

Magnetic resonance velocity imaging of gas flow in a diesel particulate filter



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ABSTRACT

Magnetic resonance (MR) velocity imaging has been used to investigate the gas flow in a diesel particulate filter (DPF), with sulphur hexafluoride (SF_6) being used as the MR-active gas. Images of the axial velocity were acquired at ten evenly spaced positions along the length of the filter, for three flow conditions corresponding to Reynolds number of $\text{Re} = 106, 254$ and 428 in the filter channels. From the velocity images, averaged axial and through-wall velocity, as a function of position along the length of the filter, have been obtained. These experimentally obtained velocity profiles are analysed and a qualitative comparison with the results of previously reported numerical simulations is made. The MR measurements were used in subsequent analysis to quantify the uniformity of the through-wall velocity profiles. From this it was observed that for higher Re flows, the through-wall velocity profile became less uniform, and the implications that this has on particulate matter deposition are discussed. The MR technique demonstrated herein provides a useful method to advance our understanding of hydrodynamics and mass transfer within DPFs and also for the validation of numerical simulations used in their design and optimization.

1. Introduction

During the operation of a diesel engine, particulate matter (PM) will form during the fuel combustion process as a result of imperfect mixing at the molecular level between the fuel droplets and oxygen (van Setten et al., 2001). In recent years diesel engine design has improved to produce lower particulate emissions, however there is still a need for onboard emission control systems to reduce the PM content in the exhaust gas. The diesel particulate filter (DPF) is currently the most widely used technology to achieve compliance with government legislation such as the EURO 6 directive which currently sets the limit of PM emissions to be 5 mg km^{-1} . At their simplest level DPFs are used to physically filter PM from the exhaust gas. However, in order to meet these increasingly stringent legislated limits, multifunctional systems incorporating a catalyst, so-called catalyzed DPFs, have also been developed (Ahmadinejad et al., 2007; York et al., 2009; Watling et al., 2012).

DPFs typically take the form of a porous ceramic substrate made from cordierite, silicon carbide or aluminium titanate. These materials are chosen for automotive applications as they are suited to the harsh environments encountered in the exhaust. Typically these materials exhibit a high mechanical strength, are resistant to high temperatures and temperature shocks and possess a low thermal expansion coefficient (Lachman and Williams, 1992; Williams, 2001). DPFs are

comprised of many parallel channels separated by permeable walls with opposite ends of the adjacent channels plugged in a 'checkerboard' pattern. Due to the structure of the DPF, the PM-laden exhaust gas enters the filter through the engine side inlet channels and because of the plugs, that are impermeable to the gas flow, the exhaust gas is forced to pass through the porous, permeable walls separating the channels. As the gas passes through this permeable wall, the PM is deposited on the wall of the inlet channel and the clean gas leaves through the outlet channel. To ensure that the filter operates for the lifetime of the vehicle and to avoid an excessive back-pressure on the engine, which results in a fuel penalty, the filter must be regenerated via a passive (continual) or active (periodic) process to remove the accumulated PM (Twigg, 2011).

It follows from the above that a good understanding of the influence of the exhaust gas flow on DPF performance is essential for the optimization of their design to achieve compliance with increasingly stringent emission legislations. To date, direct measurement of gas transport within a DPF has not been reported. Such measurements are particularly challenging because they require a non-invasive probe capable of imaging flow fields within an optically opaque medium. Instead, research has focussed on numerical modelling. The original model used to describe transport in a DPF was developed by Bissett (1984) and most subsequent models have derived in some part from this work (Koltsakis et al., 2013). Model validation has been achieved

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Nomenclature*Latin characters*

<i>d</i>	diameter, m
<i>D</i>	self-diffusion coefficient, m ² s ⁻¹
F	Fourier transform operator
G	magnetic field gradient vector, G cm ⁻¹
g	flow-encoding magnetic field gradient vector, G cm ⁻¹
<i>g</i>	flow-encoding magnetic field gradient, G cm ⁻¹
<i>J</i>	regularisation functional
<i>L</i>	length, m
<i>M</i>	number of axial velocity measurements
<i>N</i>	number of through-wall velocity calculated values
<i>NUI</i>	non-uniformity index
<i>Re</i>	Reynolds number
<i>S</i>	compressed sensing under-sampling pattern
<i>Stk</i>	Stokes number
<i>TV</i>	Total Variation
<i>t</i>	time, s
<i>v</i>	velocity, m s ⁻¹
<i>w</i>	wall thickness, m
x	matrix form of image to be reconstructed
y	matrix form of image raw data

Greek symbols

α	regularisation parameter
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γ	gyromagnetic ratio, rad s ⁻¹ T ⁻¹
Δ	difference between two values, -
Δ	flow observation time, s
δ	flow-encoding gradient encoding time, s
ε	substrate porosity, %
μ	average through-wall velocity, m s ⁻¹
μ	dynamic viscosity, Pa s
ν	experimental noise
ρ	molecular density, kg m ⁻³
σ	standard deviation of through-wall velocity, m s ⁻¹
σ	standard deviation of noise

Subscripts

ave	average
c	channel
flow	motion-encoding direction
min	minimum
max	maximum
ph1	first phase-encoding direction
ph2	second phase-encoding direction
p	pore
slice	slice of excited spins
w	wall
xy	through-wall direction
z	axial direction

indirectly by comparison of model predictions with measurements of pressure drop (Torregrosa et al., 2011; Piscaglia and Ferrari, 2009; Piscaglia et al., 2010). Schejbal et al. (2009) have compared predictions of pressure drop across the filter, inlet and outlet temperature of the gas, and outlet concentrations of NO_x with experimental data from the Extra-Urban Drive Cycle (EUDC). Although a good agreement has been reached between the empirical evidence and the models, in each of these different studies the system was treated as a 'black box' and it was therefore not possible to include any information on the flow behaviour within the channels.

Further notable numerical studies include the work of Sbrizzai et al. (2005), Soldati et al. (2010), Bensaid et al. (2009, 2010) and Yu et al. (2013a), who have reported numerical simulations of gas flow within DPF structures and used these to explore how the gas flow influences, most notably, filtration efficiency and formation of the PM layer along the length of the inlet channel during operation. In particular, the study reported by Bensaid et al. (2009) has suggested that the uniformity of the PM layer that forms during operation is determined by the relationship between the structure of the filter, the nature of the PM and the gas flow fields within these systems. The PM deposition profile that forms during operation is important as calculations suggest that this can have significant effects on the temperature profile during filter regeneration (Yu et al., 2013a,b). For instance, if there is an uneven distribution of the PM due to a non-uniform through-wall velocity profile, oxidation will occur at different rates depending on the local PM content. Consequently, high thermal stresses in the filter can form as a result of the temperature gradients produced during combustion of the uneven PM deposit. If the temperatures and thermal stresses are high enough, this can lead to melting or cracking of the filter substrate and deactivation of the catalyst in the catalyzed DPF systems. Therefore, the requirement for a good understanding of the gas flow fields on the channel scale of the DPF is of the utmost importance for their design and optimization for specific applications.

The present work employs magnetic resonance (MR) velocity imaging to measure directly the flow velocities along the DPF channels.

MR velocity imaging is particularly well suited to such measurements because it is truly non-invasive, there is no need for tracer particles and optically opaque systems can be studied. This is not the case for other flow measurement techniques such as particle imaging velocimetry (PIV), laser Doppler velocimetry (LDV) or hot wire anemometry (HWA). Further, MR velocity imaging has the advantage that it is able to acquire one, two and three-dimensional images of the flow field depending on the nature of the system and the information required. MR velocity imaging is well-established as an experimental technique for investigating liquid flows; however, imaging studies of gas flows are relatively few. This is mainly due to challenges associated with the low signal-to-noise (SNR) ratio presented to the experimentalist; in particular due to the low molecular density of the gas phase, which is typically around three orders of magnitude lower than that of liquids. Gases also exhibit a higher self-diffusion coefficient than liquids which can result in a greater degree of diffusive attenuation of the signal and blurring of the image (Sankey et al., 2009). However despite these challenges, the importance of gas phase transport processes in a range of applications, such as reaction engineering and aerodynamics research, has been an incentive to develop the capability to implement these measurements.

Newling (2008) provides a thorough review of the gas flow measurements using nuclear magnetic resonance (NMR) and magnetic resonance imaging (MRI) and a brief summary of the literature relevant to the present study is now given. Of particular relevance to the present work is the use of gas phase NMR in the study of porous materials. For example, Koptug and co-workers have demonstrated the acquisition of two-dimensional MR velocity images of hydrocarbon gases at atmospheric pressure flowing through a cylindrical pipe and alumina monoliths of different channel geometries (Koptug et al., 2000, 2001, 2002). In the monolith studies (Koptug et al., 2000), the feasibility of using MR velocity imaging to investigate the flow of thermally-polarised hydrocarbon gases (acetylene, butane, propane) at ambient pressure with reasonable detection times (20–40 min) and a spatial resolution of 400 μ m was successfully demonstrated. MR

velocity images of both gas and liquid flow have also been reported during gas-liquid trickle flow in a packed bed in which sulphur hexafluoride (SF_6) was used as the gas flow (Sankey et al., 2009). As discussed therein, thermally-polarised SF_6 was chosen as the NMR active gas to be used for velocity imaging due its favourable magnetic resonance properties which give rise to a high SNR relative to other gases. These characteristics include: (i) the SF_6 molecule has six ^{19}F nuclei per molecule and after the ^1H nucleus, the ^{19}F nucleus has the highest gyromagnetic ratio ($\gamma_{^{19}\text{F}}=0.94\gamma_{^1\text{H}}$); and (ii) SF_6 has a relatively low molecular self-diffusion coefficient, $D = 3.45 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ at 20 °C and 1 atm, which is an order or magnitude lower than other potential candidate gases such as propane. A lower self-diffusion coefficient is advantageous as it will reduce the amount of signal attenuation and blurring of the image resulting from diffusion of the gas during the image acquisition. SF_6 has also been used to acquire velocity images of high Reynolds number ($> 10^5$) turbulent flows around a bluff obstruction which is analogous to a wind tunnel measurement employed in aerodynamic research (Newling et al., 2004). In that work the MRI velocity measurements were then used to validate the flow field predicted using a CFD code.

The aim of this paper is two-fold. Firstly, to develop and implement an MR velocity measurement that will provide sufficient signal-to-noise and in-plane spatial resolution such that the velocities within the individual channels of the DPF can be accurately resolved. Secondly, to apply this method to measure the velocity at different axial positions in order to be able to characterise the evolution of the gas flow field in the channels of the DPF under different experimental conditions. From these data, the uniformity of the through-wall velocity profiles is characterised. The ability to investigate the gas flow field with sufficient in-plane and axial resolution within the DPF enables the data to be used for comparison with the simulation predictions of earlier workers and ultimately to aid in the development and validation of CFD simulations.

To achieve these aims we have used thermally-polarised SF_6 gas, as used by Newling et al. (2004) and Sankey et al. (2009) and combined this with under-sampling and compressed sensing (CS) data acquisition and reconstruction methods. Thermally-polarised SF_6 was preferred over hyperpolarised noble gases (*i.e.* ^3He or ^{129}Xe) as the MR signal associated with the latter will be subject to nuclear spin relaxation effects which are difficult to quantify. This approach enables images to be reconstructed with sufficient accuracy from significantly fewer data points than are acquired in a traditional, fully-sampled experiment, thereby resulting in a reduction in image acquisition times (Lustig et al., 2007; Holland et al., 2010; Holland and Gladden, 2014). This reduction in acquisition time can either be exploited to obtain higher temporal resolution in studying dynamic systems, or to enable greater signal averaging over a given timescale such that images of inherently low SNR systems, such as gases, may be studied. In the present study, attaining a high spatial resolution is of the utmost

importance to enable the velocity fields within each individual channel of the DPF to be resolved as accurately as possible. By employing under-sampling and CS reconstruction techniques, it has been possible to acquire images with a higher spatial resolution and SNR in a given experimental time than would have been possible using the standard MR velocity imaging methods employed previously. More specifically, two-dimensional velocity images of the gas flow within the DPF with an in-plane nominal spatial resolution of $140 \mu\text{m} \times 140 \mu\text{m}$ have been acquired. Given the channel hydraulic diameter of 1 mm, this gives between 7 and 9 pixels across each channel, depending on the orientation of the sample, which we deem sufficient to provide a rigorous test of flow profiles predicted by CFD.

The paper is structured as follows. Section 2 introduces the basic principles of compressed sensing as used in the present application. Section 3 reports the experimental methods used, and Section 4 presents the results. In particular, images of the flow velocity in the direction of superficial flow are reported. From these data the average axial and through-wall velocities on the channel-scale are calculated as a function of position along the filter.

2. Theoretical

This section provides an overview of the fundamental concepts of CS and how it has been implemented in the present study. A summary of the principles upon which the MR velocity imaging measurements implemented in the present work are based is given in the supplementary material. For a more detailed introduction to MR velocity imaging the reader is directed to the texts by Callaghan (1991) and Haacke (1999), and the review article by Gladden and Sederman (2013).

In conventional MRI acquisitions, the raw data are sampled at the Nyquist rate (Nyquist, 1928), which means that the same number of data points as there are pixels in the image must be acquired. Consequently, when high spatial resolution images are required, prohibitively-long acquisition times result. As has been discussed previously, image acquisition times can be reduced by using CS, in which the image can be reconstructed from significantly fewer data than are required using traditional MRI acquisition and reconstruction methods. The basic theory of CS and how it has been implemented in the present study will now be described.

Image compression algorithms, from which the mathematics of CS are derived, are based on the principle that the full image can be sparsely represented by just a few non-zero components in some transform domain and can therefore be stored in a 'compressed' format, such as a JPEG. CS, as employed in MRI, utilises the fact that if the image one wishes to acquire exhibits sparsity and is therefore considered to be compressible, fewer data can be acquired in the first place from which the image can be reconstructed. The successful implementation of CS in MRI is based on the following requirements: (1) the aliasing artefacts arising from the under-sampling in the linear

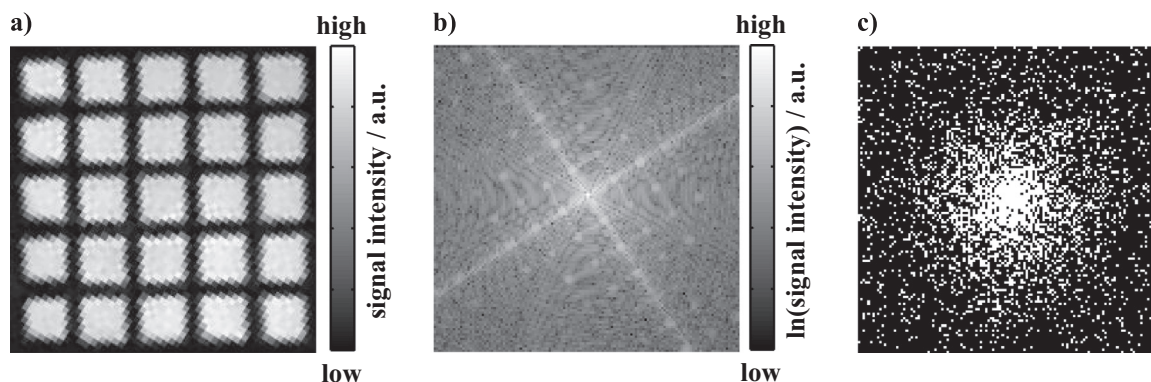


Fig. 1. a) Cross-sectional spin-density image of SF_6 inside the DPF sample, b) the corresponding fully-sampled raw data from which the image was reconstructed and c) a typical variable-density under-sampling pattern (20% of a 128×128 data matrix) used in the CS-MR acquisitions. The white pixels in the sampling pattern indicate the data to be acquired.

reconstruction must be incoherent and noise-like; (2) the desired image exhibits transform sparsity; and (3) the image is reconstructed using a non-linear algorithm that enforces sparsity and consistency with the acquired raw data (Lustig et al., 2007).

Let us consider the case where the image to be reconstructed is given by $\mathbf{x}$ which is related to the acquired data $\mathbf{y}$ via:

$$S\mathbf{F}\mathbf{x} + \nu = \mathbf{y}, \quad (1)$$

where S is the under-sampling pattern, $\mathbf{F}$ is the Fourier transform operator, ν is the normally-distributed noise (standard deviation σ and zero mean) and $\mathbf{y}$ is the vector that contains the acquired data.

To illustrate the under-sampling protocol that has been implemented in the present study, consider Fig. 1 which shows a spin-density image ($\mathbf{x}$) of SF_6 in a DPF substrate (Fig. 1a) and the fully-sampled raw data ($\mathbf{y}$) from which the image was reconstructed (Fig. 1b). For clarity, the natural logarithm of the signal intensity is shown.

From Fig. 1a, the structure of the DPF sample can clearly be seen with the bright (high signal intensity) regions corresponding to gas in the channels and the dark (low signal intensity) regions corresponding to the channel walls. To ensure that the *true* signal in the image can be recovered effectively in the CS reconstruction, it is important that the artefacts arising from sub-Nyquist rate sampling add incoherently, and appear noise-like, in the image domain (Lustig et al., 2007; Lustig et al., 2008). By sampling the raw data randomly, high levels of artefact incoherence in the linear transform can be achieved. However, considering the distribution of the signal (Fig. 1b), it has been shown that preferentially sampling the higher intensity coefficients towards the centre of two-dimensional matrix may be optimal. Therefore, in the present study the signal is sampled according to a pseudo-random, variable-density pattern sampling pattern weighted toward the centre of the data matrix. The methodology used for the generation of the under-sampling pattern in the present study is based on the Monte-Carlo incoherent sampling design approach developed by Lustig et al. (2007). A typical sampling pattern for 20% of a 128×128 data matrix is shown in Fig. 1c, where the white pixels represent the data that are sampled.

Due to under-sampling and the presence of noise, Eq. (1) is an ill-posed problem and therefore linear image reconstruction methods, such as the inverse Fourier transform, which is typically employed for the image reconstruction of fully-sampled data, will result in an image containing aliasing artefacts due to the violation of the Nyquist criterion. Therefore an approximate solution to $\mathbf{x}$ is sought by using a variational regularisation approach balancing the model and prior assumptions of $\mathbf{x}$ in terms of a regularisation functional J given by Eq. (2):

$$\begin{aligned} \mathbf{x}_\sigma &\in \arg \min_{\mathbf{x}} J(\mathbf{x}) \\ \text{subject to } \|S\mathbf{F}\mathbf{x} - \mathbf{y}\|_2 &\leq \sigma. \end{aligned} \quad (2)$$

The role of the inequality constraint enforces consistency with the

Table 1

Properties of the DPF substrate used in the present study.

Material	silicon carbide (SiC)
Filter channel length, L_c / mm	155
Channel hydraulic diameter, d_c / mm	1
Cell density/cells per square inch	300
Wall thickness, w_w / mm	0.2
Substrate porosity, ε / %	52 ± 4
Mean pore size, d_p / μm	23 ± 5

acquired data and the regularisation term J incorporates prior information on the reconstruction of $\mathbf{x}_\sigma$, which is required to counteract the ill-posedness of the problem. In the case of the CS reconstruction, the prior information is that the image can be sparsely represented either implicitly or in an appropriate transform domain. Typically, the choice of the regularisation functional that is used to map the image into the transform domain depends on the nature of image to be reconstructed. For instance, Total Variation (TV) may be more suited to an image with sharp-edges whereas the Daubechies wavelet transform lends itself well to images in which the pixel intensities change more gently (Benning et al., 2014). Due to the structure of the DPF filter with the well-defined edges of the channel walls, isotropic (direction-invariant) TV has been chosen as the regulariser due to its edge-preserving properties. The TV regulariser penalises the 1-norm of a discrete finite difference approximation of the two-dimensional gradient of the magnitude of the image data as follows,

$$J(\mathbf{x}) = \text{TV}(\mathbf{x}) = \|\nabla \mathbf{x}\|_{2,1} = \sum_{i,j \in N} \sqrt{|\nabla_1 \mathbf{x}(i,j)|^2 + |\nabla_2 \mathbf{x}(i,j)|^2}, \quad (3)$$

for $i = 1, \dots, n_1$ and $j = 1, \dots, n_2$ and $N = n_1 n_2$ which is the number of pixels in $\mathbf{x}$.

In the present work, the Tikhonov-Regularisation scheme with TV as the prior for the approximation of $\mathbf{x}_\alpha$ is used:

$$\mathbf{x}_\alpha \in \arg \min \left\{ \frac{1}{2} \|\mathbf{y} - S\mathbf{F}\mathbf{x}\|^2 + \alpha \text{TV}(\mathbf{x}) \right\}, \quad (4)$$

where the regularisation parameter α (always positive) weights the influence of the fidelity and regularisation terms in Eq. (4). The value of α used in the CS reconstructions depends on the scaling of the data. In the present study, a value of $\alpha=500$ was used for all reconstructions. The CS image reconstructions were carried out using an in-house Matlab toolbox, Object Oriented Mathematics for Inverse Problems (OOMFIP) for which the implementation is presented in Benning et al. (2014).

To demonstrate the implementation of CS in the present study, pseudo under-sampled data have been obtained from the element-by-element multiplication of the fully-sampled data (Fig. 1b) and sampling pattern (Fig. 1c). Fig. 2 shows a comparison of the spin-density images of SF_6 inside the DPF sample obtained from the Fourier transform

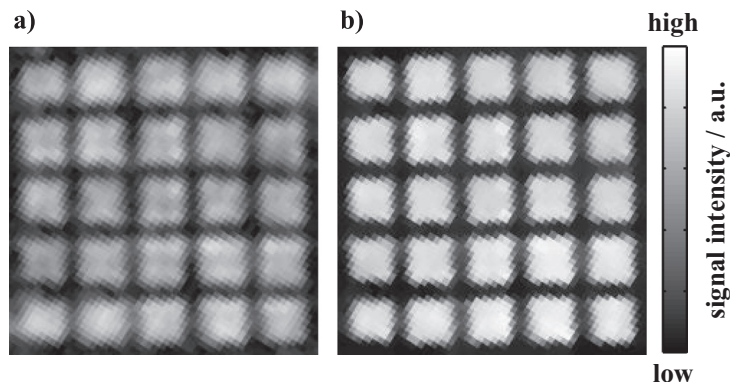


Fig. 2. Spin-density images of SF_6 inside the DPF sample obtained from the a) Fourier transform and b) CS reconstruction, of the pseudo under-sampled raw data. Only 20% of the fully-sampled raw data set was used in the reconstruction.

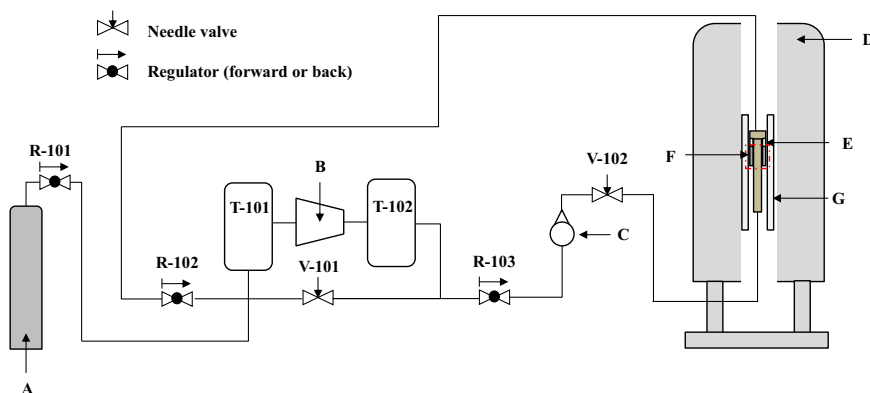


Fig. 3. Schematic of experimental setup: (A) SF_6 cylinder, (B) compressor, (C) rotameter, (D), superconducting magnet, (E) flow cell, (F) radio frequency coil, (G) magnetic field gradient coil. The red box indicates the imaging region at which the velocity measurements are acquired. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

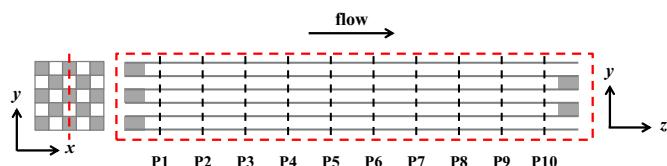


Fig. 4. Schematic of the filter substrate used in the experiments illustrating the axial (z) positions along the length of the channels at which the velocity images were acquired. The gas flow is in the positive z direction.

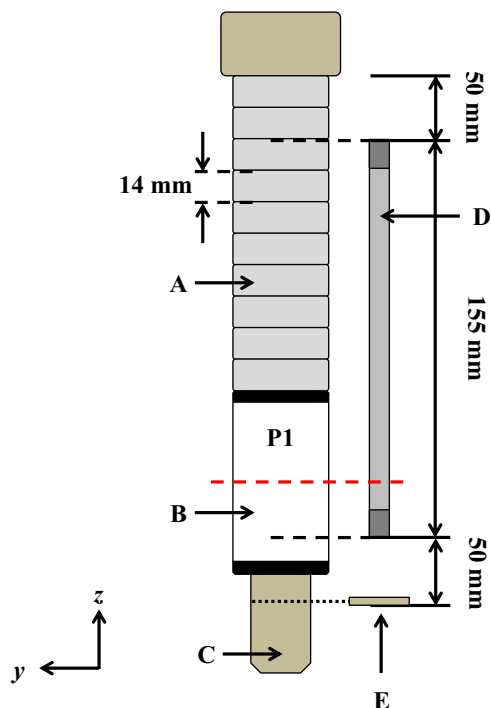


Fig. 5. Schematic diagram illustrating the method used to change the position of the DPF at the centre of the coil (indicated by the horizontal dotted line). (A) Sample spacer, (B) r.f. coil, (C) PEEK flow cell and (D) DPF substrate and (E) flow distributor. The gas exits the DPF sample through a void space of length 5 cm before leaving the flow cell. Note: drawing not to scale.

(Fig. 2a) and CS reconstruction (Fig. 2b) of the pseudo under-sampled data. From Fig. 2a it is clear that the zero-filled Fourier transform yields an image that appears to be blurred, however the structure of the DPF sample can still be resolved. This confirms that the sampling pattern is appropriate in that the information needed to reconstruct the image is acquired. The CS reconstruction (Fig. 2b) is much more

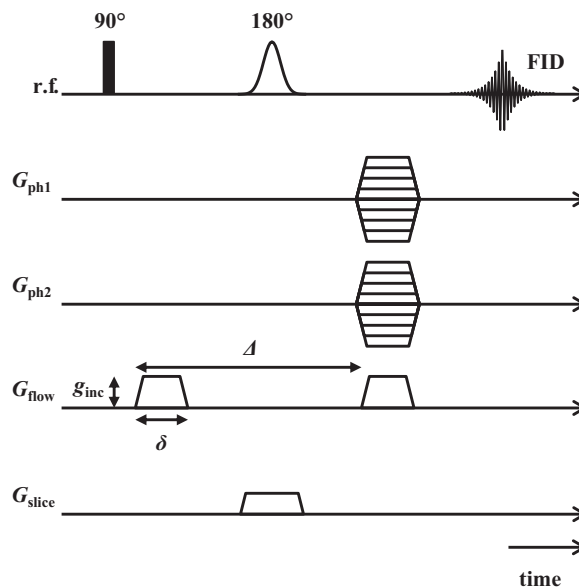


Fig. 6. The velocity encoded, slice selective spin echo single point imaging (SESP) sequence used for the acquisition of all velocity measurements. The line labelled r.f. indicates the actions performed by the radio frequency coil, *i.e.* excitation of the spins and detection of the Free Induction Decay (FID) signal. The lines labelled G_{ph1} , G_{ph2} and G_{slice} indicate the behaviour of the magnetic field gradients used for spatial-encoding and slice-selection, respectively. The line labelled G_{flow} indicates the actions performed by the magnetic field gradient used for motion encoding of the spins. The FID is the time-domain signal acquired which is the post-processed to yield the spatially-resolved velocity measurements.

visually similar to the fully-sampled image presented in Fig. 1 and the contrast between the channels and the walls has been restored, thus justifying the use of CS in this application.

3. Experimental

3.1. Materials and equipment

In the present study a 5×5 channel section of a silicon carbide (SiC) DPF substrate has been used and the relevant properties are given in Table 1. The DPF and sample holder were placed within a polyetheretherketone (PEEK) flow cell with an internal diameter of 18 mm which was designed to operate at pressures up to 900 kPa. As discussed previously, SF_6 has been used as the NMR-active gas in the present study. A schematic illustrating the closed circuit for the SF_6 gas is shown in Fig. 3. The SF_6 gas was supplied in a closed circuit consisting of a gas cylinder (BOC), high and low pressure storage

vessels, a DILO Piccolo compressor (model B022R01), upstream in-line pressure regulator, rotameter (Brooks Sho-rate 1357/D2B5D1B00000), back-pressure regulator and the PEEK flow cell holding the DPF sample.

The SF_6 from the cylinder was stepped down in pressure, from 2100 to 200 kPa, to fill the low-pressure receiver (T-101), and then passed into the compressor unit and back into the high-pressure receiver (T-102). The pressure in T-101 and T-102 was determined by the position of the needle valve on the bypass line between the two units (V-101). The gas pressure into the rotameter, which measured the flow rate, was held constant by the regulator (R-103) at 700 ± 20 kPa and the operating pressure for the system was held at 600 ± 20 kPa by the back-pressure regulator (R-102). The gas was then returned to T-101 and passed through the system in a continuous closed circuit.

A series of ten, evenly spaced velocity images were acquired at increments of 14 mm along the length of the DPF channel. A schematic illustrating the positions (P1-10) along the length of the DPF substrate where the velocity measurements were made is given in Fig. 4.

As the position of the radio frequency (r.f.) coil (*i.e.* the imaging region) is fixed within the superconducting magnet, the ability to image the velocity field at different z -positions along the DPF is achieved by moving the sample within the magnet. This has been achieved using the approach illustrated in Fig. 5. A series of spacers were made to fit between the collar of the PEEK flow cell and the top of the coil. The addition of each successive spacer changed the position of the DPF substrate at the centre of the coil (indicated by the horizontal dotted line), which is where the images were acquired.

The gas flow velocity measurements have been acquired at three different feed flow rates of SF_6 corresponding to expected average velocities of: 0.026 ± 0.003 , 0.065 ± 0.007 and 0.105 ± 0.012 m s^{-1} . The values of the velocity are based on the measured flow rate from the rotameter and the open area of the 5×5 channel section of the DPF substrate. The pressure, P , and temperature, T , at which the experiments were conducted were 600 ± 20 kPa and 23 ± 4 $^\circ\text{C}$, respectively. The uncertainty associated with the temperature represents the variation during the experiments due to heating of the gas upon compres-

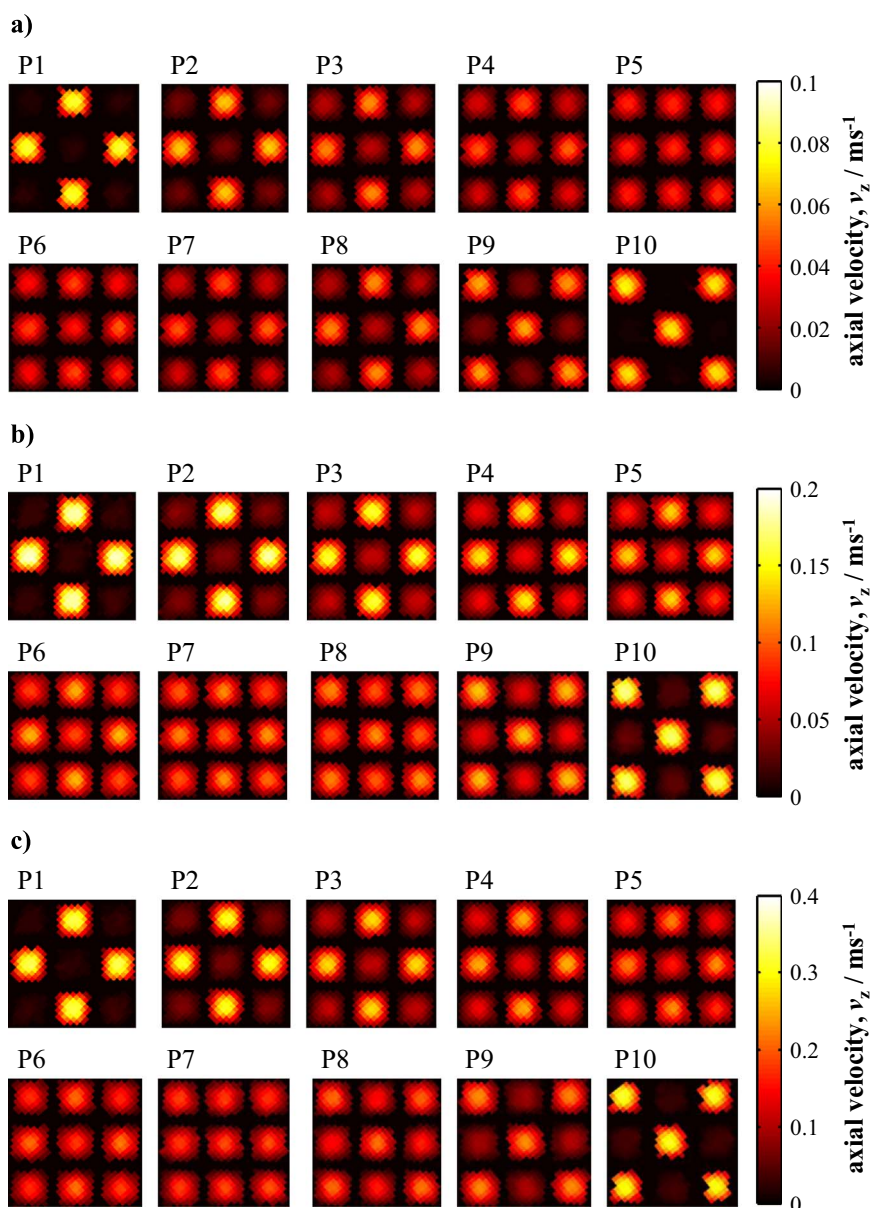


Fig. 7. Axial velocity (v_z) images of SF_6 gas flowing through the DPF substrate at gas flow rates corresponding to a) $\text{Re} = 106$, b) $\text{Re} = 254$ and c) $\text{Re} = 428$ in the inlet channels at P1. Images were acquired at 10 evenly spaced positions 14 mm apart along the length of the filter. The in-plane resolution was 0.14 mm pixel $^{-1}$ in both directions. The slice thickness was 12 mm.

sion. The gas density, ρ , and viscosity, μ , under these conditions were $35.7 \pm 1.3 \text{ kg m}^{-3}$ and $15.1 \pm 0.2 \times 10^{-6} \text{ Pa s}$, respectively.

3.2. Magnetic resonance imaging

All experiments were performed using a 9.4 T vertical bore magnet controlled by a Bruker AV-400 spectrometer operating at a ^{19}F resonance frequency of 376.6 MHz. A 25 mm diameter birdcage coil was used for r.f. excitation and signal detection. Spatial resolution was achieved with three orthogonal magnetic field gradients with a maximum strength of 146.1 G cm^{-1} .

For the velocity imaging, a slice-selective spin-echo single point imaging (SESPI) pulse sequence (Fig. 6) has been used (Petrov et al., 2011, Xiao and Balcom, 2012, Xiao and Balcom, 2013). This has been chosen for the freedom it allows in designing the data sampling patterns to maximise the incoherence, as is a requirement for the CS reconstructions (Lustig et al., 2007). The image was acquired with an isotropic field-of-view (FOV) of 18 mm and with an array of 128×128 pixels, this corresponds to an in-plane nominal spatial resolution of $0.14 \text{ mm pixel}^{-1}$ in both directions. Spatial encoding was determined by the two phase-encoding gradients (G_{ph1} and G_{ph2}) in accordance with a pre-determined sampling pattern (e.g. Fig. 1c). In the image acquisition, the raw data were under-sampled in both phase-encoding directions with only 20% of the total data points in the 128×128 data matrix (i.e. 3276 points) being acquired. Slice selection in the axial (z) direction was achieved using the 180° refocussing Gaussian-shaped r.f. pulse and slice gradient (G_{slice}). The image slice thickness was 12 mm.

Motion encoding was achieved by the two pulsed-field gradients (G_{flow}) either side of the 180° pulse using the phase-shift velocity imaging approach discussed in the supplementary material. The motion encoding parameters for the velocity measurements are as follows; $\delta = 0.74 \text{ ms}$ and $\Delta = 1.74 \text{ ms}$. For each MR velocity measurement, two images with different values of G_{flow} were acquired and g_{inc} was taken to be the difference between the two. The velocity range measured was adjusted for each gas flow rate by varying g_{inc} ($5\text{--}50 \text{ G cm}^{-1}$) accordingly. Each velocity image was corrected for inherent phase shifts caused by the gradients by acquiring each image under zero-flow conditions and then subtracting the phase map of the zero-flow measurement from the image acquired under flowing conditions. At each spatial position a free induction decay (FID) of 128 points was collected and averaged. With a recycle time of 0.032 s and four scans for the phase cycle and signal averaging, the total data acquisition time was approximately 14 min.

4. Results

The results are presented in two sections. In Section 4.1, the MR

velocity imaging data for the gas flow in the DPF are shown. For each of the flow rates studied two-dimensional images of the axial component of the velocity at ten equally-spaced positions along the length of the filter have been acquired. From these data, profiles of the axial velocity in a representative inlet and outlet channel unit have been obtained. In Section 4.2, the MR velocity imaging data reported in Section 4.1 have been used to calculate the through-wall velocity profiles and the uniformity of the through-wall distribution under the different flow conditions has been quantified.

4.1. Magnetic resonance velocity imaging

The series of axial velocity (v_z) images acquired at the ten evenly spaced positions along the length of the channel (Fig. 4) are shown for the low, medium and high flow conditions in Fig. 7(a)–(c) respectively. Each pixel is a local measurement of the axial component of the velocity vector (v_z). For each image, only the central 3×3 channel section from the full image has been shown and used for subsequent analysis as each of these individual channels are subject to the same boundary conditions in that each inlet and outlet channel is connected to four outlet and inlet channels, respectively. From the MR velocity images (Fig. 7), the average velocities for the low, medium and high flow conditions across the ten positions (P1–10) were determined to be: 0.022 ± 0.002 , 0.053 ± 0.003 and $0.093 \pm 0.005 \text{ m s}^{-1}$, respectively, where the uncertainty is a standard deviation of the ten measurements. Within their respective uncertainties, these values are in agreement with the expected average velocities stated previously (Section 3.1) for the low ($0.026 \pm 0.003 \text{ m s}^{-1}$), medium ($0.065 \pm 0.007 \text{ m s}^{-1}$) and high ($0.105 \pm 0.012 \text{ m s}^{-1}$) flow conditions. The Reynolds numbers $\text{Re} = \rho v_z d_c / \mu$ of the three flow conditions that have been investigated are: $\text{Re} = 106$, 254 and 426, based on the average axial velocity measured in the inlet channel at P1 and the properties of the SF_6 gas and DPF sample described in Section 3.1.

The two-dimensional velocity images in Fig. 7 clearly show the geometric structure of the DPF and how this influences the gas flow in these systems. Referring to the images at the front of the filter (P1) for each of the inlet conditions, the regions of higher velocity correspond to the inlet channels and the regions of lower velocity correspond to the outlet channels. It can be seen that the velocity in the inlet channel decreases, whilst increasing in the outlet channels as the gas progressively moves from the former into the latter along the length of the filter in the axial (z) direction. Finally, at the end of the filter (P10), it can be seen that the majority of the flow is now in the outlet channels. From the data shown in Fig. 7, the average axial velocity in the four inlet and five outlet channels has been determined at each position along the length of the filter and is shown in Fig. 8. As expected, the velocity in the inlet and outlet channels increase and decrease,

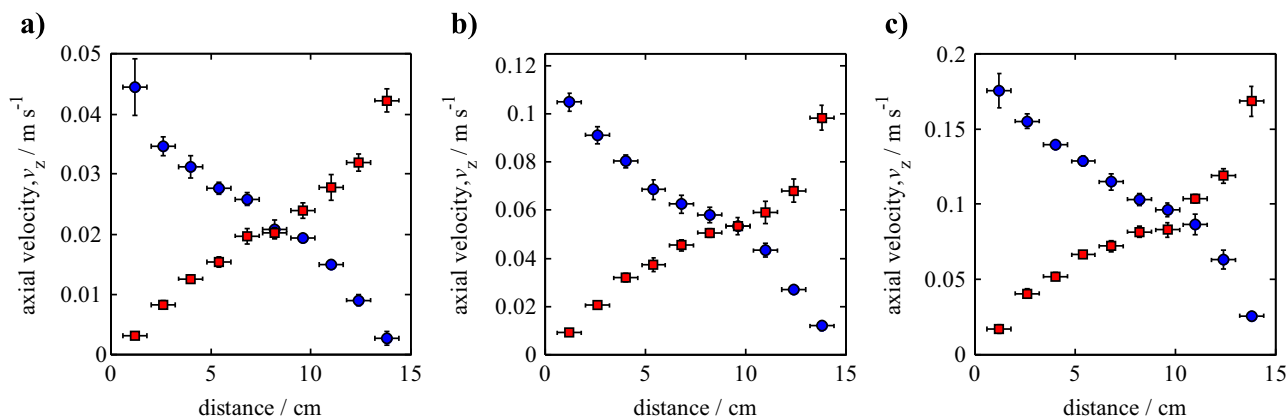


Fig. 8. Average axial velocity (v_z) in the inlet and outlet channels for the inlet conditions a) $\text{Re} = 106$, b) $\text{Re} = 254$ and c) $\text{Re} = 428$ at the 10 evenly spaced positions 14 mm apart along the length of the filter channel. The y-axis error bars correspond to the standard deviation of the velocity in the four inlet and five outlet channels and the x-axis error bars correspond to the image slice thickness.

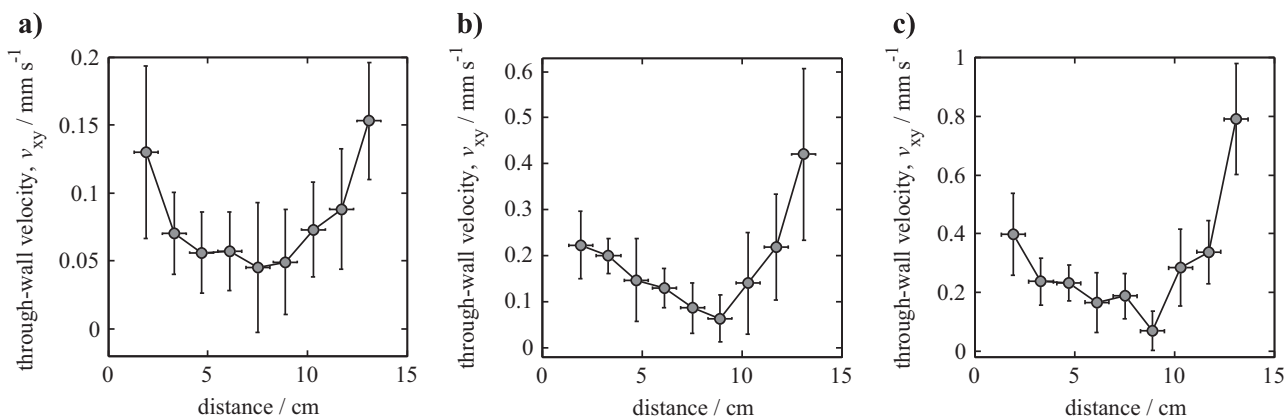


Fig. 9. Average through-wall velocity (v_{xy}) along the length of the channel for the inlet conditions a) $Re = 106$, b) $Re = 254$ and c) $Re = 428$. The y -axis error bars correspond to the standard deviation of the through-wall velocity determined from the change in volumetric flow rate in the four inlet and five outlet channels between successive spatial positions and the x -axis error bars correspond to the image slice thickness.

respectively, along the length of the filter in the direction of flow. However, it is interesting to note that these changes in velocity are not linear across the sample and this will be discussed later.

4.2. Uniformity of the through-wall velocity profile

The through-wall velocity profiles for the three flow conditions, calculated from the axial velocity profiles are shown in Fig. 9. At the flow rates used in the present study, it is expected that the pressure drop across the filter is small relative to the absolute pressure of the system. Thus it has been assumed that the gas density, and therefore volumetric flow rate, are constant along the length of the filter. Based on this assumption, the through-wall velocity profile has been obtained by calculating the average change in volumetric flow rate in the four inlet channels (Δq_{IC}) and five outlet channels (Δq_{OC}) between successive spatial positions divided by the surface area ($4d_c\Delta z$) of the channels between each measurement point, as given by Eq. (5):

$$v_{xy}(j) = \left(\frac{(\sum [\Delta q_{IC}(j)] + \sum [\Delta q_{OC}(j)])}{4N_c d_c \Delta z} \right) \quad (5)$$

where $j = 1, \dots, M$ and $M = 9$ i.e. the total number of increments between positions at which the axial velocity has been measured, Δz is the distance between successive measurements and N_c is the total number of inlet and outlet channels in the region of interest. Qualitatively, the through-wall velocity profiles for the three different inlet conditions show the same trend in that a minimum is observed around the central part of the filter with regions of higher through-wall flow toward the front and back. We note that while, in principle, it should be possible to measure the through-wall velocity it was not possible to do this in the current experiments because of the relatively low velocity of the gas flow through the wall and relatively short relaxation times of SF_6 at the experimental operating conditions (T_1 and $T_2 \approx 15$ ms at 600 kPa). The range of observation times permissible for motion encoding under the constraints of the relaxation times of the gas are insufficient to distinguish between the advective and diffusive motion of the gas.

As discussed previously, the distribution of the through-wall flow along the length of the channel is important due to the effect that it has on the PM deposition during operation. To this end, the uniformity of the through-wall velocity profile (Fig. 9) is now quantified in terms of the coefficient of variation (C_{var}). Herein, C_{var} is defined as the standard deviation (σ) divided by the average (μ) through-wall velocity (v_{xy}) along the length of the channel, for each of the flow conditions studied. The equation used to determine C_{var} is given by:

$$C_{var} = \frac{\sigma_{v_{xy}}}{\mu_{v_{xy}}} \quad (6)$$

The standard deviation (σ) and the average (μ) of the through-wall velocity were determined using Eqs. (7) and (8), respectively:

$$\sigma_{v_{xy}} = \sqrt{\frac{1}{N} \sum_{k=1}^9 [v_{xy}(k) - \mu_{v_{xy}}]^2}, \quad (7)$$

$$\mu_{v_{xy}} = \frac{1}{N} \sum_{k=1}^9 v_{xy}(k), \quad (8)$$

where $k = 1, \dots, N$ and $N = 9$ i.e. the total number of positions at which the through-wall flow has been calculated.

For comparison with the predictions of the numerical model of Yu et al. (2013a), the through-wall velocity profiles have been simulated using the expression stated therein for the same system parameters (Table 1) and experimental conditions used in the present study. From these simulated profiles, the uniformity of the through-wall velocity along the length of the filter channel is quantified in terms of the non-uniformity index (NUI), which is given by the following equation:

$$NUI = \frac{v_{xy,max} - v_{xy,min}}{v_{xy,ave}}, \quad (9)$$

where $v_{xy,max}$, $v_{xy,min}$ and $v_{xy,ave}$ are the maximum, minimum and average of the simulated through-wall velocity profiles as a function of position along the filter channels. Fig. 10 shows a direct comparison, in terms of the C_{var} and NUI , of the experimentally-determined through-wall velocity profiles (Fig. 9) and those predicted using the dimensionless

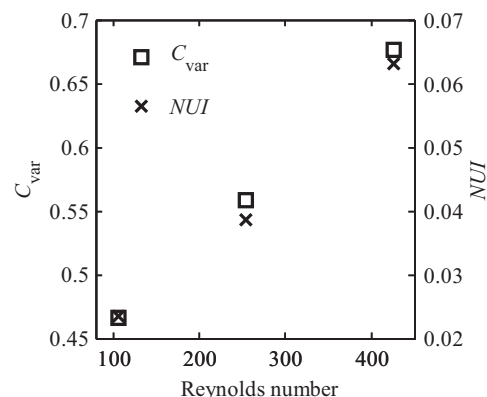


Fig. 10. Coefficient of variation (C_{var}) and non-uniformity index (NUI) as determined from the experimentally-determined and simulated through-wall velocity profiles, respectively, as a function of Reynolds number.

model of Yu et al. (2013a) for the same system parameters and experimental conditions used in the present study.

5. Discussion

Since the early work of Bissett (1984), there has been a considerable body of work in using numerical models to simulate the gas flow within DPFs. In particular, researchers have investigated how the exhaust gas flow influences physical processes taking place associated with DPFs such as PM filtration (Bensaid et al., 2009, 2010), deposition of diesel soot particles (Sbrizzai et al., 2005; Soldati et al., 2010; Zuccaro et al., 2011; Yu et al., 2013a, 2013b) and filter regeneration (Schejbal et al., 2009, 2010). It is common to all of these numerical studies that they rely on being able to accurately predict the velocity fields within these systems. The results that have been presented in Section 4 are the first direct measurements of the gas flow velocity inside the channels of a DPF. In this section, these experimentally-obtained velocity profiles will be analysed and a qualitative comparison with those published in the literature is made.

The observed non-linearity in the axial and through-wall velocity profiles (Figs. 8 and 9) are consistent with those published in the literature (Bensaid et al., 2009, 2010; Sbrizzai et al., 2005; Soldati et al., 2010; Zuccaro et al., 2011; Yu et al., 2013a). It has been suggested that the distribution of the through-wall flow along the length of the channel is determined by the pressure gradient across the channel wall at each axial position (Soldati et al., 2010; Zuccaro et al., 2011). The experimentally-observed channel-scale velocity profiles presented in Figs. 8 and 9 are consistent with these suggestions as will now be discussed.

At the front of the filter, the pressure gradient between the inlet and outlet channels is expected to be relatively high and, referring to Fig. 9, it can be seen that this corresponds to a region of relatively high through-wall flow, for each of the three feed flow rates that have been studied. The absolute pressure in both the inlet and outlet channels decreases as a result of friction along the length of the filter, but the extent to which this occurs depends on the velocity of the gas flow within the respective channels. Referring to Fig. 8, in the front portion of the filter, the axial velocity in the inlet channel is greater than in the outlet channel and therefore the absolute pressure in the former decreases at a faster rate than the latter. Consequently, the pressure gradient between the two channels progressively decreases along the length of the channel in the direction of flow thus resulting in a decrease in the through-wall flow. Referring to Figs. 8 and 9, for all three cases it can be seen that the through-wall flow decreases to a minimum which occurs at the point at which the axial velocity in the outlet channel is now greater than that in the inlet channel, as has been consistently predicted by the numerical studies discussed previously. Following the minimum, the absolute pressure in the outlet channel now decreases at a progressively faster rate than in the inlet channel. Correspondingly, this results in an increase in the pressure gradient between the channels, thus producing a higher through-wall velocity between the minima and the end of the filter, as has been observed in Fig. 9.

Figs. 8 and 9 show that the axial and through-wall flow fields for all three conditions studied exhibit the same general trend. However, it can be seen that the cross-over point in the inlet and outlet channel axial velocities and corresponding minima in the through-wall velocity occur further towards the back of the filter as Re increases. This in turn influences the distribution of the through-wall flow along the length of the filter, which has important implications for PM deposition and filtration processes during operation, as has been discussed previously. To investigate the effect that the gas flow rate has on the through-wall flow distribution, the uniformity of the experimental through-wall velocity profiles (Fig. 9) and those predicted using the computational model by Yu et al. (2013a) have been quantified in terms of the coefficient of the C_{var} (Eq. (6)) and NUI (Eq. (9)), respectively and these

data are shown in Fig. 10. From Fig. 10 it can be clearly seen that both metrics are in agreement and that the distribution of the through-wall flow becomes less uniform as the Re of the gas flow increases. These experimental observations therefore confirm the findings of the numerical modelling by Yu et al. (2013a) who have predicted that the distribution of the through-wall flow will be less uniform when the dynamic losses dominate over the frictional losses through the wall, which would be the case at higher gas flow rates. It should be noted that this validation of the NUI as predicted from numerical simulations has only been possible by having the ability to directly measure the gas flow fields within the channels of the DPF.

As discussed previously, this non-uniformity in the through-wall velocity profile will have implications for exhaust gases containing PM in the size range with a low Stokes number ($Stk \ll 1$). It has been predicted by the modelling studies of Sbrizzai et al. (2005) and Bensaid et al. (2009) that the particulates will follow the gas flow field almost exactly and therefore the resulting PM deposition profile is determined by the distribution of the through-wall flow. More specifically, it is expected that the PM deposition will be highest where the through-wall flow is the highest. Considering the results presented in Fig. 9, it is expected that as the flow rate of the gas increases, the PM would preferentially deposit towards the back of the filter thus resulting in a non-uniform distribution within the inlet channel. The application of the MR velocity imaging methods demonstrated herein to investigate the effect of PM deposition on the gas flow fields in real PM-loaded substrates will be the subject of future work.

6. Conclusions

In the present study, MR velocity imaging has been used to investigate non-invasively the gas flow in a DPF for the first time. Through the use of under-sampling data acquisitions and CS reconstructions, velocity images with an in-plane nominal spatial resolution of $140 \mu m \times 140 \mu m$ have been obtained, which is a higher spatial resolution than would have been possible with the standard MRI methods used previously, for the same acquisition time. The images of the axial velocity at ten evenly spaced positions along the length of the filter were acquired for three gas feed rates corresponding to $Re = 106$, 254 and 428 in the channels. From these data, the average axial velocity profiles of the inlet and outlet channels and through-wall velocity profiles have been calculated as a function of the position along the length of the filter. The gas flow fields within the channels of a DPF measured in this work are shown to be consistent with those that have been predicted by numerical simulations reported by other workers. More specifically, the results have confirmed the validity of the numerical studies that have suggested the variation in average velocity along the length of the filter are a consequence of the interplay between the gas flow rate and the structural properties of the filter substrate.

The MR velocity imaging measurements were used to quantify the distribution of the through-wall velocity, in terms of the coefficient of variation (C_{var}), along the length of the filter for the three flow conditions that have been investigated. From this it has been observed that as the gas flow rate increases, the through-wall velocity profile along the length of the filter became less uniform. This was in good agreement with flow uniformity indices presented in the literature, namely the non-uniformity index (NUI), predicted from numerical simulations by Yu et al. (2013a). Based on the assumption that low Stokes' number particulates will follow the streamlines of the gas flow directly, these measurements are important in predicting how the gas flow would influence PM deposition within the filter during operation.

The MR velocity imaging technique demonstrated herein should prove useful in providing new insight into the physical processes occurring within DPFs. The information obtained from these measurements will also be used for assisting in the development and validation of CFD models.

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Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at [doi:10.1016/j.ces.2016.10.017](https://doi.org/10.1016/j.ces.2016.10.017).

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