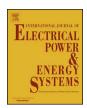
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A Markovian model for power transformer maintenance

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ABSTRACT

The condition of the insulation paper is one of the key determinants of the lifetime of a power transformer. The winding insulation paper may deteriorate aggressively and result in the unexpected failure of power transformers, especially under the presence of high moisture, oxygen, and metal contaminants. Such types of scenarios can be prevented if the deterioration is detected on time. Various types of condition monitoring techniques have been developed to detect transformer condition such as dissolved gas analysis (DGA) and frequency response analysis (FRA). They are non-intrusive and provide early warning of accelerated deterioration both chemically and mechanically. However, the accuracy of those techniques is imperfect, which means periodic inspection is still indispensable. In this paper, we discuss the value of continuous condition monitoring for power transformers and present a way to estimate this value. Towards this, a continuous-time Markov decision model is presented to optimize periodic inspections, so that the cost is minimized and the availability is maximized. We then analyze the performance based on the information from both discrete inspection and continuous condition monitoring using DGA and FRA. The result shows the dissolved gas analysis can improve the availability and operation cost, while frequency response analysis can only improve the availability of power transformers.

1. Introduction

Power transformers are critical assets in a power transmission network. A failure of a power transformer also may cause cascading failure and catastrophic blackout in the power grid. The necessity of increasing reliability and availability of power transformers can be analyzed directly from a financial point of view. Between 1997 and 2001, the total losses caused by power transformer failure in the US were over 286 million [1]. Moreover, the aging population of power transformers has increased since 1975 [2]. These imply that it is expected to have an increase in power transformer failure, and the resulting load curtailment if the maintenance strategy remains the same.

In literature, various types of maintenance models have been developed to address the problem of power transformer maintenance. Aldhubaib and Salama have developed a reliability centered maintenance and replacement approach to optimize maintenance and replacement to increase the lifetime of power transformers and reduce annual cost [3]. Dhople et al. proposed a set-theoretic method for capturing the uncertainty in Markov reliability and reward model to maximize the availability of power transformers [4]. Abu-Elanien et al. developed a decision support system to determine the life expectancy of transformers from techno-economic perspective [5]. Lima et al. designed a two-level framework of fault diagnosis and decision making for power transformers with considering the loss for life caused by overload

condition [6]. Abiri-Jahromi et al. have developed a two-stages maintenance management model that contains both mid-term and short-term maintenance to maximize the serviceability of power transformers [7]. Koksal and Ozdemir have improved the power transformer maintenance using a Markovian model [8].

As to the condition state of a power transformer is considered to be discrete, most of the developed models are based on the Markovian deterioration model. However, the deterioration of power transformers is oversimplified and modeled by Markov chain with a single deteriorating path. Such an approach is inaccurate because it overlooks the complexity of the deterioration of power transformers, such as the acceleration deterioration of insulation paper caused by high moisture. Because of this, the effective of condition-based maintenance that solely relies on the periodic inspection is over-estimated. Therefore, to re-estimate the value of continuous monitoring, it is essential to improve the deterioration model of power transformers. In practice, the accuracy of the condition monitoring is imperfect and may be interfered by operation signals and external signals. Therefore, even for the power transformers that have already installed the condition monitoring devices, periodic inspection can still provide additional value to triangulate the estimated condition information by condition monitoring. The objective of the paper is twofold: optimize the condition-based maintenance for power transformers; explore the value of online monitoring from the perspective of the lifecycle of power transformers.

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Nomer	nclature		accelerated deterioration process
		λ_{ol}	the rate of successfully detected malfunctions by online
π_{ij}	steady state probability of state (i,j)		monitoring device
λ_a	infant mortality rate of power transformers	$1/\mu_{in}$	the duration of periodic inspection
	the deterioration rate of winding insulation paper from	$1/\mu_c$	the duration of minor maintenance
	state (i,0) at normal deterioration process	$1/\mu_M$	the duration of major maintenance
$\lambda_{di,1}$	the deterioration rate of winding insulation paper from	$1/\mu_F$	the duration of corrective maintenance
	state $(i,1)$ in the accelerated chemical deterioration	$1/\mu_R$	the duration of replacement
$\lambda_{di,2}$	the deterioration rate of winding insulation paper from	C_u	the downtime penalty cost due to unexpected failure
	state (i,2) in the accelerated mechanical deterioration	C_p	The downtime penalty cost due to maintenance and in-
$\lambda_{fi,1}$	the transition rate from state (i,0) in the normal dete-		spection
	rioration to state $(i,1)$ in the accelerated chemical dete-	C'_{in}	the cost of periodic inspection
	rioration	C_C'	the cost of minor maintenance
$\lambda_{fi,2}$	the transition rate from state (i,0) in the normal dete-	$C_M^{'}$	The cost of major maintenance
	rioration to state $(i,2)$ in the accelerated mechanical de-	C_F'	the cost of corrective maintenance
	terioration	C_R'	the cost of replacement
λ_F	the sudden failure rate of power transformer in the normal	C_{ol}	the annual cost of online monitoring device
	deterioration process		Č
λ_{Fd}	the sudden failure rate of power transformer in the		

To achieve the objectives, in the second section, we use cause-effect analysis on different subsystems of the power transformer to identify the potential risk of acceleration deterioration of insulation paper caused by the malfunction of different subsystems. In section three, we develop a continuous-time Markov chain model to optimize the maintenance of power transformers based on the information from inspection and continuous monitoring. Section four analyzes the value of different types of online monitoring numerically. Section five summarizes the concluding remarks of the paper.

2. Deterioration of power transformers

To systematically analyze the deterioration of power transformers to identify the information that can be used to improve the modeling of the power transformer. According to functionality and structure, power transformers can be classified into seven subsystems: winding, magnetic core, insulation oil, bushing, tap changer, tank and cooling equipment. In practice, the condition of winding insulation paper is usually regarded as the index for power transformer condition [9] and [10]. The deterioration of the winding insulation paper may accelerate under the presence of high moisture, oxygen, and metal contaminants.

In this section, we aim to identify the potential risks of the accelerated deterioration of winding insulation paper caused by the malfunctions of other subsystems using cause-effect analysis. In general, the deterioration of winding insulation paper may accelerate in two ways: accelerated chemical aging and accelerated mechanical

aging. The accelerated chemical deterioration is a combination of three interactive processes: pyrolysis, hydrolysis, and oxidation [11]. Hydrolysis is the dominant process in the accelerated chemical degradation. The rate of hydrolysis is dependent on the content of moisture and catalyzes by the acidity [12]. The increase of acidity is caused by the sludge formation as the result of oxidation. The sludge will also increase the temperature and accelerate the pyrolysis. The accelerated chemical deterioration starts with the occurrence of contamination and moderate partial discharge. During the deterioration process, dissolved gas will be generated. Eventually, the partial discharge will result in treeing, tracking or even breakdown the winding insulation. The accelerated mechanical deterioration is usually initialized by the loss of clamping force or distortion of winding geometry, which is mainly caused by abrasion under electric-magnetic forces [13]. Under accelerated mechanical deterioration, the partial discharge will proceed to creeping and result in the breakdown of winding insulation.

The aging of winding insulation is related to the moisture, acidity, oxygen, containment level, and clamping forces. Empirically, abnormality in these factors is usually caused by malfunctions of other subsystems. For example, inelastic gasket on bushing can increase the risk of excessive moisture, oxygen, and containment level and in turn accelerate the rate of winding insulation aging and reduce the life of the power transformer. Malfunctions such as inelastic gasket can be repaired with a minor cost if it is detected on time. However, the resulting deterioration of the insulation paper is irreversible and will significantly reduce the service lifetime of the power transformer.

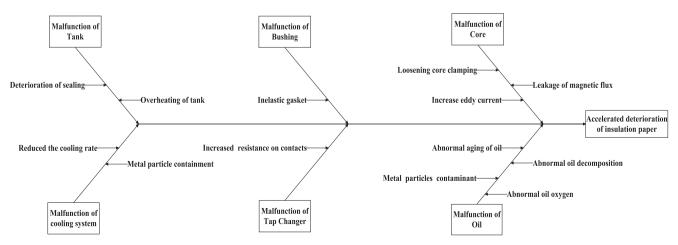


Fig. 1. Malfunctions of subsystems that can cause an accelerated deterioration of winding insulation.

Therefore, it is valuable to identify the malfunctions on different subsystems that may result in the accelerated deterioration of the insulation paper. We have identified the malfunctions from the extant literature using cause-effect analysis. The result is illustrated in Fig. 1.

In practice, the malfunction rates of the subsystems of power transformer are recorded as shown in [14]. As a result of the findings in this section, we can identify the transition rate between normal deterioration and accelerated deterioration for the winding insulation paper and refine the deterioration model of power transformers. In the next section, we will apply this knowledge to formulate the deterioration and maintenance model of power transformers in both distribution networks and transmission networks.

3. Deterioration and maintenance models for power transformers

We use continuous-time Markov chain (CTMC) to model the deterioration and maintenance of power transformers. Practically speaking, the maintenance strategy used for power transformers in distribution networks and those in transmission networks are different. We first model the deterioration and maintenance strategy for power transformers in distribution networks. The state transition diagram is illustrated in Fig. 2.

In Fig. 2, the states are represented by two indices i and j. i is the index representing the condition of the winding insulation paper and jis the index that represents the type of deterioration. We classify the condition of winding insulation i into 5 states (healthy, aged, defective, faulty and failure). Normal deterioration process is indicated with j = 0, while the accelerated chemical aging is presented as j = 1 and accelerated mechanical aging is represented as j = 2. We denote λ_{fij} as the rate of transition from condition state (i,0) to (i,j). In an accelerated deterioration process, power transformer reaches the deterioration failure state (4,0) faster than normal process. Apart from deterioration failure, we also consider sudden failure with probability λ_F , which is mainly caused by some exogenous events. In practice, if one of the power transformer subsystems has malfunctioned, the overall vulnerability of transformer will rise. We represent the probability of sudden failure during accelerated deterioration as λ_{Fd} ($\lambda_{Fd} \geqslant \lambda_F$). The infant mortality rate λ_a is also considered in the first condition state. The combination of all failure rates is a bathtub curve with diverse wear-out tails as shown in Fig. 3.

In Fig. 3, the wear-out tail is not only dependent on the type of the accelerated deterioration process it undergoes, but also on the timing of when the malfunctions result in accelerated deterioration.

In some scenarios, the main objective of maintenance of power transformers in the distribution network is to retain its basic service-ability. Corrective maintenance is implemented after the sudden failure of a power transformer. It can effectively restore the transformer to the normal operating condition just before the failure. However, not all failures are repairable. After the breakdown of winding insulation, replacement of the power transformer is necessary to preserve the serviceability of the power transformer. We denote the duration of corrective maintenance as $1/\mu_F$ with a cost C_F' , and replacement time as $1/\mu_R$ with a cost C_R' . Meanwhile, failure also results in unplanned downtime penalty C_u due to the risk of load curtailment. The availability of the power transformer A is shown in Eq. (1):

$$A = 1 - \mu_F \sum_{i=0}^{3} \pi_{i,3} + \mu_R \pi_{4,0} \tag{1}$$

where $\pi_{i,j}$ indicates the steady state probability of the state (i,j). The long-term operational cost C is shown in Eq. (2).

$$C = C_u \left(\sum_{i=0}^{3} \pi_{i,3} + \pi_{4,0} \right) + C_F' \mu_F \sum_{i=0}^{3} \pi_{i,3} + C_R' \mu_R \pi_{4,0}$$
(2)

In Eq. (2), the operation cost is a sum of three items. The first one is the downtime penalty cost for load curtailment. The second is the cost of

corrective maintenance and the third is the cost of replacement. To derive the analytic solution for Eqs. (1) and (2), we need to express the steady state probabilities analytically. Firstly, this requires expressing all states with respect to a reference state for which we choose $\pi_{0,0}$. The expressions of all the states in term of $\pi_{0,0}$ is shown in Appendix A. By recalling that the sum of all the steady state probabilities is equal to 1, we can calculate $\pi_{0,0}$ and in turn calculate the rest of steady state probabilities.

For power transformers in transmission networks, failure can be catastrophic and is associated with huge penalty costs. It is important to prevent failures and maximize the availability of power transformers. Here, condition-based maintenance is applied in addition to corrective maintenance and replacement. Condition-based maintenance is a type of maintenance strategy that recommends maintenance activities based on the information acquired through periodic inspection or online monitoring. In practice, both periodic inspection and online monitoring have their limitations. In periodic inspection, the deterioration may proceed to an unacceptable state or even failure state between the two successive inspections because of the stochastic nature of aging. The limitation of online monitoring is that not all types of malfunctions can be detected by online monitoring. Therefore, it is practical to implement both types of monitoring on power transformers in transmission networks.

The state transition diagram for condition-based maintenance of power transformers in transmission networks is shown in Fig. 4. The rate of periodic inspection is denoted as λ_{in} . If the inspection indicates that the power transformer is in an acceptable condition (i < 3 at normal deterioration), it will lead to no further action as presented by state (i,4). If any type of accelerated deterioration of the winding insulation is detected, a minor maintenance will be applied as represented by state (i,5). The malfunctioned subsystems will be repaired and deterioration will recover back to normal thereby prolonging the service lifetime of the transformer. However, because the deterioration of winding insulation paper is irreversible, minor maintenance will not restore the condition of the winding insulation. We denote the state of minor maintenance as state (i,6) where i < 3. The duration of the minor maintenance is $1/\mu_c$ with a cost C'_c . If i=3, a major maintenance (3,6) will be implemented to completely overhaul the power transformer. The major maintenance includes the replacement of the deteriorated winding insulation paper and can restore the condition of transformer back to (0,0). The duration of major maintenance is $1/\mu_M$ with a cost C'_M .

State (i,7) indicates the minor preventive maintenance that is immediately scheduled after an early warning signal of a malfunction is detected by online monitoring. The rate of such type of preventive maintenance is λ_{ol} . Because of preventive maintenance, the rates of malfunction $\lambda_{fi,1}$ or $\lambda_{fi,2}$ are reduced. When the rate $\lambda_{ol}=0$, the state diagram indicates the scenario when maintenance strategy is based on the periodic inspection only, without the support of online monitoring.

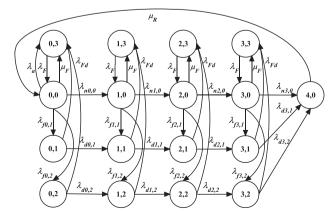


Fig. 2. State transition diagram for deterioration and maintenance of power transformers in distribution networks.

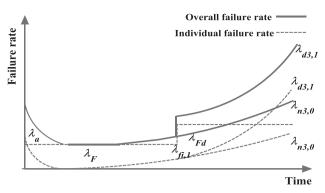


Fig. 3. A bathtub curve for the combination of different types of failure rates.

When online monitoring is implemented, early warning signals of some malfunctions can be detected on time. As a result, the malfunction can be repaired by a minor maintenance preventively before causing any non-negligible damage to the winding insulation. (i,7) represents the states for such type of minor preventive maintenances with a rate λ_{ol} . In this case, the rates of malfunctions are reduced due to the preventive maintenance that are enabled by online monitoring. Thus, λ_{ol} is equal to the reduced malfunction rate. We denote the cost of online monitoring as C_{ol} . We assume that all the maintenance and inspection activities will require the transformer to be stopped. The penalty cost per unit time for such stoppages is denoted as C_p . Because maintenance and inspection are preventive and prescheduled activities, C_p is significantly smaller than C_u , which is caused by unexpected failures. With the similar method used for the distribution network, we can express the availability and operation cost respectively as shown in Eqs. (3) and (4).

$$A = \sum_{i=0}^{3} \sum_{j=0}^{2} \pi_{i,j} \tag{3}$$

$$C = C_{u} \left(\sum_{i=0}^{3} \pi_{i,3} + \pi_{4,0} \right) + C_{p} \left(\sum_{i=0}^{2} \pi_{i,4} + \sum_{i=0}^{3} \sum_{j=5}^{7} \pi_{i,j} \right)$$

$$+ C'_{in} \mu_{in} \left(\sum_{i=0}^{2} \pi_{i,4} + \sum_{i=0}^{3} \pi_{i,5} \right) + C'_{c} \mu_{c} \left(\sum_{i=0}^{2} \pi_{i,6} + \sum_{i=0}^{3} \pi_{i,7} \right) + C'_{M} \mu_{M} \pi_{3,6}$$

$$+ C'_{F} \mu_{F} \sum_{i=0}^{3} \pi_{i,3} + C'_{R} \mu_{R} \pi_{4,0} + C_{ol}$$

$$(4)$$

In Eq. (4), the operation cost is the sum of the downtime penalty cost for unexpected failure, downtime penalty cost for preventive activities, inspection cost, minor maintenance cost, major maintenance cost, correction maintenance cost, replacement cost and the cost of online monitoring. This condition-based maintenance model opens up a successful way to find the optimal mean time between inspections to maximize the availability and minimize the operation cost with or without online monitoring. In Section 4, a numerical case study is provided to demonstrate the approach and to assess the value of different types of online monitoring.

4. Value of monitoring

In general, online monitoring is worthwhile only for power transformers in transmission networks, and we restrict our focus to those. The illustrative example is based on the parameter settings of 220 kV oil-immersed power transformers. We first aim to find the optimal mean time between inspections to maximize availability or minimize operation cost without applying online monitoring. Then, we optimize the mean time between inspections with different types of online monitoring. Using that, we assess the value of different types of online monitoring.

To ease the understanding, we assume the deterioration rates of winding insulation paper are independent of condition states. Degree of polymerization (DP) is widely used as the measurement of the condition of insulation paper. As shown in [15], 1/DP can be used as the condition boundaries for the aging insulation paper. Based on [16], the condition of insulation paper reaches the failure state when DP is below

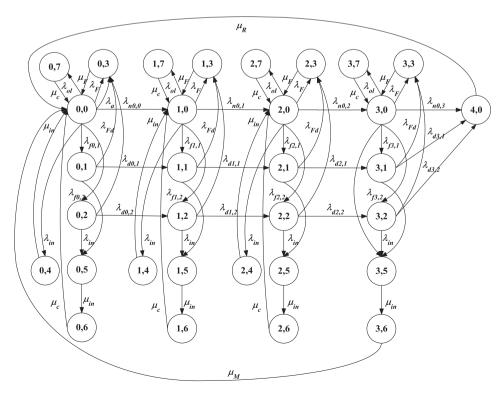


Fig. 4. State transition diagram for condition-based maintenance of power transformers in transmission networks.

200. In our case, the condition states of Markov chain are classified into five states using the DP as condition boundaries (1000–500, 500–333, 333–250, 250–200 and below 200). We reduce the number of parameters by using $\lambda_n = \lambda_{n0,0} = \lambda_{n0,1} = \lambda_{n0,2} = \lambda_{n0,3}$, $\lambda_{d1} = \lambda_{d0,1} = \lambda_{d1,1} = \lambda_{d2,1} = \lambda_{d3,1}$, $\lambda_{d2} = \lambda_{d0,2} = \lambda_{d1,2} = \lambda_{d2,2} = \lambda_{d3,2}$, $\lambda_{f1} = \lambda_{f0,1} = \lambda_{f1,1} = \lambda_{f2,1} = \lambda_{f3,1}$ and $\lambda_{f2} = \lambda_{f0,2} = \lambda_{f1,2} = \lambda_{f2,2} = \lambda_{f3,2}$. First, we consider the scenario without the online monitoring. The scenario is represented in the model as $\lambda_{ol} = 0$ and $C_{ol} = 0$. The parameters λ_{f1} and λ_{f2} are evaluated by experts according to the cause-effect analysis in Section 2. The overall numerical setting for all the parameters is shown in Table 1. In the table, the values of parameters for deterioration and maintenance duration are estimated based on the secondary data with the verification of experts. In practice, the values of failure cost and maintenance cost may vary case by case.

Based on the parameter settings, we can calculate the availability and cost of the power transformer using Eqs. (3) and (4). We plot the availability and operation cost against the mean time between inspections in Figs. 5 and 6 respectively.

Because both availability and operation cost are unimodal, we can find the optimal mean time between inspection to maximize the availability using Eq. (5).

$$\frac{dA}{d\lambda_{in}} = 0 \tag{5}$$

Likewise, we can calculate the optimal mean time between inspection to minimize the operation cost using Eq. (6).

$$\frac{dC}{d\lambda_{in}} = 0 \tag{6}$$

It can be found from Fig. 5 that the optimal mean time between inspections to maximize the availability is about 1.7 years and the maximum availability is 99.42%. From Fig. 6, the operation cost is minimized at £89,750/year, when mean time between inspections is around 0.33 years. The average rate of major maintenance is 0.0519/year.

In practice, the cost setting of C_u , C_p , C_M' and C_R' would be smaller than the default setting because protection systems, control algorithms and redundancy of power systems can significantly reduce the impact of unavailability of the power transformer. Fig. 7 provides a sensitivity analysis under different cost settings.

Fig. 7 presents the operation cost when C_u , C_p , C_M' and C_R' are scaled down to 80%, 60%, 40%, 20% of the default setting. The inspection interval is optimized to minimize the operation cost under each cost setting. The minimum operation cost and optimal inspection interval are presented in Table 2.

From Table 2, we can see that the minimum of operation cost decreases linearly when cost is reduced. In addition, the optimal inspection interval increases non-linearly.

We now examine how online condition monitoring might further improve the performance of power transformer maintenance. We consider two types of online monitoring devices: dissolved gas analysis device (DGA) and frequency response analysis device (FRA).

DGA determines the concentration of dissolved gas and moisture content. It is very sensitive to the change of key gases produced by the chemical aging of insulation paper. The accuracy of DGA can be as high as 90% [20]. It means that DGA may provide early warning for 90% of malfunctions that cause accelerated chemical aging of the insulation paper. Hence, the transition rate λ_{ol} is:

$$\lambda_{ol} = 90\% \times \lambda_{f1} = 0.0027/year$$

and λ_{f1} is reduced to 0.0003/year. According [21] and [22], we assume the annually cost for operating a condition monitoring is:

$$C_{ol} = \text{\&}3000/\text{year} \approx \text{\&}2550/\text{year}$$

With the same method, we can calculate the availability and operation cost by applying DGA. The results are illustrated in Figs. 8 and 9 respectively.

From Fig. 8, we can see that the maximum availability is 99.48% with DGA when the mean time between inspections is about 2 years. Compared to the optimized availability without DGA, the improvement by applying DGA is 0.6%. When the mean time between inspections is smaller than 0.4 years, little improvement can be made by using DGA. The improvement in availability increases with increase in mean time between inspections.

Fig. 9 shows the operation cost under different mean time between inspections with DGA. The minimum operation cost is £85,895/year with a mean time between inspection of 0.685 years. Compared to the optimized operation cost without DGA, the savings is about £3855/year. It is worthwhile to notice that when the mean time between inspection is smaller than 0.2 years, the operation cost without DGA is smaller due to the cost of online monitoring. The average rate of major maintenance is reduced to 0.0504/year.

We will then investigate the performance of FRA. The accuracy of FRA to detect the mechanical related malfunctions is shown to be 90% in [23]. In this scenario, the λ_{nl} is:

$$\lambda_{ol} = 90\% \times \lambda_{f2} = 0.0009/year$$

and the λ_{f2} is reduced to 0.0001/year. We assume C_{ol} remain the same. The resulting availability and operation cost are illustrated in the Figs. 10 and 11.

In Fig. 10, the maximized availability is 99.43% with FRA when the mean time between inspection is about 1.86 years. Compared to the optimized availability without FRA, the improvement by applying FRA is 0.01%.

Fig. 11 shows the minimized operation cost for the power transformer with FRA is £91,840/year. It is even larger than the scenario without FRA. This is because the performance improvement cannot surpass the annual cost of online monitoring. The average rate of major maintenance is reduced to 0.0517/year.

Table 3 compares the performance between DGA and FRA

As shown in Table 3, DGA can improve the availability, reduce the operation cost and reduce the average rate of major maintenance. But, FRA only provides a relatively small improvement in availability and less reduction in the average rate of major maintenance. In the example, FRA does not reduce the operation cost because the cost savings is offset by the annual operation cost of online monitoring. In practice, FRA is even less effective, because of the lack of interpretability and hence is usually applied only for post-fault tests.

5. Conclusion

In this paper, we first analyzed the malfunctions that may accelerate the deterioration of winding insulation paper. Then, we applied the information to refine the deterioration model of power transformers. We considered multiple types of failure rates of power transformers, such as infant mortality, sudden failure, and deterioration failures, so that the overall failure rate of power transformer follows a bathtub

Table 1
Parameter setting [14,17,18,19].

Parameter	Vale	Parameter	Value
λ_a	0.04/year	μ_{M}	24.39/year
λ_n	0.105/year	μ_F	12.05/year
λ_{d1}	5.333/year	μ_R	3.04/year
λ_{d2}	2.105/year	C_u	£53,000/day
λ_{f1}	0.003/year	C_p	£3200/day
λ_{f2}	0.001/year	C' _{in}	£1000
λ_F	0.008/year	C_C'	£1900
λ_{Fd}	0.048/year	C_M'	£600,000
μ_{in}	1095/year	C_F'	£5600
μ_c	91.25/year	C_R'	£1,000,000

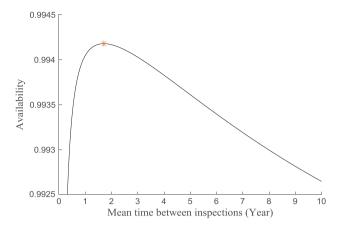


Fig. 5. Availability of the power transformer under different mean time between inspections.

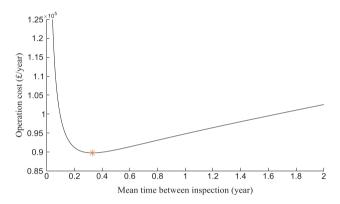


Fig. 6. Operation cost of the power transformer under different mean time between inspections.

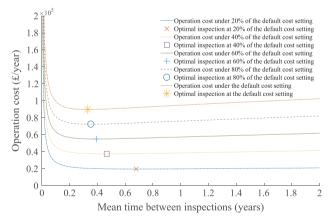
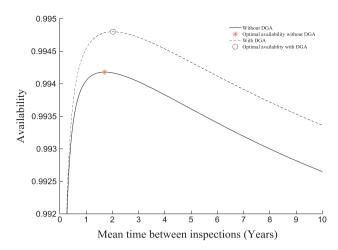


Fig. 7. Sensitivity analysis under different cost setting (C_u, C_p, C_M') and (C_n, C_p, C_M') and $(C_n,$

Table 2 Results from sensitivity analysis.

Minimum operation cost	Optimal inspection interval
£89,750/year	0.3295 year
£72,414/year	0.353 year
£55,017/year	0.392 year
£37,503/year	0.467 year
£19,705/year	0.679 year
	£89,750/year £72,414/year £55,017/year £37,503/year



 $\textbf{Fig. 8.} \ \, \textbf{Availability of the power transformer under different mean time between inspections with DGA.} \\$

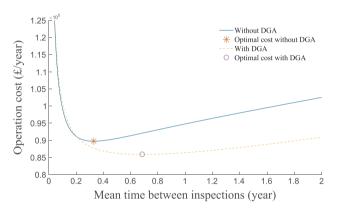


Fig. 9. Cost of the power transformer under different mean time between inspections with DGA.

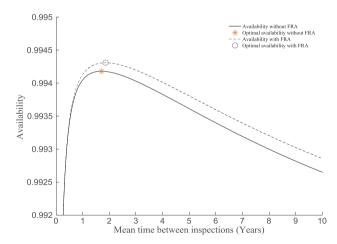


Fig. 10. Availability of the power transformer under different mean time between inspections with FRA.

curve with multiple wear-out tails. We developed analytical models for power transformer maintenance in the distribution network and transmission network using CTMC. A condition-based maintenance is designed for power transformers in the transmission network. The designed condition-based maintenance model opens up a successful way to optimize the maintenance of power transformers according to the information from both periodic inspection and online monitoring

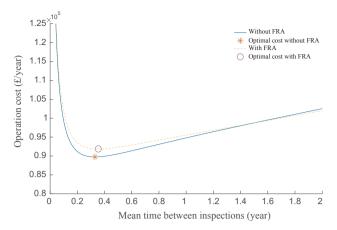


Fig. 11. Availability of the power transformer under different mean time between inspections with FRA.

devices. The value of DGA and FRA analysis were studied in a numerical case. The result shows that by applying DGA, both the availability and the operation cost of the power transformer can be improved. However, FRA can improve only the availability of the power transformer.

Appendix A

Expression of Steady States

Table 3	
performance of DGA and FI	RA.

Online monitoring device	Benefit on availability	Benefit on operation cost	Reduction in the average rate of major maintenance
DGA	0.06%	£3855/year	0.0015/year
FRA	0.01%	– £2090/year	0.0002/year

Further work will be focused on extending this model to solve the maintenance of a power system that is composed of multiples power transformers, switches, and other devices.

Acknowledgement

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State	Expression	State	Expression
$\pi_{0,0}$	1	$\pi_{2,0}$	S_2
$\pi_{0,1}$	$\frac{\lambda_{f0,1}}{\lambda_{d0,1}+\lambda_{Fd}}$	$\pi_{2,1}$	$\sum_{l=0}^{2} \prod_{i=1}^{l} rac{\lambda_{di-1,1}}{\lambda_{di,1}+\lambda_{Fd}} rac{\lambda_{f2-l,1}}{\lambda_{d2-l,1}+\lambda_{Fd}} S_{2-l}$
$\pi_{0,2}$	$\frac{\lambda_{f0,2}}{\lambda_{d0,2}+\lambda_{Fd}}$	$\pi_{2,2}$	$\sum_{l=0}^{2}\prod_{i=1}^{l}rac{\lambda_{di-1,2}}{\lambda_{di,2}+\lambda_{Fd}}rac{\lambda_{f2-l,2}}{\lambda_{d2-l,2}+\lambda_{Fd}}S_{2-l}$
$\pi_{0,3}$	$\frac{\lambda_F + \lambda_a}{\mu_F} + \frac{\lambda_{Fd}}{\mu_F} \sum_{j=1}^2 \frac{\lambda_{f0,j}}{\lambda_{d0,j} + \lambda_{Fd}}$	$\pi_{2,3}$	$\tfrac{\lambda_F}{\mu_F} S_2 + \tfrac{\lambda_{Fd}}{\mu_F} \sum_{j=1}^2 \sum_{l=0}^2 \prod_{l=1}^l \tfrac{\lambda_{dl-1,2}}{\lambda_{dl,2} + \lambda_{Fd}} \tfrac{\lambda_{f2-l,2}}{\lambda_{d2-l,2} + \lambda_{Fd}} S_{2-l}$
$\pi_{1,0}$	S_1	$\pi_{3,0}$	S_3
$\pi_{1,1}$	$rac{\lambda_{d0,1}\lambda_{f0,1}}{\prod_{i=0}^{1}\lambda_{di,1}+\lambda_{Fd}}+rac{\lambda_{f1,1}}{\lambda_{d1,1}+\lambda_{Fd}}S_{1}$	$\pi_{3,1}$	$\textstyle \sum_{l=0}^{3} \prod_{i=1}^{l} \frac{\lambda_{di-1,1}}{\lambda_{di,1} + \lambda_{Fd}} \frac{\lambda_{f2-l,1}}{\lambda_{d2-l,1} + \lambda_{Fd}} S_{3-l}$
$\pi_{1,2}$	$rac{\lambda_{d0,2}\lambda_{f0,2}}{\prod_{1=0}^{1}\lambda_{d1,2}+\lambda_{Fd}}+rac{\lambda_{f1,2}}{\lambda_{d1,2}+\lambda_{Fd}}S_{1}$	$\pi_{3,2}$	$\sum\nolimits_{l = 0}^3 {\prod\nolimits_{i = 1}^l {\frac{{{\lambda _{di - 1,2}}}}{{{\lambda _{di,2}} + {\lambda _{Fd}}}}} \frac{{{\lambda _{f2 - l,2}}}}{{{\lambda _{d2 - l,2}} + {\lambda _{Fd}}}}} {S_{3 - l}}}$
$\pi_{1,3}$	$\left(\frac{\lambda_F}{\mu_F} + \sum_{j=1}^2 \frac{\lambda_{f1,j}}{\lambda_{d1,j} + \lambda_{Fd}}\right) S_1 + \sum_{j=1}^2 \frac{\lambda_{d0,j}\lambda_{f0,j}}{\prod_{i=0}^1 \lambda_{di,j} + \lambda_{Fd}}$	$\pi_{3,3}$	$\tfrac{\lambda_F}{\mu_F} S_3 + \tfrac{\lambda_{Fd}}{\mu_F} \sum_{j=1}^2 \sum_{l=0}^3 \prod_{l=1}^l \tfrac{\lambda_{di-1,2}}{\lambda_{dl,2} + \lambda_{Fd}} \tfrac{\lambda_{f2-l,2}}{\lambda_{d2-l,2} + \lambda_{Fd}} S_{3-l}$

where

and

$$S_{1} = \frac{\lambda_{n0,0} + \sum_{j=1}^{2} \frac{\lambda_{Fd} \lambda_{d0j} \lambda_{f0j}}{\prod_{i=0}^{1} \lambda_{dij} + \lambda_{Fd}}}{\lambda_{n1,0} + \sum_{j=1}^{2} \lambda_{f1j} - \frac{\lambda_{Fd} \lambda_{f1j}}{\lambda_{d1j} + \lambda_{Fd}}},$$

$$S_{2} = \frac{\lambda_{Fd} \sum_{j=1}^{2} \left(\prod_{i=1}^{2} \frac{\lambda_{di-1,j}}{\lambda_{dij} + \lambda_{Fd}} \right) \frac{\lambda_{f0j}}{\lambda_{d0j} + \lambda_{Fd}}}{\lambda_{n2,0} + \sum_{j=1}^{2} \left(\lambda_{f2j} - \frac{\lambda_{f2j} \lambda_{Fd}}{\lambda_{d2j} + \lambda_{Fd}} \right)} + \prod_{i=1}^{2} \frac{\lambda_{ni-1,0} + \sum_{j=1}^{2} \frac{\lambda_{Fd} \lambda_{di-1j} \lambda_{fi-1j}}{(\lambda_{dij} + \lambda_{Fd})(\lambda_{di-1j} + \lambda_{Fd})}}{\lambda_{ni,0} + \sum_{j=1}^{2} \lambda_{fij} - \frac{\lambda_{Fd} \lambda_{fi,j}}{\lambda_{di,j} + \lambda_{Fd}}}$$

$$S_{3} = \frac{\lambda_{Fd} \sum_{j=1}^{2} \left(\prod_{i=1}^{3} \frac{\lambda_{di-1,j}}{\lambda_{di,j} + \lambda_{Fd}} \right) \frac{\lambda_{f0,j}}{\lambda_{d0,j} + \lambda_{Fd}}}{\lambda_{n3,0} + \sum_{j=1}^{2} \left(\lambda_{f3,j} - \frac{\lambda_{f3,j}\lambda_{Fd}}{\lambda_{d3,j} + \lambda_{Fd}} \right)} + \frac{\lambda_{Fd} \sum_{j=1}^{2} \left(\prod_{i=1}^{3} \frac{\lambda_{di-1,j}}{\lambda_{di,j} + \lambda_{Fd}} \right) \frac{\lambda_{f0,j}}{\lambda_{d0,j} + \lambda_{Fd}}}{\lambda_{n0,0} + \sum_{j=1}^{2} \frac{\lambda_{Fd}\lambda_{d0,j}\lambda_{f0,j}}{\prod_{i=0}^{1} \lambda_{di,j} + \lambda_{Fd}}} \\ + \prod_{i=1}^{3} \frac{\lambda_{ni-1,0} + \sum_{j=1}^{2} \frac{\lambda_{fd}\lambda_{di-1,j}\lambda_{fi-1,j}}{\lambda_{di,j} + \lambda_{Fd}} \frac{\lambda_{fd}\lambda_{di-1,j}\lambda_{fi-1,j}}{\lambda_{di,j} + \lambda_{Fd}}}{\lambda_{n1,0} + \sum_{j=1}^{2} \lambda_{fi,j} - \frac{\lambda_{Fd}\lambda_{f1,j}}{\lambda_{di,j} + \lambda_{Fd}}} + \frac{\lambda_{Fd} \sum_{j=1}^{2} \left(\prod_{i=1}^{2} \frac{\lambda_{di-1,j}}{\lambda_{di,j} + \lambda_{Fd}} \right) \frac{\lambda_{f0,j}}{\lambda_{d0,j} + \lambda_{Fd}}}{\lambda_{n2,0} + \sum_{j=1}^{2} \frac{\lambda_{fd}\lambda_{d2,j}\lambda_{f2,j}}{\prod_{i=2}^{3} \lambda_{di,j} + \lambda_{Fd}}} \\ + \frac{\lambda_{Fd} \sum_{j=1}^{2} \left(\prod_{i=1}^{2} \frac{\lambda_{di-1,j}}{\lambda_{di,j} + \lambda_{Fd}} \right) \frac{\lambda_{f0,j}}{\lambda_{d0,j} + \lambda_{Fd}}}{\lambda_{n2,0} + \sum_{j=1}^{2} \frac{\lambda_{Fd}\lambda_{f2,j}\lambda_{f2,j}}{\prod_{i=2}^{3} \lambda_{di,j} + \lambda_{Fd}}} \\ + \frac{\lambda_{Fd} \sum_{j=1}^{2} \left(\prod_{i=1}^{2} \frac{\lambda_{di-1,j}}{\lambda_{di,j} + \lambda_{Fd}} \right) \frac{\lambda_{f0,j}}{\lambda_{d0,j} + \lambda_{Fd}}}{\lambda_{n2,0} + \sum_{j=1}^{2} \frac{\lambda_{fd}\lambda_{f2,j}\lambda_{f2,j}}{\prod_{i=2}^{3} \lambda_{di,j} + \lambda_{Fd}}} \\ + \frac{\lambda_{Fd} \sum_{j=1}^{2} \left(\prod_{i=1}^{2} \frac{\lambda_{di-1,j}}{\lambda_{di,j} + \lambda_{Fd}} \right) \frac{\lambda_{f0,j}}{\lambda_{d0,j} + \lambda_{Fd}}}{\lambda_{n2,0} + \sum_{j=1}^{2} \frac{\lambda_{fd}\lambda_{f2,j}\lambda_{f2,j}}{\prod_{i=2}^{3} \lambda_{di,j} + \lambda_{Fd}}} \\ + \frac{\lambda_{Fd} \sum_{j=1}^{2} \left(\prod_{i=1}^{2} \frac{\lambda_{di-1,j}}{\lambda_{di,j} + \lambda_{Fd}} \right) \frac{\lambda_{f0,j}}{\lambda_{d0,j} + \lambda_{Fd}}}{\lambda_{n2,0} + \sum_{j=1}^{2} \frac{\lambda_{fd}\lambda_{f2,j}\lambda_{f2,j}}{\prod_{i=2}^{3} \lambda_{di,j} + \lambda_{Fd}}} \\ + \frac{\lambda_{Fd} \sum_{j=1}^{2} \left(\prod_{i=1}^{2} \frac{\lambda_{di-1,j}}{\lambda_{di,j} + \lambda_{Fd}} \right) \frac{\lambda_{f0,j}}{\lambda_{d0,j} + \lambda_{Fd}}}{\lambda_{n2,0} + \sum_{j=1}^{2} \frac{\lambda_{fd}\lambda_{f2,j}\lambda_{f2,j}}{\prod_{i=2}^{3} \lambda_{di,j} + \lambda_{Fd}}} \\ + \frac{\lambda_{Fd} \sum_{j=1}^{2} \left(\prod_{i=1}^{3} \frac{\lambda_{di,j} + \lambda_{Fd}}{\lambda_{di,j} + \lambda_{Fd}} \right) \frac{\lambda_{f0,j}}{\lambda_{d0,j} + \lambda_{Fd}}}{\lambda_{n1,0} + \sum_{j=1}^{2} \lambda_{f1,j} + \sum_{j=1}^{2}$$

References

- Bartley WH. Analysis of transformer failures. In: International association of engineering insurers 36th Annual Conference; 2003, p. 1–5.
- [2] Allan DM. Insulation ageing-a users perspective. In: Properties and applications of dielectric materials, 1994, Proceedings of the 4th International Conference on, 1994, vol. 2, p. 833–6.
- [3] Aldhubaib HA, Salama MM. A novel approach to investigate the effect of maintenance on the replacement time for transformers. IEEE Trans Power Deliv 2014;29(4):1603–12.
- [4] Dhople SV, Chen YC, Domínguez-García AD. A set-theoretic method for parametric uncertainty analysis in Markov reliability and reward models. IEEE Trans Reliab 2013;62(3):658–69.
- [5] Abu-Elanien AE, Salama MMA, Bartnikas R. A techno-economic method for replacing transformers. IEEE Trans Power Deliv 2011;26(2):817–29.
- [6] Lima SL, Saavedra OR, Miranda V. A two-level framework to fault diagnosis and decision making for power transformers. IEEE Trans Power Deliv 2015;30(1):497–504.
- [7] Abiri-Jahromi A, Parvania M, Bouffard F, Fotuhi-Firuzabad M. A two-stage framework for power transformer asset maintenance management—Part I: models and formulations. IEEE Trans Power Syst 2013;28(2):1395–403.
- [8] Koksal A, Ozdemir A. Improved transformer maintenance plan for reliability centred asset management of power transmission system. IET Gener Transm Distrib 2016;10(8):1976–83.
- [9] Hohlein I, Kachler AJ. Aging of cellulose at transformer service temperatures. Part 2. Influence of moisture and temperature on degree of polymerization and formation of furanic compounds in free-breathing systems. IEEE Electr Insul Mag 2005;21(5):20-4.
- [10] Liao R, Xiang B, Yang L, Tang C, Sun H. Study on the thermal aging characteristics and bond breaking process of oil-paper insulation in power transformer. In: Electrical Insulation, 2008. ISEI 2008. Conference Record of the 2008 IEEE international symposium on, 2008, p. 291–6.
- [11] Prevost TA, Oommen TV. Cellulose insulation in oil-filled power transformers: part I-history and development. IEEE Electr Insul Mag 2006;22(1):28–35.

- [12] Lundgaard LE, Hansen W, Ingebrigtsen S. Ageing of mineral oil impregnated cellulose by acid catalysis. IEEE Trans Dielectr Electr Insul 2008;15(2):540–6.
- [13] Poan X, Zhishu Z, Macalpine M, Zhijian J, Zhushi R. Estimation of the clamping pressures of transformer windings from vibration measurements. HKIE Trans 2008;15(1):30–5.
- [14] Sokolov V, Bulgakova V, Berler Z. Assessment of power transformer insulation condition. In: Electrical insulation conference and electrical manufacturing coil winding conference, 2001. Proceedings, 2001, p. 605–13.
- [15] Emsley AM, Stevens GC. Review of chemical indicators of degradation of cellulosic electrical paper insulation in oil-filled transformers. IEE Proc.-Sci. Meas. Technol. 1994: 141(5) p. 324–34
- [16] Emsley AM, Heywood RJ, Ali M, Xiao X. Degradation of cellulosic insulation in power transformers. Part 4: effects of ageing on the tensile strength of paper. IEE Proc.-Sci. Meas. Technol. 2000; 147(6), p. 285–90.
- [17] Mijailovic V. Method for effects evaluation of some forms of power transformers preventive maintenance. Electr Power Syst Res 2008;78(5):765–76
- [18] Westman T, Lorin P, Ammann PA. Fit at 50 keeping aging transformers healthy for longer. Asset Manag Maint J Apr. 2011;24(2):38.
- [19] Liang Z, Parlikad AK. A condition-based maintenance model for assets with accelerated deterioration due to fault propagation. Reliab IEEE Trans On 2015;64(3):972–82.
- [20] Muhamad NA, Phung BT, Blackburn TR, Lai KX. Comparative study and analysis of DGA methods for transformer mineral oil. In: Power Tech, 2007 IEEE Lausanne; 2007. p. 45–50.
- [21] Boss P, Lorin P, Viscardi A, Harley JW, Isecke J. Economical aspects and practical experiences of power transformers on-line monitoring. Int. Counc. Large Electr. Syst. CIGRÉ Fr., 2000.
- [22] Boss P, Horst T, Lorin P, Pfammatter K, Fazlagic A, Perkins M. Life assessment of power transformers to prepare a rehabilitation based on a technical-economical analysis. In: Cigre Session; 2002, p. 12–106.
- [23] Kim J-W, Park B, Jeong SC, Kim SW, Park P. Fault diagnosis of a power transformer using an improved frequency-response analysis. IEEE Trans Power Deliv 2005;20(1):169–78.