#### Tidal grounding-line migration modulated by subglacial 1 hydrology 2

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#### **Key Points:** 10

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11	•	Hydraulic resistance of the subglacial environment limits the speed of grounding-
12		line migration
13	•	Speed of grounding-line migration may differ between incoming and outgoing tides

- $\mathbf{s}$ ۰g.
- Asymmetric response of the grounding zone can act as a non-linear filter on the 14 tidal forcing 15

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### 16 Abstract

We present a mathematical model of the hydrology of grounding-line migration on tidal 17 timescales, in which the ice acts elastically, overlying a connected hydrological network, 18 with the ocean tides modelled by an oscillating far-field fluid height. The upstream grounding-19 line migration is driven by a fluid pressure gradient through the grounding zone, while 20 the downstream migration is limited by fluid drainage through the till. The two processes 21 are described using separate travelling-wave solutions, based on a model of fluid flow un-22 der an elastic sheet. The asymmetry between the up- and downstream motion allows the 23 grounding line to act as a non-linear filter on the tidal forcing as the pressure signal prop-24 agates upstream, and this frequency modulation is discussed in the context of velocity 25 data from ice streams across Antarctica to provide a novel constraint on till permeabil-26 ity. 27

## <sup>28</sup> Plain Language Summary

The grounding zone, where the ice sheet transitions from contact with the bed to 29 floating on the ocean, plays an important role in understanding the contribution of po-30 lar ice sheets to sea level rise. This model explores how ocean water can be pumped through 31 the grounding zone to the region underneath ice sheets as the ocean tides go in and out. 32 Water present underneath ice sheets can make the ice flow faster. We show that the dif-33 ference between how quickly the water flows into and out of the grounding zone could 34 35 explain some observations of tidal variations in glacier speed, and raises questions for the amount of melting happening underneath ice sheets. 36

#### 37 1 Introduction

The grounding zone of an ice sheet represents the region over which the ice ceases 38 to be supported by the bed and forms an ice shelf floating over the ocean. Understand-39 ing grounding-line migration is of key importance in models of glacial dynamics and global 40 climate. Ice that has passed the grounding line contributes to global sea level rise and 41 so must be considered lost in mass balance calculations (Bamber & Rivera, 2007; Shep-42 herd et al., 2012), while the incursion of ocean water at the grounding line enhances melt-43 ing from the base of ice shelves, decreasing the buttressing of grounded ice (Jenkins et 44 al., 2010). Further, the grounding zone represents a transition region where the ice lifts 45 up from its base and the basal traction dramatically decreases (Gillet-Chaulet & Durand, 46 2010), so assessing the stability of marine ice sheets relies on determining the grounding-47 line position relative to topographic pinning points (Schoof, 2007; Gudmundsson, 2013). 48

Grounding lines are not static but migrate on hourly to multi-annual timescales. 49 Over the daily tidal cycle, grounding lines move back and forth across grounding zones 50 that can be several kilometres wide (Rignot et al., 2011; Bamber et al., 2009). Ice has 51 a visco-elastic rheology, so that over timescales longer than the Maxwell time, the response 52 of the ice is mainly viscous, while on hourly to daily timescales, the response of the ice 53 is predominantly elastic (Holdsworth, 1969; Larour et al., 2005), as illustrated by flex-54 ural patterns observed close to the grounding line (Brunt et al., 2010; Vaughan, 1995). 55 For example, detailed measurements on the Rutford Ice Stream reveal a flexural wave 56 propagating across the grounding zone over the tidal cycle, characteristic of this elas-57 tic behaviour (Minchew et al., 2017). 58

Recent observations have shown that tides can affect glacial dynamics far upstream of the grounding zone. Tidal modulations of surface velocities have been observed in ice shelves and sheets across Antarctica and Greenland (Padman et al., 2018). Ocean tides are composed of diurnal and semidiurnal components. While some glaciers, such as the Bindschadler Ice Stream (Anandakrishnan et al., 2003) and the Whillans Ice Stream (Bindschadler et al., 2003) respond at the same frequency as the tidal forcing, other locations,



**Figure 1.** (a) A schematic showing an elastic ice sheet floating on the ocean, lifting off from the bed at the grounding line. (b) Close-up of the region near the grounding line showing the notation of the model.

such as the Rutford Ice Stream (Gudmundsson, 2006; Minchew et al., 2017) and Beardmore Glacier (Marsh et al., 2013), exhibit velocity variations at the fortnightly frequency
of the apparent 'beat' between the true tidal components. To generate a 14-day frequency
in the surface velocity from daily tides requires a non-linear mechanism to act between
the tidal forcing and velocity response (Rosier et al., 2015).

The origin of this non-linear response of the surface velocity remains enigmatic. Pre-70 vious authors have suggested that the tidal response is modulated by the impact of the 71 visco-elastic rheology on extensional stresses within the ice (Rosier & Gudmundsson, 2018), 72 or by a highly non-linear basal drag law (Rosier et al., 2015). The traction at the base 73 of an ice sheet is strongly dependent on the water pressure there (Tulaczyk et al., 2000; 74 Iverson, 2010). Rosier & Gudmundsson (2020) demonstrated that tidal variations in basal 75 traction at the grounding line are a dominant factor in determining the large scale ve-76 locity of marine ice sheets, suggesting the need to accurately assess lubrication by sub-77 glacial water across the grounding zone. 78

Here we explore the response of the grounding zone to ocean tidal forcing, by fo-79 cussing on the dynamics of water transport through the grounding zone over the tidal 80 cycle. Previous models of tidal grounding-line migration have neglected the hydrodynam-81 ics of the subglacial environment. Sayag & Worster (2013) found the equilibrium grounding-82 line position for a uniform elastic ice sheet at a given ocean height. Tsai & Gudmunds-83 son (2015) considered the impact of variable ice thickness on the static equilibrium po-84 sition of the grounding line, and proposed a fracture-mechanics model for the incoming 85 tide. Walker et al. (2013) suggested that glaciers could periodically 'gulp' ocean water 86 as the tide comes in. In this paper, we more closely examine the forces driving flow in 87 the subglacial cavity. We find that the asymmetry between the fast advance of the in-88 coming tide and the slow drainage during the outgoing tide produces an asymmetry in 89 grounding zone dynamics over the tidal cycle. By studying the dynamics of water trans-90 port across the grounding line over the tidal cycle we describe a self-consistent mecha-91 nism coupling the grounding-line migration and subglacial hydrology. 92

### 93 2 The model

We model the influence of subglacial hydrology on tidally-induced grounding-line 94 motion by considering a flow line ice sheet of uniform depth D resting on a bed sloping 95 upwards inland at an angle  $\theta$  (see figure 1). The base of the ice is at y = H(x, t), and 96 the depth of the subglacial cavity is  $h(x,t) = H + \theta x$  (figure 1). Upslope of the ground-97 ing line, between the bed and the base of the ice, we parametrise a distributed subglacial 98 hydrological network by an effective water film of thickness  $h_0$ , with a hydraulic trans-99 missivity proportional to  $h_0^3$  (e.g. Le Brocq et al., 2009; Bougamont et al., 2014). Far up-100 slope of the grounding line,  $h \rightarrow h_0$  as  $x \rightarrow -\infty$ , while over the ocean the ice sheet 101

is in isostatic balance, moving up and down with the ocean tides so  $H(x,t) \to H_o(t)$ as  $x \to \infty$ . Since the depth of the cavity transitions smoothly to the effective depth of the subglacial film, we arbitrarily define the grounding line to be the furthest upstream point at which the water depth is above a threshold value,  $h(x_G(t), t) = 1.5h_0$ .

We note that the role of the bedslope angle  $\theta$  is to set a hydraulic gradient in the 106 subglacial environment, and therefore can also represent the effect of a gradient in ice 107 thickness across the grounding zone. With this interpretation, the setup can also describe 108 the dynamics of grounding zones on retrograde slopes under a thinning ice sheet, where 109  $\theta$  takes the value of  $\theta - \rho_i / \rho_w \frac{dD}{dx}$  (see supporting information). In this case, we use the 110 average value of D to evaluate the bending stiffness. Similarly,  $\theta$  and  $h_0$  may be slowly 111 varying across the grounding zone without qualitatively affecting the results, but for sim-112 plicity we take representative, constant values. 113

In this paper we seek to quantify the ocean water present in the grounding zone to understand the tidal variations in basal drag on the ice above. We first consider the response of the grounding zone to idealised tides that fall or rise with constant speed, and then consider the response to more realistic tides with both solar and lunar components. Having developed a theory to account for general tidal speeds, we then force the grounding-line model with the full spectrum of tidal components.

#### 2.1 The elastic response of the ice

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On the timescale of daily tides, ice behaves predominantly elastically Vaughan (1995). For simplicity, we consider a constant ice thickness D across the grounding zone. Tidal fluctuations in height are small relative to the thickness, and hence we model the ice sheet as an elastic beam of bending stiffness  $B = ED^3/12(1 - \nu^2)$ , with Young's modulus E = 0.32 - 3.9GPa and Poisson ratio  $\nu = 0.3$  (Vaughan, 1995). Spatial variations in the profile of the ice H(x, t) thus exert a bending stress (pressure)  $B \frac{\partial^4 H}{\partial x^4}$  on the water below.

Where the ice is floating, a force balance between hydrostatic pressure and bending stresses implies that the static profile of the ice is governed by

 $\frac{\partial}{\partial x} \left( \rho g H + B \frac{\partial^4 H}{\partial x^4} \right) = 0, \tag{1}$ 

where  $\rho$  is the density of water and q the gravitational acceleration. Over the ocean, the 131 ice becomes flat, and its height is set by a hydrostatic balance with the far-field ocean 132 height. Upstream of the grounding line, the ice rests on its bed. These constraints pro-133 vide enough conditions to determine the static position of the grounding line as a func-134 tion of the ocean height, as considered by Sayag & Worster (2011) (see also supporting 135 information). The dynamic response of the grounding line when the ocean height varies 136 over tidal timescales, however, requires coupling of the ice dynamics to the motion of ocean 137 water driven in and out of the subglacial cavity by the motion of the ice. 138

## <sup>139</sup> 2.2 The dynamics of the water

For moderate bedslope at the grounding line, the subglacial cavity is much longer than it is deep, so tidally-driven flows are mainly horizontal. In this geometry the pressure within the cavity is given approximately by

$$p = \rho g(H - y) + B \frac{\partial^4 H}{\partial x^4}.$$
 (2)

Elastic and hydrostatic pressure gradients across the subglacial cavity drive a water flux through the subglacial system, resisted by drag in the hydraulic network. For simplicity, and in keeping with previous studies (Le Brocq et al., 2009; Bougamont et al., 2014), we model the conductivity of the distributed system by a laminar flow law, such that the flux through a layer of depth h is given by

$$q = -\frac{h^3}{12\mu} \frac{\partial p}{\partial x},\tag{3}$$

where  $\mu$  is the viscosity of water. Conservation of water across the grounding zone requires that

$$\frac{\partial h}{\partial t} = -\frac{\partial q}{\partial x} = \frac{\partial}{\partial x} \left[ \frac{\rho g}{12\mu} h^3 \left( \frac{\partial H}{\partial x} + \frac{B}{\rho g} \frac{\partial^5 H}{\partial x^5} \right) \right]. \tag{4}$$

As the ocean height varies over the tidal cycle,  $H(x,t) \to H_o(t)$  as  $x \to \infty$ ,  $h(x,t) \to h_0$  as  $x \to -\infty$ , and the hydrology of the grounding zone evolves according to (4).

#### **3** Results and Discussion

We proceed to highlight the asymmetry between incoming and outgoing tides by first considering each case separately. The resultant reduced model is then used to understand the non-linear response of the grounding zone to oscillatory tidal forcing.

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# 3.1 Response of the grounding zone to the rising tide

We first consider the effect of the rising tide on the subglacial system, considering a constant tidal speed. Numerical solutions of the ice-water response from integration of equation (4) are shown in figure 2a, initialised in the static equilibrium elastic position (Sayag & Worster, 2011).

For a steadily rising tide, the ice sheet quickly settles to a steady travelling-wave solution that migrates inland at the speed of the rising tide, with a series of flexural waves propagating through the grounding zone (figure 2a). As the ocean height rises, a hydrodynamic pressure gradient across the subglacial cavity forces ocean water into the grounding zone, driving the inland migration of the grounding line. This process is analogous to a fluid-driven elastic peeling, or fracturing, problem (Lister et al., 2013; Hewitt et al., 2018; Tsai & Gudmundsson, 2015).

In the small region close to the grounding line, the pressure is predominantly due to elastic flexure of the ice, leading to characteristic flexural waves. If the incoming tide drives the grounding line inwards at speed U, then from equation (4) the viscous drag balances the elastic pressure gradient over a lengthscale  $x \sim (Bh_0^3/12\mu U)^{1/5}$ . Peeling is driven by the curvature  $\kappa = \frac{\partial^2 h}{\partial x^2}$  at the grounding line,

$$\kappa \sim h_0 / x^2 = 1.35 (12\mu U/B)^{2/5} / h_0^{1/5},$$
(5)

where the constant prefactor is calculated by Lister et al. (2013). The curvature is proportional to the lag of the grounding-line position behind the equilibrium height (see supporting information), so a balance is reached in which the grounding line can rise at the same speed as the incoming tide, with a small lag of

$$\Delta H = 1.35 (12\mu U/B)^{2/5} (B/\rho g)^{1/2} / h_0^{1/5}.$$
(6)

## <sup>182</sup> 3.2 The receding ocean tide

As the tide recedes, the water at the grounding line does not all instantly drain, but is initially retained against gravity, trapped in the hydrological network. This ocean water can be retained in the subglacial environment for substantial periods of time, since the drainage speed is controlled by the effective resistance to flow through the hydrological network, and driven only by the small along-slope component of gravity. The ocean height drops over a much shorter timescale, leaving behind the trapped water (figure 2b).



Figure 2. Results of a full numerical simulation for ocean height changing at constant speed of  $2 \times 10^{-4}$ ms<sup>-1</sup> over a bedslope of  $\theta = 10^{-3}$  with  $h_0 = 0.5$ mm,  $B = 10^{16}$ kgm<sup>2</sup>s<sup>-2</sup>. Colour gradient represents passage of time, profiles shown at 1 hour intervals. Black dots show the position of the grounding line. (a) As the ocean height rises, the grounding line migrates upwards at the same speed. At the grounding line, we observe flexural waves ahead of the peeling front. (b) As the ocean height falls, the majority of the water is able to drain out, but close to the grounding line, we find that a thin film of water is retained. The top-most extent of the film begins to slowly drain in a manner governed by equation (8) - see inset.

The effective depth of this volume of retained subglacial water can be determined from a balance in equation (4) between the drag resisting water flow out of the subglacial environment, and the elastic stresses forcing water out. This elastic mechanism is analogous to the process by which surface tension keeps surfaces wet even after they are withdrawn from a fluid. The elastic stresses are proportional to  $h_{ret}/x^3$ , and have magnitude  $(\rho g/B)^{1/2}$ . The flexural wavelength during drainage,  $x \sim (Bh_{ret}^3/12\mu U)^{1/5}$ , now also depends on the retained layer height. Combining these balances, we find that

$$h_{ret} = 5.12 \frac{(\mu U)^{5/4}}{B^{1/8} (\rho g)^{5/8} \theta^{5/4}},\tag{7}$$

(8)

with the constant prefactor found by Warburton et al. (2020). For tidal values of these parameters,  $h_{ret}$  is much larger than  $h_0$ . This implies that the background hydrological system of the grounding zone is overwhelmed by the additional ocean water, and this deposited fluid forms a much deeper layer between the ice and bed. This water-saturated area lubricates the contact between the ice and the bed, lowering the basal traction over this area of the grounding zone, even though the majority of the water has been evacuated.

While the tide retreats relatively rapidly, the retained water layer drains slowly under gravity, driven by hydrostatic pressure gradients. The fluid layer both thins (driven by gradients in h), and drains downslope (driven by the bedslope  $\theta$ ) and hence from equation (4) drainage is approximately governed by

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left[ \frac{\rho g h_0^3}{12 \mu} \left( \frac{\partial h}{\partial x} - \frac{3h\theta}{h_0} \right) \right] + O(h^2).$$

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From equation (8), we see that at early times the drainage is mostly due to a thinning 209 of the top-most extent of the layer, such that the distance the drained region migrates 210 away from the high tide position is given by  $x \sim \sqrt{\rho g h_0^0 t} / \mu$ . At later times, the downs-211 lope drainage of the whole layer becomes the dominant mechanism, at constant speed 212  $\rho g h_0^2 \theta / 4 \mu$ . In both regimes, the speed of drainage is limited by the effective permeabil-213 ity of the hydrological network, parametrised by  $h_0$ . If the permeability is large, the top 214 of the fluid layer drains as quickly as it is deposited by the retreating tide. If the per-215 meability is small, there is negligible drainage, and the ocean water deposited at high 216 tide remains trapped over the tidal cycle. 217

From this continuous model for the evolution of h we can define a reduced model 218 for the motion of the grounding line over the tidal cycle. The grounding-line position as 219 defined here represents the position downstream of which a significant volume of water 220 is retained between the ice and the bed, lubricating the contact such that the till exerts 221 very little basal drag on the flow of the ice. Rosier & Gudmundsson (2020) highlight the 222 sensitive role that variations in drag in the grounding zone play in setting the velocity 223 response of ice shelves. In this way, modulations of horizontal ice velocity can be under-224 stood through the modulation of this position of decreased contact. 225

### 3.3 Grounding-line migration over the tidal cycle

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We now consider the impact of multiple daily tidal cycles on the hydrology at the grounding zone, using as an illustrative example a far-field ocean height of

$$H_o(t) = A_M \sin(\omega_M t) + A_S \sin(\omega_S t).$$
(9)

This simplified forcing, with one lunar (M) and one solar (S) component, provides the interference between the two frequencies that produces a fortnightly amplitude variation at the  $M_{sf}$  frequency. Given the asymmetry between upslope and downslope groundingline dynamics, we anticipate that the grounding zone acts as a non-linear filter on the ocean height, generating a response in the subglacial hydrological system at this 'beat' frequency.

Over the timescale of multiple tidal cycles, ice behaves visco-elastically. On the timescale 236 of a single tidal cycle, the viscous behaviour appears as a small lag in the elastic response 237 (Walker et al., 2013) or a weak modification to the bending stiffness (Reeh et al., 2003), 238 so a purely elastic description effectively captures the daily dynamics. Over the longer 239 fortnightly timescale, the ice acts passively as a boundary condition to the subglacial flow, 240 and the rheology of the ice does not appear in the expression for the drainage speed (equa-241 tion 8). Thus viscous flow of the ice is expected to have minimal impact on the asym-242 metry described here, which is set up by the short timescale elastic dynamics. 243

The grounding line moves upslope with the speed of the rising tide  $U_{rise} \sim A\omega/\theta$ , where  $A = A_M + A_S$  is the maximum tidal amplitude, but as the tide retreats, the drainage speed is independent of the changes in ocean height, and is instead limited by the properties of the subglacial environment through equation (8) to

$$U_{receed} \sim \max\left(\sqrt{\frac{\rho g h_0^3 \omega}{\mu}}, \frac{\rho g h_0^2 \theta}{\mu}\right).$$
 (10)

This fast upward motion, followed by a slower downwards retreat, leads to a groundingline position that tracks the height of the high tide as it varies over the fortnightly cycle, smoothing out the daily tidal motion. We arrive at a reduced model for groundingline migration governed only by the shape of the tidal forcing, and the ratio between downslope and upslope speeds,

$$r_{vel} = \max\left(\sqrt{\frac{\rho g h_0^3 \theta^2}{A^2 \mu \omega}}, \frac{\rho g h_0^2 \theta^2}{A \mu \omega}\right).$$
(11)

As these two parameter groupings have different dependence on  $h_0$ , we can separate out the uncertainty in  $h_0$  from the uncertainty in the other parameters, transforming variables to the phase diagram shown in figure 3. This reduced system relies on just these two ratios and is computationally much simpler than solving the full model, so the groundingline response can be easily calculated (see supporting material). This simple parameterisation of grounding-line motion could be implemented as a boundary condition in a larger scale ice sheet model, as demonstrated by Rosier & Gudmundsson (2020).



Figure 3. (a) Phase diagram showing the relative amplitudes of the 14-day  $(A_{M_{sf}})$  to 12-hour  $(A_S)$  components present in the grounding-line position, as a function of dimensionless downslope gravity  $\rho g \theta^2 A / \mu \omega$  and till permeability  $h_0 / A$ . (Reduced model, forced by two sinusoidal frequencies with  $\omega_M / \omega_S = 12.42/12$  and  $A_M / A_S = 1.5$ .) The points marked with open circles are shown in (b): Blue, tidal height, and red, grounding-line height. For very low permeability, the grounding line is pinned at the point of highest tide. As permeability increases, the high frequency components are more weakly filtered.

As a quantitative measure for the degree of non-linearity in our simple tidal exam-262 ple, we use the ratio of amplitudes between the 14-day and the 12-hour frequencies,  $A_{M_{sf}}/A_S$ , 263 in the Fourier spectrum of the grounding-line motion (for a more complete tidal model, 264 one could adapt this measure to use the amplitudes of the dominant frequencies). This 265 description clearly distinguishes between cases where the lower frequency is not gener-266 ated, cases where both frequencies are present in the motion, and cases where the response 267 is entirely at the 14-day frequency (figure 3). This measure is a function of model out-268 put alone, so can be directly applied to observations of other quantities for comparison. 269

If drainage is fast, with barely any drag exerted by the hydrological system, then 270 the grounding line can move up and down freely with the daily tides, producing a pre-271 dominantly diurnal response (figure 3i). This occurs if  $r_{vel} > 1$ , which is possible for 272 small-amplitude tides over steep bedslopes. For smaller permeability of the subglacial 273 environment, the magnitude of the daily motion is limited by the distance that the ground-274 ing line can migrate in one day (figure 3ii). This response is consistent with the obser-275 vations from the Bindschadler Ice Stream (Anandakrishnan et al., 2003) which show that 276 the amplitude of the flow fluctuations does not vary with the amplitude of the ocean tides. 277 For even smaller subglacial permeabilities, the grounding line acts as a filter on the tidal 278 signal, moving with the frequency of the amplitude envelope, generating a more dom-279 inant fortnightly component to the response (figure 3iii-iv). This ranges between a small 280 amplitude variation similar to the response observed at Beardmore Glacier (Marsh et 281 al., 2013), to the predominantly fortnightly variations seen in observations of Rutford 282 Ice Stream (Minchew et al., 2017). If the drainage is slow enough, the grounding line be-283 comes essentially fixed at the high tide position, disconnecting the response of the grounded 284 ice from the ocean tides (figure 3v). Many glaciers that exhibit no tidal response could 285 plausibly fall into this category. 286

We may apply our reduced model of grounding-line motion to the tidal model for ocean height described by Padman et al. (2002), at both the Rutford Ice Stream (flowing into the Ronne Ice Shelf) and Beardmore Glacier (flowing into the Ross). Values for



Figure 4. (i) Results of the reduced model for grounding line position with  $h_0 = 10^{-4}$  m applied to the tides at the grounding lines of (ai) Beardmore Glacier, and (bi) Rutford Ice Stream. The difference in grounding-line response can be attributed to the differing tidal amplitudes. CATS2008 tidal model described in Padman et al. (2002) used to force both. (ii) 3-day rolling average of ice horizontal surface velocity from the same locations and dates. Beardmore data (aii) from Marsh et al. (2013), (bii) Rutford data from Gudmundsson (2006).

tidal amplitude  $A \sim 3m$  and frequency  $\omega \sim 10^{-5} s^{-1}$  are taken from Padman et al. 290 (2002), and  $\rho, g$ , and  $\mu$  are well documented for water. At the grounding line of both Rut-291 ford and Beardmore, the isostatic gradient is of order  $\theta - \frac{\rho_i}{\rho} \frac{dD}{dx} \sim 10^{-2}$  (Morlighem et al., 2020), where  $\rho_i \approx 0.9\rho$  is the density of ice. From the phase diagram in figure 3a 292 293 we estimate that for Rutford, where  $\rho g \theta^2 A / \mu \omega \sim 10^8$ , we require  $h_0 / A \sim 10^{-4.5}$  for 294 strong filtering. By contrast, since tidal amplitudes are lower over the Ross Ice Shelf, that 295 same hydraulic system would lead to much weaker filtration (figure 4), consistent with 296 the primarily diurnal response of the glaciers feeding the Ross. This value of  $h_0$  repre-297 sents a permeability thickness of  $10^{-12}$ m<sup>3</sup>, consistent with estimates of till permeabil-298 ity of  $10^{-11} - 10^{-19} \text{m}^2$  for tills of thickness  $10^{-2} - 10 \text{m}$  (Fischer et al., 1998). 299

## 300 4 Conclusions

We have presented a mathematical description of the migration of ocean water through the subglacial cavity at the grounding zones of ice sheets. We have shown how an asymmetry in the physics governing incoming and outgoing migration leads to a non-linear response to the tidal forcing that is able to generate new frequencies at the grounding line itself. The model reconciles observations of ice stream velocity variations from across Antarctica and Greenland, and provides a new constraint on the effective permeability of the subglacial environment at these locations.

The model predicts that a significant amount of ocean water is retained in the subglacial environment. An outstanding question remains to determine the processes whereby this non-linear response of the subglacial water pressure is transmitted upstream of the grounding zone through the till, and to what extent this impacts on till rheology and glacial sliding. The tidal flushing of ocean water in regions conventionally understood to be grounded may also play a role in increased glacial melting as the oceans warm.

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