# From Exogenous to Endogenous Networks: Internet Applications

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# From Exogenous to Endogenous Economic Networks: Internet Applications

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**Abstract**

Economic agents' behavior is affected by their position in a network, either exogenous or endogenous, in which they interact with a subset of neighbours only. The network's links, which may be generated by vertical and/or horizontal relations, or by more complex morphologies, may explain the transition between dynamic equilibria and the instability of economic aggregates. Moreover, networks shape strategic interaction among agents by determining their strategies; the problem of access and interconnection, particularly relevant in the Internet, is perhaps the best example. A two-way feedback between strategies and network structures arises instead when links are endogenous: those features are clearly shown in the mechanism underlying the formation of peering links and R & D networks.

[ JEL Classification: L42, L14, D85, L86]

Keywords: vertical restraints; network formation; Internet; peering.

# 1. Introduction

In the last thirty years the theory of Industrial Organization has put in evidence the importance of strategic relationships among firms (Kranton and Minehart, 2000, 2001), moving away from the traditional assumption that economic agents (individuals and firms) only interact within anonymous institutions.

Very often, relationships between economic agents, within a given population, are affected by the position they have in a complex space, either real or virtual, that allows them to interact with a subset of neighbours only.

The environment in which interactions take place may be exogenous or, instead, arising from the agents' decisions; we can think about it like a multidimensional structure, whose effects on the interaction process depends on the nature of its dimensions. Such an environment, in fact, can be expressed, very simply, by a geographical dimension; or, like in Lancaster (1971) it can be modelled as a space of characteristics, describing both qualitative and horizontal differentiations. Other possible dimensions characterizing the interaction environment may be relationships affecting the production process: for instance, two firms may be in a purely vertical relationship, so that the upstream firm supplies the one downstream, or in a reciprocally vertical interaction, like two telephone companies both terminating the calls made by each and directed to the other's users. Indeed there could be many other relationships, along more complex topologies where economic agents operate.

This work aims to introduce part of the literature focusing on non-anonymous interaction between economic agents, which take place on networks that restrict the set of possible relationships. In particular, we intend to answer three main questions. In which way do networks affect economic interaction? Which network topologies are more likely to occur? When are such networks efficient?

Section two shows the importance of networks as the forum where interactions between agents take place. These relationships are essential in explaining phenomena such as non-ergodic processes, path-dependency and transition among dynamic equilibria.

In section three, networks are explicitly presented as exogenous environments in which strategic interaction takes place. The networks' topologies and the characteristics of links influence the strategies of all individuals and, through these, economic results. Technological innovations problems are analysed; we also focus on co-operation in games like the Prisoner's Dilemma, and on the issues concerning the access to essential inputs with a specific focus on Internet interconnection.

Finally, section four moves away from the idea of exogenous networks. We analyse the literature studying networks' formation, from the earliest theories to the very recent approaches, pointing out the most important elements characterizing each of them. We then present some applications concerning the formation of R&D networks and the interconnections' structure involving the firms that exchange traffic in the Internet.

Section five concludes by focusing on the debate concerning the access to essential facilities, with a special focus on the Telecommunications Industry. The problem of access, in fact, has been a key issue within the regulatory framework introduced by many super-national organizations (International Telecommunications Union, World Trade Organization, European Union, Federal Communications Commission, Organization for Economic Co-operation and Development, Asia-Pacific Economic Cooperation, Comisión Interamericana de Telecomunicaciones), each of whom played a relevant role towards the introduction of competition and the liberalization in the Telecommunications Industry.

## 2. Network externalities

### 2.1 "The snowplough metaphor"

This part focuses on the role played by networks in shaping agents' interactions. In such a context a network is intended to be an exogenous structure, which constrains and defines the potential relationships among individuals. Schelling (1978) provides some early examples where collective behaviour and local influence produce positive or negative feedbacks on individual decisions.

David (1992) introduces this interesting topic with his *"snowplough metaphor"*. He considers a circular city with N shops distributed along the main road at regular intervals. After a snowfall, every shopkeeper can make his own shop accessible again, only by sweeping away snow from his doorstep. But there are external effects to consider: "to make a customer visit a shop from the pavement, at least another shop, adjacent to the first, must have its pavement free". A shopkeeper's optimal strategy is thus affected by the actions of his next door neighbours: "to sweep away snow if his two neighbours have swept their threshold in their turn, not to clean in case they haven't, and to sweep with one half probability in case only one neighbour have cleaned its own threshold ". In this setting, thus, a set of strategies is based on local observations: every agent's decision depends on his neighbours' actions.

The snowplough model is characterized by two possible equilibrium configurations: everyone sweeps or none of them does; given any initial mixed state, the system will converge towards one of these two possible equilibria, depending on the realisation of the process. This outcome doesn't arise anymore when the structure of relations is more complex; for instance, when considering at least three spatial dimensions[1], or when the number of agents is infinite, in these cases we have continuous fluctuations between the two states characterized by uniformity of actions, without attaining convergence to either.

### 2.2 The equilibrium concept for a set of decisions incorporating local interaction

A network is defined by a finite set of individuals[2] , N, and by the links they have between each other. These links can be *one-way* or *two-way*.

a) *One way links.* Let us consider two web-sites in the Internet; it is possible that the first one has a link to the second, while the second does not have any link to the first. Such links are said *one-way*. Formally, they are represented by a binary variable, $g_{ij} \in (0,1)$, such as $g_{ij} = 1$ means that $i$ has a link with *j*, while $g_{ij} = 0$ means that $i$ doesn't have any link with *j.* A directed network $g$ is thereby defined by the set of *one-way* links:

$$g = \{g_{ij}\}_{i,j \in N}$$

b) *Two ways links*. This typology of links expresses reciprocal relationships, as in the case of family ties, or of a two-way street joining two places. In such a setting, either two agents are reciprocally connected, or they are not connected at all. Formally, a non-directed network $\overline{g}$ is outlined by the set of *two-way* links:

$$\overline{g} = \{g_{ij}\}_{i,j \in N} \quad \text{such as } g_{ij} = g_{ji} \ \forall\, i, j \in N$$

The connection between individuals[3], $i$ and $j$, in the network $g$, is required to model a potential interaction between them; in fact, the decision making process of any given agent $i$ is affected by the set of his neighbours:

$$N^d(i,g) = \{j \in N \mid g_{ij} = 1\}.$$

Since every agent's decisions can be represented by probability distributions over the set of possible actions, the situation where agents decide by only considering their neighbours strategies may be expressed by the following condition: a probability distribution $\mu$ of player ***i***'s strategies, given other players' strategies, equals the probability distribution $\mu$ given immediate neighbours' strategies only, $N^d(i,g)$".

In order to describe, both, the multiplicity of equilibria resulting from local interaction over networks and the dynamics of their selection over time, Durlauf (1993) introduces the following definition of equilibrium:
"An equilibrium exists when these conditional probabilities are compatible with a measure of joined probability for every agent at any time."

Deriving an equilibrium is thus equivalent to finding a joint probability measure, μ, matching the conditional probabilities at each node. In this context, the multiplicity of equilibria implies non-ergodicity of the process and this, in its turn, leads to the long run equilibrium depending on the realization of the history.

Spitzer (1971) focused on the uniqueness problem for such an equilibrium, and proved that: "Given a finite set of agents, $N$, a network indicating the interaction's architecture, $g = \{g_{ij}\}_{i,j \in N}$, a finite set of possible choices for each agent, $S$, and the description of the local probabilities affected only by the next door neighbours' choices; if all these probabilities differ from zero, then for each possible configuration of neighbours' choices, there is only one measure of global probability, $\mu$, compatible with every agent's conditional probabilities". In this case, thus, the introduction of spatial interaction does not affect the ergodicity of the process: independently from the initial conditions, the process' final distribution will be the one characterizing the only equilibrium.

## 2.3 Initial economic applications

A pioneering economic model formalising local interactions over networks, was formulated by Foellmer (1974). He presents an economy composed of agents reacting in the same way to an identical local environment. In his model there exist two goods and two exclusive preferences: S={+1,-1}. If $x_i = +1$ the agent *i* wants the largest possible quantity of good 1; while, if $x_i = -1$ the agent *i* wants the largest possible quantity of good 2.

Every agent's preferences are described as probabilities conditional on his direct neighbours' preferences. Moreover, Foellmer introduces the possibility of allowing a residual part of the individual's behaviour to be uninfluenced by his neighbours. The model aims to find an equilibrium price vector for the two goods clearing the market on average, as excess demands are generated by the stochastic preferences described above.

Foellmer shows that, if every agent's behaviour is -even in a small part- uninfluenced by his neighbours then, there won't be any phase transition[4] and it will be possible to determine a vector of equilibrium prices.

On the contrary, when the formation of each agent's stochastic preferences depends entirely on his neighbours' behaviour, there will be a critical value of the interaction intensity above which, for *n*-dimensional networks, with $n \geq 2$, there will be *n* equilibria compatible with the depicted microeconomic environment; in such a case, Foellmer proves that there exists no price vector which stabilises the economy. (See Allen, 1982a,b for extensions of this work on technological adoption).

Along similar lines, Durlauf (1993) introduced a stochastic growth model based on locally bounded positive externalities, whose intensity determines the likelihood of multiple equilibria. Durlauf proves that the higher is the degree of integration among industries (the number of industries that directly affect each other via technological complementarities), the greater is the likelihood of equilibrium multiplicity and non-ergodicity in the system.

**2.5 Structure and dynamics: the "sand pile" metaphor.**

We now focus on the role played by the morphology of the network interaction among productive units in explaining the instability of economic aggregates. The "sand pile model" shows the effect of aggregate individual shocks on the dynamics of the system, when the system's units transmit their own shocks through local interactions. In this model, a machine pours slowly and uniformly sand on a table, one grain at a time; first the grains lay where they fell; then they accumulate one on the other forming a pile with a small slope. Every now and then, when the slope becomes too steep, grains trickle down with small avalanches. When more sand is added, both the slope of the sand pile and the avalanches' average size increase. However, after a while, the pile does not increase anymore, because the added sand corresponds, on average, to an equal quantity of sand falling from the table. The sand pile has reached its critical state: even a single grain of additional sand might now create avalanches of any size, even catastrophic.

The self-organising feature of the system lies in its dynamics always tending to bring the pile of sand back to its critical state.[5] In this system large deviations occur very often, since their likelihood does not decline exponentially, but algebraically. Furthermore, the dynamic simulations of the model showed that while grains fall regularly, sand avalanches occur irregularly in time; according to Brock (1992), this phenomenon characterizes important economic series. Bak, Chen, Scheinkmann and Woodford (1993) consider a model of self-organised criticality, to describe an economy composed of a large number of productive units, each supplying a limited number of customers and, in turn, each supplied by a limited number of suppliers; both customers and suppliers are located near the productive unit. The graph outlining the location of productive units is a cylindrical lattice. The demand for each final good producer is characterized by stochastic fluctuations, which affects the variability of orders received by the suppliers. Such orders (and shocks) are locally and vertically correlated, as every final producer is supplied by the two upstream firms situated a line up along the network representing the productive system.

In such a context, characterized by local interaction Bak, Chen, Scheinkmann and Woodford prove that, if individual costs are non-convex, the aggregation of small independent individual shocks may lead to large aggregate fluctuations in the productive system, breaking therefore the law of large numbers.[6]

Agliardi and Giovannetti (1998) also obtained aggregate self-organised criticality when considering an economy influenced only by idiosyncratic shocks. Their model presents firms located on a mono-dimensional lattice, deciding whether to imitate the technology of an immediate neighbour. The imitation is profitable only if introduces neighbours' differentiation: such situation describes, in fact, an "anti-coordination game" where individual payoffs depend on local substitutability. Innovations, which are due to the agent obtaining the higher productive individual shock, provoke a sequence of adoptions, whose length relies on the technological configuration characterizing each node in each period. One of the model's aggregate properties, generated by the combination of micro-shocks and local interaction in the network, is the self-organisation of aggregate dynamics towards features of punctuated equilibrium (Gould and Eldredge, 1993); where, instead of regular evolution, the system is characterized by long periods of almost inactivity, followed by short periods in which catastrophic events take place.

## 3. Networks and strategic interaction

The previous sections described how the introduction of local or global interaction could induce non-ergodicity, path dependency and a possible explanation of the transition processes between dynamic equilibria. In this section we go one step further by looking at the effects of network relationships on the strategic interaction among players.

### 3.1 The role of different architectures

As previously mentioned, a network acquires its true meaning according to the relationships the players build over it. Here we begin by introducing a classification of the main network morphologies which will be used in the following sections. Let $g \in G$ be a *directed network,* according to the definition given in subsection 2.2. Such a network is said to be *complete,* and it is indicated with $g^c$, if $g_{ij} = 1 \quad \forall\, i, j \in N,$ with $i \neq j$.

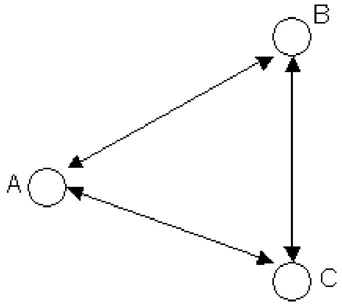



**figure 1: complete network**

In a complete network each agent is directly linked with all the others. A network $g \in G$ is said to be *empty,* and it is indicated with $g^e$, if $g_{ij} = 0$ $\forall\, i, j \in N,$ with $i \neq j$. In such a network there is no link between the agents.

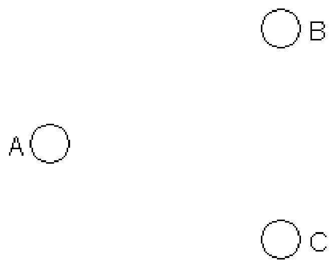



**figure 2: empty network**

Another relevant architecture is that of the *star* network $g^s$, obtained when $g_{ij} = g_{ji} = 1 \ \forall\, j \in N \setminus \{i\}$ and there are no other links.

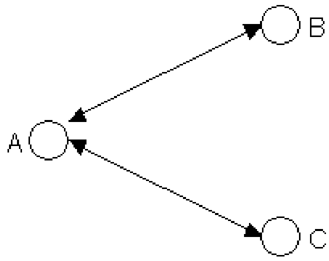



**figure 3: star network**

In a star network, all individuals are linked, but the number of links connecting any two agents is at most two, and there is, always and only, a unique path between any pair of players. Finally, a network $g \in G$ has a *wheel* architecture, $g^{w}$, when $g_{i_1 i_2} = g_{i_2 i_3} = ... = g_{i_{n-1} i_n} = g_{i_n i_1} = 1$ and there are no other links.

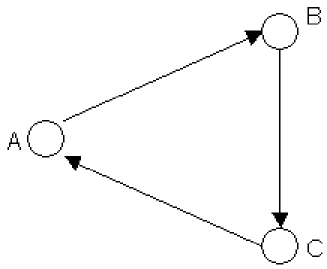
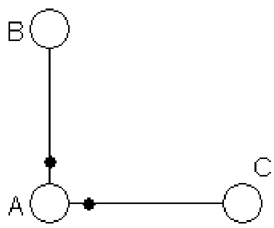



**figure 4: wheel network**

This is, for example, the circular city architecture often used in models a' la Salop of horizontal differentiation.

Among *non-directed* networks the *star network* displays some of the most interesting properties. When describing non-directed networks the star has the further specifications of being:

- *centre sponsored* when all links are built by the node-individual located at the centre of the star,
- *periphery sponsored,* when links are sponsored by the peripheral individuals.
- finally, there is a *mixed* star network, when some links are set up by the central player and some others by peripheral individuals[7].



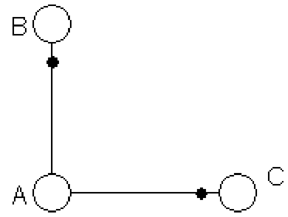




**figure 5.a: center-sponsored star** **figure 5.b: periphery-sponsored star**

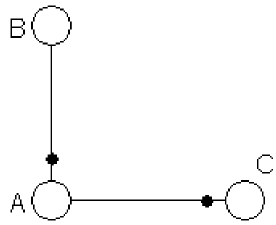



**figure 5.c: mixed-sponsored star**

Goyal (2003) studied the strategic effects of local interaction for a *coordination game* with agents exogenously located on network nodes.

In this framework the role of the network is that of providing, and limiting, the set of possible interactions among players. Indeed in this model players may only interact with others who are reachable, either directly or indirectly, along the set of links described by the network architecture. In such a context, Goyal proves that the *coordination outcome,* with each player choosing the same strategy is a Nash equilibrium for any given architecture, however in the case of a *complete network*, the *coordination outcome* is the unique Nash equilibrium of the game. With *incomplete networks*, there could instead be other set of strategies that form Nash equilibria.

Goyal also analysed the role of network architecture in the opposite scenario of *games of conflict*, that is when the players' decision variables are strategic substitutes. These games are well represented by the classical example of "the prisoner's dilemma", having only an inefficient Nash equilibrium in dominant strategies. The networks influence on the learning process and on individuals' choices is that they allow the observation of neighbours' strategies and payoffs. In this setting even in this *games of conflict* it is easier to sustain a co-operative solution without the need of introducing punitive strategies (Goyal, 2003).

Giovannetti (2000) considers a technology adoption game where, again, there is scope for *conflict* between firms competing for final demand in neighbouring markets. The market topology is à la Salop so that firms are located on a *wheel network.* This morphology, interpreted as a stylised geography, defines the conditions to obtain either symmetric equilibria, where every firms does the same thing: either adopting or non adopting the new technology, or asymmetric ones. The latter outcomes have been used to interpret the economic asymmetries often empirically observed among adjacent regions.

Lippert and Spagnolo (2004) described how co-operation, in a prisoner's dilemma game, can be supported by different network morphologies. They introduce the idea of network of relations: two agents are *related* when they co-operate, and have no incentive to deviate from co-operation in the repetition of the one shot game.[8] They study the stability of alternative network structures and their ability to support co-operation in the repeated prisoner's dilemma game. Lippert and Spagnolo especially focus on the conditions necessary for co-operation to exist even when individual incentives do not allow it (that is when the benefits from co-operation are unilateral and not bilateral). With a circular network[9], this result may be attained by aggregating punishments, to those who deviate from the co-operative agreement. The frequency of punishments will therefore depend on the information structure chosen as well as on the path length, and therefore on the network morphology.

The authors find that in a mixed *network of relations* – where reciprocally advantageous co-operative relationships coexist with unilateral ones-paradoxically agents with a bilateral benefit to the co-operation, though having a lower incentive to deviate, will also find a lower incentive to implement a punishment for a possible deviant from the co-operative agreement. A possible escape from such a paradox may be found in the construction of more articulate punishment strategies, able to limit the payoff losses from sticking to the punishment strategy for non-deviating players, making therefore co-operation more stable.

**3.2 Games on networks and Access**

One of the main issues of interest in the economics of networks concerns the definition of access prices. When networks are unidirectional, the access price is defined between who buys and who sells the access to a given production input. This happens following a precise hierarchical relation and often under monopoly conditions.

In case of a bi-directional network, that is when traffic flows in both directions of a two-ways link, each of the two firms at the edges of the link should pay the price of access to the essential input of the other firm, which often is simultaneously a competitor for the final demand. In this last case the network morphology looks like a horizontal hierarchy, with the two firms having reciprocal market power[10] expressed through the access prices.

A vast literature dealing with access prices (both for horizontal-bilateral or vertical-unidirectional relations) in strategic settings mainly related to telephony problems, and the consideration of its effects on social welfare, competition policy and regulation, is detailed in Laffont and Tirole (2001) and Armstrong (2002).

Introducing different network morphologies facilitates the economic analysis, in particular for antitrust cases, which often focused on the distinction of potential non-competitive effects between vertical and horizontal mergers among firms. A vertical merger between an essential input supplier and a firm operating in the downstream market may lead to a partial or total foreclosure[11]. The supplier may be, in fact, interested in preventing, partially or totally, its downstream division's competitors from accessing the essential input.

The reasons for such a foreclosure behaviour may be the aim of extending monopoly profits beyond the specific production segment of a wider composite goods, which is a possible goal when the monopoly comes about because of technological conditions (e.g. a bottleneck).

Ordover, Saloner and Salop (1990) studied in detail the economic incentives for a vertical merger between a (non-monopolist) seller of an upstream input and reseller of the final good. In this scenario, the new vertically integrated firm is likely to foreclose the access to its production input to the other firm which is now a competitor of its downstream division for the final demand. This foreclosure will make the production input market less competitive: non-integrated downstream firms will probably face higher input prices; hence, the integrated firm's downstream division will benefit from the input cost differential, while the final consumers will see their utility falling due to higher final prices. Thus this networked production structure allows the extension of market power to the linked markets.

The foreclosure's incentives are, however, strictly connected to what happens in the various markets: the integrated firm must take into account both the benefits produced by making its own division rivals' cost conditions worse, and the lost profits due to the decision to foreclose production inputs to these firms.

Salinger (1988) assumes that real foreclosure only occurs when vertical integration leads to higher input prices for non-integrated firms; he analyses a situation similar to that studied by Ordover, Saloner and Salop, and proves that, often, greater levels of vertical integration, may lead to decreasing input prices; and as a result, the final good's price will be lower than that under non-integration. Thus a vertical merger could be advantageous for final consumers, which differs from the findings of Ordover, Saloner and Salop.

In a similar context, but with the upstream producer being a monopolist, Rey and Tirole (2003) analysed a situation where asymmetric information leads the upstream firm to behave à la Cournot when supplying input to the downstream firms. Consequently, the profits of an essential input's producer will be lower than those obtained with symmetric information. The higher the number of downstream firms, the greater the lost profits will be; in this way upstream firm foreclosure represents now an opportunity to maintain monopoly profits and not a way to extend its monopoly power on other neighbouring markets. Rey and Tirole also prove that the incentives to engage in asymmetric information-led vertical foreclosure are inversely related to the differentiation characterizing the commodity supplied in the downstream market; thus networks may shape the relationship between market power and differentiation. Furthermore foreclosure incentives also depend on the specific location in the network of the "bottleneck" that generates monopoly power. Consider for example the case of an architecture with a reversed hierarchy from the one we just considered: i.e. composed of two upstream competitors and a monopolist in the downstream market, the latter has its monopoly power unaltered even with asymmetric information. In such a situation, the monopolist has no more incentives to vertically foreclose.

Rey and Tirole also analyse an architecture with two upstream firms and with asymmetric costs, so that production inputs for the downstream firms are no longer perfect substitutes. In this context, the top-level producer with higher costs only remains a potential competitor of the most efficient one, and the inefficient upstream producer induces a redistribution of profits through this industry even if it does not sell its input to the downstream firms. When the most efficient firm is vertically integrated, it will be encouraged to discriminate on prices when charging the downstream non-integrated firms, according to the Cournot model with asymmetric costs. Even in this occasion Rey and Tirole show how the incentives towards vertical integration increase as the upstream becomes less competitive and as the cost differential between the two upstream firms rises.

As already said, introducing the notion of network not only provides the possibility to better understand the differences between the strategic effects of vertical and horizontal mergers, but also allows to analyse behaviour based on other complex architectures as in Higgins (1997) who introduces the concept of *diagonal merger*: a merger between an upstream firm and a firm offering a good which is a substitute of the one offered by the downstream firm. Giovannetti (2003) also develops this analysis to define incentives for foreclosure, when dealing with a merger among firms simultaneously facing horizontal competition and a vertical production relationship. This kind of diagonal architecture is meant to capture the non-dedicated connectivity of Internet traffic flows. In such a context, incentives for a diagonal merger depend on the level of differentiation in the downstream market due, in turn, to consumer preferences about the method of accessing the network.

### 3.3 The Internet

The Internet is composed of many independent networks of very different size, located around the globe, all directly or indirectly interconnected with each other. This last feature guarantees the Internet's most important property: universal exchange of traffic between all end users (universal connectivity). The industry is still mainly unregulated, and networks are left completely free to decide where, how and with whom to interconnect.

Internet Service Providers (ISPs) are often rather small networks that sell Internet interconnection and related services to end users, businesses and consumers. They rely on connections to larger networks for the delivery of their customers' packets to their destinations outside the range of the ISP's own subscribers. The largest networks are called Backbones. These own or lease national or international high-speed fiber optic networks and deliver packets around the world for the many smaller networks connected to them. Backbones are not fully specialized in connecting other networks; in most cases they also reach businesses and consumers directly by operating own vertically integrated ISPs.

**Interconnection Agreements.**

Two simple types of interconnection agreements have emerged to regulate traffic at exchange points between networks: transit agreements and peering agreements.

In a *transit agreement*, a large network – the transit provider - offers access to the entire Internet to a smaller customer network against the payment of a fee often related to the capacity of the connection link. In other words, in a transit arrangement one network pays another one for interconnection and becomes a "wholesale customer" of the other networks, able to access all end users this other network can access through its other interconnection agreements.

Under a *peering agreement* two networks exchange the traffic directed to each other's end users only. Monetary settlements between peering partners used to be excluded, although recently some networks have started charging for peering (Miller 2002). Peering can be seen as a reciprocal, non-monetary exchange relationship that often implies various forms of cooperation. Peering also implies establishing direct exchange points between the two networks, and the costs of creating and maintaining the exchange points are typically shared evenly. Peering agreements may also be multilateral, and traffic exchanges may take place at private peering points or at organized exchange points such as Network Access Points (NAP) and Internet Exchange Points (IXP), specialized facilities where ISPs can connect to each other to exchange Internet traffic. NAPs and IXPs are typically 'public' internet exchange points where a switching system is provided to enable any member to exchange traffic with several other members. To peer at a NAP/IXP an ISP usually has to establish a connection and pay a membership fee, after which it can use the circuit to carry the aggregate traffic to all of the other members of the NAP/IXP with whom the ISP agree to peer. This makes Peering at a NAP/IXP of ISP cheaper than establishing a direct bilateral peering exchange point which would require installing a direct connection and many commercial deals. Being a member of a

IXP offer further advantages like sharing information and a free mutual technical help forum.

### 3.4 The Access Problem and Peering in the Internet

Prices and access conditions in network firms are relevant issues for the Internet. Here, in fact, *Internet Service Providers* (ISPs) exchange information flows with large upstream operators, *Internet backbones*[12], transferring these flows towards nodes-sites-agents representing the final destination of the original flows. The access to an essential production input is expressed, in this case, by the traffic flow transfer, following the Internet protocol (I.P.). The network of networks' success is strictly tied to the interconnection system between providers constituting the Internet: this system, in its turn, depends on the links' efficiency and reliability, on the ability to cover different geographical zones and especially on the means of access to the network physical structures.

The access problem, in the specific context of the Internet, has been studied by Crémer, Rey and Tirole (2000). They propose a duopoly where *backbones* compete to attract new customers, assuming that switching from one backbone to another is very expensive.

Crémer, Rey and Tirole consider a two-stage game. In the first stage of the game the two backbones make a free choice about the level of interconnection quality, which will be determined as the lower of the two backbone's quality levels. In the second stage, the backbones engage in Cournot quantity competition.

The model solution is immediate in the case of different market shares: the best strategy for the backbone with more customers consists in offering a low quality of interconnection to the other backbone[13]. The largest backbone will keep increasing its market share to the detriment of the smallest. The results of the model do not change if one introduces a market with more competitors, each having the same market share.

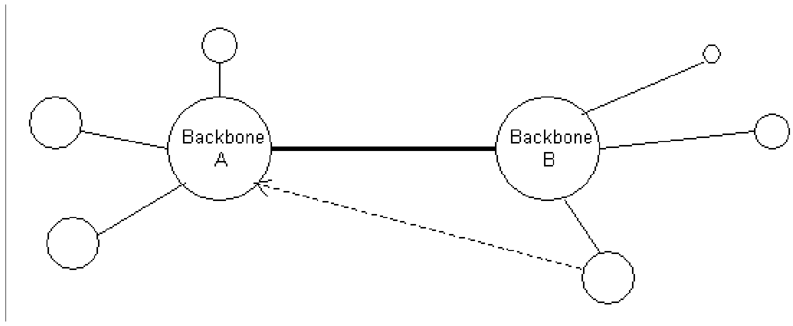



**figure 6: backbones duopoly**

Foros and Hansen (2001) introduce the hypothesis of horizontal differentiation between duopolist backbones via a Hotelling model of price competition with spatial differentiation, obtaining a totally different result (see also Roson, 2002, for a comparative evaluation of the two works).

As in Crémer, Rey and Tirole model, two operators first decide interconnection quality, then they engage in Bertrand price competition, maximizing interconnection quality, given the associated cost function.

The complex relation of complementarity and synergy between backbone's operators in the Internet, and the way in which it affects the issue of interconnection, is also analysed in the work of Laffont, Marcus, Rey and Tirole (2003). They consider a backbone competition model, allowing for two types of customers: web sites and users. The only relevant traffic is that between websites and users. Cost structure is indicated by $c_0$ and $c_1$, the cost of originating and terminating traffic, respectively. Backbones fix different prices for the websites, which produce and transmit information, and for the users, who only receive information. Thus backbones' profits are represented by:

$$\pi_i = \alpha_i \tilde{\alpha}_i [p_i + \tilde{p}_i - c] + \alpha_i \tilde{\alpha}_j \ [p_i + a - c_1] + \alpha_j \tilde{\alpha}_i (\tilde{p}_i - c_0 - a)$$

where $c = c_0 + c_1$; $\alpha_i$ and $\tilde{\alpha}_i$ define the backbones' market shares among consumers and websites, respectively; $a$ is the interconnection rate (the same for both the backbones) to distribute traffic originated from the other backbone, *off-net*, to the final users. Finally $p_i$ is the final price paid by the consumers to the backbone, in order to receive the required information; while $\tilde{p}_i$ is what websites pay to send information. The profit function measures the backbones' complementarity and synergy:

1. The first component, $\alpha_i \tilde{\alpha}_i [p_i + \tilde{p}_i - c]$, shows profits related to the traffic between consumers' demands and websites, both being customers of the same backbone. We must deduct the transmission cost, $c = c_0 + c_1$ from the revenues.
2. Then there are profits related to traffic between backbone *i*'s final users and backbone *j*'s websites: $\alpha_i \tilde{\alpha}_j \ [p_i + a - c_1]$. Revenues are given by the final price (for users) and by the terminating rate (access price), $a$, paid by the competitor backbone, minus the terminating costs already deducted.

3. Finally, we have profits related to traffic between backbone *i*'s websites and backbone *j*'s final users: $\alpha_j \tilde{\alpha}_i (\tilde{p}_i - c_0 - a)$. Backbone $i$ gets the transmission price, while paying both the termination cost and the originating cost.

First, the access price $a$ is determined, through *bargaining* or by the intervention of an exogenous authority; second, the backbones fix their prices, $p_i, \tilde{p}_i, i = 1,2$, and third, consumers choose backbones.

Laffont, Marcus, Rey and Tirole show the conditions for which there is a unique symmetric price equilibrium, such that the price for consumers equals the termination cost minus the access price, $p = c_1 - a$, while the price for websites equals origination cost plus the access price, $\tilde{p} = c_1 + a$. Thus, the price for consumers equals the opportunity cost from losing a final customer to the competitor. This result models the *business stealing*[14] effect. A second possibility of *business stealing*, concerns winning a website from the rival, which generates, for each consumer connected to such a site, a cost like $c_0 + a$[15]. Therefore, with backbones being perfect substitutes and with perfectly inelastic demand, Bertrand price competition makes profits non-existent whatever the access charge, and prices for the consumers and for the websites correspond to the traffic cost:

$$p_i + \tilde{p}_i = (c_0 + a) + (c_1 - a) = c$$

Hence the access price, $a$, determines the way websites and consumers share traffic costs. The higher the access charge, the greater will be the cost paid by the websites.

This model, where equilibrium prices satisfy the *off net* cost principle, can be extended allowing for an arbitrary number of backbones, mixed traffic flows with elastic demand (depending on price), differentiated quality for the services offered and, finally, differentiation of both origination and termination prices, either among final consumers or among websites[16]. This principle is still essential where market power produces a *mark-up* on costs, as, for instance, with horizontal differentiation, when backbones are on the opposite extremes of a linear space and with websites uniformly distributed between them.

Giovannetti (2002) studies the impact of network interconnection on prices, both retail and access ones, and on providers' profits. He focuses on networks that have no fixed hierarchies like in the Internet where two providers owning different network nodes can be at the same time vertically related, as supplier and retailer, in a routing process while being horizontally competing for the routing of different traffic.

In his paper Interconnection is represented as a change in the pre-existing network architecture. After interconnection a new dimension is added to the original network design and the former monopolist starts competing, as a retail service provider, against its downstream retailer for the routing of this transit demand. The author finds that interconnection lowers retail and access prices only when the downstream industry is poorly differentiated. Also the profitability of interconnection also depends on the degree of competitiveness of the retail sector.

Norton (1999) observes that the access to the physical structures to realize interconnection (both with *Transit* or with *Peering*) is controlled by competitors limiting the realization of economies of scale. Having said that, Norton defines an ideal model where *Carriers* sell circuits to the ISPs, which exchange traffic between them through *Transit* or *Peering* agreements. This model is a theoretical evolution of the *Exchange-based Model*, with a meeting point where ISPs may exchange traffic. The opposite scheme, alternatively represents the *Direct Circuit Interconnection Model*, where all the ISPs exchange traffic through direct links.

The first architecture is a star network, where the central player acts like a co-ordinator; the second defines a complete network.

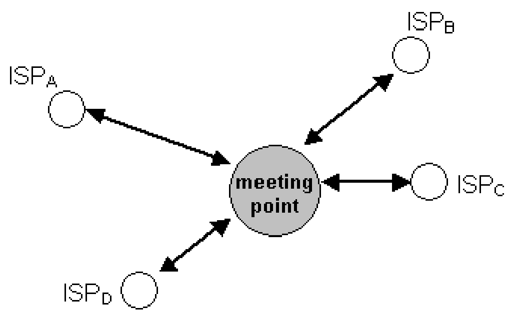



**figure 7: Exchange-based Model**

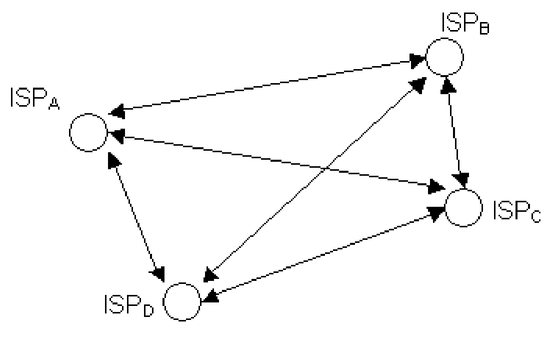



**figure 8: Direct Circuit Interconnection Model**

Norton proves that the first architecture is the most efficient, not only in solving the access problem, but also to have better interconnection efficiency (for instance, there is an advantage in managing centralized circuits in comparison with circuits located in many different places).

### 3.5 The Internet peering decisions

Earlier work has identified several factors and problems that may affect networks' decision whether and with whom to peer (see Huston 1999; Baake and Wichmann, 1999; Kende, 2000; and Filstrup 2001). A first, rather obvious factor is size. Since Peering requires establishing bilateral traffic exchange points, which entail fixed and variable technological costs, it follows that a sufficiently intense traffic flow between the end users of the two networks is a necessary precondition for peering to be economically viable. The larger two networks are, the more intense will be the traffic between their end users, and therefore networks' size is a determinant of the Peering decision.

In fact, almost all large backbone network peer with each other, the traffic being exchanged at several interconnection points homogeneously distributed on the relevant geographical areas. Somewhat smaller networks also peer with networks of comparable size, but typically have to supplement their interconnection with transit agreements with backbone networks.

Since the costs of setting up and maintaining peering points are usually shared equally by peering networks, unbalanced traffic implies an unbalanced distribution of gains from peering against a balanced distribution of costs, a rather unfair settlement. Such unbalanced situations have developed in some cases, and have led to the discontinuation of the peering arrangement, and to its replacement with a transit one. This highlights the relevance of networks asymmetries as an other determinant of the peering decision.

The theoretical approach on the analysis of peering choices when the interconnection structure is exogenous has mainly focused on the quality of the Internet services offered; this quality, in fact, largely depends on the nature of the interconnection arrangements that characterize each Internet provider's network.

Baake and Wichmann (1999) consider a situation in which two ISPs, *i* and *j*, are interconnected by a link of low quality, which can be improved by a peering agreement. Both providers are in Cournot competition with each other.

Let $\alpha_i$ and $\alpha_j$ be the users[17] connected to the respective providers, $\delta$ be the interconnection quality, and $p_i$ and $p_j$ be the prices of the services offered. The main assumptions of their model lie in the ISPs' profit functions, given by

$$\pi_i = p_i(\alpha, n_i)\alpha_i - c_i(\alpha_i, \alpha_j, \delta)$$

where $\alpha = \alpha_i + \alpha_j$ and $n_i = \alpha_i + \delta\,\alpha_j$ , with $0 < \delta \leq 1$ .

The first term of the profit function takes into account both the classic quantity effect and also a network effect: Baake and Wichmann assume, in fact, the price $p_i$ to be an inverse function of $\alpha$ , total number of internet users, and a direct function of $n_i$ , which can be considered a weighted average of the community of internet users, calculated by taking into account the quality of interconnection between them. Formally, $p_i = p_i\left(\underset{-}{\alpha}, \underset{+}{n_i}\right)$.

The costs borne by a provider, instead, are assumed to increase with the number of both its users and the other provider's users (due to their effects on traffic); the peering decision is assumed to lead to lower costs, since it avoids transit costs . Formally, $c_i = c_i\left(\underset{+}{\alpha_i}, \underset{+}{\alpha_j}, \underset{-}{\delta}\right)$

In the Cournot competition, each provider chooses its optimal network size, taking as given the network size of the second provider and the quality of interconnection. The intersection between the reaction functions $\alpha_i^* = \alpha_i^*(\alpha_j, \delta)$ and $\alpha_j^* = \alpha_j^*(\alpha_i, \delta)$ determines the market equilibrium.

Baake and Wichmann assume that the ISPs are in equilibrium, and then analyse the incentives for them to engage in a private peering. They find that peering might be advantageous even if it leads to a smaller market share. They are however unable to derive general conditions that make peering profitable; such a result is due to the fact that peering decisions affect not only costs, but also prices.

A second approach that models peering decisions through their effect on the quality of the service offered, has been introduced by, Laffont, Marcus, Rey and Tirole, who, as we have previously shown, have also analysed the access problem for Internet backbones. The authors consider a backbones' customer (a large ISP), which is connected to the backbone via a transit agreement, and analyse its incentives to substitute it with a peering relationship; they assume that backbones are perfectly replaceable for final customers (ISPs, mainly), while they are horizontally differentiated for websites. The authors examine a game where:

1. In a first stage, two backbones engage in a Peering agreement, with the access rate defined by $a$;
2. In a second stage, a particular ISP offers the two backbones a Peering agreement;
3. In a third stage, backbones make their offers to ISPs and websites.

In such a model, a backbone never loses its "interconnectivity", as it peers with the other backbone, and so though a backbone may not be directly connected with an ISP, it will be indirectly connected to it through the Peering agreement with the other backbone and through Transit between the other backbone and the ISP.

Stage two is crucial, as its outcome deeply affects the third stage of the game. In fact, if an ISP enters into a peering agreement with one of the two backbones, the backbone involved will be able to offer to the websites and other ISPs a better quality than the other backbone, which loses connectivity with the ISP. Laffont, Marcus, Rey and Tirole prove that the ISP will decide to exploit this situation where backbones have market power with respect to websites: he will thus offer a peering rate lower than the *off net* cost paid for transit. As a result, the ISP will gain from engaging in peering relationships with the backbones, whose profits, instead, decrease.

# 4. Endogenous network formation

We have now reached our last topic: the study of the incentives in forming links among economic players, and the resulting network morphologies. Three main theoretical approaches to networks formation have been studied:

a) a cooperative approach, where the link formation process is two-sided: both individuals have to agree for the link to be formed;
b) a non-cooperative approach, where the link formation process is, instead, one-sided: links can be unilaterally formed;
c) a mixed approach, which takes from both of the above theories.

In the following we discuss these three approaches and some relevant applications.

### 4.1 The cooperative approach

The cooperative approach, introduced by Myerson (1997), can be considered as a natural development of the theory that studies the formation of coalitions: this theory analyses the external structure of coalitions by focusing on the allocation of players within the various groups only; the purely cooperative approach goes further and considers the set of communication components within each coalition, where a communication component is a set of players all reciprocally connected, maintaining no links with outside players

Myerson's approach was followed by Aumann and Myerson (1988), Dutta, van den Nouweland and Tijs (1998), Calvo, Lasaga, and van den Nouweland (1999), Slikker and van den Nouweland (2000, 2001), and many others (see van den Nouweland, A., 2004, for a detailed survey). Aumann and

Myerson (1988) consider a two-stage game: in the first stage links are formed, and in the second stage the players receive payoffs, which depend on the value of the network, according to an exogenous rule. Dutta, van den Nouweland and Tijs (1998) consider a different two-stage game. Following Aumann and Myerson (1988), links are formed in the first stage, while in the second stage players get their payoffs according to an exogenous rule. However, some differences arise. The link formation process is static: accordingly, they consider a normal form game, while Aumann and Myerson use the extensive form to represent the game. In the model considered by Dutta et al., every player announces (simultaneously) to whom of the other players he wishes to link and a link is formed if both agree.

Both Aumann and Myerson (1988) and Dutta *et al*. (1998) assume that players don't bear any cost while forming links (link formation costs may be included in the allocation process). Slikker and van den Nouweland (2000), instead, introduce an explicit link formation cost variable and study how different levels of that cost influence the network formation process.

Calvo, Lasaga and van den Nouweland (1999) extend Myerson's first model by introducing the concept of a probabilistic graph, which they use in order to allow for the possibility of link failure. Slikker and van den Nouweland (2001), consider a one-stage model, where the cooperation structure and the allocation of payoffs are set simultaneously.

### 4.2 Pairwise stability

Jackson and Wolinsky (1996) introduce the concept of pairwise stability in order to analyse the network formation process. They consider a two-sided network formation process: on the one hand, both agents must agree for a link between them to be set; on the other, players can unilaterally severe links. The game is given by $(N, v, \{Y_i\})$, where $N$ is the finite set of players, $v: G \to R$ is a function that gives the value of a generic network *g*, for each $g \in G$, while $Y_i(v, g)$ is the allocation rule describing the way in which the value of the network is allocated among players. $ij \in g$ means that players *i* and *j* are linked to each other, $ij \notin g$ means the opposite. Let *g* be a network such that $ij \notin g$; then, $g + ij$ is the network obtained when the link between *i* and *j* is set and $g - ij$ means just the opposite.

A network *g* is pairwise stable if both of the following conditions hold

$(i)$ $\quad \forall\ ij \in g,\ \ Y_i(g,v) \geq Y_i(g-ij,v)$ and $Y_j(g,v) \geq Y_j(g-ij,v)$

$(ii)$ $\quad \forall\ ij \notin g,\ $ if $Y_i(g+ij,v) \geq Y_i(g,v)$ then $Y_j(g+ij,v) < Y_j(g,v)$

In words, a network *g* is pairwise stable if, given any two linked agents, none of them benefit from severing the link, and if, given any two unlinked agents, it cannot be that both of them find it convenient to form the link.

The concept of pairwise stability is very helpful to both evaluate a given network topology and determine the set of equilibria, but it says nothing about their desirability. Jackson and Wolinsky showed that pairwise stable networks are not always efficient (a network is said to be efficient if it maximizes the sum of players' payoffs). In order to prove that, they consider two different models: The Connections Model, characterized by positive network externalities, and the Co-Author Model, where externalities are negative.

**The Connections Model**

In the Connections Model the positive network externality is expressed by introducing a direct relationship between a player's payoff and the number of other players he can observe (either directly or indirectly, via the links set by other players); the benefit decays with the distance and the links are costly. The payoff function is given by

$$\pi_i(g) = w_{ii} + \sum_{j \neq i} \partial^{t_{ij}} w_{ij} - \sum_{J:ij \in g} c_{ij}$$

where $w_{ij}$ is the benefit that *i* gets when he observes *j*, $t_{ij}$ is the geodesic distance between *i* and *j* (the geodesic distance between two players is defined as the number of links in the shortest path between them), $\partial \in [0,1]$ is the information transmission decay rate and $c_{ij}$ is the cost of the link between *i* and *j*. The authors then analyse the conditions for pairwise stability and efficiency[18] in this model. The main finding obtained by Jackson and Wolinsky is that there is only one efficient network architecture, but its architecture depends on the link formation cost *c*. Particularly, in a symmetric model where

$w_{ii} = 0, c_{ij} = c, w_{ij} = w$, they show that if *c* is small the only efficient architecture is the complete network; if *c* is very high, instead, it turns to the empty network; finally, for intermediate values of *c*, the star network is the only efficient architecture. Pairwise stability depends on *c*, too: the complete network is the only one stable if *c* is small, while if *c* is high it turns to the empty network; for intermediate values of *c* there may be several stable architectures, including the star network. The set of *c*-values for which the empty network is stable is larger than the set of values for which the empty network is efficient: that outcome is due to the fact that players don't take into account the positive benefits from indirect links when they bilaterally decide to form a link.

**The Co-Author Model**

The Co-Author Model analyses an environment characterized by negative externalities. Each author is given a unit of time, which is equally allocated to $n_i$ different projects; the value of a project depends on the total time that the two co-authors put into it, $\left[\frac{1}{n_i} + \frac{1}{n_j}\right]$, and on the synergy between them (that is, how much time they spend together), $\frac{1}{n_i n_j}$; this is the factor that produces the negative externality. Player *i*'s payoff is thus given by

$$\pi_i(g) = \sum_{j:ij \in g} \left[\frac{1}{n_i} + \frac{1}{n_j} + \frac{1}{n_i n_j}\right]$$

Jackson and Wolinsky show that the efficient architecture is characterized by $N/2$ pairs of co-authors. Moreover, they prove that players agree to set links only with agents who have a smaller number of links. That typically causes a pairwise stable network to be over-connected, like the complete network; efficient architectures, such as the star network, are not pairwise stable.

The Co-Author Model shows that individual incentives (expressed by the concept of pairwise stability) don't lead to an efficient outcome. Such a result is due to the nature of both the value function $v$ and the allocation rule $Y$. The analysis of the relationship between stability and efficiency is a challenge for the part of economic research whose main goal is to find a set of conditions that assures outcomes that are both stable and efficient. In reply, several papers study the properties of given classes of value functions (see Jackson, 2003), while others focus on different concepts of efficiency.

### 4.3 Extensions

The concept of pairwise stability asks us to test for the conditions (i) or (ii) for each pair of players[19] separately. Let $g$ be a pairwise stable network and $i$ a player that has many links in $g$: by definition, this player never finds it profitable to remove one of his links only, but he may find it profitable to remove more links simultaneously. By definition, again, there are no pairs of players who might wish to change their strategies, but there may exist a coalition $S$ of players who all take advantage from revising their strategies simultaneously. In both of these cases, conditions (i) and (ii) hold, but the network $g$ is some way from being stable.

Dutta and Mutuswami (1997) and Jackson and van den Nouweland (2001) have strengthened the concept of pairwise stability, allowing for coalitions other than just pairs of players to deviate.

The following concept of strong stability is due to Jackson and van den Nouweland. Let $g$ be a network and $S \subset N$ be a coalition of players; a network $g' \in G$ is obtainable from $g \in G$ via deviations by $S$ if any new link involves players in $S$ only, and at least one player for any deleted link is in $S$.

A network $g$ is strongly stable if, given a value function $v$ and an allocation rule $Y$, for any $S \in N$, $g'$ that is obtainable from $g$ via deviations by $S$, and $i \in S$ such as $Y_i(g',v) > Y_i(g,v)$, there exists $j \in S$ such that $Y_j(g',v) < Y_j(g,v)$.

While pairwise stability considers degenerate coalitions, formed by two players only, strong stability focuses on coalitions in general, without any restriction on the number of players involved, providing a considerably stronger concept of equilibrium. Of course, the use of this refinement makes sense only when players can coordinate their actions, thus it is especially useful when we wish to examine small networks.

Watts (2001) extends Jackson and Wolinsky's static model by introducing a dynamic network formation game. In particular, given an initial network $g$, she considers a sequential process where links are formed or severed one at time; players only look at their payoffs when they decide whether to add or delete a link. In such a situation each equilibrium is pairwise stable, while not always efficient. Jackson and Watts (2002) introduced stochastic perturbations to the network formation process: in this setting, the process goes on forever, and never converges to architectures that may be locally optimal.

A further development is due to Johnson and Gilles (1999); they introduce the hypothesis of players' spatial heterogeneity, by assuming the link formation cost is positively correlated with the geodesic distance between the two players involved.

### 4.4 Non-Cooperative Models of Networks Formation

In the non-cooperative approach to network formation, introduced by Bala and Goyal (2000a,b), players can add or delete links unilaterally; thus there is non need for an agreement between the players in order for a link to be set: this is the main difference between this approach and the previous ones.

Bala and Goyal (2000a) consider a set of players and assume that each of them possesses some information, which can be augmented by connecting to other players (information is assumed not rival). Hence, a player *i*'s benefit increases with the number of players that he can observe, either directly or indirectly, via the links formed by other players. Links are costly for the players who form them[20]. Let *i* be a player who has a link with *j*. Bala and Goyal consider two different patterns of information transmission: in the one way flow models, the information goes from *j* to *i*, while in the two-way flow models, the information goes from and to both players.

**Nash Networks**

Bala e Goyal (2000a) introduce the network game $\Gamma = (N, \{G_i\}, \{\Pi_i\})$, where $\{N = 1,...,n\}$, with $n \geq 3$, is the set of players $G_i$ the set of strategies for the generic player $i$. A pure strategy for the generic player $i$ is a vector $g_i = (g_{i1},...,g_{ii-1},g_{ii+1},...,g_{in}) \in G_i$, such that $g_{ij} \in \{0,1\}$ $\forall j \in N \setminus \{i\}$ [21]; $\Pi_i$ is the payoff.

Every profile of strategies $g \in G$, where $G = G_1 \times ... \times G_n$, identifies a network. Thus, a *Nash Network* is a profile of strategies $g^* \in G$ which is a Nash equilibrium for $\Gamma$, that is

$$\Pi_i(g_i^*, g_{-i}^*) \geq \Pi_i(g_i, g_{-i}), \quad \forall g_i \in G_i \quad \forall i \in N$$

The payoff function of each player $i$ depends on both the number of players that are observed by $i$ and the number of links that he has formed. Namely, let $N^d(i,g)$ be the set of players with whom $i$ maintains a link, and $N(i,g)$ the set of players observed by $i$. The number of elements in these sets is respectively given by $|N^d(i,g)| = \mu_i^d(g)$ and $|N(i,g)| = \mu_i(g)$. Player $i$'s payoff is thus given by $\Pi_i(g,) = \Phi(\mu_i(g), \mu_i^d(g))$.

In particular, the total cost that player $i$ bears is given by $c\mu_i^d(g)$, where c is the cost for a single link (see Vergara Caffarelli, 2004 for an extension of the model, which considers also a network maintenance cost).

**Properties of Nash Networks**

Without information decay, Bala and Goyal prove that:

- In the one-way flow models, a Nash Network is either empty or minimally connected.[22]

- In the two-way flow models, a Nash Network is either empty or minimally pairwise-connected.

Thus, regardless of the information transmission pattern, in a Nash Network either each player observes no one or each player observes every other player and there are no redundant links.

Bala e Goyal show that, in general, the number of Nash Networks increases very rapidly with *n*. That leads them to focus on Strict Nash Networks, that is Networks supported by a Strict Nash Equilibrium for $\Gamma$. The set of Strict Nash Networks is significantly more restrictive. Bala and Goyal prove that:

- In the one-way flow models, a Strict Nash Network is either empty or a wheel.
- In the two-way flow models, a Strict Nash Network is either empty or a center-sponsored star.

The importance of these results is weakened by the strong assumption that information obtained through indirect links has the same value as that obtained through direct links. Hence, Bala and Goyal (2000a) move away from this assumption, allowing for information decay. In particular, they introduce the decay parameter $\delta \in (0,1]$ and assume that the value of information possessed by each agent is 1. Thus, the value of agent *j*'s information to *i* is $\delta^{d(i,j;g)}$, where $d(i,j;g)$ is the geodesic distance between *i* and *j*.

Bala and Goyal show that in the one-way flow model with information decay, non empty Nash Networks are not necessarily connected. The analysis is more complex and they have to consider a less general model to obtain restricted classes of equilibrium architectures. Hence, they assume the payoff function to be linear, given by

$$\Pi_i(g) = 1 + \sum_{j \in N(i,g) \setminus \{i\}} \partial^{d(i,j,g)} - \mu_i^d(g)c$$

and prove that a Strict Nash Network is either connected or empty. Similarly, in the two-way information flow pattern, they consider the payoff function

$$\Pi_i(\bar{g}) = 1 + \sum_{j \in N(i,\bar{g}) \setminus \{i\}} \partial^{d(i,j,\bar{g})} - \mu_i^d(\bar{g})c$$

and show that a Strict Nash Network is once again either connected or empty.

Bala and Goyal (2000b) consider a two-way information flow pattern and extend Bala and Goyal (2000a), moving away from the assumption of perfect reliability of the links and allowing for each of them to fail with positive probability. In order to represent the assumption of imperfect reliability, to add realism to the model, they introduce a parameter $p \in (0,1)$: each link works with probability $p$, while fails with probability $1-p$. Hence, a profile of strategies $g \in G$ now identifies a stochastic network.

The main findings of this model show that imperfect reliability leads to links' redundancy, and the greater the number of players, the greater level of superconnectivity will be present in the resulting equilibrium architectures. [23]

Bala and Goyal (2000a,b) assume homogeneous agents. Haller and Sarangi (2003) generalize Bala and Goyal's model with imperfect reliability, allowing for the heterogeneity of agents[24]. Like in Bala and Goyal, a network is a set of probabilistic links[25], but they go further and assume that each link's failure probability depends on the players involved. Haller and Sarangi prove that Bala and Goyal's (2000a) result (a Nash Network is either connected or empty) holds only when the failure probabilities are not very different among the links. Otherwise, Nash Networks may be partially connected.[26] Moreover, Haller and Sarangi deal with some of the disputed aspects of the non-cooperative approach. In particular, they extend Bala and Goyal's model, introducing the assumption of imperfect information: they prove that redundant links are formed when player's beliefs about the reliability of indirect links are lower than the effective probabilities.

A different approach to the heterogeneity of players is due to Galeotti and Goyal (2002). They consider Bala and Goyal's two-way flow model without information decay, and assume that the link formation costs and the related benefits depend on the players involved; hence, the payoff function is given by $\pi_i(g) = \sum_{j \in N(i,g)} b_{ij} - \sum_{j \in N^d(i,g)} c_{ij}$. They prove that, in such a situation, every Strict Nash is minimal.[27]

Galeotti and Goyal show also that, given any network *g*, then there exists a list of individual costs and benefits for each player such that the network *g* is Nash.[28] In order to get restricted classes of equilibrium architectures, Galeotti and Goyal consider a less general model, in which the link formation costs and the related benefits depend on the player who formed the link only: in this new setting they show that a Strict Nash Network is either a center-sponsored star, a collection of center-sponsored stars, or the empty network.

**Further developments**

The importance of the non-cooperative approach is weakened by two considerations about how much it fits reality. According to Jackson (2003), in most social and economic interactions, links are formed when both agents agree; moreover, when the information flow is two-way, Nash Networks may have redundant links.

A different approach, which borrows from both the non-cooperative and cooperative models, has been suggested recently by Goyal and Joshi (2003b); they introduce the concept of pairwise stability as a refinement of Nash Equilibrium.

In their model, homogeneous players decide simultaneously with which of the remaining players, they wish to link[29]: a link is effectively constructed only if both players agree; the set of links therefore identifies a non-directed network. Goyal and Joshi consider the network game introduced by Bala and Goyal (2000a), with the only difference that a pure strategy for player $i$ is a vector of desired links $s_i = (s_{i1},...,s_{ii-1},s_{ii+1},...,s_{in}) \in \mathrm{S_i}$. The originality of their approach lies in the concept of stability they suggest. Particularly, given the network game $\Gamma(f)$, a profile of strategies $s^* = \{s_1^*, s_2^*,...,s_n^*\}$ is a Nash equilibrium if $\Pi_i(s_i^*, s_{-i}^*) \geq \Pi_i(s_i, s_{-i}^*) \quad \forall s_i \in S_i, \quad \forall i \in N$. Unfortunately, in such a context, the empty network is always Nash, as it is always mutually optimal not to offer to form a link. The authors get through this problem by adding a second condition for a network to be stable. A network $g$ is a pairwise stable equilibrium when both of the following hold:

a) $g$ is a Nash equilibrium for $\Gamma(f)$

b) $\forall\ g_{i,j} = 0$, $\Pi_i(g + g_{i,j}) - \Pi_i(g) > f \ \Rightarrow\ \Pi_j(g + g_{i,j}) - \Pi_j(g) < f$ where $f$ is the link formation cost.

It is worth noting that (b) represents only the (ii) condition for Jackson and Wolinksy's pairwise stability. In fact, the condition (i) is included in the Nash equilibrium concept: if *i* and *j* are linked in *g*, and *g* is Nash, then, by definition, no player gains any advantage from severing the link. On the other hand, the condition (b) is very important, because it prevents the empty network from always being an equilibrium.[30]

In general, a Nash Network where at least one link is missing is not stable, according to Goyal and Joshi, as there exists a pair of non-linked players who would both benefit from the link. Thus, pairwise stable equilibrium networks constitute a subset of the Nash equilibria of the game $\Gamma(f)$.

**4.5 Research and Development Networks**

Perhaps, today one of the most important productive input is information. The issues concerning how firms share their information, thus, can be considered as an application of the problem of access: in particular firms can be seen in a mutually vertical relationship, where each firm possesses important information, which is relevant to the other firms.

The literature focusing on R&D links, thus, encompasses both the issue of access and the endogenous networks formation.

The benefit from joining a Research and Development network can be expressed by a reduction in marginal cost, so that

$$c_i(g) = \beta_0 - \beta\eta_i(g), i \in N$$

where $\eta_i(g)$ is the number of firms directly linked to firm *i*.

In such a situation, where the network affects the firms' costs, the formation of a cooperative structure among the firms is strongly (and inversely) related to the level of competitiveness characterizing the market in which they operate. When firms are Bertrand's competitors, firm *i*'s profit function is given by

$$\begin{aligned} &\pi_i(g) = 0, \text{if} \quad c_i(g) \geq c_j(g) \text{ for at least one } j \neq i \\ &\pi_i(g) > 0, \text{if} \quad c_i(g) < c_j(g)\ \forall j \neq i \end{aligned}$$

Such a level of competitiveness nullifies any incentives to collaborate, and so the only stable architecture is the empty network (reciprocal foreclosure).

Goyal and Joshi (2003) assume, instead, that firms are Cournot competitors, and obtain different results: each stable network is characterized by a dominant group architecture, that consists of a complete component with at least two nodes and other single-node components only.[31] Hence, there will be a set of firms *N'* such that

$$\begin{aligned} &g_{ij} = 1 \ldots \forall i, j \in N' \\ &\text{while} \quad g_{k,l} = 0 \quad \forall k \in N \setminus N', \forall l \in N \setminus \{k\} \end{aligned}$$

(the foreclosure is thus expressed by the presence of a group of *N'* integrated firms, and *N-N'* non integrated firms)

Goyal and Moraga-Gonzales (2001) consider a more complex model, where the network effects on the marginal cost are endogenous: the benefit from joining a R&D network depends now not only on the network's architecture, but also on the firms' individual efforts (investments) in R&D. Formally, the marginal cost function is given by

$$c_i(g) = \beta_0 - \sum_{k \in N(i,g)} \gamma_k \, , i \in N \, , \beta_0 > 0$$

where $\gamma_k$ is the level of investments of firm *k*.

When the firms operate in independent markets, Goyal and Moraga-Gonzales show that the complete network is the only stable[32] architecture. When firms are, instead, Cournot competitors, they obtain more interesting results[33]: among symmetric networks (each firm maintains the same number of links), the complete network is stable, while the empty network is not.[34] They then generalize their model, allowing for networks to be asymmetric and introducing an element that represents the spillovers between non connected firms; the marginal cost function is given by

$$c_i(g) = \beta_0 - \sum_{k \in N(i,g)} \gamma_k - \beta_1 \sum_{l \notin N(i,g)} \gamma_l \, , i \in N \, , \; 0 < \beta_1 < \beta_0$$

In this new setting, Goyal and Moraga-Gonzales consider a model with three firms only, and obtain results close to Goyal and Joshi: the complete network is always stable, while a partially connected network is stable only if $\beta_1$ is small (that is, incentives to foreclose increase when the spillovers from non connected firms are small).

**4.6 Endogenous Peering in the Internet**

We now analyse peering in the context of endogenous network formation. Badasyan and Chakrabarti (2003) have proposed an interesting model that studies the peering agreements between symmetric Internet backbones through the theory of social network formation. Each backbone is connected to the National Access Point (NAP) through which it is connected to the other backbones in the Internet. Badasyan and Chackrabarti consider a four stage non-cooperative model. In the first stage each backbone signals its willingness to peer with the others[35]: a peering agreement materializes when both of the backbones wish to peer; then, backbones fix the capacity level for their networks. In the third stage each backbone involved in a peering agreement decides how much to invest in the capacity of that link[36]; finally, in the fourth stage, backbones compete à la Bertrand price competition.

The total demand for the backbones is exogenous (thus the model focuses on the backbones' market shares), and the (homogeneous) customers decide by taking into account both the price and the quality of the services offered. When the investment in public infrastructure is pretty low, the model has two Subgame Perfect Nash Equilibria: in the first case each backbone peers with all the others (the complete network), while in the second, opposite, case there are no peering agreements at all (the empty network) and each backbone is connected to the others via the NAP only.

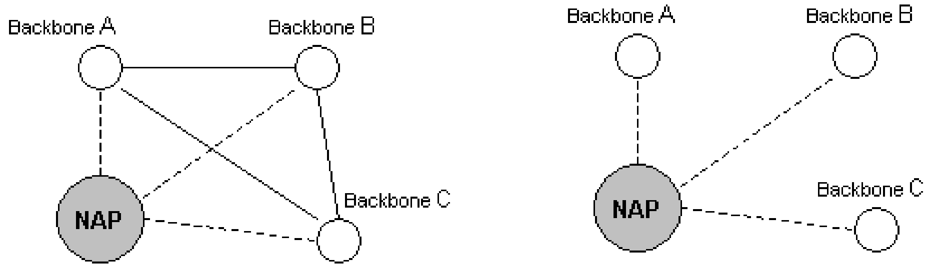



**figure 9: equilibria when the investment in public infrastructure is low**

Moreover, the empty network is Pareto superior to the complete network. An explanation for such a result lies in the assumptions about the benefits of a private peering agreement: it only enhances the quality of the service offered by reducing the possibility of traffic congestion (because two peering backbones may connect their customers using either the private direct link or the NAP), while there is no reduction in the costs borne by the backbones to exchange traffic. The backbones offering a better quality service may charge a greater price: in any case, when there is complete connection among the backbones, the qualitative differentiation among the services offered disappears, and customers only take advantage from peering agreements (a typical example of coordination failure).

When the investment in public infrastructure is large enough, the risk of traffic congestion decreases; as a result, backbones' incentives to engage in private peering disappear. Badasyan and Chakrabarti show that in this case profits are lower compared than those in the complete and empty networks with congestion, while customers' payoff increases.

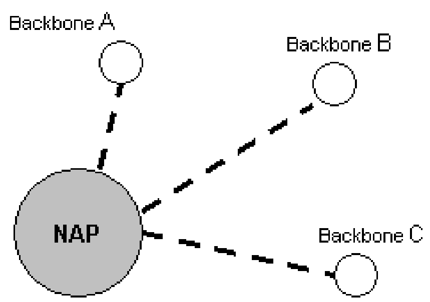


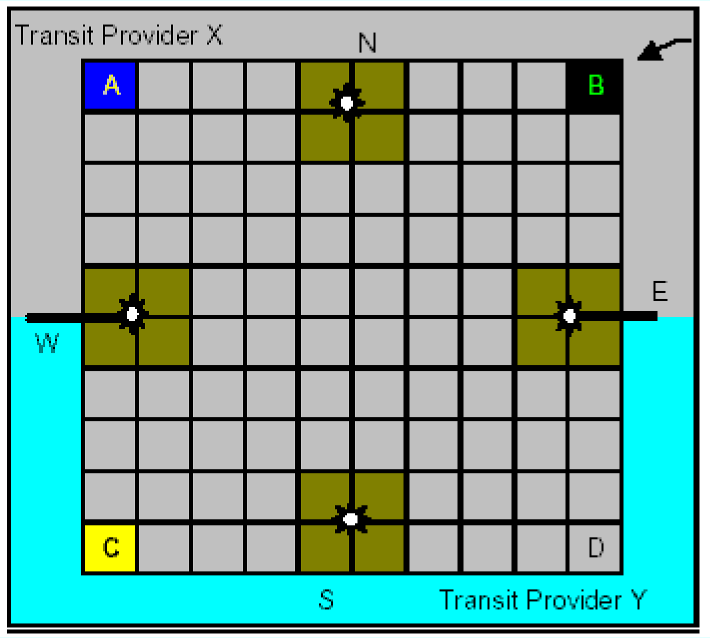

**figure 10: equilibrium when the investment in public infrastructure is large**

Finally, they move past this problem by analysing the formation of peering agreements in a cooperative setting: they consider Jackson and Wolinsky's pairwise stability concept and show that only the complete network is stable, while the empty network is efficient (it maximizes the backbones profits). Moreover a limiting factor of their work is that Badasyan and Chakrabarti do not allow for the possibility of transit, which is the natural alternative to peering. A more suitable setting could be the following: first, the backbones decide whether to join an Internet Exchange Point (IXP); then, the IXP's members decide whether to engage in bilateral peering taking into account the transit costs.

An interesting work focusing on the mechanism underlying the formation of peering agreements is due to Norton (2002a) who, rather than developing a theoretical model, set up a simple simulation environment, in order to observe the process "live", step by step. He considers an environment where four ISPs (namely A, B, C, D) are located in just as many equidistant locations. Each ISP's profit depends on both the number of its customers and that of the ISPs to which it is connected, either indirectly, via a *transit provider*, or directly, via a *peering* agreement. *Peering* agreements, however, may only occur when the ISPs' both reach the same *Internet Exchange Point* (there are four IXPs, equidistant from the ISPs). Each ISP, in turns, tries to extend its own internal network by developing effective marketing strategies, and to engage in *peering* relationships if these are both possible and profitable[37]. Each ISP aims to obtain more profit than the others. Norton assumes that they do not compete to acquire direct customers[38]; hence, they only focus on extending their internal networks and on saving traffic costs by peering with the others.

**The Peering Simulation Game**

The virtual game board consists of a matrix of squares, each representing a territory of customers (and an associated quantity of internet traffic). There are four Exchange Points in which ISPs can negotiate peering, called N, E, S, W. The Transit Providers (Y & X) shown on the board are not real players in the game, represent the upstream ISPs selling transit access to the other ISPs.
ISPs compete to maximize their bank account. In particular, revenues directly depend on the number of squares the ISP sits on. On the other hand, transit fees are proportional to the number of squares that other ISPs occupy. Those costs can be reduced by establishing peering relationships with other players, eliminating transit fees to access each other squares.




The game is played as follows: each ISP, in turn, rolls the die and acquires the number of squares indicated by the die (ISPs can only acquire squares adjacent to or diagonally a square the ISP occupies). If the ISP is an Exchange Point with other ISPs, he can attempt to establish a peering relationship.

Source: http://www.cs.berkeley.edu/~randy/Courses/cs294.s02/PeeringGame.pdf

Simulations' results show that *peering* agreements are, in general, essential to earn high profits; nevertheless, it comes out that the adoption of proper marketing strategies may be even more advantageous than *peering* agreements. Moreover the desire to win often prevents the ISPs from profitable *peering*.

Finally, the model proves the existence of a strong first-move advantage: it is crucial to reach soon a relevant market share.

Norton (2002b) provides further findings: the tactic chosen to obtain peering (he distinguish between nineteen different tactics) is crucial; moreover, he suggest another important result, expressed by the sentence "Once a Customer, never a Peer": it is very difficult to transform a transit agreement into a peering agreement; thus, the opportunity to sign a transit agreement must be weighted very carefully, especially with potential peers.

# 5. Conclusions

Our analysis has shown so far that relationships between firms operating in network industries are very complex: on the one hand, they need to be interconnected to offer their services; on the other hand, they often compete on the same market. This two-faced complementarity-competitiveness relationship may lead to suboptimal outcomes.

In the most typical of such network industries, Telecommunications, the level of competitiveness is very often restricted by the advantages enjoyed by well-established network operators against entrant firms: incumbent operators often own "essential facilities", enjoy "economies of scale and scope", are able to cross-subsidize some activities from others, and take advantage from the inertia that characterizes consumers' decisions about which carrier to subscribe to. The introduction of the access pricing mechanism played a key role in the process towards competition in network industries. Such a price, which is paid by the entrant firm to the dominant one in order to access its infrastructure, made possible the transition towards competitive environments where firms can still benefit from the economies of scale characterizing the telecommunications industry.

The crucial importance of the access price mechanism in easing the transition from monopoly to competition shows very clearly the relevance of the literature we have presented in order to set adequate competition policies in network industries. Such policies, in fact, have to consider both the strategic incentives that lead operators to interconnect, and their decisions within a given network. In particular, the competition policies dealing with the access to essential facilities are very relevant, since incumbent operators have many ways to restrict competition from new entrants. The *Green Book on Telecommunications*, published in 1987, first introduced some measures within the European Union; in 1997, the *Interconnection Directive* strengthened the regulatory framework, providing for interconnection agreements to be public and non discriminatory and for the access fee to be cost-based. This regulation was then harmonized by the *New Interconnection Directive*, in 2002. Within other regional organizations, such as APEC and CITEL in Latin America, the regulatory framework provides basic principles and useful information in order to develop an adequate interconnection policy, but it is not binding for the member States. A very important result concerning interconnection policy was achieved in 1997 with the *WTO Reference Paper*. This agreement, binding for over 60 member States, provides for non-discriminatory arrangements and for the access fee to be cost-based; besides, it implements the unbundling of the local loop, that is the possibility of accessing separately the various components of a network. In any case, this regulatory framework, only targets the operators in a dominant position (the former monopolists): such a solution, which characterizes the European Commission's policy, reflects the view that maintaining interconnection duties and obligations on all operators leads to an inefficient outcome. Earlier in the USA, the local loop unbundling had been a crucial issue of the 1996 *Telecommunications Act*: just three years later more than 70 per cent of operators offered separate access to theirs networks. In Europe, the regulatory framework for unbundled access was introduced in 2000, with *Regulation n. 2887*. The World moved towards unbundling very slowly. By late 2001, only 41 ITU members States and 25 in Latin America required local loop unbundling. On the other hand, unbundling was a key regulatory issue for many OECD Countries between 2000 and 2002: by April 2002, separate access to the networks' components was provided, or at least regulated, in 23 member States, while three years earlier they were just 11. Networks operators typically enjoy an effective market power when other operators ask to reach their subscribers (for example, to terminate a telephone call): thus, a subscriber is a bottleneck. Competition authorities have tried to get through this problem, but it still looks considerable for fixed-line to mobile-line call termination and for international telephony (see Valletti, 2000).

Accordingly, in Europe, the *New Interconnection Directive* attributed greater power to the mobile operators National Regulation Authorities (NRA) in order to better watch over their termination tariffs and agreements.

**Networks and markets contiguity**

The analysis of strategic interaction in a network environment gets even more interesting when we consider the relationships between firms that operate in different markets who are neighbours in a complex network. A firm that dominates one market, for instance, may offer very high prices (without affecting significantly its market share) to subsidize lower prices in the more competitive markets in which it operates.

The *WTO Reference Paper* specifically prohibits such a practice, which is highly anti-competitive. In many countries, the NRA have confirmed the prohibition of cross-subsidization, but a further regulatory framework was needed to make it effective. Article 8 of 1997 EU *Interconnection Directive* provided the "accounting separations", which applies to operators in a dominant position; they are required to separate accounts for their interconnection related activities and their other commercial activities. The 2002 *New Interconnection Directive* strengthened this regulatory framework by attributing greater power to the NRA.

Sometimes Regulation Authorities used stronger measures in cases of serious anti-competitive practices. The 1999 EU *Cable Ownership Directive*, for instance, provided for the operators in dominant position to place their cable television operations in a structurally separate company. The separation of

AT&T from the Regional Bell Operating Companies, in 1984, was more drastic, as the ownership, and not only the structure, of the two groups was separated.

**The Internet**

Our survey considered several applications concerning the interconnection problem in the Internet. In particular, we focused on different approaches to analyse both peering and transit relationships, which represent two alternative access agreements, and thus pricing, models for the Internet.

Although the Internet developed without a dedicated regulatory framework (whose implementation is quite complicated due Internet's super-national nature), it has been particularly affected by the regulation targeting the neighbouring telephony market, where Internet providers access their final customers.

In any case, some measures especially targeting the Internet have been provided by the Antitrust Authority, expressing the prohibition or the (conditional) authorization of mergers between large market share backbones, such as the ones between MCI and Worldcom and between MCI-Worldcom and Sprint.

Within the ITU, the debate about the opportunity to regulate Internet international interconnection fees still goes on even while this is written (June 2004). In particular, the ITU Recommendation D. 50 suggests basing such fees on the cost and features of networks, routes and traffic. Backbones with large market shares and most western countries have described this recommendation, which is not binding, as too strict; on the other hand, countries like China, Vietnam and Australia, which do not want the Internet international interconnection fees to be driven only by commercial concerns, consider the recommendation to be weak. Those countries complain about bearing the entire links' formation investments and maintenance costs for accessing the network, while those links are two-way, and thus they can also be used from the countries hosting the larger backbones.

Another question which still needs to be answered, concerns the identification of the main elements determining high Internet access prices in many developing countries. On the one hand, asymmetric interconnection policies, and thus asymmetric fees, are very likely to be the right answer: large backbones engage in peering relationships, while they sign transit agreements with the smaller ones; thus, small peripheral countries end up by paying the whole interconnection cost. On the other hand, very often such high prices are caused by the endurance of local access monopolies and by the failure to liberalize the telecommunications market which still holds in many developing countries. (Many of them are, for instance, WTO members, but did not ratify the Reference Paper, thus they still protect their market from foreign competitors). Probably, the above mentioned causes, price discrimination in the upstream market and monopoly power in the downstream market, both keep Internet access prices high, strengthening the digital divide. This problem is very important, as the debate in the current international community considers the digital divide as one of the main forces that allows underdevelopment to hold. The first stage of the World Summit on the Information Society (WSIS), which took place in Geneva in December 2003, argued for this problem to be solved as top priority for the world.

The Internet is the typical network industry, thus liberalization policies targeting the downstream markets are not effective when a serious attempt to oversee the access policies in the upstream markets is not undertaken; unfortunately, authorities are experiencing many difficulties when attempting to observe those relationships, as most bilateral interconnection agreements are confidential.

Hence, greater transparency in the access policies, involving either the transit prices and the peering agreements, is needed by the national and international regulation authorities.

Without the right information, in fact, it is not possible to undertake proper market's analysis, which are very important especially when there are firms that may enjoy remarkable market power. When the industry consists of firms headquartered in different countries, or when the technologic progress follows a peculiar dynamic, the introduction of an ex-ante regulation turns out to be quite difficult, sometimes even impossible; hence, a crucial role is played by the possibility to monitor, and in the event, punish anticompetitive practices, which may express in a price and/or quality discrimination. The last one, in particular, is very effective when the profitability of the applications offered is strictly related to the quality and the speed characterizing the information transmission process, and thus to the interconnections' structure and the access agreements between the firms.

# Notes

[1] The snowplough model is an application of the elector model (Kindermann, R.P. e Snell, J.L., 1980). Following this model an elector's political opinions change at some random point, after having observed the opinion of a neighbour. The test on the opinions' asymptotic uniformity, as on the decision to sweep or not, is based on the percolation theory (Grimmet, G.R. 1989).
[2] There should be at least three individuals.
[3] It could be a direct or an indirect link, using other players' links (in this last case, one may say that there is a "path" between two individuals).
[4] This is an application of Spitzer theorem, Spitzer F. (1971).
[5] In fact in a subcritical state, the sand pile's slope rises, because there are on average, small avalanches; while in a supercritical state, there are broader avalanches, bringing slope back to a critical state.
[6] The link with the sand pile metaphor comes to the fore when fluctuations in the orders of final goods causes a "productive avalanche" along the productive process, only applying complementarities in space and time.
[7] This classification may be adapted to the case of two individuals agreeing with the constitution of links. In such a situation the "active" individual, that is the one proposing the link or the one meeting the greater costs in its realisation, will become relevant.
[8] These pairs of agents create a *network of relations*.
[9] According to Lippert and Spagnolo, circularity is essential for the transmission of information and for the realisation of collective punishments.
[10] Competition policy analyses either the collusive potential of the access price – in the case of horizontal networks- or the discrimination policies in the access conditions – in the case of networks with a vertical hierarchy.
[11] Such foreclosure has different modes of implementation: downstream integration and exclusive rights deals. See Tirole , J. (1988), chapter 8.
[12] These hierarchies are not so strict in the Internet. One may notice an overturning of the initial hierarchies, since connection does not occur through a dedicated line (see Giovannetti, E. 2004).For this reason there are several ways to agree, in a hierarchy as with Transit or co-operatively as with Peering.
[13] Analytically, this may be proved by defining the backbone $i$ service quality as $s_i = k(d_i + \theta\, d_j)$, where $k$ is a positive parameter, $d_i$ stands for the backbone $i$'s customers and $\theta$ , with$\theta \in [0,1]$ indicates interconnection quality. We can easily prove that $s_i - s_j = k(1-\theta)(d_i - d_j)$.
[14] If backbone 1 wins a customer from backbone 2, traffic originating from backbone 2 websites and required by that consumer, will no longer only belong to backbone 2 . In fact backbone 1 will have the same profit from the access price $a$, minus the ending cost, $c_1$. In this way $c_1 - a$ represents a part of that consumer's opportunity cost for backbone 1. Moreover there will be traffic from the backbone 1 website, costing $c_0 + a$ , with the customer belonging to backbone 2 and now costing $c = c_0 + c_1$. Thus the lower opportunity cost (related to this other traffic component) depending on the same consumer, is $c_1 - a$ .
[15] This happens because $c_0$ is the traffic originating cost, while $a$ applies for all consumers. In fact, the termination cost must be paid by all the customers of the other backbone, but it corresponds also to what is lost for the final customers, because backbone 2 should also have paid charge $a$, in order to allow backbone 1 to terminate its own customers traffic.
[16] The previous results totally change when considering the possibility of having genuine asymmetric access charges. The authors prove that in this case, there is no equilibrium in pure strategies.

[17] We have slightly modified the notation to make it uniform among this model and the one introduced by Laffont *et al*. (2003); however, while in Laffont *et al*. the coefficients $\alpha_i$ and $\alpha_j$ indicate the market shares, in this model they show instead the number of customers.
[18] A network is said to be strongly efficient if, given the function *v*, its overall total value is the maximum among all possible networks.
[19] If players are linked we test for (i), while if they are not we test for (ii).
[20] Bala and Goyal assume *free riding* on the links formed by others.
[21] $g_{ij}$ = 1 if *i* maintains a link whit *j*, while $g_{ij}$ = 0 means the opposite. As each player *i* has to decide about $n-1$ potential links, the cardinality of his set of strategies is $|G_i| = 2^{n-1}$.
[22] A network *g* is said to be connected if each player observes every other player, directly or indirectly via a path. Furthermore, *g* is said to be minimally connected if it is connected and there is only a path between each pair of players.
[23] A connected network is said to be super-connected if it is connected and there exist links after whose deletion the network is still connected.
[24] Only a small part of the literature focused on agents' heterogeneity, first introduced by Johnson, C. and Gilles, R.P. (1999). While they use the concept of pairwise stability, Haller, H. and Sarangi, S. (2003) consider agents' heterogeneity in the non-cooperative approach.
[25] The concept of a random graph had been previously used to represent the communication opportunities between firms in large markets (Kirman, A.P., 1983, Kirman, A.P. et al,1986, Haller, H. 1990, Ioannides, Y.M., 1990); Calvo, E., Lasaga, J. and van den Nouweland, A. (1999) introduce the assumption of random graphs in a purely cooperative setting.
[26] Haller, H. and Sarangi, S. (2003) show that there exists a maximum link formation cost, depending on *P*, for which the complete network is a Strict Nash Equilibrium. Moreover, if the link formation cost exceeds the maximum probability for a link to fail among *P*'s elements, that is $c > \max\{p_{ij}\}$, then the empty network is a Strict Nash. To get through intermediate cases more assumptions are requested.
[27] Given a network *g*, we define a component of *g*, $C(g)$, a set $C(g) \subset N$ such that there is a path between any pair of players in $C(g)$ and there does not exist any path between a player in $C(g)$ and a player who is not in $C(g)$. Let $\#C(g)$ be the number of components in *g*. A network *g* is said to be minimal if $\#C(g) < \#C(g - g_{i,j})$ $\forall g_{i,j} = 1$, where $\#C(g - g_{i,j})$ is the number of components in *g* after the deletion of the link between *i* and *j*.
[28] Thus, a large class of architectures can be a Nash Equilibrium (not Strict) for an appropriate list of individual costs and benefits.
[29] The assumption that players announce with whom they wish to form a link was first introduced by Dutta, B., van den Nouweland, A. and Tijs, S. (1998).
[30] Obviously, for particular payoff functions or values of $f$, the empty network may be a pairwise stable equilibrium; that result, of course, is not pathologic.
[31] Thus, the empty network is never a Nash equilibrium.
[32]The stability concept used is the one defined by Jackson and Wolinsky.
[33] Those results reflect the overall effect of two opposite elements: on the one hand, each link leads to a reduction in the marginal cost; on the other hand it enhances the competitiveness of rival firms (information flow is two-way).
[34] The complete network is the only stable network when considering four firms only; with a larger number of firms other architectures may be stable, but it is quite difficult to select them.
[35] Badasyan and Chakrabarti initially assume that each backbone cannot discriminate among the others.
[36] The capacity of a link is given by the sum of the investments by the two peers and a component which reflects the level of public infrastructure that can be used by the backbones.
[37] It is a relative concept, as shown by the model's results.
[38] This strategy is in fact independent of the links' structure.

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