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Adaptive Block Compressive Sensing: Toward a Real-Time and Low-Complexity Implementation

JOSEPH ZAMMIT^{ID}, (Member, IEEE), AND IAN J. WASSELL^{ID}

Computer Laboratory, University of Cambridge, Cambridge CB3 0FD, U.K.

Corresponding author: Joseph Zammit (jz390@cl.cam.ac.uk)

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ABSTRACT Adaptive block-based compressive sensing (ABCS) algorithms are studied in the context of the practical realisation of compressive sensing on resource-constrained image and video sensing platforms that use single-pixel cameras, multi-pixel cameras or focal plane processing sensors. In this paper, we introduce two novel ABCS algorithms that are suitable for compressively sensing images or intra-coded video frames. Both use deterministic 2D-DCT dictionaries when sensing the images instead of random dictionaries. The first uses a low number of compressive measurements to compute the block boundary variation (BBV) around each image block, from which it estimates the number of 2D-DCT transform coefficients to measure from each block. The second uses a low number of DCT domain (DD) measurements to estimate the total number of transform coefficients to capture from each block. The two algorithms permit reconstruction in real time, averaging 8 ms and 26 ms for 256×256 and 512×512 greyscale images, respectively, using a simple inverse 2D-DCT operation without requiring GPU acceleration. Furthermore, we show that an iterative compressive sensing reconstruction algorithm (IDA), inspired by the denoising-based approximate message passing algorithm, can be used as a post-processing, quality enhancement technique. IDA trades off real-time operation to yield performance improvement over state-of-the-art GPU-assisted algorithms of 1.31 dB and 0.0152 in terms of PSNR and SSIM, respectively. It also exceeds the PSNR performance of a state-of-the-art deep neural network by 0.4 dB and SSIM by 0.0126.

INDEX TERMS Adaptive block compressive imaging, adaptive block compressive sensing, compressed sensing, deterministic sensing matrices, iterative reconstruction.

I. INTRODUCTION

Our interest in low-power, autonomous image and video sensors, for example, for use in wireless sensor networks [1], has led us to explore compressive sensing (CS) as a means of reducing complexity and power requirements at the sensor. In these scenarios, a large number of limited energy and storage encoders are usually deployed under the control of a relatively more complex decoder [2]. CS has been touted as an excellent data acquisition technique because it departs from the classical digital signal processing paradigm of sampling at the Nyquist frequency, source coding, channel coding, and transmission. In CS, the aim is to push computational complexity to the decoder, leading to simple encoders but complex reconstruction at the decoder [3]. When applied

to image acquisition, the disadvantages of current schemes are that they do not allow for real-time reconstruction of images or require specialized hardware, such as a graphical processing unit (GPU), which increases complexity such that they might not be feasible or economical in a wireless sensor network with a large number of nodes.

The theory of CS developed by Candès *et al.* [4] and Donoho [5], allows us to acquire a significantly lower number of measurements than classical theory suggests. The developed theory has been backed up by the development of the single-pixel camera for image and video signals [6]. CS theory has also been applied in multi-pixel cameras [7] and focal plane processing image sensors [8].

In CS, an unknown, sparse vector $\mathbf{f} \in \mathbb{R}^N$ is sensed by multiplying it with a measurement matrix $\Phi \in \mathbb{R}^{M \times N}$. The M resulting measurements comprise vector $\mathbf{y} \in \mathbb{R}^M$. In image and video compression, the signal of interest is

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sparse in a transform domain with transform Ψ , such that $\mathbf{y} = \Phi\Psi\mathbf{x}$ forms a set of underdetermined linear equations. In this case, $\mathbf{A} = \Phi\Psi$ is called the sensing matrix. CS theory shows that a sparse signal can be completely recovered from $M \ll N$ measurements, provided the sensing matrix satisfies some condition, such as the restricted isometry property (RIP) [9].

Reconstructing the image from the measurements requires the solution of $\mathbf{y} = \mathbf{A}\mathbf{x}$. The direct approach is to solve the underdetermined system using L_0 minimization, which is, however, NP-hard. L_1 and total variation (TV) minimization recast the problem as a linear programming (LP) problem, which can be practically solved using state-of-the-art LP solvers. This, in itself, is nontrivial, and a significant number of reconstruction techniques have been proposed to accelerate the solution, such as matching pursuit [10], Bayesian [11], approximate message passing (AMP) [12], and recently denoising AMP (D-AMP) [13] and neural networks [14].

In image and video applications, the difficulty in collecting and reconstructing measurements is compounded by the large dimensions of the images (or frames in video). Therefore, block-based CS (BCS) has been proposed to ease the problem [15]–[17]. The image is partitioned into $B \times B$ pixel blocks, where B is significantly smaller than the height H or width W of the image. The size of the measurement matrix is substantially smaller and hence easier to store. Computing the measurements requires inner products of size $B \times B$ rather than $H \times W$ and hence much more energy efficient, and each block can be measured independently, for example, using a multi-pixel camera, thereby accelerating the measurement and reconstruction of the image. These benefits come at the expense of reconstruction quality, for example, as measured using the peak signal-to-noise ratio (PSNR) and structural similarity index measure (SSIM) [18].

Another problem with BCS is that whereas an image is always highly sparse in a transform domain, this is not always the case with the sub-image blocks. Since the sparsity varies significantly, either the compression ratio $C_R = M/N$ is compromised or some of the blocks are not reconstructed successfully when Gaussian measurement matrices are employed. This is not the case when the block measurement matrix comprises deterministic 2D-DCT basis functions in zigzag order. Although the ensuing measurement matrix is not RIP compliant, the compressed sensing measurements can always recover the image blocks, trivially in tens of milliseconds, using the inverse 2D-DCT, because the location of the transform coefficients is known. We refer to compressed sensing using low-pass, 2D-DCT matrices as the L-DCT-ZZ algorithm.

A number of authors have proposed adapting the number of measurements m per block, depending on the sparsity of the sub-image blocks, to solve some of the problems of BCS and increase reconstruction quality. Their work is summarized below. The L-DCT-ZZ algorithm can also benefit

from adapting m block by block. Specifically, in this paper, we propose two adaptive algorithms to estimate sparsity - block boundary variation (BBV) and DCT domain (DD) - that can be applied to compressive image sensing.

A. CONTRIBUTIONS

The main contribution in this paper is the development of two novel adaptive block compressive sensing (ABCS) algorithms that employ the deterministic partial, low-pass, 2D-DCT sensing matrix, namely, (i) adaptive linear block boundary variation estimation (AL-DCT-BBV), and (ii) adaptive linear 2D-DCT with DCT Domain estimation (AL-DCT-DD). These algorithms estimate block sparsity in the 2D-spatial and 2D-DCT transform domains and vary the number of compressive measurements in each block, respectively, as shown in figure 1. The number of measurements in each block is increased when the sparsity is low (where low sparsity means that the block has more non-zero transform coefficients in the 2D-DCT domain).

Extensive MATLAB simulations show that our algorithms achieve state-of-the-art performance amongst CPU reconstructed, real-time algorithms that can be reconstructed from measurements collected from single-pixel cameras, multi-pixel cameras and focal plane processing image sensors. These algorithms are reconstructed using one iteration of the inverse 2D DCT transform, in under 8 ms and 30 ms for 256×256 and 512×512 images, respectively. Another advantage is that the 2D-IDCT will always reconstruct an image block correctly. When CS is used to reconstruct blocks sensed using random measurement matrices, successful reconstruction is probabilistic, and the failure probability increases as the blocks become less sparse.

Our second contribution is the iterative denoising algorithm (IDA), derived from the denoising approximate message passing (D-AMP) algorithm [13]. IDA can be used as a post-processing step to real-time AL-DCT-BBV and AL-DCT-DD algorithms, as shown in figure 1, albeit with a large decoding time penalty. However, IDA reconstruction achieves state-of-the-art (SOTA) PSNR and SSIM results amongst ABCS results published in the literature.

B. STRUCTURE OF THE PAPER

section II discusses related work in the field of ABCS. section III introduces CS theory and reconstruction algorithms. Non-adaptive deterministic measurement algorithms are introduced in section IV. The AL-DCT-BBV and AL-DCT-DD algorithms are introduced and analysed in section V, with the post-processing IDA algorithm presented in section VI. The algorithms are investigated empirically in section VII, first comparing them with other adaptive CS algorithms, then with two state-of-the-art CS algorithms proposed in the literature: the GPU-assisted CREAM [19] and the deep neural network (DNN) BCS-Net [20]. Section VIII draws conclusions from the study.

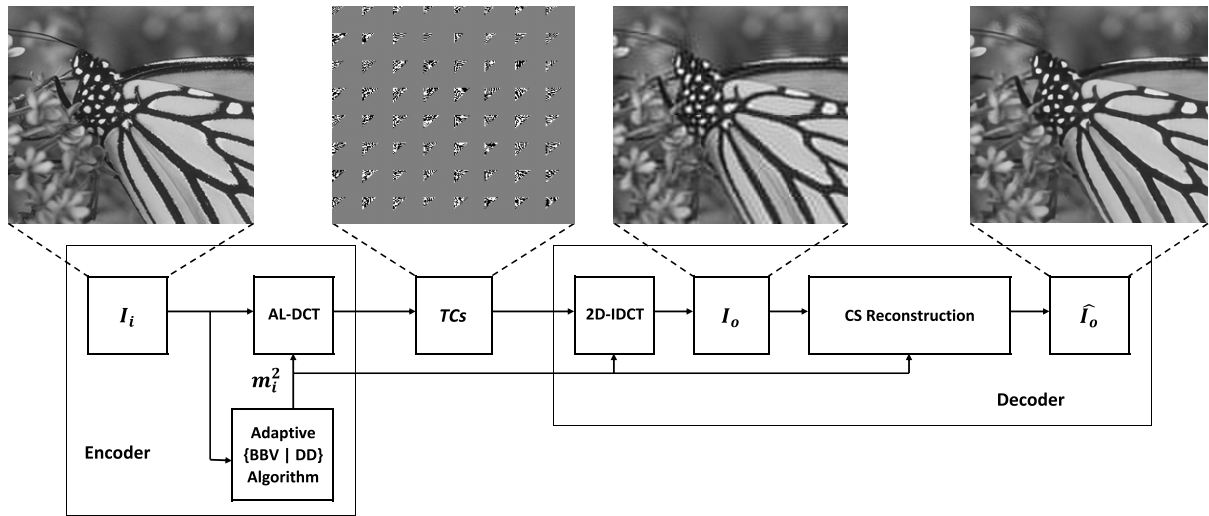


FIGURE 1. Adaptive AL-DCT-BBV and AL-DCT-DD system architecture. I_i is the input image, I_o is the image output in real time by the inverse 2D-DCT and $\hat{I}_o$ is the higher quality image output by the CS reconstruction, for example, using the IDA algorithm. AL-DCT acquires m_i^2 adaptive CS measurements per block where each measurement is a 2D-DCT transform coefficient (TC).

C. NOTATION USED IN THIS PAPER

Matrices are designated by bold capital letters, with vectors in bold lower case. Scalars are represented by normal letters, both upper and lower case, sometimes accompanied by subscripts. The superscript i designates the value of vectors or scalars in the i th iteration. Subscripts also designate the scalar components of vectors and matrices.

The non-adaptive linear L-DCT algorithm with low-pass transform coefficients collected in JPEG zigzag style is denoted L-DCT-ZZ. The adaptive versions of this algorithm are termed AL-DCT-BBV and AL-DCT-DD, with block boundary variation and DCT domain transform coefficient estimation, respectively. If the adaptive algorithms are further processed by the IDA reconstruction algorithm, we add the hyphenated suffixes -IDA-DnCNN or -IDA-BM3D to show whether the IDA algorithm uses the DnCNN [21] or BM3D [22] denoiser.

II. RELATED WORK

Image BCS was first proposed by Lu Gan [15]. In this paper, the author proposed the development of spatially adaptive reconstruction algorithms as further work. Other seminal work on BCS of images and video was conducted by Fowler, Mun, Chen and Tramel [16], [17], [23], [24].

The non-adaptive, linear 2D-DCT CS technique, referred to here as L-DCT-ZZ, was inspired by the work of Romberg [25], who observed that CS of images is difficult and that a number of low-pass, linear 2D-DCT transform measurements, acquired in JPEG zigzag order [26], aid reconstruction significantly. In [25], Romberg compared low-pass, linear DCT acquisition in JPEG zigzag order (referred to in this paper as L-DCT-ZZ) with a scheme that combines some linear DCT measurements with noiselet measurements and concluded that the latter achieves better results.

Recently, Yuan and Haimi-Cohen showed that L-DCT-ZZ can be reconstructed using CS reconstruction techniques, referred to as the compressive sensing-based image compression system (CSbIC) in [27], and can achieve better SSIM results when compared with JPEG, especially at high compression ratios, though PSNR was reduced.

A significant body of literature has concentrated on the development of ABCS for images. Several authors estimate the number of measurements per block from measurements captured at the encoder. We refer to these as encoder-side (ES) techniques.

In some encoder-side papers, it is assumed that a full set of image pixels are already available to the encoder prior to compressive sensing and are used to perform the classification task. We refer to these as encoder-side full sensing (ES-FS) techniques. They are not feasible because in CS, it is assumed that capturing a few measurements from which we infer the pixel values is preferable to capturing the pixels themselves. We include these ES-FS schemes as benchmarks. Due to their inherent advantage in assuming that all pixels and/or transform coefficients are available before an image is adaptively sensed, they should achieve optimal results. A significant number of authors have published ES-FS techniques [28], [29] [30], [31] [32], [33] [34] and [35].

The BBV and DD algorithms proposed in this paper do not assume that a full set of pixels or transform measurements is available and are referred to as encoder-side compressive sensing (ES-CS) techniques. Other authors have proposed ES-CS schemes, such as [36], [37] [38] and [39].

Averbuch *et al.* proposed a departure from using random dictionaries that are incoherent with the sparse basis as used in classical CS [37]. Their dictionaries, instead, are nonrandom transform coefficient measurements from the sparsifying basis, for example, wavelets. They showed that these

nonrandom measurements can still be captured on a DMD. This scheme is not block-based, however.

An alternative to ES-CS is to first transmit a reduced resolution image using normal BCS techniques, reconstruct it at the decoder, perform the analysis there, and feed back the substrate information to the encoder. We refer to these as decoder-side compressive sensing (DS-CS) techniques. [36], [40] and [41] proposed DS-CS ABCS techniques.

Several encoder-side papers ignore the measurements required to classify blocks prior to allocating the substrate to each block. Others include these measurements as part of the overall compressive measurements. Our BBV and DD algorithms include these estimation measurements as part of the block measurement allocation.

This paper uses SOTA reconstruction algorithms to increase PSNR and SSIM quality, with a reasonable reconstruction time. Donoho, Maleki and Montanari developed message passing algorithms for compressed sensing [12] with reduced reconstruction time. This led to the work on D-AMP by Metzler [13], [42], using block-matching and 3D filtering (BM3D) and DnCNN denoisers. The BM3D and DnCNN denoisers have also been used in recent work on image CS reconstruction using the plug-and-play method [43], [44] [45].

III. COMPRESSIVE IMAGE SENSING AND RECONSTRUCTION

A. COMPRESSIVE IMAGE SENSING

In sensing applications, it is often the case that information needs to be extracted from a set of measured data points. Taking $\mathbf{f} \in \mathbb{R}^N$ as the signal of interest and $\mathbf{y} \in \mathbb{R}^M$ as the measured signal, the relationship between the two can be written as:

$$\mathbf{y} = \Phi \mathbf{f} \quad (1)$$

where $\Phi \in \mathbb{R}^{M \times N}$ is the measurement matrix sampling signal $\mathbf{f}$ to give $\mathbf{y}$. If linear measurements of $\mathbf{f}$ are taken, then the reconstruction problem is reduced to solving a set of linear equations. The classical Nyquist-Shannon sampling theorem sets the sampling rate to twice the highest frequency for $\mathbf{f}$ to be reconstructed.

In a CS framework, the signal of interest $\mathbf{f}$ has a discrete representation $\mathbf{x} \in \mathbb{R}^N$ in some transform domain with basis $\Psi \in \mathbb{R}^{N \times N}$ such that $\mathbf{f} = \Psi \mathbf{x}$. Vector $\mathbf{x}$ is defined as sparse if it contains S non-zero elements such that $S \ll N$. The measured signal $\mathbf{y}$ is therefore given by:

$$\mathbf{y} = \Phi \Psi \mathbf{x} \quad (2)$$

Grouping the measurement matrix and sparsifying basis matrix gives:

$$\mathbf{y} = \mathbf{A} \mathbf{x} \quad (3)$$

where $\mathbf{A} = \Phi \Psi \in \mathbb{R}^{M \times N}$ is defined as the sensing matrix. The goal in CS is thus to represent the original signal using $M < N$ samples, substantially below the Nyquist rate. As the

locations of the S sparse non-zero values of $\mathbf{x}$ are not known in advance, reconstruction involves solving the undetermined set of equations represented by (3) [46].

Two main questions arise from equation (3): (i) How should the measurement and sensing matrices be chosen; and (ii) how does one solve for $\mathbf{x}$? As the solution of (3) involves locating the non-zero components in vector $\mathbf{x}$, the most natural approach is an L_0 minimization, which can be written as:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_0 \quad s.t. \mathbf{A} \mathbf{x} = \mathbf{y} \quad (4)$$

As L_0 minimization is NP-hard and thus computationally intractable, a common approach is to rewrite the problem as an L_1 minimization:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad s.t. \mathbf{A} \mathbf{x} = \mathbf{y} \quad (5)$$

Solving equation (5) can now be achieved via linear programming (LP), which is tractable [46]. This is commonly referred to as basis pursuit [46]. Faster reconstruction methods have been proposed in the literature, such as matching pursuit [10], approximate message passing (AMP) [12], denoising AMP (D-AMP) [13] and neural networks [14].

In real-world applications, noise is always present, and rewriting (3) to take this into account gives:

$$\mathbf{y} = \mathbf{A} \mathbf{x} + \mathbf{n} \quad (6)$$

where $\mathbf{n} \in \mathbb{R}^M$ represents Gaussian noise. The convex relation in (5) does not hold in this instance. However, it can be replaced by basis pursuit denoising (BPDN) [46] giving:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad s.t. \|\mathbf{A} \mathbf{x} - \mathbf{y}\|_2^2 < \epsilon \quad (7)$$

where ϵ depends on $\mathbf{n}$.

An important feature of the sensing matrix is that it should retain the information pertinent to the sparse signal $\mathbf{x}$. There are several conditions that can determine whether a sensing matrix possesses this feature. Three well-known properties are (i) the restricted isometry property (RIP), (ii) the null space property, and (iii) mutual coherence. The sensing matrix that satisfies any of these three properties possesses the required information preservation feature [46].

It has been shown that when $\mathbf{A}$ is a random Gaussian matrix, it satisfies the RIP property [46] with high probability. Gaussian matrices are often used as the measurement matrices in the image CS literature. When the sensing matrix Φ is Gaussian, the resulting sensing matrix $\mathbf{A} = \Phi \Psi$ is also Gaussian (because of the affine property of multivariate distributions) and benefits from satisfying the RIP with high probability. Simpler matrices, such as random (scrambled) Hadamard matrices, are also RIP compliant (studied as structured random sampling matrices in [46]) and have added implementation advantages, especially on DMD cameras [6]. However, optimal performance can be obtained using Gaussian or Bernoulli i.i.d matrices [4].

B. RECONSTRUCTION ALGORITHMS

Many compressive sensing reconstruction algorithms exist to recover sparse signals from compressive samples. Recently, Pilastrri and Tavares presented a taxonomy of reconstruction algorithms in compressive sensing [47]. In this taxonomy, the algorithms are grouped into six clusters: convex relaxation, greedy, non-convex, iterative, Bregman iterative, and combinatorial.

Reconstruction algorithms are characterized by their inherent complexity and the minimum number of compressive samples from which they can reliably recover the sparse coefficients and hence reconstruct the signal.

In image and video coding, we encounter signals whose transform in some domain is only approximately sparse; hence, we need reconstruction algorithms that can recover the predominant S sparse components in the presence of “noise” consisting of a long tail of small, non-zero components. If the sorted coefficients x_i decay with a power law, such that:

$$|x_i| \leq c \cdot i^{-p} \quad (8)$$

where c is a constant and $p \geq 1$; then, x_i is called p -compressible and can be approximated by a sparse signal (definition 2.2 in [46]). The transform coefficients of the 2D image and video data fit this model.

1) APPROXIMATE MESSAGE PASSING

In 2009, Donoho *et al.* [12] proposed approximate message passing (AMP) as a faster alternative to LP as a reconstruction technique for CS. AMP adds a correction term to iterative soft-thresholding (ISTA) [48] to vastly improve convergence.

In ISTA, the inverse noisy problem $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$ is solved for $\mathbf{x}$ iteratively. Let $\mathbf{A}$ be the $M \times N$ sensing matrix with $M < N$, $\mathbf{y}$ a vector with M measurements, and $\mathbf{x}^t \in \mathbb{R}^N$ the current estimate of the sparse solution. Then, starting from an initial guess $\mathbf{x}^0 = 0$, $\mathbf{x}^t$ is computed by solving equations (9) and (10) iteratively:

$$\mathbf{x}^{t+1} = \eta_t(\mathbf{A}^* \mathbf{z}^t + \mathbf{x}^t) \quad (9)$$

$$\mathbf{z}^t = \mathbf{y} - \mathbf{A}\mathbf{x}^t \quad (10)$$

where $\mathbf{z}^t$ is the current error, $\mathbf{A}^*$ is the transpose of $\mathbf{A}$, and η_t is a scalar threshold function that is applied component-wise. This simple iterative scheme can be applied to larger-scale applications than standard LP solvers because of the very low cost per iteration and low storage requirements [12].

It is shown that adding the Onsager correction term $1/\delta \mathbf{z}^{t-1} \langle \eta'_{t-1}(\mathbf{A}^* \mathbf{z}^{t-1} + \mathbf{x}^{t-1}) \rangle$ to (10), derived from belief propagation in graphical models, improves the convergence of the ISTA solution. The operator $\langle \mathbf{u} \rangle$ performs the component-wise mean of the vector $\mathbf{u}$, that is, $\langle \mathbf{u} \rangle \equiv \sum_{i=1}^N u_i/N$, η'_t is the derivative of η_t and $\delta = M/N$ (equivalent to C_R in this paper).

The AMP algorithm then finds $\mathbf{x}$ iteratively by solving (11) and (12) below:

$$\mathbf{x}^{t+1} = \eta_t(\mathbf{A}^* \mathbf{z}^t + \mathbf{x}^t) \quad (11)$$

$$\mathbf{z}^t = \mathbf{y} - \mathbf{A}\mathbf{x}^t + \frac{1}{\delta} \mathbf{z}^{t-1} \langle \eta'_{t-1}(\mathbf{A}^* \mathbf{z}^{t-1} + \mathbf{x}^{t-1}) \rangle \quad (12)$$

The authors further show that the AMP solution has the same theoretical sparsity-undersampling trade-off as the LP-based reconstruction.

2) DENOISING AMP

Metzler *et al.* [13] extended the AMP framework by replacing the thresholding nonlinearity η_t with a denoiser D_σ , which can be applied to reduce the Gaussian noise in the estimate $\mathbf{x}_0 + \sigma \mathbf{z}$, where $\mathbf{z} = \mathbf{N}(\mathbf{0}, \mathbf{I})$ is Gaussian with zero mean and unit variance. They do this to leverage the rich literature on image denoisers, which has led to sophisticated algorithms that can remove Gaussian noise with large variance σ^2 from images. Their D-AMP iterations are given by equations (13) to (15):

$$\mathbf{x}^{t+1} = D_{\hat{\sigma}^t}(\mathbf{x}^t + \mathbf{A}^* \mathbf{z}^t) \quad (13)$$

$$\mathbf{z}^t = \mathbf{y} - \mathbf{A}\mathbf{x}^t + \frac{1}{\delta} \mathbf{z}^{t-1} \frac{\text{div}[D_{\hat{\sigma}^{t-1}}(\mathbf{x}^{t-1} + \mathbf{A}^* \mathbf{z}^{t-1})]}{m} \quad (14)$$

$$(\hat{\sigma}^t)^2 = \frac{\|\mathbf{z}^t\|_2^2}{m} \quad (15)$$

where $\text{div}[D(\mathbf{x})] = \sum_{i=1}^n \frac{\partial D(\mathbf{x})}{\partial x_i}$, x_i is the i^{th} element of $\mathbf{x}$, n is the size of $\mathbf{x}$, and m is the number of measurements.

The authors claimed that “D-AMP offered state-of-the-art CS recovery performance while operating tens of times faster than competing methods”. Indeed, AMP combined with a BM3D [22] denoiser recovers a 128×128 pixel image, with close to state-of-the-art PSNR, in tens of seconds at a sampling rate of 10%. Other algorithms have since used the GPU to improve and accelerate reconstruction, such as CREAM [19] and BCS-Net [20]. D-AMP has also been upgraded to use the GPU-accelerated DnCNN denoiser by Metzler *et al.* [42].

IV. IMAGE BCS USING DETERMINISTIC 2D-DCT SENSING MATRICES

In this paper, we focus on the use of deterministic, linear 2D transform sensing matrices such as 2D-DCT. The 2D discrete Walsh-Hadamard transform can also be used and has a simpler implementation because the measurement matrix consists of $\{+1/-1\}$ elements. However, the PSNR and SSIM performance yielded by our simulations is inferior to 2D-DCT and is not reported here.

To accomplish compressive sensing, capitalizing on the energy compaction property of the 2D transform, M low-pass transform coefficients are sensed and sorted in increasing order of frequency using the zigzag scanning procedure present in JPEG [26]. We refer to this as the L-DCT-ZZ algorithm.

If the objective is not compressive sensing but compression, all the transform coefficients may be collected (full sensing), and then only those exceeding some threshold are retained. It is possible to set one threshold for the whole image (THI) or one threshold per block (THB). In the

first case, the number of retained transform coefficients per block varies block by block so that the THI algorithm is inherently adaptive. The number of retained transform coefficients in the THB algorithm is the same in each block, but the positions of the transform coefficients in the DCT domain are not known. The THI algorithm sets an upper bound to what can be achieved using adaptive 2D-DCT techniques. These two algorithms are defined formally in supplementary section III [49].

The advantage of deterministic sensing in L-DCT-ZZ is that the signals can be decoded very rapidly at the decoder, such that the collected images or frames can be used in real-time image and video transmission, for example, or to decrease the measurement time in magnetic resonance imaging (MRI). Block-based L-DCT-ZZ, similar to other block-based image compression algorithms, suffers from blocking effects at $C_R \leq 0.2$. Although the adaptive algorithms described below were found to attenuate blocking effects caused by the block-based measurements, some blocking is still visible when $C_R \leq 0.2$. Blocking can be removed in real time using the zigzag scan filter (ZZF) and thresholding filter (THF) described in supplementary section VI [49]. Better quality deblocking and image quality enhancement can be achieved using the CS reconstruction algorithms proposed and is studied in this paper, derived from Metzler's D-AMP algorithm [13].

V. ADAPTIVE IMAGE BCS

The non-adaptive L-DCT-ZZ algorithm measures a fixed number of transform coefficients per block. In the adaptive schemes, the number of transform coefficients collected varies according to the sparsity of each block. Figure 1 shows a block diagram of the proposed adaptive system.

Two techniques for adapting the number of linear 2D-DCT transform coefficients in block compressive sensing are proposed, one in the spatial domain and the other in the 2D-DCT transform domain. Both techniques use two phases to encode an image adaptively. In the first phase, m_i^1 measurements are collected from each block i and used to determine the number of additional transform coefficients, m_i^2 , to be collected from all blocks in the second phase.

In the case of the DD technique, the m_i^1 measurements are collected as the DC and low-frequency components, as shown in figure 2, and determine the next m_i^2 measurements to be collected in phase 2.

All the measurements collected from both phases are considered for reconstruction purposes and hence the total number of measurements collected $M = \sum_{i=1}^{n_B} (m_i^1 + m_i^2)$, where n_B is the number of blocks in the image.

The methods used to estimate m_i^2 must not be computationally intensive because they are implemented at the encoder side, and a target application for the methods described in this paper is for autonomously powered wireless image and video sensors. However, it is also possible to transmit the m_i^1 measurements to a decoder and use more elaborate algorithms at the decoder to estimate m_i^2 per block. This information

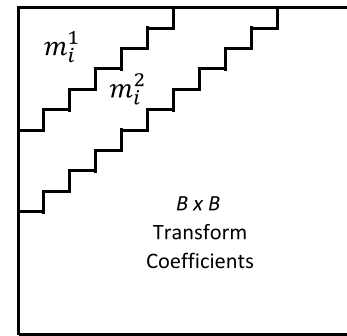


FIGURE 2. Adaptive L-DCT-DD phase 1 and 2 measurements.

is then fed back to the encoder as has been proposed in the literature [36], [40], [41]. In power-constrained applications, it is necessary to consider whether the power required to operate a receiver module to receive the m_i^2 feedback information is greater than the power required to just compute m_i^2 at the encoder. The requirement to wait for the reception of feedback information will also add to the reconstruction time of the image, although the increase may be contained if the sensing node is close to the receiver and the data rate of the link is sufficiently high.

Figure 1 shows the block AL-DCT measurements reconstructed in real time at the decoder using an inverse 2D-DCT transform and reconstituted into the output image I_o . The number of phase-two transform coefficients, m_i^2 , is transmitted as side information. If better reconstruction quality is required, full-image CS reconstruction is invoked to output image $\hat{I}_o$, albeit with a large reconstruction time penalty.

A. ADAPTIVE L-DCT-ZZ USING BLOCK BOUNDARY VARIATION

The adaptive L-DCT-ZZ algorithm using BBV, AL-DCT-BBV, is described in Algorithm 1. The idea behind this technique is to adapt the number of transform coefficients collected in proportion to the total variation (TV) of pixel values around the perimeter of a block. It is inspired by the observation that if a block includes significant texture features, then these will likely cross-block boundaries and can be detected by measuring the TV at the block border. Since the number of pixels in the boundary is only $4B$, determining m_i^2 from just these pixels is more efficient than having to determine it from all B^2 pixels ($4B$ is the boundary constructed from efficient adjacent block measurements, as depicted in figure 3, it is not the $(4B - 4)$ boundary of a block in isolation). When the compression factor $C_F = N/M$ in the block is high, the number of transform coefficients that can be collected is low, and the measurements required to calculate the BBV become a significant fraction of the whole budget for the block.

The measurement of the BBV can either be treated as a necessary overhead, or it is accounted for by reducing the number of transform coefficients that can be collected. This paper takes the latter view and attempts to minimize

Algorithm 1 AL-DCT-BBV

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- Input:** Image $\mathcal{I}$, compression factor C_F , block size B ,
Reconstruction Algorithm =
{IDCT2|D-AMP|DAMP-D|IDA }.
- Output:** Image $\mathcal{O}$ sensed using AL-DCT-BBV and
reconstructed using a reconstruction algorithm.
- 1 Crop and partition image, $\mathcal{I}$ into $n_B = \lfloor H/B \rfloor \cdot \lfloor W/B \rfloor$,
 $B \times B$ blocks;
 - 2 Collect $m_i^1 = 2 \cdot n_S$ samples per block where
 $n_S = \lfloor B/C_F \rfloor$ measurements are equally spaced in the
top and left-hand block border;
 - 3 Estimate the number m_i^2 of additional samples to collect
from each block i where m_i^2 is given by equation (22);
 - 4 Collect the m_i^2 transform coefficients from each block i ;
 - 5 Transmit $M = \sum_{i=1}^{n_B} (m_i^1 + m_i^2)$ measurements;
 - 6 Reconstruct $\mathcal{O}$ from the M received samples using the
selected reconstruction algorithm.
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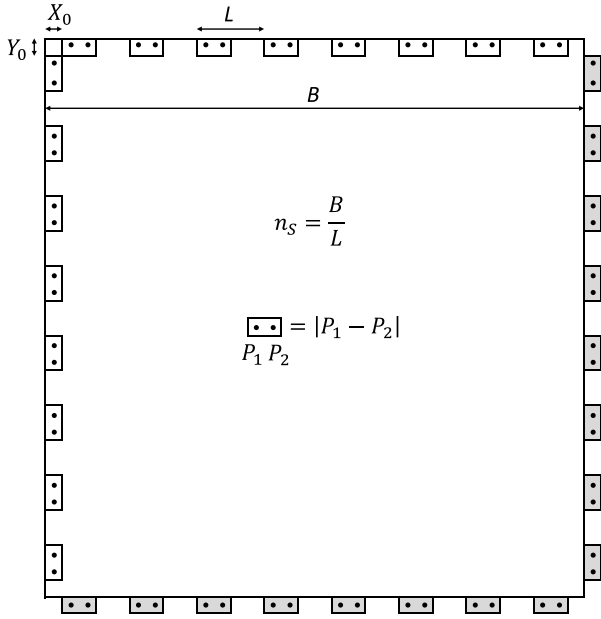


FIGURE 3. Block boundary variation measurements for a 32×32 block. The shaded measurements are imported from adjacent blocks.

the number of measurements to estimate the BBV. This is accomplished in three ways. First, only the TV in the top row and left-hand column borders are measured in each block. Since the blocks stack to form an image, the right-hand TV is obtained from the next block to the right, and the bottom row TV is obtained from the block below the current row. This reduces the number of pixels that need to be measured per block to $(2B - 1)$. Second, rather than collecting pixel values, it is possible to collect pixel difference measurements, which involve only two adjacent pixels at a time. Thus, it is possible to reduce the measurements to approximately B per block. Third, it is possible that not all adjacent pixel measurements are necessary and that the BBV can be estimated from fewer

measurements. It is thus possible to collect a pixel difference measurement every L pixels around the block border.

The right-hand blocks and the bottom blocks do not have right-hand and bottom neighbours that can contribute to the missing edges of the boundary. In these cases, the right-hand and bottom TVs can be measured from the current block.

Figure 3 shows BBV measurements in a 32×32 pixel block. The variation measurements in the current block, which consist of the absolute differences $|P_1 - P_2|$ of two adjacent pixels P_1 and P_2 , are shown as rectangles outlined in black. X_0 is the pixel offset of the first variation measurement in a row, Y_0 is the pixel offset of the first measurement in a column, and L is the stride between pixel variation measurements. In the figure, $X_0 = 1$, $Y_0 = 1$ and $L = 4$. We acquire phase 1 measurements that decrease as C_F increases. Hence, we set L empirically:

$$L = \lfloor C_F \rfloor \quad (16)$$

and

$$X_0 = Y_0 = \lfloor L/2 \rfloor \quad (17)$$

The number of measurements n_S per block side is given by:

$$n_S = \lfloor B/L \rfloor \quad (18)$$

and the number of measurements used to calculate the BBV is $4 \cdot n_S$, with half of them in the current block and the other half in adjacent blocks, shaded in grey in figure 3. BBV_i is given by:

$$BBV_i = \sum_{j=0}^{n_S-1} \{ |P_{(1, X_0+j \cdot L+1)} - P_{(1, X_0+j \cdot L)}| + |P_{(Y_0+j \cdot L+1, 1)} - P_{(Y_0+j \cdot L, 1)}| + |P_{(B+1, X_0+j \cdot L+1)} - P_{(B+1, X_0+j \cdot L)}| + |P_{(Y_0+j \cdot L+1, B+1)} - P_{(Y_0+j \cdot L, B+1)}| \} \quad (19)$$

If $C_F > B$, n_S is zero according to equations (18) and (16). In this case, the algorithm reverts to being non-adaptive.

The number of blocks n_B in an image of H by W pixels is given by:

$$n_B = \lfloor H/B \rfloor \cdot \lfloor W/B \rfloor \quad (20)$$

where the floor operation indicates that the image dimensions are cropped to be multiples of block size B . Then, the number of BBV measurements M_{BBV} required in phase one is equal to the number of measurements in each block $2 \cdot n_S$ multiplied by the number of blocks n_B and adding the measurements at the right and bottom edges and is given by:

$$M_{BBV} = 2 \cdot n_S \cdot n_B + (H + W)/L \quad (21)$$

These BBV measurements, in addition to being used to calculate the number of measurements m_i^2 required in phase two, can also be transmitted to the decoder to serve as additional reconstruction measurements. Then, assuming the total number of measurements used in image reconstruction is equal to M , the number of measurements in block i during

phase 2, m_i^2 , is given by the number of remaining measurements ($M - M_{BBV}$) distributed amongst the blocks as a proportion of the block boundary variation measurement in block BBV_i to the total block boundary measurements from all blocks and is hence given by:

$$m_i^2 = \left\lfloor (M - M_{BBV}) \cdot \frac{BBV_i}{\sum_{i=1}^{n_B} BBV_i} \right\rfloor \quad (22)$$

where BBV_i is the BBV of block i , which is the sum of the absolute difference of two adjacent pixels at each measurement point around the boundary of the block. Half of the measurements are collected from block i , and the other half are collected from the adjacent blocks to the right and below, as shown in figure 3. m_i^2 has to be capped at $B^2 - m_i^1$, and any remaining measurements can be collected from other non-fully measured blocks.

Algorithm 2 AL-DCT-DD

Input: Image $\mathcal{I}$, compression factor C_F , block size B ,
Reconstruction Algorithm = {IDCT2 |
D-AMP|DAMP-D|IDA}}

Output: Image $\mathcal{O}$ sensed using AL-DCT-DD and
reconstructed using reconstruction algorithm.

- 1 Crop and partition, $\mathcal{I}$ into $n_B = \lfloor H/B \rfloor \cdot \lfloor W/B \rfloor$, $B \times B$ blocks;
 - 2 Calculate the number of non-adaptive measurements per block $m_i^1 = \lfloor B^2/(2 \cdot C_F) \rfloor$;
 - 3 Collect m_i^1 L-DCT-ZZ coefficients from each block;
 - 4 Estimate the number of L-DCT-ZZ transform coefficients to collect per block in phase 2, m_i^2 , from equation (23);
 - 5 Collect m_i^2 additional coefficients as required;
 - 6 Transmit $M = \sum_{i=1}^{n_B} (m_i^1 + m_i^2)$ measurements;
 - 7 Reconstruct $\mathcal{O}$ from the M received samples using the selected reconstruction algorithm.
-

B. ADAPTIVE L-DCT-ZZ IN THE DCT DOMAIN

The adaptive L-DCT-ZZ algorithm in the DCT domain, AL-DCT-DD, described in Algorithm 2, uses m_i^1 linear 2D DCT measurements per block i , in zigzag scan order, to compute the number of measurements m_i^2 to collect in phase 2, as shown in figure 2. Several authors have considered first capturing a full set of measurements ($M = N$) to compute the value of all the pixels at the encoder and then adapting the number of measurements to transmit per block, as reported in section II. These full sensing techniques are feasible if the power required to sense the transform coefficients directly in the optical domain is less than the power required to capture pixels, convert them from analogue to digital, and then perform the transformation using digital computations. Zhu *et al.* [36] also consider a similar *significance-based allocation factor*, but this requires a whole block of transform coefficients. CS techniques, such as our AL-DCT-DD algorithm, collect fewer measurements ($M < N$) and hence require less sensing power than full sensing techniques.

The best PSNR performance by an adaptive algorithm in the DCT domain is achieved when using the full sensing THI algorithm that selects the highest valued transform coefficients from all the blocks. However, if compressive sensing is used, the strategy is to assign m_i^2 in proportion to the number n_i of the m_i^1 measurements whose absolute value exceeds a threshold T . We found empirically that collecting half the number of available measurements in phase 1, that is, $n_B \cdot m_i^1 = M/2$, allows us to distribute a remaining equal number in proportion to n_i such that m_i^2 is given by:

$$m_i^2 = \left\lfloor n_B \cdot m_i^1 \cdot \frac{n_i}{\sum_{i=1}^{n_B} n_i} \right\rfloor \quad (23)$$

The selection of T is crucial to the performance of the algorithm. The best values were found empirically (refer to supplementary material, section IV [49]) and are tabulated in table 1 for different image heights H and compression factors C_F .

TABLE 1. Values of T based on image height H and C_F . The dynamic range of the pixel values is [0 255].

	$H = 256$	$H = 512$
$T = 15$	$C_F \leq 2$	$C_F \leq 2$
$T = 30$	$2 < C_F < 3.33$	$2 < C_F < 10$
$T = 60$	$C_F \geq 3.33$	$C_F \geq 10$

C. ANALYSIS OF THE BBV AND DD ADAPTIVE TECHNIQUES

Using the 2D DCT as the sparsifying domain, the number of transform coefficients in each block, m_i , that maximize PSNR, can be found by measuring all transform coefficients in all the blocks and selecting the M largest transform coefficients, irrespective of the blocks in which they occur. This follows from the orthogonal property of the DCT [50], meaning that it preserves the inner product. Since the sum of the square of the image pixels is equal to the sum of the square of the transform coefficients, if we select the highest valued transform coefficients, we are guaranteed to collect the highest signal energy.

The resulting algorithm is referred to as the THI algorithm in this paper. The higher the number of transform coefficients in a block i , the lower the sparsity in the block, and the more measurements that are required from that block. It is required to show that the number of measurements per block $\hat{m}_i$ estimated by the BBV and DD algorithms are well correlated with the number of measurements m_i that maximize PSNR.

The correlation coefficient $\rho_{m_i, \hat{m}_i}$ and the confidence level $(1 - p)$, where p is the probability of the null hypothesis (that there is no correlation, i.e., $\rho_{m_i, \hat{m}_i} = 0$), were measured for the Set256 and Set512 image sets in figures 4 and 5 for $C_R = \{0.1 \dots 0.5\}$. In all cases, the probability of the null hypothesis was always below 0.0001, indicating strong confidence in the correlation results. The average correlation results $\rho_{m_i, \hat{m}_i}$ for the 256×256 and 512×512 image sets

TABLE 2. Correlation coefficient $\rho_{m_i, \hat{m}_i}$ for 256×256 and 512×512 image sets for both BBV and DD adaptive algorithms with varying C_R .

$\rho_{m_i, \hat{m}_i}$	Adaptive Algorithm = DD						Adaptive Algorithm = BBV					
C_R	0.1	0.2	0.3	0.4	0.5	Av	0.1	0.2	0.3	0.4	0.5	Av
256 x 256 Image Set	0.779	0.805	0.781	0.835	0.887	0.817	0.645	0.764	0.789	0.775	0.811	0.757
512 x 512 Image Set	0.741	0.818	0.818	0.807	0.852	0.807	0.719	0.784	0.795	0.786	0.802	0.777

**FIGURE 4.** Set256: 256×256 image set.**FIGURE 5.** Set512: 512×512 image set.

are tabulated in table 2 for both the BBV and DD algorithms (original image sets are available in [49]). The full results are reported in the supplementary material, section V [49].

For both image sets, the best correlation was obtained with the DD algorithm. The correlation was strongest at $C_R = 0.5$. The lowest correlation occurs for BBV at $C_R = 0.1$ and may be due to the low level of measurements acquired to compute m_i , since the number of block boundary measurements is proportional to C_R . This indicates that BBV should probably have a minimum number of boundary measurements from a correlation perspective, although this could impact PSNR and SSIM, because each phase one measurement reduces the number of DCT measurements.

VI. RECONSTRUCTING DETERMINISTIC CS USING ITERATIVE DENOISING-BASED ALGORITHMS

In this section, we build on the work of Metzler [13] to design iterative algorithms that better reconstruct images compressively sensed using AL-DCT-BBV and AL-DCT-DD. The iterative denoising-based reconstruction algorithm is represented in block diagram form in figure 6. In D-AMP:

$$\alpha^t = \frac{1}{\delta} \frac{\text{div}[D_{\hat{\sigma}^{t-1}}(\mathbf{x}^{t-1} + \mathbf{A}^* \mathbf{z}^{t-1})]}{m} \quad (24)$$

so that $\alpha^t \mathbf{z}^{t-1}$ is the Onsager term. The Δ block introduces a delay of one iteration, $D_{\hat{\sigma}^t}$ is the denoiser block, and $\mathbf{r}^t$ is given by:

$$\mathbf{r}^t = \mathbf{y} - \mathbf{A} \mathbf{x}^t \quad (25)$$

The other variables are as defined for D-AMP above. To use D-AMP on deterministic, block-based CS, deterministic forward transform is cast as a sensing matrix $\mathbf{A}$ which has a sparse representation:

$$\mathbf{A} = \begin{bmatrix} \mathbf{B}_{1,1} & & & \\ & \ddots & & \\ & & \mathbf{B}_{i,i} & \\ & & & \ddots \\ & & & & \mathbf{B}_{n_B, n_B} \end{bmatrix} \quad (26)$$

where $\mathbf{B}_{i,i}$ are $m \times n$ basis matrices with $n = B \times B$, $m = \lfloor n \times C_R \rfloor$, C_R is the compression ratio M/N , B is the size of one side of the image block and n_B is the number of blocks in the image. In non-adaptive CS, m and n are constant since there are equal measurements per block. In adaptive CS, m varies per block. $\mathbf{A}^*$ is the transpose of $\mathbf{A}$.

When $\mathbf{B}$ is a random Gaussian block matrix, we empirically found that D-AMP frequently failed to recover some of the blocks correctly, leading to very significant blocking effects. This never happens when the row vectors of $\mathbf{B}$ are taken from the 2D-DCT basis functions, in zigzag order, with the first basis function being that for the DC component.

1) MODIFIED D-AMP - DAMP-D

D-AMP was derived based on a Gaussian distribution of the residual reconstruction error at each iteration [13]. When the measurement matrix is a low-pass deterministic 2D-DCT matrix, this assumption no longer holds, and the reconstruction quality deteriorates. Figure 6 shows that the Onsager term implements an adaptive integration of the reconstruction error, akin to integral control in a closed-loop feedback system. This inspires us to vary the integral control loop gain by introducing a damping factor D_F . Then, α^t in equation (24) becomes:

$$\alpha^t = \frac{1}{\delta} \frac{\text{div}[D_{\hat{\sigma}^{t-1}}(\mathbf{x}^{t-1} + \mathbf{A}^* \mathbf{z}^{t-1})]}{m} \frac{1}{D_F} \quad (27)$$

The modified algorithm is referred to as D-AMP Damped (DAMP-D) with damping factor D_F .

2) SIMPLIFIED D-AMP - IDA

Since the Onsager term requires the residual error to be Gaussian, it was hypothesized that a simplified version of the iterative denoising algorithm might provide better performance. Indeed, simplifying α^t to:

$$\alpha^t = \frac{1}{D_F} \quad (28)$$

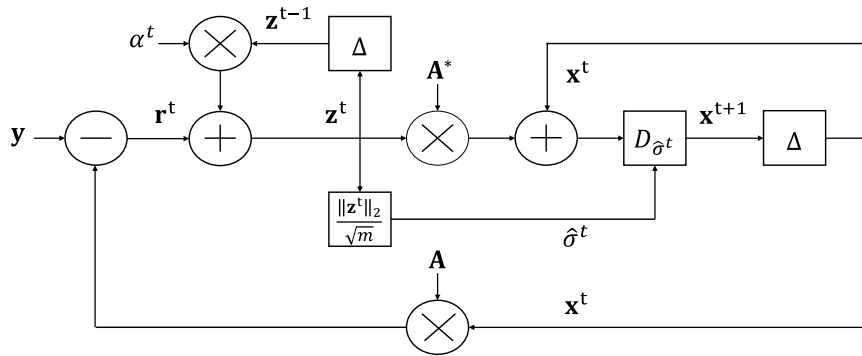


FIGURE 6. Generic block diagram for iterative denoising reconstruction algorithms: D-AMP, DAMP-D and IDA. This depicts equations 13, 14 and 15 in the text. The delay blocks Δ impart a one-iteration delay such that the output is the vector in the previous iteration. The Onsager term is $\alpha^t \mathbf{z}^{t-1}$ with α^t given by equation 24 and is derived from the divergence of the $\mathbf{x}^t$ vector from the previous iteration.

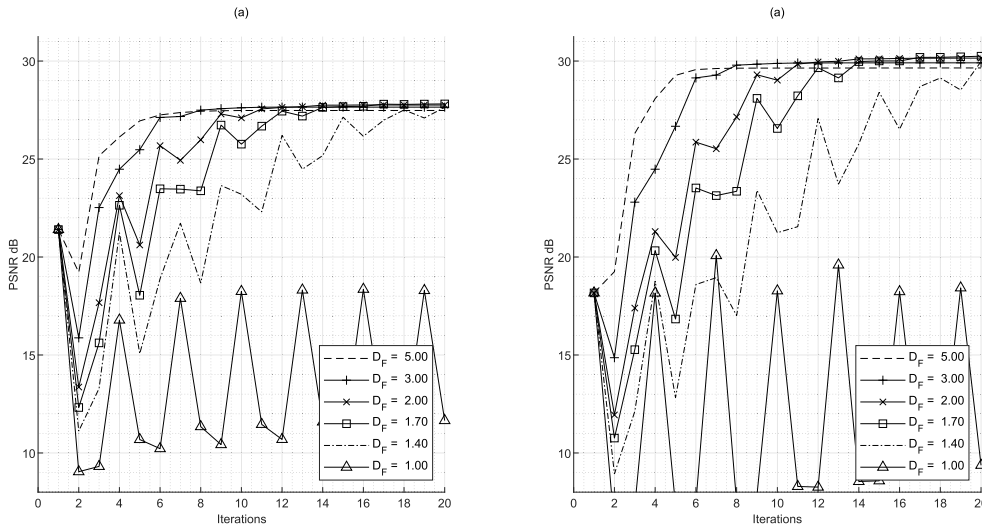


FIGURE 7. Reconstructing AL-DCT-DD using (a) D-AMP (when $D_F = 1.00$) and IDA (when $D_F > 1.00$) with a BM3D denoiser and (b) using D-AMP and IDA with a DnCNN denoiser. The results shown for the 256×256 Monarch image sensed at $C_R = 0.1$.

was found to provide better performance than DAMP-D. The ensuing algorithm is called the IDA with damping factor D_F .

Figure 7 shows that IDA without damping, i.e., $D_F = 1$, did not reconstruct the AL-DCT-DD-sensed Monarch image from image set 256×256 satisfactorily using either of the denoisers. The PSNR decreased on the first iteration and then oscillated around a level that was 7.5 dB to 10 dB lower. However, when D_F increased above 1, following the initial dip, the PSNR tended to increase. With $D_F \geq 1.4$, when employing the BM3D denoiser, IDA improved the initial AL-DCT-DD PSNR by approximately 2 dB, the maximum value after 20 iterations being reached with $D_F = 2$. When employing the DnCNN denoiser, IDA improved the AL-DCT-DD PSNR by 4.4 dB after 20 iterations. The results for the other images in the 256×256 image set are reported in supplementary [49] figure 9.

As IDA using the DnCNN denoiser is both substantially faster than with the BM3D denoiser (refer to table 9 in [49] for run-time comparison) and achieves superior performance, we present only results for this algorithm in this paper.

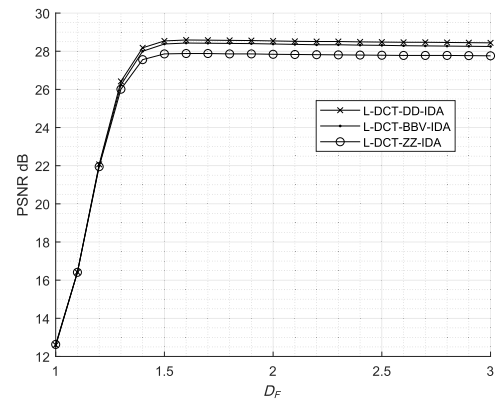


FIGURE 8. PSNR versus D_F for L-DCT-ZZ, AL-DCT-BBV and AL-DCT-DD reconstructed using IDA for the 256×256 image set, with compression ratio $C_R = 0.1$.

3) TUNING D_F

To find the optimal value of D_F , the PSNR was plotted for D_F between 1.0 and 3.0 for various algorithms, image sets and compression ratios. For example, figure 8 shows

TABLE 3. L-DCT-ZZ, AL-DCT-BBV and AL-DCT-DD with IDCT, D-AMP and IDA reconstruction on the 256×256 and 512×512 image sets, using the DnCNN denoiser. Maximum PSNR and SSIM values for all compression ratios are in bold.

C_R	256x256 Image Set						512x512 Image Set					
	L-DCT-ZZ		L-DCT-ZZ-D-AMP		L-DCT-ZZ-IDA		L-DCT-ZZ		L-DCT-D-AMP		L-DCT-IDA	
	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM
0.01	20.09	0.5187	4.82	0.0000	20.79	0.5831	22.85	0.5343	4.99	0.0000	23.53	0.5753
0.02	21.68	0.5955	5.86	0.0000	22.65	0.6739	24.23	0.5925	5.71	0.0049	24.97	0.6327
0.04	23.40	0.6806	6.71	0.1530	24.87	0.7687	25.70	0.6685	6.97	0.1738	26.63	0.7102
0.10	26.44	0.8096	19.84	0.7432	28.42	0.8733	28.63	0.7940	19.67	0.6853	29.60	0.8204
0.20	29.37	0.8889	28.89	0.8828	31.95	0.9281	31.62	0.8767	28.25	0.8162	32.63	0.8923
0.30	31.95	0.9308	31.06	0.9208	34.85	0.9563	33.82	0.9164	29.64	0.8629	34.74	0.9273
0.40	34.38	0.9562	31.96	0.9282	37.31	0.9725	35.90	0.9411	30.16	0.8680	36.68	0.9487
0.50	36.93	0.9728	33.41	0.9568	39.69	0.9823	38.06	0.9593	32.31	0.9094	38.73	0.9643
0.01 - 0.04	21.72	0.5983	5.80	0.0510	22.77	0.6752	24.26	0.5984	5.89	0.0596	25.04	0.6394
0.10 - 0.50	31.81	0.9117	29.03	0.8864	34.45	0.9425	33.61	0.8975	28.01	0.8284	34.47	0.9106
C_R	AL-DCT-BBV		AL-DCT-BBV-D-AMP		AL-DCT-BBV-IDA		AL-DCT-BBV		AL-DCT-BBV-D-AMP		AL-DCT-BBV-IDA	
	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM
	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM
0.01	20.09	0.5187	4.82	0.0000	20.79	0.5831	22.85	0.5343	4.99	0.0000	23.53	0.5753
0.02	21.68	0.5955	5.86	0.0000	22.65	0.6739	24.23	0.5925	5.71	0.0049	24.97	0.6327
0.04	23.00	0.6629	6.71	0.1661	24.38	0.7436	25.74	0.6636	7.10	0.1292	26.74	0.7035
0.10	26.70	0.7982	15.63	0.6863	28.73	0.8672	29.18	0.7923	13.96	0.5735	30.30	0.8216
0.20	30.54	0.8930	29.36	0.8891	33.37	0.9359	32.72	0.8805	22.77	0.8033	33.85	0.8976
0.30	33.32	0.9334	30.93	0.9112	36.42	0.9607	35.26	0.9191	27.63	0.8533	36.32	0.9303
0.40	35.90	0.9561	33.15	0.9403	39.01	0.9733	37.36	0.9419	31.03	0.8860	38.25	0.9492
0.50	38.29	0.9702	33.88	0.9459	41.19	0.9810	39.36	0.9571	33.08	0.9080	40.02	0.9612
0.01 - 0.04	21.59	0.5924	5.79	0.0554	22.61	0.6669	24.27	0.5968	5.93	0.0447	25.08	0.6372
0.10 - 0.50	32.95	0.9102	28.59	0.8746	35.74	0.9436	34.78	0.8982	25.69	0.8048	35.75	0.9120
C_R	AL-DCT-DD		AL-DCT-DD-D-AMP		AL-DCT-DD-IDA		AL-DCT-DD		AL-DCT-DD-D-AMP		AL-DCT-DD-IDA	
	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM
	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM
0.01	20.32	0.5271	5.32	0.0000	21.05	0.5918	23.09	0.5417	5.06	0.0000	23.81	0.5837
0.02	22.02	0.6085	5.64	0.0087	23.09	0.6880	24.60	0.6042	5.83	0.0019	25.35	0.6434
0.04	23.99	0.6998	7.76	0.2146	25.43	0.7810	26.33	0.6826	7.36	0.1784	27.13	0.7170
0.10	27.07	0.8173	20.38	0.7697	29.13	0.8784	29.40	0.8000	17.20	0.6771	30.31	0.8227
0.20	30.48	0.8953	29.92	0.8944	33.14	0.9308	32.50	0.8797	27.14	0.8156	33.39	0.8938
0.30	33.46	0.9350	31.19	0.9228	36.13	0.9563	34.94	0.9179	30.00	0.8556	35.67	0.9269
0.40	36.10	0.9588	32.67	0.9324	38.78	0.9722	37.08	0.9408	30.68	0.8741	37.70	0.9467
0.50	38.58	0.9735	33.72	0.9524	40.97	0.9811	39.19	0.9595	33.21	0.9083	39.77	0.9634
0.01 - 0.04	22.11	0.6118	6.24	0.0744	23.19	0.6869	24.67	0.6095	6.08	0.0601	25.43	0.6480
0.10 - 0.50	33.14	0.9160	29.57	0.8943	35.63	0.9438	34.62	0.8996	27.64	0.8261	35.37	0.9107

the PSNR versus D_F plotted for L-DCT-ZZ, AL-DCT-BBV and AL-DCT-DD reconstructed using IDA for the 256×256 image set, with $C_R = 0.1$. $D_F = 2.0$ was empirically found to be a good compromise value across the compression ratios of interest.

4) BM3D AND DnCNN DENOISERS

Metzler *et al.* [42] implemented their D-AMP algorithm as an unrolled deep neural network, replacing the BM3D [22] with DnCNN [21], which is known to be a more accurate and faster denoiser. In this paper, we present the performance of iterative reconstruction using the DnCNN denoiser due to its superior performance.

The version of DnCNN used in this paper consists of 17 layers. The first layer consists of $64, 3 \times 3 \times 1$ convolutional filters. The next fifteen layers consist of $64, 3 \times 3 \times 64$ convolutional filters, followed by batch normalization [51] and a rectified linear unit (ReLU) [52]. The final layer consists of $3 \times 3 \times 64$ convolutional filters.

VII. EMPIRICAL INVESTIGATION

In this section, L-DCT-ZZ, AL-DCT-BBV and AL-DCT-DD are first compared empirically when reconstructed directly using the inverse 2D DCT (IDCT) and using CS reconstruction with the D-AMP and IDA algorithms on the 256×256 and 512×512 image sets. The CS algorithms were implemented using a DnCNN denoiser as detailed in section VI-4. The two adaptive algorithms were then compared with other published encoder-side and decoder-side adaptive CS algorithms, as well as with full sensing techniques. Finally, IDA was compared with two state-of-the-art non-adaptive algorithms, CREAM [19] and BCS-Net [20]. Since none of the techniques published in the literature were accompanied by source code, the method we used to compare with these published results was to repeat the simulation on the same image test sets and at the same compression ratios.

The simulations were executed on a server equipped with an Intel Xeon CPU E5-160 v3 clocked at 3.50 GHz, with 32.00 GB of RAM, running MATLAB version 2019a on Windows 10. The D-AMP algorithm was downloaded

together with the D-AMP toolbox from [53]. The DnCNN code and models were downloaded from <https://github.com/ricedsp/prDeep> and require the MatConv-Net package from <https://www.vlfeat.org/matconvnet/>.

A. COMPARING THE DETERMINISTIC ALGORITHMS USING DIRECT AND CS RECONSTRUCTION

Table 3 presents the reconstruction results of the baseline linear L-DCT-ZZ algorithm and the two adaptive algorithms presented in this paper: AL-DCT-BBV and AL-DCT-DD. The results are also presented when using the original D-AMP algorithm and the IDA algorithm presented in this paper as a post-processing step.

When no CS reconstruction post-processing was utilised, AL-DCT-DD achieved the best results at all compression ratios (0.01 – 0.5) on 256×256 images. PSNR performance improved by an average of 0.97 dB over the baseline L-DCT-ZZ algorithm across all compression ratios, and SSIM improved by an average of 0.0077. This improvement suffered no penalty in reconstruction execution time, as shown in table 8. At an image resolution of 512×512 , this average performance increased in PSNR and SSIM by 0.79 dB and 0.0054, respectively. However, AL-DCT-BBV achieved a marginally higher PSNR increase for compression ratios in the range 0.1 – 0.5 than AL-DCT-DD.

The original D-AMP as a post-processing reconstruction algorithm failed to reconstruct images at very low compression ratios (0.01 – 0.04). This is as expected, as this algorithm was not designed to handle images that have been sensed using our deterministic sensing matrix. Interestingly, D-AMP achieved some success in reconstructing images at higher compression ratios, but the quality degraded in all cases.

IDA successfully improved the performances of all three algorithms. PSNR improved by an average of 2.04 dB and 0.83 dB across all compression ratios for 256×256 and 512×512 image resolutions, respectively. Similarly, SSIM improved by 0.0475 and 0.0229, respectively. It is interesting to note that AL-DCT-BBV-IDA achieved the best performance amongst the higher compression ratios (0.10 – 0.50), while AL-DCT-DD-IDA was the best amongst the low compression ratios (0.01 – 0.04). This has interesting implications in future schemes that utilise these algorithms in video transmission, mainly those that employ the group of picture structure [54] wherein key frames are sensed at a relatively high compression ratio and non-key frames at a substantially lower compression ratio.

B. COMPARISON WITH ADAPTIVE CS RESULTS IN THE LITERATURE

L-DCT-ZZ and the two adaptive algorithms achieved state-of-the-art performance, in PSNR and SSIM terms, across many adaptive CS algorithms proposed in the literature. Table 4 summarizes the PSNR results, where our real-time algorithms perform better than the ES-CS, DS-CS and ES-FS BCS algorithms in the literature. Each row in table 4 reports

TABLE 4. Comparison of L-DCT-ZZ, AL-DCT-DD and AL-DCT-BBV with ABCS results in the literature derived on the image sets in the literature, using the compression ratios in the referenced publication. Best results are highlighted in bold. The underlined results are the only inferior L-DCT-ZZ results.

Algorithm	Type	PSNR dB	L-DCT-ZZ	AL-DCT-BBV	AL-DCT-DD
			PSNR dB	PSNR dB	PSNR dB
ABCS-TVAL3 [38]	ES-CS	30.82	37.38	39.35	38.68
JRW-BCS [36]	ES-CS	33.50	<u>33.25</u>	34.26	34.19
ABCS-Zhang [40]	DS-CS	29.21	30.06	33.73	31.21
Proposed in [41]	DS-CS	31.08	32.88	34.16	34.44
JRW-BCS-Sol2-EB [36]	DS-CS	34.08	<u>33.25</u>	34.19	34.26
JRW-BCS-Sol2-VB [36]	DS-CS	34.04	<u>33.25</u>	34.19	34.26
ABCS-Wang [28]	ES-FS	28.36	35.22	36.85	36.54
Var-reg OP3 [29]	ES-FS	29.18	30.39	31.02	31.64
Proposed [32]	ES-FS	27.08	31.48	32.93	32.41
ABCS-Canh [33]	ES-FS	32.16	34.37	35.59	35.62
ABCS-SF-D [34]	ES-FS	32.64	<u>32.26</u>	33.84	33.19
JND [35]	ES-FS	28.35	32.13	33.34	33.36

the PSNR result of a published ABCS scheme and those of our algorithms on the same image set.

Only JRW-BCS, JRW-BCS-Sol2-EB and JRW-BCS-Sol2-VB from [36] and ABCS-SF-D [34] achieve better results than L-DCT-ZZ, with our AL-DCT-BBV and AL-DCT-DD algorithms achieving the best performance in all cases.

TABLE 5. Comparison of AL-DCT-BBV-IDA and AL-DCT-DD-IDA using the DnCNN denoiser with adaptive CS algorithms in the literature using the indicated reconstruction algorithm. The results in each PSNR column were derived using the image set in the cited publication, using the compression ratios in the referenced publication.

Algorithm	Type	Image Set	[37]	[39]	[36]
		RA	PSNR dB	PSNR dB	PSNR dB
StatACS [37]	ES-CS	IWT	31.57		
InVDS-WT [39]	ES-CS	DAMP		34.72	
InVDS-DCT [39]	ES-CS	DAMP		34.83	
InVDS-HT [39]	ES-CS	DAMP		34.40	
JRW-BCS-Sol1-EB [36]	ES-FS	NESTA			34.59
JRW-BCS-Sol1-VB [36]	ES-FS	NESTA			35.41
JRW-BCS-Sol1-SB [36]	ES-FS	NESTA			35.48
JRW-BCS-Sol2-SB [36]	DS-CS	NESTA			34.53
AL-DCT-BBV-IDA-DnCNN	ES-CS	IDA-DnCNN	32.12	35.06	35.12
AL-DCT-DD-IDA-DnCNN	ES-CS	IDA-DnCNN	32.30	34.41	35.07
L-DCT-THB	ES-FS	IDCT			39.03
L-DCT-THI	ES-FS	IDCT			40.87

Table 5 compares AL-DCT-BBV and AL-DCT-DD, reconstructed using IDA with the DnCNN denoiser, against seven adaptive algorithms whose performance cannot be matched by our real-time algorithms. All our CS reconstructed algorithms outperformed StatACS and JRW-BCS-Sol2-SB in terms of PSNR. StatACS is not block-based, although it can be reconstructed in real time. AL-DCT-BBV-IDA outperformed the three InVDS algorithms in [36].

None of our CS reconstructed algorithms exceed the PSNR performance of the full sensing, Sol1-VB and Sol1-SB variants presented in [36]. A comparison with the full sensing THB and THI algorithms is fairer. In this case, THB achieved 39.03 dB and THI 40.87 dB, exceeding the best Sol1 variant by 3.55 dB and 5.39 dB, respectively.

TABLE 6. Comparing CREAM PSNR and SSIM with L-DCT-ZZ, AL-DCT-BBV and AL-DCT-DD reconstructed using IDA with the DnCNN denoiser, with $D_F = 2.0$ and 15 iterations. Maximum PSNR/SSIM values are in bold and second largest underlined.

C_R	0.1		0.2		0.3		0.4		Average	
Algorithm	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM	PSNR dB	SSIM
CREAM	27.31	0.8206	31.67	0.9014	34.14	0.9318	36.11	0.9510	32.31	0.9012
L-DCT-ZZ	26.92	0.8030	29.67	0.8795	31.82	0.9182	33.71	0.9429	30.53	0.8859
AL-DCT-BBV	27.28	0.7982	30.64	0.8823	33.34	0.9226	35.57	0.9459	31.71	0.8873
AL-DCT-DD	27.67	0.8131	30.78	0.8865	33.19	0.9223	35.22	0.9446	31.72	0.8916
L-DCT-ZZ-IDA	28.41	<u>0.8506</u>	31.61	0.9105	33.82	<u>0.9385</u>	35.70	<u>0.9571</u>	32.39	0.9142
AL-DCT-BBV-IDA	<u>28.85</u>	0.8486	32.73	0.9158	35.43	0.9433	37.48	0.9591	33.62	0.9167
AL-DCT-DD-IDA	29.28	0.8575	<u>32.68</u>	<u>0.9128</u>	<u>34.89</u>	<u>0.9385</u>	<u>36.76</u>	0.9554	<u>33.40</u>	<u>0.9161</u>

TABLE 7. Comparing BCS-Net PSNR and SSIM with L-DCT-ZZ, AL-DCT-BBV and AL-DCT-DD reconstructed using IDA with the DnCNN denoiser, with $D_F = 2$ and 15 iterations. Maximum PSNR values are in bold and second largest underlined.

C_R	0.01		0.03		0.05		0.1		0.2		0.3		0.4		Average	
Algorithm	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
BCS-Net	20.88	0.5505	<u>24.47</u>	0.7087	26.04	0.7723	<u>29.43</u>	0.8676	33.06	0.9283	35.60	<u>0.9554</u>	36.70	0.9662	29.45	0.8213
L-DCT-ZZ	20.33	0.5072	22.82	0.6369	24.60	0.7260	27.06	0.8193	30.17	0.8953	32.70	0.9340	35.01	0.9569	27.53	0.7822
AL-DCT-BBV	20.33	0.5072	22.91	0.6424	24.17	0.7031	27.29	0.8112	31.08	0.8980	33.90	0.9363	36.29	0.9563	28.00	0.7792
AL-DCT-DD	20.56	0.5163	23.27	0.6519	25.03	0.7366	27.68	0.8272	31.16	0.9013	34.06	0.9376	36.53	0.9591	28.33	0.7900
L-DCT-ZZ-IDA	20.99	0.5615	24.09	0.7153	25.81	0.7864	28.78	0.8721	32.29	0.9275	34.90	0.9544	37.05	0.9698	29.13	0.8267
AL-DCT-BBV-IDA	20.99	0.5615	24.09	0.7153	25.58	0.7676	29.09	0.8676	33.29	0.9316	36.11	0.9567	38.35	<u>0.9694</u>	29.64	0.8242
AL-DCT-DD-IDA	21.25	0.5717	24.48	0.7240	26.30	0.7934	29.45	0.8769	<u>33.27</u>	<u>0.9296</u>	35.95	0.9543	38.23	<u>0.9694</u>	29.85	0.8313

C. COMPARISON WITH TWO STATE-OF-THE-ART, NON-ADAPTIVE ALGORITHMS

In this section, we compare our algorithms with CREAM [19] and BCS-Net [20]. Table 6 shows that the real-time AL-DCT-DD algorithm achieved a better PSNR than CREAM at $C_R = 0.1$, but CREAM was superior to our real-time algorithms in all other cases. When our adaptive algorithms were reconstructed using IDA, the situation reversed, with both always performing better. The best PSNR and SSIM results were achieved by AL-DCT-BBV-IDA followed by AL-DCT-DD-IDA for $D_F = 2$ and with 15 iterations.

The results in table 6 were obtained with an average reconstruction time of 7.85 s over all compression ratios, with 15 iterations. L-DCT-ZZ, AL-DCT-BBV and AL-DCT-DD were reconstructed in fewer than 40 ms. AL-DCT-BBV-IDA reconstructed an image in 5.2 s on average, with 15 iterations at a compression ratio of 0.4.

Comparing our algorithms with BCS-Net, Table 7 shows that the best average PSNR and SSIM results were obtained by AL-DCT-DD-IDA with $D_F = 2$ and 15 iterations. AL-DCT-BBV-IDA achieves better results at higher compression ratios. The average reconstruction time over all compression ratios was approximately 3.5 s for 15 iterations. The reconstruction time for a compression ratio of 0.1 was approximately 2.37 s for 15 iterations.

D. COMPLEXITY ANALYSIS

Block-based L-DCT-ZZ and AL-DCT-DD acquire M measurements using $M \times B^2$ operations, whereas normal CS algorithms require $M \times (H \times W)$ operations. AL-DCT-BBV

requires $M_{BBV} + (M - M_{BBV}) \times B^2$ operations. These analogue computations occur in the optical domain on single-pixel and multi-pixel cameras. The adaptive algorithms also need to compute the number of phase-2 measurements per block in the digital domain. These total $n_B \times n_S$ computations in the case of AL-DCT-BBV and $n_B \times B^2$ in the case of AL-DCT-DD.

The direct reconstruction of L-DCT-ZZ, AL-DCT-BBV and AL-DCT-DD using the inverse 2D DCT requires M inversions of a $B \times B$ matrix. Implemented as a separable inverse transform, 2D-IDCT requires $\mathcal{O}(B^2 \log B)$ operations. Therefore, IDCT reconstruction requires $\mathcal{O}(MB^2 \log B)$ operations.

With reference to figure 6 and equation (27), D-AMP has a computational complexity dominated by the three matrix multiplications and the denoiser computation and is of order $\mathcal{O}(MN^2 + 2NM^2) + \mathcal{O}(D_{\hat{\sigma}})$, where $\mathcal{O}(D_{\hat{\sigma}})$ is the complexity of the denoiser.

With reference to equation (27), D-AMP has the same order of complexity because it merely divides the Onsager term by α every iteration. However, equation (28) shows that IDA avoids the computation of the Onsager term altogether and has complexity $\mathcal{O}(MN^2 + NM^2) + \mathcal{O}(D_{\hat{\sigma}})$. Simulation results show that D-AMP and D-AMP-D reconstruct a 256×256 image in approximately 2.4 s when using a DnCNN denoiser, whereas IDA-DnCNN requires 1.8 s. These results are in agreement with the theoretical complexity results.

On our platform, the L-DCT-ZZ, AL-DCT-BBV and AL-DCT-DD algorithms reconstruct 512×512 images in less than 30 ms. The improvement in performance with CS reconstruction comes at the expense of reconstruction time. Table 8 compares the reconstruction times of these algorithms

TABLE 8. Comparison of reconstruction times for L-DCT-ZZ, AL-DCT-BBV and AL-DCT-DD using 2D-IDCT and IDA reconstruction with the DnCNN denoiser on the 256×256 image set, $D_F = 2$ and 20 iterations.

Algorithm	L-DCT-ZZ	AL-DCT-		L-DCT-	AL-DCT-	
	ZZ	BBV	DD	ZZ-IDA	BBV-IDA	DD-IDA
C_R	Time s	Time s	Time s	Time s	Time s	Time s
0.01	0.0069	0.0069	0.0067	0.69	0.69	0.69
0.02	0.0080	0.0074	0.0070	0.79	0.79	0.80
0.04	0.0067	0.0069	0.0066	1.05	1.02	1.07
0.10	0.0067	0.0071	0.0067	1.86	1.75	1.84
0.20	0.0067	0.0068	0.0066	3.20	2.89	3.11
0.30	0.0067	0.0069	0.0065	4.42	4.09	4.38
0.40	0.0068	0.0069	0.0067	5.89	5.38	5.72
0.50	0.0067	0.0067	0.0066	7.86	6.50	7.08

with and without IDA post-processing using the DnCNN denoiser on the 256×256 image set.

VIII. CONCLUSIONS AND FURTHER WORK

Two adaptive algorithms, AL-DCT-BBV and AL-DCT-DD, have been described that adapt the number of deterministic 2D-DCT measurements collected from block-based compressive image sensors. Both the BBV and the DD techniques proposed have been shown to estimate the number of transform coefficients per block that correlates well with the number of transform coefficients estimated by the full 2D DCT analysis. AL-DCT-BBV and AL-DCT-DD achieve state-of-the-art performance in adaptive block CS of images that can be used in non-GPU-assisted, real-time applications, such as image and video capture with single-pixel cameras, multi-pixel cameras or focal plane processing image sensors. The algorithms reconstruct the compressively sensed images in real time, for example, less than 8 ms and 30 ms for 256×256 and 512×512 images, respectively.

The original D-AMP algorithm frequently fails to reconstruct or improve our algorithms using deterministic sensing matrices. A block diagram representation of the D-AMP algorithm has been derived and used to interpret D-AMP as a feedback system with integral control. A modified version, DAMP-D, which damps the Onsager term with a factor D_F and a simplified version, called the iterative denoising algorithm (IDA), has been derived that performs better when reconstructing our deterministic CS algorithms. The IDA algorithm simplifies the Onsager term to $\mathbf{z}^t/D_F$ and achieves a considerable reduction in computational complexity and better results than D-AMP and DAMP-D. Both modified algorithms can be used as a post-processing technique on the AL-DCT-BBV and AL-DCT-DD algorithms. Setting $D_F = 2.0$ has been empirically found to be close to optimal for the 256×256 and 512×512 image sets used in our simulations.

The two adaptive algorithms reconstructed using IDA with a DnCNN denoiser outperform two state-of-the-art algorithms in terms of PSNR and SSIM. AL-DCT-BBV-IDA outperforms the GPU-assisted CREAM PSNR and SSIM by 1.31 dB and 0.0155, respectively. AL-DCT-DD-IDA outperforms the DNN BCS-Net PSNR and SSIM by 0.40 dB and

0.01, respectively. Even the non-CS reconstructed, real-time algorithms perform close to CREAM and BCS-Net, with low to moderate PSNR penalties, for example, within 0.59 dB of the CREAM PSNR and within 1.12 dB of BCS-Net, making them suitable for real-time compressive image and video sensing.

We also note that by utilising DCT measurements, the proposed algorithms can leverage the well-established source and channel coding techniques developed for hybrid DPCM/DCT encoding standards. The AL-DCT-BBV algorithm has better performance at C_R values greater than 0.1, with AL-DCT-DD performing better at lower C_R values. This means that in video coding applications, the AL-DCT-BBV algorithm will be more suitable for high-quality intra-coded frames and AL-DCT-DD for the more compressed predicted frames.

The improvement in performance by damping and simplifying the Onsager term was examined empirically. The development of a theoretical framework to exploit the interpretation of IDA as feedback systems with an integral component is of interest for further work.

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JOSEPH ZAMMIT (Member, IEEE) received the B.Sc. degree in communications and computer engineering from the University of Malta, Msida, Malta, in 2012, and the M.Sc. degree in wireless and optical communications from University College London, London, U.K., in 2013. He is currently pursuing the Ph.D. degree with the Computer Laboratory, University of Cambridge, Cambridge, U.K. His research interests include sparse representations, wireless networks, deep learning, and distributed systems.



IAN J. WASELL received the B.Sc. and B.Eng. degrees from the University of Loughborough, Loughborough, U.K., in 1983, and the Ph.D. degree from the University of Southampton, Southampton, U.K., in 1990. He is currently a Senior Lecturer at the Computer Laboratory, University of Cambridge, Cambridge, U.K. He has more than 15 years of experience in the simulation and design of radio communication systems gained via a number of positions in industry and higher education. He has published more than 180 articles concerning wireless communication systems. His research interests include fixed wireless access, sensor networks, cooperative networks, propagation modeling, compressive sensing, and cognitive radio. He is a member of the IET and a Chartered Engineer.

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