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1 Introduction

The recent boom-bust episodes concerning asset prices, such as those in South-East Asia and Latin America in the late 1990s or the U.S. dot-com and housing booms in the early 2000s, were preceded by massive in- and outflows of foreign speculative investments. These large fluctuations in asset prices are usually referred to as "bubbles" as they are difficult to be explained by economic fundamentals (Shiller, 2000). This paper argues that, in assessing the effects of financial globalization on bubbles, the conditions of financial market play an important role.

Our analysis is motivated by the following empirical facts.

1. *Asymmetric Financial development:* Figure 1(a) and 1(b) plot the IMF financial development index for major advanced and emerging economies in 1980 and 2018.[1] We learn that the overall degree of financial development is higher in the advanced economies than the emerging ones. Moreover, there is a significant variation in financial development even among the advanced economies. In particular, the United States has one of the most developed, if not the most, financial markets in the world.[2]

2. *Rising International capital flows:* In the past few decades, both advanced and emerging economies experienced large movements in international capital flows. Figures 2(a) and 2(b) plot the current account over GDP ratio in the U.S. and Thailand, as a representative emerging market. The U.S. experienced persistent current account deficit since 1990s until reaching its peak in 2006. Thailand also had current account deficit but it turned to a sudden capital outflow, *i.e.* it experienced a sudden stop, in 1997.

3. *Boom and bust in asset prices:* Figures 3(a) and 3(b) show the stock market and house price indices in the U.S. and Thailand. Since 1980s, the U.S. experienced a surge in stock and house prices, especially the house price, until they plummeted due to the global financial crisis in 2008. Thailand also experienced an increase in stock price until the Asian financial crisis in 1997 caused a precipitous drop in stock price.

This paper shows that the bubble-like dynamics of asset prices (fact 3) can be an equilibrium outcome of financial integration among economies with different degrees of financial development (fact 1) generating large and sometimes jittery capital flows (fact 2). In particular, we address three interrelated questions. First, does financial globalization favor the emergence of bubbles? Second, if so, how would bubbles affect economic growth during the process of financial globalization? Last but not least, how would bubbles affect welfare in different countries? To address these questions, we construct a two-country model of rational bubbles with financial frictions, where each country

[1] The index was developed combining the depth, access, and efficiency of financial institutions and markets. See Svirydzenka (2016) for detailed methodology.

[2] In 2018, the U.S. financial market the was third developed in the world, whereas Switzerland was ranked the first and the United Kingdom the second.

is at a different level of financial development. Financial development is defined by the degree with which creditors can enforce the debt contract to debtors. In particular, the key indicator of financial development is the fraction of the debtors' future income which they can credibly pledge to the creditors (Hart and Moore, 1994).

Our main result is as follows. First, we show that the effect of financial globalization on the existence of bubbles depends on both the absolute and relative degrees of financial development in the two countries. It is traditionally thought that bubbles cannot exist in an equilibrium when the agents are infinitely lived (Tirole, 1982). However, when the financial market is imperfect, bubbles can circulate even when the agents are infinitely lived.[3] Our first result shed novel theoretical light on how financial globalization changes the existence condition of bubbles.

We show that, when the domestic financial market is either sufficiently developed or underdeveloped relative to the rest of the world, bubbles in the domestic country cannot arise under financial autarky but can arise under financial globalization. This is because capital inflows following liberalization suppress the domestic interest rate relative to the growth rate, making it possible for bubbles to be sustained. Financial globalization thus promotes the emergence of bubbles. On the contrary, bubbles are unlikely to arise if the domestic financial development is in the middle range and the foreign financial market is either sufficiently developed or underdeveloped. This is because, capital outflows increase the domestic interest rate relative to the growth rate, making it difficult for bubbles to exist. Financial globalization thus prevents the emergence of bubbles. Overall, the main conclusion is that the effect of financial globalization on the existence condition of bubbles is non-monotonic.

Our results are consistent with the evidence put forward by the large body of literature studying recent financial crises. According to a leading hypothesis, the United States, which had one of the most developed financial markets worldwide, absorbed excess savings in emerging economies in the aftermath of the Asian currency crisis. The low interest environment generated by persistent current account deficits during the 2000s is commonly believed to have fueled bubbles in real estate and stocks as investors searched for higher yields (Obstfeld and Rogoff, 2009; Yellen, 2009; Bernanke, 2011). Among emerging countries, the lack of institutional quality and regulatory framework made them more fragile against short-term external financial flows and credit booms followed by currency crises during the 1990s (Kaminsky and Reinhart, 1999; Stiglitz, 2000; Caballero and Krishnamurthy, 2001). This paper contributes to the literature showing how the integration of economies with different degrees of financial development can trigger bubble-like dynamics of asset prices.

Second, we show that, with financial globalization, the effect of bubbles on economic growth is also non-monotonic. Theory suggests that bubbles have two offsetting effects on economic

[3] See the literature review for details.

growth. On one hand, bubbles crowd out resources for capital investment and suppress economic growth. On the other hand, they expand the producers' balance sheet and increase their net worth, which crowds in capital investment. The model in this paper is tuitable to investigate how financial globalization affects the relative sizes of these two effects.[4]

Namely, when the domestic financial market is either developed or underdeveloped relative to the foreign one, financial globalization improves the crowd-in effect relative to crowd-out effect. The growth-enhancing effect of bubbles becomes stronger. However, when the degree of domestic financial market development is in the middle range, financial globalization mitigates the crowd-in effect relative to the crowd-out effect. The growth effect of bubbles becomes weaker.

Our result can explain a key empirical fact concerning the effects of financial crises on economic growth. Historically, financial crises induced a persistent drop in the output growth rate compared to the pre-crisis trend (Cerra and Saxena, 2008). Our result suggests that, as long as the domestic financial market is either developed or underdeveloped relative to the foreign one, bubbles increase the economic growth rate greatly under financial globalization, which implies their collapse in turn leads to a large decline in economic growth. In other words, financial globalization not only facilitates the emergence of bubbles but also strengthens the magnitude of the associated booms and busts.

Finally, we show that the welfare effects of bubbles can be significantly asymmetric across countries. Based on a closed-economy setting, bubbles have been shown to be welfare-improving (Samuelson, 1958; Tirole, 1985). Reconsidering this result using a rigorous full welfare analysis under two-country framework, we find that bubbles in one country can reduce welfare in the foreign country. In particular, this is the case when bubbles emerge only in one country. In line with the literature, bubbles tend to be welfare-improving for the bubble-holding country. This is because, when agents are financially constrained and they cannot consume smoothly against idiosyncratic productivity risks, the circulation of bubbles works as an insurance device against the idiosyncratic risks (Bewley, 1980). However, different from the literature, bubbles in a sufficiently well-developed financial market are welfare-reducing for the non-bubble-holding country. This is because the negative effect of bubbles on economic growth is transmitted abroad through general equilibrium effect. Our result suggests a justification for "lean-against-the-wind" policy since policymakers need to take into account this negative externality of bubbles on other countries.

Related Literature. There is a strand of literature that studies the relationship between financial development, international capital flows, and financial crises. On the advanced economies' side,

[4]In previous papers, such as Martin and Ventura (2015a, 2015b) and Ikeda and Phan (2018), crowd-in effect does not exist without assuming exogenous bubble creation shock. However, in our model, crowd-in effect arises endogenously through general equilibrium effect. See the literature review for details.

Matsuyama (2004), Caballero et al. (2008), and Mendoza et al. (2009) built a model of "global imbalances," in which the countries with most developed financial markets experience persistent current account deficits. On the emerging markets' side, Caballero and Krishnamurthy (2001) argued that tight domestic and international borrowing constraints increased their vulnerability to sudden stops of capital flows and downturns in asset prices. In particular, our model without bubbles is based on Aoki et al. (2007, 2009, 2010), who showed that both financially developed and underdeveloped economies can experience capital inflows.[5] While their main focus is on the economy without bubbles, we consider the existence of asset bubbles under two-country economy.

Our paper studies the existence condition of bubbles under infinite horizon economy. It is traditionally thought that bubbles cannot exist in an equilibrium when agents are infinitely lived (Tirole, 1982). The Tirole's model assumes perfect financial market, *i.e.,* agents can borrow and lend freely with each other. On the other hand, when the financial market is imperfect, bubbles (or fiat money) can circulate even when agents are infinitely lived (Bewley, 1980; Scheinkman and Weiss, 1986; Woodford, 1990; Kocherlakota, 1992; Santos and Woodford, 1997; Kiyotaki and Moore, 2019). Intuitively, when financial friction limits the agents' borrowing capacity, the interest rate is suppressed relative to the growth rate. Then, the growth rate of bubble assets does not exceed the economic growth rate so that bubbles can be sustained in equilibrium.

In particular, our model is based on Hirano and Yanagawa (2017), who showed that bubbles cannot exist when the financial market is either sufficiently developed or underdeveloped but they can only exist when the degree of financial development is in the intermediate range.[6] Their model is based on closed economy settings. We extend their model to a two-country framework where the two countries face different degrees of financial development and discuss how financial globalization changes this existence condition of bubbles.

There are some papers on bubbles and international capital flows (Martin and Ventura, 2015a, 2015b; Ikeda and Phan, 2018). There are several important differences between extant studies and ours. First, their studies are based on overlapping generations (OLG) model. As suggested by Samuelson (1958) and Tirole (1985), under OLG framework, financial market imperfection is not essential for the existence of bubbles. This is because, as long as the economy is dynamically inefficient, bubbles can exist in equilibrium even if financial market is perfect. However, in an infinitely-lived agents model, bubbles can only exist when financial market is imperfect. We study how financial globalization affects their existence condition.

The second point is regarding the effect of bubbles on capital investment. In the previous papers, crowd-in effect occurs due to the assumption of exogenous bubble creation shock by young agents. In their models, agents invest in capital when they are young and consume when they are old. Even

[5] I thank Kosuke Aoki for sharing the slides of Aoki et al. (2007).

[6] We will discuss the intuition for this result in Section 2.4.1.

if young agents cannot borrow, they can create bubble assets, which increases their wealth directly and crowd in capital investment. This implies that the crowd-in effect arises because they assume bubbles expand agents' wealth during good times. Without this assumption, crowd-in effect does not exist but only crowd-out effect exists.[7] However, in our model, bubbles generate crowd-in effect endogenously through general equilibrium effect. The agents buy bubbles when they are unproductive and sell bubbles when they are productive. This speculative investment increases the asset return and expands the borrowers' net worth. This relaxes the agents' credit constraint and generates crowd-in effect.[8] We consider how financial globalization changes the relative size of crowd-in and crowd-out effects.

Third, in the literature, without bubbles, capital always flows from financially underdeveloped to developed economies. This implies that financial globalization prevents bubbles in emerging markets, which contradicts the episodes of emerging markets' credit booms and currency crises in 1990s. By contrast, in our model, financial globalization can expand the existence region of bubbles and promotes the growth-enhancing effects of bubbles even under tighter financial market conditions. Finally, our infinite-horizon model does not stray far away from the standard real business cycle (RBC) model. As Farhi and Tirole (2012) point out, this framework is more suitable for quantitative studies on the welfare implications of bubbles or even further policy analyses.

In addition, our work is also related to papers on the effect of bubbles on investment and economic growth. For instance, Tirole (1985) showed that bubbles crowd out capital investment. Grossman and Yanagawa (1993) extended this framework to the endogenous growth model and showed that bubbles crowd savings out of capital investment and reduce economic growth. Conversely, Woodford (1990), Caballero and Krishnamurthy (2006), Hirano and Yanagawa (2017), and Kiyotaki and Moore (2019) developed a framework where bubbles serve as liquidity and crowd in capital investment. We study how financial globalization changes the relative size of these two effects and how the effect of financial globalization depends on the degrees of financial development in the two countries.

Finally, to the best of my knowledge, little work has been done on welfare implication of bubbles in a two-country framework.[9] The original work by Hirano and Yanagawa (2017) show that, even if bubbles reduce economic growth or they are expected to collapse, bubbles tend to be welfare-improving due to consumption-smoothing effect. Their analysis is based on closed economy settings. However, we find that, under two-country framework, although bubbles are welfare-improving for the bubble-holding country, they are welfare-reducing for the non-bubble-holding country under some parameter condition.

[7] Their assumption of exogenous bubble creation is based on Martin and Ventura (2012).

[8] Bernanke and Gertler (1989) showed that borrowers' balance sheet plays an important role in business cycles.

[9] Martin and Ventura (2015a, 2015b) assume linear utility function and hence volatility of bubbles is irrelevant to the utility.

The rest of this paper is organized as follows. Section 2 presents the two-country model, both with and without bubbles. Section 3 shows the effects of financial globalization on the existence condition of bubbles. Section 4 examines how financial globalization changes the effects of bubbles on economic growth. Section 5 conducts a full welfare analysis of asset bubbles under financial globalization. Section 6 concludes the paper.

2 Model

We consider a two-country economy that consists of entrepreneurs and final good producers.[10] We assume a discrete-time and infinite-horizon economy. The two countries are large open economies, Home and Foreign, each having a continuum of entrepreneurs and final good producers. The Home and Foreign entrepreneurs produce differentiated intermediate goods.[11] The final goods producers, either Home or Foreign, purchase the two intermediate goods to produce a homogeneous final good.

Let us start with final good producers. In each period, final good producers combine two differentiated intermediate goods to produce one single final good:

$$z_t = \left[\omega^{\frac{1}{\sigma}} m_{ht}^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} m_{ft}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where z_t is the final output at date t and m_{ht} and m_{ft} are the demands for Home and Foreign intermediate goods at date t, respectively. σ is the elasticity of substitution between the two intermediate goods and ω the weight on Home intermediate goods.[12]

Regarding entrepreneurs, we focus on the Home entrepreneurs, as the Foreign entrepreneurs face similar conditions. A typical entrepreneur have the following expected discounted utility:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \log c_t^i\right], \tag{2}$$

where i is the index of each entrepreneur and c_t^i his/her consumption at date t. $\beta \in (0,1)$ is the subjective discount factor and $\mathbb{E}_0$ is the expectation conditional on date 0 information. At each date,

[10]Alternatively, we can do a similar analysis using small open economy model. Although the small open economy setup is simple enough so that we can derive a fully analytical solution, it also has limitations. See Appendix A.1.

[11]This assumption is imposed so that, even if the two countries face different rates of return on savings, in the long run, the economic growth rates across countries are equalized through the adjustment of terms of trade (intermediate good prices). In Section 2.4.1, we will describe this mechanism in detail.

[12]Following Acemoglu and Ventura (2002), we assume $\sigma > 1$ in the simulation. However, whether σ is greater or smaller than one does not affect our main result. In Appendix A.12, we conduct a comparative statistics on the value of σ.

each entrepreneur meets high-productivity investment projects (H-projects) with probability s, and low-productivity ones (L-projects) with probability $1 - s$. We call investment in H-projects "H-investment (L-investment)." The probability s is exogenous and independent across entrepreneurs. At the beginning of each period t, entrepreneurs know whether they have H- or L-projects. We call entrepreneurs with H-projects (L-projects) "H-types" (L-types). The production function of intermediate goods is:

$$m^i_{ht+1} = a^i_t k^i_t,\ a^i_t \in \{\alpha, \gamma\}, \tag{3}$$

where k^i_t is the investment level at date t and m^i_{ht+1} the output at date $t + 1$. a^i_t is the marginal productivity of investment at date t. $a^i_t = \alpha$ holds if the entrepreneur has H-projects and $a^i_t = \gamma$ if he/she has L-projects. We assume $\alpha > \gamma$.[13]

The entrepreneurs can borrow from other entrepreneurs, either domestically or abroad, to finance their investment. Let b^i_t and b^{wi}_t be the domestic and international borrowings at date t, respectively. Moreover, let r_t and r^*_t be the gross interest rates in the Home and Foreign countries at date t, respectively, and we define $r^w_t = \min\{r_t,\ r^*_t\}$ as the world interest rate.[14] We assume that, due to the financial friction, creditors can seize only a certain fraction of the pledgeable assets. Based on Aoki et al. (2010), for debt contracts to be fulfilled, foreign creditors restrict their lending so that the debt repayment cannot exceed a $\phi\theta$ fraction of the future output:

$$r^w_t b^{wi}_t \leq \phi\theta \mathbb{E}_t p_{ht+1} a^i_t k^i_t, \tag{4}$$

and domestic creditors restrict their lending so that the sum of domestic and foreign debt repayments cannot exceed a θ fraction of the future output:[15][16]

$$r_t b^i_t + r^w_t b^{wi}_t \leq \theta \mathbb{E}_t p_{ht+1} a^i_t k^i_t. \tag{5}$$

Parameters $\theta \in [0, 1]$ and $\phi \in [0, 1)$, both assumed to be exogenous, represent the degrees of

[13]In the main text, we assume that the only one source of asymmetry between the two countries is the degrees of financial development. Hence, the values of α and γ are common across the two countries. In Appendix A.14, we allow for the possibility where the two countries have different technologies and discuss the effect of worldwide technological progress on the emergence of bubbles.

[14]If r^w_t was higher than r_t, Home L-types would lend all their savings abroad while Home H-types want to borrow domestically. This contradicts the credit market clearing condition.

[15]For simplicity, we assume bubbles cannot be collateralized. In our framework, even if bubbles are not collateralized, they affect the economic growth rate by expanding the entrepreneurs' balance sheets. Hirano and Yanagawa (2017) also discussed the case where bubbles can be collateralized. They showed that, under closed economy, when the value of bubbles as collateral is sufficiently small, H-types do not buy bubbles but only L-types buy them. (See Section 2.2 for the entrepreneurs' behavior.)

[16]In equilibrium, since m_{ht+1} is predetermined at date t, p_{ht+1} is also predetermined at date t as equation (30).

financial development and financial openness in the Home country, respectively. An economy with a developed financial system has a developed legal system to enforce the debt contract so that agents can use large fraction of future income as collateral to borrow. We consider θ the overall degree of financial development and ϕ the relative inability of foreign creditors to enforce the contract.[17] When ϕ is positive, the Home country can borrow abroad. Similarly, we define $\theta^* \in [0, 1]$ and $\phi^* \in [0, 1)$ as the degrees of financial imperfection and financial openness in the Foreign country, respectively. We investigate how an exogenous increase in ϕ and ϕ^* affects the region of (θ, θ^*) where bubbles can exist.

Our main interest is in the economy where bubbles can exist. Following Tirole (1985), we define bubble assets as those with no fundamental returns.[18] Let x_t^i be the amount of bubble assets purchased by entrepreneur i at date t and let Q_t^x be the per unit price of bubble assets in terms of consumption goods at date t. Let X be the aggregate supply of bubbles, assumed to be constant over time. Following Weil (1987), we assume stochastic bubbles. In each period, bubbles survive (bubble price is positive) with some exogenous probability π and bubbles collapse (bubble price becomes zero) with probability $1 - \pi$ conditional on survival in the previous period. Once bubbles collapse, their reappearance in the future is not expected *ex ante*. Formally, at each date t, $Q_t^x = Q_t > 0$ if bubbles survive with probability π and $Q_t^x = 0$ if they collapse with probability $1 - \pi$. Moreover, in order to capture the key intuition on the existence condition and growth effect of bubbles, we focus on the case where bubbles exist only in the Home country and cannot be traded internationally.[19]

Each entrepreneur faces the following four constraints: the flow of funds constraint,

$$c_t^{\mu i} + k_t^{\mu i} + Q_t^x x_t^i = p_{ht}^{\mu} m_{ht}^{\mu i} - r_{t-1}^{\mu} b_{t-1}^{\mu i} - r_{t-1}^{\mu w} b_{t-1}^{\mu w i} + b_t^{\mu i} + b_t^{\mu w i} + Q_t^x x_{t-1}^i, \tag{6}$$

the foreign and domestic borrowing constraints (4) and (5), and the short-sale constraint:[20]

$$x_t^i \geq 0. \tag{7}$$

Here, subscript μ denotes the variable in the economy with bubbles. We define the net worth of

[17] See Hart and Moore (1994) for microfoundation of this setting.

[18] Santos and Woodford (1997) showed that bubbles cannot exist on productive assets in a rational expectations equilibrium. However, even if we could introduce bubbles for productive assets, we expect that the main implication for the existence condition of bubbles and their effect on economic growth would be similar.

[19] Caballero and Krishnamurthy (2006) also assume bubble assets, such as real estate, can only be traded domestically. In our framework, even if bubbles cannot be traded across nations, they can affect international capital flows through the change in interest rates. In Appendix A.15, we also consider the case where bubbles can be traded internationally (e.g., stock markets).

[20] Kocherlakota (1992) showed that, in endowment economies with infinitely lived agents, short-sale constraint is important for the existence of bubbles. This is because the short-sale constraint plays the role of no-Ponzi-game condition. Without the constraint, agents can obtain infinitely large profit by short-selling bubble assets.

the entrepreneur at date t as $e_t^{\mu i} = p_{ht}^{\mu} m_{ht}^{\mu i} - r_{t-1}^{\mu} b_{t-1}^{\mu i} - r_{t-1}^{\mu w} b_{t-1}^{\mu w i} + Q_t x_{t-1}^{i}$.

2.1 Final Good Producers

We first derive the equilibrium behavior of final good producers. In each period, final good producers maximize their profit:

$$\pi_t = \left[\omega^{\frac{1}{\sigma}} m_{ht}^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} m_{ft}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - p_{ht} m_{ht} - p_{ft} m_{ft}. \tag{8}$$

By taking the first-order condition, we obtain:

$$\frac{m_{ht}}{m_{ft}} = \frac{\omega}{1-\omega}\left(\frac{p_{ht}}{p_{ft}}\right)^{-\sigma}. \tag{9}$$

Let final good be the numeraire good. Then, the Dixit-Stiglitz price index can be expressed as:

$$P_t \equiv \left[\omega p_{ht}^{1-\sigma} + (1-\omega) p_{ft}^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = 1. \tag{10}$$

2.2 Entrepreneurs

We then characterize the equilibrium behaviors of entrepreneurs. We focus on Home entrepreneurs, since Foreign entrepreneurs face similar constraints. Let

$$\overline{r}_t^{\mu} = \begin{cases} \alpha p_{ht+1}^{\mu}, & \text{if } r_t^{\mu} = r_t^{\mu w}, \\ \frac{\alpha(1-\phi\theta) p_{ht+1}^{\mu}}{1-\frac{\alpha\phi\theta}{r_t^{\mu w}} p_{ht+1}^{\mu}}, & \text{if } r_t^{\mu} > r_t^{\mu w}, \end{cases} \text{ and } \underline{r}_t^{\mu} = \begin{cases} \gamma p_{ht+1}^{\mu}, & \text{if } r_t^{\mu} = r_t^{\mu w}, \\ \frac{\gamma(1-\phi\theta) p_{ht+1}^{\mu}}{1-\frac{\gamma\phi\theta}{r_t^{\mu w}} p_{ht+1}^{\mu}}, & \text{if } r_t^{\mu} > r_t^{\mu w}, \end{cases}$$

where $\frac{a_t^i(1-\phi\theta) p_{ht+1}^{\mu}}{1-\frac{a_t^i\phi\theta}{r_t^{\mu w}} p_{ht+1}^{\mu}}$, $a_t^i \in \{\alpha, \gamma\}$ is the leveraged rate of return of investment when they borrow abroad up to the limit. We focus on the equilibrium where $\underline{r}_t^{\mu} \leq r_t^{\mu} < \overline{r}_t^{\mu}$. This is because no entrepreneur in the Home country lends to domestic agents when $r_t^{\mu} < \underline{r}_t^{\mu}$, while no entrepreneur produces when $r_t^{\mu} > \overline{r}_t^{\mu}$.[21]

In our model, entrepreneurs have an incentive to invest in bubbles when they become L-types, and sell them when they become H-types. For H-types, when $r_t^{\mu} < \overline{r}_t^{\mu}$, the foreign and domestic

[21] When $r_t^{\mu} = \overline{r}_t^{\mu}$, bubbles cannot exist in equilibrium. This is because, since agents are risk-averse, for stochastic bubbles to exist, the expected return on bubbles must be strictly greater than the interest rate, r_t, which is equal to H-types' leveraged rate of return on investment. Hence, bubbles grow faster than the economic growth rate so that they cannot be sustained in the long run in equilibrium. See Hirano et al. (2015). Hence, we focus on the case where $r_t^{\mu} < \underline{r}_t^{\mu}$ so that the domestic borrowing (5) is binding.

borrowing constraints, (4) and (5), are binding, and they never invest in bubbles, that is, (7) is also binding. Since we adopt a logarithmic utility function, entrepreneurs consume a $1-\beta$ fraction of their net worth in each period, that is, $c_t^{\mu i} = (1-\beta)e_t^{\mu i}$.[22] Then, by using equations (4), (5), (6), and (7), the investment function for H-types can be written as:

$$k_t^{\mu i} \leq \frac{\beta e_t^{\mu i}}{1 - \frac{\alpha\phi\theta}{r_t^{\mu w}} p_{ht+1}^{\mu} - \frac{\alpha(1-\phi)\theta}{r_t^{\mu}} p_{ht+1}^{\mu}}, \tag{11}$$

where the equality holds if $r_t^{\mu} < \bar{r}_t^{\mu}$. The numerator represents the net savings of entrepreneurs and the denominator the required downpayment per unit of investment.[23] When $r_t^{\mu} > r_t^{\mu w}$, (4) binds so that H-types borrow abroad up to the limit. When $r_t^{\mu} = r_t^{\mu w}$, (4) does not bind so that H-types are indifferent between borrowing from domestic or foreign agents. Moreover, since $e_t^{\mu i} = p_{ht}^{\mu} m_{ht}^{\mu i} - r_{t-1}^{\mu} b_{t-1}^{\mu i} - r_{t-1}^{\mu w} b_{t-1}^{\mu w i} + Q_t x_{t-1}^i$, we learn that bubbles increase the entrepreneurs' net worth. When they have an opportunity to invest in H-projects, they sell bubbles to increase their capital investment.

For L-types, since $c_t^{\mu i} = (1-\beta)e_t^{\mu i}$, the flow of funds constraint (6) becomes:

$$k_t^{\mu i} + (-b_t^{\mu i}) + (-b_t^{\mu w i}) + Q_t x_t^i = \beta e_t^{\mu i}. \tag{12}$$

We learn that L-types have four options to allocate their savings: capital ($k_t^{\mu i}$), domestic and foreign bonds ($-b_t^{\mu i}$ and $-b_t^{\mu w i}$), and bubbles (x_t^i). Solving the maximization problem in Appendix A.2, We can derive the demand function for bubbles as:

$$Q_t x_t^i = \frac{\pi \frac{Q_{t+1}}{Q_t} - r_t^{\mu}}{\frac{Q_{t+1}}{Q_t} - r_t^{\mu}} \beta e_t^{\mu i}. \tag{13}$$

To determine how to split the remaining savings, we need the following two complementary slackness conditions. First, L-types choose whether to produce or not. When $r_t^{\mu} > \underline{r}_t^{\mu}$, L-types never produce but lend all their savings to either domestic or foreign borrowers. When $r_t^{\mu} = \underline{r}_t^{\mu}$, L-types are indifferent to whether they lend their net worth or borrow abroad to produce. Hence, the following condition must hold in equilibrium:[24]

$$(r_t^{\mu} - \underline{r}_t^{\mu})k_t^{\mu i} = 0, \; r_t^{\mu} \geq \underline{r}_t^{\mu}, \text{ and } k_t^i \geq 0.$$

Second, L-types decide whether they borrow from domestic or foreign agents. As in H-types' case,

[22]See Sargent (1988).

[23]See, for example, Kiyotaki and Moore (1997) and Kiyotaki (1998).

[24]We can formally check that this condition holds using the optimization conditions described in Appendix A.2.

the international borrowing constraint (4) binds when $r_t^{\mu} > r_t^{\mu w}$, while (4) does not bind when $r_t^{\mu} = r_t^{\mu w}$.

2.3 Equilibrium

We denote the variables in the Foreign country with a star ($*$). Let C_t^{μ} and $C_t^{\mu\prime}$ be the aggregate consumption of Home H- and L-types, and $C_t^{\mu*}$ and $C_t^{\mu*\prime}$ those of Foreign H- and L-types, respectively. Similarly, let K_t^{μ} and $K_t^{\mu\prime}$ be the aggregate investments of Home H- and L-types, and $K_t^{\mu*}$ and $K_t^{\mu*\prime}$ those of Foreign H- and L-types, respectively. Moreover, we define B_t^{μ}, $B_t^{\mu\prime}$, $B_t^{\mu*}$, and $B_t^{\mu*\prime}$ as the aggregate borrowing from domestic agents, and $B_t^{\mu w}$, $B_t^{\mu w\prime}$, $B_t^{\mu w*}$, and $B_t^{\mu w*\prime}$ as the aggregate borrowing from foreign agents, respectively. Additionally, let M_{ht+1}^{μ} and M_{ft+1}^{μ} be the aggregate supplies of intermediate goods in the Home and Foreign countries, respectively.

Then, the market clearing conditions for final goods, intermediate goods, domestic and foreign credit, and bubbles can be respectively written as:[25]

$$p_{ht}^{\mu} M_{ht}^{\mu} = C_t^{\mu} + C_t^{\mu\prime} + K_t^{\mu} + K_t^{\mu\prime} - (B_t^{\mu w} + B_t^{\mu w\prime} - r_t^{\mu w} B_{t-1}^{\mu w} - r_t^{\mu w} B_{t-1}^{\mu w\prime}), \tag{14}$$

$$p_{ft}^{\mu} M_{ft}^{\mu} = C_t^{\mu*} + C_t^{\mu*\prime} + K_t^{\mu*} + K_t^{\mu*\prime} - (B_t^{\mu w*} + B_t^{\mu w*\prime} - r_t^{\mu w} B_{t-1}^{\mu w*} - r_t^{\mu w} B_{t-1}^{\mu w*\prime}), \tag{15}$$

$$M_{ht}^{\mu d} = M_{dt}^{\mu s}, \tag{16}$$

$$M_{ft}^{\mu d} = M_{ft}^{\mu s}, \tag{17}$$

$$B_t^{\mu} + B_t^{\mu\prime} = 0, \tag{18}$$

$$B_t^{\mu*} + B_t^{\mu*\prime} = 0, \tag{19}$$

$$B_t^{\mu w} + B_t^{\mu w\prime} + B_t^{\mu w*} + B_t^{\mu w*\prime} = 0, \text{ and} \tag{20}$$

$$X_t = X. \tag{21}$$

The competitive equilibrium is defined as the set of prices $\left\{r_t^{\mu}, r_t^{\mu*}, p_{ht+1}^{\mu}, p_{ft+1}^{\mu}, Q_t^x\right\}_{t=0}^{\infty}$ and quantities $\left\{c_t^{\mu i}, k_t^{\mu i}, b_t^{\mu i}, b_t^{\mu w i}, z_t^{\mu i}, m_{ht+1}^{\mu}, m_{ft+1}^{\mu}, C_t^{\mu}, C_t^{\mu\prime}, C_t^{\mu*}, C_t^{\mu*\prime}, K_t^{\mu}, K_t^{\mu\prime}, K_t^{\mu*}, K_t^{\mu*\prime}, B_t^{\mu}, B_t^{\mu\prime}, B_t^{\mu*}, B_t^{\mu*\prime},\right.$ $\left.B_t^{\mu w}, B_t^{\mu w\prime}, B_t^{\mu w*}, B_t^{\mu w*\prime}, M_{ht+1}^{\mu}, M_{ft+1}^{\mu}\right\}_{t=0}^{\infty}$ such that (i) each final good producer solves (8); (ii) each entrepreneur chooses consumption, investment, domestic and foreign borrowing, and bubble assets to maximize their expected discounted utility (2) under the constraints (3), (4), (5), (6), and (7); and (iii) market clearing conditions, (14), (15), (16), (17), (18), (19), (20), and (21) are satisfied.

[25] The terms inside the brackets in (14) and (15) correspond to current account deficit.

2.4 Bubbleless Economy

We first derive the equilibrium without bubbles, *i.e.,* $Q_t^x = 0$ for all t. Let the variables without the subscript μ be those under bubbleless economy. Aggregating (11), the investment function for the Home and Foreign H-types can be written as:

$$K_t \leq \frac{\beta s Y_t}{1 - \frac{\alpha\phi\theta}{r_t^w} p_{ht+1} - \frac{\alpha(1-\phi)\theta}{r_t} p_{ht+1}} \ (r_t < \overline{r}_t \text{ when the equality holds), and}$$
$$K_t^* \leq \frac{\beta s Y_t^*}{1 - \frac{\alpha\phi^*\theta^*}{r_t^w} p_{ft+1} - \frac{\alpha(1-\phi^*)\theta^*}{r_t^*} p_{ft+1}} \ (r_t^* < \overline{r}_t^* \text{ when the equality holds),}$$

where $Y_t = p_{ht}M_{ht} - r_{t-1}^w B_{t-1}^w - r_{t-1}^w B_{t-1}^{w\prime}$ and $Y_t^* = p_{ft}M_{ft} - r_{t-1}^w B_{t-1}^{*w} - r_{t-1}^w B_{t-1}^{w*\prime}$ are the aggregate outputs net of international debt repayment in the Home and Foreign countries at date t, respectively. Note that, since every entrepreneur becomes H-type with probability s at the beginning of each period, the aggregate net worth of H-types is a fraction s of the aggregate output of all entrepreneurs.

Regarding L-types, since Home L-types never produce when $r_t > \underline{r}_t$ and Foreign L-types never produce when $r_t^* > \underline{r}_t^*$, we have:

$$(r_t - \underline{r}_t)\, K_t' = 0,\ r_t \geq \underline{r}_t, \text{ and } K_t' \geq 0,$$
$$\left(r_t^* - \underline{r}_t^*\right) K_t^{*\prime} = 0,\ r_t^* \geq \underline{r}_t^*, \text{ and } K_t^{*\prime} \geq 0.$$

Moreover, aggregating the consumption function and using the market clearing conditions for final goods, (14) and (15), the international borrowing constraints for the Home and Foreign entrepreneurs can be rewritten as:

$$K_t + K_t' \leq \beta Y_t + \frac{\phi\theta p_{ht+1}}{r_t^w}(\alpha K_t + \gamma K_t') \ (r_t = r_t^w \text{ when the inequality holds), and}$$
$$K_t^* + K_t^{*\prime} \leq \beta Y_t^* + \frac{\phi^*\theta^* p_{ft+1}}{r_t^w}(\alpha K_t^* + \gamma K_t^{*\prime}) \ (r_t^* = r_t^w \text{ when the inequality holds).}$$

From the goods market clearing conditions (14) and (15), the Home and Foreign net savings can be written as $K_t + K_t' - B_t^w - B_t^{w\prime} = \beta Y_t$ and $K_t^* + K_t^{*\prime} - B_t^{w*} - B_t^{w*\prime} = \beta Y_t^*$, respectively. Using the international credit market clearing condition (20), we learn that the world investment and savings are equal:

$$K_t + K_t' + K_t^* + K_t^{*\prime} = \beta(Y_t + Y_t^*). \tag{22}$$

Moreover, aggregating (9) and using the intermediate goods market clearing condition, we obtain:

$$\frac{\alpha K_t + \gamma K_t'}{\alpha K_t^* + \gamma K_t^{*\prime}} = \frac{\omega}{1-\omega}\left(\frac{p_{ht+1}}{p_{ft+1}}\right)^{-\sigma}. \tag{23}$$

Additionally, we have the Dixit-Stiglitz price index (10).

2.4.1 Equilibrium under Financial Autarky

We then derive the equilibrium interest and growth rates without bubbles. To understand how they are determined, we begin with financial autarky case, that is, $\phi = \phi^* = 0$. Under financial autarky, we can derive the interest and growth rates analytically. The aggregate output in the Home country can be written as:

$$Y_{t+1} = (\alpha K_t + \gamma K_t')p_{ht+1}. \tag{24}$$

Moreover, using $c_t^i = (1-\beta)e_t^i$ and the goods market clearing condition (14) in the Home country, the aggregate investment in the Home country becomes equal to its net savings: $K_t + K_t' = \beta Y_t$. Hence, the economic growth rate can be expressed as:

$$g_t \equiv \frac{Y_{t+1}}{Y_t} = \beta[\alpha h_t + \gamma(1-h_t)]p_{ht+1}, \tag{25}$$

where $h_t \equiv \frac{K_t}{\beta Y_t}$ is the fraction of H-investment out of aggregate net savings. When θ is sufficiently low, not all savings are allocated to H-projects but L-types produce themselves so that $r_t = \gamma p_{h+1}$ and $h_t = \frac{s}{1-\frac{\alpha\theta}{\gamma}} < 1$. This reduces the economic growth rate compared to the case under perfect financial market, *i.e.,* $\theta = 1$. When θ is in the middle range, all savings are allocated to H-projects and r_t is adjusted so that $h_t = \frac{s}{1-\frac{\alpha\theta}{r_t}p_{ht+1}} = 1$. When θ is sufficiently high, the domestic borrowing constraint (5) is no longer binding and the interest rate becomes equal to the rate of return on savings by H-types, *i.e.,* $r_t = \alpha p_{ht+1}$. Then, the equilibrium interest and growth rates in the Home country are determined as:

$$r_t = \begin{cases} \gamma p_{ht+1}, & \text{if } 0 \le \theta < \tilde{\theta}^a \equiv \frac{\gamma}{\alpha}(1-s), \\ \frac{\alpha\theta}{1-s}p_{ht+1}, & \text{if } \tilde{\theta}^a \le \theta < \hat{\theta}^a \equiv 1-s, \text{ and} \\ \alpha p_{ht+1}, & \text{if } \hat{\theta}^a \le \theta \le 1, \end{cases} \tag{26}$$

and

$$g_t \equiv \frac{Y_{t+1}}{Y_t} = \beta[\alpha H^a(\theta) + \gamma(1 - H^a(\theta))]p_{ht+1}, \tag{27}$$

where $H^a(\theta) \equiv \text{Min}\left[\frac{s}{1-\frac{\alpha\theta}{\gamma}}, 1\right]$. The interest and growth rates in the Foreign country can be expressed similarly as functions of θ^*.

To examine how the intermediate goods prices, p_{ht+1} and p_{ft+1}, are determined, we define the total factor productivities (TFPs) in the Home and Foreign countries as $a_{ht} = \frac{\alpha K_t + \gamma K_t'}{K_t + K_t'}$ and $a_{ft} = \frac{\alpha K_t^* + \gamma K_t^{*\prime}}{K_t^* + K_t^{*\prime}}$, respectively. The TFP in the Home country is expressed as a function of θ:

$$a_{ht} = a_{ht}(\theta) = \alpha H^a(\theta) + \gamma(1 - H^a(\theta)), \tag{28}$$

and the TFP in the Foreign country can be expressed similarly as a function of θ^*: $a_{ft} = a_{ft}(\theta^*)$.

As in Acemoglu and Ventura (2002), in the long run, p_{ht+1} and p_{ft+1} adjust so that the growth rates are equalized between the two countries. In fact, even if the two countries face different degrees of financial development, the economy is on the balanced growth path (BGP) if and only if the two countries share the same growth rates.[26] Hereafter, we focus on this BGP equilibrium.

Using a_{ht} and a_{ft}, the growth rates in the Home and Foreign countries can be expressed as:

$$g_t \equiv \frac{Y_{t+1}}{Y_t} = \beta a_{ht} p_{ht+1} = \beta a_{ft} p_{ft+1} = \frac{Y_{t+1}^*}{Y_t^*} \equiv g_t^*. \tag{29}$$

Substituting (29) into (10), we have:

$$p_{ht+1} = \left[\omega + (1-\omega)\left(\frac{a_{ft}}{a_{ht}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}, \tag{30}$$

which implies that, when productivity is high in the Home country, the price of Home intermediate good decreases since the marginal return on investment is low. In other words, when the Home country is accumulating capital faster than the Foreign, its commodity price decreases so that further capital accumulation is discouraged. Hence, in the long run, the growth rates are equalized between the two countries. Note that, since the production function (3) is linear in capital, there is no transition dynamics and the economy achieves the BGP immediately when given the initial output, Y_0.

Combining (26), (27), and (30), we can solve for the interest and growth rates analytically. Then, we obtain the following Proposition, in which subscript a represents the variables under

[26] We can prove this formally under financial autarky. See Appendix A.3.1.

financial autarky. Hereafter, the proofs of all Propositions are given in Appendix. Unless otherwise stated, the proofs of Propositions are similar to Hirano and Yanagawa (2017).

Proposition 1. *Given θ^*, the Home interest rate, r_t^a, depends non-monotonically on θ. When $\theta < \tilde{\theta}^a \equiv \frac{\gamma}{\alpha}(1-s)$, r_t^a is a decreasing function of θ. When $\theta > \tilde{\theta}^a$, r_t^a is an increasing function of θ. Furthermore, the economic growth rate, g^a, is a monotonically increasing function of θ.*

Figure 4(a) depicts a numerical example of Proposition 1.[27] We take θ on the horizontal axis and g_t and r_t on the vertical one. The parameter values are set as follows: $\beta = 0.98$, $\alpha = 1.1$, $\gamma = 1.0$, $s = 0.18$, $\omega = 0.5$, and $\sigma = 2$. The Foreign financial development is fixed at $\theta^* = 0.5$.

Figure 4(a) has two important features. First, the economic growth rate is larger than the interest rate only in the intermediate range of θ. When θ is low, since most savings are invested in L-projects, the growth rate becomes close to $\beta\gamma p_{ht+1}^a$, which is lower than the interest rate, γp_{ht+1}^a. When θ is in the middle range, the growth rate becomes relatively high because most funds are allocated to H-types, while the interest rate is suppressed because L-types still produce. When θ is high, the interest rate becomes αp_{ht+1}^a, which is higher than the growth rate, $\beta\alpha p_{ht+1}^a$. This result is crucial for the existence condition of bubbles. As Tirole (1985) suggested, bubbles can exist as long as the growth rate is larger than the interest rate. We will show later that our result regarding the existence condition of bubbles is consistent with the Tirole's argument.

Second, as shown in Proposition 1, the interest rate is non-monotonic in the degree of financial development. When θ is sufficiently high ($\theta > \tilde{\theta}^a$), the interest rate is increasing in θ. This is because higher θ increases H-types' borrowing demand. However, when θ is sufficiently low ($\theta \leq \tilde{\theta}^a$), the interest rate is decreasing in θ. When θ is low, since L-types have an incentive to produce, the interest rate is determined as $r_t = \gamma p_{ht+1}$. When θ becomes relatively high, the TFP in the Home country, a_{ht}, increases. As implied by (30), higher a_{ht} leads to lower p_{ht+1}, which decreases the interest rate.

Figure 4(b) expresses the relative values of the Home and Foreign interest rates under financial autarky. We take θ on the horizontal axis and θ^* on the vertical one. Due to the non-monotonicity of interest rates, the Home interest rate is higher than the Foreign one when θ is either sufficiently high or low and θ^* takes an intermediate value. On the other hand, the Foreign interest rate is higher than the Home one when θ^* is either sufficiently high or low and θ takes an intermediate value. As we will discuss later, this is important for understanding the direction of capital flows since capital flows toward the country with higher rate of return on savings.

We add two remarks to this result. First, for bubbles to exist in the middle range of θ, the non-monotonicity of the interest rate is not necessary. In fact, as long as there are heterogeneous

[27] Aoki et al. (2007) showed this result numerically.

investment opportunities, bubbles can exist only in the middle range of θ even if the interest rate is monotonically increasing (see Hirano and Yanagawa, 2017). This model features the non-monotonicity of the interest rate so that even financially underdeveloped economies can experience capital inflows following financial globalization.

Second, the assumption of two types of productivity is not crucial to the non-monotonicity of the interest rate. As we will see in Appendix A.16, this non-monotonicity holds even with a continuum of productivity under specific plausible parameter conditions. In the main text, we focus on the case with two types of productivities because we can obtain analytical results regarding the existence condition of bubbles. This analytical tractability gives us a clearer intuition on the main features of our model.

2.4.2 Equilibrium under Financial Globalization

Next, we derive the expression for economic growth rate under financial globalization. We focus on the growth rate in the Home country. We first begin with the case where $r_t = r_t^w$ so that the international borrowing constraint does not bind. Let $Y_{t+1} = (\alpha K_t + \gamma K_t')p_{ht+1} - r_t^w(B_t^w + B_t^{w\prime})$ be the aggregate output net of foreign repayment.[28] Using $K_t + K_t' - B_t^w - B_t^{w\prime} = \beta Y_t$, we obtain:

$$Y_{t+1} = \alpha p_{ht+1} K_t + r_t(\beta Y_t - K_t),$$

where $r_t \geq \underline{r}_t = \gamma p_{ht+1}$, and $K_t = \frac{\beta s Y_t}{1-\frac{\alpha\theta}{r_t}p_{ht+1}}$. When $r_t = \gamma p_{ht+1}$, both H- and L-types invest in capital. When $r_t > \gamma p_{ht+1}$, only H-types invest in capital but L-types engage in lending abroad. The first term is the return on H-investment and the second term is the sum of returns on L-investment and net foreign lending. The growth rate of Y_t becomes:

$$\frac{Y_{t+1}}{Y_t} = \beta[\alpha p_{ht+1} h_t + r_t(1 - h_t)],$$

where $h_t = \frac{K_t}{\beta A_t} = \frac{s}{1-\frac{\alpha\theta}{r_t}p_{ht+1}}$.[29]

Next, we consider the case where $r_t > r_t^w$. Since the international borrowing constraint binds,

[28]Under financial autarky ($B_t^w = B_t^{w\prime} = 0$), this is equal to (24).

[29]We consider the growth rate of aggregate output net of foreign repayment instead of the growth rate of output itself. This is because, when $r_t = r_t^w$, Home H-types (L-types) are indifferent between borrowing (lending) domestically and abroad. This makes the output growth rate indeterminate at each country level and it is determined implicitly in equilibrium. However, under the BGP, those two growth rates become equal.

Y_{t+1} can be written as:

$$\frac{Y_{t+1}}{Y_t} = \begin{cases} \beta[\overline{r}_t h_t^{\phi} + \underline{r}_t(1 - h_t^{\phi})], & \text{if } r_t = \underline{r}_t, \\ \beta\overline{r}_t, & \text{if } r_t > \underline{r}_t, \end{cases}$$

where $\overline{r}_t = \frac{\alpha(1-\phi\theta)p_{ht+1}}{1-\frac{\alpha\phi\theta}{r_t^w}p_{ht+1}}$ and $\underline{r}_t = \frac{\gamma(1-\phi\theta)p_{ht+1}}{1-\frac{\gamma\phi\theta}{r_t^w}p_{ht+1}}$ are H- and L-types' leveraged rate of return on investment and:

$$h_t^{\phi} = \frac{K_t - B_t^w}{\beta A_t} = \left(1 - \frac{\alpha\phi\theta}{r_t^w}p_{ht+1}\right)\frac{K_t}{\beta A_t} = \frac{s\left(1 - \frac{\alpha\phi\theta}{r_t^w}p_{ht+1}\right)}{1 - \frac{\alpha\phi\theta}{r_t^w}p_{ht+1} - \frac{\alpha(1-\phi)\theta}{r_t}p_{ht+1}}$$

is the fraction of H-investment net of foreign borrowing as a share of aggregate net savings. When $r_t = \underline{r}_t$, both H- and L-types borrow abroad up to the limit and produce. When $r_t > \underline{r}_t$, L-types neither produce nor lend abroad but engage in lending to domestic H-types. When $r_t > \underline{r}_t$, $K_t - B_t^w = \beta A_t$ or $h_t^{\phi} = 1$ holds since L-types no longer borrow and produce. The growth rate in the Foreign country can be derived in a similar way.

Alternatively, we can derive a more general form of economic growth rate by aggregating the laws of motion of net worth for H- and L-types in Appendix A.2. We learn that the growth rate can be written as the weighted average of H- and L-types' rate of return on savings:

$$\begin{aligned} g_t \equiv \frac{Y_{t+1}}{Y_t} &= \beta\left[s\frac{\alpha(1-\theta)p_{ht+1}}{1 - \frac{\alpha\phi\theta}{r_t^w}p_{ht+1} - \frac{\alpha(1-\phi)\theta}{r_t}p_{ht+1}} + (1-s)r_t\right] \\ &= \beta\left[s\frac{\alpha(1-\theta^*)p_{ft+1}}{1 - \frac{\alpha\phi^*\theta^*}{r_t^w}p_{ft+1} - \frac{\alpha(1-\phi^*)\theta^*}{r_t^*}p_{ft+1}} + (1-s)r_t^*\right] = \frac{Y_{t+1}^*}{Y_t^*} \equiv g_t^*. \end{aligned} \tag{31}$$

Based on this numerical example, we obtain the following Proposition. Under financial globalization, it is difficult to obtain an analytical solution in general. This is because the interest rates in the two countries are determined implicitly to satisfy the international credit market clearing condition (22). Hence, in the following part of this paper, to proceed as analytically as possible, each Proposition focuses on the region of (θ, θ^*) where we can derive an analytical result. However, we can check numerically that the statements in each Proposition hold for all values of (θ, θ^*) that we consider (see Appendix A.3.2 for computational method.). As before, the variables with subscript a represent those under financial autarky.

Proposition 2. *We focus on the region of (θ, θ^*) where we can derive the relative values of interest and growth rates explicitly under financial globalization. (i) When $r_t^a > r_t^{a*}$, there exist parameter values (θ, θ^*) such that globalization decreases decreases the Home interest rate relative to the growth rate. (ii) When $r_t^a < r_t^{a*}$, there exist parameter values (θ, θ^*) such that globalization increases the Home interest rate relative to the growth rate. (iii) When $r_t^a = r_t^{a*}$, globalization does not change the relative values between the growth and interest rates.*

This Proposition shows that, when the Home interest rate is higher than the Foreign interest rate under financial autarky, globalization increases the growth rate relative to the Home interest rate. On the other hand, when the Home interest rate is lower than the Foreign interest rate under financial autarky, globalization increases the Home interest rate relative to the growth rate.

Then, we discuss how financial globalization affects the relative values of interest and growth rates in the Home country, which is crucial in understanding the existence of bubbles under financial globalization. As discussed in 2.4.1, due to the non-monotonicity of the interest rates, when θ is either sufficiently high or low relative to θ^*, the Home interest rate is higher than the Foreign one. Hence, financial globalization leads to capital inflows, which suppresses the interest rate relative to the growth rate. On the other hand, when θ is in the middle range and θ^* is either sufficiently high or low, financial globalization leads to capital outflows, which increases the interest rate relative to the growth rate.

Figure 5(a), 5(b), and 5(c) show the values of interest and growth rates under financial globalization when the Foreign financial development is high ($\theta^* = 0.9$), in the middle range ($\theta^* = 0.5$), and low ($\theta^* = 0.1$), respectively. $\overline{\theta}^a$ and $\underline{\theta}^a$ are the upper and lower values of θ where the growth rate is larger than the interest rate under financial autarky.[30] $\overline{\theta}^g$ and $\underline{\theta}^g$ are the corresponding values of θ under financial globalization. We learn that, when θ^* is in the middle range, financial globalization expands the region of θ where the growth rate is larger than the interest rate ($\underline{\theta}^g < \underline{\theta}^a$ and $\overline{\theta}^a < \overline{\theta}^g$). On the other hand, when θ^* is either sufficiently high or low, financial globalization shrinks the corresponding region of θ ($\underline{\theta}^g = \underline{\theta}^a$ and $\overline{\theta}^g < \overline{\theta}^a$). As will discuss in Section 3, $\overline{\theta}^a$ and $\underline{\theta}^a$ correspond to the upper and lower bounds of the existence region of bubbles under financial autarky and $\overline{\theta}^g$ and $\underline{\theta}^g$ correspond to the upper and lower bounds under financial globalization. We will later use the result in this section to discuss the effect of financial globalization on the existence condition of bubbles.[31]

[30]The formal values of $\overline{\theta}^a$ and $\underline{\theta}^a$ will be given in Section 3.

[31]The relative values between the Home and Foreign interest rates under financial globalization is shown in Figure 6. From the borrowers' perspective, although there is an equalizing force on the Home and Foreign interest rates, they can be different when the international borrowing constraint is binding. On the other hand, from the lenders' perspective, the interest rates when they lend domestically and abroad are always the same. This is because our model focuses on the friction that arises from borrowers' limited ability to repay their debt but abstracts away from the cost of lending abroad.

2.5 Economy with Bubbles

Next, we consider an economy with bubbles. We focus on the dynamics of an economy when bubbles survive, *i.e.,* $Q_t^x = Q_t > 0$. As mentioned earlier, we assume bubbles only exist in the Home country and cannot be traded across nations.

We begin with H-types. The aggregate investment functions for Home and Foreign H-types are given as:

$$K_t^\mu \leq \frac{\beta s A_t^\mu}{1 - \frac{\alpha\phi\theta}{r_t^{\mu w}} p_{ht+1}^\mu - \frac{\alpha(1-\phi)\theta}{r_t^\mu} p_{ht+1}^\mu} \ (r_t^\mu < \overline{r}_t^\mu \text{ when the equality holds), and} \tag{32}$$

$$K_t^{\mu*} \leq \frac{\beta s A_t^{\mu*}}{1 - \frac{\alpha\phi^*\theta^*}{r_t^{\mu w}} p_{ft+1}^\mu - \frac{\alpha(1-\phi^*)\theta^*}{r_t^{\mu*}} p_{ft+1}^\mu} \ (r_t^{\mu*} < \overline{r}_t^{\mu*} \text{ when the equality holds),} \tag{33}$$

where $A_t^\mu = p_{ht}^\mu M_{ht}^\mu - r_{t-1}^{\mu w} B_{t-1}^w - r_{t-1}^{\mu w} B_{t-1}^{\mu w\prime} + Q_t X$ and $A_t^{\mu*} = p_{ft}^\mu M_{ft}^{\mu*} - r_{t-1}^{\mu w} B_{t-1}^{\mu w*} - r_{t-1}^{\mu w} B_{t-1}^{\mu w*\prime}$ are the aggregate net worth of all the Home and Foreign entrepreneurs at date t.

Next, we turn to L-types. Since Home L-types never produce when $r_t^\mu > \underline{r}_t^\mu$ and Foreign L-types never produce when $r_t^{\mu*} > \underline{r}_t^{\mu*}$, we have:

$$\left(r_t^\mu - \underline{r}_t^\mu\right) K_t^{\mu\prime} = 0, \ r_t^\mu \geq \underline{r}_t^{L\prime}, \text{ and } K_t^{\mu\prime} \geq 0, \tag{34}$$

$$\left(r_t^{\mu*} - \underline{r}_t^{\mu*}\right) K_t^{\mu\prime} = 0, \ r_t^{\mu*} \geq \underline{r}_t^{\mu*}, \text{ and } K_t^{*\prime} \geq 0. \tag{35}$$

The international borrowing constraints for the Home and Foreign entrepreneurs can be written as:

$$K_t^\mu + K_t^{\mu\prime} + Q_t X \leq \beta A_t^\mu + \frac{\phi\theta p_{ht+1}^\mu}{r_t^{\mu w}} (\alpha K_t^\mu + \gamma K_t^{\mu\prime}) \ (r_t^\mu = r_t^{\mu w} \text{ when the inequality holds), and} \tag{36}$$

$$K_t^{\mu*} + K_t^{\mu*\prime} \leq \beta A_t^{\mu*} + \frac{\phi^*\theta^* p_{ft+1}^\mu}{r_t^{\mu w}} (\alpha K_t^{\mu*} + \gamma K_t^{\mu*\prime}) \ (r_t^{\mu*} = r_t^{\mu w} \text{ when the inequality holds).} \tag{37}$$

From (14) and (15), the Home and Foreign net savings can be written as $K_t^\mu + K_t^{\mu\prime} - B_t^{\mu w} - B_t^{\mu w\prime} + Q_t X = \beta A_t^\mu$ and $K_t^{\mu*} + K_t^{\mu*\prime} - B_t^{\mu w*} - B_t^{\mu w*\prime} = \beta A_t^{\mu*}$, respectively. Using (20), we obtain:

$$K_t^\mu + K_t^{\mu\prime} + K_t^{\mu*} + K_t^{\mu*\prime} + Q_t X = \beta(A_t^\mu + A_t^{\mu*}). \tag{38}$$

Additionally, the intermediate goods market clearing condition implies that:

$$\frac{\alpha K_t^{\mu} + \gamma K_t^{\mu\prime}}{\alpha K_t^{\mu *} + \gamma K_t^{\mu *\prime}} = \frac{\omega}{1-\omega}\left(\frac{p_{ht+1}^{\mu}}{p_{ft+1}^{\mu}}\right)^{-\sigma}. \tag{39}$$

Finally, we have the Dixit-Stiglitz price index:

$$1 = \left[\omega (p_{ht}^{\mu})^{1-\sigma} + (1-\omega)(p_{ft}^{\mu})^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{40}$$

2.5.1 Equilibrium

We then derive the equilibrium interest and growth rates under the economy with bubbles. We focus on the Home country since bubbles only exist in the Home country.

To capture the intuition, we begin with the financial autarky case. The derivation under financial autarky is based on Hirano and Yanagawa (2017). Using (14), the aggregate net savings of the Home country can be written as:

$$K_t^{\mu} + K_t^{\mu\prime} + Q_t X = \beta A_t^{\mu}.$$

or equivalently, using $A_t^{\mu} = Y_t^{\mu} + Q_t X$,

$$K_t^{\mu} + K_t^{\mu\prime} = \beta Y_t^{\mu} - (1-\beta) Q_t X. \tag{41}$$

This implies that bubbles crowd out aggregate resources used for real investment. On the other hand, H-types' investment can be expressed as:

$$K_t^{\mu} = \frac{\beta s A_t^{\mu}}{1 - \frac{\alpha\theta}{r_t} p_{ht+1}} = \frac{\beta s Y_t^{\mu}}{1 - \frac{\alpha\theta}{r_t} p_{ht+1}} + \frac{\beta s Q_t X}{1 - \frac{\alpha\theta}{r_t} p_{ht+1}}.$$

The second term implies that bubbles expands the entrepreneurs' balance sheet and crowd in capital investment. Moreover, since H-types use leverage to invest, the increase in capital is greater than the direct expansion in net worth.

Next, we discuss how the interest rate is determined. Let $\mu_t = \frac{Q_t X}{\beta A_t^{\mu}}$ be the size of bubble defined as the share of bubble assets out of net savings. Moreover, let $L^a(\theta) = 1 - H^a(\theta) = \text{Max}\left[1 - \frac{s}{1-\frac{\alpha\theta}{\gamma}}, 0\right]$. When bubble size is small enough, that is:

$$Q_t X \leq \text{Max}\left[\beta A_t^{\mu} - \frac{\beta s A_t^{\mu}}{1 - \frac{\alpha\theta}{\gamma}}, 0\right],\ i.e.,\ \mu_t \leq L^a(\theta),$$

both H- and L-types invest in capital and $r_t^\mu = \gamma p_{ht+1}^\mu$. On the other hand, when bubble size is large, that is, $\mu_t \geq L^a(\theta)$, only H-types invest in capital and r_t^μ satisfies:

$$\mu_t = 1 - \frac{K_t^\mu}{\beta A_t^\mu} = 1 - \frac{s}{1 - \frac{\alpha\theta}{r_t^\mu} p_{ht+1}^\mu}, \text{ i.e. } r_t^\mu = \frac{\alpha\theta(1-\mu_t)p_{ht+1}^\mu}{1 - s - \mu_t}.$$

To determine the economic growth rate, substituting (41) into $Y_{t+1}^\mu = (\alpha K_t^\mu + \gamma K_t^{\mu\prime})p_{ht+1}^\mu$, we obtain:

$$\frac{Y_{t+1}^\mu}{Y_t^\mu} = \begin{cases} \quad \beta[\alpha H^a(\theta) + \gamma(1 - H^a(\theta))]p_{ht+1}^\mu & \\ +[\beta(\alpha-\gamma)H^a(\theta) - (1-\beta)\gamma]p_{ht+1}^\mu \dfrac{Q_t X}{Y_t^\mu}, & \text{if } \mu_t \leq L^a(\theta), \\ \beta\alpha p_{ht+1}^\mu - (1-\beta)\alpha p_{ht+1}^\mu \dfrac{Q_t X}{Y_t^\mu}, & \text{if } \mu_t \geq L^a(\theta), \end{cases} \tag{42}$$

where $\frac{Q_t X}{Y_t^\mu} = \frac{\beta\mu_t}{1-\beta\mu_t}$. We will discuss how μ_t is determined in the next subsection. From (42), we learn the relationship between the bubble size and the economic growth rate. When bubbles are small ($\mu_t \leq L^a(\theta)$), both H- and L-types invest in capital. On one hand, the term $\beta(\alpha-\gamma)H^a(\theta)$ represents the additional rate of return when resource is allocated to H-project. Suppose that L-types buy bubbles at date t. Then, if they become H-types at date $t+1$, they can sell bubbles and use their increased net worth to invest in H-projects. In this sense, bubbles crowd in productive investment by reallocating resource from L- to H-investment. On the other hand, if L-types buy bubbles, they cut back L-investment and reduce economic growth by $(1-\beta)\gamma p_{ht+1}^\mu$. When bubbles are large ($\mu_t \geq L^a(\theta)$), L-types are no longer producing. Hence, bubbles crowd out even H-investment and reduce economic growth by $(1-\beta)\alpha p_{ht+1}^\mu$.[32]

Next, we derive the economic growth rate under financial globalization. Although we cannot solve for the interest rate explicitly under financial globalization in a way that it depends on the interest rate and terms of trade which are determined exogenously in equilibrium.

We first begin with the case where $r_t^\mu = r_t^{\mu w}$. Since the international borrowing constraint does not bind, H-types (L-types) are indifferent between borrowing (lending) domestically and abroad. Using $Y_{t+1}^\mu = (\alpha K_t^\mu + \gamma K_t^{\mu\prime})p_{ht+1}^\mu - r_t^{\mu w}(B_t^{\mu w} + B_t^{\mu w\prime})$ and $K_t^\mu + K_t^{\mu\prime} - B_t^{\mu w} - B_t^{\mu w\prime} = \beta Y_t^\mu - (1-\beta)Q_t X$,

[32]Another way to interpret the crowd-in and crowd-out effects is that bubbles increase the rate of return on savings by L-types, which expands the entrepreneurs' net worth and increases their investment. At the same time, the higher interest rate limits the borrowing capacity for H-types, which reduces their leveraged rate of return on investment. This interpretation corresponds to the liquidity effect and the leverage effect in Farhi and Tirole (2012). See Hirano and Yanagawa (2017) for discussion.

the growth rate can be written as:

$$\frac{Y^{\mu}_{t+1}}{Y^{\mu}_t} = \beta[\alpha p^{\mu}_{ht+1} h^{\mu}_t + r^{\mu}_t(1-h^{\mu}_t)] + [\beta(\alpha p^{\mu}_{ht+1} - r^{\mu}_t)h^{\mu}_t - (1-\beta)r^{\mu}_t]\frac{Q_t X}{Y^{\mu}_t}, \tag{43}$$

where $r^{\mu}_t \geq \underline{r}^{\mu}_t = \gamma p^{\mu}_{ht+1}$ and $h^{\mu}_t = \frac{K^{\mu}_t}{\beta A^{\mu}_t} = \frac{s}{1-\frac{\alpha\theta}{r^{\mu}_t}p^{\mu}_{ht+1}}$. When $r^{\mu}_t = \gamma p^{\mu}_{ht+1}$, both H- and L-types invest in capital. When $r^{\mu}_t > \gamma p^{\mu}_{ht+1}$, only H-types invest in capital. The term $\beta(\alpha p^{\mu}_{ht+1} - r^{\mu}_t)h^{\mu}_t$ implies that bubbles crowd in H-investment by reallocating resource from L- to H-investment, while $(1-\beta)r^{\mu}_t$ implies that bubbles crowd out L-investment and net foreign lending.

Next, we consider the case where $r^{\mu}_t > r^{\mu w}_t$. Since the international borrowing constraint binds, the entrepreneurs borrow abroad up to the limit to invest in capital. The growth rate can be written as:

$$\frac{Y^{\mu}_{t+1}}{Y^{\mu}_t} = \begin{cases} \beta[\overline{r}^{\mu}_t h^{\mu\phi}_t + \underline{r}^{\mu}_t(1-h^{\mu\phi}_t)] & \\ +[\beta(\overline{r}^{\mu}_t - \underline{r}^{\mu}_t)h^{\mu\phi}_t - (1-\beta)\underline{r}^{\mu}_t]\frac{Q_t X}{Y^{\mu}_t}, & \text{if } r^{\mu}_t = \underline{r}^{\mu}_t, \\ \beta\overline{r}^{\mu}_t - (1-\beta)\overline{r}^{\mu}_t\frac{Q_t X}{Y^{\mu}_t}, & \text{if } r^{\mu}_t > \underline{r}^{\mu}_t, \end{cases} \tag{44}$$

where $\overline{r}^{\mu}_t = \frac{\alpha(1-\phi\theta)p^{\mu}_{ht+1}}{1-\frac{\alpha\phi\theta}{r^{\mu w}_t}p^{\mu}_{ht+1}}$, $\underline{r}^{\mu}_t = \frac{\gamma(1-\phi\theta)p^{\mu}_{ht+1}}{1-\frac{\gamma\phi\theta}{r^{\mu w}_t}p^{\mu}_{ht+1}}$, and:

$$h^{\mu\phi}_t = \frac{K^{\mu}_t - B^{\mu w}_t}{\beta A^{\mu}_t} = \left(1 - \frac{\alpha\phi\theta}{r^{\mu w}_t}p^{\mu}_{ht+1}\right)\frac{K^{\mu}_t}{\beta A^{\mu}_t} = \frac{s\left(1-\frac{\alpha\phi\theta}{r^{\mu w}_t}p^{\mu}_{ht+1}\right)}{1-\frac{\alpha\phi\theta}{r^{\mu w}_t}p^{\mu}_{ht+1} - \frac{\alpha(1-\phi)\theta}{r^{\mu}_t}p^{\mu}_{ht+1}}.$$

When $r^{\mu}_t = \underline{r}^{\mu}_t$, both H- and L-types borrow abroad up to the limit and produce. When $r^{\mu}_t > \underline{r}^{\mu}_t$, L-types neither produce nor lend abroad but engage in lending to domestic H-types. The term $\beta(\overline{r}^{\mu}_t - \underline{r}^{\mu}_t)h^{\mu\phi}_t$ captures the crowd-in effect and $(1-\beta)\underline{r}^{\mu}_t$ and $(1-\beta)\overline{r}^{\mu}_t$ capture the crowd-out effect.

Then, we discuss how financial globalization changes the relative size of crowd-in and crowd-out effects. As we learn from the second terms of (43) and (44) (the first line), in determining the effect of bubbles on economic growth, the relative rates of return on savings between H- and L-types $(\overline{r}^{\mu}_t - \underline{r}^{\mu}_t)$ plays a crucial role. We discuss how financial globalization changes this relative return. When capital inflows occur, H-types' leveraged return on investment increases relative to L-types' one because agents can borrow abroad at cheaper rate to invest in capital. (When ϕ increases, $\overline{r}^{\mu}_t$ increases relative to $\underline{r}^{\mu}_t$.) Since bubbles crowd in investment by reallocating resource from L- to H-project, this higher return on H-project strengthens the crowd-in effect of bubbles. On the other hand, when capital outflows occur, capital outflows increases the interest rate. This increases L-types' return on savings relative to H-types' one and mitigates the crowd-out effect of bubbles.

As we will discuss in Section 4, the degree of financial development in the two countries is a key determinant for bubbles' growth effect as they affect the direction of capital flows and thus the relative size of crowd-in and crowd-out effects.

To this point, we derived the growth rate in the Home country. Since bubbles only exist in the Home country, the Foreign growth rate can be expressed similarly to the bubbleless case. As in Ventura (2012), the effect of bubbles on economic growth is propagated abroad via terms-of-trade adjustment so that the two countries share the same growth rates in the long run.[33][34]

Finally, we note that, using the laws of motion of net worth for H- and L-types in Appendix A.2, we can derive the growth rate of aggregate net worth as the weighted average of H- and L-types' rate of return on savings:

$$\begin{aligned} g_t^{\mu} = \frac{A_{t+1}^{\mu}}{A_t^{\mu}} &= \beta\left[s\frac{\alpha(1-\theta)p_{ht+1}^{\mu}}{1-\frac{\alpha\phi\theta}{r_t^{\mu w}}p_{ht+1}^{\mu}-\frac{\alpha(1-\phi)\theta}{r_t^{\mu}}p_{ht+1}^{\mu}} + (1-s)r_t^{\mu} + \left(\frac{Q_{t+1}}{Q_t}-r_t^{\mu}\right)\mu_t\right] \\ &= \beta\left[s\frac{\alpha(1-\theta^*)p_{ft+1}^{\mu}}{1-\frac{\alpha\phi^*\theta^*}{r_t^{\mu w}}p_{ft+1}^{\mu}-\frac{\alpha(1-\phi^*)\theta^*}{r_t^{\mu *}}p_{ft+1}^{\mu}} + (1-s)r_t^{\mu *}\right] = \frac{A_{t+1}^{\mu *}}{A^{\mu *}} = g_t^{\mu *}, \end{aligned} \tag{45}$$

instead of the growth rates of output itself or output net of international debt repayment. However, under the BGP, all of those growth rates are equal.

2.5.2 Dynamics of bubbles

Finally, we consider the dynamics of bubbles. Since $\mu_t = \frac{Q_t X}{\beta A_t^{\mu}}$, μ_t evolves over time according to:

$$\mu_{t+1} = \frac{\frac{Q_{t+1}}{Q_t}}{\frac{A_{t+1}^{\mu}}{A_t^{\mu}}}\mu_t. \tag{46}$$

[33]Ventura (2012) assumes free trade of intermediate goods but abstracts away international capital flows.

[34]To understand how the growth effect of bubbles is propagated via terms of trade, suppose bubbles crowd in H-investment and increase productivity a_{ht} in the Home country. Then, (10) and (29) imply that p_{ht+1} decreases and p_{ft+1} increases. This creates an upward pressure on the Foreign growth rate. We will formalize this idea in Appendix A.8.

Aggregating the demand function (13) of bubbles and solving it for $\frac{Q_{t+1}}{Q_t}$, the rate of return on bubbles can be written as:[35]

$$\frac{Q_{t+1}}{Q_t} = \frac{r_t^{\mu}(1 - s - \mu_t)}{\pi(1 - s) - \mu_t}. \tag{47}$$

Note that, when $\pi < 1$, the rate of return on bubbles, $\frac{Q_{t+1}}{Q_t}$, is strictly greater than that of safe asset, r_t^{μ}, due to the risk premium.

We first focus on the dynamics of bubbles under financial autarky. Combining equations (45), (46), and (47), the dynamics of bubbles can be written as:

$$\mu_{t+1} = \begin{cases} \dfrac{\frac{1-s-\mu_t}{\pi(1-s)-\mu_t}}{\beta\left[1 + \frac{\alpha-\gamma}{\gamma-\alpha\theta}s + \frac{(1-\pi)(1-s)}{\pi(1-s)-\mu_t}\mu_t\right]}\mu_t, & \text{if } \mu_t \leq L^a(\theta), \\ \dfrac{\theta}{\beta}\dfrac{1}{\pi(1-s) - (1-\theta)\mu_t}\mu_t, & \text{if } \mu_t \geq L^a(\theta). \end{cases} \tag{48}$$

As discussed in Weil (1987), depending on the bubble size, μ_0, in the initial period, there are three possible patterns of bubble dynamics that are consistent with equilibrium. First, bubbles explode to infinity and cannot be sustained in equilibrium. Second, bubble size converges to zero (asymptotically bubbleless path). Third, bubble size is strictly positive and constant over time. We focus on the BGP where bubble size, μ_t, is positive and constant and the output, net worth, and price of bubbles share the same constant growth rates:

$$\frac{Y_{t+1}^{\mu}}{Y_t^{\mu}} = \frac{A_{t+1}^{\mu}}{A_t^{\mu}} = \frac{Q_{t+1}}{Q_t}. \tag{49}$$

Especially, under financial autarky, we can formally prove that there is a unique positive value μ^* such that $\mu_t = \mu^* > 0$ for all t (see Hirano and Yanagawa, 2017 for the proof.). Using (45), (46), and (47), the steady-state bubble size under financial autarky can be written as a function of θ:

$$\mu^{a*}(\theta) = \begin{cases} \dfrac{\pi - \frac{1-\pi\beta(1-s)}{\beta s\left[1+\frac{\alpha-\gamma}{\gamma-\alpha\theta}\right]}}{1 - \frac{1-\pi\beta(1-s)}{\beta s\left[1+\frac{\alpha-\gamma}{\gamma-\alpha\theta}\right]}}(1-s), & \text{if } \mu_t \leq L^a(\theta), \\ \dfrac{\pi\beta(1-s) - \theta}{\beta(1-\theta)}, & \text{if } \mu_t \geq L^a(\theta). \end{cases} \tag{50}$$

Let θ^{am} be the value of θ which satisfies $\mu^{a*}(\theta) = L^a(\theta)$. We have $\mu^{a*}(\theta) < L^a(\theta)$ if $\theta < \theta^{am}$ and

[35]To derive (47), we use the relationship that the aggregate net worth of L-types is a fraction $1 - s$ of the aggregate output of all entrepreneurs.

$\mu^{a*}(\theta) > L^a(\theta)$ if $\theta > \theta^{am}$.

3 Effects of Financial Globalization on the Existence of Bubbles

Here, we examine the effect of financial globalization on the existence condition of bubbles.[36] We first examine the financial autarky case, under which we can derive an analytical expression of the existence condition. We derive the parameter condition where the steady state bubble size (50) under financial autarky is strictly positive. Hereafter, we assume (a) $\pi\beta\alpha \geq \gamma$ and (b) $\gamma(1-\pi\beta(1-s)) - \pi s\beta\alpha > 0$.[37] We define $\underline{\theta}^a$ and $\overline{\theta}^a$ as the lower and upper bounds of θ where bubbles can exist under financial autarky.

Proposition 3. *Under financial autarky, bubbles can exist if the following condition is satisfied:*

$$\underline{\theta}^a \equiv \frac{\gamma(1-\pi\beta(1-s)) - \pi s\beta\alpha}{\alpha(1-\pi\beta)} < \theta < \pi\beta(1-s) \equiv \overline{\theta}^a.$$

Figure 7(a) shows the existence region of bubbles under financial autarky in Proposition 3. This Proposition implies that bubbles can only exist in the intermediate range of financial development. This corresponds to the fact that the growth rate is larger than the interest rate only in the intermediate range of θ, as shown in Figure 4(a). The existence condition is characterized the same as Hirano and Yanagawa (2017) and it does not depend on p_{ht+1} and p_{ft+1}. This is because introducing terms of trade do not change the relative allocation between H- and L-types.

Moreover, we can use the structure of the bubbleless economy to characterize the existence condition of bubbles. The necessary condition for the existence of bubbles is that, under the bubbleless economy, the economic growth rate is not lower than the interest rate. This result is consistent with the existence condition of bubbles in Tirole (1985).[38]

Proposition 4. *Under financial autarky, the existence condition of bubbles is satisfied if, under the bubbleless economy, the growth rate is not lower than the interest rate.*

[36] While the main text focuses on how financial globalization affects the emergence of bubbles, in appendix A.13, we also discuss how bubbles in turn affect international capital flows and how this change affects worldwide production efficiency.

[37] Under assumption (a), $\overline{\theta}^a$ is greater than $\underline{\theta}^a$ so that the region of θ where bubbles can exist under financial autarky is non-empty. Under assumption (b), bubbles cannot exist under financial autarky when θ is sufficiently low. We will discuss how globalization changes this existence condition in the low-θ economy.

[38] Note that $\overline{\theta}^a$ is increasing in π and that $\underline{\theta}^a$ is decreasing in π. This implies that, when bubbles are risky (π is small), the existence region of bubbles shrinks since agents are risk-averse. When bubbles are deterministic ($\pi = 1$), their existence region becomes identical to the region of θ where the growth rate is larger than the interest rate under financial autarky.

Next, we consider the existence condition of bubbles under financial globalization. In general, for bubbles to exist, the expected return on bubbles must be strictly greater than the return on the safe assets, *i.e.,* $\pi \frac{Q_{t+1}}{Q_t} > r_t^{\mu}$, as implied by equation (13). We define $\underline{\theta}^g$ and $\overline{\theta}^g$ as the lower and upper bounds of θ where bubbles can exist under financial globalization. For given level of θ^*, $\underline{\theta}^g$ and $\overline{\theta}^g$ are derived as the upper and lower bounds of θ such that the steady-state bubble size μ^* is positive and equations (32), (33), (34), (35), (36), (37), (38), (39), (40), (45), (47), and (49) are satisfied. Although it is difficult to derive the values of $\underline{\theta}^g$ and $\overline{\theta}^g$ analytically in general, they can be derived numerically (see Appendix A.3.2 for details).

In the following Proposition, we assume deterministic bubbles, *i.e.,* $\pi = 1$, and focus on the parameter region where we can derive an analytical result. However, we can check numerically that the statements in each Proposition hold for all values of (θ, θ^*) even under stochastic bubbles.

Proposition 5. *Assume $\pi = 1$. We focus on the region of (θ, θ^*) where we can derive the values of $\overline{\theta}^g$ and $\underline{\theta}^g$ explicitly. When $\underline{\theta}^a < \theta^* < \overline{\theta}^a$, there exist parameter values (θ, θ^*) such that $\underline{\theta}^g < \underline{\theta}^a$ and $\overline{\theta}^a < \overline{\theta}^g$ holds so that financial globalization expands the region of (θ, θ^*) where bubbles can exist. When $\theta^* < \underline{\theta}^a$ or $\overline{\theta}^a < \theta^*$, $\underline{\theta}^a = \underline{\theta}^g$ and $\overline{\theta}^g < \overline{\theta}^a$ holds so that financial globalization shrinks the region of (θ, θ^*) where bubbles can exist.*

Figure 7(b) shows the existence region of bubbles under financial globalization.[39] Proposition 5 shows that the effects of financial globalization on the existence condition of bubbles depend on both absolute and relative degrees of financial development in the Home and Foreign countries, respectively. As discussed in Section 2.4.2, when θ is sufficiently high and θ^* is in the middle range, financial globalization leads to capital inflows and suppresses the interest rate relative to the growth rate. This increases the demand for bubbles with higher returns and expands their existence region. This implies that bubbles cannot exist under financial autarky but they can exist under financial globalization. On the other hand, when θ is in the middle range and θ^* is either sufficiently high or low, financial globalization leads to capital outflows, which increases the interest rate relative to the growth rate and shrinks the existence region of bubbles. This implies that bubbles can exist under financial autarky but they cannot exist under financial globalization.

This implies that financial globalization facilitates the emergence of bubbles when the domestic financial market is either developed or underdeveloped relative to the foreign one. Our result is consistent with the advanced economies' housing booms followed by global financial crisis and emerging economies' credit booms and currency crises.

Moreover, we can show that, even under financial globalization, the necessary condition for the

[39]In the numerical simulation, the survival probability of bubbles is set to $\pi = 0.99$.

existence of bubbles is that, under the bubbleless economy, the economic growth rate is not lower than the interest rate. Again, we assume deterministic bubbles to obtain an analytical result.

Proposition 6. *Assume $\pi = 1$. We focus on the region of (θ, θ^*) where we can derive the analytical results in Propositions 2 and 5. Under financial globalization, bubbles can exist if, under the bubbleless economy, the growth rate is not lower than the interest rate.*

4 Effects of Bubbles on Economic Growth

Next, we consider how the effects of bubbles on economic growth differ comparing before and after financial globalization. In other words, we examine how globalization affects the boom-bust magnitude of bubbles.

We begin with the case under financial autarky. Comparing the growth rates under bubbleless and bubble economies, we obtain the following Proposition.

Proposition 7. *Under financial autarky, there exists $\theta^{a\prime} \in (\underline{\theta}^a, \overline{\theta}^a)$ which satisfies the following: (i) If $\underline{\theta}^a < \theta < \theta^{a\prime}$, the growth rate under the bubble economy is higher than that of the bubbleless economy. (ii) If $\theta^{a\prime} < \theta < \overline{\theta}^a$, the growth rate under the bubbleless economy is higher than that of the bubble economy.*

Proposition 7 implies that bubbles have both crowd-in and crowd-out effects on economic growth and that which effect dominates the other depends on the degree of financial development. When θ is low, crowd-in effect dominates crowd-out effect. However, when θ is high, crowd-out effect dominates the crowd-out effect. This result is the same as Hirano and Yanagawa (2017), as introducing terms of trade does not change the relative returns on savings for H- and L-types.

The intuition is as follows. As discussed in Section 2.5.1, bubbles crowd in capital investment by reallocating resources from L- to H-investment, while they crowd out aggregate resources for capital investment. Without bubbles, $L^a(\theta)$ is invested in L-projects. When θ is small ($\underline{\theta}^a \leq \theta < \theta^{am}$), $L^a(\theta)$ is large ($\mu^{a*}(\theta) < L^a(\theta)$). This implies that bubbles reallocate large share of aggregate net savings from L- to H-investment. Hence, the crowd-in effect always dominates the crowd-out effect. However, when θ is large ($\theta^{am} \leq \theta < \overline{\theta}^a$), $L^a(\theta)$ is small ($\mu^{a*}(\theta) > L^a(\theta)$). Then, bubbles crowd out not only L-investment but also H-investment. In this case, the crowd-in effect dominates the crowd-out effect when θ is relatively low ($\theta^{am} \leq \theta < \theta^{a\prime}$), while the crowd-out effect dominates the crowd-in effect when θ is relatively high ($\theta^{a\prime} \leq \theta < \overline{\theta}^a$). As we will show later, this result is important when discussing welfare effect of bubbles under two-country settings.

Next, we examine how globalization changes the bubbles' effect on economic growth, *i.e.,* how the relative sizes of the crowd-in and crowd-out effects change comparing before and after financial globalization. Here, we measure the growth effect of bubbles by taking the ratio between growth rates of the bubble and bubbleless economies (g_t^μ / g_t) and compare it before and after financial globalization.

To capture the key intuition, it is better to start with a result under deterministic bubbles, *i.e.,* $\pi = 1$. (The result under stochastic bubbles is described in Appendix A.9.) Figure 8 compares the growth effects of bubbles between financial autarky and globalization. Here, we focus on the colored region of (θ, θ^*), in which bubbles can exist both under financial autarky and globalization.[40]

In the red region, the growth effect of bubbles is larger under financial globalization, *i.e.,* $(g_t^\mu / g_t)^{global} > (g_t^\mu / g_t)^{autarky}$, while in the green region, their growth effect is larger under financial autarky, *i.e.,* $(g_t^\mu / g_t)^{global} < (g_t^\mu / g_t)^{autarky}$. We learn that the effects of financial globalization on bubbles' growth effect depends on both absolute and relative degrees of financial development in the two countries. Specifically, when θ is either sufficiently high or low and θ^* is in the middle range, financial globalization strengthens the growth effect of bubbles, *i.e.,* $(g_t^\mu / g_t)^{global} > (g_t^\mu / g_t)^{autarky}$. However, when θ^* is either sufficiently high or low and θ is in the middle range, financial globalization weakens the growth effect of bubbles, *i.e.,* $(g_t^\mu / g_t)^{global} < (g_t^\mu / g_t)^{autarky}$.

The intuition for this result is as follows. As discussed in Section 2.5.1, the relative rates of return on savings between H- and L-types play a crucial role in determining the relative size of crowd-in and crowd-out effects. When θ is either sufficiently high or low relative to θ^*, financial globalization leads to capital inflows. This improves H-types' rate of return on investment relative to L-types' one. Hence, the crowd-in effect becomes large relative to the crowd-out effect. On the other hand, when θ is in the middle range and θ^* is either sufficiently high or low relative to θ^*, financial globalization leads to capital outflows. This suppresses H-types' rate of return on investment relative to L-types' one. Hence, the crowd-out effect becomes large relative to the crowd-in effect.

Combining this result with Propositions 5, we learn that, when domestic financial market is either developed or underdeveloped relative to the foreign one, financial globalization not only facilitates the emergence of bubbles but also strengthens the growth effect of bubbles. In other words, financial globalization magnifies the boom-bust magnitude of bubbles.

We add a remark to this result. In this Section, in order to compare the growth effect of bubbles, we focus on the region of (θ, θ^*) where bubbles can exist both under financial autarky and globalization. When θ is high, there exists a region of (θ, θ^*) where bubbles cannot exist under financial autarky but can exist under financial globalization.[41] When θ becomes sufficiently high,

[40] In Figure 8, since we assume deterministic bubbles ($\pi = 1$), the existence region of bubbles is smaller than the case under stochastic bubbles ($\pi < 1$) in Figures 7(a) and 7(b). See footnote 38.

[41] This region is plotted in Figure 7, which shows the existence condition of bubbles under financial globalization. However, it is not plotted in Figures 8 and 12, which show the growth effect of bubbles under deterministic and

there exists a region of (θ, θ^*) where bubbles reduce the economic growth rate even under financial globalization. (the logic is similar to Proposition 7.) This region is our main focus in the next Section. We will show that bubbles in sufficiently well-developed financial market lead to welfare losses in the other country due to their negative effect on economic growth.

5 Welfare Analysis

Finally, we conduct a full welfare analysis regarding the asset bubbles under financial globalization. To this end, we compute a *ex-ante* welfare of entrepreneurs in the two countries under bubbleless and bubble economy and compare how bubbles in one country affect the welfare in the two countries.

We first derive the welfare under bubbleless economy. Let $V_t(e_t)$ and $V_t^*(e_t^*)$ be the value functions of the Home and Foreign entrepreneurs under the bubbleless economy and e_t and e_t^* be their net worth at the beginning of date t. Since the entrepreneurs face idiosyncratic productivity risk, we compute the *ex-ante* welfare before entrepreneurs knows their type. Solving for the value function, we obtain:

$$V_t(e_t) = W(\theta,\, \theta^*,\, \phi,\, \phi^*) + \frac{1}{1-\beta}\log(e_t), \text{ and} \tag{51}$$

$$V_t^*(e_t^*) = W^*(\theta,\, \theta^*,\, \phi,\, \phi^*) + \frac{1}{1-\beta}\log(e_t^*), \tag{52}$$

where the derivation is given in the Appendix A.10. The first term is a function of the degrees of financial development and financial openness in the Home and Foreign countries and the second term is a linear function of $\log(e_t)$ and $\log(e_t^*)$, respectively. Note that $W(\theta,\, \theta^*,\, \phi,\, \phi^*)$ and $W^*(\theta,\, \theta^*,\, \phi,\, \phi^*)$ depend on the interest rates and the terms of trade in the two countries. Under some region of (θ, θ^*), we cannot solve for the factor prices analytically since they are determined implicitly to satisfy the international credit market clearing condition (22). However, for each given (θ, θ^*), we can solve for the factor prices numerically and pin down the values of $W(\theta,\, \theta^*,\, \phi,\, \phi^*)$ and $W^*(\theta,\, \theta^*,\, \phi,\, \phi^*)$ uniquely as functions of $\theta,\ \theta^*,\ \phi$, and ϕ^*.

Next, we derive the *ex-ante* welfare under bubble economy. In order to understand how bubbles have different welfare effects on the domestic and foreign countries, we continue to assume that bubbles only exist in the Home country.[42] Let $V_t^{\mu}(e_t^{\mu})$ and $V_t^{\mu*}(e_t^{\mu*})$ be the value functions of the Home and Foreign entrepreneurs under bubble economy and e_t^{μ} and $e_t^{\mu*}$ be their net worth at

stochastic bubbles, respectively.

[42]It would also be interesting to consider welfare implications when both countries hold bubble assets. To this end, we will need to take into account the coexistence of bubbles with different risks and returns. We leave this issue for future research. (Matsuyama et al., 1993 discuss equilibrium where different types of currencies can coexist.)

the beginning of date t. Since entrepreneurs are risk-averse, they take into account the increased volatility caused by bubble burst. Solving for the value function, we obtain:

$$V_t^{\mu}(e_t^{\mu}) = W^{\mu}(\theta, \theta^*, \phi, \phi^*) + \frac{1}{1-\beta}\log(e_t^{\mu}), \text{ and} \tag{53}$$

$$V_t^{\mu*}(e_t^{\mu*}) = W^{\mu*}(\theta, \theta^*, \phi, \phi^*) + \frac{1}{1-\beta}\log(e_t^{\mu*}), \tag{54}$$

where the derivation is given in the Appendix A.10. The first term is a function of the degrees of financial development and financial openness in the Home and Foreign countries and the second term is a linear function of $\log(e_t^{\mu})$ and $\log(e_t^{\mu*})$, respectively.

Then, we compute the *ex-ante* welfare under bubbleless and bubble economies. Following Hirano and Yanagawa (2017), we compute the value function evaluated at $t = 0$. Moreover, to compute the welfare, we make the following assumptions: (i) each country is equally endowed with a unit measure of entrepreneurs, (ii) in the initial period, each country is equally endowed with aggregate output Y_0 and each entrepreneur in each country is equally endowed with output, that is, $y_0^i = y_0^{*i} = y_0 = Y_0$, (iii) the aggregate supply of bubble assets in the Home country is normalized to one, that is, $X = 1$, and (iv) in the initial period, each entrepreneur in the Home country is equally endowed with one unit of bubble assets, that is, $x_t^i = x_t = 1$. Under these assumptions, the entrepreneurs' initial net worth can be expressed as $e_0 = e_0^* = Y_0$ under bubbleless economy and $e_t^{\mu} = \frac{Y_0}{1-\beta\mu^*}$ and $e_0^{\mu*} = Y_0$ under bubble economy. Here, the term $\frac{1}{1-\beta\mu^*}$ captures the initial wealth effect of bubbles. If the entrepreneurs hold bubble assets at the initial period, their initial net worth increases. Moreover, this increased net worth also expand their future net worth as they grow over time, so that their lifetime consumption and thus welfare increase.

Finally, we compare the welfare under bubbleless and bubble economies. We first show the result under financial autarky, that is, $\phi = \phi^* = 0$. We assume that the initial endowment of aggregate output in each country is $Y_0 = 1$. The next Proposition shows how bubbles in the Home country affect the welfare in the Foreign country.[43]

Proposition 8. *Let $\theta^{a\prime}$ be defined in Proposition 7. Under financial autarky, Home bubbles increase Foreign welfare if $\underline{\theta}^a \leq \theta < \theta^{a\prime}$ and Home bubbles decrease Foreign welfare if $\theta^{a\prime} \leq \theta < \overline{\theta}^a$.*

Moreover, combining Propositions 7 and 8, we obtain the following Corollary.

[43]Regarding the Home country's welfare, it is difficult to obtain an analytical result because bubbles affect H- and L-types' rates of return on savings through terms of trade. However, we can numerically check that Home bubbles increase the Home country's welfare under all (θ, θ^*) in their existence region, regardless of whether bubbles increase or decrease the economic growth rate or whether bubbles are expected to collapse or not.

Corollary 1. *Home bubbles increase Foreign welfare if and only if they increase the world economic growth rate and Home bubbles decrease Foreign welfare if and only if they decrease the world economic growth rate.*

Figure 9(a) shows how bubbles in the Home country affects welfare in the two countries. In the red region, bubbles improve welfare in the Home country but reduce welfare in the Foreign country, while in the green region, bubbles improve welfare in both Home and Foreign countries. This implies that, while Home bubbles always improve welfare in the Home country, it can reduce welfare in the Foreign country as long as the financial market in the Home country is sufficiently well-developed. Moreover, the region of θ where Home bubbles reduce Foreign welfare is identical to its region where Home bubbles reduce the world growth rate.

Intuitively, as in Hirano and Yanagawa (2017), bubbles are welfare-improving for the bubble-holding country even if bubbles reduce economic growth and even if they are expected to collapse in the future. The reason is twofold. The first reason is the initial wealth effect of bubbles as discussed before. The second reason is the consumption smoothing effect of bubbles. While bubbles increase the interest rate and thus the rate of return on savings for the savers (L-types), they decrease the leveraged rate of return on investment for the investors (H-types) due to high borrowing cost. Therefore, bubbles reduce the difference in the rates of return on savings for H- and L-types. This lower volatility increases the entrepreneurs' welfare because they are risk-averse. Since the initial net worth effect and consumption-smoothing effect of bubbles dominate their negative effects, the overall welfare effect of bubbles is positive.

However, we also found that, bubbles can be welfare-reducing for the non-bubble-holding country. The intuition is as follows. When the financial market is sufficiently well-developed, bubbles crowd out H-types' productive investment and depress the economic growth. This growth-depressing effect is transmitted abroad through the terms-of-trade adjustment. Hence, bubbles also reduce the economic growth rate in the non-bubble holding country and reduce its welfare.

On the other hand, under financial autarky, consumption smoothing effect does not work in the Foreign country. This is because the consumption smoothing effect is caused since bubbles increase the interest rate. Since there is no capital flow, the consumption smoothing effect is not transmitted abroad through the adjustment in interest rates. In this sense, bubbles in one country can have an asymmetric welfare effects on the two countries depending on the degree of financial development.

Next, we show the result under financial globalization. Here, we assume $\phi = \phi^* = 0.5$. Figure 9(b) shows the effect of Home bubbles on two countries' welfare. In contrast with the previous case, we have the blue region, in which Home bubbles reduce the world growth rate but improve the welfare in both Home and Foreign countries. This is because capital flow has an equalizing force on the two countries' interest rates. Hence, consumption smoothing effect, which is caused by the higher interest rate, is also working in the Foreign country.

However, even under financial globalization, there still remains the red region, in which Home bubbles reduce both Foreign country's growth rate and welfare. Figure 9(b) shows that the blue region only exists when both θ and θ^* are sufficiently high. This is because, when the financial market is close to perfect, capital flows smoothly across countries and the equalization force of the Home and Foreign interest rates is strong. However, except for this special case, the growth-reducing effect of bubbles dominates their consumption-smoothing effect so that Home bubbles reduce Foreign welfare.

According to the traditional literature, bubbles are generally regarded as welfare-improving (Samuelson, 1958; Tirole, 1985). Our novel result suggests that, although bubbles are welfare-improving for the bubble-holding countries, they are welfare-reducing for the non-bubble-holding countries under some parameter conditions. Our result provides an important justification for policy intervention against bubbles, as policymakers need to take into account the bubbles' negative pecuniary externality on Foreign welfare.

6 Conclusion

In this paper, we developed a two-country model of rational bubbles with asymmetric degrees of financial development. We examined (i) how an integration of international financial markets affects the existence condition of bubbles, and (ii) how bubbles' effects on economic growth is different comparing before and after financial globalization.

We found that the effect of globalization on emergence of bubbles and their growth effects depends on both absolute and relative degrees of financial market development between the two countries. In particular, when the domestic financial market is either developed- or underdeveloped, financial globalization expands the existence condition of bubbles and strengthens the their growth-enhancing effect. On the other hand, when the domestic financial market is in the middle range and foreign financial market is either developed or underdeveloped, financial globalization shrinks the existence condition of bubbles and weakens the their growth-enhancing effect. This non-monotonic relationship implies that financial globalization facilitates the emergence of bubbles and strengthens their boom-bust magnitudes when the domestic financial market is either developed or underdeveloped relative to the rest of the world.

Moreover, we conducted a full welfare analysis of asset bubbles under two-country framework. As in the traditional view, bubbles improve welfare of the bubble-holding countries because of consumption-smoothing effect. However, we found that, bubbles in sufficiently well-developed financial markets reduce welfare of the non-bubble holding countries because the negative effect of bubbles on economic growth is transmitted abroad via general equilibrium effect.

An important topic for future research is an empirical relationship between financial globalization and bubbles. This is a challenging task due to the identification problem: we have to distinguish whether the asset price hike is driven by economic fundamentals or market expectations. Although there is little consensus regarding how to identify bubbles, this is an essential direction of research for macro-finance literature.[44] Finding an empirical evidence of bubbles will open up new avenues for future research, including the econometric analysis on financial globalization and bubbles.

[44]Guerron-Quintana et al. (2020) proposed a model of recurrent bubbles and used Bayesian estimation to identify the time periods with and without bubbles.

A Appendix

A.1 Small Open Economy

In this section, we consider the small open economy version of our model. This small open economy setup is simple enough so that we can derive a fully analytical solution and helps us to capture the key intuition of the two-country model. However, we also point out that this setup has two limitations. First, as we discuss below, since there is no terms-of-trade effect, capital always flow from financially under- to well-developed country. This implies that financial globalization prevents bubbles in financially underdeveloped economies, which contradicts the emerging markets' experience of capital account liberalization and currency crisis. Second, we cannot discuss how bubbles in one country affect the other country's welfare.

The small open economy model has the same structure as the two-country model, except that the Home entrepreneurs produce one homogeneous final good, that the Foreign country is represented by the world interest rate r^w, and that there is no market for international credit. We focus on the entrepreneurs' behavior in the Home country. Unless otherwise stated, the results under financial autarky and proofs of Propositions are similar to Hirano and Yanagawa (2017).

We consider a small open economy which consists a continuum of entrepreneurs and one homogeneous good. A typical entrepreneur has an expected utility same as (2). The entrepreneurs have a following production function:

$$z_{t+1}^i = a_t^i k_t^i,$$

where z_{t+1}^i is the output of final goods at date $t+1$. The foreign and domestic credit constraints are expressed as:

$$r^w b_t^{wi} \leq \phi\theta a_t^i k_t^i, \text{ and} \tag{A.1}$$

$$r_t b_t^i + r^w b_t^{wi} \leq \theta a_t^i k_t^i, \tag{A.2}$$

where the world interest rate, r^w, is fixed. In the economy with bubbles, each entrepreneur faces the following four constraints: the flow of funds constraint:

$$c_t^{\mu i} + k_t^{\mu i} + Q_t^x x_t^i = z_t^{\mu i} - r_{t-1}^{\mu} b_{t-1}^{\mu i} - r^w b_{t-1}^{\mu wi} + b_t^{\mu i} + b_t^{\mu wi} + Q_t^x x_{t-1}^i, \tag{A.3}$$

the foreign and domestic borrowing constraints, (A.1) and (A.2), and the short-sale constraint (7). We define the net worth of the entrepreneur at date t as $e_t^{\mu i} = z_t^{\mu i} - r_{t-1}^{\mu} b_{t-1}^{\mu i} - r^w b_{t-1}^{\mu wi} + Q_t^x x_{t-1}^i$.

A.1.1 Entrepreneurs

We then characterize the equilibrium behavior of entrepreneurs. We assume $\gamma < r^w < \alpha$.[45] Let:

$$\bar{r}_t^\mu = \begin{cases} \alpha, & \text{if } r_t^\mu = r^w, \\ \frac{\alpha(1-\phi\theta)}{1-\frac{\alpha\phi\theta}{r^w}}, & \text{if } r_t^\mu > r^w, \end{cases}$$

where $\frac{\alpha(1-\phi\theta)}{1-\frac{\alpha\phi\theta}{r^w}}$ is the leveraged rate of return of investment when H-types borrow abroad up to the limit. On the other hand, since we assume $r_w > \gamma$, L-types never borrow abroad in order to produce. We focus on the equilibrium where

$$\gamma \leq r_t^\mu < \bar{r}_t^\mu.$$

For H-types, since $c_t^{\mu i} = (1-\beta)e_t^{\mu i}$, using the equations (7), (A.1), (A.2), and (A.3), the investment function can be written as:

$$k_t^{\mu i} \leq \frac{\beta e_t^{\mu i}}{1 - \frac{\alpha\phi\theta}{r^w} - \frac{\alpha(1-\phi)\theta}{r_t^\mu}}, \tag{A.4}$$

where the inequality holds if $r_t^\mu = \bar{r}_t^\mu$. When $r_t^\mu > r^w$, (A.1) binds so that H-types borrow abroad up to the limit. When $r_t^\mu = r^w$, (A.1) does not bind so that H-types are indifferent between borrowing from domestic or foreign agents. For L-types, since $c_t^{\mu i} = (1-\beta)e_t^{\mu i}$, the flow of funds constraint (A.3) becomes the same as (12). Their demand function for bubbles is given by (13). Moreover, L-types face the following two complementary slackness conditions. First, L-types choose whether to produce or not. When $r_t^\mu > \gamma$, k_t^i must be 0. When $r_t^\mu = \gamma$, L-types are indifferent between lending their net worth and borrowing abroad to produce. Hence:

$$(r_t^\mu - \gamma)k_t^{\mu i} = 0, \; r_t^\mu \geq \gamma, \text{ and } k_t^i \geq 0.$$

Second, L-types decide whether they borrow from domestic or foreign agents. As in H-types' case, the international borrowing constraint (4) binds when $r_t^\mu > r_t^{\mu w}$, while (A.1) does not bind when

[45] We implicitly assume that (i) the productivities of Foreign H- and L-types, α and γ, are the same as the Home ones and that (ii) the Foreign country is also facing a financial friction so that $r^w < \alpha$. When $\gamma < r^w < \alpha$, the Home country can experience both capital in- and outflows depending on the degree of financial development. Hereafter, we focus on this parameter region. On the other hand, when $r^w < \gamma$, the Home country always experiences capital inflows regardless of the degree of financial development. When $r^w = \gamma$, Home L-types are indifferent between borrowing abroad to produce and lending abroad.

$r_t^{\mu} = r_t^{\mu w}$.

A.1.2 Equilibrium

Let Z_t^{μ} be the aggregate output at date t. Then, the market clearing condition for final goods can be written as:

$$Z_t^{\mu} = C_t^{\mu} + C_t^{\mu\prime} + K_t^{\mu} + K_t^{\mu\prime} - (B_t^{\mu w} + B_t^{\mu w\prime} - r^w B_{t-1}^{\mu w} - r^w B_{t-1}^{\mu w\prime}), \tag{A.5}$$

and the market clearing condition for domestic credit and bubbles are as the same as (18) and (21). The competitive equilibrium is defined as the set of prices $\{r_t^{\mu}, Q_t^x\}_{t=0}^{\infty}$ and quantities $\{c_t^{\mu i}, k_t^{\mu i}, b_t^{\mu i}, b_t^{\mu w i}, z_t^{\mu i}, C_t^{\mu}, C_t^{\mu\prime}, K_t^{\mu}, K_t^{\mu\prime}, B_t^{\mu}, B_t^{\mu\prime}, B_t^{\mu w}, B_t^{\mu w\prime}, Z_t^{\mu}\}_{t=0}^{\infty}$, such that (i) each entrepreneur chooses consumption, investment, domestic and foreign borrowing, and bubble assets to maximize their expected discounted utility (2) under the constraints (7), (A.1), (A.2), and (A.3), and (ii) market clearing conditions, (18), (21), and (A.5), are satisfied.

A.1.3 Bubbleless Economy

We first derive the equilibrium without bubbles, *i.e.,* $Q_t^x = 0$ for all t. Aggregating (A.4), the investment function for H-types can be written as:

$$K_t \leq \frac{\beta s Y_t}{1 - \frac{\alpha\phi\theta}{r^w} - \frac{\alpha(1-\phi)\theta}{r_t}} \text{ } (r_t < \bar{r}_t \text{ when the equality holds}),$$

where $Y_t = Z_t - r^w B_{t-1}^{w} - r^w B_{t-1}^{w\prime}$ is the aggregate output net of foreign repayment at date t. Regarding L-types, since they never produce when $r_t > \gamma$, we have:

$$(r_t - \gamma)K_t' = 0, \; r_t \geq \gamma, \text{ and } K_t' \geq 0.$$

Moreover, aggregating the international borrowing constraint (A.2), we obtain:

$$K_t + K_t' \leq \beta Y_t + \frac{\phi\theta}{r^w}(\alpha K_t + \gamma K_t') \text{ } (r_t = r^w \text{ when the inequality holds}). \tag{A.6}$$

We then characterize the steady state interest and growth rates. We begin with the financial autarky, that is, $\phi = 0$. Using the same logic as in Section 2.4.1, the interest and growth rates become:

$$r_t = \begin{cases} \gamma, & \text{if } 0 \le \theta < \tilde{\theta}^a \equiv \frac{\gamma}{\alpha}(1-s), \\ \dfrac{\alpha\theta}{1-s}, & \text{if } \tilde{\theta}^a \le \theta < \hat{\theta}^a \equiv 1-s, \text{ and} \\ \alpha, & \text{if } \hat{\theta}^a \le \theta \le 1, \end{cases}$$

and

$$g_t = \beta a_{ht} = \begin{cases} \beta\left[s\dfrac{\alpha(1-\theta)}{1-\frac{\alpha\theta}{\gamma}} + (1-s)\gamma\right], & \text{if } 0 \le \theta < \tilde{\theta}^a, \text{ and} \\ \beta\alpha, & \text{if } \tilde{\theta}^a \le \theta \le 1, \end{cases}$$

where a_{ht} is defined the same as (28).

Next, we consider the case under financial globalization. In equilibrium under financial globalization, the Home interest rate must be higher or equal to the world interest rate, i.e.[46]

$$r_t \ge r^w.$$

Since we assume $r^w > \gamma$, L-types never produce, *i.e.*, $K_t' = 0$. Then, the international borrowing constraint (A.6) implies $B_t^{w\prime} \le 0$, i.e. L-types never borrow abroad but either lend their savings to Home H-types or the Foreign country.

The aggregate output net of foreign repayment can be written as::

$$Y_{t+1} = \alpha K_t - r^w B_t^w - r^w B_t^{w\prime}. \tag{A.7}$$

Using $c_t^i = (1-\beta)e_t^i$ and the goods market clearing condition (A.5), we learn that the aggregate investment net of foreign borrowing is equal to the net savings:

$$K_t - B_t^w - B_t^{w\prime} = \beta Y_t. \tag{A.8}$$

We can use (A.7) and (A.8) to characterize the economic growth rate. When θ is sufficiently low, L-types do not lend all their savings to the domestic H-types but they also lend abroad so that $r_t = r^w$ and $h_t \equiv \frac{K_t}{\beta Y_t} = \frac{s}{1-\frac{\alpha\theta}{r^w}} < 1$. When θ is in the middle range, L-types only lend to H-types and

[46]If the Home interest rate were lower than the world interest rate in equilibrium, Home L-types would lend all their net savings to Foreign country, while Home H-types would borrow from domestic L-types. This contradicts the domestic credit market clearing condition.

$r_t > r^w$ is adjusted so that:

$$h_t^{\phi} \equiv \frac{K_t - B_t^{w\prime}}{\beta Y_t} = \frac{s\left(1 - \frac{\alpha\phi\theta}{r^w}\right)}{1 - \frac{\alpha\phi\theta}{r^w} - \frac{\alpha(1-\phi)\theta}{r_t}} = 1.$$

When θ is sufficiently high, the international borrowing constraint (A.1) binds but the domestic borrowing constraint (A.2) does not. Hence, the interest rate is equal to H-types' leveraged rate of return on investment:

$$r_t = \frac{\alpha(1-\phi\theta)}{1 - \frac{\alpha\phi\theta}{r^w}}.$$

Hence, the equilibrium interest and growth rates can be expressed as:

$$r_t = \begin{cases} r^w, & \text{if } 0 \leq \theta < \tilde{\theta}^g \equiv \frac{r^w}{\alpha}\frac{1-s}{1-\phi s}, \\ \dfrac{\alpha(1-\phi)\theta}{(1-s)\left(1 - \frac{\alpha\phi\theta}{r^w}\right)}, & \text{if } \tilde{\theta}^g \leq \theta < \hat{\theta}^g \equiv \frac{r^w}{\alpha}\frac{1-s}{1-\phi s}, \text{ and} \\ \dfrac{\alpha(1-\phi\theta)}{1 - \frac{\alpha\phi\theta}{r^w}}, & \text{if } \hat{\theta}^g \leq \theta \leq 1, \end{cases}$$

and

$$g_t = \begin{cases} \beta\left[s\dfrac{\alpha(1-\theta)}{1-\frac{\alpha\theta}{r^w}} + (1-s)r^w\right], & \text{if } 0 \leq \theta < \tilde{\theta}^g, \text{ and} \\ \beta\dfrac{\alpha(1-\phi\theta)}{1 - \frac{\alpha\phi\theta}{r^w}}, & \text{if } \tilde{\theta}^g \leq \theta \leq 1. \end{cases}$$

The following proposition shows how globalization affects the relative values of interest and growth rates.

Proposition A1. *There exists $\theta^b \in (\tilde{\theta}^a, \hat{\theta}^a)$ such that the following is satisfied. When $0 \leq \theta < \theta^b$, globalization increases the interest rate relative to the growth rate, i.e. $\frac{r_t^g}{r_t^a} > \frac{g_t^g}{g_t^a}$. When $\theta^b \leq \theta < \hat{\theta}^g$, globalization increases the growth rate relative to the interest rate, i.e. $\frac{g_t^g}{g_t^a} > \frac{r_t^g}{r_t^a}$. When $\hat{\theta}^g \leq \theta \leq 1$, globalization does not change the relative value of growth and interest rates, i.e. $\frac{g_t^g}{g_t^a} = \frac{r_t^g}{r_t^a}$.*

Proof. We first assume $\phi > \frac{\alpha - r^w}{\alpha s}$ so that $\hat{\theta}^a < \tilde{\theta}^g$.

(i) When $0 \leq \theta < \tilde{\theta}^a$, $\frac{r_t^g}{r_t^a} > \frac{g_t^g}{g_t^a}$ is equivalent to $r^w > \gamma$, which holds by assumption.

(ii) When $\tilde{\theta}^a \leq \theta < \hat{\theta}^a$, $\frac{g_t^g}{g_t^a} > \frac{r_t^g}{r_t^a}$ is equivalent to:

$$f(\theta) = \alpha\theta^2 - [\alpha + (1-s)r^w]\theta + (1-s)r^w < 0.$$

From case (i), we learn that $f(\tilde{\theta}) > 0$ holds. Moreover, $f(\hat{\theta}^a) = -s(1-s)(\alpha - r^w) < 0$. Hence, there exists $\theta^b \in (\tilde{\theta}^a, \hat{\theta}^a)$ such that $f(\theta) > 0$ for $\tilde{\theta}^a \leq \theta < \theta^b$ and $f(\theta) < 0$ for $\theta^b < \theta < \hat{\theta}^a$.

(iii) When $\hat{\theta}^a \leq \theta < \tilde{\theta}^g$, $\frac{g_t^g}{g_t^a} > \frac{r_t^g}{r_t^a}$ is equivalent to $\alpha > r^w$, which holds by assumption.

(iv) When $\tilde{\theta}^g \leq \theta < \hat{\theta}^g$, $\frac{g_t^g}{g_t^a} > \frac{r_t^g}{r_t^a}$ is equivalent to $\theta < \hat{\theta}^g$, which holds obviously.

(v) When $\hat{\theta}^g \leq \theta \leq 1$, $\frac{g_t^g}{g_t^a} = \frac{r_t^g}{r_t^a} = \frac{1-\phi\theta}{1-\frac{\alpha\phi\theta}{r^w}}$.

Next, we assume $\phi > \frac{\alpha - r^w}{\alpha s}$ so that $\tilde{\theta}^g < \hat{\theta}^a$. The case where (i) $0 \leq \theta < \tilde{\theta}^a$, (ii) $\tilde{\theta}^g \leq \theta < \hat{\theta}^a$, (iv) $\hat{\theta}^a \leq \theta < \hat{\theta}^g$, and (v) $\hat{\theta}^g \leq \theta \leq 1$ can be proven similarly to before. Under the case where (iii) $\tilde{\theta}^g \leq \theta < \hat{\theta}^a$, $\frac{g_t^g}{g_t^a} > \frac{r_t^g}{r_t^a}$ is equivalent to $\theta < 1$, which holds obviously. □

This proposition shows that globalization increases the interest rate relative to the growth rate in a financially underdeveloped economy, while globalization increases the growth rate relative to the interest rate in a financially well-developed economy. As discussed later, this relationship is essential in understanding how globalization changes the existence condition of bubbles.

A.1.4 Economy with Bubbles

Next, we consider the economy with bubbles, *i.e.,* $Q_t^x = Q_t > 0$. The aggregate investment function H-types is given as:

$$K_t^\mu \leq \frac{\beta s A_t^\mu}{1 - \frac{\alpha\phi\theta}{r^w} - \frac{\alpha(1-\phi)\theta}{r_t^\mu}} \ (r_t^\mu < \overline{r}_t^\mu \text{ when the equality holds}),$$

where $A_t^\mu = Z_t^\mu - r^w B_{t-1}^w - r^w B_{t-1}^{\mu w\prime} + Q_t X$ is the aggregate net worth of the entrepreneurs. Regarding L-types, since they never produce when $r_t^\mu > \gamma$, we have:

$$(r_t^\mu - \gamma)K_t^{\mu\prime} = 0,\ r_t^\mu \geq \gamma,\ \text{and } K_t^{\mu\prime} \geq 0.$$

Moreover, the international borrowing constraint can be written as:

$$K_t^\mu + K_t^{\mu\prime} + Q_t X \leq \beta A_t^\mu + \frac{\phi\theta p_{ht+1}^\mu}{r^w}(\alpha K_t^\mu + \gamma K_t^{\mu\prime}) \ (r_t^\mu = r_t^w \text{ when the inequality holds}).$$

Using (A.5), the aggregate net savings can be written as:

$$K_t^{\mu} - B_t^{\mu w} - B_t^{\mu w\prime} + Q_t X = \beta Y_t^{\mu}. \tag{A.9}$$

Moreover, aggregating the international borrowing constraint (A.2), we obtain:

$$K_t^{\mu} + Q_t X \leq \beta Y_t^{\mu} + \frac{\phi\theta}{r^w}\alpha K_t^{\mu}, \tag{A.10}$$

where $r_t^{\mu} = r^w$ when the inequality holds.

Then, we derive the equilibrium interest and growth rates. First, under financial autarky, using the same logic as Section 2.5.1, the interest and growth rates can be derived as:

$$r_t^{\mu} = \begin{cases} \gamma, & \text{if } \mu_t \leq L^a(\theta), \\ \dfrac{\alpha\theta(1-\mu_t)}{1-s-\mu_t}, & \text{if } \mu_t \geq L^a(\theta), \end{cases} \tag{A.11}$$

and

$$\frac{Y_{t+1}^{\mu}}{Y_t^{\mu}} = \begin{cases} \beta[\alpha H^a(\theta) + \gamma(1-H^a(\theta))] + [\beta(\alpha-\gamma)H^a(\theta) - (1-\beta)\gamma]\dfrac{Q_t X}{Y_t^{\mu}}, & \text{if } \mu_t \leq L^a(\theta), \\ \beta\alpha - (1-\beta)\alpha\dfrac{Q_t X}{Y_t^{\mu}}, & \text{if } \mu_t \geq L^a(\theta). \end{cases} \tag{A.12}$$

Next, we consider the case under financial globalization. Let $L^g(\theta) = \text{Max}\left[1 - \frac{s\left(1-\frac{\alpha\phi\theta}{r^w}\right)}{1-\frac{\alpha\theta}{r^w}}\right]$. When:

$$Q_t X > \text{Max}\left[\beta Y_t^{\mu} - \frac{\beta s\left(1-\frac{\alpha\phi\theta}{r^w}\right)Y_t^{\mu}}{1-\frac{\alpha\theta}{r^w}}, 0\right],\ i.e.,\ \mu_t > L^g(\theta),$$

L-types do not lend abroad but invest in bubbles. Hence, r_t^{μ} satisfies:

$$\mu_t = 1 - \frac{K_t^{\mu} - B_t^{w\mu}}{\beta Y_t^{\mu}} = 1 - \frac{s\left(1-\frac{\alpha\phi\theta}{r^w}\right)}{1-\frac{\alpha\phi\theta}{r^w} - \frac{\alpha(1-\phi)\theta}{r_t^{\mu}}},\ \text{i.e. } r_t^{\mu} = \frac{(1-\mu_t)\alpha(1-\phi)\theta}{(1-s-\mu_t)\left(1-\frac{\alpha\phi\theta}{r^w}\right)}. \tag{A.13}$$

On the other hand, when $\mu \leq L^g(\theta)$, L-types lend some of their net worth abroad and $r_t = r^w$. Since $Y_{t+1}^{\mu} = \alpha K_t^{\mu} - r^w B_t^w - r^w B_t^{w\prime} + Q_{t+1}X$, using the international borrowing constraint (A.10), the economic growth rate under financial globalization can be written as:

$$\frac{Y_{t+1}^{\mu}}{Y_t^{\mu}} = \begin{cases} \beta[\alpha H^g(\theta) + \gamma(1 - H^g(\theta))] + [\beta(\alpha - \gamma)H^g(\theta) - (1-\beta)r^w]\dfrac{Q_t X}{Y_t^{\mu}}, & \text{if } \mu_t \leq L^g(\theta), \\ \beta\bar{r} - (1-\beta)\bar{r}\dfrac{Q_t X}{Y_t^{\mu}}, & \text{if } \mu_t \geq L^g(\theta), \end{cases} \tag{A.14}$$

where $H^g(\theta) = \frac{s}{1-\frac{\alpha\theta}{r^w}}$ and $\bar{r} = \frac{\alpha(1-\phi\theta)}{1-\frac{\alpha\phi\theta}{r^w}}$. Alternatively, we can derive the growth rate of aggregate net worth as:

$$\frac{A_{t+1}^{\mu}}{A_t^{\mu}} = \begin{cases} \beta\left[\alpha(1 - L^a(\theta)) + \gamma(L^a(\theta) - \mu_t) + \dfrac{Q_{t+1}}{Q_t}\mu_t\right], & \text{if } \mu_t \leq L^a(\theta), \\ \beta\left[\alpha(1-\mu_t) + \dfrac{Q_{t+1}}{Q_t}\mu_t\right], & \text{if } \mu_t \geq L^a(\theta). \end{cases}$$

under financial autarky and:

$$\frac{A_{t+1}^{\mu}}{A_t^{\mu}} = \begin{cases} \beta\left[\alpha(1 - L^g(\theta)) + r^w(L^g(\theta) - \mu_t) + \dfrac{Q_{t+1}}{Q_t}\mu_t\right], & \text{if } \mu_t \leq L^g(\theta), \\ \beta\left[\alpha\dfrac{1-\phi\theta}{1-\frac{\alpha\phi\theta}{r^w}}(1-\mu_t) + \dfrac{Q_{t+1}}{Q_t}\mu_t\right], & \text{if } \mu_t \geq L^g(\theta). \end{cases}$$

under financial globalization. Note that, under the BGP, the growth rates of Y_t^{μ} and A_t^{μ} are equal.

Finally, we consider the dynamics of bubbles. Since $\mu_t = \frac{Q_t X}{\beta A_t^{\mu}}$, μ_t evolves over time according to (46). Aggregating the demand function (13) of bubbles, the rate of return on bubbles can be written as (47), which is strictly greater than r_t^{μ} when $\pi < 1$. The dynamics of bubbles can be written as (48) under financial autarky and:

$$\mu_{t+1} = \begin{cases} \dfrac{\frac{1-s-\mu_t}{\pi(1-s)-\mu_t}}{\beta\left[1 + \frac{\alpha - r^w}{r^w - \alpha\theta}s + \frac{(1-\pi)(1-s)}{\pi(1-s)-\mu_t}\mu_t\right]}\mu_t, & \text{if } \mu_t \leq L^g(\theta), \\ \dfrac{\theta}{\beta}\dfrac{1-\phi}{\pi(1-s)(1-\phi\theta) - (1-\theta)\mu_t}\mu_t, & \text{if } \mu_t \geq L^g(\theta), \end{cases}$$

under financial globalization. We focus on the steady state where the bubble size μ_t is constant over time, i.e. $\mu_t = \mu^*$. The steady-state value μ^* can be expressed as (50) under financial autarky, and:

$$\mu^{g*}(\theta) = \begin{cases} \dfrac{\pi - \frac{1-\pi\beta(1-s)}{\beta s\left[1+\frac{\alpha - r^w}{r^w - \alpha\theta}\right]}}{1 - \frac{1-\pi\beta(1-s)}{\beta s\left[1+\frac{\alpha - r^w}{r^w - \alpha\theta}\right]}}(1-s), & \text{if } \mu_t \leq L^g(\theta), \\ \dfrac{\pi\beta(1-s)(1-\phi\theta) - \theta(1-\phi)}{\beta(1-\theta)}, & \text{if } \mu_t \geq L^g(\theta), \end{cases} \tag{A.15}$$

under financial globalization. Let θ^{gm} be the value of θ which satisfies $\mu^{g*}(\theta) = L^g(\theta)$. We have $\mu^{g*}(\theta) < L^g(\theta)$ if $\theta < \theta^{gm}$ and $\mu^{g*}(\theta) > L^g(\theta)$ if $\theta > \theta^{gm}$.

A.1.5 Effect of Financial Globalization on the Existence of Bubbles

We examine the effect of financial globalization on the existence condition of bubbles. We can prove that, under both financial autarky and globalization, bubbles exist in the intermediate range of financial development.

Proposition A2. *Under financial autarky, bubbles can exist if and only if the following condition is satisfied:*

$$\underline{\theta}^a \equiv \frac{\gamma(1-\pi\beta(1-s)) - \pi s\beta\alpha}{\alpha(1-\pi\beta)} < \theta < \pi\beta(1-s) \equiv \overline{\theta}^a,$$

and under financial globalization, bubbles can exist if and only if the following condition is satisfied:

$$\underline{\theta}^g \equiv \frac{r^w(1-\pi\beta(1-s)) - \pi s\beta\alpha}{\alpha(1-\pi\beta)} < \theta < \frac{\pi\beta(1-s)}{1-(1-\pi\beta(1-s))\phi} \equiv \overline{\theta}^g.$$

Furthermore, under financial globalization, the necessary and sufficient condition for the existence of bubbles is that the economic growth rate under the bubbleless economy, which is consistent with Tirole (1985).

Proposition A3. *Under both financial autarky and globalization, the existence condition of bubbles is satisfied if and only if the growth rate under the bubbleless economy is not lower than the interest rate under the bubbleless economy.*

Finally, we learn from Propositions A2 that globalization increases both upper and lower bounds of the existence condition of bubbles.

Proposition A4. *Under the small open economy, $\underline{\theta}^a < \underline{\theta}^g$ and $\overline{\theta}^a < \overline{\theta}^g$.*

This implies that, in a financially well-developed economy relative to Foreign, financial globalization expands the region of θ where bubbles can exist, while in a financially underdeveloped economy relative to Foreign, financial globalization shrinks the region of θ where bubbles can exist. This is because, as we learned in Proposition A1, in a financially well-developed economy, globalization increases the economic growth rate relative to the interest rate due to capital inflow. Hence, bubbles cannot arise under financial autarky but can arise under financial globalization. On the other hand, in a financially underdeveloped economy, globalization increases the interest rate relative to the growth rate due to capital outflow. Hence, bubbles can arise under financial autarky but cannot arise under financial globalization.

However, this result from small open economy model contradicts the emerging market's experience, because in reality, emerging countries with underdeveloped financial market often experience sudden capital inflows and asset bubbles following capital account liberalization. Proposition 5 shows that, when taking into account the terms-of-trade adjustment, financial globalization expands the existence region of bubbles even if the domestic financial market is underdeveloped relative to the foreign one. This is because, when the financial market is underdeveloped, a high marginal return on investment, which is reflected in better terms of trade, causes capital inflows. This suppresses the interest rate relative to the growth rate.

A.1.6 Effect of Bubbles on Economic Growth

Next, we examine the effect of bubbles on the economic growth rate. Combining (47), (50), (A.12), (A.14), and (A.15), the economic growth rate in the bubble economy becomes:

$$g^{\mu a}(\theta) = \begin{cases} \beta\gamma\left[1 + \dfrac{\alpha-\gamma}{\gamma-\alpha\theta}s + \dfrac{\pi - x^a}{x^a}(1-s)\right], & \text{if } \underline{\theta}^a \le \theta < \theta^{am}, \\ \alpha\dfrac{\beta[1-\pi(1-s)] + (1-\beta)\theta}{1-\pi\beta(1-s)}, & \text{if } \theta^{am} \le \theta < \overline{\theta}^a, \end{cases} \tag{A.16}$$

under financial autarky, and:

$$g^{\mu g}(\theta) = \begin{cases} \beta r^w\left[1 + \dfrac{\alpha - r^w}{r^w - \alpha\theta}s + \dfrac{\pi - x^g}{x^g}(1-s)\right], & \text{if } \underline{\theta}^g \le \theta < \theta^{gm}, \\ \alpha\dfrac{\beta[1-\pi(1-s)] + [1-\beta-\{1-\pi\beta(1-s)\}\phi]\theta}{[1-\pi\beta(1-s)]\left(1-\frac{\alpha\phi\theta}{r^w}\right)}, & \text{if } \theta^{gm} \le \theta < \overline{\theta}^g, \end{cases} \tag{A.17}$$

where $x^a \equiv \frac{1-\pi\beta(1-s)}{\beta s\left[1+\frac{\alpha-\gamma}{\gamma-\alpha\theta}\right]}$ and $x^g \equiv \frac{1-\pi\beta(1-s)}{\beta s\left[1+\frac{\alpha-r^w}{r^w-\alpha\theta}\right]}$

Then, we consider how financial globalization affects the relative size of crowd-in and crowd-out effects, and how it changes the effect of bubbles on economic growth. We assume (a) $r^w < \beta\alpha$, (b) $\phi > \frac{\alpha - r^w}{\alpha(1-\beta+s\beta)}$ and (c) $\frac{\alpha-\gamma}{r^w-\gamma} > 1 + \frac{1-\beta}{s\beta}$.[47] In this section, we focus on the region $\underline{\theta}^g < \theta < \overline{\theta}^a$, where bubbles can exist under both financial autarky and globalization. We measure the growth effect of bubbles by taking the ratio between the growth rates under financial autarky and globalization. We compare the values of $(g_t^\mu/g_t)^{autarky}$ and $(g_t^\mu/g_t)^{global}$, and examine how their relative values depend on θ. We obtain the following proposition:

Proposition A5. *Assume (a), (b), and (c). Then, there exists $\theta'' \in (\underline{\theta}^g, \theta^{a\prime})$ which satisfies the following. (i) If $\underline{\theta}^g \leq \theta < \theta''$, the growth-enhancing effect of bubbles is stronger under financial autarky, i.e., $(g_t^\mu/g_t)^{global} < (g_t^\mu/g_t)^{autarky}$. (ii) If $\theta'' < \theta \leq \overline{\theta}^a$, the growth-enhancing effect of bubbles is stronger under financial globalization, i.e., $(g_t^\mu/g_t)^{global} > (g_t^\mu/g_t)^{autarky}$.*

Proof. Since $\underline{\theta}^a < \underline{\theta}^g < \theta^{a\prime} < \overline{\theta}^a$, $\frac{g_t^{\mu a}}{g_t^a} > 1$ when $\underline{\theta}^g \leq \theta < \theta^{a\prime}$, while $\frac{g_t^{\mu a}}{g_t^a} < 1$ when $\theta^{a\prime} < \theta < \overline{\theta}^a$. On the other hand, since $\overline{\theta}^a < \theta^{g\prime}$, $\frac{g_t^{\mu g}}{g_t^g} = 1$ when $\theta = \underline{\theta}^g$, and $\frac{g_t^{\mu g}}{g_t^g} > 1$ when $\underline{\theta}^g < \theta \leq \overline{\theta}^a$. Hence, there exist $\theta'' \in (\underline{\theta}^g, \theta^{a\prime})$ which satisfies (i) $\frac{g_t^{\mu g}}{g_t^g} > \frac{g_t^{\mu a}}{g_t^a}$ when $\underline{\theta}^g \leq \theta < \theta''$ and (ii) $\frac{g_t^{\mu g}}{g_t^g} < \frac{g_t^{\mu a}}{g_t^a}$ when $\theta'' < \theta \leq \overline{\theta}^a$. □

Proposition A5 implies that, in a financially well-developed economy, financial globalization strengthens the growth-enhancing effect of bubbles, while in a financially underdeveloped economy, financial globalization mitigates the growth-enhancing effect of bubbles. Intuitively, when θ is high, financial globalization leads to capital inflows. This improves H-types' rate of return on investment relative to L-types' one and increase the demand for bubbles, as discussed in Section 4. Hence, crowd-in effect of bubbles becomes large relative to crowd-out effect. On the other hand, when θ is high, financial globalization leads to capital outflows. This suppresses H-types' rate of return on investment relative to L-types' one and decrease the demand for bubbles. Hence, crowd-out effect of bubbles becomes large relative to crowd-in effect.

We can check numerically that the similar results holds under stochastic bubbles. The parameter values are set as follows: $\beta = 0.98$, $\alpha = 1.1$, $\gamma = 1.0$, $s = 0.18$, $\omega = 0.5$, $\sigma = 2$, $r^w = 1.04$, $\phi = 0.5$, and $\pi = 0.99$. Using (A.12), (A.14), (A.16), and (A.17), we can check that the growth-enhancing effect of bubbles becomes stronger under financial autarky if $\underline{\theta}^g = 0.6258 \leq \theta < \theta'' =$

[47] Under assumption (a), $\underline{\theta}^g < \overline{\theta}^a$ holds so that there exists a region of θ where bubbles can exist under both financial autarky and globalization. Under assumption (b), $\overline{\theta}^a < \theta^{g\prime}$ holds. This ensures that when the degree of financial globalization is sufficiently large, the growth-enhancing effect of bubbles under financial globalization is larger than that under financial autarky when θ is large enough. Under assumption (c), $\underline{\theta}^g < \theta^{a\prime}$ holds so that the growth-enhancing effect of bubbles under financial autarky is larger than that under financial globalization when θ is low enough. These assumptions are satisfied under standard parameter settings.

0.6671, and the growth-enhancing effect of bubbles becomes stronger under financial globalization if $\theta'' < \theta < \overline{\theta} = 0.7956$.

Again, this result does not hold in the two-country model, in which the growth-enhancing effect of bubbles can be stronger even under financially underdeveloped economies following financial globalization.

A.1.7 Effects of Bubbles on Capital Flows

Finally, we examine the effect of bubbles on capital flows. Similarly to Ikeda and Phan (2018), we measure capital flows by the share of H- and L-types' capital inflows out of net savings, expressed by $(B_t^w + B_t^{w\prime})/(\beta Y_t)$. We first consider the economy without bubbles. We use (A.8) to pin down the level of capital flows. When $0 \le \theta < \tilde{\theta}^g$, we have $\frac{K_t}{\beta Y_t} = \frac{s}{1-\frac{\alpha\theta}{r^w}}$ and

$$\frac{B_t^w + B_t^{w\prime}}{\beta Y_t} = \frac{s}{1 - \frac{\alpha\theta}{r^w}} - 1.$$

We can derive $\frac{B_t^w + B_t^{w\prime}}{\beta Y_t} < 0$ if $0 \le \theta < \frac{r^w}{\alpha}(1-s)$, while $\frac{B_t^w + B_t^{w\prime}}{\beta Y_t} > 0$ if $\frac{r^w}{\alpha}(1-s) < \theta < \tilde{\theta}^g$. When $\tilde{\theta}^g \le \theta < 1$, since $B^{w\prime} = 0$, we have $\frac{K_t}{\beta Y_t} = \frac{1}{1-\frac{\alpha\phi\theta}{r^w}}$ and

$$\frac{B_t^w}{\beta Y_t} = \frac{\frac{\alpha\phi\theta}{r^w}}{1 - \frac{\alpha\phi\theta}{r^w}} > 0.$$

This implies that at an aggregate level, capital outflow occurs in a financially underdeveloped economy, while capital inflow occurs in a financially well-developed economy.

Next, we consider the economy with bubbles. When $\underline{\theta}^g < \theta < \overline{\theta}^g$, since $r_t > r^w$ and $K_t' = 0$, aggregating (A.1), we have $B_t^{w\prime} = 0$. Using (A.9) and (A.10), we obtain:

$$\frac{B_t^{\mu w}}{\beta Y_t^{\mu}} = \frac{\frac{\alpha\phi\theta}{r^w}}{1 - \frac{\alpha\phi\theta}{r^w}}(1 - \mu^{g*}(\theta)).$$

where $\mu^{g*}(\theta)$ is obtained in (A.15).

Figure 10 plotted the capital flows under bubbleless and bubble economies. The blue line shows the capital flow under bubbleless economy, and the red line shoes the capital flows under the bubble economy. The positive value implies capital inflows and the negative value implies capital outflows. Figure 10 shows that bubbles increase capital inflows when the economy is financially underdeveloped, while bubbles decrease capital inflows when the economy is financially well-developed. This is because, when the economy is financially underdeveloped, since bubbles increase the rate of return on savings relative to the world interest rate, some L-types stop lending abroad and start to invest in domestic bubbles. Hence, bubbles increase net capital inflows. On the other hand, when the economy is financially well-developed, the higher interest rate increases H-types' borrowing cost and crowds out their investment. This limits H-types' foreign borrowing and reduces capital inflows.

In contrast, under the two-country setup, bubbles can reduce capital inflows even under financially underdeveloped economies, as will be discussed in Appendix A.13. Under the small open economy, L-types lend abroad at higher world interest rate. Hence, they never produce under financial globalization. On the other hand, under the two-country model, when the economy is financially underdeveloped, the Home interest rate can be higher than the Foreign one due to terms of trade effect. Hence, even L-types can borrow abroad to produce. When bubbles exist, L-types stop borrowing abroad to produce and start to invest in domestic bubbles. This implies that bubbles increase the worldwide TFP by limiting capital inflows into inefficient production sectors.[48]

A.2 Maximization Problem for the Entrepreneurs

In this section, we describe the maximization problem faced by the entrepreneurs. We focus on the equilibrium with bubbles because the equilibrium under bubbleless economy can be derived in a similar way. We first set up the Lagrangian:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \Bigg[\sum_{t=0}^{\infty} \beta^t \Big\{ \log c_t^{\mu i} &+ \lambda_t^{\mu i} (e_t^{\mu i} - c_t^{\mu i} - k_t^{\mu i} + b_t^{\mu i} + b_t^{\mu w i} - Q_t x_t^{\mu i}) \\ &+ \psi_t^{\mu w i} (a_t^{\mu i} \phi \theta p_{h,t+1} k_t^{\mu i} - r_t^w b_t^{\mu w i}) + \psi_t^{\mu i} (a_t^{\mu i} \theta p_{h,t+1} k_t^{\mu i} - r_t^w b_t^{\mu w i} - r_t b_t^{\mu i}) \\ &+ \zeta_t^{\mu i} k_t^{\mu i} + \delta_t^{\mu i} x_t^{\mu i} \Big\} \Bigg],\end{aligned}$$

where $\lambda_t^{\mu i}$, $\psi_t^{\mu w i}$, $\psi_t^{\mu i}$, $\zeta_t^{\mu i}$, and $\delta_t^{\mu i}$ are the Lagrangian multipliers on the budget constraint, the international and domestic borrowing constraints, the non-negativity constraint on production, and the short-sale constraint, respectively. Then, we have the first order conditions:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c_t^{\mu i}} &= \frac{1}{c_t^{\mu i}} - \lambda_t^{\mu i} = 0, \\ \frac{\partial \mathcal{L}}{\partial k_t^{\mu i}} &= -\lambda_t^{\mu i} + \beta \mathbb{E}_t \lambda_{t+1}^{\mu i} a_t^i p_{ht+1} + \psi_t^{\mu w i} a_t^i \phi \theta p_{ht+1} + \psi_t^{\mu i} a_t^i \theta p_{ht+1} + \zeta_t^{\mu i} = 0, \\ \frac{\partial \mathcal{L}}{\partial b_t^{\mu w i}} &= \lambda_t^{\mu i} - \beta \mathbb{E}_t \lambda_{t+1}^{\mu i} r_t^w - \psi_t^{\mu w i} r_t^w - \psi_t^{\mu i} r_t^w = 0,\end{aligned}$$

[48]Under the small open economy, since L-types never produce, TFP is always $a_{ht} = \alpha$ after financial globalization, regardless of whether bubbles exist or not. Hence, bubbles do not change the TFP. However, under the two-country model, bubbles can increase the TFP because they reduce capital flows into inefficient production sectors.

$$\frac{\partial \mathcal{L}}{\partial b_t^{\mu i}} = \lambda_t - \beta \mathbb{E}_t \lambda_{t+1}^{\mu i} r_t - \psi_t^{\mu i} r_t = 0, \text{ and}$$
$$\frac{\partial \mathcal{L}}{\partial x_t^i} = -Q_t \lambda_t^{\mu i} + \beta \mathbb{E}_t \lambda_{t+1}^{\mu i} Q_{t+1}^x + \delta_t^{\mu i} = 0,$$

and the complementary slackness conditions:

$$\begin{aligned}
&\lambda_t^{\mu i} \geq 0,\ c_t^{\mu i} + k_t^{\mu i} + b_t^{\mu i} + b_t^{\mu w i} + Q_t x_t^{\mu i} \leq e_t^{\mu i}, \\
&\lambda_t^{\mu i}(e_t^{\mu i} - c_t^{\mu i} - k_t^{\mu i} + b_t^{\mu i} + b_t^{\mu w i} - Q_t x_t^{\mu i}) = 0, \\
&\psi_t^{\mu w i} \geq 0,\ r_t^w b_t^{\mu w i} \leq a_t^{\mu i} \phi \theta p_{h,t+1} k_t^{\mu i},\ \psi_t^{\mu w i}(a_t^{\mu i} \phi \theta p_{h,t+1} k_t^{\mu i} - r_t^w b_t^{\mu w i}), \\
&\psi_t^{\mu i} \geq 0,\ r_t^w b_t^{\mu w i} + r_t b_t^{\mu i} \leq a_t^{\mu i} \theta p_{h,t+1} k_t^{\mu i},\ \psi_t^{\mu i}(a_t^{\mu i} \theta p_{h,t+1} k_t^{\mu i} - r_t^w b_t^{\mu w i} - r_t b_t^{\mu i}) = 0, \\
&\zeta_t^{\mu i} \geq 0,\ k_t^{\mu i} \geq 0,\ \zeta_t^{\mu i} k_t^{\mu i} = 0, \text{ and} \\
&\delta_t^{\mu i} \geq 0,\ x_t^{\mu i} \geq 0,\ \delta_t^{\mu i} x_t^{\mu i} = 0,
\end{aligned}$$

and the transversality conditions for capital, domestic and foreign bond, and bubble assets.[49] Note that the expectational term captures that the bubble-holders take into account the idiosyncratic productivity risk and the possibility where bubbles collapse.

Next, we derive H- and L-types' laws of motion of net worth. Regarding H-types, we focus on the equilibrium where the domestic borrowing constraint is binding, *i.e.,* $\psi_t^{\mu i} > 0$. Moreover, we guess $\delta_t^{\mu i} > 0$ so that H-types do not hold bubbles in equilibrium. (Later, we will check the condition where this holds in equilibrium.) Using $e_{t+1}^{\mu i} = \alpha k_t^{\mu i} - r_t^{\mu} b_t^{\mu i} - r_t^{\mu w} b_t^{\mu w i}$, we can derive the law of motion of H-types' net worth as:

$$e_{t+1}^{\mu i} = \frac{\alpha(1-\theta) p_{ht+1}^{\mu}}{1 - \frac{\alpha\phi\theta}{r_t^{\mu w}} p_{ht+1}^{\mu} - \frac{\alpha(1-\phi)\theta}{r_t^{\mu}} p_{ht+1}^{\mu}} \beta e_t^{\mu i}. \tag{A.18}$$

Regarding L-types, since (i) $r_t^{\mu} \geq \underline{r}_t^{\mu}$ and $r_t^{\mu} \geq r_t^{\mu w}$ hold, (ii) L-types never produce ($k_t^{\mu i} = 0$) when $r_t^{\mu} > \underline{r}_t^{\mu}$, and (iii) the international borrowing constraint (4) binds when $r_t^{\mu} > r_t^{\mu w}$, we obtain the following relationship:

$$\gamma k_t^{\mu i} - r_t^{\mu} b_t^{\mu i} - r_t^{\mu w i} b_t^{\mu w i} = r_t^{\mu}(\beta e_t^{\mu i} - Q_t x_t^i). \tag{A.19}$$

Using (A.19) and $e_{t+1}^{\mu i} = \gamma k_t^{\mu i} - r_t^{\mu} b_t^{\mu i} - r_t^{\mu w} b_t^{\mu w i} + Q_{t+1}^x x_t^i$, the law of motion of L-types' net worth

[49]The transversality condition for bubbles can be written as $\lim_{t\to\infty} \beta^t Q_t x_t^i / c_t^{\mu i} = 0$. Using $c_t^{\mu i} = (1-\beta) e_t^{\mu i}$ and the demand function (13) of bubbles, we can check that this condition is satisfied under the balanced growth path.

can be expressed as:

$$e_{t+1}^{\mu i} = r_t^{\mu}\beta e_t^{\mu i} + \left(\frac{Q_{t+1}}{Q_t} - r_t^{\mu i}\right) Q_t x_t^i. \tag{A.20}$$

Since the entrepreneurs' productivity a_t^i follows an iid process, aggregating (A.18) and (A.20), the law of motion of aggregate net worth can be expressed as (43).

Finally, we derive H- and L-types' demand for bubbles. We first check that H-types do not demand bubbles. We can show that $\delta_t^{\mu i} > 0$ holds for H-types if:

$$\frac{\alpha(1-\theta)p_{ht+1}^{\mu}}{1 - \frac{\alpha\phi\theta}{r_t^{\mu w}}p_{ht+1}^{\mu} - \frac{\alpha(1-\phi)\theta}{r_t^{\mu}}p_{ht+1}^{\mu}} > \pi\frac{Q_{t+1}}{Q_t}.$$

In our numerical example, this condition is satisfied for all pairs of (θ, θ^*) where bubbles can exist in equilibrium. Hence, H-types do not hold bubbles in equilibrium.

Next, we derive L-types' demand function for bubbles. For L-types to buy bubbles in equilibrium, $\delta_t^{\mu i} = 0$ holds. Combining the first order conditions with respect to $c_t^{\mu i}$, x_t^i, and b_t^i, we obtain the Euler equations:

$$\frac{1}{c_t^{\mu i,\pi}} = \pi\beta\frac{Q_{t+1}}{Q_t}\frac{1}{c_{t+1}^{\mu i,\pi}} \text{ and} \tag{A.21}$$

$$\frac{1}{c_t^{\mu i,\pi}} = \pi\beta\frac{r_t^{\mu}}{c_{t+1}^{\mu i,\pi}} + (1-\pi)\beta\frac{r_t^{\mu}}{c_{t+1}^{\mu i,1-\pi}}, \tag{A.22}$$

where $c_{t+1}^{\mu i,\pi} = (1-\beta)(\gamma k_t^{\mu i} - r_t^{\mu} b_t^{\mu i} - r_t^{\mu wi} b_t^{\mu wi} + Q_{t+1}x_t^i)$ and $c_{t+1}^{\mu i,1-\pi} = (1-\beta)(\gamma k_t^{\mu i} - r_t^{\mu} b_t^{\mu i} - r_t^{\mu wi} b_t^{\mu wi})$ are the consumption when the bubbles survive and collapse at date $t+1$, respectively. Combining equations (A.19), (A.21), and (A.22), we can prove that the demand function of bubbles is given by (13).

A.3 Characterization of Balanced Growth Path Equilibrium

A.3.1 Case under Financial Autarky

In this section, we describe the characterization of balanced growth path (BGP) equilibrium. in which the growth rates of all variables are constant. We first begin with the financial autarky case, *i.e.,* $\phi = \phi^* = 0$ so that we can prove the existence of BGP analytically. Especially, we can show that, even if the two countries face different degrees of financial development, the BGP exist if and only if the two countries share the same growth rate. The sketch of proof is as follows.

First, under the stochastic steady state where the bubble size μ_t is constant, we can check that

(i) the aggregate quantities in the Home country $\{C_t^{\mu}, C_t^{\mu\prime}, K_t^{\mu}, K_t^{\mu\prime}, B_t^{\mu}, B_t^{\mu\prime}, M_{ht+1}^{\mu}\}$ share the same growth rate as the aggregate net worth A_t^{μ} in the Home country, that (ii) the aggregate quantities in the Foreign country $\{C_t^{\mu*}, C_t^{\mu*\prime}, K_t^{\mu*}, K_t^{\mu*\prime}, B_t^{\mu*}, B_t^{\mu*\prime}, M_{ht+1}^{\mu*}\}$ share the same growth rate as the aggregate net worth $A_t^{\mu*}$ in the Foreign country, and that (iii) the growth rates of bubble price and aggregate net worth are the same, *i.e.,* $\frac{Q_{t+1}}{Q_t} = \frac{A_{t+1}^{\mu}}{A_t^{\mu}}$.

We next show that the interest rates and the terms of trade in the two countries are constant if and only if the growth rates in the two countries are equalized. Suppose that the growth rates in the two countries are equalized. Using (42) and (47), the growth rate in the Home country can be rewritten as:

$$g_t^{\mu} = \begin{cases} \beta\left[\alpha(1-L^a(\theta)) + \gamma L^a(\theta) + \dfrac{\gamma(1-s)(1-\pi)}{\pi(1-s)-\mu^a(\theta)}\mu^a(\theta)\right]p_{ht+1}^{\mu}, & \text{if } \mu_t \leq L^a(\theta), \\ \beta\alpha\left[1 + \left(\dfrac{\theta(1-\mu^a(\theta))}{\pi(1-s)-\mu^a(\theta)} - 1\right)\mu^a(\theta)\right]p_{ht+1}^{\mu}, & \text{if } \mu_t \geq L^a(\theta), \end{cases}$$

where the bubble size, $\mu_t = \mu^{a*}(\theta)$, is given by (50). The growth rate in the Foreign country is given by $g_t^{\mu*} = \beta a_{ft} p_{ft+1}^{\mu}$. Letting $g_t^{\mu} = g_t^{\mu*}$ and using (40), we can pin down the values of p_{ht+1} and p_{ft+1} uniquely. Hence, the terms of trade in the two countries are constant and so are the interest rates. This also implies that the growth rates of all aggregate quantities are constant and equalized across the two countries, which is consistent with BGP.

On the other hand, suppose that the growth rate in the Home country is larger that in the Foreign country. Since the growth rate of capital and thus aggregate output is equal to that of net worth in each country, we can use (39) to obtain:

$$\frac{g_t^{\mu}}{g_t^{\mu}} = \frac{A_{t+1}^{\mu}/A_t^{\mu}}{A_{t+1}^{\mu*}/A_t^{\mu*}} = \left(\frac{p_{ht+1}^{\mu}/p_{ht}^{\mu}}{p_{ft+1}^{\mu}/p_{ft}^{\mu}}\right)^{-\sigma}.$$

Using $\frac{A_{t+1}^{\mu}}{A_t^{\mu}} > \frac{A_{t+1}^{\mu*}}{A_t^{\mu*}}$ and (10), we learn that the terms of trade is increasing over time in the Home country and decreasing over time in the Foreign country $\left(\frac{p_{ht+1}^{\mu}}{p_{ht}^{\mu}} < 1 < \frac{p_{ft+1}^{\mu*}}{p_{ft}^{\mu*}}\right)$. Since g_t^{μ} and $g_t^{\mu*}$ are linear in p_{ht+1}^{μ} and p_{ft+1}^{μ}, respectively, this implies that the growth rate is increasing over time in the Home country and decreasing over time in the Foreign country. This is inconsistent with BGP. The case where $\frac{A_{t+1}^{\mu}}{A_t^{\mu}} < \frac{A_{t+1}^{\mu*}}{A_t^{\mu*}}$ cam be proven similarly. Hence, the BGP exists if and only if the economic growth rates are equalized between the two countries.

A.3.2 Case under Financial Globalization

Next, we consider the case under financial globalization, *i.e.,* $\phi, \phi^* > 0$. Since the interest rates are determined implicitly to satisfy the international credit market clearing condition, (38), we cannot solve for the model analytically. Instead, we assume that the two countries share the same growth rate and use a computational method to calculate the BGP equilibrium.

To solve the model, we first discretize the parameter space of (θ, θ^*) and for each (θ, θ^*), we use the Matlab's fsolve.m routine to calculate the equilibrium. We solve for the BGP equilibrium where (i) the interest rates and the terms of trade $\left\{r_t^\mu, r_t^{\mu*}, p_{ht+1}^\mu, p_{ft+1}^\mu\right\}$, (ii) the shares of capital out of net worth $\left\{\frac{K_t^\mu}{\beta A_t^\mu}, \frac{K_t^{\mu\prime}}{\beta A_t^\mu}, \frac{K_t^{\mu*}}{\beta A_t^{\mu*}}, \frac{K_t^{\mu*\prime}}{\beta A_t^{\mu*}}\right\}$, (iii) the world economic growth rates $\left\{g_t^\mu\right\}$, (iv) the ratio between Home and Foreign net worth $\left\{\frac{A_t^\mu}{A_t^{\mu*}}\right\}$, (v) the bubble size $\{\mu_t\}$, and (vi) the growth rate of bubbles' price $\left\{\frac{Q_{t+1}}{Q_t}\right\}$ are all constant.

We use equations (32), (33), (34), (35), (36), (37), (38), (39), (40), (45), (47), and (49) to solve the model. Especially, for each (θ, θ^*), we check (i) whether L-types in each country produce or not, (ii) whether the domestic borrowing constraint binds or not for H-types in each country, and (iii) whether the international borrowing constraint binds or not for L-types in each country. We proceed by trial and error. Finally, we check if the short-sale constraint (7) binds for H-types, which is actually satisfied for all (θ, θ^*) in the existence region of bubbles in our numerical example.

A.4 Proof of Proposition 1

If $0 \le \theta < \tilde{\theta}^a$, since $\alpha > \gamma$, (28) implies $\frac{\partial a_{ht}}{\partial \theta} > 0$. By differentiating (30) with respect to θ, we obtain:

$$\frac{\partial p_{ht+1}}{\partial \theta} = -\left[\omega + (1-\omega)\left(\frac{a_{ft}}{a_{ht}}\right)^{\sigma-1}\right]^{\frac{2-\sigma}{\sigma-1}} (1-\omega) a_{ft}^{\sigma-1} a_{ht}^{-\sigma} \frac{\partial a_{ht}}{\partial \theta} < 0.$$

Note that under financial autarky, a_{ft} does not depend on θ. Then, (26) implies $\frac{\partial r_t}{\partial \theta} = \gamma \frac{\partial p_{ht}}{\partial \theta} < 0$. Moreover, by totally differentiating (27) with respect to θ, we obtain:

$$\frac{dg_t}{d\theta} = \beta\omega \left[\omega + (1-\omega)\left(\frac{a_{ft}}{a_{ht}}\right)^{\sigma-1}\right]^{\frac{2-\sigma}{\sigma-1}} \frac{\partial a_{ht}}{\partial \theta} > 0.$$

If $\tilde{\theta}^a \le \theta \le 1$, since $a_{ht} = \alpha$, (27) and (30) imply that p_{ht} is constant, and so is g_t. Moreover, (26) implies that r_t is increasing in θ if $\tilde{\theta}^a \le \theta < \hat{\theta}^a$ and constant if $\hat{\theta}^a \le \theta \le 1$.

A.5 Proof of Proposition 2

We focus on how globalization changes the relative values of the growth and interest rates, g_t and r_t, in order to match them with the existence condition of Home bubbles later in Proposition A.7. Although the model itself cannot be solved explicitly, we can derive the relative values of growth and interest rates under some values of (θ, θ^*).

A.5.1 Case where Home Financial Market is Underdeveloped $(\theta < \theta^*)$

First, we consider the case where the Home country is less financially developed than Foreign country, *i.e.,* $\theta < \theta^*$. First, we consider the parameter region of (θ, θ^*) where $r_t^a > r_t^{a*}$ under financial autarky so that capital flows into the Home country following financial globalization. There are three possible cases regarding the equilibrium behaviors of Home and Foreign L-types and the relative values of r_t and r_t^* under financial globalization.

(2.1-i) When $K_t' > 0,\ K_t^{*\prime} > 0$, and $r_t > r_t^*$, since both Home H- and L-types can borrow abroad to produce, $\{g_t,\ r_t,\ r_t^*,\ p_{ht+1},\ p_{ft+1}\}$ are determined by $r_t = \underline{r}_t,\ r_t^* = \gamma p_{ft+1}$, and equations (10) and (31). Note that $g_t^* = \beta a_{ft} p_{ft}$. Since $\frac{g_t}{r_t} = \frac{g_t}{r_t^*}\frac{r_t^*}{r_t}$, we have:

$$\frac{d\left(g_t/r_t\right)}{d\phi} = -\beta\frac{a_{ft}}{\gamma}\frac{1}{\left(r_t/r_t^*\right)^2}\frac{d\left(r_t/r_t^*\right)}{d\phi}.$$

Note that the ratio between the growth rate and the Foreign interest rate, $\frac{g_t}{r_t^*} = \beta\frac{a_{ft}}{\gamma}$, does not depend on ϕ. Letting $x_t = \frac{p_{ht}}{p_{ft}}$, we have

$$\frac{d\left(r_t/r_t^*\right)}{d\phi} = \frac{\partial\left(r_t/r_t^*\right)}{\partial x_{t+1}}\frac{dx_{t+1}}{d\phi} + \frac{\partial\left(r_t/r_t^*\right)}{\partial\phi}.$$

The first term indicates that capital inflows suppress the rate of return on savings in the Home country relative to the Foreign one due to terms of trade adjustment. The second term indicates that, since the Home entrepreneurs can borrow abroad at a cheaper interest rate, higher ϕ increases the Home interest rate due to the higher leveraged rate of return on investment.

Since $\frac{r_t}{r_t^*} = \frac{\gamma(1-\phi\theta)x_{t+1}}{1-\phi\theta x_{t+1}}$, we obtain $\frac{\partial\left(r_t/r_t^*\right)}{\partial x_{t+1}} = \frac{\gamma(1-\phi\theta)}{(1-\phi\theta x_{t+1})^2} > 0$ and $\frac{\partial\left(r_t/r_t^*\right)}{\partial\phi} = \frac{\gamma\phi x_{t+1}(x_{t+1}-1)}{(1-\phi\theta x_{t+1})^2} > 0$.[50] We prove $\frac{dx_{t+1}}{d\phi} < 0$. Combining $r_t = \underline{r}_t,\ r_t^* = \gamma p_{ft+1}$, and equation (31), the relative terms of trade, x_{t+1}, is pinned down by the quadratic equation:

$$f(x_{t+1}, \phi) = \left[\gamma(1-\phi\theta) + a_{ft}\phi\theta\right]\alpha\phi\theta(1-\theta)x_{t+1}^2$$

[50]Under financial autarky ($\phi = \phi^* = 0$), since $x_t = \frac{p_{ht}}{p_{ft}} = \frac{a_{ft}}{a_{ht}}$, we have $x > 1$ if $\theta < \theta^*$, and $x < 1$ if $\theta > \theta^*$, as long as either the Home or Foreign H-types are financially constrained. We obtain the same result numerically under financial globalization.

$$
\begin{aligned}
&-\big[a_{ft}\phi\theta[\gamma(1-\phi\theta)+\alpha(1-2\theta+\phi\theta)] \\
&\quad+\gamma(1-\phi\theta)\{s\alpha(1-\theta)+(1-s)[\gamma(1-\phi\theta)-\alpha(1-\phi)\theta]\}\big]x_{t+1} \\
&+a_{ft}\big[\gamma(1-\phi\theta)-\alpha(1-\phi)\theta\big]=0.
\end{aligned}
$$

By the implicit function theorem, we obtain:

$$
\frac{dx_{t+1}}{d\phi}=-\frac{\partial f/\partial\phi}{\partial f/\partial x_{t+1}}.
$$

Since it is difficult to evaluate this terms of trade effect analytically, we focus on the case where $\theta=0$ and $\phi=0$. First, by differentiating $f(x_{t+1},\phi)$ with respect to x_{t+1} and evaluating it at $\theta=0$, we obtain:

$$
\left.\frac{\partial f}{\partial x_{t+1}}\right|_{\theta=0}=-\gamma[s\alpha+(1-s)\gamma]<0.
$$

Next, we prove $\frac{\partial f}{\partial\phi}<0$. To this end, we first observe that:

$$
\frac{\partial f}{\partial\phi}=(2\theta a\phi+b)\theta, \tag{A.23}
$$

where:

$$
\begin{aligned}
a&=[\alpha(a_{ft}-\gamma)(1-\theta)x_{t+1}-(\alpha-\gamma)a_{ft}+\gamma(1-s)(\alpha-\gamma)]x_{t+1},\ \text{and} \\
b&=\alpha\gamma(1-\theta)x_{t+1}^2-a_{ft}[\gamma+\alpha(1-2\theta)]x+\gamma[s\alpha(1-\theta)+(1-s)(\gamma-\alpha\theta)]x_{t+1} \\
&\quad-\gamma(1-s)(\alpha-\gamma)x+a_{ft}(\alpha-\gamma).
\end{aligned}
$$

We first prove $b<0$ when $\theta=0$ and $\phi=0$. Note that $a_{ht}=s\alpha+(1-s)\gamma$ when $\theta=0$. Then, $b<0$ is equivalent to:

$$
a_{ft}>a_{ht}. \tag{A.24}
$$

This implies that the productivity in the Foreign country is greater than the Home one. Since (A.24) is equivalent to $\theta^*>\theta=0$, we learn $b<0$ always holds when $\theta=0$ and $\phi=0$. Then, by continuity, (A.23) implies $\frac{\partial f}{\partial\phi}<0$ for θ and ϕ small enough. Combining these results, we obtain $\frac{dx_{t+1}}{d\phi}<0$. Intuitively, the first term is positive since capital inflow dampens the terms of trade and thus the interest rate in the Home country. On the other hand, the second term is positive since Home L-types can borrow abroad at cheaper world interest rate and enjoy higher leveraged return on investment, which is reflected to higher interest rate. This implies that $\frac{d(g_t/r_t)}{d\phi}<0$ as long as the former effect dominates the latter. Taking $\phi > 0$, we can numerically check that $\frac{d(g_t/r_t)}{d\phi} < 0$ always holds if $\theta < \theta^*$.

(2.1-ii) When $K_t' > 0,\ K_t^{*\prime} = 0$, and $r_t > r_t^*$, we cannot solve for r_t analytically because the Foreign interest rate, r_t^*, is determined implicitly to satisfy the international credit market clearing condition (20) in equilibrium and the Home interest rate, $r_t = \underline{r}_t$, depends on r_t^*. However, we can numerically check that, due to the terms of trade adjustment, the increase in g_t is greater than that in r_t.

(2.1-iii) When $K_t' > 0,\ K_t^{*\prime} = 0$, and $r_t = r_t^*$, we have $r_t = \gamma p_{ht+1},\ g_t = \beta a_{ht} p_{ht+1}$. We learn that $\frac{g_t}{r_t} = \beta \frac{a_{ht}}{\gamma}$ does not depend on ϕ. Hence, an increase in ϕ does not change the relative values of g_t and r_t.

Next, we consider the parameter region of (θ, θ^*) where $r_t^a < r_t^{a*}$ under financial autarky so that capital flows into the Foreign country following financial globalization. There are four possible cases as follows.

(2.1-iv) When $K_t' > 0,\ K_t^{*\prime} = 0$, and $r_t = r_t^*$, an increase in ϕ does not change the relative values of g_t and r_t, similarly to case (2.1-iii).

(2.1-v) When $K_t' > 0,\ K_t^{*\prime} = 0$, and $r_t < r_t^*$, we have $r_t = \gamma p_{ht+1}$ and $g_t = \beta a_{ht} p_{ht+1}$. Hence, an increase in ϕ does not change the relative values of g_t and r_t.

(2.1-vi) When $K_t' = 0,\ K_t^{*\prime} = 0$, and $r_t < r_t^*$, similarly to case (2.1-i), the interest rates are determined implicitly in equilibrium. We can numerically check that the increase in r_t is greater than that in g_t.

(2.1-vii) When $K_t' = 0, K_t^{*\prime} = 0$, and $r_t = r_t^*$, when H-types are constrained, the interest rates are determined implicitly in equilibrium. We can numerically check that the increase in r_t is greater than that in g_t following financial globalization, since capital outflows occur to satisfy $r_t = r_t^*$. When H-types are unconstrained, we have $g_t = \beta \alpha p_{ht+1}$ and $r_t = \alpha p_{ht+1}$. We learn that $\frac{g_t}{r_t} = \beta \frac{\alpha}{\gamma}$ does not depend on ϕ. Hence, an increase in ϕ does not change the relative values of g_t and r_t.

Finally, the region of (θ, θ^*) where $r_t^a = r_t^{a*} = \alpha$ under financial autarky corresponds to case (2.1-vii) so that an increase in ϕ does not change the relative values of g_t and r_t.

A.5.2 Case where Home Financial Market is Well-developed ($\theta > \theta^*$)

We then consider the case where $\theta > \theta^*$. First, we consider the parameter region of (θ, θ^*) where $r_t^a < r_t^{a*}$ under financial autarky.

(2.2-i) When $K_t' > 0,\ K_t^{*\prime} > 0$, and $r_t < r_t^*$, we have $r_t = \gamma p_{ht+1}$ and $g_t = \beta a_{ht} p_{ht+1}$. Hence, globalization does not change the relative values of g_t and r_t.

(2.2-ii) When $K_t' > 0,\ K_t^{*\prime} > 0$, and $r_t < r_t^*$, the interest rates are determined implicitly in equilibrium. We can numerically check that the increase in r_t is greater than that in g_t.

(2.2-iii) When $K_t' = 0,\ K_t^{*\prime} > 0$, and $r_t = r_t^*$, we have $g_t = g_t^* = \beta a_{ft} p_{ft+1}$ and $r_t = r_t^* = \gamma p_{ft+1}$. Hence, globalization does not change the relative values of g_t and r_t.

Next, we consider the parameter region of (θ, θ^*) where $r_t^a > r_t^{a*}$ under financial autarky.

(2.2-iv) When $K_t' = 0,\ K_t^{*\prime} > 0$, and $r_t = r_t^*$, globalization does not change the relative values of g_t and r_t, similarly to case (2.2-iii).

(2.2-v) When $K_t' = 0,\ K_t^{*\prime} > 0$, and $r_t > r_t^*$, the Home interest and growth rates are respectively given by:

$$r_t = \begin{cases} \frac{\alpha(1-\phi)\theta p_{ht+1}}{(1-s)\left(1-\frac{\alpha\phi\theta}{r_t^*}p_{ht+1}\right)}, & \text{if Home H-types are constrained, and} \\ \frac{\alpha(1-\phi\theta)p_{ht+1}}{1-\frac{\alpha\phi\theta}{r_t^*}p_{ht+1}}, & \text{if Home H-types are unconstrained,} \end{cases}$$

and

$$g_t = \beta\frac{\alpha(1-\phi\theta)p_{ht+1}}{1-\frac{\alpha\phi\theta}{r_t^*}p_{ht+1}}.$$

When Home H-types are unconstrained,

$$\frac{g_t}{r_t} = \beta(1-s)\frac{1-\phi\theta}{(1-\phi)\theta}.$$

Hence,

$$\frac{d\left(g_t/r_t\right)}{d\phi} = \beta(1-s)\frac{1-\theta}{\theta(1-\phi)^2} > 0.$$

This implies that globalization depresses the interest rate relative to the growth rate due to capital inflow. On the other hand, when Home H-types are unconstrained, $\frac{g_t}{r_t} = \beta$. Hence, globalization does not change the relative values of g_t and r_t^*.

(2.2-vi) When $K_t' = 0,\ K_t^{*\prime} > 0$, and $r_t > r_t^*$, the interest rates are determined implicitly in equilibrium. We can numerically check that the increase in g_t is greater than that in r_t when Home H-types are constrained but that the relative values of g_t and r_t does not change when Home H-types are unconstrained.

(2.2-vii) When $K_t' = 0, K_t^{*\prime} = 0$, and $r_t = r_t^*$, as in case (2.1-vii), when H-types are constrained, the interest rate r_t becomes high relative to g_t following financial globalization. On the other hand, when H-types are unconstrained, globalization does not change the relative values between g_t and r_t.

Finally, under the region of (θ, θ^*) where $r_t^a = r_t^{a*} = \alpha$ under financial autarky corresponds to case (2.2-vii) so that globalization does not change the relative values between g_t and r_t.

From these observations, we learn financial globalization increases the growth rate relative to the Home interest rate when the Home interest rate is higher than the Foreign one under the financial autarky, while globalization decreases the growth rate relative to the Home interest rate when the Home interest rate is lower than the Foreign one under financial autarky.

Figure 11 shows the numerical results. In the red region, the increase in g_t is greater than that in r_t following financial globalization, while in the green region, the increase in g_t is smaller than the increase in r_t. In the yellow region, globalization does not change the relative values of g_t and r_t. We can also observe that the increase in g_t is greater than that in r_t when θ is either sufficiently high or low and θ^* is in the middle range, and vice versa when θ is in the middle range and θ^* is either sufficiently high or low.

A.6 Proof of Proposition 5

For bubbles to exist in equilibrium, the following two conditions must be satisfied: (i) the interest rate must be between H- and L-types' return on investment ($\underline{r}_t^\mu \leq r_t^\mu < \overline{r}_t^\mu$) and (ii) the bubble size must be strictly positive ($\mu_t > 0$). We consider three cases where the Foreign financial development, θ^*, is in the high, middle, and low ranges.

A.6.1 Case where Foreign financial market is well-developed ($\theta^* \geq \overline{\theta}^a$)

(5-i) When $K_t^{\mu*\prime} = 0$ and $r_t^\mu < r_t^{\mu*}$, the interest and growth rates can be respectively given by:

$$r_t^{\mu*} = \begin{cases} \dfrac{\alpha(1-\phi^*)\theta^* p_{f\,t+1}^\mu}{(1-s)\left(1-\frac{\alpha\phi^*\theta^*}{r_t^\mu}p_{f\,t+1}^\mu\right)}, & \text{if Foreign H-types are constrained, and} \\ \dfrac{\alpha(1-\phi^*\theta^*)p_{f\,t+1}^\mu}{1-\frac{\alpha\phi^*\theta^*}{r_t^\mu}p_{f\,t+1}^\mu}, & \text{if Foreign H-types are unconstrained,} \end{cases}$$

$$g_t^\mu = \beta\left[s\frac{\alpha(1-\theta)p_{ht+1}^\mu}{1-\frac{\alpha\theta}{r_t^\mu}p_{ht+1}^\mu} + (1-s)r_t^\mu\right] = \beta\frac{\alpha(1-\phi^*\theta^*)p_{ft+1}^\mu}{1-\frac{\alpha\phi^*\theta^*}{r_t^\mu}p_{ft+1}^\mu} = g_t^{\mu*}, \text{ and } g_t^\mu = r_t^\mu.$$

Hence, $r_t^\mu \geq \underline{r}_t^\mu$ if $\theta \geq \frac{\gamma(1-\beta(1-s))-s\beta\alpha}{\alpha(1-\beta)} \equiv \underline{\theta}^g = \underline{\theta}^a$, which implies that globalization does not change the lower bound of θ where bubbles can exist. Next, we consider the value of μ_t. Let $x_t = \frac{p_{ht}^\mu}{p_{f\,t}^\mu}$. Since $g_t = g_t^*$ in equilibrium, we have

$$x_{t+1} = [\beta + (1-\beta)\phi^*\theta^*]\,\frac{1-\beta(1-s)}{(1-\beta)\theta + s\beta}.$$

From (32) and (37), H-types' investment (measured in terms of net savings) can be expressed as:

$$\frac{K_t^{\mu}}{\beta A_t^{\mu}} = \frac{s}{1 - \frac{\alpha\theta}{r_t^{\mu}} p_{ft+1}^{\mu}} \text{ and } \frac{K_t^{\mu*}}{\beta A_t^{\mu*}} = \frac{1}{1 - \frac{\alpha\phi^*\theta^*}{r_t^{\mu}} p_{ft+1}^{\mu}}.$$

Using (23), the ratio of Home and Foreign net worth, $\zeta_t^{\mu} \equiv \frac{A_t^{\mu}}{A_t^{\mu*}}$, can be expressed as:

$$\zeta_t^{\mu} = \frac{(K_t^{\mu*}/\beta A_t^{\mu*})}{(K_t^{\mu}/\beta A_t^{\mu})} \frac{\omega}{1-\omega} x_{t+1}^{1-\sigma}.$$

From (38), we obtain the bubble size:

$$\begin{aligned} \mu_t &= \left(1 - \frac{K_t^{\mu}}{\beta A^{\mu}}\right) + \frac{1}{\zeta_t^{\mu}} \left(1 - \frac{K_t^{\mu*}}{\beta A_t^{\mu*}}\right) \\ &= \frac{1}{\beta(1-\theta)} \left[\beta(1-s) - \theta - (1-\beta(1-s))\phi^*\theta^* \frac{1-\omega}{\omega} x_{t+1}^{\sigma-2}\right]. \end{aligned}$$

Hence, $\mu_t > 0$ as long as:

$$\theta < \beta(1-s) - (1-\beta(1-s))\phi^*\theta^* \frac{1-\omega}{\omega} x_{t+1}^{\sigma-2} \equiv \overline{\theta}^g.$$

Comparing this result with Proposition 3, we learn $\overline{\theta}^g < \overline{\theta}^a = \beta(1-s)$, i.e. an increase in ϕ^* reduces the upper bound of θ where bubbles can exist.

(5-ii) When $K_t^{\mu*\prime} = 0$ and $r_t^{\mu} = r_t^{\mu*}$, the interest and growth rates can be expressed as:

$$\begin{aligned} g_t^{\mu} &= \alpha \frac{(1-\beta)\theta + s\beta}{1-\beta(1-s)} p_{ht+1}^{\mu} = r_t^{\mu}, \text{ and} \\ g_t^{\mu*} &= \alpha \frac{(1-\beta)\theta^* + s\beta}{1-\beta(1-s)} p_{ft+1}^{\mu} = r_t^{\mu*}. \end{aligned}$$

By the same discussion as case (5-i), $\underline{\theta}^g = \underline{\theta}^a$. Moreover, $\mu_t > 0$ as long as:

$$\theta < \beta(1-s) + \frac{\beta(1-s) - \theta^*}{(1-\beta)\theta^* + s\beta} \frac{(1-\beta)\theta + s\beta}{\beta(1-\theta)} \frac{1-\omega}{\omega} x_{t+1}^{\sigma-1} \equiv \overline{\theta}^g. \quad \text{(A.25)}$$

Since $\theta^* \geq \overline{\theta}^a = \beta(1-s)$ by assumption, the second term of (A.25) is negative. Hence, we have $\overline{\theta}^g < \overline{\theta}^a$.

A.6.2 Case where Foreign financial market development is in the middle range ($\underline{\theta}^a \leq \theta^* < \overline{\theta}^a$)

(5-iii) When $K_t^{\mu*\prime} = 0$ and $r_t^{\mu} = r_t^{\mu*}$, $\underline{\theta}^g = \underline{\theta}^a$ as in case (5-i). Moreover, $\mu_t > 0$ if (A.25) holds. Since $\theta^* < \overline{\theta}^a = \beta(1-s)$ by assumption, the second term of (A.25) is positive. Hence, we have $\overline{\theta}^g > \overline{\theta}^a$.

(5-iv) When $K_t^{\mu*\prime} = 0$ and $r_t^{\mu} > r_t^{\mu*}$, bubbles exist if the following conditions are satisfied: $\underline{r}_t^{\mu} \leq r_t^{\mu} \leq \overline{r}_t^{\mu}$, $g_t^{\mu} = r_t^{\mu}$, $\mu_t > 0$, and equations (40) and (45). Since the international borrowing constraint (32) is binding, we obtain $\mu_t > 0$ if $\theta < \overline{\theta}^g$, where:

$$\overline{\theta}^g = \frac{\beta(1-s)}{1-(1-\beta(1-s))\phi} > \beta(1-s) = \overline{\theta}^a.$$

Hence, we have $\overline{\theta}^g > \overline{\theta}^a$. We cannot derive $\underline{\theta}^g$ analytically since the Foreign interest rate, $r_t^{\mu*}$, is determined implicitly to satisfy the international credit market clearing condition (38), and the Home interest rate, $r_t^{\mu} = \underline{r}_t^{\mu}$, depends on $r_t^{\mu*}$. However, we can numerically check that $\underline{\theta}^g < \underline{\theta}^a$ holds.

(5-v) When $K_t^{\mu*\prime} > 0$ and $r_t^{\mu} > r_t^{\mu*}$, bubbles exist if the following conditions are satisfied: $\underline{r}_t^{\mu} \leq r_t^{\mu} \leq \overline{r}_t^{\mu}$, $r_t^{\mu*} = \gamma p_{ft+1}^{\mu}$, $g_t^{\mu} = r_t^{\mu}$, $\mu_t > 0$, and equations (40) and (45). Note that $g_t^{\mu*} = \beta a_{ft} p_{ft+1}^{\mu}$. We can prove $\overline{\theta}^g > \overline{\theta}^a$ similarly to case (5-iv). To see $\underline{\theta}^g < \underline{\theta}^a$, we will prove that there exists $\underline{\theta}^g \in (0, \underline{\theta}^a)$ such that $r_t^{\mu} > \underline{r}_t^{\mu}$ for all $\theta \in (\underline{\theta}^g, \overline{\theta}^g)$.[51] By direct calculation, we can show that $r_t^{\mu} > \underline{r}_t^{\mu}$ holds if:

$$\begin{aligned} f(\theta,\phi) = &\{(1-\beta(1-s))(a\phi^2 + b\phi) - \alpha(1-\beta)^2\}\theta^2 \\ &+ \{(1-\beta(1-s))c\phi + [\gamma(1-\beta(1-s)) - 2s\beta\alpha](1-\beta)\}\theta \\ &+ s\beta[\gamma(1-\beta(1-s)) - s\beta\alpha] < 0, \end{aligned}$$

where:

$$\begin{aligned} a &= (1-\beta(1-s))(\alpha-\gamma)\left(\beta\frac{a_f}{\gamma} - 1\right), \\ b &= (1-\beta)\left[\alpha - \gamma - \alpha\left(\beta\frac{a_f}{\gamma} - 1\right) + \beta a_f\right], \text{ and} \\ c &= [\gamma(1-\beta(1-s)) - s\beta\alpha]\left(\beta\frac{a_f}{\gamma} - 1\right) - (1-\beta)\beta a_f + s\beta(\alpha-\gamma). \end{aligned}$$

We first observe $f(0,\phi) = s\beta[\gamma(1-\beta(1-s)) - s\beta\alpha] > 0$ for all ϕ by assumption. Next, we prove

[51] Here, we prove that the lower bound $\underline{\theta}^g$ of the existence region of bubbles under globalization is smaller than autarky. However, it is difficult to derive analytically the parameter conditions where $\underline{\theta}^g$ is decreasing in ϕ.

$f(\underline{\theta}^a, \phi) < 0$ for all ϕ. To prove this, we first obtain $f(\underline{\theta}^a, 0) = 0$. Using this, we learn that, for all $\phi \in (0, 1)$, $f(\underline{\theta}^a, \phi) = f(\underline{\theta}^a, \phi) - f(\underline{\theta}^a, 0) < 0$ is equivalent to:

$$\tilde{f}(\underline{\theta}^a, \phi) = [(a\phi + b)\underline{\theta}^a + c]\phi < 0, \tag{A.26}$$

Note that, since $\theta^* > \frac{\gamma(1-\beta(1-s))-s\beta\alpha}{\alpha(1-\beta)} = \underline{\theta}^a$, we have $a_f \equiv s\frac{\alpha(1-\theta^*)}{1-\frac{\alpha\theta^*}{\gamma}} + (1-s)\gamma > \frac{\gamma}{\beta}$. This implies $a > 0$ so that the quadratic function $\tilde{f}(\underline{\theta}^a, \phi)$ is a U-shaped parabola with respect to ϕ. Moreover, $\tilde{f}(\underline{\theta}^a, 0) = 0$. Hence, in order to prove (A.26) holds for all $\phi \in (0, 1)$, it is sufficient to prove $\tilde{f}(\underline{\theta}^a, 1) = (a + b)\underline{\theta}^a + c < 0$. This condition is equivalent to $a_f > \frac{\gamma}{\beta}$, which holds if $\theta^* > \underline{\theta}^a$. Therefore, there exists $\underline{\theta}^g \in (0, \underline{\theta}^a)$ such that $r_t^\mu > \underline{r}_t^\mu$ for all $\theta \in (\underline{\theta}^g, \overline{\theta}^g)$. This implies $\underline{\theta}^g < \underline{\theta}^a$.

A.6.3 Case where Foreign financial market is underdeveloped ($\theta^* < \underline{\theta}^a$)

(5-vi) When $\theta^* < \underline{\theta}^a$, $\underline{\theta}^g = \underline{\theta}^a$ as in case (5-i). Although we cannot derive $\overline{\theta}^g$ explicitly, we can numerically check that $\overline{\theta}^g < \overline{\theta}^a$ holds.

From these observations, globalization expands the parameter region of (θ, θ^*) where bubbles can exist if $\underline{\theta}^a \leq \theta^* < \overline{\theta}^a$, while globalization shrinks the parameter region of (θ, θ^*) where bubbles can exist if $\theta^* < \underline{\theta}^a$ or $\theta^* \geq \overline{\theta}^a$.[52]

A.7 Proof of Proposition 6

For each case in Proposition 2, we can check the condition of θ where $g_t \geq r_t$ holds and compare the result with the existence condition of bubbles obtained in Proposition 5.

A.7.1 Case where Home Financial Market is Underdeveloped ($\theta < \theta^*$)

Firstly, the following cases determine the lower bound of θ which satisfies $g_t \geq r_t$.

Under case (2.1-i), we can characterize the value of θ which satisfies $g_t = r_t$ implicitly using the following equations: $r_t = \underline{r}_t$, $r_t^* = \gamma p_{ft+1}$, equations (10) and (31), and $g_t = r_t$. Comparing this with the existence condition of bubbles under case (5-v), we learn that $g_t \geq r_t$ if and only if $\theta \geq \underline{\theta}^g$, where $\underline{\theta}^g$ is obtained in case (5-v).

Under case (2.1-ii), we cannot derive the value of $\underline{\theta}^g$ analytically since the interest rates are determined implicitly in equilibrium. We can numerically check $g_t \geq r_t$ as long as $\theta \geq \underline{\theta}^g$, where $\underline{\theta}^g$ is obtained in cases (5-iv).

[52] We have discussed the case where $\frac{\gamma(1-\beta(1-s))-s\beta\alpha}{\alpha(1-\beta)} > 0$. When $\frac{\gamma(1-\beta(1-s))-s\beta\alpha}{\alpha(1-\beta)} \leq 0$, the condition $r_t^\mu \geq \underline{r}_t^\mu$ is always satisfied since $\theta \geq \underline{\theta}^g = 0$, Hence, bubbles can exist as long as $\theta < \overline{\theta}^g$, where $\overline{\theta}^g > \overline{\theta}^a$ if $\underline{\theta}^a \leq \theta^* < \overline{\theta}^a$, and $\overline{\theta}^g \geq \overline{\theta}^a$ if $\theta^* < \underline{\theta}^a$ or $\theta^* \geq \overline{\theta}^a$.

Under cases (2.1-iii), (2.1-iv), and (2.1-v), we can derive $g_t = r_t$ at $\theta = \underline{\theta}^g = \underline{\theta}^a$. Moreover, as we learned in Proposition 1, g_t is increasing and r_t is decreasing in θ. Hence, $g_t \geq r_t$ as long as $\theta \geq \underline{\theta}^g = \underline{\theta}^a$, where $\underline{\theta}^g$ is obtained in the cases (5-i), (5-ii), and (5-iii).

Next, the following case determines the upper bound of θ which satisfies $g_t > r_t$. Under case (2.1-vi), we cannot derive the value of $\overline{\theta}^g$ analytically. We can numerically check $g_t \geq r_t$ as long as $\theta \leq \underline{\theta}^g$, where $\overline{\theta}^g$ is obtained in case (5-i).

Finally, case (2.1-vii) does not corresponds to any parameter regions where bubbles can exist in equilibrium. Especially, when H-types are unconstrained, $\frac{g_t}{r_t} = \beta < 1$ holds so that bubbles cannot exist.

A.7.2 Case where Home Financial Market is Well-developed ($\theta > \theta^*$)

Firstly, the following case determines the lower bound of θ which satisfies $g_t \geq r_t$ when θ^* is low. Under case (2.2-i), the result is similar to cases (2.1-iii), (2.1-iv), and (2.1-v): $g_t \geq r_t$ as long as $\theta \geq \underline{\theta}^g = \underline{\theta}^a$, where $\underline{\theta}^g$ is obtained in case (5-vi).

Next, the following cases determine the upper bound of θ which satisfies $g_t > r_t$ when θ^* is low ($\theta^* < \underline{\theta}^a$).

Under case (2.2-ii), similarly to case (2.1-ii), we cannot derive the value of $\overline{\theta}^g$ analytically. We can numerically check $g_t > r_t$ as long as $\theta \leq \overline{\theta}^g$, where $\overline{\theta}^g$ is obtained in cases (5-vi).

Under case (2.2-iii) and (2.2-iv), using $a_{ft} = s\frac{\alpha(1-\theta^*)}{1-\frac{\alpha\theta^*}{\gamma}} + (1-s)\gamma$, we can derive $g_t \geq r_t$ if $\theta^* \geq \underline{\theta}^a$, while $g_t < r_t$ if $\theta^* < \underline{\theta}^a$. Hence, the region where $\theta^* \geq \underline{\theta}^a$ corresponds to the existence region of bubbles in case (5-i), while the region where $\theta^* < \underline{\theta}^a$ corresponds to the region where bubbles cannot exist.

Finally, the following cases determine the upper bound of θ which satisfies $g_t > r_t$ when θ^* is high ($\theta^* \geq \underline{\theta}^a$).

Under cases (2.2-v) and (2.2-vi), when H-types are constrained, $g_t > r_t$ as long as $\theta < \frac{\beta(1-s)}{1-(1-\beta(1-s))\phi} = \overline{\theta}^g$, where $\overline{\theta}^g$ is obtained in cases (5-iv) and (5-v). When H-types are unconstrained, $g_t < r_t$ holds so that bubbles cannot exist in equilibrium.

Finally, case (2.2-vii) does not corresponds to any parameter regions where bubbles can exist in equilibrium.

A.8 Proof of Proposition 7

Combining (42) and (50), the growth rates under bubble economy can be simplified as:

$$g_t^\mu \equiv \frac{Y_{t+1}^\mu}{Y_t^\mu} = \beta a_{ht}^\mu p_{ht+1}^\mu = \beta a_{ft} p_{ft+1}^\mu = \frac{Y_{t+1}^{\mu*}}{Y_t^{\mu*}} \equiv g_t^{\mu*}. \tag{A.27}$$

where:

$$a^{\mu}_{ht} = a^{\mu}_{h}(\theta) = \begin{cases} \dfrac{\gamma s \frac{\alpha(1-\theta)}{1-\frac{\alpha\theta}{\gamma}}}{1-\pi\beta(1-s)} \equiv \underline{a}^{\mu}_{h}(\theta), & \text{if } \underline{\theta}^{a} \le \theta < \theta^{am}, \\ \alpha\dfrac{\beta[1-\pi(1-s)]+(1-\beta)\theta}{1-\pi\beta(1-s)} \equiv \overline{a}^{\mu}_{h}(\theta), & \text{if } \theta^{am} \le \theta < \overline{\theta}^{a}, \end{cases} \tag{A.28}$$

where θ^{am} is the value of θ that solves $\underline{a}^{\mu}_{h}(\theta) = \overline{a}^{\mu}_{h}(\theta)$. Note that Home bubbles do not affect the Foreign TFP, a_{ft}. This is because, although Home bubbles affect the terms of trade, this does not change the relative rates of return on savings between Foreign H- and L-types.

Then, we compare the growth rates under bubbleless and bubble economies. Substituting $g_t = \beta a_{ht} p_{ht+1}$ into (30), the growth rate under bubbleless economy can be expressed as:

$$g_t = \beta \left[\omega a_{ht}^{1-\sigma} + (1-\omega) a_{ft}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

Similarly, the growth rate under bubble economy can be expressed as:

$$g^{\mu}_t = \beta \left[\omega (a^{\mu}_{ht})^{1-\sigma} + (1-\omega) a_{ft}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

We learn that $g^{\mu}_t > g_t$ if and only if $a^{\mu}_{ht} > a_{ht}$, while $g^{\mu}_t < g_t$ if and only if $a^{\mu}_{ht} < a_{ht}$. Moreover, comparing (28) and (A.28), we learn that $a^{\mu}_{ht} > a_{ht}$ if and only if $\underline{\theta}^{a} \le \theta < \theta^{a\prime}$ and $a^{\mu}_{ht} < a_{ht}$ if and only if $\theta^{a\prime} < \theta < \overline{\theta}^{a}$, where $\theta^{a\prime}$ is the value of θ that solves:

$$\alpha \frac{s}{1-\frac{\alpha\theta}{\gamma}} + \gamma\left(1 - \frac{s}{1-\frac{\alpha\theta}{\gamma}}\right) = \frac{\gamma s \frac{\alpha(1-\theta)}{1-\frac{\alpha\theta}{\gamma}}}{1-\pi\beta(1-s)}.$$

Hence, $g^{\mu}_t > g_t$ if and only if $\underline{\theta}^{a} \le \theta < \theta^{a\prime}$ and $g^{\mu}_t < g_t$ if and only if $\theta^{a\prime} < \theta < \overline{\theta}^{a}$.

A.9 Effect of bubbles on economic growth when bubbles are stochastic

In Section 4, we learned that, under deterministic bubbles ($\pi = 1$), financial globalization strengthens the growth-enhancing effect of bubbles when θ is either high or low and θ^* is in the middle range. In this section, we describe how assuming stochastic bubbles ($\pi < 1$) affects this result. Figure 12(a) shows the case where $\pi = 0.999$ and Figure 12(b) shows the case where $\pi = 0.99$.[53] Under stochastic bubbles, we have an exceptional case: even if θ is small and θ^* is in the middle range so that capital inflows occur, financial globalization can mitigate the growth-

[53] When bubbles are risky (π is small), their existence region shrinks since the agents are risk-averse.

enhancing effect of bubbles. This is because, under deterministic bubbles, since bubbles increase the return on savings, L-types never produce but either invest in bubbles or lend their savings. On the other hand, under stochastic bubbles, since agents are risk-averse, L-types invest some of their net worth in capital and demand less bubbles even if bubbles increase the return on savings. When capital inflows occur, L-types borrow abroad to increase their production, which reduces their demand for bubbles. This mitigates the crowd-in effect of bubbles. When π is close to one, the effect of L-types' production becomes smaller. Hence, the region of (θ, θ^*) where crowd-in effect dominates the crowd-out effect becomes similar to the one under deterministic bubbles.

A.10 Derivation of Value Function

A.10.1 Bubbleless Economy

In this section, we describe how to derive the value function of the entrepreneurs. We first derive the value function under bubbleless economy. We focus on welfare of the Home entrepreneurs. Given the optimal decision rules, the value function can be written as:

$$V_t(e_t) = \log(c_t) + \beta[sV_t(e^H_{t+1}) + (1-s)V_t(e^L_{t+1})],$$

where $e^j_{t+1} = R^j_t \beta e_t$ $(j = H, L)$. R^H_t and R^L_t are the rates of return on net savings and e^H_{t+1} and e^L_{t+1} are the net worths of the entrepreneurs when they are H- and L-types at date t, respectively. The detailed expressions for R^H_t and R^L_t will be given below. Moreover, entrepreneurs consume a $1-\beta$ fraction of their net worth in each period, that is, $c^i_t = (1-\beta)e^i_t$, regardless of whether they are H- or L-types.

We guess that the value function $V_t(e_t)$ is a linear function of $\log(e_t)$:

$$V_t(e_t) = W + \vartheta \log(e_t).$$

We can use the method of undetermined coefficients to obtain:

$$W = \frac{1}{1-\beta}\left[\log(1-\beta) + \frac{\beta}{1-\beta}\log\beta + \frac{\beta}{1-\beta}J_0(\theta, \theta^*, \phi, \phi^*)\right] \equiv W(\theta, \theta^*, \phi, \phi^*), \tag{A.29}$$
$$\vartheta = \frac{1}{1-\beta},$$

with

$$J_0(\theta, \theta^*, \phi, \phi^*) = s\log(R^H_t) + (1-s)\log(R^L_t). \tag{A.30}$$

Regarding H-types' return on savings, when $r_t = r_t^w$,

$$R_t^H = \begin{cases} \frac{\alpha(1-\theta)p_{ht+1}}{1-\frac{\alpha\theta}{r_t^w}p_{ht+1}}, & \text{if } r_t < \alpha p_{ht+1}, \text{ and} \\ \alpha p_{ht+1}, & \text{if } r_t = \alpha p_{ht+1}, \end{cases}$$

and when $r_t > r_t^w$,

$$R_t^H = \begin{cases} \frac{\alpha(1-\theta)p_{ht+1}}{1-\frac{\alpha\phi\theta}{r_t^w}p_{ht+1}-\frac{\alpha(1-\phi)\theta}{r_t}p_{ht+1}}, & \text{if } r_t < \frac{\alpha(1-\phi\theta)p_{ht+1}}{1-\frac{\alpha\phi\theta}{r_t^w}p_{ht+1}}, \text{ and} \\ \frac{\alpha(1-\phi\theta)p_{ht+1}}{1-\frac{\alpha\phi\theta}{r_t^w}p_{ht+1}}, & \text{if } r_t = \frac{\alpha(1-\phi\theta)p_{ht+1}}{1-\frac{\alpha\phi\theta}{r_t^w}p_{ht+1}}. \end{cases}$$

For L-types, their rate of return on investment is always equal to the domestic interest rate, *i.e.,* $R_t^L = r_t$, for the same reason as explained in Appendix A.2. Although the value function depends on the factor prices which cannot be solved for explicitly under some (θ, θ^*), we can use a numerical method to pin them down uniquely as functions of θ, θ^*, ϕ, and ϕ^*.

A.10.2 Bubble Economy

Next, we derive the value function under bubble economy. We assume that bubbles only exist in the Home country and derive the value function of the Home entrepreneurs. Given the optimal decision rules, the value function can be written as:

$$V_t^\mu(e_t^\mu) = \log(c_t^\mu) + \beta\pi[sV_t^\mu(e_{t+1}^{\mu H}) + (1-s)V_t^\mu(e_{t+1}^{\mu L})] + \beta(1-\pi)[sV_t^\mu(e_{t+1}^{\mu H}) + (1-s)V_t(e_{t+1}^{L\prime})],$$

where $e_{t+1}^{\mu H} = R_t^{\mu H}\beta e_t^\mu$, $e_{t+1}^{\mu H} = R_t^{\mu H}\beta e_t^\mu$, and $e_{t+1}^{L\prime} = R_t^{L\prime}\beta e_t^\mu$. $R_t^{\mu H}$ is the rate of return on H-projects and $R_t^{\mu L}$ and $R_t^{L\prime}$ are the rates of returns on L-projects at date t when the bubbles survive and collapse at date $t+1$, respectively. The detailed expressions for $R_t^{\mu H}$, $R_t^{\mu L}$, and $R_t^{L\prime}$ will be given below. $e_{t+1}^{\mu H}$, $e_{t+1}^{\mu L}$, and $e_{t+1}^{L\prime}$ are the net worths of the entrepreneurs in the corresponding cases. Note that, if bubbles collapse at date $t+1$, L-types lose their savings they invested at date t. Hence, $e_{t+1}^{\mu L} > e_{t+1}^{L\prime}$. On the other hand, since H-types do not hold bubbles in equilibrium, their rate or return, $R_t^{\mu H}$, is independent of whether bubbles collapse or not.

We guess that the value function $V_t(e_t^\mu)$ is a linear function of $\log(e_t^\mu)$:

$$V_t(e_t^\mu) = W^\mu + \vartheta^\mu \log(e_t^\mu).$$

We can use the method of undetermined coefficients to obtain:

$$\begin{aligned}
W^{\mu} = & \frac{\beta(1-\pi)}{1-\beta\pi}\left[W(\theta,\theta^*,\phi,\phi^*) + \log(1-\beta) + \frac{\beta}{1-\beta}\log\beta \right. \\
& \left. + \frac{\beta}{1-\beta}[\pi J_1(\theta,\theta^*,\phi,\phi^*) + (1-\pi)J_2(\theta,\theta^*,\phi,\phi^*)]\right] \\
& \equiv W^{\mu}(\theta,\theta^*,\phi,\phi^*), \text{ and} \\
\vartheta^{\mu} = & \frac{1}{1-\beta},
\end{aligned}$$

where $W(\theta,\theta^*,\phi,\phi^*)$ is obtained in the bubbleless economy case and

$$\begin{aligned}
J_1(\theta,\theta^*,\phi,\phi^*) &= s\log(R_t^{\mu H}) + (1-s)\log(R_t^{\mu L}), \text{ and} \\
J_2(\theta,\theta^*,\phi,\phi^*) &= s\log(R_t^{\mu H}) + (1-s)\log(R_t^{L\prime})
\end{aligned}$$

Regarding H-types' return on savings, since the domestic borrowing constraint (5) does not bind for H-types when bubbles exist (see footnote 21 in Section 2.2), their return on savings is given by:

$$R_t^{\mu H} = \frac{\alpha(1-\theta)p_{ht+1}^{\mu}}{1 - \frac{\alpha\phi\theta}{r_t^{\mu w}}p_{ht+1}^{\mu} - \frac{\alpha(1-\phi)\theta}{r_t^{\mu}}p_{ht+1}^{\mu}}.$$

On the other hand, regarding L-types, using (47) and (A.20), their return on savings can be written as:

$$R_t^{\mu L} = \frac{r_t^{\mu}\pi(1-s-\mu_t)}{\pi(1-s)-\mu_t}, \text{ and } R_t^{L\prime} = \frac{r_t^{\mu}\pi(1-s-\mu_t)}{1-s}.$$

Hence, the value function for the Home entrepreneurs can be defined as (53). Regarding the value function (54) for the Foreign entrepreneurs, the functional form is the same as the bubbleless case (52) since bubbles do not exist in the Foreign country. However, Home bubbles affect the Foreign country's welfare indirectly by changing the terms of trade and thus the rate of return on savings for the Foreign entrepreneurs.

A.11 Proof of Proposition 8

First, the Foreign entrepreneurs' value function $V_t^*(e_t^*)$ can be expressed as (52), where $W(\theta,\theta^*,\phi,\phi^*)$ and $J_0^*(\theta,\theta^*,\phi,\phi^*)$ can be expressed similarly to (A.29) and (A.30). We learn that $V_t^*(e_t^*)$ is increasing in R_t^{H*} and R_t^{L*}. Moreover, under financial autarky, the rate of return on savings by the Foreign entrepreneurs is given by:

$$R_t^{H*} = \begin{cases} \frac{\alpha(1-\theta^*)p_{ft+1}}{1-\frac{\alpha\theta^*}{\gamma}}, & \text{if } 0 \leq \theta^* < \tilde{\theta}^a, \\ \frac{\alpha(1-\theta^*)p_{ft+1}}{s}, & \text{if } \tilde{\theta}^a \leq \theta^* < \hat{\theta}^a, \text{ and} \\ \alpha p_{ft+1}, & \text{if } \hat{\theta}^a \leq \theta^* \leq 1, \end{cases}$$

and

$$R_t^{L*} = \begin{cases} \gamma p_{ft+1}, & \text{if } 0 \leq \theta^* < \tilde{\theta}^a, \\ \frac{\alpha\theta}{1-s} p_{ft+1}, & \text{if } \tilde{\theta}^a \leq \theta^* < \hat{\theta}^a, \text{ and} \\ \alpha p_{ft+1}, & \text{if } \hat{\theta}^a \leq \theta^* \leq 1, \end{cases}$$

where $\tilde{\theta}^a$ and $\hat{\theta}^a$ are defined in (26). Since R_t^{H*} and R_t^{F*} are increasing in p_{ft+1}, we learn that the Foreign country's welfare, $V_t^*(e_t)$ is increasing in p_{ft+1}.

Next, to examine how bubbles in the Home country affect the terms of trade in the two countries. we combine (A.27) and (40), we obtain:

$$p^{\mu}_{ht+1} = \left[\omega + (1-\omega)\left(\frac{a_{ft}}{a^{\mu}_{ht}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}. \tag{A.31}$$

Comparing (30) and (A.31), we learn that $p^{\mu}_{ht+1} < p_{ht+1}$ if $a^{\mu}_{ht} > a_{ht}$ and $p^{\mu}_{ht+1} > p_{ht+1}$ if $a^{\mu}_{ht} < a_{ht}$. Moreover, (40) implies that $p^{\mu}_{ft+1} > p_{ft+1}$ if $p^{\mu}_{ht+1} < p_{ht+1}$ and $p^{\mu}_{ft+1} < p_{ft+1}$ if $p^{\mu}_{ht+1} > p_{ht+1}$. Hence, $p^{\mu}_{ft+1} > p_{ft+1}$ if $a^{\mu}_{ht} > a_{ht}$ and $p^{\mu}_{ft+1} < p_{ft+1}$ if $a^{\mu}_{ht} < a_{ht}$. Since $V_t^*(e_t)$ is increasing in p_{ft+1}, $V_t^{\mu*}(e_t^{\mu*}) > V_t^*(e_t^*)$ if and only if $\underline{\theta}^a \leq \theta < \theta^{a\prime}$ and $V_t^{\mu*}(e_t^{\mu*}) < V_t^*(e_t^*)$ if and only if $\theta^{a\prime} < \theta < \overline{\theta}^a$.

A.12 Comparative Statistics on the Elasticity of Substitution between the Home and Foreign Intermediate Goods

In this section, we examine how different values on the elasticity of substitution between the Home and Foreign intermediate goods affect the properties of equilibrium. First, we check that the non-monotonicity of interest rate holds regardless of the value of σ. As we showed in Appendix (A.4), r_t is decreasing in θ if $0 \leq \theta < \tilde{\theta}^a$, increasing in θ if $\tilde{\theta}^a \leq \theta < \hat{\theta}^a$ and constant if $\hat{\theta}^a \leq \theta \leq 1$, and this result does not depend on whether σ is larger or smaller than one. Figure 13(a) shows how r_t depends on the value of σ. We plot the cases under $\sigma = 0.5,\ 2$, and 5. We learn that, whether σ is greater or smaller than one, the interest rate is non-monotonic with respect to θ.

Next, we examine how the existence region of bubbles changes depending on the value of σ. We compare with the case under $\sigma = 2$ (low σ) and $\sigma = 20$ (high σ). Figure 13(b) shows the existence region of bubbles under financial globalization with different values of σ. In the red region, bubbles can only exist under low σ, while in the blue region, bubbles can only exist under high σ. In the green region, bubbles can exist under both high and low σ. The important case is the red region. We learn that, as long as both Home and Foreign countries are financially well-developed, an increase in σ shrinks the region of (θ, θ^*) where bubbles can exist. This is because, when σ is high, Home and Foreign goods are highly substitutable, financial globalization promotes reallocation of capital between the two countries through international capital flow. As a result, the interest rates in both countries increase. For the country which experiences capital inflow, interest rate increases since the leveraged rate of return on investment for productive agents increase. For the country which experiences capital outflow, capital outflow itself increases the interest rate. While higher interest rate increases the return on savings for L-types, it decreases the return on investment for H-types due to higher borrowing cost. Hence, the interest rate becomes high relative to the growth rate so that bubbles cannot exist in equilibrium.[54]

We finally check that, under perfect substitution, the equilibrium does not exist in general. To see this, we focus on the case under financial autarky and assume bubbles do not exist. When $\sigma = \infty$, the production function (1) becomes linear:

$$y_t = m_{ht} + m_{ft}.$$

Hence, the profit maximization problem (8) implies $p_{ht} = p_{ft} = 1$. Using (23), we obtain:

$$\frac{\alpha K_t + \gamma K_t'}{\alpha K_t^* + \gamma K_t^{*\prime}} = \frac{\omega}{1-\omega}.$$

Under financial autarky, this is equivalent to:

$$\frac{a_{ht}(\theta)}{a_{ft}(\theta^*)} = \frac{\omega}{1-\omega}\left(\frac{A_t}{A_t^*}\right)^{-1}, \tag{A.32}$$

[54]An exception is that, as indicated by the blue region, as long as the Home or Foreign country is financially well-developed and the other is underdeveloped, an increase in σ shrinks the region of (θ, θ^*) where bubbles can exist. This is because, since capital flows into financially underdeveloped economy, the effect of increase in interest rates is limited. However, when σ is high, both p_{ht} and p_{ft} increase so that equation (10) is satisfied. This increases the growth rate relative to the interest rate so that bubbles cannot exist in equilibrium.

where the TFP is defined as (28). Since A_t and A_t^* are given at date t, (A.32) implies that the equilibrium does not exist unless (θ, θ^*) takes the value so that the ratio between the Home and Foreign TFPs becomes equal to the ratio of production weights times the inverse of the ratio between Home and Foreign net worth. Therefore, we learn that, for an equilibrium to exist, it is essential that each country specializes in producing its own intermediate good and that the prices for intermediate goods are adjusted so that their markets clear.

A.13 Effect of Bubbles on International Capital Flows

While the main text focuses on how financial globalization affects the emergence of bubbles, in this section, we consider how bubbles affect international capital flows. We can also discuss how this change in international capital allocation affects the worldwide production efficiency.

Regarding the bubbleless economy case, Aoki et al. (2007) showed that the effects of financial globalization on capital flows and TFP depend on both absolute and relative degrees of financial development between the two countries. When domestic financial development is sufficiently high compared to the rest of the world, globalization increases TFP, since the Home country provides a better savings vehicle and absorbs inefficient production in the Foreign country. Conversely, when domestic financial development is sufficiently low, globalization decreases TFP because even unproductive agents in the Home country can borrow abroad to produce, which crowds out productive investments in the Home country.

We investigate the effect of bubbles on international capital allocation and TFP. We focus on how bubbles affect borrowing and lending between domestic and foreign entrepreneurs. As in the previous section, we assume bubbles exist only in the Home country and can only be traded domestically.

As in Appendix A.1.7, we measure capital flows by the share of H- and L-types' capital inflows out of the net savings, expressed by $\frac{B_t^w + B_t^{w*}}{\beta A_t}$. Figure 14 compares the amount of capital inflows between financial autarky and globalization. In the red region, the capital inflows into the Home country are larger in the bubble economy, *i.e.,* $\left(\frac{B_t^w + B_t^{w*}}{\beta A_t}\right)^{global} > \left(\frac{B_t^w + B_t^{w*}}{\beta A_t}\right)^{autarky}$, while in the green region, the capital inflows into the Home country are larger in the bubbleless economy, *i.e.,* $\left(\frac{B_t^w + B_t^{w*}}{\beta A_t}\right)^{global} < \left(\frac{B_t^w + B_t^{w*}}{\beta A_t}\right)^{autarky}$. Figure 14 shows that, when θ is high, bubbles increase capital inflows. However, when θ is low and θ^* is in the middle range, bubbles decrease capital inflows.[55]

The intuition for this result is as follows. In our framework, entrepreneurs stop producing and buy bubbles when they are L-types and sell bubbles to invest when they are H-types. On one

[55]When θ is too high, bubbles decrease the capital flows. This is because when θ is high, the crowd-out effect of bubbles dominates the crowd-in effect. Since productive agents in the Home country produce less, the amount of foreign borrowing reduces. However, we do not focus on this parameter region.

hand, when θ is high, since Home bubbles increase the rate of return on savings in the Home country relative to the Foreign one, bubbles increase capital flows toward Home H-types and absorb inefficient production by Foreign L-types. On the other hand, when θ is low, Home L-types can borrow abroad to produce. However, when bubbles exist, L-types stop producing and start investing in bubbles because they provide a better savings vehicle. Hence, bubbles improve worldwide production efficiency by directing capital flows to more efficient production sectors.

Finally, I analyze the effects of bubbles on worldwide TFP under financial globalization. Worldwide TFP is defined by:

$$a_t = \frac{Y_t}{K_t + K_t' + K_t^* + K_t^{*\prime}},$$

where

$$Y_t = \left[\omega^{\frac{1}{\sigma}} (\alpha K_t + \gamma K_t')^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}} (\alpha K_t^* + \gamma K_t^{*\prime})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

is the aggregate output. Figure 15 shows the change in TFP between financial autarky and globalization. We can numerically check that bubbles improve TFP in almost all parameter regions of (θ, θ^*) since bubbles direct capital flows into efficient production sectors.[56]

A.14 Effect of Worldwide Technological Progress

In this section, we discuss how worldwide technological progress affects the existence condition of bubbles. Historically, asset price bubbles often coincide with the periods of technological progress (Scheinkman, 2014). Especially, technological innovation in one country often affects the emergence of bubbles in other countries, as represented by the U.S. dot-com bubble or recent artificial intelligence (AI) bubbles.

In the main text, we assumed that the Home and Foreign countries had the same levels of productivity: $\alpha = 1.1$ and $\gamma = 1.0$. However, we now assume that the productivity of Foreign H-types increases to $\alpha^F = 1.15$, while the productivity of the Home entrepreneurs remains the same. We consider how this change in α in the Foreign country affects the existence region of bubbles in the Home country.

Figure 16 compares the existence conditions of Home bubbles before and after the Foreign technological progress. In the green region, bubbles can exist both before and after the increase in α^F. In the red region, bubbles can only exist after the increase in α^F, while in the blue region, bubbles can only exist before the increase in α^F. We learn that, when the Home financial market

[56]As an exception, bubbles can decrease TFP when θ sufficiently high and θ^* is sufficiently low. This is because Home bubbles crowd out Home H-types' investment and thus reduce capital flows into efficient production sectors.

is sufficiently well- or underdeveloped, an increase in α^F expands the existence region of bubbles in the Home country. Intuitively, since Foreign technological progress depresses the terms of trade in the Foreign country, the terms of trade in the Home country improves and the Home growth and interest rates become high relative to the Foreign one. Since capital flows into the country with higher rate of return on savings, capital flows into the Home country and its interest rate is depressed relative to the growth rate. Hence, bubbles cannot arise under financial autarky but they can arise under financial globalization.[57]

Hirano and Yanagawa (2017) showed that, under closed-economy settings, there is a positive feedback relationship between technological innovation and bubbles. Our result indicates that, under financial globalization, technological progress in one country can also facilitate the emergence of bubbles in other countries, since the effect of productivity increase is transmitted to other countries via general equilibrium effect.

One exception is that, when the Home financial development is in the middle range and the Foreign financial development is sufficiently high, an increase in α^F shrinks the existence region of bubbles in the Home country. This is because higher α^F increases Foreign H-types' borrowing demand and accelerates capital outflow from the Home country. Hence, the Home interest rate increases relative to the growth rate, making it difficult for bubbles to exist.

A.15 International Trading of Bubbles

Here, we allow for international trading of bubbles and discuss their existence condition. We assume that bubbles can arise in both countries and that the Home (Foreign) entrepreneurs can purchase bubbles which arise in the Foreign (Home) country.

For simplicity, we consider the case with deterministic bubbles. Let X be the total supply of bubbles and X^h and X^f the supply of bubbles in Home and Foreign countries, respectively. We assume X, X^h, and X^f are constant over time.[58] Moreover, let Q_t^h and Q_t^f be the prices of bubbles in the Home and Foreign countries, respectively. We define $\mu_{ht} = \frac{Q_t^h X^h}{\beta A_t^\mu}$ and $\mu_{ft} = \frac{Q_t^f X^f}{\beta A_t^{\mu*}}$ as the bubble size in the Home and Foreign countries and let $\mu_t = \frac{Q_t X}{\beta(A_t^\mu + A_t^{\mu*})}$ be the world bubble size. Let $\zeta_t = \frac{A_t^\mu}{A_t^{\mu*}}$ be the relative wealth of the two countries. Using $X = X^h + X^f$, we obtain:

$$\frac{\zeta_t \mu_{ht} + \mu_{ft}}{\zeta_t + 1} = \mu_t.$$

[57]Under financial autarky, Foreign technological progress has no effects on the existence condition of bubbles in the Home country. This is because, while Foreign technological progress affects the terms of trade, this change in terms of trade itself does not affect the relative values of H-types' and L-types' rate of returns on savings. Hence, the relative values of growth and interest rates remain unchanged.

[58]Since we do not impose any restrictions on international trading of bubbles, their disaggregated allocations between the two countries are indeterminate (See Ikeda and Phan, 2018.). We assume both X^h and X^f are fixed exogenously.

The international credit market clearing condition (38) can be rewritten as:

$$\frac{\zeta_t}{\zeta_t+1}\frac{K_t^{\mu}}{\beta A_t^{\mu}}+\frac{1}{\zeta_t+1}\frac{K_t^{\mu*}}{\beta A_t^{\mu*}}+\mu_t=1.$$

We then derive the existence conditions of bubbles in each country. Figures 17(a) and 17(b) show the existence region of bubbles in the Home country and Figures 17(c) and 17(d) show their existence region in the Foreign country. We learn that the effect of financial globalization on bubbles depends on both absolute and relative degrees of financial development in the two countries. When θ is either sufficiently high or low but θ^* is in the middle range, financial globalization expands the existence region of Home bubbles but shrinks that of Foreign bubbles. On the other hand, when θ is in the middle range but θ^* is either sufficiently high or low, globalization shrinks the existence region of Home bubbles but expands that of Foreign bubbles. When both θ and θ^* are in the middle range, bubbles can exist in both countries.

A.16 Case with a Continuum of Productivity

Here, we derive the parameter conditions under which the non-monotonicity of the interest rate with respect to domestic financial development holds even with a continuum of productivity. Here, we focus on the financial autarky case. First, since investment and savings are equalized in the Home country, we have:

$$\int_{\frac{r_t}{p_{ht+1}}}^{\alpha}\frac{1}{1-\frac{\theta a}{r_t}p_{ht+1}}f(a)da=1. \tag{A.33}$$

By totally differentiating (A.33) with respect to θ, we obtain:

$$\frac{dr_t}{d\theta}=\frac{\frac{p_{ht+1}}{r_t}\int_{\frac{r_t}{p_{ht+1}}}^{\alpha}\frac{a}{\left(1-\frac{\theta a}{r_t}p_{ht+1}\right)^2}f(a)da}{\frac{f(r/p_{ht+1})}{1-\theta}\frac{1}{p_{ht+1}}+\frac{\theta p_{ht+1}}{r_t^2}\int_{\frac{r_t}{p_{ht+1}}}^{\alpha}\frac{a}{\left(1-\frac{\theta a}{r_t}p_{ht+1}\right)^2}f(a)da}+\frac{r_t}{p_{ht+1}}\frac{dp_{ht+1}}{d\theta}. \tag{A.34}$$

The first term captures the leverage effect: when θ is high, the borrowing demand by highly productive agents increases so that the interest rate increases. The second term captures the terms-of-trade effect: when θ is high, the high economic growth rate depresses the terms of trade and thus the interest rate. Equation (A.34) implies that $\frac{dr_t}{d\theta} < 0$ if and only if:

$$\frac{dp_{ht+1}}{d\theta} < -\frac{\left(\frac{p_{ht+1}}{r_t}\right)^2 \int_{\frac{r_t}{p_{ht+1}}}^{\alpha} \frac{a}{\left(1-\frac{\theta a}{r_t}p_{ht+1}\right)^2} f(a)da}{\frac{f(r/p_{ht+1})}{1-\theta}\frac{1}{p_{ht+1}} + \frac{\theta p_{ht+1}}{r_t^2}\int_{\frac{r_t}{p_{ht+1}}}^{\alpha} \frac{a}{\left(1-\frac{\theta a}{r_t}p_{ht+1}\right)^2} f(a)da}. \tag{A.35}$$

This implies that the interest rate is a decreasing function of θ when the terms of trade effect is sufficiently larger compared to the leverage effect.

Next, the economic growth rates in the Home and Foreign countries are given by:

$$g_t = \beta p_{ht+1}\int_{\frac{r_t}{p_{ht+1}}}^{\alpha} \frac{a}{1-\frac{\theta a}{r_t}p_{ht+1}} f(a)da, \text{ and } g_t^* = \beta p_{ft+1}\int_{\frac{r_t^*}{p_{ft+1}}}^{\alpha} \frac{a}{1-\frac{\theta^* a}{r_t^*}p_{ft+1}} f(a)da.$$

Since the growth rates are equalized between the two countries, we obtain $\frac{dg_t}{d\theta} = \frac{dg_t^*}{d\theta}$. Furthermore, using price index (10), we have:

$$\frac{dp_{ft+1}}{d\theta} = -\frac{\omega}{1-\omega}\left(\frac{p_{ht+1}}{p_{ft+1}}\right)^{-\sigma}\frac{dp_{ht+1}}{d\theta}.$$

Then, we learn that $\frac{dp_{ht+1}}{d\theta}$ can be expressed as:

$$\begin{aligned}\frac{dg_t}{d\theta} =& \beta\frac{dp_{ht+1}}{d\theta}\int_{\frac{r_t}{p_{ht+1}}}^{\alpha} \frac{a}{1-\frac{\theta a}{r_t}p_{ht+1}} f(a)da + \beta\frac{p_{ht+1}}{r_t}\frac{\frac{f(r/p_{ht+1})}{1-\theta}\int_{\frac{r_t}{p_{ht+1}}}^{\alpha} \frac{a}{\left(1-\frac{\theta a}{r_t}p_{ht+1}\right)^2} f(a)da}{\frac{f(r/p_{ht+1})}{1-\theta}\frac{1}{p_{ht+1}} + \frac{\theta p_{ht+1}}{r_t^2}\int_{\frac{r_t}{p_{ht+1}}}^{\alpha} \frac{a}{\left(1-\frac{\theta a}{r_t}p_{ht+1}\right)^2} f(a)da} \\ =& -\frac{\omega}{1-\omega}\left(\frac{p_{ht+1}}{p_{ft+1}}\right)^{-\sigma}\beta\frac{dp_{ht+1}}{d\theta}\int_{\frac{r_t^*}{p_{ft+1}}}^{\alpha} \frac{a}{1-\frac{\theta^* a}{r_t^*}p_{ft+1}} f(a)da = \frac{dg_t^*}{d\theta}.\end{aligned} \tag{A.36}$$

To capture the intuition, we begin with the analytical explanation before proceeding to numerical experiments. For simplicity, we consider the case where $\theta = \theta^* = 0$, because when $\frac{dr_t}{d\theta} < 0$ at $\theta = 0$, it is likely r_t is decreasing in θ when θ is low. Note that when $\theta = \theta^* = 0$, $r_t = r_t^* = \gamma$ and $p_{ht+1} = p_{ft+1} = 1$. Then, (A.36) implies that:

$$\frac{dp_{ht+1}}{d\theta} = -(1-\omega)\frac{\int_\gamma^\alpha a(a-\gamma)f(a)da}{\gamma\int_\gamma^\alpha af(a)da} < 0. \tag{A.37}$$

Using (A.35) and (A.37), $\frac{dr_t}{d\theta} < 0$ if and only if:

$$(1-\omega)\int_{\gamma}^{\alpha} a(a-\gamma)f(a)da > \frac{1}{\gamma f(\gamma)}\left[\int_{\gamma}^{\alpha} af(a)da\right]^2.$$

We learn that r_t is a decreasing function of θ when (i) α is sufficiently high compared to γ; (ii) the fraction of entrepreneurs with low productivity, $f(\gamma)$, is sufficiently large, but not too large; and (iii) the weight of production in the Home country, ω, is sufficiently small.

Intuitively, when α is relatively large compared to γ, an increase in θ is likely to cause higher economic growth because more resources are allocated to productive investments. This depresses the terms of trade and thus the interest rate. Conversely, when the population share of entrepreneurs with low productivity is large, even if θ increases, the interest rate remains suppressed, since the productivity of marginal investor is low. This strengthens the terms-of-trade effect compared to the direct one so that the interest rate becomes a decreasing function in θ. Furthermore, (A.37) shows that when the weight of production is small in the Home country, the Home intermediate good price decreases strongly in response to the increase in θ, making the terms-of-trade effect relatively large.

Next, we provide a numerical example in which the non-monotonicity of the interest rate holds with a continuum of productivity. Here, we adopt the standard assumption that the productivity follows a power law distribution.[59] As in Hirano and Yanagawa (2017), we consider:

$$f(a) = \chi a^{-\eta-1}.$$

Since $\int_{\gamma}^{\alpha} f(a)da = 1$, we have $\chi = \frac{\eta}{\frac{1}{\gamma^\eta}-\frac{1}{\alpha^\eta}}$. Then, the form of the density function becomes:

$$f(a) = \frac{\eta}{\frac{1}{\gamma^\eta}-\frac{1}{\alpha^\eta}}a^{-\eta-1}, \tag{A.38}$$

Note that, when η is large, the population share of entrepreneurs with low productivity is large. The parameter values are set as follows: $\alpha = 1.2$, $\gamma = 0.4$, $\omega = 0.1$, $\eta = 3$, and $\sigma = 2$. Moreover, we assume deterministic bubbles, *i.e.,* $\pi = 1$. Figure 18(a) represents r_t as a function of θ given $\theta^* = 0.3$ and Figure 18(b) represents r_t^* as a function of θ^* given $\theta = 0.2$.

Figure 18(a) shows that the equilibrium Home interest rate r_t is a decreasing function when θ is low, and increasing function when θ is high. Figure 18(c) shows that, when θ is either sufficiently high or low compared to θ^*, r_t is larger than r_t^* so that capital flows into the Home country following the capital account liberalization.

[59] See, for example, di Giovanni and Levchenko (2012).

We make two remarks on this result. First, as shown in Figure 18(b), for given θ, r_t^* is monotonically increasing with respect to θ^*, since the weight of production in the Foreign country, $1 - \omega$, is large. Although the non-monotonicity of the interest rate does not hold in both countries at the same time, our result implies that emerging countries with relatively tight financial markets and low weights of production can face higher interest rates compared to advanced countries.

Second, (A.36) implies that $\frac{dp_{ht+1}}{d\theta} < 0$ and $\frac{dg_t}{d\theta} > 0$, that is, the Home intermediate good price is decreasing in θ, while the world growth rate is increasing in θ regardless of the parameter values and the distributional form of productivity. Intuitively, an improvement in the financial market allocates more resource to the productive agents and facilitates growth, while it also suppresses the terms of trade and hinders growth. Our result shows that the former effect always dominates the latter so that the world growth rate is increasing in θ.

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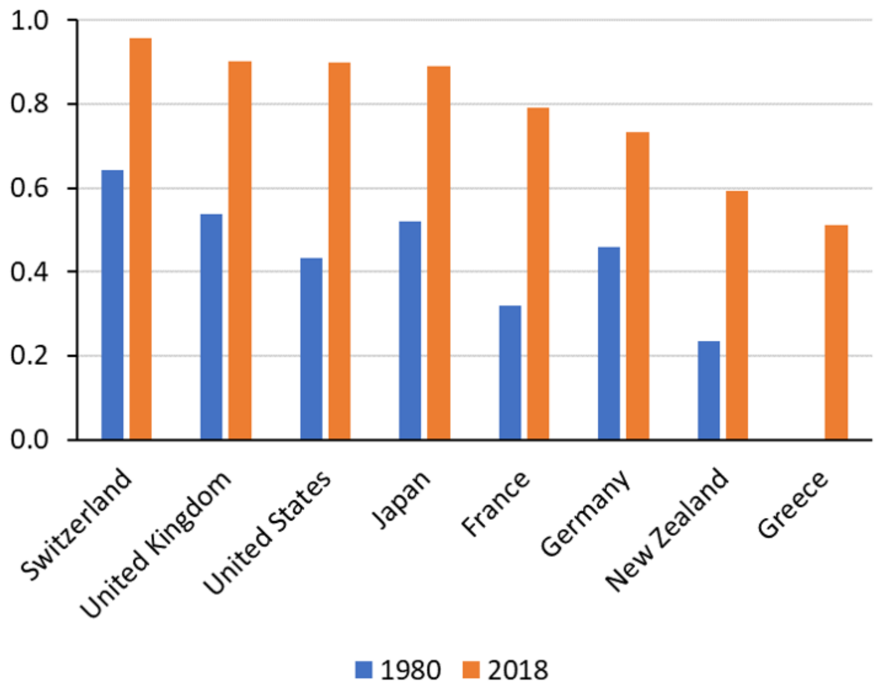
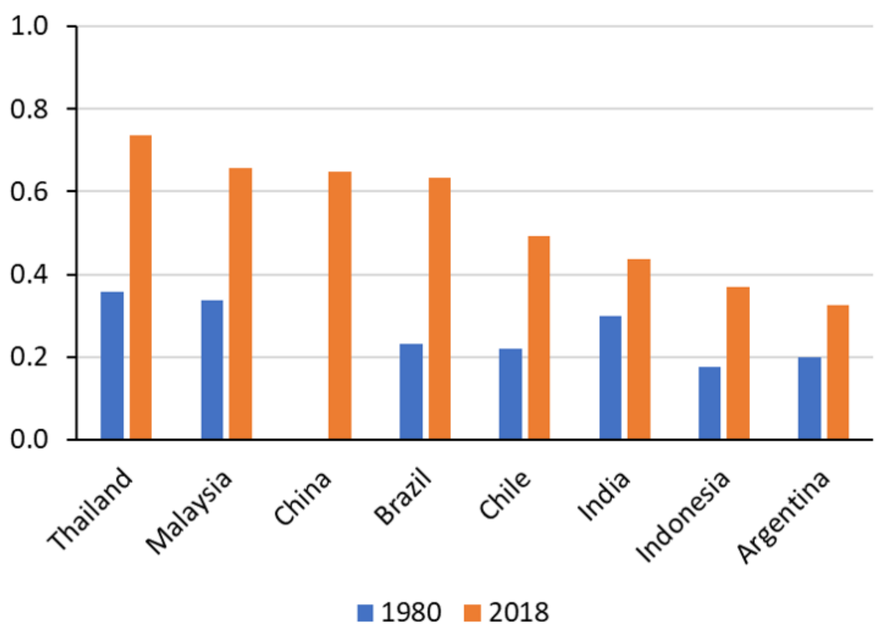
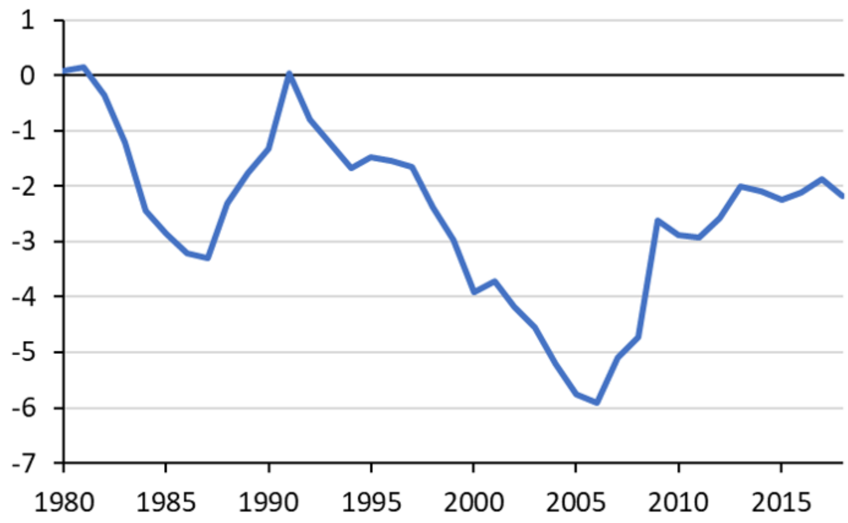


(a) Advanced markets

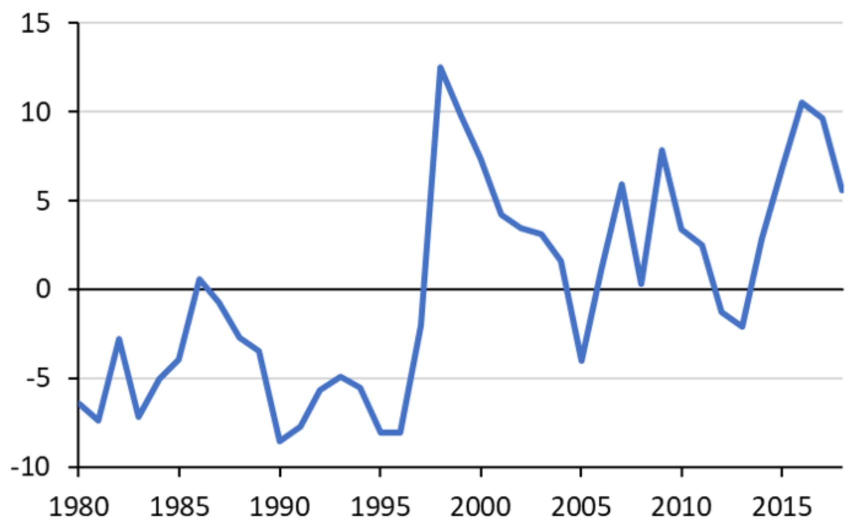


(b) Emerging markets

Figure 1: Financial development index by country (source: IMF).

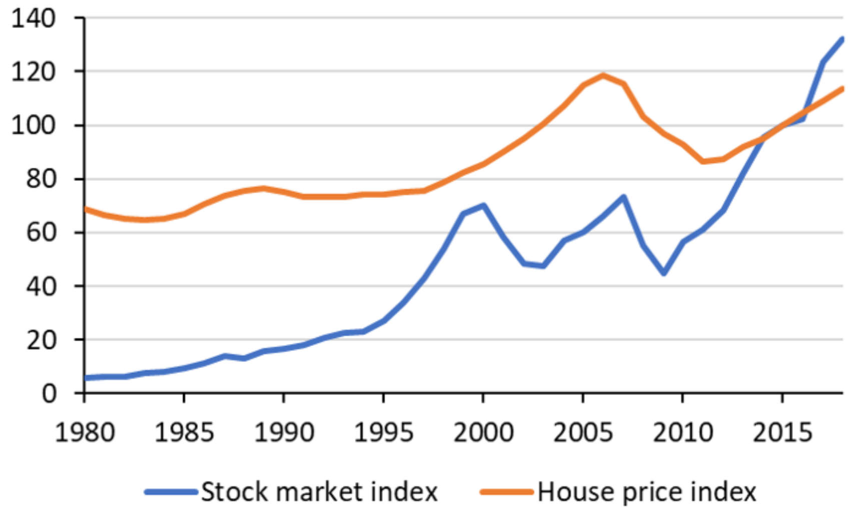


(a) United States

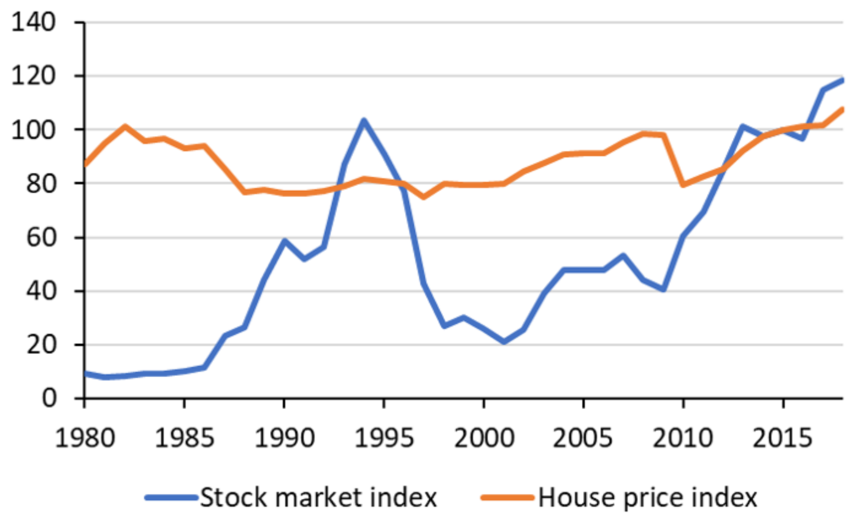


(b) Thailand

Figure 2: Current account as a percentage of GDP (source: World Bank).

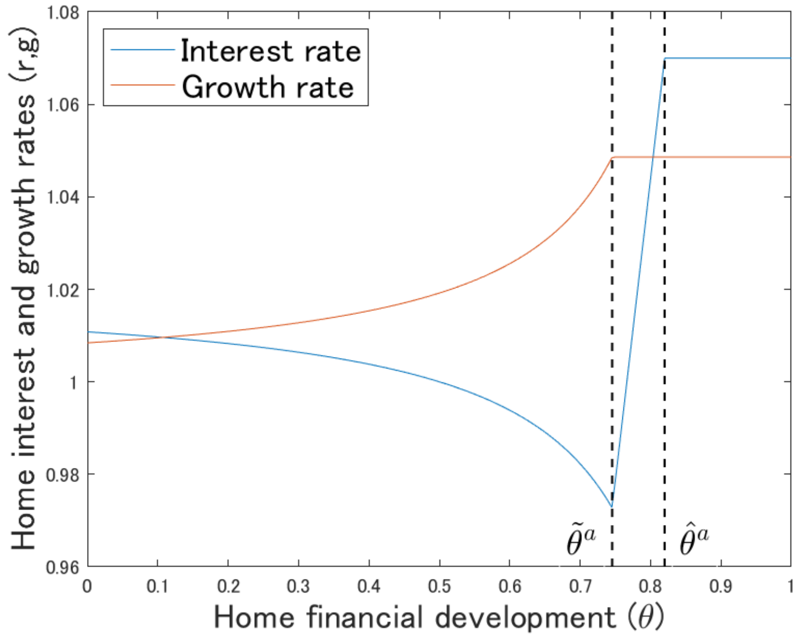


(a) United States

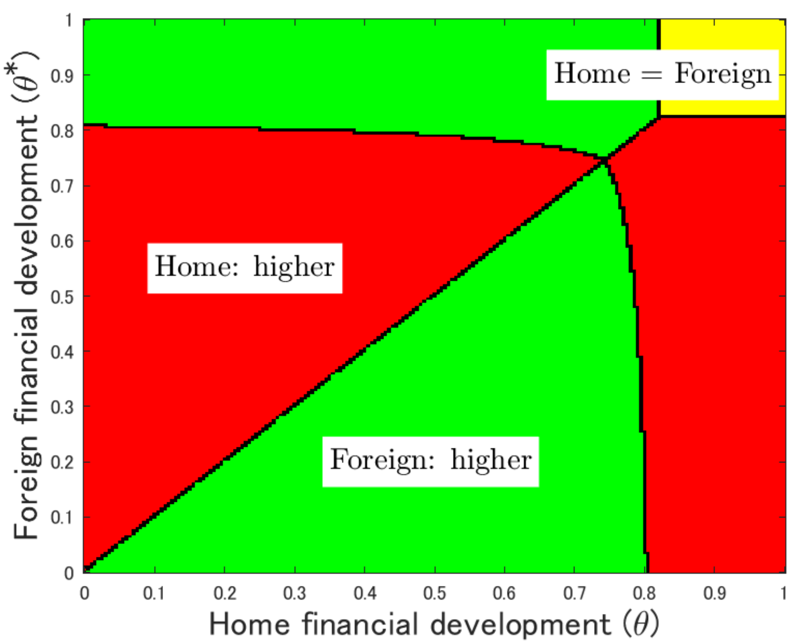


(b) Thailand

Figure 3: Stock market and house price indices. Note: The stock market indices are retrieved from Global Financial Data. We used S&P500 and Cowles composite price index (US) and SET general index (Thailand). The house price indices are retrieved from OECD (US) and BIS (Thailand). The stock market and house price indices are inflation-adjusted.

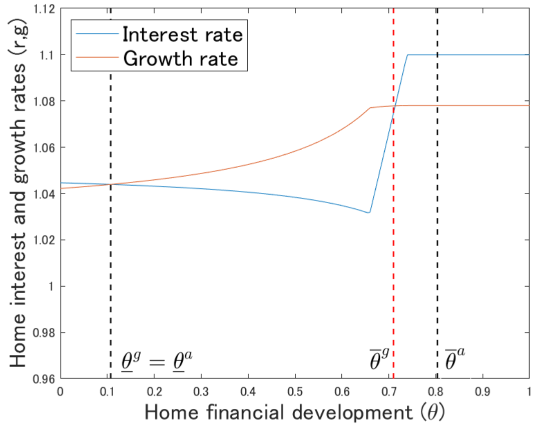


(a) Home interest and growth rates

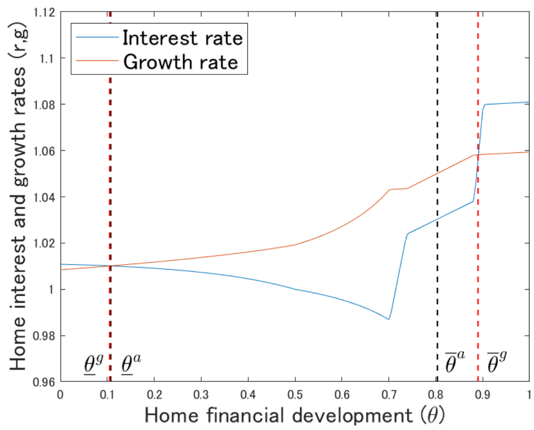


(b) Comparison of Home and Foreign interest rates

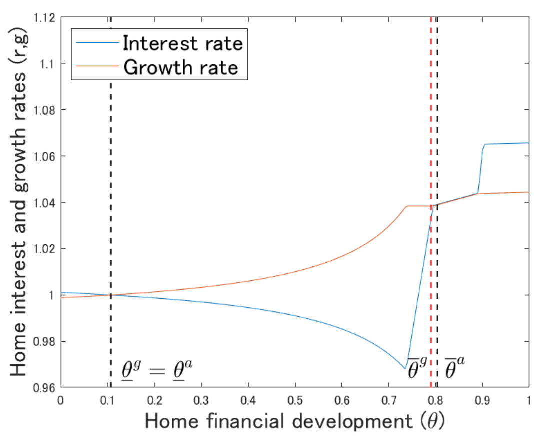
Figure 4: Interest and growth rates under financial autarky.



(a) High θ^* ($\theta^* = 0.9$)



(b) Middle θ^* ($\theta^* = 0.5$)



(c) Low θ^* ($\theta^* = 0.1$)

Figure 5: Interest and growth rates under financial globalization.

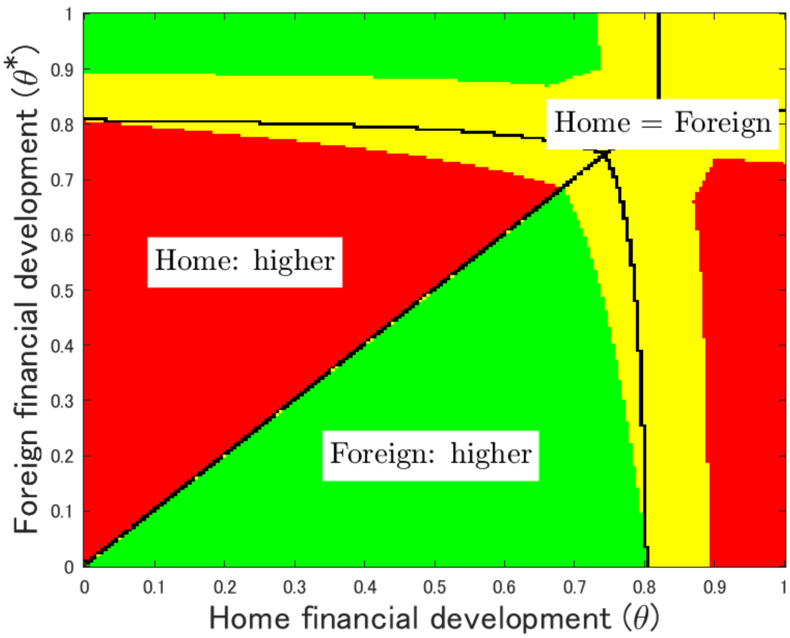
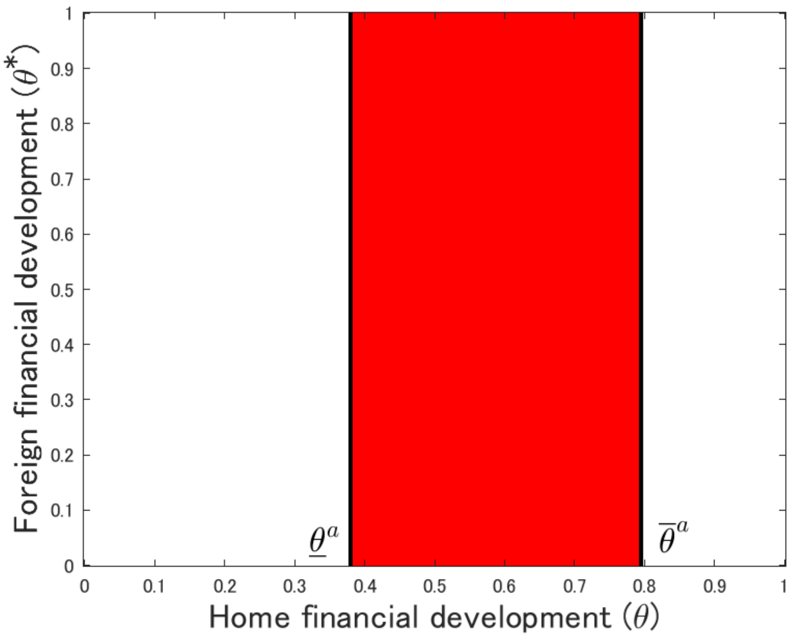
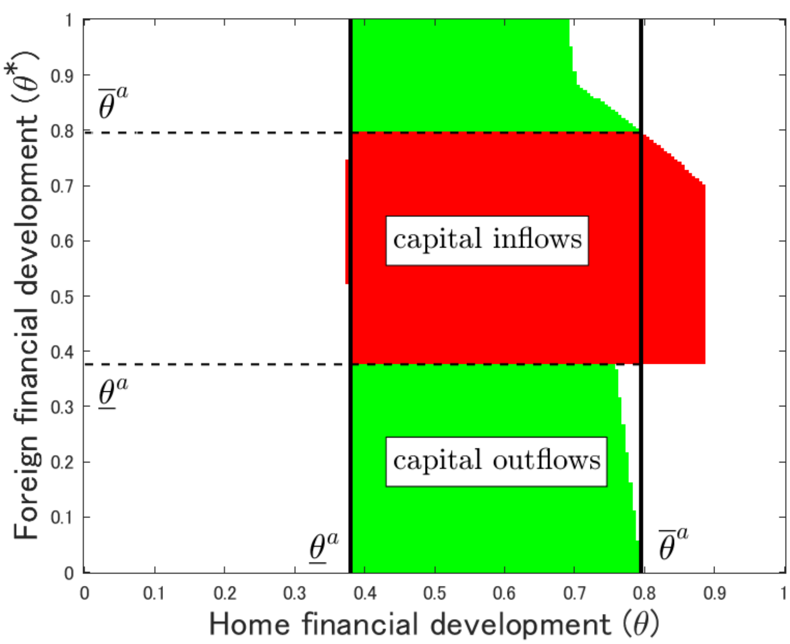


Figure 6: Comparison of Home and Foreign interest rates under financial globalization.



(a) Financial autarky



(b) Financial globalization

Figure 7: Existence conditions of bubbles.

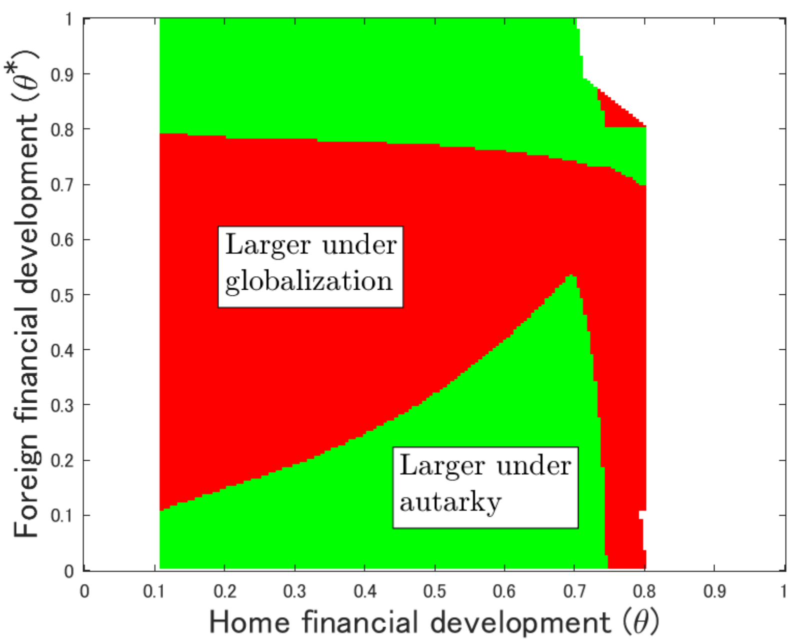
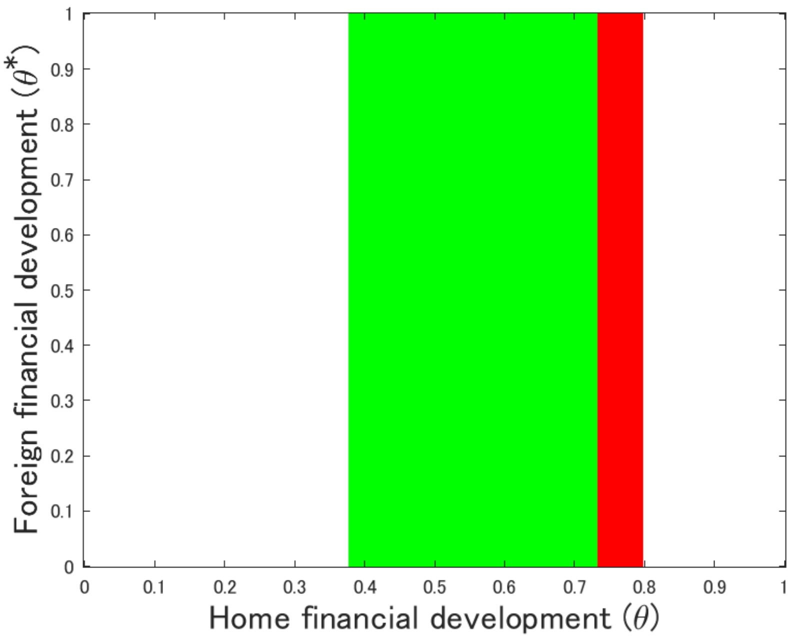
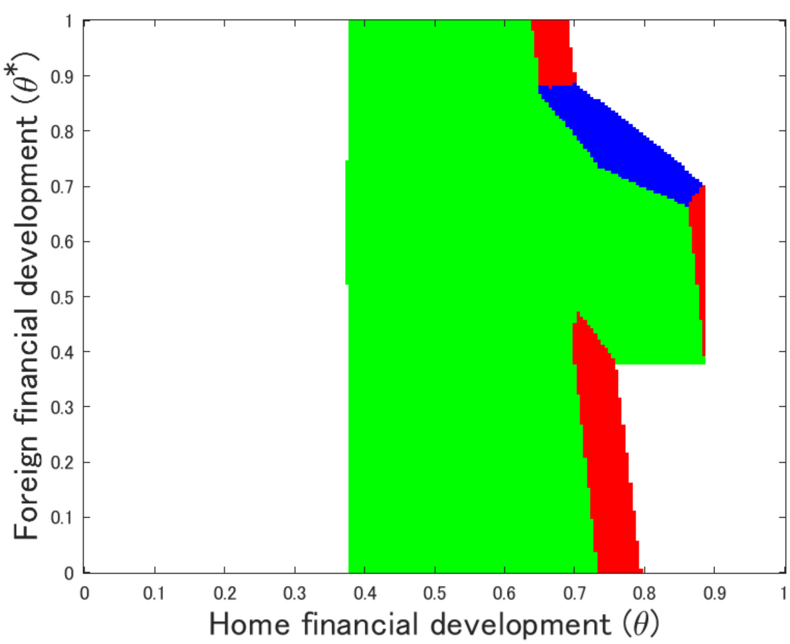


Figure 8: Bubbles' growth-enhancing effect: deterministic bubbles ($\pi = 1$). Note: we only plot the region of (θ, θ^*) where bubble can exist under both financial autarky and globalization.



(a) Financial Autarky ($\phi = \phi^* = 0$)



(b) Financial Globalization ($\phi = \phi^* = 0.5$)

Figure 9: Welfare Effect of Bubbles.
Red: Home bubbles decrease both growth rate and welfare in the Foreign country.
Green: Home bubbles increase both growth rate and welfare in the Foreign country.
Blue: Home bubbles decrease the growth rate but increase welfare in the Foreign country.

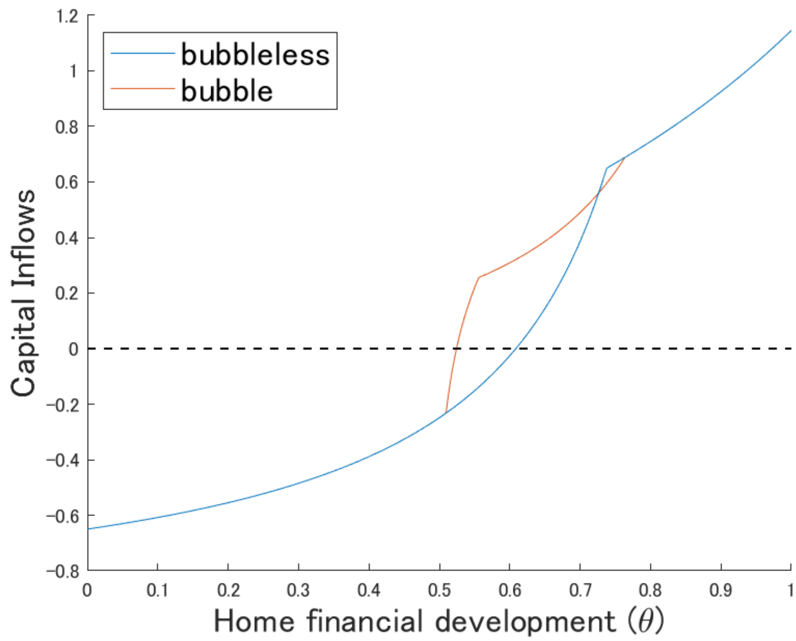


Figure 10: Effect of bubbles on capital inflows (small open economy).

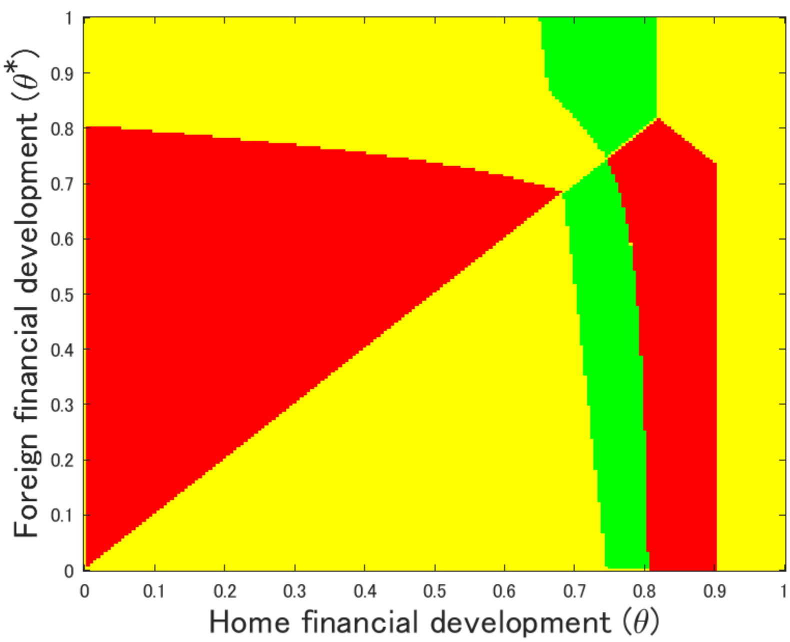
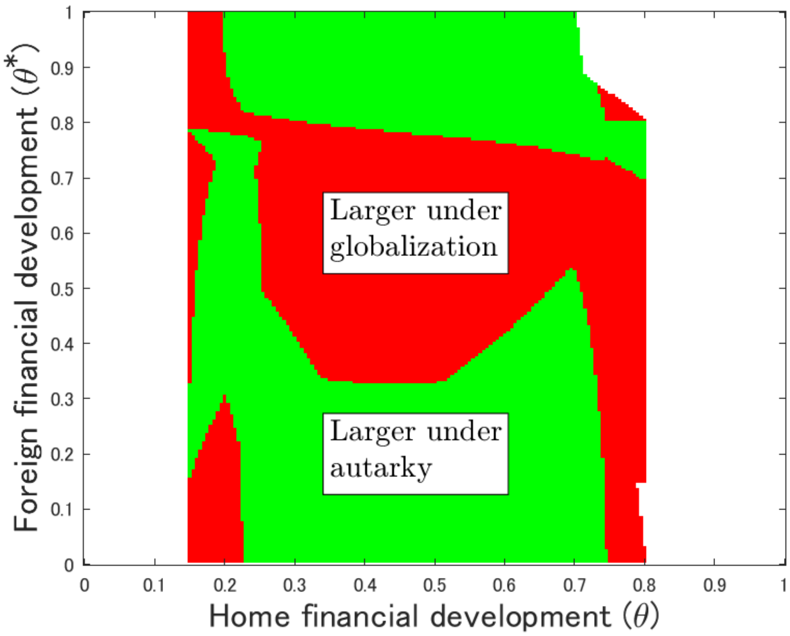
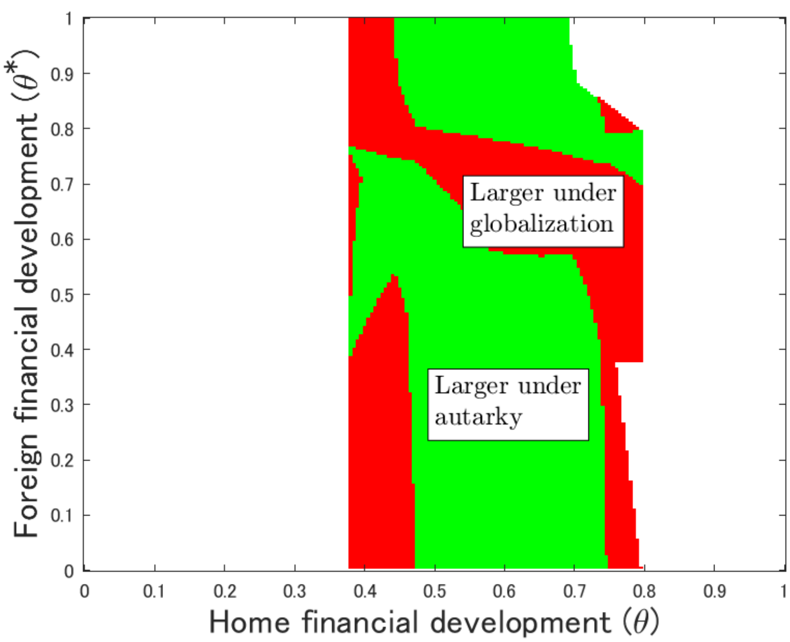


Figure 11: Effect of financial globalization on growth and interest rates.
Red: globalization increases the growth rate relative to the interest rate.
Green: globalization decreases the growth rate relative to the interest rate.
Yellow: globalization does not change the relative size between growth and interest rates.



(a) $\pi = 0.999$



(b) $\pi = 0.99$

Figure 12: Bubbles' growth-enhancing effect: stochastic bubbles ($\pi < 1$).
Note: we only plot the region of (θ, θ^*) where bubble can exist under both financial autarky and globalization.

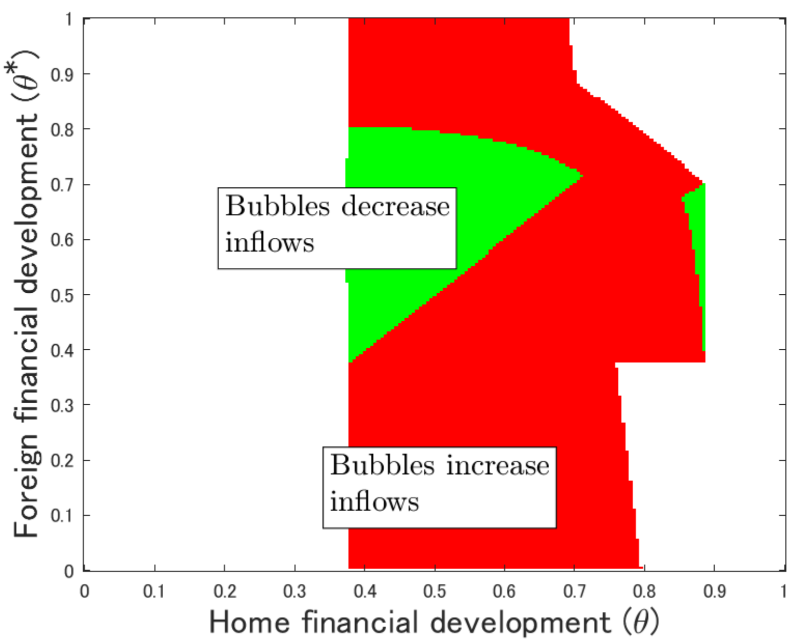
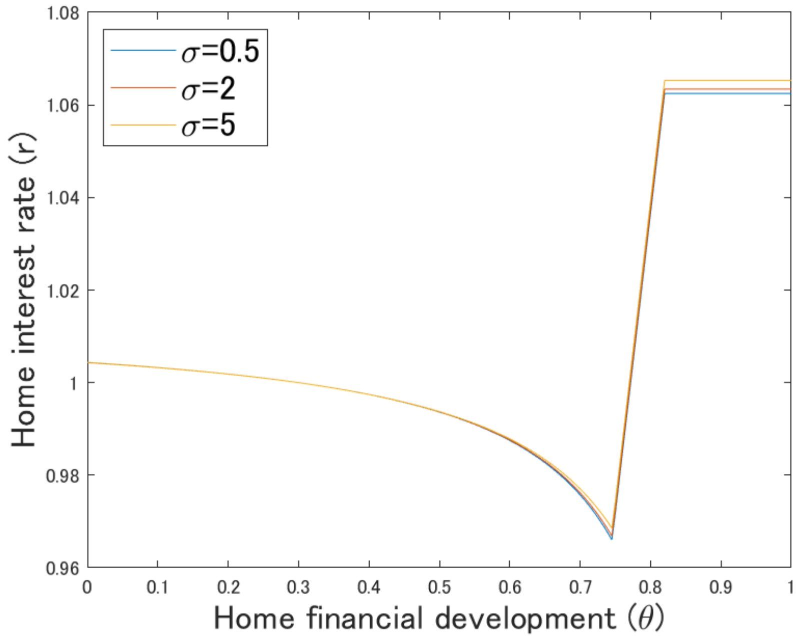
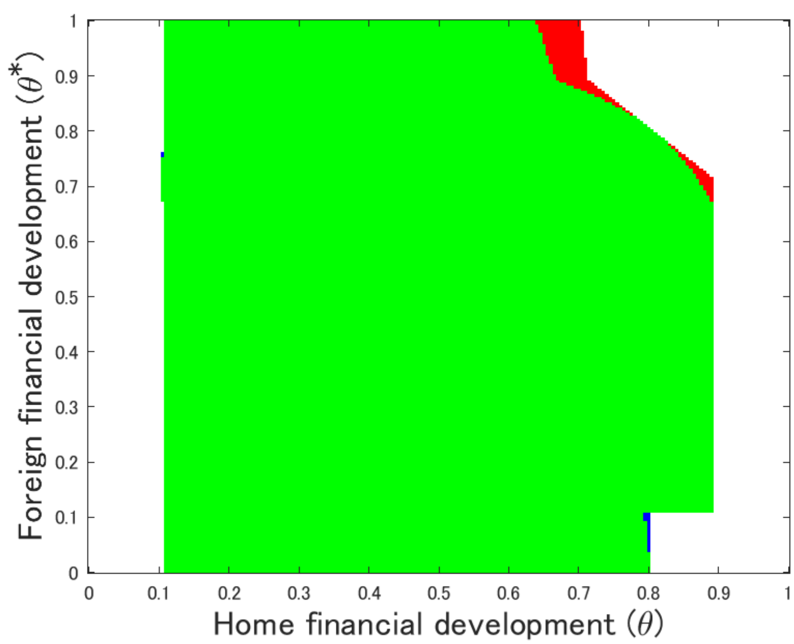


Figure 13: Effect of bubbles on international capital flows (two-country model).
Red: bubbles increase net capital inflows into Home entrepreneurs.
Green: bubbles decrease net capital inflows into Home entrepreneurs.



(a) Home interest rate under different values of σ



(b) Effect of an increase in σ on the existence region of bubbles.
Red: bubbles can only exist under low σ.
Blue: bubbles can only exist under high σ.
Green: bubbles can exist under both low and high σ.

Figure 14: Comparative statics on the values of σ

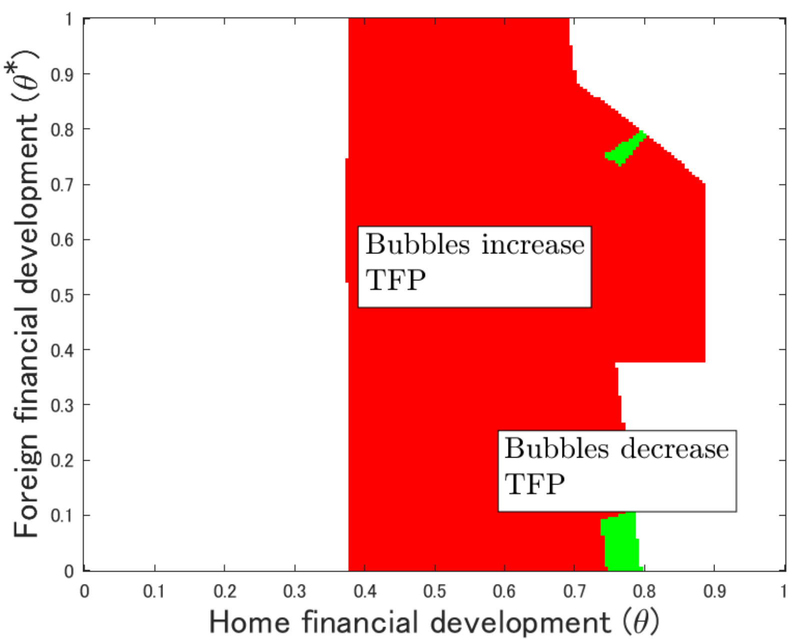


Figure 15: Effect of bubbles on total factor productivity.
Red: Bubbles increase TFP.
Green: Bubbles decrease TFP.

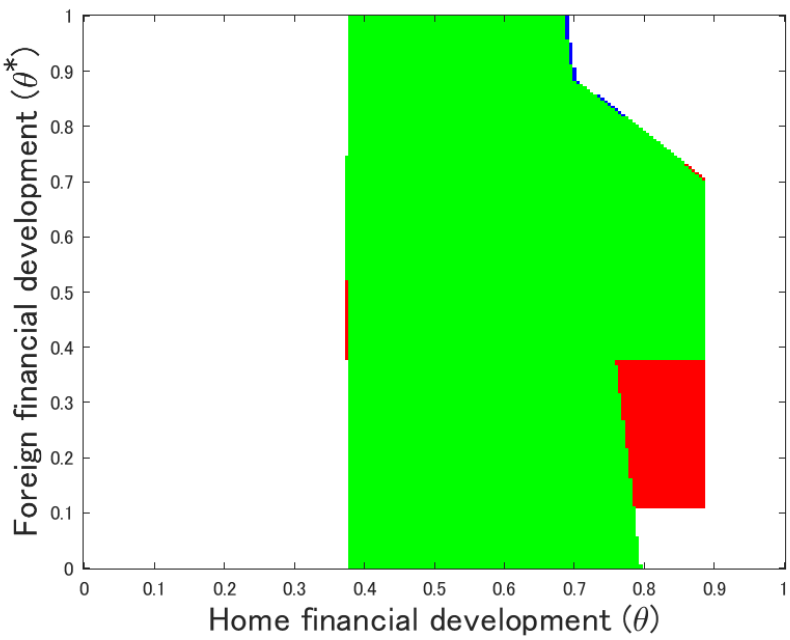
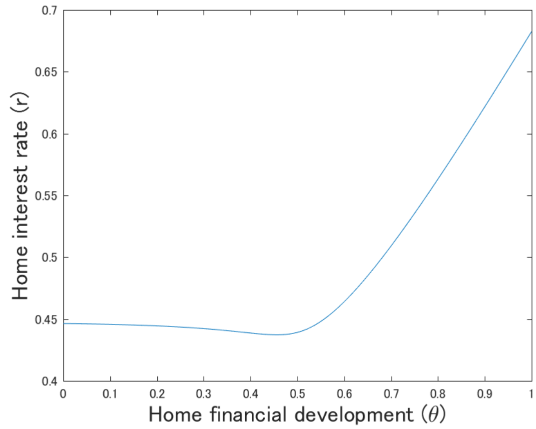
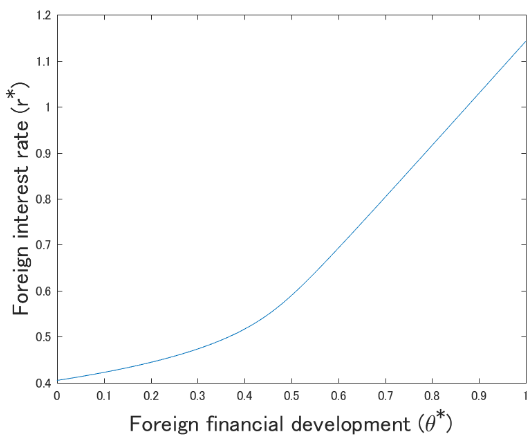


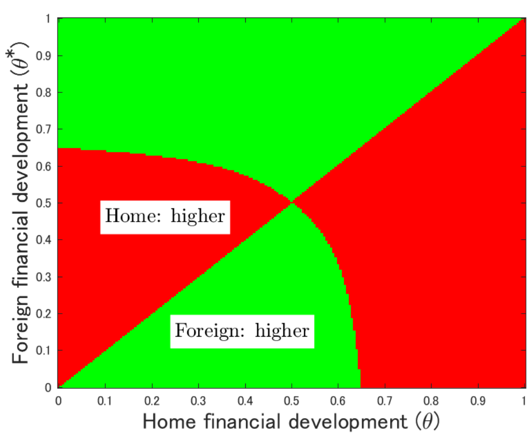
Figure 16: Effect of Foreign technological progress on the existence region of Home bubbles.
Red: bubbles can only exist after the increase in α^F.
Blue: bubbles can only exist before the increase in α^F.
Green: bubbles can exist both before and after the increase in α^F.



(a) Home interest rate

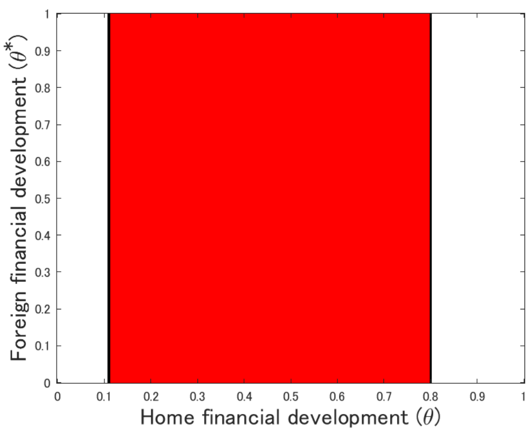


(b) Foreign interest rate

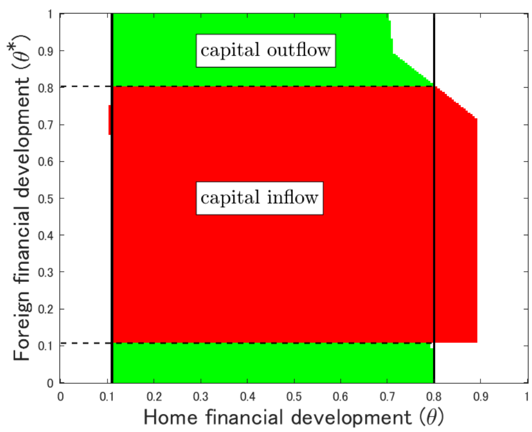


(c) Comparison of Home and Foreign interest rates

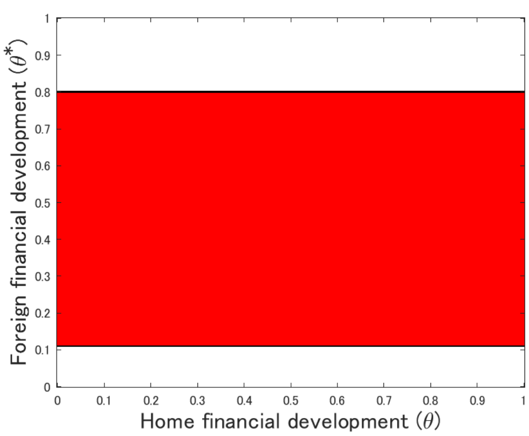
Figure 17: Interest rates under financial autarky (continuum of productivity).



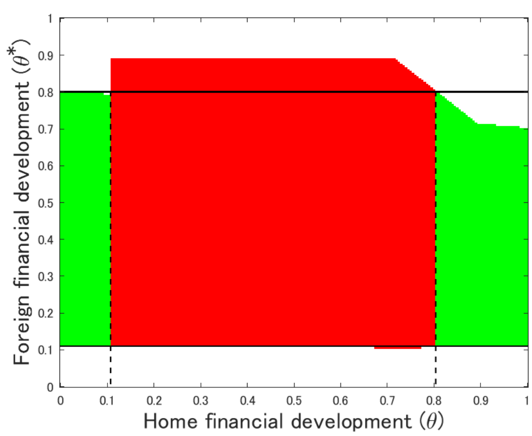
(a) Existence of Home bubbles
(financial autarky)



(b) Existence of Home bubbles
(financial globalization)



(c) Existence of Foreign bubbles
(financial autarky)



(d) Existence of Foreign bubbles
(financial globalization)

Figure 18: Existence conditions of bubbles with international trading of bubbles.