The use of suction or blowing to prevent separation of a turbulent boundary layer

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Summary

The work described divides itself into three parts. The first of these describes an experimental investigation into the influence of a line sink on a turbulent boundary layer, the object of which was to ascertain the overall effect on the values of boundary layer thickness and mean velocity profile shape factor of removing a given amount of fluid. To this end, an axisymmetric boundary layer duct was constructed. Within the limitations of the experimental investigation, which was restricted to the case of only one initial value of shape-factor, it was found possible to represent the effect of a suction strip on a boundary layer in a semi-empirical manner. It was also apparent that the transient effects as represented by the lack of universality of the mean velocity distribution only persisted for a limited extent downstream of the suction strip.

The second part of this work considers the problem of the optimum distribution of suction in order to suppress the separation of a boundary layer. A fairly comprehensive theoretical treatment of this problem is presented which can be used to define the distribution of suction for any surface over which the boundary layer flow is essentially two-dimensional. The basis of this approach is that the suction distribution can be defined by specifying an upper limit on the value of either one of the two parameters which are normally taken as defining the state of a boundary layer, i.e., momentum thickness and shape factor. The precise value of this upper limit is defined by the condition that the suction power required should be a minimum. A series of calculations have been

undertaken which illustrate the general validity of this approach and which further result in a prediction of the minimum suction quantity necessary in order to obtain a given lift coefficient. These results may be used as the basis for a project study of an aircraft which utilises this type of boundary layer control, and also as a starting point for an experimental investigation which would introduce the influence of the various methods of attaining an idealised porous surface in practice.

The third part of the work considers the alternative of boundary layer control by tangential blowing. Experimental measurements on a plane wall jet are compared with Glauert's theoretical predictions and it is noted that, whereas the basic idea behind Glauert's approach is confirmed, some of the detailed predictions show significant discrepancies. The existence of a region of universal mean velocity distribution near the surface is confirmed.

A method of calculating the streamwise variation of the maximum velocity of a wall jet is proposed which is based on the principles of similarity of the mean velocity distribution, continuity and variation of momentum due to the action of the surface shearing force. Consideration is made of the effects of surface curvature and the superposition of a free stream on the development of a wall jet and it is noted that the latter effect is small. In the case of flap blowing, it is shown that the non-dimensional blowing momentum coefficient can be interpreted directly in terms of the value of the ratio of maximum jet velocity to local stream velocity where both are measured at the trailing edge.

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Except where stated to the contrary, the work presented in this dissertation is entirely that of the author.

Contents

<u>Appendices</u>

LIST OF SYMBOLS

$u(y)$	streamwise velocity
U	velocity outside the boundary layer or maximum jet velocity
v_s	suction velocity <u>into</u> surface
x	distance along surface
y	distance perpendicular to surface
δ	boundary layer thickness to $\frac{u}{U} = 0.99$
δ^*	boundary layer displacement thickness. $\delta_2^* = \int_0^\infty \left(1-\frac{u}{U}\right) dy$, $\delta_3^* = \int_0^\infty \left(1-\frac{u}{U}\right)\left(1-\frac{y}{r}\right) dy$
θ	boundary layer momentum thickness. $\theta_2 = \int_0^\infty \left(1-\frac{u}{U}\right)\frac{u}{U} dy$, $\theta_3 = \int_0^\infty \left(1-\frac{u}{U}\right)\frac{u}{U}\left(1-\frac{y}{r}\right) dy$
$H=\frac{\delta^*}{\theta}$	boundary layer shape factor
H'	$\left(\frac{\delta-\delta^*}{\theta}\right)$
R_θ	$U\frac{\theta}{\nu}$, $R_c = \frac{U_c}{\nu}$
Θ	θR_θ^m
C_Q	generalised suction quantity coefficient defined by equation (2.11)
C_L	$\frac{L}{\frac{1}{2}\rho U^2 c}$ sectional lift coefficient
c_f	$\frac{\tau_0}{\frac{1}{2}\rho U^2} = 2\left(\frac{u_*}{U}\right)^2 = \frac{2G}{R_\theta^m}$ local skin friction coefficient
u_*	$\sqrt{\frac{\tau_0}{\rho}}$ friction velocity
A, B	coefficients of the universal logarithmic mean velocity distribution
Γ	$-\frac{\Theta}{U}\frac{dU}{dx}$
F	rate of entrainment of fluid into the boundary layer per unit area
λ	$\frac{\varepsilon}{\delta}$ line sink suction quantity coefficient defined in equation (1.4)

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t	$\frac{U}{pU_o}$ = (local velocity outside the boundary layer)/(maximum velocity at the peak suction
p	(maximum velocity outside the boundary layer at the upstream limit of a region of suction)/(minimum velocity outside the boundary layer at the downstream limit of a region of suction) = Recovery factor
q, r	parameters defining the streamwise distribution of velocity outside the boundary layer (Equation 2.9)
μ, λ	geometrical parameters defining the pressure distribution over the upper surface of a flapped wing section
K	correction factor relating the variation in momentum thickness due to a line sink to that estimated by the "chop off" approach. See Equation (1.5)
δ_1	thickness of wall jet from surface to point of maximum velocity
δ_t	thickness of wall jet from point of maximum velocity to point where $u = \frac{U}{2}$
β	$\frac{\delta_1}{\delta_t}$
α	Glauert's (1956) wall jet shape factor
ε	coefficient of eddy viscosity (Equation 7.8G)
κ	Prandtl eddy viscosity constant (Equation 3.5)
σ	constant defining the lateral spread rate of a jet (Equation 3.12b)
f	non-dimensionalised stream function (defined in Glauert (1956))
η	non-dimensionalised lateral coordinate (defined in Glauert (1956))
C_μ	$\frac{J}{\frac{1}{2}\rho U^2 c}$ jet momentum coefficient
J	jet momentum/unit span
p	static pressure
$R_{(x)}$	radius of curvature

Survey of previous work on the effect of a line sink on a turbulent boundary layer

The first part of this work describes an experimental investigation into the effect of a narrow suction strip on a turbulent boundary layer. Of particular interest is the influence on momentum thickness and shape factor of removing a given proportion of the boundary layer.

Extensive experimental investigations have been undertaken which demonstrate the effectiveness of slot suction as a means of increasing the maximum lift coefficient of conventional aerofoils. Some of this work, which extends as for back as 1940 or earlier, has been described in a series of papers by Regenscheit (1946), by Regenscheit and Schrenk (1947) and by Walz and Ehlers (1947). Although these tests covered a wide range of aerofoil parameters, they were basically of an ad hoc nature and it is not possible to deduce the effect of the suction slot on the boundary layer in any detail. As far as the author is aware, there are only two investigations described in the literature which follow similar lines to the one to be outlined. Wallis (1950) describes an experimental investigation of the effect of a suction slot on a turbulent boundary layer in zero pressure gradient. Pierpoint(1949) also considered the problem but did so mainly from the point of view of the optimum shape for an efficient suction slot. Although both these investigations offered a useful background to the problem, neither were sufficiently comprehensive to present a basis for predicting the effect of removing a given proportion of a boundary layer of an arbitrary shape factor

1. Outline of an experimental investigation into the influence of a line sink on a turbulent boundary layer.

1.1. Introduction.

Once one accepts the concept of a "friction" or boundary layer of fluid of reduced streamwise momentum in the vicinity of a surface, the qualitative explanation of flow separation becomes self-evident. Thus, surface friction dissipates the momentum and should such a flow meet a region of rising static pressure, the fluid velocity tends to decrease yet further and may reverse direction in the immediate vicinity of the surface. In this event, the flow is said to have separated from the surface.

If boundary layer separation is to be avoided, two alternative approaches suggest themselves.

(i) Removal of the fluid in the immediate vicinity of the surface.

(ii) Re-energisation of the flow in the immediate vicinity of the surface by means of a high energy jet directed tangentially along the surface. Such a jet will be capable of withstanding more severe adverse pressure gradients without separation.

In addition, any device which increases the rate of transport of momentum across the boundary layer can be used to delay separation. This is a less powerful technique than (i) or (ii) and will not be considered further.

The object of this investigation is to consider the problem of how to control the development of a boundary layer in order to avoid separation. The problem can be stated as follows:-

(a) What is the optimum distribution of suction, or correspondingly, the optimum configuration of tangential blowing slots? The optimum distribution will be assumed to be that which requires minimum power.

(b) What is the minimum amount of suction (or blow) necessary in order to suppress a flow separation? This pre-supposes that the answer to (a) is already known.

Section (1) describes an experimental investigation of the effect of a line sink on a turbulent boundary layer and as a result, a simple analytical representation is proposed which describes this effect. The problem of the optimum distribution of suction is then considered in Section (2) and it is shown that the two equations which are normally sufficient, (i.e., momentum equation for the boundary layer thickness and auxiliary equation for the mean velocity profile shape factor) to describe the development of a turbulent boundary layer on an impervious surface must be supplemented if suction is applied through the surface. The third equation must clearly be a statement of how the suction is to be distributed. This distribution must be such that separation is avoided and, furthermore, that the power required to do this is a minimum. The analysis of the optimum suction distribution which follows is used to predict the suction power necessary to achieve a given lift coefficient. This should be useful as a standard to be compared with experimental results.

Section (3) considers the alternative approach of tangential blowing. Experimental measurements of the development of a plane wall jet are compared with Glauert's (1956) theoretical predictions. Introducing experimentally the additional complication of a free stream superposed on the wall jet it is shown that it has only a second order effect on the development of the wall jet.

1.2. Design and development of a boundary layer duct.

Whereas continuously distributed suction is an idealisation which must be accepted in the interests of analytical simplicity, it will rarely be achieved in practice. In a practical boundary layer control installation, the suction will probably be distributed in the form of discrete spanwise strips.

Suction systems have been designed utilising spanwise slots, the main advantages being simplicity, reduced possibility of blockage and reduced pressure drop. At an early stage in the investigation, the author concluded that there was a requirement for an experimental investigation of the effect of a spanwise suction strip on a turbulent boundary layer. The aim of this investigation was twofold.

1. It was hoped to establish a simple analytical representation of the effect of a line sink on a boundary layer. Using this, it would be possible to extend the equations governing the development of a boundary layer in an adverse pressure gradient on an impervious surface, to cover the case of a seriesof discrete suction strips distributed such that they suppress a flow separation.

2. The secondary object was to investigate the transient behaviour of a boundary layer in the vicinity of a line sink, and in particular, to determine:-

(a) How the universal logarithmic mean velocity distribution re-establishes itself, and what kind of transient mean velocity distributions occurred during this process. Associated with this is the problem of how the surface shearing force varies immediately downstream of a suction slot.

(b) How far downstream of the slot the boundary layer mean velocity distribution and general development characteristics again become normal.

In order to facilitate such investigation, it was necessary to construct a boundary layer duct consisting of a variable entry length of roughly zero pressure gradient followed by a region of rising static pressure designed to encourage a boundary layer separation. A variable entry length provides a direct means of controlling the thickness of the boundary layer at the point where it enters the region of rising pressure. It was initially proposed to investigate the boundary layer in the vicinity of a single suction strip at a fixed station in the region of rising pressure. Firstly, it was necessary to decide whether to design the equipment in order to approach effectively two-dimensional flow conditions, or alternatively, to construct an axisymmetric duct and thereby eliminate "end" effects. A two-dimensional duct of suitable aspect ratio was found to be physically large and to demand an excessive amount of power and hence the relative simplicity and smaller size of an axisymmetric duct was thought to satisfy the requirements more adequately. The disadvantage of the axisymmetric system is that any small asymmetry in the flow is difficult to eliminate, and, as small physical deviations from a truly axisymmetric duct seemed to cause serious asymmetries in the flow, the problem of obtaining axisymmetric flow conditions presented some difficulty. The full significance of this was not appreciated until the duct had been completed, and boundary layer mean velocity measurements taken at various positions around the duct, and at a number of streamwise stations. Several months of development work were directed towards improving the steadiness and axialsymmetry of the flow. It became clear that it is increasingly difficult to maintain axisymmetric flow conditions in a diffuser as the boundary layer approaches separation, and it is virtually impossible once the flow has separated.

Fig. (1.1) shows the boundary layer duct as finally constructed. The whole system was erected vertically and suitably supported on multi-purpose slotted angle. Fig. (1.2) shows the suction strip in more detail and also the positions of the traversing stations down the diffuser. A suction unit positioned at the downstream end of the system was prefered to a "blow-down" system in order to eliminate the problems associated with smoothing the highly turbulent flow downstream of a fan or blower.

An outline will now be given of the main considerations which contributed to the design of each of the major components of the duct.

1. <u>Bell mouth entry.</u>

As originally conceived, the introduction of the flow into the parallel entry length was to be effected using a Borda mouthpiece.

A considerable amount of time was spent in an attempt to obtain steady flow conditions both in the potential core, and in the boundary layer of the entry length, but no amount of screening against room draughts or, alternatively, judicious positioning of transition wires, proved acceptable. The decision was therefore taken to construct a bell mouthed entry in fibreglass. Measurements of the surface static pressure variation down the bell mouth entry indicated a favourable pressure gradient and as no large scale velocity fluctuations could be found in the core of the entry length, the design was considered satisfactory.

2. Entry length.

In order to achieve the required standard of precision it was found necessary to construct the entry length out of seamless cold rolled tubing. Machined rings were attached at intervals in order to maintain an accurately circular section.

3. Diffuser.

The diffuser was constructed from longitudinal sections of wood, attached onto a 1 in. thick machined circular steel plate and machined to size. Immediately following the machining process, the wood was sealed from the atmosphere with wax polish in an attempt to minimise distortion. The circular metal disc was found to be necessary for mounting in the lathe, but was also useful as a means of minimising distortion of the wooden section over a period of time. Without the metal disc the pressure necessary to hold the wooden diffuser in the jaws of the chuck was such that the wood distorted and left no fixed reference surface for control of the accurate machining process.

At its downstream end, the diffuser fitted into a recess bored in a mild steel plate which further assisted in maintaining the circularity of the section. In spite of the precautions taken the wood tended to distort, particularly at the seams. It is considered that the construction of this diffuser in wood represented one of the major shortcomings of the equipment. However, the construction of a diffuser from a metal casting would have presented such serious problems from the point of view of the difficulty of obtaining suitable castings, difficulty of machining such a shape, and general considerations of expense and complication, that the deficiencies of the wooden diffuser were accepted. As mentioned previously, it is just such distortion which, in conjunction with an incipiently separating flow condition, can result in a large degree of asymmetry in the flow. The area ratio of the diffuser was designed such that, in the absence of suction through the slot and without any centre bodies present, the boundary layer was almost separated at the downstream end of the diffuser. Mean velocity traverses to be described, indicate this to be the case.

4. Retractable settling length.

A settling length was incorporated in the equipment for two reasons:-

(a) In order that it could be removed and thereby enable an operator to set up traversing equipment on the inside surface of the diffuser.

(b) In order that a boundary layer in an incipient state of separation at the downstream end of the diffuser could be given an opportunity to recover and become more firmly attached to the surface before entering the collector box. This was mainly directed towards reducing the feedback of large scale turbulence from the collector box to the diffuser.

The perspex window which is designed to facilitate visual flow observations is detachable, thereby allowing the removal of the diffuser centrebody in sections. This is necessary before the settling section can be withdrawn.

5. Collector box and fan.

Honeycomb and a fine pressure reducing gauze separate the settling section from the collector box. The object of this gauze is to reduce feedback to the diffuser of large scale turbulence in the collector box. A centrifugal fan driven by a constant speed motor is used to induce flow through the system and a variable area vent into the collector box is used as a means of controlling the flow velocity down the duct.

6. Centrebodies.

The essential requirement of the boundary layer duct was to facilitate measurements of the effect of a suction slot on a boundary layer in various stages of development towards separation. As an alternative to variable slot position or a variable angle diffuser, it was decided that a centrebody positioned in the entry length through which flow could be removed or injected into the potential core would provide a direct means of controlling the local surface static pressure distribution without altering the geometry of the equipment. Any resulting rise in the static pressure along the surface upstream of the suction strip would be reflected in an increase in the value of boundary layer shape factor at this point. Furthermore, in the case where fluid is removed through the centrebody any boundary layer developed on the centrebody would pass through the surface and there would be no wake downstream of this centrebody, which might otherwise have caused a certain amount of unsteadiness in the diffuser. Due to lack of time, this inlet length centrebody was in fact, never used.

The main object of the diffuser centrebody was to counteract the effect of the annular suction strip on the streamwise static pressure distribution down the diffuser. Thus, with a thick boundary layer, the streamwise pressure distribution down the diffuser is strongly affected, not only by the variations of geometric cross-sectional area of the diffuser, but also the effective variations of cross-section area due to the variation in the displacement thickness of the boundary layer. It was proposed to alter the effective streamwise thickness distribution of the diffuser centrebody by choosing the distribution and quantity of air emitted from the centrebody so that the static pressure distribution along the walls of the diffuser remained unaffected by removal of air through the annular suction slot. It is interesting to note that it was found to be possible to compensate for a wide range of suction quantities through the annular slot, using a fixed distribution of porosity along the centrebody, simply by adjusting the amount of air withdrawn through the centrebody.

1.3. Construction of the suction strip and experimental details

Non-dimensional slot suction coefficient used

The porous strip was constructed from a fine gauze* available under the proprietary name of 'Perflec'. It was mounted flush with the surface in a recess in a perspex ring and the ring was in turn mounted in an annular slot in the diffuser (see Fig.1.2). The gauze used had a large resistance

* The gauze used had an open area ratio of approximately 0.2 and a "q" drop of the order of 40.

to flow through it, which tended to minimise local inflow and outflow in the condition of nominally zero suction. The perforations of the gauze were so fine that it was prone to blockage after a prolonged period of operation. An indication of axialsymmetry of the suction distribution was obtained from twelve static tappings registering the pressure around the annular suction chamber. This pressure distribution could be adjusted by means of screw clips on the twelve flexible tubes, which connect the annular suction chamber to a common collecting chamber and thence to the suction pump.

Traverses across the boundary layer were taken at a series of stream-wise positions along, and angular positions around, the duct (Fig. 1.2). Total pressure traverses were followed immediately by static pressure traverses, both pressures being referenced with respect to a flush static pressure tapping in the vicinity of the traversing station. The position of these static tappings is not shown in Fig. (1.2) as it is arbitrary, and does not materially affect the results. The total head probes were manufactured by reducing the wall thickness of hypodermic tubing to 0.005 in. and hammering it out on a piece of shim steel 0.004 in. thick. The static probe was a standard unit made of hypodermic tubing (O.D. = 0.034 in.). No corrections were made to the results for displacement effects, Reynolds number effects or the effects of turbulence. When not in use, all holes in the surface used for inserting traversing probes were filled flush to the surface with plasticine.

The probe was traversed across the boundary layer using a micrometer screw with a range of three inches, and positioned relative to the surface electrically. As the surface of the diffuser was non-conducting it was necessary to coat it locally with a thin layer of graphite, and the indication of contact was obtained using an oscilloscope. This was desirable as the electrical resistance of the thin conducting film was very large. Extraneous electrical pick-up in the open circuit condition was damped out immediately the circuit was completed by contact of the probe with the surface. Between 30 and 40 readings throughout the traverse were found necessary to define accurately the mean velocity distribution. As separation is approached, it becomes increasingly important to consider the variation of static pressure across the boundary layer in the vicinity of the surface, especially if it is required to define accurately the universal velocity distribution.* In some of the traverses near separation, the variation of static pressure in this region was at least as significant as the variation of total head.

An accurate indication of the airspeed down the duct was obtained by measuring the mean pressure registered by three interconnected flush static tapping points at a given streamwise station (Y) (Fig. (1.1)) on the entry length just upstream of the diffuser. This pressure was referenced with respect to a static tapping in the bellmouth entry immediately downstream of the gauze and honeycomb, thereby eliminating any error introduced by progressive blockage of the gauzes.

The suction quantity removed per unit length of sink can be non-dimensionalised using the local velocity outside the boundary layer and a length scale representative of boundary layer thickness. Boundary layer thickness is a somewhat ill-defined quantity and displacement thickness would appear to represent a logically acceptable thickness parameter. However, such a non-dimensional expression cannot easily be interpreted in terms of the proportion of boundary layer removed, as the numerical value appropriate to complete removal of the boundary layer varies with shape

* See Fig 1.11

factor. An alternative non-dimensional suction coefficient, and one which is more compatible with the proposed semi-theoretical representation of the effects of suction on a boundary layer, is the parameter (λ) defined thus

$$\lambda = \frac{\varepsilon}{\delta} \qquad \text{where } q = \int_0^{\varepsilon} u\,dy$$

and $u(y)$ refers to the velocity profile without suction.

It is noted that $\lambda=1$ implies removal of 99% of the boundary layer and hence the value of λ can be interpreted as the proportion of the boundary layer removed. For a singly infinite family of mean velocity distributions which can be represented by a power law, the alternative suction coefficients are related thus,

$$q/U\delta^* = \frac{\int_0^{\varepsilon} u\,dy}{U\delta^*} = \frac{\delta}{\delta^*} \cdot \int_0^{\lambda} \left(\frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \frac{\delta}{\delta^*} \cdot \int_0^{\lambda} \left(\frac{y}{\delta}\right)^{\frac{H-1}{2}} d\left(\frac{y}{\delta}\right) = \frac{2}{H+1}\lambda^{\frac{H+1}{2}}$$

from which it can be seen that $\frac{q}{U\delta^*} = \frac{2}{(H-1)}$ implies virtually complete removal of the boundary layer.

Values of skin friction coefficient are obtained by comparing the mean velocity distribution in the inner twenty percent of the boundary layer with the universal mean velocity distribution. The sublayer will normally be too thin for accurate measurement using standard traversing equipment, and hence the appropriate part of the universal velocity distribution is the logarithmic region

$$\frac{u}{u_*} = A \log\left(\frac{u_* y}{\nu}\right) + B \tag{1.1}$$

Differentiating equation (1.1) for a given value u(y), in order to determine the errors in u_* introduced by errors in the assumed value of A and B, we have

$$\frac{du_*}{u_*} = - \frac{(u/u_* - B)}{(u/u_* + A)} \cdot \frac{dA}{A} - \frac{B}{(u/u_* + A)} \cdot \frac{dB}{B} \tag{1.2}$$

Assuming values of A = B = 5.6 proposed by Ross, (1956) after an extensive investigation of the coefficients which have been proposed, and as $\frac{u}{u_*} > 10$ outside the sublayer, the coefficients of dA/A and dB/B are both of the order 1/3 or less. Hence, for the worst case of an error in B adding to that in A, the resulting error in u_* is still less than the individual errors in A or B. Also, from various interpretations of experimental data it would appear that a high value of the coefficient (A) is normally associated with a low value of (B) and vice versa, thereby indicating that there has been a tendency to cancel an error in the value of one coefficient by a further error of opposite sign in the value of the other coefficient. This can easily be understood once it is realised that the logarithmic region of the mean velocity distribution is hardly more than a point of inflexion between the sublayer and the outer region of flow. The implication of this insofar as it concerns the error in u_* as defined by equation (1.2) is that the errors in (A) and (B) can be expected to cancel and it is not unreasonable to expect the resulting error to be of an order one half of that which can be expected for either. Hence, if it is assumed that the values of both A and B are known to within 10 per cent., it should be possible to estimate the value of the friction velocity to within 2 per cent. accuracy, and the corresponding value of skin friction coefficient to within 4 per cent. This order of accuracy is well within that of the experimental measurements of u and y. Although experimental inaccuracies due to random scatter can be eliminated by defining the velocity distribution by means of a large number of experimental measurements u(y), the inaccuracies due to displacement effects, Reynolds number effects and turbulence which are consistent from reading to reading, cannot be eliminated easily. However, it is thought that an overall accuracy on skin friction coefficient of between 5 per cent. and 10 per cent. may reasonably be expected. The whole basis of the method is invalid immediately downstream of a suction slot where the boundary layer is in a transitional state.

The technique used to define u_* from a given experimental measurement of u(y) is basically that proposed by Ross (1956). Equation (1.1) can be written,

$$\frac{u}{u_*} + A \log \frac{u}{u_*} = A \log \left(\frac{uy}{\nu}\right) + B$$

or,

$$\frac{u}{u_*} = f\left\{ \left(\frac{uy}{\nu}\right), A, B \right\} \tag{1.3a}$$

Hence,

$$u_* = \sqrt{\frac{\tau_0}{\rho}} = \frac{u}{f(\frac{uy}{\nu}, A, B)} \tag{1.3b}$$

The relationship of equation (1.3a) can be expressed graphically and hence for any one experimental reading u(y) it is possible to define (uy/ν), u/u_* and hence u_*. Having calculated the effective value

u_* for all the experimental points on the inner part of the mean velocity profile, it is possible to define the value of u_* characteristic of the velocity profile by plotting u_* as a function of (y) and taking the best horizontal straight line through the points. Visual inspection of the tabulated values of u_* is often sufficient and plotting was usually found to be unnecessary. Fig 1.12 shows a typical variation of u_* across the boundary layer.

Using the relationship $c_f = \frac{\tau_0}{\frac{1}{2}\rho U^2} = 2\left(\frac{u_*}{U}\right)^2$, it is then possible to calculate the appropriate value of the skin friction coefficient (c_f).

Equation (2.2) with $(v_s/U) = 0$ represents the empirical skin friction relationship proposed by Ludwieg and Tillmann (1949) for an impervious surface. Hence $c_f . R_\theta^{0.268}$ is a parameter which should be independent of local boundary layer Reynolds number (R_θ) and for an impervious surface should be purely a function of mean velocity profile shape factor (H). Fig. (1.3) compares the experimentally defined variation of $c_f . R_\theta^{0.268}$ with shape factor with that predicted from the empirical formula of Ludwieg and Tillmann and the trends which the two curves follow are very similar over a wide range of values of shape factor (H) varying between $1.3 < H < 3.0$. There is, however, an apparent difference in magnitude at all values of shape factor (H) which gives a mean value of $\frac{c_f \text{ experimental}}{c_f \text{ calculated}}$ of approximately 0.7. Recalculating the values of momentum thickness and shape factor as two-dimensional rather than axisymmetric parameters has only a second order effect on the value of this ratio and, in any case, accentuates rather than reduces the discrepancy between calculated and experimentally defined values of skin friction coefficient. Without any direct measurements of the value of skin friction, it is not possible to say whether this discrepancy arises from an error in the experimentally defined or the calculated value of surface shearing force. However, the very good qualitative agreement is satisfactory and confirms the essential validity of the mean velocity traverses taken across a boundary layer which is close to separation.

1.4.1. Preliminary evaluation of boundary layer duct without a diffuser centrebody.

The initial phase of the experimental work was undertaken without a centre body in the diffuser and had the following objects.

(i) To investigate the symmetry of the flow with various amounts of applied suction.

(ii) To investigate the combined effect on a boundary layer of a suction slot which tends to reduce the value of the boundary layer shape factor and the additional pressure recovery which the boundary layer has to sustain as a result of the removal of fluid from the diffuser.

Fig. (1.4) shows the streamwise variation of static pressure down the diffuser, from which it is clear that the effect of suction is to introduce a constant increment in pressure recovery factor. The pressure recovery factor was non-dimensionalised by means of the dynamic pressure at station Y immediately upstream of the entry to the diffuser. The variation of static pressure around the diffuser at a given streamwise station is small and not indicative of the degree of asymmetry subsequently found from measurements of the mean velocity profile.

Figs. (1.5a) and (1.5b) indicate the degree of asymmetry for the cases $\lambda = 0$ and $\lambda = 0.223$ respectively. Thus it can be seen that, whereas the degree of asymmetry without suction is reasonable, the effects of suction are deleterious. It mustbe be remembered that the suction coefficient $\lambda = 0.223$ is quite large and probably in excess of the range of values which are of most interest. This value of suction coefficient was chosen in order to illustrate the effect of suction under the worst conditions and lesser values will have a correspondingly reduced effect on the symmetry of the flow. A range of boundary layer parameters which have been derived from the velocity distribution of Figs. (1.5a) and (1.5b) are presented in tabular form in Table I. It can be seen that, without suction, the asymmetry becomes progressively worse as the boundary layer develops. The influence of suction on the symmetry of the flow was minimised by balancing the static pressure around the annular collector chamber behind the porous strip by individually adjusting screw clips on the twelve rubber tubes which connected the annular slot to a plenum chamber. The asymmetry introduced by suction can therefore only be attributed to variation of the effective area around the annular suction strip which was kept to a minimum during construction.

This degree of asymmetry both with and without suction was the best that could be obtained within the time scale available and was only achieved as a result of extensive modifications to the various components of the duct. In view of this, it follows that the boundary layer development cannot strictly be discussed using the well known streamwise development equations for axisymmetric flow, as cross-flow effects will modify the development to an extent which will be difficult to determine.

The variation of the mean velocity distribution down the diffuser at a fixed angular station is shown in Fig. (1.6) for $\lambda = 0$ and $\lambda = 0.223$ and an indication is given of the transient type of mean velocity distribution which occurs downstream of a suction slot.

It is further noted from Fig. (1.5) that, in the absence of suction, the boundary layer is close to separation at the downstream end of the diffuser. This was one of the features aimed at in the initial design of the equipment.

1.4.2. Investigation of the effects of a suction strip on a turbulent boundary layer using a diffuser centrebody.

The investigation described in the previous section indicated that, although the symmetry of the flow was not good, the transient effects of the suction strip persisted for only a short distance downstream, if expressed in terms of local boundary layer thickness. As the object of the experimental programme was to investigate both the transient effects immediately downstream of the strip on the boundary layer shape factor and the momentum thickness, it was thought that the asymmetry, although clearly not desirable, might not seriously detract from the validity of the results.

The diffuser centrebody was mounted as shown in Fig. (1.1) and the appropriate distribution of porosity was obtained by trial and error, using a 42 tube inclined manometer to obtain a pictorial representation of the distribution of static pressure down the diffuser. Using a fixed distribution of porosity along the centrebody it was found to be possible to compensate for the rise in the general level of the static pressure downstream of the suction strip simply by adjusting the centrebody blowing pressure according to the slot suction pressure. There is a local variation of static pressure in the immediate vicinity of the suction slot which is the normal sink effect and which is not associated with the removal of fluid from a bounded system. This local variation of static pressure persists even with the diffuser centrebody operating.

Fig. (1.7) shows the development of the mean velocity profile at two stations upstream and five downstream of the suction strip for a range of values of the suction coefficient. It can be seen that the universal logarithmic mean velocity distribution is apparently re-established at Station T5 (2") which is only $1\frac{1}{4}$ in. downstream of the aft end of the suction strip. Fig. (1.8) shows the streamwise variation of various boundary layer parameters which are derived from the mean velocity distributions of Fig. (1.7). Assuming that universality of the mean velocity distribution is indicative of the end of the transient phase of development, the variation of shape factor and momentum thickness at Station T5(2") with suction coefficient, Fig. (1.9), is representative of the overall effect of a suction strip on a boundary layer.

Assuming that the immediate effect of a suction strip on a boundary layer can be represented by simply removing the appropriate amount of fluid from the inner part of the mean velocity profile, it is possible to calculate the shape factor and momentum thickness immediately downstream of the strip (i.e., at Station T5a $3\frac{1}{4}$"). Thus, representing the mean velocity distribution without suction by a power law

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{H_1-1}{2}}$$

one obtains (1.4)

$$\frac{\theta_2}{\theta_1} = \frac{\int_\varepsilon^\delta \left(1-\frac{u}{U}\right)\frac{u}{U}dy}{\int_0^\delta \left(1-\frac{u}{U}\right)\frac{u}{U}dy} = 1 - \frac{2H_1}{(H_1-1)}\lambda^{\frac{H_1+1}{2}} + \frac{(H_1+1)}{(H_1-1)}\lambda^{H_1}$$

$$\frac{H_2}{H_1} = \frac{\int_\varepsilon^\delta \left(1-\frac{u}{U}\right)dy}{\int_0^\delta \left(1-\frac{u}{U}\right)dy} \cdot \frac{\int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy}{\int_\varepsilon^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy} = 1 - \left\{\frac{(H_1+1)\lambda^{H_1} + (H_1+1)\lambda - 2(H_1+1)\lambda^{\frac{H_1+1}{2}}}{(H_1-1) - 2H_1\lambda^{\frac{H_1+1}{2}} + (H_1+1)\lambda^{H_1}}\right\}$$

where,

$\int_0^\varepsilon u\,dy = q$, H_1, θ_1 refer to conditions without suction

$\lambda = \varepsilon/\delta$ H_2, θ_2 refer to conditions with suction applied

The transient effects immediately downstream of the suction strip will modify the values of momentum thickness and shape factor in a manner which, in the light of the present lack of understanding of these transient effects, can only be defined empirically. From Fig. (1.9) it is clear that, whereas the shape factor is approximately constant throughout the transitional region, the value of momentum thickness decreases for small values and increases for larger values of suction coefficient. The increase of momentum thickness throughout the transitional region can possibly be explained in terms of the large transient surface shearing forces associated with the larger values of slot suction coefficient (λ). The decrease in momentum thickness on an impervious surface under the influence of a relatively high surface shearing force and in a region of adverse pressure gradient, is worthy of note. Wallis (1950) has noted this behaviour and attempted to describe it in terms of transfer of energy from the turbulent to mean flow. Clearly a more comprehensive investigation is necessary before it is possible to present a coherent theory which quantitatively describes the overall effect of a line sink on the momentum thickness. For the purpose of the step-by-step calculations which are undertaken in Section (2), an empirical factor (K) will be used to represent the effect of a line sink on the boundary layer momentum thickness. (K) is defined by equation (1.5),

$$K = \left[\frac{1 - \left(\frac{\theta_2}{\theta_1}\right)_{exp.}}{1 - \left(\frac{\theta_2}{\theta_1}\right)_{calc.}}\right] \quad \text{where} \left(\frac{\theta_2}{\theta_1}\right)_{calc.} = \frac{\int_{\varepsilon}^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy}{\int_{0}^{\delta} \left(\frac{u}{U}\right)\left(1 - \frac{u}{U}\right) dy} \qquad (1.5)$$

The value of (K) will be a function of local boundary layer shape factor (H) but as experimental information is limited to the case H=1.7, this dependence must be neglected for the purpose of the calculations of section (2.4.2.2).

The reason for the success of this simple minded approach of removing the part of the boundary layer profile next to the surface, insofar as it appears to predict the variation of mean velocity profile shape factor, may lie in the fact that for small values of λ (Equation 1.4) the effect of a line sink on shape factor is an order greater than the effect on momentum thickness. Hence, the effect of variation of shape factor throughout the transient region will be less. The experimental investigation described is limited to the value of shape factor (H) = 1.7

Further investigation covering a range of values of shape factor is clearly required.

1.5. Conclusions.

The investigation described in this thesis, divides itself very clearly into three parts and it is, therefore, convenient to summarise the main conclusions after each part.

Section (1) is a description of an experimental investigation which has been undertaken in order to establish the effect of a discrete suction strip on a turbulent boundary layer. It outlines the considerations which contributed to the design and development of an axisymmetric boundary layer duct and describes some of the disadvantages of such a set-up if used to investigate a boundary layer near separation. The basic difficulty was simply that of preserving axisymmetric flow conditions as separation was approached.

The main point of interest with regard to the experimental measurements was the extensive use which was made of the universal logarithmic mean velocity distribution as a means of estimating the surface shearing stress. Estimates of skin friction coefficient obtained using this technique for a range of boundary layer conditions, varying from zero pressure gradient to almost separated flow, were compared with those estimated using the empirical formula of Ludwieg and Tillmann (1949). The correlation as a function of (H) and R_Θ ~~agreement obtained~~ was good and one is left in some doubt as to whether it is an error in the experimental value of skin friction coefficient or the value predicted by the Ludwieg and Tillmann relationship which ~~is in error~~ accounts for the numerical discrepancy.

The preliminary evaluation of the boundary layer duct was undertaken without centrebody and was directed primarily towards an investigation of the axial symmetry of the flow. It was found that the transient effects downstream of the narrow suction strip were of limited streamwise extent and hence the significance of the lack of axial symmetry of the flow was somewhat reduced. Further work was undertaken using a centrebody to counteract the static pressure rise associated with the removal of fluid from a bounded duct, in order to investigate more precisely the effect of a suction strip on a turbulent boundary layer. A simple analytical expression has been derived which defines the effect of removing a given proportion of the boundary layer momentum thickness and shape factor. The derivation of such a relationship was the prime object of the experimental investigation and has made it possible to programme a step-by-step calculation of the development of a boundary layer along a surface, with suction strips distributed according to a predetermined pattern in order to suppress a flow separation.

The optimum distribution of suction to suppress flow separation

Outline of previous work

The second part of this investigation considers the problem of the optimum distribution of suction necessary to suppress separation of a boundary layer in an adverse pressure gradient.

The problem of increasing the maximum lift coefficient of a wing has been considered by several investigators (Ref. Williams 1960), but the optimisation of the suction distribution has in general only been approached in an ad hoc manner. Raspet (1958) considered the optimum distribution of suction to be that which limited the value of boundary layer momentum thickness. Dutton (1955), Sarnecki (1958) and others have considered the development of an incompressible turbulent boundary layer with uniformly distributed suction on a flat plate in zero pressure gradient. This work was not sufficiently general to be of direct use in the investigation to be described, although work by Sarnecki did offer some guidance concerning the influence of porous suction on the skin friction coefficient for a turbulent boundary layer.

Two basic equations are necessary in order to predict the development of a boundary layer in an adverse pressure gradient. These are the Von Karman momentum equation which govers the variation of the boundary layer thickness parameter, and the auxiliary equation which defines the variation of shape factor. The Von Karman momentum equation is of the standard form of equation (2.1). Many attempts have been made to construct an auxiliary equation all of which are basically empirical. Various forms of this equation have been presented by Von Doenhoff and Tetervin, Garner, Maskell, Schuh and other workers (Ref. Spence 1956).

All these various forms of the auxiliary equation have been derived for an impervious surface and because of their empirical nature, they do not lend themselves to further development in order to consider the additional effect of suction through the surface. For the purpose of the step by step calculations on the impervious surface between suction strips (2.4.2.2.) the auxiliary equation presented by Spence (1956) was used. This particular form of the equation was chosen as it was derived in a physically plausible manner, and further because it appeared to give reasonable agreement with experimental measurements and some of the more accepted methods of calculation. The decision was however somewhat arbitrary. A skin friction relationship based on that derived by Ludwieg and Tillmann (1949) was used as this represented the variation with Reynolds Number and shape factor.

As far as the author is aware, there are only two forms of the auxiliary equation which consider the influence of flow through a porous surface. The one used in the calculations of Section (2.4.2.1) is that derived by Head (1958). More recently Pechau (1960) has used a form of the boundary layer energy equation as the basis for some calculations which are similar to these presented by the author. The equation presented by Head(1958) was based on a simple physical idea, was of a simple form and appears to predict the development of shape factor on an impervious surface at least as well as most of the alternative methods. At the time when this part of the investigation was initiated, Head's equation was the only one published which could consider the effect of porous suction or injection.

2. Consideration of the problem of the optimum distribution of suction in order to suppress a flow separation.

2.1. Introduction.

The use of suction as a means of suppressing a flow separation is a logical consequence of Prandtl's boundary layer hypothesis. It has practical applications both as a means of preventing stall of lifting surfaces and also in the design of efficient large angle diffusers.

There are two separate problems associated with the application of boundary layer control using suction in order to increase the maximum lift coefficient of a wing. They are:-

(i) To design the optimum aerofoil section and wing planform from considerations of boundary layer control.

(ii) To determine the most efficient suction distribution for any given wing.

The investigation to be outlined is largely directed towards consideration of (ii) but this inevitably results in certain general ideas as to the optimum wing configuration from the point of view of boundary layer control. The approach to problem (ii) must necessarily be largely experimental, but any guidance which may be forthcoming from theoretical considerations is valuable as a means of reducing wind-tunnel and flight development programmes. Much the same applies to the problem of designing efficient large-angle diffusers by removing boundary layer fluid through the diffuser wall and thereby suppressing a flow separation.

For the case of a "high-lift" wing, two suction systems will be considered as representative of the extremes of the large number of possible arrangements. The first case, which is more realistic, considers the aircraft as having only one suction unit, the pressure being such that it maintains a reasonable margin below the lowest static pressure on the wing surface. For this case, the optimum distribution of suction can be obtained by suitably varying the effective porosity of the surface and as there will be regions over which the pressure drop through the surface is large, this approach is intrinsically inefficient. One obvious means of improving the efficiency is to use a multi-stage suction unit in which each stage extracts fluid from the boundary layer at the appropriate region of static pressure. With this arrangement, the pressure drop through the surface can be reduced to a minimum consistent with the need to maintain a margin in order to prevent outflow for "off-design" conditions of incidence, flap angle and air speed. The ultimate extreme of this approach, although clearly impracticable due to the complexity of the associated ducting system, is to define the local duct pressure according to the local wing surface static pressure. This system is considered as it represents an ideal in terms of efficiency. The condition defining the optimum distribution of suction is again that of minimum overall power required, but this now differs from the condition of minimum suction quantity. A further reason for considering the two suction systems is to confirm that the validity of the arguments which contribute to the discussion of the optimum suction distribution is not dependent on the precise definition of the word "optimum".

In practice, pressure losses down the duct will represent a large proportion of the pumping power necessary. These losses can easily be calculated once the ducting system and flow quantities are defined and are not considered further as they do not materially affect the consideration of the optimum suction distribution.

Conditions similar to the above are applicable to the design of efficient "large-angle" diffusers which incorporate boundary layer control by suction. If the static pressure along the diffuser exceeds the external pressure, a suction system can be devised by distributing the effective surface porosity in such a way that the minimum total quantity of fluid is extracted. Even after extraction from the boundary layer this fluid need not necessarily be at atmospheric pressure, in which case it can possibly be utilised in the auxiliary services. If the pressure in the diffuser is less than atmospheric then a suction pump is necessary and in principle, the problem becomes identical with that discussed above.

Although in practice, the removal of the boundary layer may be achieved by a series of spanwise holes or slots, it represents a considerable simplification from the analytical point of view if the problem of the optimum suction distribution is treated by considering the idealisation of suction continuously applied through a smooth porous surface. This idealisation eliminates the discontinuities associated with slot suction and the complicated three-dimensional effects associated with the flow into discrete holes.

2.2. The boundary layer momentum equation and skin friction relationship.

The von Karman integral equation which represents the balance of the mean flow momentum is the basis of many of the well known approximate methods of predicting the development of a boundary layer. For a porous surface, this equation can be written thus,

$$\frac{d\theta}{dx} = \frac{\tau_o}{\rho U^2} + (H+2) \cdot \left(-\frac{\theta}{U}\frac{dU}{dx}\right) - \frac{v_s}{U} \tag{2.1}$$

As virtually no systematic information is available from which it is possible to deduce the dependence of surface shearing force on suction velocity, it is necessary to make a somewhat arbitrary extension of one of the empirical relationships which have been established for an impervious surface. One such equation is as follows:

$$c_f = \frac{\tau_o}{\frac{1}{2}\rho U^2} = \frac{2G(\frac{v_s}{U},H)}{R_\theta^{\,m}} \tag{2.2a}$$

where

$$G\left(\frac{v_s}{U},H\right) = \left[1 + \frac{v_s/U}{0.01}\right] \cdot 0.123.10^{-0.678H}, \quad m = 0.268 \tag{2.2b}$$

The Ludwieg Tillmann (1949) skin friction relationship Eq. (2.2) was chosen as the basic equation as it considers the dependence of surface shearing force on shape factor as well as Reynolds number. As a first approximation, the dependence of $G(v_s/U)$ is assumed to be linear and it is further assumed that a value $(v_s/U) = 0.01$ effectively doubles the value of G. This relationship is approximate, but its essential validity is confirmed by a limited amount of experimental measurement undertaken by Sarnecki (1958) on a turbulent boundary layer in zero pressure gradient. Although these admittedly rather crude assumptions with regard to skin friction coefficient may be somewhat in error, it will be shown that such an error does not invalidate the general argument from which the optimum suction distribution is derived. Such an error will, however, have a direct influence on the predicted suction quantity required and to a lesser degree on the manner in which this suction is distributed.

Inhomogeneity of the suction distribution may also have a profound effect on the effective value of skin friction coefficient and thereby modify the suction quantity necessary to achieve a given lift coefficient. It is this aspect of the problem which can only be treated in an ad hoc manner and which makes the problem of the optimum suction distribution one which, within the forseeable future, must be largely experimental.

2.3. Discussion of the problem of the optimum suction distribution.

It is necessary to determine the distribution of suction which requires minimum suction power in order to maintain a given lift coefficient from a wing, or pressure recovery factor from a diffuser. For the region considered the local velocity outside the boundary layer can vary from $p\,U_o$ to U_o $(p > 1)$ by any path in a streamwise distance $0 < x/c < 1$. It is necessary to specify the state of the boundary layer at the beginning of this region. In the most general case, the boundary layer at $x/c = 0$ may be either laminar or turbulent, but the additional complications associated with quantitative prediction of transition preclude consideration of the former case. This is not a serious limitation however, since transition must be achieved as near to the beginning of the region of adverse pressure gradient as is possible in order to avoid the possibility of a laminar boundary layer separation. Assuming the boundary layer to be turbulent at the beginning of the region $(x/c = 0)$, it is necessary to state the initial values of momentum thickness (θ_o) and shape factor (H_o). A chararacteristic Reynolds number must also be defined.

Ideally, the criterion defining the optimum distribution of suction might be expressed directly as a relation for suction velocity, but it is diffult to conceive how this could be generalised to the case of an arbitrary distribution of velocity outside the boundary layer. A simple and effective way of expressing this criterion is as a limit on some characteristic of the boundary layer. The two parameters which it is normally assumed, define the state of a turbulent boundary layer are momentum thickness (θ) and shape parameter (H). Any criterion which is to be generally applicable must be non-dimensional in order that it should be unaffected by the length scale of any specific system. Shape factor (H) satisfies this condition, but the momentum thickness must be non-dimensionalised by parameters describing the severity of the adverse pressure gradient and possibly the physical properties of the fluid. For a region of limited overall pressure recovery, for which variations in the streamwise extent of impervious surfaces can have a significant influence on the overall suction power required, it cannot be stated a priori that these non-dimensionalising parameters must be local ones. However, if one considers the limiting case of a region over which the pressure recovery factor (p) is large, the condition of minimum overall suction power required must reduce to a local condition on the boundary layer which ensures that the contribution to the total suction power from every point throughout the region is minimum. Hence, as whatever parameters are used to non-dimensionalise local momentum thickness must still be applicable in the limit as the recovery factor (p) approaches infinity, it will be assumed that the same local parameters are appropriate for finite values of recovery factors. Hence a suitable non-dimensional form of momentum thickness will be established by consideration of the asymptotic case for which $p \rightarrow \infty$ whilst the optimum numerical value of this criterion must be established from considerations of the minimum total suction for any specific distribution of velocity outside the boundary layer.

One advantage of expressing the criterion as a limit on the value of a local boundary layer parameter is that it simplifies the problems associated with arbitrary initial conditions. Thus, if the boundary layer is initially thin, there will be greater extent of impervious surface before the optimum value of the criterion is attained and a corresponding saving in total suction quantity required.

One necessary condition for a suction criterion is that it must preclude the possibility of a boundary layer separation. A criterion defined as a limit of the value of mean velocity profile shape factor ($H < 2.6$) satisfies this condition. Consider now, a suitable non-dimensional form of the momentum thickness. A well known parameter of boundary layer theory is the ratio $\left(\dfrac{-\frac{\theta}{U}\frac{dU}{dx}}{\frac{1}{2}c_f}\right)$ which is virtually identical with Polhausen's parameter used in laminar boundary layer theory (Polhaushen 1921) and also with the parameter used by Buri to calculate the development of a turbulent boundary layer in a pressure gradient (Buri 1931). This parameter represents the ratio of overall pressure forces to surface shearing forces acting on the boundary layer and it suggests itself as a possible non-dimensional form of momentum thickness. By limiting the maximum value of this ratio, the boundary layer would develop primarily under the influence of the applied suction and the surface shearing forces, whilst contributions from the pressure gradient terms in the boundary layer development equations could be restrained. Whilst the value of $\left(\frac{-\frac{\theta}{U}\frac{dU}{dx}}{\frac{1}{2}c_f}\right)$ is finite, the boundary layer will remain attached to the surface.

In order to obtain the order of magnitude of the value of the parameter $\left(\frac{-\frac{\theta}{U}\frac{dU}{dx}}{\frac{1}{2}c_f}\right)$ necessary to maintain the boundary layer in a condition far from separation, consider the auxiliary equation proposed by Spence (1958) for an impervious surface

$$\Theta\frac{dH}{dx} = \varphi(H)\Gamma - \psi(H)$$

where,

$$\varphi(H) = 9.524\ (H-1.21)(H-1) \tag{2.3}$$

$$\psi(H) = 0.00307\ (H-1)^2$$

$$\Theta = \theta R_\theta^m,\ \Gamma = -\frac{\Theta}{U}\frac{dU}{dx}$$

Spence used the skin friction relationship proposed by Young (1953) for an impervious flat plate in zero pressure gradient as defined in equation (2.2a) with $G = 0.00885$, $m = 0.2$.
It is noted that

$$\left(\frac{\Gamma}{G}\right) = \frac{-\frac{\theta}{U}\frac{dU}{dx}}{\frac{1}{2}c_f} = \frac{-\frac{\Theta}{U}\frac{dU}{dx}}{G} = \frac{\text{Pressure forces}}{\text{Surface shearing forces}} \tag{2.4}$$

Spence derived his auxiliary equation (2.3) such that $\psi(H)$ represented the decrease in shape factor (H) with increasing Reynolds number on a flat plate in zero pressure gradient. $\varphi(H)$ represents the influence of the adverse pressure gradient which tends to increase the value of shape factor (H). As removal of fluid from the boundary layer will always tend to decrease the value of shape factor (H), it is clear that a maximum numerical value of the parameter (Γ/G) given by equation (2.5) will imply $dH/dx \leqslant 0$ even with suction applied and consequently will maintain a boundary layer in a state which is far from separation at a value of shape factor of the order $(H) \leqslant 1.4$.

$$\left(\frac{\Gamma}{G}\right) = \left(\frac{-\frac{\Theta}{U}\frac{dU}{dx}}{G}\right) = \frac{\psi(H)}{G\varphi(H)} = 0.079 \text{ with } \left.\begin{array}{l} H = 1.4 \\ G = 0.00885 \end{array}\right\} \qquad (2.5)$$

It is noted that for a turbulent boundary layer, the relationship of (Γ/G) and H as discussed above, is roughly that of cause and effect. The term (Γ/G) represents the contribution of the pressure gradient term in the equations defining the streamwise development of boundary layer thickness and shape factor. Shape factor (H) is an indication of the state of the boundary layer with respect to separation and increases as a result of the boundary layer having to sustain unfavourable values of (Γ/G) for a prolonged streamwise extent. For a laminar boundary layer, the two expressions are even more intimately related in that the local value of (Γ/G) defines the shape of the velocity profile and hence the local value of H.

The suitability of the condition (Γ/G) = constant as a criterion which can be used to define the optimum suction distribution, stems from considerations of the limiting case for which the overall recovery factor (p) is large. For this case, the total suction power required will be large and any possible saving derived from a limited extent of impervious surface prior to the commencement of suction which is associated with arbitary initial conditions can be neglected. From this point of view, it is reasonable that the condition of minimum overall suction power might be replaced by a local condition. It is also reasonable that, for this asymptotic case, the appropriate criterion is that which represents a constraint on the value of (Γ/G), thereby limiting the term which causes a boundary layer to separate rather than allowing the boundary layer thickness to increase to such an extent that the pressure gradient terms in the auxiliary equation for shape factor (H) become important, after which the thicker boundary layer demands correspondingly greater amounts of suction in order to prevent separation. Consider now which local condition might be used to replace the overall condition of minimum total suction power for the asymptotic case, in which $p \to \infty$. If the boundary layer is allowed to become unduly thick, the tendency to thicken further will be increased due to the term $(H + 2) \cdot \left(- \frac{\theta}{U} \frac{dU}{dx} \right)$ in the momentum equation. If the boundary layer is too thin, the rate of growth due to adverse pressure gradient will be small, but the increased surface shearing stress associated with a thinner boundary layer will tend to offset any gains which might otherwise be obtained. As a compromise it is plausible to assume that the optimum suction distribution is the one which controls the boundary layer thickness such that it maintains a balance between these two extremes. Using the momentum equation (2.1) it is possible to express this condition thus,

$$\frac{d}{d\theta}\left(\frac{d\theta}{dx}\right) = \frac{d}{d\theta}\left[\frac{\tau_o}{\rho U^2} + (H+2) \cdot \left(-\frac{\theta}{U}\frac{dU}{dx}\right) - \frac{v_s}{U}\right] = 0$$

Using equation (2.2) and assuming that H = constant $\approx$ 1.4 we obtain

$$\frac{d}{d\theta}\left(\frac{d\theta}{dx}\right) = -m \frac{G}{R_\theta^{\ m+1}} \left(\frac{U}{\nu}\right) + \left(-\frac{1}{U}\frac{dU}{dx}\right)(H+2) = 0 \qquad (2.6)$$

It is noted that the value of momentum thickness (θ) is not dependent on the local value of suction velocity as long as the latter remains finite.

Equation (2.6) can be written

$$\left(\frac{\Gamma}{G}\right) = \frac{-\frac{\theta}{U}\frac{dU}{dx} R_\theta^{\ m}}{G} = \left(\frac{m}{H+2}\right) \qquad (2.7a)$$

Inserting numerical values

$$m = 0.268$$

$$H = 1.4$$

one obtains

$$\left(\frac{\Gamma}{G}\right)_{opt} = 0.078 \qquad (2.7b)$$

Thus, not only does this argument suggest the condition (Γ/G) = constant as a possible criterion for the optimum distribution of suction for large values of recovery factor (p), but it also predicts a numerical value of an order which is satisfactory if boundary layer separation is to be avoided. (Eq. 2.5).

The above argument provides an indication that (Γ/G) is at least a possible non-dimensional form of the local boundary layer momentum thickness which might be suitable as a criterion for defining the optimum suction distribution in a region over which the pressure recovery is large. Ultimately however, the suitability of a suction criterion must depend on whether it predicts the suction distribution which requires minimum suction power in order to maintain unseparated flow. As already stated for the case of a region of very severe pressure recovery for which the initial conditions have a negligible influence on the total suction power required, it is reasonable to suppose that the condition of minimum total suction power might be replaced by a local condition. This supposition was the basis for the proposed condition $\frac{d}{d\theta}\left(\frac{d\theta}{dx}\right) = 0$. If it is assumed that (Γ/G) is constant, the local suction velocity ratio can be expressed using equation (2.1) as

$$\frac{v_s}{U} = (H+2) \cdot \left(-\frac{\theta}{U}\frac{dU}{dx}\right) + \frac{G}{R_\theta^{\,m}} - \frac{d\theta}{dx} \qquad (2.1)$$

$$= \left(H+2+\frac{G}{\Gamma}\right) \cdot -\frac{1}{U}\frac{dU}{dx}\left[\frac{\Gamma/G \cdot G}{-\frac{1}{U}\frac{dU}{dx}\,(U/\nu)^m}\right]^{1/m+1} - \frac{d}{dx}\left\{\frac{(\Gamma/G)\cdot G}{-\frac{1}{U}\frac{dU}{dx}\,(U/\nu)^m}\right\}^{1/m+1} \qquad (2.8)$$

Assuming that, as $p\to\infty$, the condition of minimum overall suction power can be replaced by that of minimum suction per unit chord, one can write an equation defining $(\Gamma/G)_{opt}$ thus,

$$\frac{\partial v_s}{\partial(\Gamma/G)} = 0 \qquad (2.8a)$$

The term involving $\frac{d}{dx} \cdot G(v_s/U, H) = \frac{dG}{d(\frac{v_s}{U})} \cdot \frac{d(\frac{v_s}{U})}{dx} + \frac{dG}{dH} \cdot \frac{dH}{dx}$ will be neglected as a first approximation on the basis that the derivatives of both suction velocity ratio and shape factor with respect to streamwise position are small. For the case in which (Γ/G) is the appropriate criterion, the shape factor will be constant at approximately the value appropriate to a flat plate ($H=1.4$) and it can be seen a posteriori from calculations in Section (2.4) that the suction velocity ratio is approximately constant once suction is established.

From Eq. (2.8) it can be seen that

$$\left(\frac{\Gamma}{G}\right)_{\substack{opt. \\ p\to\infty}} = \frac{m}{\left[H+2+\frac{1}{(m+1)}\left\{1-m-\frac{U\frac{d^2U}{dx^2}}{\left(\frac{dU}{dx}\right)^2}\right\}\right]} \qquad (2.8\cancel{a}\,b)$$

The similarity between equations (2.7a) and (2.8b) is noted. The two equations predict values of $(\Gamma/G)_{\substack{opt. \\ p\to\infty}}$ which are in close agreement if the chordwise velocity distribution is such that the numerical value of $\left(\frac{U\ddot{U}}{\dot{U}^2}\right)$ is of the order of unity or less. The chordwise variation of $\left(\frac{U\ddot{U}}{\dot{U}^2}\right)$ can only be investigated for a specific variation of streamwise velocity outside the boundary layer. The doubly infinite family of velocity distributions of Eq. (2.9) which join the end points $U = pU_o$, $x/c = 0$ and $U = U_o$, $x/c = 1$ can be used to represent an approximation to the large variety of possible monotonic velocity distributions on the upper surface of a wing. Eq. (2.9) is plotted in Fig. (2.1) for a range of values of q and r and fixed value of $p = 7$.

$$t = \frac{U}{pU_o} = \left[1-\left(\frac{p^{1/q}-1}{p^{1/q}}\right)\left(\frac{x}{c}\right)^r\right]^q \qquad (2.9)$$

r will in general, be greater than unity corresponding to a maximum in the local velocity distribution at $x/c = 0$, and q will **normally** be negative. Using Eq. (2.9) $(\Gamma/G)_{\substack{opt \\ p\to\infty}}$ can be written

$$\left(\frac{\Gamma}{G}\right)_{\substack{opt \\ p\to\infty}} = \frac{m}{H+2+\frac{(1-m)}{(1+m)}-\frac{1}{(m+1)}\cdot\left\{\left(\frac{q-1}{q}\right)-\frac{1}{q^{r}}\cdot\frac{(r-1)}{(t^{-1/q}-1)}\right\}} \quad (2.10)$$

Whereas the value of $(\Gamma/G)_{opt}$ as defined by Eq. (2.10) is dependent on the streamwise position, it can easily be shown that this dependence is only weak and that, for a wide range of values of q and r, the values of $(\Gamma/G)_{\substack{opt \\ p\to\infty}}$ predicted by equations (2.7a) and (2.10) are similar. For the case of $r = 1$ the dependence in Eq. (2.10) on streamwise position (t) vanishes.

The preceding analysis which is based on the alternative conditions $\frac{d}{d\theta}\left(\frac{d\theta}{dx}\right) = 0$ or $\frac{dv_s}{d(\Gamma/G)} = 0$, is indicative of the acceptability of $(\Gamma/G) =$ constant as a criterion, at least for the case of a prolonged and severe pressure gradient. In order to consider the problem more rigorously it is necessary to derive the optimum value of (Γ/G) by minimising the overall suction power required. In order to discuss the two cases of minimum idealised suction power and minimum suction quantity within the framework of a single analysis, a generalised suction coefficient (C_Q) will be considered as defined by Eq. (2.11),

$$C_Q = \int\left(\frac{v_s}{U}\right)\cdot\left(\frac{U}{U_\infty}\right)^{2n+1}\cdot\left(1 - n\,\frac{U_\infty^2}{U^2}\right) d\left(\frac{x}{c}\right) \quad (2.11)$$

For $n = 0$, C_Q refers to the suction quantity coefficient and for n=1, to the suction power coefficient of the ideal system discussed in Section (2.1).

Consider again, the case of α monotonic and asymptotically severe and prolonged adverse pressure gradient. As previously stated the extent of impervious surface before suction begins will be small and will not significantly affect the optimum value of (Γ/G). Thus, using Eq. (2.11) to define C_Q and neglecting the term involving dG/dx as before, the value of $(\Gamma/G)_{opt}$ for a large pressure recovery factor (p) can be derived as in Eq. (2.11) using limits of integration $x/c = 0$ to $x/c = 1$. For smaller values of recovery factor (p), the dependence of the limits of integration on the value of (Γ/G) must be considered

$$\frac{\partial C_Q}{\partial(\Gamma/G)} = \frac{\partial}{\partial(\Gamma/G)}\int_{\frac{x}{c}=0}^{1}\left[\left(\frac{G}{\Gamma} + H + 2\right)\cdot\left(-\frac{1}{U}\frac{dU}{dx}\right)\left\{\frac{\Gamma/G.G}{\frac{1}{U}.\frac{dU}{dx}(U/\nu)^m}\right\}^{1/m+1} - \frac{d}{dx}\left\{\frac{\Gamma/G.G}{-\frac{1}{U}\frac{dU}{dx}(U/\nu)^m}\right\}^{1/m+1}\right]\left(\frac{U}{U_\infty}\right)^{2n+1}\left(1-n\frac{U_\infty^2}{U^2}\right)d\left(\frac{x}{c}\right) = 0 \tag{2.12a}$$

$$\text{Writing } \frac{1}{I} = \frac{\int_0^1\left\{\frac{1}{-\frac{1}{U}.\frac{dU}{dx}(U/\nu)^m}\right\}^{1/m+1}\left(\frac{U}{U_\infty}\right)^{2n+1}\left[1-n\left(\frac{U_\infty}{U}\right)^2\right]d\left(+\frac{U}{U_\infty}\right)}{\int_0^1\left(\frac{U}{U_\infty}\right)^{2n+1}.\left\{1-n\left(\frac{U_\infty}{U}\right)^2\right\}.d\left\{\frac{1}{-\frac{1}{U}\frac{dU}{dx}(U/\nu)^m}\right\}^{1/m+1}} \tag{2.12b}$$

the suction power coefficient of the ideal system discussed in Section (2.1).

Consider again, the case of a monotonic and asymptotically severe and prolonged adverse pressure gradient. As previously stated the extent of impervious surface before suction begins will be small and will not significantly affect the optimum value of (Γ/G). Thus, using Eq. (2.11) to define C_Q and neglecting the term involving dG/dx as before, the value of $(\Gamma/G)_{opt}$ for a large pressure recovery factor (p) can be derived as in Eq. (2.11) using limits of integration $x/c = 0$ to $x/c = 1$. For smaller values of recovery factor (p), the dependence of the limits of integration on the value of (Γ/G) must be considered

$$\frac{\partial C_Q}{\partial(\Gamma/G)} = \frac{\partial}{\partial(\Gamma/G)} \int_{\frac{x}{c}=0}^{1} \Bigg[\left(\frac{G}{\Gamma} + H + 2\right) \cdot \left(-\frac{1}{U}\frac{dU}{dx}\right) \left\{\frac{\Gamma/G . G}{-\frac{1}{U}\frac{dU}{dx}(U/\nu)^m}\right\}^{1/m+1} - \frac{d}{dx}\left\{\frac{\Gamma/G . G}{-\frac{1}{U}\frac{dU}{dx}(U/\nu)^m}\right\}^{1/m+1} \Bigg] \left(\frac{U}{U_\infty}\right)^{2n+1} \left(1 - n\frac{U_\infty^2}{U^2}\right) d\left(\frac{x}{c}\right) = 0 \tag{2.12a}$$

Writing

$$\frac{1}{I} = \frac{\int_0^1 \left\{\frac{1}{-\frac{1}{U}\frac{dU}{dx}(U/\nu)^m}\right\}^{1/m+1} \left(\frac{U}{U_\infty}\right)^{2n+1} \left[1 - n\left(\frac{U_\infty}{U}\right)^2\right] d\left(\frac{U}{U_\infty}\right)}{\int_0^1 \left(\frac{U}{U_\infty}\right)^{2n+1} \cdot \left\{1 - n\left(\frac{U_\infty}{U}\right)^2\right\} \cdot d\left\{\frac{1}{-\frac{1}{U}\frac{dU}{dx}(U/\nu)^m}\right\}^{1/m+1}} \tag{2.12b}$$

we have from Eq. (2.12)

$$\left(\frac{\Gamma}{G}\right)_{\substack{opt\\ p\to\infty}} = \frac{m}{(H+2+I)} \tag{2.13}$$

In order to evaluate I and show that it is small compared to (H+2), it is necessary to assume a form for the streamwise variation of local velocity outside the boundary layer. Using Eq. (2.9) and Eq. (2.12b) it is possible to calculate the value of the parameter I as a function of n, p, q and r. This integration is undertaken in Section (2.4) for n = 0 and 1 and specific values of p, q and r and in general, it is found that the value of I never greatly exceeds (H+2) thereby confirming the order of $(\Gamma/G)_{\substack{opt\\ p\to\infty}}$

The preceding analysis shows that the value of $(\Gamma/G)_{\substack{opt\\ p\to\infty}}$ predicted for a region of prolonged and severe pressure recovery by minimising the generalised suction quantity coefficient is closely related to that derived from the intuitive condition $d/d\theta\,(d\theta/dx) = 0$. There are also strong indications that this value of $(\Gamma/G)_{opt}$ controls the boundary layer such that it will not separate. Consider now, the implications of reducing the severity of the overall recovery factor (p). If the value of (Γ/G) were increased, it would result in an increase in the streamwise extent of impervious surface before suction begins, with a consequent saving in C_Q. This increased value of (Γ/G) will result in a correspondingly increased value of suction velocity once suction is established but, if this is more than offset by the saving introduced by prolonging the extent of impervious surface, a nett reduction in C_Q will result. Thus, a reduction in the severity of the overall recovery factor results in an increase in the optimum value of (Γ/G). Mathematically, this effect is introduced as shown in Section (2.4) by considering the limits of the integral expression for C_Q as variables which are dependent on (Γ/G) thereby introducing an additional term into the Eq. (2.12) defining $(\Gamma/G)_{opt}$. As the value of $(\Gamma/G)_{opt}$ increases so does the influence of the adverse pressure gradient relative to the skin friction term and with it the tendency for the shape factor to increase, until ultimately the value of $(\Gamma/G)_{opt}$ is such that it does not preclude the possibility of separation. At this stage (Γ/G) ceases to represent a suitable criterion. Boundary layer separation is possible as soon as the tendency for the shape parameter (H) to increase under the influence of the adverse pressure gradient exceeds that for it to decrease due to suction. Under the influence of suction the boundary layer shape factor will be maintained at approximately the value appropriate to a flat plate as long as (Γ/G) is of the order of 0.079 or less as defined by Eq. (2.5). It is clear that the development of a boundary layer under the condition (Γ/G) = constant at a numerical value such that $\left(\frac{dH}{dx}\right)$ is equal to, or just greater than zero, is closely related to the boundary layer development under the condition of constant shape factor of about 1.4.

As the severity of the adverse gradient is further reduced, the appropriate criterion defining the optimum suction distribution becomes (H) = constant. The greater the value of H_{opt} at which the boundary layer is maintained, the greater will be the extent of impervious surface before suction begins, but the boundary layer thickness throughout the region of suction will be correspondingly greater. As a result of this increased boundary layer thickness the suction velocities will be proportionately increased, but if the streamwise extent of suction is sufficiently small, it is possible that a nett saving in suction power could result. Thus, as the severity of the overall pressure gradient decreases so the optimum value of shape factor increases. Ultimately, this value of the shape factor approaches that appropriate to separation, but simultaneously the streamwise extent of the suction decreases to zero. Hence, in the limiting case which corresponds to a large value of shape factor, no suction is necessary as the region of impervious surface has extended to the downstream end of the region of adverse gradient.

The argument so far can be summarised as follows. The criterion defining the optimum distribution of suction is stated as a limit on the value of a local boundary layer parameter. The optimum distribution of suction is taken as that which requires the minimum suction power in order to achieve a given value of lift coefficient. It is assumed that there are only two independent parameters which define the state of a turbulent boundary layer. These are momentum thickness (θ) and mean velocity profile shape factor (H). Any suitable boundary layer parameter must be non-dimensional in order that it should be independent of the linear scale of the suction system and must also control the boundary layer such that a separation is avoided. Numerical values of shape factor $H < 2.6$ satisfy both these conditions. Momentum thickness might logically be non-dimensionalised using suitable derivatives of the adverse streamwise pressure distribution. The ratio $(\Gamma/G) = \frac{\text{pressure forces}}{\text{surface shearing forces}}$ as defined by Eq. (2.4) suggests itself as a possible non-dimensional form of momentum thickness. An indication of the suitability of such a criterion is obtained from consideration of the case in which the overall recovery factor (p) is large. In this case, the condition of minimum total suction power might logically be replaced by a local condition which ensures that the contribution to the total suction power from every streamwise position is a minimum. Also, for such a value of recovery factor $(p \to \infty)$ the arbitrary initial conditions do not significantly affect the suction power required and hence do not influence the optimum suction distribution. Consideration of which local condition might be suitable as a replacement for that of minimum total suction quantity leads one to Eq. (2.6) which states $\frac{d}{d\theta}\left(\frac{d\theta}{dx}\right) = 0$. This condition is suggested as it maintains a balance between the rate of growth of boundary layer thickness due to the adverse pressure gradient and that due to the surface shearing force. In order to confirm that the value of $(\Gamma/G)_{opt}$ predicted by this means will in fact maintain unseparated flow, reference is made to the auxiliary Eq. (2.3) derived by Spence for an impervious surface from consideration of the variation of shape factor in zero pressure gradient and in a severe adverse pressure gradient. Considering further the case of large overall pressure recovery factor $(p \to \infty)$ it is shown that the values of $(\Gamma/G)_{opt}$ derived from the conditions $\partial v_s/\partial(\Gamma/G) = 0$ (Eq. 2.8a) or $\partial C_Q/\partial(\Gamma/G) = 0$ (Eq. 2.12) are both closely related to that predicted by the intuitive condition $\frac{d}{d\theta}\left(\frac{d\theta}{dx}\right) = 0$. It is then shown that a reduction in the value of recovery factor (p) results in an increase in the value of $(\Gamma/G)_{opt}$ as defined from considerations of minimum C_Q until the tendency for shape factor to increase under the influence of the adverse pressure gradient exceeds that for it to decrease due to suction. Shape factor (H) = constant at a value defined by the condition $dC_Q/dH = 0$ then becomes the appropriate criterion. As the value of (p) decreases so H_{opt} increases until the case is reached in which the boundary layer is about to separate at the downstream end of the region of pressure recovery and therefore no suction is required. This argument does not consider the possibility of stall due to a leading edge laminar separation. Under these circumstances it is simply necessary to precipitate transition using any of the accepted techniques and no extra suction is required.

Consider now, the case of a flapped aerofoil section. The pressure distribution on the upper surface of a flapped aerofoil is composed of two separate regions of adverse pressure gradient connected by a region of favourable gradient immediately upstream of the flap knuckle. In this case, the condition of the boundary layer and in particular, the initial values of shape factor (H) and momentum thickness (θ) at the beginning of the second region of adverse pressure gradient is directly dependent on the value of the appropriate criterion considered optimum for the first region of pressure recovery immediately downstream of the leading edge suction peak. Thus, there is an interpendence of the two regions of suction which, although it may not be strong, does mean that the optimum suction distribution for each region cannot be defined without reference to the other region. The degree of interdependence decreases as the peak suction over the flap knuckle increases due to the isolating influence of the region of favourable pressure gradient forward of the flap knuckle. There is also the well known interdependence of the local surface pressure distributions in that the increased circulation associated with a flapped aerofoil results in an increase of induced incidence which modifies the pressure distribution in the vicinity of the leading edge suction peak. This is not a boundary layer effect and is not considered further.

The proposed approach of assuming H or (Γ/G) to be constant in the region of suction has one apparent shortcoming in that, considering any numerical value of H_{opt} or $(\Gamma/G)_{opt}$ as determined by minimising C_Q, it is possible to relax the suction in the immediate vicinity of the trailing edge and thereby allow the value of the shape factor to increase to that appropriate to separation at the trailing edge. There is no means of allowing for this potential reduction in suction quantity within the framework of the present analysis, but this may not be important for two reasons:-

(i) If the suction is designed to give a large increment of C_L in excess of the normal stalled value, the proportional saving in suction power will be small. If, on the other hand, the suction distribution is designed to give a small increment of C_L above the normal stalled value, the value of H_{opt} will not differ greatly from that appropriate to a separated boundary layer.

(ii) The loss of lift coefficient (or diffuser pressure recovery factor) caused by the additional thickening of the boundary layer due to the relaxation of suction towards the downstream end of the region of adverse gradient will tend to offset the saving in suction power.

2.4. Detailed calculations for a streamwise variation of velocity outside the boundary layer appropriate to an unflapped wing or simple diffuser.

This section describes a series of calculations which illustrate the more general arguments of the previous section. These calculations are undertaken for streamwise variations of velocity outside the boundary layer which are special cases of Eq. (2.9). The initial calculations are directed towards a closer investigation of the value of $(\Gamma/G)_{opt}$ for large values of recovery factor (p), taking into consideration the interdependence of the limits of the integration on the value of (Γ/G). It is shown that the case $r = 1$ gives somewhat misleading results due to the initial condition $dU/dx \neq 0$ at $x = 0$ which is physically unrealistic. For $r > 1$, $dU/dx = 0$ at $x = 0$ and this anomaly disappears in that $(\Gamma/G)_{opt}$ approaches the value predicted by assuming fixed limits in the integration of suction velocity to obtain C_Q.

Further calculations are restricted to the special case $r = 1$ (Eq. 2.9) which results in a considerable degree of analytical simplification in that the majority of the integrals can be obtained in closed form. The case $r = 1$ can only be representative of an actual velocity distribution by referring it to an origin aft of the peak suction and the initial values of momentum thickness and shape factor would then have to be adjusted accordingly. C_Q is calculated as a function of C_L for the velocity distributions corresponding to $r = 1$, and $q = \pm 1$ in order to investigate the dependence of this relationship on the streamwise velocity distribution.

2.4.1. Consideration of regions over which the pressure recovery factor is large and hence (Γ/G) is the appropriate criterion.

Using the following assumptions:

(a) (Γ/G) = constant throughout a region of suction is the appropriate suction criterion.

(b) As the first stage of an iterative process, the terms derived from the variation of the coefficient (G) of the skin friction relationship with streamwise position can be neglected.

(c) The streamwise variation of velocity outside the boundary layer is defined by Eq. (2.9). Consideration will be restricted to the values $r \geqslant 1$, $q \leqslant 1$ in that these are more representative of velocity distributions which are practically significant.

With these assumptions, Appendix (1) shows that the suction velocity ratio can be written

$$\left(\frac{v_s}{U}\right) = \left[\frac{(-qr)\left(p^{-\frac{1}{q}}-1\right)^{1/r}}{pR_c}\right]^{m/m+1} \cdot \left(\frac{\Gamma}{G}\right)^{1/m+1} \cdot G^{1/m+1} \cdot t^{-\frac{1}{q}\left\{1-\frac{(1-qm)}{m+1}\right\}} \cdot \left(t^{\frac{1}{q}}-1\right)^{(m/m+1)\left(\frac{r-1}{r}\right)} \quad x$$

$$\left[\left(H+2+\frac{\Gamma}{G}+\frac{\Gamma}{r}\right) + \frac{(1-qm)}{q(m+1)} + \frac{(1-1/r)}{q(m+1)\left(t^{-1/q}-1\right)}\right] \qquad (2.14)$$

It can be seen from Eq. (2.14) that as the boundary layer develops (i.e., as the value of t decreases) so the term $\frac{(1-1/r)}{q(m+1)(t^{-\frac{1}{q}}-1)}$ decreases in magnitude and it follows that, once suction has been established, it is necessary for it to extend to the trailing edge ($x/c = 1$, $t = 1/p$). Thus, the only variable limit of integration to be considered is the upstream one corresponding to the beginning of suction. This statement cannot be made generally for any monotonically decreasing external velocity distribution but will be so for the majority of chordwise velocity distributions found in practice.

Eq. (2.11) is used to define the generalised suction quantity coefficient (C_Q) and substituting for (v_s/U) from Eq. (2.14), the equation defining the optimum value of (Γ/G) can be expressed as Eq. (A1.5) (Appendix I). Appendix I shows the details of the calculation from which can be derived the value of $(\Gamma/G)_{opt}$ by means of a laborious process of iteration. It also shows that for $r > 1$ and $p \to \infty$, the first ~~second~~ term of Eq. (A.1.5) ~~(2.15)~~ vanishes and the optimum value of $(\Gamma/G)_{opt}$ can be obtained by neglecting the dependence of the limits of integration on the value of (Γ/G). It can also be seen that the anomalous behaviour of the case $r = 1$, $p \to \infty$ in this respect is derived from the fact that for $r = 1$, $\frac{dt}{d(x/c)} \to \infty$ at $x/c = 0$ as $p \to \infty$ and hence the contribution of the second term in Eq. (A1.5) no longer vanishes for large values of recovery factor (p).

Due to the complexity of the integration processes associated with the case of an arbitrary value of $r > 1$, further numerical calculations of $(\Gamma/G)_{opt}$ have been restricted to the case $r = 1$. Fig. (2.2) shows the decrease in $(\Gamma/G)_{opt}$ with increasing values of recovery factor (p), for $q = \pm 1$ and $n = 0$ and 1. For the purpose of these calculations the dependence of $G(v_s/U)$ on suction velocity is neglected as the first stage of an iterative process. The considerable difference between the case $n = 0$ and $n = 1$, which correspond to a minimum suction quantity and minimum idealised suction power respectively, is due to the strong sensitivity of the latter to any flow removed from a region of low static pressure (i.e., high local velocity outside the boundary layer). Hence, the optimum suction distribution for minimum idealised suction power corresponds to a larger value of $(\Gamma/G)_{opt}$ which implies that the onset of suction is delayed and correspondingly, less fluid is removed from a region of low static pressure.

Fig. (2.3) shows the variation of generalised suction quantity coefficient with recovery factor (p) for $q = \pm 1$, $n = 0$ and 1 obtained by inserting values of $(\Gamma/G)_{opt}$ in the expression for C_Q. Fig. (2.3) also shows the value of C_Q obtained by assuming $(\Gamma/G)_{opt} = m/(H+2)$ and it can be seen that the error introduced by using this approximation is small. The insensitivity of C_Q to the precise value of (Γ/G) is clearly of considerable practical significance.

It is noted that an approximate relationship between $C_Q(n = 0)$ and $C_Q(n = 1)$ can be obtained by assuming that suction begins at $x/c = 0$ and noting from equation (2.14) that $(v_s/U) = t^{-(m/m+1).(q+1)/q}$ for $r = 1$. Thus,

$$\frac{\text{Idealised suction power required}}{\text{Power required for single stage suction unit}} = \frac{C_Q(n{=}1)}{C_Q(n{=}0).(p^2-1)} \simeq \frac{\int_0^1 \frac{v_s}{U}.\left(\frac{U}{U_\infty}\right)\left\{\left(\frac{U}{U_\infty}\right)^2 - 1\right\} d\left(\frac{x}{c}\right)}{(p^2-1)\int_0^1 \left(\frac{v_s}{U}\right).\left(\frac{U}{U_\infty}\right) d\left(\frac{x}{c}\right)}$$

$$= \frac{p^2}{(p^2-1)} \cdot \left\{ \frac{\frac{1}{q}+1 - \left(\frac{m}{m-1}\right)\left(\frac{1+q}{q}\right)}{\frac{1}{q}+3 - \left(\frac{m}{m+1}\right)\left(\frac{1+q}{q}\right)} \right\} \cdot \left[\frac{1 - \left(\frac{1}{p}\right)^{3+\frac{1}{q} - \left(\frac{m}{m+1}\right)\left(\frac{1+q}{q}\right)}}{1 - \left(\frac{1}{p}\right)^{1+\frac{1}{q} - \left(\frac{m}{m+1}\right)\left(\frac{1+q}{q}\right)}} \right] - \frac{1}{(p^2-1)}$$

The value of this ratio may be as low as 1/3 which gives an indication of the potential economies in suction power that are ideally possible by using a multi-stage suction unit. However, as the duct pressure losses have a considerable influence on the suction pressure required, this apparent economy cannot be realised to its full extent in practice.

The calculation so far has been pursued on the assumption that the skin friction coefficient is independent of suction velocity. This is clearly not the case and, in order to be able to correct for this, it is necessary to consider the dependence of C_Q on the effective value of skin friction coefficient. From Eq. (2.14) we see that $C_Q \propto \left(\frac{v_s}{U}\right)\alpha(G)^{1/m+1}$. It was further assumed in Eq. (2.2b) that (G) was linearly dependent on suction velocity ratio. From Eq.(2.14) it can be seen that for $r = 1$, the streamwise variation of suction velocity ratio $\frac{v_s}{U} \propto t^{-\frac{m}{m+1}\left(\frac{1+q}{q}\right)}$ is small and, in the interests of simplicity, it was decided to readjust the value of (G) on the basis of a mean suction velocity ratio $\overline{(v_s/U)}$ defined thus,

$$\overline{\left(\frac{v_s}{U}\right)} \cdot \int \left(\frac{U}{U_\infty}\right)^{2n+1} \left\{1 - n\left(\frac{U_\infty}{U}\right)^2\right\} d\left(\frac{x}{c}\right) = \int \frac{v_s}{U} \cdot \left(\frac{U}{U_\infty}\right)^{2n+1} \left\{1 - n\left(\frac{U_\infty}{U}\right)^2\right\} \cdot d\left(\frac{x}{c}\right)$$

Hence,

$$\left(\frac{\overline{v_s}}{U}\right) = \frac{C_Q}{\dfrac{p^{2n+1+\frac{1}{q}}}{q\left(p^{\frac{1}{q}}-1\right)} \displaystyle\int_{1/p}^{t_1} t^{2n+\frac{1}{q}}\left(1 - n/p^2t^2\right) dt} \tag{2.16}$$

Using Eq. (2.2b) to define the dependence of (G) on suction velocity ratio, one obtains

$$G'\left(\frac{v_s}{U}\right) = \left[1 + \left(\frac{\bar{v}_s}{U}\right)\Big/ 0.01\right] \cdot G\left(\frac{v_s}{U} = 0\right) \tag{2.17}$$

and hence the corrected value of C_Q' can be written

$$C_Q' = \left(\frac{G'}{G}\right)^{1/m+1} \cdot \; C_Q$$

$$= \left[1 + \frac{C_Q/0.01}{\left\{\dfrac{p^{2n+1+\frac{1}{q}}}{q\left(p^{\frac{1}{q}}-1\right)} \displaystyle\int_{1/p}^{t_1} t^{2n+\frac{1}{q}}\left(1 - n/p^2t^2\right) dt\right\}}\right]^{1/m+1} \cdot \; C_Q$$

The effect on the value of C_Q of incorporating the dependence of skin friction coefficient on suction velocity ratio is seen to be quite significant (Fig. 2.3). However, the order of C_Q remains the same. As systematic experimental results defining the dependence of skin friction coefficient on suction velocity ratio become available, this relationship can be predicted more accurately.

2.4.2. Less prolonged and severe regions of pressure rise for which shape factor (H) = constant is the appropriate criterion.

As the severity of the overall pressure recovery is progressively reduced, it becomes profitable to increase the extent of impervious surface prior to the beginning of suction, at the expense of a higher value of suction velocity for the limited extent over which suction is applied. As is seen in the previous Section (2.4.1), this results firstly in an increase in the optimum numerical value of $(\Gamma/G)_{opt}$ until ultimately, the value of this parameter is so large that it does not safeguard the boundary layer against separation.

At this stage, it is necessary to replace this criterion by one which restricts the cumulative effect of the pressure gradient term (Γ/G) on shape factor, rather than its numerical value. Thus, shape factor (H) becomes the appropriate criterion and its optimum numerical value increases as the overall recovery factor (p) is relaxed. This increase in the value of H_{opt} corresponds to a protracted extent of impervious surface which more than offsets the higher suction velocities which are necessary over the limited extent of porous surface.

Two alternative approaches are made to the problem of predicting the streamwise development of a turbulent boundary layer under the influence of suction distributed such that the value of the shape factor (H) remains constant. The first approach is based on a recent publication by M.R. Head (1958) in which he derives an extended form of the auxiliary equation for shape factor which includes terms associated with a porous surface. The second approach is based on the effect of a discrete suction strip on a boundary layer as derived experimentally in Section (1). In the limit it is assumed that, a large number of line sinks spaced closely together approaches the case of continuous suction. The advantage of this approach is that it offers a means of investigating the effects of relatively widely spaced suction slots with correspondingly large intermediate variations in shape factor. Practically, this is an important case.

2.4.2.1. Calculations of the distribution of suction such that the shape factor is constant using Head's method (1958).

The auxiliary equation presented by M.R. Head (1958) for the development of a turbulent boundary layer under the influence of suction can be written,

$$\theta \frac{dH}{dx} = \left(\frac{dH}{dH'}\right) \cdot \left[F - \left(\frac{V_s}{U}\right) + H' \cdot \left(-\frac{\theta}{U}\frac{dU}{dx}\right) - H' \frac{d\theta}{dx} \right] \qquad (2.18a)$$

where F is the entrainment/per unit area of non-turbulent fluid and is defined empirically as a function of shape factor (H). Assuming a single parameter family of mean velocity profiles, H' and hence (dH'/dH)

are functions solely of H. Eq. (2.18a) together with the momentum equation (2.1) and the condition $dH/dx = 0$ if $(v_s/U) \neq 0$ provide sufficient equations to define the distribution of suction for an arbitrary variation of static pressure along the surface. Thus, substituting into Eq. (2.18a) from (2.1) one obtains,

$$\theta \frac{dH}{dx} = \left(\frac{dH}{dH'}\right) \cdot \left[F + \frac{v_s}{U} \cdot (H'-1) - H'(H+1) \cdot \left(-\frac{\theta}{U}\frac{dU}{dx}\right) - H' \frac{\tau_o}{\rho U^2}\right]$$

Using the condition $dH/dx = 0$ where $v_s/U \neq 0$, and as $dH'/dH \neq 0$, the auxiliary equation can be written

$$F - H'(\tau_o/\rho U^2) + \frac{v_s}{U}(H'-1) - H'(H+1) \cdot \left(-\frac{\theta}{U}\frac{dU}{dx}\right) = 0 \qquad (2.18b)$$

Substituting Eq. (2.18b) into the momentum equation one obtains,

$$\frac{d\theta}{dx} = -\frac{\theta}{U}\frac{dU}{dx} \cdot (H+2) - \left(\frac{v_s}{U}\right) + \left(\frac{\tau_o}{\rho U^2}\right) \qquad (2.1)$$

or

$$\frac{d\theta}{dx} = \left\{\frac{H'-H-2}{H'-1}\right\} \cdot \left(-\frac{\theta}{U}\frac{dU}{dx}\right) + \left(\frac{F - H'\frac{\tau_o}{\rho U^2}}{H'-1} + \frac{\tau_o}{\rho U^2}\right) \qquad (2.19)$$

At this stage it is necessary to make an assumption with regard to the value of skin friction coefficient. As the boundary layer will develop under these circumstances primarily under the influence of the adverse pressure gradient and the suction velocity, it is proposed as a first approximation to represent the contribution of the skin friction terms by assuming it to be constant along the chord at a value appropriate to the $R_e$ynolds number and shape factor considered. As a first approximation, the value used is that predicted by the equation (2.2) of Ludwieg and Tillmann (1949) for an impervious surface, using the appropriate values of shape factor (H) and local boundary layer Reynolds number based on momentum thickness. Once the suction quantity required has been calculated, a second approximation is obtained by suitably modifying the value of the skin friction coefficient according to Eq. (2.17).

Integrating Eq. (2.19) for momentum thickness one obtains

$$\theta = \theta_1\left(\frac{U_1}{U}\right)^{\alpha} + \left(\frac{F - H'\tau_0/\rho U^2}{H'-1} + \frac{\tau_0}{\rho U^2}\right)\cdot\frac{1}{U^{\alpha}}\cdot\int_{x_1}^{x} U^{\alpha}\,dx \qquad (2.20a)$$

where,

$$\alpha = \left(\frac{H'-H-2}{H'-1}\right)$$

Suffix (1) refers to conditions at the beginning of suction. Substituting this expression for momentum thickness into Eq. (2.18b), one obtains an expression for suction velocity in terms of the distribution of velocity outside the boundary layer and the conditions at the beginning of the region of suction.

$$\left(\frac{v_s}{U}\right) = \left(-\frac{1}{U}\frac{dU}{dx}\right)\cdot\frac{H'(H+1)}{(H'-1)}\left[\theta_1\left(\frac{U_1}{U}\right)^{\alpha} + \left(\frac{F - H'\frac{\tau_0}{\rho U^2}}{H'-1} + \frac{\tau_0}{\rho U^2}\right)\cdot\frac{\int_{x_1}^{x} U^{\alpha}\,dx}{U^{\alpha}}\right] - \left\{\frac{F - H'(\tau_0/\rho U^2)}{H'-1}\right\}$$

(2.21a)

Again considering the special case of the infinite family of streamwise velocity distributions of Eq. (2.9) with r = 1, Eqs. (2.20a) (2.21a) can be written

$$\left(\frac{\theta}{c}\right) = \left(\frac{\theta_1}{c}\right)\left(\frac{t_1}{t}\right)^{\alpha} + \left\{\frac{(F-H'\tau_0/\rho U^2)}{(H'-1)} + \frac{\tau}{\rho U^2}\right\}\frac{p^{1/q}}{q(p^{1/q}-1)} \cdot \frac{t^{1/q}}{(\alpha+1/q)}\left\{\left(\frac{t_1}{t}\right)^{\alpha+\frac{1}{q}} - 1\right\} \tag{2.20b}$$

$$\left(\frac{v_s}{U}\right) = \frac{H'(H+1)}{(H'-1)} \cdot \frac{q(p^{1/q}-1)}{p^{1/q}} \cdot t^{-1/q} \cdot \left(\frac{\theta}{c}\right) - \left(\frac{F-H'\tau_0/\rho U^2}{H'-1}\right)$$

(2.21b) ~~(2.2b)~~

It follows from Eq. (2.11) that the generalised suction quantity coefficient can be written

$$C_Q = \int \left(\frac{v_s}{U}\right)\left(\frac{U}{U_\infty}\right)^{2n+1}\left[1 - n\left(\frac{U_\infty}{U}\right)^2\right] d\left(\frac{x}{c}\right)$$

or,

$$\frac{C_Q}{\dfrac{p^{(1/q)+1+2n}}{q(p^{1/q}-1)}} = \frac{H'(H+1)}{(H'-1)} \cdot t_1^{2n+1} \cdot \left[\left\{\frac{1-(1/pt_1)^{2n+1-\alpha}}{2n+1-\alpha}\right\} - \frac{n}{p^2 t_1^2}\left\{\frac{1-(1/pt_1)^{2n-1-\alpha}}{2n-1-\alpha}\right\}\right] \times$$

$$\times \left[\left(\frac{\theta_1}{c}\right)\frac{q(p^{1/q}-1)}{p^{1/q}} + \frac{F-\tau_0/\rho U^2}{H'-1}\,\frac{t_1^{1/q}}{(\alpha+1/q)}\right]$$

$$-t_1^{2n+1+1/q}\left[\frac{H'(H+1)}{(H'-1)} \cdot \frac{(F-\tau_0/\rho U^2)}{(H'-1)}\,\frac{1}{(\alpha+1/q)} + \left\{\frac{F-H'(\tau_0/\rho U^2)}{H'-1}\right\}\right] \times$$

$$\times \left[\left\{\frac{1-(1/pt_1)^{2n+1+1/q}}{2n+1+1/q}\right\} - \frac{n}{p^2 t_1^2}\left\{\frac{1-(1/pt_1)^{2n-1+1/q}}{2n-1+1/q}\right\}\right] \tag{2.22}$$

The initial values of t_1 and (θ_1/c) for a given value of shape factor (H) can be derived using any of the well known methods which describe the development of a boundary layer on an impervious surface, once the initial conditions of H_o and $(\theta/c)_o$ at $(x/c) = 0$ are given. In order to be consistent, Eq. (2.18a) is used as the auxiliary equation whilst Eq. (A2.1) derived by Spence (1958) is used to calculate the development of the momentum thickness.

Rather than differentiate the expression for C_Q in Eq. (2.22), which is difficult due to the empirical relationships between H, H' and F which are expressed in graphical form only (Head, 1958), it was thought preferable to estimate values of C_Q for a range of values of shape factor (H) and overall recovery factor (p). From Fig. (2.4), it is possible to define the optimum numerical value of shape factor and the associated minimum value of suction quantity coefficient as a function of recovery factor (p) for a series of streamwise velocity distributions corresponding to values of $q = \pm 1$.

From Fig. (2.4) it can be seen that, for a given value of recovery factor (p), the optimum numerical value of shape factor is dependent on the chordwise distribution of velocity outside the boundary layer, (i.e., the value of q) but the general trend of a decreasing value of H_{opt} with increasing recovery factor (p) is confirmed.

2.4.2.2. Calculation of the boundary layer development with discrete suction strips distributed such that the value of shape factor (H) varies between pre-determined limits.

As a result of the experimental investigation of the effect of a suction slot on a boundary layer as described in Section (1), it was shown that:-

(i) The effect of a line sink on the value of shape factor (H) can be represented by removing from the inner part of the mean velocity profile, an amount corresponding to the quantity of flow actually withdrawn through the surface. The value of shape factor is constant through the transient region. The corresponding overall change in momentum thickness calculated in this way must be modified by an empirical factor which represents the variation of momentum thickness in the transient region immediately downstream of the suction strip. It was shown that, for small suction quantities, the momentum thickness decreased through this transient region whereas for larger suction quantities it appeared to increase.

(ii) The transient effects of abnormally high values of surface shearing force and non-universality of the inner velocity profile only extended a short distance downstream of the suction strip.

Hence, assuming the power law type of mean velocity distribution, the effect of removing a given quantity of fluid is defined by Eq. (1.4) and (1.5).

The representation of the boundary layer mean velocity distribution by a power law is not sufficiently accurate to be used in the calculation of the amount of fluid removed. This is due to the infinite value of velocity gradient at the surface which is predicted by such a power law.

A better representation of the mean velocity distribution in this region is given by Eq. (2.23)

$$\frac{u}{U} = 5.6\sqrt{\frac{c_f}{2}}\left[\log_{10}\left\{\left(\frac{Uy}{\nu}\right)\sqrt{\frac{c_f}{2}}\right\} + 1\right] \tag{2.23}$$

where,

$$c_f = 0.246.R_\theta^{-0.268}\,.\,10^{-0.678H}$$

The suction quantity coefficient ΔC_{Q_n} at ~~each~~ the n^{th} suction strip can be derived from Eq. (2.23) thus,

$$\Delta C_{Q_n} = \frac{q_n}{U_o c} =$$

$$\frac{U}{U_o}.\left(\frac{\theta_n}{c}\right)\frac{1.97.10^{-0.339H}\lambda\left(\frac{H+1}{H-1}\right).H\left\{0.111+\log_{10}\frac{H(H+1)}{(H-1)} - 0.339H - \log\left(\frac{1}{\lambda}\right) + 0.866\log R_n\right\}}{R_n^{0.134}}$$

The equations describing the development of the boundary layer along the region of impervious surface between the suction strips are based on those presented by Spence (1958). For the streamwise velocity distribution of Eq.(2.9) with $r = 1$, these can be written as in Eqs. (A2.3) and (A2.5) of the Appendix.

Given initial values of shape factor (H_o) and momentum thickness $(\theta/c)_o$ at the station $x/c = 0$, it is possible to calculate the development of the boundary layer on an impervious surface to a given value of shape factor (H) as shown in Appendix II~~I~~ and thereafter a step-by-step calculation can be used to determine the streamwise distribution of suction. Such calculations have been programmed for a Ferranti Pegasus digital computer for $q = \pm 1$ and a range of values of overall recovery factor (p). The resultant variation of suction quantity coefficient (C_Q) is compared with that predicted in the previous Section (2.4.2.1) in Figs. (2.4a) and (2.4b) for values of $q = +1$ and -1 respectively.

In these cases the variation in the value of shape factor between slots is limited to H = 0.1 which is approaching continuous suction as shown by Fig. (2.5). From Fig. (2.5) it is interesting to note the unfavourable effect of finite spacing of suction strips as compared to continuous suction. This effect might possibly be expected on the grounds that a finite suction strip will extract fluid from the boundary layer which has a finite streamwise momentum which might otherwise have assisted the boundary layer against the adverse pressure gradient. In other words, slot suction is less efficient than continuous suction as only the latter removes fluid which has been completely de-energised with respect to motion in the streamwise direction. This intuitive idea is confirmed by Fig. (2.5).

The major discrepancy between the two alternative approaches as shown in Figs. (2.4a) or (2.4b) is derived from the assumed form of the boundary layer development equations for the impervious surface prior to the beginning of suction. In order to be consistent, the development of the boundary layer up to the onset of suction was calculated using the appropriate form of the auxiliary equation, viz., Eq. (2.18a) for Section (2.4.2.1) or Eq. 2.3 for Section (2.4.2.2). The same initial conditions at the leading edge (x/c = 0) were assumed in each case as derived in Appendix II. Fig. (2.6) illustrates the differences in the predicted variation of boundary layer shape factor on the impervious surface for $p = 2$, $q = \pm 1$. Spence's (1958) form of the momentum Eq. (A 2.1) was used in each instance so that the difference can only be associated with the assumed form of the auxiliary equation. The precise technique used for the calculation of boundary layer growth prior to the onset of suction was either that recommended by Head (1958) for Section (2.4.2.1), or alternatively, the iterative process of Appendix II Eq. (A2.7) for the step-by-step approach of Section (2.4.2.2). Whereas the resulting discrepancy introduces a considerable difference in the amount of suction predicted for a small value of recovery factor (p), it can be seen from Figs. (2.4a) and (2.4b) that, considering the radically different approach and assumptions, the agreement between the two methods of calculating the suction quantity required is good at the larger values of recovery factor (p). Furthermore, by initiating the step-by-step calculation at the same streamwise station as in Section (2.4.2.1), in order that the two $C_Q(p)$ relationships are made to agree at the point of zero suction, the agreement will clearly be improved.

2.5. Calculation of the suction quantity coefficient (C_Q) required in order to achieve a given lift coefficient (C_L) on an unflapped or a flapped wing.

It is possible to approximate to the streamwise distribution of velocity outside the boundary layer by means of Eq. (2.9) by suitably choosing the values of the parameters (p) and (q). For simplicity, it will be assumed that the velocity measured at the trailing edge of a wing ($x/c = 1$) is equal to the free stream velocity at infinity. Furthermore, for values of the overall lift coefficient of the order which are of interest, the contribution to wing lift derived from the pressure distribution on the under surface of a wing will be small and hence a relatively crude approximation to this velocity distribution can be used in order to calculate the overall lift coefficient, without introducing any serious inaccuracy. The assumed distribution of velocity along the under surface of the wing is a linear variation from zero at the forward stagnation point which is assumed coincident with the leading edge to $U=U_\infty$ at the trailing edge.

2.5.1. Unflapped wing.

The lift coefficient associated with the velocity distribution of Eq. (2.9), Fig. (2.1) with r=1 is given by Eq. (2.26)

$$C_L = \int_0^1 \frac{(P_L - P_U)}{\frac{1}{2}\rho U_\infty^2} . d\left(\frac{x}{c}\right) = \int_0^1 \left(\frac{U_u^2 - U_L^2}{U_\infty^2}\right) d\left(\frac{x}{c}\right) = -\frac{1}{3} + \frac{p^2 . p^{1/q}}{q(p^{1/q}-1)} . \frac{q}{(2q+1)} \left[1 - \left(\frac{1}{p}\right)^{\frac{(1+2q)}{q}} \right] \tag{2.26}$$

It is possible to eliminate the recovery factor (p) between C_L (p, q) as defined by Eq. (2.26) and C_Q (p, q) from Eq. (2.22) or Eq. (A1.4) and thereby obtain a direct relationship between C_Q and lift coefficient. From Fig. (2.7) it can be seen that for larger values of lift coefficient this relationship is not strongly dependent on the type of streamwise velocity distribution (i.e., the value of q), a fact which is of course, of considerable practical significance. However, the value of C_L corresponding to $C_Q = 0$ would appear to be ~~unrealistically~~ rather large and it is possible that better overall agreement with experiment will be obtained by replotting Fig. (2.7) as a relationship between C_Q

and incremental lift coefficient above the normal stalled value without suction.

2.5.2. Flapped wing.

It would not be normal practice for an unflapped wing to provide large values of lift coefficient for two reasons.

(i) A low value of static pressure corresponding to large local stream velocities in the vicinity of the leading edge is inefficient in that the maximum suction pressure must be defined by this and hence the suction power require~~s~~d will be correspondingly greater.

(ii) The attitude of a fixed wing aircraft necessary to achieve a large value of lift coefficient without the use of trailing edge flaps would detract from its use as a landing aid from the pilot's point of view.

The use of a flapped wing alleviates these difficulties.

It is necessary to make some assumption concerning the optimum proportion of flap angle to wing incidence which are to be used. Assuming that a single-stage suction unit is employed, the condition of minimum suction power is effectively the same as that of minimum suction quantity removed through the surface. The optimum ratio of flap angle to wing incidence for this case is clearly obtained when the two suction peaks are of equal magnitude as otherwise it would be the magnitude of the larger suction peak which would define the necessary pump pressure.

It is proposed to simplify the treatment of the suction distribution over a flapped wing by utilising the relationships already derived for a monotonic distribution of the velocity outside the boundary layer as represented by equation (2.27)

Upper surface.

$$0 < \frac{x}{c} < \mu, \quad t = \frac{U}{pU_o} = \left[1 - \frac{(p/p_1)^{1/q} - 1}{(p/p_1)^{1/q}} \left(\frac{x}{\mu c} \right) \right]^q$$

$$\mu < \frac{x}{c} < (1-\lambda), \quad t = \frac{U}{pU_o} = \left[1 - \frac{(p/p_1)^{1/q} - 1}{(p/p_1)^{1/q}} \left\{ \frac{1 - \lambda - x/c}{1 - \lambda - \mu} \right\} \right]^q$$

$$(1-\lambda) < \frac{x}{c} < 1, \quad t = \frac{U}{pU_o} = \left[1 - \frac{p^{1/q} - 1}{p^{1/q}} \left\{ \frac{-1 + \lambda + x/c}{\lambda} \right\} \right]^q \tag{2.27}$$

Lower surface.

$$t = \frac{U}{pU_o} = \frac{1}{p} \cdot \left(\frac{x}{c}\right)$$

The total suction quantity required and the corresponding value of lift coefficient achieved can be expressed in terms of the results derived for an unflapped wing as shown in equation (2.28)

$$C_{Q_f} = \int_{x/c=0}^{\mu} \left(\frac{v_s}{U}\right)\left(\frac{U}{U_\infty}\right) d\left(\frac{x}{c}\right) + \int_{x/c=1-\lambda}^{1} \left(\frac{v_s}{U}\right)\left(\frac{U}{U_\infty}\right) d\left(\frac{x}{c}\right)$$

$$= p_1 \mu C_Q(p/p_1, q) + \lambda C_Q(p, q) \tag{2.28}$$

$$C_{L_f} = (1-\lambda) p_1^2 \,.\, C_L(p/p_1, q) + \lambda . C_L(p, q)$$

In equation (2.28), $C_Q(p, q)$ and $C_L(p, q)$ refer to values calculated for a monotonic variation of velocity outside the boundary layer.

The following values are taken as typical of a flapped wing, $p_1 = \frac{(p+4)}{5}$, $\mu = 0.4$, $\lambda = 0.35$ and calculations are undertaken for $q = \pm 1$. As in general for a flapped wing $p \neq p_1$, there will be a different optimum value of the appropriate suction criterion for the forward as compared to the trailing edge region of suction. Indeed, it is possible that the severity of the pressure recovery factor of the leading edge region is such that suction is only required over the more severe trailing edge region aft of the flap knuckle. However, due to the somewhat peculiar and discontinuous manner in which the value of H_{opt} varies with recovery factor (p) as shown by Fig. (2.4), and also in order to simplify the calculations, it was decided to assume the same value of H_{opt} for both regions of pressure recovery. Fig. (2.7) shows the resulting variation of C_Q in terms of the lift coefficient (C_L) attained for a range of values of shape factor (H_{opt}) or $(\Gamma/G)_{opt} = \frac{m}{(H+2)}$ for $q = \pm 1$. From Fig. (2.7), it is possible to define the variation of the optimum value of shape factor with recovery factor (p) and hence lift coefficient and it is seen that the type of variation is much the same as that derived for an unflapped wing section. The two sets of curves refer to estimates of suction quantity coefficient derived using the skin friction relationship for an impervious surface or alternatively the modified skin friction relationship of Eqs. (2.2b) and (2.17) which introduces a dependence on suction velocity.

Fig. (2.8) shows the minimum value of C_Q as a function of lift coefficient as derived from Fig. (2.7) for an unflapped or a flapped section with a chordwise velocity distribution corresponding to $q = \pm 1$. It can be seen that the variation of C_Q as a function of lift coefficient is similar for an unflapped and a flapped wing, but the numerical value of C_Q for a flapped wing is considerably less. In order to deduce the relationship for an arbitrary chordwise distribution of velocity is is only necessary to express this in terms of an equivalent value of the factor (q). It is also noted from Fig. (2.8) that there is a considerable advantage to be gained by using a flapped section as compared to an unflapped one. The superiority of the flapped section can be accounted for as follows:-

(i) The region of favourable pressure gradient immediately forward of the flap knuckle contributes to the lift without requiring any suction to combat the possibility of separation.

(ii) As the value of $p_1 > 1$, there is a resultant reduction in the effective recovery factor (p/p_1) downstream of the leading edge suction peak and a corresponding reduction in the amount of suction required over this region.

(iii) As $p_1 > 1$, the general level of the static pressure over the forward part of the upper surface of the wing is correspondingly reduced, thereby providing an increase in the lift coefficient of the wing.

The optimum streamwise pressure distribution, defined in terms of the maximum lift for a given suction quantity, is given by a wing which is cambered in such a way that the static pressure over the upper surface is constant for the greater part of the chord and only increases to a value of the order of the free stream static pressure in the immediate vicinity of the trailing edge. The flapped wing section is a better approach to this ideal than is an unflapped one.

2.6. The experimental approach to the determination of the optimum suction distribution on a given wing.

The approach to the problem of the optimum suction distribution which has been presented is an idealised one. It is formulated using a tentative skin friction relationship Eq. (2.2), which is based on the one derived empirically for an impervious surface. Furthermore the boundary layer development equations will almost certainly require modification in the light of further experience. However, it is believed that the analysis is essentially sound in principle and will be useful as a starting point for an experimental investigation.

Any experimental investigation must approach the optimum suction distribution from the "over-sucked" condition in order that flow separation is avoided and as a result the chordwise pressure distribution is independent of the distribution of suction to the first order. Under these circumstances the boundary layer will be thin and this pressure distribution will, except in the immediate vicinity of the trailing edge, be closely represented by that derived using the assumptions of potential flow. Hence, the first stage of an investigation is to compare the potential flow velocity distribution with that of Eq.(2.9) or Fig. (2.1) and to define an equivalent value of the recovery factor (p) and the factor (q), for any monotonic region of adverse gradient. Hence, for given boundary layer conditions at the beginning of each region of adverse gradient, it is possible to define the optimum numerical value of the appropriate suction criterion (H) or (Γ/G). The appropriate suction distribution can then be used as a first approximation, and following this with boundary layer measurements at a series of chordwise stations, it is possible to readjust the suctiondistribution in order to maintain the suction criterionat its appropriate value. Further controlled variations of the numerical value of this criterion will enable its optimum value to be defined experimentally, thereby compensating for the somewhat tentative assumptions of the theoretical approach.

It is only possible to define a distribution of suction for a given wing which is correct for a given free stream velocity, incidence and flap angle. However, by suitably arranging the distribution of porosity over a flap such that, as the flap is deflected, it uncovers further porous surface, it is clearly possible to approximate to the optimum suction distribution for a range of values of flap angle. The problem of catering for "off design" conditions of wing incidence and airspeed is more difficult and needs further consideration as these quantities are closely related for a given aircraft weight and flap configuration.

2.7. Conclusions.

Consideration of the problem of the optimum suction distribution over a wing in order to attain a given value of lift coefficient has led to the formulation of a theoretical approach in terms of a limit on the value of a local boundary layer parameter. The optimum suction distribution is defined as that which requires minimum suction power to maintain unseparated flow. The main conclusions of this approach can be summarised as follows:

(i) The parameter $(\Gamma/G) = \dfrac{(-\frac{\theta}{U}\frac{dU}{dx})}{\frac{1}{2}c_f}$ is the appropriate non-dimensional form of local boundary layer momentum thickness which must be restrained below a given numerical value for the case in which the recovery factor (p) is considerably in excess of that necessary to cause flow separation in the absence of suction. For the asymptotic case $p \rightarrow \infty$ the optimum numerical value can be written $(\Gamma/G)_{\substack{opt \\ p\rightarrow\infty}} \simeq (\frac{m}{H+2})$. Maintaining this value of (Γ/G) restricts the growth of the boundary layer so that it develops primarily under the influence of the suction and the surface shearing forces; consequently there is little tendency for the value of the shape factor to increase significantly above that appropriate to a boundary layer in zero pressure gradient.

(ii) Progressive reductions in the value of recovery factor (p) result in increased values of $(\Gamma/G)_{opt}$, until ultimately the boundary layer is sufficiently influenced by the pressure gradient for the value of shape factor to increase towards that appropriate to separation. At this stage (Γ/G) ceases to be a satisfactory criterion.

(iii) For further reductions in the value of the recovery factor (p), the optimum value of shape factor $(H)_{opt}$ increases and ultimately approaches that appropriate to separation. Thus, in the limiting case for which the pressure recovery is just insufficient to cause separation, the optimum value of shape factor is large and the streamwise extent of suction approaches zero.

These general principles are demonstrated by reference to an infinite family of streamwise velocity distributions. The skin friction relationship which is tentatively assumed for these calculations has a dependence on Reynolds number (R_θ) and shape factor (H) which has been defined for an impervious surface by Ludwieg and Tillman (1949). The coefficient of this relationship is assumed to be dependent on the local suction velocity according to Eq. (2.2b). From these calculations it is noted that,

(a) Over the range of values of recovery factor (p) for which (Γ/G) is the appropriate criterion, the dependence of overall suction quantity coefficient on the value of (Γ/G) is weak, and it is possible to assume $(\Gamma/G)_{opt} \simeq (\Gamma/G)_{opt_{p\to\infty}} \simeq \left(\frac{m}{H+2}\right)$ without introducing any significant error.

(b) The calculations of the development of a turbulent boundary layer with shape factor (H) constant using the method of M.R. Head (1958) are confirmed using the step-by-step approach based on the author's experimental investigation as described in Section (1).

(c) Relationships defining C_Q as a function of C_L have been calculated and these are not strongly dependent on the path taken by the velocity between its maximum and minimum values. Hence, these C_Q/C_L relationships should be a useful standard for comparison with experimental results.

Thus, it can be seen that consideration of the problem of the optimum distribution of suction for "high lift" has led to the derivation of a C_Q/C_L relationship which can be used in initial feasibility studies and also as a starting point for an experimental investigation which would lead to a refinement of some of the more tentative assumptions of the theoretical approach.

The arguments with regard to the optimum suction distribution are equally applicable to the design of a large angle diffuser which utilises this method of boundary layer control in order to suppress flow separation and thereby increase the pressure recovery factor.

2.8. Suggestions for further work.

The following is a summary of the various aspects of the present investigation which would benefit from further consideration.

1. A more extensive investigation of the effect of a suction strip on a boundary layer is required. A range of values of shape factor should be considered. The transitional behaviour downstream of a suction strip demands closer examination with particular reference to the variation of momentum thickness throughout the transitional region.
2. Further work is necessary in order to refine the boundary layer auxiliary equation proposed by M.R. Head (1958), as this is the basis of a large part of the calculations. Particular attention should be paid to the predictions of the variation of shape factor on an impervious surface which would appear to be somewhat in error. Consideration might be given to an alternative approach using the energy integral equation suitably extended to cover the influence of suction through the surface.

3. Measurements of the development of a boundary layer in a severe adverse pressure gradient and under the influence of suction distributed as predicted would be useful as confirmation of the basic assumptions of the calculations. This would also provide an indication of the effect of the various ways of approaching an idealised porous surface.

4. It would be instructive to undertake a design study of the compatibility of the suction distributions on the upper surface of a wing for low drag in the cruise configuration and "high lift" in the low speed condition. The implications of a compromise suction distribution might then be investigated. It is suggested that by carrying a small amount of aileron droop in the cruise condition, it might be possible to obtain a chordwise pressure distribution which is similar to that in the low speed condition. This might alleviate the problems of a compromise suction distribution in that it would at least maintain a favourable pressure gradient immediately forward of the flap knuckle in the cruise condition and therefore reduce the tendency for transition over this impervious region.

5. Consideration must be made of the effect of "off design" conditions in order to define a suitable speed margin above the stall at which it would be safe to fly an aircraft which utilises this means of boundary layer control.

   Calculation of the value of shape factor at the downstream end of the region of pressure recovery for a range of values of airspeed and incidence would present a first stage in this investigation.

6. Consideration has so far been restricted to the application of suction over the upper surface of a wing. However, a small amount of suction suitably distributed on the underside of a flapped wing in the vicinity of the wing/flap junction might introduce a very acceptable increase in lift coefficient. This requires further consideration.

3. Boundary Layer Control by Tangential Blowing.

3.1. Previous work and outline of present approach.

A considerable amount of work, largely of an ad hoc nature, has been undertaken ref. Williams (1960), the main object of which has been to determine the influence of boundary layer control by tangential blowing on the performance and control characteristics of an aircraft. Measurements of the effect of tangential blowing on lift and drag have been invaluable as a means of indicating the improvements in performance offered by this means of boundary layer control. However, little fundamental work has been published which might act as a basis for theoretical predictions of the effects of blowing. This is largely explained by the difficult nature of the problem normally involving considerations of the development of a wall jet/turbulent boundary layer combination in an adverse pressure gradient. A comprehensive appraisal of Glauert's theoretical treatment of the wall jet is given in section 3.2.2.2.. Sigalla has presented an experimental investigation of a plane wall jet.

The aim of the present approach is to analyse the problem in the light of current boundary layer theory and to obtain an appreciation of the factors governing the development of a wall jet/boundary layer combination in an adverse pressure gradient. The breakdown of the problem is represented diagrammatically in Fig. (3.1).

3.1. Development of a Wall Jet along a plane surface.

The object of the present wall jet investigation is threefold.

(i) To determine experimentally the nature of the flow in the immediate vicinity of the surface. Of particular interest is the existence of some degree of universality of the mean velocity distribution which is typical of a turbulent boundary layer.

(ii) To evaluate Glauert's (1956) theory of the wall jet.

(iii) To establish a means of estimating the streamwise variation of the maximum jet velocity with distance from the slot, taking into consideration the momentum losses associated with the surface shearing force.

3.2.1. Equipment and experimental details.

Fig. (3.2) shows the equipment used and indicates the salient dimensions. Static and pitot pressure traverses were undertaken at six streamwise stations. The pitot tube was manufactured from hypodermic steel tubing the end of which had been annealed, the wall thickness was reduced to 0.005" and the tubing hammered out on a piece of shim steel 0.004" thick to give an overall thickness 0.013". No corrections to the results were made for displacement of the centre of pressure or Reynolds Number effects. The pitot tube was positioned relative to the surface by electrical contact and traversed with a micrometer screw. The pressures were measured relative to atmospheric pressure using a micro-manometer with a range of 14 cms alcohol. An indication of the stagnation pressure in the plenum chamber was used in order to control the jet momentum and frequent checks were made to ensure that the velocity distribution at the datum station (x = 3"), and in particular the maximum velocity at this station, remained constant. The two-dimensionality of the flow was confirmed by measuring velocity profiles at stations 4" either side of the centre line and within the limits of experimental error, these profiles were identical with those measured on the centreline. End plates were considered as a means of preventing three dimensional "end" effects but the vortex structure associated with the induced velocities was considered to be more detrimental to the two-dimensionality of the flow than the lateral spread of the jet which would ensue in their absence. The maximum possible height of the end plates was limited to approximately 6 inches by the traversing gear mounting system and it was therefore decided to undertake the measurements without end plates. Due to the large aspect ratio it is unlikely that the shape of the velocity profile measured on the centreline will be significantly affected but the absence of end plates may significantly modify the streamwise variation of peak jet velocity and jet thickness.

3.2.2. Results of the experimental investigation of a plane wall jet.

3.2.2.1. Universal distribution of mean velocity in the vicinity of the surface. Evaluation of the skin friction coefficient.

For a streamwise station less than 12" from the slot, the extent of the velocity profile between the surface and the velocity maximum is considered insufficient for it to be possible to obtain accurate measurements of the velocity distribution with available probes and traversing gear. For stations x = 12", 18", 24" and 30" the points on the measured velocity distribution at a distance of an order $\frac{\delta_1}{10}$ (where δ_1 is measured from the surface to the point of maximum velocity) were assumed to lie on the curve

$$u/u_* = A \log\left(\frac{u_* y}{\nu}\right) + B \qquad (3.1)$$

Assuming the values A = B = 5.6 as in section (1.3) it is possible to determine u_* and by replotting the profile as $\frac{u}{u_*}$ against $\log \frac{u_* y}{\nu}$ and from Fig. (3.3) it can be seen that up to a distance of the order of $\delta_1/5$ the curves are universal. The precise technique for deducing u_* is described in section (1.3). It can be shown (Schlichting 1955) that for a flat plate or pipe, a $^1/7$th power law type of mean velocity distribution can be deduced from the Blasius skin friction law for pipe flow by replacing (U) and (r) by (u) and (y) respectively. Thus

$$C_f = \frac{\tau_0}{\frac{1}{2}\rho U^2} = 0.045\left(\frac{U r}{\nu}\right)^{-1/4} \qquad (3.2.)$$

where r = pipe radius or boundary layer thickness

U = velocity at centreline or in freestream

This can be justified on the grounds that the surface shearing force is defined by flow conditions near to the surface. Writing $\frac{\tau}{\rho} = u_*^2$ and rearranging Eq. (3.2) it becomes

$$\frac{u}{u_*} = 8.74\left(\frac{u_* y}{\nu}\right)^{1/7} \qquad (3.3)$$

From Fig. (3.3) it can be seen that Eq. (3.3) is a ~~good~~ fair approximation to the universal logarithmic mean velocity profile near the surface. For the outer part of the mean velocity profile of Fig. (3.3) there is a discrepancy between a boundary layer type of velocity profile, as exemplified by Eq. (3.3), and the experimental results for a wall jet. For a boundary layer the outer part of the velocity profile is associated with a universal velocity defect law.

It is an accepted concept of boundary layer theory that the eddies defining the turbulent shearing stresses and hence the mean velocity profile in the outer region of a boundary layer have a characteristic dimension of the order of the boundary layer thickness. In the case of a wall jet however, the boundary condition at the velocity maximum differs from that for a turbulent boundary layer and the factors governing the velocity distribution in the outer part are associated with the eddy structure of the flow region outside the position of maximum velocity. Thus the outer "boundary layer" region of a wall jet is a blending region between the flow near the surface which can be defined in terms of a universal mean velocity distribution and the jet-like flow in the outer part. It is not surprising therefore that there is a considerable difference between the velocity distributions for a boundary layer and a wall jet in this region. One important consequence of this difference between a wall jet and a boundary layer is that, although Eq. (3.3) is valid for the inner part of a wall jet or boundary layer, Eq. (3.2) is no longer an acceptable skin friction relationship. Fig. (3.4) shows $2\left(\frac{u_*}{U}\right)^2 \cdot \left(\frac{U\delta_1}{\nu}\right)^{1/4}$ as a function of distance from the slot and indicates that the coefficient C = 0.045 in Eq. (3.2) must be replaced by C = 0.058 which represents a considerable increase in the effective value of skin friction coefficient. Thus for a wall jet the skin-friction relationship can be written .

$$c_f = \frac{\tau_0}{\frac{1}{2}\rho U^2} = 0.058 \left(\frac{U\delta_1}{\nu}\right)^{-1/4} \qquad (3.4)$$

Measurements by Sigalla (1958) at a slightly different value of Reynolds number using Preston tubes indicated a value of C = 0.0565 which is identical to within the limits of experimental error with the coefficient of Eq. (3.4). In the absence of detailed measurements of the velocity profile Sigalla was apparently unable to explain the difference between this value and that proposed by Blasius for a flat plate boundary layer.

3.2.2.2. Comparison of experimental results with Glauert's theory of the wall jet.

3.2.2.2.1. Outline of Glauert's theory.

The basic assumption of Glauert's (1956) theory is that the wall jet resembles a turbulent boundary layer in the immediate vicinity of the surface and a free jet in its outer region. The seventh power law velocity profile derived from the Blasius skin friction equation corresponds to a variation of eddy viscosity $\epsilon_1 \simeq R^{3/4}$ (Eqs. 6.1 G, 6.2 G, 6.3 G and 6.5 G).* On the other hand, using Prandtl's assumption for a free jet, the eddy viscosity varies linearly with Reynolds number. In order to derive a solution in similarity variables it is necessary to assume the same Reynolds number dependency for each region and by relaxing either one of these conditions Glauert arrives at two alternative sets of exponents which describe the streamwise variation of maximum velocity and jet width. (Eqs. 6.11 G or 9.1 G for the radial case and Eqs. 9.7 G and 9.8 G for the plane wall jet). Having assumed the same Reynolds number dependence for the two regions, the eddy viscosity is assumed to be constant across the outer region and for the inner region it is assumed to vary with distance from the surface according to Eqs. 6.3 G or 6.4 G. The two boundary-layer type equations, one for each region, expressed in terms of similarity variables(Eqs. 7.2 G and 7.3 G) are solved independently and it is shown that the solutions over the appropriate regions are independent of Reynolds number except for scaling factors. By considering the boundary conditions at the junction of the two regions (stream function ψ and streamwise velocity (u) are continuous, at the point where u = U)

* Footnote: In the text that follows an equation marked "G" refers to Glauert's (1956) original paper.

Glauert derives a relationship between Reynolds number, the Prandtl eddy viscosity constant (κ) and a parameter α. α defines both the shape of the velocity profile and also the streamwise development of the jet (Eq. 8.8 G).

3.2.2.2.2. Estimate of the value of α from experimental data.

The mean velocity profiles at the various traversing stations are non-dimensionalised with respect to maximum velocity and a characteristic lateral dimension. Fig. (3.5) compares the experimental results with velocity distributions predicted by Glauert for $\alpha = 1.1$ and $\alpha = 1.2$. The discrepancy at the outer edge of the wall jet is to be expected as the assumption of constant eddy viscosity is invalid in this region due to intermittency and also the accuracy of measurement in this region will be limited by relatively large cross flow velocities. It can also be seen that there is discrepancy between theory and experiment in the vicinity of the velocity maximum. This discrepancy was noted by Sigalla (1958) and would appear to represent a weakness in Glauert's theory.

3.2.2.2.3. Estimate of the value of Prandtl's constant of eddy viscosity (κ) from the experimental results.

Prandtl (1942) first introduced the concept of eddy viscosity which he assumed constant across a free jet and from dimensional considerations to be proportional to the lateral jet dimension and characteristic velocity.

$$\varepsilon = \kappa U \delta_t \qquad (3.5)$$

where U is the peak jet velocity

δ_t is a characteristic lateral dimension of the outer part of the jet defined as the distance from the peak velocity to half this velocity.

κ is the coefficient of eddy viscosity which is analogous to kinematic viscosity in laminar flow.

The basis for the assumption of constant eddy viscosity is derived from the hypothesis that for a free jet or wake the eddies which govern the transport of momentum and thereby define the turbulent shearing stress, have a characteristic dimension of the same order as the jet width.

As in the case of a free laminar jet, for which the kinematic viscosity defines the rate of spread and, through the conditions of similarity and constant momentum, the rate of streamwise variation of jet velocity, so for a turbulent jet the value of (κ) defines the lateral rate of spread. This is well recognised in the case of a free jet and the value of (κ) is defined experimentally in terms of the spread rate. In the investigations of Bakke (1957) and Sigalla (1958) this appears to have been overlooked and the value of (κ) has been estimated from Eq. (8.8 G) of Glauert's (1956) paper using values of α obtained from mean velocity profiles. As will be shown, this practice has introduced considerable errors into the estimated value of (κ), partly because Eq. (8.8 G) appears to be somewhat inaccurate and also because of the great sensitivity of (κ) to the value of α chosen as the best match with experimental data. An experimental investigation into the validity of Eq. (8.8 G) is given in section 3.2.2.2.5.

The relationship between (K) and the lateral spread rate is deduced in appendix III for both a plane and a radial wall jet from Glauert's theory. Thus, for a plane wall jet Glauert's theory predicts two alternative relationships depending on whether the Reynolds number dependence of the eddy viscosity is defined by the inner boundary layer type flow or the outer jet type flow. These relationships can be written

$$K = \frac{\frac{4}{(5+4\alpha)}\left(\frac{\delta_t}{x}\right)}{f'_{o_{max}}\,\eta_t^2} \qquad (3.6\text{ a})$$

or

$$K = \frac{\frac{1}{(1+\alpha)}\cdot\left(\frac{\delta_t}{x}\right)}{f'_{o_{max}}\,\eta_t^2} \qquad (3.6\text{ b})$$

$f_{o_{max}}$ and η_t are functions of α defined in Glauert's (1956) paper. The corresponding relationships for a radial wall jet can be written

$$K = \frac{\frac{9}{(5+4\alpha)}\cdot\left(\delta_t/x\right)}{f'_{o_{max}}\,\eta_t^2} \qquad (3.7\text{ a})$$

or

$$K = \frac{\frac{2}{(\alpha+1)}\left(\frac{\delta_t}{x}\right)}{f'_{o_{max}}\,\eta_t^2} \qquad (3.7\text{ b})$$

It is noted that as (α) approaches unity (i.e. Reynolds number approaches infinity) so Eq. (3.6 b) approaches the well known relationship for a free jet (Schlichting 1956) as in this case

$$\alpha \to 1, \quad f'_{o_{max}} \to \tfrac{1}{2}, \quad \eta_t \to 2\tanh^{-1}(1/\sqrt{2})$$

and hence

$$K \to \frac{(\delta_t/x)}{\{2\tanh^{-1}(1/\sqrt{2})\}^2} = \frac{(1.125)^2}{4}\cdot\left(\frac{\delta_t}{x}\right)$$

Eq. (3.6 a) also approaches the same result as $\alpha \to 1$ to within a good degree of approximation. The same comments apply equally in the case of a radial wall jet. The equation defining (K) is still dependent on α, as is Eq. (8.8 G) from which (K) has been estimated by Bakke (1957) and Sigalla (1958), but this dependence is now weak and an approximate value of α is sufficient to obtain a precise value of (K).

Fig. (3.6) shows jet width (δ_t) as a function of distance from the slot. The lack of linearity is probably due to lateral spread effects in the absence of end plates and the initial slope is taken as a basis for calculation of (K). The effective origin of (x) is 1.2" behind the jet exit and the value of $\left(\frac{\delta_t}{x}\right) = 0.068$. ? From the mean velocity distribution $\alpha = 1.1$, and hence (K) can be estimated using the values of $f_{o_{max}}$ and η_t as functions of α from Glauert's (1956) paper. The estimated value for a plane wall jet is $K = 0.0212$. If the relationship derived for a free jet had been used, the predicted value would be $K = 0.0218$. These two values are identical to within the limits of accuracy of the measurement. " Eqn 3.6 b.

This experimental value of (K) for a plane wall jet is interesting when compared to the corresponding figure for a plane free jet for which $K = 0.037$ according to Schlichting (1955). The ratio of the two figures clearly reflects the ratio of the characteristic dimensions of the eddies which are responsible for momentum transfer and thereby govern the level of the turbulent shearing stress. It would be interesting to compare the value of (K) for a radial free jet with that for a radial wall jet but no results for the former case appear to be available.

Values of (K) determined by the author in terms of the lateral rate of spread from published information by Bakke (1957) for a radial wall jet ($\alpha = 1.3$, $R = 3.5 \times 10^3$) and Sigalla (1958) for a plane wall jet ($\alpha = 1.1$, $R = 5 \times 10^4$) indicate values of $\left(\frac{\delta_t}{x}\right) = 0.0565$, $(\kappa) = 0.040$ and $\left(\frac{\delta_t}{x}\right) = 0.0514$, $\kappa = 0.0156$ respectively. The discrepancy between the results of Sigalla and the author in the experimental determination of lateral spread rate can probably be accounted for by the small aspect ratio of the boundary layer channel used by the former. The width of the plate used by the author was 30" which compares with a width of 6" used by Sigalla. Hence, although Sigalla used "end plates" to limit lateral flow effects, the results of the author must be accepted as a better approach to two dimensional flow conditions. It is noted that these values of (κ) when compared with those obtained from the same data using Eq. (8.8 G) and quoted by the authors (Bakke radial wall jet $\kappa = 0.012$ and Sigalla plane wall jet $\kappa = 0.035$), indicated a considerable discrepancy. This can be accounted for by the inaccuracy of Eq. (8.8 G) and its conditioning with respect to α, which results in excessive sensitivity of (κ) to the value of α chosen as the best match between theoretical and experimental velocity profiles.

3.2.2.2.4. <u>Experimental investigation of Glauerts Eq. (8.8 G).</u>

$$\kappa R^{1/4} = \frac{0.0275}{(\alpha + 0.07)} \cdot \left(f_{o_{max}}\right)^{-5/4} \cdot \left(f'_{o_{max}}\right)^{1/4} \eta_t^{-3/4} \qquad (8.8\ G)$$

In the previous two subsections the parameters κ and α were determined from experimental data, the former in terms of lateral spread rate and the latter by matching the experimental mean velocity profiles to predicted profiles. α is effectively the shape parameter of the velocity profile. Using the experimental values of (κ) and

α as functions of Reynolds number it is possible to undertake a direct experimental verification of equation (8.8 G).

Eq. (8.8 G) is an expression of the relationship between the eddy viscosities of the inner and outer regions and is derived from an assumed variation of eddy viscosity for these two regions together with assumed boundary conditions at their junction. The parameter (K) defines the magnitude of the turbulent stresses in the outer region whilst the behaviour in the immediate vicinity of the surface is defined by Eq. (7.8 G).

$$\frac{\tau}{\rho} = \varepsilon \frac{du}{dy} = \frac{0{,}045}{2} \cdot u^2 \left(\frac{uy}{\nu}\right)^{-1/4} \qquad (7.8\ G)$$

Although the effective skin friction relationship expressed in terms of the maximum velocity (U) and the distance from the surface to the point of maximum velocity (δ_1) has a coefficient C = 0.058 as shown in section 3.2.2.1, the Blasius coefficient C = 0.045 correctly determines the velocity distribution in the immediate vicinity of the surface and it is this value which must be used in the derivation of Eq. (8.8 G). This is contrary to a statement by Sigalla (1958) in which the coefficient of Eq. (8.8 G) is erroneously modified to correspond to C = 0.0565.

Fig. (3.7) shows $\left(1/_{K R^{1/4}}\right)$ as a function of α according to Eq. (8.8 G) and comparison with experimental results indicates considerable scatter. Further experimental information, particularly at a reduced value of Reynolds number, is required in order to complete this investigation.

3.2.2.2.5. Comparison of theoretical predictions of streamwise development with experiment.

Glauert's theory of the wall jet seeks a restricted type of similarity solution with the streamwise velocity and jet thickness depending on the streamwise position according to Eq. (3.8)

$$u \simeq x^{a} \qquad y \simeq x^{b} \tag{3.8}$$

a and b are related to the wall jet mean velocity profile shape factor (α) which is a function of Reynolds number (R) and the eddy viscosity coefficient (κ). Two alternative relations between "a" and "b" can be derived, depending on whether the eddy viscosity is linked to the inner or the outer region of the flow. For a radial wall jet these alternative relationships differ very little for any practical value of α, and for the limit $\alpha \Rightarrow 1$ the two sets of values of "a" and "b" converge numerically and agree with the values for a radial free jet (a = -1, b = 1). This behaviour can be accounted for by the slow variation of Reynolds number with streamwise position as shown in Eqs. (3.9 a) and (3.9 b)

$$R = \frac{U\delta_{+}}{\nu} \simeq x^{a+b} \simeq x^{\left(\frac{1-\alpha}{1+\alpha}\right)} \tag{3.9 a}$$

or

$$R \simeq x^{\left(\frac{1-\alpha}{1.25+\alpha}\right)} \tag{3.9 b}$$

Eq. (3.9 a) is deduced on the assumption that the variation of (ϵ) is linked to the outer region and equation (3.9 b) if linked to the inner region. The value of α will, for practical values of Reynolds number, normally be just greater than unity. It is a basic assumption of Glauert's theory that the eddy viscosity must have the same dependence on Reynolds number for inner and outer regions and, as the

Reynolds number is a very weak function of (x), this approximation is acceptable.

For a plane wall jet however, the streamwise variation of Reynolds number is more severe as indicated in Eq. (3.10a) for (ε) linked to the outer region and Eq. (3.10b) if linked to the inner region.

$$R = \left(\frac{U\delta_t}{\nu}\right) \simeq x^{a+b} \simeq x^{\frac{1}{1+\alpha}} \tag{3.10a}$$

or

$$R \simeq x^{\frac{1}{1.25+\alpha}} \tag{3.10b}$$

The stronger dependency of Reynolds number on streamwise position tends to invalidate the basic assumption of the analysis. One result of this is that the values of "a" and "b" obtained by the two alternative approaches differ considerably. Furthermore Eq. (3.10b) does not approach the case of a "half free jet" as α approaches unity. In view of the stronger dependency of Reynolds number on streamwise position (x), the plane wall jet must be regarded as an essentially more difficult problem than a radial wall jet and the limitations of Glauert's approach recognised.

$0.468 < a < .524$ From Eqs. (9.7 G) and 9.8 G) the values of "a" and "b" corresponding to $\alpha = 1.1$ can be determined and vary between $0.469 < a < 0.525$

$0.89 < b < 1$ and $0.85 < b < 1$. As there are, a priori, no grounds for preferring one of these alternative results to the other it must be accepted that Glauert's theory does not satisfactorily predict the streamwise development of a plane wall jet.

Any experimental determination of "a" and "b" for a plane wall jet will be susceptible to three-dimensional cross flow effects and an effective system of end plates together with a large ratio of

$\frac{\text{plate width}}{\text{boundary layer thickness}}$ is necessary to reduce these effects to an acceptable level. Fig. (3.8) shows log(U) and log (δ_t) as a function of log(x), where x is measured from the effective origin obtained by linear extrapolation, and the following values are noted (a) = 0.63, (b) = 0.965. (a -ve?) The effect of three-dimensional cross flow will be to increase the value of "a" and decrease "b". It would appear that the experimental information tended to support Eq. (3.10a) rather than the alternative Eq. (3.10b).

3.2.3. Predictions of the streamwise development of a wall jet based on momentum principles.

For a free jet the conditions of similarity and constant streamwise momentum enable estimates of peak jet velocity to be made at any streamwise station in terms of the slot width and slot exit velocity. For a wall jet however, the momentum is reduced by the surface shearing force. Glauert's theorem of constancy of "flux of exterior momentum flux", which is deduced for a viscous wall jet by considering the equation of motion and the associated boundary conditions as an eigenvalue problem, is not valid for a turbulent wall jet as it is necessary to postulate a different form for the lateral variation of turbulent eddy viscosity for the inner and outer regions.

The present approach to the problem is one which considers the loss of momentum, using the experimentally determined skin friction relationship. Considering unit width of a plane wall jet on a flat plate, the rate of loss of momentum is equal to the surface shearing stress

$$\frac{dJ}{dx} = -\tau_0 \tag{3.11}$$

In order to calculate the momentum associated with a given velocity profile, the latter is represented by a 1/7th power law boundary layer type profile from the surface to the velocity maximum, together with a plane free jet type profile outside this region. Thus,

$$0 < y < \delta_1 \qquad \frac{u}{U} = \left(\frac{y}{\delta_1}\right)^{1/7} \tag{3.12a}$$

$$\delta_1 < y < \infty \qquad \frac{u}{U} = (1 - \tanh^2 \eta) \tag{3.12b}$$

$$\text{where} \qquad \eta = \sigma\left(\frac{y - \delta_1}{x}\right)$$

σ is a parameter which defines the rate of spread of the outer region and is related to the Prandtl constant of eddy viscosity (κ). Thus from Eq. (3.12b)

$$\frac{\sigma \delta_t}{x} = \tanh^{-1} \frac{1}{\sqrt{2}} = 0.89$$

where δ_t is the width from the point where u = U, to u = U/2. From Eqs. (3.6a) and (3.6b) we have a relation between (κ) and (σ) thus :

$$\kappa = \frac{\frac{1}{1.25 + \alpha}\left(\frac{\delta_t}{x}\right)}{f'_{0max}\eta_t^2} = \frac{\frac{1}{\frac{1.125}{1.25} + \alpha}\cdot\left(\frac{0.89}{\sigma}\right)}{f'_{0max}\eta_t^2} \tag{3.13a}$$

$$\text{or} \qquad \kappa = \frac{\frac{1}{1+\alpha}\cdot\left(\frac{\delta_t}{x}\right)}{f_{0max}\eta_t^2} = \frac{\frac{1}{1+\alpha}\cdot\left(\frac{0.89}{\sigma}\right)}{f'_{0max}\eta_t^2} \tag{3.13b}$$

Accepting the mean of these two results as representative, κ is determined as a function of α using Glauert's theory. It can be seen that for small values of (α) just greater than unity, the error introduced by assuming α = 1 and accepting the value $\sigma\kappa = \frac{1.125}{4}$ deduced by Schlichting for a free jet, is not much larger than the difference between Eq. (3.13a) and (3.13b). Thus using this value of $\kappa\sigma$ and the value of κ = 0.0218 from Section 3.2.2.2.4. the corresponding value of σ = 12.9 and this value will be assumed to be independent of Reynolds number to the first order.

From Eqs. (~~2.15a~~ 3.12a) and (~~2.15b~~ 3.12b) the streamwise momentum associated with a wall jet is

$$\frac{J}{\rho} = U^2\delta_1\int_0^1\left(\frac{y}{\delta_1}\right)^{2/7} d\left(\frac{y}{\delta_1}\right) + \frac{U_\cdot^2 x}{\sigma}\int_0^\infty\left(1-\tanh^2\eta\right)^2 d\eta$$

or
$$\frac{J}{\frac{2}{3}\rho U^2 \frac{x}{\sigma}} = 1 + \frac{7}{6}\left(\frac{\delta_t\sigma}{x}\right)\left(\frac{\delta_1}{\delta_t}\right) = 1 + 1.04\beta \qquad (3.14)$$

where $\beta = \frac{\delta_1}{\delta_t}$ can be derived as a function of Reynolds number experimentally or as a function of ($\propto$) (i.e., Reynolds number and κ) from Glauert's theory. β is a slowly varying function of Reynolds number and can be assumed constant along the length of a wall jet.

Substituting Eq. (3.14) into Eq. (3.11) and using Eq. (3.4) as the appropriate skin friction relationship we have

$$\tau_0 = \tfrac{1}{2}\rho U^2\; 0.058\left(\frac{U\delta_1}{\nu}\right)^{-1/4} = -\frac{d}{dx}\left\{\frac{2}{3}\rho U^2\frac{x}{\sigma}(1+1.04\beta)\right\} \qquad (3.15)$$

The integration of this equation can strictly only be taken from the streamwise station for which similarity of mean velocity profile is achieved but if taken to the jet nozzle will give an approximation to the streamwise momentum loss and hence the peak jet velocity at any ~~streamwise momentum loss and hence the peak jet velocity at any~~ streamwise station. Hence integrating Eq. (3.15) from $x=x_o$, $U=U_o$ to any arbitrary streamwise station we have

$$1-\left(\frac{J}{J_o}\right)^{1/8} = 1-\left(\frac{U^2 x}{U_o^2 x_o}\right)^{1/8} = \frac{3}{2}\cdot\frac{\sigma}{(1+1.04\beta)}\cdot\frac{0.058}{2}\left\{\frac{\beta}{\sigma}\cdot\frac{U_o x_o}{\nu}\tanh^{-1}\frac{1}{\sqrt{2}}\right\}^{-1/4}\left(1-\left(\frac{x_o}{x}\right)^{1/8}\right)$$

$$= 0.0438\cdot\frac{\sigma}{(1+1.04\beta)}\cdot\left(\frac{U_o\delta_{1o}}{\nu}\right)^{-1/4}\left\{1-\left(\frac{x_o}{x}\right)^{1/8}\right\} \qquad (3.16a)$$

In order to estimate the magnitude of cross flow effects in the wall jet measurements of the author, the streamwise variation of jet momentum (J) determined experimentally from the mean velocity traverses was compared with that predicted by Eq. (3.16) using the station 3 in. downstream of the jet exit as the initial station. The equilibrium mean velocity profile is already established at this station. The streamwise rate of loss of momentum flux was found to be considerably in excess of that which could be accounted for by the surface shearing force and it is presumably associated with lateral spread effects in the absence of end plates.

As the momentum loss $\frac{\Delta J}{J_o}$ is normally small, Eq.(3.16) can be written using a binomial expansion thus

$$\frac{\Delta J}{J_o} = 8 \times 0.0438 . \frac{\sigma}{(1+1.04\beta)} . \left(\frac{U_o \delta_o}{\nu}\right)^{-1/4} \left\{1 - \left(\frac{x_o}{x}\right)^{1/8}\right\} \qquad (3.16b)$$

Normally with thin slits, only values of (x) much greater than (x_o) are of interest. In order to consider conditions up to the slit exit, it is reasonable to assume $x_o \doteq 10\ t$, $\delta_o \doteq t$, $U_o = U_{slot}$. Then taking $\sigma = 12.9$ and $\beta = 0.2$ at normal scale Reynolds numbers it is possible to define $\frac{\Delta J}{J_o}$. This factor is of considerable interest in relation to the surface friction losses associated with a jet flap scheme which uses a significant percentage chord flap as a means of directing the jet. It is shown in Section (3.4) that the streamwise development of a wall jet and in particular that of the peak jet velocity is to the first order not affected by the presence of a free stream superposed on ~~top of~~ the wall jet. Thus Eq. (3.16) can be assumed to define correctly the behaviour in these circumstances as long as the maximum jet velocity exceeds the local free stream velocity.

3.3. Effect of surface curvature on the structure and development of a wall jet.

The detailed analysis of (3.2) has not been extended to the case of a curved surface. Before further progress can be made in this respect there are two outstanding problems to be considered.

(i) Assuming that the outer part of the velocity profile can be represented by a "half" jet mean velocity profile, it is necessary to investigate the dependence of the coefficient of eddy viscosity on surface curvature.

(ii) It is necessary to investigate the separation of the wall jet from the curved surface and, possibly by analogy with the approach used for a turbulent boundary layer, establish a criterion for this separation. This will involve detailed measurements of the velocity distribution in the immediate vicinity of the surface which can only be obtained by an experimental investigation undertaken on a large scale.

Neither of these points have been considered experimentally but it is thought that the analysis which follows might be used as a means of predicting the static pressure distribution on a curved surface and hence as a basis for an investigation of wall jet separation on a convex surface.

It is assumed that the velocity distribution across the wall jet is adequately represented by Eq. (3.12a) and (3.12b) and the jet momentum by Eq. (3.14). Assuming also that the streamlines are approximately parallel to the surface, the equation for the pressure gradient across a streamline is given by the expression

$$\frac{dp}{dy} = \frac{\rho u^2}{\{R(x) + y\}} \qquad (3.17)$$

Hence from Eqs. (3.12) and (3.17) one obtains

$$p_\infty - p_w = \frac{2}{3} \cdot \frac{\rho U^2}{R(x)} \cdot \frac{x}{\sigma} \int_0^\infty \frac{3}{2} \frac{(1-\tanh^2\eta)^2}{(1+\eta/R')} d\eta + \frac{\rho U^2 \delta_1}{R(x)} \int_0^1 \frac{(y/\delta_1)^{2/7}}{\left[1+\frac{y}{\delta_1}\cdot\frac{\delta_1}{R}\right]} d\left(\frac{y}{\delta_1}\right)$$

and if

$$\delta_1 << R, \quad p_\infty - p_w = \frac{2}{3} \frac{\rho U^2}{R} \cdot \frac{x}{\sigma} \cdot \left[\int \frac{3}{2} \frac{(1-\tanh^2\eta)^2}{(1+\eta/R')} d\eta + 1.04\beta\right]$$

where

$$\eta = \frac{\sigma(y-\delta_1)}{x}, \quad R' = \frac{\sigma R}{x} \qquad \text{—— (3.17)}$$

As β is normally small compared with unity, Eq. (3.17) may be written

$$\frac{(p_\infty - p_w)}{J/R(x)} = \int_0^\infty \frac{3}{2} \cdot \frac{(1-\tanh^2\eta)^2}{(1+\eta/R')} d\eta + 1.04\beta\left[1 - \int_0^\infty \frac{3}{2} \cdot \frac{(1-\tanh^2\eta)^2}{(1+\eta/R')} d\eta\right] - (1.04\beta)^2 \qquad (3.18)$$

Furthermore, the last two terms on the right hand side of Eq. (3.18) are normally of second order and can be neglected. The value of $\int_0^\infty \frac{(1-\tanh^2\eta)^2}{(1+\eta/R')} d\eta$ as a function of R' has been evaluated numerically and is shown in Fig. (3.9). It is therefore possible to obtain an estimate of the value of (σ) from the static pressure distribution on a curved surface. The streamwise variation of the momentum of the wall jet J(x) must be considered.

3.4. Effects of free stream on the development of a wall jet.

An experimental investigation of the interaction of a wall jet and a turbulent boundary layer was undertaken by replacing the suction system in the boundary layer duct described in section (1) by a tangential blowing slot. Velocity profiles were taken at stations T6, T7 and T8 [Fig. (1.2)] for various combinations of jet exit velocity and local free stream velocity.

Intuitively it seems that the effect of a free stream on the development of a wall jet might be twofold.

(i) The jet spread rate might be expected to decrease due to the reduced lateral dimension and difference between peak jet velocity and velocity at the effective edge of the wall jet. According to Prandtl's hypothesis it is this dimension and the associated velocity difference which define the effective eddy viscosity and hence the lateral spread rate of the jet.

(ii) The high degree of turbulence at the edge of the jet might be expected to exaggerate the lateral spread rate by producing a more vigorous entrainment process.

With reference to Figs. (3.10 a, b and c) it would appear that the net result of these two opposing effects is to produce a spread rate only slightly less than that corresponding to a simple wall jet, (i.e. Zero free stream case).

3.5. Prediction of C_μ as a non-dimensional blowing parameter.

In section 3.4 it was shown that the streamwise variation of the maximum velocity of a wall jet/boundary layer combination could be calculated in terms of the case of zero free stream velocity. In addition section (3.2) shows that the spread of a wall jet on a plane surface can be predicted in similar manner to a normal free jet emerging into still air, the main difference being in the value of the effective jet mixing constant and the dissipation of momentum by surface shearing forces. It is further assumed that the local surface radius of curvature is large compared to the characteristic lateral jet dimension, from which it is reasonable to neglect the direct effects of local surface curvature on the spread rate. Firstly, consider the case in which the adverse pressure gradient is insufficient to cause a separation of the jet from the surface. Under these circumstances the lift coefficient achieved is determined by the displacement thickness of the wall jet/boundary layer combination at the trailing edge and also the velocity profile at this station which can be characterised by the ratio $\left(\frac{U_{jet\,TE}}{U_{free\,stream\,TE}}\right)$. This ratio is directly related to the non-dimensional momentum blowing coefficient C_μ thus

$$C_\mu = \frac{m V_J}{\frac{1}{2}\rho U_\infty^2 c} = \frac{4}{3} \cdot \frac{\lambda}{\sigma \mu} \cdot (1 + \cdot 04\beta) \cdot \left(\frac{U_{jet\,TE}}{U_{freestream\,TE}}\right)^2$$

where μ is the streamwise momentum loss factor = $\frac{\text{Momentum at trailing edge}}{\text{Momentum at jet exit}}$

$\lambda = \frac{\text{flap chord}}{\text{wing chord}}$

σ = 12.9 = lateral spread coefficient of a wall jet

$\beta = (\delta_1/\delta_t)$

Inserting representative values of $\mu = 0.8$, $\beta = 0.2$, $\lambda = 0.3$ we have

$$C_\mu = 0.0466 \cdot \left(\frac{U_{jet\,TE}}{U_{freestream\,TE}}\right)^2$$

Thus for a given configuration and for a small flap deflection ($\xi < 25^\circ$ say) which is insufficient to induce a flow separation, the value of C_μ can be interpreted directly in terms of velocity ratio at the trailing edge. It would be interesting to analyse the lift coefficient of such a flapped aerofoil relative to the ideal potential flow value, in order to determine the significance of this trailing edge velocity ratio. Intuitively one might expect that a value of trailing edge velocity ratio of an order 1.0 would be necessary in order to realise the potential flow lift coefficient for a small value of flap deflection.

As the flap angle is increased, the influence of the adverse pressure gradient associated with the free stream becomes increasingly important until ultimately the wall jet will separate from the surface. The adverse pressure gradient which the wall jet has to sustain has a contribution from the wall jet itself as well as from the free stream. The contribution of the wall jet to this pressure gradient is partly derived from the variation of local curvature along the surface but as was seen in Section (3.3), the lateral spreading of a wall jet round a surface of constant curvature induces a pressure gradient around the surface. It is difficult to generalise as to the relative proportions of each of these contributions to the surface pressure variation but it seems probable that the self induced pressure field will be more localised in the vicinity of the flap knuckle where the local curvature is large and the wall jet boundary layer thin. Hence the major factor governing separation will be the free stream pressure distribution which is spread over the greater part of the flap chord and hence acts on the wall jet boundary layer at a later stage in its development when it is thicker and therefore more prone to separation.

By analogy with the behaviour of a normal turbulent boundary layer in an adverse pressure gradient, the parameter representing the effects of an adverse pressure gradient on a wall jet boundary layer might be written $\frac{\delta_1}{\tau_0} \times \frac{dp}{dx}$ where $\frac{dp}{dx}$ is the streamwise pressure gradient along the surface. In accordance with normal practice, the surface shearing stress might be defined by the flat plate value of Eq. (3.4). Following Spence's (1958) approach to the derivation of an auxiliary equation one might seek to establish an equation for the shape factor of the wall jet boundary layer mean velocity profile of the form

$$\left(\frac{U\delta_1}{\nu}\right)^{1/4} \delta_1 \frac{dH}{dx} = \psi_1(H).\left[-\frac{\delta_1}{\frac{1}{2}\rho U^2}\cdot\frac{dp}{dx}\right] + \phi_1(H)$$

where ϕ_1(H) is associated with the variation of shape factor with Reynolds number in zero pressure gradient

ψ_1 (H) is derived from the behaviour of a thick boundary layer in a severe adverse pressure gradient.

The momentum equation which is used to define the streamwise variation of the boundary layer thickness parameter in "normal" boundary layer development calculations can be replaced by the condition of constant lateral rate of spread as determined by the eddy structure of the outer wall jet flow. This expression can be written

$$\frac{d\delta_1}{dx} = F\left\{\frac{U\delta_1}{\nu}, \kappa\right\}$$

and the form of the function (F) might be determined experimentally or from Glauert's (1956) theory of the wall jet. Using the empirical auxiliary equation in conjunction with this lateral spread equation it might be possible to predict wall jet separation and hence the value of C_μ necessary to ensure attached flow for large flap angles.

3.6. Conclusions.

The work presented is not a complete investigation of the problem but it provides an assessment of it and may be useful as a basis for further work. The conclusions can be summarised briefly as follows.

(i) The existence of a region of universal mean velocity distribution in the immediate vicinity of the surface which is normally associated with a turbulent boundary layer, is established for the case of a plane wall jet in zero pressure gradient.

(ii) An empirical skin friction relationship for the wall jet Eq. (3.4) is derived from values of surface shearing stress which are deduced from the universality of the inner part of the mean velocity distribution.

(iii) A comparison of the results of an experimental investigation with Glauert's (1956) theory of the wall jet shows the following results:

(a) The predicted mean velocity profiles agree reasonably well with experiment, although there is a significant discrepancy which has been noted previously by Sigalla (1958)

(b) Glauert's theory predicts a relationship between the eddy viscosity constant (k), local Reynolds number (R) and the Glauert velocity profile shape factor (α) which is not in agreement with experiment.

(c) Glauert's theory does not adequately predict the streamwise variation of maximum jet velocity or lateral spread rate for a plane wall jet.

It is noted that, in the case of a free jet, the constant of eddy viscosity (k) is best determined from the lateral spread rate.

(iv) An expression is derived for the variation of the maximum velocity of a wall jet using momentum principles. A significant proportion of the jet momentum may be dissipated by the surface shearing force immediately downstream of the slot.

(v) An expression is derived for the surface static pressure distribution for a curved wall jet. Separation of a wall jet has not been considered.

(vi) The superposition of a free stream does not greatly modify the streamwise variation of maximum jet velocity. It is shown that C_μ can be interpreted directly in terms of the ratio $\left(\frac{U_{jet\ TE}}{U_{free\ stream\ TE}}\right)$.

3.7. Suggestions for further work.

It is clear from the conclusions that the investigation is not complete. There are, in the opinion of the author, two further lines of investigation which would complete this work and thereby leave one with a clearer understanding of the mechanism of blowing as a means of suppressing a boundary layer separation.

Firstly, a comprehensive experimental investigation is required of the separation of a wall jet from a curved surface both with and without a free stream. Attention should be directed towards the development of the wall jet boundary layer region up to separation with particular reference to the behaviour of the mean flow parameters such as shape factor and skin friction coefficient. Interpreting such measurements against a general background of information with regard to the separation of a normal turbulent boundary layer, it ought to be possible to establish an auxiliary equation for wall jet boundary layer shape factor as suggested in section 3.5. This would facilitate the treatment of the problem of trailing edge blowing at large flap angles for which the local adverse pressure gradient along the flap might be sufficient to cause separation of the jet from the surface. It would also be possible to state quantitatively how much blowing is required at the leading edge of a wing in order to suppress a leading edge separation. In this case the optimum jet momentum required is clearly that which just maintains the wall jet in an unseparated condition in spite of the adverse pressure gradient aft of the leading edge suction peak.

The second investigation suggested by the author is a confirmation in practical terms of the separation criterion which one might hope to deduce from the above work. Thus it is suggested that a two dimensional wing with a blown trailing edge flap should be pressure plotted in a wind tunnel or in flight. Mean velocity traverses perpendicular to the local surface and at various chordwise stations would show the streamwise development of the wall jet/boundary layer combination and in particular the variation of the ratio

$$\frac{\text{Maximum jet velocity}}{\text{Local velocity outside the boundary layer}}$$

. It is this velocity ratio together with the effective displacement thickness of the wall jet/ boundary layer combination at the trailing edge of the flap which replaces the well known Kutta Jowkouski condition at a sharp trailing edge with potential flow. It is therefore the condition which governs the circulation round the section and hence the lift coefficient which can be achieved.

Appendix I

Calculation of $\left(\frac{\Gamma}{G}\right)_{opt.}$ for the doubly infinite family of streamwise velocity distributions of Equation (2·9)

Using the assumptions of section (2·4·1) and assuming $U_0=U_\infty$, the suction velocity ratio and quantity coefficient can be written

$$\frac{v_s}{U} = \left(H+2+\frac{G}{\Gamma}\right)\cdot\left(\frac{-1}{U}\frac{dU}{dx}\right)\cdot\left[\frac{\left(\frac{\Gamma}{G}\right)G}{-\frac{1}{U}\frac{dU}{dx}\left(\frac{U}{\nu}\right)^m}\right]^{\frac{1}{m+1}} - \frac{d}{dx}\left[\frac{\left(\frac{\Gamma}{G}\right)G}{-\frac{1}{U}\frac{dU}{dx}\left(\frac{U}{\nu}\right)^m}\right]^{\frac{1}{m+1}} \qquad \text{A1-1}$$

and

$$CQ = \int\left(\frac{v_s}{U}\right)\left(\frac{U}{U_\infty}\right)^{2n+1}\left[1-n\left(\frac{U_\infty}{U}\right)^2\right]d\left(\frac{x}{c}\right) \qquad \text{A1-2}$$

Substituting for $\left(\frac{v_s}{U}\right)$ in equation A1-2 one obtains

$$CQ=\left(\frac{\Gamma}{G}\right)^{\frac{1}{m+1}}\int\left[\left(H+2\right)\left(-\frac{1}{U}\frac{dU}{dx}\right)\left\{\frac{G}{-\frac{1}{U}\frac{dU}{dx}\left(\frac{U}{\nu}\right)^m}\right\}^{\frac{1}{m+1}} - \frac{d}{dx}\left[\frac{G}{-\frac{1}{U}\cdot\frac{dU}{dx}\left(\frac{U}{\nu}\right)^m}\right]^{\frac{1}{m+1}}\right]$$

$$\times\left(\frac{U}{U_\infty}\right)^{2n+1}\left\{1-n\frac{U_\infty^2}{U^2}\right\}\frac{dx}{c}$$

$$+\left(\frac{\Gamma}{G}\right)^{\frac{1}{m+1}-1}\int\left(-\frac{1}{U}\frac{dU}{dx}\right)\left[\frac{G}{\left(-\frac{1}{U}\frac{dU}{dx}\right)\left(\frac{U}{\nu}\right)^m}\right]^{\frac{1}{m+1}}\cdot\left(\frac{U}{U_\infty}\right)^{2n+1}\left\{1-n\left(\frac{U_\infty}{U^2}\right)^2\right\}\frac{dx}{c}$$

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Substituting for U(x) from equation (2·9), the suction velocity ratio can be expressed

$$\frac{v_s}{U} = \left[\frac{(-qr)(p^{-1/q}-1)^{1/r}}{pR_c}\right]^{\frac{m}{m+1}} \cdot \left(\frac{\Gamma}{G}\right)^{\frac{1}{m+1}} \cdot (G)^{\frac{1}{m+1}} \cdot t^{-1/q\left\{1-\left(\frac{1-qm}{m+1}\right)\right\}} \times$$

$$\times \left(t^{1/q}-1\right)^{\frac{m}{m+1}\cdot\left(\frac{r-1}{r}\right)} \cdot \left[\left(H+2+\frac{G}{\Gamma}\right) + \frac{1-qm}{q(m+1)} + \frac{(1-1/r)}{q(m+1)}\frac{1}{(t^{-1/q}-1)}\right]$$

-A1-3

Substituting into Equation A1-2 this value of $\left(\frac{v_s}{U}\right)$ one obtains

$$\frac{CQ(-qr)(p^{-1/q}-1)^{1/r}}{G^{1/m+1}\left[\frac{(-qr)(p^{-1/q}-1)^{1/r}}{pR_c}\right]^{m/m+1} p^{2n+1}} = \int_{1/p}^{t_1} \left(\frac{\Gamma}{G}\right)^{\frac{1}{m+1}} \cdot t^{2n+\frac{(1-qm)}{q(m+1)}} \times$$

$$\times \left(t^{1/q}-1\right)^{-\frac{(1-1/r)}{m+1}} \cdot \left(1-\frac{n}{p^2t^2}\right) \cdot \times$$

$$\times \left[\left(H+2+\frac{G}{\Gamma}\right) + \frac{(1-qm)}{q(m+1)} - \frac{(1-1/r)}{q(m+1)}\frac{1}{(1-t^{-1/q})}\right] dt \qquad \text{-A1-4}$$

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Hence as $\left(\frac{\Gamma}{G}\right)_{opt}$ is defined from the condition $\frac{dCQ}{d\left(\frac{\Gamma}{G}\right)} = 0$ one can write

$$\frac{dCQ}{d\left(\frac{\Gamma}{G}\right)} = 0 = t_1^{\,2n+\frac{(1-qm)}{q(m+1)}} \cdot \left(t_1^{\,1/q} - 1\right)^{-\frac{(1-1/r)}{m+1}} \cdot \left(1 - \frac{n}{p^2 t_1^2}\right) \times$$

$$\times \left\{ H+2+\frac{G}{\Gamma} + \frac{(1-qm)}{q(m+1)} - \frac{(1-1/r)}{q(m+1)} \times \frac{1}{(1-t_1^{-1/q})} \right\} \cdot \frac{dt_1}{d\left(\frac{\Gamma}{G}\right)} \cdot \left(\frac{\Gamma}{G}\right)^{\frac{1}{m+1}} \qquad \text{A1.5}$$

$$+ \int_{1/p}^{t_1} t^{\,2n+\frac{1-qm}{q(m+1)}} \cdot \left(t^{\,1/q} - 1\right)^{-\frac{(1-1/r)}{(m+1)}} \cdot \left(1 - \frac{n}{p^2 t^2}\right)$$

$$\times \left[\left\{ H+2 + \frac{(1-qm)}{q(m+1)} - \frac{(1-1/r)}{q(m+1)} \cdot \frac{1}{(1-t^{-1/q})} \right\} \cdot \left(\frac{\Gamma}{G}\right)^{\frac{1}{m+1}-1} \cdot \frac{1}{(m+1)} + \left(\frac{\Gamma}{G}\right)^{\frac{1}{m+1}-2} \left(\frac{1}{m+1} - 1\right) \right] dt$$

$$\text{or } \left(\frac{\Gamma}{Gm}\right)_{opt} = \left(\frac{\Gamma}{Gm}\right)_{\infty} \times \frac{\left[1 + \frac{m+1}{m} Y\right]}{\dfrac{\left[1-(m+1).Y.\left\{(H+2) + \frac{1}{q}\frac{(1-qm)}{(m+1)} - \frac{(1-\frac{1}{r})}{q(m+1)} \frac{1}{(1-t_1^{-1/q})}\right\}\right]}{\left(\frac{\Gamma}{Gm}\right)_{\infty}^{-1}}} \qquad \text{A1.6}$$

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where

$$\left(\frac{G_m}{\Gamma}\right)_\infty = H+2+\frac{(1-qm)}{q(m+1)} - \frac{1-1/r}{q(m+1)} \cdot \frac{\int_{1/p}^{t_1} f(t).\frac{1}{(1-t^{-1/q})}\,dt}{\int_{1/p}^{t_1} f(t)\,dt}$$

$$Y = \left(\frac{\Gamma}{G}\right).\left[-\frac{dt_1}{d\left(\frac{\Gamma}{G}\right)}\right].\frac{f(t_1)}{\int_{1/p}^{t_1} f(t)\,dt} \qquad \text{A1·6}$$

and $f(t) = t^{\,2n+\frac{1-qm}{q(m+1)}}\left(t^{1/q}-1\right)^{-\frac{(1-1/r)}{(m+1)}}\left(1-\frac{n}{p^2t^2}\right)$

The growth of $\left(\frac{\Gamma}{G}\right)$ up to the beginning of suction can be derived from the momentum equation thus

$$\frac{d\theta}{dx} = \frac{G}{R\theta^m} + \left(H+2\right).\left(-\frac{\theta}{U}\frac{dU}{dx}\right)$$

This equation can be rewritten

$$\frac{d\Theta}{dx} = (1+m)\left[G + \left(H+\frac{2+m}{1+m}\right)X - \frac{\Theta}{U}\frac{dU}{dx}\right]$$

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and integrating one obtains

$$\Theta U^{\beta} = (1+m)G \int^{x} U^{\beta}\, dx + \text{constant} \qquad \text{A1·7}$$

$$\text{where } \beta = (1+m)\left[H + \left(\frac{2+m}{1+m}\right)\right]$$

putting in the initial limits $U = pU_0$, $\Theta = \Theta_0$

one obtains

$$\frac{\Theta}{\Theta_0} = \left(\frac{pU_0}{U}\right)^{\beta} + \frac{(1+m)G}{\Theta_0 . U^{\beta}} \int_0^{x} U^{\beta}\, dx$$

Hence $$\frac{\Gamma}{G} = - \frac{\frac{\Theta}{U}\frac{dU}{dx}}{G} = -\frac{1}{U} \cdot \frac{\frac{dU}{dx}}{c}\left(\frac{pU_\infty}{U}\right)^{\beta}\left[\frac{1}{G} . \frac{\Theta_0}{c} . \cancel{\left(\frac{pU_\infty}{U}\right)^{\beta}} + (1+m)\int_0^{x} \frac{U^{\beta}}{(pU_\infty)^{\beta}} \frac{dx}{c}\right]$$

Substituting from Equation (2·9), this equation can be written

$$\left(\frac{\Gamma}{G}\right) = \frac{(-qr) . (p^{-1/q} - 1)^{1/r} (t_1^{1/q} - 1)^{1 - \frac{1}{r}}}{t_1^{\beta + \frac{1}{q}}} \left[\left(\frac{\Theta_0}{c}\right)\frac{1}{G} + (1+m)\, I\right] \qquad \text{A1·8}$$

$$\text{where } I = \int_{t_1^{-\frac{1}{q}}}^{1} \frac{y^{-\beta q - \frac{1}{r} - 1}(1-y)^{\frac{1}{r} - 1}\, dy}{r\left(p^{-\frac{1}{q}} - 1\right)^{\frac{1}{r}}} \qquad y = t^{-1/q}$$

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Differentiating $\frac{\Gamma}{G}$ with respect to t_1 one obtains

$$\left(\frac{\Gamma}{G}\right)\cdot-\frac{dt_1}{d\left(\frac{\Gamma}{G}\right)}=\frac{t_1}{\left[\left(\beta+\frac{1}{q}\right)+\frac{(1+m)}{\frac{\Gamma}{G}}-\frac{\left(1-\frac{1}{r}\right)}{q}\,\frac{1}{\left(1-t_1^{-\frac{1}{q}}\right)}\right]}$$

Hence from equation (A1·6)

$$Y=\frac{-\frac{\Gamma}{G}\cdot\frac{dt_1}{d\Gamma/G}\cdot f(t_1)}{\int_{1/p}^{t_1}f(t)\,dt}=\frac{t_1\cdot t_1^{\,2n+\frac{1-qm}{q(m+1)}}\cdot\left(t_1^{\,1/q}-1\right)^{-\frac{(1-1/r)}{m+1}}\cdot\left(1-\frac{n}{p^2t_1^2}\right)}{\left[\left(\beta+\frac{1}{q}\right)+\frac{(1+m)}{\left(\frac{\Gamma}{G}\right)}-\frac{\left(1-\frac{1}{r}\right)}{q}\,\frac{1}{\left(1-t_1^{-\frac{1}{q}}\right)}\right]\int_{1/p}^{t_1}f(t)\,dt}$$

(A1·9)

Consider now the asymptotic case for which $p\rightarrow\infty$. From equation (A1·8) we have $p\rightarrow\infty$, $I\rightarrow 0$, $t_1\rightarrow 1$ for finite $\left(\frac{\Gamma}{G}\right)$. Also from equation (A1·9) assuming $\int f(t)dt$ finite, $Y\rightarrow 0$ and hence from equation (A1·6) $\left(\frac{\Gamma}{G}\right)_{opt}\rightarrow\left(\frac{\Gamma}{G}\right)_{\infty}$. The expression of equation (A1·6) for $\left(\frac{\Gamma}{G_m}\right)_{\infty}$ includes integrals (Beta functions) which can not be obtained in closed form. However, assuming that both

contd.

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$\int \frac{f(t)}{\left(1-t^{-\frac{1}{q}}\right)}$ dt and $\int f(t)dt$ are finite, then their ratio although

numerically greater than unity will be of order unity. Thus

the expression for $\left(\frac{\Gamma}{G\,m}\right)_{\infty}$ is of order (H + 2) which confirms the

results derived, somewhat less rigorously, in section (2·4·1)

If one considers the special case r = 1, all the

integrals can be expressed in closed form. However, one looses

an essential condition of any suitable streamwise velocity

variation in that $\frac{dU}{dx} \neq 0$ at $\frac{x}{c} = 0$ for r = 1. One result of

this is that the case r = 1 gives somewhat misleading predictions

of the value of $\left(\frac{\Gamma}{Gm}\right)_{opt_{\infty}}$. Thus for r = 1, Equations (A1·8) and

(A1·6) reduce to

$$\left(\frac{\Gamma}{G}\right) = \frac{(-q)\left(p^{-\frac{1}{q}} - 1\right)}{t_1^{\beta + \frac{1}{q}}} \times \left[\frac{\Theta_0}{c}\,\frac{1}{G} + \frac{(1+m)}{\left(p^{-\frac{1}{q}} - 1\right)} \cdot \frac{1}{(-\beta q - 1)}\left(1 - t_1^{\beta + \frac{1}{q}}\right)\right] \qquad \text{A1·10}$$

contd

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$$\left(\frac{\Gamma}{G m}\right)_{opt} = \left[H+2+\frac{(1-qm)}{q(m+1)}\right]^{-1} \cdot \frac{\left[1+\frac{m+1}{m}Y\right]}{\left[1-(m+1)Y\right]}$$

$$Y = \frac{t_1^{\,2n+1+\frac{1-qm}{q(m+1)}}\left(1-\frac{n}{p^2 t_1^2}\right)}{\left[\beta+\frac{1}{q}+\frac{(1+m)}{\frac{\Gamma}{G}}\right]\int_{1/p}^{t_1} t^{\,2n+\frac{(1-qm)}{q(m+1)}}\left(1-\frac{n}{p^2t^2}\right)dt}$$ A1·10 cont.

Hence as $p \to \infty$, $t_1 \to 1$ for $\left(\frac{\Gamma}{G}\right)$ finite, but the value of Y does not necessarily approach zero. Hence the value of $\dfrac{\left(\frac{\Gamma}{G m}\right)_{\infty}}{\left[H+2+\left\{\frac{1-qm}{q(m+1)}\right\}\right]^{-1}}$ does not approach unity.

In section (2·3), the strong similarity is noted between the value of $\left(\frac{\Gamma}{G}\right)_{opt}$ for large values of recovery factor (p) deduced from considerations of minimum generalised suction quality coefficient as in equation (A1·6) and minimum local suction velocity ratio. Thus using Equation (A1-1) and the condition $\dfrac{d\left(\frac{v_s}{U}\right)}{d\left(\frac{\Gamma}{G}\right)} = 0$ one

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obtains the equation

$$\left(\frac{\Gamma}{Gm}\right)_{opt} = \frac{1}{\left[H+2+\frac{1}{(m+1)}\left\{ 1-m-\frac{U\ddot{U}}{\dot{U}^2} \right\} \right]}$$

$$= \frac{1}{H+2+\frac{(1-qm)}{q(m+1)} - \frac{(r-1)}{qr(m+1)}\left(\frac{1}{1-t^{-\frac{1}{q}}}\right)}$$

The strong resemblance between this equation and equation (A1·6) is confirmation of the suggestion made intuitively that the condition of minimum overall generalised suction coefficient might, in the case $p \rightarrow \infty$, be replaced by a condition of minimum local suction velocity.

Equation (A1·10) was used as the basis for an iterative method for estimating the value of $\left(\frac{\Gamma}{G}\right)_{opt}$ for the restricted case $(r = 1)$ as stated in section (2·4·1)

Appendix II

Step by step calculation of the distribution of suction with the value of shape factor (H) varying between predefined limits.

Following Spence (1956), it was shown in appendix I that the momentum integral equation for an impervious surface can be written

$$\Theta U^{\beta} = (1+m)G \int^{x} U^{\beta}\, dx + \text{constant} \qquad \text{A2·1}$$

$$\text{where } \beta = (1+m)\left[H + \frac{2+m}{1+m}\right]$$

Assuming values of $m = 0{\cdot}2$, $G = \frac{0{\cdot}0177}{2}$, $H = 1{\cdot}5$ we find $\beta = 4$

The auxiliary equation can be written

$$\Theta \frac{dH}{dx} = 9{\cdot}542(H - 1{\cdot}21)(H-1)\left(-\frac{\Theta}{U}\frac{dU}{dx}\right) - 0{\cdot}00307(H-1)^2$$

which can be integrated to the form

$$U^2\left[2{\cdot}105 - \frac{0{\cdot}442}{(H-1)}\right] = \text{constant} - \int^{x} \frac{U^2 dx}{\theta\left(\frac{U\theta}{\nu}\right)^{\frac{1}{5}}} . 0{\cdot}00135 \qquad \text{(A2·2)}$$

For the monotonic variation of velocity outside the boundary layer as represented by equation (2·9) with r = 1, equation (A2·1) can be written

Appendix II

$$\frac{\theta_{n+1}}{\theta_n'} = \left(\frac{t_n}{t_{n+1}}\right)^{\frac{1}{6}} \left[\left(\frac{t_n}{t_{n+1}}\right)^4 + \frac{0{\cdot}0106.\, p^{\frac{1}{q}}}{(4q+1)\left(p^{\frac{1}{q}} - 1\right)\left(\frac{\theta_n'}{c}\, R\theta_n'^{\,1/5}\right)} \frac{\left(t_n^{4+\frac{1}{q}} - t_{n+1}^{4+\frac{1}{q}}\right)}{t_{n+1}^4}\right]^{\frac{5}{6}} \qquad \text{A2·3}$$

where θ_n' and $R\theta_n' = \dfrac{pU_o\, t_n\, \theta_n'}{\nu}$ refer to conditions downstream

of the nth suction strip and θ_{n+1} to conditions immediately

upstream of the $(n+1)^{th}$ strip.

Substituting this expression for momentum thickness $\left(\frac{\theta}{c}\right)$ into the integral term of the auxiliary equation (A2·2) we have

$$\int_{x_n}^{x_{n+1}} \frac{U^2 dx}{\theta\left(\frac{U\theta}{\nu}\right)^{1/5}} = -\,(pU_\infty)^2\, \frac{(4q+1)}{0{\cdot}0106\, q} \times \cancel{\theta_n' \left(\frac{U_n\theta_n'}{\nu}\right)^{\frac{1}{5}}} \times$$

$$\times \int_{t_n}^{t_{n+1}} \frac{t^{5+1/q}\, dt}{\left[\frac{4q+1}{0{\cdot}0106}\, t_n^4\, \left(\frac{p^{+1/q} - 1}{p^{1/q}}\right) . \frac{\theta_n}{c} . \left(\frac{t_n\, pU_\infty\theta_n}{\nu}\right)^{\frac{1}{5}} + t_n^{4+\frac{1}{q}} - t^{4+\frac{1}{q}}\right]}$$

Introducing the new variable

$$S = \frac{t}{\left[\left(\frac{4q+1}{0{\cdot}0106}\right) \cdot \frac{\theta_n'}{c} \cdot \left(\frac{pU_\infty\, \theta_n'\, t_n}{\nu}\right)^{\frac{1}{5}} \left(\frac{p^{1/q} - 1}{p^{1/q}}\right) t_n^4 + t_n^{4+\frac{1}{q}}\right]^{\frac{1}{4+1/q}}}$$

contd.

Appendix II

This integral becomes

$$\int_{x_n}^{x_{n+1}} \frac{U^2 dx}{\theta\left(\frac{U\theta}{\nu}\right)^{1/5}} = (\rho U_\infty)^2 \left(\frac{4q+1}{0\cdot 0106}\right) \left[\frac{4q+1}{0\cdot 0106}\ \frac{\theta_n}{c}\left(\frac{\rho U_o\ t_n \theta_n'}{\nu}\right)^{\frac{1}{5}} \left(\frac{p^{1/q}-1}{p^{1/q}}\right) t_{n+1}^{4} + t_n^{4+\frac{1}{q}}\right]^{\frac{2}{4+1/q}}$$

$$\times \int_{S_n}^{S_{n+1}} \frac{S^{5+\frac{1}{q}}\ ds}{(1-S^{4+1/q})}$$

This integral in terms of (S) cannot be expressed in closed form.

However, for relatively closely spaced steps for which the value of (S) is just less than unity the denominator can be written approximately thus

$$\left(1-S^{4+\frac{1}{q}}\right) \simeq \frac{\left(4+\frac{1}{q}\right)}{\left(6+\frac{1}{q}\right)} \left(1 - S^{6+\frac{1}{q}}\right)$$

and the integral can be written

$$\int_{S_n}^{S_{n+1}} \frac{S^{5+\frac{1}{q}}}{1-S^{4+1/q}}\ ds \simeq \frac{\left(6+\frac{1}{q}\right)}{\left(4+\frac{1}{q}\right)} \int_{S_n}^{S_{n+1}} \frac{S^{5+\frac{1}{q}}}{1-S^{6+1/q}}\ ds = \frac{-1}{\left(4+\frac{1}{q}\right)} \times \log_e\left(\frac{1-S_{n+1}^{\ 6+\frac{1}{q}}}{1-S_n^{\ 6+1/q}}\right)$$

contd.

Appendix II

And hence the auxiliary equation (A2·2) can be written

$$\left(\frac{t_{n+1}}{t_n}\right)^2 = \left\{\frac{\chi(H_n)}{\chi(H_{n+1})}\right\}\left[1 - \frac{0{\cdot}00135}{0{\cdot}0106\,\chi(H_n)}\cdot F_n^{\,2}\,\log_e\left[\frac{F_n^{\,6+\frac{1}{q}} - \left(\frac{t_{n+1}}{t_n}\right)^{6+\frac{1}{q}}}{F_n^{\,6+1/q} - 1}\right]\right]$$

$$\text{where } F_n = \left[\frac{\frac{4q+1}{0{\cdot}0106}\cdot\frac{\theta_n'}{c}\cdot\left\{\frac{pU_o}{\nu}\,c.\,\frac{\theta_n'}{c}\cdot t_n\right\}^{\frac{1}{5}}\frac{(p^{1/q}-1)}{p^{1/q}}\,t_n^{\,4} + t_n^{\,4+\frac{1}{q}}}{t_n^{\,4+\frac{1}{q}}}\right]^{\frac{1}{4+1/q}}$$

A2·4

For steps which are closely spaced with respect to the local boundary layer thickness, the logarithmic term of equation (A2·4) is small compared with unity and hence equation (A2·4) can be written

$$\left(\frac{t_{n+1}}{t_n}\right) = \left\{\frac{\chi(H_n)}{\chi(H_{n+1})}\right\}^{\frac{1}{2}}\cdot\left[1 - \frac{0{\cdot}00135\,F_n^{\,2}}{2\times0{\cdot}0106\,\chi(H_n)}\cdot\log_e\left\{\frac{F_n^{\,6+\frac{1}{q}} - \left\{\frac{\chi(H_n)}{\chi(H_{n+1})}\right\}^{\frac{1}{2}(6+\frac{1}{q})}}{F_n^{\,6+1/q} - 1}\right\}\right]$$

A2·5

Appendix II

A modified form of equation (A2·5) was used to calculate the values of (t_1) & $\left(\frac{\theta_1}{c}\right)$ at the onset of suction in terms of conditions H_0 and $\left(\frac{\theta_0}{c}\right)$ at the leading edge $\left(\frac{x}{c} = 0\right)$. Representative values of $\left(\frac{\theta_0}{c}\right)$ have been estimated using the approximate method of A. Walz which is described by Schlichting (1955). The relevant equation which governs the growth of a laminar boundary layer can be written

$$\frac{U\theta^2}{\nu} = \frac{0{\cdot}470}{U^5} \int_{x=0}^{x} U^5 dx$$

Assuming that the velocity gradient is constant from the stagnation point $(U = 0)$ to the peak suction $(U = pU_0)$, the value of θ_0 can be written

$$\theta_0^{\,2} = \frac{0{\cdot}47}{6} \cdot \left(\frac{c'\nu}{pU_0}\right), \quad c' = \text{distance from stagnation point}$$

to peak suction.

Inserting nominal values of $c' = 0{\cdot}5$ft, $U_\infty = 100$ft/sec, $c = 4$ft and $\frac{1}{\nu} = 6400$ we have $\left(\frac{\theta}{c}\right) = \frac{6{\cdot}2 \times 10^{-5}}{\sqrt{p}}$ (A2·6·a)

Appendix II

Also $\left(\frac{\Theta_o}{c}\right) = \left(\frac{pU_o . t . \theta}{\nu}\right)^m \left(\frac{\theta}{c}\right)$

$$= (159)^m \times 6{\cdot}2 \times 10^{-5} \; p^{-\frac{1}{2}(1-m)} \qquad \text{A2}{\cdot}6{\cdot}\text{b)}$$

Assuming that momentum thickness is constant throughout the transitional region, these values of $\left(\frac{\theta_o}{c}\right)$ and $\left(\frac{\Theta_o}{c}\right)$ together with $H_o = 1{\cdot}4$ are used as nominal values which define the condition of the turbulent boundary layer at the peak suction $(x/c = 0)$.

The modified form of Equation (A2$\cdot$5) used to calculate the values of (t_1) and $\left(\frac{\theta_1}{c}\right)$ is based on the assumption that, in this case, the departure of the value of (S) from unity will be larger and a better approximation to the integral expression in terms of (S) can be written

$$\int_{S_{LE}}^{S_1} \frac{S^{5+\frac{1}{q}}\,ds}{(1-S^{4+1/q})} \simeq \int_{S_{LE}}^{S_1} \frac{S^{5+\frac{1}{q}}}{(1-S^{6+1/q})}\,ds = \frac{1}{(6+\frac{1}{q})} \log_e \left\{ \frac{1-S_{LE}^{\,6+\frac{1}{q}}}{1-S_1^{\,6+1/q}} \right\}$$

contd.

Appendix II

Hence the values of (t_1) and $\left(\frac{\theta_1}{c}\right)$ at the onset of suction have been estimated by iterating equation (A2·7) for t_1

$$t_1 = \left\{\frac{\chi(H_{LE})}{\chi(H_1)}\right\}^{\frac{1}{2}} \left[1 - \frac{1}{2} \cdot \frac{(4q+1)}{(6q+1)} \cdot \frac{0{\cdot}00135}{0{\cdot}0106} \cdot \frac{F_{LE}^{2}}{\chi(H_{LE})} \log_e \left\{\frac{F_{LE}^{6+\frac{1}{q}} - t_1^{6+\frac{1}{q}}}{F_{LE}^{6+1/q} - 1}\right\}\right] \qquad \text{A2·7}$$

It is noted from fig(2·6) there are quite large differences between the value of t_1 corresponding to the onset of suction as calculated by this method or alternatively using the auxiliary equation of Head (1958). This results in a significant difference in the predictions of suction quantity (C_Q) required for a small value of lift coefficient which is just in excess of that required to stall the lifting surface without suction.

Evaluation of the constant of eddy viscosity (κ) in terms of the lateral spread rate using Glauerts' (1956) theoretical approach

Glauert assumes that the mean velocity profiles are similar and that

$$u \;\alpha\; x^a$$

$$y \;\alpha\; x^b$$

where u(y) is the local velocity parallel to the surface and y is measured normal to the surface.

Considering firstly the two dimensional case and introducing the dimensionless variables

$$\bar{u} = \frac{u}{U}, \; \bar{v} = \frac{v}{U}, \; \bar{y} = \frac{yU}{\nu}, \; \bar{x} = \frac{xU}{\nu}, \; \bar{\psi} = \psi\nu$$

equation (6·7) G can be rewritten (G refers to Glauerts' (1956) paper)

$$\bar{\psi} = \bar{x}^{a+b} f(\eta)$$

$$\bar{u} = \left(\frac{a+b}{\lambda}\right) \bar{x}^{a} f'(\eta)$$

$$\eta = \left(\frac{a+b}{\lambda}\right) \frac{\bar{y}}{\bar{x}^{b}}$$

Appendix III

Considering first the case in which the Reynolds number dependence of the eddy viscosity is defined from the boundary layer region, then

$$\varepsilon_1 = A\lambda \bar{x}^{\frac{3}{4}(a+b)} f'^{6} \nu \qquad \text{A3·1}$$

$$\varepsilon_0 = \lambda \bar{x}^{\frac{3}{4}(a+b)} \nu$$

Substituting into the boundary layer equation

$$u\frac{du}{dx} + v\frac{du}{dy} = \frac{d}{dy}\left(\varepsilon \frac{du}{dy}\right) \qquad \text{5·1 G}$$

then both sides vary with x in the same way if

$$a + 5b = 4 \qquad \text{6·6 G}$$

Hence equation (5·1 G) reduces to

$$\frac{d}{d\eta}\left(A f_1'^{6} f_1''\right) + f_1 f_1'' + \alpha f_1'^{2} = 0$$

$$f_0''' + f_0 f_0'' + \alpha f_0'^{2} = 0$$

where $a = \dfrac{-4\alpha}{5+4\alpha}$, $b = \dfrac{4+4\alpha}{5+4\alpha}$ 9·7 G

Appendix III

For the outer region we have

$$\varepsilon_o = \kappa U_{max}\, \delta t \qquad 3\cdot5$$

Substituting from Equation(A3•1) and Equation (6•6 G)

into Equation (3•5) we have

$$\kappa = \frac{(4 - 4b)}{f{o'}_{max}\, \eta_t^2} \left(\frac{\delta t}{x} \right) \qquad A3\cdot2$$

where η_t refers to the interval between the point where

$u = U_{max}$ and $u = \frac{1}{2} \cdot U_{max}$.

Substitute in Equation (2•7) for b in terms of α from

Equation (9•7 G) we have

$$\kappa = \frac{\left(\frac{4}{5+4\alpha} \right)}{f{o'}_{max}\, \eta_t^2} \left(\frac{\delta t}{x} \right) \qquad 3\cdot6a$$

A similar analysis applies if the streamwise

variation of eddy viscosity ε_o is assumed to be linked to

the outer part of the velocity profile in which case Equation

(A3•1) is replaced by

Appendix III

$$\varepsilon_1 = A \lambda \bar{x}^{(a+b)} f_1'^{6} \nu$$

$$\varepsilon_0 = \lambda \bar{x}^{(a+b)} \nu$$

From this b = 1 for similarity in Equation(5·1G).

Equation(9·7G)is replaced by

$$a = - \frac{\alpha}{1+\alpha} , \; b = 1$$

and hence

$$\kappa = \frac{\frac{1}{(1+\alpha)} \left(\frac{\delta t}{x} \right)}{f'_{o_{max}} \; \eta_t^{2}} \qquad 3\cdot6b$$

The corresponding results for a radial wall jet with ε linked to the inner and outer regions are respectively

$$\kappa = \frac{\frac{9}{5 + 4\alpha} \left(\frac{\delta t}{x} \right)}{f'_{o_{max}} \; \eta_t^{2}} \qquad 3\cdot7a$$

$$\kappa = \frac{\frac{2}{1+\alpha} \left(\frac{\delta t}{x} \right)}{f'_{o_{max}} \; \eta_t^{2}} \qquad 3\cdot7b$$

APPENDIX III

It is noted that Equation (3·6b) predicts the relation given by Schlichting (1955) for a two dimensional free jet as the value of α approaches unity, as in this case

$$f_{o_{max}}' = \tfrac{1}{2}, \quad \eta_t = 2 \tanh^{-1} \frac{1}{\sqrt{2}}$$

and hence

$$\kappa = \frac{\left(\dfrac{\delta t}{x}\right)}{\left(2 \tanh^{-1} \dfrac{1}{\sqrt{2}}\right)^2} = \frac{(1\cdot125)^2}{4} \left(\frac{\delta t}{x}\right)$$

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TABLE 1. Summary of Experimental Results.

PHASE I. Without diffuser centrebody

Station (Ref. Fig.1.2)	Local radius of curvature r thou.	Suction coeff. (λ)	U_1 ft/sec.	u^* ft/sec.	$C_f = 2\left(\frac{u^*}{U_1}\right)^2$	Axisymmetric				Twodimensional			
						δ_3^* thou.	θ_3 thou.	H_3	c_{f_3}' CALC.	δ_2^* thou.	θ_2 thou.	H_2	c_{f_2}' CALC.
T2(a)(2")	4020	Zero	133.5	4.83	26.2	162	123	1.3	27.5	176	136	1.29	28.1
(c)	"	"	133.5	5.00	28.2	124	96	1.29	30.7	132	103	1.28	30.6
(e)	"	"	133.1	5.00	28.2	120	93	1.29	31.0	128	101	1.27	31.4
(g)	"	"	133.1	4.85	26.4	136	102	1.32	28.5	145	112	1.29	29.6
T3(a)(2")	4230	Zero	124.4	3.66	17.2	191	131	1.46	22.1	205	143	1.43	22.8
T5(a)(3¼)	4620	Zero	113.5	2.09	6.78	399	224	1.78	11.9	452	259	1.75	12.0
	"	0.132	109.7	-	-	398	227	1.75	-	-	-	-	-
	"	0.223	108.6	-	-	406	234	1.74	-	-	-	-	-
T5(a)(2")	4740	Zero	110.5	2.07	7.00	411	220	1.86	10.5	457	254	1.80	11.3
(c)	"	"	-	-	-	-	-	-	-	-	-	-	-
(e)	"	"	-	-	-	-	-	-	-	-	-	-	-
(g)	"	"	110.4	2.15	7.6	399	217	1.84	11.0	-	-	-	-
T5(a)(2")	"	0.132	105.4	2.27	9.25	385	217	1.77	13.1	-	-	-	-
T5(a)(2")	"	0.223	101.2	2.37	10.8	359	209	1.72	13.7	-	-	-	-
(c)	"	0.223	101.6	2.9	16.2	279	179	1.56	18.5	305	199	1.53	18.8
(e)	"	0.223	100.0	-	-	-	-	-	-	-	-	-	-
(g)	"	0.223	102.3	1.68	5.38	488	231	2.11	7.24	542	266	2.04	7.82
T6(a)(2")	4970	Zero	106.3	1.55	4.26	555	262	2.12	6.85	625	308	2.03	7.54
T6(a)(2")	"	0.223	96.8	1.86	7.38	493	259	1.90	9.91	-	-	-	-

Note. The factor 10^{-4} is omitted from the column of c_f values.

C_f' IS CALCULATED USING THE EQUATION OF LUDWIG AND TILLMANN. (1949)

PHASE I. Without diffuser centrebody (cont)

Station (Ref. Fig.1.2)	Local radius of curvature r thou	Suction coeff. (λ)	U_1 ft/sec.	u_* ft/sec.	$C_f=2\left(\frac{u_*}{U_1}\right)^2$	Axisymmetric δ_3^* thou.	θ_3 thou.	H_3	c_{f_3}' CALC.	Twodimensional. δ_2^* thou.	θ_2 thou.	H_2	c_{f_2}' CALC.
T7(a)(2")	5200	Zero	102.6	1.29	3.18	716	311	2.31	4.98	-	-	-	-
T7(a)(2")	"	0.132	97.8	1.25	3.25	694	295	2.35	4.70	-	-	-	-
T7(a)(2")	"	0.223	93.8	1.34	4.08	672	302	2.20	5.37	-	-	-	-
T8(a)(2")	5450	Zero	100.9	0.945	1.76	876	324	2.70	2.65	1017	410	2.48	3.51
T8(a)(2")	"	0.223	91.2	0.89	1.91	905	336	2.70	2.73	-	-	-	-
T9(a)	5690	Zero	97.7	0.625	0.82	1054	342	3.08	1.45	1246	440	2.83	2.02
T9(c)	"	"	98.5	1.22	3.02	857	347	2.47	3.74	990	424	2.33	4.43
(e)	"	"	99.1	1.02	2.12	911	325	2.82	2.29	1067	429	2.49	3.43
(g)	"	"	99.0	1.29	3.38	799	321	2.49	3.70	912	384	2.38	4.19
T9(a)	"	0.223	88.0	0.63	1.03	1094	347	3.15	1.34	-	-	-	-
(c)	"	0.223	88.6	1.85	8.74	572	304	1.88	10.1	645	353	1.83	10.5
(e)	"	0.223	88.7	1.05	2.78	843	344	2.45	4.01	979	421	2.33	4.56
(g)	"	0.223	89.0	0.86	1.87	973	330	2.95	1.85	1131	411	2.75	2.38

Note. The factor 10^{-4} is omitted from the column of c_f values.

TABLE 1. Summary of Experimental Results.

PHASE II. With diffuser centrebody.

Station (Ref. Fig.1.2)	Local radius of curvature r thou.	Suction coeff. (λ)	U_1 ft/sec.	u_* ft/sec.	$C_f = 2\left(\frac{u_*}{U_1}\right)^2$	Axisymmetric δ_3^* thou.	Axisymmetric θ_3 thou.	Axisymmetric H_3	Axisymmetric c_{f_3}' CALC.	Twodimensional δ_2^* thou.	Twodimensional θ_2 thou.	Twodimensional H_2	Twodimensional c_{f_2}' CALC.
T3(2")	4230	Zero	123.6	3.4	15.1	238.5	163	1.46	21.0	261	182	1.43	21.3
T3(2")	"	0.0294	123.15	-	-	-	-	-	-	-	-	-	-
T3(2")	"	0.0955	121.79	-	-	-	-	-	-	-	-	-	-
T5(4¾)	4500	Zero	114.7	2.55	9.86	338	201	1.68	14.2	373	229	1.63	15.0
T5(4¾)	"	0.0294	113.7	2.73	11.5	308	191	1.61	16.3	339	215	1.58	16.4
T5(4¾)	"	0.0955	113.25	3.06	14.6	270	174	1.55	18.3	295	195	1.51	18.9
T5(3¼)	4620	Zero	112.1	2.5	9.94	345	202	1.68	13.7	382	233	1.64	14.8
"	"	0.0175	112.1			316	197	1.6					
"	"	0.0294	111.8	Not		335	197	1.69					
"	"	0.055	112.7			300	181	1.65		Not			
"	"	0.0955	112.0	in		336	212	1.59					
"	"	0.163	111.8	equalibrium		278	183	1.52		Calculated			
T5(2¾)	4680	Zero	109.9	2.32	8.9	355	208	1.71	13.7	392	237	1.65	14.6
"	"	0.0175	110.2			332	205	1.62					
"	"	0.0294	110.9	Not		320	198	1.62					
"	"	0.055	110.9	in		301	189	1.58		Not			
"	"	0.0955	112.0			291	188	1.55		Calculated			
"	"	0.163	110.5	Equalibrium		276	183	1.51					

Note. The factor 10^{-4} is omitted from the column of c_f values.

PHASE II. <u>With diffuser centrebody</u> (cont)

Station (Ref. Fig.1.2)	Local radius of curvature r thou.	Suction coeff (λ)	U_1 ft/sec.	u_* ft/sec.	$C_f = 2\left(\frac{u_*}{U_1}\right)^2$	Axisymmetric δ_3^* thou.	Axisymmetric θ_3 thou.	Axisymmetric H_3	Axisymmetric c_{f_3}' CALC.	Twodimensional δ_2^* thou.	Twodimensional θ_2 thou.	Twodimensional H_2	Twodimensional c_{f_2}' CALC.
T5(2")	4740	Zero	108.2	2.24	8.56	363	209	1.74	13.1	400	238	1.68	13.9
"	"	0.0175	109.0	2.61	11.4	350	210	1.67	14.6	386	239	1.62	15.3
"	"	0.0294	109.4	2.74	12.5	330	203	1.63	15.7	362	228	1.59	16.1
"	"	0.055	109.0	2.90	14.2	304	193	1.58	17.1	332	215	1.54	17.7
"	"	0.0955	108.6	3.05	15.8	288	186	1.55	18.3	315	208	1.51	18.7
"	"	0.163	109.0	3.4	19.5	260	175	1.49	20.3	282	196	1.44	21.3
T6(2")	4970	Zero	103.0	1.68	5.32	503	253	1.99	8.55	562	294	1.91	9.32
"	"	0.0175	102.9	2.04	7.84	438	238	1.84	10.9	485	272	1.78	11.7
"	"	0.0294	102.9	2.16	8.82	417	233	1.78	11.9	460	266	1.73	12.6
"	"	0.055	103.5	2.39	10.7	371	222	1.69	14.5	419	250	1.68	13.8
"	"	0.0955	104.2	2.65	12.9	336	210	1.60	16.4	378	236	1.6	15.9
"	"	0.163	105.0	3.03	16.7	287	187	1.53	19.0	312	207	1.51	18.9
T7($3\frac{3}{4}$")	5100	Zero	101.7	1.36	3.60	617	276	2.24	5.69	696	327	2.13	6.41
"	"	0.0175	101.2	1.75	5.98	521	260	2.02	8.4	581	300	1.94	8.87
"	"	0.0294	100.8	1.84	6.7	487	247	1.97	8.93	539	283	1.90	9.61
"	"	0.055	100.9	2.02	8.0	450	238	1.89	10.2	496	271	1.83	10.8
"	"	0.0955	101.0	2.36	11.0	387	220	1.74	12.7	420	247	1.70	13.6
"	"	0.163											
T7(2")	5200	Zero	100.4	1.16	2.7	693	291	2.38	4.5	790	350	2.26	5.15
"	"	0.0175	100.0	1.52	4.62	580	273	2.13	6.85	651	320	2.03	7.60
"	"	0.0294	99.7	1.74	6.12	533	265	2.01	8.26	595	307	1.94	8.80
"	"	0.055	99.7	1.97	7.84	471	249	1.89	10.1	522	286	1.83	10.7
"	"	0.0955	99.8	2.24	10.0	406	230	1.76	12.4	444	259	1.71	13.2
"	"	0.163	100.0	2.77	15.3	300	188	1.6	17.1	322	207	1.56	17.8

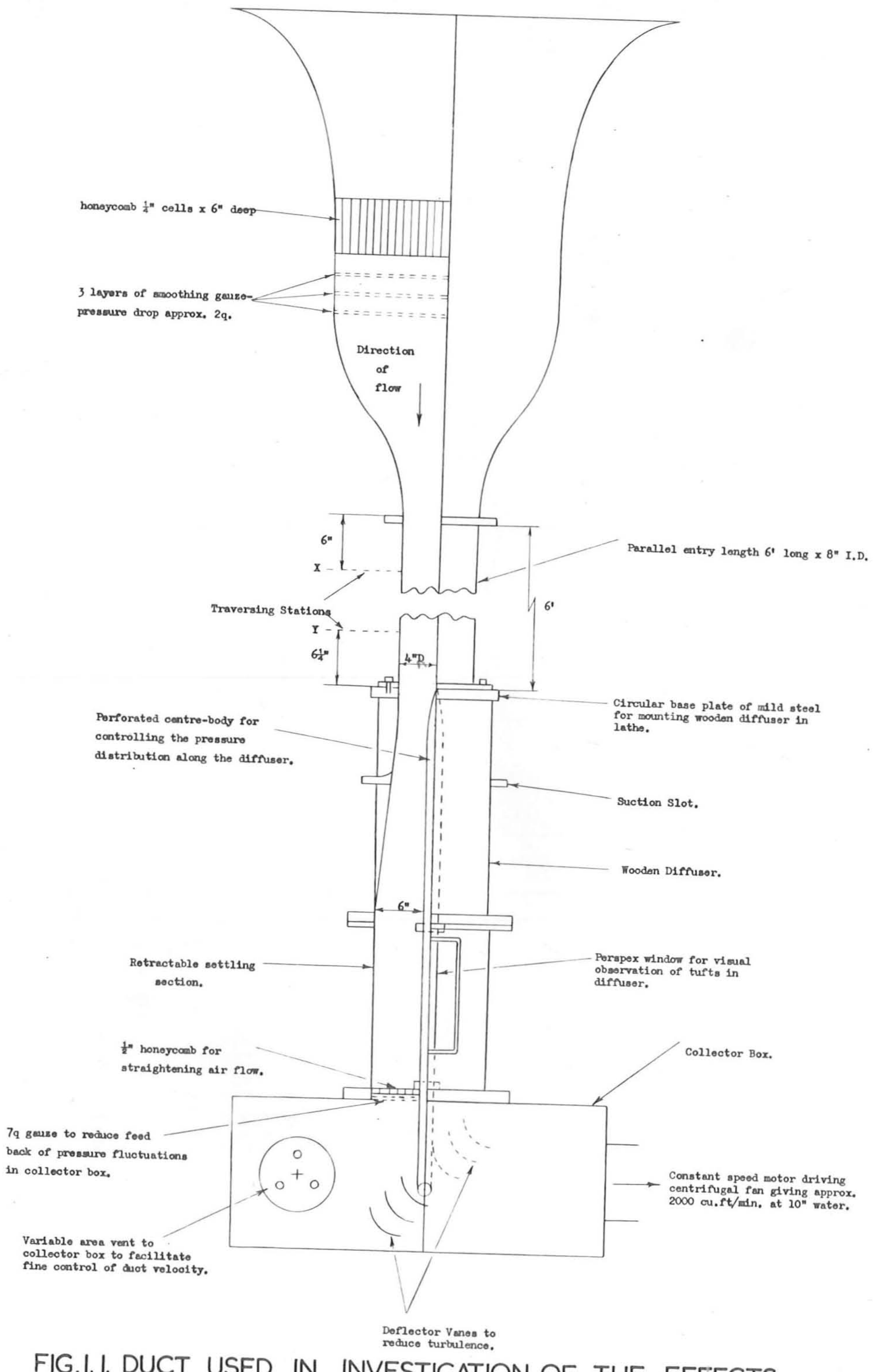


FIG. 1.1. DUCT USED IN INVESTIGATION OF THE EFFECTS OF A SUCTION STRIP ON A BOUNDARY LAYER.

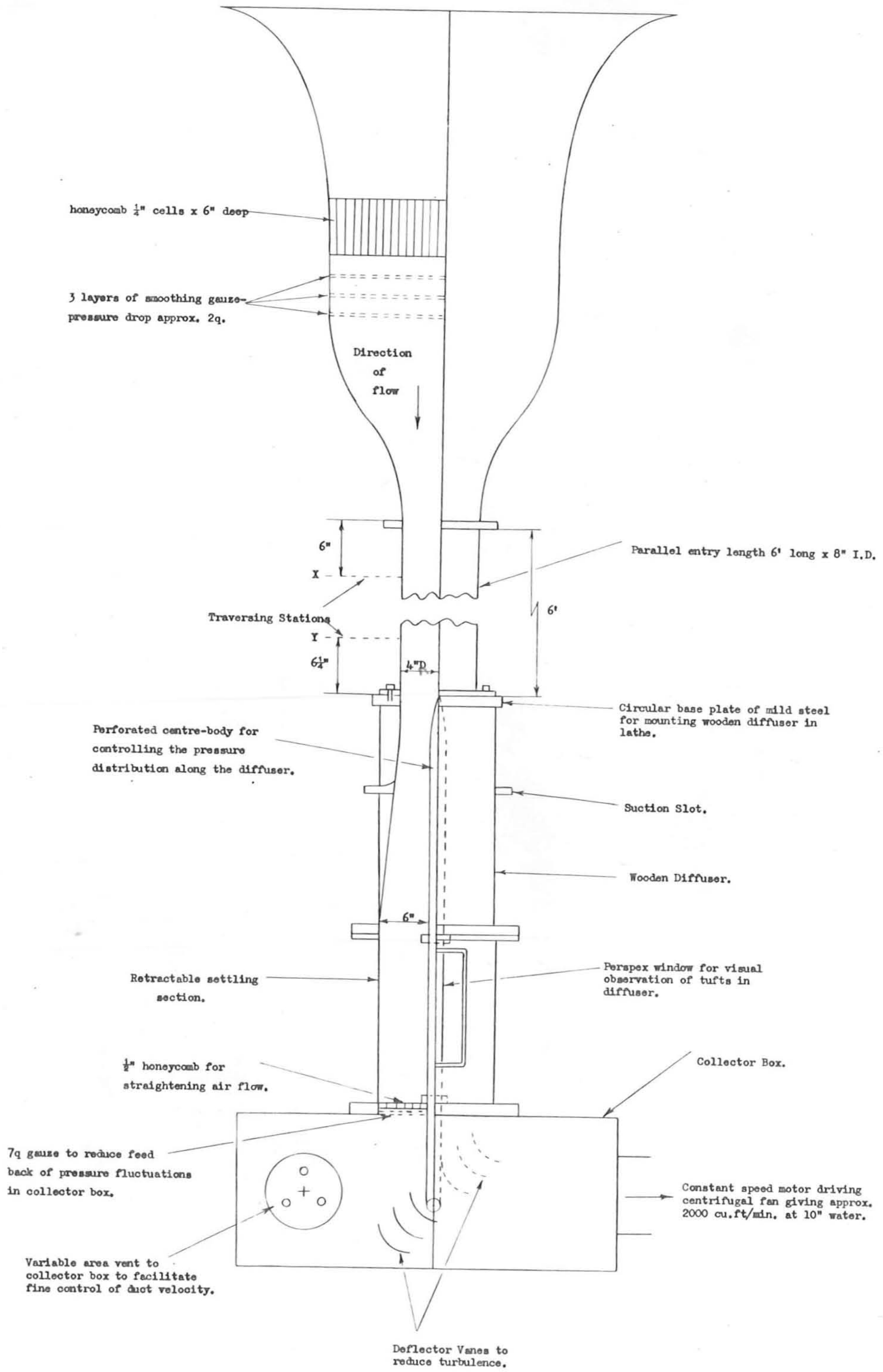


FIG. I. I. DUCT USED IN INVESTIGATION OF THE EFFECTS OF A SUCTION STRIP ON A BOUNDARY LAYER.

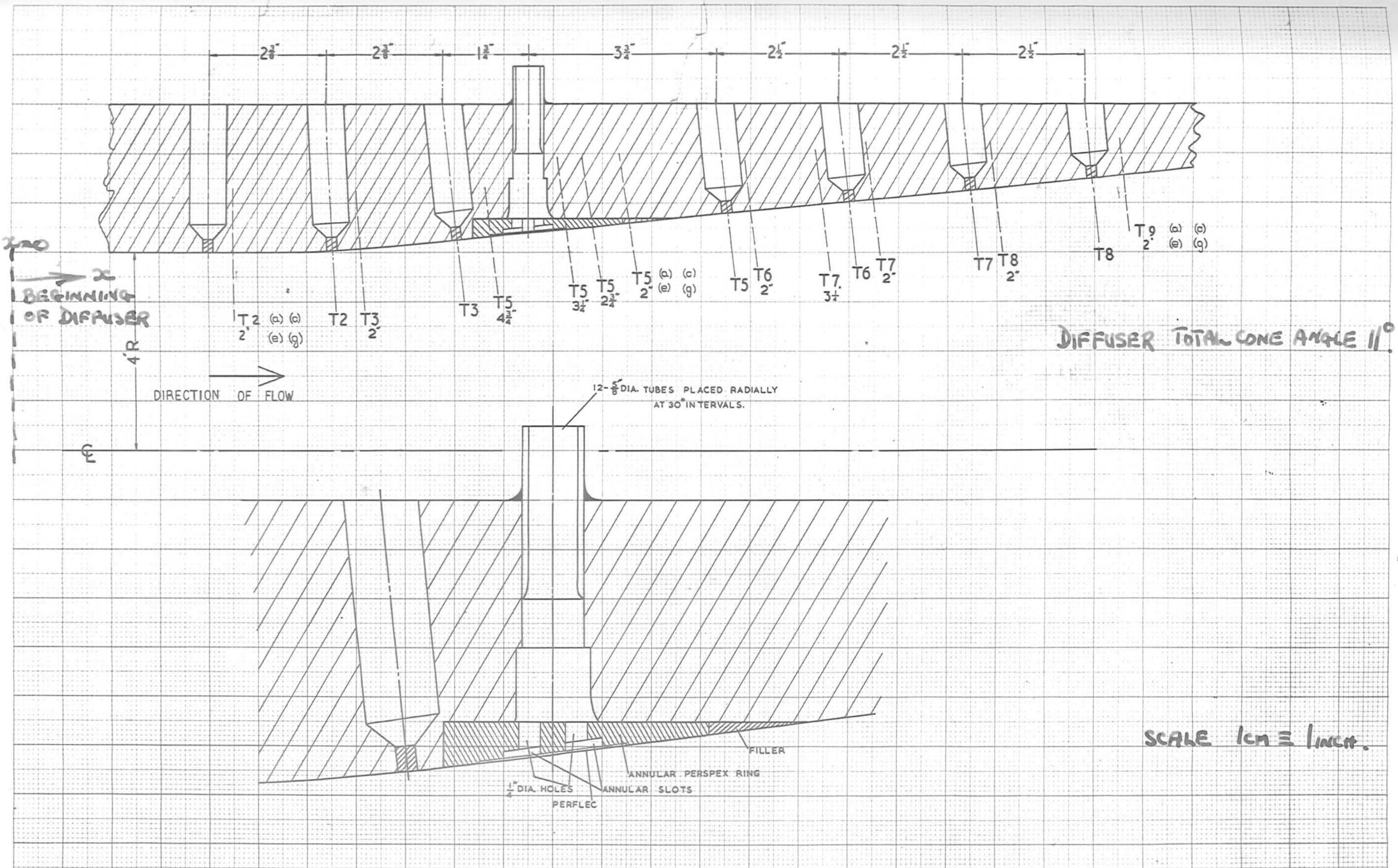


FIG. 1.2 TRAVERSING STATIONS DOWN DIFFUSER OF BOUNDARY LAYER DUCT

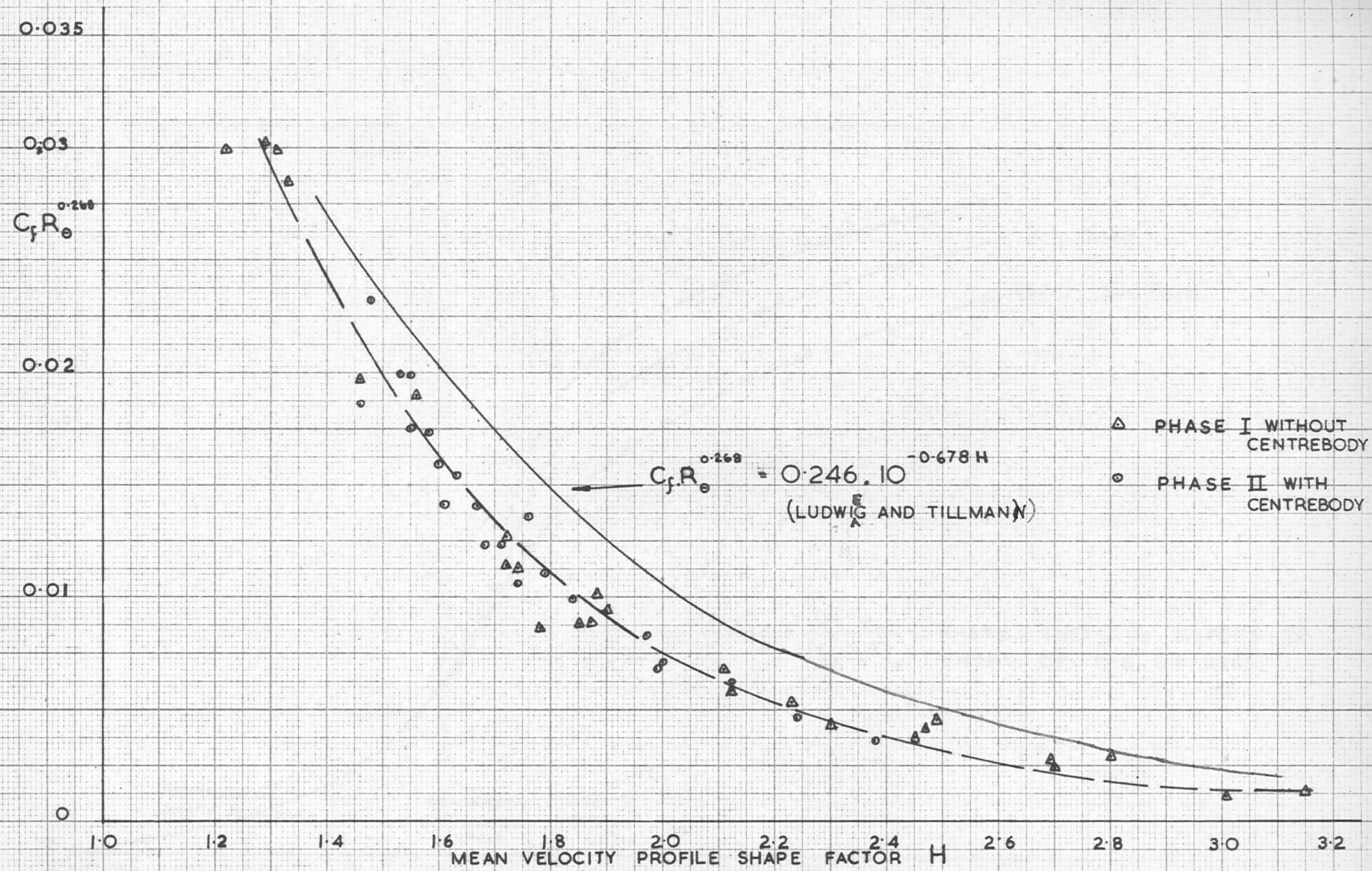


FIG 1·3 COMPARISON OF THE SKIN FRICTION COEFFICIENT ESTIMATED FROM THE UNIVERSALITY OF THE INNER PART OF THE MEAN VELOCITY PROFILE WITH THE VALUE PREDICTED BY THE EMPIRICAL FORMULA OF LUDWIEG AND TILLMANN (1949)

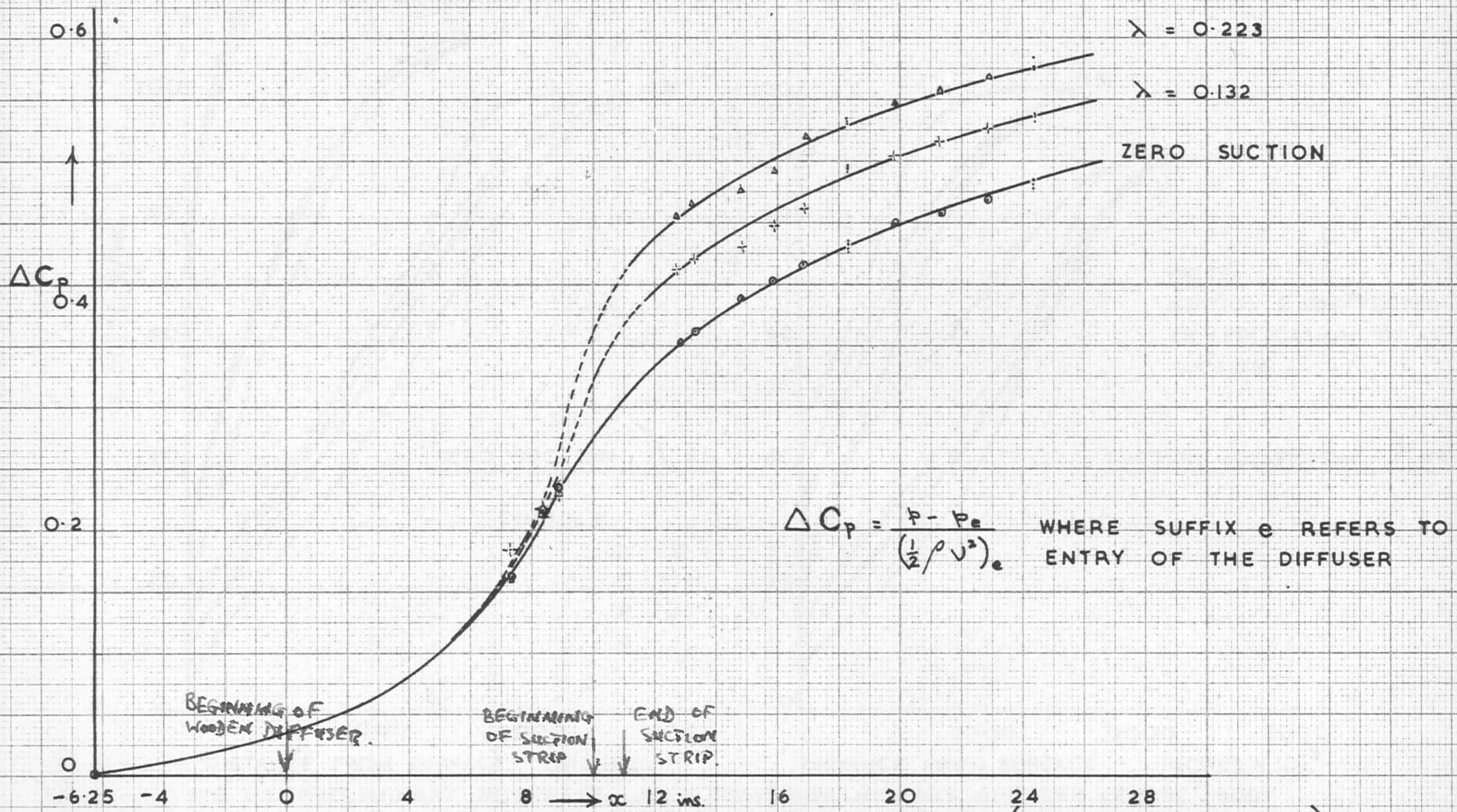


FIG 1·4 VARIATION OF STATIC PRESSURE ALONG THE DIFFUSER (WITHOUT CENTREBODY)

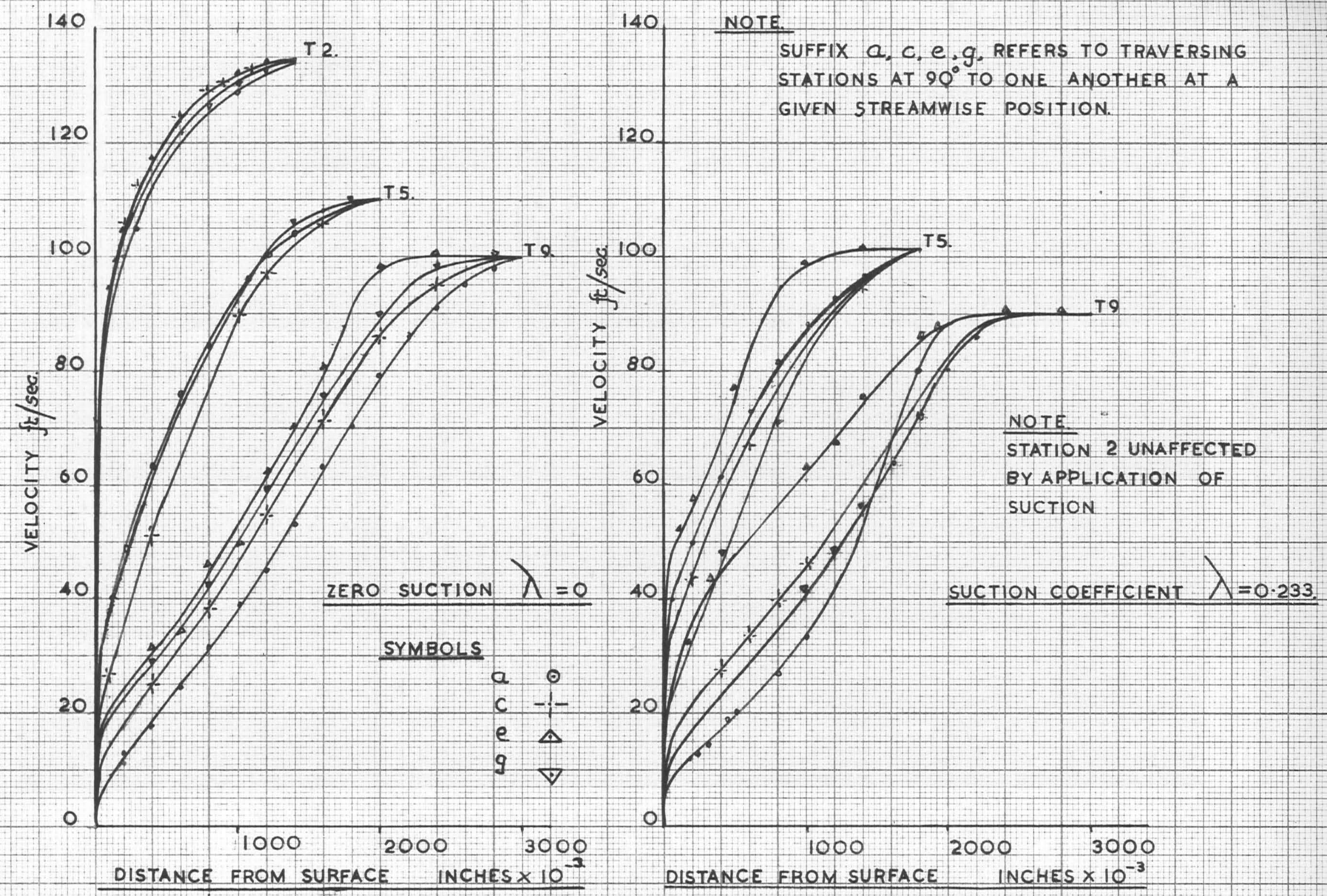


FIG. 1·5 PRELIMINARY BOUNDARY LAYER TRAVERSES WITHOUT DIFFUSER CENTRE BODY TO ESTABLISH THE DEGREE OF ASYMMETRY WITH AND WITHOUT SUCTION APPLIED.

FIG. 1·6 DEVELOPMENT OF BOUNDARY LAYER DOWN DIFFUSER WITH AND WITHOUT SUCTION (NO CENTREBODY)

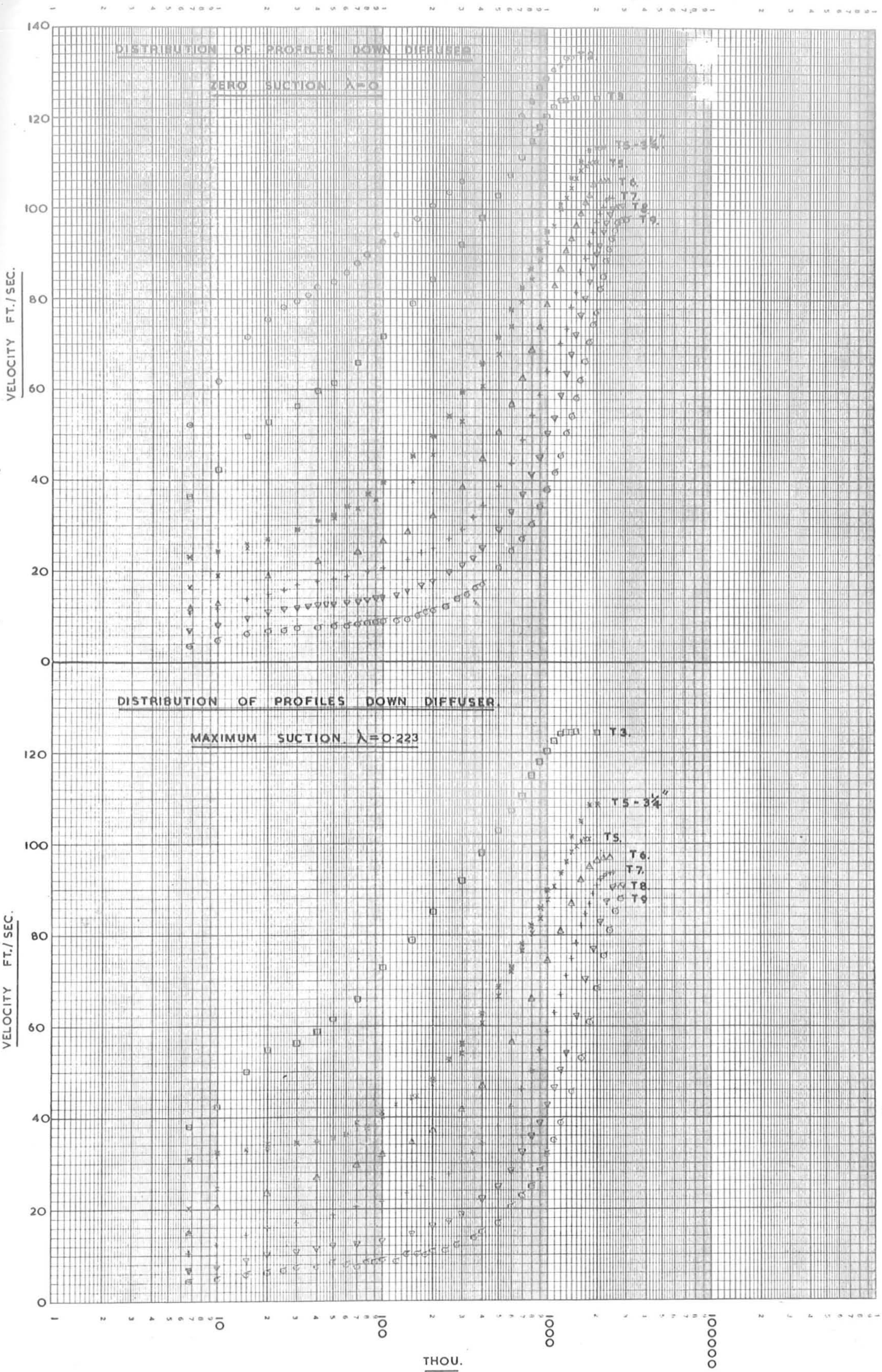
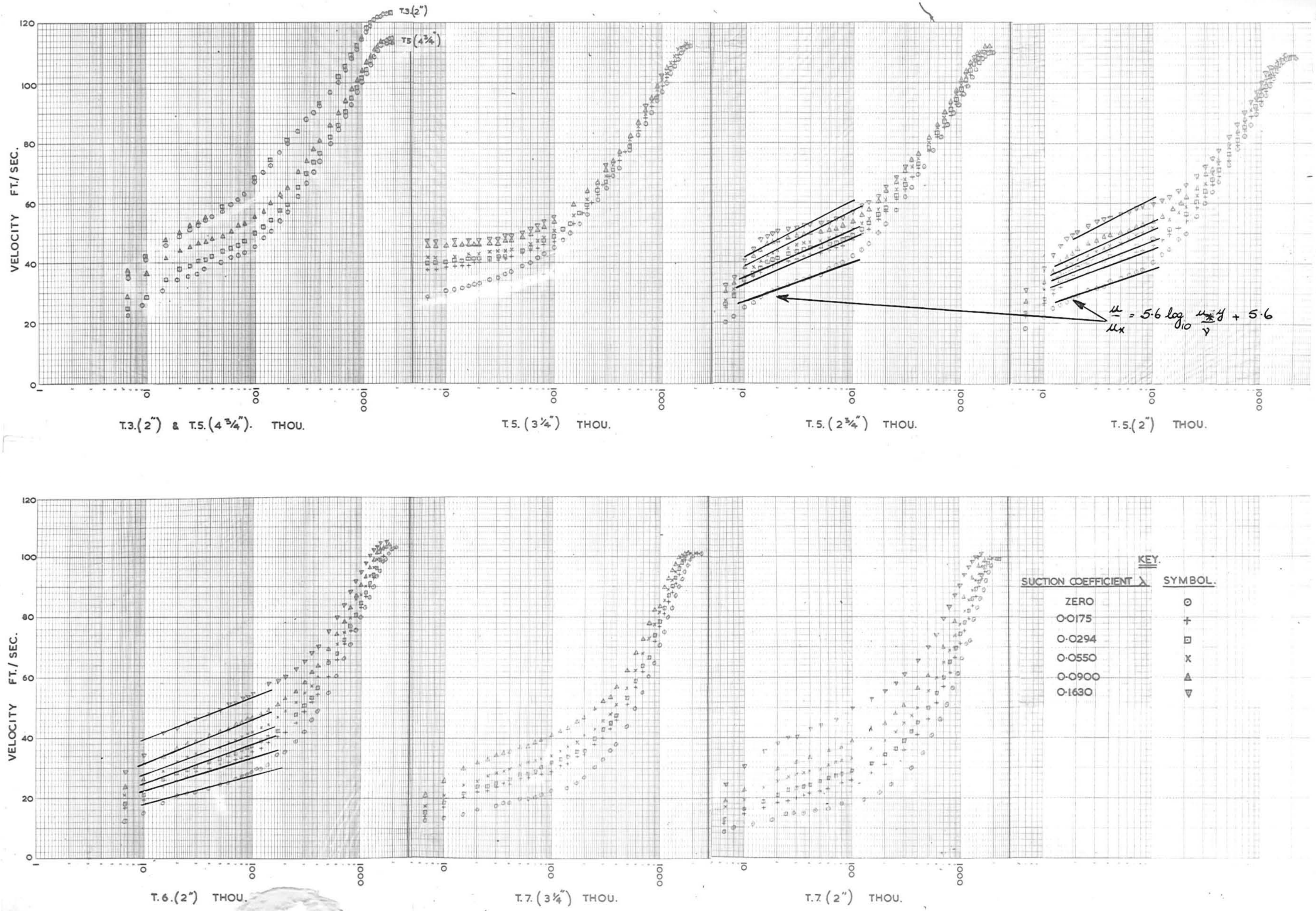
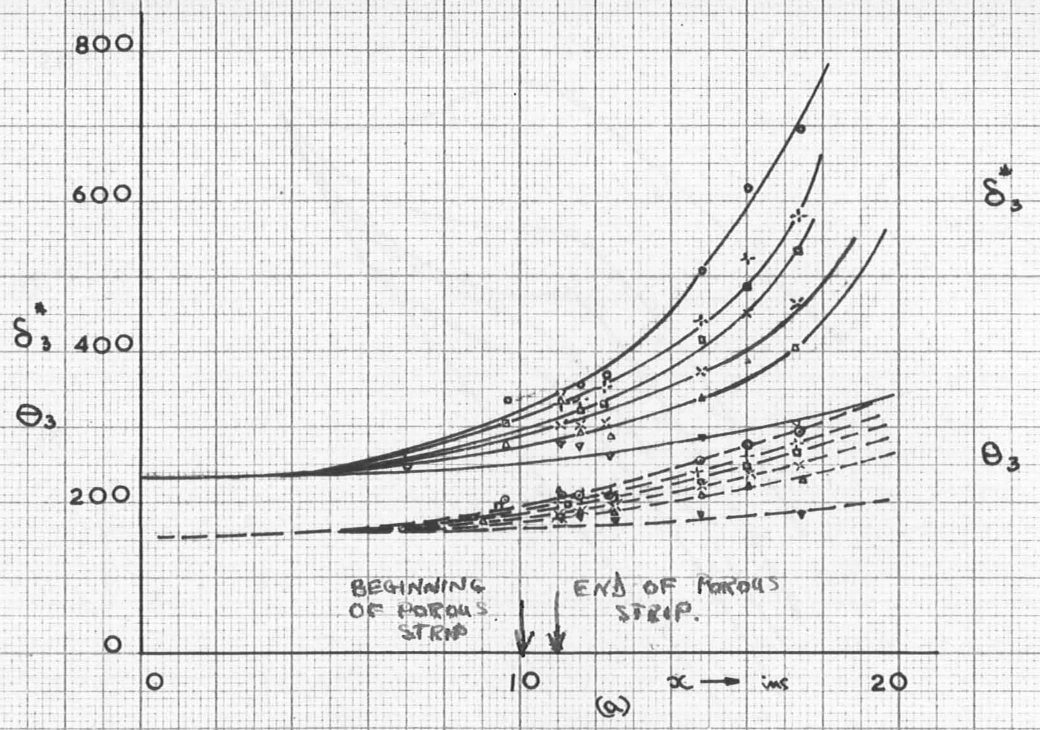


FIG. (1·7) DEVELOPMENT OF THE BOUNDARY LAYER IN THE VICINITY OF A SUCTION SLOT. (PHASE II. WITH CENTREBODY)





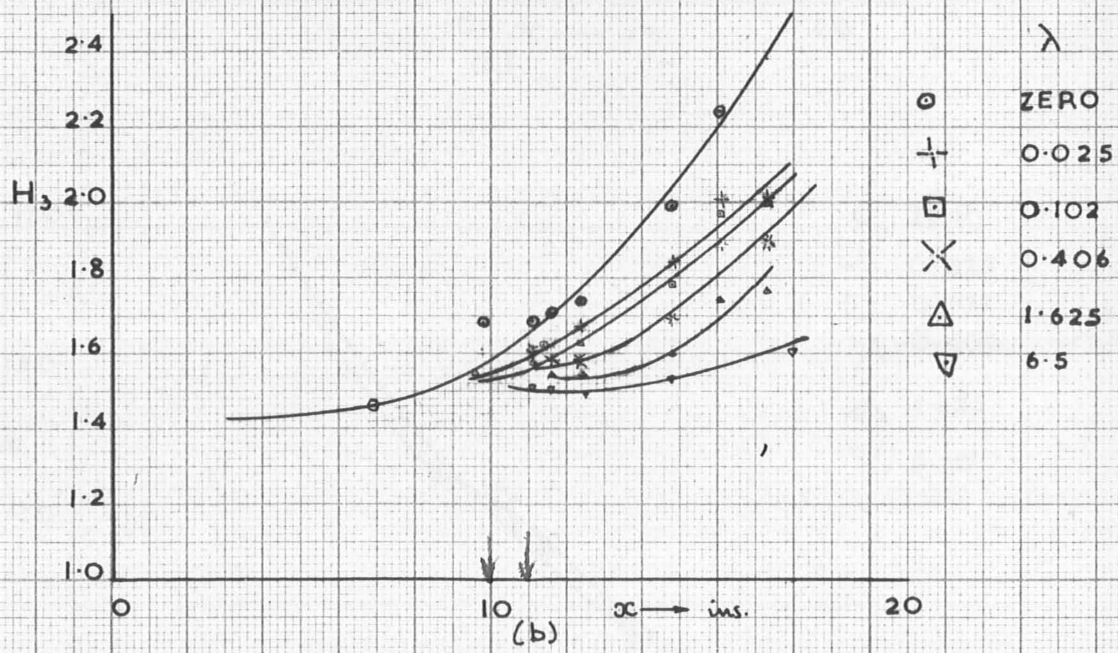


FIG 1. 8 VARIATION OF SOME LOCAL BOUNDARY LAYER PARAMETERS IN THE VICINITY OF A SUCTION STRIP

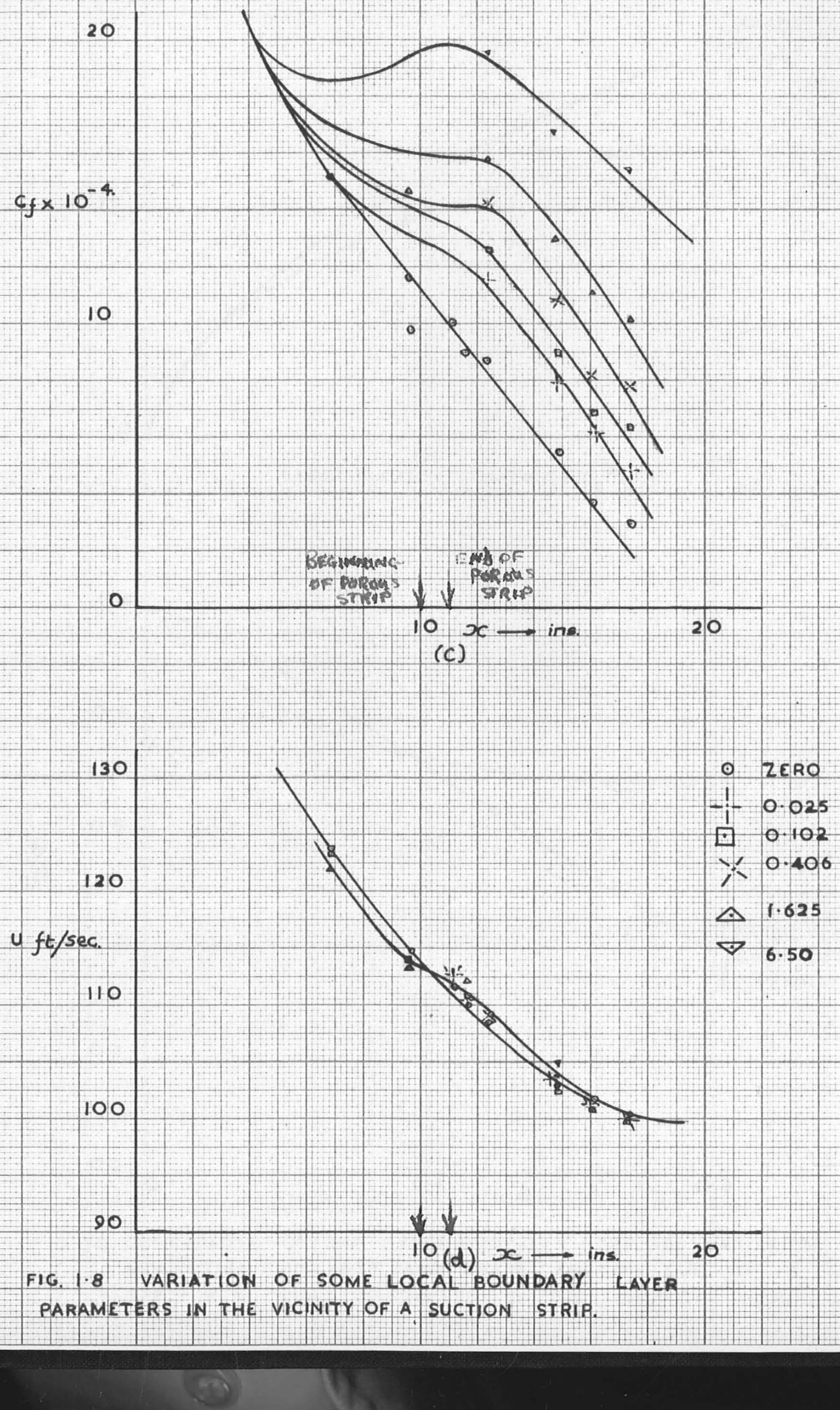


FIG. 1·8 VARIATION OF SOME LOCAL BOUNDARY LAYER PARAMETERS IN THE VICINITY OF A SUCTION STRIP.

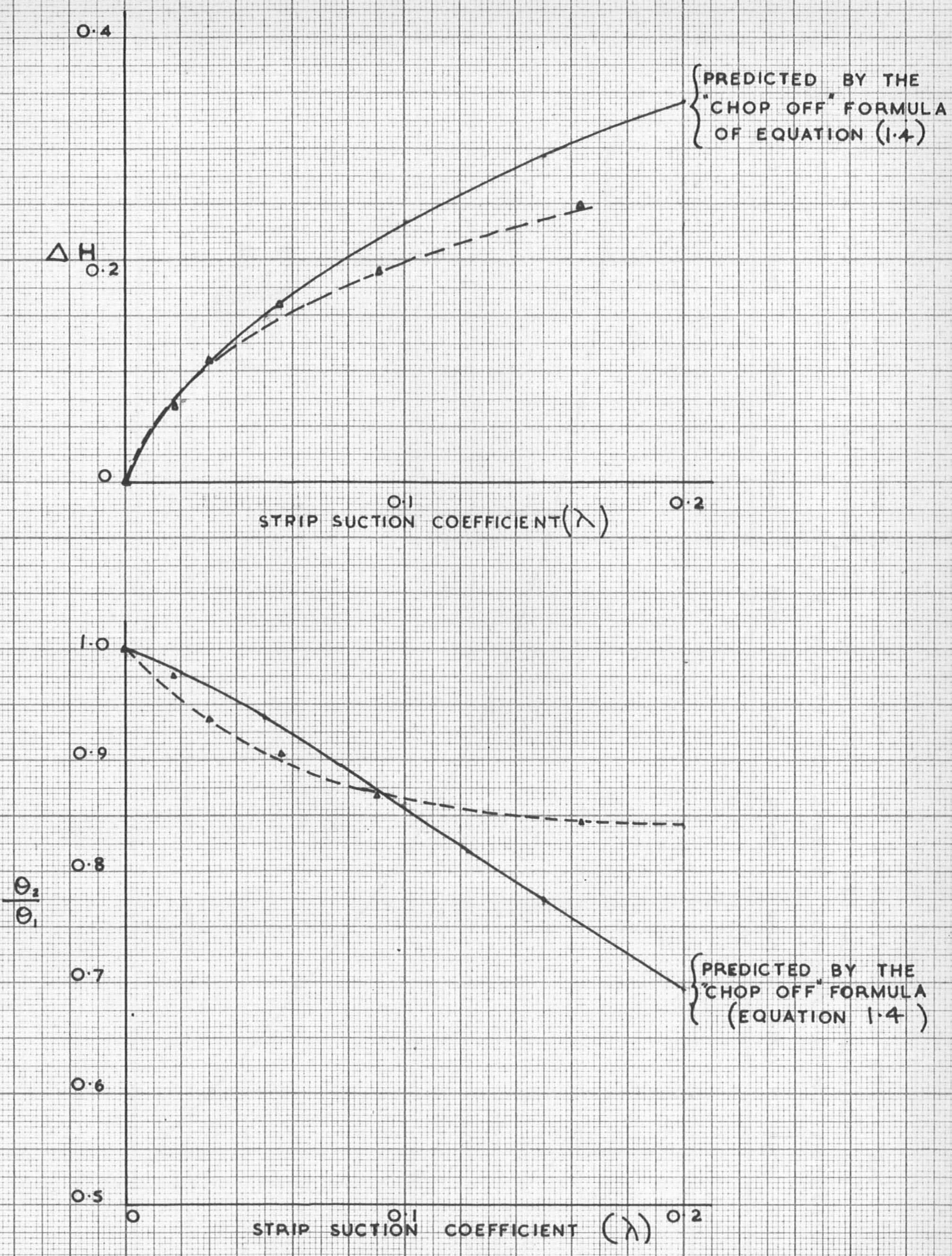


FIG 1·9 COMPARISON OF THE EXPERIMENTALLY DEFINED VARIATION OF MOMENTUM THICKNESS AND SHAPE FACTOR DUE TO A SUCTION STRIP WITH THAT PREDICTED BY THE "CHOP OFF" FORMULA

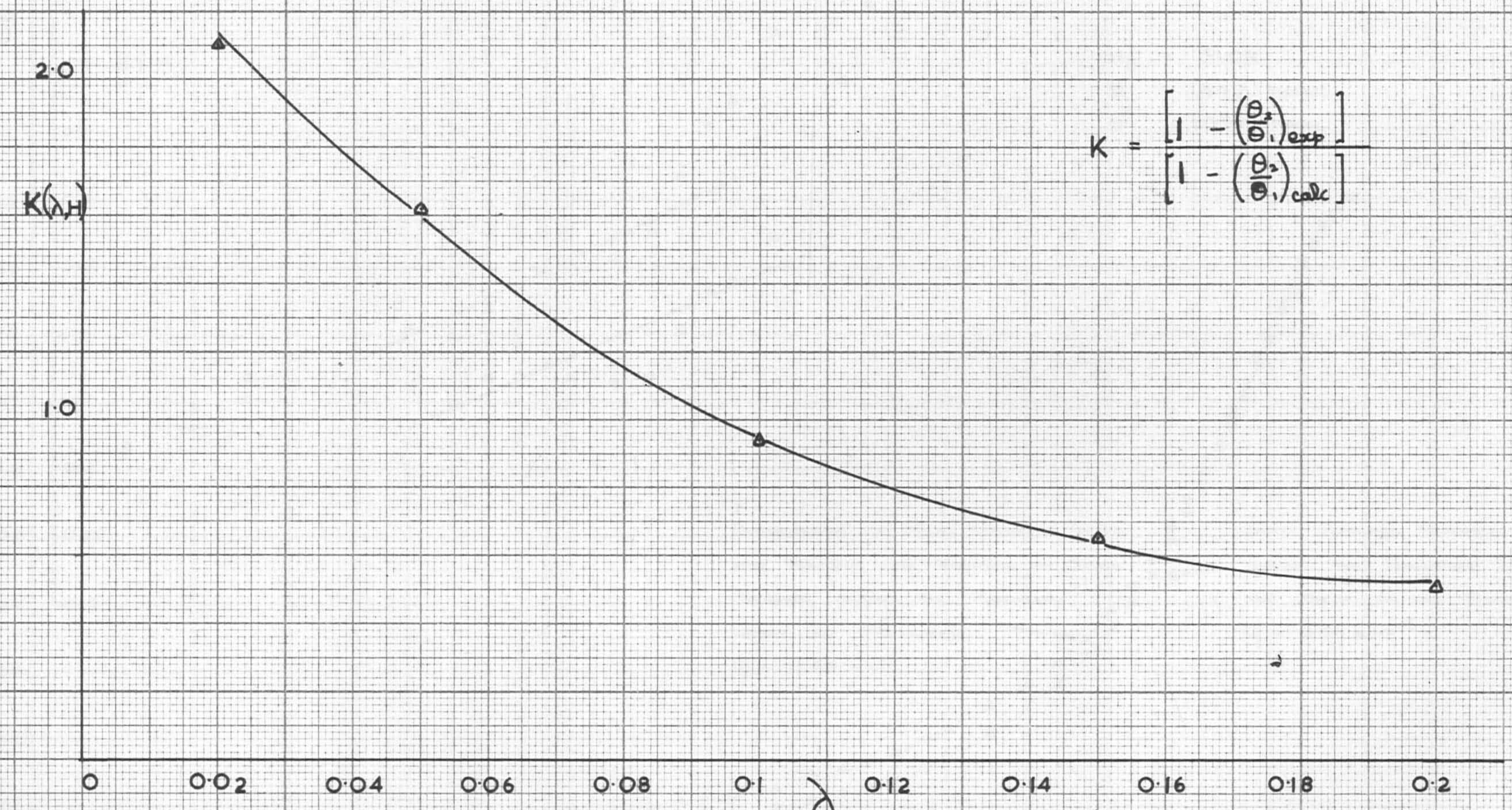


FIG.1.10 VARIATION OF EMPIRICAL CORRECTION FACTOR $K(\lambda_H)$ WHICH ADJUSTS THE VARIATION OF MOMENTUM THICKNESS AS PREDICTED BY THE "CHOP OFF" FORMULA TO THAT MEASURED EXPERIMENTALLY AT A VALUE OF SHAPE FACTOR $(H) = 1{\cdot}7$

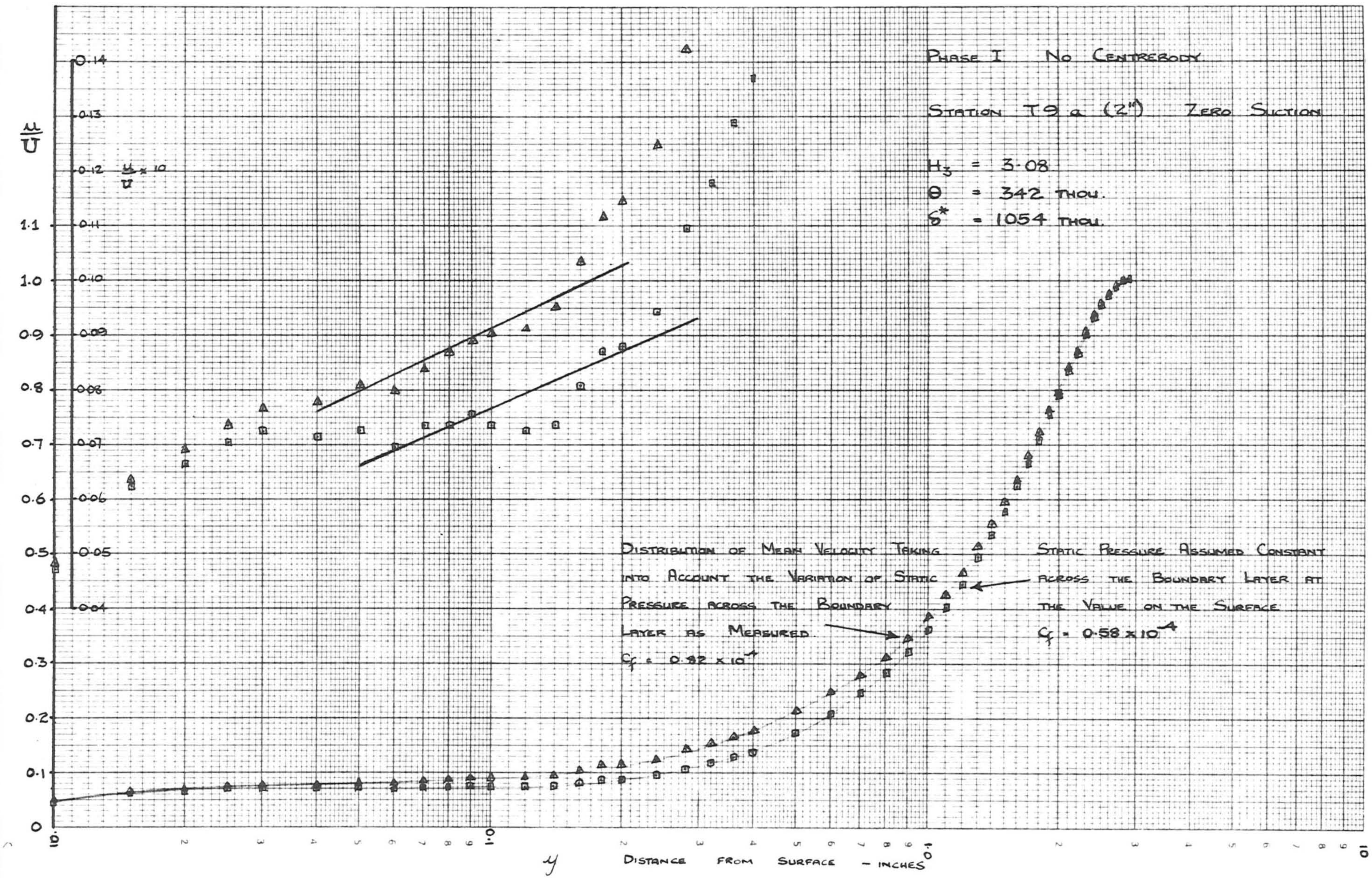
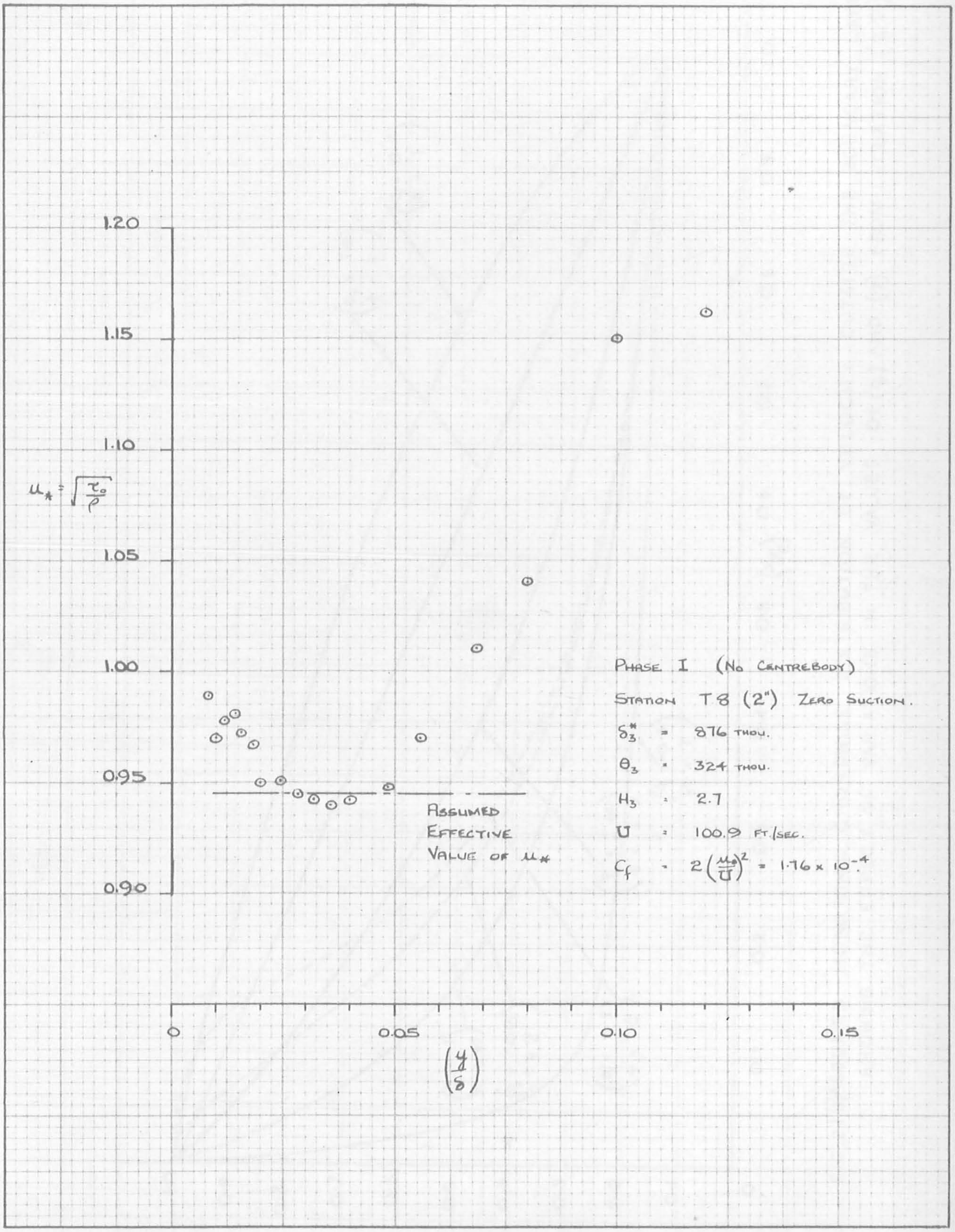


Fig. 1. 11. The Effect of Neglecting the Variation of Static Pressure Across a Turbulent Boundary Layer Near Separation.

Fig. 1.12. Typical Variation of the Value of Friction Velocity Deduced from Measurements of $u(y)$ using the Universality of the Inner Part of the Mean Velocity Profile.



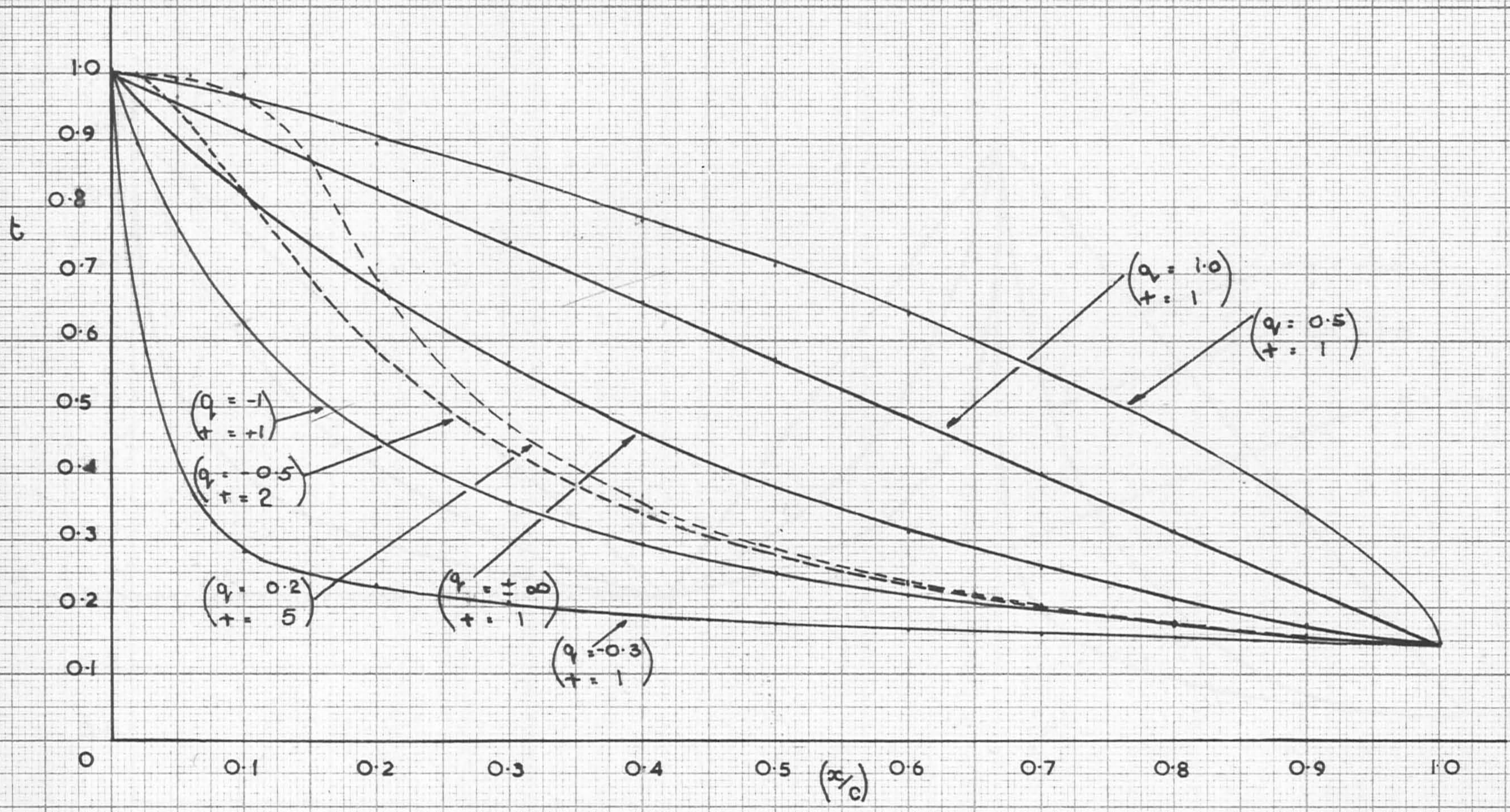


FIG 2·1 (a) ASSUMED MONOTONIC CHORDWISE DISTRIBUTION OF VELOCITY OUTSIDE THE BOUNDARY LAYER SHOWING THE EFFECT OF VARIATIONS IN THE VALUES OF (q) AND (t) FROM EQUATION (2·9).

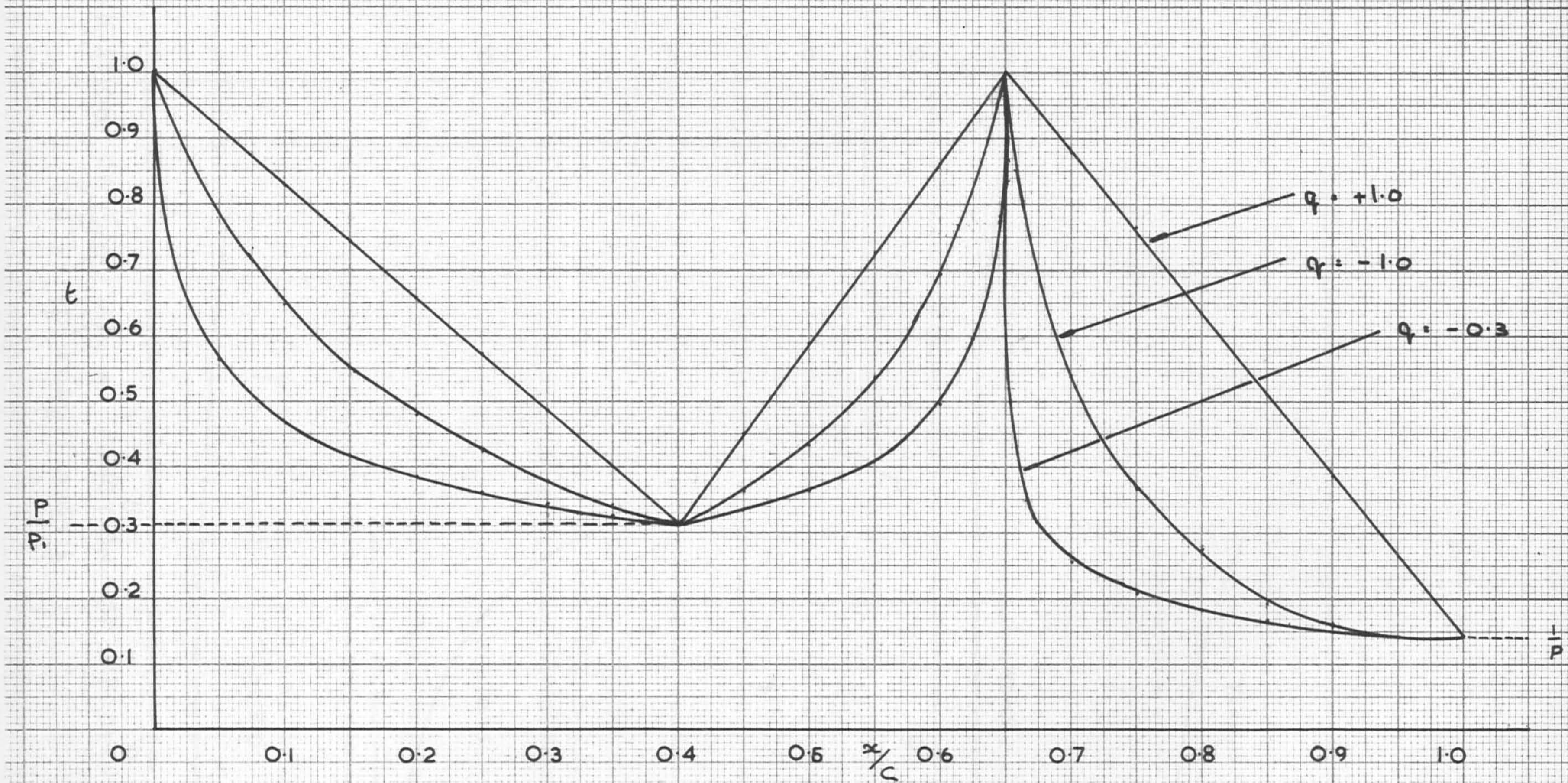


FIG 2·1(b) THE ASSUMED DISTRIBUTION OF VELOCITY OUTSIDE THE BOUNDARY LAYER FOR A FLAPPED WING WITH $p = 7$ $t = 1$ [EQUATION (2·9)]

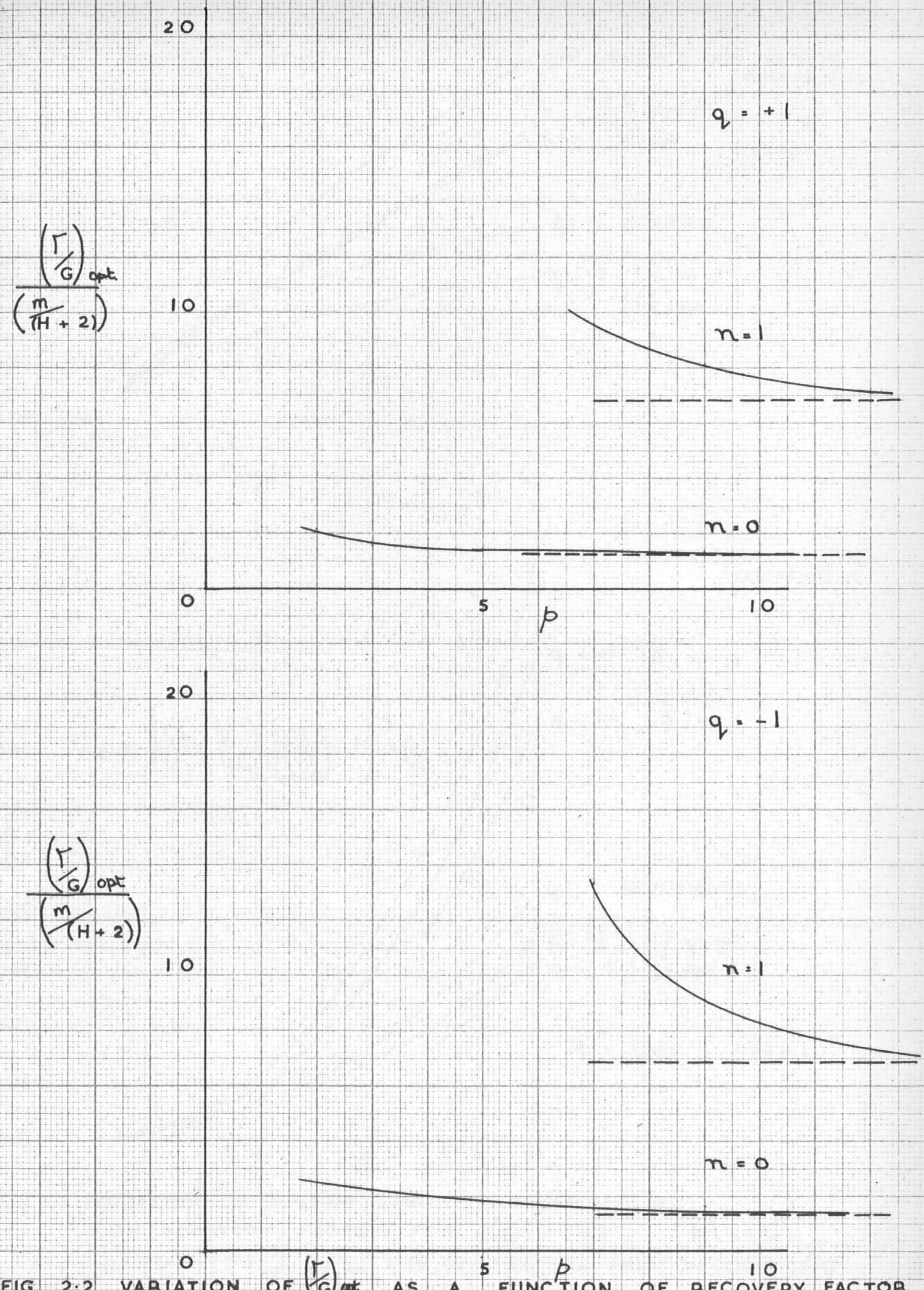


FIG. 2·2 VARIATION OF $\dfrac{\left(\Gamma/G\right)_{opt}}{\left(\frac{m}{H+2}\right)}$ AS A FUNCTION OF RECOVERY FACTOR (p) FOR $q = \pm 1$. AND $r = 1$.

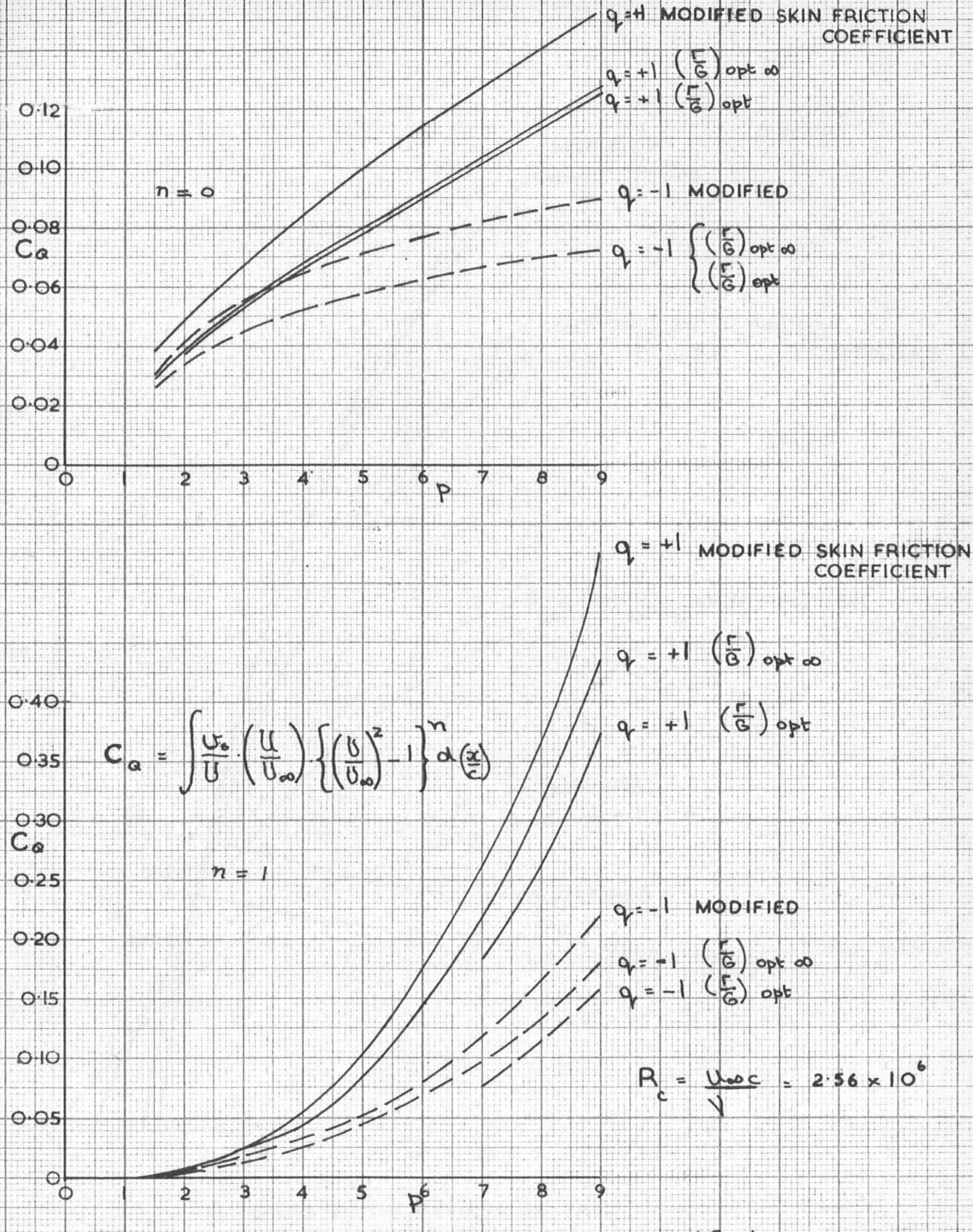
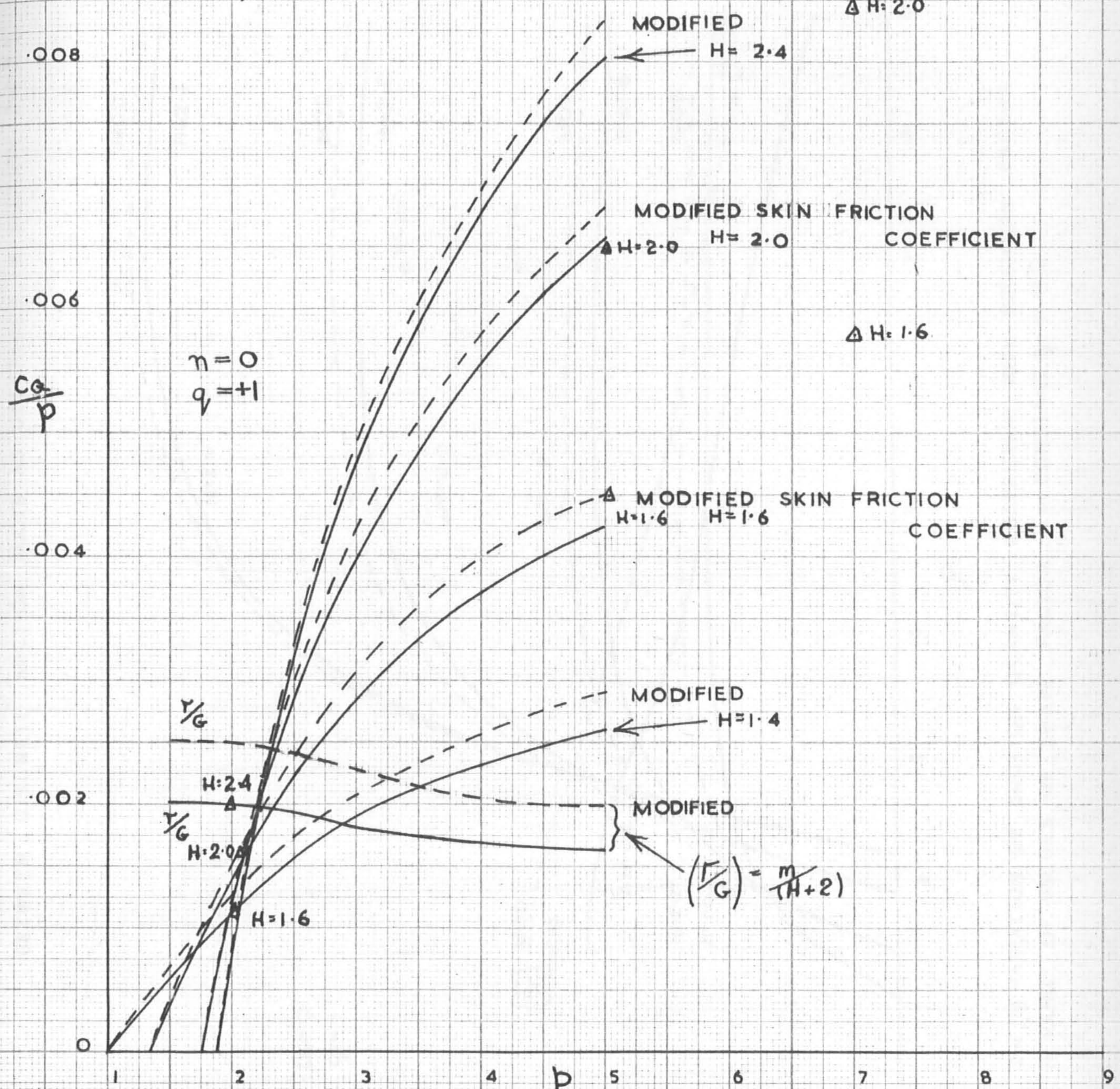


FIG 2·3 VARIATION OF SUCTION QUANTITY COEFFICIENT (C_Q) WITH RECOVERY FACTOR FOR $\left(\frac{r}{G}\right) = \left(\frac{r}{G}\right)_{opt}$, $\left(\frac{r}{G}\right) = \left(\frac{r}{G}\right)_{opt\ p=\infty}$ AND $\left(\frac{r}{G}\right) = \left(\frac{r}{G}\right)_{opt.\ p=\infty}$ WITH MODIFIED SKIN FRICTION COEFFICIENT



Δ REFERS TO THE RESULTS OF THE STEP BY STEP CALCULATIONS BETWEEN DISCRETE SUCTION SLOTS AS DESCRIBED IN SECTION 2·4·22

FIG 2·4(a) VARIATION OF SUCTION QUANTITY COEFFICIENT WITH RECOVERY FACTOR FOR A RANGE OF VALUES OF SHAPE FACTOR H=CONSTANT q=+1

△ REFERS TO THE RESULTS OF THE STEP BY STEP CALCULATIONS BETWEEN DISCRETE SUCTION SLOTS AS DESCRIBED IN SECTION 2·4·2·2

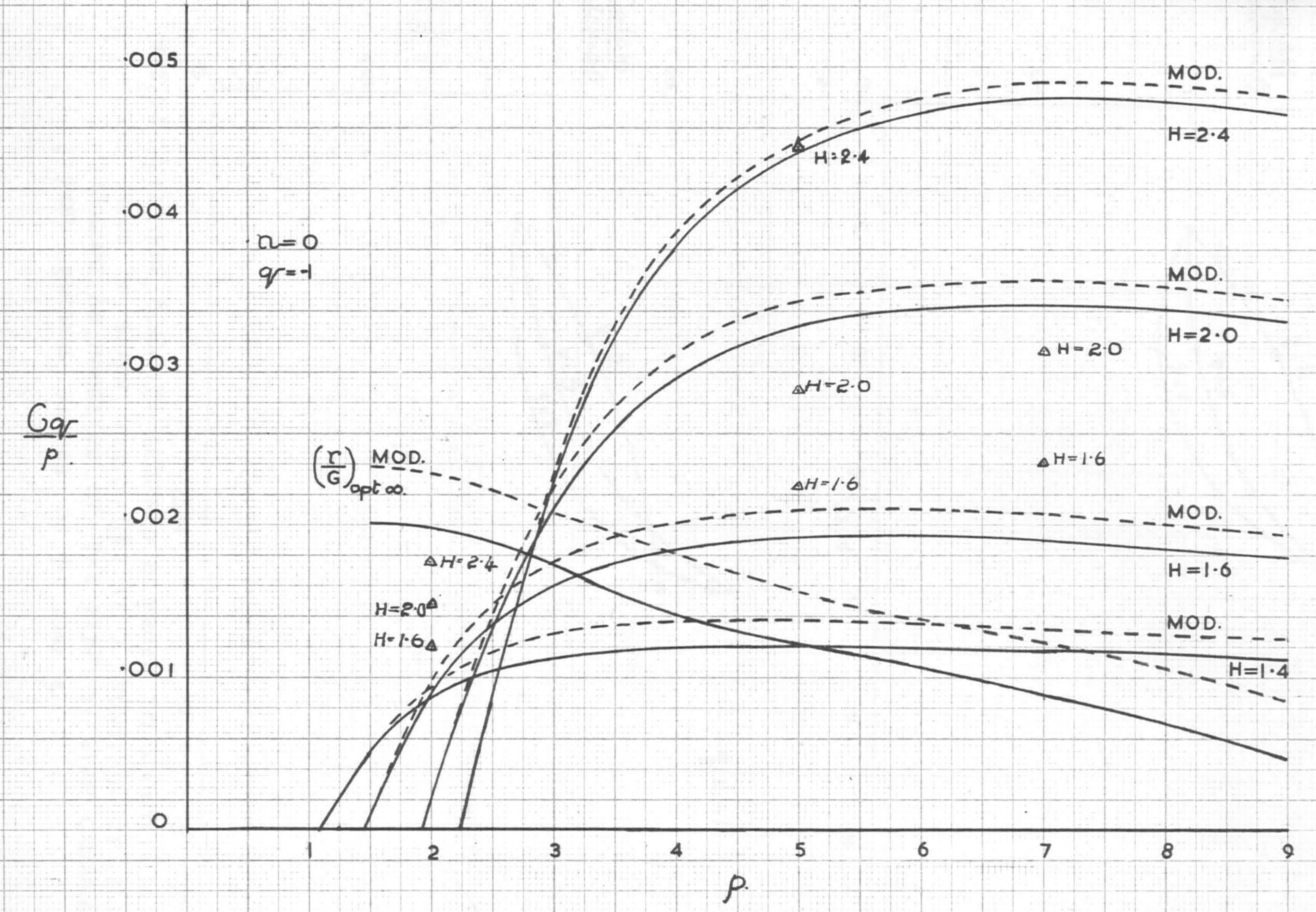


FIG. 2·4 (B) VARIATION OF SUCTION QUANTITY COEFFICIENT WITH RECOVERY FACTOR FOR A RANGE OF VALUES OF SHAPE FACTOR H = CONSTANT $q = -1·0$

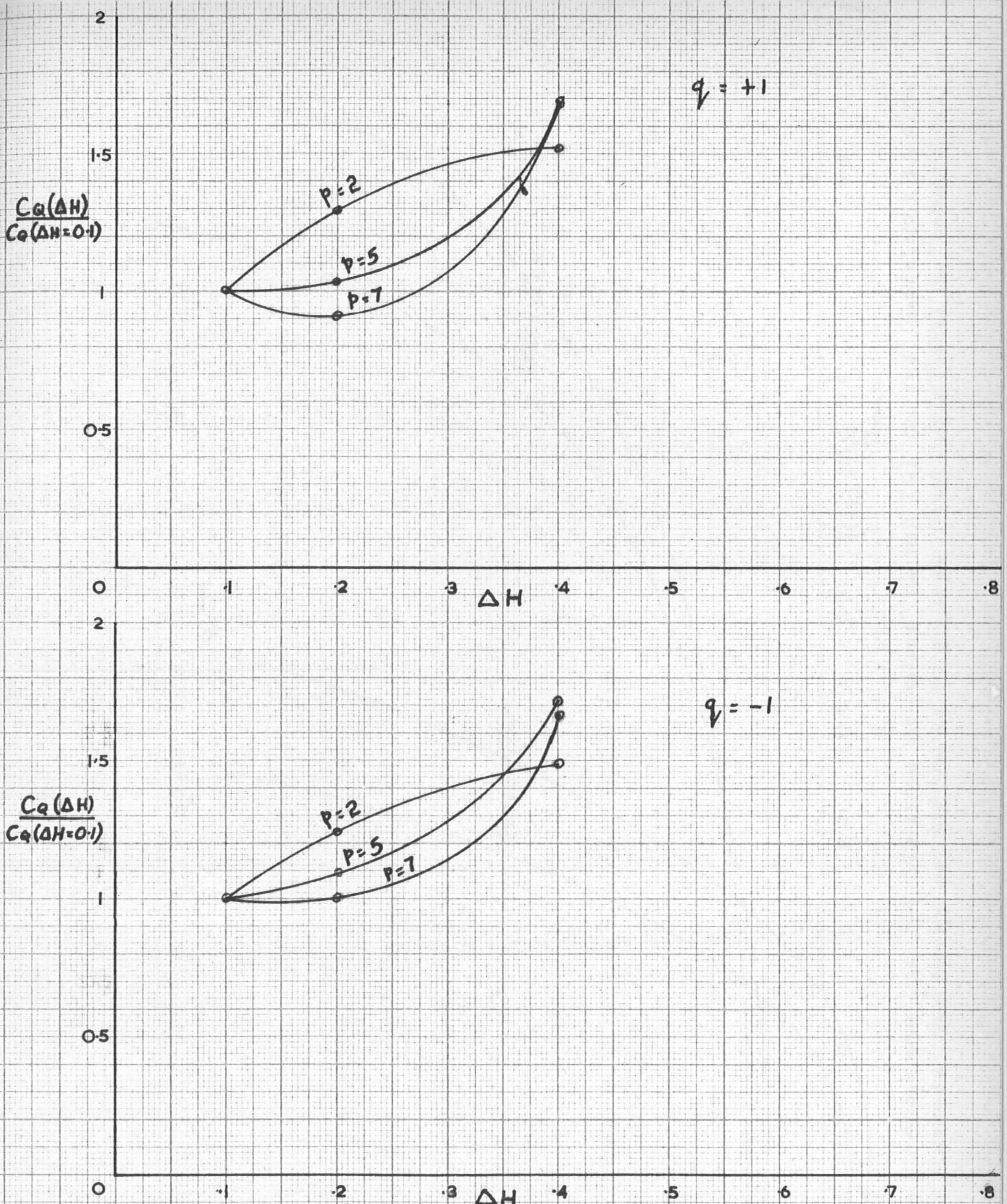


FIG 2·5 SUCTION QUANTITY REQUIRED IN ORDER TO MAINTAIN A MEAN VALUE OF SHAPE FACTOR (H)=2 AS A FUNCTION OF SLIT SPACING AS INDICATED BY THE STEP BY STEP CALCULATIONS FOR VARIOUS VALUES OF p AND q.

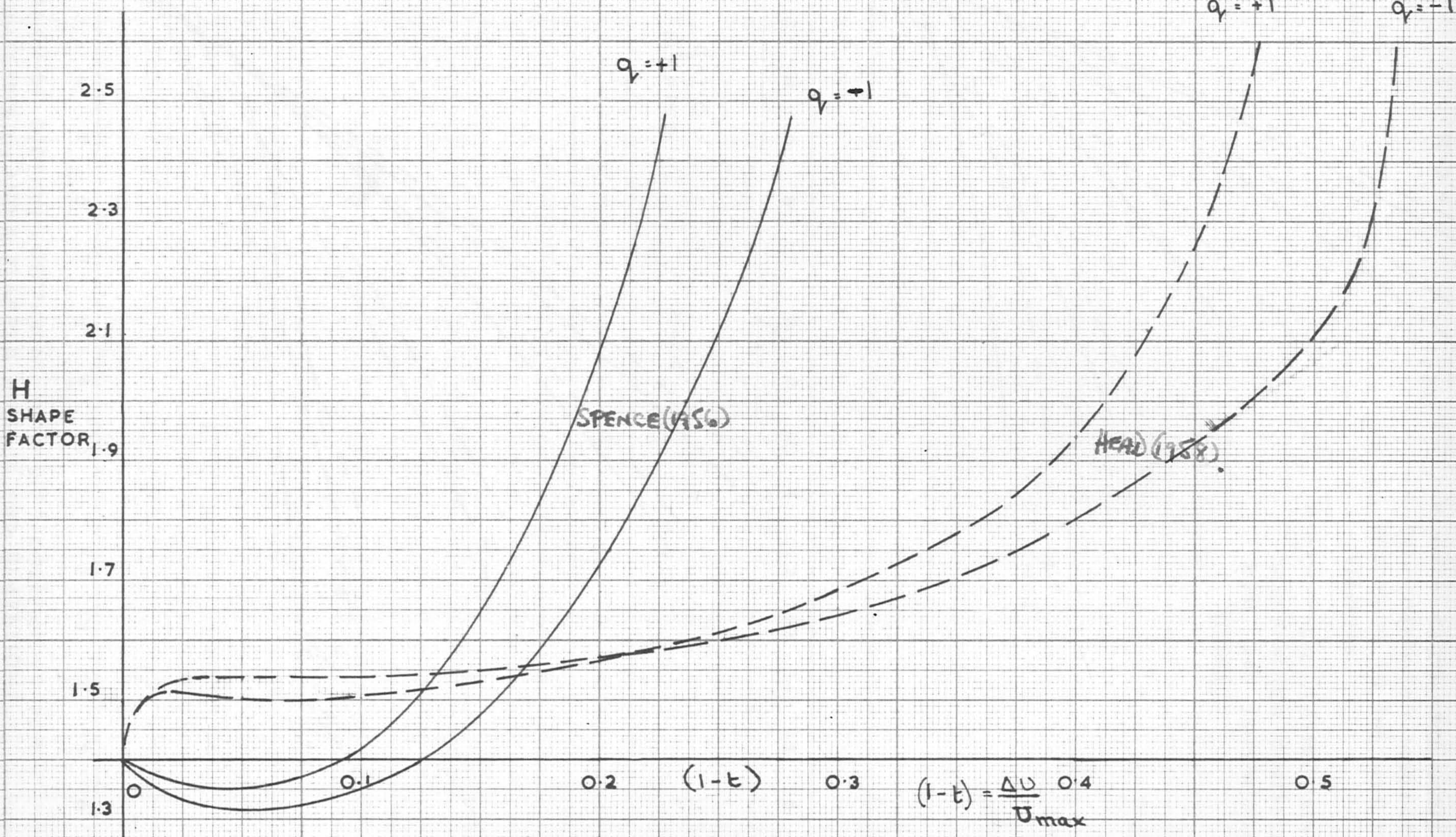


FIG. 2·6 COMPARISON OF THE VARIATION OF SHAPE FACTOR (H) ON THE IMPERVIOUS SURFACE BEFORE THE ONSET OF SUCTION AS CALCULATED USING HEAD (1958) OR SPENCE (1956)

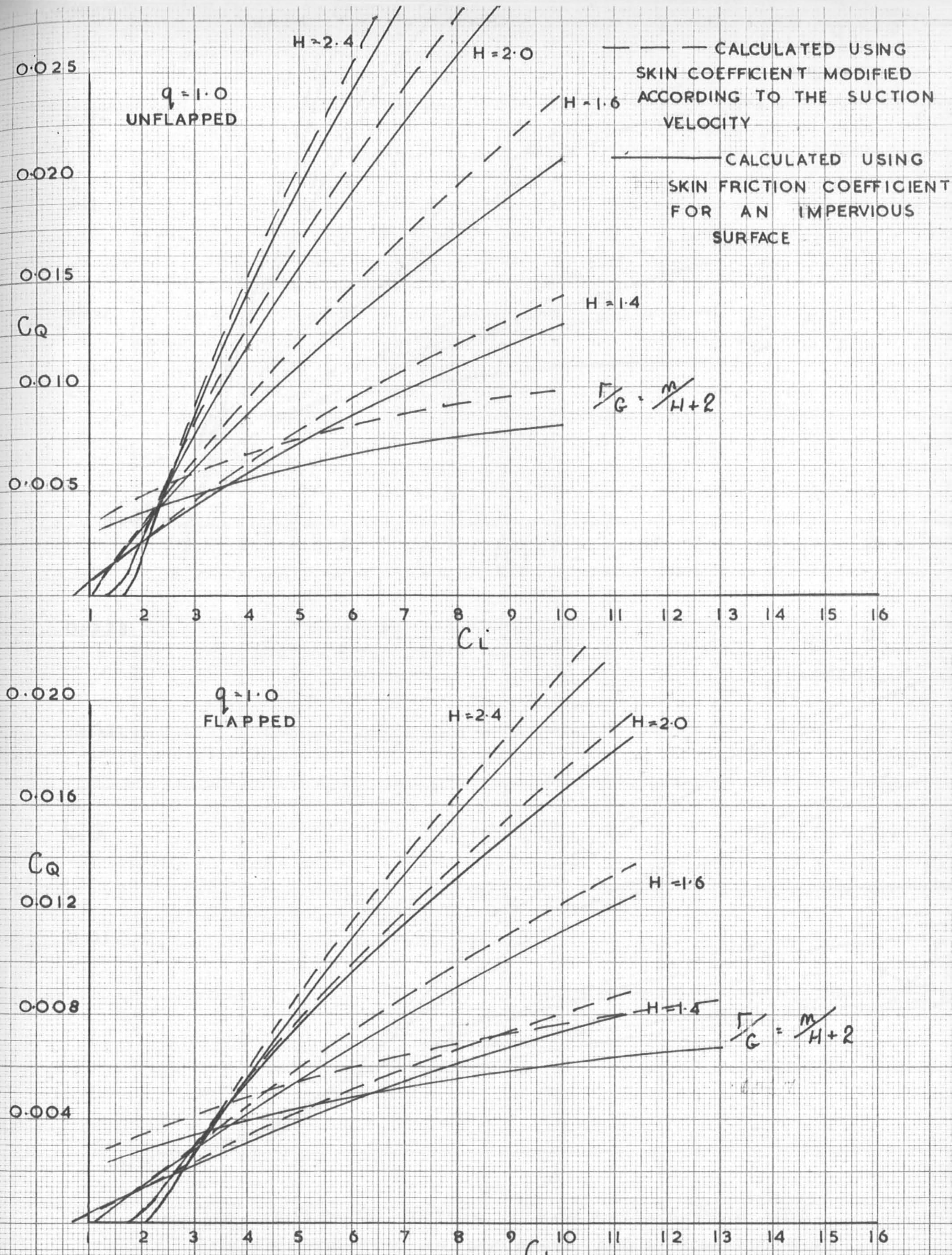


FIG 2·7(a) VARIATION OF SUCTION COEFFICIENT WITH LIFT COEFFICIENT FOR AN UNFLAPPED AND A FLAPPED WING IN TWO DIMENSIONS

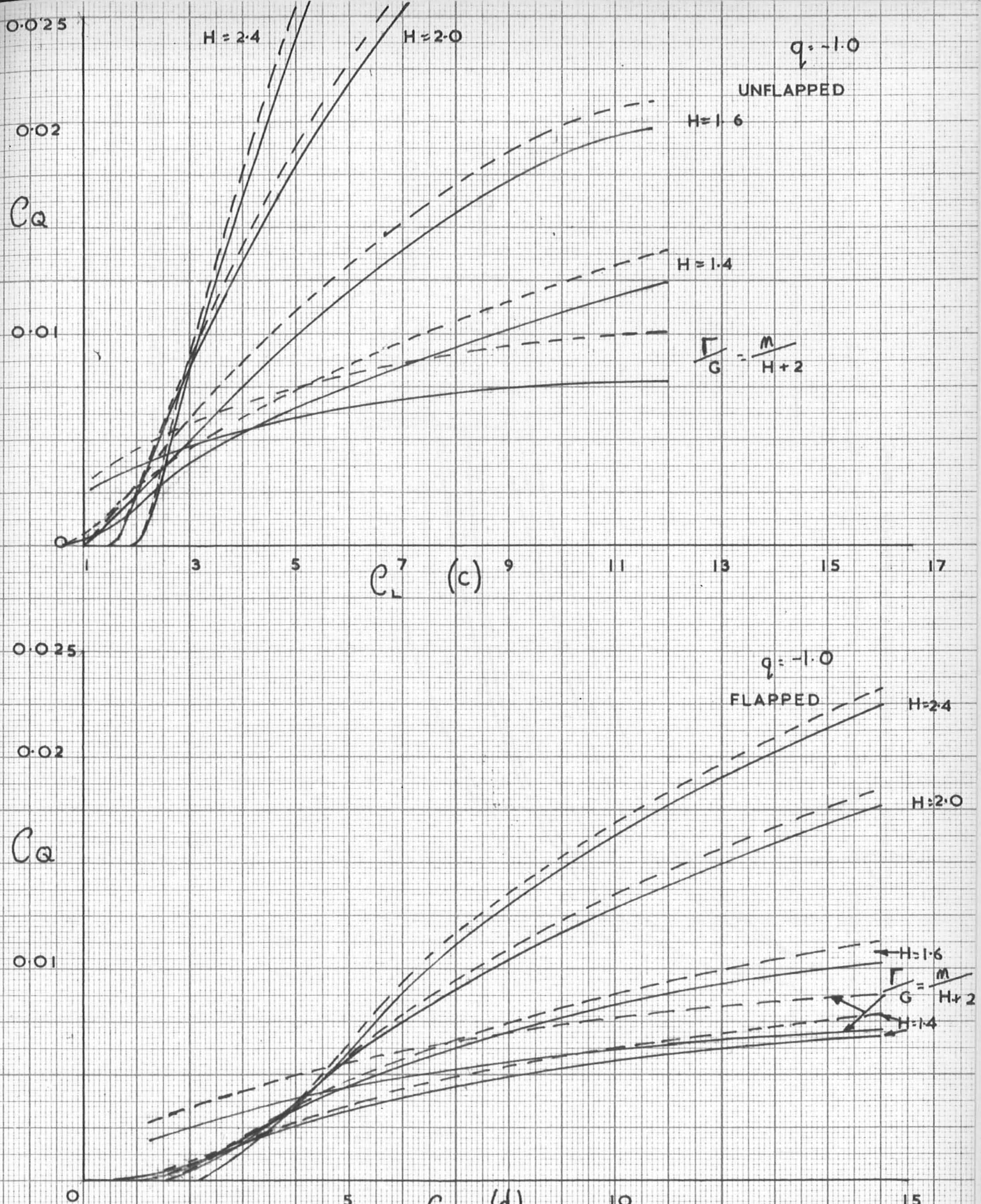


FIG 2·7(b) VARIATION OF SUCTION COEFFICIENT C_Q (n·o) WITH LIFT COEFFICIENT (C_L) FOR AN UNFLAPPED AND A FLAPPED WING IN TWO DIMENSIONS

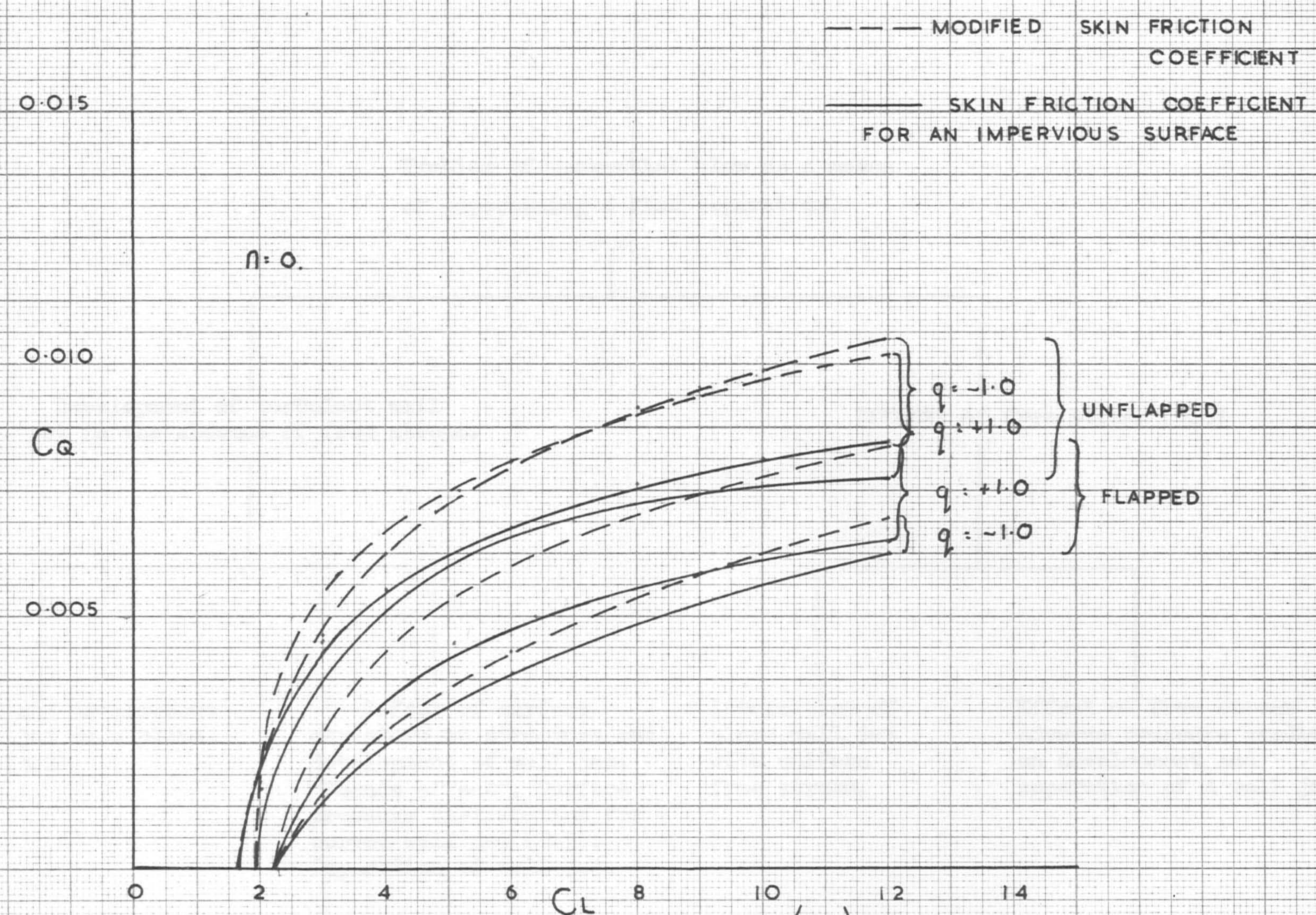


FIG 2·8 VARIATION OF THE OPTIMUM VALUES OF (C_Q) WITH LIFT COEFFICIENT (C_L) FOR A FLAPPED AND UNFLAPPED WING

Problem of tangential blow as a means of suppressing a flow separation.

Development of turbulent wall jet in the absence of free stream.

Effects of free stream (3.5)

Structure and development of a wall jet on a plane surface. (3.2)

Effect of surface curvature on structure and development of wall jet. "Coanda effect". Surface pressure distribution for curved wall jet (3.3). Separation of wall jet.

Interaction of plane wall jet with uniform free stream.

Effect of free stream adverse pressure gradient on development and separation of a wall jet.

FIG. 3.1. BREAKDOWN OF THE PROBLEM OF TANGENTIAL BLOWING.

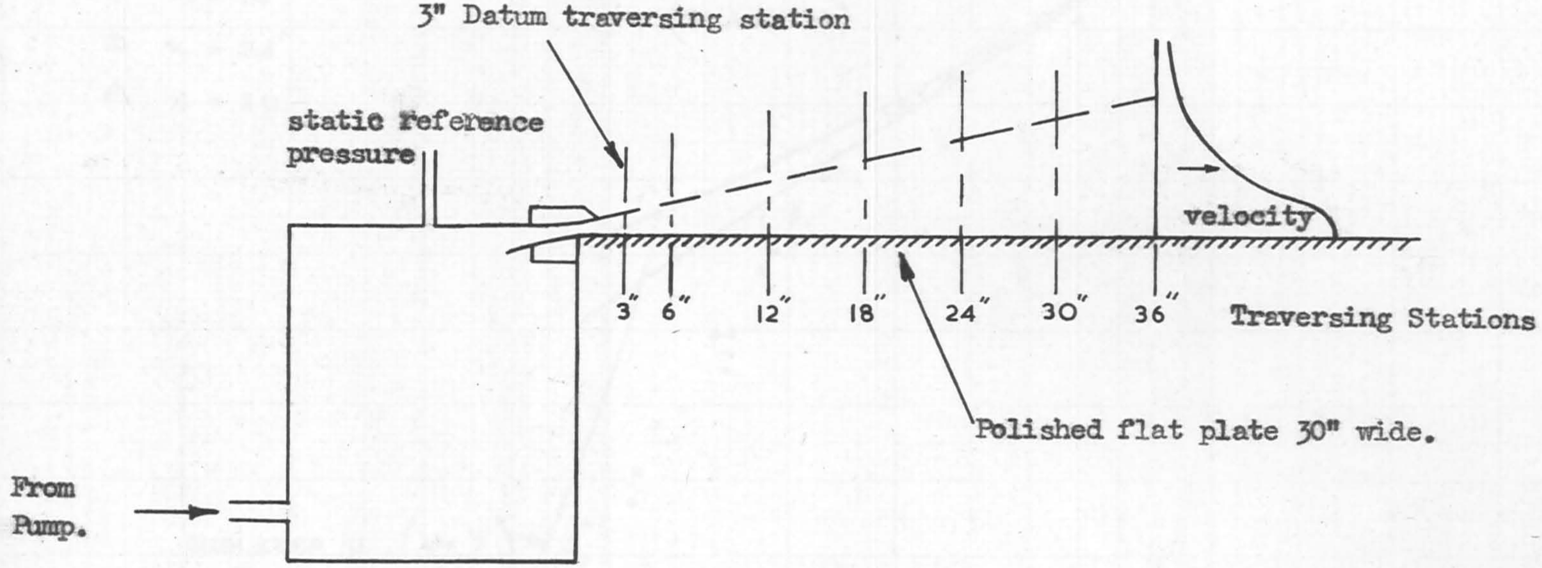


FIG. 3.2 EQUIPMENT USED TO INVESTIGATE THE DEVELOPMENT OF A WALL JET ALONG A PLANE SURFACE.

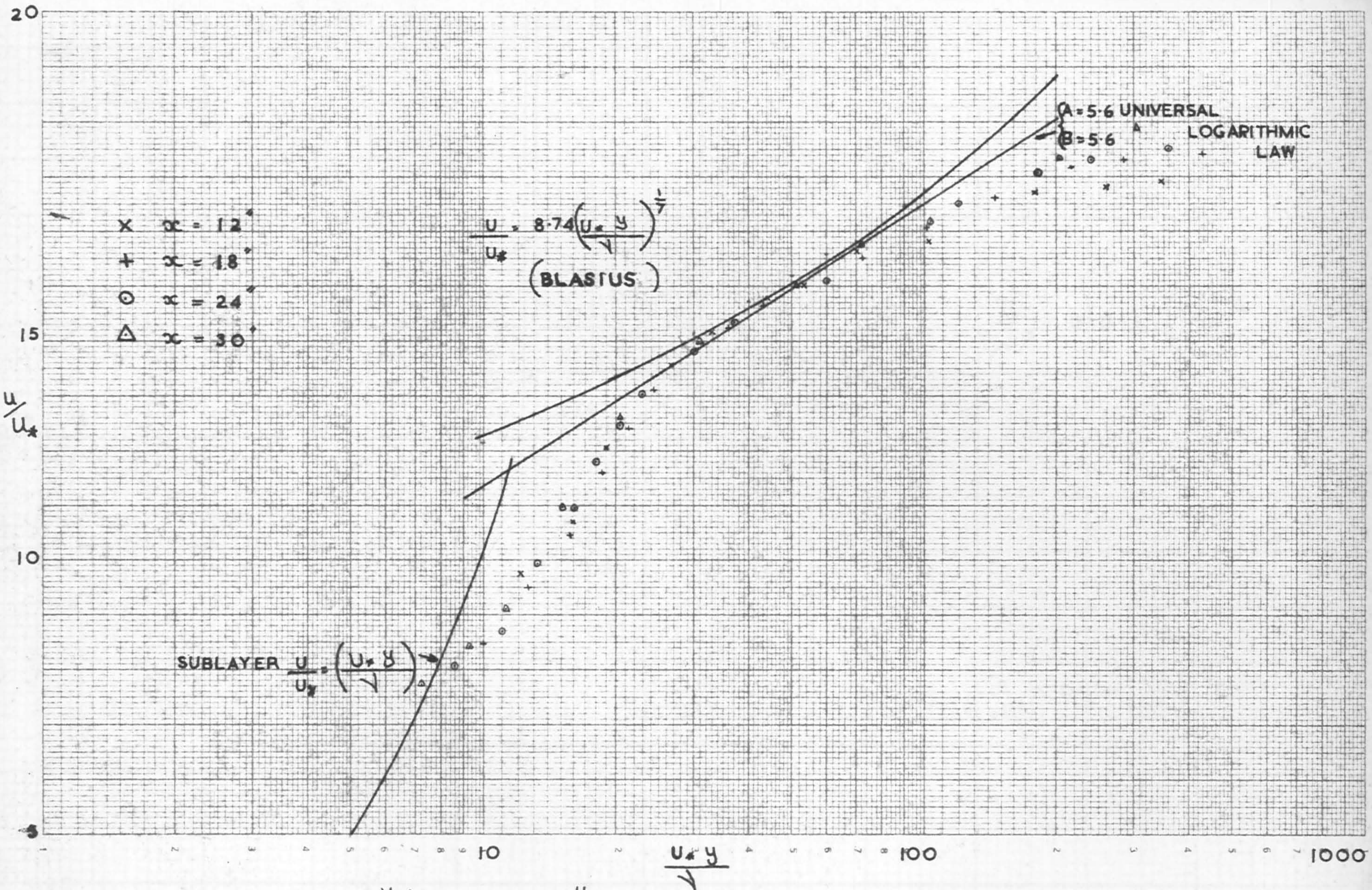


FIG. 3.3 VARIATION OF U/U_* WITH $\text{Log}\,\frac{U_* y}{\nu}$ FOR THE INNER PART OF A WALL JET.

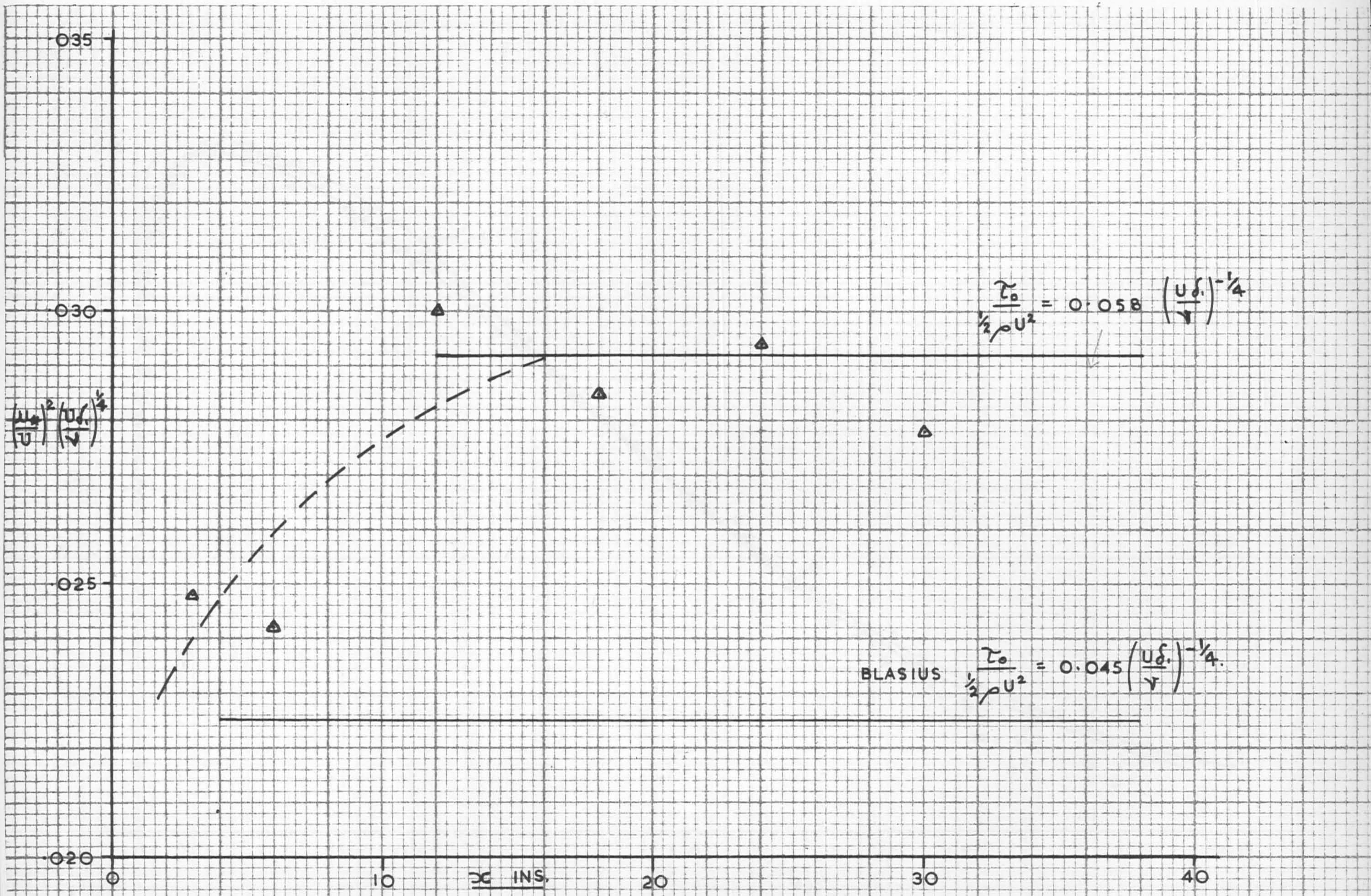


FIG. 3.4 SKIN FRICTION DISTRIBUTION FOR WALL JET

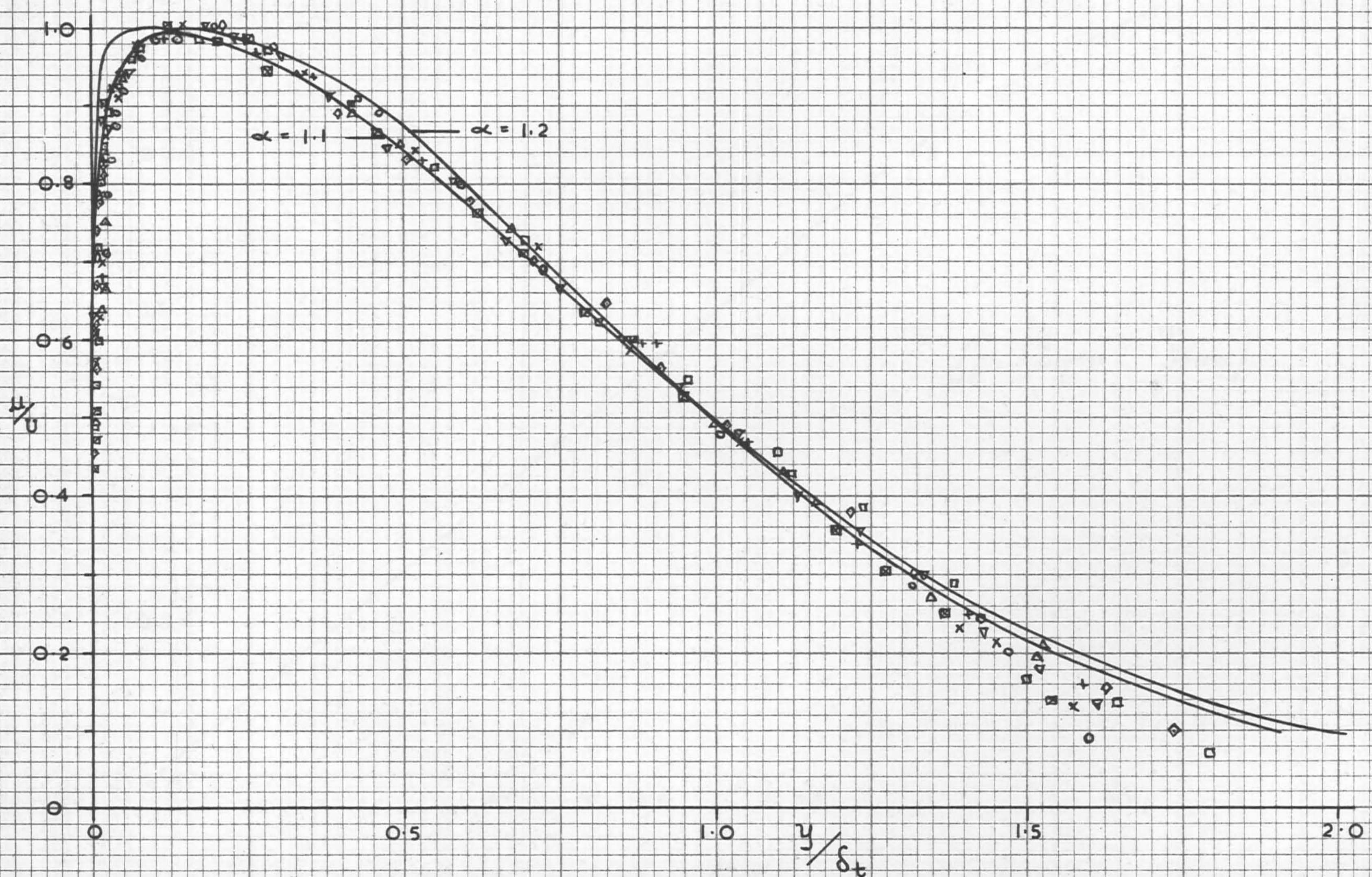


FIG. 3.5 COMPARISON OF EXPERIMENTAL RESULTS WITH GLAUERTS (1956) THEORY

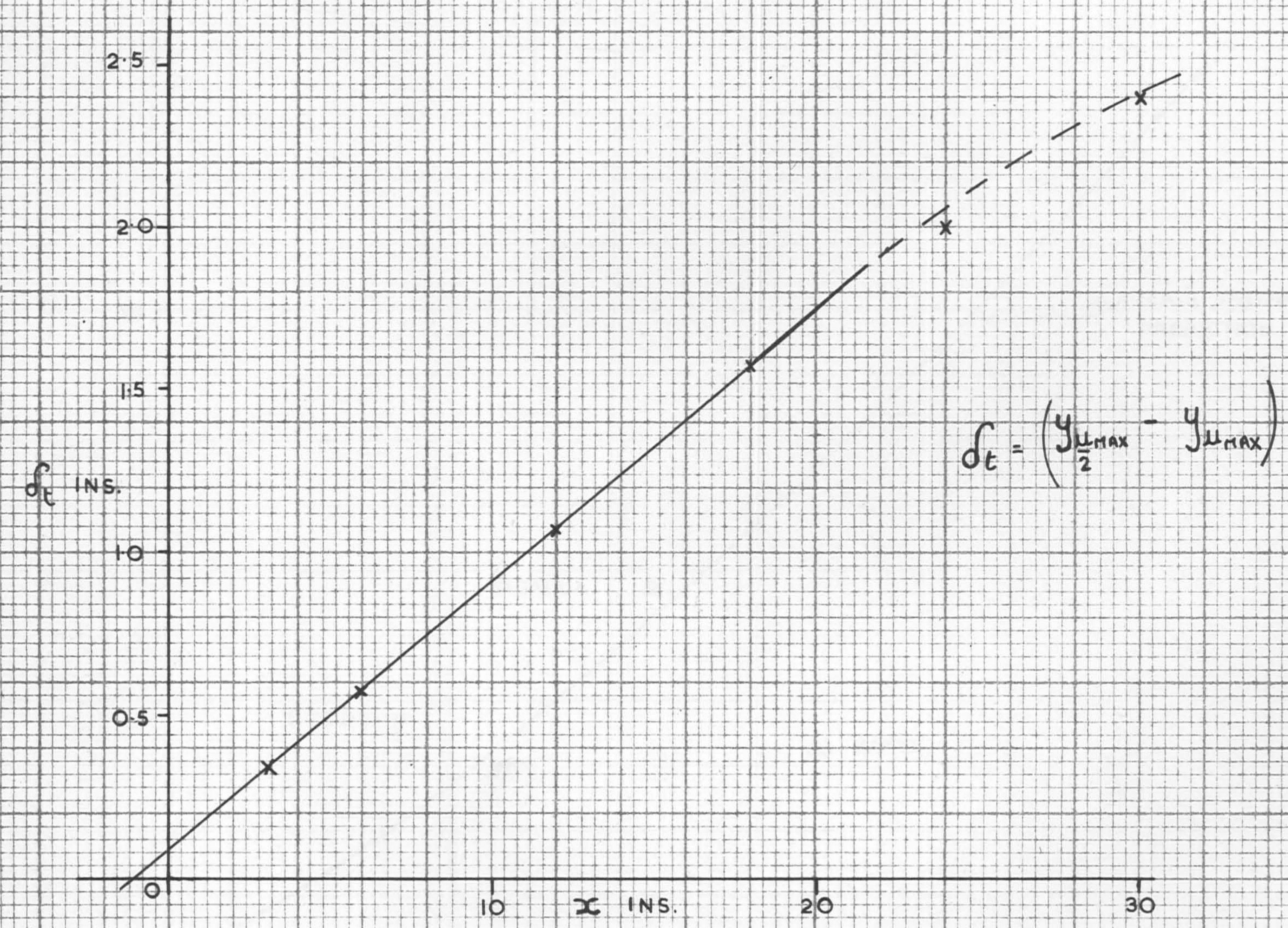


FIG. 3.6 LATERAL SPREAD OF WALL JET.

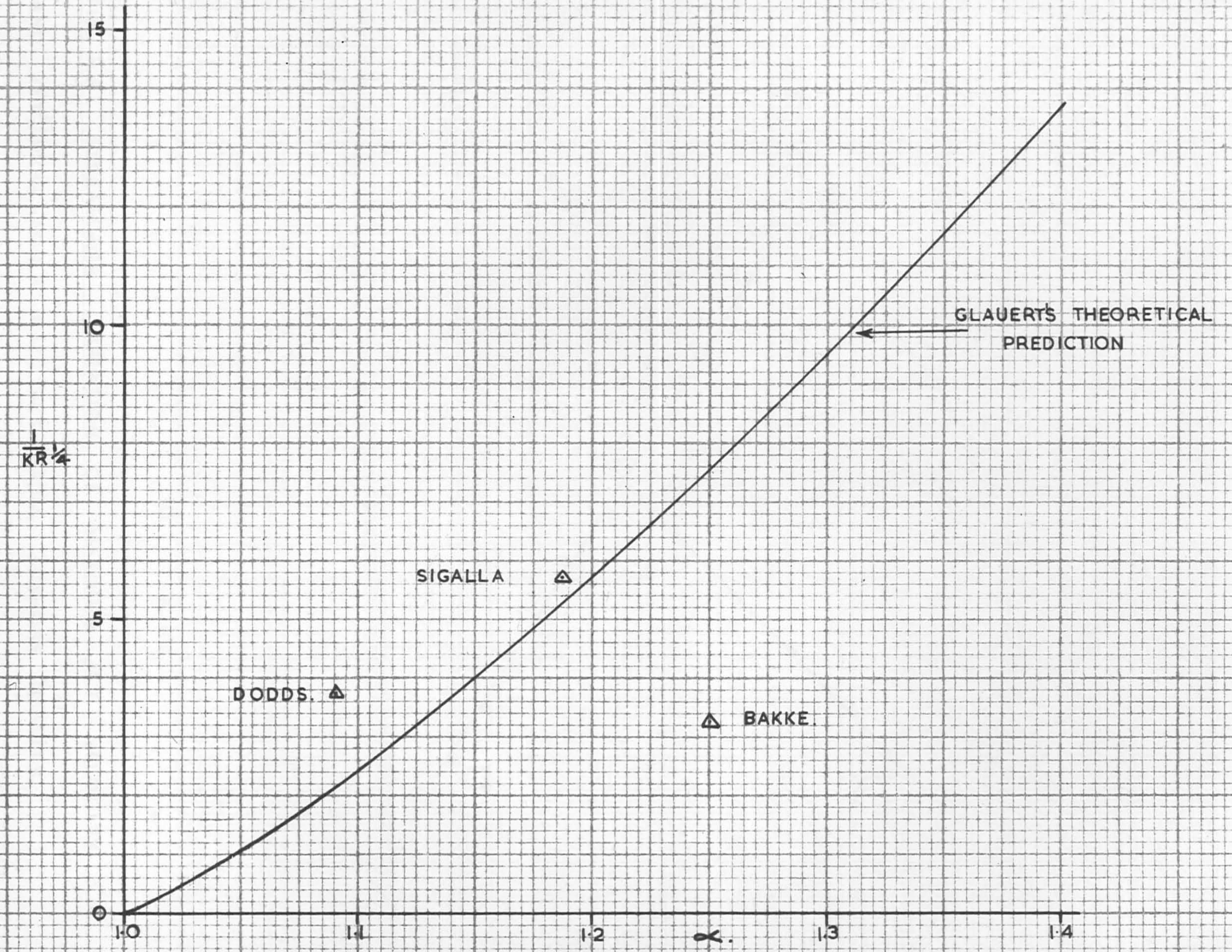


FIG. 3.7 $(KR^{\frac{1}{4}})^{-1}$ AS A FUNCTION OF α.

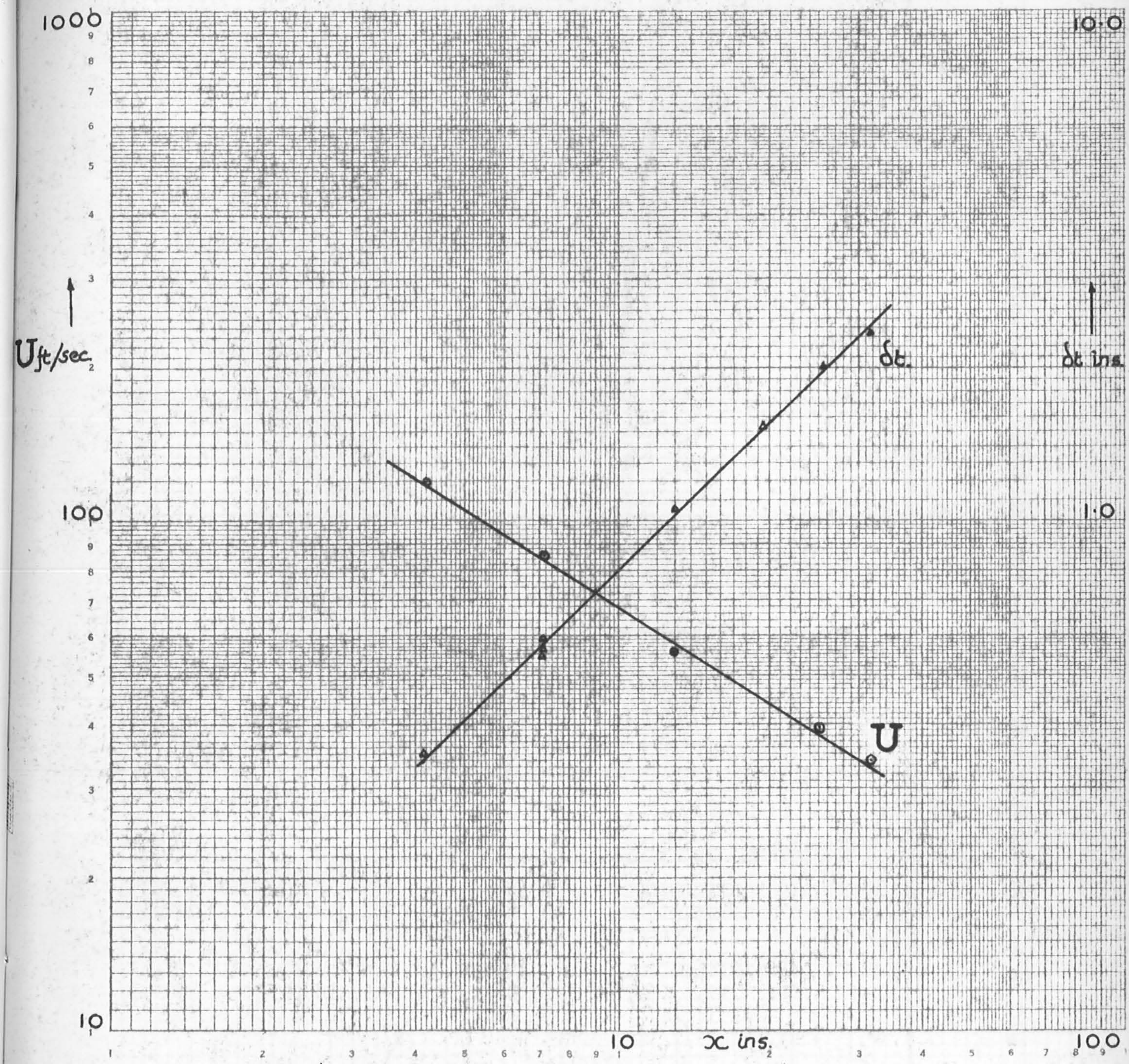


FIG. 3.8 STREAMWISE VARIATION OF MAXIMUM JET VELOCITY AND THICKNESS FOR A WALL JET FROM AUTHOR'S RESULTS.

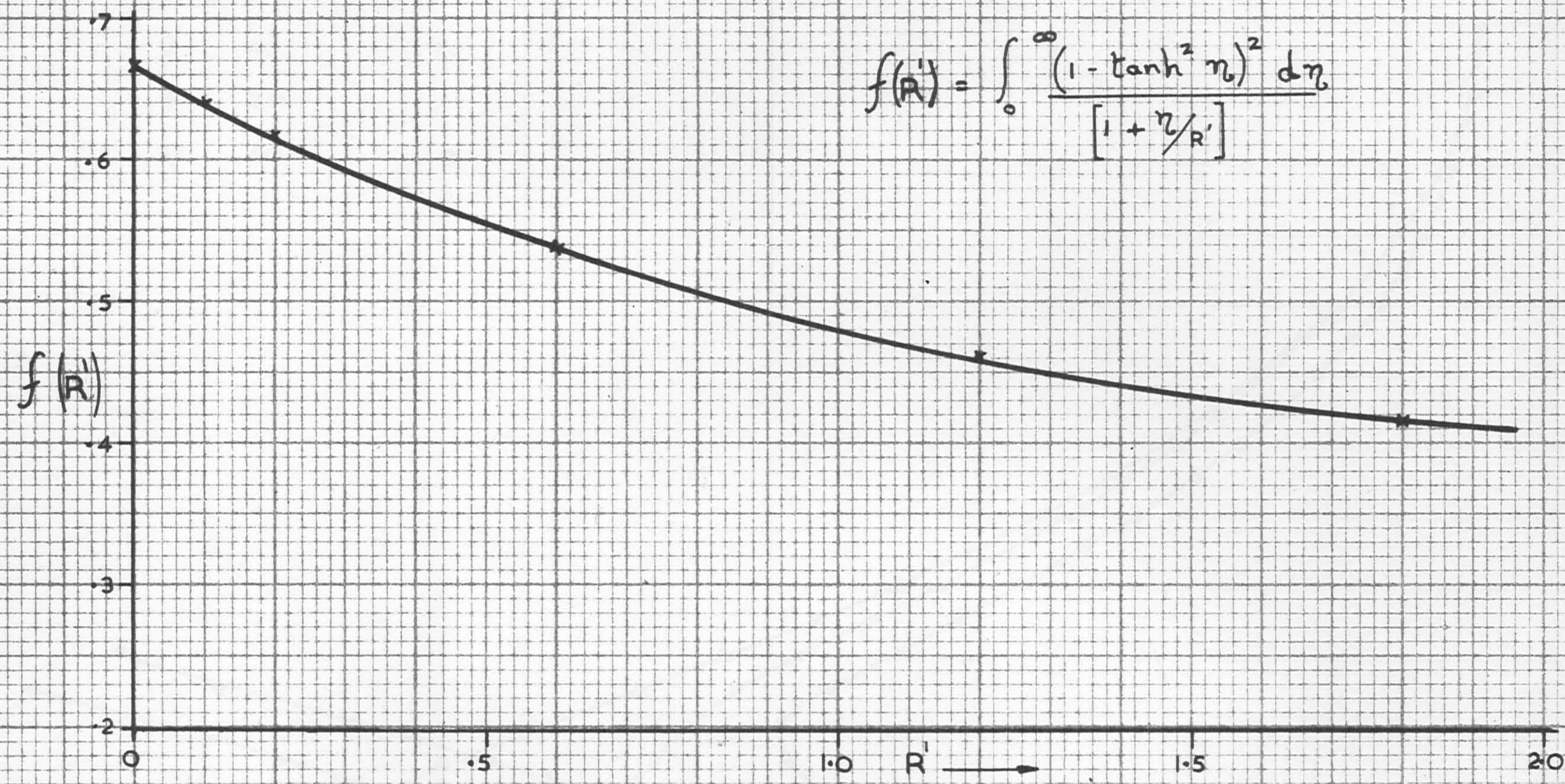


FIG. 3.9 VARIATION OF THE FUNCTION $f(R')$ OBTAINED BY NUMERICAL INTEGRATION

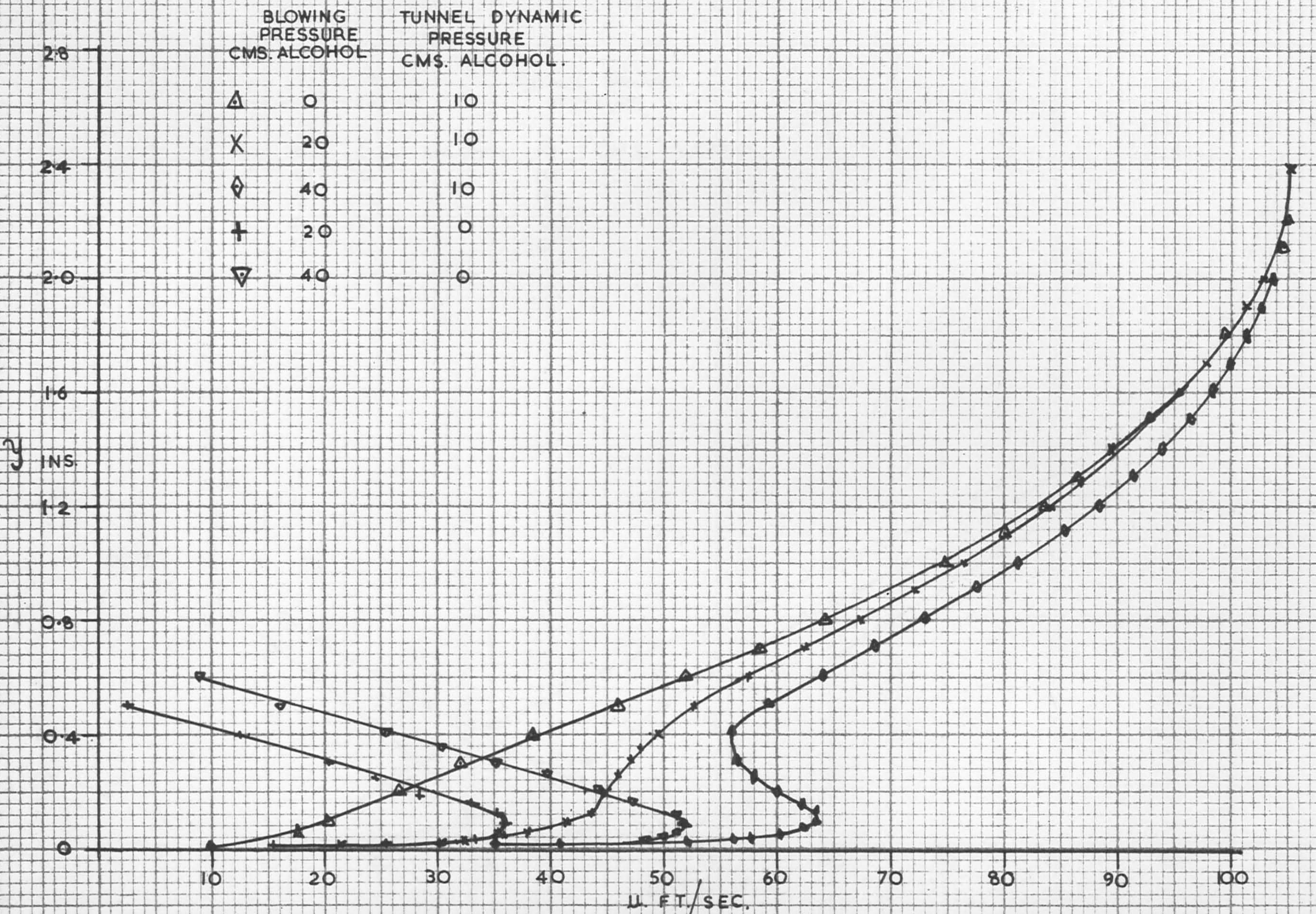


FIG. 3.10 (a) STATION T6

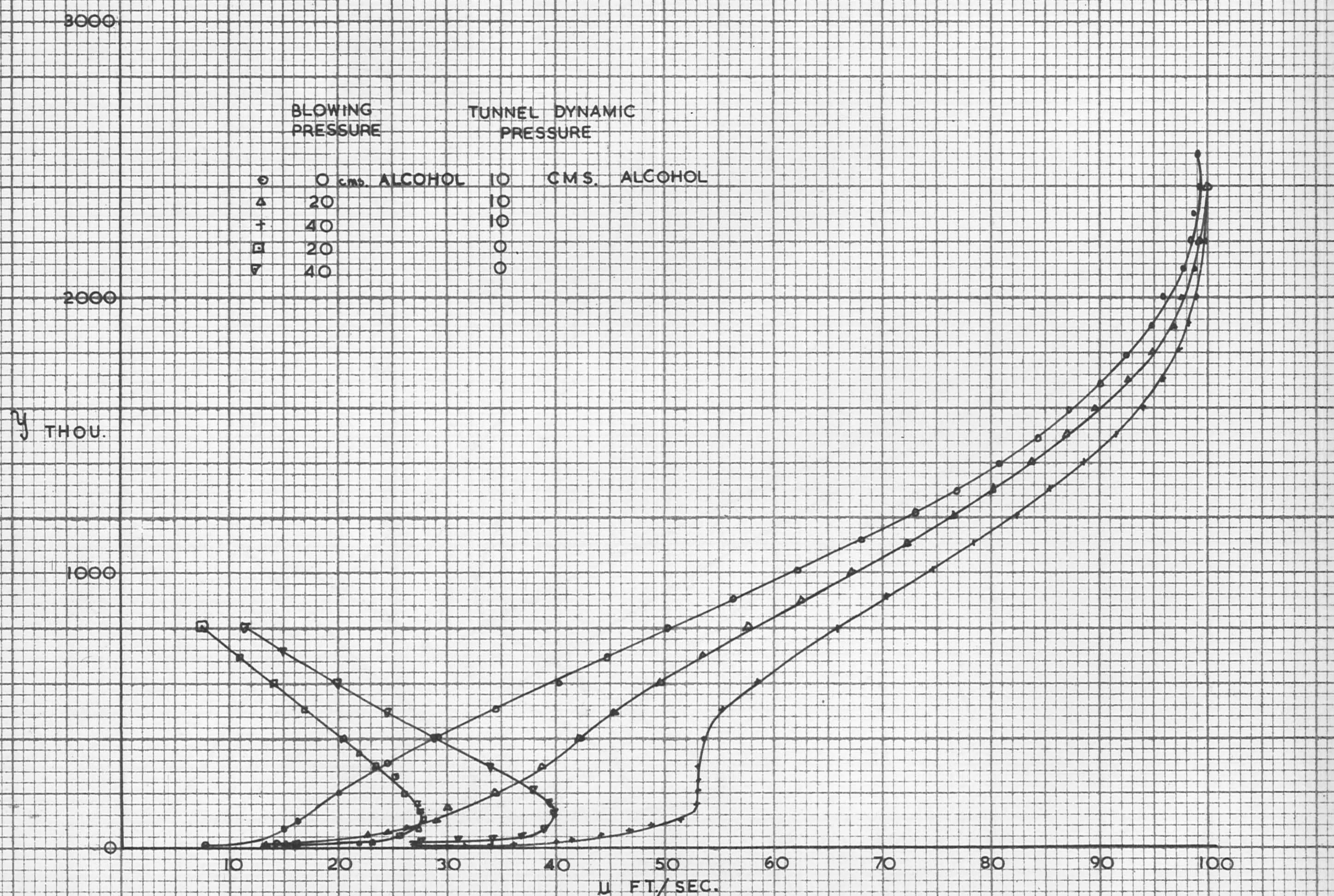


FIG. 3.10. (b) STATION T7

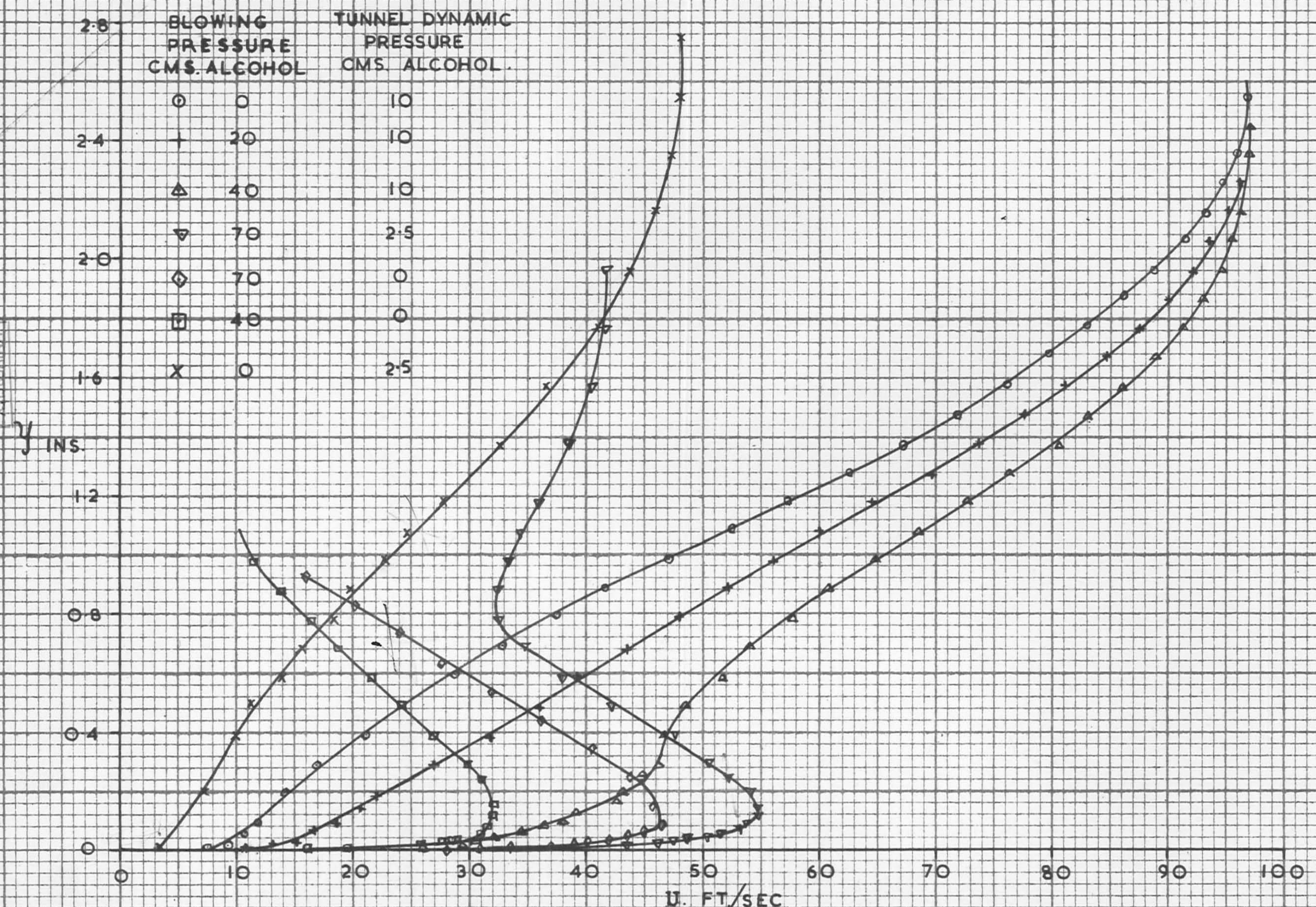


FIG. 3. 10c STATION 8