UNIVERSITY OF
CAMBRIDGE

Cambridge Working Papers in Economics

Public Finances, Business Cycles and Structural Fiscal Balances

Kai Liu

CWPE 1411

Public Finances, Business Cycles and Structural Fiscal Balances*

Kai Liu†

First version: January 2013; this version: March 2014

Abstract: This paper proposes a new framework to analyze and estimate structural fiscal balances. Stochastic trends are properly incorporated, and the numerical solution of the DSGE model serves as part of the Kalman smoother to extract structural fiscal balances. For the UK, a setting of an integrated random walk for the underlying stochastic trends fits the date best. The response of nominal fiscal revenue to the technology shock is small. The shocks to foreign demand and to foreign goods price both have positive effects on fiscal revenue. An expansionary monetary policy shock has a great positive short-run impact on fiscal revenue, but the influence is not persistent because of the open-economy characteristic of the UK. An expansion in government spending can also increase fiscal revenue, but the effect is not persistent as well due to the domestic and external crowd-out effects. A contractionary fiscal policy (cutting government expenditure or increasing the lump-sum tax temporarily), rather than an expansionary one, will benefit economic recovery and also improve fiscal stance. Compared to a temporary increase of the lump-sum tax, cutting government spending is relatively more effective and it alleviates the two kinds of crowd-out effects.

Key words: business cycles, structural fiscal balances, DSGE model, Kalman filter and smoother

JEL classification codes: C54, E32, E62, H62

* I thank Sean Holly, Andrew Harvey, Paul Levine, Pontus Rendahl, and participants at the macroeconomics workshop in the University of Cambridge and 2013 Royal Economics Society Easter School.

† Faculty of Economics, University of Cambridge/School of Economics, Renmin University of China; Email: kl363@cam.ac.uk.

1. INTRODUCTION

The world economy has still being struggling to recover since the 2007-2008 world-wide financial and economic crises. At this stage, besides slow economic recovery and high unemployment rate, many countries have encountered problems of public finances as well. This provokes economists and policy makers to think further about the relationship between public finances and business cycles, and also try to forecast the long-run trend of fiscal balances for these countries.

The long-run trend of fiscal balances, formally known as structural fiscal balances (SFB), are calculated when transitory components are removed from fiscal revenues and expenditures. *Cyclically adjusted fiscal balances* are commonly employed, which refers to the difference between the trend levels of fiscal aggregates when only the cyclical effect of output gap is considered. Recently especially after the Great Recession, more other factors are supposed to have significant influences on fiscal aggregates, such as inflation, asset prices and terms of trade (Bornhorst *et al.*, 2011). In this paper, *structural fiscal balances* are defined as the difference between the trend levels of aggregate fiscal revenue and aggregate government expenditure. For nominal government expenditure, I only adjust it one-to-one to the cycle of aggregate price level and ignore other cyclical factors' influences on it, since real government expenditure is discretionary and fiscal economists always assume zero elasticity of it with respect to cyclical factors, such as in Bornhorst *et at.* (2011).

This paper criticizes the traditional elasticity-trend methodology, and proposes a new framework to analyze structural fiscal balances and the relationship between public finances and business cycles. The effects of various shocks, including the technology shock, the stock market shock, foreign real and nominal shocks and domestic policy shocks, on aggregate fiscal revenue and their propagation mechanisms are examined in detail. Trends and cycles of aggregate variables, including structural fiscal balances, can be extracted from data simultaneously. To be specific, a small open-economy New Keynesian DSGE model with exogenous growth, stock market, and fiscal and monetary policies is constructed, stochastic trends are properly incorporated, Bayesian method is employed to estimate the model, and the numerical solution of the DSGE model serves as part of the Kalman smoother to do the signal extraction.

The UK economy is taken as an example in this paper. Bayesian estimation results indicate that for the UK economy a setting of integrated random walk for the underlying stochastic trends of the economy fits the date best. The impulse response analysis reveals the basic relationship between aggregate fiscal revenue and business cycles. The transmission mechanism of various shocks' effect on nominal fiscal revenue is explained by two main channels: the real channel through real GDP which can be viewed as the real tax base of fiscal revenue, and the nominal channel through the aggregate price level. Although in the medium term the response of nominal fiscal revenue to the technology shock is positive, the effect is not big. Both of the foreign shocks, the shock to foreign demand and the shock to foreign goods price, have positive effects on fiscal revenue. An expansionary monetary policy shock would have a great positive short-run impact on nominal fiscal revenue, but the influence is not persistent because of the open-economy characteristic of the UK. An expansion in government spending can also increase nominal fiscal revenue to a certain degree, but the effect is not persistent as well due to two kinds of crowd-out effects generated by an increase of government spending: it crowds out domestic investment, and it pushes up the price of domestic goods and simultaneously crowds out foreign demand. The shock to the stock price has no effect on fiscal revenue. The forecast error variance decomposition of the fiscal revenue cycle tells that: the shocks to the nominal interest rate, foreign output and the government spending are the three major contributors to the variation of the fiscal revenue cycle; the shock to the foreign price makes some contribution to the fluctuation of the fiscal revenue cycle; and the shocks to the temporary productivity and stock price are of very minor importance.

We also discuss the public finances of the UK in the post Great Recession period when both the economic recovery and the fiscal sustainability should be taken into consideration. Generally speaking, it is not an appropriate choice to adopt an expansionary fiscal policy by either increasing the government expenditure or cutting the lump-sum tax. An expansionary fiscal policy will deteriorate the fiscal stance (higher government debt-GDP ratio or higher fiscal deficit) as well as harm the economic recovery in the medium term. On the contrary, a contractionary fiscal policy (cutting the government expenditure or a temporary increase of the lump-sum tax) will benefit both the economic recovery and the fiscal stance. Compared to a temporary increase of the lump-sum tax, cutting the government spending is relatively more effective and it alleviates both the domestic and external crowd-out effects generated by the government spending.

The rest of this paper is organized as follows: Section 2 is a review of relevant literature. Section 3 is the theoretical framework of our approach to analyze structural fiscal balances. In section 4 we do the Bayesian estimation of our DSGE model, and then the signal extraction of the fiscal revenue cycle and structural fiscal balances is achieved. Section 5 explores the relationship between fiscal revenue and business cycles in detail, and tries to uncover the transmission mechanisms of various shocks' impacts on fiscal revenue. Robustness checks are implemented in Section 6, in order to see whether the results about signal extraction, impulse response analysis and variance decomposition are sensitive or not to our calibration. Section 7 is a policy evaluation of public finances for the UK in the post Great Recession period. Finally in section 8 we conclude.

2. LITERATURE REVIEW

Estimating structural fiscal balances is of special interest to national treasuries of many countries and international institutions such as IMF, ECB, and OECD. Normally there are three steps to estimate structural fiscal balances: first, identify and remove one-off fiscal operations such as public expenditure on a natural disaster; secondly, assess the impact of the business cycle (output gap) on fiscal revenue and expenditure; finally, estimate the effects of other factors. In practice, the second and third steps can be done together, using the elasticity-trend approach.

The elasticity-trend approach consists of three further steps: first estimate the fiscal revenue and expenditure elasticities with respect to real output and other factors; then do the trend-cycle decomposition for these factors; finally calculate the trend levels of fiscal revenue and expenditure and then structural fiscal balances, using the estimated elasticities and the trend levels of real output and other factors.

Among the existing literature, fiscal elasticities with respect to different factors, such as output gap, asset prices, commodity price and terms of trade, and inflation, are investigated broadly. Girouard and André (2005) discuss the cyclically adjusted budget balances for OECD countries in detail. Aydin (2010) studies the case of South Africa, with an emphasis on the effects of commodity and asset prices, and the credit cycle as well. Terms of trade[1] may have a negative effect on fiscal revenue, especially for commodity exporter countries. Price and Dang (2011) explain the necessity of incorporating the asset prices effects when removing the transitory components of fiscal balances, and provides an econometric method to estimate structural fiscal balances for OECD countries. This econometric approach is an $ARDL(1,1,1,\ldots,1)$ model, which can be used to estimate both the short-run and long-run fiscal elasticities with respect to output and asset prices.

Table 1. Short run and long run fiscal revenue elasticities for the UK

Elasticities	EC parameter	Real GDP	GDP deflator	Terms of trade	Financial stress
Short run	0	1.30	1.16	0	0
Long run		0.35	1.48	-0.44	0.005

Source: author's calculation.
Note: zero in the table means the corresponding estimated elasticity (or parameter) is not significant at the significance level of 10%.

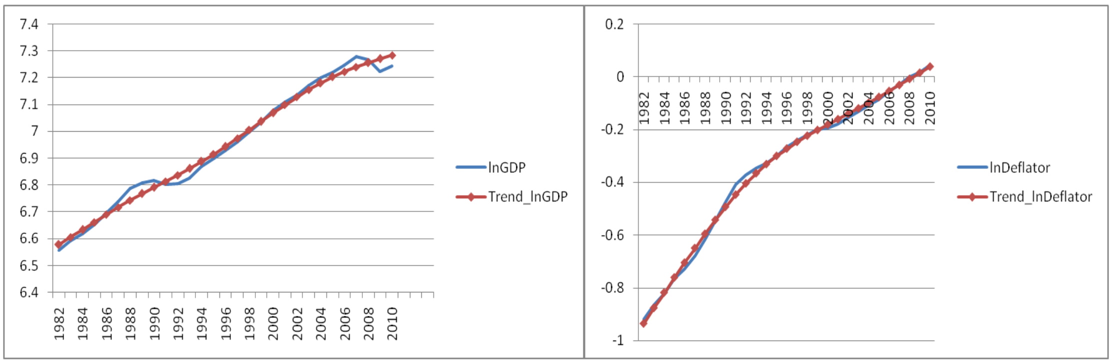


Fig. 1. The HP trends of real GDP and GDP deflator for the UK according to the HP filtering method (lambda=100 for annual data)

Table 1 lists the short run and long run fiscal revenue elasticities with respect to various factors according to an $ARDL$ model, for the UK. Figure 1 depicts the trends of real GDP and GDP deflator resulting from the HP filtering method. Consequently,

[1] In this paper, terms of trade are defined as the ratio of the imported goods price over the exported goods price.

the trend and the cycle of fiscal revenue (and then structural fiscal balances) can be calculated and are shown in Figure 2, following the elasticity-trend approach.

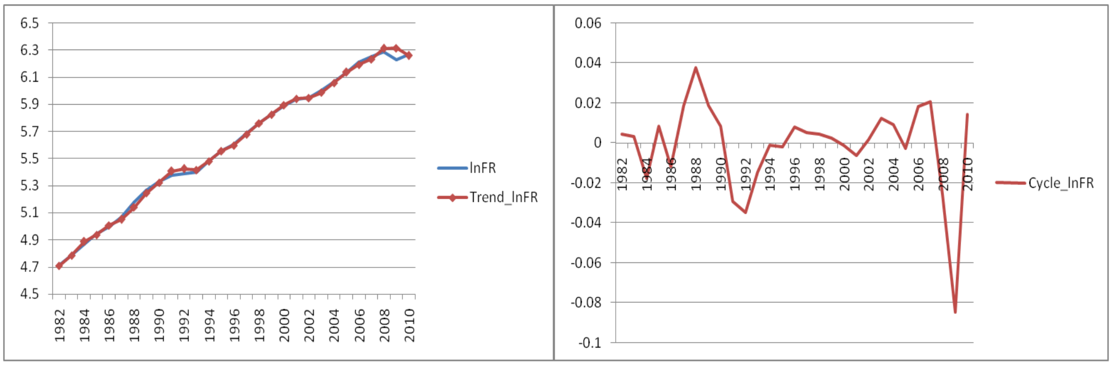


Fig. 2. The trend and the cycle of fiscal revenue for the UK according to the elasticity-trend approach

Besides the elasticity-trend approach which is most widely used, some other econometric methods such as structural VAR are sometimes used to estimate structural fiscal balances as well. There are several obvious shortcomings of using such empirical models to estimate fiscal elasticities and calculate structural fiscal balances. First of all, the propagation mechanisms of transitory effects on fiscal aggregates are not clear. Secondly, there is a risk of over adjusting. Third, fiscal elasticities and trend levels are estimated separately, and there is a problem of inconsistency. For example, the HP filter is broadly used to get the trend levels of influencing factors, such as in the example given above, in order to calculate the trend level of fiscal revenue. The problem here is that: if one can justify the usage of the HP filter to de-trend GDP and other aggregate variables, why not using it to de-trend fiscal revenue directly? Figure 3 depicts the cycles of fiscal revenue for the UK, using the HP and structural time series model (STM) methods respectively,[2] which are quite different from the cycle in Figure 2.

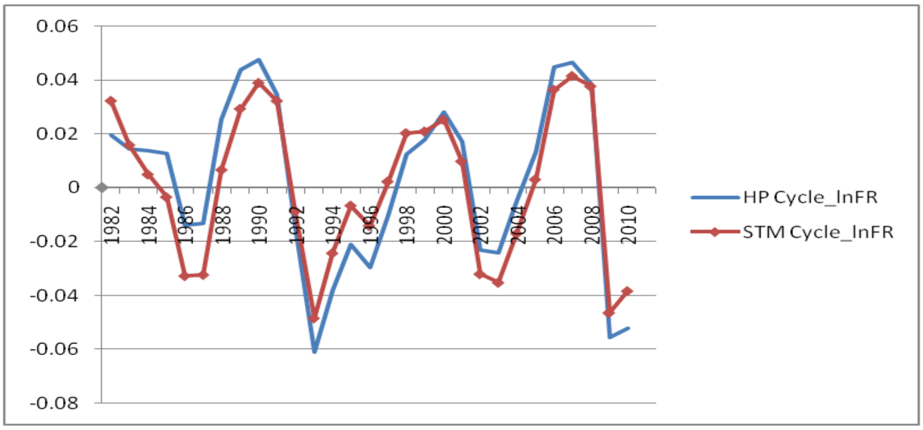


Fig. 3. The cycles of fiscal revenue for the UK according to the HP and STM methods

[2] For STM, refer to Harvey (1989) and Harvey and Jaeger (1993), and a smooth trend is employed here. We will discuss STM in detail in a coming part of the paper. The HP filtering method is in fact a special kind of STM.

Garratt *et al*. (2006) also criticize univariate pre-filtering procedures (such as the HP filter), and they present a derivation of multivariate Beveridge-Nelson trends from cointegrating vector autoregressive model. The multivariate Beveridge-Nelson trends are interpreted as conditional cointegrating equilibrium values, and the nature of the permanent and transitory components can be related to the nature of the error correction process, at both finite and infinite horizons. Similar to Garratt *et al*. (2006), Dees *et al*. (2010) use the long-horizon forecasts, provided by a global VAR model which takes account of unit roots and cointegration in the global economy, to model the permanent (trend) components of variables. Cointegrating relationships between trends can be modeled within the framework of multivariate structural time series model with common trends as well, as illustrated in Harvey (1989). And a multivariate structural time series model can be also utilized to explore the possible linear relationships between cycles, using common cycle settings. Either the multivariate Beveridge-Nelson approach of Garratt *et al*. (2006) or the multivariate structural time series model with common trends and common cycles is still a pure statistical model, and when they are used to de-trend nominal fiscal revenue and analyze structural fiscal balances, the propagation mechanisms of influencing factors on fiscal revenue are still a black box.

So in this paper we will analyze and do the signal extraction of structural fiscal balances in a framework of DSGE modeling. In order to get rid of the de-trending problems explained above, we will incorporate stochastic trends in our DSGE model, and the cyclical components of aggregate variables and structural fiscal balances will be obtained from the data simultaneously.

Why should we incorporate stochastic trends into a DSGE model? The common practice of bridging the DSGE models and the data is to eliminate trends altogether in the data --- “pre-filter” them using such as the HP filter --- before estimating a model. This two-step approach is very problematic and criticized by Fukac and Pagan (2005), Ferroni (2011), Canova (2012) and Lafourcade and Wind (2012). In contrast to the two-step approach, An and Schorfheide (2007), Lafourcade and Wind (2012) and many other Bayesian DSGE papers incorporate stochastic trends into DSGE models and estimate the parameters regarding the whole system altogether, using Bayesian techniques. According to Ferroni (2011) and Canova (2012), joint estimation is unambiguously preferable to the two-step approach, since it can avoid problems ranging from trend misspecification to wrong cross-frequency correlation. They both suggest developing a flexible specification for trends in the observation equations and estimating them jointly with the cyclical theoretical model summarized in the transition equations.

Trends can be brought into a DSGE model in several ways. In the literature of Bayesian DSGE models, a common practice is to assume a drifted random walk for the permanent technology (or productivity), as in An and Schorfheide (2007), Lafourcade and Wind (2012). And in the stochastic steady state, aggregate variables such as real GDP and consumption are driven by this same drifted-random-walk stochastic trend, which is an I(1) process. After removing this common stochastic trend from the model economy, the system would be put into a stationary representation. Here it is in fact implicitly assumed that the balanced growth property of aggregate variables in the deterministic steady state is maintained in the stochastic steady state. Another way is as in Smets and Wouters (2007), assuming that all real variables in the model expand at the same deterministic rate, which directly reflects the property of a balanced growth path. This deterministic-trend approach can neither capture the observable fluctuations of the data properly nor reflect the complexity of the economic reality. Another alternative way to incorporate stochastic trends is using an integrated random walk model or local linear trend model, as in literature of the structural time series model (or unobserved components model). An integrated random walk or a local linear trend is an I(2) process. It is more flexible than a drifted-random-walk trend, and under some circumstance it can be reduced to a drifted random walk.

While the Bayesian DSGE literature aims to better estimate structural parameters of the DSGE models by incorporating stochastic trends, the aim of this paper by incorporating stochastic trends is to better extract trends and cycles (particularly structural fiscal balances) from data and make the signal extraction consistent with the macroeconomic theory. In this paper, stochastic trends will be incorporated formally in a general equilibrium framework, and cross-equation restrictions that link transition and observation equations more tightly will be generated, thus resulting in a richer and theory-consistent correlation structure in the model. Stochastic trends are incorporated not only in the process of DSGE model estimation, but also embedded in the mechanism of the signal extraction of structural fiscal balances. It is the data that determines whether the underlying stochastic trends of the model economy are I(2) or I(1) processes. Section 3 below provides a theoretical open-economy DSGE model of structural fiscal balances.

3. A MODEL OF STRUCTURAL FISCAL BALANCES

The model is an open-economy DSGE model with economic growth, stock market, tax system and fiscal and monetary policies. And nominal fiscal revenue and then fiscal balances are endogenously determined. There is a continuum, with unity measure, of countries in the world, and the home country is one of them and thus of zero measure.

3.1. Households

3.1.1. Inter-temporal optimization

In the home country it is assumed there is a continuum, with unity measure, of infinitely-living households. A representative household seeks to maximize his life time utility:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \, [\phi \cdot lnC_t + (1-\phi) \cdot \ln(1-L_t)]$$

where E is the expectation operator, β is the utility discount factor, ϕ is the utility weight for consumption C_t, and labour supply is L_t. For tractability, we assume additively separable logarithmic utility.

The representative household can invest in three assets: real capital K_t which is used as a production factor with real rental rate r_t^1; government bond B_t with nominal interest rate R_t^2; and equity shares $S_t(i)$ of the firm i which produces an intermediate good i with dividend $D_t(i)$ and its share price $Q_t(i)$. As we will explain, there is a unit-measure continuum of differentiated intermediate goods, and each intermediate-goods firm has monopoly profit due to the monopolistic competition of intermediate-goods market.

The government collects six kinds of taxes from households: labour income tax with rate τ_L, capital rental income tax with rate τ_K, consumption tax with rate τ_C, bond interest income tax with rate τ_B, share dividend tax with rate τ_D, and nominal lump-sum tax LST_t. Therefore, the budget constraint for the household is:

$$\begin{aligned}(1+\tau_C) \cdot P_t \cdot C_t + P_t \cdot K_{t+1} + B_{t+1} + LST_t + \int_0^1 S_{t+1}(i) \cdot Q_t(i)\, di \\ = [1-\delta+(1-\tau_K) \cdot r_t^1] \cdot P_t \cdot K_t + (1-\tau_L) \cdot W_t \cdot L_t \\ +[1+(1-\tau_B) \cdot R_{t-1}^2] \cdot B_t + \int_0^1 S_t(i) \cdot [Q_t(i) + (1-\tau_D) \cdot D_t(i)] di \quad (1)\end{aligned}$$

where P_t is the aggregate price level of the final good, δ is the capital depreciation rate, and W_t is the nominal wage rate. It is assumed that the government bond is paid according to the nominal interest rate of its previous period.

The household's problem is to choose the consumption level C_t, labour supply L_t, capital stock for the next period K_{t+1}, the quantity of government bond for the next period B_{t+1}, and the stock shares of each intermediate firm $S_{t+1}(i)$, in order to maximize his life time utility, subject to the budget constraint of each period, equation (1), given the price levels of the final good and stock shares, various tax rates, real capital rental rate and nominal bond interest rate. The first order conditions (FOCs) of the utility maximization problem are:

$$\frac{(1-\phi) \cdot C_t}{\phi \cdot (1-L_t)} = \frac{(1-\tau_L) \cdot W_t}{(1+\tau_C) \cdot P_t} \quad (2)$$

$$E_t \left\{\beta \cdot \frac{C_t}{C_{t+1}} \cdot [1-\delta+(1-\tau_K) \cdot r_{t+1}^1]\right\} = 1 \quad (3)$$

$$E_t \left\{\beta \cdot \frac{C_t}{C_{t+1}} \cdot \frac{P_t}{P_{t+1}} \cdot [1+(1-\tau_B) \cdot R_t^2]\right\} = 1 \quad (4)$$

$$E_t \left\{\beta \cdot \frac{C_t}{C_{t+1}} \cdot \frac{P_t}{P_{t+1}} \cdot \frac{Q_{t+1}(i) + (1-\tau_D) \cdot D_{t+1}(i)}{Q_t(i)}\right\} = 1 \quad (5)$$

3.1.2. Financial wealth and human wealth of households

We define the financial wealth Ω_t and human wealth H_t of the household as follows:

$$\Omega_t \triangleq [1-\delta+(1-\tau_K)\cdot r_t^1]\cdot P_t\cdot K_t+[1+(1-\tau_B)\cdot R_{t-1}^2]\cdot B_t + \int_0^1 S_t(i)\cdot[Q_t(i)+(1-\tau_D)\cdot D_t(i)]di \tag{6}$$

$$\mathrm{H}_t \triangleq E_t\left\{\sum_{k=0}^{\infty} F_{t,t+k}\cdot[(1-\tau_L)\cdot W_{t+k}\cdot L_{t+k}-LST_{t+k}]\right\} \tag{7}$$

where $F_{t,t+k}$ is the equilibrium discount factor of wealth and defined recursively by $F_{t,t}=1$, equation (8) and (9) as follows. The second part of equation (8) is derived from equation (4).

$$F_{t,t+1} \triangleq \frac{1}{1+(1-\tau_B)\cdot R_t^2} = E_t\left\{\beta\cdot\frac{C_t}{C_{t+1}}\cdot\frac{P_t}{P_{t+1}}\right\} \tag{8}$$

$$F_{t,t+k} \triangleq \prod_{i=0}^{k-1} F_{t+i,t+i+1} = \prod_{i=0}^{k-1}\frac{1}{1+(1-\tau_B)\cdot R_{t+i}^2} \tag{9}$$

The so-called no-ponzi game condition is given by the following equation:

$$\lim_{k\to\infty} E_t\left\{F_{t,t+k}\cdot\Omega_{t+k}\right\}=0 \tag{10}$$

Combining equation (6) with equation (1), (3), (4) and (5) gives the following first-order difference equation of financial wealth Ω_t:

$$(1+\tau_C)\cdot P_t\cdot C_t + E_t\left\{F_{t,t+1}\cdot\Omega_{t+1}\right\} = (1-\tau_L)\cdot W_t\cdot L_t - LST_t + \Omega_t \tag{11}$$

Then combining equation (7), (9), (10) and (11) yields an equation in which the equilibrium nominal consumption is a fixed proportion of household's total wealth (financial wealth plus human wealth), as below:

$$P_t\cdot C_t = \frac{1}{1+\tau_C}\cdot(1-\beta)\cdot(\Omega_t+\mathrm{H}_t)$$

So the stock price $Q_t(i)$ may affect the household's consumption and thus the whole economy through a wealth effect.

3.2. Stochastic trends in the economy and unobserved components model

3.2.1. Definition of trends and cycles and stochastic trends

In this paper, as explained, both trends and cycles are taken into consideration when estimating the model and doing signal extraction, rather than using the normal but problematic methods, such as the HP filter.

The trends of aggregate variables are defined when they are on the steady state balanced growth path of our DSGE model. Then the cyclical components (or cycles) of aggregate variables are defined as the gaps between their actual and trend levels. The steady state balanced growth path itself is assumed to be stochastic. In our model there are two basic stochastic trends driving the stochastic steady state: the trend in the permanent technology which drives real output and the trend in the aggregate price level (for example, due to the increasing supply of fiat money). All other variables' trends are composed of these two basic trends. These two basic trends are exogenously given. Therefore, the whole dynamic economic system in our model are divided into two uncorrelated parts: one is the stochastic steady state driven by these two underlying trends, and the other is the cyclical fluctuation around the steady state, which will be represented by the log-linearized version of our DSGE model.

3.2.2. Stochastic trends and unobserved components model

In the literature of unobserved components model (UCM) or structural time series model (STM), such as Harvey (1989) and Harvey and Jaeger (1993), stochastic trends and cycles can be directly modeled as unobserved components.

For a seasonally-adjusted time series y_t, the measurement equation can be given as follows:[3]

$$y_t = \overline{y_t} + \widehat{y_t}$$

The stochastic trend component $\overline{y_t}$ can be normally assumed to follow a local linear trend model:

$$\overline{y_t} = \overline{y_{t-1}} + \gamma_t + \eta_t$$
$$\gamma_t = \gamma_{t-1} + \zeta_t$$

where γ_t can be viewed as, even though not strictly, the growth rate, and it is a random walk process; the disturbances, η_t and ζ_t are white noises, and serially uncorrelated.

If the variance of the disturbance term ζ_t is zero, the growth rate γ_t would be a constant, and then the above local linear trend model becomes a drifted random walk:

$$\overline{y_t} = \overline{y_{t-1}} + \gamma + \eta_t$$

where γ is then the average growth rate. An and Schorfheide (2007), Lafourcade and Wind (2012) and many others use such a drifted random walk to model the stochastic trend of permanent technology.

Either local linear trend model or drifted random walk represents a growing trend. Then how to deal with the "trend" which is neither upward growing nor stationary, such as the trend component of unemployment rate or inflation rate for some countries? The answer is to simply use a random walk. Take the time series of inflation rate Π_t for instance:

$$\Pi_t \triangleq lnP_t - lnP_{t-1}$$
$$\Pi_t = \overline{\Pi_t} + \widehat{\Pi_t}$$
$$\overline{\Pi_t} = \overline{\Pi_{t-1}} + \eta_t$$

where P_t is the aggregate price level, $\overline{\Pi_t}$ is the trend inflation (sometimes called core inflation), and $\widehat{\Pi_t}$ is the cyclical component of inflation (or inflation gap). If we

[3] In the UCM literature, usually another residual component is included in the measurement equation. But in this paper, we follow the convention of macroeconomics and do not distinguish between cyclical component and residual component, and the gap between actual time series and its trend is the cycle. For aggregate variables such as GDP, consumption, and so on, commonly take logarithm first; but for inflation rate or interest rate, do not take logarithm.

map the trend inflation to the trend price level, then in fact the above random walk model for trend inflation is equivalent to assuming an integrated random walk model, or smooth trend model, for the aggregate price level:

$$\overline{lnP_t} - \overline{lnP_{t-1}} = \overline{\Pi}_t$$
$$\overline{\Pi}_t = \overline{\Pi}_{t-1} + \eta_t$$

This is a special case of local linear trend model, in which the disturbance term of the trend is assumed to have zero variance.

The HP filter method is a special kind of smooth trend model with the signal-noise ratio being a fixed number. Whether using a drifted random walk or the smooth trend model (or local linear trend model) to model stochastic trends depends on the integration property of the trends. If a trend is an I(1) process or its growth rate is stationary, a drifted random walk is a good choice; however, if the trend is an I(2) process or its growth rate is not stationary, then the smooth trend model or local linear trend model is a better alternative. Therefore, different from Lafourcade and Wind (2012) and many others who directly use a drifted random walk to model the stochastic trend of permanent technology without checking its integration property, we do the integration tests first for the two basic trends of our model: trends in the permanent technology and the aggregate price level, which could be reflected by the time series of real GDP and GDP deflator index according to the steady state analysis of our model in a later section. Since in this paper the UK economy will be taken as an example, the following figure depicts the growth rates of real GDP and GDP deflator for the UK.

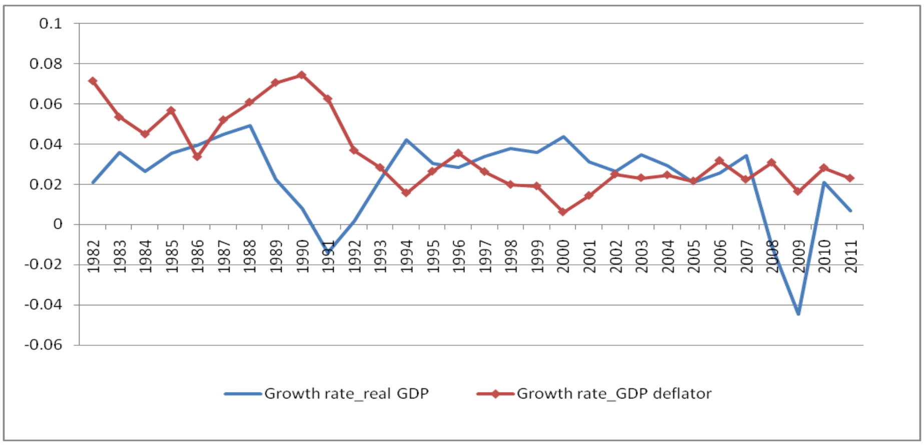


Fig. 4. Growth rates of real GDP and GDP deflator for the UK (annual data)

Table 2. Unit root tests for growth rates of real GDP and GDP deflator

Growth rate	ADF statistic	1% and 5% critical values	P value
Real GDP	-2.90	-3.68 and -2.97	0.06
GDP deflator	-2.30	-3.68 and -2.97	0.18

Note: the null hypothesis is that the tested growth rate has a unit root.

Either Figure 4 or Table 2 tells the same story: the growth rates of real GDP and GDP deflator of the UK are both non-stationary at the significance level of 5%. So in this paper, both of the two basic trends (the trend in the permanent technology and the trend in the aggregate price level) are assumed to be an integrated random walk (a smooth trend)[4] as follows:

$$\overline{y_t} = \overline{y_{t-1}} + \gamma_t$$
$$\gamma_t = \gamma_{t-1} + \zeta_t$$

Note that for the growth rate of real GDP, it is stationary at the significance level of 10%. So in a later part other alternative specifications for the stochastic trends will be examined, and it will prove the smooth trend specification here is best.

3.2.3. Cycles and cyclical DSGE representation

In the literature of UCM, the transition equation for the cycle is always exogenously given as follows:

$$\begin{bmatrix} \widehat{y_t} \\ \widehat{y_t}^* \end{bmatrix} = \rho \begin{bmatrix} cos\lambda_y & sin\lambda_y \\ -sin\lambda_y & cos\lambda_y \end{bmatrix} \begin{bmatrix} \widehat{y_{t-1}} \\ \widehat{y_{t-1}}^* \end{bmatrix} + \begin{bmatrix} k_t \\ {k_t}^* \end{bmatrix}$$

where λ_y is frequency in radians, ρ is a damping factor with $0 < \rho < 1$ and k_t and ${k_t}^*$ are two mutually independent white noise disturbances with zero means and common variance. The disturbances are serially and mutually uncorrelated. The reduced form for the cycle is, in fact, an ARMA(2,1) process in which the autoregressive part has complex roots. Different from the convention of UCM, the transition equation for the cycles in this paper will be endogenously given by a DSGE model.

For the cyclical components of aggregate variables in DSGE models, log-linearization of a model around its steady state will give a vector difference equation (with expectation term) of the cyclical components, and its solution is (or can be numerically approximated as) a first-order vector autoregressive (VAR) process. This VAR(1) process is then the transition equation for cyclical components, which, combined with the transition equations for trend components and the measurement equation, will put the system into the traditional state space and Kalman-filter framework.

3.3. Final good producer and price indices

Final good producers first produce home good $Y_{H,t}$ by combining a continuum of home-made intermediate goods $Y_{H,t}(i)$, and foreign good $Y_{F,t}$ by combining a continuum of imported foreign intermediate goods $Y_{j,t}(i)$; and then combine home good and foreign good to produce the final good Y_t, which can be used for households' consumption, capital investment and government's expenditure.

The final good producers are perfectly competitive and there is zero profit for them. The technologies of producing home and foreign good, and then final good are all CES technologies as follows:

[4] Here we employ the smooth trend model rather than the local linear trend model because this not only makes the trend smoother but also simplifies the estimation of the model.

$$Y_{H,t} = \left[\int_0^1 Y_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$Y_{F,t} = \left[\int_0^1 Y_{j,t}{}^{\frac{\xi-1}{\xi}} dj\right]^{\frac{\xi}{\xi-1}}$$

$$Y_{j,t} = \left[\int_0^1 Y_{j,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$Y_t = \left[(1-\rho)^{\frac{1}{\omega}} \cdot Y_{H,t}{}^{\frac{\omega-1}{\omega}} + \rho^{\frac{1}{\omega}} \cdot Y_{F,t}{}^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$$

where i represents the brand of intermediate goods, j is the country index, $Y_{j,t}$ is the foreign goods bundle from country j, ε denotes the elasticity of substitution between the differentiated intermediate goods within one single country, ξ measures the substitutability between goods produced in different foreign countries, $\omega(>0)$ represents the elasticity of substitution between domestic and foreign goods, and ρ refers to the share of domestic aggregate demand allocated to imported goods and is thus a natural index of openness of the small open economy in our model.

Then given the price levels of goods, the cost minimization problem of the representative final good producer yields the following demand functions:

$$Y_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \cdot Y_{H,t} \tag{12}$$

$$Y_{j,t}(i) = \left(\frac{P_{j,t}(i)}{P_{j,t}}\right)^{-\varepsilon} \cdot Y_{j,t} \tag{13}$$

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_{F,t}}\right)^{-\xi} \cdot Y_{F,t} \tag{14}$$

$$Y_{H,t} = (1-\rho) \cdot \left(\frac{P_{H,t}}{P_t}\right)^{-\omega} \cdot Y_t \tag{15}$$

$$Y_{F,t} = \rho \cdot \left(\frac{P_{F,t}}{P_t}\right)^{-\omega} \cdot Y_t \tag{16}$$

where $P_{H,t}(i)$, $P_{j,t}(i)$, $P_{j,t}$, $P_{H,t}$, $P_{F,t}$, and P_t are respectively the price levels of home-made intermediate good i, imported intermediate good i from country j, imported goods bundle from country j, home good, foreign good and final good, all denominated in domestic currency. So in fact $P_{H,t}$ is the GDP deflator. Since the final good producers are perfectly competitive and there is no profit for them, we can easily derive the following price index formulas:

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \tag{17}$$

$$P_{j,t} = \left[\int_0^1 P_{j,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \tag{18}$$

$$P_{F,t} = \left[\int_0^1 P_{j,t}{}^{1-\xi} dj\right]^{\frac{1}{1-\xi}} \tag{19}$$

$$P_t = \left[(1-\rho) \cdot P_{H,t}{}^{(1-\omega)} + \rho \cdot P_{F,t}{}^{(1-\omega)}\right]^{\frac{1}{1-\omega}} \tag{20}$$

Now we give some definitions and derive some identities which link the price levels, exchange rates and terms of trade. For simplicity and tractability, we assume *symmetric steady state* for all the countries in our model, which means in the steady state many variables of the home country and foreign countries share the same properties, and some of them have the same values or dynamics.

The effective terms of trade (TOT) is defined by:

$$TOT_t \triangleq \frac{P_{F,t}}{P_{H,t}} = \left[\int_0^1 TOT_{j,t}{}^{1-\xi} dj\right]^{\frac{1}{1-\xi}} \tag{21}$$

where $TOT_{j,t} = P_{j,t}/P_{H,t}$ is the terms of trade for country j, and the second part of the above equation is derived from equation (18). Equation (21) can be approximated around the symmetric steady state when $TOT_{j,t} = 1$ for all $j \in [0,1]$ by:

$$tot_t \triangleq ln(TOT_t) \approx \int_0^1 tot_{j,t} dj$$

where $tot_{j,t} \triangleq ln(TOT_{j,t})$. Hereafter, for a variable denoted by a capital letter, say X_t, the corresponding small letter x_t is defined as its logarithm: $x_t \triangleq lnX_t$.

Similarly, log-linearization of equation (20) around the symmetric steady state when $P_{j,t} = P_{F,t} = P_{H,t} = P_t$, together with the definition of the effective terms of trade, equation (21), will result in the following equations:

$$lnP_t = lnP_{H,t} + \rho \cdot tot_t \tag{22}$$

$$lnP_t = lnP_{F,t} - (1-\rho) \cdot tot_t \tag{23}$$

Given the definition of inflation rate ($\Pi_t \triangleq \Delta\, lnP_t$), equation (22) gives:

$$\Pi_t = \Pi_{H,t} + \rho \cdot \Delta\, tot_t$$

where Δ is the difference operator.

The law of one price (LOOP) is assumed to hold, so:

$$P_{j,t}(i) = E_t^j \cdot P_{j,t}^j(i), \text{for } \forall \text{ j, i} \in [0,1]$$

where E_t^j is the exchange rate, price of country j's currency denominated in domestic currency; and $P_{j,t}^j(i)$ is the price of intermediate good i imported from country j, denominated in country j's currency. Combining this LOOP condition with equation (18) leads to:

$$P_{j,t} = E_t^j \cdot P_{j,t}^j, where\ P_{j,t}^j = \left[\int_0^1 P_{j,t}^j(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$$

Log-linearization of equation (19) around the symmetric steady state when $P_{j,t} = P_{F,t} = P_{H,t} = P_t$, together with the above equation, will give the result below:

$$lnP_{F,t} = \int_0^1 (lnE_t^j + lnP_{j,t}^j)dj = e_t + p_t^*$$

where $e_t \triangleq \int_0^1 (lnE_t^j)dj$ is the log aggregate exchange rate, and $p_t^* = \int_0^1 (lnP_{j,t}^j)dj$ is the log world price index.[5] Equation (23) and the above equation can yield:

$$tot_t = \frac{1}{1-\rho} \cdot (e_t + p_t^* - lnP_t)$$

If the aggregate exchange rate and world price level are defined as $EX_t \triangleq exp(e_t)$, and $P_t^* \triangleq exp(p_t^*)$, then the equation above gives the relationship between the real exchange rate, $REX_t = EX_t \cdot P_t^*/P_t$, and terms of trade, tot_t:

$$ln(REX_t) = (1-\rho) \cdot tot_t$$

This equation tells us that: although the LOOP holds for each individual intermediate good, the purchasing power parity (PPP) does not hold; and the real exchange rate may fluctuate over time as a result of variations in the relative price of home and foreign good.

3.4. Intermediate-goods firms and price setting

Intermediate goods market is monopolistically competitive. Firm i produces a differentiated intermediate good i with a Cobb-Douglas production function:

$$Y_t(i) = \epsilon_t \cdot K_t(i)^{1-\alpha} \cdot [A_t \cdot L_t(i)]^\alpha \tag{24}$$

where total factor productivity is decomposed into a temporary shock ϵ_t and a permanent stochastic trend A_t, whose stochastic processes are respectively:

$$ln\epsilon_t = \rho_a \cdot ln\epsilon_{t-1} + v_t^a, v_t^a \sim N(0, \sigma_a^2)$$

and

$$lnA_t = lnA_{t-1} + \gamma_t^A$$
$$\gamma_t^A = \gamma_{t-1}^A + \eta_t^A, \eta_t^A \sim N(0, \sigma_A^2)$$

When the economy reaches its stochastic steady state, the temporary shock is equal to its mean value one, and the total factor productivity is $(A_t)^\alpha$.

Then FOCs of the cost minimization problem are given by:

$$r_t^1 = (1-\alpha) \cdot \epsilon_t \cdot \left[\frac{K_t(i)}{A_t \cdot L_t(i)}\right]^{-\alpha} \cdot mc_t(i) \tag{25}$$

$$\frac{W_t}{P_t} = \alpha \cdot \epsilon_t \cdot A_t \cdot \left[\frac{K_t(i)}{A_t \cdot L_t(i)}\right]^{1-\alpha} \cdot mc_t(i)$$

where $mc_t(i)$ is the real marginal cost. The above two equations can imply:

$$\frac{W_t}{P_t \cdot r_t^1} = \frac{\alpha}{1-\alpha} \cdot \frac{K_t(i)}{L_t(i)} \tag{26}$$

Then combining the above equation and equation (25), we can get:

$$mc_t(i) = \frac{1}{\epsilon_t} \cdot \frac{(1-\alpha)^{\alpha-1}}{\alpha^\alpha} \cdot \left(\frac{W_t}{P_t \cdot A_t}\right)^\alpha \cdot (r_t^1)^{1-\alpha} \equiv mc_t \tag{27}$$

Following the staggered price setting of Calvo (1983), we assume each

[5] Here domestic price does not affect the world price index, since we assume each country is of zero measure.

intermediate-goods firm may re-optimize its nominal price only with probability $1-\theta$ in any given period. With probability θ, instead, the firm automatically and costlessly adjusts its price according to an indexation rule. There are two types of indexation rules usually employed in the literature: to steady state inflation, such as Yun (1996); and to past inflation rates, such as Christiano *et al.* (2005).[6] For simplicity, the steady-state-inflation indexation rule is adopted. Combining the fact that all firms resetting prices will choose an identical price $P_{H,t}^S$ and the equation (17), we can get:

$$P_{H,t}=\left[\theta\cdot\left((1+\overline{\Pi})\cdot P_{H,t-1}\right)^{1-\varepsilon}+(1-\theta)\cdot {P_{H,t}^{S}}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \qquad (28)$$

At the deterministic steady state, $P_{H,t}=P_t$, and $P_t/P_{t-1}=1+\overline{\Pi}$, where $\overline{\Pi}$ is the deterministic steady-state inflation. So at the steady state $P_{H,t}^S=P_{H,t}$.

The price-resetting firm sets price $P_{H,t}^S$ to maximize the current market value of the profits generated while that price remains effective, which means it solves the following optimization problem:

$$\max_{P_{H,t}^S}\sum_{k=0}^{\infty}\theta^k\cdot E_t\left\{F_{t,t+k}\cdot\left[P_{H,t}^S\cdot(1+\overline{\Pi})^k\cdot Y_{t+k|t}-\Phi_{t+k}(Y_{t+k|t})\right]\right\} \qquad (29)$$

subject to the sequence of demand constraints:

$$Y_{t+k|t}=\left[\frac{P_{H,t}^S\cdot(1+\overline{\Pi})^k}{P_{H,t+k}}\right]^{-\varepsilon}\cdot Y_{t+k}^d \qquad (30)$$

where $F_{t,t+k}$ is the discount factor for nominal payoffs, Φ_{t+k} is the nominal cost function, $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last freely reset its price in period t, and Y_{t+k}^d is the total demand (including domestic and foreign) for domestic goods. FOC of the above problem is given by:

$$\sum_{k=0}^{\infty}\theta^k\cdot E_t\left\{F_{t,t+k}\cdot Y_{t+k|t}\cdot\left[P_{H,t}^S\cdot(1+\overline{\Pi})^k-\boldsymbol{\kappa}\cdot\Phi'_{t+k|t}\right]\right\}=0 \qquad (31)$$

where $\Phi'_{t+k|t}=\Phi'_{t+k}(Y_{t+k|t})$ is the nominal marginal cost in period $t+k$ for a firm that last reset its price in period t and $\Phi'_{t+k|t}=P_{t+k}\cdot mc_{t+k|t}$, and $\boldsymbol{\kappa}=\varepsilon/(\varepsilon-1)$ which can be interpreted as the desired or frictionless markup.

We guess (and will prove later on) that in the deterministic steady state, consumption and output grow at the same rate as the average growth rate of permanent technology, γ_A. And in deterministic the steady state, price levels grow at the rate of $\overline{\Pi}$. So given equation (8) and (9), a first-order Taylor expansion of equation (31) around the constant inflation steady state yields:

$$lnP_{H,t}^S-lnP_{H,t-1}=(1-\beta\cdot\theta)\cdot\sum_{k=0}^{\infty}(\beta\cdot\theta)^k\cdot E_t\left\{\begin{matrix}\widehat{mc_{t+k|t}}+lnP_{t+k}\\-lnP_{H,t-1}-\boldsymbol{\kappa}\cdot\overline{\Pi}\end{matrix}\right\} \qquad (32)$$

where $\widehat{mc_{t+k|t}}=ln(mc_{t+k|t})-ln(mc)$ is the log deviation of real marginal cost

[6] Ascari *et al.* (2010) estimate and compare New-Keynesian DSGE monetary models of the business cycle derived under two different pricing schemes – Calvo and Rotemberg – under a positive trend inflation rate. Their empirical findings provide evidence in favor of the statistical superiority of the Calvo setting. That is one reason why in our model we choose the Calvo pricing scheme.

from its steady state value mc, and $mc = 1/\boldsymbol{\kappa} = (\varepsilon - 1)/\varepsilon$.

Equation (27) says that the real marginal cost is independent of the level of production, and hence $\widehat{mc_{t+k|t}} = \widehat{mc_{t+k}}$ in equation (32), which can then imply that:

$$lnP_{H,t}^{S} - lnP_{H,t-1} = \beta \cdot \theta \cdot E_t\left(lnP_{H,t+1}^{S} - lnP_{H,t}\right)$$

$$+(1 - \beta \cdot \theta) \cdot \left(\widehat{mc}_t + lnP_t - lnP_{H,t}\right) + \Pi_{H,t} - \beta \cdot \theta \cdot \overline{\Pi}$$

The above equation, together with equation (28) and (22), will give the following open-economy New-Keynesian Phillips curve (NKPC):

$$\Pi_{H,t} = (1 - \beta) \cdot \overline{\Pi} + \beta \cdot E_t\left(\Pi_{H,t+1}\right) + \frac{(1 - \theta) \cdot (1 - \beta \cdot \theta)}{\theta} \cdot (\widehat{mc}_t + \rho \cdot tot_t) \quad (33)$$

When the steady-state inflation rate is higher (probably driven by higher growth of money supply), or the expectation of future home-good price inflation increases, the home-good inflation in the current period will increase as well. When real marginal cost is higher than its steady state value, home-good inflation is going to increase. When the foreign-good price inflation increases, the terms of trade will increase and thus lead to a higher home-good inflation (the so called “imported inflation”). The greater the degree of the economy openness (ρ) is, the bigger this kind of imported-inflation effect will be. In Galí and Monacelli (2005) and Funke *et al.* (2010), the terms of trade do not appear in their open economy NKPC, because in their models nominal marginal cost is calculated as the real marginal cost times home good price level, rather than final good price level used in our model. This is not reasonable, since it is an open economy and the households consume final good that is synthesized by both home good and foreign good. So the nominal wage rate should correspond to the final good price level, rather than only the home good price. The foreign good price can affect the home good inflation through nominal marginal cost (such as nominal wage rate for workers). Finally, the sticker the price setting is (higher θ), the smaller the effects of real marginal cost and terms of trade on home-good price inflation are. This is because the price adjustment of firms is now more inertial and less sensitive to the changes of market environment.

3.5. Government, taxation, and fiscal and monetary policies

For the government of our small open economy, fiscal revenue consists of six components: labour income tax with rate τ_L, capital rental income tax with rate τ_K, consumption tax with rate τ_C, bond interest income tax with rate τ_B, share dividend tax with rate τ_D, and nominal lump-sum tax LST_t. Therefore, the nominal aggregate fiscal revenue FR_t is equal to:

$$FR_t = \tau_L \cdot W_t \cdot L_t + \tau_K \cdot r_t^1 \cdot P_t \cdot K_t + \tau_B \cdot R_{t-1}^2 \cdot B_t$$

$$+\tau_C \cdot P_t \cdot C_t + \int_0^1 S_t(i) \cdot \tau_D \cdot D_t(i) di + LST_t$$

Government debt B_t evolves according to:

$$B_{t+1} = (1 + R_{t-1}^2) \cdot B_t - FB_t$$

where $FB_t = FR_t - G_t$ denotes fiscal balances (surplus), and G_t is the nominal government expenditure. We define the debt-GDP ratio b_t and government expenditure-GDP ratio g_t as follows:

$$b_t = B_t / (P_{H,t} \cdot GDP_t) \tag{34}$$

$$g_t = G_t / (P_{H,t} \cdot GDP_t) \tag{35}$$

We assume that government expenditure-GDP ratio g_t follows an $AR(1)$ process:[7]

$$ln(g_t) = (1 - \rho_G) \cdot ln(g) + \rho_G \cdot ln(g_{t-1}) + v_t^G, v_t^G \sim N(0, \sigma_G^2) \tag{36}$$

We also assume the lump-sum tax rule of the government is to react to deviations from the target debt-GDP ratio b with a lag. Define the lump-sum tax-GDP ratio lst_t as:

$$lst_t = LST_t / (P_{H,t} \cdot GDP_t)$$

Then the lump-sum tax rule has the following format:

$$\frac{lst_t}{lst} = \left(\frac{b_{t-1}}{b}\right)^{\varepsilon_{LST}} \tag{37}$$

where lst is the steady-state level of lump-sum tax-GDP ratio, and ε_{LST} is the elasticity.

In terms of monetary policy, we assume a Taylor type empirical monetary policy rule for nominal interest rate R_t^2 as follows:

$$\begin{aligned} R_t^2 = (1 - \rho_R) \cdot [\bar{R} + \varphi_1 \cdot (\Pi_t - \bar{\Pi}) + \varphi_2 \cdot \mathrm{GAP}_t] \\ + \rho_R \cdot R_{t-1}^2 + v_t^R, v_t^R \sim N(0, \sigma_R^2) \end{aligned}$$

where $\bar{R}$ is the steady-state level of nominal interest rate, $\bar{\Pi}$ is the deterministic steady-state inflation rate and is also assumed to be the target inflation rate of the monetary authority, and GAP_t is the real GDP gap.

3.6. Equilibrium

3.6.1. Resources constraints and aggregate demand

For labour market and capital market, we have the following market clearing conditions:

$$L_t = \int_0^1 L_t(i)\, di \tag{38}$$

$$K_{t+1} = \int_0^1 K_{t+1}(i)\, di = (1 - \delta) \cdot K_t + I_t \tag{39}$$

We define the real GDP in the way below:

$$GDP_t \triangleq \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

So given the budget constraint $P_{H,t} \cdot GDP_t = \int_0^1 P_{H,t}(i) \cdot Y_t(i) di$, the demand of the

[7] Here we actually assume in the steady state government expenditure is a fixed proportion of GDP. This is to make our model solvable, but not meaning that we care about the "trend" of real government expenditure. As we explained previously, real government expenditure is discretionary in reality and fiscal economists usually do not consider its cyclical property. This is why in this paper when we calculate structural fiscal balances; we only adjust nominal government expenditure one-to-one to the cycle of aggregate price level, but ignore the cyclicality of real government expenditure.

intermediate good i is given by:

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \cdot GDP_t \tag{40}$$

Integrating the above equation over all the intermediate-goods firms, given equation (38), (39), (24) and (26), will yield the following aggregate production function:

$$GDP_t \cdot Disp_t = \epsilon_t \cdot {K_t}^{1-\alpha} \cdot [A_t \cdot L_t]^\alpha \tag{41}$$

where $Disp_t$ is the price dispersion defined as follows, and it is greater than or equal to one.

$$Disp_t = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} di \tag{42}$$

For the final good demand Y_t, we have the following identity:

$$Y_t = C_t + I_t + G_t/P_t$$

The market clearing condition for each intermediate good is :

$$Y_t(i) = Y_{H,t}(i) + \int_0^1 Y_{H,t}^j(i) dj$$

where $Y_{H,t}^j(i)$ is the demand of home-made intermediate good i from country j. According to equations (12)-(16) and their counterparts for each foreign country j, the above equation is equivalent to the following:

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \cdot \begin{bmatrix} (1-\rho) \cdot \left(\frac{P_{H,t}}{P_t}\right)^{-\omega} \cdot Y_t + \\ \rho \cdot \int_0^1 \left(\frac{P_{H,t}}{E_t^j \cdot P_{F,t}^j}\right)^{-\xi} \cdot \left(\frac{P_{F,t}^j}{P_t^j}\right)^{-\omega} \cdot Y_t^j \, dj \end{bmatrix}$$

where $P_{F,t}^j$, P_t^j and Y_t^j are respectively foreign good price, aggregate price, and aggregate demand of country j.

Plugging the equation above into equation (40), we can obtain:

$$GDP_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\omega} \cdot \left[(1-\rho) \cdot Y_t + \rho \cdot \int_0^1 \left(TOT_t^j \cdot TOT_{j,t}\right)^{\xi-\omega} \cdot \left(REX_t^j\right)^{\omega} \cdot Y_t^j \, dj\right]$$

where $TOT_{j,t} = P_{j,t}/P_{H,t}$ represents the bilateral terms of trade between the home country and foreign country j, and TOT_t^j is the effective terms of trade of country j.

First order log-linear approximation of the equation above around the symmetric steady state when $TOT_t^j = TOT_{j,t} = 1$ and $Y_t^j = Y_t$ can lead to the following:

$$gdp_t = (1-\rho) \cdot y_t + \rho \cdot y_t^* + \rho \cdot [\xi + \omega \cdot (1-\rho)] \cdot tot_t \tag{43}$$

where $gdp_t = ln(GDP_t)$, $y_t = ln(Y_t)$, and $y_t^* = \int_0^1 ln(Y_t^j) dj$ is the log world aggregate demand.

Since symmetry is assumed in our model, a condition analogous to equation (43) will hold for all countries. So we can derive a world market clearing condition as below:

$$gdp_t^* \triangleq \int_0^1 gdp_t^j \, dj = \int_0^1 (1-\rho) \cdot y_t^j \, dj + \rho \cdot y_t^*$$

$$+\rho \cdot [\xi + \omega \cdot (1-\rho)] \cdot \int_0^1 tot_t^j dj = y_t^* \quad (44)$$

where $\int_0^1 tot_t^j dj = 0$ can be easily proved. Equation (44) tells an intuitive result: for the world as a whole, aggregate product (supply) equals aggregate demand. Substituting equation (44) into equation (43), we can obtain:

$$gdp_t = (1-\rho) \cdot y_t + \rho \cdot gdp_t^* + \rho \cdot [\xi + \omega \cdot (1-\rho)] \cdot tot_t$$

3.6.2. Stock market

Euler equation (5) of the stock market can give the equation below:

$$Q_t(i) = E_t\{F_{t,t+1} \cdot [Q_{t+1}(i) + (1-\tau_D) \cdot D_{t+1}(i)]\}$$

We define the aggregate dividend and aggregate stock price as follows:

$$\begin{cases} D_t \triangleq \int_0^1 D_t(i) di \\ Q_t \triangleq \int_0^1 Q_t(i) di \end{cases}$$

Then we can get the following aggregate relationship:

$$Q_t = E_t\{F_{t,t+1} \cdot [Q_{t+1} + (1-\tau_D) \cdot D_{t+1}]\} = \sum_{k=1}^{\infty} (1-\tau_D) \cdot E_t\{F_{t,t+k} \cdot D_{t+k}\} \quad (45)$$

This result is intuitive, saying that the aggregate stock price is equal to the present value of all the future after-tax aggregate dividends. Since the dividend comes from the monopolistic profit of intermediate-goods firms, we can get:

$$D_t = P_{H,t} \cdot GDP_t - W_t \cdot L_t - P_t \cdot r_t^1 \cdot K_t \quad (46)$$

3.6.3. Aggregate supply: marginal cost and NKPC

Combining equation (25) with equation (26) and (24) will lead to:

$$r_t^1 \cdot K_t(i) + \frac{W_t}{P_t} \cdot L_t(i) = Y_t(i) \cdot mc_t$$

Given equation (40) and (42), integrating the equation above over intermediate-goods firms will give the following aggregate equation:

$$r_t^1 \cdot K_t + \frac{W_t}{P_t} \cdot L_t = GDP_t \cdot mc_t \cdot Disp_t \quad (47)$$

Combining the equation above with the equation for aggregate dividend, equation (46), we can get:

$$D_t = GDP_t \cdot (P_{H,t} - P_t \cdot mc_t \cdot Disp_t) \quad (48)$$

Log-linearization of equation (48) around the steady state gives the following:

$$lnD_t = gdp_t + \varepsilon \cdot lnP_{H,t} - (\varepsilon - 1) \cdot [lnP_t + ln(mc_t)] + \alpha_5 \quad (49)$$

where $ln(Disp_t) = 0$ because $Disp_t$ is of second order and is equal to zero up to a first order approximation (see Galí and Monacelli, 2005; and others) .

As said previously, it is assumed there is a stochastic trend in the aggregate price level P_t. Now assume the measurement equation and transition equation of the aggregate price level P_t take the following form:

$$lnP_t = lnM_t + lnN_t$$
$$lnM_t = lnM_{t-1} + \gamma_t^P$$
$$\gamma_t^P = \gamma_{t-1}^P + \eta_t^P, \eta_t^P \sim N(0, \sigma_P^2)$$

where M_t is the stochastic trend of the aggregate price level, and N_t is the cycle.

Equation (49), written in a cycle form, can lead to:

$$\widehat{D_t} = \mathrm{GAP}_t + \varepsilon \cdot \widehat{P_{H,t}} - (\varepsilon - 1) \cdot (\widehat{P_t} + \widehat{mc_t})$$

where variables with hat denote the corresponding cyclical components: percentage deviation from variables' steady state levels, and GAP_t is the real GDP gap. Combining the equation above with equation (33) and (22), given the fact that $\widehat{P_t} = lnN_t = \widehat{N_t}$, we can get the following NKPC:

$$\Pi_{H,t} = (1-\beta) \cdot \overline{\Pi} + \beta \cdot E_t(\Pi_{H,t+1})$$
$$+ \frac{(1-\theta) \cdot (1 - \beta \cdot \theta)}{\theta \cdot (\varepsilon - 1)} \cdot \left(\mathrm{GAP}_t - \widehat{D_t} - \rho \cdot tot_t + \widehat{N_t}\right)$$

Therefore, the GDP deflator inflation $\Pi_{H,t}$ depends on trend inflation, inflation expectation, GDP gap, stock dividend (monopolistic profit) gap, terms of trade, and the aggregate price cycle.

3.7. Trend-cycle decomposition of the model: a state-space representation

Log-linearization of our model around its deterministic steady-state balanced growth path will lead to a set of equations linking the variables' cyclical components. Here when one variable is bond interest rate, capital rental rate, or inflation rate, its cyclical component means the difference between its level and its steady state value. Otherwise, cyclical component of one variable denotes the percentage deviation from its steady state value. All the cyclical components of the relevant variables are denoted by hat, except that the terms of trade cycle and the output gap are denoted by tot_t and GAP_t respectively.

In Appendix A, 24 log-linearized equations linking all the aggregate variables of interest are provided. As a whole, this linear cyclical DSGE system can be written in the following form:

$$E_t\left[\mathcal{F}_\Theta\left(\widehat{X_{t+1}}, \widehat{X_t}, \widehat{X_{t-1}}, \Xi_t\right)\right] = 0$$

where X_t is a vector of all the endogenous variables in the system, $\widehat{X_t}$ is its corresponding cyclical component vector, $\Xi_t \sim iid(0, \Sigma)$ is a random vector of structural shocks, and $\mathcal{F}_\Theta$ is a real function parameterized by a real vector Θ gathering the deep parameters of the model. The whole system is stochastic, forward looking and linear.

When a unique, stable and invariant solution of the system exists, it can be given by a stochastic vector difference equation:

$$\widehat{X_t} = \mathcal{H}_\Theta\left(\widehat{X_{t-1}}, \Xi_t\right) \tag{50}$$

Then the endogenous variables are written as a function of their lags and the contemporaneous structural shocks. $\mathcal{H}_\Theta$ collects the corresponding policy rules and transition functions. Generally, it is not possible to get a closed form solution, and usually an approximation of the true solution (50) is considered. A local approximation of equation (50) around the steady state when $\widehat{X_t} = 0$ can be used, and then solution (50) is approximately linear.

Suppose X_t^O is the vector of observable variables, and then the trend-cycle decomposition of the data should take the following form:

$$X_t^O = \overline{X_t^O} + \widehat{X_t^O}$$

where $\overline{X_t^O}$ is the vector of trend components, and in this paper it is a vector integrated random walk as follows:

$$\overline{X_t^O} = \overline{X_{t-1}^O} + \Gamma_t$$

$$\Gamma_t = \Gamma_{t-1} + \Lambda_t$$

And $\widehat{X_t^O}$ is the vector of the observables' cyclical components, and it is a linear function of $\widehat{X_t}$:

$$\widehat{X_t^O} = \mathrm{Z} \cdot \widehat{X_t}$$

Therefore, we can put the whole model together with the observable variables into a state space representation as below:

$$\begin{cases} X_t^O = \overline{X_t^O} + \widehat{X_t^O} \\ \overline{X_t^O} = \overline{X_{t-1}^O} + \Gamma_t \\ \Gamma_t = \Gamma_{t-1} + \Lambda_t \\ \widehat{X_t^O} = \mathrm{Z} \cdot \widehat{X_t} \\ \widehat{X_t} = \mathcal{H}_\Theta\left(\widehat{X_{t-1}}, \Xi_t\right) \end{cases}$$

Since there are unit root processes (integrated random walks), from the computational point of view we need to take second-order difference to put the above state space form into a stationary representation as below:

$$\begin{cases} \Delta^2 X_t^O = \widehat{X_t^O} - 2\widehat{X_{t-1}^O} + \widehat{X_{t-2}^O} + \Lambda_t \\ \widehat{X_t^O} = \mathrm{Z} \cdot \widehat{X_t} \\ \widehat{X_t} = \mathcal{H}_\Theta\left(\widehat{X_{t-1}}, \Xi_t\right) \end{cases} \tag{51}$$

3.8. Steady state

It is necessary to solve and analyze the steady state of our model for two reasons. First of all, the steady state is one of the key elements of DSGE paradigm, and in our framework the steady state corresponds to the trend of the whole economy. The steady state balanced growth property determines the co-integration relationships among the trend components in the above state space model. Secondly, some parameters of the log-linearized equation system linking the cyclical components of aggregate variables in the above section depend on the steady state property of the model.

We first solve for the steady state balance growth path when the steady state is deterministic, i.e. $\eta_t^A \equiv 0$, and $\eta_t^M \equiv 0$. Thus, all the growth rates on the steady state balanced growth path are the functions of the following two basic constant growth rates:

$$\begin{cases} BG_A = \gamma_A \\ BG_M = \overline{\Pi} \end{cases}$$

Henceforth, we use BG_x to denote the growth rate of variable x at the steady state.

The "guess and verify" method is employed to obtain steady state growth rates for aggregate variables. Equations (26), (27), (34), (35), (41), (2), and (48) & (45) can respectively lead to the following relationships among growth rates:

$$\begin{cases} BG_W - BG_M - BG_{r_t^1} = BG_K - BG_L \\ BG_{mc} = \alpha \cdot (BG_W - BG_M - BG_A) + (1-\alpha) \cdot BG_{r_t^1} \\ BG_B = BG_M + BG_{GDP} \\ BG_G = BG_M + BG_{GDP} \\ BG_{GDP} = \alpha \cdot (BG_A + BG_L) + (1-\alpha) \cdot BG_K \\ BG_C = BG_W - BG_M \\ BG_D = BG_Q = BG_P + BG_{GDP} \end{cases}$$

Since at the steady state, $mc = (\varepsilon - 1)/\varepsilon$, we can get $BG_{mc} = 0$. Equation (3) and (4) can yield the following:

$$\begin{cases} R_t^2 = \dfrac{1}{1-\tau_B} \cdot (BG_C - ln\beta + BG_M) \\ r_t^1 = \dfrac{1}{1-\tau_K} \cdot (BG_C - ln\beta + \delta) \end{cases}$$

Therefore, $BG_{R_t^2} = 0$, and $BG_{r_t^1} = 0$ as well.

Since there is no population growth in our model, it is reasonable to guess that $BG_L = 0$. And we also *guess*:

$$\begin{cases} BG_{GDP} = BG_C = BG_I = BG_K = BG_A \\ BG_W = BG_A + BG_M \\ BG_{FR} = BG_A + BG_M \\ BG_B = BG_G = BG_A + BG_M \end{cases} \tag{52}$$

It is easy to *verify* that: given the above guess, the whole system is self-consistent at the steady state.

In the above log-linearized DSGE system linking the cyclical components, there are some unknown parameters which are determined by the steady state balanced growth path. To be specific, four ratios need to be pinned down:

$$\begin{cases} RT_D \triangleq \overline{D_t}/(\overline{P_{H,t}} \cdot \overline{GDP_t}) \\ RT_Q \triangleq \overline{Q_t}/(\overline{P_{H,t}} \cdot \overline{GDP_t}) \\ RT_K \triangleq \overline{K_t}/\overline{GDP_t} \\ RT_{FR} \triangleq \overline{FR_t}/(\overline{P_{H,t}} \cdot \overline{GDP_t}) \end{cases}$$

The ratio RT_D can be given by equation (48): $RT_D = 1/\varepsilon$, if ε is known. The ratio RT_Q can be computed by RT_D and the steady-state price-earnings ratio of the stock market, $\overline{Q_t}/\overline{D_t}$. RT_{FR} can be given by the average nominal fiscal revenue-GDP ratio of the data. And finally RT_K and the elasticity of substitution between intermediate goods within one single country ε, given the capital depreciation rate δ, can be derived by equation (47) and (39) as follows:

$$RT_K = \frac{\varepsilon - 1}{\varepsilon \cdot (\gamma_A - ln\beta + \delta)} \cdot (1-\alpha) \cdot (1-\tau_K) \tag{53}$$

and

$$\varepsilon = 1 + \frac{1}{1 - \frac{(\delta + \gamma_A) \cdot (1-\alpha) \cdot (1-\tau_K)}{RT_I \cdot (\delta + \gamma_A - ln\beta)}} \tag{54}$$

where RT_I denotes the investment-GDP ratio in the steady state, which can be given by the average investment-GDP ratio of the data. Note that in this paper the elasticity of substitution between intermediate goods is endogenously, rather than exogenously determined.

Now we come to the stochastic steady state when the two basic exogenous growth rates are random walks and given by:

$$\begin{cases} BG_A = \gamma_t^A = \gamma_{t-1}^A + \eta_t^A \\ BG_M = \gamma_t^P = \gamma_{t-1}^P + \eta_t^P \end{cases} \tag{55}$$

As in the literature, for simplicity we assume that the balanced growth property in the deterministic steady state situation is maintained in the setting of stochastic steady state. For example, in the stochastic steady state: GDP, consumption and investment will all grow at the rate BG_A; nominal aggregate fiscal revenue and government expenditure will both grow at the rate $BG_A + BG_M$.

4. BAYESIAN ESTIMATION AND SIGNAL EXTRACTION

4.1. UK as an example

We take the UK as an example of the small open economy in our model. Six macroeconomic time series are considered: real GDP (GDP_t), GDP deflator ($P_{H,t}$), nominal primary fiscal revenue (FR_t), nominal primary government expenditure (G_t), stock price index (Q_t), and terms of trade index (tot_t). Here terms of trade index is normalized to make its mean equal to one. The data, from IMF WEO database, is annual data and the sample period is from 1981 to 2011. It is worth pointing out that we do not use the data for real consumption, real investment and nominal government bond even though they are available, because for the Bayesian estimation implemented in this paper the number of observable variables must be smaller than or equal to the number of exogenous shocks. As shown by equation (51), all the six time series except terms of trade should be transformed into their second-order difference before the estimation. We plot the data in Appendix C1.

Some unknown parameters of our open economy DSGE model will be estimated by Bayesian method, conditional on prior information concerning the values of parameters. We abandon the standard but problematic practice which at the very beginning removes trend components from the observed macroeconomic variables simply using such as the HP filter. Instead, we do the model estimation and signal extraction of trend-cycle decomposition simultaneously. Advances have been made during recent years in estimating DSGE models, shifting emphasis in quantitative macroeconomics from calibration exercises to directly estimating the parameters of a structural model and letting the data speak. The so called Bayesian technique, as strongly claimed by An and Schorfheide (2007) and others, is currently the standard tool to estimate DSGE models. Just as shown in this paper, linear approximation of a DSGE model can lead to a state space representation that could be analyzed using the Kalman filter. Given the specification of prior distributions for the parameters and the likelihood based on the data, the state space representation can then yield the parameter's posterior distribution. Bayesian estimation is to maximize the likelihood of the posterior distributions. Metropolis-Hastings Markov Chain Monte Carlo algorithm is employed to numerically obtain a sequence from the unknown posterior distributions. Once we get the posterior distributions of parameters, we use their posterior means to pin down the whole model. After that, we can do the impulse response analysis and variance decomposition as well, especially for the variable that we are mostly interested: nominal fiscal revenue FR_t. Then we can also do the signal extraction, using the Kalman smoother, to get a time series for the cyclical component of nominal fiscal revenue, $\widehat{FR}_t$, which is an unobserved endogenous variable in the state space representation of our framework. Therefore, the trend component of the nominal fiscal revenue can be obtained. Finally we can calculate the structural fiscal balances.

Bayesian estimation is in fact the middle-of-the-road line between the traditional calibration procedure and econometrically maximum likelihood estimation. On one hand, it lets data speak and can fully utilize the information in the data. On the other hand, reasonable prior distributions guarantee that the estimation result does not deviate too much away from macroeconomics theories.

4.2. Calibration

There are some parameters that remain fixed during the estimation procedure, and need to be calibrated. This kind of parameters fall into two categories: one includes those parameters that are difficult to estimate, such as substitution elasticities between home and foreign goods and Frisch wage elasticity of labour supply; the other category is a collection of parameters that are better identified using other information. To account for these calibrated parameters' influence on the estimation results, robustness checks will be executed at last, by using alternative values for these parameters. We first present the baseline calibration of these fixed parameters as follows.

Table 3 lists the baseline calibrated values for some parameters. The production function parameter α is set to 0.69, consistent with Faccini *et al.* (2011), Bhattarai and Trzeciakiewicz (2012), and many other studies on the UK economy. Nevertheless, Dicecio and Nelson (2007) use a value of 0.64. We will do a robustness check for this aspect later on. The annual utility discount factor β is set to be 0.96, indicating that the quarterly discount factor is equal to 0.99. This follows Dicecio and Nelson (2007), Moons (2009), Faccini *et al.* (2011), and Bhattarai and Trzeciakiewicz (2012). The weight of consumption in the utility, ϕ, is set to be 0.50, following Paetz (2011). Two parameters in the Taylor rule, φ_1 and φ_2, are set to be 1.5 and 0.25 respectively. The former is based on the suggestion of Moons (2009) and Faccini *et al.* (2011), even thought Dicecio and Nelson (2007) employ a relatively smaller value, 1.28. The latter is the average number of those in Dicecio and Nelson (2007), and Moons (2009), since there is no commonly used setting.

Table 3. Baseline calibration

Parameter	Calibrated value	Based on
α	0.69	Faccini *et al.* (2011), Bhattarai and Trzeciakiewicz (2012)
β	0.96	Bhattarai and Trzeciakiewicz (2012), Faccini *et al.* (2011), Moons (2009), and Dicecio and Nelson (2007)
ϕ	0.50	Paetz (2011)
φ_1	1.50	Moons (2009), Faccini *et al.* (2011)
φ_2	0.25	Average of Dicecio and Nelson (2007) and Moons (2009)
δ	0.10	Faccini *et al.* (2011), where quarterly rate equals 0.025
ξ	1.5	Collard and Dellas (2002) suggest a value between 1 and 2
ω	6.0	Micro data suggest 5-10; Obstfeld and Rogoff (2000)
ε_{LST}	0.0005	Lafourcade and Wind (2012)
τ_C	0.169	Carey and Tchilinguirian (2000), consumption tax rate
τ_K	0.384	Carey and Tchilinguirian (2000), capital income tax rate
τ_L	0.210	Carey and Tchilinguirian (2000), labour income tax rate
τ_B	0.15	UK tax rates on savings income of the year 2012-2013
τ_D	0.15	UK dividend tax rates of the year 2012-2013

Now we look at several elasticities in our model: elasticity of substitution between intermediate goods within one single country ε, elasticity of substitution between goods from different foreign countries ξ, elasticity of substitution between domestic and foreign goods ω, and lump-sum tax feedback elasticity for government debt ε_{LST}.

In Funke *et al.* (2010) and many other New Keynesian DSGE models, ε is often calibrated to be 11, leading to a 10% steady-state markup over marginal cost. But in this paper, we make ε endogenously determined by other parameters, especially by the physical capital depreciation rate δ. Because, otherwise (if we set ε to be 11), according to equation (54) the capital depreciation rate can be a very unreasonable value, and the same happens for some other parameters whose values depend on δ. Therefore, first we set δ to be 0.10, consistent with Faccini *et al.* (2011), in which the quarterly rate of capital depreciation is equal to 0.025. Then equation (54) yields a value of 17.76 for ε, which means a 6%, rather than 10%, steady-state markup over marginal cost. For ξ, Collard and Dellas (2002) suggest a value between one and two, so we use 1.5, although Paetz (2011) adopts a smaller value, 1. For ω, micro data typically indicates a value in the range of 5 to 10 (Funke *et al.*, 2010); and Obstfeld and Rogoff (2000) have shown that such high elasticity can explain an observed large home bias in trade. So we set it to be 6 at the beginning. However, Moons (2009) and Paetz (2011) use a value of 1 and 1.5 respectively for UK. We will analyze it further in the section of robustness checks. $\varepsilon_{LST} = 0.0005$ is directly from Lafourcade and Wind (2012).

Tax rates need to be pinned down by referring to specialized tax studies. In this paper, as in many other macroeconomics papers, the so called "average effective tax rates (AETRs)", "implicit tax rates" or "tax ratios" are used to measure the effective overall tax burden from the major taxes and are consistent with the concept of aggregate tax rates at the national level and with the assumption of representative agent as well. Some estimation strategies have been proposed to "combine information on various statutory tax schedules, tax returns and tax codes with data on income distribution, household surveys, and projections of real present values for investment projects in specific industries" (Carey and Tchilinguirian, 2000), in order to provide suitable measure of aggregate taxation, AETRs. Mendoza *et al.* (1994) propose a method to compute aggregate tax rates for large industrial countries, and for UK their estimation is: $\tau_C = 0.14$, $\tau_K = 0.56$, and $\tau_L = 0.27$ for the period 1965-1988. Bhattarai and Trzeciakiewicz (2012) provide a different estimate for UK, and their calibration result is: $\tau_C = 0.2008$, $\tau_K = 0.4071$, and $\tau_L = 0.2844$. In this paper we use the result of OECD. Carey and Tchilinguirian (2000) study the tax systems of OECD countries and estimate average effective tax rates on capital, labour and consumption for these countries. According to their estimation, during the period of 1991-1997, UK has the following AETRs: $\tau_C = 0.169$, $\tau_K = 0.384$, and $\tau_L = 0.21$. In this paper, we do not consider the progressive property of the labour income tax, and just use the average rate. For the average tax rates on government bond interest and stock dividend, we cannot find professional and reliable studies. According to a website of UK government (http://www.hmrc.gov.uk/taxon/uk.htm), in the year of 2012-2013 dividend tax rates are from the lowest 10% to the highest 42.5%, depending on one's overall taxable income; and tax rates on savings income are from 10% to 50%. Thus we use a relatively conservative value, 15%, to calibrate the average tax rates on bond interest income and dividend income, τ_B and τ_D.

Table 4. Steady state values and ratios

Value/Ratio	Calibrated value	Based on
$\bar{\Pi}$	0.03535	Data, average inflation rate
γ_A	0.02383	Data, average real GDP growth rate
g	0.37990	Data, average government expenditure-GDP ratio
b	0.45490	Data, average government bond-GDP ratio
L	0.50	Faccini *et al.* (2011), Paetz (2011)
RT_D	1/17.76	$RT_D = 1/\varepsilon$, and $\varepsilon = 17.76$
RT_{FR}	0.36604	Data, average fiscal revenue-GDP ratio
RT_I	0.17459	Data, average investment-GDP ratio
RT_{IC}	0.38184	Data, average investment-consumption ratio
RT_{QD}	13.6	(Average stock price-to-earnings ratio), an initial guess

Besides above parameters, several steady state values and ratios need to be pinned down as well, since some coefficients of the log-linearized DSGE equations rely on them. Table 4 gives the calibrated values for these steady state values and ratios.

Since at the steady state, real GDP grows at the same rate as permanent technology, according to equation group (99), we use the average real GDP growth rate to calibrate the parameter γ_A. Following Faccini *et al.* (2011) and Paetz (2011), we assume the equilibrium labour supply is 0.50, in order to ensure that the Frisch elasticity, $L\ /\ (1-L)$, is equal to 1. The ratio of dividend to GDP is set to 1/17.76, given the value of the elasticity of substitution between intermediate goods within one single country ε. RT_{QD} denotes the steady state ratio of stock price to gross dividend, and is set to be 13.6 as an initial guess. Here in fact we implicitly assume that the stock market P/E ratio (price-to-net-earnings ratio) of UK is 16, and then the ratio of stock price to gross dividend is calculated as 16 times $(1-\tau_D)$. All other ratios in Table 2 are computed as the average values of the data. RT_Q, stock price-GDP ratio at the steady state, is then computed as RT_{QD} times RT_D. The steady state capital stock-GDP ratio, RT_K, can be got by equation (53) and (54), given that other parameters are already known.

4.3. Prior and posterior distributions

Table 5 summarizes a detailed description of the prior distributions for some structural parameters in our DSGE model.

Table 5. Prior and posterior distributions

	Prior distribution			Posterior distribution		
Parameter	Type	Mean	St. dev.	Mean	Confidence	interval
ρ	beta	0.3	0.1	0.4413	[0.2110	0.6206]
θ	beta	0.2	0.1	0.7033	[0.5897	0.8681]
ρ_a	beta	0.7	0.1	0.7071	[0.5586	0.8506]
ρ_Q	beta	0.7	0.1	0.8397	[0.7437	0.9294]
ρ_{gap}^*	beta	0.7	0.1	0.8000	[0.6588	0.9300]
ρ_P^*	beta	0.7	0.1	0.7057	[0.5578	0.8602]
ρ_R	beta	0.7	0.1	0.8359	[0.5461	0.9755]
ρ_G	beta	0.7	0.1	0.7253	[0.6289	0.8183]
σ_a	invg	0.01	2	0.0134	[0.0024	0.0330]
σ_Q	invg	0.02	2	0.0262	[0.0139	0.0378]
σ_{gap}^*	invg	0.10	2	0.0671	[0.0364	0.0962]
σ_P^*	invg	0.01	2	0.0142	[0.0104	0.0179]
σ_R	invg	0.01	2	0.0104	[0.0036	0.0155]
σ_G	invg	0.03	2	0.0393	[0.0304	0.0479]
σ_A	invg	0.01	2	0.0160	[0.0040	0.0286]
σ_P	invg	0.01	2	0.0122	[0.0090	0.0152]

The parameter ρ, representing the weight of foreign good in the aggregate consumption, is an indicator of the degree of openness. It is bounded between zero (autarky) and one (complete integration). The prior mean value is chosen to be 0.30, as in Moons (2009) and Paetz (2011), to indicate a relatively high home bias of international trade for the UK. For the Calvo price rigidity parameter θ, we set its prior mean to be 0.20, which means that 20 percent of domestic firms cannot freely reset their prices within each year, and is equivalent to the circumstance that 50 percent of domestic firms cannot freely reset their prices within each quarter. This is a common setting for the UK economy, such as in Faccini *et al.* (2011), Paetz (2011) and Bhattarai and Trzeciakiewicz (2012). [8] Because ρ and θ are both bounded by the unit interval $[0, 1)$, we use beta distributions as their priors.

There are six persistence parameters for AR(1) processes: ρ_a, ρ_Q, ρ^*_{gap}, ρ^*_P, ρ_R, and ρ_G. Following Funke *et al.* (2010), the prior means of these persistence parameters are all set to be a standard value 0.7, and they are all subject to a beta distribution with stand deviation equal to 0.1. Bhattarai and Trzeciakiewicz (2012) also adopt this kind of prior distribution for the Taylor rule parameter ρ_R. For the corresponding six parameters of standard deviations, we use inverse-gamma distributions as their priors according to the standard convention, such as in Smets and Wouters (2007), Del Negro and Schorfheide (2008), Castelnuovo and Nisticó (2010) and Lafourcade and Wind (2012). And their prior means are chosen based on trials with a very weak prior, while the degrees of freedom of the inverse-gamma distributions are equal to 2, which corresponds to a rather loose prior.[9] σ_A and σ_P are the standard deviations of the innovations to the growth rates of stochastic permanent technology and trend aggregate price level respectively. Their prior means are both set to be 1%.

The posterior distributions of the parameters on the UK sample are obtained by using Dynare's MCMC algorithm. Two chains of 30, 000 draws are run, with the last 70% retained. The scale used for the jumping distribution is set to a value consistent with an acceptance rate in the neighborhood of 25% to ensure that the tails of the distributions are correctly identified. According to the Brooks and Gelman diagnostics (see Appendix C2), convergence of the MCMC algorithm is well behaved.

Table 5 provides the posterior distributions for the parameters, including the posterior means and 90% confidence intervals. Figure C4.1 and C4.2 in the appendix depict the prior and posterior distributions of the parameters listed in Table 5. Overall, these parameters seem to be well identified, given the fact that their posterior distributions are either not centered on the prior or they are centered but with a smaller dispersion implying high significance of the estimates. It is a common practice in Bayesian DSGE modeling to compare posteriors to priors as informal indicators of identification. However, this may be misleading, since priors can differ from posteriors even for unidentified parameters, as illustrated in Koop *et al.* (2011). Along the line of Iskrev (2010), identification analysis can be performed in Dynare toolbox and the result indicates that all the parameters of the benchmark model are identified. The picture in the Appendix C3 plots the measures of this kind of analysis,

[8] For other countries, θ=0.7 is often assumed in a quarterly model, such as in Christiano *et al.* (2005) for US, and in Genberg and Pauwels (2005) and Funke *et al.* (2010) for Hong Kong.

[9] For the degrees of freedom of the inverse-gamma distributions, we also tried +infinity rather than 2, and the Bayesian estimation results are similar.

where large bars imply strong identification, while low bars signal potential weak identification for the respective parameter.

The degree of openness, ρ, has a posterior mean of 0.4413, indicating a medium degree of openness. $\theta = 0.7033$ means that on average 70.33% of the firms in the UK cannot re-optimize the prices of their products within one year, showing a rather higher price rigidity compared with US (Christiano *et al.*, 2005) or Hong Kong (Funke *et al.*, 2010). All the six autocorrelation parameters have posterior means greater than 0.7, indicating strong persistence of these variables or shocks. Top three most persistent shocks are: the stock market shock, the nominal interest rate shock, and the shock to the foreign output gap. The estimated standard deviations of shocks can give us a first impression about their relative magnitudes and what kind of shocks are likely to drive the cyclical variations in the macroeconomic time series. Most volatile shocks include: the shock to the foreign output gap, the government spending shock, and the stock market shock. In the following sections we will further explore the driving forces of the UK business cycles, especially for fiscal aggregates.

In order to judge whether or not our benchmark model fits the data well, one-step-ahead predictions can be implemented in the Bayesian estimation procedure. Figure C4.3 in the appendix shows the posterior mean of one-step-ahead predictions for the six time series explored by this paper.[10] In general, our model fits the data well, except that the one-step-ahead prediction error for the stock price is sometimes big because of the high volatility of its actual time series.

4.4. Signal extraction of structural fiscal balances

As explained previously, the structural fiscal balance (SFB_t) in this paper is defined as the difference between the trend level of nominal fiscal revenue and the trend level of nominal government expenditure; and to calculate trend government expenditure, we only adjust nominal government expenditure one-to-one to the cycle of aggregate price level. So we have:

$$SFB_t = \overline{FR_t} - \overline{G_t}$$
$$\overline{G_t} = \exp[ln(G_t) - \widehat{N_t}]$$

And the trend level of fiscal revenue can be calculated as follows once we get a smoothed time series for its cyclical component ($\widehat{FR_t}$):

$$\overline{FR_t} = \exp[ln(FR_t) - \widehat{FR_t}]$$

So how to get the smoothed time series of the nominal aggregate fiscal revenue cycle (together with the aggregate price cycle) is the key to do the signal extraction of structural fiscal balances. We use the posterior means of the Bayesian estimation to pin down the whole model. Given the state space representation of our model, equation (51), the signal extraction of the fiscal revenue cycle can be done by using the Kalman smoother, demonstrated in Durbin and Koopman (2001). The smoothed time series we get for the cyclical component of fiscal revenue is depicted in Figure 5 (the red line, "Fiscal revenue cycle 1", is the fiscal revenue cycle obtained by the

[10] For those five unstationary variables (terms of trade excluded), we first get the one-step-ahead predictions for their second-order differences, and then recover the one-step-ahead predictions for their levels.

elasticity-trend approach, as shown in Figure 2).

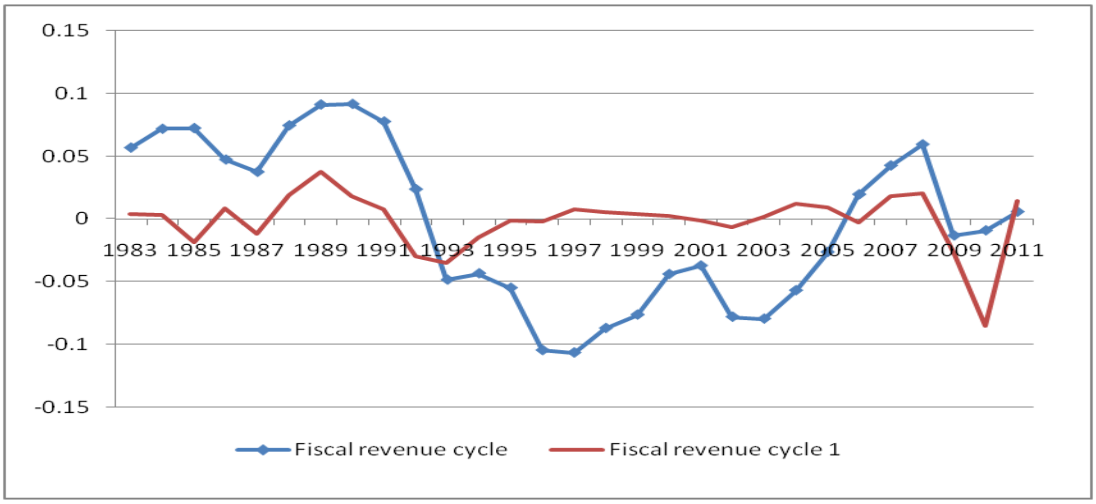


Fig. 5. Smoothed time series of the fiscal revenue cycle (blue line)

Fig. 6. The trend level of fiscal revenue (in logarithm) and structural fiscal balances (as a fraction of nominal GDP)

Figure 6 depicts the trend level of fiscal revenue and structural fiscal balances (as a fraction of nominal GDP) extracted for the UK, as long as the time series for the cyclical component of fiscal revenue has been obtained already.

4.5. Re-examine the stochastic trends: model comparison

As shown previously, for the UK economy the growth rates of real GDP and GDP deflator are not stationary, so in the benchmark model we adopt a reasonable setting for two basic stochastic trends: an integrated random walk (or a smooth trend model). Thus we want to see, from an empirical point of view, whether our specification for the stochastic trends is superior to other alternative settings, such as the deterministic trend assumption in Smets and Wouters (2007) and drifted random walk assumption in many Bayesian DSGE papers, say, Lafourcade and Wind (2012). Table B1 in the appendix gives the Bayesian estimation results of three alternative models together with the benchmark model, including posterior means of parameters and the log likelihood values of model variants. M1 is for deterministic trend model, M2 is for drifted-random-walk trend model, and M3 is for the model in which real GDP trend is a drifted random walk while aggregate price trend is an integrated random walk. Then equation (M1), (M2) and (M3) below replace equation (55) in the benchmark model. To make results comparable, the priors used in the estimations are exactly the same as the one used for the benchmark model, and the numbers of MCMC chains and draws for each chain are the same as well.

$$\begin{cases} BG_A = \gamma_A \\ BG_M = \overline{\Pi} \end{cases} \tag{M1}$$

$$\begin{cases} BG_A = \gamma_t^A = \gamma_A + \eta_t^A \\ BG_M = \gamma_t^P = \overline{\Pi} + \eta_t^P \end{cases} \tag{M2}$$

$$\begin{cases} BG_A = \gamma_t^A = \gamma_A + \eta_t^A \\ BG_M = \gamma_t^P = \gamma_{t-1}^P + \eta_t^P \end{cases} \tag{M3}$$

Bayesian inference allows a framework for comparing alternative and potentially misspecified models based on their marginal likelihood. To compare models (say, $M_i \in M$ and $M_j \in M$) we calculate the Bayes factor $BF_{i,j}$ which is the ratio of their posterior likelihoods ($L(y|M_i)$ and $L(y|M_j)$):

$$BF_{i,j} = \frac{L(y|M_i)}{L(y|M_j)} = \frac{exp(LL(y|M_i))}{exp(LL(y|M_j))}$$

where $LL(y|M_i)$ is the log likelihood for model M_i. To assess rival models, we can compute the model probabilities p_1, p_2, $\ldots p_n$ for n models, given Bayes factors. Since $\sum_{i=1}^{n} p_i = 1$, we have that:

$$p_i = \begin{cases} 1/\sum_{i=1}^{n} BF_{i,1}\,, & if\ i = 1 \\ p_1 \cdot BF_{i,1}\,, & if\ i > 1 \end{cases}$$

Table 6 gives the marginal log likelihood values and posterior model odds (probabilities) for the benchmark and three alternative models. The log likelihood for our benchmark model is greater than any of the alternatives, indicating the superiority of the benchmark model. The model odd of the benchmark model is about 83%, confirming that our benchmark specification (assuming an integrated random walk for both two basic stochastic trends) fits the date best.

Table 6. Marginal log likelihood values and posterior model odds

Model	Benchmark	M1	M2	M3
Log likelihood	308.82	298.22	304.01	307.21
Model odds	0.8278	0.0000	0.0067	0.1655

5. BUSINESS CYCLES AND STRUCTURAL FISCAL BALANCES

5.1. Impulse response analysis

When we use the posterior mean of the Bayesian estimation to pin down the system linking the cyclical components of endogenous variables, it can be used to do the impulse response analysis, forecast error variance decomposition and historical shock decomposition. For impulse responses, we pay special attention to the time profiles of the fiscal revenue cycle to different exogenous shocks, since according to the definition in this paper the structural fiscal balances is mainly related to the fiscal revenue cycle and the cyclicality of real government expenditure is ignored.

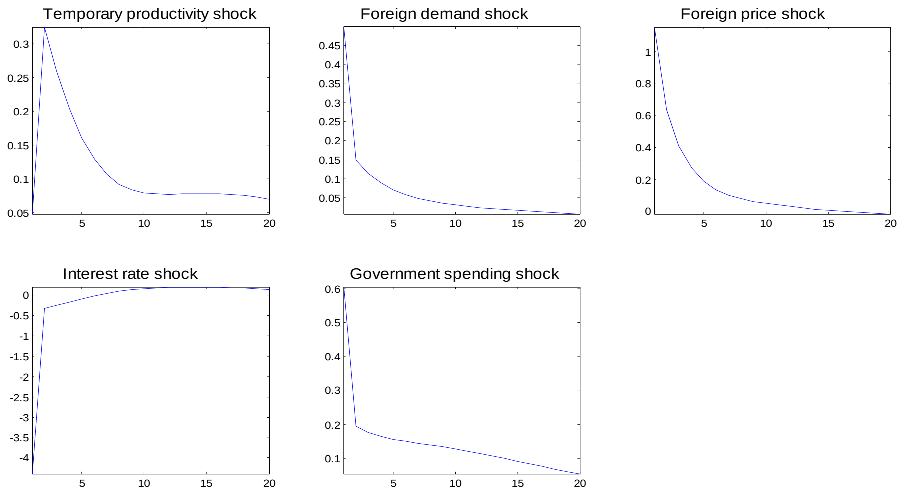


Fig. 7. Impulse responses of the fiscal revenue cycle (in percent) to one-percent shocks to the temporary technology $ln\epsilon_t$, foreign output gap $\widehat{GDP_t^*}$, foreign price gap $\widehat{P_{F,t}}$, nominal interest rate R_t^2, and the government expenditure-GDP ratio g_t.

Figure 7 depicts the orthogonalized impulse response functions[11] of the fiscal revenue cycle to one-percent exogenous shocks. Other things equal, when there is a one-percent shock to the temporary technology, on impact there would be a 0.05% increase of aggregate nominal fiscal revenue. Two years later fiscal revenue will be 0.32% above its trend level, and afterwards the fiscal revenue cycle decreases gradually to zero. The positive effect on fiscal revenue of the temporary technology shock is intuitive and consistent with the literature, because a positive technology shock will lead to an increase of GDP and thus an increase of tax base, for either consumption tax or production factors' income taxes. The interesting phenomenon is that the short-run elasticity of nominal fiscal revenue with respect to the technology shock is only 0.05%, which is much smaller than a conventional expectation (close to one).

Figure 8 gives the impulse responses of the output gap to one-percent exogenous shocks. In fact, the short-run elasticity of the real GDP with respect to the technology shock is only 0.75, less than one. According to equation (41), a one-percent shock to the temporary technology will lead to a one-percent increase of real output, if both the capital stock and labor input would not fluctuate much. However, the optimization

[11] In this paper exogenous shocks are not correlated, so there is no difference between orthogonalized impulse response functions and generalized impulse response functions (Pesaran and Shin, 1998).

behaviors of firms and households will result in an increase of investment and a decline of labor supply, in response to this positive technology shock (shown in Figure C4.4 in the appendix). And for our benchmark model here the labor supply cycle decreases much larger than the increase of the capital stock cycle. As a result, the real output responses less than one-to-one to the technology shock. Furthermore, the taxation system is nominally defined, and a real shock such as this one-percent technology shock may influence aggregate price levels such as GDP deflator and then influence nominal fiscal revenue. Figure C4.4 in the appendix shows that a one-percent temporary technology shock will lead to a 0.22% decrease of GDP deflator on impact. This is possible because: given the world's total demand for the UK's real GDP is unaffected much, the supply increase of the UK's real GDP resulted from a positive technology shock will lead to a relatively cheaper price of the UK's goods in the global market, indicating larger terms of trade if the foreign price does not change. This is explained by equation (43) and confirmed by the Figure C4.4 in the appendix. This negative response of GDP deflator to the technology shock, together with the decrease of labor supply in the short run, partly explains the very small short-run elasticity of nominal fiscal revenue with respect to the technology shock.

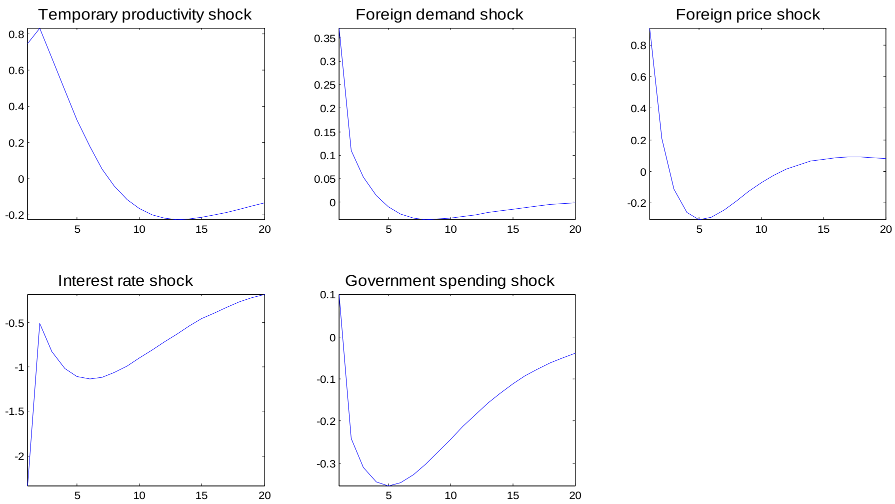


Fig. 8. Impulse responses of the output gap (in percent) to one-percent shocks to the temporary technology $ln\epsilon_t$, the foreign output gap $\widehat{GDP_t^*}$, the foreign price gap $\widehat{P_{F,t}}$, the nominal interest rate R_t^2, and the government expenditure-GDP ratio g_t.

The responses of the fiscal revenue cycle to foreign shocks are exhibited in Figure 7 as well. The short-run elasticity of nominal fiscal revenue with respect to the real GDP of the rest of the world is 0.50. According to equation (43), a positive shock to the foreign output gap is likely to increase the domestic real GDP and then domestic fiscal revenue. Given the value of the degree-of-openness parameter, ρ, which is 0.4413, a one-percent foreign output increase corresponds to about 0.44% increase of domestic real output, keeping domestic aggregate demand and terms of trade unchanged. However, Figure 8 shows that the short-run elasticity of the domestic output with respect to the foreign demand shock is only 0.35, less than 0.44. This is because the terms of trade is also affected by this foreign demand shock. Intuitively, a positive foreign demand shock will lead to an increase of domestic goods' price and then a lower level of the terms of trade. This nominal effect will result in a negative effect on the domestic output, shown by equation (43). Figure C4.5 in the appendix proves this transmission mechanism. On the other hand, a higher aggregate price level will push up the fiscal revenue which is nominally defined.

The short-run elasticity of nominal fiscal revenue with respect to the foreign goods price is 1.15. Here the foreign goods price is denominated in domestic currency, so its fluctuation can come from either exchange rate movement or the variability in the foreign goods price denominated in foreign currency. A positive shock to the foreign goods price also has two channels to affect the nominal fiscal revenue of the UK: one is the real channel through real output, and the other is the nominal channel through the aggregate price level. As we will explain, both of these two channels have positive effects on fiscal revenue in the short run and thus result in a relatively large short-run elasticity, 1.15. A positive shock to the foreign goods price will directly increase the terms of trade and thus the world's demand for domestic output, since the domestic goods become relatively cheaper. Again equation (43) tells that this will lead to an increase of domestic GDP, confirmed by Figure 8 showing that the short-run elasticity of real output with respect to the foreign goods price is about 0.8. Since the import price increases, the aggregate price level of the UK economy will increase as well, shown by Figure C4.6 in the appendix. As a whole, both the increase of real GDP and the raise of the aggregate price level, resulted from the positive foreign price shock, will lead to an increase of nominal fiscal revenue.

The short-run elasticity[12] of nominal fiscal revenue with respect to the nominal interest rate is -4.39, which is negative and quite big in absolute value. However, the negative effect of this monetary policy shock is not persistent, and one year after the shock hitting the economy nominal fiscal revenue is only 0.3% below its trend level. The negative effect of a positive nominal interest rate shock is easy to understand: this contractionary monetary policy will suppress consumption and investment, and then lead to a reduction of real output. Figure 8 shows that the short-run elasticity of real output with respect to the nominal interest rate is about -2.2, indicating a strong contractionary effect on the real economy. Furthermore, the aggregate price level will decrease as a result as well, and this is why the nominal fiscal revenue will be reduced much in the short run. The non-persistence of the negative effect of monetary policy shock on fiscal revenue should be explained by the open-economy characteristic of the UK. The immediate disinflation effect of a positive monetary policy shock will push up the terms of trade, and this relative price effect will increase the world's demand for domestic output to a certain degree and can avoid persistent declines of domestic real output and fiscal revenue. The transmission mechanism is clearly shown in Figure C4.7 in the appendix.[13] A positive nominal interest rate shock is likely to

[12] Here "elasticity" is not defined in a conventional way, but defined as the ratio of the percentage change of nominal fiscal revenue to the level change (not the percent change) of nominal interest rate.

[13] The large response of the capital stock in the figure is caused by the large response of the investment. If one wants to achieve a smaller reaction of investment and capital stock, investment adjustment cost can be

raise the fiscal revenue from bond interest income in the short run, but our result proves that this positive effect seems to be very small.

Figure 7 also shows that when the government expenditure-GDP ratio increases by 1% (it is equivalent to a 1% increase of nominal government expenditure, assuming nominal GDP keeps unchanged), nominal fiscal revenue will be 0.61% above its trend level on impact. Similarly to the monetary policy shock, the effect of the government spending shock is not persistent as well. One year later nominal fiscal revenue will be only about 0.2% above its trend level. To make clear of the transmission mechanism of the government spending shock, we first pay attention to the effect on the real output of the government spending shock. Figure 8 says that the immediate effect on the real output is slightly positive, indicating a small positive government spending multiplier in the short run; but afterwards the real output is always below its trend level, indicating a strong crowd-out effect of this expansionary fiscal policy. This can be also proved by the behavior of the investment cycle in Figure C4.8 in the appendix. Figure C4.8 depicts the responses of GDP-deflator inflation and aggregate-price inflation to this one-percent government spending shock as well. Initially the increase of the aggregate internal demand caused by the positive government spending shock will lead to an increase of domestic goods price (GDP deflator) and then the aggregate price. This positive nominal effect is persistent, and the aggregate price will continue to increase in the next five years. The increase of the aggregate price level will raise the nominal fiscal revenue. Nevertheless, the reduction of the terms of trade (shown in Figure C4.8), caused by the increase of domestic goods price, will lead to a decrease of real GDP (according to equation (43)) and then fiscal revenue. To conclude, a positive short-run elasticity of nominal fiscal revenue to the government spending shock can be explained by a slightly positive short-run government spending multiplier together with the increase of aggregate price caused by this government spending shock. The non-persistence of the effect can be explained by the fact that in the medium term real output will be below its trend level due to two kinds of crowd-out effects generated by this government spending shock: it crowds out domestic investment in the medium term, and it pushes up the price of domestic goods and then crowds out foreign demand as well.

An interesting result is that: a shock to the stock price gap has no effect on the nominal fiscal revenue. If we look at the policy and transition equation for the fiscal revenue cycle, given by equation (50), it is found that both the stock price gap and the shock to the stock price gap do not enter into the policy and transition function. This is consistent with the result of Funke *et al.* (2010). In the following section of variance decomposition analysis, we will see that the shock to the stock price gap explains the biggest part of stock price non-fundamental variation. But it has no effect on dividend. Equation (49) shows that the nominal dividend is determined by real output, real marginal cost, and the aggregate prices of domestic goods and final goods. Stock price cannot influence dividend in our model, and thus cannot influence the fiscal revenue from the dividend and the aggregate nominal fiscal revenue.

Table 7 below summarizes all the above impulse response analysis for the UK

incorporated into our benchmark model.

nominal fiscal revenue.

Table 7. Summary of the impulse response analysis for nominal fiscal revenue

		Impulse response properties	
		Positive or negative effect	Short run elasticity
Shock to	Temporary productivity	positive	0.05
	Foreign output gap	positive	0.50
	Foreign price gap	positive	1.15
	Nominal interest rate	negative	-4.39
	Government spending	positive	0.61
	Stock-price gap	no effect	0.00

5.2. Forecast error variance decomposition

Forecast error variance decomposition is computed as in the VAR literature through a Cholesky decomposition of the covariance matrix of the exogenous shocks. It is computed relative to the sum of the contribution of each shock to the forecast error (mean square error) of some endogenous variable, and all the contributions normally sum up to the aggregate variance of this variable. [14] When the shocks are correlated, the variance decomposition depends upon the order of the variables. However, in this paper all the exogenous shocks are assumed to be uncorrelated.

Table 8. Forecast error (unconditional) variance decomposition (in percent)

	Shocks					
	Temporary productivity	Stock price gap	Foreign demand	Foreign price	Interest rate	Government spending
$\widehat{FR}_t$	1.33	0	27.38	8.18	42.58	20.53
GAP_t	9.40	0	15.13	5.54	37.35	32.58
$\widehat{\Pi_{H,t}}$	7.47	0	19.59	14.23	28.96	29.75
$\widehat{\Pi}_t$	3.19	0	8.36	63.41	12.35	12.69
tot_t	4.46	0	16.02	7.79	19.88	51.84
$\widehat{Q_t}$	0.48	85.1	0.34	0.50	5.69	7.88

Table 8 lists the unconditional forecast error variance decompositions of several aggregate variables' cyclical components. Our main interest is the variance decomposition of the nominal fiscal revenue cycle. Overall, the nominal interest rate shock is the major driving force of the fiscal revenue cycle, and more than 40% of its volatility is explained by this monetary policy shock. It is consistent with the biggest (in absolute value) short run elasticity of fiscal revenue with respect to the nominal interest rate, shown in Table 7. Table B2 in the appendix also shows the conditional forecast error variance decompositions of the fiscal revenue cycle for different forecasting horizons. When the forecasting horizon goes to infinity, the conditional

[14] For detailed explanations and computation formulas of variance and historical decomposition in VAR models, refer to Canova (2007).

forecast error variance decomposition will converge to the unconditional one.

The shocks to the foreign output gap and the government spending-GDP ratio play the second and third most important roles, and more than 20% of the volatility of the fiscal revenue cycle comes from each of these two shocks. Although the short run elasticities of fiscal revenue with respect to these two shocks do not have big absolute values, their estimated standard errors are the greatest two among all the shocks, given by Table 5.

Since the UK is an open economy and the value of the parameter ρ indicates a medium degree of openness, the foreign price shock should contribute to the variation of aggregate fiscal revenue. Indeed, about 8% of the variability in the fiscal revenue cycle can be explained by the foreign price shock. An interesting finding is that the shock to the temporary productivity has almost no contribution to the cyclical movement of the aggregate nominal fiscal revenue: it only explains 1.33% of the whole forecast error variance. Finally, the stock price shock plays no role in explaining the volatility of fiscal revenue.

It is found that the temporary productivity shock does not contribute much to the variation of many aggregate variables' cycles, as shown in Table 8. It only explains 9.40% and 7.47% of the total forecast error variances of the output gap and the GDP-deflator inflation gap respectively. In our framework, the trends of aggregate variables (such as GDP and fiscal revenue) are stochastic themselves, and the stochastic permanent technology has already explained much of the variation in the data. So relatively the role of the temporary technology shock is weakened.

For the forecast error variance decompositions of both the output gap and the GDP-deflator inflation gap, the nominal interest rate shock and the government expenditure shock are the biggest two contributors. These imply that monetary and fiscal policies in the UK play a big role in the business cycles of the real economy.

Foreign shocks, including the shocks to foreign output and foreign goods price, are of a certain importance to explain the UK's business cycles, except for the stock price fluctuation. Particularly, more than 50% of the fluctuation of the inflation gap owes to the foreign goods price shock. These results again are consistent with the open-economy characteristic of the UK economy. And not surprisingly, the fluctuation of the stock price gap is substantially driven by the stock price shock.

5.3. Historical shock decomposition

We turn to the historical contribution of each type of shock over the sample period for the UK. It this paper, while impulse responses trace out how the cyclical components of aggregate variables respond to various shocks and the variance decomposition measures the contribution of each shock to the variability of the cycles of aggregate variables, the historical shock decomposition describes the contribution of each shock to the deviations of the cycles of aggregate variables from their baseline forecasted paths. To be specific, the historical shock decomposition for the nominal fiscal revenue cycle is shown in Figure 9. It can be a fruitful exercise to analyze this historical shock decomposition, because it allows us to identify the main sources of specific booms and recessions in the fiscal revenue cycle.

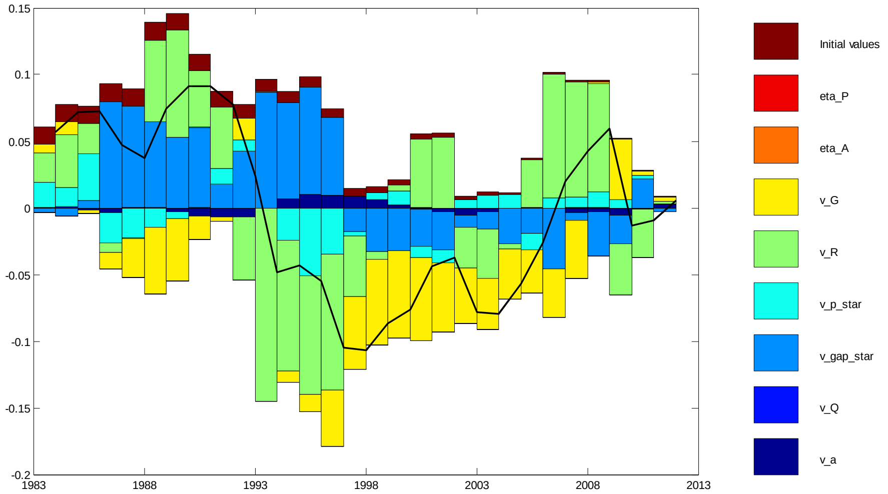


Fig. 9. Historical shock decomposition of fiscal revenue cycle

Overall, the results in Figure 9 confirm our previous finding that the shocks to the nominal interest rate, foreign output and government spending are the three largest contributors in explaining the variability of the fiscal revenue cycle. Nevertheless, some new results show up. For example, while in the period 1986-1987 the shock to the foreign demand played the major role, in the period 2007-2008 the nominal interest rate shock was the dominant factor determining the fiscal revenue cycle. Table 9 provides the historical shock decompositions of the fiscal revenue cycle in four sub periods when the fiscal revenue cycle glided down. We can see that: for the period 1985-1987, the glide of the fiscal revenue cycle was mainly generated by the change in the realized foreign price shock: from a positive effect on fiscal revenue in 1985 to a negative effect in 1987. However, the big drop of the fiscal revenue cycle during the period 1990-1997 (from +9% to about -10%) was driven by both the nominal interest rate shock and the foreign demand shock.

Table 9. Historical shock decomposition of the UK fiscal revenue cycle (percentage)

Sub-period	Observed change	Contributions of various shocks					
		v_t^a	v_t^Q	$v_{gap,t}^*$	$v_{P,t}^*$	v_t^R	v_t^G
1985-1987	-3.48	0.19	0	7.00	-5.76	-2.30	-2.64
1990-1997	-19.80	1.48	0	-7.77	-0.43	-8.76	-3.68
2001-2003	-4.23	-0.02	0	1.58	1.89	-9.01	1.41
2008-2009	-7.26	-0.27	0	1.18	-0.53	-11.98	4.38

6. ROBUSTNESS CHECKS

Now we do some robustness checks to see whether the results above are sensitive or not to our calibration of some parameters, which are not part of the Bayesian estimation result. 7 cases are considered as below:

(1) The production function parameter α is changed to be 0.64, following Dicecio and Nelson (2007).
(2) The elasticity of substitution between domestic and foreign goods, ω, is changed to be a smaller value, 4.
(3) The lump-sum tax feedback elasticity for government debt ε_{LST} is set to be a larger number, 0.01.
(4) The tax rates on consumption, capital rental income and labour income are altered to be the calibration result of Bhattarai and Trzeciakiewicz (2012) for the UK: $\tau_C = 0.2008$, $\tau_K = 0.4071$, and $\tau_L = 0.2844$.
(5) The tax rates on bond interest income and dividend (τ_B and τ_D) are changed to be a greater rate, 20%.
(6) The steady state ratio of stock price to gross dividend, RT_{QD}, is set to be a larger number, 17, equivalent to that the stock market P/E ratio (price-to-net-earnings ratio) is 20.
(7) The steady state ratio of stock price to gross dividend, RT_{QD}, is set to be a smaller number, 11.9, equivalent to that the stock market P/E ratio is 14.

For all these 7 cases, we re-do the Bayesian estimation of our model, and then use the corresponding posterior means to pin down the model. Three things interest us most: first, whether the smoothed time series of the fiscal revenue cycle we extract from the data are sensitive or not to the alternative calibrations; second, the impulse responses of the fiscal revenue cycle to various shocks are robust or not; and third, the variance decomposition (the contributions of each exogenous shock to the forecast error) of the fiscal revenue cycle is sensitive or not. In the appendix, Table B3 and B4 respectively provide the Bayesian estimation results and the variance decomposition results for the benchmark model as well as 7 alternative settings. Figure C4.9 and C4.10 respectively show the smoothed time series of the fiscal revenue cycle we extract from the data and the impulse responses of the fiscal revenue cycle to various shocks.

The Bayesian estimation results are quite similar, and the three most volatile shocks are always the shock to the foreign output gap, the government spending shock, and the stock market shock. For the forecast error variance decomposition, the following results are robust: (a) the shocks to the nominal interest rate, foreign output and the government spending are the three major contributors of the variation of the fiscal revenue cycle, and their contribution as a whole is about 90%; (b) the shock to the foreign price makes some contribution to the fluctuation of the fiscal revenue cycle, above 5%; (c) the shock to the temporary productivity is of very minor importance; and (d) the shock to the stock price gap explains nothing of the fluctuation of the fiscal revenue cycle.

The smoothed time series of the fiscal revenue cycle extracted from the data, in general, are bounded within a narrow range around the benchmark result, except for case 5 which exhibits a certain deviation from the benchmark before the year 2000.

In terms of the impulse responses of the fiscal revenue cycle to various shocks, the results are quite robust shown by Figure C4.10, except that the short-run impacts of the temporary productivity shock differ from a negative impact to a positive one. As explained for the benchmark model, the response of real GDP to the technology shock is positive, but the response of GDP deflator to the shock is negative due to the immediate increase of aggregate supply. Therefore, the difference in the short-run impacts of the temporary productivity shock on nominal fiscal revenue can be easily explained by the relative size of these two effects: the positive real effect and the negative nominal effect. If the former is bigger, then the overall impact is positive. If the latter is bigger, the overall impact is negative.

7. Public finances in the post Great Recession period

During the current post Great Recession period, fiscal policy makers in many countries are confronted with a dilemma: on one hand, the slow recovery during the liquidity trap urges the government to adopt stimulating policies such as increasing the government expenditure; on the other hand, fiscal crisis or fiscal unsustainability exists and the government does not have enough fiscal space for doing this.

For the UK, Figure 6 tells that: both the fiscal balances and the structural fiscal balances decreased a lot after 2007 and reached a level of about -8% of nominal GDP in 2009; and in 2011 the fiscal deficit was still about 6% of nominal GDP. Regarding the public finances for the UK in the post Great Recession period, several interesting questions can be asked: is it a suitable policy to increase the government expenditure? Is there a role for tax cuts? To evaluate these policy questions, we need to pay attention to their effects not only on real output but also on fiscal stance indicated by such as fiscal deficit, and government debt-GDP ratio, which reached a level of about 82% of nominal GDP in 2011.

Although we ignore the fluctuation of government expenditure when we define and calculate the structural fiscal balances in previous sections of this paper, we must take it into account when we discuss the consequence of a policy in a period of a high fiscal deficit and a high debt-GDP ratio, especially when we explore an expansionary policy in government spending. The primary fiscal deficit as a fraction of nominal GDP is defined as below:

$$FD_t = (G_t - FR_t)/(P_{H,t} \cdot GDP_t)$$

If the cyclical component of the primary fiscal deficit (as a fraction of nominal GDP) is defined as its deviation from its steady-state value, then we have the following log-linearized equation:

$$\widehat{FD}_t = (fr - g) \cdot \left(\mathrm{GAP}_t - \rho \cdot tot_t + \widehat{N}_t\right) - \left(fr \cdot \widehat{FR}_t - g \cdot \widehat{G}_t\right)$$

where fr and g are the steady-state fiscal revenue-GDP ratio and government expenditure-GDP ratio, respectively.

7.1. Monetary policy and zero lower bound of nominal interest rate

As have been explained, an expansionary monetary policy (reducing the nominal interest rate) has positive and strong short-run effects on both real output and nominal fiscal revenue in normal times, even though the effects are not persistent. However, in the post Great Recession period for the UK, it is not feasible to implement such an expansionary monetary policy, because the nominal interest rate has reached its zero lower bound already.

Therefore, an unconventional monetary policy called quantitative easing (QE) or large-scale asset purchase program is employed to stimulate the economy in the UK, like in the US and the Euro area. Since there is no feature of QE in our benchmark model, we cannot provide insight of this unconventional monetary policy's effect on fiscal stance here. We will focus on fiscal policies, including increasing the government spending and tax cuts, for the UK in the post Great Recession period.

7.2. An expansion in government spending

We have shown that: although the short-run elasticity of nominal fiscal revenue with respect to the government spending is positive, this positive effect is not persistent due to two kinds of crowd-out effects generated by the increased government spending: it crowds out domestic investment in the medium term, and it pushes up the price of domestic goods and then crowds out foreign demand as well. As another result, in the medium term real output will also be below its trend level, although the immediate effect on the real output is slightly positive.

Therefore, due to its strong crowd-out effect, an expansion in government spending is likely to deteriorate the fiscal stance (to enlarge the fiscal deficit and increase the debt-GDP ratio), and simultaneously harm the economic recovery (to make real output below its trend level) in the medium term. This is confirmed by the figure below.

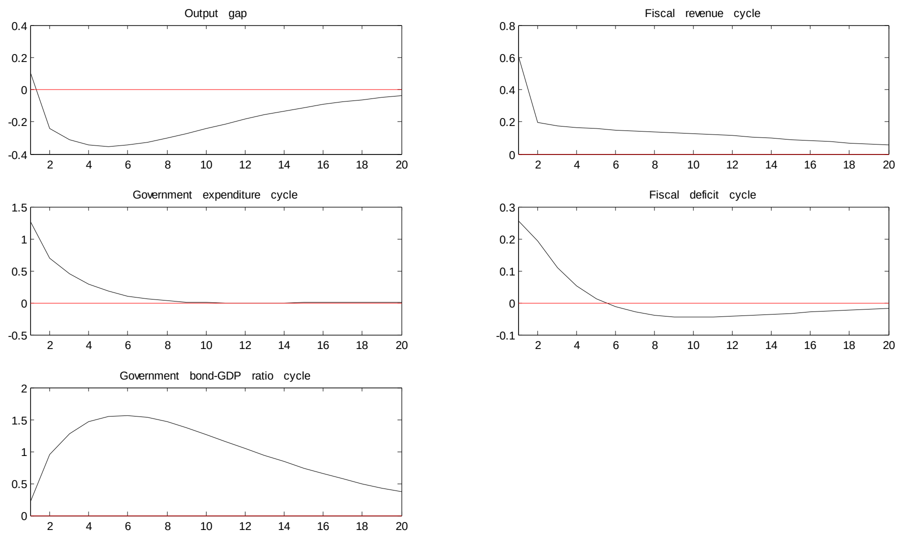


Fig. 10. Impulse responses of the output gap, the fiscal revenue cycle, the government expenditure cycle, the fiscal deficit cycle and the cycle of the government bond-GDP ratio to a positive one-percent shock to the government expenditure-GDP ratio g_t.

In spite of the positive effect of the government spending expansion on nominal fiscal revenue, the fiscal deficit will be expanded because the increase of nominal fiscal revenue is less than the increase of nominal government spending. Consequently, the government bond-GDP ratio is enlarged either, shown by Figure 10. Although five years after the government spending expansion, the fiscal deficit will be slightly below its steady-state level, it is overall not a suitable stimulating policy for the UK in the current post Great Recession period with a high fiscal deficit and a high debt-GDP ratio.

On the contrary, cutting the government spending would be a good alternative policy, which will in the medium term improve the fiscal stance (lower fiscal deficit and lower government debt-GDP ratio) as well as benefit the economic recovery (higher real output), even though the real output will be negatively affected slightly on impact.

7.3. Tax cuts

Tax cuts can be analyzed in two dimensions: one is to reduce the tax rate of either consumption tax or production factors' income taxes; the other is to reduce the lump-sum tax (can also be viewed as to increase the transfer to households). Since for our benchmark model the tax rates are fixed and are structural parameters determining the model's steady state, cutting the tax rates will result in a transition from one steady state to another. We leave this for further studies, and mainly discuss the effect of cutting the lump-sum tax.

In the benchmark model, there is no uncertainty about the lump-sum tax rule and it is given by equation (37). As dealing with the government spending shock in equation (36), we add a lump-sum tax shock (with no persistence) into equation (37) and then we have the following:

$$\widehat{\mathrm{LST}_t} = \varepsilon_{LST} \cdot \left(\widehat{\mathrm{B}_{t-1}} - \mathrm{GAP}_{t-1} - \widehat{N_{t-1}} + \rho \cdot tot_{t-1}\right) + \left(\mathrm{GAP}_t + \widehat{N_t} - \rho \cdot tot_t\right) - v_t^{LST}$$

which replaces the seventh equation in Appendix A. By incorporating this new shock into the benchmark model, our results presented in previous sections will not change much and qualitatively unchanged at all.

Figure 11 shows the impulse responses of the output gap, the fiscal revenue cycle, the government expenditure cycle, the fiscal deficit cycle and the cycle of the government bond-GDP ratio to a negative one-percent shock to the lump-sum tax-GDP ratio. On impact, the nominal fiscal revenue cycle is negative, which is reasonable since cutting the lump-sum tax (or increasing the fiscal transfer to households) will immediately reduce the nominal fiscal revenue. Meanwhile real output and government expenditure are nearly not affected in the short run. As a result, the fiscal deficit together with the government debt-GDP ratio will be deteriorated in the short run. In the medium term, both the economic recovery (indicated by the response of real output) and the government debt-GDP ratio will be deteriorated, while the fiscal deficit is improved to a very small and insignificant degree. To conclude, cutting the lump-sum tax is not an appreciate fiscal policy for the UK in the current post Great Recession period, taking into account both the economic recovery and the fiscal stance improvement.

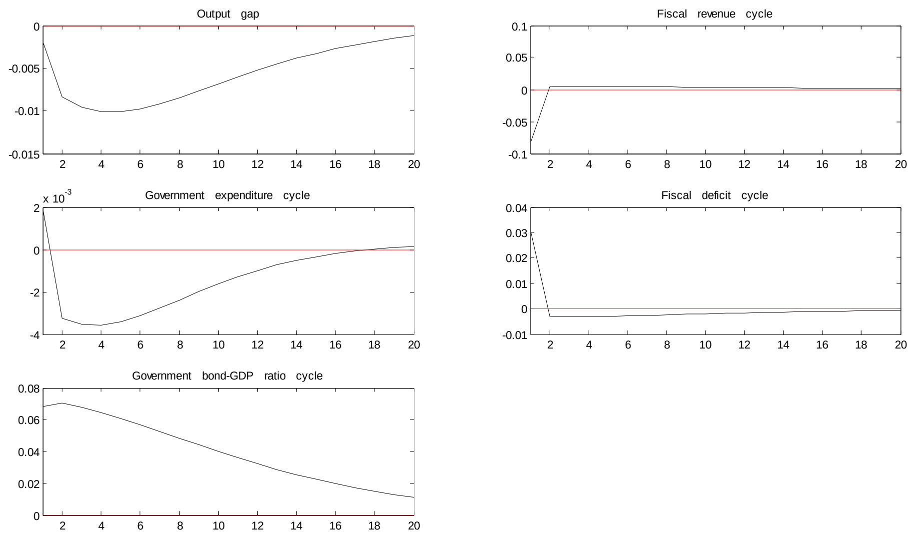


Fig. 11. Impulse responses of the output gap, the fiscal revenue cycle, the government expenditure cycle, the fiscal deficit cycle and the cycle of the government bond-GDP ratio to a negative one-percent shock to the lump-sum tax-GDP ratio lst_t.

On the contrary, increasing the lump-sum tax temporarily would be a suitable alternative, which will in the medium term improve the fiscal stance (lower the government debt-GDP ratio and nearly not affect the fiscal deficit) as well as benefit the economic recovery (increase real output), while the fiscal deficit will also be improved in the short run. However, the amplitude of the economy's response to this temporary positive lump-sum tax shock is much smaller, especially compared to the case of a negative government spending shock.

Generally speaking, for the UK economy it seems not an appropriate choice to adopt an expansionary fiscal policy by either increasing the government expenditure or cutting the lump-sum tax, in the current post Great Recession period. An expansionary fiscal policy will deteriorate the fiscal stance (higher government debt-GDP ratio or higher fiscal deficit) as well as harm the economic recovery in the medium term. On the contrary, a contractionary fiscal policy (cutting the government expenditure or a temporary increase of the lump-sum tax) will benefit both the economic recovery and the fiscal stance. Cutting the government spending is relatively more effective, and it alleviates both the domestic and external crowd-out effects generated by the government spending.

8. CONCLUDING REMARKS

In this paper, we propose a new framework to extract structural fiscal balances from data consistently with the macroeconomic theory, and to analyze the relationship between public finances and business cycles. The UK economy is taken as an example.

The impulse response analysis reveals the basic relationship between aggregate fiscal revenue and business cycles. The transmission mechanism of various shocks' effect on nominal fiscal revenue is explained by two main channels: the real channel through real GDP which can be viewed as the real tax base of fiscal revenue, and the nominal channel through the aggregate price level. Although in the medium term the response of nominal fiscal revenue to the technology shock is positive, the effect is not large, partly due to the less than one-to-one response of real output and the negative response of GDP deflator. Both of the foreign shocks, the shock to foreign demand and the shock to foreign goods price, have positive effects on fiscal revenue. An expansionary monetary policy shock (to lower nominal interest rate) would have a great positive short-run impact on nominal fiscal revenue, but the influence is not persistent because of the open-economy characteristic of the UK. An expansion in government spending can also increase nominal fiscal revenue to a certain degree, but the effect is not persistent as well due to two kinds of crowd-out effects generated by an increase of government spending: it crowds out domestic investment in the medium term, and it pushes up the price of domestic goods and simultaneously crowds out foreign demand. The shock to the stock price has no effect on fiscal revenue, since in our model stock price cannot influence nominal dividend and other aggregate variables from where the aggregate nominal fiscal revenue comes.

The analysis of forecast error variance decomposition of the fiscal revenue cycle tells that: the shocks to the nominal interest rate, foreign output and the government spending are the three major contributors to the variation of the fiscal revenue cycle, and their contribution as a whole is about 90%; the shock to the foreign price makes some contribution to the fluctuation of the fiscal revenue cycle, above 5%; and the shocks to the temporary productivity and stock price are of very minor importance. Robustness checks are implemented, and the above results concerning the signal extraction, impulse responses and forecast error variance decomposition are robust.

We discuss the public finances of the UK in the post Great Recession period when both the economic recovery and the fiscal sustainability should be taken into consideration. Generally speaking, it is not an appropriate choice to adopt an expansionary fiscal policy by either increasing the government expenditure or cutting the lump-sum tax. An expansionary fiscal policy will deteriorate the fiscal stance (higher government debt-GDP ratio or higher fiscal deficit) as well as harm the economic recovery in the medium term. On the contrary, a contractionary fiscal policy (cutting the government expenditure or a temporary increase of the lump-sum tax) will benefit both the economic recovery and the fiscal stance. Compared to a temporary increase of the lump-sum tax, cutting the government spending is relatively more effective and it alleviates both the domestic and external crowd-out effects generated by the government spending.

Possible further extensions of this paper can be achieved in several ways. First of all, forecasting exercises of our framework can be done, especially for the forecasts of aggregate fiscal variables. Secondly, in our theoretical model we have not considered the bracket-creep effect of inflation on tax revenues. This issue is important in the field of public finances, since inflation alters the distributive properties of nominally defined tax systems. If one can incorporate this issue into our DSGE model, he may find richer results of the inflation's effect on fiscal revenue and structural fiscal balances. Last but not least, the zero lower bound of nominal interest rate and unconventional monetary policy can be introduced into our framework. In the last three years of our data sample, the nominal interest rate of the UK reached its zero lower bound. Then some unconventional monetary policy such as QE, rather than a Taylor-type interest rate rule, was implemented. One may ask: what is the effect of QE on fiscal revenue and structural fiscal balances?

REFERENCES

Abiad, A. and J.D. Ostry (2005). Primary Surpluses and Sustainable Debt Levels in Emerging Market Countries. IMF, Policy Discussion Paper, No. 05/6.

An, S. and F. Schorfheide (2007). Bayesian Analysis of DSGE Models. *Econometric Reviews*, 26, 113–172.

Ascari, G., E. Castelnuovo, and L. Rossi (2010). Calvo vs. Rotemberg in a Trend Inflation World: An Empirical Investigation. University of Pavia, Department of Economics and Quantitative Methods, Working Paper, No. 108.

Aydin, B. (2010). Cyclicality of Revenue and Structural Balances in South Africa. IMF, Working Paper, No. 10/216.

Bhattarai K. and D. Trzeciakiewicz (2012). Macroeconomic Impacts of Fiscal Policy Shocks in UK: A DSGE Analysis. Unpublished manuscript, Business School, University of Hull.

Bornhorst, F., G. Dobrescu, A. Fedelino, J. Gottschalk, and T. Nakata (2011). When and How to Adjust Beyond the Business Cycle? A Guide to Structural Fiscal Balances. IMF Fiscal Affairs Department, Technical notes and Manuals 11/02.

Calvo, G.A. (1983). Staggered Prices in a Utility-maximizing Framework. *Journal of Monetary Economics*, 12(3), 383-398.

Canova, F. (2007). *Method for Applied Macroeconomic Research*. Princeton University Press.

Canova, F. (2012). Bridging Cyclical DSGE Models and the Raw Data. Unpublished manuscript.

Carey, D. and H. Tchilinguirian (2000). Average Effective Tax Rates on Capita, Labour and Consumption. OECD Economics Departmetn Working Papers, No. 258, OECD Publishing.

Castelnuovo, E. and S. Nisticó (2010). Stock Market Conditions and Monetary Policy in a DSGE Model for the US. Research Discussion Papers, 11/2010, Bank of Finland.

Christiano, L., M. Eichenbaum, and C. Evans (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113(1), 1-45.

Collard, F. and H. Dellas (2002). Exchange Rate Systems and Macroeconomic Stability. *Journal of Economic Dynamics and Control*, 49, 571–599.

Dees, S., M.H. Pesaran, L.V. Smith, and R.P. Smith (2010). Supply, Demand and Monetary Policy Shocks in a Multi-country New Keynesian Model. European Central Bank, Working Paper Series No. 1239.

Del Negro, M. and F. Schorfheide (2008). Forming Priors for DSGE Models (and How It Affects the Assessment of Nominal Rigidities). *Journal of Monetary Economics*, 55, 1191–1208.

DiCecio, Riccardo and E. Nelson (2007). An Estimated DSGE Model for the United Kingdom. *Federal Reserve Bank of St. Louis Review*, July/August 2007, 89(4), 215-231.

Durbin, J. and S.J. Koopman (2001). *Time Series Analysis by State Space Methods*.

Oxford University Press.
Escolano, Julio (2010). A Practical Guide to Public Debt Dynamics, Fiscal Sustainability, and Cyclical Adjustment of Budgetary Aggregates. IMF Fiscal Affairs Department, Technical notes and Manuals 10/02.
Faccini, R., S. Millard, and F. Zanetti (2011). Wage Rigidities in an Estimated DSGE Model of the UK Labour Market. Bank of England, Working Papers No. 408.
Farrington, S., J. McDonagh, C. Colebrook, and A. Gurney (2008). Public Finances and the Cycle. Working Paper No. 5 (Washington: U.S. Department of Treasury).
Ferroni, F. (2011). Trend Agnostic One-step Estimation of DSGE Models. *B.E. Journal of Macroeconomics*, 11(1): 25.
Fukac, M. and A. Pagan (2005). Issues in Adopting DSGE Models for Policy Decisions. Unpublished manuscript.
Funke, M., M. Paetz, and E. Pytlarczyk (2010). Stock Market Wealth Effects in an Estimated DSGE Model for Hong Kong. *Economic Modelling*.
Gal í, J. and T. Monacelli (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies*, 72, 707–734.
Garratt A., D. Robertson, and S. Wright (2006). Permanent vs. Transitory Components and Economic Fundamentals. Journal of Applied Econometrics, 21, 521-542.
Genberg, H. and L. Pauwels (2005). An Open-economy New Keynesian Phillips Curve: Evidence from Hong Kong. *Pacific Economic Review*, 10, 261–277.
Girouard, N. and C. Andr é (2005). Measuring Cyclically Adjusted Budge Balances for OECD Countries. OECD Economics Department, Working Paper, No. 434 (Paris: OECD).
Harvey, A.C. (1989). *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge: Cambridge University Press.
Harvey, A.C. and A. Jaeger (1993). Detrending, Stylized Facts and the Business Cycle. *Journal of Applied Econometrics*, 8, 231-247.
IMF (2012). *Fiscal Monitor April 2012: Balancing Fiscal Policy Risks*.
Immervoll, H. (2000). The Impact of Inflation on Income Tax and Social Insurance Contributions in Europe. EUROMOD, Working Paper, No. EM2/00.
Iskrev, N. (2010). Local Identification in DSGE Models. *Journal of Monetary Economics*, 57(2), 189–202.
Kanda, D. (2010). Asset Booms and Structural Fiscal Positions: The Case of Ireland. IMF, Working Paper, No. 10/57.
Koop, G., M.H. Pesaran, and R.P. Smith (2011). On Identification of Bayesian DSGE Models. Unpublished manuscript.
Lafourcade, P. and J.d. Wind (2012). Taking Trends Seriously in DSGE Models: An Application to the Dutch Economy. DNB Working Papers, No. 345.
Mendoza, E.G., A. Razin and L.L. Tesar (1994). Effective Tax Rates in Macroeconomics: Cross-country Estimates of Tax Rates on Factor Incomes and Consumption. *Journal of Monetary Economics*, 34, 297-323.
Moons, C. (2009). Monetary and Fiscal Policy Rules in an Estimated Two Country DSGE Model: Micro-economic Foundations and Application to the Euro Area

and the UK. HUB Research Paper, 2009/03.
Obstfeld, M. and Rogoff, K. (2000). The Six Major Puzzles in International Macroeconomics: Is There a Common Cause? In: Bernanke, B., Rogoff, K. (Eds.), *NBER Macroeconomics Annual*, 15: 339-390. MIT Press.
Paetz, M. (2011). Stock Price Dynamics and the Business Cycle in an Estimated DSGE Model for the UK. Unpublished manuscript, Hamburg University.
Pesaran M.H. and Y. Shin (1998). Generalized Impulse Response Analysis in Linear Multivariate Models. *Economics Letters*, 58: 17–29.
Price, R. and T. Dang (2011). Adjusting Fiscal Balances for Asset Price Cycles. OECD Economics Department, Working Paper No. 868.
Rodriguez, J.C., C.R. Tokman and A.C. Vega (2007). Structural Balance Policy in Chile. *Studies in Public Finance of the Budget Office of the Finance Ministry* (Santiago).
Smets, F. and R. Wouters (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3), 586-606.
Turner, D. (2006). Should Measures of Fiscal Stance be Adjusted for Terms of Trade Effects? OECD Economics Department, Working Paper, No. 519 (Paris: OECD).
Yun, T. (1996). Nominal Price Rigidity, Money Supply Endogeneity and Business Cycle. *Journal of Monetary Economics*, 37, 345-370.
Zeng, L., (2012). Determinants of the Primary Fiscal Balance—Evidence from a Panel of Countries. IMF, Working Paper.

APPENDICES

Appendix A: the cyclical DSGE representation of the benchmark model:

$$\widehat{\Pi_t} = \widehat{\Pi_{H,t}} + \rho \cdot \Delta\, tot_t$$

$$\mathrm{GAP}_t = (1-\rho)\cdot \widehat{Y_t} + \rho \cdot \widehat{GDP_t^*} + \rho \cdot [\xi + \omega \cdot (1-\rho)] \cdot tot_t$$

$$\widehat{C_t} = E_t\left(\widehat{C_{t+1}}\right) - \left[(1-\tau_B)\cdot \widehat{R_t^2} - E_t\left(\widehat{\Pi_{t+1}}\right)\right]$$

$$\widehat{Q_t} = \alpha_2 \cdot E_t\left(\widehat{Q_{t+1}}\right) + (1-\alpha_2)\cdot E_t\left(\widehat{D_{t+1}}\right) - (1-\tau_B)\cdot \widehat{R_t^2} + \eta_t^Q$$

$$\widehat{\Pi_{H,t}} = \beta \cdot E_t\left(\widehat{\Pi_{H,t+1}}\right) + \frac{(1-\theta)\cdot(1-\beta\cdot\theta)}{\theta\cdot(\varepsilon-1)} \cdot \left(\mathrm{GAP}_t - \widehat{D_t} - \rho \cdot tot_t + \widehat{N_t}\right)$$

$$\widehat{R_t^2} = (1-\rho_R)\cdot\left[\varphi_1 \cdot \widehat{\Pi_t} + \varphi_2 \cdot \mathrm{GAP}_t\right] + \rho_R \cdot \widehat{R_{t-1}^2} + v_t^R$$

$$\widehat{\mathrm{LST}_t} = \varepsilon_{LST}\cdot\left(\widehat{\mathrm{B}_{t-1}} - \mathrm{GAP}_{t-1} - \widehat{N_{t-1}} + \rho \cdot tot_{t-1}\right) + \mathrm{GAP}_t + \widehat{N_t} - \rho\cdot tot_t$$

$$\widehat{\mathrm{G}_t} = \rho_G \cdot\left(\widehat{\mathrm{G}_{t-1}} - \mathrm{GAP}_{t-1} - \widehat{N_{t-1}} + \rho\cdot tot_{t-1}\right) + \mathrm{GAP}_t + \widehat{N_t} - \rho \cdot tot_t + v_t^G$$

$$\widehat{\mathrm{B}_{t+1}} = \frac{g}{b\cdot(1+\overline{\Pi}+\gamma_A)}\cdot\widehat{G_t} + \frac{1+R}{(1+\overline{\Pi}+\gamma_A)}\cdot\left(\widehat{\mathrm{B}_t} + \widehat{R_{t-1}^2}\right) - \frac{g + b\cdot(R-\overline{\Pi}-\gamma_A)}{b\cdot(1+\overline{\Pi}+\gamma_A)}\cdot\widehat{FR_t}$$

$$\widehat{\mathrm{FR}_t} = \frac{\tau_L \cdot \overline{\mathrm{P}_{H,t}}\cdot\overline{\mathrm{GDP}_t}}{\overline{\mathrm{FR}_t}}\cdot\left(\mathrm{GAP}_t + \widehat{N_t} - \rho\cdot tot_t\right) + \frac{(\tau_D - \tau_L)\cdot\overline{\mathrm{D}_t}}{\overline{\mathrm{FR}_t}}\cdot\widehat{D_t} + \frac{(\tau_K-\tau_L)\cdot r\cdot\overline{\mathrm{P}_t}\cdot\overline{\mathrm{K}_t}}{\overline{\mathrm{FR}_t}}\cdot\left(\frac{\widehat{r_t^1}}{r} + \widehat{N_t} + \widehat{K_t}\right) + \frac{\tau_B\cdot R\cdot\overline{\mathrm{B}_t}}{\overline{\mathrm{FR}_t}}\cdot\left(\frac{\widehat{R_{t-1}^2}}{R} + \widehat{B_t}\right) + \frac{\tau_C\cdot\overline{\mathrm{P}_t}\cdot\overline{\mathrm{C}_t}}{\overline{\mathrm{FR}_t}}\cdot\left(\widehat{N_t} + \widehat{C_t}\right) + \frac{\overline{\mathrm{LST}_t}}{\overline{\mathrm{FR}_t}}\cdot\widehat{LST_t}$$

$$(1-\tau_K)\cdot E_t\left(\widehat{r_{t+1}^1}\right) = (1-\tau_B)\cdot\widehat{R_t^2} - E_t\left(\widehat{\Pi_{t+1}}\right)$$

$$\widehat{\mathrm{Y}_t} = \frac{\overline{\mathrm{C}_t}}{\overline{\mathrm{Y}_t}}\cdot\widehat{C_t} + \frac{\overline{\mathrm{I}_t}}{\overline{\mathrm{Y}_t}}\cdot\widehat{I_t} + \left(1 - \frac{\overline{\mathrm{C}_t}}{\overline{\mathrm{Y}_t}} - \frac{\overline{\mathrm{I}_t}}{\overline{\mathrm{Y}_t}}\right)\cdot\left(\widehat{G_t} - \widehat{N_t}\right)$$

$$\widehat{K_{t+1}} = \frac{1-\delta}{1+\gamma_A}\cdot\widehat{K_t} + \frac{\gamma_A+\delta}{1+\gamma_A}\cdot\widehat{I_t}$$

$$\widehat{W_t} = \widehat{C_t} + \frac{L}{1-L}\cdot\widehat{L_t} + \widehat{N_t}$$

$$\widehat{D_t} = \mathrm{GAP}_t + \widehat{N_t} - \varepsilon\cdot\rho\cdot tot_t - (\varepsilon-1)\cdot\widehat{mc_t}$$

$$\widehat{mc}_t = \alpha \cdot \left(\widehat{W_t} - \widehat{N_t}\right) + (1-\alpha) \cdot \frac{\widehat{r_t^1}}{r} - \widehat{\epsilon_t}$$

$$\mathrm{GAP}_t = (1-\alpha) \cdot \widehat{K_t} + \alpha \cdot \widehat{L_t} + \widehat{\epsilon_t}$$

$$ln\epsilon_t = \rho_a \cdot ln\epsilon_{t-1} + v_t^a, v_t^a \sim N(0, \sigma_a^2)$$

$$\widehat{\Pi_t} = \widehat{N_t} - \widehat{N_{t-1}}$$

$$\widehat{K_t} - \widehat{L_t} = \widehat{W_t} - \widehat{N_t} - \frac{\widehat{r_t^1}}{r}$$

$$\widehat{N_t} = \widehat{P_{F,t}} - (1-\rho) \cdot tot_t$$

In the fourth equation above, we add a shock, η_t^Q, to account for the high volatility in the stock price gap, following Funke *et al.* (2010). The high volatility of the UK stock price time series can be seen in Appendix C1. For identification, we need three more equations. We assume AR(1) processes for η_t^Q, $\widehat{GDP_t^*}$ and $\widehat{P_{F,t}}$ as below:

$$\eta_t^Q = \rho_Q \cdot \eta_{t-1}^Q + v_t^Q, v_t^Q \sim N(0, \sigma_Q^2)$$

$$\widehat{GDP_t^*} = \rho_{gap}^* \cdot \widehat{GDP_{t-1}^*} + v_{gap,t}^*, v_{gap,t}^* \sim N(0, (\sigma_{gap}^*)^2)$$

$$\widehat{P_{F,t}} = \rho_P^* \cdot \widehat{P_{F,t-1}} + v_{P,t}^*, v_{P,t}^* \sim N(0, (\sigma_P^*)^2)$$

The 24 equations above give a set of linear equations linking all the cyclical components of the model's aggregate variables.

Appendix B: supplementary tables

Table B1. Bayesian estimation results for benchmark and alternative settings of stochastic trends

	Posterior means of models			
Parameter	Benchmark	M1	M2	M3
ρ	0.4413	0.6173	0.5025	0.5036
θ	0.7033	0.5943	0.6131	0.7046
ρ_a	0.7071	0.8105	0.7349	0.7122
ρ_Q	0.8397	0.8582	0.8592	0.8450
ρ^*_{gap}	0.8000	0.8056	0.8301	0.7967
ρ^*_P	0.7057	0.8919	0.7594	0.7180
ρ_R	0.8359	0.8843	0.9365	0.9414
ρ_G	0.7253	0.7456	0.7539	0.7567
σ_a	0.0134	0.0238	0.0087	0.0092
σ_Q	0.0262	0.0277	0.0257	0.0263
σ^*_{gap}	0.0671	0.0643	0.0741	0.0619
σ^*_P	0.0142	0.0272	0.0165	0.0152
σ_R	0.0104	0.0160	0.0112	0.0100
σ_G	0.0393	0.0389	0.0384	0.0384
σ_A	0.0160		0.0189	0.0139
σ_P	0.0122		0.0164	0.0120
Log likelihood	308.82	298.22	304.01	307.21

Note: M1 is for deterministic trend model, M2 is for drifted-random-walk trend model, and M3 is for the model in which real GDP trend is a drifted random walk while aggregate price trend is an integrated random walk.

Table B2. Conditional forecast error variance decompositions for the fiscal revenue cycle and output gap (in percent)

		Fiscal revenue cycle			Output gap		
	Horizon (year)	1	3	10	1	3	10
Shocks	v^a_t	0.01	0.70	1.08	6.69	14.34	9.00
	v^Q_t	0	0	0	0	0	0
	$v^*_{gap,t}$	27.62	28.52	28.34	41.62	32.52	17.03
	$v^*_{P,t}$	6.65	8.59	8.52	11.20	8.44	6.00
	v^R_t	51.69	47.11	43.49	39.42	32.69	36.38
	v^G_t	14.02	15.07	18.57	1.07	12.01	31.58

Table B3. Bayesian estimation results for robustness checks

	Posterior means of the models							
Parameter	Benchmark	R1	R2	R3	R4	R5	R6	R7
ρ	0.4413	0.3961	0.4414	0.4049	0.3636	0.5892	0.4165	0.4016
θ	0.7033	0.7045	0.7592	0.7475	0.7085	0.6570	0.7259	0.7464
ρ_a	0.7071	0.7031	0.7168	0.7122	0.7050	0.7033	0.7129	0.7097
ρ_Q	0.8397	0.838	0.8304	0.8311	0.8395	0.8539	0.8393	0.8288
ρ_{gap}^*	0.8000	0.8021	0.7692	0.7709	0.7797	0.82110	0.7864	0.7685
ρ_P^*	0.7057	0.7249	0.7115	0.6965	0.7020	0.7045	0.7015	0.6966
ρ_R	0.8359	0.8778	0.7849	0.815	0.8095	0.9273	0.8327	0.8026
ρ_G	0.7253	0.7406	0.7155	0.7167	0.7241	0.7527	0.7203	0.7154
σ_a	0.0134	0.0136	0.0199	0.0187	0.0132	0.0061	0.0163	0.0197
σ_Q	0.0262	0.0249	0.0256	0.0262	0.0249	0.0273	0.0248	0.0267
σ_{gap}^*	0.0671	0.0743	0.0527	0.0642	0.0645	0.0655	0.0667	0.0641
σ_P^*	0.0142	0.0146	0.0139	0.0142	0.0146	0.0137	0.0141	0.0142
σ_R	0.0104	0.0089	0.0089	0.0095	0.0086	0.0136	0.0104	0.0094
σ_G	0.0393	0.0384	0.0395	0.0388	0.0391	0.0383	0.0393	0.0394
σ_A	0.0160	0.0146	0.0169	0.0158	0.0160	0.0130	0.0162	0.0180
σ_P	0.0122	0.0118	0.0121	0.0120	0.0120	0.0130	0.0122	0.0121

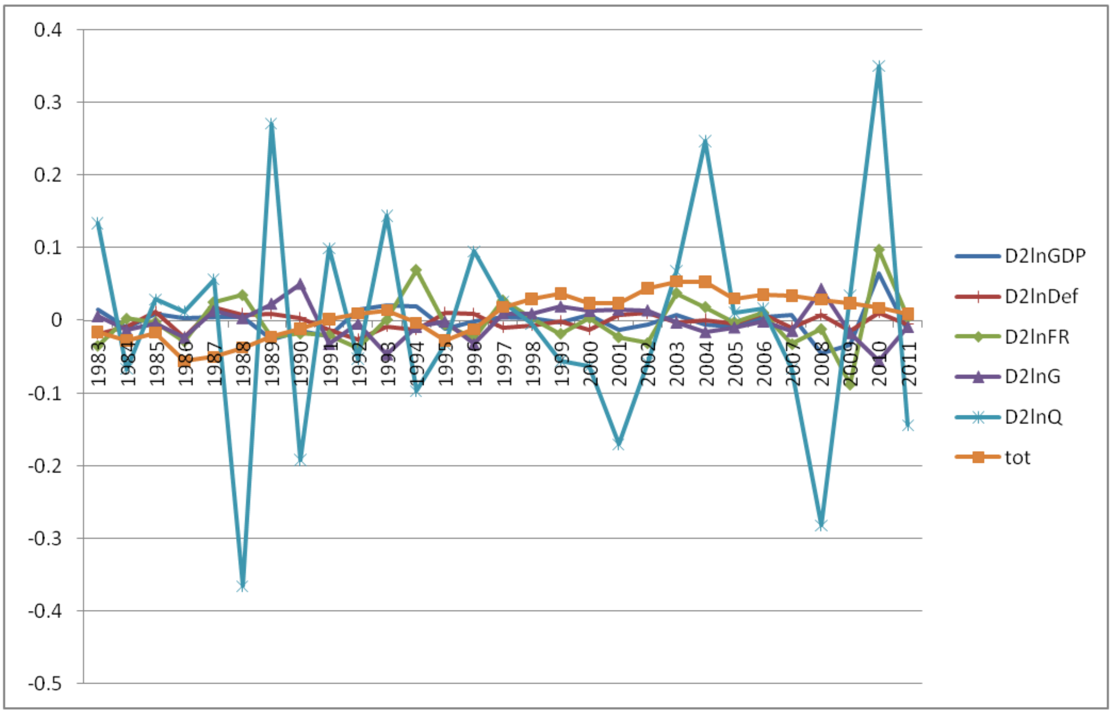
Note: R1, R2… R7 denote seven alternative calibrations of the model, for robustness checks.

Table B4. Robustness checks: forecast error variance decompositions of the fiscal revenue cycle (in percent) under the benchmark and 7 alternative settings

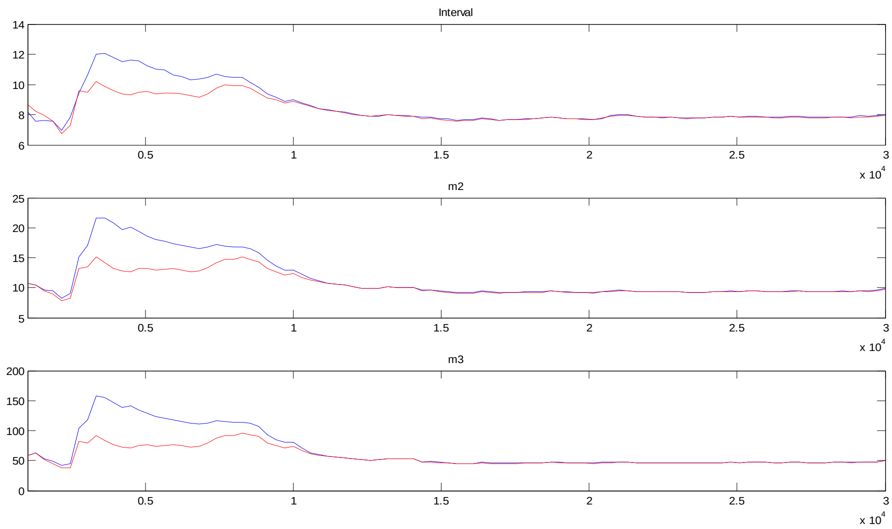
Different Settings of the model	Shocks to					
	Temporary productivity	Stock price	Foreign demand	Foreign price	Interest rate	Government spending
Benchmark	1.33	0.00	27.38	8.18	42.58	20.53
$\alpha = 0.64$	1.25	0.00	25.99	7.90	42.08	22.79
$\omega = 4$	4.23	0.00	21.33	6.91	28.95	38.58
$\varepsilon_{LST} = 0.01$	2.78	0.00	22.76	9.45	38.85	26.17
$\tau_C = 0.201$, $\tau_K = 0.407$, $\tau_L = 0.284$	1.64	0.00	19.10	8.83	38.79	31.64
$\tau_B = 0.20$, $\tau_D = 0.20$	0.36	0.00	33.04	5.71	52.52	8.36
$RT_{QD} = 17$	1.89	0.00	23.76	8.14	43.85	22.35
$RT_{QD} = 11.9$	3.13	0.00	22.48	9.51	36.93	27.95

Appendix C: supplementary figures

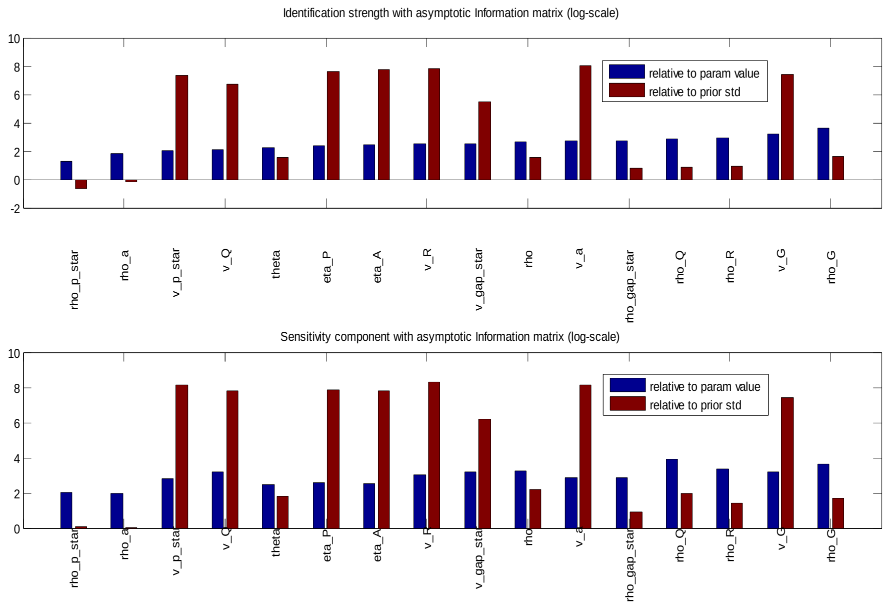
Appendix C1: data plot



Appendix C2: multivariate Brooks and Gelman diagnostics



Appendix C3: identification of the benchmark Bayesian DSGE model



Appendix C4: other figures

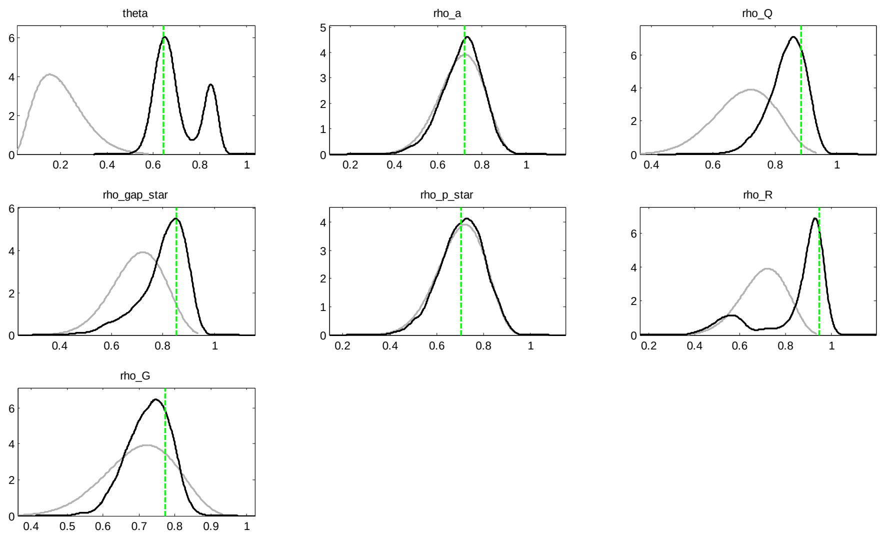


Fig. C4.1. Prior vs. posterior distributions in the Metropolis-Hastings procedure: part 1 (dashed grey line: prior; solid black line: posterior; vertical green line: posterior mean)

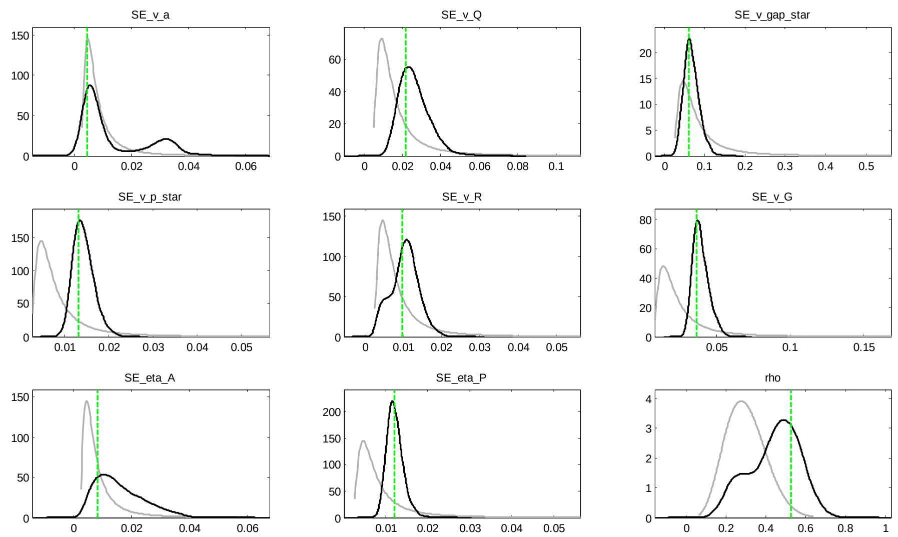


Fig. C4.2. Prior vs. posterior distributions in the Metropolis-Hastings procedure: part 2 (dashed grey line: prior; solid black line: posterior; vertical green line: posterior mean)

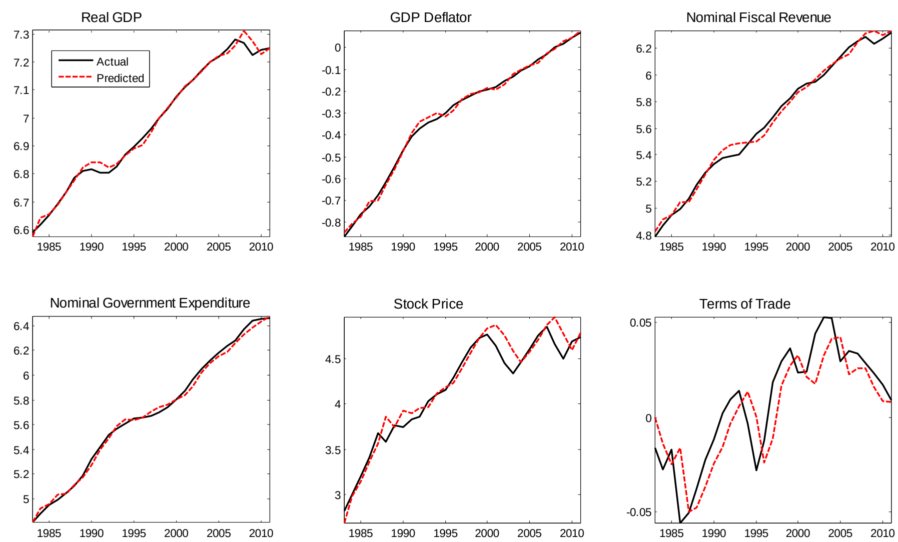


Fig. C4.3. Fit of the benchmark model: dashed red lines are the posterior mean of one-step-ahead predictions, and solid black lines are the actual data (in logarithm).

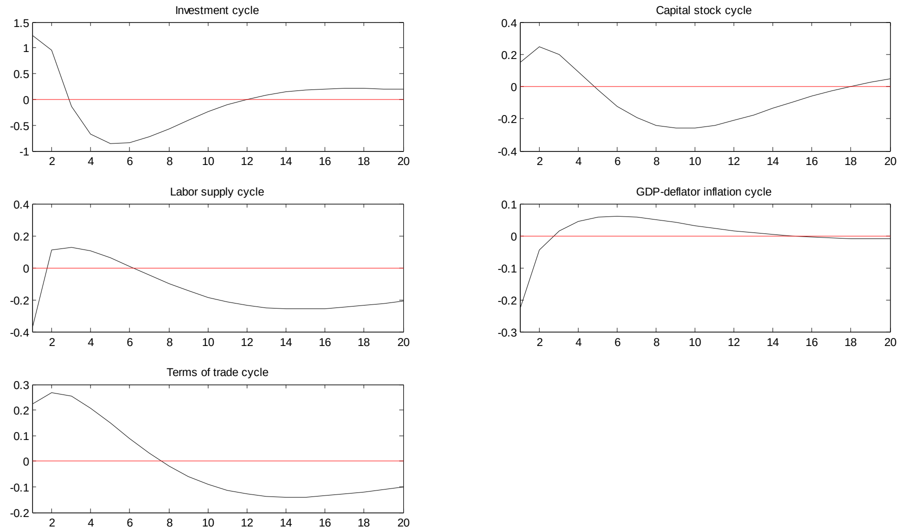


Fig. C4.4. The transmission of the temporary technology shock: impulse responses of the investment cycle, the capital stock cycle, the labor supply cycle, the GDP-deflator inflation cycle and the terms of trade cycle to a one-percent temporary technology shock.

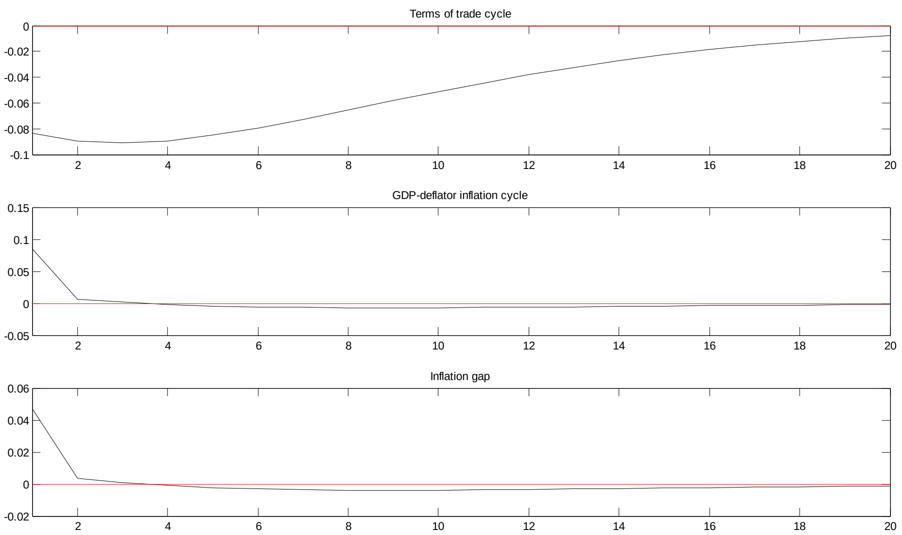


Fig. C4.5. The transmission of the foreign demand shock: impulse responses of the terms of trade cycle, GDP-deflator inflation cycle and the inflation gap to a one-percent foreign output shock.

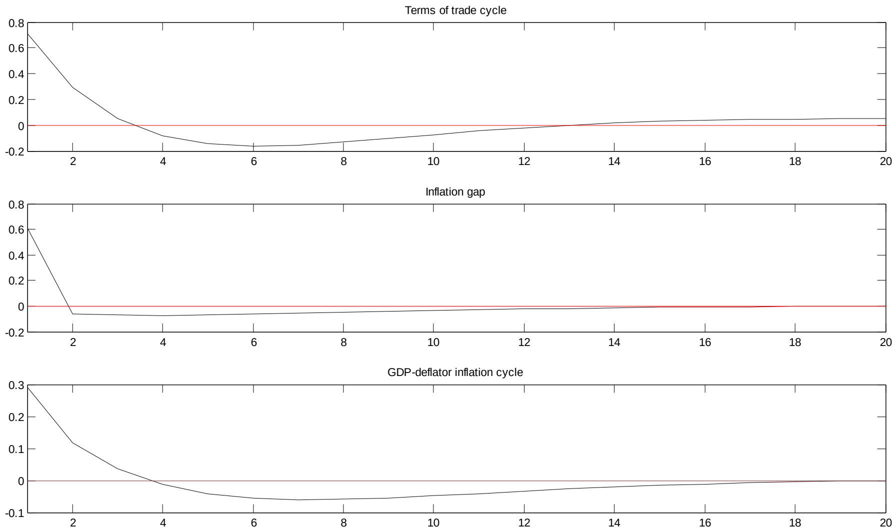


Fig. C4.6. The transmission of the foreign price shock: impulse responses of the terms of trade cycle, the inflation gap, and the GDP-deflator inflation cycle to a one-percent foreign price shock.

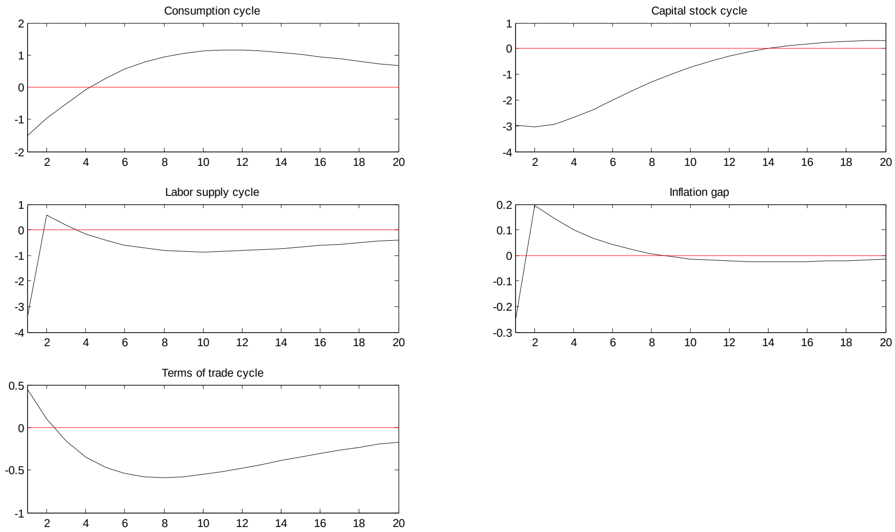


Fig. C4.7. The transmission of the monetary policy shock: impulse responses of consumption, capital stock, labor supply, the inflation gap and the terms of trade to a one-percent nominal interest rate shock.

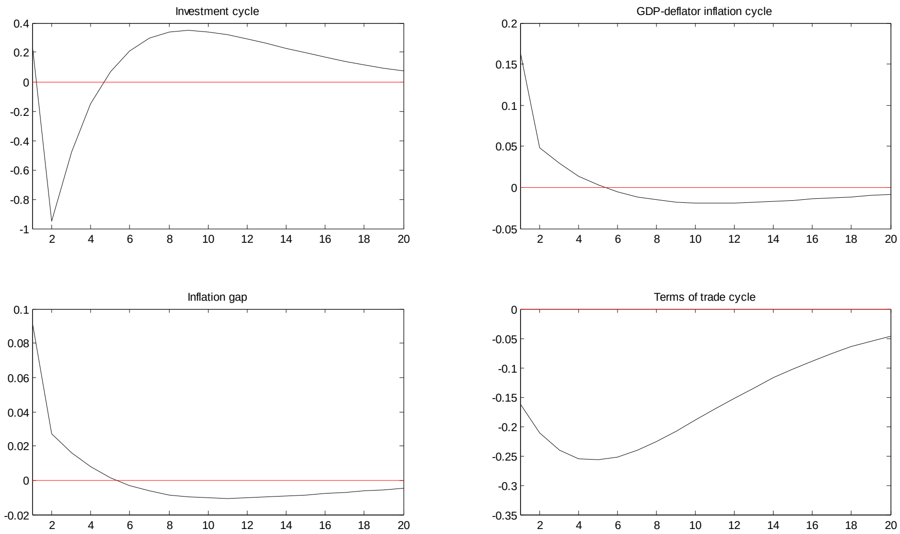


Fig. C4.8. The transmission of the government spending shock: impulse responses of the investment cycle, the GDP-deflator inflation cycle, the inflation gap, the terms of trade cycle to a one-percent government spending shock.

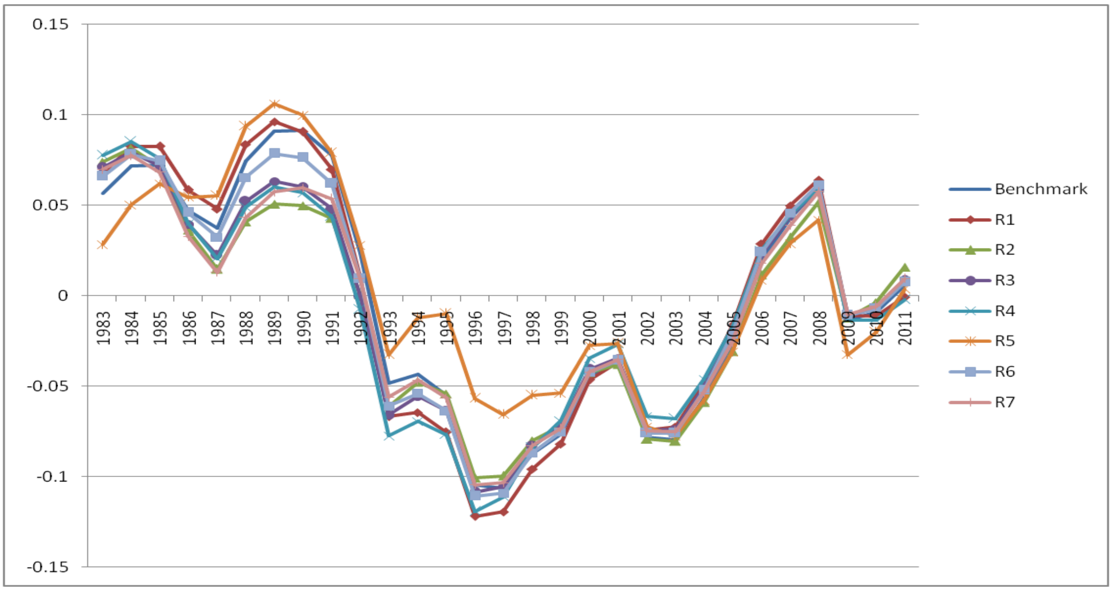


Fig. C4.9. The smoothed time series of the fiscal revenue cycle under different calibration settings for robustness checks: benchmark model and 7 alternative settings

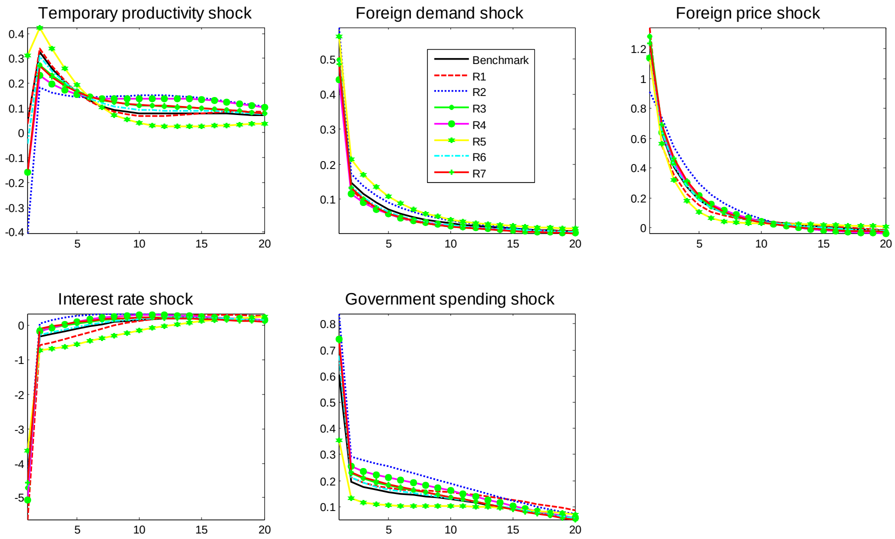


Fig. C4.10. Impulse responses of the fiscal revenue cycle (in percent) to one-percent shocks to the temporary technology $ln\epsilon_t$, the foreign output gap $\widehat{GDP}_t^*$, the foreign price gap $\widehat{P_{F,t}}$, nominal interest rate R_t^2, and the government expenditure-GDP ratio g_t, under different calibration settings for robustness checks: benchmark model and 7 alternative settings