Regulated competition under increasing returns to scale

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Abstract

In the classical models of regulation economics, a mechanism that secures truthful revelation involves paying a subsidy to the firm. In this paper, we investigate whether it is possible to create a regulatory mechanism under a no-subsidy constraint that induces the firm to report its private information truthfully. We consider a number of firms operating under regulated competition and with increasing returns to scale technology. It is shown that in equilibrium, each firm chooses to report truthfully without receiving any subsidy. The use of competition may give rise to an efficiency loss due to the increasing returns to scale. However, we show that our mechanism may still be better, from a social welfare point of view, than the case of monopoly regulation that involves no subsidy.

1. Introduction

The interest in regulation of markets characterized by either monopoly or oligopoly has a long history, see e.g. the survey by Armstrong and Sappington (2007), and the privatization of formerly government owned and managed industries, mainly in the field of public utilities, has promoted an increased concern about regulation, often in a context of market failures in the form of increasing returns to scale or asymmetric information.

In the present paper we consider such a case, where there are increasing returns to scale in production. In the context of public utilities, this may occur for companies providing services to the consumer (such as electricity or natural gas) which they either produce themselves or acquire in a market where large buyers obtain considerable rebates. As is well-known, optimal allocation in a market with increasing returns to scale occurs at a level of output where price equals marginal cost, so that pricing according to marginal cost (see e.g. Joskow (1976)) can be sustained only if the producers are paid a subsidy.

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Throughout this paper, we assume that subsidization of producers is not possible, so that the welfare optimum cannot be achieved. We shall not discuss reasons for or justification of this assumption, which is taken as given here as in many other contexts, such as e.g. the theory of Ramsey pricing (cf. Hagen and Sheshinski (1986)). Even so, regulation serves the purpose of achieving an allocation which is welfare superior to monopoly or to an oligopolistic equilibrium for the case with more than one firm. In particular, we shall compare regulation of a single firm, a monopoly, with a regulated competition between two firms, under the conditions of increasing returns to scale and no subsidization. This comparison is not altogether trivial, since on the one hand, the increasing returns to scale tend to favor production in a single plant, but on the other hand the competition between two firms may lead to lower prices and larger output.

As mentioned above, we shall assume that there is asymmetric information about production cost which is private information for the firm in the case of monopoly and for each firm if there are several firms. Thus the regulator must try to overcome the informational advantage of the firm (or firms). In the case of a single firm, this takes us to the theory of regulated monopoly, now under conditions of increasing returns to scale.

There is a considerable literature on regulation of monopolies under asymmetric information. In Loeb and Magat (1979), subsidies are used as incentives to surpass the information asymmetry, and Baron and Myerson (1982) present a mechanism that induces the regulated monopoly to report its cost truthfully. We shall use the ideas of Baron and Myerson in our treatment of regulated monopoly, but since subsidization is not allowed, the regulation which emerges cannot implement a welfare optimum, and truthful revelation is obtained by allowing the firm a profit which is no less than what could be achieved by misinformation of the regulator. The loss of welfare due to smaller output and to the deliberate closing down of production when cost is too high, may be considerable.

As an alternative to the regulated monopoly, we then consider a case with two firms both engaged in production, a regulated duopoly. Since we have increasing returns to scale, a marginal cost equilibrium, where price equals marginal cost in each of the two firms, would entail negative profits in both firms, and negative profits in at least one firm if the regulator is free to redistribute profits between firms, thus, we cannot achieve a welfare optimum in a regulated duopoly.[1] Accepting this, the regulation must come so close to the welfare maximum as possible without subsidies, meaning that market revenue should equal total cost of production, possibly with an acceptable profit margin. We consider one such regulation, where the firms are receiving regulated incomes which allow for transfers of market profits from one firm to another but no overall subsidization. However, the regulation depends on the level of cost of the two firms, which is private information, and it is assumed that firms

[1]The result of Gradstein (1995) about the possibility of achieving a welfare optimum in regulated oligopoly does not apply when there are increasing returns to scale, since it uses only the first order conditions for profit maximization, not taking into account the nonnegativity condition.

send messages about their true cost. Having received the messages, the regulator proceeds according to the largest of the two messages, but using regulated incomes which favor the firm with the smallest message. It is shown that truthful revelation is an equilibrium, however, the firm with the lowest cost can maintain an informational rent since the regulator acts as if both firms have the higher cost.

The two approaches to regulation of a market with increasing returns to scale and no subsidies are compared using an assessment of their welfare loss (relative the welfare optimum). In regulated monopoly, this welfare loss is caused by the cases of either producing nothing (when cost is high) or producing too little (in order to ensure truthful revelation of cost). In the regulated duopoly, there is also a loss from producing too small an output, even if much smaller than under monopoly, but to this must be added the loss arising from producing in two firms instead of in a single firm. The final result depends on the parameter values, so that the superior market structure must be determined from the circumstances in each practical application.

As mentioned already, the literature on regulation of monopolistic and oligopolistic markets is quite considerable, even if only few contributions deal with our particular combination of increasing returns to scale and asymmetric information about cost. Problems of asymmetric information models have been studied in a variety of contexts, see e.g. Lewis and Sappington (1988), Laffont and Tirole (1993) and Laffont and Martimort (2002). Also the problems of regulating an oligopolistic market, and in particular a duopoly, has been considered in the literature. Auriol and Laffont (1992) consider a duopoly where the efficiency parameters are private information, and the regulation uses yardstick competition in order to lower the informational rents. Other contributions are Dalen (1998) and Tangerås (2009), showing that asymmetric information involves informational rents or costs that have to be accepted by the regulator.

Regulated oligopoly was treated by Wolinsky (1997) in the context of spacial competition, The idea of selecting from potential producers is exploited by Wang (2000) in a model of regulating an oligopoly with unknown cost, where only firms reporting low cost are allowed to stay in the market. In Evrenk and Zenginobuz (2010), the duopolists are regulated through a revenue contest, whereby the firm with lower revenue must transfer some of its profits to the other firm. The work of Sengupta and Tauman (2011) deals with an oligopolistic market with increasing returns to scale technology, where the incentive mechanism is based on a bidding contest, the outcome of which is a contract with subsidies to only one firm, while the other firms exit the market. An alternative approach to regulation was proposed by Koray and Sertel (1988), allowing firms to state their cost under the obligation to produce in accordance with their statements.

As it can be seen, the approach followed in the present paper is one which builds on an established tradition, even if it differs in combining the three issues of increased return to scale, asymmetric information, and no subsidies. However, the main results of the paper are those dealing with the comparison of alternative market structures, showing that under suitable conditions on parameters regulated duopoly may be superior to monopoly, even with increasing returns to scale.

This paper is organized as follows. In Section 2 we set up the basic model with its underlying general assumptions. In the following Section 3, we treat the model of regulated monopoly, following the ideas in Baron and Myerson (1982) but with the additional conditions of increasing returns to scale and no subsidization; the model is illustrated by a numerical example showing that no output is produced when cost is too high. The main result of this section is a lower bound for the welfare loss occurring under regulated monopoly. In Section 4, we turn to the case of regulated duopoly, presenting first the rules for income formation when the cost parameter has been determined, and then the method for determining this parameter; for comparison, the same numerical example as in the previous section is used to illustrate the results of regulated duopoly. The welfare assessment is carried on in Section 5, where we present an upper bound for the welfare loss, which may then be used together with previous results to compare regulated monopoly and duopoly. This comparison is reconsidered in Section 6 under the assumption that the monopolist is selected as one of two potential producers according to lowest reported cost. Finally, Section 7 contains some concluding comments. Proofs of lemmas and propositions are collected in an appendix to the paper.

2. The model

We consider an industry with a technology admitting increasing returns to scale. The number of firms operating in this industry all have access to the same technology, defined by a cost function $\theta C(q)$, depending on the level of output q and a multiplicative productivity parameter θ. The function C is twice differentiable and concave, meaning that there are nondecreasing returns to scale. The productivity parameter θ of the firm, also to be mentioned as its type, is observed only by the firm itself. It is assumed to be randomly distributed in an interval $\Theta = \left[\underline{\theta}, \overline{\theta}\right] \subset \mathbb{R}_+$, with probability distribution function F.

The demand for the output of the industry is described by an inverse demand function $P(q)$, where $P(q)$ is the price at which the output q can be sold in the market. We assume that $P(q)$ is non-increasing and convex in q, and that for each $\theta \in \Theta$, there is a unique q such that $P(q)q = \theta C(q)$. All of these assumptions are satisfied in standard models of industrial organization.

Consumer satisfaction at production q is given by

$$V(q) = \int_0^q P(s)\, ds, \tag{1}$$

and consumer surplus is $S(p) = V(q) - P(q)q$. The net welfare given that the firm has type θ is

$$W(q, \theta) = V(q) - \theta C(q). \tag{2}$$

For each $\theta \in \Theta$, $q^*(\theta)$ denotes the production which maximizes $W(q, \theta)$. The maximal expected welfare is then

$$U^* = \int_\Theta W(q^*(\theta), \theta)\, dF(\theta).$$

Due to the increasing returns to scale, we have that $P(q^*(\theta))q^*(\theta) < C(q^*(\theta))$, so that revenue does not cover costs of production. If there is only one producer, a regulation which results in the welfare maximizing production is chosen, meaning that subsidies are needed if the welfare maximizing production is to be sustained.

In the sequel, two different ways of regulating this market will be compared, one where only a single firm is operating, being a regulated monopoly, and another one with two firms in the market, operating under regulated competition. We are considering only regulation that does not involve subsidization, the welfare maximizing production cannot be achieved, and a comparison of the two regimes of regulation must be based on their expected efficiency losses.

3. Regulated monopoly

Regulation of a monopoly has been investigated, at least for the case of non-increasing returns to scale. Here we follow the approach of Baron and Myerson (1982), but as we consider regulation under a no-subsidy constraint, their model is adapted to the purpose.

In a regulated monopoly with no subsidies, the regulator chooses a policy, that is a triple (δ, q, r) of functions of θ, where

(a) $\delta(\theta)$ is the probability that the firm is allowed to market a nonzero output,

(b) $q(\theta)$ is the output which the firm should produce, and

(c) $r(\theta)$ is the revenue assigned to the producer.

The firm announces a (productivity) type θ, which may or may not be the true value.

The objective of the regulator is assumed to be maximization of expected net welfare

$$U(\delta, q, r) = \int_\Theta W(q(\theta), \theta)\delta(\theta)\, dF(\theta) \tag{3}$$

subject to the no-subsidy condition

$$r(\theta) \le P(q(\theta))q(\theta). \tag{4}$$

Since θ cannot be observed by the regulator, the mechanism must be incentive compatible.

For each pair (θ, θ) let

$$\pi(\hat{\theta}, \theta) = \delta(\hat{\theta}) \left[r(\hat{\theta}) - \theta C(q(\hat{\theta})) \right],$$

be the net income of the producer of type θ who states the type $\hat{\theta}$, given the regulatory policy (δ, q, r). The incentive compatibility constraint is then

$$\pi(\theta, \theta) \geq \pi(\hat{\theta}, \theta), \quad \text{all } \theta, \hat{\theta} \in \Theta. \tag{5}$$

Also, the mechanism is voluntary in the sense that it satisfies the participation constraint

$$r(\theta) \geq \theta C(q(\theta)), \ \text{all } \theta \in \Theta. \tag{6}$$

It is convenient to reformulate the participation and incentive compatibility constraints using the following result, where $\pi(\theta) = \pi(\theta, \theta)$ denotes the net income of the producer announcing true type.

Lemma 1 *Let (δ, q, r) be a regulatory policy which satisfies the constraints* (4)-(6). *Then*

$$\pi(\theta) = \pi(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} \delta(t) C(q(t))\, dt$$

and

$$\delta(\theta) \left[P(q(\theta)) q(\theta) - \theta C(q(\theta)) \right] \geq \pi(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} \delta(t) C(q(t))\, dt \geq 0, \ \text{all } \theta \in \Theta. \tag{7}$$

The proof of Lemma 1 (and of the propositions to follow) can be found in the appendix. The inequality in (7) is useful when assessing the welfare loss arising from a monopoly, regulated under the no-subsidy constraint: For any given θ the output $q(\theta)$ falls short of maximizing net surplus $V(q) - \theta C(q)$, since $P(q)q < \theta C(q)$ at the maximum of this surplus due to increasing returns to scale. Moreover, the incentive constraint shows that $P(q(\theta))q(\theta)$ typically must exceed $\theta C(q(\theta))$ by a considerable amount, needed to ensure the truthful revelation of type.

A regulatory policy is optimal if it maximizes (3) subject to the constraints (4)-(6). Some properties of optimal regulatory policies are listed below.

Lemma 2 *Let (δ, q, r) be an optimal regulatory policy. Then*

(i) $\delta(\theta) \in \{0, 1\}$ *for all* $\theta \in \Theta$,

(ii) there is $\theta^0 \in \Theta$, *such that* $\{\theta \mid \delta(\theta) = 1\} = [\underline{\theta}, \theta^0]$,

(iii) for each θ *with* $\delta(\theta) = 1$, $q(\theta) \in [q^0, q^1]$, *where* q^0 *is defined by* $P(q^0)q^0 = \theta^0 C(q^0)$ *and* $q^1 = \sup \{q \mid P(q)q - \theta C(q) \geq (\theta^0 - \theta) C(q^0), \theta \leq \theta^0\}$.

As shown by the lemma, in an optimal regulatory policy either production is not carried out at all or the firm will be asked to produce in the interval $[q^0, q^1]$ defined in part (iii). The specific value θ^0 for the largest admissible type and the productions q^0 and q^1 will be used repeatedly in the sequel.

As mentioned above, even the optimal regulatory policy will result in a welfare loss as compared to U^*. The size of this welfare loss can be assessed using the results stated in the two preceding lemmas.

Proposition 1 *Let* (δ, q, r) *be a regulatory policy which maximizes welfare subject to the constraints* (4)-(6). *Then*

$$P(q(\theta)) - \theta C'(q(\theta)) \geq \frac{\theta^0 - \theta}{q^1} C(q^0)$$

for each θ *with* $\delta(\theta) = 1$, *and the welfare loss associated with this policy satisfies the inequality*

$$U^* - U(\delta, q, r) \geq \left(\frac{C(q^0)}{q^1}\right)^2 \frac{1}{2|P'(q^0))|} \int_{\underline{\theta}}^{\theta^0} (\theta^0 - \theta)^2 \, dF(\theta). \tag{8}$$

Example. Suppose that the demand relationship is

$$P(q) = 3.4 - 0.2q,$$

and that the cost function C is given by

$$C(q) = 2q + \ln(5 + q) - \ln 5,$$

which is increasing and concave. We let the interval of types be $\Theta = [0.5, 1.5]$ and take θ to be uniformly distributed in this interval.

In the regulated monopoly, truthful reporting of type implies that if there is nonzero production at some θ, then the monopolist must be at least as well off at any $\theta' < \theta$ as if θ was reported, cf. Lemma 2. In Table 1, this level of output is shown for alternative values of θ. The third column shows expected welfare given that the type is $\leq \theta$. It is found by adding the welfare at θ in the second column the average difference between cost at θ and actual cost (and then dividing by the length of the interval of type). It is seen that it achieves its maximum at $\theta = 1.4$, so that it is better for society not to produce when $\theta > 1.4$, and average welfare is then 8.51.

For comparison, the optimal production choices under unregulated monopoly have been inserted for the selected values of θ. Output and welfare will be considerably smaller than what is achieved under the regulation. Approximating average welfare using the means of the intervals in the table gives a value of 7.65, which should be compared to the 8.51 achieved under regulation. Thus, regulating a monopoly, even with no subsidization, makes sense from a welfare point of view.

Table 1. Production and welfare under regulated and unregulated monopoly

	Regulated			Unregulated	
θ	Break-even output at θ	Welfare at θ	Average welfare over all $\theta' \leq \theta$	$q_m(\theta)$	Welfare (monopoly)
0.5	13.51	18.25	0	6.77	13.60
0.75	11.76	13.78	3.96	5.90	10.24
1.0	9.95	9.90	6.70	5.00	7.30
1.25	8.13	6.61	8.20	4.10	4.84
1.4	7.05	4.93	8.51	3.58	3.62
1.5	6.28	3.94	8.43	2.15	2.84

It is not surprising that an incentive scheme under which the producer must be granted a share in the profits sufficiently big to induce truthful revelation of preferences will result in a welfare loss when compared to marginal cost pricing which even without incentive schemes is sustainable only with subsidies paid to the firm. The point is not so much that there is a welfare loss; rather we want to compare this welfare loss with what obtains if there is some regulated competition in the market. In the next section, we consider one such situation.

4. Regulated duopoly

In this section, we consider a method of regulating the market which differs markedly from the case treated above, since the regulator allows for more than one firm operating. In the context of increasing returns to scale, this entails an efficiency loss; on the other hand, the absence of subsidies will in some situations make the duopoly solution preferable from a social welfare point of view. As before, the technology of the firms is described by their type $\theta \in \Theta$. For the time being, we assume that θ is determined at the industry level, so that the two firms have the same θ, which however is not observable to the regulator. At a later stage, we consider the case of firm-specific productivity parameters.

Since there are increasing returns to scale, the regulation must counteract the inherent tendency towards concentrating production in a single unit, so that it must treat the two firms differently as long as one firm produces more than another. Let q_1 and q_2 be the production choices of the two firms, with $q_1 \geq q_2$. Then the market revenue is $P(q_1 + q_2)(q_1 + q_2)$ and total cost is $\theta C(q_1) + \theta C(q_2)$; the regulator is constrained by an overall no-subsidy constraint

$$P(q_1 + q_2)(q_1 + q_2) - [\theta C(q_1) + \theta C(q_2)] \geq 0, \tag{9}$$

meaning that profits may be transferred from one producer to another as long as the overall non-subsidy constraint (9) is not violated. We let $\mathcal{D}$ denote the set of all (θ, q_1, q_2) with $\theta \in \Theta$ and $q_1 \geq q_2$ for which (9) is satisfied.

In order to induce firms to choose productions which are as large as possible, the regulator remunerates the firms according to cost plus a percentage markup, as long as this is compatible with (9). Let $\lambda > 0$ be a percentage markup on cost in the two firms, kept fixed throughout this section. If $(\theta, q_1, q_2) \in \mathcal{D}$, then the regulated income of firm 1 (the firm with the largest output) is determined as

$$r_1(q_1, q_2, \theta) = \min\left\{(1+\lambda)\theta C(q_1), P(q_1+q_2)q_1\right\}, \tag{10}$$

so that the firm receives a reimbursement of its cost plus the proportional allowance for profits, as long as this does not exceed the revenue obtained from selling its output. The smaller firm 2 is regulated in a slightly different way, namely by

$$r_2(q_1, q_2, \theta) = \begin{cases} (1+\lambda)\theta C(q_i) - c & \text{if } P(q_1+q_2)(q_1+q_2) \geq (1+\lambda)\theta\left[C(q_1)+C(q_2)\right], \\ P(q_1+q_2)q_2 - c & \text{otherwise,} \end{cases} \tag{11}$$

where $c \geq 0$ is a constant which will be specified in the sequel. Thus, firm 2 is reimbursed according to its cost plus allowed profits as long as this remuneration is consistent with the principle of no subsidization of the market. This regulation gives a certain advantage to the smaller firm, allowing for expansion of its production if the large producer gets a profit which exceeds than λ times its cost.

When regulated according to (10)-(11), the net income of firm i is

$$\pi_i(q_1, q_2, \theta) = r_i(q_1, q_2, \theta) - \theta C(q_i), i = 1, 2. \tag{12}$$

The pair (q_1^0, q_2^0) of productions is an equilibrium for the regulation if $q_1^0 \geq q_2^0 > 0$ (so that both firms are producing nonzero output), and no firm has a profitable deviation,

$$\pi_1(q_1^0, q_2^0, \theta) \geq \pi_1(q_1, q_2^0, \theta), \text{ all } q_1 \geq 0, \tag{13}$$

$$\pi_2(q_1^0, q_2^0, \theta) \geq \pi_2(q_1^0, q_2, \theta), \text{ all } q_2 \geq 0, \tag{14}$$

so that (q_1^0, q_2^0) is a Nash equilibrium in the game $\Gamma_d(\theta)$ with strategies $q_i \geq 0$, $i = 1, 2$, and payoffs given by (12). Thus, for the purposes of regulation, a particular value of θ has been selected. We are particularly interested in the case where this selected value is the true type of firm 2 but not necessarily of firm 1.

Proposition 2 *Let $\theta \in \Theta$. Then the following hold:*

(i) If both firms have type θ, then there is a unique equilibrium (q_1^0, q_2^0), it satisfies q_1^0 =

$q_2^0 = q_d$ and

$$P(2q_d^0)q_d^0 = (1+\lambda)\theta C(q_d^0).$$

(ii) If firm 1 has type $\theta' \leq \theta$, while firm 2 has type θ, then (q_d^0, q_d^0) is also a Nash equilibrium in the game where firm 2 has payoff given by

$$r_1(q_1, q_2, \theta) - \theta' C(q_1).$$

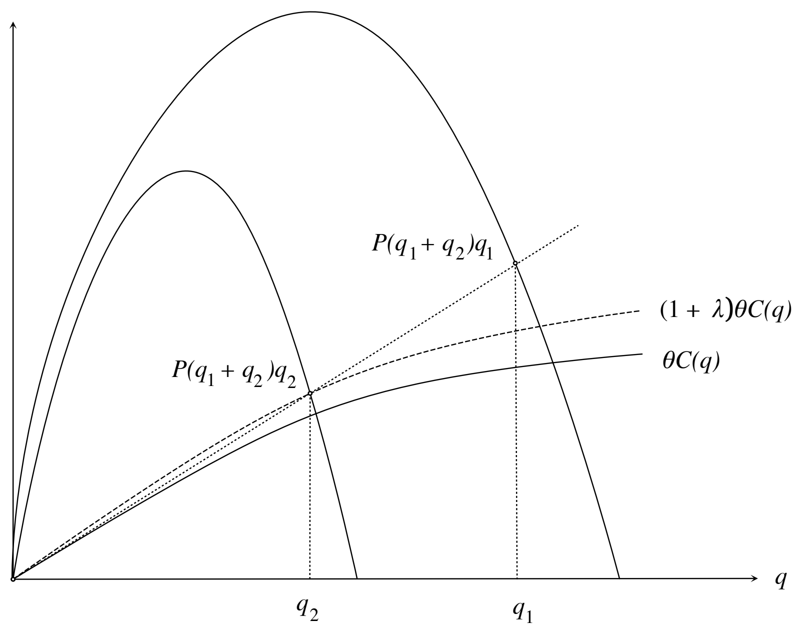

Fig. 1

The workings of the regulation are in Fig.1. The situation depicted is *not* an equilibrium; while firm 1 cannot do better given the choice of production q_2 of the other firm, there is room for improvement for firm 2. Indeed, production can be expanded as long as the profits of the first firm exceeds λ times its cost, as it does in the case depicted. It is easily seen that firm 2 will retain this inducement to expand production until its output equals that of the other firm.

In the sequel, we let $q_d(\theta)$ denote the common output in the symmetric equilibrium at $\theta \in \Theta$. We notice that $q_d(\theta)$ is a continuous function of the type parameter. Furthermore, it is seen that the equilibrium payoffs

$$\pi_i(\theta) = \pi_i(q_d(\theta), q_d(\theta), \theta), i = 1, 2,$$

are nondecreasing function of θ: When θ gets smaller, output is increased and revenue falls while cost increases.

We now extend the income regulation scheme to a mechanism for determining production in the case where types are firm-specific so that the two firms may have different type. In this case, the regulator must select a suitable value of θ. We assume that firms submit messages about their type, after which they are assigned numbers 1 or 2 to the firms according to their stated productivity parameters. The regulation of income is performed according to the largest of the stated values, but with firm 1 receiving a larger income than firm 2. Formally, we define the *two-stage regulated duopoly* as follows: In the first stage, the firms A and B submit types $\hat{\theta}_A$, $\hat{\theta}_B$. The messages submitted induce an ordering of the firms, whereby the number 1 is assigned to the firm having submitted the smallest type (using random selection with equal probability in the case that $\hat{\theta}_A = \hat{\theta}_B$). The firms are then regulated according to the largest of the two submitted types. Letting

$$\theta_1 = \min\{\hat{\theta}_A, \hat{\theta}_B\}, \theta_2 = \max\{\hat{\theta}_A, \hat{\theta}_B\},$$

in the second stage the two firms choose productions subject to the regulations $r_1(q_1, q_2, \theta_2)$ for the first firm and $r_2(q_1, q_2, \theta_2)$ for the second firm, with the constant in (11) defined as

$$c(\theta_1, \theta_2) = \pi_1(\theta_2) - \pi_1(\theta_1)$$

so that in the equilibrium with outputs $(q_d(\theta_2), q_d(\theta_2))$, firm 2 gets the payoff $\pi_1(\theta_1)$ which is smaller than $\pi_1(\theta_2)$.

Since the game is one of incomplete information, with firms observing only their own productivity type but not that of the other firm, the relevant equilibrium concept is that of a Bayesian Nash equilibrium: For each firm, strategies are functions which for each possible type specify the message to be submitted and the production to be chosen given the messages submitted in the first stage. A pair of strategies constitute a Bayesian Nash equilibrium if for each player and each possible type of this player, the strategy chosen maximizes expected payoff (where expectation is taken over the types of the other player), given the strategy of the other player.

Proposition 3 *Truth-telling, $\hat{\theta}_i = \theta_i$ for $i = A, B$, followed by selection of the symmetric duopoly production $q_d(\theta_2)$, for each player, is a Bayesian Nash equilibrium in the two-stage regulated duopoly.*

This result tells us that our mechanism goes some way in the direction of revealing true effectivity parameters; for each firm, stating the true productivity type is as good a message as any other one no matter what the other firm might state. In order to obtain the true types, some inefficiency has to be accepted, since the firm with the largest output is regulated according to a productivity parameter which has a higher value than the true one. This efficiency loss looks much the one which occurred when regulating a monopoly, but there is a crucial difference: In the duopoly, the firm can hide the true productivity parameter only up to the value disclosed by the other firm, whereas the monopolist can hide it more thoroughly, as no other values are revealed.

Example (continued). With the specifications of P, C, Θ and F given in the example of the previous section, we can find the equilibrium outputs $q(\theta)$ of the two firms at each θ solving the equation

$$P(2q(\theta))2q(\theta) - 2\theta C(q(\theta)) = 0.$$

The solutions are shown in Table 2.

Table 2. Production and welfare under regulated and unregulated duopoly

	Regulated			Unregulated	
θ	$q_d(\theta)$	Welfare at $2q_d(\theta)$	Average welfare given $\theta_2 \leq \theta$	Duopoly output	Welfare (duopoly)
0.5	6.72	18.04	18.04	4.50	16.01
0.75	5.85	13.51	17.18	3.91	11.98
1.0	4.90	9.61	14.42	3.30	8.49
1.25	3.98	6.32	12.15	2.69	5.58
1.5	3.05	3.69	10.20	2.06	3.24

As was to be expected, there is a welfare loss connected with producing in two firms instead of a single one when the true parameter θ is known to the regulator, but the regulated duopoly makes up for this loss by allowing for nonzero output for all values of the type parameter. On the other hand, since regulation follows θ_2, the largest of the two type parameters, low values of θ_2 occur only with a small probability, and the average welfare obtained must be computed with respect to the distribution of the largest of to draws. This gives an average of 10.20, which is greater than the 8.51 obtained in the regulated monopoly.

As in the case of monopoly, Table 2 shows the outputs in the symmetric Cournot-Nash equilibrium for each of the given values of the productivity parameter θ, and once more it is seen that welfare is higher when the market is regulated by our mechanism. We have shown only the results obtained when the two firms have the same type, which however give an impression of the general situation. For all values of θ, welfare is lower without regulation than in the regulated case.

A more detailed comparison of the efficiency losses occurring under regulated monopoly and duopoly will be given in the next section.

5. Welfare assessment of regulated duopoly and monopoly

In the previous section, a model of regulated duopoly was developed where competition between firms results in revelation of productivity, at least insofar that the largest of the two productivity parameters can be used to regulate output. However, producing in two firms instead of in a single one entails an efficiency loss, which may offset the gain from competiton. So in order to compare the two ways of organizing the production, a more detailed analysis will be needed.

As a first step, we derive a counterpart of Proposition 1 for the regulated duopoly. For this, we shall use an assumption of *moderately* increasing returns to scale: Although marginal cost is decreasing, it does not decrease too fast. More specifically, we assume that there is a constant $\eta > 0$ such that

$$\left| \frac{C(q)}{q} - C'(q^0) \right| \leq \eta \quad \text{for } q \geq q^0, \tag{15}$$

where q^0 is the quantity defined in Lemma 2.

Under this assumption, we may prove the following result.

Lemma 3 *Assume that increasing returns to scale are moderate in the sense of (15). Let $\theta_1 \leq \theta_2$ be the types of the two firms. If the markup λ is small enough, then*

$$P(2q_d(\theta_2)) - \theta_2 C'(2q_d(\theta_2)) \leq \theta_2 \eta. \tag{16}$$

In the following, we shall apply Lemma 3 to bound the welfare loss arising under regulated duopoly, and therefore we assume from now on that the markup λ is small. The assessment in (16) provides us with an upper bound for the difference between price and marginal cost in the duopoly equilibrium. With suitable assumptions on demand and cost functions, this may be used to show that the duopoly solution comes closer to optimum than does regulated monopoly if the productivity parameter has the value θ_2. However, such a comparison is insufficient for several reasons: It uses the largest of the two parameters stated by the two firms as the relevant parameter in the monopoly case, and in addition it neglects the allocative efficiency caused by producing in two instead of one firm.

A comparison of the two ways of arranging production must be based on averages, since two types are drawn randomly in the duopoly case. We denote by G the distribution of the largest of the two randomly drawn types.

Proposition 4 *Assume that increasing returns to scale are moderate. Then expected welfare loss of the regulated duopoly is smaller than that of regulated monopoly if*

$$\left[\frac{C(q^0)}{q^1} \right]^2 \frac{1}{|P'(q^0)|} \int_{\underline{\theta}}^{\theta^0} (\theta^0 - \theta)^2 \, dF(\theta) > \frac{(3\eta)^2}{|P'(q^1)|} \int_{\Theta} \theta^2 \, dG(\theta), \tag{17}$$

where η is defined by (15) and the quantities q^0, q^1 in Lemma 2.

The constants preceding the integral signs on the two sides of the inequality (17) are of roughly comparable size, in particular if both demand $P(\cdot)$ and marginal cost $C(\cdot)$ are approximately constant. In that case the value of integrals will be decisive for whether inequality in (17) holds or not. Since small values of θ contribute more to the integral on the right-hand side than to that on the left-hand side, we expect regulated duopoly to be superior to regulated monopoly when the small values of θ are more likely to occur than larger values.

It may be added that using the method of regulated duopoly, which selects productions which balance cost and revenue, yields a better result from a welfare point of view than the unregulated monopoly, where output is further reduced so as to support maximal profit, at least as long as the efficiency loss from producing in two firms instead of in a single firm is not too large. Thus, the regulation considered will be justified in terms of improvement of welfare. An additional advantage of regulated duopoly will emerge if consumer welfare is weighted higher than profits of producers. In the regulated monopoly, the producer is left with a considerable profit earned as informational rent. In the regulated duopoly, only one of the firms gets this type of profits, and prices tend to be lower, consumer surplus larger than under monopoly.

6. Welfare assessment with firms competing for monopoly

It might be argued that the assessment which was made in the previous section was inherently unfavorable to the monopoly solution, since basically we compare a two-firm market with another one where only single firm is present. Taking averages over types we introduce an asymmetry, drawing two types at random in one case and only one in the other case.

This may be remedied if in our treatment of regulated monopoly we add another feature, namely a competition between two potential producers for obtaining the right to produce as a regulated monopolist. Formally, there are now two firms A and B, and both submit messages of the form $\hat{\theta} \in \Theta$. Based on the messages submitted, the producing firm is appointed as the firm submitting the smallest message, $\theta_1 = \min\{\hat{\theta}_A, \hat{\theta}_B\}$, and this firm is regulated according to the regulatory policy of section 3, but based on the largest of the two messages, $\theta_2 = \max\{\hat{\theta}_A, \hat{\theta}_B\}$. More specifically, production and regulated revenue for the winning firm are set to

$$\begin{aligned} &(q(\theta_2), r(\theta_2)) \text{ if } \theta_2 \leq \theta^0 \text{ or } \theta_1 > \theta^0 \\ &(q(\theta^0), r(\theta^0)) \text{ if } \theta_1 \leq \theta^0 < \theta_2, \end{aligned} \tag{18}$$

and nothing is produced if $\theta_1 > \theta^0$.

It may be considered as somewhat unrealistic that the regulator, having obtained information about productivities, chooses to proceed as if the true value of θ for the winning firm could be anything below θ^0, which indeed is the principle behind the optimal regulatory policy (δ, q, r). The main advantage of our approach is an equilibrium strategy.

Proposition 5 *In the mechanism for selecting a monopolist according to smallest reported type and regulating selected firm according to (18), stating the true type is a Bayesian Nash equilibrium.*

Due to collection of messages from the potential firms, the regulator obtains some information about true productivity parameters, in particular since the true types are equilibrium messages, as is indeed the case, see below. However, the regulator is committed by the rules of the game to follow the policy (δ, q, r), acting as if the producing firm could conceal its true productivity. Therefore, this mechanism should be considered as a somewhat artificial one, being used here mainly for the purpose of comparison, and it cannot be considered as a candidate for an optimal regulation policy when a single producer is selected from several candidates.

With these reservations we state our last result on comparing monopoly and duopoly under regulation with subject to the no-subsidy constraint. Repeating the steps of the reasoning leading to Proposition 4, taking into consideration that the regulation of the monopolist takes the range of type parameters to be bounded at the largest of the two types, we get an assessment of the following form.

Proposition 6 *Assume that increasing returns to scale are moderate. Then expected welfare loss of the regulated duopoly is smaller than that of regulated monopoly with selection if*

$$\left(\frac{C(q^0)}{q^1}\right)^2 \frac{1}{|P'(q^0)|} \int_{\underline{\theta}}^{\theta^0} \left\{ \int_{\underline{\theta}}^{\theta_2} \frac{1}{F(\theta_2)} (\theta^0 - \theta_1)^2 \, dF(\theta_1) \right\} dF(\theta_2) > \frac{(3\eta)^2}{|P'(q^1)|} \int_{\Theta} \theta^2 \, dG(\theta), \quad (19)$$

where η is the constant defined in (15) and q^0, q^1 in Lemma 2.

The inequality (19), which gives conditions for the duopoly solution to be better than monopoly with selection of active firm, does not differ much from (17), and this may seem surprising given that we have introduced an element of competition between potential producers. However, the revelation of true types is not exploited fully since the subsequent choice of output and remuneration of producers follows the original regulatory policy (δ, q, r), based on the absence of information about true types, and a more elaborate version of monopoly with selection may perform better. On the other hand, the monopoly with selection may not be an option if there are costs of initiating production, as the presence of such costs would deter potential producers from participating and create inefficiencies if the monopoly rights are shifted repeatedly between different producers. In such cases the regulated duopoly appears as a reasonable alternative, retaining the element of competition while keeping the inefficiency within bounds.

7. Concluding comments

In the preceding sections, we have discussed two different methods of regulating production in a market with increasing returns to scale in production, given that the cost of production is private information of the firms, and assuming that producers cannot be subsidized by the regulator. In the case of a single producer, the method of regulation was that proposed by Baron and Myerson (1982) adapted to the present situation, and in the case of two producers, we introduced a method of regulated duopoly, where firms report their type and the regulation of producer incomes is based on the larger of the reported types but with favors to the low-cost producer. It was shown that for suitable parameter values, the regulated duopoly may yield an outcome which is better for society than that of regulated monopoly.

The superiority of duopoly, even under conditions of increasing returns to scale, is a consequence of the competition between the producers, which induces truthful revelation of cost. This cannot be obtained under regulated monopoly, so that the regulator must accept that the monopolist is as well off when stating true cost as when stating any higher level of cost. Thus, the main results derived can be seen as emphasizing the role of competition as a simple and reasonably robust way of obtaining if not efficient, then at least less inefficient allocation. The particular version of regulated duopoly may not be the most adequate one in some cases. We have opted for a regulation which gives rise to an equal division of total output among firms, but the cost conditions might be such that an asymmetric solution, with a small and a large producer, would be less costly to society, and the regulation which we have considered might be adapted to this case as well.

As was mentioned in the previous section, the comparison of duopoly and monopoly may seem incomplete without introducing another firm into the context of regulated monopoly, and therefore a comparison was made of regulated duopoly and a regulated monopoly which included a selection of the monopolist. In the latter case, an element of competition has been introduced, and it should be expected that also in this case, competition would be at work to reduce the informational rent appropriated by the monopolist. This happened only to a very limited extent, showing that the addition of a selection procedure must be accompanied by a regulatory policy adapted to this procedure, something which however would be beyond the scope of the present paper.

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Appendix: Proofs

In this section, we give the proofs of the lemmas and propositions stated in the previous sections.

Proof of Lemma 1: The function $\pi(\hat{\theta},\theta)$ has bounded partial derivative $C(q(\theta))$ with respect θ, so by the envelope theorem (cf. e.g. Milgrom and Segal (2002)), (5) implies that $\pi(\theta)$ can be written as

$$\pi(\theta) = \pi(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} \delta(t)C(q(t))\,dt$$

for each θ. Now (4) and (6) give that

$$\delta(\theta)P(q(\theta))q(\theta) \geq \pi(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} \delta(t)C(q(t))\,dt + \theta C(q(\theta)) \geq \delta(\theta)\theta C(q(\theta)),$$

and subtracting $\theta C(q(\theta))$ everywhere we get (7). □

Proof of Lemma 2: (i) An optimal regulatory policy (δ, q, r) maximizes $\int_{\Theta} W(q(\theta),\theta)\delta(\theta)\,dF(\theta)$ subject to the constraint in (7). Suppose that $0 < \delta(\theta) < 1$ is some small interval. Then increasing $\delta(\theta)$ will increase expected welfare without violating the constraint, a contradiction, so that $\delta(\theta)$ must be either 0 or 1. If $\theta^0 = \inf\{\theta \mid \delta(\theta) = 1\}$ we get from continuity of objective and constraint that also $\delta(\theta^0) = 1$.

(ii) Suppose that $\delta(\theta) = 0$ and that $\delta(\theta') = 1$ for some $\theta' > \theta$. Then $r(\theta') \geq \theta' C(q(\theta') > \theta C(q(\theta'))$, and we would have a violation of incentive compatibility. Thus, $\{\theta \mid \delta(\theta) = 0\}$ is an interval in Θ containing $\overline{\theta}$, and by (i), its complement is an interval containing $\underline{\theta}$.

(iii) Let $q^0 = q(\theta^0)$. Since $P(\theta^0)q(\theta^0) \geq \theta^0 C(q^0))$ and (δ, q, r) is optimal, we have that $P(\theta^0)q^0 = \theta^0 C(q^0)$. For $\theta' < \theta^0$, $q(\theta') \geq q(\theta^0)$ by incentive compatibility, and clearly $q(\theta') \leq q^1$, where q^1 is such that $P(q^1)q^1 - \theta C(q^1)$ is at least equal to the profit to be obtained by stating θ^0 instead of the true type θ, all $\theta \leq \theta^0$.

Proof of Proposition 1: For each $\theta \in \Theta$, the welfare maximizing production $q^*(\theta)$ satisfies $P(q^*(\theta)) = \theta C'(q^*(\theta))$, whereas $q(\theta)$ must be such that profits $P(q(\theta))q(\theta) \geq r(\theta) \geq \theta C(q(\theta))$ whenever $\delta(\theta) \neq 0$, since (δ, q, r) is individually rational.

By Lemma 1,

$$P(q(\theta))q(\theta) - \theta C(q(\theta)) \geq r(\theta) - \theta C(q(\theta)) = \pi(\theta^0) + \int_{\theta}^{\theta^0} \delta(t)C(q(t))\,dt > (\theta^0 - \theta)C(q^\delta),$$

where we have used that $\pi(\theta^0) \geq 0$ by (6) and assessed the integral by the length of the interval for which $\delta(\theta) = 1$, multiplied by a value $C(q)$, which is as small as any value $C(q(\theta))$ taken in this interval. Dividing on both sides by $q(\theta) \neq 0$, we get

$$P(q(\theta)) - \theta\frac{C(q(\theta))}{q(\theta)} \geq \frac{\theta^0 - \theta}{q(\theta)}C(q^0) \geq \frac{\theta^0 - \theta}{q^1}C(q^0), \tag{20}$$

where $q^1 \geq q^0$ are defined in Lemma 2. Since $C'(q) \leq \dfrac{C(q)}{q}$ for all q on the left-hand side (the inequality holds due to the non-decreasing returns to scale), one gets that

$$P(q(\theta)) - \theta C'(q(\theta)) \geq \frac{\theta^0 - \theta}{q^1} C(q^0), \tag{21}$$

which gives an assessment of the difference between price and marginal cost at the production $q(\theta)$.

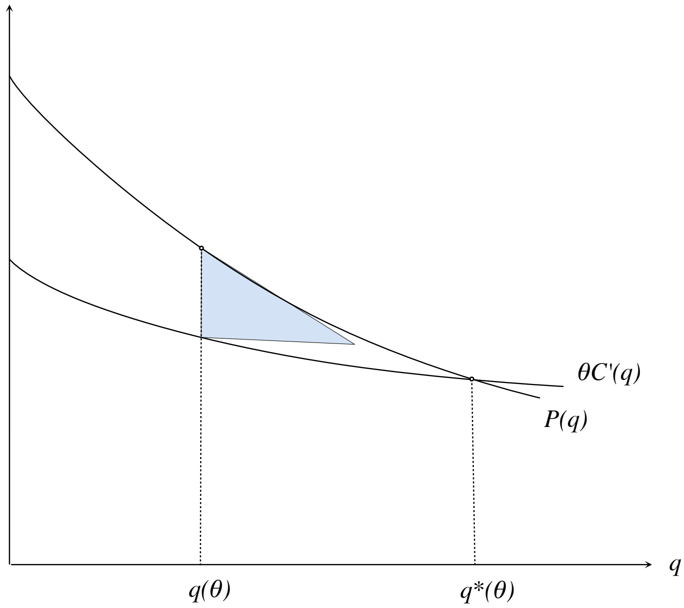

Fig. 2

For each $\theta \in [\underline{\theta}, \theta^0]$, the welfare loss $W(q^*(\theta), \theta) - W(q(\theta), \theta)$ from producing $q(\theta)$ rather than $q^*(\theta)$ can be found as the area between the demand and the marginal cost curves from θ to the welfare optimum (the intersection of the two curves). Since $C'(\cdot)$ is decreasing, this area contains the right-angled triangle with height equal to $P(q(\theta)) - \theta C'(q(\theta))$ and with the tangent to θC at $q(\theta)$ as its hypothenuse, see Figure 2. The remaining side of this triangle has length $(P(q(\theta)) - \theta C'(q(\theta)))|P'(q(\theta))|^{-1}$, so the area of the triangle is

$$\frac{1}{2} \frac{P(q(\theta)) - \theta C'(q(\theta))}{|P'(q(\theta))|} (P(q(\theta)) - \theta C'(q(\theta))),$$

and we get that

$$W(q^*(\theta), \theta) - W(q(\theta), \theta) \geq \frac{[P(q(\theta)) - \theta C'(q(\theta))]^2}{2|P'(q(\theta))|} \geq \left(\frac{\theta^0 - \theta}{q^1} C(q^0) \right)^2 \frac{1}{2|P'(q^0)|}, \tag{22}$$

where we have used that marginal cost is non-increasing together with our assumption on P. Now the assessment in (8) is obtained by integration over θ. □

Proof of Proposition 2: (i) Define $q_d(\theta)$ by

$$P(2q_d(\theta)) 2q_d(\theta) = 2(1 + \lambda)\theta C(q_d(\theta)).$$

Then $q_d(\theta)$ is well-defined since total revenue $P(q)q$ is decreasing and $C(q)$ increasing in q. We check that $(q_d(\theta), q_d(\theta))$ is an equilibrium: Indeed, since no firm is earning profits above the allowed markup, none of them can improve their payoff by changing output.

To show uniqueness, let (q_1^*, q_2^*) be another equilibrium with $q_1^* \geq q_2^* > 0$. If $q_1^* = q_2^* = q^*$, then either $q^* > q_d(\theta)$, in which case $P(2q^*)2q^* < P(2q_d(\theta))2q_d(\theta)$, so that total profits are smaller than at $q_d(\theta)$, or $q^* < q_d(\theta)$, then $P(2q^*)2q^* > P(2q_d(\theta))2q_d(\theta)$ and $P(2q^*)q^* > (1+\lambda)\theta C(q^*)$, so that each firm has an incentive to expand production, and (q^*, q^*) cannot be an equilibrium. We may suppose therefore that $q_1^* > q_2^*$. If $P(q_1^* + q_2^*) < (1+\lambda)\theta C(q_2^*)$, then $\pi_2(q_1^*, q_2^*, \theta)$ can be increased by reducing q_2, so $P(q_1^* + q_2^*)q_2^* \geq (1+\lambda)\theta C(q_2^*)$, and from this and the concavity of C it follows that $P(q_1^* + q_2^*)q_1^* > (1+\lambda)\theta C(q_1^*)$. But then firm 2 will be reimbursed according to $(1+\lambda)\theta C(q_2)$ when increasing q_2 from q_2^*, contradicting that (q_1^*, q_2^*) is an equilibrium.

(ii) If the true type of firm 1 is $\theta' < \theta$, then since remuneration follows market revenue if output is increased and is proportional to cost for decreasing output, we conclude that payoff for firm 1 is maximal at $q_d(\theta)$. □

Proof of Proposition 3: Assume that firm B has chosen the message $\hat{\theta}_B$, and consider the choice problem of firm A with productivity parameter θ_A.

Suppose first that $\theta_A \geq \hat{\theta}_B$. If A sends a message $\hat{\theta}_A \geq \hat{\theta}_B$, then payoff to firm A will be $\pi(\hat{\theta}_B)$. If $\hat{\theta}_A < \hat{\theta}_B$, then payoff to firm A is $\pi(\hat{\theta}_A)$, and by monotonicity of π, this is inferior to $\pi(\hat{\theta}_B)$. We conclude that message $\hat{\theta}_A = \theta_A$ is an optimal message in this situation.

Next, let $\theta_A < \hat{\theta}_B$. If firm A chooses a message $\hat{\theta}_A \geq \theta_B$, it will be given the number 2, and if it chooses $\hat{\theta}_A < \hat{\theta}_B$, it will be given the number 2, but in either case, it will get the payoff $\pi(\hat{\theta}_B)$, meaning that also in this case the message $\hat{\theta}_A = \theta_A$ is an optimal choice. □

Proof of Lemma 3: According to the regulation, the productivity parameter chosen is θ_2, and $q_d(\theta_2)$ is produced in each firm. Since $q_d(\theta_2)$ is determined by $P(2q_d(\theta_2))2q_d(\theta_2) = 2(1+\lambda)\theta_2 C(q_d(\theta_2))$, we get that

$$P(2q_d(\theta_2))2q_d(\theta_2) - (1+\lambda)\theta_2 C(2q_d(\theta_2)) \leq 0$$

for sufficiently small values of λ. Using that

$$\frac{\theta_2 C(2q_d(\theta_2))}{2q_d(\theta_2)} - \theta_2 C'(2q_d(\theta_2)) \leq \theta_2 \eta$$

by (15), we obtain the inequality

$$P(2q_d(\theta_2)) - \theta_2 C'(2q_d(\theta_2)) \leq \theta_2 \eta,$$

which is (16). □

In the proof of Proposition 4, we need the following lemma.

Lemma 4 *Assume that increasing returns to scale are moderate in the sense of (15). If $q^0 \leq q \leq q^*$, then*

$$|C'(q) - C'(q^*)| \leq 2\eta. \tag{23}$$

Proof: From (15) we get that

$$C'(q^0)q - \eta q \leq C(q) \leq C'(q^0) + \eta q$$

for all $q \geq q^0$, and after differentiation we get that

$$C'(q^0) - \eta \leq C'(q) \leq C'(q^0) + \eta$$

for all $q \geq q^0$, from which (23) follows. □

PROOF OF PROPOSITION 4: For $\theta \in \Theta$, let $\Delta W^m(\theta)$ and $\Delta W^d(\theta)$ be the welfare loss under monopoly and duopoly when the type is θ, respectively, as compared to the welfare maximizing production $q^*(\theta)$ where θ_2 the largest of the two parameters θ_1 and θ_2 chosen randomly. In the duopoly case, $\Delta W^d(\theta)$ consists not only in allocative loss from producing $2q_d(\theta)$ rather than $q^*(\theta)$, but also in efficiency loss from producing in two firms rather than in a single firm.

We have from (22) that in the monopoly case,

$$\Delta^m(\theta) = W(q^*(\theta), \theta) - W(q(\theta), \theta) \geq \left(\frac{\theta^0 - \theta}{q^1} C(q^0) \right)^2 \frac{1}{2|P'(q^0)|}$$

where the welfare loss was bounded from below by the area of a suitable triangle, as shown in Fig. 2.

We now bound the allocative welfare loss at θ in the duopoly case by the area of the right-angled triangle defined by the points $(2q_d(\theta_2), P(2_d(\theta_2)))$, $(q^*(\theta_2), P(q^*(\theta_2)))$, and $(2q_d(\theta_2), P(2q_d(\theta_2))$, giving that

$$\begin{aligned}
\Delta^d W(\theta_2) &= W(2q(\theta_2), \theta_2) - W(q^*(\theta_2), \theta_2) \leq \frac{1}{2} \left[P(2q_d(\theta_2)) - P(q^*(\theta_2)) \right] \left[q^*(\theta_2) - 2q_d(\theta_2) \right] \\
&\leq \frac{1}{2} \left[P(2q_d(\theta_2)) - \theta_2 C'(q^*(\theta_2)) \right] \left[q^*(\theta_2) - 2q_d(\theta_2) \right] \\
&= \frac{1}{2} [[P(2q_d(\theta_2)) - \theta_2 C'(2q_d(\theta_2)] + \theta_2 [C'(2q_d(\theta_2) - C'(q^*(\theta_2))] \left[q^*(\theta_2) - 2q_d(\theta_2) \right] \\
&\leq \frac{1}{2} 3\eta\theta_2 \left[q^*(\theta_2) - 2q_d(\theta_2) \right]
\end{aligned}$$

where we have used first the definition of $q^*(\theta_2)$ and then (16) and Lemma 4. Assessing now the distance from $2q_d(\theta_2)$ to $q^*(\theta_2)$ by

$$q^*(\theta_2) - 2q_d(\theta_2) \leq |P'(q^1)|^{-1} (P(2q_d(\theta_2)) - P(q^*(\theta_2))) \leq |P'(q^1)|^{-1} 3\eta\theta_2,$$

we finally get that

$$\Delta^d W(\theta_2) \leq \frac{1}{2} |P'(q^1)|^{-1} (3\eta)^2 \theta_2^2.$$

Taking expectations over $\theta \in \Theta$ we obtain that expected loss from producing in monopoly exceeds expected loss from duopoly,

$$\int_\Theta \Delta W^m(\theta)\, dF(\theta) \geq \int_\Theta \Delta W^d(\theta)\, dG(\theta),$$

if

$$\int_{\underline{\theta}}^{\theta^0} \left[\frac{C(q^0)(\theta^0 - \theta)}{q^1} \right]^2 \frac{1}{|P'(q^0)|}\, dF(\theta) > \int_\Theta \frac{(3\eta)^2}{|P'(q^1)|} \theta^2\, dG(\theta),$$

or equivalently if

$$\left[\frac{C(q^0)}{q^1} \right]^2 \frac{1}{|P'(q^0)|} \int_{\underline{\theta}}^{\theta^0} (\theta^0 - \theta)^2\, dF(\theta) > \frac{(3\eta)^2}{|P'(q^1)|} \int_\Theta \theta^2\, dG(\theta),$$

which is (17). □

PROOF OF PROPOSITION 5: Let the types of the two firms be θ_A and θ_B, and assume that firm B has submitted the true type θ_B. If $\theta_A \geq \theta_B$, then firm A will be selected only if it submits a type $\hat{\theta}_A \leq \theta_B$, and in this case its payoff when producing will be zero or negative. If $\theta_A < \theta_B$, firm A is selected and may get a positive payoff if it announces $\hat{\theta}_A \leq \theta_B$ and regulated according to (δ, q, r) with θ_B as largest feasible type, and by incentive comparability of the regulation, firm A is at least as well off by announcing θ_A as any other $\hat{\theta}_A < \theta_B$. □

PROOF OF PROPOSITION 6: The right-hand side in (19) equals that of (17), so we consider only the left-hand side, which arises from integrating the expression for $\Delta^m W(\theta)$, first over all values of θ_1 smaller that θ_2,

$$\left[\frac{C(q^0)}{q^1}\right]^2 \frac{1}{|P'(q^0)|} \int_{\underline{\theta}}^{\theta_2} \frac{1}{F(\theta_2)} (\theta^0 - \theta_1)^2 \, dF(\theta_1),$$

giving the average welfare loss when the largest of the two types is θ_2, and then integrating over θ_2. □