in equation (1.7). Indeed, similar to previous studies, the beta coefficient of the recession indicator dummy (based on NBER recession markings) was found to be positive and significant[31] for the various sample periods studied.

$$\ln(h_{t,SPX}) = \alpha + \beta_1 \, Recession\ Dummy + z_t \qquad (1.7)$$

In particular, estimation results show that the beta of the recession dummy seems to have increased over the 2000-2011 sample period compared to its historical average.[32] That is to say that the difference between equity returns volatility during recession and non-recession periods has increased compared to its historical norm – although, as discussed above, this sample is most exposed to endogeneity bias - that said, the direction of the impact of recessionary periods in explaining equity returns volatility appears to be independent of the sample period considered. Specifically, since 2000, empirical results indicate that conditional equity returns volatility was around 30% higher during recession periods compared to non-recession periods, as opposed to a differential of 12% observed since the start of the 20th century.

Table 1.3 Conditional Equity Returns Volatility and Recessions

Period	Recession (NBER)	Adj R-Sq
Jan 1900 - Aug 2011	0.12**	2%
Jan 1950 - Aug 2011	0.18**	5%
Jan 2000 - Aug 2011	0.30*	9%

**, * indicate beta coefficients that are significant at 1%
and 5% level of confidence respectively.

Overall, equity market volatility is clearly related to the general health of the economy and it appears that this relationship has strengthened over the last decade. As discussed above, the main explanation

[31] Using, at least 5% level of significance after using Newey-West adjustment.

[32] Recession dummy takes the value 1 during quarters marked as recession by NBER, 0 otherwise.

behind this finding appears to be the sharp increase in financial leverage and a credit boom, with total debt-to-GDP ratio[33] in the US rising from 2.66 in 1999 to 3.47 in 2007. Indeed, the ensuing private sector de-leveraging triggered by the sub-prime crisis played an important role in generating a much deeper-than-average economic contraction during the 2008-9 period. This in turn was associated with a much higher-than-average rise in equity returns volatility, a dynamic the regression estimates shown in table 1.3 appear to be capturing.

Empirical Results – UK, Germany and Japan

In this sub-section, table 1.4 shows the empirical results derived from estimating equation (1.6)[34] for the UK, Germany and Japan. Similar to the case of the US, there is indeed strong statistical evidence[35] of a relationship between conditional volatility of macro factors and that of conditional equity volatility over the various sample periods studied for the three countries in question.

Focusing on the UK's case, the relatively high adjusted R-square statistic across various formulations of equation (1.6) clearly stands out. Again, similar to the case of the US, the link between the bond volatility factor and the equity returns volatility factor comes out significant at the 1% level using the dataset starting from the 18th century. Moreover, there is clearly an increase in the goodness-of-fit measure during the 1965-1980 sample period (which includes the two oil shocks) with inflation and unemployment factors coming out as strongly significant, registering p-values less than 1%.

Turning to the more recent sample period which starts from January 2000, it appears that the bond volatility and ip volatility factor have been statistically important in terms of their relationship with conditional equity volatility, while the inflation and unemployment volatility factors come out as statistically insignificant. The clear decrease in the relevance of the inflation volatility factor in recent times is similar to the case of the US. As discussed in the previous sub-section, this can be attributed to a shift towards credible inflation targeting by key central banks (including the Bank of England), which leads to lower and more stable inflation rates in advanced countries.

Table 1.4 also shows the results for Japan and Germany, where the focus has been on the post-World War II period, due to data limitations. Here, once again, there is clear evidence of a statistically significant relationship between domestic macro volatility factors and the conditional equity returns volatility for both these countries. However, judging by the signs of various coefficients of the conditional macro volatility factors and the associated adjusted R-square statistics for the various

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[33] Source: http://www.relooney.info/SI_FAO-Asia/Global-Crisis_23.pdf

[34] See Appendix 1.1 for details on the relevant equity and macro data series employed in the estimation procedure for Germany, Japan and the UK.

[35] As with the US, the Newey-West adjusted covariance matrix was used to assess statistical significance, given the incidence of auto-correlation in error-terms.

sample periods studied, the empirical results appear to be less uniform compared to the case of the US and the UK.

For instance, in Japan's case, the 1965-1980 period estimation throws up a counter-intuitive result in the form of a negative coefficient (with a p-value of less than 1%) for the conditional bond volatility factor. While in Germany's case, the 1990-2011 period estimation registers adjusted R-square of just 8%, compared to an average of 40% for other periods. Data distortions caused by the German re-unification appear to be causing the sharp fall in goodness-of-fit measures.

Moreover, in Japan's case the bond volatility factor seems to be quite irrelevant in shaping conditional equity volatility. This result is not that surprising given Japan's struggle with deflation and the resultant ultra-low bond yields driven by the Bank of Japan's zero interest rate and quantitative easing policy in recent years. However, the conditional volatility of business cycle variables UE and IP does appear to be particularly important, with both factors showing statistically significant coefficients for the various time periods studied. In addition, the influence of conditional volatility of inflation on conditional equity volatility appears to have strengthened over the last decade or so, with the coefficient rising to +0.82 over the 2000-2011 period.

Turning to Germany's case, the lack of importance of business cycle variables in influencing conditional equity volatility over the various sample periods studied is striking. That said, when it comes to Germany, both inflation and bond volatility factors appear to be important drivers of conditional equity volatility based on statistical significance metrics. In addition, the relevance of inflation factor appears to have strengthened over recent years, a situation which resembles that of Japan.

As shown in table 1.5, empirical results obtained from estimating equation (1.7) show a clear and statistically significant increase in conditional equity volatility during recession periods for these countries, much like in the US.[36] Furthermore, the link appears to have strengthened in recent years as well, with the period since 2000 capturing two sharp downturns in economic activity in the UK, Germany and Japan. Specifically, results indicate an 11%, 13% and 31% increase in conditional equity volatility during recession periods compared to non-recession periods for the UK, Japan and Germany, respectively. This shows that the importance of macro factors in shaping equity volatility is borne out of data for other OECD countries (i.e. in addition to the US) as well.

[36] Recession marking is based on OECD data. Dummy variable for recession takes a value 1 during recession period and 0, otherwise.

Table 1.4 Empirical Results for UK, Japan and Germany

Period	Π	Ip	Ue	Y	Adj R-Sq
UK					
Aug 1729- Aug 2011	-	-	-	0.62**	37%
Feb 1914 - Aug 2011	0.15**	-	-0.14	0.45**	40%
Jan 1956 - Aug2011	0.29**	-0.02	0.02	0.38**	43%
Jan 1965 - Dec 1980	0.47**	-0.02	0.42**	0.20	69%
Jan 1990 - Aug 2011	0.12	0.17*	0.21	0.48**	45%
Jan 2000 - Aug 2011	0.11	0.22*	-0.18	0.55**	48%
Japan					
Jan 1949 - Aug 2011	0.15*	-	0.08	0.03	12%
Feb 1953 - Aug 2011	0.02	0.21*	0.00	0.06	6%
Jan 1965 - Dec 1980	0.21**	-0.08	0.24**	-0.26**	58%
Jan 1990 - Aug 2011	0.28	0.15*	0.68**	0.02	39%
Jan 2000 - Aug 2011	0.82**	0.24**	0.08	0.01	43%
Germany					
Jan 1948 - Aug 2011	0.34**	-	-0.06	0.22*	27%
Feb 1958 - Aug 2011	0.09	0.04	-0.03	0.24*	12%
Jan 1965 - Dec 1980	0.17**	0.19**	0.02*	0.26**	42%
Jan 1990 - Aug 2011	0.20*	0.03	-0.07	0.27*	8%
Jan 2000 - Aug 2011	0.57**	-0.01	-0.12	0.16*	17%

**, * indicate beta coefficients that are significant at 1%
and 5% level respectively.

Table 1.5 Recessions and Conditional Equity Volatility

Period	Recession Indicator	F-Stat
UK		
Sep 1955 - Aug 2011	0.04*	3.78*
Jan 1990 - Aug 2011	0.06*	5.80*
Jan 2000 - Aug 2011	0.11**	8.82**
Japan		
Dec 1961 - Aug 2011	0.13*	32.52**
Jan 1990 - Aug 2011	0.16**	36.38**
Jan 2000 - Aug 2011	0.13*	10.45**
Germany		
Nov 1960 - Aug 2011	0.11*	27.98**
Jan 1990 - Aug 2011	0.15*	21.51**
Jan 2000 - Aug 2011	0.31**	52.30**

**, * indicate beta coefficients that are significant at 1%
and 5% level respectively.

Undershoot of Actual Equity Volatility (Pre-Great Recession Period)

Focusing on the period leading up to the Great Recession, the undershooting behaviour of actual equity volatility is also borne out for non-US countries as well. Figures 1.4, 1.5 and 1.6 clearly show that actual equity volatility was systematically below macro-based equity volatility (or fitted values) during the 2004-2007 period for these three countries as well and was also found to be independent of the sample size considered. This finding lends further weight to the potential mispricing of macro risk in global equity market hypothesis.

Figure 1.4 Germany – Actual vs Fitted Conditional Equity Volatility

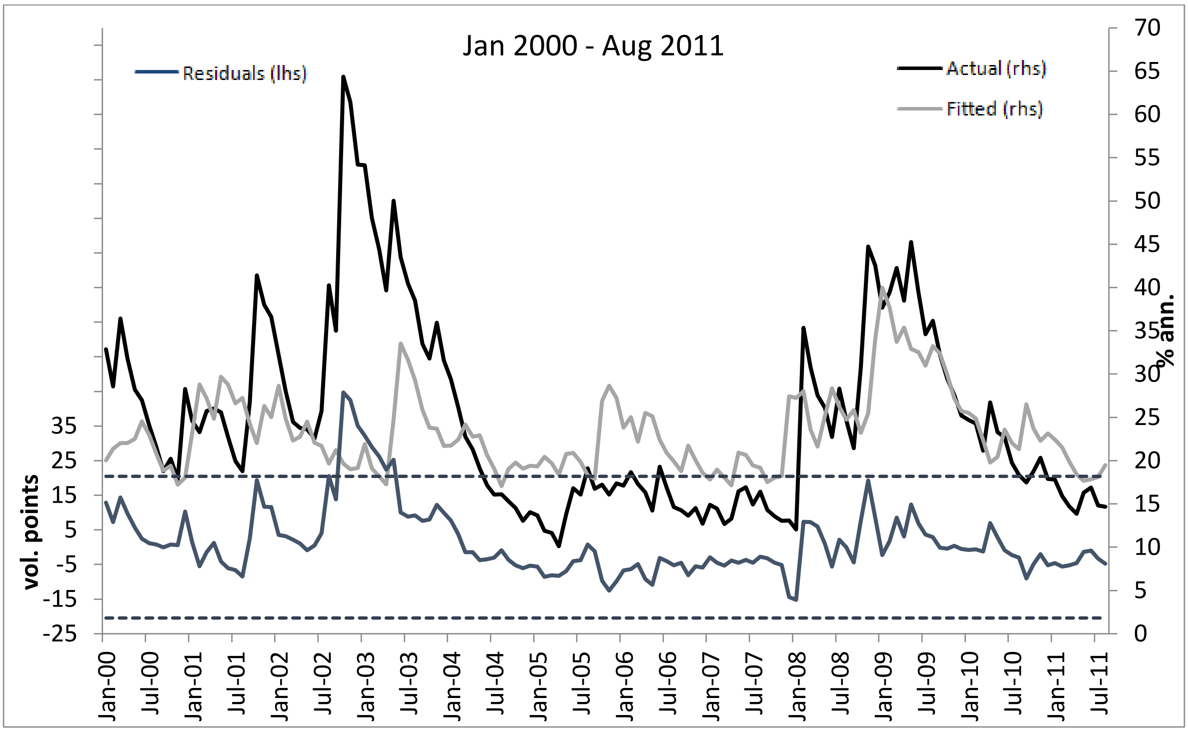


Figure 1.5 UK – Actual vs Fitted Conditional Equity Volatility

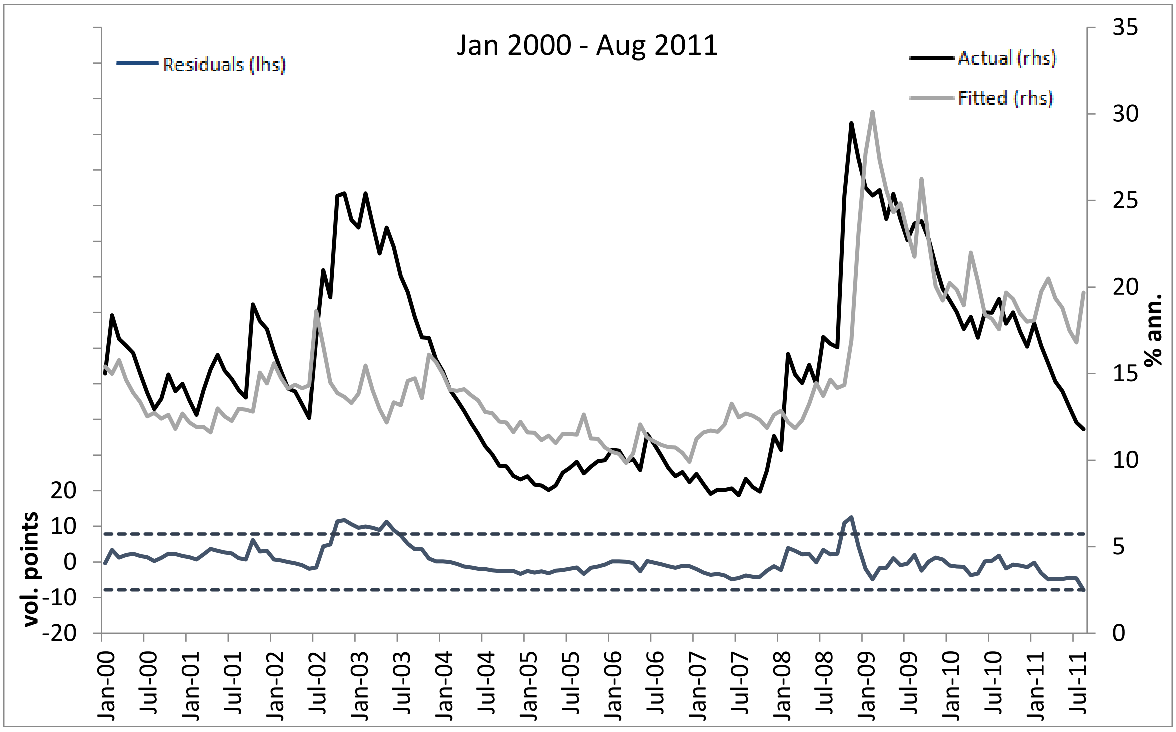
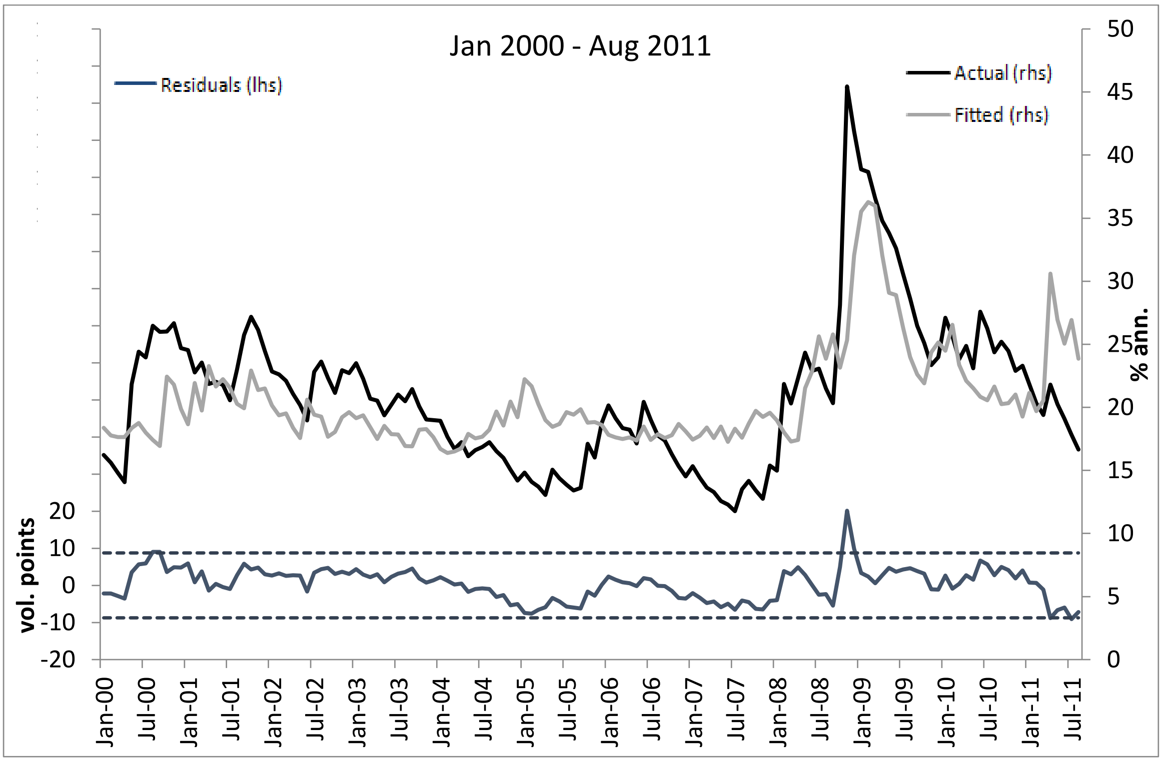


Figure 1.6 Japan – Actual vs. Fitted Conditional Equity Volatility



1.5 MACRO DRIVERS OF VIX AND VDAX

Following on from the empirical analysis that focused on the relationship between the conditional volatility of equity returns and macro volatility, this sections presents a fair-value model for the VIX (which represents implied volatility of the S&P 500) derived from the variability of key macro-state variables.

As a function of macro-state variability, the modelling of implied volatility was first introduced by Ahmed et al (2008). They highlighted the crucial role played by macro-state variables in driving the implied volatility of equity markets during the US sub-prime crisis months. In this chapter, the framework has been expanded to add additional variables together with a larger sample period. In addition, the relationship between VDAX, which represents the implied volatility of the DAX index, and the conditional variability of German macro-state variables is also studied.

Specifically, the following regression is run in order to derive the empirical relationship between spot VIX and macro-state volatility:

$$\ln(VIX_t) = \alpha + \beta_1 \ln(h_{t,\pi}) + \beta_2 \ln(h_{t,ip}) + \beta_3 \ln(h_{t,ue}) + \beta_4 \ln(h_{t,y}) + z_t \quad (1.8)$$

assuming, z_t is I.I.D and $h_{t,i}$ is the conditional volatility of a given variable i derived from the relevant GARCH model estimation.

Estimation results using data since 1986[37] show that with the exception of inflation, all other macro-state variables play an important role in shaping variations in implied volatility of the US equity market. Indeed, for the 1986-2011 sample period, estimation results show a healthy adjusted R-square of 23% with beta coefficients of ip, ue and 10-year bond volatility showing p-values of at least 5%. The standard battery of diagnostic tests show stationary residuals (the presence of unit root is rejected at 1% level of significance), but a clear incidence of autocorrelation in error terms. [38] As in the case of conditional equity volatility model, Newey-West adjustment has been used to account for this.

As before, turning to the actual against fitted picture (1986-2011), empirical results show actual implied volatility of the S&P 500 was significantly below the fair-value spanned by the macro-state variables during the 2004 to late-2007 period. Here, as discussed before, the reduction in implied volatility can be connected with the sharp compression in credit spreads witnessed during these years, as pricing of credit spreads was pushed to unreasonably low levels.

Moreover, the model clearly picks up the macro nature of the Great Recession with both macro-state volatility and implied equity volatility rising sharply during the 2008/9 period. However, unlike

[37] VIX futures data starts from 1986, hence the starting point for the sample.
[38] For instance, see the Augmented Dickey Fuller test.

conditional equity volatility, there is evidence of a short-lived overshoot of implied equity volatility relative to its macro-based fair value during the Great Recession period. In addition, evidence of overshoot weakens in formulations including the ISM variables, thus bringing it in line with the conditional equity volatility results.

In contrast, the 1987 crash and the accompanying rise in implied volatility is seen largely as a non-macro event compared with the 2008-2009 experience, with the macro-based volatility only showing a slight uptick rather than a sustained rise. This is similar to the conditional volatility results.

Another interesting observation that emerges from the estimation results is the consistently positive spread between actual implied equity volatility and macro-based fair value over the 1996-2001 period. This was later normalised by a rise in fair value during the tech-burst years, with macro-derived volatility catching-up with higher implied volatility.

Turning to the 2000-2011 sub-sample period, estimation results show a higher adjusted R-square (34% compared with 23% for 1986-2011 period) with slightly higher betas for the IP and UE conditional volatility factors. Once again, inflation volatility is found to be insignificant in explaining the shifts in implied equity volatility. Moreover, similar to the 1986-onwards sample, the 2004-2007 period shows a deep negative disconnect between actual and macro-based implied volatility, which was then corrected, so to speak, during the 2008-2009 period.

The importance of ISM Manufacturing Index in Shaping Implied Volatility

In terms of the composition of the macro-state variable set, the significance of Institute of Supply Management's (ISM) manufacturing survey in explaining the shifts in the VIX index has also been tested. Indeed, the high correlation between the manufacturing survey index and US IP, coupled with its timely availability, makes the ISM release one of the key data points in the economic data calendar[39].

Estimation results (see table 1.6) show that conditional volatility of the ISM manufacturing index has a positive impact on the US equity market's implied volatility. However, based on goodness-of-fit measures, the model incorporating the ISM volatility factor slightly underperforms the one including the IP factor, though the difference is quite small (adjusted R-square of 20% vs. 23% for the 1986-2011 case).

Another formulation of the model attempts to explore the relationship between a threshold level of ISM and the VIX index. Specifically, this regression replaces the ISM volatility factor with a dummy

[39] For instance, see Haris et al (2004), "Using Manufacturing Surveys to Assess Economic Conditions".

variable, which records the level of the ISM manufacturing index (1 for below 50 reading and 0 otherwise).[40] Using this specification, empirical results show that periods in which the ISM is below 50 (i.e. the index indicates a contraction phase), the implied volatility of the S&P 500 tends be on average 13% higher than periods in which the ISM index is higher than 50. Moreover, the beta appears to have strengthened during the 2000-2011 period (from 0.13 to 0.28) coupled with better goodness-of-fit measures over this period. Again, these empirical results are similar to the recession period results presented in section 1.3b, which highlighted the role played by the transitions in business cycle on conditional volatility of equity returns.

Finally, another formulation of the model including the actual level of the ISM index[41] (rather than its conditional volatility or the below 50 threshold captured using a dummy variable) is also estimated. The results again show a clear negative relationship between the level of the ISM manufacturing index and the VIX, which appears to have strengthened, since the turn of the century.

Table 1.6 Fair-Value Model Results for the VIX index

Period	Π	IP	UE	Y	ISM	ISM Dummy	ISM Level	Adj R-Sq
US								
Jun 1986 - Aug 2011	-0.01	0.35**	0.57**	0.21*	-	-	-	23%
Jan 2000 - Aug 2011	-0.16	0.42**	0.70*	0.22	-	-	-	34%
Jun 1986 - Aug 2011	0.03	-	0.69**	0.21**	0.10**	-	-	20%
Jan 2000 - Aug 2011	-0.08	-	0.94**	0.23**	0.09*	-	-	29%
Jun 1986 - Aug 2011	0.02	-	0.62**	0.29**	-	0.13**	-	20%
Jan 2000 - Aug 2011	-0.07	-	0.61**	0.33**	-	0.28**	-	35%
Jun 1986 - Aug 2011	0.00	-	0.53**	0.31**	-	-	-0.70*	20%
Jan 2000 - Aug 2011	-0.16*	-	0.23	0.40**	-	-	-1.85**	47%

**, * indicate beta coefficients that are significant at 1% and 5% level respectively.

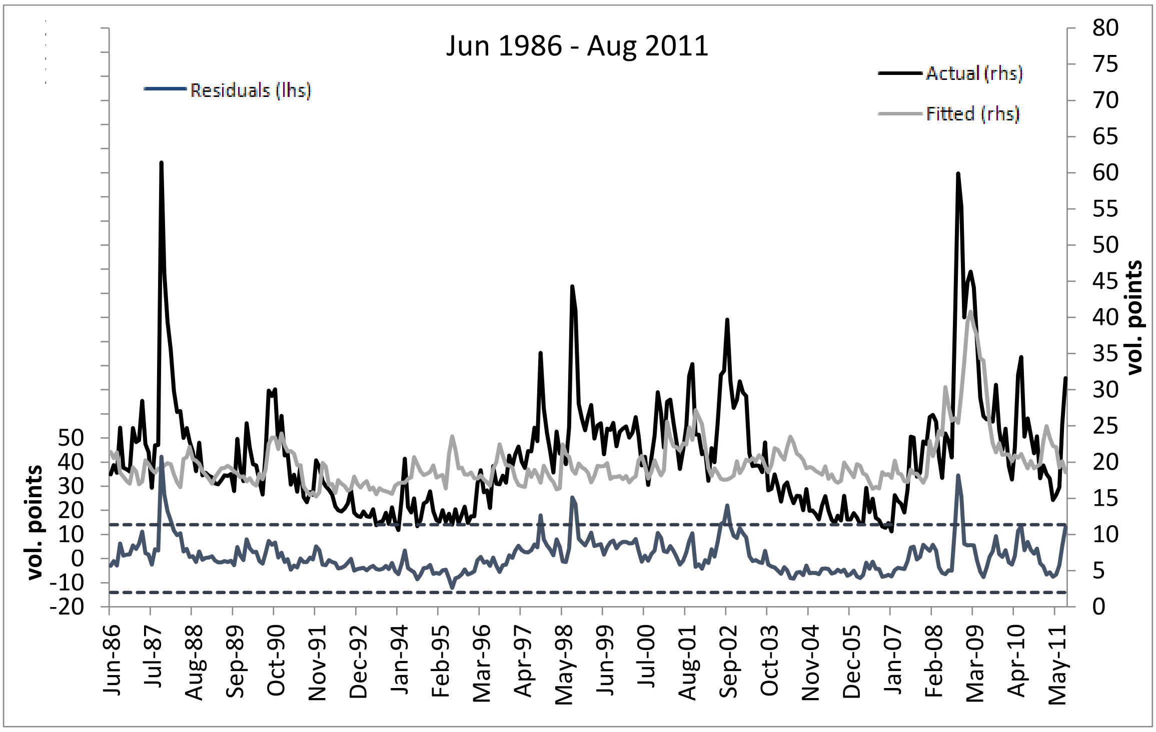
Mean-Reversion of Residuals

Focusing on the diagnostics, the mean-reverting property of the residual term implies that the macro-based fair value acts an important anchor point for market-based equity implied volatility. Overall, the residual term was found to be stationary for the all the various formulations shown in table 1.6, using ADF tests, the null of a unit root was rejected at 1% level of significance.

[40] A reading of above 50 is consistent with growth in the manufacturing sector, while a reading of below 50 is consistent with a contraction. See www.ism.ws for more details.
[41] Using the natural log functional form.

Given the stationarity property of the residual term, instances of extreme level of the spread between actual and macro-based fair value (relative to its historical average) indicates potential for future normalisation. This is an important result for both long and short-term investors with the macro-based fair values providing a fundamentally driven reference level for market-determined implied equity volatility.

Figure 1.7 Actual vs Fitted (VIX against conditional vol of UE, IP, Inflation and Y)



Macro Drivers of VDAX

In this sub-section, estimation results for the German VDAX index are provided. Similar to the VIX case, the following regression is estimated:

$$\ln(VDAX_t\ ) = \alpha + \beta_1 \ln(h_{t,\pi}) + \beta_2 \ln(h_{t,ip}) + \beta_3 \ln(h_{t,ue}) + \beta_4 \ln(h_{t,y}) + z_t \quad (1.9)$$

assuming, z_t is i.i.d and $h_{t,i}$ is the conditional volatility estimate of variable i

The results presented in table 1.7 show the importance of conditional volatility of inflation and bond volatility in driving implied volatility of the DAX index over the 1992-2011 period.[42] However, there is a clear counter-intuitive result in the form of negative sign of the ue conditional volatility factor (showing p-value of less than 1%) over the sample period. Here, the distortions created by the re-

[42] VDAX series starts from 1992.

unification of Germany in 1991 appear to be the driving factor behind the odd result, as the shortening of the sample period to 2000-2011 leads to an insignificant ue volatility factor.

Another specification of the model, which produces better goodness-of-fit measures, replaces the IP volatility factor with various specification of the Ifo index. Similar to the ISM manufacturing index, the Ifo business climate index is a widely followed monthly indicator of economic activity in Germany. The empirical results indicate that the level of the Ifo index[43], as opposed to its conditional volatility, has been an important driver of implied equity volatility, especially over the 2000-2011 period, with an increase in the Ifo level consistent with a reduction in implied equity volatility.

The dummy variable specification (Ifo=1, when reading Ifo<100), as expected, shows a positive beta. This is consistent with an increase in volatility during activity contraction phases. Again, there is a notable increase in the beta since the turn of the century, showing the stronger role of macro-state factors in shaping the implied volatility of German equities.

Broadly, the results for VDAX were found to be less stable compared to the VIX case, especially during the 1990s period. That said, as shown in the actual vs. fitted plot in figure 1.8, the under-pricing of equity risk is also visible during the pre-2008-2009 years, with actual implied volatility running significantly lower than its macro-based estimate.

[43] Ifo index was used in the regression in natural log form.

Figure 1.8 Actual vs Fitted – VDAX against conditional vol of Inflation, ue, y and level of Ifo index

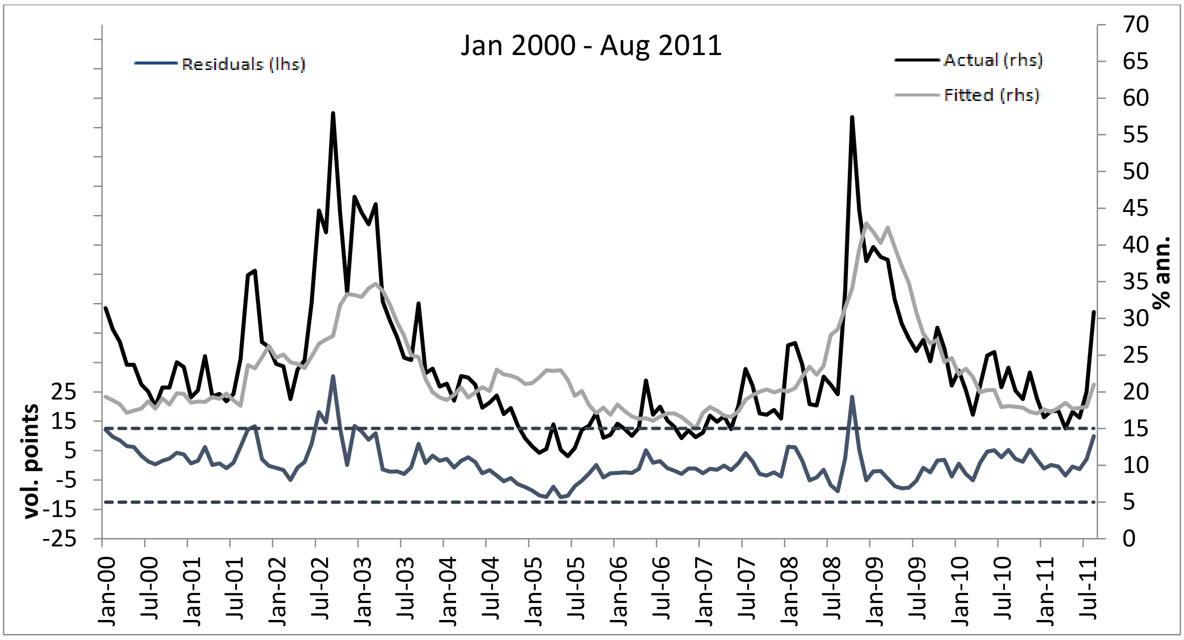


Table 1.7 Fair-Value Model Results for VDAX

Period	Π	Ip	Ue	Y	Ifo	Ifo Dummy	Ifo Level	Adj R-Sq
Germany								
Jan 1992 - Aug 2011	0.46**	0.07	-0.43**	0.47**	-	-	-	21%
Jan 2000 - Aug 2011	0.59*	0.02	0.04	0.16*	-	-	-	12%
Jun 1992 - Aug 2011	0.47**	-	-0.45**	0.45**	0.05	-	-	22%
Jan 2000 - Aug 2011	0.59**	-	-0.03	0.14	0.04	-	-	12%
Jan 1992 - Aug 2011	0.44*	-	-0.41**	0.51**	-	0.04*	-	22%
Jan 2000 - Aug 2011	0.37*	-	0.43*	0.01	-	0.35**	-	32%
Jan 1992 - Aug 2011	0.34*	-	-0.38**	0.53**	-	-	-1.00	24%
Jan 2000 - Aug 2011	0.00	-	0.64**	0.17*	-	-	-3.44**	49%

**, * indicate beta coefficients that are significant at 1% and 5% level respectively.

1.6 CONCLUSION

The key findings of this chapter are the following:

1. Conditional equity return volatility can be explained by a variability in the macro state environment, even after taking into account lagged equity return volatility variables, which are known at time t and captured using the GARCH formulation. Using detailed empirical analysis on a long-term historical data-set, this study shows that this relationship holds across

different countries and sample periods (even when endogeneity of macro variables appears to become a concern).

2. Moreover, this study finds that the behaviour of conditional equity returns volatility, when assessed against the variability in macro environment, displayed different properties during the Great Recession (2008/9) and Great Depression (1929/39) periods. Here, the sharp rise in conditional equity volatility during the 2008-9 period was more or less in line with estimates derived from the macro-based model. On the other hand, conditional equity volatility significantly overshot the relevant macro-based estimates during the Great Depression period. Results pertaining to the Great Recession period appear to be independent of the sample period under consideration.

3. The above empirical result appears to strengthen the case of policy interventionists such as the former Federal Reserve Chairman Ben Bernanke and former Bank of England governor, Sir Mervyn King, who both argued that the unprecedented monetary policy easing played an important role in stabilising the economic situation in advanced economies, and by extension, equity prices, in the aftermath of the Lehman bankruptcy in 2008.

4. Turning to fair-value model results of conditional equity returns volatility, it appears that equity returns volatility was significantly "undervalued", when judged against macro-driven fair values over the 2004-2007 period. This disconnect was visible in all four countries studied and across different sample periods, thus lessening concerns related to simultaneity bias, which appear to be more of a concern since the early 1990s as the size of equity markets relative to the economy grew strongly. It appears that factors such as an increased leverage in the household and financial sector, low global real rates and deregulation (which incentivized excess lending) may have caused this disconnect to appear before the onset of the Great Recession, which corrected this "valuation" gap.

5. In addition, macro-based fair values of implied volatility of the S&P 500 and DAX indices (namely, VIX and VDAXX) showed a similar disconnect during the 2004-2007 period.

6. Overall, this study shows that macro-driven fair values of both realized and implied volatility should form an important part of volatility assessment tool kit for policy makers focused on financial stability.

APPENDIX 1.1 Data series used and sources

The equity indices and relevant macro data used in this paper are sourced from Global Financial Data (GFD).[44] For the macro-state variable set, business cycle indicators such as industrial production and unemployment rate are used. In addition, consumer price index data is used to estimate inflation volatility and 10yr bond yields data is deployed as a proxy for financial asset and central bank policy variability.

Equity Indices

US: The S&P 500 composite market capitalization index of stock prices has been used. The original indices were constructed by S&P in 1923 consisting of 233 stocks. From 1790-1801, GFD has calculated an equal-weighted index using data from 7 banks and two transport companies. Beginning in 1871, the Cowles/S&P index of stocks is used. More information is available in Standard and Poor's, 2000 and Cowles Commission for Research in Economics, Common-Stock Indexes, 2nd ed., Bloomington: Principia Press, 1939.

UK: A range of different sources have been chain-linked by GFD using 1962 as base. Only East Indies stock were included before 1694. The Bank of England was added in 1695 and the South Sea Company was added in 1711. A capitalization-weighted index for 287 UK equities is used from 1907 to May 1933. The Actuaries General Index is used from 1933 to 1952, which precedes the Financial Times-Actuaries All-Share Index. Rostow's Total Index of Share Prices is used until 1850 and the London and Cambridge Economic Service Index is used thereafter.

Japan: GFD uses the Fisher Index from September 1948 through April 1949. The Nikkei 225, which employs an average price calculation method similar to the Dow formula, is used from May 1949 onwards.

Germany: The data starts with the 300-share monthly index from July 1948. This is followed by the inclusion of the Commerzbank Index from 1956-1969. Thereafter, the CDAX Price Index is used which includes all stocks traded on the Deutsche Borse. Data referring to unified Germany is only after 1993, before which all data pertains to Western Germany.

[44] GFD, http://www.globalfinancialdata.com/index.html

Inflation[45]

US: The Consumer Price index is based on a combination of three indices. From 1820 through to 1874, the annual cost-of-living index calculated by the Federal Reserve Bank is used. From 1875 until 1912, a monthly Index of General Prices calculated by the Federal Reserve Bank of New York is used. From 1913 on, the Bureau of Labor's Consumer Price Index is used.

UK: Data before 1900 are taken from Brown and Hopkins' paper in *Economica* (February 1959), which follows the construction of the monthly chain-linked Consumer Price Index with 1996 as base. It is compiled using a sample of over 650 goods and services for which movements in price are regularly measured around the UK. The weights of the items used are derived from the annual UK Expenditure and Food Survey (EFS) and Household Final Consumption Expenditure (HHFCE) data.

Japan: GFD uses the Bureau of Statistics' numbers which are available from 1946. The CPI is constructed as the weighted arithmetic mean using year 2000 as base, covering monthly price data on 598 items from 167 sample cities, towns and villages across Japan.

Germany: The official Consumer Price Index (CPI) is used. It starts from 1948 and uses 2000 as base. It covers all population groups and regions of Germany. Data used is compiled by GFD from Wirtschaft, Statistik and Bunesamt (1948-).

Industrial Production

US: Data from 1790 to 1915 uses Joseph H. Davis's "An Annual Index of U.S. Industrial Production" from *The Quarterly Journal of Economics* (2004). Data from 1915 to 1921 use John W. Kendrick, Productivity Trends in the United States (Princeton, 1961) and Federal Reserve data is used from 1921 onwards.

UK: Office of National Statistics (ONS)

Japan: Monthly data for 1948 onwards is compiled from the United Nations, Monthly Statistical Bulletin and Eurostat. This series has been seasonally adjusted for the purpose of this study (using the standard US Census Bureau X-12 program).

Germany: GFD has sourced data for 1948 onwards from the United Nations, Monthly Statistical Bulletin and Eurostat. This series has been seasonally adjusted for the purpose of this study.

[45] All inflation series were seasonally adjusted using the US Census Bureau's X-12 program.

Unemployment

US: The data are compiled using information from the monthly Current Population Survey (CPS) which covers the civilian non-institutional population 16 years and older. The data is collected from the FRED database and distributed by GFD. The FRED database uses data from a number of US government entities including the Federal Reserve and Bureau of Labour Statistics (BLS).

UK: The unemployment series for the UK uses unemployment numbers from the building trades from 1888 to May 1923 and include insured workers from June 1923 to 1948. Following this period, data has been compiled on the number of individuals between 16 and 60 claiming unemployment benefits at Jobcentre Plus local offices. GFD uses data that national bodies have disseminated regularly to Eurostat.

Japan: Data are compiled and disseminated by the Labour Force Survey (LFS) covering all persons 15 years or older, in accordance with the international guidelines set out by the International Labour Organization (ILO). GFD sources the data disseminated by the LFS from the Eurostat database.

Germany: The series provided by GFD covers total unemployment figures, covering unemployed job-seekers or those employed for less than 15 hours per week in the age group 15-64. The data is based on submissions by the jobless to German employment offices throughout the country. The source used by GFD is the Eurostat database.

Bond Yields

US: The historical data has been sourced by GFD from Richard E. Sylla, Jack Wilson and Robert E. Wright, Price Quotations in Early U.S. Securities Markets, 1790-1860; *Hunt's Merchants Magazine* (1843-1853); *The Economist* (1854-1861); *The Financial Review* (1862-1918); Federal Reserve Bank, National Monetary Statistics (New York: FRB, 1941, 1970 (annually thereafter); and Salomon Brothers, *Analytical Record of Yields and Yield Spreads* (New York: Salomon Brothers, 1995). The 'constant maturity' yield was sourced from FRB, H-15 tables, which are available from 1953.

UK: The benchmark 10-year bond is used for this series, sourced by the GFD from the Central Statistical Office, Annual Abstract of Statistics (London: CSO, 1853-); the *Financial Times* and the Bank of England.

Japan: GFD sources bond yield data from the Tokyo Stock Exchange Monthly Statistical Report (1937-1946), the Industrial and Commercial Semi-Annual Report (1948-1957) and the Bank of Japan, Economic Statistics Monthly (1969-). The Bank of Japan benchmark bond has been used for this entire series.

Germany: The data complied are from Bayerisches Statistisches Landesamt (Munchen: Bayerisches Statistisches Landesamt, 1946-1947); Statistisches Bundesamt, Wirtschaft und Statistik and Bundesbank (1948-55) and the Deutsche Bundesbank‘s Monthly Report (1956-).

CHAPTER 2: MISSPECIFICATION IN AN ASYMMETRICALLY DEPENDENT WORLD: IMPLICATIONS FOR VOLATILITY FORCASTING

In this chapter, we assess the relative abilities of GARCH and stochastic volatility models (SV) to forecast volatility in a world where the true volatility data exhibit asymmetric dependence. To avoid problems of data dependence, we shall assume that we know the true model and use artificially generated data to assess the competing models' forecasting abilities. Specifically, we initially assume that the true model is EGARCH(1,2). Our analysis confirms the superiority of the SV model under the normal distribution assumption. However, using t-distributed shocks, results vary and appear to depend on the value of β, which we believe is related to the behaviour of the relevant volatility models when β is close to 1. We also find that, based on conventional measures of forecasting accuracy such as MSE, SV forecasts are very exposed to outliers relative to GARCH. This is partially a consequence of the need to exponentiate the SV forecasts (since SV is a model of log-volatility). We show how the presence of non-normality maps onto the time series structure. We show that exponentiation under some circumstances leads to the non-existence of population moments.

2.1 INTRODUCTION

Applied economists are often uncertain as to which of the common volatility models is better to use, especially in the context of forecasting. In this paper, our contribution to the literature takes the form of an assessment of the relative abilities of GARCH and Stochastic Volatility (SV) models to forecast volatility. To avoid problems of data dependence, we shall assume that we know the true model and use artificially generated data to assess the competing models' forecasting abilities. This has the advantage that volatility is observable from the point of view of the simulator. Thus, we can avoid using variations of realised volatility which is difficult to calculate in cases where the data are generated by processes with jumps and other irregularities.

In the practitioner community, use of GARCH(1,1) as a model to forecast volatility remains wide-spread. For example, dominant risk management vendor software such as the MSCI BARRA Global Equity Model, MSCI BARRA Predicting Risk at Short-Horzions module and the Bloomberg factor model used in PORT (an up and coming risk management tool) all use a GARCH(1,1) formulation to improve upon the Exponentially Weighted Moving Average scheme. In addition, studies such as Hansen and Lunde (2005) have documented that GARCH(1,1) performed better than more than 300 different ARCH-type models (using IBM stock returns and DM/S data) and continues to set a high bar to replace GARCH(1,1) as the preferred volatility forecasting model in practitioner circles. Indeed, given increased computing power available driven by advances in modern technology, the trade-off between computational complexity arising from using a more complicated model versus any forecast improvement/or lack of is an interesting area of study with practical implications. In addition, as the focus of this study is on assessing the efficacy of volatility forecasts being generated from different candidate models, our emphasis is on the volatility equation rather than the mean equation in this chapter.

We shall initially assume that the true model is EGARCH(1,2) based on convincing empirical work by Pagan and Schwert (1990). This also recognises the importance of asymmetric dependence in financial data. We further extend their analysis through an up-to-date dataset. Their analysis was based on the US equity market. We also apply it to the US 10 year bonds. The difficulty with any simulation is that, through the adept choice of the true model, we can tilt the simulation to favour our preferred method. We would argue that we have fixed the true model to be different from both alternative models which the econometrician assumes are GARCH(1,1) and SV(1,1). Here the choice of using SV(1,1) as part of the candidate model set is driven by the observation that both the true model and the SV(1,1) model are log volatility models which may confer an advantage to SV. The SV possesses two sources of noise whilst the GARCH has only one, which may also favour SV. However EGARCH(1,2) has only one noise, so it is entirely possible that this could help GARCH. Neither assumed model has the more complex asymmetric lag structure of the EGARCH(1,2).

In section 2.2, we present a survey of the literature and section 2.3 describes the models in order to analyse their statistical attributes. In section 2.4, we provide context by considering and analysing equity and bond return data and the relevant markets to motivate our choice of "true" parameter values. This involves an inspection of the economic history of US financial macroeconomics. Turning to section 2.5, we discuss the various metrics used to assess the forecasting abilities of the two competing models. In section 2.6, we outline in detail the exact simulation method deployed and present our results with analysis. Section 2.7 concludes the chapter.

2.2 LITERATURE REVIEW

Research activity that is focused on constructing, analyzing and evaluating non-linear time-series models of variance and covariance has increased significantly over the last two decades. The importance of variance in both theory and application is paramount. For instance, variance is the only unknown variable that drives the pricing of contingent assets, such as the European and American options which are often used by market participants for both hedging and speculation purposes[46]. In essence, options value the volatility of the return of underlying security rather than its mean.

[46]Trading of option contracts in modern financial markets encompasses a broad range of underlying securities (both financial and real). See semi-annual BIS Survey (May 2014) for more details. Moreover, exchanges such as the CME Group which facilitate trading of listed-option contracts predominantly offer European or American type products.

Moreover, volatility of the return of risky securities plays an important role in the calibration of various risk management frameworks, such as those based on the Value-at-Risk methodology[47]. Here, the objective of the risk management exercise is to estimate the *ex-ante* risk profile of portfolios of risky assets, where the consolidated tracking error or absolute volatility is the key variable of interest.[48]

Given the forward-looking nature of both option pricing/trading and risk management exercises, it is imperative to model the dynamics of variance. This is especially true given that financial time series, such as equity, foreign exchange and fixed income returns, rarely exhibit constant variance[49]. Turning to additional empirical stylised facts exhibited by the volatility of key financial time series, studies such as Shephard (1996) note the presence of heavy tails (which are reflected in very large standardised fourth moments) and clustering[50] (in which periods of large moves tend to be followed by periods of similar characteristics). This is linked to the presence of strong autocorrelation characteristic displayed by squared returns at extended lags.[51]

Indeed, the existence of stylised attributes listed above, when considering the behaviour of volatility with respect to time can create serious issues with the usage of simple specifications. These specifications include random walk and historical moving averages (including those which are exponentially-weighted) as reliable volatility forecasting mechanisms.

Again, as noted by Shephard (1996), there are various variance modelling methodologies, which attempt to explicitly account for stylised characteristics displayed by the behaviour of variance of financial and economic time series. Following Cox (1981), these methodologies can be conceptually divided into either belonging to observation-driven or parameter-driven categories.

In the observation-driven category, the autoregressive conditional heteroscedasticity (ARCH) model developed by Engle (1982) dominates the field. Specifically, the ARCH model allows the variance of the return process to be a linear function of lagged squared returns. Not surprisingly, ARCH-type models have attracted significant attention in recent years, especially given their similarity with the moving-average type models used for capturing changing means.

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[47] The usage of VaR as a risk assessment metric really took off during the late-1990s, when JP Morgan released estimates of variance and covariance of various securities and asset classes. Given its intuitive appeal, over the last decade, VaR has become the established measure of market risk exposure in the global financial industry.

[48] *Ex ante* absolute volatility is the target risk assessment variable for absolute return strategies, such as global macro, equity long/short etc and is important for financial institutions, such as banks and insurance companies which have a regulatory duty to measure market risk embedded in their balance sheets.

[49] For example, see Taylor (1986).

[50] This was first studied by Mandelbrott (1963).

[51] This feature is visible in the correlogram of squared returns of financial time series, such as equity returns.

An important extension of the ARCH framework is the Generalised ARCH model (or GARCH)[52], which models the variance of the underling return process to be a linear function of both lagged squared observations and variance. The GARCH (1,1) model (see section 2.3 for exact specification) has had tremendous success in empirical work and, as Shephard (1996) outlines, is usually considered as the benchmark model by many econometricians.

In terms of further important extensions of the ARCH framework, the EGARCH model developed by Nelson (1991) has also had a significant impact on the preferred method of modelling and forecasting financial time series volatility. Specifically, the EGARCH specification (see section 2.3 for details on exact specification) allows the variance process to respond to asymmetric shocks to the underlying stochastic series. The ability to let the variance process respond differently to rise or fall in financial time series (such as equity returns) is particularly useful. For instance, as noted first by researchers such as Black (1976), Schwert (1989a, 1989b) and Sentana (1991), equity return volatility tends to be significantly higher during periods of negative returns compared to periods when relative price changes are positive[53].

Focusing on the fundamental drivers of this important asymmetry, the leverage skew argument discussed by Geske et al (1984)–whereby a firm's value can be seen as net present value of future income plus assets minus liabilities– can explain part of the irregularity seen in the behaviour of equity returns volatility during periods of rising and falling stock prices. These various components have different volatilities and can lead to leverage-related skew.

Moreover, as noted by Schwert (1989b), firms operating in high fixed cost environments can also lead to an operational leverage effect, as the sensitivity of near-term earnings to business cycle gyrations increases during recession periods, with final sales falling and the cost base responding with a lag. Furthermore, in asset markets such as equities, it is significantly more important to hold downward protection, given the systematic long held by long-term investors such as pension funds and insurance companies coupled with inability of certain type of investors to undertake outright short positions, such as retail investors.

Finally, regulatory and risk management frameworks can exacerbate volatility during negative return periods. This situation forces position liquidation as certain thresholds are hit. For example, forced portfolio shifts on the back of changes in ratings of underlying securities. For instance, Gande and Parsley (2004) showed that the asymmetric impact of sovereign rating changes on the size and

[52] This observation is usually attributed to Bollerslev (1986), but was developed simultaneously by Taylor (1986).
[53] This is commonly referred to as the "leverage effect".

direction of equity capital flows in 85 countries using the 1996-2002 sample period. Their empirical study found that rating downgrades led to significantly higher capital outflows, while the response to upgrades was more muted. In addition, they also reported that lower levels of corruption decreased the response, whereby countries with less corruption experienced smaller outflows around rating downgrade actions.

In asset management space, which essentially embeds a principal-agent set-up, the tendency to use Value-at-Risk mechanisms to transparently manage market exposure risk can increase volatility in falling markets and attenuate it during rising markets. For example, Basak and Shapiro (2001) use a utility maximising framework to show that investors using a Value-at-Risk method to manage market exposures tend to take larger risk positions and as a result, incur heavier losses when markets turn against them.

In FX markets, clear fundamental drivers of this type of asymmetry often exist in emerging markets, in the form of a stronger tendency of relevant authorities to intervene in foreign exchange markets[54]. These markets tend to view the FX rate as a policy tool. Likewise, in interest rate markets central bank actions and communication can also lead to an asymmetry of response in a certain direction.

Turning to the parameter-driven variance modelling category, state-space models allow the variance to be a function of some unobserved or latent component. The stochastic volatility model (see section 2.3 for exact specification of the SV model) is an example of such a state-space set-up. The usage of this model within econometrics is usually associated with the work of Harvey (1996). Specifically, the stochastic volatility technique fits a model to the variance of the series of interest, by treating it as an unobserved random variable which follows a stochastic process. To ensure that the variance is always positive, a stochastic process is set up for the logarithm of variance. Despite the difficulty in estimating an exact likelihood function, the key attraction of the SV model lies in its connection with the Orstein-Uhlenbeck diffusion process used in finance theory. Indeed, using the Edgeworth expansions, Dassios (1992) shows that the volatility formulation depicted in the SV model is a better approximation of the continuous time Ornstein-Ulhenbeck process observed at discrete intervals, rather than the EGARCH model. Within the literature on options pricing, increased attention has recently been directed at examining the implications of non-linear volatility models on option prices. Duan (1995) developed the option pricing framework using ARCH in an equilibrium setting, which was further augmented by Kallsen et al (1995) in an arbitrage free continuous time setting. In terms of evaluating the ARCH based

[54] The Chinese Yuan is a good example of a heavily managed currency with frequent Central bank-induced changes in the exact mechanics of the managed float.

framework, studies such as Satchell et al (1993) and Amin et al (1993) found that the GARCH based option pricing model produced a better fit to market prices than the Black-Scholes model. Although GARCH-type models do a good job of depicting foreign exchange dynamics, as noted above, the presence of “volatility skew” in equity space requires additional assessment. Here, studies such as Schmidt (1996) extend the option pricing framework further to incorporate EGARCH effects in the volatility process. Furthermore, the stochastic volatility process has also been deployed to improve the option pricing framework, with studies such as Heston (1993) providing a neat closed-form solution for options with stochastic volatility.

Given the central role of volatility calibration in option pricing/trading and risk management systems discussed above, evaluating the forecasting ability of various volatility models also forms an important area of research. Poon & Granger (2003) provide a summary of 93 research papers which focus on the forecasting performance of various volatility methodologies. Conclusions based on comparison exercises carried out in the different studies vary and also depend on the nature of the asset class studied coupled with the exact forecasting evaluation metric(s) used. All in all, as Poon et al (2003) noted, given the complexity of the issues involved and importance of volatility measure, volatility forecasting continues to remain a specialist subject area that attracts a vigorous research focus.

2.3 MODEL SPECIFICATIONS

We first describe our true model, which is the EGARCH (1,2) model:

$$x_t = \sigma_t \varepsilon_t \tag{2.1}$$

$$ln(\sigma_t^2) = a + \beta ln\,(\sigma_{t-1}^2) + \sum_{k=1}^{2} \alpha_k \left[\lambda \varepsilon_{t-k} + \delta\left(|\varepsilon_{t-k}| - E(|\varepsilon_{t-k}|)\right)\right] \tag{2.2}$$

Essentially, the δ(.) function in equation (2.2) allows both the size and sign of its argument to affect its value. Given the addition of δ(.) in the variance term, when

$\varepsilon_{t-k} > 0$, $d\sigma_t/d\varepsilon_{t-k} = \sum_{k=1}^{2} \alpha_k [\lambda + \delta]$, while the derivative is $\sum_{k=1}^{2} \alpha_k [\lambda - \delta]$, when $\varepsilon_{t-k} < 0$.

The explicit ability of the model to incorporate an asymmetric response of variance to the sign of the underlying stochastic disturbance is particularly useful. As discussed in detail in section 2.2, the irregularity which is observed in the behaviour of the volatility is an important empirical stylised fact of several asset markets ad is driven by a number of fundamental and behavioural factors.

In terms of statistical properties, if ε_{t-1} is distributed i.i.d, therefore δ (.) is also i.i.d. Equation (2.2) also has a constant mean and variance. In addition, ε_t is uncorrelated with ($|\varepsilon_t| - E|\varepsilon_t|$) , given the symmetry of ε_t. As a result, it is clear to see that (2.2) is an autoregression and σ_t is stationary as long as $|\beta| < 1$ (this condition allows asymptotic normality to be achieved as well).

We now examine returns which are modelled via stochastic volatility, considering both popular specifications. These specifications only differ in the way that the return is correlated with the latent volatility, which is itself a stochastic process.

Model A – Contemporaneous correlation

$$x_t = \sigma_t \varepsilon_t \tag{2.3}$$

$$\ln(\sigma_t^2) = \alpha + \beta \ln(\sigma_{t-1}^2) + \nu_t \tag{2.4}$$

where:

$$\begin{pmatrix} \varepsilon_t \\ \nu_t \end{pmatrix} \sim \text{iid N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \right) \tag{2.5}$$

Model B – Lagged inter-temporal correlation

Here we merely replace equation (2.4) with the below and leaving equations (2.3) and (2.5) unchanged.

$$\ln(\sigma_{t+1}^2) = \alpha + \beta \ln(\sigma_t^2) + \nu_t \tag{2.6}$$

We notice immediately that for Model B, ε_t and σ_t are independent irrespective of the value of ρ, given ε_t and ν_t are i.i.d. In what follows, we use model B. In this paper, we consider the case where ρ = 0, so that in either model ε_t and σ_t are independent.

We now consider Model B, where the dynamic properties of the SV model become clearer after using a log transformation on x^2_t:

$$\ln x^2_t = \ln \sigma^2_t + \ln \varepsilon^2_t \tag{2.7}$$

From (2.6) and (2.7) we notice that $\ln(x_{t+1}^2)$ is given as the sum of two components an AR(1) and a white noise. Consequently, its ACF is equivalent to that of an ARMA(1,1).

In fact the precise form of the ARMA(1,1), in terms of (2.6) and (2.7) is given by (2.8):

$$\ln(x_{t+1}^2) = \alpha + \beta \ln(x_t^2) + \nu_t + l\,n(\varepsilon_{t+1}^2) - \beta \ln(\varepsilon_t^2) \tag{2.8}$$

We now examine this equation under different distributional assumptions on the two errors v_t and ε_t. In particular, it shows how distributional assumptions feed into the ARMA structure.

Theorem 1.

Assuming $v_t \sim iid\ (0, \sigma_v^2)$, $\varepsilon_t \sim iid\ (0,1)$ and the mean and variance of $\ln(\varepsilon_t^2)$ are given by μ and δ^2, where $\mu = \mathrm{K}'(0)$ and $\delta^2 = \mathrm{K}''(0)$, where$\mathrm{K}^i(s)$ is the cumulant generating function of $\ln(\varepsilon_t^2)$.

Using the above notation, the ARMA(1,1) representation of (2.8) is:

$$\ln(x_{t+1}^2) = \alpha \ + K'(0)\,(1-\beta) + \beta \ln(x_t^2) + w_{t+1} - q w_t \qquad (2.9)$$

where w_t and w_{t+1} are white noise processes with zero mean and variance equal to

$$d^2 = \frac{\sigma_v^2 + (1+\beta^2)K''(0) + (\sigma_v^4 + (1-\beta^2)^2 (K''(0))^2 + 2\sigma_v^2 (1+\beta^2) K''(0))^{.5}}{2}$$

And $q = \dfrac{2\,\beta K''(0)}{\sigma_v^2 + (1+\beta^2)K''(0) + (\sigma_v^4 + (1-\beta^2)^2 (K''(0))^2 + 2\sigma_v^2 (1+\beta^2) K''(0))^{.5}}$

Proof: See Appendix 2.3.

In the following corollaries, we specialise the result of the Theorem for particular distributions of v_t and ε_t. Their proofs are also in Appendix 2.3, which is noted above.

Corollary 1

When v_t and ε_t are both normally distributed but independent, $\varepsilon_t^2 \sim \chi_{(1)}^2$ and consequently $\ln(\varepsilon_t^2) \sim (-1.27, 4.93)$. The result in the lemma holds with $\mu = -1.27$ and $\delta^2 = 4.93$.
The above results verify the assertions of Harvey, Ruiz, and Sheppard (1994).

Corollary 2

When v_t and ε_t have independent t-distributions with n and m degrees of freedom, we need to respectively scale them to ensure that their variances are σ_v^2 and 1. The results in the lemma now hold with $\mu = K'(0) = \ln(m-2) + \psi\left(\frac{1}{2}\right) - \psi\left(\frac{m}{2}\right)$ and $\delta^2 = \psi'\left(\frac{1}{2}\right) + \psi'\left(\frac{m}{2}\right)$

Where $\psi(\cdot)$ and $\psi'(\cdot)$ are the digamma and trigamma functions respectively. ■

It is worth investigating how q changes with $\delta^2 = \mathrm{K}''(0)$ - We see that an increase will lead to an increase in q. This supports the notion that the more non-normality exists in the underlying process, the more auto-correlation appears in the derived dynamic process. Of course, these remarks are predicated by a number of implicit assumptions; namely, that q is less than one and that an increase in δ^2 is an increase in non-normality. We also note that an increase in δ^2 leads to an increase in d.

For the case of normality, we discuss how we would forecast volatility. Whilst we can recover our original parameters from this structure, as the three parameters are identified from a forecasting perspective, we need only to consider the conditional mean one period ahead of $\ln(x_{t+1}^2)$ which under normality is given by (2.10):

$$\alpha - 1.27(1-\beta) + \beta \ln(x_t^2) - q\hat{z}_t \tag{2.10}$$

Since our model is ARMA(1,1), there are appropriate formulae for k-period ahead forecasts. To recover our forecast of $\ln(\sigma_{t+k}^2)$ we simply add 1.27 to our original forecasts. Finally, we exponentiate our answer to convert our forecast to a volatility forecast. There is an issue here which pertains to whether we adjust the bias, which we do not address. If the ε_t and v_t terms are allowed to be correlated with each other via ρ, then in a manner similar to the EGARCH model, the SV model also allows asymmetric response of variance to the sign of the innovation in the underlying series of interest, see Harvey and Shepherd (1996). In fact, a negative correlation coefficient between v_t and ε_t generates the "leverage effect". More generally, $\ln(\sigma_t)$ can follow any stationary ARMA process, in which case x_t is also stationary and its properties depend on the dynamic properties of $\ln(\sigma_t)$. Alternatively, $\ln(\sigma_t)$ can also be allowed to follow a random walk process:

$$\ln(\sigma_{t+1}^2) = \alpha + \ln(\sigma_t^2) + \nu_t \ , \ v_t \sim \ \mathrm{NID}(0, \sigma^2_v) \tag{2.11}$$

In the above case, log x^2_t has two components: a random walk and a white noise. This specification is very similar to the Integrated GARCH[55] model, as both models share the same best linear unbiased predictor (see Harvey *et al* (1994)). However, the crucial difference between the two is that the variance in the random walk SV model is an unobserved component, whereas in the IGARCH model, it is exactly known.

[55] For further details on this model, see Nelson (1990).

Model C. We contrast the SV(1,1) model with the GARCH(1,1) model which, in our current notation is given by:

$$\sigma_t^2 = \alpha + \beta \varepsilon_{t-1}^2 \sigma_{t-1}^2 + \delta \sigma_{t-1}^2 \tag{2.12}$$

As noted in section 2.1, the GARCH model is an important extension of the ARCH modelling methodology, which includes moving average terms in the variance process. The main advantage of using the GARCH formulation compared to ARCH lies in its ability to capture serial correlation in ε^2_t terms with a smaller number of parameters.

It is important to note that that in the GARCH model, the response of conditional variance to underlying innovations depends on the latter's size and not its sign, unlike in the EGARCH formulation.

The Existence of Moments

The issue of the existence of moments turns out to be highly relevant when forecasting fat-tailed distributions. We show in this sub-section exactly why this is important in the context of our problem. Considering log-volatility processes generally:

$$r_t = \vartheta + u_t \tag{2.13}$$

$$ln(\sigma_t^2) = a + \beta ln\,(\sigma_{t-1}^2) + V_t \tag{2.14}$$

say so $|\beta| < 1$

$ln(\sigma_t^2) = \frac{a}{1-\beta} + \sum_{j=0}^{\infty} \beta^j\, V_{t-j}$ is a representation of the steady-state distribution of $\ln(\sigma_t^2)$. This case covers both EGARCH(1,1) and SV(1) models. This also covers EGARCH (1, 2) models as used by Pagan and Schwert (1990):

$$ln(\sigma_t^2) = a + \beta ln\,(\sigma_{t-1}^2) + \sum_{k=1}^{2} \alpha_k \left[\lambda \varepsilon_{t-k} + \delta\left(|\varepsilon_{t-k}| - E(|\varepsilon_{t-k}|)\right)\right] \tag{2.15}$$

Where $\varepsilon_t = \frac{u_t}{\sigma_t}$ and u_t is the error in the returns equation and $E(u_t^2/F_t) = \sigma_t^2$.

The calculations that we present below will follow for those models, once we compute the moving average representation of the process. We present the following details.

Using the independence of V_{t-j},

$$\sigma_t^2 = ex\,p\left(\frac{a}{1-\beta}\right), exp(\textstyle\sum_{j=0}^{\infty} \beta^j\, V_{t-j}) \tag{2.16}$$

$$= exp\left(\frac{a}{1-\beta}\right) \prod_{j=0}^{\infty} exp(\beta^j V_{t-j}) \tag{2.17}$$

And hence $E(\sigma_t^{2s})= exp\left(\frac{sa}{1-\beta}\right)\prod_{j=0}^{\infty} E(exp(\ s\beta^j V_{t-j}))$ (2.18)

Now, if the moment generating function (mgf) of V_t exists then $M_V(s\beta^j)$ = $E\ (exp\big(s\beta^j V_{t-j}\big))$ exists with possibly some restrictions on s and β.

Therefore:

$$E(\sigma_t^{2s})= exp\left(\frac{sa}{1-\beta}\right)\prod_{j=0}^{\infty} M_V(s\beta^j) \tag{2.19}$$

We now consider the EGARCH(1,2) under normality and derive $E(\sigma_t^{2s})$ for that case.

Here;
$$V_t = \sum_{k=1}^{2} \alpha_k(\lambda\varepsilon_{t-k} + \delta(|\varepsilon_{t-k}| - E(|\varepsilon_{t-k}|)))$$

Under normality, the existence of the moment generating function implies the existence of moments of all orders for both the SV model and the EGARCH model. In particular, for EGARCH we need expressions for half-normal moment generating functions, i.e. if $V \sim N(0, \sigma_v^2)$.

$$M_v^+(t) = \ E(ex\,p(tV)\,/V > 0) \tag{2.20}$$

$$= 2exp(\frac{\sigma_v^2 t^2}{2})(1 - \Phi(-\sigma_v t)) \tag{2.21}$$

And $M_v^-(t)$ = $2exp(\frac{\sigma_v^2 t^2}{2})(1 - \Phi(\sigma_v t))$

Using such calculations, we arrive at the formula below, see Appendix 2.4.

Theorem 2.

For EGARCH(1,2), the sth moment of SV, where

$$\gamma_1 = \alpha_1,\ \gamma_2 = (\alpha_1\beta + \alpha_2)\ \gamma_j = (\alpha_1\beta + \alpha_2)\beta^{j-2},\ j \geq 3$$

Is given by:

$$E(\sigma_t^{2s}) = \exp(\frac{s\alpha}{1-\beta})\exp(\frac{-s\delta\sqrt{2/\pi}(\alpha_1+\alpha_2)}{1-\beta})\prod_{j=0}^{\infty}[\sum_{k=1}^{2}\exp(\frac{b_k^2\gamma_j^2}{2})\Phi(b_k\gamma_j)] \tag{2.22}$$

However, for the mgf to exist and be finite, all moments of V_t must be finite and if V_t does not have finite moments of all orders, $E(\sigma_t^{2s})$ will not be finite. This has implications for using MSE and similar measures to evaluate forecasting accuracy. In the case of, for example, the EGARCH model having t-distributed errors, we may need to use a different criterion for assessing forecast accuracy.

2.4 ESTIMATING "TRUE" PAREMETER VALUES

In this section, we use certain financial series of interest to extract parameter values for the true model–namely EGARCH(1,2)–which are then used in the forecast quality assessment exercise carried out in section 2.6. Given their unmatched importance in global asset markets, we focus on S&P 500 (which is the key US equity market benchmark) and US 10-year bond market returns.

We have included bond market data in our analysis as well, given the scant attention which is paid to this asset class in volatility forecasting literature. The main focus in this literature has traditionally been based on equity and foreign exchange universes. For bond returns, we have used the zero-coupon 10-year bond yield and converted it into price to generate returns[56] using (2.23):

$$y_t = \left[\left(\frac{F}{PV}\right)^{1/n}\right] - 1 \qquad (2.23)$$

where, y_t is the zero coupon bond yield, F is the face value of the bond, PV is the present value or the current price and n = number of periods.

US Equity Returns

First, we hall begin by focusing on data sources. We have used Global Financial data and Bloomberg databases[57] to extract S&P 500 and US 10-year bond yield data since September 1791. The relevance of using pre-war data in analyzing current asset price dynamics has increased in the post-2007/8 crisis world. For instance, high profile asset managers such as Pacific Investment Management Company (PIMCO) have characterized the post-crisis world as "the new normal", whereby advanced economies such as the US are likely to experience lower trend economic growth for an extended period of time. Put another way, PIMCO's main contention is that historical analysis carried out using post-war economic and market data is losing relevance, as a result of structural shifts in the global economy seen over the last six years. Given this backdrop, a key objective of our empirical analysis is to highlight and discuss any shift, when compared to recent and long-term historical data, in model parameters that are visible in post-2007/8 crisis data.

Starting with S&P 500 monthly returns, we fitted both EGARCH(1,1) and EGARCH(1,2) and compared the log-likelihood statistic of the two competing models in order to ascertain the appropriate specification. The resultant log-likelihood goodness-of-fit ratio test statistic yielded clear preference for the EGARCH(1,2) model[58].

[56] We also take account of carry in bond return calculation.
[57] See Appendix 1.1 for additional details on underlying data sources.
[58] Modelling exercises shows log likelihood of 5199.4 for EGARCH(1,2) vs 5191.9 for EGARCH(1,1) using the September 1791-May 2014 sample period under the normal distribution assumption. Performing the Log-likelihood ratio test yielded a statistic of 15.0 vs 5% critical value of 5.991 for Chi-square-2 distribution.

Table 2.1 S&P 500 Returns – normal distribution specification[59], EGARCH(1,2)

Period	θ_1	θ_2	λ_1	λ_2	β	Log Likelihood	AIC
September 1791 - May 2014	**0.25**	-0.01	**-0.13**	**0.1**	**0.98**	5199.4	-3.88
	3.2	-0.17	-2.4	1.95	101.4		
January 1834 - December 1925	**0.38**	**-0.11**	**-0.08**	0.02	**0.92**	2104.8	-3.8
	4.04	-1.2	-2.4	0.4	22.9		
January 1925 - December 1939	-0.23	**0.67**	**-0.32**	**0.24**	**0.96**	215.5	-2.32
	-1.5	3.6	-3.0	2.0	44.9		
September 1791 - December 1949	**0.39**	-0.17	**-0.04**	0.01	**0.98**	3821.3	-4.0
	3.7	-1.6	-2.3	0.73	133.2		
January 1950 - May 2014	-0.03	**0.25**	**-0.33**	**0.26**	**0.90**	1407.4	-3.6
	-0.32	2.8	-4.4	2.7	20.5		
January 2000 to Dec 2007	-0.44	**0.2**	**-0.06**	**-0.27**	**0.88**	191.6	-3.8
	2.1	1.0	-0.6	-2.5	31.2		
January 2008 to May 2014	-0.28	0.09	**-0.7**	**0.5**	**0.95**	138.6	-3.4
	-1.2	0.38	4.6	3.2	45.1		

*bold enteries display coefficients that are significant at 5% level

Table 2.2 S&P 500 – t-distribution specification, EGARCH(1,2)

Period	θ_1	θ_2	λ_1	λ_2	β	Log Likelihood	T dist D.o.F	AIC
September 1791 - May 2014	**0.23**	0.00	**-0.15**	**0.12**	**0.98**	5276.8	6.2	-3.94
	4.7	0.0	-4.3	3.7	242.9			
January 1834 - December 1925	**0.37**	**-0.09**	**-0.07**	0.01	**0.90**	2120.8	8.0	-3.8
	5.0	-1.1	-1.5	0.23	27.9			
January 1925 - December 1939	-0.14	**0.53**	**-0.35**	**0.26**	**0.96**	216.0	10.6	-2.31
	-0.75	2.6	-3.1	2.5	35.90			
September 1791 - December 1949	**0.34**	**-0.12**	**-0.07**	0.05	**0.98**	3894.4	5.1	-4.1
	5.7	-2.1	-2.0	1.30	257.1			
January 1950 - May 2014	-0.03	**0.25**	**-0.29**	**0.2**	**0.89**	1414.6	11.6	-3.6
	-0.3	2.3	-4.9	2.9	20.0			
January 2000 to Dec 2007	-0.4	0.33	-0.07	-0.24	**0.88**	190.3	311.1	-3.8
	-1.06	1.0	-0.32	-0.93	18.8			
January 2008 to May 2014	-0.31	0.2	**-0.66**	0.45	**0.94**	137.1	257.8	-3.35
	-0.6	0.4	-2.2	1.9	27.1			

*bold enteries display coefficients that are significant at 5% level

Focusing on the full sample period results (September 1791 – May 2014) shown in tables 2.1 & 2.2, a number of salient points emerge:

1. First, we find evidence of t-distribution in the error process with degrees- of-freedom of around 6 with a p-value of 0.0%. This result confirms the presence of a fat-tail in the error process.

2. There is also clear evidence of "leverage effects" in the volatility process with positive shocks associated with a coefficient of 0.20 vs 0.26 for negative shocks using the t-distribution specification and 0.21 vs 0.27 respectively, for the normal distribution specification. Put simply, the results confirm that negative shocks have a stronger impact on equity return volatility compared to positive return shocks. This highlights the usefulness of using EGARCH methodology when modelling the equity market volatility process.

[59] This is based on using HAC Adjustment for hypothesis testing.

3. Finally, there is evidence of high autocorrelation in the volatility process of equity returns with an estimated beta parameter of 0.98. However, standard Wald restriction test rejected the β = 1 null hypothesis for both normal and t-distribution specification at 5% level of significance.

Turning to the various sub-sample period results, we first focus on the January 1834 to December 1925 sample, which is the period studied by Pagan and Schwert (1990). The magnitude and sign of leverage effects estimates are similar to Pagan et al (1990)'s findings, though the size of the beta parameter was ascertained to be higher.

Comparing the t-distribution specification's estimation results from the pre- to the post-war period, the fall in the size of the beta parameter (from 0.98 to 0.89 respectively) is quite striking. The numerous episodes of deep equity market corrections captured during the pre-war period (particularly driven by the inclusion of the Great Depression period within the sample) may explain this finding. Indeed, comparing the pre-(2000/2007) with the post-Great Recession period (2008/2014) yields a similar pattern. For instance, using the 2000/2007 period, the β parameter is estimated to be 0.88 compared to 0.94 for the subsequent January 2008- May 2014 sample.

Focusing on the volatility dynamics observed during the Great Depression period (Jan 1925 - December 1939), the beta parameter is estimated as 0.96[60], with the standard Wald statistic failing to reject the β =1 null hypothesis resulting in a p-value of 14.5%. This result is in line with observations made by Pagan et al (1990), who noted that the stationarity property of the volatility process seems to be rejected by the data during the Great Depression period.

By looking at more recent data, we found evidence that the volatility process remained stationary over the January 2008-May 2014 period, unlike the Great Depression period. As noted above, however, we did find a sharp rise in the degree of autocorrelation in the volatility process, when compared to the pre-crisis period.

Studying the size and sign of leverage effects using the pre- and post- Great Recession period data also generated a number of interesting observations. First, the leverage effect parameters (λ and δ) were ascertained to be statistically insignificant using the pre-Great Recession period. Second, in terms of magnitude, it appears that a positive return shock had a negative impact on volatility (coefficient of -0.38), while a negative return shock was associated with a coefficient of +0.24, suggesting an increase in volatility. Shifting to the January 2008- May 2014 period, the coefficient associated with a positive

[60] Using the t-distribution specification. The same conclusion holds for normal distribution specification as well.

shock was ascertained to be -0.32, while it was estimated as +0.10 for negative return shocks. It is worth noting, however, that only λ_1 was determined to be statistically significant at the 5% level.

Taking into account the statistical significance of the parameters, it appears that leverage effects were missing during the pre-crisis period (where all coefficients are statistically insignificant), while they were estimated as -0.66 (λ_1) for positive shocks and +0.66 for negative shocks during the January 2008 to May 2014 period.

If we compare the above results with both the full sample and post-war sample-based estimates, it is interesting to note that the impact of positive return shocks on equity return volatility appears to have shifted in its sign. Specifically, using recent data, the empirical exercise shows that positive return shocks are no longer associated with an increase in volatility, and may actually be consistent with a reduction in volatility. Another noticeable attribute of the sample period observed since the start of the 21st century is the absence of fat-tails in the error process.

In our view, these attributes appear to be connected to the emergence of major boom-bust type cycles in the price of equities in a number of advanced economies including the US since the 1980s (see Borio et al (1994)). The main property of asset markets (in our case, equities) experiencing boom-bust cycles is that they undergo periods of sustained gain, thus creating a “bubble” followed by subsequent price corrections. The sustained and steady nature of the returns experienced during the boom-phase of the cycle can explain the reduction or lack of response of volatility to positive shocks. Furthermore, the relative lack of absence of very sharp corrections, such as the October 1987 crash in the more recent sample period, can explain the absence of t-distribution type effects in the error process. For instance, table 2.3 below shows the estimated kurtosis of squared returns (as a proxy for unconditional variance) for the various sample periods studied. Here, what is clear is the relative decrease in the magnitude of kurtosis visible in the more recent periods, which corroborates with evidence of lack of fat-tails found in the volatility process.

Table 2.3 Kurtosis of squared equity returns

Kurtosis of Squared Returns	
September 1791 - May 2014	175.7
January 1950 - May 2014	108.3
January 2000 - December 2007	8.9
January 2008 - May 2014	29.9

"*Stability is destabilising*". These three words succinctly convey a view first put forward by Hyman Minsky (1975, 1982, 1986). The basic thesis of this idea is that the institutional support provided to backstop and stabilise asset price discovery mechanisms in the aftermath of a crisis can change behaviour in such a manner that it supports the creation of future speculative bubbles. We think this idea may be at play, when volatility falls in response to positive return shocks. For instance, if the Sharpe ratio associated with equity investing, aided by both the magnitude of the return as well as reduction in volatility, starts to rise sharply during bull market periods then it can potentially create a view reinforcement mechanism, which then attracts additional demand for the risky asset. Indeed, the resultant self-enforced view based flow can then contribute to a bubble creation[61], which is then eventually corrected.

US 10-year Bond Returns

Tables 2.4 & 2.5 show the parameter estimates of fitting a EGARCH(1,2) model to US 10-year bond returns data. As noted above, bond returns were calculated using the 10 year zero coupon bond yield data available from Global Financial Data and Bloomberg. Similar to the case of equities, data availability means we are able to run the regression from September 1791 to May 2014 using monthly data.

[61] A bubble creation is defined as a situation when the valuation of the asset class in question starts to show a de-link with underlying fundamentals.

Table 2.4 US 10yr Bond Returns - normal distribution specification, EGARCH (1,2)

Period	θ_1	θ_2	λ_1	λ_2	β	Log Likelihood	AIC
September 1791 - May 2014	**0.55**	-0.44	**-0.37**	**0.29**	**0.99**	8169.0	-6.1
	4.8	-3.8	-3.6	2.9	447.3		
January 1834 - December 1925	**0.6**	**-0.44**	**-0.6**	**0.52**	**1.00**	3810.3	-6.9
	3.9	-2.6	-4.6	3.9	286.4		
September 1791 - December 1949	**0.57**	**-0.45**	**-0.49**	0.4	**0.99**	6352.9	-6.7
	4.81	-3.9	-4.3	3.60	396.0		
January 1950 - May 2014	**0.34**	-0.06	-0.003	-0.07	**0.98**	1857.6	-4.8
	3.4	-0.6	-0.05	-1.1	144.1		
January 2000 to Dec 2007	-0.37	0.13	**0.22**	**0.46**	-0.21	220.8	-4.45
	-1.9	0.5	2.0	2.41	-0.7		
January 2008 to May 2014	0.09	**-0.58**	0.22	0.31	**0.63**	173.3	-4.32
	0.28	-1.96	1.1	1.2	3.4		

*bold enteries display coefficients that are significant at 5% level or lower

Table 2.5 US 10-year Bond Returns- t-distribution specification EGARCH (1,2)

Period	θ_1	θ_2	λ_1	λ_2	β	Log Likelihood	T dist D.o.F	AIC
September 1791 - May 2014	**0.42**	**-0.23**	**-0.12**	**0.07**	**0.99**	8659.4	3.1	-6.47
	8.4	-4.8	-3.3	2.0	636.1			
January 1834 - December 1925	**0.46**	**-0.27**	-0.06	0.01	**0.99**	4204.8	2.4	-7.6
	4.2	-2.97	-0.96	0.16	317.7			
September 1791 - December 1949	**0.45**	**-0.23**	**-0.17**	**0.12**	**0.99**	6822.8	2.6	-7.2
	6.2	-3.6	-3.2	2.50	345.9			
January 1950 - May 2014	**0.33**	-0.07	0.03	**-0.1**	**0.99**	1874.1	7.0	-4.8
	4.04	-0.8	0.5	-1.9	154.8			
January 2000 to Dec 2007	-0.36	0.14	0.22	**0.46**	-0.20	220.8	340.8	-4.4
	-1.1	0.35	0.84	2.89	-0.44			
January 2008 to May 2014	0.11	**-0.56**	0.22	0.3	**0.63**	173.4	27.9	-4.3
	0.31	-1.9	1.03	1.05	3.2			

*bold enteries display coefficients that are significant at 5% level or lower

We now turn to the model's parameter estimates. Diagnostic tests show evidence of better goodness-of-fit for the EGARCH(1,2) specification when compared to EGARCH(1,1).[62] As in the equities case, the goodness-of-fit is here assessed on the basis of the log-likelihood ratio statistic. This approach is appropriate in this setting given the nested nature of the two competing models of interest.

When analysing the EGARCH(1,2) fitted model parameters, three key points emerge on the basis of the exercise carried out using the September 1791 to May 2014 sample period (or the full sample). These three points are the following:

1. The t-distribution error process version of the EGARCH(1,2) estimation leads to a better in-sample model fit, when assessed using the minimum AIC criterion. The degrees-of-freedom

[62] The EGARCH(1,1) results are available upon request.

of the underling t-distribution is ascertained to be around 3 which thus indicates the presence of fat-tails in the error process.

2. Second, there is evidence of leverage effects in the bond market with negative return shocks (or an increase in yields). This yields a coefficient of 0.19 vs 0.03 for positive return shocks using the normal error distribution specification and 0.24 vs 0.14 respectively for the t-distributed error specification.

3. The β parameter is estimated to be around 0.99 for both normal and t- distributed specifications, indicating evidence of high autocorrelation in bond market volatility process. However, standard Wald statistic based hypothesis testing shows that the null hypothesis of β =1 restriction has a p-value of 0.0%, thus rejecting it at 5% level of significance.

Turning to the various sub-sample estimation results, it is interesting to note that during the pre-Great Financial Crisis period (2000 to 2007), the beta parameter was found to be statistically insignificant for both the normal and t-distribution specifications, unlike both the full and post-war period sample based estimates. Meanwhile, evidence of leverage effects was also found to be statistically weak. In addition, there was no statistical evidence supporting a t-distribution error process during this period, as t-distribution's degrees-of-freedom was ascertained to be 341 with a p-value of 99%.

The absence of autocorrelation in the volatility process during the 2000-2007 period is quite striking. In our view, this attribute of bond market volatility process can be explained by the "global savings glut" dynamic and the relatively steady nature of Federal Reserve's monetary policy witnessed over this period.

The global savings glut hypothesis was explained in considerable detail by former Federal Reserve chairman Ben Bernanke in a speech delivered at the Sandridge Lecture for the Virginia Association of Economists in 2005.[63] In his remarks, Bernanke postulated that a significant increase in the flow of international savings had been finding its way into US debt markets during that period, thus creating a fundamental de-link between domestic US macro fundamentals and the yield curve. The flipside of this dynamic was the large current account deficit being run by the US economy over this period.[64] Bernanke pointed to two important drivers behind this crucial development. The first of these was an enhanced saving motive for rich countries with aging populations. The second was an increase in desired savings by developing countries as they switched from a net user to net supplier of funds to

[63] Full text of the speech available on Federal Reserve website.
http://www.federalreserve.gov/boarddocs/speeches/2005/200503102/
[64] Using IMF Data

international capital markets during the aftermath of the Asian crisis and Russian default during the late-1990s.

In our view, the impact of this structural increase in the desired level of international savings can explain the shift in the nature of the bond market volatility. This volatility manifested itself via the absence of high autocorrelation in the volatility process. Put another way, we think that the significant increase in structural flow into the US debt markets witnessed during this period had a stabilizing effect on bond return dynamics, thus reducing the persistence of exogenous shocks.

In addition, we think that during this period the steady nature of the Federal Reserve's policy decisions also played a stabilizing role as policy uncertainty fell. Looking back, the Central bank ran an incredibly steady hiking cycle, when compared to historical experience, as the economy started to turn around in 2004. Indeed, the Federal Reserve hiked its funds rate by 25bp per meeting almost continuously over the 2004 to 2006 period as the base rate reached 5.25% in mid-2006, from a low of 1% in mid-2004. Indeed, in the post-crisis literature, the highly predictable nature of Federal policy during this period has been identified as one of the driving forces behind the formation of the US housing bubble (see for instance Obstfeld and Rogoff (2009)).

Overall, we think that a combination of these two factors –that is, a structural increase in global savings flow to US debt markets with an extremely steady (and thereby, largely predictable) Federal policy path– can explain the neutralization of the high autocorrelation attribute of bond market volatility. Furthermore, shocks to the returns process show a significantly reduced persistence during this period when compared to historical experience.

Shifting to the post-December 2007 period estimation, empirical results show evidence of an increase in the magnitude of beta to 0.63, although it is still assessed to be below the 0.99 level estimated using both the full and post-war sample periods). Meanwhile, evidence of the presence of leverage effects still comes out as statistically weak. That said, in terms of the magnitude and direction of the estimated asymmetric effects, it appears that a positive return shock is still driving an increase in volatility (coefficient of 0.04), while a negative return shock now appears to be consistent with a reduction in volatility (coefficient of -1.02).

This "odd" leverage effect behaviour appears to be capturing numerous episodes of sharp falls in bond yields (which are generating positive returns), as witnessed over the 2008/9 and 2011/12 period. This occurred as key Central banks led by the Federal Reserve embarked on a series of unconventional monetary policies, that generally took the form of outright purchases of government bonds, in an effort to provide stimulus to their respective economies after the US housing bubble bust. A number of prominent central bankers, including the former Federal chairman Ben Bernanke himself, went on to note that the unconventional monetary policy framework adopted (when zero-bound in short rates was hit) by key central banks such as the Federal Reserve and Bank of England as the great recession hit in 2008/9 was designed to work through the “portfolio rebalance effect” (see Bernanke & Reinhart (2004) for instance). Specifically, the Central bank’s suppression of risk-free real interest rates on the back of outright asset purchases was designed to force investors to buy risky assets. Looking back, this shift in future asset return expectations, arriving on the back of the policy noted above, appears to have happened suddenly. This led to episodes of sharp fall in nominal bond yields as the easing action (in the case of late-2008) or the communication of easing intention (in the case of quantitative easing phase two done in late-2010) was transmitted by Central bank officials to market participants. All in all, the rapid fall in aggregate real demand and the accompanying monetary/fiscal policy response was the key bond market return shaping force during this period. The reaction to these developments was also visible in the sharp fall in bond yields as lower inflation/growth dynamics and consequently an easier monetary policy path was priced-in by the market.

Focusing on the increase in the magnitude of the β parameter visible in the post-December 2007 period estimation results, when compared to the 2000-2007 sample period, it would appear that the bond return volatility process started to “normalize” towards historical average. This is evident from the re-emergence of higher autocorrelation in the volatility process.

2.5 EVALUATING FORECASTING PERFORMANCE

A number of subjective decisions along various dimensions have to be made in forecasting volatility and the evaluation of the model’s forecast. As noted in section 2.2, Poon and Granger (2003) provide a helpful summary of forecasting-related decisions, as well as the problem’s different dimensions, using information gleaned from more than 90 papers. Specifically, the two most important dimensions of the forecast assessment exercise are: the proxy used for realized volatility (which is a latent variable) and the treatment of the data set with either an in-sample/out-sample bifurcation or the usage of a rolling scheme, under which the model parameter estimates are updated with each additional observation. As we discuss in detail in section 2.6, in this study we use the in/out sample data division on each iteration of the true model’s simulation–using the EGARCH(1,2) specification depicted in equations (2.1) & (2.2)–in order to provide the relevant data points. We then apply various forecast evaluation techniques. Here, we use the true model’s underlying volatility as the benchmark to assess the quality of forecasts.

The nature of the different metrics used to compare forecasting ability needs to be guided by a combination of statistical considerations and the required application of the forecast. For instance, in options space, which is specifically used to guide trading decisions, the forecaster may prefer to take into account the asymmetry of the forecast error and therefore to penalize over- or under- prediction.

The ability to assess or penalize under-prediction is useful within the context of risk management systems as well. This is because the forecaster can have an incentive or preference, driven by regulation, to apply a heavier penalty to under-prediction. This is especially true in the post- 2008/9 crisis world, which has seen a number of new financial sector-focused regulations come into effect, such as for example, Dodd-Frank and Basel III. These new regulation regimes embed a shift towards using more conservative risk assessment methodologies within the banking sector. Indeed, this change in preference towards using more conservative methodologies has been driven by the severity of the recession, in terms of loss of output and a sharp rise in unemployment seen globally, as well as the crucial role played by the global financial sector in amplifying the original US housing-centric shock.[65] Moreover, this important change in the regulatory landscape was further strengthened in 2011/12 as the European debt crisis situation came to the fore. This situation led to the emergence of severe funding pressures on key European financial institutions.[66]

However, despite the context-specific appeal of studying asymmetric prediction error, the analysis of symmetric forecast errors, which applies the same weight to under- and over- predictions, is a more appropriate benchmark for assessing the overall goodness-of-fit and allows relevant comparison with other studies in this research area.

When it comes to symmetric prediction error assessment, the two widely used forecast evaluation metrics, Mean and Root Mean Square Error (RMSE), are deployed in this study. In addition, we also use Mean Absolute Error (MAE) in order to undertake a comprehensive assessment of the forecast ability of the two models.

The exact specifications of the above forecast evaluation statistics are given below:

$$\text{RMSE} = \sqrt{\frac{1}{n}\ \sum_{i=1}^{n}(h_t - \ \overline{h_t}\ )^2} \qquad (2.25)$$

$$\text{MAE} = \frac{1}{n}\ \sum_{i=1}^{n} abs(h_t - \ \overline{h_t}\ ) \qquad (2.26)$$

where h_t is the true volatility, $\overline{h_t}$ is the relevant model's forecast generated by minimising the mean-square error forecast function and n is the total number of forecasts assessed.

[65] For example, Aiyar (2012) explores how the funding market shock to globalised banks was transmitted to the real economy via reduced domestic credit supply.
[66] For example, Neri et al (2013) carried out an analysis of the macroeconomic effects of the European sovereign debt crisis.

Turning to asymmetric forecasting error evaluation metrics, we use the Mean Mixed Error-Under (MME-U) and Mean Mixed Error-Over (MME-O) statistics to assess tendency to under- or over- predict true volatility:

MME – U = $\frac{1}{n}$ $(\sum_{i=1}^{k} abs\left(h_t - \overline{h_t}\right) + \sum_{i=1}^{l} abs(h_t - \overline{h_t})^{0.5})$ (2.27)[67]

MME – O = $\frac{1}{n}$ $(\sum_{i=1}^{l} abs\left(h_t - \overline{h_t}\right) + \sum_{i=1}^{k} abs(h_t - \overline{h_t})^{0.5})$ (2.28)

In addition, we also use the **Diebold-Mariano (DM)** test statistic for comparing predictive accuracy of the various candidate models and the formulation of the test is described below:

As above, let $\overline{h_{t,1}}$ and $\overline{h_{t,2}}$ denote competing forecasts of h_t based on the two candidate models. The forecast errors from the two models at forecast length i is given below;

$$\epsilon_{t+i|t}^{1} = h_{t+i} - \overline{h_{t+\iota,1}}$$

$$\epsilon_{t+i|t}^{2} = h_{t+i} - \overline{h_{t+\iota,2}}$$

The accuracy of each forecast is then assessed by using a squared loss function (i.e. a = 1 or 2)

Squared error loss: $L\left(\epsilon_{t+i|t}^{a}\right) = \left(\epsilon_{t+i|t}^{a}\right)^2$

(where a = 1 or 2 reflecting the two competing models)

To determine if one model predicts better than another we may test the null hypothesis

$$H_0 : E\left[L\left(\epsilon_{t+i|t}^{1}\right)\right] = E\left[L\left(\epsilon_{t+i|t}^{2}\right)\right]$$

against the alternative

$$H_1 : E\left[L\left(\epsilon_{t+i|t}^{1}\right)\right] \neq E\left[L\left(\epsilon_{t+i|t}^{2}\right)\right]$$

The Diebold-Mariano test is based on the loss differential

$$d_t = L\left(\epsilon_{t+i|t}^{1}\right) - L\left(\epsilon_{t+i|t}^{2}\right)$$

The null of equal predictive accuracy is then

$$H_0 : E[d_t] = 0$$

[67] Where k +l = n and k is the number of under predictions and l is the number of over-predictions, respectively.

The Diebold-Mariano test statistic is calculated as

$$S = \frac{\bar{d}}{\left(\widehat{avar}(\bar{d})\right)^{1/2}} = \frac{\bar{d}}{\left(\widehat{LRV}_{\bar{d}}/T\right)^{1/2}}$$

where

$$\bar{d} = \frac{1}{T}\sum_{t=t_0}^{T} d_t$$

Where T is the total number of forecasts calculated

$$LRV_{\bar{d}} = \gamma_0 + 2\sum_{j=1}^{\infty} \gamma_j, \quad \gamma_j = cov(d_t, d_{t-j})$$

and $\widehat{LRV}_{\bar{d}}$ is a consistent estimate of the asymptotic (long-run) variance of $\sqrt{T}\bar{d}$. The long-run variance is used in the statistic because the sample of loss differentials $\{d_t\}_{t_0}^{T}$ are serially correlated for $i > 1$. Diebold and Mariano (1995) show that under the null of equal predictive accuracy

$$S \overset{A}{\sim} N(0,1)$$

So we reject the null of equal predictive accuracy at the 5% level if

$$|S| > 1.96$$

2.6 SIMULATION METHOD AND RESULTS

In this section, we outline the method of simulation used in our study. We deploy our agreed true model, i.e. an EGARCH(1,2) calibrated using the parameters estimated in section 2.4. These empirical estimations are an extension of Pagan and Schwert's work (1990). We then use a random generator to generate a time series by assuming that the estimated parameters are true parameters. In the first instance, it will be normal and hence all moments will exist. We also experiment with a t- distribution specification with 5, 10 and 50 degrees-of-freedom in order to compare the results.

In terms of the mechanics of the simulation method used, and assuming that our initial estimates satisfy stationarity conditions, we let the true model run for 30,000 periods, so that the resultant time series is stationary. We then used the 27000 th observation as the first observation of the sample set to be used. The above exercise generates a true return and volatility series which should not suffer from initial value problems.

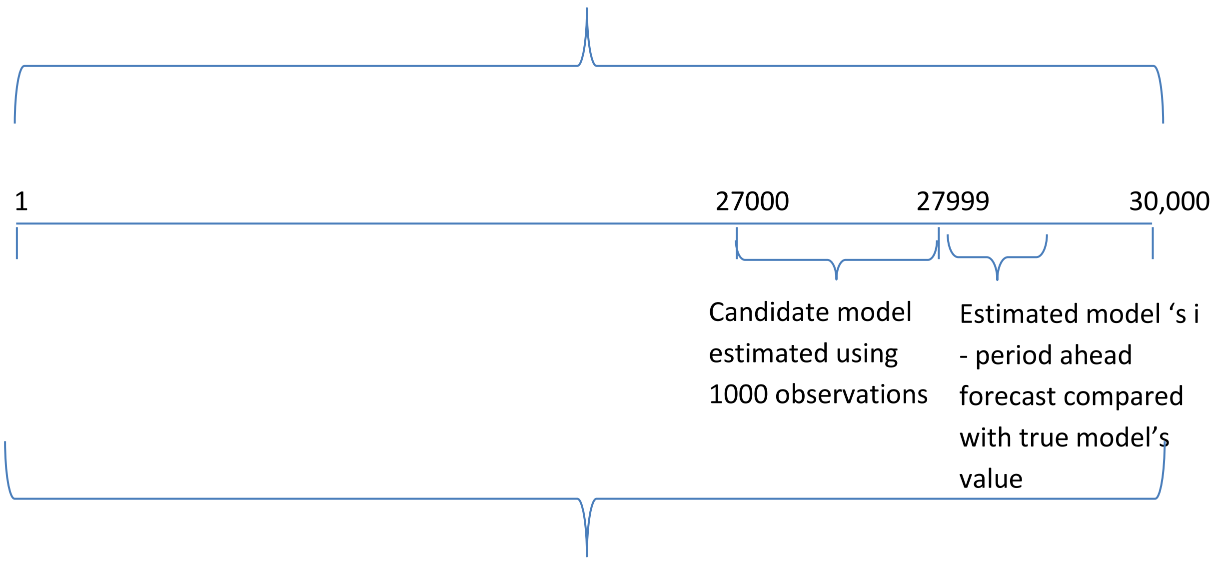
Then, we take the 27000^{th} to 27999^{th} observation as the sample set for T =1000 and 27000^{th} to 27499^{th} for T= 500 and so on to estimate the SV(1,1), GARCH(1,1) and EGARCH(1,2) models over different sample sets T. In this study, we have considered the case for T=50 and T=100 as well to assess the importance of the size of the sample set used in driving the results.

Next, we use the estimated models to forecast the next 20 periods, which are then stored in order to estimate their absolute and relative forecasting accuracy, using the various forecasting assessment metrics discussed in section 2.5. We then assess the forecast accuracy of the various candidate models at period length 1,5,10 and 20 to provide a more granular read on the relative abilities of the various models under consideration.

The entire exercise is repeated 100 times. Here, we keep the "true" model intact but take a new "true" sample of T observations followed by a re-estimation of the SV and GARCH models in each iteration. The point of the procedure is to make true volatilities known, which is not the case with historical volatility. This allows us to compare forecasts with true underlying volatility.

Figure 2.1 Simulation Schematic using the case for T =1000 [68]

True Model (EGARCH(1,2)) Simulated to Generate 30,000 observations:[69]



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[68]This has been included for illustrative purposes only, and is not drawn to scale. Same exercise is carried out using T =50,100 and 500 as well, where T is the size of the sample set.

[69] The entire exercise was repeated 100 times.

Results

The results of the simulation exercise are shown in Appendix 2.1, which tabulates the various forecast assessment metrics estimated using various distribution and β assumptions. The quantities shown in these tables are Monte-Carlo averages based on 100 replications (as noted above) for a maximum 20-period forecast length for each set (as noted above, we have assessed the relative forecast abilities of the various models at forecast period 1,5,10 and 20 individually).

The forecast assessment for each model is done on the minimum mean square error (MMSE) forecast, which is the forecast $\overline{y_{t+1}}$ that minimizes the expected square loss. The forecasts are generated by using the forecast function in Matlab, which estimates minimum mean square error (MMSE) forecasts recursively, following Baillie and Bollerslev (1992) and Box et. al (1994). These forecasts are then used to generate conditional mean and variance forecasts for the SV and GARCH models respectively in our simulation code.

In terms of the true model's parameters used to simulate the EGARCH model, the Pagan and Schwert (1990) study reported a β estimate of 0.74 using US equity returns data (1834 to 1925 sample period), compared to the 0.98 we have estimated using the 1793-2014 period (see tables 2.1 & 2.2). For our simulation exercise, we use the 1791-2014 period, true model parameters (i.e. the true model) which have been estimated using S&P 500 data (see section 2.4) and consider the case for β = 0.75 as well.

The simulation results indicate that the SV model generally outperformed the GARCH model under the normal distribution assumption on the basis of the various metrics considered. This result appears to generally hold irrespective of the value of the β studied and is strongly visible when T =1000 (on the basis of the DM test). However, exceptions to this general conclusion show-up when T is small (i.e. T =50) or when β is 0.75 and forecast quality assessment is done at the 10-period length (here, DM test shows statistically significant difference between the GARCH and SV forecasts at 5% level of significance, especially, when T =500 or 1000). Also, using the DM test, at forecast period 20, the difference in the forecast of the two models was found to be statistically insignificant across the various cases considered which shows that the volatility forecasts from the two models asymptotically converge as the forecast period increases.

Turning to the comparison with the EGARCH(1,2) model generated forecasts (i.e. using an EGARCH(1,2) model fitted over the T sample set and then generating the corresponding MMSE forecasts), SV turned out to be better as well on the basis of the metrics considered, when the forecast length is 1 or 5 and β = 0.98 (in other cases, the difference is statistically insignificant, though a few times in favour of EGARCH(1,2) model), while the difference between GARCH (1,1) and EGARCH(1,2) was generally found to be statistically insignificant (based on the DM test statistic) except when β is 0.75 and the assessment was done at forecast period 1 (under this case, GARCH was found to be better when T is small). Here, such cases seem to be driven by parameter uncertainty even though the true model is EGARCH(1,2).

However, under the t-distribution assumption the relative assessment results start to become more varied, with the GARCH model starting to outperform the SV model on almost all assessment metrics considered as lower values of β are used. For instance, using the $\beta = 0.75$ assumption coupled with a t(5) distribution assumption, the GARCH model's superiority stretches across all metrics including the ones designed to capture forecast error's accuracy using asymmetric weighting schemes (i.e. MME-U and MME-O) and remains intact (to a lesser degree) even when T rises. This switch in performance is also visible in DM test results, especially when T and forecast length are low.

However, as the β value is increased, simulation results start to once again converge towards normal distribution results. Specifically, for $\beta = 0.98$ assumption, we find that the SV model generated relatively lower relative forecast error statistics, irrespective of the specific t-distribution assumption used, although, the DM test showed statistically insignificant difference between the two forecasts considered.

When comparing with the results with the EGARCH(1,2) model, there was some weak evidence that GARCH(1,1) is the better model, though it was generally found to be statistically insignificant, irrespective of the value of β and size of T considered. Comparing the SV model with EGARCH(1,2), the results are similar, though in some cases such as when T =1000, EGARCH(1,2) was found to be better at forecast length 5 ($\beta = 0.75$).

We think that the quality of stationarity under GARCH/EGARCH and SV model structures, when β is close to 1, can help shed light on the driving factors behind the findings discussed above. Specifically, for the GARCH/EGARCH model, a β value of 0.98 is close to the stationarity-bound depending on the ARCH parameter. However, for the SV model, stationarity is only dependent on the β parameter, as we know that for AR(1), we often need to be closer to 1 than 0.98 for the bound to be hit. However, developing a deeper understanding may involve analyzing the behaviour of near-random walk processes for which, as far as we are aware, a theory for volatility models has yet to be developed. Our intuition is that as we get close to an I(1) process, it is only the first two moments that matter and the non-existence of higher moments is irrelevant. Some results that support this intuition are due to Boswijk (2001) and Ling and Li (1998). They show that with near-integrated volatility, maximum likelihood estimators converge to distributions all of whose moments exist. Whilst no results have been proven for forecasts it is likely that these results will imply less dependence on fat tails.

Finally, in order to compare the relative forecast abilities of the various models with varying computational complexity (in terms of estimation processing time), figure 2.2 below shows the relevant metric based on 1000 replications each. All in all, on average, SV appears to require around 14x more time to fit compared to the GARCH formulation, while around 5x more time compared to the EGARCH model using a standard 2.7GHz intel core processor. Indeed, the heavy computational complexity of SV needs to be carefully considered when assessing the importance of the gains in forecasting quality (especially, under the normal distribution case) achieved versus the GARCH and EGARCH models.

Figure 2.2: Computational Complexity of the Various Models

Model	Time per fit
GARCH	0.026s
SV	0.372s
EGARCH	0.073s

Using Macbook Pro (2.7 GHz intel Core i7 processor)

2.7 CONCLUSION

Chapter 2 addresses the question of what type of simple volatility model an econometrician should use when confronted by empirical data in the forecasting process. We argue that the best true model might be the one for which past empirical work has been the most convincing. We believe to be the EGARCH(1,2) model in the context of US equity and bond markets. The econometrician considers only GARCH(1,1), SV(1,1) and EGARCH(1,2) as the three competitors. We then compare the relative performances of the candidate models when the true model is EGARCH(1,2). We also derive the moments of the EGARCH(1,2), which can be offered as a comparison to the moments of the other two models.

To generate artificial data from the true model, it needs to be estimated. We estimated the true model parameters of EGARCH(1,2) using long-term returns data for both S&P 500 and US 10-year bonds. We also connect the observed shifts in model parameters during the various sub-samples studied with the broader macroeconomic situation prevalent in the US economy.

Our analysis confirms the superiority of the SV model under the normal distribution assumption. However, using t-distributed shocks, results vary and appear to depend on the value of β, which we believe is related to the behaviour of the given volatility models when β is close to 1.

Finally, we find that based on conventional measures of forecast accuracy such as MSE, SV forecasts are very exposed to outliers relative to GARCH. This is partially a consequence of the need to exponentiate the SV forecasts, since SV is a model of log-volatility. Furthermore, we show how the presence of non-normality maps onto the time series structure, and that exponentiation under some circumstances leads to the non-existence of population moments.

It seems that simple estimators which ignore asymmetric dependence in volatility will forecast satisfactorily, depending upon particular circumstances related to the actual distributions of errors. While we have concerned ourselves with misspecification, we acknowledge that the best procedure here is to forecast with the actual asymmetrically dependent process. However, this has numerous challenges which we shall discuss in future work.

APPENDIX 2.1: Simulation Results

Appendix Table 2.1.1a

β = 0.98, Gaussian, for T = 50,100,500 & 1000, forecast length 1, 5, 10 & 20

RMSE

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	**5**	**10**	**20**	**1**	**5**	**10**	**20**	**1**	**5**	**10**	**20**
50	1.48	1.70	2.11	2.60	2.27	2.14	2.33	2.59	4.41	2.41	2.66	2.98
100	2.07	2.29	2.58	2.84	1.58	1.48	1.66	1.74	2.11	1.86	2.14	2.49
500	4.43	4.64	4.85	5.07	2.96	2.82	2.70	2.42	4.35	4.47	4.70	5.06
1000	4.50	4.71	4.91	5.11	1.99	1.78	1.70	1.51	3.79	3.35	3.14	2.81

MAE

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	**5**	**10**	**20**	**1**	**5**	**10**	**20**	**1**	**5**	**10**	**20**
50	1.07	1.09	1.35	1.65	1.45	1.14	1.29	1.46	2.09	1.27	1.38	1.53
100	1.41	1.35	1.59	1.84	1.24	0.97	1.07	1.18	1.29	1.06	1.23	1.46
500	1.84	1.84	2.13	2.52	1.46	1.19	1.23	1.28	1.79	1.66	1.85	2.08
1000	2.04	1.87	2.08	2.43	1.35	0.97	0.94	0.98	1.87	1.55	1.62	1.72

MME-U

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	**5**	**10**	**20**	**1**	**5**	**10**	**20**	**1**	**5**	**10**	**20**
50	0.98	0.88	0.95	1.07	1.14	0.80	0.88	0.97	1.34	0.86	0.87	0.95
100	1.07	0.89	1.01	1.15	1.09	0.77	0.82	0.91	1.06	0.80	0.89	1.00
500	1.11	0.97	1.12	1.32	1.06	0.79	0.84	0.94	1.10	0.92	1.04	1.17
1000	1.19	0.91	1.04	1.27	1.06	0.71	0.75	0.87	1.14	0.88	0.98	1.14

MME-O

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	**5**	**10**	**20**	**1**	**5**	**10**	**20**	**1**	**5**	**10**	**20**
50	1.04	1.15	1.41	1.67	1.42	1.23	1.35	1.49	1.99	1.36	1.46	1.57
100	1.41	1.41	1.63	1.85	1.20	1.07	1.13	1.21	1.26	1.14	1.28	1.48
500	1.82	1.92	2.16	2.52	1.44	1.28	1.27	1.29	1.77	1.73	1.88	2.09
1000	2.03	1.96	2.11	2.44	1.34	1.07	0.99	0.99	1.87	1.64	1.64	1.73

DM Statistics – Significance level 1.96 (in absolute terms). +/- statistic implies inferiortiy/superiority of the left-side model. For T = 50,100,500 and 1000 and forecast length = 1,5,10 & 20.

	1	**5**	**10**	**20**		
50	-0.60	0.61	1.16	1.05	GARCH(1,1)	SV(1,1)
100	2.66	2.49	1.92	1.68		
500	2.27	2.38	2.42	1.23		
1000	2.62	2.69	2.72	1.27		

50	-1.49	-0.52	0.34	0.35	GARCH(1,1)	EGARCH(1,2)
100	-0.09	1.03	1.02	0.98		
500	0.98	1.57	0.69	0.03		
1000	1.09	1.38	1.46	1.16		

50	-1.60	-0.89	-2.04	-1.48	SV(1,1)	EGARCH(1,2)
100	-0.82	-0.85	-1.23	-1.55		
500	-1.26	-1.21	-1.19	-1.07		
1000	-2.07	-2.13	-2.03	-1.61		

Appendix Table 2.1.1b

β = 0.75, Gaussian, for T = 50,100,500 & 1000, forecast length 1,5,10 & 20

RMSE

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	5	10	20	1	5	10	20	1	5	10	20
50	0.40	0.39	0.43	0.52	1.37	0.47	0.51	0.53	0.77	0.41	0.66	0.52
100	0.29	0.35	0.37	0.49	0.52	0.39	0.46	0.45	0.67	0.33	0.51	0.39
500	0.26	0.32	0.35	0.43	0.20	0.25	0.42	0.37	0.32	0.33	0.37	0.42
1000	0.19	0.29	0.34	0.43	0.16	0.24	0.41	0.37	0.22	0.31	0.35	0.42

MAE

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	5	10	20	1	5	10	20	1	5	10	20
50	0.26	0.29	0.38	0.44	0.70	0.36	0.44	0.42	0.53	0.29	0.60	0.33
100	0.22	0.30	0.33	0.45	0.39	0.31	0.41	0.38	0.46	0.26	0.46	0.33
500	0.16	0.29	0.34	0.42	0.15	0.21	0.41	0.35	0.21	0.29	0.36	0.40
1000	0.13	0.28	0.33	0.43	0.12	0.22	0.40	0.36	0.18	0.28	0.34	0.42

MME-U

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	5	10	20	1	5	10	20	1	5	10	20
50	0.37	0.46	0.41	0.62	0.59	0.49	0.49	0.57	0.54	0.39	0.59	0.47
100	0.34	0.51	0.33	0.65	0.48	0.47	0.41	0.57	0.49	0.41	0.47	0.54
500	0.31	0.53	0.34	0.65	0.27	0.43	0.41	0.59	0.35	0.52	0.36	0.63
1000	0.31	0.52	0.33	0.65	0.28	0.45	0.40	0.59	0.34	0.52	0.34	0.64

MME-O

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	**5**	**10**	**20**	**1**	**5**	**10**	**20**	**1**	**5**	**10**	**20**
50	0.35	0.31	0.56	0.45	0.80	0.41	0.59	0.45	0.64	0.38	0.75	0.37
100	0.30	0.30	0.53	0.45	0.48	0.35	0.60	0.39	0.58	0.31	0.65	0.34
500	0.20	0.29	0.57	0.42	0.22	0.22	0.63	0.35	0.27	0.30	0.59	0.40
1000	0.16	0.28	0.57	0.43	0.17	0.22	0.63	0.36	0.22	0.29	0.58	0.42

DM Statistics – Significance level 1.96 (in absolute terms). +/- statistic implies inferiortiy/superiority of the left-side model. For T = 50,100,500 and 1000 and forecast length = 1,5,10 & 20.

	1	**5**	**10**	**20**		
50	-1.25	-2.00	-1.25	0.00	**GARCH(1,1)**	**SV(1,1)**
100	-3.07	-1.37	-1.36	0.99		
500	0.96	2.05	-2.10	1.54		
1000	1.95	3.09	-2.26	1.67		

50	-2.38	0.82	-1.78	-0.09	**GARCH(1,1)**	**EGARCH(1,2)**
100	-2.90	-0.10	-1.79	1.21		
500	-1.39	-0.56	-2.13	1.45		
1000	-1.26	-0.96	-0.90	1.26		

50	1.48	1.18	-1.99	0.07	**SV(1,1)**	**EGARCH(1,2)**
100	-1.31	1.60	-1.51	1.51		
500	-2.55	-1.60	2.03	-1.62		
1000	-2.63	-1.78	2.09	-1.64		

Appendix Table 2.1.2a

β = 0.98, t (5), for T = 50,100,500 & 1000, forecast length 1,5,10 & 20

RMSE

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	5	10	20	1	5	10	20	1	5	10	20
50	1.88	2.05	1.97	2.26	3.83	2.16	2.26	4.02	2.10	2.08	2.32	4.10
100	3.82	3.97	3.79	3.84	3.56	3.52	3.43	3.90	3.75	3.81	3.70	3.98
500	2.13	2.22	2.00	1.93	1.24	1.17	0.86	0.83	1.87	1.75	1.33	1.08
1000	4.21	4.33	4.15	4.11	2.22	1.85	1.27	0.93	3.62	3.19	2.52	1.85

MAE

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	5	10	20	1	5	10	20	1	5	10	20
50	1.08	1.23	1.15	1.32	1.75	1.23	1.15	1.42	1.15	1.23	1.14	1.52
100	1.86	2.06	1.96	2.07	1.58	1.59	1.52	1.72	1.69	1.68	1.53	1.57
500	1.04	1.13	1.00	0.99	0.78	0.75	0.62	0.71	0.98	1.02	0.81	0.75
1000	1.70	1.87	1.67	1.68	1.24	1.13	0.83	0.70	1.60	1.60	1.24	1.03

MME-U

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	5	10	20	1	5	10	20	1	5	10	20
50	0.81	0.89	0.87	0.97	0.96	0.84	0.85	0.98	1.07	1.09	0.81	1.02
100	1.00	1.08	1.08	1.17	0.89	0.90	0.90	1.03	0.90	0.93	0.91	0.98
500	0.75	0.82	0.76	0.79	0.67	0.67	0.59	0.71	0.74	0.79	0.70	0.70
1000	0.93	1.05	0.94	0.98	0.84	0.83	0.70	0.68	0.92	1.00	0.84	0.80

MME-O

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	**5**	**10**	**20**	**1**	**5**	**10**	**20**	**1**	**5**	**10**	**20**
50	1.15	1.29	1.21	1.35	1.84	1.29	1.22	1.44	1.20	1.30	1.23	1.54
100	1.93	2.11	2.02	2.08	1.66	1.65	1.60	1.75	1.77	1.76	1.60	1.60
500	1.14	1.17	1.09	1.07	0.87	0.81	0.73	0.80	1.07	1.07	0.90	0.83
1000	1.77	1.90	1.75	1.75	1.32	1.18	0.93	0.79	1.67	1.63	1.31	1.11

DM Statistics – Significance level 1.96 (in absolute terms). +/- statistic implies inferiortiy/superiority of the left-side model. For T = 50,100,500 and 1000 and forecast length = 1,5,10 & 20.

	1	**5**	**10**	**20**		
50	-1.03	-0.49	-0.83	-1.94	GARCH(1,1)	SV(1,1)
100	0.27	0.58	0.51	-0.23		
500	1.53	1.59	1.49	1.35		
1000	2.12	2.19	2.16	1.13		

50	-1.04	-0.73	-0.89	-0.95	GARCH(1,1)	EGARCH(1,2)
100	0.04	0.11	0.04	-0.16		
500	2.01	1.41	1.42	1.31		
1000	2.08	1.18	1.16	1.43		

50	-1.00	-1.00	-1.11	-1.19	SV(1,1)	EGARCH(1,2)
100	-0.36	-0.80	-0.53	-0.11		
500	-1.45	-1.76	-1.62	-1.63		
1000	-1.14	-1.19	-1.15	-1.16		

Appendix Table 2.1.2b

β = 0.75, t (5), for T = 50,100,500 & 1000, forecast length 1,5,10 & 20

RMSE

	GARCH(1,1)				**SV(1,1)**				**EGARCH(1,2)**			
	1	**5**	**10**	**20**	**1**	**5**	**10**	**20**	**1**	**5**	**10**	**20**
50	0.38	0.42	0.56	0.52	0.89	0.48	0.61	0.48	0.96	0.66	0.41	0.41
100	0.51	0.33	0.61	0.40	0.60	0.36	0.51	0.33	0.51	0.40	0.45	0.28
500	0.20	0.20	0.51	0.30	0.24	0.28	0.46	0.25	0.30	0.34	0.51	0.30
1000	0.20	0.20	0.50	0.28	0.18	0.26	0.44	0.23	0.24	0.21	0.45	0.29

MAE

	GARCH(1,1)				**SV(1,1)**				**EGARCH(1,2)**			
	1	**5**	**10**	**20**	**1**	**5**	**10**	**20**	**1**	**5**	**10**	**20**
50	0.30	0.34	0.48	0.36	0.51	0.40	0.46	0.34	0.56	0.53	0.32	0.27
100	0.29	0.23	0.56	0.33	0.36	0.31	0.45	0.25	0.36	0.33	0.41	0.22
500	0.16	0.18	0.50	0.28	0.17	0.25	0.44	0.22	0.22	0.25	0.49	0.27
1000	0.17	0.19	0.50	0.27	0.14	0.24	0.43	0.21	0.19	0.20	0.49	0.27

MME-U

	GARCH(1,1)				**SV(1,1)**				**EGARCH(1,2)**			
	1	5	10	20	1	5	10	20	1	5	10	20
50	0.42	0.37	0.64	0.51	0.52	0.44	0.59	0.46	0.53	0.54	0.48	0.37
100	0.42	0.27	0.73	0.54	0.44	0.34	0.64	0.43	0.45	0.34	0.62	0.40
500	0.35	0.19	0.71	0.52	0.33	0.26	0.66	0.45	0.37	0.26	0.69	0.50
1000	0.38	0.19	0.70	0.52	0.32	0.24	0.65	0.45	0.37	0.20	0.70	0.52

MME-O

	GARCH(1,1)				SV(1,1)				EGARCH(1,2)			
	1	5	10	20	1	5	10	20	1	5	10	20
50	0.40	0.50	0.50	0.41	0.61	0.54	0.48	0.41	0.68	0.67	0.37	0.36
100	0.33	0.39	0.56	0.33	0.45	0.48	0.46	0.28	0.44	0.52	0.41	0.25
500	0.18	0.41	0.50	0.28	0.22	0.47	0.44	0.22	0.27	0.47	0.49	0.28
1000	0.18	0.42	0.50	0.27	0.18	0.48	0.43	0.21	0.21	0.42	0.49	0.27

DM Statistics – Significance level 1.96 (in absolute terms). +/- statistic implies inferiortiy/superiority of the left-side model. For T = 50,100,500 and 1000 and forecast length = 1,5,10 & 20.

	1	5	10	20		
50	-2.83	-2.35	-0.49	1.34	GARCH(1,1)	SV(1,1)
100	-0.78	-0.51	1.40	0.95		
500	-1.48	-2.80	1.93	1.44		
1000	1.17	-2.89	2.16	1.57		

50	-1.59	-1.77	1.58	1.24	GARCH(1,1)	EGARCH(1,2)
100	0.25	-1.41	1.56	1.18		
500	-1.20	-1.26	-0.56	-1.30		
1000	-1.26	-1.65	1.53	-0.63		

50	-0.26	-1.84	1.73	0.66	SV(1,1)	EGARCH(1,2)
100	0.51	-0.97	1.48	1.52		
500	-1.38	-1.36	-1.65	-1.46		
1000	-1.71	2.85	-1.24	-1.65		

APPENDIX 2.2: ADDITIONAL DETAILS REGARDING UNDERLYING DATA SOURCES USING GLOBAL FINANCIAL DATA (GFD) AND BLOOMBERG

US 10-year Government Bond Yields: The historical data has been sourced by GFD from: Richard E. Sylla, Jack Wilson and Robert E. Wright, *Price Quotations in Early U.S. Securities Markets, 1790-1860; Hunt's Merchants Magazine* (1843-1853)*; The Economist* (1854-1861); *The Financial Review* (1862-1918); Federal Reserve Bank; National Monetary Statistics (New York: FRB, 1941, 1970 annually thereafter); and Salomon Brothers, *Analytical Record of Yields and Yield Spreads* (New York: Salomon Brothers, 1995). The 'constant maturity' yield was sourced from FRB, H-15 tables, which are available from 1953.

US Equity returns

The original S&P indices were introduced by the Standard Statistics Corporation in 1923, which covers 233 stocks in 26 sectors. Data were calculated on a weekly basis dating back to 1918. The daily indices were introduced in 1928 and consisted of a 90-stock average including 50 industrials, 20 rails and 20 utilities.

The Standard and Poor's Composite combines a number of different indices. From 1791 to 1801, GFD has calculated an equal-weighted index using data from seven banks, three insurance companies and two transport companies. The banks are the Union National Bank of Boston; the Massachusetts National Bank of Boston; the First Bank of the United States; the Bank of the State of New York; the Bank of Pennsylvania; the Bank of South Carolina; and the Bank of America. The three insurance companies are the New York Insurance Company; the Insurance Company of Pennsylvania and the Insurance Company of North America; and the two transport companies are the Philadelphia and Lancaster Turnpike Company and the Schuylkill Permanent Bridge Company.

Using Smith and Cole's index in *Fluctuations in American Business, 1790-1860*, the index combines the monthly price indexes of bank stocks (from 1802-1815); bank and insurance stocks (from February 1815 to December 1845), and rails (from 1834-1862) gleaned from Smith and Cole. Furthermore, the stocks from railroads (1863-1870) comes from Frederick R. Macaulay, *The Movements of Interest Rates, Bond Yields and Stock Prices in the United States Since 1856* (1938). Where these indices overlap, they have been weighted according to the number of stocks included in the indices. Beginning in 1871, the Cowles/Standard and Poor's Composite index of stocks is used. The Standard and Poor's indices were first calculated in 1918, and the Cowles Commission back-calculated the series to 1871 using the *Commercial and Financial Chronicle*.[70]

APPENDIX 2.3: PROOF OF THEOREM

By assumption $v_t \sim \ (0, \sigma_v^2)$ but we now need the moments of $\ln(\varepsilon_t^2)$. Using the moment generating function, we have immediately that

$$E\big[\exp\big(s\ln(\varepsilon_t^2)\big)\big] = E[(\varepsilon_t^2)^s] = M(s)$$

Now the mean and variance of $\ln(\varepsilon_t^2)$ will be given as functions of the derivatives of M(s), evaluated at s=0.

Letting

$$K(s) = \ln M(s)$$

We have

$$K'(s) = \frac{M'(s)}{M\ (s)}$$

$$K''(s) = \frac{M\ (s)M''(s) - \big(M'(s)\big)^2}{\big(M\ (s)\big)^2}$$

Consequently,

$$\ln(\varepsilon_t^2)\ \sim\ \big(K'(0), K''(0)\big)$$

Thus

$$\mu = K'(0) \text{ and } \delta^2 = K''(0)$$

Examining the composite error in our ARMA(1,1) representation, we have

$$v_t\ -\ \beta\ln(\varepsilon_t^2)\ \sim\ (-\beta\mu, \sigma_v^2 +\ \beta^2\,\delta^2)$$

Also,

[70] For more information, see Standard and Poor's, *Security Price Index Record*, New York: Standard and Poor's, 2000; and Cowles, *Commission for Research in Economics*, Common-Stock Indexes, 2nd ed., Bloomington: Principia Press, 1939.

$\ln(\varepsilon_{t+1}^2) \sim (\mu, \delta^2)$

So, letting w_t and w_{t+1} be white noise processes such that

$w_t \sim (0, d^2)$

We have immediately that

$g_{t+1=}\ln(\epsilon_{t+1}^2)+v_t - \beta\ln(\epsilon_t^2) = w_{t+1} - qw_t$ is an MA(1) process and we need to solve for q and d.

In particular;

Var(g_{t+1}) $= d^2(1+q^2)== \sigma_v^2$ +(1+β^2)$K"(0)$)

And Cov(g_{t+1}, g_t)=-qd^2=-$\beta K"(0)$

Therefore; solving the two equations, q= $\beta K"(0)/d^2$

And $d^2(1+(\beta K"(0)/d^2)^2)== \sigma_v^2$ +(1+β^2)$K"(0)$

$d^4 - d^2(\sigma_v^2$ +(1+β^2)$K"(0)) + (\beta K"(0))^2) = 0$

Substituting we see that the resulting quadratic has reciprocal roots. Taking a positive solution

We arrive, after some calculation, at

$$d^2 = \frac{\sigma_v^2 + (1+\beta^2)K"(0)+(\sigma_v^4+(1-\beta^2)^2(K"(0))^2 + 2\sigma_v^2(1+\beta^2)K"(0))^{.5}}{2}$$

And q=$\frac{2\,\beta K"(0)}{\sigma_v^2+(1+\beta^2)K"(0)+(\sigma_v^4+(1-\beta^2)^2(K"(0))^2+2\sigma_v^2(1+\beta^2)K"(0))^{.5}}$

Different distributional assumptions on v_t and ε_t will generate different μ and δ^2 but with

$v_t \sim (0, \sigma_v^2)$.

Proof of Corollary 1

Under normality

$$M(s) = E\left[\left(x_{(1)}^2\right)^s\right] = \frac{2^s\Gamma\left(\frac{1}{2}+s\right)}{\Gamma\left(\frac{1}{2}\right)}$$

With $K(s) = s\ln 2 + \ln\Gamma\left(\frac{1}{2}+s\right) - \ln\Gamma\left(\frac{1}{2}\right)$

giving

$$\mu = K'(0) = \ln 2 + \psi\left(\frac{1}{2}\right) \approx -1.27$$

$$\delta^2 = K''(0) = \psi'\left(\frac{1}{2}\right) \approx 4.93$$

Where $\psi(\cdot)$ and $\psi'(\cdot)$ are the digamma and trigamma functions respectively.

Proof of Corollary 2

Since v_t and ε_t have zero mean we need to scale them to have the correct variance of σ_v^2 for v_t and 1 for ε_t Consequently, we let

$$v_t^2 = \frac{\sigma_v^2(n-2)x_{(1)}^2}{x_n^2} \text{ and } \varepsilon_t^2 = \frac{(m-2)x_{(1)}^2}{x_n^2}$$

Therefore

$$\ln(\varepsilon_t^2) = \ln(m-2) + \ln x_{(1)}^2 - \ln x_{(m)}^2$$

and thus

$$M(s) = (m-2)^s E\left[\left(x_{(1)}^2\right)^s\right] E\left[\left(x_{(m)}^2\right)^{-s}\right]$$

$$= (m-2)^s 2^s \frac{\Gamma\left(\frac{1}{2}+s\right) 2^{-s}\Gamma\left(\frac{m}{2}-s\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{m}{2}\right)}$$

and

$$K(s) = s\ln(m-2) + \ln\Gamma\left(\frac{1}{2}+s\right) + \ln\Gamma\left(\frac{m}{2}+s\right) - \ln\Gamma\left(\frac{1}{2}\right) - \mathrm{in}\Gamma\left(\frac{m}{2}\right)$$

Consequently,

$$K'(s) = \ln(m-2) + \psi\left(\frac{1}{2} + s\right) - \psi\left(\frac{m}{2} - s\right)$$

$$K''(s) = \psi'\left(\frac{1}{2} + s\right) + \psi'\left(\frac{m}{2} - s\right)$$

Therefore

$$\mu = K'(0) = \ln(m-2) + \psi\left(\frac{1}{2}\right) - \psi\left(\frac{m}{2}\right)$$

$$\delta^2 = \psi'\left(\frac{1}{2}\right) + \psi'\left(\frac{m}{2}\right)$$

APPENDIX 2.4: PROOF OF EGARCH(1,2) MOMENT GENERATING FUNCTION

$$\ln\sigma_t^2 = \frac{\alpha}{1-\beta} + \sum_{j=0}^{\infty} \beta^j V_{t-j}$$

$$\sigma_t^2 = e^{\alpha/(1-\beta)} \prod_{j=0}^{\infty} \exp(\beta^j V_{t-j}).$$

$$E[\sigma_t^{2s}] = \exp\left(\frac{s\alpha}{1-\beta}\right) \prod_{j=0}^{\infty} E\left[\exp(s\beta^j V_{t-j})\right]$$

$$V_{t-j} = \sum_{k=1}^{2} \alpha_k \left(\lambda\varepsilon_{t-j-k} + \delta\left(|\varepsilon_{t-j-k}| - E(|\varepsilon_{t-j-k}|)\right)\right)$$

$$\beta^j V_{t-j} = \beta^j \sum_{k=1}^{2} \alpha_k \left(\lambda\varepsilon_{t-j-k} + \delta\left(|\varepsilon_{t-j-k}| - E(|\varepsilon_{t-j-k}|)\right)\right)$$

$$= \beta^j \left[\alpha_1 \left(\lambda\varepsilon_{t-1-j} + \delta\left(|\varepsilon_{t-1-j}| - E(|\varepsilon_{t-1-j}|)\right)\right)\right]$$

$$+\beta^j \left[\alpha_2 \left(\lambda\varepsilon_{t-2-j} + \delta\left(|\varepsilon_{t-2-j}| - E(|\varepsilon_{t-2-j}|)\right)\right)\right]$$

Consider

$$\sum_{j=0}^{\infty} \beta^j\left(\alpha_1(\lambda\varepsilon_{t-1-j}) + \alpha_2\lambda\varepsilon_{t-2-j}\right)$$

$$= \lambda\left[\alpha_1 \sum_{j=0}^{\infty} \beta^j\, \varepsilon_{t-1-j} + \alpha_2 \sum \beta^j \varepsilon_{t-2-j}\right]$$

$$= \lambda\left[\alpha_1\varepsilon_{t-1} + \alpha_1 \sum_{j=1}^{\infty} \beta^j\, \varepsilon_{t-1-j} + \alpha_2 \sum_{j=0}^{\infty} \beta^j \varepsilon_{t-2-j}\right]$$

$$= \lambda\left[\alpha_1\varepsilon_{t-1} + \alpha_1 \sum_{j=0}^{\infty} \beta^{j+1}\, \varepsilon_{t-1-(j+1)} + \alpha_2 \sum_{j=0}^{\infty} \beta^j \varepsilon_{t-2-j}\right]$$

$$= \lambda \left[\alpha_1 \varepsilon_{t-1} + (\alpha_1 \beta + \alpha_2) \sum_{j=0}^{\infty} \beta^j \, \varepsilon_{t-2-j} \right]$$

Also,

$$\sum_{j=0}^{\infty} \beta^j \left(\alpha_1 \delta\big(|\varepsilon_{t-1-j}| - E\big[|\varepsilon_{t-1-j}|\big]\big) + \alpha_2 \delta\big(\varepsilon_{t-2-j}| - E\big[|\varepsilon_{t-2-j}|\big]\big) \right)$$

$$= \delta \left[\alpha_1 (|\varepsilon_{t-1}| - E[|\varepsilon_{t-1}|]) + (\alpha_1 \beta + \alpha_2) \sum_{j=0}^{\infty} \beta^j \left(|\varepsilon_{t-2-j}| - E\big(|\varepsilon_{t-2-j}|\big) \right) \right]$$

Putting together:

$$\sum_{j=0}^{\infty} \beta^j V_{t-j} = \alpha_1 \left(\lambda \varepsilon_{t-1} + \delta\big(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)\big) \right)$$

$$+(\alpha_1 \beta + \alpha_2) \sum_{j=0}^{\infty} \beta^j \left[\lambda \varepsilon_{t-2-j} + \delta \left(|\varepsilon_{t-2-j}| - E\big(|\varepsilon_{t-2-j}|\big) \right) \right]$$

So

$$exp\left(s \sum_{j=0}^{\infty} \beta^j V_{t-j} \right)$$

$$= exp\left[s\alpha_1 \left(\lambda \varepsilon_{t-1} + \delta\big(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)\big) \right) \right]$$

$$exp\left[s(\alpha_1 \beta + \alpha_2) \textstyle\sum_{j=0}^{\infty} \beta^j \left[\lambda \varepsilon_{t-2-j} + \delta \left(|\varepsilon_{t-2-j}| - E\big(|\varepsilon_{t-2-j}|\big) \right) \right] \right]$$

$$= exp\left[s\alpha_1 \left(\lambda \varepsilon_{t-1} + \delta\big(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)\big) \right) \right] \qquad \textstyle\prod_{j=0}^{\infty} exp\left(s(\alpha_1 \beta + \alpha_2) \beta^j \left[\lambda \varepsilon_{t-2-j} + \delta \left(|\varepsilon_{t-2-j}| - E\big(|\varepsilon_{t-2-j}|\big) \right) \right] \right)$$

Thus:

$$E\left[exp\left(s \sum_{j=0}^{\infty} \beta^j V_{t-j} \right) \right]$$

$= E[exp(s\alpha_1 W_{t-1})] \prod_{j=0}^{\infty} E\left[exp(\alpha_1\beta + \alpha_2)\beta^j W_{t-2-j})\right]$

Where

$W_{t-j} = \lambda\varepsilon_{t-j} + \delta\left(\left|\varepsilon_{t-j}\right| - E\left(|\varepsilon_{t-j}\right)\right)$

We now consider:

$E\left[exp\left(s\gamma_j W_{t-j}\right)\right]$, *j= 0,1,2,…*

$\gamma_1 = \alpha_{1,}\, \gamma_2 = (\alpha_1\beta + \alpha_2)$

$\gamma_j = (\alpha_1\beta + \alpha_2)\beta^{j-2}, j \geq 3$

$E\left[exp\left(s\gamma_j W_{t-j}\right)\right]$

$= E\left[exp\left(s\gamma_j\left[\lambda\varepsilon_{t-j} + \delta\left(\left|\varepsilon_{t-j}\right| - E\left(|\varepsilon_{t-j}|\right)\right)\right]\right)\right]$

$= E\left[exp\left(\theta_1\varepsilon_{t-j} + \theta_2\left(\left|\varepsilon_{t-j}\right| - E\left(|\varepsilon_{t-j}|\right)\right)\right)\right]$

$= exp\left(-\theta_2 E\left[|\varepsilon_{t-j}|\right]\right)E\left[exp\left(\theta_1\varepsilon_{t-j} + \theta_2|\varepsilon_{t-j}|\right)\right]$

Now let $\varepsilon_{t-j} \sim iid\ N\ (0,1)$ and thus we can easily show that:

$E\left(|\varepsilon_{t-j}|\right) = \sqrt{2/\pi}$ and

$E\left[exp\left(\theta_1\varepsilon_{t-j} + \theta_2|\varepsilon_{t-j}|\right)\right]$

$= \int_{-\infty}^{\infty} exp\left(\theta_1\varepsilon_{t-j} + \theta_2|\varepsilon_{t-j}|\right)\frac{1}{2\pi}e^{-\varepsilon^2_{t-j/2}}d\varepsilon_{t-j}$

$$= \int_{-\infty}^{\infty} exp\left(\theta_1\varepsilon_{t-j} - \theta_2\varepsilon_{t-j}\right)\frac{1}{2\pi}e^{-\varepsilon^2_{t-j/2}}d\varepsilon_{t-j}$$

$$= \int_{0}^{\infty} exp\left(\theta_1\varepsilon_{t-j} + \theta_2\varepsilon_{t-j}\right)\frac{1}{2\pi}e^{-\varepsilon^2_{t-j/2}}d\varepsilon_{t-j}$$

$= \int_{-\infty}^{\infty} exp\left(-(\theta_2 - \theta_1)\varepsilon_{t-j}\right)\frac{1}{2\pi}e^{-\varepsilon^2_{t-j/2}}d\varepsilon_{t-j}$ (A)

$+ \int_{0}^{\infty} exp\left((\theta_2 + \theta_1)\varepsilon_{t-j}\right)\frac{1}{2\pi}e^{-\varepsilon^2_{t-j/2}}d\varepsilon_{t-j}$ (B)

Integral A

Consider the exponent

$$-\frac{\varepsilon_{t-j}^2}{2} - (\theta_2 - \theta_1)\varepsilon_{t-j}$$

$$= -\frac{1}{2}\left(\varepsilon_{t-j}^2 + 2(\theta_2 - \theta_1)\varepsilon_{t-j} + (\theta_2 - \theta_1)^2 - (\theta_2 - \theta_1)^2\right)$$

$$= \tfrac{1}{2}(\theta_2 - \theta_1)^2 - \tfrac{1}{2}\left(\varepsilon_{t-j} + (\theta_2 - \theta_1)\right)^2$$

Thus:

(A) → $$e^{(\theta_2-\theta_1)^2/2} \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} exp\left(-\tfrac{1}{2}\left(\varepsilon_{t-j} + (\theta_2 - \theta_1)\right)^2\right) d\varepsilon_{t-j}$$

$$\varepsilon_{t-j} \rightarrow u = \epsilon_{t-j} + (\theta_2 - \theta_1)$$

i.e., $\varepsilon_{t-j} = u - (\theta_2 - \theta_1)$

$$d\varepsilon_{t-j} = du$$

(A) → $e^{(\theta_2-\theta_1)^2/2} \int_{-\infty}^{(\theta_2-\theta_1)} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$

$$= e^{(\theta_2-\theta_1)^2/2}\Phi(\theta_2 - \theta_1)$$

Similarly, we have:

(B) →

$$= e^{(\theta_2-\theta_1)^2/2}\Phi(\theta_2 + \theta_1)$$

Therefore:

$$E\left[exp\left(\theta_1\varepsilon_{t-j} + \theta_2|\varepsilon_{t-j}|\right)\right]$$

$$= e^{(\theta_2-\theta_1)^2/2}\Phi(\theta_2 - \theta_1) + e^{(\theta_2+\theta_1)^2/2}\Phi(\theta_2 + \theta_1)$$

and finally,

$$E\left[exp\left(s\gamma_j w_{t-j}\right)\right]$$

$$= e^{-\theta_2\sqrt{2/\pi}}\left[e^{(\theta_2-\theta_1)^2/2}\Phi(\theta_2 - \theta_1) + e^{(\theta_2+\theta_1)^2/2}\Phi(\theta_2 + \theta_1)\right]$$

Since:

$\theta_1 = s\gamma_j\lambda$ and $\theta_2 = s\gamma_j\delta$

we have

$(\theta_2 - \theta_1) = s\gamma_j(\delta - \lambda)$

$(\theta_2 + \theta_1) = s\gamma_j(\delta + \lambda)$

Thus

$E\left[exp\left(s\gamma_j w_{t-j}\right)\right]$

$$= exp\left(-s\delta\sqrt{\frac{2}{\pi}}\gamma_j\right)\left[exp\left(\frac{1}{2}\left(s^2\gamma_j^2(\delta-\lambda)^2\right)\right)\Phi\left(s\gamma_j(\delta-\lambda)\right)\right.$$
$$\left. + \ exp\left(\frac{1}{2}\left(s^2\gamma_j^2(\delta+\lambda)^2\right)\right)\Phi\left(sy_j(\delta+\lambda)\right)\right]$$

$exp(s\alpha_1 W_{t-1})\ exp(s(\alpha_1\beta + \alpha_2)w_{t-2})\ \ exp\left(s\left(\alpha_{1\beta} + \alpha_2\right)\beta W_{t-3}\right)exp(s(\alpha_1\beta + \alpha_2)\beta^2 W_{t-4})$

$\gamma_1 = \alpha_1$

$\gamma_2 = (\alpha_1\beta + \alpha_2)$

$\gamma_j = (\alpha_1\beta + \alpha_2)\beta^{j-2},\ J \geq 3$

$E[exp(s\ln\sigma_t^2)] = E[\sigma_t^{2s}]$

$$= exp\left(\frac{s\alpha}{1-\beta}\right)E\left[exp\left(s\sum_{j=0}^{\infty}\beta^j V_{t-j}\right)\right]$$

$$= exp\left(\frac{s\alpha}{1-\beta}\right)E\left[exp\left(s\sum_{j=0}^{\infty}\gamma_j W_{t-j}\right)\right]$$

$$= exp\left(\frac{s\alpha}{1-\beta}\right)\prod_{j=o}^{\infty}E\left[exp\left(s\gamma_j W_{t-j}\right)\right]$$

Where

$$E\left[exp\left(s\gamma_j W_{t-j}\right)\right] = exp\left(-s\delta\sqrt{\frac{2}{\pi}}\gamma_j\right)\left[exp\left(\frac{1}{2}\left(s^2\gamma_j^2(\delta-\lambda)^2\right)\right)\Phi\left(s\gamma_j(\delta-\lambda)\right) +\right.$$

$$\left. exp\left(\frac{1}{2}\left(s^2\gamma_j^2(\delta+\lambda)^2\right)\right)\Phi\left(s\gamma_j(\delta+\lambda)\right)\right]$$

with

$$\gamma_{1=\alpha_{1,}}\,\gamma_2 = (\alpha_1\beta+\alpha_2), \gamma_j = (\alpha_1\beta+\alpha_2)\beta^{j-2}, \qquad j \geq 3$$

$$\prod_{j=0}^{\infty}\left[exp(-\alpha_1\gamma_j)\square\left[exp\left(\frac{b_2^2}{2}\gamma_j^2\right)\Phi\left(b_1\lambda_j\right) + exp\left(\frac{b_2^2}{2}\gamma_j^2\right)\Phi\left(b_2\gamma_j\right)\right]\right]$$

$$e^{\alpha_1\gamma_2}\left[exp\left(\frac{b_1^2}{2}\gamma_1^2\right)\Phi(b_1y_2) + exp\left(\frac{b_2^2}{2}\gamma_1^2\right)\Phi(b_2y_1)\right]$$

$$e^{\alpha_1\gamma_2}\left[exp\left(\frac{b_1^2}{2}\gamma_2^2\right)\Phi(b_1y_2) + exp\left(\frac{b_2^2}{2}\gamma_2^2\right)\Phi(b_2y_2)\right]$$

$$= e^{-a_1\sum\gamma j}\prod_{j=0}^{\infty}\left(e^{\frac{b_1^2y_j^2}{2}}\Phi\left(b_1\gamma_j\right) + e^{\frac{b_2^2y_j^2}{2}}\Phi\left(b_2\gamma_j\right)\right)$$

$$a_1 = -s\delta\sqrt{\frac{2}{\pi}}$$

$$b_1 = s(\delta+\lambda)$$

$$b_2 = s(\delta+\lambda)$$

$$\sum_{j=0}^{\infty}\gamma_j = \alpha_1 + \sum_{j=2}^{\infty}(\alpha_1\beta+\alpha_2)\beta^{j-2}$$

$$= \alpha_1 + (\alpha_1\beta+\alpha_2)\sum_{j=2}^{\infty}\beta^{j-2}\; 1+\beta+\beta^2+\beta^3+\ldots \qquad \begin{matrix} l=j-2 \\ j=l+2 \end{matrix}$$

$$= \alpha_1 + (\alpha_1\beta+\alpha_2)\sum_{l=0}^{\infty}\beta^l = \alpha_1 + \frac{(\alpha_1\beta+\alpha_2)}{(1-\beta)}$$

$$\sum_{j=0}^{\infty}\gamma_j = \alpha_1 + \frac{(\alpha_1\beta+\alpha_2)}{1-\beta}$$

$$= \frac{\alpha_1(1-\beta)+\alpha_1\beta+\alpha_2}{1-\beta}$$

$$= \frac{\alpha_1+\alpha_2}{1-\beta}$$

$$E[\sigma_t^{2s}] = exp\left(\frac{s\alpha}{1-\beta}\right) exp\left(\frac{-s\delta\sqrt{2/\pi}(\alpha_1+\alpha_2)}{1-\beta}\right) \ \prod_{j=0}^{\infty}\left(e^{\frac{b_1^2\gamma_j^2}{2}}\Phi\left(b_1\gamma_j\right) + e^{\frac{b_1^2\gamma_j^2}{2}}\Phi\left(b_2\gamma_j\right)\right)$$

CHAPTER 3: DOES INFLATION MATTER FOR EQUITY RETURNS?

This chapter explores the relationship between equity returns and inflation using long-term historical data for four of the largest economies in the world: the US, Japan, the UK and Germany. Unlike most previous studies, the study explores both the long-term and the short-term dimension of the bi-variate relationship between equity returns and growth in consumer prices in order to ascertain if equities are a hedge against inflation. In general, mixed support was found for the hypothesis of a stable long-run equilibrium relationship, while the short term analysis showed evidence of an asymmetric behaviour during different inflationary environments, which could not simply be explained in terms of different economic growth environments. For a long-term investor such as a pension fund the key implication of these results is that short-term dynamics cannot be ignored in the belief that the stock market will turn out to be a perfect inflation hedge in the long-run - an attribute which is highly desirable when liabilities are inflation-linked.

3.1 INTRODUCTION

In principle an asset is regarded as an inflation hedge when it protects investors against changes in the price index on a period-by-period basis. Reilly, Johnson and Smith (1970) define an asset to be a complete inflation hedge if the real rate of return is at least as great in inflationary periods as it is in non-inflationary periods. A partial hedge would on the other hand be an asset whose nominal rate of return is greater in inflationary as opposed to during non-inflationary periods.

An inflation hedge is clearly a valuable asset to hold especially in a context in which future liabilities are indexed to the consumer price index. The extent to which financial assets are effective inflation hedges ultimately depends on the effects of inflation on the real economy. If inflation is neutral and all contracts are automatically indexed to the price index, then both financial assets and real assets are likely to be effective inflation hedges. The price of equities would compensate investors for the rise in consumer prices, because it would discount higher nominal dividends. Moreover, if the real interest rate were not very volatile, the price of nominal bonds would not move significantly because inflation would be expected and incorporated in the yield offered to investors at the outset.

The basic theoretical concept in this area is commonly attributed to Fisher (1930), who postulated that nominal financial returns reflects full information concerning the possible future values of the rate of inflation. This effect is known as the “Fisher effect” and is widely accepted. To elaborate further, the Fisher hypothesis states, expected nominal risky asset returns move one for one with expected inflation such that expected real returns are independent of expected inflation. A related implication is that assets which represent claims to real payments, such as equity, should offer a hedge against unexpected inflation, while assets which represent claims to nominal assets, such as bonds, should not be expected to offer such hedging characteristics. Although, this theoretical framework could in principle hold independently of the holding period, previous studies have found different results depending on whether a shorter or longer horizon was considered.

This study uses a two-step empirical hypothesis testing process to explore in detail the bi-variate relationship between equity market dynamics and consumer price inflation from a pension fund investment's point of view. Note, that the objective of the study carried out in this chapter is to understand further the inflation hedging properties of equities (within the Fisher framework) in various countries rather than to explore the exact empirical drivers of the relationship. Indeed, this focus on inflation hedging properties of equity returns justifies the use of the bi-variate set-up employed in this chapter. The argument is that, from a pension fund's perspective, if over long horizons equity returns do adjust to inflation, even with a lag, the short term dynamics become almost irrelevant unless there is a clear mismatch between liability maturity and the equity/inflation adjustment cycle. On the other hand, if the above hypothesis fails to hold, there is an even stronger argument for exploring the short-term dynamics of the equity-inflation relationship and to look for stable patterns, if they exist. These patterns could be then exploited empirically for forecasting and potentially exploited by pension funds using tactical overlay strategies.

Equity returns and inflation historical data were obtained by Global Financial Data (www.globalfindata.com) and are described in section 3.3. Both monthly and annual data series were employed.

In the next two sections, we discuss first the academic literature in this area, and then review the sources of the data. In section 3.4, we present the results of the long term analysis and section 3.5 discusses the empirical examination focused on short term dynamics.

The key finding of this report is that over the long-run there is mixed support for a stable equilibrium relationship between equity and consumer prices in the countries examined. Moreover, the investigation of the short-term dynamics highlights the asymmetric behaviour of equity markets during different inflationary environments.

3.2 INFLATION AND EQUITY RETURNS: LITERATURE REVIEW

The impact of inflation on returns of financial assets especially equity has been an important theoretical and empirical question for many years. The very high and volatile inflation years of 1970s and the subsequent convergence to the present low inflation environment, now potentially exposed to the threat of deflation, have challenged economic theory, while at the same time increasing the practical relevance of the debate. There have been many academic studies since the mid-1970s, both theoretical and empirical, which attempted to explore the relationship between equity and inflation. In general, the empirical results (mostly using US data) were mixed and highly sensitive to the sample period used, the choice between multi-period (rolling) and single-period financial asset returns and the econometric methodology employed.

The first challenge to the "Fisher effect" paradigm came from the work of Jaffe and Mandelker (1976), one of the earliest empirical studies in this area. They found a negative (statistically significant) relationship between US equity returns and concurrent inflation (sample 1953-1971), although over longer periods (using lagged inflation as regressors) this relationship was found to be positive. In addition, Jaffe and Mandelker (1976), examined the relationship between anticipated inflation (proxied by short interest rates or ARIMA model forecasts) and equity returns. The result was a significant negative relationship between the aforementioned variables. This finding was clearly inconsistent with the Fisher hypothesis. Later empirical studies by Fama and Schwert (1977), Gultekin (1983) and Nelson (1975) reported similar conclusions.

Nelson (1975)'s study on the relationship between the two variables (using US data) followed an empirical methodology similar to Jaffe and Mandelker (1976). Nelson (1975), based on the empirical results, argued that the observed negative relationship between equity and inflation, although counterintuitive, may not imply a departure from market efficiency notion per se, whereby valuable information is fully reflected in market prices. However, the observation that ex-ante equity prices could be below the risk free rate and sometimes negative was more worrying.

Gultekin (1983) used data for 26 countries including UK and USA, for the period 1946-1979. He used the panel regression methodology to test the relationship. The results, he found were unstable and differed across countries.

Solnik (1978) solved the CAPM model in the real mean variance space and derived expression for the efficient frontier. Solnik (1978) showed that investors subject to different inflation rates (tastes) will never hold the same portfolios, whatever their risk aversion. The market portfolio will not be efficient for all or most investors nor will it be nominal efficient.

Various theoretical frameworks and empirical studies have been developed to explain the negative correlation between observed inflation and equity returns. Fama (1981), Geske and Roll (1983), Ram and Spencer (1983), James et al (1985) and Stulz (1986) all attempted to explain the negative association between equity returns and price acceleration.

Fama (1981) hypothesised that the negative correlation between equity returns and inflation is not a causal relationship but it is proxying for a positive relationship between real activity and equity returns and is induced by a negative relationship between real activity and inflation. Geske and Roll (1983) argued that equity prices signal changes in inflationary expectations because of a chain of macroeconomics events.

Ram and Spencer (1983) using restricted Vector Autoregressive Regressions (VAR), found evidence of unidirectional causality from inflation to equity returns. While James et al (1985) using a Vector Autoregressive Moving Average Model (VARMA) simultaneously modelled the causal links between equity returns, real activity, money supply and inflation. They found evidence that equity returns signal both changes in real activity and changes in monetary base. This suggests a link between money supply and real activity signalled by equity returns. This observation would be consistent with Geske and Roll's explanation.

Lee (1992) using 4 variable unrestricted VAR model examined the causal relationships between inflation, interest rates, real activity and real equity returns for post war USA economy. He also studied the dynamic interactions using error decomposition and impulse responses. His major empirical findings were more close to Fama's (1981) explanation, rather than the Geske and Roll (1983) theory.

Finally, Barnes, Boyd and Smith (1999) analyzed data from a panel of countries and supported the view of a direct negative relationship between inflation and equity returns, due to capital markets inefficiencies caused by high inflation such as financial markets frictions, reduction in liquidity and credit extension as well as reduced physical and human capital investment.

The above-mentioned studies, both theoretical and empirical, focused almost exclusively on short term asset returns and inflation with time horizons of one year or less. These empirical studies implicitly postulated that the Fisher model would hold at all horizon lengths. However, there are institutional investors such as pension funds and life insurance companies in the market, with longer investment horizons and these are very important players in most developed countries.

Boudoukh and Richardson's (1993) study attempted to fill this void by exploring the relationship between ex-ante equity returns and ex-ante inflation over longer periods. They used the instrumental variable (IV) econometric approach to model ex-ante long term inflation, using past inflation rates, short and long term interest rates as measures of ex-ante inflation. The results showed a positive relationship between nominal equity returns and both ex-ante and ex-post long term inflation (sample 1802-1990). These results were robust to the different sub periods used and were valid both for the US and the UK. The bottom line was that, although the theoretical expectation of a positive relationship does not hold empirically over short term, there was evidence that such was not the case over a longer term horizon.

Engsted and Tanggaard (2001) analysed the relationship between expected equity and bond returns an expected inflation at short and long horizons using vector autoregression (VAR) model involving only one period variables. They employed the estimated a 1-period VAR model to project the inflation and equity series over longer term horizons, thus avoiding the near-non stationarity problem of multi-period returns and inflation. Unlike, Boudoukh and Richardson (1993), Engsted and Tanggaard (2001) found a positive, albeit weak relationship between expected returns and inflation over both short and long horizons.

Hess and Lee (1999) investigated the relationship by splitting the causes of inflation into demand shocks and supply shocks. They argued that supply shocks reflect real output shocks and cause a negative relationship between equity returns and inflation, while demand shocks are mainly due to monetary reasons and induce a positive relation between equity returns and inflation. They also constructed a theoretical model to show, how the two different shocks can result in totally opposite relationships between equity returns and inflation. By using time series decomposition technique devised by Blanchard and Quah (1989), Hess and Lee disentangled the two types of disturbances. They used empirical data from both post war and pre-war US to test the theoretical hypothesis, the "shock dependency" of the relationship between inflation and equity returns. They reported evidence from post-war US, UK, Japan and Germany, showing the relative predominance of supply shocks, thus the observed negative relation.

Lothian and McCarthy (2001) explored the relationship using a different angle. Following Cagan (1974), they employed panel methodology to 14 OECD countries (sample 1949-1999) and compared the results with US and UK (1702-1999). They also found some support for the long term inflation hedge hypothesis. However, there are puzzlingly long adjustment lags of around 10 years before the "Fisher effect" relationship can be detected.

More recently, this subject has been approached using industry level data by Ciner (2014), who examined the relationship between equity returns and inflation in a frequency dependent framework. The analysis carried out showed that a positive relation in fact exists between equity returns and high frequency inflation shocks for commodity and technology-related industries. In addition, Austin & Dutt (2015) using more recent data found no evidence that equity returns hedge inflation at long-horizons even after correcting for endogeneity and overlapping observations. In Emerging Markets (EM) space, Spyrou(2004) investigated the relationship between inflation and equity returns for ten major emerging market countries. The results indicated a positive and statistically significant connection between stock returns and inflation dynamics for three countries in the sample, while it was positive but statistically insignificant for a further three, while only for one country, the relationship was found to be negative and statistically insignificant.

The discussion above gives a brief overview of the literature and the evolution of thought in this area over the years. However, the debate remains far from settled.

3.3 DATA

This section provides more detail on historical stock market and consumer prices indices used in the analysis. All the series were calculated by Global Financial Data (www.globalfindata.com).

For US equity returns two alternative series are used: the Wilshire 5000 and the S&P 500 indices, both adjusted for dividends. The Standard and Poor’s indices were first calculated in 1918, and the Cowles Commission back-calculated the series to 1871 using the Commercial and Financial Chronicle. The 90-stock Composite was calculated from 1926 through February 1957 when S&P introduced the S&P 500 stock average including 425 industrials, 25 rails and 50 utilities, weighting the index substantially in favour of the industrials. S&P did not calculate the 500-stock index prior to March 1957, but used the old 90-share index (as well as the old 50 industrials, 20 rails and 20 utilities indices) to extend the data back to 1928.

The Wilshire 5000 Equity Index measures the performance of all U.S. headquartered equity securities with readily available price data. Approximately 6,800 capitalization-weighted securities are used in the index. Before December 1974, when the Russell 5000 total return index was first calculated, this series uses Schwert (1990)'s methodology to provide an index of United States stocks dating back to 1802. This index combines the monthly price indexes of mainly bank, insurance and railroad stocks.

The All-Share index contains the historical data for the United Kingdom. East Indies Stock is used for 1693. The index is an unweighted arithmetic average of Bank of England and East Indies stock from 1694 to August 1711, and of Bank of England, East Indies and South Sea stock from September 1711 to January 1811. Rostow's Total Index of Share Prices is used from 1811 to 1850. Hayek's index was taken from Rostow and excludes banks, insurance and bridge stocks, but includes industrial stocks. This index is linked to the London and Cambridge Economic Service index, which begins in July 1867 and continues until 1906. The L&CES index consisted of 25 stocks in 1867 and had grown to 75 stocks by 1914.

The Banker's Magazine kept a capitalisation-weighted index of 287 stocks, which gave the total capital values of the companies that were included. This was the broadest index of London shares at the time and the index is used beginning in 1907. Although this index was calculated beginning in 1887, the Banker's Magazine usually omitted calculating the index for one month during the summer, and for this reason it is excluded until 1907 when calculations were made for every month. The London market closed in August 1914 and reopened in January 1915. The Banker's Magazine Index is used through May 1933. Beginning in June 1933, the Actuaries General Index is used. This index included financial stocks, commodities and utilities, but excluded debentures and preferred shares.

Beginning in April 1962, the Financial Times-Actuaries All-Share Index is used. All indexes have been chain linked to one another to create a continuous index with the All-Share index's base of April 10, 1962 used as the base for the entire index. The All-Share Index is a capitalisation-weighted price index and covers about 98-99% of the capital value of all UK companies. It uses the Paasche formula, adjust for capitalisation changes, and has its components reviewed in December. It combines the FT-SE 100, FT-SE Mid-250 and FT-SE Small Cap indices, but excludes the Fledgling and AIM index components.

For Germany the stock index uses the CDAX composite in 1970. Data prior to that have been calculated using the Reichsamt/Bundesamt index through 1954 and the Commerzbank index thereafter as well as dividend yield data from the Statistiches Reichsamt/Bundesamt (annual through 1928, monthly through April 1942, no data from 1942 through 1952, annual from 1952 through 1955 and monthly thereafter). No yield data are available from April 1942 through 1952, so it was assumed that the dividend payout of March 1942 continued through 1952 without a change. No calculations are made from 1914 through 1925.

For Japan the stock index uses the Nikko Securities Composite beginning in 1980, and the JSRI total return index from 1952 through 1979. Data prior to that have been recalculated using historical data on yields and the Japan National Bank index through 1932, the Oriental Economist Index from 1933 through 1945, the Fisher Index from 1946 through 1949 and the Nikkei-225 index from May 1949 through 1951. Yield data are annual from 1921 through

August 1926, and no yield data was available from 1942 through April 1949, so it was assumed that the dividend pay-out in 1941 continued until April 1949.

Consumer prices in the US (Mitchell, 1998) before the official CPI was first calculated by the Bureau of Labor are based upon a combination of three indices. From 1820 through 1874, the annual cost-of-living index calculated by the Federal Reserve Bank is used. From 1875 until 1912, it uses a monthly Index of General Prices calculated by the Federal Reserve Bank of New York, which was weighted between wholesale commodity prices (20%) Wage payments (35%), the Cost of Living (35%) and Rents (10%). From 1913 on, the Bureau of Labor's Consumer Price Index is used. For information on how the prices of individual goods have changed, see Derks (1999).

In the UK retail prices are calculated since 1914 by the Central Statistical Office (Office of National Statistics after 1996). Data from before 1900 and the key sources are Brown and Hopkins (1956) and Brown and Hopkins (1959). Annual data are used through 1914 with monthly food prices used for August through December 1914.

In Japan the official CPI index since 1946 and before then the source is Monthly Statistical Bulletin published by the League of Nations.

Germany's official CPI began in February 1920, but prior to that a consumer price index calculated by Gielen (1994) is used. This is also compared with an index of the average level of foodstuff prices in 200 German cities is available (Calwer, 1960). The base is July 1914 = 100. Prices stabilized in December 1923 after the Weimar hyperinflation, and a new series in gold Reichsmark was introduced.

The series are allowed to overlap for comparison. The official German CPI series continued until February 1945 when it was halted. The data for March 1945 through June 1948 is from Munich where a CPI index used the old 1913/14=100 gold marks base. In July 1948 the Bundesamt (the German federal statistical office) once again began calculating an official CPI Index for the entire country, and this index is used currently.

3.4 INFLATION AND EQUITY: LONG-TERM RELATIONSHIP

The first step towards testing the long-run relationship between equity returns and inflation is to examine contemporaneous correlation between equity returns and inflation using 1-year data. In general, correlation is not high in nominal terms and real returns are lower when inflation is high except for Weimar Germany (this is due to one exceptional outlier). If equities were a perfect hedge over a 1-year period, one would have expected a correlation around 100% in nominal terms and not significantly different from zero in real terms. The latter hypothesis clearly does not encounter much support in the data.

Table 3.1 Contemporaneous correlation between equity returns and inflation (1-year)

Country	Period	Nominal returns/inflation	Real returns/inflation
US (Wilshere)	1840-2001	6.21%	-23.99%
US (S&P500)	1900-2001	-1.81%	-24.44%
UK	1800-2001	8.67%	-47.13%
UK	1900-2001	5.56%	-35.12%
Japan	1921-2001	-1.28%	-47.28%
Germany	1870-2001	-50.00%	44.75%
Germany	1930-2002	-34.49%	-43.59%

The next step is to test the causal relationship between equity returns and inflation, assessing the predictive power of historical data of one variable for subsequent values of a second variable (this is the Granger causality framework, Granger, 1969).

More formally, if past values of inflation explain equity returns then inflation Granger causes equity returns. This does not imply causality in any behavioural sense, it just says that past inflation help predict the future pattern of equity returns. This is a case of direct causality supporting the inflation hedge hypothesis, although one should be careful in drawing implications because the test gives no indication on the sign of the relationship.

At the same time, past values of equity returns could also contribute to explain today's inflation, suggesting reverse causality and a more complex pattern. Lagged adjustment of equity returns to inflation can also be captured by the Granger causality framework. Reverse causality could be for instance consistent with Geske and Roll (1983) interpretation of anticipated response of equity prices to worsening economic conditions and higher expected inflation.

Finally, there could also be a bi-directional feedback between inflation and equity with both past equity returns contributing to explain future inflation and equity returns adjusting to past inflation.

Table 3.2 shows the results of the Granger causality test using data for USA, UK, Japan, and Germany.

Table 3.2 Granger causality tests for Equity Returns and Inflation

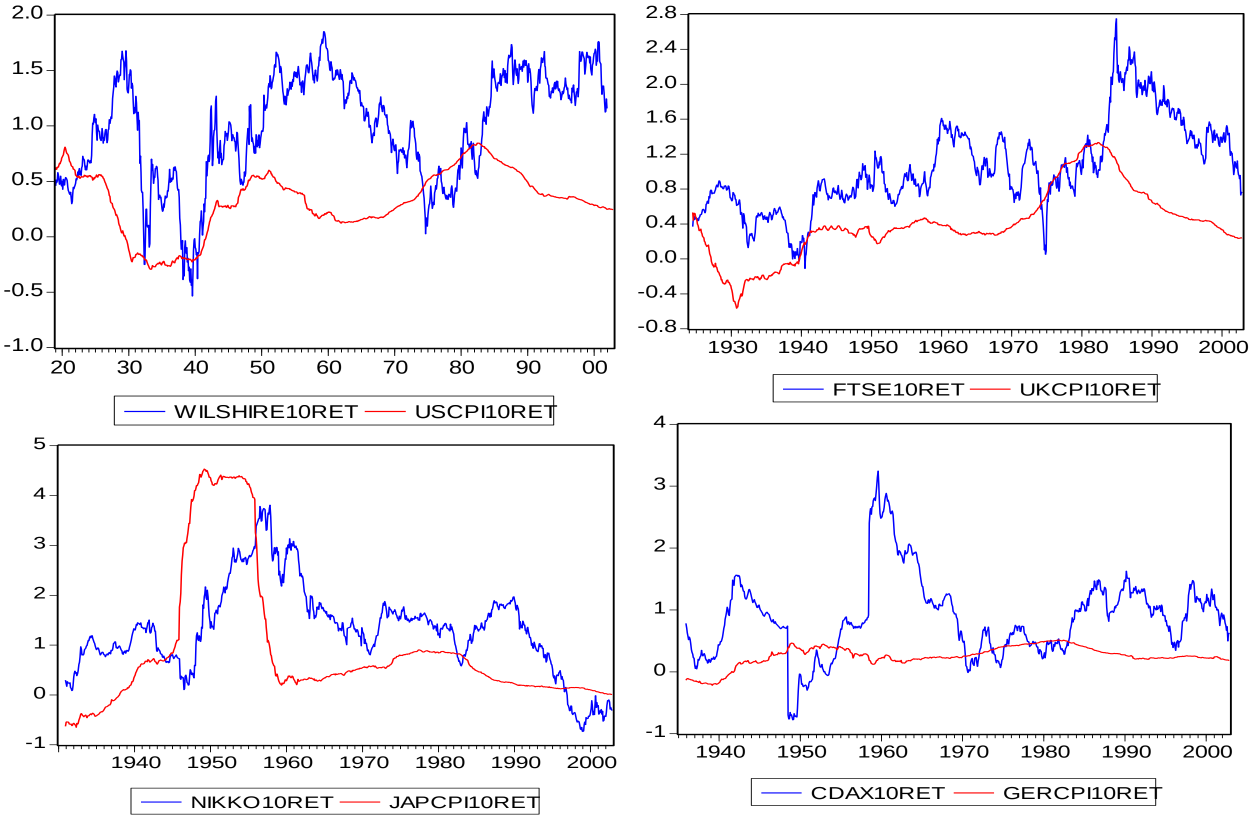
Country	**Period**	**Lag**	**Inflation does not Granger cause equity returns (P<0.05 reject)**	**Equity returns do not Granger cause inflation (P<0.05 reject)**
US (Wilshire)	1820-2002	10 years	0.19	0.00
US (S&P500)	1870-2002	5 years	0.01	0.01
US (S&P500)	1926-2002	60 months	0.06	0.01
UK	1694-2002	10 years	0.00	0.96
UK	1900-2002	10 years	0.06	0.98
UK	1926-2002	60 months	0.21	0.01
Japan	1921-2002	4 years	0.01	0.89
Japan	1926-2002	60 months	0.04	0.00
Germany	1870-2002	10 years	0.63	0.00
Germany	1926-2002	60 months	0.00	0.00

Except for Japan, the results of the test are at best mixed and sensitive to the length of the lags postulated. For US, the inflation hedge hypothesis is only supported for 5-year lag specification. Whilst for UK and Germany, the results are not stable (in the UK the hypothesis seems to encounter less support with more recent data), in Japan there is stronger support for the hypothesis of an adjustment of equity returns to inflation at 5% level of significance. However, even in Japan the results are driven by the only significant experience of inflation in the Japanese history: the immediate aftermath of WWII.

At the same time, the results show incidence of reverse causality particularly in the US, but also in Japan and Germany, symptomatic of a bi-directional feedback system and a more complex pattern of relationship than a simple inflation hedging hypothesis would suggest.

Correlation and Granger causality analysis investigated a pattern of association between equity returns and inflation (both contemporaneous and past). However, limited support for the Fisher hypothesis is based on the analysis of inflation and returns over a 1-year horizon, although taking into account lagged effects. It could well be that returns measured on a longer timeframe do adjust to inflation. A simple direct test could be performed using rolling 10-year returns calculated on underlying monthly data. Figure 3.1 below shows their distribution in the four countries:

Figure 3.1 Rolling 10-years returns in USA, UK, Japan and Germany



In the four countries,10-year equity returns have generally compensated investors for inflation, but this has not occurred in all economic environments. At times of higher than normal inflation returns measured on a 10-year period have been negative in real terms: Japan in the 1950s and the 1970s in all other countries. In addition to that, real returns have been negative in the US and the UK during a period of severe economic recession and deflation (the 1930s).

Table 3.3 shows the results of regressions of rolling 10-year equity returns over 10-year accumulated inflation. Standard errors are corrected for autocorrelation using the standard

Newey-West methodology. The same methodology was also applied to rolling 5-year returns for comparison (Table 3.4).

Table 3.3 Regression results of 10-year rolling equity returns over 10-year inflation

Country	Period	Beta Coefficient	Significance	R-square
US (S&P)	1919-2002	0.16	0.33	0.82%
US (S&P)	1919-2002 If CPI>=Median (3.47% pa)	-1.34	0.00	16.3%
US (S&P)	1919-2002 If CPI<Median (3.47% pa)	0.87	0.00	31.6%
UK	1924-2002	0.77	0.00	28.52%
UK	1924-2002 If CPI>=Median (3.56% pa)	0.28	0.05	3.30%
UK	1924-2002 If CPI<Median (3.56% pa)	0.77	0.00	26.56%
Japan	1930-2002	0.24	0.02	14.09%
Japan	1930-2002 If CPI>=Median (4.6% pa)	0.17	0.00	11.20%
Japan	1930-2002 If CPI<Median (4.6% pa)	1.46	0.00	18.77%
Germany	1935-2002	-0.81	0.07	3.81%
Germany	1935-2002 If CPI>=Median (2.53% pa)	-2.65	0.00	13.36%
Germany	1935-2002 If CPI<Median (2.53% pa)	1.99	0.00	18.83%

Table 3.4 Regression results of 5-year rolling equity returns over 5-year inflation

Country	Period	Beta Coefficient	Significance	R-square
US (S&P)	1915-2002	0.23	0.40	0.96%
US (S&P)	1915-2002 If CPI>=Median (3.47% pa)	-0.73	0.00	9.99%
US (S&P)	1915-2002 If CPI<Median (3.47% pa)	2.62	0.00	33.74%
UK	1919-2002	0.30	0.03	4.00%
UK	1919-2002 If CPI>=Median (3.56% pa)	-0.10	0.68	0.20%
UK	1919-2002 If CPI<Median (3.56% pa)	0.94	0.00	19.85%
Japan	1925-2002	0.15	0.02	4.67%
Japan	1925-2002 If CPI>=Median (4.6% pa)	0.09	0.17	2.36%
Japan	1925-2002 If CPI<Median (4.6% pa)	1.20	0.00	10.14%

Germany	1930-2002	-0.31	0.63	0.40%
Germany	1930-2002 If CPI>=Median (2.53% pa)	-4.03	0.00	19.50%
Germany	1930-2002 If CPI<Median (2.53% pa)	3.49	0.00	43.99%

The key result is that equity returns appear to be an imperfect inflation hedge even on a long-horizons, as they fail to compensate investors for growth in consumer prices when hedging properties would be most needed (at times of high inflation). Over the entire sample only in the UK and with a 10-year horizon the hypothesis of a one-for-one relationship encounters some empirical support. Moreover, when the sample is broken in two according to whether inflation is above or below its long term median, in all countries the estimated relationships become negative at times of higher than normal inflation.

There is however a more formal test to judge whether the hypothesis of a long-run one-for-one equilibrium finds empirical support, which does not require to postulate ex-ante what the length of the adjustment period should be. In fact, many economic and financial series drift apart in the short run, only to be brought together by market corrections over the long-run. The technique utilised to capture such dynamics is called cointegration, and was introduced by Granger (1981) as a way of statistically characterising equilibrium between two or more economic series. Cointegration in itself does not imply equilibrium in any behavioural sense, it only describes the tendency of two or more economic variables to move towards a particular region of the possible outcome space.

The concept of cointegration is an extension of the theory of non-stationary time series. The starting point is that most economic variables are characterised by the presence of a stochastic trend or, in other words, they exhibit systematic variations over time, which are hardly predictable (Maddala and In-Moo Kim, 1998). This leads to the famous problem of spurious regression first mentioned by Yule (1926), the fact standard regression analysis is not applicable to judge dependency between two non-stationary series.

However, Engle and Granger (1987) showed that, if a linear combination of two or more non-stationary series (i.e. $y-\lambda x-c$) displays a mean reverting behaviour, then there is a long-term equilibrium between the series as they share a common stochastic trend. It has also been showed that the cointegrating coefficient λ can be efficiently estimated using ordinary least squares. Stock (1987) showed that not only least squares is consistent for the true cointegrating coefficient but also that it converges to its true value faster than a coefficients estimated with stationary variables because of the infinite variance of all other linear combinations

Cointegration between two variables implies that, if the system is to return to its long-run equilibrium, at least one of the two variables responds to the magnitude of the disequilibrium. For instance, if we believe consumer prices and the stock market are cointegrated, then, when there is a positive gap between the two, or, in other words, when prices are higher than their long-run level relative to the stock market, at least one of the following must be true: 1) consumer prices will decrease and/or stock prices will increase, 2) consumer prices will decrease more than stock prices, 3) stock prices will increase more than consumer prices.

Using standard literature, this intuition can be formalised using a full error-correction model of the form:

$$\Delta y_t = \alpha_1 + \alpha_y (y_t - \lambda x_t) + \sum_{i=1}^{L} \alpha_{11}(i) \Delta y_{t-i} + \sum_{i=1}^{L} \alpha_{12}(i) \Delta x_{t-i} + \varepsilon_{yt} \quad (3.1)$$

where y is the stock market index (in log terms), x is the consumer prices level (in log terms) and αy is the adjustment coefficient capturing the speed at which variable y converge to its long term equilibrium position. Bearing in mind the example above, it is clear that not all adjustments coefficients need to be significantly different from zero, that is not all the variables in the system necessarily respond to deviations from the equilibrium (if they do not they are said to be block exogeneous).

There continues to be a strong interest in cointegration models to study the behaviour of financial markets. An early reference in this area is Campbell and Shiller (1986), who estimated a long term relationship between long term and short term interest rates as well as between stock prices and dividends. Tokat, Rachev and Schwatz (2003) estimate a long-run cointegrating relationship between the S&P 500 price index, consumer prices, the dividend yield (under the assumption that it is non-stationary), Treasury bill and bond rates. Bessler and Young (2003) extended Kasa's (1992) work using cointegration and error correction models to estimate dynamic relationships between nine major stock markets. Finally, Cassola and Morana (2002) investigate, among others, the relationship between stock market and economic growth in the Euro area.

In the actuarial literature Sherris, Tedesco and Zenwirth (1999) worked on cointegration with Australian data, exploring whether there was evidence of a long-run equilibrium relationship between short and long term interest rates, dividends and consumer prices as well as the stock market and consumer prices.

With a similar methodology, Cardinale (2003a) investigated the relationship between wages and asset returns in the UK and Cardinale (2003b) the inflation hedging properties of British residential and commercial real estate. In carrying out the tests of cointegration, we have used the Johansen (1988)'s technique, in line with most of the empirical literature in this area and have kept the bi-variate focus of the exercise (as noted above, our aim is to understand the inflation hedging properties of the equity market rather than study the empirical drivers of the relationship). Detailed results are presented in Appendix 3.1.

However, before discussing the results of the cointegration analysis exercise in detail, it is important to investigate the stationarity properties of the underlying data series at hand – results of which are shown in table 3.5 below. Overall, both consumer price index and equity index level series for the various countries considered (all in log form) were found to be I(1), which makes them admissible to be used when carrying out cointegration analysis. That said, it is important to note that sample choice in the case of Germany is important for the above result to hold as including the early-1920s Weimar years showed evidence of a break, which was detected by using the unit root with break test that identified 1922/23 as the period of the break.

Table 3.5: Stationarity Properties of CPI and Equities (all series in log terms, annual data, p-values shown below estimated using the Augmented Dicky-Fuller test)

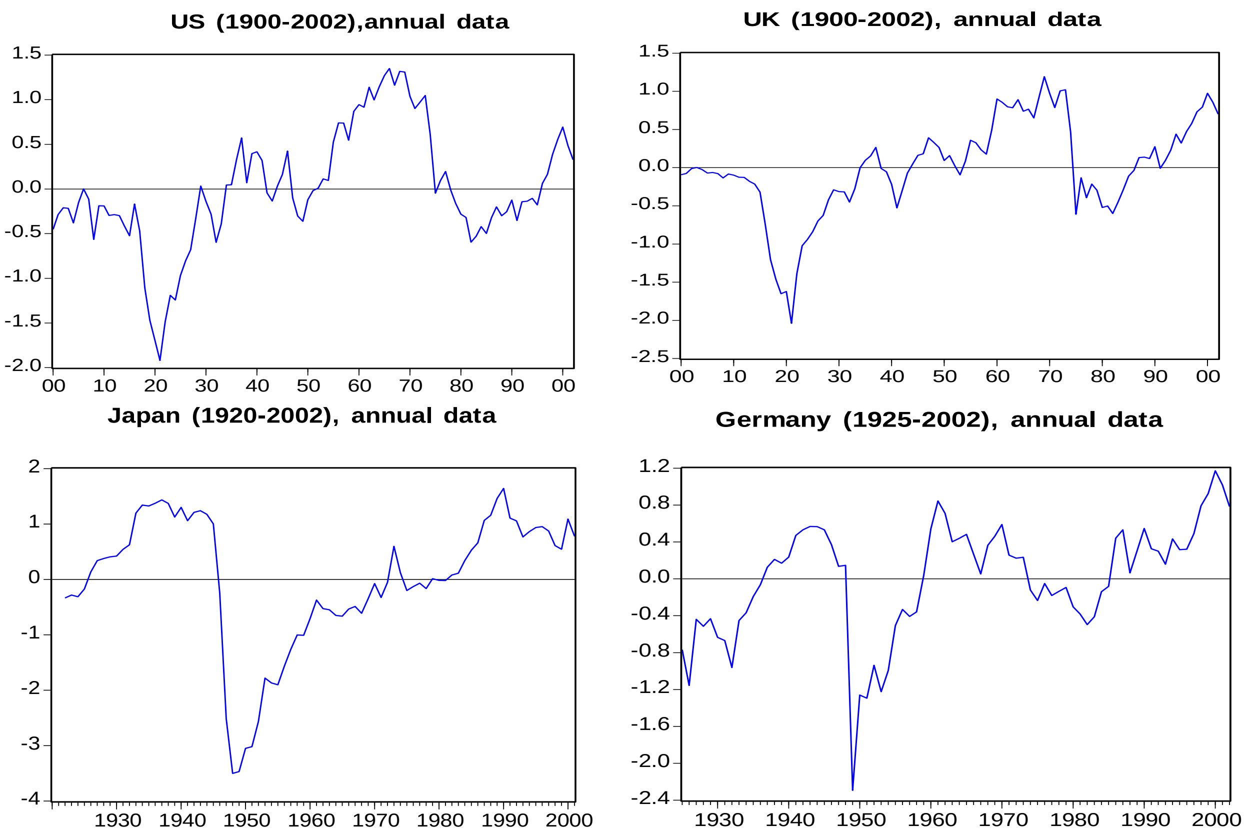
	Level	First Difference
1900-2002		
US CPI	96.90%	1.1%
S&P	99.50%	0.0%
1900-2002		
UK CPI	99.10%	1.6%
FTSE	99.30%	0.0%
1920-2002		
Japan CPI	72.80%	0.0%
Nikkei	69.60%	0.0%
1925-2002		
Germany CPI	94.50%	0.0%
CDAX	92.10%	0.0%

Figure 3.2 presents the estimated equilibrium errors and permits to visually judge the strength of the equilibrium relationship or, in other words, whether deviations from the equilibrium path are later corrected, as well as the length of the adjustment period. The models were estimated with annual data and 10 lags (except for Japan where the null of no cointegration could not be rejected even with 1 lag). As usual, a standard log transformation to the original series was applied before estimating the cointegrating models.

For the US, standard Johansen (1988) tests (see appendix 3.1) reject the null of no cointegration with 10 lags. However, evidence of long-term equilibrium relationship is weak as residuals diverged from zero for significant periods of time and the estimated coefficient (with 1900-2002 data) is higher than one (2.78). In addition, there is no evidence supporting cointegration, when a linear-trend term is assumed, when carrying out the test.

However, some evidence of a long-term adjustment pattern can be found in figure 3.2, as rising inflation in the 1970s helped to restore equilibrium after 20 years of post-war rising nominal and real equity returns during the 1950s-60s. Further, evidence comes from the 1980s when rising equity returns and lower inflation reversed the trend of the 1970s but at the end of the 1990s it was the equity bear market and not inflation which brought the system back towards its long-run equilibrium level. Indeed, the dot-com bubble burst episode is out-of-line with the pattern observed over the 1900-2002 period, in which adjustment in consumer prices rather than stock market dynamics has played a bigger role in restoring the equilibrium (formally, this can be seen in the error correction model from the magnitude of the coefficient of the residuals term estimated from the cointegrating equation[71] - See Appendix 3.1.2).

Figure 3.2 Equilibrium errors from cointegrating relationships between equities and prices[72]



In the UK's case, standard tests do not reject the null of no cointegration with 10 lags, but overall the evidence is similar to the US case. The estimated coefficient (with 1900-2002 data) is substantially higher than one (it is in fact 2.0) and in general, similar to the US, inflation has played a stronger role in restoring the equilibrium with evidence of adjustment taking place during the 1920s (residuals were deeply negative) and the 1970s (residuals were strongly positive).

Turning to Japan, as shown in appendix 3.2, the evidence of a long-run equilibrium is mixed as well, although the Johansen test rejects the null of no cointegration with as low as one lag. A visual inspection of the data in figure 3.2 does not provide support for the hypothesis of regular periodic cycles, but rather suggests the dominance of an adjustment which took place during 1950s, when a peak in inflation was followed by price stabilisation and a stock market boom, which forced the residual back towards 0. To partially control for this effect, a post-WWII dummy variable is used, when estimating error-correction models – however, even then the importance of the 1950s adjustment in driving the mean-reverting behavior of the residuals remains intact.

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[71] In the case of the US the sensitivity is 0.07 for the stock market and more than twice as much for inflation, which is also statistically significant (0.17).

[72] Equilibrium errors are the residuals from the cointegrating regression (the difference y- λ x-c measured throughout the sample and shown in figure 3.2 above.

The estimated long-run coefficient (with 1920-2002 data) is marginally but still significantly higher than one (1.30)[73] and, unlike in the UK and the US, it was stock market dynamics rather than shifts in consumer price index that drove the shift towards the equilibrium (even after the period post WWII is controlled for using a dummy variable – note that only the stock market adjustment coefficient is significant in the error correction model)[74]. In addition, even in Japan's case, there is no evidence of cointegration, when a linear-trend term is assumed, when carrying out the test.

Finally, in Germany's case, standard Johansen tests reject the null of no cointegration with lags higher than 8. However, like in all other countries, inclusion of a linear-trend term changed the result of the test. As with Japan, a post-WWII dummy variable is introduced when estimating error-correction models. The estimated coefficient (with 1925-2002 data) is significantly higher than one (2.26) and similar to Japan's case, it was the stock market dynamics which played a bigger role in restoring the equilibrium as captured by the magnitude of the stock market adjustment coefficient in the error-correction model (-0.29).

Overall, evidence presented above show mixed support for the one-for-one long-run equilibrium hypothesis. Specifically, estimated equilibrium relationships appears to be an outcome of one-off events rather than a result of sustained or more frequent corrections. Furthermore, in all cases, estimated coefficients in the cointegration equation are significantly higher than the theoretical value of one and the test is highly sensitive to the inclusion of the trend term. Also, no general conclusion can be derived on what forces drive the system back to the equilibrium. In the US and the UK, it was the behavior of consumer prices, while in Japan and Germany it was mainly the equity market dynamics. All in all, these observations shed doubt on the existence of a stable long-run equilibrium and a common adjustment process across the different countries studied.

3.5 Inflation and Equity Returns: Short-term dynamics

The discussion in the previous section gives mixed support to the notion of long-run equilibrium between equity market prices and consumer prices. In particular, although there appears to be some evidence of a long term equilibrium relationship, diagnostic results cast doubt on its stability at different points in history and across countries. Therefore, following the two-step hypothesis framework, outlined in section 3.1, we also investigated short term dynamics of the relationship between equity returns and inflation to gain additional insights and to identify recurring patterns, if they exist.

The analysis below is carried out using the longest available series published on a monthly basis, but equity returns and inflation are defined on a year-on-year basis (e.g. December 1967 to December 1968). The sample starts at 1910 for the US, 1915 for the UK, 1922 for Japan and 1927 for Germany, thus excluding the Weimar hyperinflation experience.

The inflation series is split into six buckets which are: Deflation (inf <0% p.a, D), Very Low inflation (up to 1.5% pa, VL), Low inflation (1.5% up to 3%, L), Moderate Inflation (3% up to 6% p.a, M), High (6% up to 10% p.a, H) and Very High inflation (10% p.a and above, VH). The six regimes are chosen to get a diverse carve out of the inflation series in order to differentiate asymmetric behaviour of equity returns, if there is any, during different inflationary environments. We believe that this is a good starting point to explore the short-term dynamic

[73] This can be formally test with the LR Test for Binding restrictions (null hypothesis is coefficient equal to one). In this case the test statistic is equal to 3.98, which corresponds to a p-value (under a Chi-square distribution) of 0.046
[74] Coeff of -0.09 with a t-stat of -3.03.

and investigate the presence of any regimes impacting the relationship between equity returns and inflation.

The choice of splitting the sample alongside the various inflation buckets might appear arbitrary at first sight but it was preferred to compare frequencies across countries using a common definition, rather than using quartiles or quintiles dependent on each underlying distribution especially in the context of the widespread adoption of inflation targeting frameworks adopted by key central banks since 1996/7, which are all centered around the absolute 2% target[75].

Table 3.6 below shows the basic statistical features of the year on year nominal equity series when sorted out on the basis of different inflation regimes.

Table 3.6 Statistical features of YoY nominal equity returns by inflation buckets

US

Inflation environment	Obs.	% over Total	Mean	Median	Standard deviation	95% confidence interval
Deflation	163	14.62%	5.10%	16.62%	33.97%	(-0.1%;10.3%)
Inflation lower than 1.5% pa	198	17.76%	12.02%	13.15%	17.60%	(9.6%;14.5%)
Inflation between 1.5% and 3% pa	252	22.60%	12.85%	14.08%	15.67%	(10.9%;14.8%)
Inflation between 3% and 6% pa	285	25.56%	10.50%	11.29%	14.29%	(8.8%;12.2%)
Inflation between 6% and 10% pa	111	9.96%	8.56%	6.37%	15.84%	(5.6%;11.5%)
Inflation above 10%	106	9.51%	1.25%	3.14%	17.10%	(-2.0%;4.5%)
Overall sample	1115		9.44%	11.47%	19.89%	(8.3%;10.6%)

UK

Inflation environment	Obs.	% over Total	Mean	Median	Standard deviation	95% confidence interval
Deflation	152	14.49%	4.41%	8.61%	15.24%	(2.0%;6.8%)
Inflation lower than 1.5% pa	141	13.44%	13.15%	12.25%	13.40%	(10.9%;15.4%)
Inflation between 1.5% and 3% pa	187	17.83%	11.86%	12.82%	13.66%	(9.9%;13.8%)
Inflation between 3% and 6% pa	271	25.83%	10.71%	10.32%	15.65%	(8.8%;12.6%)

[75] For example, see http://www.imf.org/external/pubs/ft/fandd/basics/target.htm for more details on the history of inflation targeting.

Inflation between 6% and 10% pa	134	12.77%	12.71%	12.18%	15.93%	(10.0%;15.4%)
Inflation above 10%	164	15.63%	4.83%	3.76%	26.69%	(0.8%;8.9%)
Overall sample	1049		9.67%	10.42%	17.56%	(8.6%;10.7%)

Japan

Inflation environment	**Obs.**	**% over Total**	**Mean**	**Median**	**Standard deviation**	**95% confidence interval**
Deflation	203	20.88%	1.67%	0.00%	21.78%	(-1.3%;4.7%)
Inflation lower than 1.5% pa	134	13.79%	13.93%	10.53%	23.21%	(10.9%;17.9%)
Inflation between 1.5% and 3% pa	140	14.40%	9.19%	12.18%	22.08%	(5.5%;12.9%)
Inflation between 3% and 6% pa	197	20.27%	15.90%	14.19%	20.49%	(13.0%;18.8%)
Inflation between 6% and 10% pa	159	16.36%	14.06%	10.15%	19.32%	(11.1%;17.1%)
Inflation above 10%	139	14.30%	17.37%	5.66%	35.82%	(11.4%;23.3%)
Overall sample	972		11.60%	10.00%	24.53%	(10.1%;13.1%)

Germany

Inflation environment	**Obs.**	**% over Total**	**Mean**	**Median**	**Standard deviation**	**95% confidence interval**
Deflation	88	9.65%	10.10%	9.24%	36.89%	(2.4%;17.8%)
Inflation lower than 1.5% pa	190	20.83%	10.76%	11.16%	22.50%	(7.6%;14.0%)
Inflation between 1.5% and 3% pa	317	34.76%	12.40%	10.79%	20.46%	(10.2%;14.7%)
Inflation between 3% and 6% pa	245	26.86%	6.56%	4.39%	20.30%	(4.0%;9.1%)
Inflation between 6% and 10% pa	46	5.04%	-5.26%	1.82%	35.75%	(-15.6%;5.1%)
Inflation above 10%	26	2.85%	-33.81%	3.23%	86.40%	(-67.0%;-0.6%)
Overall sample	912		8.06%	8.03%	28.75%	(6.2%;9.9%)

Interestingly, frequency of inflation regimes has been rather different across countries, with deflation being far more frequent in Japan (21% of total observations) and high inflation more frequent in the UK and Japan (around 15%). Finally, in the US and UK inflation between 3% and 6% has been the most prevalent regime, while in post-Weimar Germany it was inflation between 1.5% and 3% which was the most frequent. In Japan, deflation and inflation between 3% and 6% displayed the highest frequency over the sample period considered.

In the overall sample mean returns were higher in Japan (11.60%), but Sharpe ratios were better in the UK and US because of lower volatility. Medians are very different from means which indicates departure from log-normality (in general if the median is lower than the mean this indicates negative skewness, higher frequency in the left tail, while if the median is higher, higher frequency in the right tail is expected).

Japan has had higher than normal frequency in the right tail (positive surprises), while the UK and the US in the left tail (negative surprises). In Germany, mean and median were very close. Confidence intervals capture 95% of return frequency when the distribution is normal, but clearly when the mean is very far from the median this is no longer true. Under the assumptions of normality, the difference in means is significant if intervals do not overlap. Confidence intervals presented here are not adjusted for autocorrelation (which arises by design because of rolling returns). The adjustment would make intervals wider but change none of the implications (i.e. in deflation, US interval would be [-9%,-19%] instead of [-0.1%,-10%]).

From the tables, it can be observed that during deflation mean returns were significantly lower than in low inflation environments in the UK and Japan. In particular, average returns in Japan were only 1.67% in deflation as opposed to 13.93% in low inflation periods. In the US, the confidence interval is very wide because of high standard deviation but the lower-bound is negative during deflation. Volatility is also substantially higher in deflation except in Japan and departure from log-normality are substantial both in the UK & the US.

In low inflation environments, mean returns are highest (up to 1.5% consumer prices growth in Japan and UK and between 1.5% and 3% in US and Germany), while at the same time standard deviations are low and means are closer to medians, with the exception of Japan. Inflation higher than 3% has brought lower mean returns in Germany and US, while Japan and UK have higher resilience towards inflation up to 10%. Inflation above 10% on the other hand has brought significantly higher volatility in all countries and significantly lower mean returns in all countries except Japan. The last observation ties in well with the conventional macroeconomic belief that very high inflation does have a real impact on the economy and by extension equity market behaviour, even though in theory inflation should be neutral. Table 3.6 below replicates the analysis using real instead of nominal returns to shed further light on these patterns.

Table 3.6 Statistical features of YoY real equity returns by inflation buckets

US

Inflation environment	Obs.	Mean	Median	Standard deviation	95% confidence interval
Deflation	163	9.09%	19.21%	34.04%	(3.9%;14.3%)
Inflation lower than 1.5% pa	198	11.04%	11.83%	17.51%	(8.6%;13.5%)
Inflation between 1.5% and 3% pa	252	10.30%	11.30%	15.34%	(8.4%;12.2%)
Inflation between 3% and 6% pa	285	6.09%	7.20%	13.77%	(4.5%;7.7%)

Inflation between 6% and 10% pa	111	0.71%	-1.71%	14.73%	(-2.0%;3.4%)
Inflation above 10%	106	-10.92%	-10.81%	15.35%	(-13.8%;-8.0%)
Overall sample	1115	6.21%	7.11%	20.19%	(5.0%;7.4%)

UK

Inflation environment	Obs.	Mean	Median	Standard deviation	95% confidence interval
Deflation	152	10.66%	12.13%	18.23%	(7.8%;13.6%)
Inflation lower than 1.5% pa	141	12.37%	11.79%	11.37%	(10.1%;14.6%)
Inflation between 1.5% and 3% pa	187	9.41%	10.42%	13.43%	(7.5%;11.3%)
Inflation between 3% and 6% pa	271	6.19%	6.10%	15.08%	(4.4%;8.0%)
Inflation between 6% and 10% pa	134	4.53%	4.68%	14.97%	(2.0%;7.1%)
Inflation above 10%	164	-9.28%	-9.80%	22.64%	(-12.7%;-5.8%)
Overall sample	1049	5.61%	7.29%	17.84%	(4.5%;6.7%)

Japan

Inflation environment	Obs.	Mean	Median	Standard deviation	95% confidence interval
Deflation	203	6.53%	4.98%	21.78%	(3.5%;9.5%)
Inflation lower than 1.5% pa	134	13.19%	9.70%	23.06%	(9.3%;17.1%)
Inflation between 1.5% and 3% pa	140	6.82%	9.58%	21.64%	(3.2%;10.4%)
Inflation between 3% and 6% pa	197	11.13%	9.35%	19.54%	(8.4%;13.9%)
Inflation between 6% and 10% pa	159	5.96%	1.84%	17.96%	(3.2%;8.7%)
Inflation above 10%	139	-11.01%	-10.60%	31.20%	(-16.2%;-5.8%)
Overall sample	972	5.82%	5.34%	23.71%	(4.3%;7.3%)

Germany

Inflation environment	Obs.	Mean	Median	Standard deviation	95% confidence interval
Deflation	88	15.52%	13.15%	38.17%	(7.5%;23.5%)
Inflation lower than 1.5% pa	190	9.85%	10.22%	22.28%	(6.7%;13.0%)
Inflation between 1.5% and 3% pa	317	10.00%	8.26%	20.10%	(7.8%;12.2%)
Inflation between 3% and 6% pa	245	2.19%	0.01%	19.44%	(-0.2%;4.6%)
Inflation between 6% and 10% pa	46	-11.35%	-4.97%	33.22%	(-21.0%;-1.8%)
Inflation above 10%	26	-40.06%	-7.02%	77.01%	(-69.7%;-10.5%)
Overall sample	912	5.90%	5.05%	28.25%	(4.1%;7.7%)

In the overall sample, real equity returns have been higher in the US (over 6%), just below 6% in Germany (excluding Weimar hyperinflation periods) and Japan, and 5.6% in the UK. In deflation, real equity returns have been on average substantially lower only in Japan, while standard deviations have been very high in all other countries. In the US, the UK and Japan real returns have been highest when inflation was below 1.5%, while in Germany deflation had highest mean returns (early 1950s) but very high volatility. Interestingly, in Japan not only deflation but also inflation between 1.5% and 3% brought lower mean returns (early 1990s). In the US, real returns have been significantly lower when inflation was higher than 6%, in Germany when it was higher than 3%, while in the UK and Japan only above 10%.

In conclusion, while results are mixed for deflation, the empirical analysis shows that inflation up to 3% has generally been good for equity returns (except for the rather unique pattern of Japan in the early-1990s when low inflation preceded a decade of sustained deflationary pressures). Very high inflation is associated with negative real equity returns across all countries, providing support for to Barnes, Boyd and Smith’s (1999) capital markets inefficiency argument or Hess and Lee (1999) hypothesis, especially, if real output shocks (such as the oil crisis) are the prevalent source of higher inflation.

At the same time, very high inflation periods tend to be associated with higher volatility of returns and this could be interpreted as higher uncertainty over the true value of underlying earnings and discount rates. In the US, volatility is higher also during deflation periods (notably, the Great Depression), indicating a relationship between equity market volatility and major economic and financial crises (which was explored in more detail in chapter 1). More generally, more volatile real returns appear to be associated with low average returns and moderate/high inflation periods in the UK are other examples of high volatility and lower equity returns outcomes.

Transitions in Inflationary Environment and Equity Returns

So far, we have examined contemporaneous patterns by looking at the behavior of equity returns during different inflationary environments. The next step of the analysis is to explore the dynamics of equity returns when there is a transition from one inflation bucket to another.

Table 3.7 shows the frequency of inflation based transitions (i.e. from state i at time t to state j at time t+1) and mean equity returns over the t to t+1 period using annual data. To take into account, the smaller sample size (as we are using annual data), we have combined the very low and low inflation buckets used earlier into a single category (denoted as L).

Table 3.7 Frequency and mean nominal equity returns in the transition from inflation buckets (using annual data)

US (1900-2002) Index: S&P500

	D	L	M	H	VH
D to	5	5	1	0	0
L to	5	33	9	1	2
M to	1	9	11	3	0
H to	0	2	3	2	2
VH to	0	1	0	3	4

	D	L	M	H	VH
D to	5.3%	12.8%	26.7%	N/A	N/A
L to	14.4%	10.2%	8.1%	-12.5%	0.0%
M to	13.4%	10.8%	12.5%	-10.7%	N/A
H to	N/A	13.9%	14.9%	12.7%	-6.8%
VH to	N/A	-20.8%	N/A	10.6%	9.9%

UK (1900-2002)

	D	L	M	H	VH
D to	8	6	0	0	1
L to	4	22	10	1	2
M to	1	9	11	2	0
H to	0	1	1	4	3
VH to	1	2	1	2	10

	D	L	M	H	VH
D to	1.6%	10.4%	N/A	N/A	6.8%
L to	10.9%	5.8%	10.2%	5.0%	-0.6%
M to	-9.6%	14.0%	8.8%	4.0%	N/A
H to	N/A	20.9%	18.9%	18.8%	-9.2%
VH to	3.8%	16.8%	25.7%	4.7%	10.6

Japan (1920-2002)

	D	L	M	H	VH
D to	8	4	1	1	1
L to	6	15	3	1	1
M to	1	3	6	5	2
H to	0	2	6	2	1
VH to	0	2	1	2	7

	D	L	M	H	VH
D to	-10.8%	10.6%	4.2%	68.2%	60.5%
L to	22.7%	9.6%	-0.2%	3.9%	4.7%
M to	2.0%	6.6%	23.0%	12.0%	-8.0%
H to	N/A	17.6%	10.8%	2.2%	4.8%
VH to	N/A	38.9%	47.9%	2.2%	14.6%

Germany (1900-2002)

	D	L	M	H	VH
D to	5	4	1	1	1
L to	3	35	10	1	1
M to	2	11	8	3	0
H to	0	1	3	1	1
VH to	2	0	1	1	6

	D	L	M	H	VH
D to	-6.4%	9.7%	-29.2%	7.6%	N/A
L to	-1.8%	12.2%	3.8%	6.0%	-77.6%
M to	9.7%	9.3%	10.8%	5.9%	N/A
H to	N/A	7.4%	6.9%	-18.5%	-75.4%
VH to	127.6%	N/A	-18.0%	-93.2%	0.2%

From the results in table 3.7, in all countries, transition towards lower inflation has generally brought higher equity returns compared to a switch towards higher inflation. However, the transition from low to medium inflation (up to 6%) has been relatively good in terms of nominal returns in the US (+8.1%) and the UK (+10.2%). On the other hand, a deflationary environment has not been good for equity returns (i.e. deflation to deflation transition) and this was particularly true in Japan (-10.8% average return) while interestingly, the transition towards deflation (low to deflation transition) has been a relatively good environment for equity market performance, with the exception of Germany, but equity returns across the four countries studied appear to drop significantly as deflation sets in (i.e. during D to D transition).

Overall, the transition analysis carried out uncovers common features across the various countries with transition to falling inflation being consistent with higher equity market returns – a pattern which changes completely as deflation sets in. Furthermore, the differentiated behavior of equity market performance during different inflation environments imply that usage of regime switching-type models is an interesting avenue of future research from the point of view of investors with the goal to understand the bi-variate relationship between equity returns and inflation using the Fisher framework.

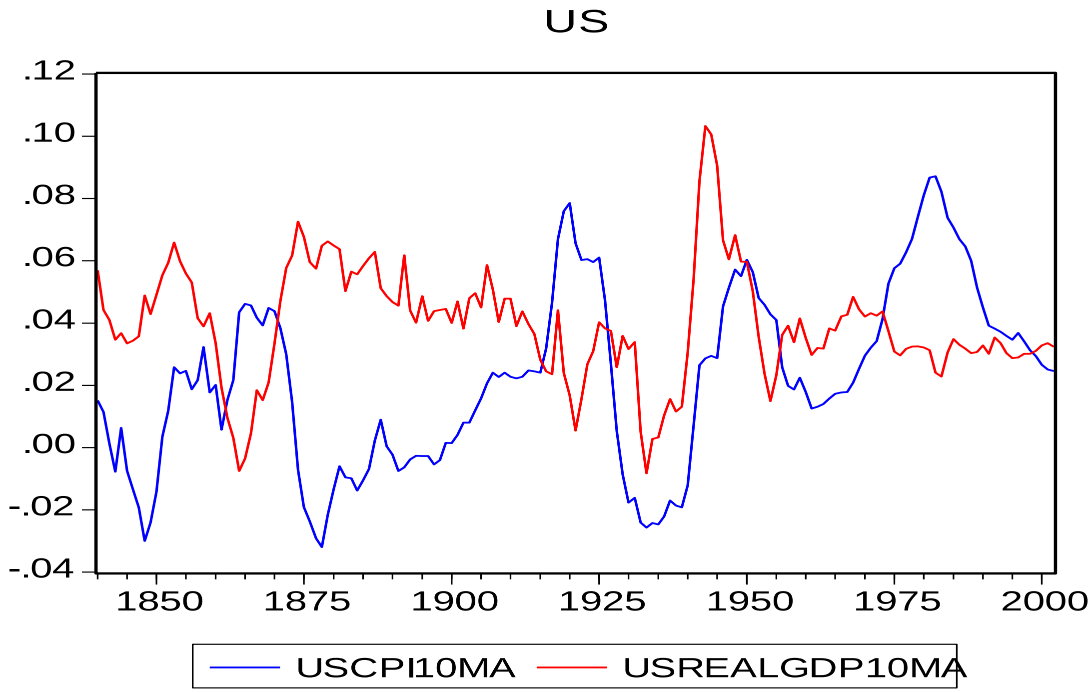
Inflation, Equity returns and Economic Growth

Following Fama (1981)'s approach, we have also investigated the behaviour of equity returns in the real GDP growth and inflation space to capture any interactions between the two variables (i.e equity returns and inflation) during different economic growth settings.

In this sub-section, we have defined inflation/growth regimes on the basis of two variables: Inflation and real GDP growth. The analysis is carried out only for the UK and the US because only for those countries GDP data was available since 1840 from the Annual Files of the Global Financial Database (www.globalfindata.com). Since 1929, the US series corresponds to the GDP in billions of chained 2000 dollars series calculated by the Bureau of Economic Analysis (BEA) while the UK series corresponds to the Gross Domestic Product (£ million at chained volume measures) calculated by the Central Statistical Office (Office of National Statistics after 1996).

The inflation/growth regimes are defined by comparing annual GDP growth and inflation rates to the corresponding 10-year moving averages to capture a transition using the 1840-2002 sample period. Figure 3.3 shows the key features of the economic growth and inflation dynamics in the two countries.

Figure 3.3 Inflation/Growth regimes in the US and UK (1840-2002, based on 10-year moving averages)



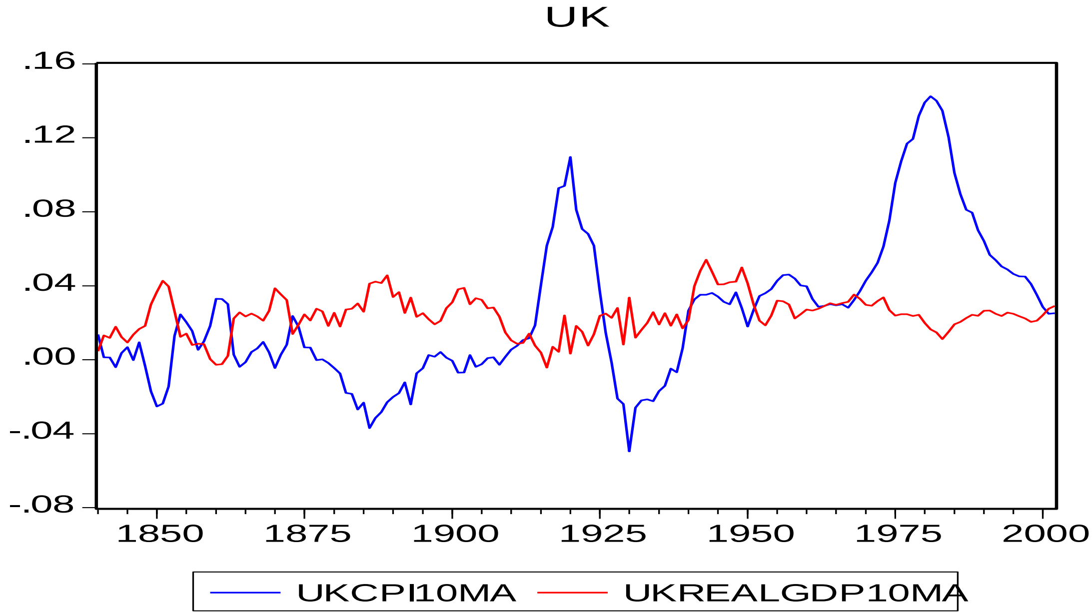


Table 3.8 shows sample statistics for equity returns, inflation and GDP growth for both countries. Over the sample period studied, the US economy has grown on average 1.5 percentage points (ppt) more than the UK economy with a 1 ppt lower inflation per annum. This has translated into 1.5ppt higher equity return per annum, both in nominal and real terms for the US, albeit with a 3ppt higher standard deviation. The results showing the behavior of US and UK equity markets (nominal and real returns) during different inflation/growth environments are shown in table 3.9.

Table 3.8 Real GDP, inflation and equity returns (annual data, 1840-2002)

	Inflation		Real GDP		Equity returns		Real equity returns	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
US	2.01%	5.44%	3.89%	5.49%	8.25%	18.04%	6.34%	18.31%
UK	2.25%	6.69%	2.43%	4.97%	6.89%	15.07%	4.95%	15.23%

Table 3.9 Equity returns categorised by dynamic Inflation/Growth regimes (annual data, 1840-2002)

US: Nominal Wilshire

	Mean	Standard deviation	Skewness	95% confidence interval	
Rising inflation - Rising growth	6.3%	17.0%	-0.31	1.0%	11.6%
Rising growth - Falling inflation	12.5%	10.4%	-0.53	9.3%	15.6%
Falling growth - Falling inflation	7.3%	22.1%	-0.46	0.9%	13.6%
Falling growth - Rising inflation	6.7%	20.3%	-0.25	-0.1%	13.5%
Overall	8.3%	18.0%	-0.54	5.5%	11.0%

US: Real Wilshire (1840-2002)

	Mean	Standard deviation	Skewness	95% confidence interval	
Rising inflation - Rising growth	1.5%	16.6%	-0.23	-3.7%	6.6%
Rising growth - Falling inflation	12.2%	9.8%	-0.72	9.3%	15.2%
Falling growth - Falling inflation	8.9%	22.2%	-0.36	2.5%	15.2%
Falling growth - Rising inflation	1.3%	20.1%	-0.22	-5.5%	8.1%
Overall	6.3%	18.3%	-0.43	3.5%	9.2%

UK: Nominal FTSE

	Mean	Standard deviation	Skewness	95% confidence interval	
Rising inflation - Rising growth	1.8%	12.5%	-0.14	-3.1%	6.7%
Rising growth - Falling inflation	9.9%	11.1%	0.48	7.0%	12.9%
Falling growth - Falling inflation	10.9%	13.7%	0.03	5.7%	16.0%
Falling growth - Rising inflation	4.5%	18.7%	0.72	-0.3%	9.3%
Overall	6.9%	15.1%	0.37	4.6%	9.2%

UK: Real FTSE

	Mean	Standard deviation	Skewness	95% confidence interval	
Rising inflation - Rising growth	-2.8%	12.8%	-0.49	-7.8%	2.2%
Rising growth - Falling inflation	12.4%	11.0%	0.60	9.5%	15.4%
Falling growth - Falling inflation	10.5%	12.7%	-0.25	5.7%	15.2%
Falling growth - Rising inflation	-1.1%	16.6%	-0.84	-5.4%	3.2%
Overall	5.0%	15.2%	-0.67	2.6%	7.3%

As shown in table 3.9, both in the US and the UK falling inflation brought significantly higher real returns over the sample period. In addition, nominal returns over 11% appear to be typical of rising growth/falling inflation environments. At the same time, at least in the UK, rising inflationary environments have been characterized by a higher frequency of negative surprises (negative skewness is greater in absolute value terms). However, in the US, the latter is not observed because the transition to deflation of the Great Depression is considered as a falling inflation environment.

While inflation clearly matters for equity returns (i.e. rising inflation leads to lower real/nominal returns), the additional impact of real growth is not statistically significant. Although, both in the US and the UK, rising growth/falling inflation environment has been characterized by higher mean real returns when compared to the falling growth/falling inflation periods, but at the 95% level, the difference in mean equity returns is not significant. However, this conclusion is mitigated somewhat in the UK, as the rising growth/falling inflation environment was characterized by positive surprises (evidence of positive skewness in equity returns), while the reverse was true for the falling growth/falling inflation regime. On the other hand, during rising inflation periods, real equity returns have on average been very low (indeed, negative in the UK) and the impact of real GDP growth appears to be quite marginal (equity returns are still negative in the UK, even during the rising inflation/rising growth environment).

In conclusion, falling inflation has brought higher equity returns (real and nominal) both in the UK and the US, which appears to be the case, irrespective of the economic growth environment. Quite surprisingly, growth matters significantly less than inflation, and it seems to matter more within a falling inflation environment. Finally, there is evidence that real equity returns around 10% were a bonus-linked to a falling inflation environment in the US and the UK, while, when inflation rises, equity returns have on average been around 5% lower in nominal terms and even more so in real terms.

3.6 Conclusions

The main findings of this study are:

1. There appears to be mixed support for a long-run equilibrium relationship between stock market prices and consumer prices for the different countries studied. Firstly, estimated equilibrium relationships appears to be an outcome of one-off events rather than sustained or more frequent corrections. Secondly, in all cases estimated coefficients are significantly higher than the theoretical value of one. Also, no general conclusion can be driven on what forces drive the system back to the equilibrium across the different countries.

2. With a short-term horizon (1-year), equity returns have been significantly higher at times of inflation up to 3%., with the exception of Japan where moderate inflation (3% up to

6%) has been better for stock market performance. Deflation and higher inflation periods have been generally bad for equity returns across the various countries studied.

3. Short-term analysis shows asymmetric behaviour of equity markets during different inflationary regimes and the transition from one regime to another. Transition to lower inflation environment has historically brought better performance for equity markets in all countries.

4. At least in the UK and the US (using the 1840-2002 sample period), GDP growth has been less important than inflation in explaining the behaviour of equity returns. However, GDP growth does appear to play a role in explaining the asymmetric behaviour of equity markets during the different inflationary environments, although it appears to matter only within falling inflation periods.

5. A related implication of the results shown in this chapter is that there is a case for modelling equity returns over the long-run without exante postulating equilibrium relationships, whilst taking into account short-term dynamics. In addition, the results derived (using a bi-variate setting) also shed doubt on the commonly held belief that equity is necessarily a good hedge for inflation from the point-of-view of pension fund investment.

APPENDIX 3.1 COINTEGRATION RESULTS

Appendix table 3.1.1 USA Johansen cointegration test results

Sample: 1900 2002
Included observations: 103
Series: LOG(WILSHIREIND) LOG(USCPIIND)
Lags interval: 1 to 10

Data Trend:	None	None	Linear	Linear	Quadratic
Rank or No. of CEs	No Intercept No Trend	Intercept No Trend	Intercept No Trend	Intercept Trend	Intercept Trend
Selected (5% level) Number of Cointegrating Relations by Model (columns)					
Trace	1	1	0	0	0
Max-Eig	1	1	0	0	0
Log Likelihood by Rank (rows) and Model (columns)					
0	247.0864	247.0864	254.7121	254.7121	256.2508
1	253.7517	257.6991	259.4605	259.9588	260.3007
2	254.7272	259.4607	259.4607	261.7017	261.7017
Akaike Information Criteria by Rank (rows) and Model (columns)					
0	-4.021094	-4.021094	-4.130333	-4.130333	-4.121375
1	-4.072849	-4.130081	-4.144864*	-4.135122	-4.122344
2	-4.014120	-4.067197	-4.067197	-4.071878	-4.071878
Schwarz Criteria by Rank (rows) and Model (columns)					
0	-2.997899	-2.997899	-3.055977*	-3.055977*	-2.995859
1	-2.947334	-2.978985	-2.968189	-2.932867	-2.894509
2	-2.786285	-2.788203	-2.788203	-2.741724	-2.741724

Appendix Table 3.1.2 USA, Vector Error Correction Estimation Results

Vector Error Correction Estimates Sample: 1900 2002 Included observations: 103 Standard errors in ( ) & t-statistics in [ ]		
Cointegrating Eq:	CointEq1	
LOG(WILSHIREIND(-1))	1.000000	
LOG(USCPIIND(-1))	-2.778344 (0.25655) [-10.8296]	
C	4.227769	
Error Correction:	D(LOG(WILSHIREIND))	D(LOG(USCPIIND))
CointEq1	0.006840 (0.03516) [0.19454]	0.016931 (0.00613) [2.76277]
D(LOG(WILSHIREIND(-1)))	-0.107050 (0.11656) [-0.91838]	0.040168 (0.02032) [1.97699]
D(LOG(WILSHIREIND(-2)))	-0.327308 (0.11689) [-2.80024]	-0.049292 (0.02037) [-2.41938]
D(LOG(WILSHIREIND(-3)))	-0.093551 (0.11847) [-0.78967]	-0.007722 (0.02065) [-0.37394]
D(LOG(WILSHIREIND(-4)))	-0.144207 (0.11662) [-1.23658]	-0.036158 (0.02033) [-1.77878]
D(LOG(WILSHIREIND(-5)))	-0.207145 (0.11800) [-1.75546]	-0.045129 (0.02057) [-2.19412]
D(LOG(WILSHIREIND(-6)))	-0.150876 (0.12150) [-1.24173]	-0.031616 (0.02118) [-1.49280]
D(LOG(WILSHIREIND(-7)))	-0.034198 (0.12194) [-0.28045]	-0.004219 (0.02125) [-0.19852]

D(LOG(WILSHIREIND(-8)))	-0.096186	-0.005572
	(0.12227)	(0.02131)
	[-0.78670]	[-0.26146]
D(LOG(WILSHIREIND(-9)))	-0.005882	-0.042117
	(0.11566)	(0.02016)
	[-0.05085]	[-2.08912]
D(LOG(WILSHIREIND(-10)))	0.017387	-0.018229
	(0.11772)	(0.02052)
	[0.14770]	[-0.88840]
D(LOG(USCPIIND(-1)))	-0.195522	0.784963
	(0.61684)	(0.10752)
	[-0.31697]	[7.30062]
D(LOG(USCPIIND(-2)))	0.244408	-0.134469
	(0.78880)	(0.13749)
	[0.30985]	[-0.97801]
D(LOG(USCPIIND(-3)))	-0.102664	-0.218855
	(0.78237)	(0.13637)
	[-0.13122]	[-1.60484]
D(LOG(USCPIIND(-4)))	-0.242171	0.251032
	(0.79370)	(0.13835)
	[-0.30512]	[1.81451]
D(LOG(USCPIIND(-5)))	0.495404	0.075618
	(0.79806)	(0.13911)
	[0.62076]	[0.54359]
D(LOG(USCPIIND(-6)))	-0.160353	-0.062386
	(0.80062)	(0.13955)
	[-0.20029]	[-0.44704]
D(LOG(USCPIIND(-7)))	-0.111523	0.075601
	(0.78648)	(0.13709)
	[-0.14180]	[0.55148]
D(LOG(USCPIIND(-8)))	0.981618	0.039656
	(0.76157)	(0.13275)
	[1.28895]	[0.29874]
D(LOG(USCPIIND(-9)))	0.142214	0.022939
	(0.75474)	(0.13156)
	[0.18843]	[0.17437]

D(LOG(USCPIIND(-10)))	1.237060	0.011332
	(0.59179)	(0.10315)
	[2.09038]	[0.10985]
C	0.135686	0.024118
	(0.04977)	(0.00868)
	[2.72630]	[2.78017]
R-squared	0.264877	0.627507
Adj. R-squared	0.074289	0.530935
Sum sq. resids	2.797022	0.084981
S.E. equation	0.185826	0.032391
F-statistic	1.389789	6.497799
Log likelihood	39.56728	219.5020
Akaike AIC	-0.341112	-3.834991
Schwarz SC	0.221645	-3.272234
Mean dependent	0.091048	0.031099
S.D. dependent	0.193138	0.047294
Determinant Residual Covariance		3.60E-05
Log Likelihood		259.4605
Log Likelihood (d.f. adjusted)		234.7117
Akaike Information Criteria		-3.664305
Schwarz Criteria		-2.487630

Appendix table 3.1.3 UK Johansen cointegration test results

Sample: 1900 2002
Included observations: 103
Series: LOG(FTSEIND) LOG(UKCPIIND)
Lags interval: 1 to 10

Data Trend:	None	None	Linear	Linear	Quadratic
Rank or No. of CEs	No Intercept No Trend	Intercept No Trend	Intercept No Trend	Intercept Trend	Intercept Trend
Selected (5% level) Number of Cointegrating Relations by Model (columns)					
Trace	0	0	0	0	0
Max-Eig	0	0	0	0	0
Log Likelihood by Rank (rows) and Model (columns)					
0	195.3608	195.3608	197.7976	197.7976	201.4435
1	197.1756	202.6903	203.5109	205.1889	205.5422
2	197.3955	203.5182	203.5182	208.3807	208.3807

Akaike Information Criteria by Rank (rows) and Model (columns)					
0	-3.016715	-3.016715	-3.025196	-3.025196	-3.057156
1	-2.974283	-3.061947	-3.058464	-3.071628*	-3.059072
2	-2.900883	-2.980936	-2.980936	-3.036519	-3.036519
Schwarz Criteria by Rank (rows) and Model (columns)					
0	-1.993520*	-1.993520*	-1.950841	-1.950841	-1.931641
1	-1.848768	-1.910852	-1.881789	-1.869373	-1.831237
2	-1.673048	-1.701941	-1.701941	-1.706365	-1.706365

Appendix table 3.1.4 UK, Vector Error Correction Estimation Results

Vector Error Correction Estimates Sample: 1900 2002 Included observations: 103 Standard errors in ( ) & t-statistics in [ ]		
Cointegrating Eq:	CointEq1	
LOG(FTSEIND(-1))	1.000000	
LOG(UKCPIIND(-1))	-2.006089 (0.21780) [-9.21065]	
C	3.222689	
Error Correction:	D(LOG(FTSEIND))	D(LOG(UKCPIIND))
CointEq1	0.010414 (0.04443) [0.23441]	0.043736 (0.01436) [3.04649]
D(LOG(FTSEIND(-1)))	-0.200475 (0.12275) [-1.63315]	-0.068693 (0.03967) [-1.73172]
D(LOG(FTSEIND(-2)))	-0.236103 (0.12451) [-1.89620]	-0.076772 (0.04024) [-1.90805]
D(LOG(FTSEIND(-3)))	-0.265535 (0.12563) [-2.11356]	-0.065937 (0.04060) [-1.62414]

D(LOG(FTSEIND(-4)))	-0.002338	-0.004288
	(0.12004)	(0.03879)
	[-0.01947]	[-0.11054]
D(LOG(FTSEIND(-5)))	-0.070916	-0.036714
	(0.11492)	(0.03714)
	[-0.61708]	[-0.98861]
D(LOG(FTSEIND(-6)))	-0.196432	-0.040959
	(0.11301)	(0.03652)
	[-1.73825]	[-1.12164]
D(LOG(FTSEIND(-7)))	0.020246	-0.031056
	(0.11207)	(0.03622)
	[0.18065]	[-0.85754]
D(LOG(FTSEIND(-8)))	0.109768	-0.037701
	(0.11009)	(0.03557)
	[0.99709]	[-1.05975]
D(LOG(FTSEIND(-9)))	0.120375	-0.036990
	(0.10816)	(0.03495)
	[1.11294]	[-1.05833]
D(LOG(FTSEIND(-10)))	0.157962	-0.003656
	(0.10878)	(0.03515)
	[1.45211]	[-0.10402]
D(LOG(UKCPIIND(-1)))	-0.097451	0.332665
	(0.32380)	(0.10464)
	[-0.30096]	[3.17929]
D(LOG(UKCPIIND(-2)))	0.408323	0.301633
	(0.34196)	(0.11050)
	[1.19408]	[2.72966]
D(LOG(UKCPIIND(-3)))	0.078942	0.004012
	(0.36023)	(0.11641)
	[0.21914]	[0.03447]
D(LOG(UKCPIIND(-4)))	0.104971	0.192394
	(0.35473)	(0.11463)
	[0.29592]	[1.67840]
D(LOG(UKCPIIND(-5)))	0.013570	0.027933
	(0.35722)	(0.11544)
	[0.03799]	[0.24198]
D(LOG(UKCPIIND(-6)))	-0.052012	-0.101131

	(0.34857)	(0.11264)
	[-0.14922]	[-0.89784]
D(LOG(UKCPIIND(-7)))	0.624714	-0.031542
	(0.33683)	(0.10885)
	[1.85467]	[-0.28978]
D(LOG(UKCPIIND(-8)))	0.368113	0.149947
	(0.34241)	(0.11065)
	[1.07506]	[1.35515]
D(LOG(UKCPIIND(-9)))	0.359811	0.170352
	(0.34237)	(0.11063)
	[1.05095]	[1.53977]
D(LOG(UKCPIIND(-10)))	0.443413	0.051245
	(0.34965)	(0.11299)
	[1.26815]	[0.45353]
C	0.049776	0.031228
	(0.03659)	(0.01182)
	[1.36042]	[2.64117]
R-squared	0.266808	0.400320
Adj. R-squared	0.076721	0.244848
Sum sq. resids	2.595208	0.271001
S.E. equation	0.178996	0.057842
F-statistic	1.403612	2.574861
Log likelihood	43.42404	159.7779
Akaike AIC	-0.416001	-2.675299
Schwarz SC	0.146757	-2.112541
Mean dependent	0.081667	0.038401
S.D. dependent	0.186285	0.066562
Determinant Residual Covariance		0.000107
Log Likelihood		203.5109
Log Likelihood (d.f. adjusted)		178.7621
Akaike Information Criteria		-2.577904
Schwarz Criteria		-1.401229

Appendix table 3.1.5 Japan Johansen cointegration test results

Date: 02/10/04 Time: 10:58
Sample: 1920 2001
Included observations: 80
Series: LOG(NIKKOIND) LOG(JAPCPIIND)
Exogenous series: DUMPOSTWW2
Warning: Rank Test critical values derived assuming no exogenous series
Lags interval: 1 to 1

Data Trend:	None	None	Linear	Linear	Quadratic
Rank or No. of CEs	No Intercept No Trend	Intercept No Trend	Intercept No Trend	Intercept Trend	Intercept Trend
Selected (5% level) Number of Cointegrating Relations by Model (columns)					
Trace	1	1	0	0	0
Max-Eig	1	1	0	0	0
Log Likelihood by Rank (rows) and Model (columns)					
0	49.45258	49.45258	55.59003	55.59003	56.49371
1	58.26079	59.18880	60.28625	60.60184	61.36425
2	58.46473	61.37729	61.37729	62.30746	62.30746
Akaike Information Criteria by Rank (rows) and Model (columns)					
0	-1.136315	-1.136315	-1.239751	-1.239751	-1.212343
1	-1.256520	-1.254720	-1.257156*	-1.240046	-1.234106
2	-1.161618	-1.184432	-1.184432	-1.157687	-1.157687
Schwarz Criteria by Rank (rows) and Model (columns)					
0	-1.017213	-1.017213	-1.061099*	-1.061099*	-0.974140
1	-1.018317	-0.986742	-0.959403	-0.912517	-0.876802
2	-0.804314	-0.767578	-0.767578	-0.681281	-0.681281

Appendix table 3.1.6 Japan, Vector Error Correction Estimation Results

Vector Error Correction Estimates Date: 02/10/04 Time: 10:52 Sample(adjusted): 1922 2001 Included observations: 80 after adjusting endpoints Standard errors in ( ) & t-statistics in [ ]		
Cointegrating Eq:	CointEq1	
LOG(NIKKOIND(-1))	1.000000	
LOG(JAPCPIIND(-1))	-1.301705	
	(0.10367)	
	[-12.5568]	
C	1.009112	
Error Correction:	D(LOG(NIKKOIND))	D(LOG(JAPCPIIND))
CointEq1	-0.088886	0.014578
	(0.02927)	(0.01768)
	[-3.03694]	[0.82472]
D(LOG(NIKKOIND(-1)))	-0.010040	-0.029618
	(0.10755)	(0.06496)
	[-0.09335]	[-0.45597]
D(LOG(JAPCPIIND(-1)))	-0.044116	0.945637
	(0.13577)	(0.08199)
	[-0.32494]	[11.5329]
C	0.117223	0.021222
	(0.03056)	(0.01846)
	[3.83527]	[1.14971]
DUMPOSTWW2	0.083408	-0.198424
	(0.15721)	(0.09495)
	[0.53053]	[-2.08983]
R-squared	0.221627	0.678551
Adj. R-squared	0.180113	0.661407
Sum sq. resids	3.697446	1.348599
S.E. equation	0.222034	0.134094
F-statistic	5.338696	39.57960
Log likelihood	9.460288	49.80334
Akaike AIC	-0.111507	-1.120083
Schwarz SC	0.037369	-0.971207
Mean dependent	0.117740	0.079597
S.D. dependent	0.245213	0.230447
Determinant Residual Covariance		0.000864
Log Likelihood		60.28625

Log Likelihood (d.f. adjusted)	55.12316
Akaike Information Criteria	-1.078079
Schwarz Criteria	-0.720775

Appendix table 3.1.7 Germany Johansen cointegration test results

Sample: 1925 2002 Included observations: 78 Series: LOG(CDAXIND) LOG(GERCPIIND) Lags interval: 1 to 8					
Data Trend:	None	None	Linear	Linear	Quadratic
Rank or No. of CEs	No Intercept No Trend	Intercept No Trend	Intercept No Trend	Intercept Trend	Intercept Trend
Selected (5% level) Number of Cointegrating Relations by Model (columns)					
Trace	1	1	0	0	2
Max-Eig	1	1	0	0	0
Log Likelihood by Rank (rows) and Model (columns)					
0	153.6288	153.6288	163.1386	163.1386	163.1822
1	163.7298	163.9971	167.1313	169.2843	169.3046
2	163.8505	167.2771	167.2771	172.5640	172.5640
Akaike Information Criteria by Rank (rows) and Model (columns)					
0	-3.118687	-3.118687	-3.311246	-3.311246	-3.261083
1	-3.275124	-3.256335	-3.311059	-3.340623*	-3.315503
2	-3.175655	-3.212233	-3.212233	-3.296513	-3.296513
Schwarz Criteria by Rank					

(rows) and Model (columns)					
0	-2.151832	-2.151832	-2.283963*	-2.283963*	-2.173371
1	-2.187412	-2.138409	-2.162918	-2.162268	-2.106935
2	-1.967086	-1.943236	-1.943236	-1.967088	-1.967088

Appendix table 3.1.8 Germany, Vector Error Correction Estimation Results

Vector Error Correction Estimates
Sample: 1925 2002
Included observations: 78
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOG(CDAXIND(-1))	1.000000	
LOG(GERCPIIND(-1))	-2.283944	
	(0.21811)	
	[-10.4713]	
C	5.475950	
Error Correction:	**D(LOG(CDAXIND))**	**D(LOG(GERCPIIND))**
CointEq1	-0.295410	0.016260
	(0.09365)	(0.00954)
	[-3.15426]	[1.70516]
D(LOG(CDAXIND(-1)))	-0.133864	0.032261
	(0.14795)	(0.01507)
	[-0.90476]	[2.14144]
D(LOG(CDAXIND(-2)))	-0.013509	0.014248
	(0.14586)	(0.01485)
	[-0.09262]	[0.95935]
D(LOG(CDAXIND(-3)))	-0.214488	-0.028505
	(0.14519)	(0.01478)
	[-1.47733]	[-1.92823]
D(LOG(CDAXIND(-4)))	0.138108	-0.006480
	(0.13315)	(0.01356)
	[1.03725]	[-0.47794]
D(LOG(CDAXIND(-5)))	0.151133	0.010695
	(0.13022)	(0.01326)
	[1.16062]	[0.80660]
D(LOG(CDAXIND(-6)))	-0.078950	-0.014605
	(0.12137)	(0.01236)

	[-0.65046]	[-1.18174]
D(LOG(CDAXIND(-7)))	0.022791	0.008309
	(0.12259)	(0.01248)
	[0.18591]	[0.66569]
D(LOG(CDAXIND(-8)))	0.105550	0.008873
	(0.11616)	(0.01183)
	[0.90867]	[0.75021]
D(LOG(GERCPIIND(-1)))	0.377764	0.278772
	(1.21477)	(0.12369)
	[0.31098]	[2.25380]
D(LOG(GERCPIIND(-2)))	0.220535	0.005432
	(0.10108)	(0.01029)
	[2.18187]	[0.52782]
D(LOG(GERCPIIND(-3)))	-0.005619	-0.006776
	(0.09512)	(0.00969)
	[-0.05907]	[-0.69964]
D(LOG(GERCPIIND(-4)))	0.166024	0.005332
	(0.08802)	(0.00896)
	[1.88626]	[0.59497]
D(LOG(GERCPIIND(-5)))	0.208591	0.017287
	(0.08615)	(0.00877)
	[2.42127]	[1.97070]
D(LOG(GERCPIIND(-6)))	0.130519	0.005960
	(0.07783)	(0.00792)
	[1.67698]	[0.75205]
D(LOG(GERCPIIND(-7)))	0.175772	0.018342
	(0.07183)	(0.00731)
	[2.44691]	[2.50769]
D(LOG(GERCPIIND(-8)))	0.172209	0.012887
	(0.06590)	(0.00671)
	[2.61303]	[1.92042]
C	0.110658	0.012773
	(0.06891)	(0.00702)
	[1.60588]	[1.82048]
DUMPOSTWW2	-0.686978	0.031152
	(0.18652)	(0.01899)
	[-3.68309]	[1.64028]

R-squared	0.410189	0.472642
Adj. R-squared	0.230246	0.311753
Sum sq. resids	5.561995	0.057665
S.E. equation	0.307036	0.031263
F-statistic	2.279556	2.937687
Log likelihood	-7.687880	170.5056
Akaike AIC	0.684305	-3.884758
Schwarz SC	1.258375	-3.310688
Mean dependent	0.065701	0.023066
S.D. dependent	0.349956	0.037684
Determinant Residual Covariance		6.74E-05
Log Likelihood		174.9835
Log Likelihood (d.f. adjusted)		153.2081
Akaike Information Criteria		-2.902772
Schwarz Criteria		-1.694203

CHAPTER 4 – HETEROGENEOUS INVESTORS AND ASSET MARKET EQUILIBRIUM

This chapter theoretically illustrates how heterogeneous expectations and quality of information of investors with different investment objectives affect asset market demand and pricing in an equilibrium setting. Within the traditional finance paradigm, arbitrageurs absorb the demand shocks thus ensuring that asset prices remain at their "fundamental price". Theoretical work by De Long et al. (1990) and Shleifer and Vishny (1997) have shown how perfect arbitrage can break down, thereby allowing demand shifts to impact asset prices.

In addition, when it comes to investor behaviour under uncertainty, the rise of behavioural economics first popularised by Kahneman and Tversky (1972) has, in recent years, shown an increased appreciation of the flaws in the classical expected utility theory framework: a framework that has reigned for several decades as the dominant normative and descriptive model of decision making under uncertainty. According to Machina (1982), this is mainly due to the simplicity and normative appeal of its axioms, the familiarity of the ideas it employs and the elegance of its characterizations of various types of behaviour in terms of the of properties of the utility function it deploys. However, there is now general agreement that the theory does not provide an adequate description of behaviour under uncertainty as a substantial body of evidence shows that decision-makers systematically violate its basic tenets. Indeed, one of the main weaknesses of the expected utility framework is the existence of heterogeneous investor types (both individuals and institutional) with different investment objectives, preferences and information signals and the related implications on asset market price equilibrium (for example, see Hey (1997) for a list of the major alternative theories to the classical expected utility framework). It is important to note that whilst heterogeneity does not directly contradict expected utility theory (EUT), EUT struggles to deal with these ideas when it comes to practitioner models. The contribution of this chapter is to provide an asset pricing model that displays heterogeneity and is usable by practitioners. This distinction is even more important in the post-Great Recession era, where the tightening of regulations, such as Basel III, Volcker rule and Dodd-Frank, is driving an even stronger wedge between the objective functions of regulated and non-regulated investors.

Specifically, herding as a form of connected behaviour takes place when investors copy and follow other investors' decisions while overriding their own private information and beliefs (see Devenow and Welch (1996) and Avery and Zemsky (1998)). The factors that determine herding can emanate from different sources depending on investor types (these may be individual or institutional) and their respective objective functions.

Indeed, the IMF Global Financial Stability Report (April 2015), which uses Lakonishok et al.'s (1992a) measure of herding estimated using CRSP and the University of Chicago data-set, shows that herding has been on the rise across the majority of asset classes, across both equities and bonds and investor types (both individual and institutional) over the 2006-2014 period.

In this paper, we propose a theoretical model in which the sources of demand shocks are perturbations to parameters that reflect preferences that are incorporated in the structure of the wealth maximisation function, information and the subjective beliefs of different types of investors. In addition, analytical exercises are carried out to demonstrate the impact of various parametric changes on the equity risk premium.

All in all, this chapter extends Wagner's (2002) work on portfolio selection under a benchmark-based wealth maximisation framework. In addition, the analytical findings of this chapter also augment the work done by Kapur & Timmermann (2005), who have analysed the impact of using relative performance contracts (when it comes to delegated investment management) on equity risk premium and herding behaviour in an equilibrium setting.

It is important to note that for the sake of model tractability, we have made a number of simplifying assumptions which are clearly laid out in the relevant sections in order to emphasise and highlight the role played by the difference in the structure of utility maximisation in driving investor behaviour. That said, we have incorporated a number of realistic real-world attributes such as heterogeneous information and considered subjective beliefs (especially, volatility) to understand the interaction among the various model parameters and their impact on investor demand.

4.1 INTRODUCTION

Most academic studies differentiate between individual and institutional investor categories on the basis of different investment goals and the way they generate and trade on information. Relative to institutions, individual investors are thought to be less informed and thus more sensitive to the influence of psychological biases, market sentiment and attention-grabbing events such as market shocks (see Kaniel et al. (2008) and Barber et al. (2008)).

Examples of institutional investors include entities such as bank trusts, mutual funds, pension funds and insurance companies, which operate under a different legal environment compared to individuals. Institutions also tend to engage more in active asset management and spend significant resources to identify securities. According to researchers such as Falkenstein (1996) and Del Guercio (1996) these securities often tend to be mispriced relative to their fundamental values and have other characteristics deemed to be favourable for investments, such as when, for example, stock prices reach a certain price level and/or market cap.

In terms of the importance of institutional investors in Europe, out of the total Euro 19.7 trillion assets under management, around 75% can be regarded as belonging to the institutional investor category. This category covers banks, sovereign wealth funds, pension funds and insurance companies[76].

Similarly, in the US, the importance of institutions in terms of corporate equity ownership has increased dramatically over the last 60 years. For example, the proportion of US public equities managed by institutions has risen steadily from about 7 or 8% of market capitalisation in 1950 to about 67% in 2010. Institutional investor ownership is an even stronger factor in the largest corporations with 73% of outstanding equities in the 1000 largest corporations being owned by institutional investors in 2009[77].

Given the underlying shift in investor type, a change in the structure of investment ownership from individuals to institutions implies a shift in the demand patterns of the "representative agent" under a wealth maximisation framework, given differences in information set preferences and objective functions. Indeed, in this paper, we assume that in addition to heterogeneous expectations regarding asset returns and information, following Wagner (2002) we have motivated both analytically and intuitively a form of expected utility function which directly incorporates the widespread use of benchmarks, which incorporates the emphasis placed on relative performance by a variety of institutional investors.

If we turn to the theoretical literature, a common theme emerges in which individual investors are treated as ignorant and uninformed when compared to institutional investors and appear to make trading decisions which are frequently based on sentiment. For example, Shiller (1984) and De Long et al. (1990) argue that fads and fashion may influence individual investor behaviour. Similarly, Shleifer and Summers (1990) suggest that individual investors might herd if they follow common signals (such as brokerage house recommendations, common news source or forecasters). In addition, Copeland and Galai (1983), Glosten and Milgrom (1985) and Easely and O'Hara (1987) and Back (1992)) hypothesize individual investors as uninformed traders who merely add noise to the price formation

[76] Source: Spence Johnson Institutional Intelligence (2014) and Spence Johnson European Insurance Asset Management (2015).

[77] Source: US Securities and Exchange Commission Report, April 2013.

process. In addition, recent studies such as Witte (2013)[78] explore the long-standing question about the survival of noise traders in relation to informed participants in financial markets using agent-based modelling and show how typical stylised features such as volatility clustering, fat-tailed returns and bubbles/crashes can be reproduced within this framework.

More recently, empirical studies such as those of Li et al. (2016), which use a trading volume-based measure, show that better informed institutional investors' trade more selectively, whereas less-informed individuals allocate their investments even across stocks. Their examination is conducted using trading volume data from the Chinese stock market (using Shanghai Stock Exchange data), which is widely known to exhibit dominance of individual investors and limited arbitrage opportunities. For instance, according to Shanghai Stock Exchange data, around 81% of tradeable shares by market capitalisation are owned by individual investors, as of July 2014. In addition, work by Cohen (2009)[79] evaluated the impact of loyalty on individual investors portfolio choce and found that cost to employees of showing loyalty to their firm (as measured by investment in company stock) is large, amounting to nearly a 20% loss in retirement income.

In terms of subjective beliefs regarding the behaviour of risky asset returns, Levy et al. (1996) demonstrate the importance of heterogeneous expectations in determining risky asset prices using simulation analysis. They deal with this research question by investigating a stock market model with and without heterogeneous expectations. They show that by introducing diverse beliefs, market inefficiencies disappear and dynamics become more realistic. Therefore, on the basis of practical observations, it is plausible to assume that institutional and individual investors, who rely on different sources of information and analysis, have different return expectations. For instance, Yale School of Management's Confidence Indices for US equity markets[80] show marked differences in the one-year ahead equity return expectations of institutional and individual investors. Moreover, the importance of heterogeneous information can increase further during times of high volatility. For instance, Christie and Huang (1995) argue that individuals are more likely to suppress their beliefs and follow the market consensus during periods of market stress. In addition, studies such as those of Ivković and Weisbenner (2007) and Hong, Kubik and Stein (2005) empirically measure the "word of mouth" effect,

[78] Witte (2013) – "Fundamental traders 'tragedy of the commons": information costs and other determinants,,,,

[79] Cohen, L. 2009. Loyalty-based portfolio choice. Review of Financial Studies 22(3): 1213

[80] See http://icf.som.yale.edu/financial_data/confidence_index/explanation.shtml#year.

whereby individuals and mutual funds respectively show similarities in stock holdings that could be linked to a commonality in geographical residence.

Moreover, the difference in information sets, which might lead to heterogeneous expectations between the institutional and individual investors highlighted above, is strongly backed by several empirical studies. For instance, a number of empirical papers report a positive relationship between changes in the institutional ownership of individual stocks and concurrent period return. Wermers (1999) found that stocks bought by mutual funds tend to experience positive abnormal returns. Meanwhile, Gibson et al. (2003) reported a positive relationship between institutional ownership and stock market returns using US institutions data with over $100 million or more in exchange traded securities. Nofsinger et al. (1999) found that, at the margin, institutional investors are better informed than other investors. They found econometric evidence to support the hypothesis that institutional investors buy undervalued and sell overvalued securities. That said, studies such as Lakonishok et al.'s (1992a) found no relationship between institutional ownership and stock market returns using data for 341 US pension funds. This study explores the relationship between extreme movement in ownership and the return on assets held by institutional investors (although, transaction costs and fees are ignored) and therefore test the superior information production of institutional investors at the margin.

It is important to note that in the case of institutional investors, the legal and often liability-constrained environment that they face as fiduciaries can give rise to investment objectives that are very different from individual investors; this is particularly apparent for example in the case of pension funds and insurance companies. Specifically, the use of performance benchmarks (such as peer group universes or the market cap used by the majority of pension funds in the UK to decide asset allocation policies), as well as the importance placed on relative risk are two key features of institutional investor's preference.[81] We believe that these two features have direct roots in the investor's utility function.

Turning to a visible example of common information shock, the post 2008/9 crisis period is an important sample to consider. This period saw direct central bank interventions rise exponentially in fixed income markets in a bid by policy makers to support growth and inflation. For instance, compared to almost negligible holdings pre-2008, key Central banks (such as the Federal Reserve,

[81] See"Performance Benchmarks for Institutional Investors: Measuring, Monitoring and Modifying Investment Behaviour" by Blake and Timmermann (2002).

European Central Bank, Bank of England, and Bank of Japan) now own around 25% of all outstanding government debt, a trend which is likely to strengthen going forward.[82]

From the perspective of monetary policy transmission, the "portfolio rebalancing effect" induced by these direct asset purchases works through by central banks keeping risk-free rates low across the entire yield curve, which in turn forces investors to take more risk in order to reach their investment objectives[83]. Moreover, the policy signalling function (such as forward guidance for example) which has also been in action in recent years also has quite a powerful influence on the nature and quality of information signals received by both institutional and individual investors. In this situation the Central bank can signal future policy action, which leads to a change in investment behaviour of various investors in the concurrent period.

To conclude, we analytically structure distinct utility functions for individual and institutional investors respectively by bringing in different preferences (such as, for example, the stronger tendency of institutional investors to benchmark) and factors which incorporate heterogeneous expectations, and by extension, information signal quality. The detailed analytical exercises which follow adds to the literature by furthering our understanding of the drivers of herding. The herding phenomenon either emanates via common benchmarks, in the case of institutional investors, or stronger correlated information, in the case of both individual and institutional investors. This thereby motivates the various sources of commonality in portfolio holdings, such as those determined by fads and fashions, market sentiment and central bank behaviour in the case of individual investors coupled with a higher incidence of agency problems, in the case of institutional investors.[84]

Prior to detailing the model assumptions, we list some general features of our model focusing on the heterogeneity in informational content and wealth maximisation function structure. There are two classes of "investors", institutional and private. Within each class, they differ only in risk aversion and initial wealth.

Furthermore, we also show how work in this area can be extended by employing bayesian analysis if one assumes that the two classes of investors (private and institutional) differ in their prior beliefs, specifically with respect to prior covariance matrices (see assumption 5, equation 4.6).

Each class also receives a separate information signal (which is common within each investor class), specified by $\tilde{Z}_i$ and $\tilde{Z}_p$ respectively. The signals received by each class are independent of each other

[82] Bruegel database of sovereign bond holdings developed by Merler and Pisani-Ferry (2012).
[83] See Joyce et al (2012)

[84] Nofsinger and Sias (1999).

(details are given in Assumption 6, equation 4.7). In addition, the signals are observed prior to portfolio formation. Lastly, we assume that agents do not observe, nor act on, each other's risky asset demands (partially, because of signal independence).

Note, in order to focus on the consequences (for risky asset demand) of heterogeneity in wealth functions, we assume a much simpler form of information differentiation between the two classes of investors. Indeed, this assumption simplifies from Keynes (1936) observation long ago, who argued that financial markets are excessively volatile because professional investors are more focussed on forecasting the forecasts of others rather than with understanding the drivers of fundamental value of assets they trade. Elements of this "beauty contest" attribute of financial markets as noted by Angeletos et al (2008), Angeletos et al (2007) and Bacchetta and Wincoop (2005). Specifically, Angeletos et al (2007) analysed the equilibrium and welfare implications for a tractable class of economies (games) that have externalities, strategic complementarity and heterogeneous information. The paper concluded with a few relevant applications, focussing on production externalities, beauty contests, business cycles, and large Cournot and Bertrand games.

4.2 MODEL SPECIFICATION

Assumption 1; Wealth Distribution. Cross-Sectional and Temporal.

To model the effects of different types of investors with different perceptions of the stock market on the asset market equilibrium, we formulate a two period model of portfolio choice. Time is denoted by $t = t_0, t_1$. There are *I* institutional investors with initial wealth $W_{t_0,i}$ and *P* individual or private investors with initial wealth $W_{t_0,p}$.We also assume that $\sum_{\mathrm{i}=1}^{I} W_{t_0,i} > \sum_{p=1}^{P} W_{t_0,p}$ to capture the distribution of wealth between institutional and individual investors which is observed in practice. We assume all wealth processes are Gaussian.

End of Assumption 1,.

Assumption 2; Private Investors' Utility Functions

All agents in this class are assumed to be risk averse and make choices in order to maximize their expected utility of end-period wealth. In this model, we assume the structure of asset returns to be normally distributed and assume that the *p*th private investor has the following utility function

$$\mathrm{U_p} = -\exp\left[-\rho_{1,p}\, W_{t_1,p}\right] \quad . \tag{4.1}$$

Where $\rho_{1,k}$ is the risk aversion coefficient and is positive for all private investors. Each private investor maximizes the expected utility of end-period wealth conditional on information$\mathcal{F}_p$.

End of Assumption 2.

Assumptions 1 and 2 have some immediate implications.

Since $W_{t_1,p}$ is normally distributed, then

$$\mathbb{E}\left(\mathrm{U}_p(W_{1,p})\right) = -\exp\left[-\rho_{1,p}\ \left[\mathbb{E}(W_{t_1,p}) - \tfrac{1}{2}\ \rho_{1,p}\ \mathrm{Var}(W_{t_1,p})\right]\right] \tag{4.2}$$

It follows that to maximize (4.2) is equivalent to maximizing

$$\mathrm{V}_p = \mathbb{E}(W_{t_1,p}) - \tfrac{1}{2}\ \rho_{1,p}\ \mathrm{Var}(W_{t_1,p}) \tag{4.3}$$

The above model is the Mean Variance Model of Asset Demand[85] (MVMAD) and has been used as a basis for various representative agent models. The theoretical studies of Grossman (1975), Grossman and Stiglitz (1980) and De Long et al. (1990) offer a few examples of studies which use MVMAD as the base model.

Institutional Investors

The other class of investors in this stylized world are the institutional owners and/ or managers of risky assets. As briefly noted above, empirical evidence across advanced and emerging economies continues to show the increasing dominance of institutional investors as asset owners and managers. For instance, according to a Towers Watson study[86], pension fund assets alone in major economies stand at around USD 36 trillion level (end-2014) or around 84% of world GDP compared to around USD 19 trillion 10 years ago.

As we briefly discuss in the introduction, there are strong reasons to expect an institution's investment objective function to differ from that of an individual's. One major driver of this difference is the legal environment that institutions face as fiduciaries. The organizational set-up, the decision-making dynamics and the presence of different stakeholders may give rise to investment objectives that are more complicated than a straightforward wealth maximisation goal.

For example, occupational pensions in UK are organized on a trust basis, with a board of trustees responsible for deciding the asset allocation of funds. Other important players in this set-up are the sponsors (generally a company), the beneficiaries (who tend to be either active, deferred and pensioners), the actuaries, investment consultants and external asset managers. Although the final objective of the pension plan is to deliver pensions to scheme members, there can be instances when different parties have differing priorities and time horizons.

[85] Tobin's Risk Aversion Model

[86] Towers Watson Global Pension Assets Study - 2015

Diving into the world of pension plans, it is interesting to note a counter trend reflected in a shift towards what is called a defined contribution (DC) design, when it comes to pension fund management. This emerges from the historical practice of using a defined benefit (DB) framework when it comes to managing of pension fund assets. This shift is visible in a number of advanced and major emerging economies[87] with DC assets growing at a pace of 7% per annum over the last 10 years, compared to a 4.3% per annum growth in DB assets. That said, countries such as the Netherlands, the UK, Canada and Japan still remain predominately DB, despite the sharp rise in DC assets seen recently.

The two designs of pension provision have different characteristics with respect to the balance of risk faced by employers and employees respectively. Specifically, in a DC scheme, the investment risk is borne by the employees (see Merton et al. (1985)), while in DB, the employees have guaranteed benefits, which is a function of years of service and wage history. Therefore, the investment risk in a DB's case directly enters a plan sponsor's decision equation.

One can think of individual or private investors in our stylized world as DC members, who are responsible for their own investment choices, and institutional investors as DB scheme trustees who share the ultimate responsibility of deciding the fund's overall asset allocation policy and have fiduciary duties.

As part of widespread investment practice since the early 1980s, DB schemes have been setting themselves an objective of outperforming their peer group median. Not surprisingly, academic research has argued that this objective has little apparent relation with the ultimate goal of pension funds and in addition, the practice introduces incentives for herding (see Blake et al. 2002). The proponents of peer group type benchmarks argue that it represents the "distilled wisdom" of investment management firms. Another dominant practice which is especially apparent in fixed income investing is the usage of market-capitalisation based benchmarks. In the case of fixed income, market-capitalisation benchmark reflects the precise issuance structure of the debt market with the largest issuers (whether countries or corporates) getting the highest weight in the index. While in the case of equities, market-capitalisation weighs the portfolio on the basis of the final market-cap (number of shares outstanding x price) of individual securities.

An important behavioural explanation behind the use of peer group benchmarks or market-cap based reference points is referred to as "regret risk" (see Shefrin (1999)). Trustees may experience "regret" if they use an asset allocation policy which is different from others and thus opens up the possibility

[87] Towers Watson Global Pension Assets Study - 2015

of extreme deviation from the norm. Shefrin (2000) argues that in the real world investors are partly guided by their emotions and these emotions are reflected in the use of benchmarks. Agents experience "regret" when they compare their decisions to a better course of action *ex post*. Therefore, by adopting benchmarks (whether these are peer-group or market-cap) the plan sponsors can "hedge" against this type of non-financial payoff in the short term. Benchmarks affect portfolio choice in various ways. Benchmarks provide a reference point and can lead the investors to change their positions in order to achieve a desired place relative to the reference point. In a principle agent setting, which is mostly the case in the institutional world, both regret and responsibility are closely related (see Shefrin 1999). The incidence of "regret risk" is likely to be higher in situations where investors are directly responsible for their decisions.

More specifically, regret theory specifies a two-attribute utility function where the investor faces a trade-off between two attributes, both impacting perceived utility under a choice-based framework (see Loomes and Sugden (1982) for more details). Here, payoff from an investor's decision is compared to a hypothetical alternative choice, whereby *ex ante* if realised wealth is lower/higher than the outcome of the alternative choice (i.e. hypothetical wealth generated by a benchmark portfolio), then the investor experiences regret or rejoices.

Turning to studies focussed on multi-attribute utility functions, while multivariate generalisations of risk aversion have been extensively developed (e.g. Karni (1979); Pratt (1986); Gollier and Pratt (1996)), studies such as Li and Ziemba (1989), Finkelshtain & Chalfant (1993) and Grant & Satchell (2016) have also developed portfolio choice models using multi-variate utility functions.

The above mentioned agency behaviour is quite wide-spread for investment advisors, pension funds and mutual funds. Once individuals have delegated their investment decisions to an institution, they can imperfectly monitor the agents' choices. The agents' incentives may often differ from those of the principals, which in this case are individuals. Furthermore, a costless discretion over the choice of investment agent and complete control are usually not possible. Therefore, an imperfect control over the investment decisions leads to different incentives which can result in different demand patterns between the two groups. The use of benchmarks can be thought of as an example of incentives towards institutional agents which they employ in order to mitigate regret risk. In addition, the use of benchmarks can therefore be thought of as a manifestation of those fiduciary motives which are linked to "prudence" in order to avoid "regret risk".

Del Guercio (1996) examined the issue of prudence as it relates to equity ownership of banks and mutual funds, providing explanation and evidence to show that different types of institutions are affected by prudence restrictions to varying degrees. Empirical studies and survey analysis suggest that many non-bank institutions consider prudence characteristics (see Del Guercio (1996), Longstreth (1986) and Badrinath et al. (1989)). Although standards of prudence vary depending on the institution under consideration, Del Guercio constructed a prudence proxy for S&P stocks which was a function of firm age, dividend yield and security price volatility. In terms of the asset allocation decision process, these various prudence characteristics can be embedded in the weights of various benchmark schemes (peer group benchmark is one of the possible schemes). On the other hand, investing in the largest issuers (both equity and debt as part of market-capitalisation benchmarks) may also reflect an avenue to express this behavioural characteristic.

Another explanation stems from the usage of relatively short-term evaluation techniques and how these impact on the perceived utility of institutions such as mutual funds and asset managers. Lakonishok et al. (1992b) found a positive correlation between the relative performance of funds and the in-flow of new investment funds. Similarly, Chevalier and Ellison (1998) and Sirri et al. (1998) found a positive, non-linear relationship between performance and the in-flow of new money to mutual funds. Since fees within the asset management industry are an increasing function of fund size, particularly in the UK, outperforming the market thereby results in higher fee income. Therefore, on the basis of both behavioural and empirical evidence, it is more realistic to assume that the objective function of institutional investors spans both absolute and relative wealth maximisation, unlike the case of individual investors.

We here attempt to capture the impact of benchmark based asset allocation behaviour on risky asset demand (equities in our world) by directly embedding it in institutional investors' objective function.

Assumption 3; Institutional Investors' Utility Functions

Specifically, using Wagner's (2002) approach, we have assumed the following expected utility function for institutional class of investors in our model. This assumption explicitly recognises the role of benchmarks and relative performance.

Following Wagner (2002), institutional investors in our stylised world possess a multi-attribute utility function:

$$\mathrm{U} = \mathrm{U}(W_A, W_B)$$

Where: $W_A =$ Wealth under λ weights on different securities and $W_B =$ Wealth under $\overline{\lambda}_0$ (benchmark; assumed exogenous) weights on different securities such as peer group or market capitalisation.

Equivalent to a classical setting, utility is assumed to be a strictly increasing concave function of final wealth W_A; that is:

$\partial \mathrm{U}\,(W_A, W_B)/\partial W_A \;> 0$ (I)

$\partial^2 \mathrm{U}\,(W_A, W_B)/\partial W_A^2 < 0$ (II)

With respect to wealth under benchmark, i.e. W_B, we assume the following restrictions:

$\partial \mathrm{U}\,(W_A, W_B)/\partial \mathrm{W}_B \;\leq 0$ (III)

$\partial^2 \mathrm{U}\,(W_A, W_B)/\partial W_A W_B \;\geq 0$ (IV)

The institutional investor is assumed to maximise the expected value of the utility function $\mathrm{U}(W_A, W_B)$, where a second order Taylor Series expansion is assumed a sufficient description of investor preferences as we are in a Gaussian setup.

End of Assumption 3.

The economic interpretations of the first two restrictions in Assumption 3 are non-satiation and risk averse behaviour respectively.

Under the assumed model, an institutional decision maker is choosing λ to maximize his wealth in period one under the budget constraint. Since we have assumed $\overline{\lambda}_0$ is exogenous, $\mathrm{E}(W_B)$ and $\mathrm{Var}(W_B)$ are given constants. The institutional investor is assumed to maximise the expected value of the utility function $\mathrm{U}(W_A, W_B)$, where a second order Taylor Series expansion is a sufficient description of investor preferences as we are in a Gaussian setup.

Forming a Taylor series expansion of the utility function around the value of $\mathrm{U}(E(W_A), E(W_B))$, we take expectations and evaluate the derivatives at $\mathrm{E}(W_A)$ and $\mathrm{E}(W_B)$; this gives the following problem formulation:

$$\begin{aligned}\mathbb{E}[\mathrm{U}(W_A, W_B)] = {} & \mathrm{U}\big(\mathbb{E}(W_A), \mathbb{E}(W_B)\big) \\ & + 1/2\, \partial^2 \mathrm{U}(W_A, W_B)/\partial W_A^2 \times Var(W_A) \\ & + 1/2\, \partial^2 \mathrm{U}(W_A, W_B)/\partial W_B^2 \times Var(W_B) \\ & + \partial^2 \mathrm{U}(W_A, W_B)/\partial W_A, W_B \;\times Cov(W_A, W_B)\end{aligned} \tag{4.4}$$

Assumption 4; Institutional Investors' Benchmark

We choose the common benchmark with exogenous weights which all institutional investors hold (assuming no short positions in the benchmark).

End of Assumption 4.

Defining h_i (which varies between 0 and 1) as the proportion of wealth invested by i_{th} investor in the common benchmark, we define the benchmark-weighted average wealth of institutional investor category in period 0 as $\overline{W}_{t_0,\mathrm{I}} = \frac{1}{I}\ (\sum_{i=1}^{I} h_i\ W_{t_0,i})$, where I is the total number of institutional investors (as defined above). For ease of notation, we will set $W_B = \overline{W}_{t_0,\,I}$

Using a non-satiation condition and algebraic manipulations we yield the following expected utility function for *i*th institutional investor using the above defined benchmark (see appendix 4.1 for details).

$$\mathrm{V}_i = \mathbb{E}\big(W_{t_1,i}\big) - \frac{1}{2}\rho_{1,i}Var\big(W_{t_1,i}\big) - \frac{1}{2}\rho_{2,i}Var\big(W_{t_1,i} - \overline{W}_{t_0,I}\big) \qquad (4.5)$$

$\rho_{1,i}$ can be interpreted as expected utility penalty, due to volatility of absolute wealth

$\rho_{2,i}$ can be interpreted as expected utility penalty, due to volatility of relative wealth

The reasoning behind using $\overline{W}_0$ rather than $\overline{W}_1$ in the relative term is to capture the time lag effect in reporting. In addition, using $\overline{W}_1$ introduces unrealistic endogeneity into the system. The relative magnitude of $\rho_{1,i}$ and $\rho_{2,i}$ in the objective function discussed above can also be linked to the time-horizon of investment decision. Here, it is plausible to assume that the shorter the horizon of the institutional investor the larger the weight attached to the relative risk term.

Indeed, the utility formulation given in (4.4), which is a combination of absolute and relative risk minimization can be thought of as being based on a behavioural approach to decision-making developed independently by Bell (1982) and Loomes and Sugden (1982).

As noted above, according to the above authors, utility is defined as a psychological perception which is measured in a bi-attribute utility setting. Regret theory postulates that the two attribute felicity function, whereby the decision-maker faces a trade-off between absolute and relative wealth, both of which influence the perceived utility. In this framework, utility is associated with the outcome of the investment decision and is measured in reference to the outcome of the hypothetical alternative choice; wealth in period one is achieved using benchmark weights.

Among others, Wagner (2001), Roll (1992) and Chow et al. (1999) have introduced a quadratic tracking approach, which is an application of "regret behaviour" in portfolio selection framework. The above expected utility formulation captures the two key features of institutional investment, namely the use of benchmarks and the role of relative performance risk in asset allocation.

Institutional investors such as pension funds usually delegate investment decisions to professional money managers, which are often judged by the total return performance relative to an agreed benchmark. This is a sensible approach, in the sense that the sponsor's direct alternative to active management is the index fund. Therefore, an active manager is worth the extra fee only if relative performance is on average positive. Risky asset returns are however exceedingly noisy, and it may not be possible to ascertain with considerable confidence the extent of the manager's value added. This has led many sponsors to focus on the volatility of tracking error, i.e. the variability of return above a certain benchmark. We suggest that a focus on tracking error can also motivate the role of short-term relative performance risk in the institutional investor utility function.

In addition to the arguments discussed above, we believe that the utility formulation specified above also captures a realistic decision process dynamic, whereby investment decision is delegated from the principal to the agent and relative performance evaluation is directly employed. Such a setting can explain the delegation of investment decisions by pension fund trustees to professional managers. In addition, it is reasonable to think that trustees are themselves working under such a setting in which the principals are the pension beneficiaries. Kapur and Timmermann (2005) studied the impact of relative performance evaluation on the equity risk premium in a principal-agent setting. They concluded that an emphasis on relative performance based contracts lowers the equity risk premium and can create tendencies to herd. We postulate that for institutional investors, absolute risk matters in the long term whilst relative risk matters in the short term. This reflects the "double benchmarking" common to pension funds, whereby decisions are based on both short term and long term performance.

On the basis of the arguments discussed above, we believe that a combination of factors, such as the prudence motives which are linked to regret behaviour; the relative performance evaluations which are employed by sponsors; and the positive connection between relative performance and income (for mutual funds and asset managers), can explain the link between short term relative performance risk and institution's investment equation.

4.3 HETEROGENEOUS EXPECTATIONS

Heterogeneous expectations can play a very important role in the price determination of risky assets and can explain the dynamics of markets more realistically (see Levy et al. (1996)). The existence of investor heterogeneity goes beyond the framework of asymmetric information to include diversity in prior beliefs. The capital asset pricing model, the Black-Scholes model option valuation, and the majority of economic analyses rely on the assumption of homogenous expectations. Levy et al. (1996) studied the dynamics of equity markets in the presence of agents with heterogeneous beliefs regarding asset market returns. On the basis of simulation results, they concluded that the assumption of heterogeneous expectations, dramatically changed market dynamics and moreover, the equity price patters obtained were more realistic and similar to the observed price patterns.

In practice, it is true that investors form their expectations by very different methods. Some investors focus on accounting data, others look at price ratios for clues, while others may have sophisticated time series prediction models to estimate the *ex-ante* distribution of returns. Therefore, it is realistic to assume that institutional and individual investors have different beliefs regarding the returns of risky assets in addition to the differences in their respective investment objectives. This is because they rely on different sources of information and thus employ different methods of information production.

Furthermore, institutional investors such as pension funds exhibit strong degrees of home bias in their security holdings (see French and Poterba (1991), Cooper and Kaplanis (1994) and Tesar and Werner (1995)). Several explanations have been offered for investor home bias in the literature; these include hedge characteristics against the domestic risks of domestic equity; foreign returns implicit in the domestic equity returns; government restrictions and asymmetric information (as seen in the "familiarity breeds investment" argument offered by Huberman (2001)). In addition to this, institutions such as pension funds differ from the rest because of the key role played by pension liabilities. DB plan's liability is predominantly affected by demographic and economic factors. The demographic factors include mortality, termination, disability and retirement. Economic factors include inflation, productivity increases and capital market performance, which affect pension liabilities through wage growth and discount rates. It is plausible that the home bias is embedded in subjective beliefs regarding the distribution of returns, which in turn shape the portfolio choices of institutional investors.

Assumption 5, Belief Assumption.

In this model, we assume that agents allocate their wealth across two types of assets, namely risk-free debt and equity[88].

The distribution of price of equity is given by:

$$\widetilde{\Pi}_{t_1} = \overline{\Pi}_{t_1} + \epsilon_j \text{ where } \epsilon_j \sim \mathcal{N}(0,\ \Sigma_{\pi\pi}) \qquad (4.6a)$$

The bond pays fixed rate of return r_f. In addition, let Π_0 be the *S x 1* vector of prices per share of the securities at time t_0 and $\widetilde{\Pi}_{t_1}$ the *S x 1* vector of stochastic prices per share at time t_1 (where S is the number of risky securities) and $\Sigma_{\pi\pi}$ the S x S covariance variance matrix, while $\overline{\Pi}_{t_1}$ is the Sx1 vector of mean prices at t=1.

End of Assumption 5.

In terms of further direction of work in this area, we can also introduce heterogeneity in beliefs. One specification of such heterogeneity can be introduced by dividing the equity universe into two categories: type *X* and type *Y*. Both institutional and private agents have common beliefs regarding the type *X* equity, but their beliefs vary regarding type *Y* securities.

In this case, consider a capital market with S_1 type *X* risky assets, S_2 type *Y* risky assets and one risk free asset (bond). The bond pays fixed rate of return r_f. Denote Π_0 the *S x 1* vector of prices per share of the securities at time t_0 and $\widetilde{\Pi}_{t_1}$ the *S x 1* vector of stochastic prices per share at time t_1 (where $S = S_1 + S_2$, total number of risky assets).

Here, the prior distribution of price of equity can be denoted by:

$$\widetilde{\Pi}_{t_1} = \overline{\Pi}_{t_1} + \epsilon_j \text{ where } \epsilon_j \sim \mathcal{N}\left(0,\ \Sigma_{jj}\right) \qquad (4.6a)$$

where *j* is investor type (institutional or private, *i* or *p*) and Σ_{jj} is the *S x S* covariance matrix assumed by the different types of investors.

Furthermore, we can assume that Σ_{ii} has the following structure:

$$\Sigma_{ii} = \Sigma_{pp} + \Sigma_{dd} \qquad (4.6b)$$

[88] Note N-1 = S, where S is the number of risky assets and N as the total number of securities including one risk free asset.

where Σ_{dd} is the *S x S* diagonal matrix with the first S_1 (corresponding to type *X* stocks) rows null and the following S_2 rows with negative diagonal entries, indicating a better prior knowledge for institutional investors.

The above structure assumes that the private investor's perception of *Y* type stocks is different from the institutional investors on the absolute risk dimension rather than the mean of returns dimension. This is an approximation in a world where mean forecasts are well established and agreed but volatility forecasts are not. In the literature, more attention has been paid to the role of heterogeneous beliefs regarding mean forecasts rather than volatility forecasts. The latter can be thought of as an outcome of a single dominant risk management system. However, examples of heterogeneous volatility beliefs do occur in derivatives literature, where they play a crucial role in converting Black-Scholes “no-trade” equilibria into equilibria where actual trades occur[89].

In addition, the specific type of risk belief based heterogeneity outlined above is relevant to those cases where different agents view the equity universe in categories and groups on the basis of a shared connection among them. The classification of stocks in different categories clearly exists in the financial markets. When making portfolio decisions, investors class the stocks in broad classes such as large cap, value and venture capital, and then proceed to decide how much funds to allocate in each category (see Swensen (2000)). On the basis of this observation, one can think of stock type *X* and stock type *Y* as different "styles" which differing types of investors view differently.

On the basis of econometric evidence, Gompers and Metrick (2001) argue that the demands of institutional investors for different equity characteristics is different from that of other investors. They note that institutional investors invest in stocks which are larger and liquid. They show that a change in ownership from the " individual to institutional" can explain the return advantage of large cap stocks over small cap stocks since 1980. Hence, in the case of large cap versus small cap, the assumption of different beliefs of the agents regarding the risk of each category holds more weight, since it was well documented in 1981 that from 1926-1979, small equity held a clear return advantage over large cap stocks in US. As a result, it can be argued that the mean return forecast was well established, while one of the motivations behind the demand for different stock characteristics on behalf of institutions since 1981, derives from the basis of differences in perceived risk assessment.

[89] For instance, see A general equilibrium analysis of option and stock market interactions. J Detemple, L Selden - International Economic Review, (1991)

4.4 INFORMATION STRUCTURE

The model detailed above is further extended to incorporate information heterogeneity between the different types of investors. Admati (1985, 1987) studied the impact of diverse information on asset market equilibrium. The theory has been extended to explain delegated investment management within institutional investment (see Bhattacharya et al. (1985)).

Assumption 6, Informational Structure.

We therefore assume that before making portfolio choices, the private investors observe a common (*S x 1)* signal $\tilde{Z}_p$ and institutional investors observe a common (*S x 1)* signal $\tilde{Z}_i$.

In addition, we assume that the signal and price of risky assets in period 1 are jointly multi normally distributed.

$$\begin{pmatrix} \tilde{\pi}_{t_1} \\ \tilde{Z}_i \\ \tilde{Z}_p \end{pmatrix} \sim N \left[\begin{pmatrix} \bar{\pi}_{t_1} \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \Sigma_{\pi\pi} & \Sigma_{\pi z_i} & \Sigma_{\pi z_p} \\ \cdot & \Sigma_{z_i z_i} & 0 \\ \cdot & 0 & \Sigma_{z_p z_p} \end{bmatrix} \right] \qquad (4.7)$$

We also assume that covariance matrices are positive definite and institutional investors are better informed than private investors (all diagonal entries of $\Sigma_{z_i z_i}$are smaller than diagonal entries of $\Sigma_{z_p z_p}$).

End of Assumption 6.

The part of Assumption 6 that institutional investors are better informed than private investors is a much stronger assumption than a case where institutions have informational advantage for only a subset of the equity universe.

4.5 EQUILIBRIUM

Given this model formulation, an asset market equilibrium can be defined as the market clearing condition.

We also need some assumption about who knows what.

Assumption 7, Informational Knowledge.

We assume that the structure of preferences, information and subjective beliefs are common knowledge but that no account is taken of other investor's decisions when maximizing expected utility.

End of Assumption 7.

Investors choose their respective portfolios to maximise their respective expected investment objective function.

Let $\Lambda_D = (\lambda_1, \lambda_2, \lambda_3 \dots \lambda_D)'$ be the *S x D* matrix of equity demand where $D = \{I + P\}$ for the *I and P* institutional (private) investors, where demand is a function of initial price $\Pi_{t_0}$and the total number of investors. Given the aggregate demand for equity securities and their fixed supply Q, the price Π_{t_0} is determined through market clearing:

$$\sum_{i=1}^{I} \lambda_i \left(\Pi_{t_0}\right) + \sum_{p=1}^{P} \lambda_p \left(\Pi_{t_0}\right) = Q \qquad (4.8)$$

The equilibrium solution is subject to the information revelation problem first studied by Grossman (1975): investors may be able to infer information received by other investors from the equilibrium price. This problem can be addressed by allowing Q to be random with a large variance. Such a specification might reflect the impact of liquidity traders (see Kapur & Timmermann (2005)). We have ignored the issue in this paper, as this simplifies the algebra without significantly affecting the conclusions.

4.6 PRIVATE INVESTOR OPTIMAL DEMAND

The portfolio choice of *k*th private investor is examined below. The return to investment is given by:

$$\tilde{R} = \lambda \tilde{K} + r_f \qquad (4.9)$$

where $\tilde{K}$ is the(stochastic) excess price. Also, for any Π_{t_0}, let $\overline{K} = \overline{K}\left(\Pi_{t_0}\right) \equiv E\left[\tilde{K}\left(\Pi_{t_0}\right)\right] = \overline{\Pi}_{t_1} - \Pi_{t_0}(1 + r_f)$ be the mean value of excess prices which is the equity risk premium times the initial price Π_{t_0}. We have the following result:

Proposition 1 Consider a private investor *p* with coefficient of absolute risk aversion $\rho_{1,p}$ and information signal Z_p. The optimal portfolio demand conditional on receiving signal Z_p is

$$\lambda_{p,z_p} = \left(\Sigma_{\pi\pi} - \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1}\Sigma_{z_p \pi}\right)^{-1}\left(\overline{K} + \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1} Z_p\right)/\rho_{1,p} \qquad (4.10)$$

(see Appendix 4.2 for details).

This is the standard demand for a multi security setup and assumed mean variance structure of preferences. Equity holding is increasing in $\left(\overline{K} + \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1} Z_p\right)$, which is the expected value of $\tilde{K}\left(\Pi_{t_0}\right)$ conditional on the signal. Demand is decreasing in the risk aversion parameter $\rho_{1,p}$ and conditional variances by which we mean the diagonal elements of $\left(\Sigma_{\pi\pi} - \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1}\Sigma_{z_p \pi}\right)$. Notice that across individuals who are private investors, the demands differ only in risk aversion.

Note that we have not ruled out short sales as these do not affect our results in any significant way.

4.7 INSTITUTIONAL OPTIMAL DEMAND

We analyse the portfolio choice of institutional investor in this subsection. Given the assumed preferences and parameters, the *i*th institutional investor chooses the following optimal portfolio to maximise his/her expected utility function.

$$\lambda_{i,z_i} = \frac{\left[\left(\Sigma_{\pi\pi} - \Sigma_{\pi z_i}\Sigma_{z_i z_i}^{-1}\Sigma_{\pi z_i}'\right)^{-1}\left(\overline{K} + \Sigma_{\pi z_i}\Sigma_{z_i z_i}^{-1}Z_i\right) + \rho_{2,i}\overline{\lambda}_0\right]}{(\rho_{1,i} + \rho_{2,i})} \tag{4.11}$$

where $\overline{\lambda}_0$ are peer group benchmark weights on different securities (see appendix 4.3 for details).

For an institutional investor, the optimal demand includes a benchmark link term $\rho_{2,i}\overline{\lambda}_0/(\rho_{1,i} + \rho_{2,i})$, in addition to the mean and variance terms. The importance of the benchmark in shaping the optimal demand depends on the weight ($\rho_{2,i}$) institutional investor places on the relative risk term, in the expected utility function. It is straightforward to see that institutional demand collapses to private investor demand for $\rho_{2,i} = 0$. As with the private investor, the equity holding is increasing in the signal adjusted expected excess return and decreasing in the risk aversion parameters and conditional variance. Whilst these institutional demands differ only with respect to risk aversion within this group, they differ in other ways from private investor demands.

4.8 AGGREGATION AND EQUILIBRIUM

In this stylised world, the aggregate demand for equity holding is given by the following Lemma. For ease of notation, we have simplified the expression (see appendix 4.4 for details).

Lemma 1 *Consider a market with I institutional investors and P private investors, the optimal aggregate equity holding is given by*

$$\begin{aligned}\Sigma_{i=1}^{I}\lambda_i + \Sigma_{p=1}^{P}\lambda_p = &\left[\left[A_I^{-1}\left[\overline{K} + \Delta_i Z_i\right]\right]\sum_{i=1}^{I} 1/(\rho_{1,i} + \rho_{2,i})\right] \\ &+ \sum_{i=1}^{I}\rho_{2,i}\overline{\lambda}_0/(\rho_{1,i} + \rho_{2,i}) + \left[A_p^{-1}\left[\overline{K} + \Delta_p Z_p\right]\right]\Big/\sum_{p=1}^{P} 1/\rho_{1,p}\end{aligned} \tag{4.12}$$

Note that the role of benchmark in shaping aggregate demand depends both on the weight placed on relative risk term and number of institutional investors.

Equilibrium price Π_{t_0} is determined by using the market clearing condition.

Lemma 2 *Consider a market with I institutional investors and P private investors with optimal demand holdings given by 4.12. The equilibrium price, given the optimal portfolio choice of all investors is*

$$\Pi_{t_0} = M^{-1}\left[B_p + B_i - Q\right] \quad (4.13)$$

where

$$M = r_f\left[A_i^{-1}\Sigma_{i=1}^{I} 1/\left((\rho_{1,i} + \rho_{2,i}) + A_p^{-1}\Sigma_{p=1}^{P} 1/\rho_{1,p}\right)\right]$$

$$B_p = \left(A_p^{-1}\Delta_p Z_p\right)\Sigma_{p=1}^{P} 1/\rho_{1,p} + A_p^{-1}\overline{\Pi}_{t_1}\Sigma_{p=1}^{P} 1/\rho_{1,p}$$

$$B_i = \left[\overline{\lambda}_0\Sigma_{i=1}^{I}\rho_{2,i}/(\rho_{1,i} + \rho_{2,i})\right] + \left[\left(A_i^{-1}\Delta_i Z_i\right)\Sigma_{i=1}^{I} 1/(\rho_{1,i} + \rho_{2,i})\right] + \left[A_i^{-1}\overline{\Pi}_{t_1}\Sigma_{i=1}^{I} 1/(\rho_{1,i} + \rho_{2,i})\right]$$

A necessary and sufficient condition for a well-behaved downward sloping demand curve is that $M^{-1}$is a positive definite matrix. Appendix 4.5 presents a formal proof which shows that this is indeed the case.

4.9 COMPARATIVE STATICS

Case 1: Impact of change in relative and absolute risk tolerance of institutional investors

The values of $\rho_{1,i}$ and $\rho_{2,i}$ capture the emphasis that different institutional investors place in risk aversion terms; that is, on long and short term performance respectively. For example, consider the case in which $\sum_{i=1}^{I}\rho_{2,i}$ increases by constant x and $\sum_{i=1}^{I}\rho_{1,i}$ decreases by the same constant x, *ceteris paribus*. This specific perturbation means that the institutional investor universe is putting more weight on relative performance at the expense of absolute performance; here, the total risk aversion of the institutional investor universe remains constant.

The impact on Π_{t_0} of this specific perturbation is via B_i. Within B_i the influence comes through $\left[\overline{\lambda}_0\sum_{i=1}^{I}\rho_{2,i}/\left(\rho_{1,i} + \rho_{2,i}\right)\right]$, which increases as a result of the change. Therefore, an increase in the weight of relative risk aversion, at the expense of absolute risk aversion, increases Π_{t_0}, if the benchmark $\overline{\lambda}_0$ does not consist of short positions. However, if the benchmark consists of short positions for some risky assets, the influence on the price of short position assets will be negative.

The exact change in Π_{t_0} as result of this perturbation is $\left[x/\sum_{i=1}^{I} 1/\left(\rho_{1,i} + \rho_{2,i}\right)\right]M^{-1}\overline{\lambda}_0$. In terms of portfolio selection, it is easy to see that the higher the increase in ρ_2, the closer the portfolio holdings get to the benchmark.

Case 2: Impact of change in subjective beliefs of investors

The second case is impact on Π_{t_0} due to a change in investor beliefs about the parameters of the price distribution ceteris paribus. This perturbation shows that Π_{t_0} depends on the weighted average of return and variance beliefs of the two types of investors. Consider a simple case where A_i^{-1} and A_p^{-1} are diagonal matrices. An increase in the variance of all risky assets prices ($\widetilde{\Pi}_{t_1}$) for both investor types will affect Π_{t_0} via three channels. The impact of such a change on B_j (where *j* is investor type) is negative. On the hand, the impact on M^{-1} of such a change is positive, therefore the net effect on Π_{t_0} is ambiguous. The impact depends on the value of signal realized and risk aversion coefficients of the different investor universe.

Case 3: Impact of change in the parameters of signal distribution

The third case is the impact of change in information quality on Π_{t_0}. Consider a simple case, where Δ_i and Δ_p are diagonal matrices. An increase in the variance of signal, ∀*N* risky assets (increase in all diagonal entries of signal variance matrices $\Sigma_{z_p}$and $\Sigma_{z_i}$by a constant x) reduces the diagonal entries of Δ_j (where *j* is investor type) ceteris paribus. This change has downward impact on Π_{t_0} (if realized signal *Z* is positive) via B_i and B_p. The positive signal variance perturbation reduces the influence of the signal in shaping the demand of the risky assets. Lower the signal variance, more confidence the investors have in the signal they observe before making portfolio choices.

A positive change in diagonal entries of $\Sigma_{\pi z_p}$and $\Sigma_{\pi z_i}$increase's the diagonal entries of Δ_p and Δ_i respectively. Higher the covariance between price of risky assets in period 1 and the signal, more informative the signal. In such a perturbation, the signal plays a relatively bigger role in shaping the demand of the risky assets. The influence on Π_{t_0} depends on the sign of the signal *Z* realized.

4.10 CONCLUSIONS

In this chapter, we explore the equilibrium consequences of heterogeneous expectations, information and investor type on risky asset demand and pricing using a simple two-period and two asset model.

Specifically, we examine the impact of introducing benchmark-driven investors in a stylised wealth maximisation setting and introduce stochastic expectations (subjective beliefs) and information, which are allowed to vary according to investor type.

We show how changes in risk aversion parameters (both absolute and relative) have a direct impact on risky asset demand/pricing and how an increase in relative risk aversion can lead to herding towards the benchmark. In addition, both the nature of expectations and quality of information can impact portfolio selection and can also lead to similar portfolio holdings if actual or perceived variance of signal received by various investors decreases or there is a reduction in the heterogeneity of distribution of price expectations. For instance, in the case of Central bank driven interventions (such as the decision of European Central Bank to buy certain corporate bonds announced in April 2016), such an action can be interpreted as creating homogenous expectations and information signals which are independent of investor type, given the buying programme’s size, scope and transparency.

APPENDIX 4.1 FORMAL PROOF OF INSTITUTIONAL UTILITY FUNCTION

Non-satiation implies that maximising the first term in (4.4) is equivalent to maximising expected wealth W_A. In addition, the third term in equation (4.4) is constant, since $Var(W_B)$ is a given constant. A reformulation of the problem is:

$$\text{Max: } \lambda'\mathbb{E}\left(\widetilde{K}(\Pi_{t_0})\right) - \tfrac{1}{2}\alpha_1\lambda' Var\left(\widetilde{K}(\Pi_{t_0})\right)\lambda + \alpha_2\lambda' Var\left(\widetilde{K}(\Pi_{t_0})\right)\overline{\lambda}_0 + C \qquad (4.1.1a)$$

where $\widetilde{K}(\Pi_{t_0}) \equiv \widetilde{\Pi}_{t_1} - \Pi_{t_0}(1 + r_f)$

also assuming, $\alpha_1, \alpha_2 \geq 0$ & $\alpha_1 > \alpha_2$

and $C \in \mathbb{R}$

(4.1.1a) is equivalent to

$$\begin{aligned}\lambda'\mathbb{E}\left(\widetilde{K}(\Pi_{t_0})\right) - \frac{1}{2}\alpha_1\lambda' Var\left(\widetilde{K}(\Pi_{t_0})\right)\lambda + \alpha_2\lambda' Var\left(\widetilde{K}(\Pi_{t_0})\right)\overline{\lambda}_0 - \frac{1}{2}\alpha_2\lambda' Var\left(\widetilde{K}(\Pi_{t_0})\right)\lambda \\ - \frac{1}{2}\overline{\lambda}'_0 Var\left(\widetilde{K}(\Pi_{t_0})\right)\overline{\lambda}_0 + \frac{1}{2}\alpha_2\lambda' Var\left(\widetilde{K}(\Pi_{t_0})\right)\lambda + \frac{1}{2}\overline{\lambda}'_0 Var\left(\widetilde{K}(\Pi_{t_0})\right)\overline{\lambda}_0 \\ + C\end{aligned} \qquad (4.1.1b)$$

Note that $1/2\,\overline{\lambda}'_0 Var(\widetilde{K}(\Pi_{t_0}))\overline{\lambda}_0$ is a given constant and α_2 is positive (using IV) and

$$\begin{aligned}(\lambda - \overline{\lambda}_0)' Var\left(\widetilde{K}(\Pi_{t_0})\right)(\lambda - \overline{\lambda}_0) = \lambda' Var\left(\widetilde{K}(\Pi_{t_0})\right)\lambda - 2\,\lambda' Var\left(\widetilde{K}(\Pi_{t_0})\right)\overline{\lambda}_0 \\ + \overline{\lambda}'_0 Var\left(\widetilde{K}(\Pi_{t_0})\right)\overline{\lambda}_0\end{aligned} \qquad (4.1.2)$$

where we have used the fact that $Var\left(\widetilde{K}(\Pi_{t_0})\right)$ is symmetric. Using (4.1.2), (4.1.1b) can be written as

$$\lambda'\mathbb{E}\left(\widetilde{K}(\Pi_{t_0})\right) - \frac{1}{2}(\alpha_1 - \alpha_2)\lambda' Var\left(\widetilde{K}(\Pi_{t_0})\right)\lambda - \frac{1}{2}\alpha_2(\lambda - \overline{\lambda}_0)' Var\left(\widetilde{K}(\Pi_{t_0})\right)(\lambda - \overline{\lambda}_0) + C \qquad (4.1.3)$$

If we assume $(\alpha_1 - \alpha_2) = \rho_1$ and $\alpha_2 = \rho_2$ then (4.1.3) is the same form of utility function which we are using in our model for institutional investors.

APPENDIX 4.2: DERIVATION OF PRIVATE INVESTORS DEMAND FUNCTION

Key Assumptions

Consider a capital market with S risky assets and one risk-free asset.

Referring to the belief system, we assume the following:

$$\widetilde{\Pi}_{t_1} \sim \mathcal{N}\left(\overline{\Pi}_{t_1}, \Sigma_{\pi\pi}\right)$$

Information Structure

$$\begin{pmatrix} \tilde{\pi}_{t_1} \\ \tilde{Z}_i \\ \tilde{Z}_p \end{pmatrix} \sim N\left[\begin{pmatrix} \overline{\pi}_{t_1} \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} \Sigma_{\pi\pi} & \Sigma_{\pi z_i} & \Sigma_{\pi z_p} \\ \cdot & \Sigma_{z_i z_i} & 0 \\ \cdot & 0 & \Sigma_{z_p z_p} \end{bmatrix}\right]$$

Assuming, that the common signal $\tilde{Z}_p$ is observed by an private investor (before portfolio formation) and the prices of risky assets in period one are jointly normally distributed (note, that as specified in equation (4.7), $\tilde{Z}_p$ is independent of $\tilde{Z}_i$):

Matrix solution

We assume that the investor decides to hold λ_p' risky assets and $W_{t_0,p} - \lambda_p' \Pi_{t_0}$ in a risk-free asset. A private investor's stochastic wealth at time 1 is given by the following equation:

$$\widetilde{W}_{t_1,p} = r_f\left(W_{t_0,p} - \lambda_p' \Pi_{t_0}\right) + \lambda_p' \widetilde{\Pi}_{t_1} \qquad \text{(4.2.1a)}$$

Using that $\widetilde{K}\left(\Pi_{t_0}\right) \equiv \widetilde{\Pi}_{t_1} - \Pi_{t_0} r_f$, we have

$$\widetilde{W}_{t_1,p} = r_f W_{t_0,p} + \lambda_p' \widetilde{K}\left(\Pi_{t_0}\right) \qquad \text{(4.2.1b)}$$

The Utility Maximisation Problem

$$\text{Max } \mathbb{E}\left(\widetilde{W}_{t_1,p}\right) - \frac{\rho_{1,p}}{2} \text{Var}\left(\widetilde{W}_{t_1,p}\right) \qquad \text{(4.2.2)}$$

$$\text{s.t } \widetilde{W}_{t_1,p} = r_f W_{t_0,p} + \lambda_p' \widetilde{K}\left(\Pi_{t_0}\right)$$

Differentiating (4.3.2) w.r.t to λ_p' results

$$\mathbb{E}\left(\widetilde{K}\right) = Var\left(\widetilde{K}\left(\Pi_{t_0}\right)\right) \lambda_p \rho_{1,p} \qquad \text{(4.2.3)}$$

Solving for λ_p yields

$$\lambda_p = \left(Var\left(\widetilde{K}\left(\Pi_{t_0}\right)\right)\right)^{-1} \mathbb{E}\left(\widetilde{K}\left(\Pi_{t_0}\right)\right)/\rho_{1,p} \tag{4.2.4}$$

Using standard multi-normal analysis (Morrison (1976)) gives the following result:

$$\mathbb{E}\left(\widetilde{\Pi}_{t_1}|Z_p\right) = \overline{\Pi}_{t_1} + \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1} Z_p$$

$$\mathrm{Var}\left(\widetilde{\Pi}_{t_1}|Z_p\right) = \Sigma_{\pi\pi} - \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1}\Sigma_{z_p\pi}$$

The results are equivalent to the following:

$$\mathbb{E}\left(\widetilde{K}|Z_p\right) = \overline{K} + \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1} Z_p \tag{4.2.5}$$

$$\mathrm{Var}\left(\widetilde{K}|Z_p\right) = \Sigma_{\pi\pi} - \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1}\Sigma_{z_p\pi} \tag{4.2.6}$$

where $\overline{K}\left(\Pi_{t_0}\right) = \overline{\Pi}_{t_1} - (1 + r_f)\Pi_{t_0}$

Replacing the above expressions in 4.2.4 yields the conditional demand for risky assets based on private investor's belief

$$\lambda_{p,z_p} = \frac{\left(\Sigma_{\pi\pi} - \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1}\Sigma_{z_p\pi}\right)^{-1}\left(\overline{K} + \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1} Z_p\right)}{\rho_{1,p}} \tag{4.2.7}$$

APPENDIX 4.3 DERIVATION OF INSTITUTIONAL INVESTORS DEMAND FUNCTION

Assuming that the investor decides to hold λ_i' risky assets and $\mathrm{W}_{t_0,i} - \lambda_i'\Pi_{t_0}$ in risk-free asset

An institutional investor's stochastic wealth at time t_1 is given by the following equation:

An institutional investor's wealth at period 1 is given by:

$$\widetilde{W}_{t_1,i} = r_f\left(W_{t_0,i} - \lambda_i'\Pi_{t_0}\right) + \lambda_i'\widetilde{\Pi}_{t_1} \tag{4.3.1a}$$

$$\overline{W}_{t_0} = r_f\left(W_{t_0,i} - \lambda_i'\Pi_{t_0}\right) + \overline{\lambda}_0'\widetilde{\Pi}_{t_1} \tag{4.3.1b}$$

Using the definition of $\widetilde{K}\left(\Pi_{t_0}\right)$

$$\widetilde{W}_{t_1,i} = r_f W_{t_0,i} + \lambda_i'\widetilde{K}\left(\Pi_{t_0}\right) \tag{4.3.1c}$$

$$\overline{W}_{t_0} = r_f W_{t_0,i} + \overline{\lambda}_0'\widetilde{K}\left(\Pi_{t_0}\right) \tag{4.3.1d}$$

The Utility Maximization Problem

$$\mathrm{V}_j = \mathbb{E}\left(\widetilde{W}_{t_1,i}\right) - \tfrac{\rho_{1,i}}{2}\mathrm{Var}\left(\widetilde{W}_{t_1,i}\right) - \tfrac{\rho_{2,i}}{2}\mathrm{Var}\left(\widetilde{W}_{t_1,i} - \overline{W}_{t_0}\right) \tag{4.3.2}$$

Replacing the (4.3.1c) and (4.3.1d) in (4.3.2) gives

$$\begin{aligned}\mathrm{V}_j = \mathbb{E}\left(C_i + \lambda_i'\widetilde{K}\left(\Pi_{t_0}\right)\right) - \frac{\rho_{1,i}}{2}\mathrm{Var}\left(C_i + \lambda_i'\widetilde{K}\left(\Pi_{t_0}\right)\right)\\ - \frac{\rho_{2,i}}{2}\mathrm{Var}\left(\left(C_i + \lambda_i'\widetilde{K}\left(\Pi_{t_0}\right)\right) - \left(C_i + \overline{\lambda}_0'\widetilde{K}\left(\Pi_{t_0}\right)\right)\right)\end{aligned} \tag{4.3.3}$$

where $C_i = r_f W_{t_0,i}$

Differentiating (4.3.3) w.r.t λ_i' and assuming $\overline{\lambda}_0'$ as exogenous gives the following F.O.C

$$\mathbb{E}\left(\widetilde{K}\left(\Pi_{t_0}\right)\right) - \lambda_i'\rho_{1,i}\,\mathrm{Var}\left(\widetilde{K}\left(\Pi_{t_0}\right)\right) - \rho_{2,i}\left(\lambda_i - \overline{\lambda}_0\right)'\,\mathrm{Var}\left(\widetilde{K}\left(\Pi_{t_0}\right)\right) = 0$$

Solving for λ_i:

$$\lambda_i = \left(\mathrm{Var}\left(\widetilde{K}\left(\Pi_{t_0}\right)\right)^{-1}\mathbb{E}\left(\widetilde{K}\left(\Pi_{t_0}\right)\right) + \rho_{2,i}\overline{\lambda}_0\right)/\left(\rho_{1,i} + \rho_{2,i}\right) \tag{4.3.4}$$

Information Structure

Using standard multi-normal analysis (Morrison (1976)) gives the following result:

$$\mathbb{E}\left(\widetilde{\Pi}_{t_1}|Z_i\right) = \overline{\Pi}_{t_1} + \Sigma_{\pi z_i}\Sigma_{z_i z_i}^{-1}Z_i \tag{4.3.5a}$$

$$\mathrm{Var}\left(\widetilde{\Pi}_{t_1}|Z_i\right) = \Sigma_{\pi\pi} - \Sigma_{\pi z_i}\Sigma_{z_i z_i}^{-1}\Sigma_{z_i\pi} \tag{4.3.6a}$$

This is equivalent to:

$$\mathbb{E}\left(\widetilde{K}|Z_i\right) = \overline{K} + \Sigma_{\pi z_i}\Sigma_{z_i z_i}^{-1}Z_i \tag{4.3.5b}$$

$$\mathrm{Var}\left(\widetilde{K}|Z_i\right) = \Sigma_{\pi\pi} - \Sigma_{\pi z_i}\Sigma_{z_i z_i}^{-1}\Sigma_{z_i\pi} \tag{4.3.6b}$$

Where $\overline{K} = \overline{K}\left(\Pi_{t_0}\right) = \overline{\Pi}_{t_1} - (1 + r_f\,)\,\Pi_{t_0}$

Substituting (4.3.5b) and (4.3.6b) in (4.3.4) yields the conditional demand function by institutional investors for risky assets

$$\lambda_{i,z_i} = \frac{(\left(\Sigma_{\pi\pi} - \Sigma_{\pi z_i}\Sigma_{z_i z_i}^{-1}\Sigma_{z_i\pi}\right)^{-1}\left(\overline{K} + \Sigma_{\pi z_i}\Sigma_{z_i z_i}^{-1}Z_i\right) + \rho_{2,i}\overline{\lambda}_0)}{(\rho_{1,i} + \rho_{2,i})} \tag{4.3.7}$$

APPENDIX 4.4 EQUILIBRIUM SOLUTION FOR PRICE OF RISKY ASSETS IN PERIOD 0 OR Π_{t_0}

Demand vector for kth private investor:

$$\lambda_{p,Z_p} = \frac{\left(\Sigma_{\pi\pi} - \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1}\Sigma_{z_p\pi}\right)^{-1}\left(\overline{K} + \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1}Z_p\right)}{\rho_{1,p}} \tag{4.4.1}$$

Demand vector for the *j*th institutional investor:

$$\lambda_{i,Z_i} = \frac{[\left(\Sigma_{\pi\pi} - \Sigma_{\pi z_i}\Sigma_{z_i z_i}^{-1}\Sigma_{z_i\pi}\right)^{-1}\left(\overline{K} + \Sigma_{\pi z_i}\Sigma_{z_i z_i}^{-1}Z_i\right) + \rho_{2,i}\overline{\lambda}_0]}{(\rho_{1,i} + \rho_{2,i})} \tag{4.4.2}$$

Let:

$$\left(\Sigma_{\pi\pi} - \Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1}\Sigma_{z_p\pi}\right)^{-1} = A_p^{-1}$$

$$\left(\Sigma_{\pi\pi} - \Sigma_{\pi z_i}\Sigma_{z_i z_i}^{-1}\Sigma_{z_i\pi}\right)^{-1} = A_i^{-1}$$

$$\Sigma_{\pi z_p}\Sigma_{z_p z_p}^{-1} = \Delta_p$$

$$\Sigma_{\pi z_i}\Sigma_{z_i z_i}^{-1} = \Delta_i$$

Using the above notation, aggregate demand for private investors is given by:

$$\textstyle\sum_{p=1}^{P} \lambda_{p,Z_p} = \left[A_p^{-1}\left[\overline{K} + \Delta_p Z_p\right]\right] / \sum_{k=1}^{P} 1/\rho_{1,k} \tag{4.4.3}$$

and aggregate demand for institutional investors is given by:

$$\textstyle\sum_{i=1}^{I} \lambda_{i,Z_i} = \left[\left[A_I^{-1}\left[\overline{K} + \Delta_i Z_I\right]\right] \sum_{i=1}^{I} 1/\left(\rho_{1,i} + \rho_{2,i}\right)\right] + \sum_{i=1}^{I} \rho_{2,i}\,\overline{\lambda}_0/\left(\rho_{1,i} + \rho_{2,i}\right) \tag{4.4.4}$$

In equilibrium

The total supply of risky assets equals the total demand. Assuming Q as the supply *S x 1* vector of risky assets, then the market clearing condition is given by the following equation:

$$\textstyle\sum_{i=1}^{I} \lambda_{i,Z_i} + \sum_{p=1}^{P} \lambda_{p,Z_p} = Q \tag{4.4.5}$$

Let

$$M = r_f \left[A_I^{-1} \sum_{i=1}^{I} \frac{1}{\rho_{1,i} + \rho_{2,i}} + A_p^{-1} \sum_{p=1}^{P} \frac{1}{\rho_{1,p}}\right]$$

$$B_p = A_p^{-1}\Delta_p Z_p \sum_{p=1}^{P} 1/\rho_{1,p} + A_p^{-1}\overline{\Pi}_{t_1} \sum_{p=1}^{P} 1/\rho_{1,p}$$

$$B_i = \left[\overline{\lambda}_0 \sum_{j=1}^{N} \frac{\rho_{2,j}}{\rho_{1,j}+\rho_{2,j}}\right] + \left[\mathrm{A}_I^{-1}\Delta_i Z_i \sum_{i=1}^{I} \frac{1}{\rho_{1,i}+\rho_{2,i}}\right] + \left[A_I^{-1}\overline{\Pi}_{t_1} \sum_{i=1}^{I} \frac{1}{\rho_{1,i}+\rho_{2,i}}\right]$$

Solving for Π_{t_0} using (4.4.5) gives

$$\Pi_{t_0} = M^{-1}\left[B_p + B_i - Q\right] \tag{4.4.6}$$

APPENDIX 4.5 PROOF THAT M^{-1} IS POSITIVE DEFINITE MATRIX

By construction A_p and A_i are non-singular, positive definite matrices. By proposition 2 below, their weighted sum will be positive definite as long as the weights are non-negative (this requires that $\alpha_1 > \alpha_2$)

General definition of positive definite matrix

If x is an arbitrary *n x 1* vector, if $x'Cx > 0$,and x is not equal to the zero vector, then C is a positive definite matrix.

Definition of Symmetric Matrix (*n x n*)

If $C' = C$ then C is a symmetric matrix.

Proposition 1

If C is symmetric positive definite matrix so is C^{-1}.

See Rau et al. (p. 240) for proof.

Proposition 2

If Y and C are symmetric positive definite matrices, so is $C + Y$.

Proof

Assume $C + Y$ is not positive definite then $x'(C + Y)x \leq 0$ for some *n x 1* vector x.

This implies that $x'(C + Y)x = x'Cx + x'Yx \leq 0$:

However, by assumption C and Y are positive definite which implies $x'Cx$ and $x'Yx$ are individually positive: Therefore $x'(C + Y)x \leq 0$ cannot hold. Thus $C + Y$ has to be a positive definite matrix. Proposition 1 & 2 together imply that M^{-1} is positive definite matrix, since M^{-1} is a positively weighted sum of positive definite matrices (see above). Thus the demand curve for risky assets is downward sloping.

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