Learning Stability for Monetary Policy Rules in a Two-country Model*

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Abstract

This work evaluates whether or not the interest rate rules under different exchange rate regimes lead to a REE that is both locally determinate and stable under adaptive learning by private agents. I find that monetary interdependence among countries is crucial for the determinacy and learning stability of the economy in the open economy case, even without the coordination of the policymakers. Under floating exchange rate regime, both countries should follow aggressive interest rate rules simultaneously, in order to obtain determinate and learnable REE. Furthermore, the openness diminishes the regions for the determinate and learnable rules relative to its closed economy counterpart under the floating regime, while in other exchange rate regime, the additional reaction towards the level or change of nominal exchange rate will enlarge this region.

Key words: Adaptive learning, interest rate rules, open economy, exchange rate regime, determinacy, learnability

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1 Introduction

1.1 Overview and main results

The implementation of monetary policy rules, particularly interest rate feedback rules, has been extensively studied in many aspects within the forward-looking New Keynesian models recently. The current standard methodology for modelling expectations in these models is to assume *rational expectations* (RE), where agents fully understand the structure of the economy. However, the RE methodology implicitly makes some rather strong assumptions, since expectations can be out of equilibrium due to exogenous events and therefore might result in instability of the equilibrium.[1] Therefore, adaptive learning approaches, as a relaxation of the assumption of RE, have been introduced into New Keynesian literature, by assuming private agents follow a learning process to form expectations. It has been recognized that *learning stability* (learnability) is a necessary additional criterion for evaluating alternative monetary policy feedback rules, and in particular, only policy rules inducing learnable *rational expectations equilibrium* (REE) could be advocated.

Most of these discussions on monetary policy rules under adaptive learning are based on the standard New Keynesian model within a closed economy context. However, in practice, most monetary authorities have to face an open economy environment. Therefore, it is natural and necessary to extend these discussions into open economy models to study how the consideration of open economy will affect the design of monetary policy. Ever since the work of Obstfeld and Rogoff (1995), there has been a large body of literature on New Open-economy Macroeconomics. They are normally fully optimizing models with monopolistic competition and nominal rigidities in an open economy framework. Incorporating elements from both these New Open-economy Macroeconomic literature and the standard New Keynesian literature, some researchers extend the New Keynesian model to an open economy context, such as Clarida, Gali and Gertler (2002) (CGG (2002) hereafter), Gali and

[1] See Bullard and Mitra (2002), and Evans and Honkapohja (2003a, 2006) for the discussions.

Monacelli (2005), and Benigno and Benigno (2006b) (BB (2006b) hereafter).[2] They follow Calvo (1983) to model nominal rigidities, which allows richer dynamic effects of monetary policy and longer periods of fluctuation around the equilibrium than those with only one-period advanced price-setting rule in early literature of New Open-economy Macroeconomics.[3] This characteristic makes it natural and reasonable to conduct dynamic analysis under adaptive learning within open economies, providing new insights for the design of monetary policies.

Based on BB (2006b), this paper discusses the learning stability for monetary policy rules in a two-country model, which is one of the extensions of the New Keynesian model to open economies. In particular, a special open-economy Phillips curve with the terms of trade is introduced. Therefore, there is explicit interaction between countries in this open economy, even without instrument rules reacting towards international variables. Furthermore, the economy is not fully forward-looking, which is different from the New Keynesian literature in closed economies, since the terms of trade are state variables and depend on the past values. Adopting the methods developed by Evans and Honkapohja (1999, 2001), this paper analyzes the determinacy and learnability within this two-country model for different interest rate rules under three exchange rate regimes: (1) a floating exchange rate regime, (2) a fixed exchange rate regime, and (3) two cases of managed exchange rate regimes. The current results show that the conditions for equilibrium determinacy are always sufficient for stability under adaptive learning under three regimes, which is in line with McCallum (2006). In particular, it is proved that the conditions for determinacy and learnability coincide under the floating regime and fixed regime.[4]

Another important finding is that under the floating regime, the interest

[2] BB (2006b) is a revised version of their previous working paper "Monetary Policy Rules and the Exchange Rate" 2001 and "Exchange Rate Determination under Interest Rate Rules" 2004.

[3] Such as the Redux model in Obstfeld and Rogoff (1995), Chari, Kehoe, and McGrattan (1998), Engel (2002), Devereux and Engel (2000), Obstfeld and Rogoff (2000, 2002), Tille (2001), and others.

[4] The numerical results suggest that the conditions for determinacy and learnability also coincide under managed regimes.

rate rules followed by two countries are required to be simultaneously aggressive, in order to guarantee the REE determinate and learnable. The failure to satisfy the determinacy and learnability conditions in one country can result in the indeterminate and unstable REE under learning for the whole economy. Monetary interdependence, therefore, across countries is crucial for the dynamics of the economy in the open economy case, even without the coordination of the policymakers, as Corsetti and Pesenti (2001), CGG (2002), and BB (2006b) discussed under rational expectations. Moreover, it is found that the openness diminishes the region for determinate and learnable interest rate rules relative to its closed economy counterpart under the floating regime. However, additional reaction to the change or the level of the nominal exchange rate in policy rules of one country will enlarge the region for determinacy and learnability under the fixed regime and managed regimes relative to the floating regime. The change of the region is due to the terms of trade effects in open economy environments. This paper also introduces a new methodology to simplify the analysis of high-dimension system by partitioning its matrix form into several subsystems. This methodology will be discussed in detail in Section 2.5.

1.2 Related literature

Adopting the methods developed by Evans and Honkapohja (1999, 2001), Bullard and Mitra (2002) have discussed learnability for four typical variants of Taylor's interest rate feedback rules based on the model by Woodford (1999) and Rotemberg and Woodford (1998, 1999). They found that the Taylor principle is closely linked with learnability for all four specifications of the policy rules, and the determinacy conditions are sufficient for learnability when policy rules react to current values or future forecasts of inflation and output deviations, while it does not hold for policy rules under the lagged data specification. McCallum (2006) obtains the same results for a general class of linear models considered by Evans and Honkapohja (2001). His analysis shows that for this broad class of models, determinacy condition is sufficient but not necessary for learnability if current information is available for individuals, while it is not sufficient if instead only lagged information can be observed in the learning process. Evans and Honkapohja (2003a) review the recent work on interest rate setting, and emphasize that the design of monetary policy needs to take into account the possibility that the economy may not always be in a REE, and therefore the learnability constraint is a key criterion in designing a good policy rule.

The recent paper by Bullard and Schaling (2006) (BS (2006) hereafter) also discusses the learnability in a two country New Keynesian framework, based on a different model by CGG (2002) from this paper. The framework of CGG (2002) is a straightforward extension of the standard New Keynesian model to two countries, in which there is a natural separation between countries, and the difference from its closed economy counterpart comes only from the parameters of the model and the natural rate of variables. Using the model by CGG (2002), BS (2006) have studied the determinacy and learnability for both instrument rules and targeting rules with or without concerns of international economic conditions, which leads to their results with some differences from this paper. They show that monetary policy rule in each country must satisfy the determinacy and learnability conditions independently if policymaker focuses only on domestic inflation and output gap. On the other hand, if policymakers consider international economic variables in their policy rules, it will induce international feedback between the two economies, and therefore determinacy and learnability conditions will depend on joint policies and economies of two countries.

Llosa and Tuesta (2005) (LT (2005) hereafter) have discussed the learnability for monetary policy rules in a small open economy case, based also on the model of CGG (2002).[5] Following Bullard and Mitra (2002), they discuss the determinacy and learnability for three simple monetary policy rules, a domestic inflation (DI) targeting Taylor-type rule, a consumer price inflation (CPI) targeting Taylor-type rule, and a policy rule for the managed exchange regime.[6] They argue that conditions for determinacy and E-stability in the

[5] Differently, they assume there is home bias in preferences.

[6] In the managed exchange regime, the policy rule reacts to CPI and output gap as well as the nominal exchange rate.

small open economy model are isomorphic to those in a closed economy case when policymakers follow contemporaneous data rules. However, they warn that policymakers should be more careful about forward looking rules, which can easily induce indeterminacy and instability under learning.

I introduce the two-country model and the methodologies used in this paper in section 2, analyze the determinacy and learnability under different exchange rate regimes in section 3, 4, and 5, and then draw the conclusions in the last section.

2 The model and methodologies

2.1 The baseline model

The baseline model is outlined as a fully optimizing two-country model with monopolistic competition and nominal rigidities, based on work of BB (2006b).[7] This model incorporates elements from both the large body of New Keynesian literature in a closed economy and the recent literature on New Open Economy Macroeconomics.

This economy is made up of two countries, the home country (H) and the foreign country (F). The whole economy is populated by a continuum of households on the interval of $[0, 1]$, in which the households over the $[0, n)$ live in the home country and the households over $(n, 1]$ live in the foreign country. A representative agent is both a producer and a consumer, who produces a single differentiated product but consumes all the goods produced in both countries. The markets are complete both domestically and internationally, by assuming that agents can trade complete contingent one-period nominal bonds denominated in the Home currency.

As in the literature on New Open Economy Macroeconomics, both monopolistic competition and nominal rigidities are introduced into the model. The former assumption rationalizes the existence of price stickiness without violating the producers' constraints. The latter assumption is introduced in a different way from the most in the literature of New Open Economy

[7] See Benigno and Benigno (2006b) for more details of the model.

Macroeconomics, by assuming a *Calvo*-style (1983) price-setting rule, which says that each firm has a fixed probability of $1-\alpha$ to set a new price in each period.[8] This probability is not related to how long it has been since the price adjustment last time, and it is the same for all agents.

This paper considers only the case where the instrument of monetary policy is the domestic short-run nominal interest rate. Accordingly, the exchange rate regimes are modelled by designing different interest rate rules followed by the central banks in both countries. Finally, there are two kinds of country-specific fluctuations, the demand shocks $\left(g^H, g^F\right)$ and the supply shocks $\left(a^H, a^F\right)$.

In the following sections, I mainly present the log-linear approximation form of the model.

2.1.1 AD block

In the aggregate demand block, it is assumed that each agent derives utility from consuming an index of the consumption goods and the holding of money, while derives disutility from producing the products. Therefore, a representative consumer chooses the consumption allocation by maximizing the expected discounted value of the utility flow

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U\left(C_s^j\right) + L\left(\frac{M_s^j}{P_s^i}, \xi^i\right) - V\left(y_s^j, z_s^i\right) \right],$$

subject to the relevant budget constraints, with

$$i = \begin{cases} H,\ if\ j \in [0, n) \\ F,\ if\ j \in (n, 1] \end{cases},$$

where the upper index j denotes the individual agent j; the lower index i denotes the specific country i; U is an increasing concave function of consumption index of agent j; L is an increasing concave function of real money balances $\frac{M}{P}$; y is a production function; β is the intertemporal discount factor

[8] For the simplicity, the degrees of rigidity $1-\alpha$ are assumed to be the same across countries in this paper.

in the consumption preference, with $0 < \beta < 1$; and ξ and z are country-specific shocks to the money demand and productivity respectively.

C^j, C_H^j, and C_F^j are defined as

$$
\begin{aligned}
C^j &\equiv \frac{\left(C_H^j\right)^n \left(C_F^j\right)^{1-n}}{n^n \left(1-n\right)^{1-n}}, \\
C_H^j &\equiv \left[\left(\frac{1}{n}\right)^{1/\sigma} \int_0^n c^i\left(h\right)^{\frac{\sigma-1}{\sigma}} dh\right]^{\sigma/(1-\sigma)}, \\
C_F^j &\equiv \left[\left(\frac{1}{1-n}\right)^{1/\sigma} \int_n^1 c^i\left(f\right)^{\frac{\sigma-1}{\sigma}} df\right]^{\sigma/(1-\sigma)},
\end{aligned}
$$

where C_H^j and C_F^j are indexes of consumption across the continuum of differentiated goods produced respectively in the home country and the foreign country; $\sigma > 1$ is the elasticity of substitution across differentiated goods produced within a country; and in this economy the elasticity of substitution between the C_H and C_F is assumed as one. For simplicity, the parameter n is assumed to denote not only the population size but also the share of the bundle of goods produced within that country in the consumption index, i.e. the economic size.

From the log-linear approximation to the first-order conditions of the representative consumers in the countries H and F, we can derive the equilibrium conditions for the aggregate demand block. Due to the same of population size and economic size, as well as the assumptions of complete international markets and law of one price, the Euler equation can be described by only one equation in the log-linear form[9]

$$E_t\hat{C}_{t+1} = \hat{C}_t + \rho^{-1}n\left(\hat{\imath}_t^H - E_t\pi_{t+1}^H\right) + \rho^{-1}\left(1-n\right)\left(\hat{\imath}_t^F - E_t\pi_{t+1}^F\right), \quad (1)$$

where $\hat{C}$ is the consumption index; $\hat{\imath}_t^H$ and $\hat{\imath}_t^F$ are the nominal interest rates in the home country and foreign country respectively; π_t^H and π_t^H are the producer inflation rates in the two countries respectively, with $\pi_t^H =$

[9]See Benigno and Benigno (2006b) and their Technical Appendix for the details of derivation.

$\ln P_{H,t}/P_{H,t-1}$ and $\pi_t^F = \ln P_{F,t}/P_{F,t-1}$; and ρ is the inverse of the intertemporal consumption elasticity of substitution. Therefore, the expected consumption growth depends on the weighted average of real return in units of the Home and Foreign goods indices.

The assumption of monopolistic competition implies that when prices are fixed the output is demand determined by

$$\hat{Y}_t^H = (1-n)\,\hat{T}_t + \hat{C}_t + g_t^H,\ \hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t + g_t^F, \tag{2}$$

where $\hat{Y}_t^H$ and $\hat{Y}_t^F$ are output produced in the home country and foreign country respectively; and $\hat{T}_t$ is the terms of trade, which is the relative price of imported goods to exported goods. Even though the aggregate consumption index $\hat{C}_t$ has the same impacts on both countries, the terms of trade and the country-specific demand shocks induce the dispersion of output across the two countries.

In the following sections, I define

$$X^W \equiv nX^H + (1-n)X^F,\ X^R \equiv X^F - X^H,$$

where $\tilde{X}$ is the natural rate of X, and $\hat{X}$ is the sticky-price equilibrium of X, given a variable X^H for the home country and a variable X^F for the foreign country.

From (2), the world output can be written as the sum of world consumption and world demand shock

$$\hat{Y}_t^W = \hat{C}_t + g_t^W.$$

If define the world output gap y^W as the difference between the sticky-price equilibrium and flexible-price equilibrium (the natural rate) of the world output

$$y_t^W = \hat{Y}_t^W - \tilde{Y}_t^W,$$

equation (1) can be written in terms of world output gap

$$y_t^W = E_t y_{t+1}^W - \rho^{-1} n \left(\hat{\imath}_t^H - E_t \pi_{t+1}^H \right) - \rho^{-1} (1-n) \left(\hat{\imath}_t^F - E_t \pi_{t+1}^F \right) + \rho^{-1} \tilde{R}_t^W, \quad (3)$$

where $\tilde{R}_t^W$ is the Wicksellian rate, which is the perturbation to the world natural real interest rate and is a combination of world demand and supply shocks.

Equation (3) is the microfounded "open-economy" IS curve for this model. It is similar to the new IS curve in the New Keynesian model in a closed economy. Actually, the IS curve for each country can be derived as

$$\begin{aligned} y_t^H &= E_t y_{t+1}^H - \rho^{-1} n \left(\hat{\imath}_t^H - E_t \pi_{t+1}^H \right) - \rho^{-1} (1-n) \left(\hat{\imath}_t^F - E_t \pi_{t+1}^F \right) && \text{(3-1)} \\ &\quad - (1-n) \left(E_t \hat{T}_{t+1} - \hat{T}_t \right) + rr_t^H, \\ y_t^F &= E_t y_{t+1}^F - \rho^{-1} n \left(\hat{\imath}_t^H - E_t \pi_{t+1}^H \right) - \rho^{-1} (1-n) \left(\hat{\imath}_t^F - E_t \pi_{t+1}^F \right) && \text{(3-2)} \\ &\quad + n \left(E_t \hat{T}_{t+1} - \hat{T}_t \right) + rr_t^F, \end{aligned}$$

where y_t^H, y_t^F are the output gaps in the home country and foreign country respectively; and $rr_t^H$and rr_t^F are functions of the natural rate of interest, demand and supply shocks.

The log-linear approximation of the terms of trade definition is

$$\hat{T}_t = \hat{T}_{t-1} + \triangle S_t + \pi_t^F - \pi_t^H, \quad (4)$$

which implies the terms of trade are sluggish state variables, since they depend on past values of the change of the nominal exchange rate and the inflation rate differential. Furthermore, the different decomposition of the adjustment of the terms of trade between the change of nominal exchange rate and inflation rate differential can lead to different equilibria, depending on the substitutability of goods produced within and across countries as well as the monetary policies followed by both central banks.

From the uncovered interest parity, the expected depreciation of the exchange rate depends on the nominal short-run interest rate differential

$$E_t \triangle S_{t+1} = \hat{\imath}_t^H - \hat{\imath}_t^F, \text{ where } \triangle S_{t+1} = \hat{S}_{t+1} - \hat{S}_t. \tag{5}$$

2.1.2 AS block

Under the *Calvo*-pricing assumption, each firm can set a new price with a fixed probability $1-\alpha$ in each period, by maximizing its expected discounted value of profits.[10] From the optimizing behaviour of firms, two aggregate supply equations in the AS block are obtained[11]

$$\pi_t^H = \lambda \widehat{mc}_t^H + \beta E_t \pi_{t+1}^H, \ \pi_t^F = \lambda \widehat{mc}_t^F + \beta E_t \pi_{t+1}^F,$$

where $\widehat{mc}$ is the deviation of the real marginal costs from the steady state. Together with the labour supply decisions of the households, the AS curves for the two countries become

$$\begin{aligned} \pi_t^H &= \lambda \left[(1-n)(1+\eta)\left(\widehat{T}_t - \widetilde{T}_t\right) + (\rho+\eta)\left(y_t^W\right) \right] + \beta E_t \pi_{t+1}^H \quad (6) \\ &= (1-n) k_T \left(\widehat{T}_t - \widetilde{T}_t\right) + k_C y_t^W + \beta E_t \pi_{t+1}^H, \\ \pi_t^F &= \lambda \left[-n(1+\eta)\left(\widehat{T}_t - \widetilde{T}_t\right) + (\rho+\eta)\left(y_t^W\right) \right] + \beta E_t \pi_{t+1}^F \quad (7) \\ &= -n k_T \left(\widehat{T}_t - \widetilde{T}_t\right) + k_C y_t^W + \beta E_t \pi_{t+1}^F, \end{aligned}$$

where define $k_C \equiv \lambda(\rho+\eta)$ and $k_T \equiv \lambda(1+\eta)$; $\widetilde{T}_t$ is the natural rate of the terms of trade under flexible prices, which is a combination of relative demand and supply shocks; and η is the elasticity of the disutility of producing the differentiated goods.

Equations (6) and (7) are the dynamic forward-looking AS curves in an open economy, in which the terms of trade have direct impacts on the real marginal costs and thus influence the domestic inflation. Therefore, these open-economy AS curves are different from the standard New Phillips curves in the New Keynesian literature within the closed economy context, due to

[10] This model assumes complete pass-through for firms setting prices.

[11] Define $\lambda \equiv [(1-\alpha\beta)(1-\alpha)/\alpha] \cdot [1/(1+\sigma\eta)]$.

the existence of additional channel of the terms of trade for the shock transmission mechanism. For example, for the home country, an increase in the terms of trade or a depreciation of the domestic currency results in an increase in the relative price of the goods produced in the foreign country, i.e. the domestically produced goods turn to be cheaper, which boosts the demand for the goods produced in the home country and therefore pushes up the domestic inflation in the home country. Additionally, the rise in the domestic production and the price of the imported goods cause the reduction in the marginal utility of nominal income and the increase in the optimal prices faced by the firms in the home country, which pushes up the domestic inflation more. Therefore, in the open-economy Phillips curves, there is positive influence of the terms of trade on the domestic inflation in the home country, while there is negative influence of the terms of trade on the inflation in the foreign country.

2.2 Interest rate rules under different monetary regimes

As in the standard New Keynesian literature, this model is closed by the specification of the monetary policy rules followed by both central banks. The short-term nominal interest rate is assumed as the instrument of monetary policies. Normally, the interest rate rule in a closed economy is set to react towards domestic inflation and output gap, according to the classical Taylor rule. In an open economy, however, the specification of the monetary rules is much more controversial, due to more international variables to concern.[12] I start by specifying the simple and reasonable interest rate rules to analyze the determinacy and learnability in this two-country model.

Following the BB (2006b), three different exchange rate regimes will be discussed: a floating exchange rate regime, a fixed exchange rate regime and the managed exchange rate regimes. Under different monetary regimes, different interest rate rules are designed for the two countries, H and F, respectively.

[12] For example, Ball (1998), Ghironi (1998), McCallum and Nelson (1998), Monacelli (1998), Svensson (2000), and Weeparana (1998) have recently analyzed the monetary policies in open-economy models.

1. Under a floating exchange rate regime, interest rate rules in both countries do not react directly to the nominal exchange rate

$$\begin{aligned} \hat{\imath}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{\imath}_t^F &= \phi^*\pi_t^F + \psi^* y_t^F. \end{aligned} \tag{8}$$

2. Under a fixed exchange rate regime, the exchange rate of foreign currency is assumed to peg to the home currency. To get a determinate and fixed nominal exchange rate, the interest rate rules are designed as[13]

$$\begin{aligned} \hat{\imath}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{\imath}_t^F &= \hat{\imath}_t^H - \mu\hat{S}_t, \text{ where } \mu > 0. \end{aligned} \tag{9}$$

3. Under a managed exchange rate regime, it is assumed that the interest rate rule in one country partly reacts to the nominal exchange rate, either in level or in the deviations of the level.

   Under managed exchange rate regime (I), the rule followed by the central bank of the foreign country reacts to the level of the nominal exchange rate

$$\begin{aligned} \hat{\imath}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{\imath}_t^F &= \phi^*\pi_t^F + \psi^* y_t^F - \mu_1\hat{S}_t, \end{aligned} \tag{10}$$

   where $\hat{S}_t \equiv \ln(S_t/S^*)$, and S^* is the exchange rate target.

   Under managed exchange rate regime (II), the rule followed by the central bank of the foreign country reacts to the deviations in the level of the nominal exchange rate from a defined target

$$\begin{aligned} \hat{\imath}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{\imath}_t^F &= \phi^*\pi_t^F + \psi^* y_t^F - \mu_2\triangle S_t. \end{aligned} \tag{11}$$

[13] There are many fixed exchange rate regimes existent depending on the different specification of policy rules. Here is a simple one of them.

For simplicity, this paper at first considers the case that two countries have the identical parameters in the policy rules, i.e. $\phi = \phi^*$, and $\psi = \psi^*$, in order to obtain analytical results. Afterwards, it discusses numerically the interdependence across the two countries with different values of parameters in the policy rules. All the parameters in the policy rules are non-negative, i.e. $\phi, \phi^*, \psi, \psi^*, \mu, \mu_1, \mu_2 \geq 0$.

2.3 Summary of the structure

The structure of the model is characterized by the microfounded "open-economy" IS curve (3), the open-economy AS curves (Phillips curves) (6) and (7), the terms of trade equation (4), the uncovered interest parity (5), and the interest rate rules (8), (9), (10), or (11) under different exchange rate regimes

$$y_t^W = E_t y_{t+1}^W - \rho^{-1} n \left(\hat{\imath}_t^H - E_t \pi_{t+1}^H \right) - \rho^{-1} (1-n) \left(\hat{\imath}_t^F - E_t \pi_{t+1}^F \right) + \rho^{-1} \tilde{R}_t^W, \quad (3)$$

$$\begin{aligned} y_t^H &= E_t y_{t+1}^H - \rho^{-1} n \left(\hat{\imath}_t^H - E_t \pi_{t+1}^H \right) - \rho^{-1} (1-n) \left(\hat{\imath}_t^F - E_t \pi_{t+1}^F \right) \quad \text{(3-1)} \\ &\quad - (1-n) \left(E_t \hat{T}_{t+1} - \hat{T}_t \right) + rr_t^H, \\ y_t^F &= E_t y_{t+1}^F - \rho^{-1} n \left(\hat{\imath}_t^H - E_t \pi_{t+1}^H \right) - \rho^{-1} (1-n) \left(\hat{\imath}_t^F - E_t \pi_{t+1}^F \right) \quad \text{(3-2)} \\ &\quad + n \left(E_t \hat{T}_{t+1} - \hat{T}_t \right) + rr_t^F, \end{aligned}$$

$$\pi_t^H = (1-n) k_T \left(\hat{T}_t - \tilde{T}_t \right) + k_C \left(y_t^W \right) + \beta E_t \pi_{t+1}^H, \quad (6)$$

$$\pi_t^F = -n k_T \left(\hat{T}_t - \tilde{T}_t \right) + k_C \left(y_t^W \right) + \beta E_t \pi_{t+1}^F, \quad (7)$$

$$\hat{T}_t = \hat{T}_{t-1} + \triangle S_t + \pi_t^F - \pi_t^H, \quad (4)$$

$$E_t \triangle S_{t+1} = \hat{\imath}_t^H - \hat{\imath}_t^F. \quad (5)$$

It is obvious that the two countries are correlated in this economy, even without reaction towards the international variables in policy rules. In particular, by introducing the terms of trade into the Phillips curve, the terms of trade and the nominal exchange rate have direct and implicit impacts on the relative prices of goods and thus affect the demand of the goods produced in the different countries, which will in turn increase or decrease the producer price inflation. Furthermore, there is another channel where the exchange rate and thus the international variables have impacts on the domestic economy if the policy rules responds directly towards the nominal exchange rate in level or the deviations. Similarly, Guender (2006) introduces a model with the existence of real exchange rate in the Phillips curve within a small open economy. He discusses that the pricing decisions of domestic firms are, therefore, affected by movements of the exchange rate and other international variables, and this feature will thus affect the conduct of monetary policy in open economy. CGG (2002) introduce another fully optimizing two-country model with monopolistic competition and nominal rigidities. Differently, the model by CGG (2002) is a simple extension of the standard New Keynesian model, with the same forms of the IS curves and Phillips curves as in its the closed economy counterpart, and with the only difference in parameters and natural rate of interests. Therefore, the interaction of the two economies in CGG (2002) comes only from the instrument rules with reaction towards international variables followed by monetary authorities. If the instrument rules only react towards domestic variables, the two countries are separated naturally, and one country does not affect the other directly or implicitly. Furthermore, generally the model in this paper is not in a fully forward-looking form as standard New Keynesian model, because the terms of trade are state variables, which depend on past values of the change of the nominal exchange rate and the inflation rate differential, as function (4) shows. Therefore, the slow adjustment of prices in this two-country model not only comes from the nominal rigidities but also comes from the inertia of the terms of trade, which lead to the slow adjustment of the real marginal costs. One exception is that the inertia of the terms of trade is completely eliminated under the floating exchange rate regime.

2.4 Flexible price equilibrium

The natural values of real variables are described by following

$$\begin{aligned}\tilde{C}_t^W &= \frac{\eta}{\eta+\rho}\left(a_t^W - g_t^W\right),\ \tilde{T}_t = \frac{\eta}{1+\eta}\left(g_t^R - a_t^R\right), \\ \tilde{Y}_t^H &= (1-n)\,\tilde{T}_t + \tilde{C}_t^W + g_t^H,\ \tilde{Y}_t^F = -n\tilde{T}_t + \tilde{C}_t^W + g_t^F.\end{aligned}$$

From above, it is world shocks that have effects on the natural rate of world consumption, and relative shocks affect the natural rate of the terms of trade.

In the equilibrium, the terms of trade equation (4) becomes

$$\tilde{T}_t = \tilde{T}_{t-1} + \triangle\tilde{S}_t + \tilde{\pi}_t^F - \tilde{\pi}_t^H.$$

By assuming zero producer inflation rates in this particular flexible price equilibrium, it implies

$$\tilde{T}_t = \tilde{T}_{t-1} + \tilde{S}_t - \tilde{S}_{t-1}.$$

Given the initial values of $\tilde{T}_{-1} = \tilde{S}_{-1} = 0$, it means

$$\tilde{T}_t = \tilde{S}_t.$$

From the uncovered interest parity equation (5), the natural nominal interest rates differential can be expressed as

$$\tilde{R}_t^R \equiv \tilde{\imath}_t^H - \tilde{\imath}_t^F = E_t\triangle\tilde{S}_{t+1} = E_t\left(\tilde{T}_{t+1} - \tilde{T}_t\right). \tag{12}$$

From the world average Euler equation (1), the world natural nominal interest rate is determined by world shocks, where

$$\tilde{R}_t^W \equiv n\tilde{\imath}_t^H + (1-n)\,\tilde{\imath}_t^F = \rho\left(E_t\tilde{C}_{t+1} - \tilde{C}_t\right). \tag{13}$$

The natural nominal interest rate for each country is therefore derived from (12) and (13) as

$$\tilde{\imath}_t^H = \tilde{R}_t^W + (1+n)\,\tilde{R}_t^R,\ \tilde{\imath}_t^F = \tilde{R}_t^W - n\tilde{R}_t^R.$$

2.5 Methodology

2.5.1 Equilibrium determinacy

The implementation of monetary policy rules, particularly interest rate feedback rules, has been extensively studied in many aspects within the New Keynesian literature recently. The first major issue in the literature is whether or not the designed monetary policy rule guarantees real determinacy. When the system has unique stationary REE, the model is said to be determinate. When there are multiple stationary solutions for REE, including "sunspot solutions", the model suffers from indeterminacy, which is plainly undesirable.[14] Bernanke and Woodford (1997), Svensson and Woodford (1999, 2003) and Woodford (1999b, 2000) demonstrated that some policy rules can result in indeterminacy of equilibrium or multiple RE solutions in a closed economy framework. Bullard and Mitra (2002), Evans and Honkapohja (2003a, 2003b, 2006) have further investigated this issue. Accordingly, this paper first discusses the equilibrium determinacy for the specific interest rate rules under three different exchange rate regimes in this two-country model.

Consider a standard system of

$$\begin{aligned} y_t &= \alpha + \beta E_t y_{t+1} + \delta y_{t-1} + \kappa \omega_t, \qquad (14) \\ \omega_t &= \varphi \omega_{t-1} + e_t, \quad |\varphi| < 1, \end{aligned}$$

where y_t is an $n \times 1$ vector of endogenous variables, α is an $n \times 1$ vector of constants, β, δ, κ and φ are $n \times n$ matrices of coefficients, ω_t is an $n \times 1$ vector of exogenous variables following a stationary VAR, and e_t is an $n \times 1$ vector of white noise terms.

To study determinacy, expectations can be replaced by their actual values and the system can be written as

$$Y_{t+1} = JY_t + other, \qquad (15)$$

[14] In the case of "sunspot solutions", REE depends on extraneous random variables that influence the economy solely through the expectations of the agents.

where $Y_t \equiv \begin{pmatrix} Y_{1,t+1} & Y_{2,t+1} \end{pmatrix}'$, $J \equiv \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}$, and $Y_{i,t} \equiv \begin{pmatrix} \tilde{y}_{i,t} & \hat{y}_{i,t} & \tilde{y}^l_{i,t} \end{pmatrix}'$ with $i = 1, 2$; $\tilde{y}_{i,t}$ denotes the endogenous variables in $y_{i,t}$; $\hat{y}_{i,t}$ denotes the predetermined variables in $y_{i,t}$, including exogenous variables and lagged endogenous variables; and *other* denotes the innovations arising from replacing the expectations in the structural model by their actual values. The condition for equilibrium determinacy is therefore that the number of eigenvalues of J outside the unit circle is equal to the dimension of endogenous variables in Y_t.

In some model, the system (15) can be expressed in the form of

$$Y_{1,t+1} = J_{11}Y_{1,t} + other_1, \tag{15-1}$$

$$Y_{2,t+1} = J_{21}Y_{2,t} + J_{22}Y_{2,t} + other_2, \tag{15-2}$$

in which the matrix J becomes lower block triangular, which is called as the case (DE1). The eigenvalues of J are therefore all of the eigenvalues of J_{11} and J_{22}. A necessary condition for equilibrium determinacy is therefore that the number of all of the eigenvalues of J_{11} and J_{22} outside the unit circle is equal to the dimension of endogenous variables in Y_t.

Moreover, in order to get the determinate equilibrium for the full system (15), the subsystem (15-1) should also be determinate, which means that the number of eigenvalues of J_{11} outside the unit circle should be also exactly equal to the dimension of endogenous variables in $Y_{1,t}$. Therefore, the necessary and sufficient condition for the determinacy of the full system (DE1) is that each of the number of eigenvalues of J_{11} and J_{22} outside the unit circle should be exactly equal to the dimension of endogenous variables in $Y_{1,t}$ and $Y_{2,t}$ respectively.

Otherwise, if only the number of the eigenvalues of J (or J_{11} and J_{22}) outside the unit circle is equal to the dimension of endogenous variables $\tilde{y}_t$, it is possible for the full system (15) indeterminate in some cases. For example, when the number of all the eigenvalues of J outside the unit circle happens to be equal to the dimension of endogenous variables $\tilde{y}_t$, but the number of eigenvalues of J_{11} outside the unit circle is less than the dimension of endogenous variables in $Y_{1,t}$, the subsystem (15-1) will be indeterminate, which implies multiple stationary sunspot solutions for REE of $Y_{1,t}$. For each solution, the subsystem (15-2) has a corresponding stationary solution, and therefore the full system (15) is indeterminate.

If J_{21} in (15-2) is null matrix, the matrix J in (15) becomes block diagonal, which is called as the case (DE2). In this case, the system can be divided into two independent subsystems. In order to get the determinate equilibrium for the full system (15), the necessary and sufficient condition is that the two systems must satisfy the condition for determinacy separately. That means the number of eigenvalues of J_{11} and J_{22} outside the unit circle should be respectively equal to the dimension of endogenous variables in $Y_{1,t}$ and $Y_{2,t}$.

Therefore, if the system is in the form of (DE1), to obtain the determinacy condition for full system (15) is equivalent to calculate the determinacy condition for a simplified system (DE2) or two independent subsystems (15-3) and (15-4).

Following this methodology, the analysis for large systems can be simplified to derive the conditions for determinacy under three exchange rate regimes.

2.5.2 Learning stability

By adopting the methods developed by Evans and Honkapohja (1999, 2001), I analyze the learnability for the interest rate rules under different exchange rate regimes in this two-country model. It is assumed that the expectations of agents are not fully rational any more; instead, they use an adaptive learning rule to form their expectation values and update their forecasting in each period when new data becomes available. In particular, I will discuss the *recursive least squares* (RLS) learning rules and conditions for *expectational stability* (E-stability) for *minimal state variable* (MSV) solutions. As Evans and Honkapohja (1998, 2001) showed, the RLS learning is convergent to REE if and only if E-stability conditions are satisfied.[15]

[15] Marcet and Sargent (1989a, 1989b) discuss this issue for general linear models with a unique REE. Evans and Honkapohja (1994, 1998) make an improvement mathematically to discuss the local stability in univariate and multivariate linear models with multiple

Following Evans and Honkapohja (2001), E-stability is defined considering the standard model (14). The MSV solution is assumed in the form of

$$y_t = a + by_{t-1} + c\omega_t, \tag{16}$$

where $a, b,$and c are to be determined by the method of undetermined coefficients. By assuming time-t information set $(1, y_t', \omega_t')$, the corresponding expectations are calculated as

$$E_t y_{t+1} = a + by_t + c\varphi\omega_t.$$

Insert it into equation (14), and thus the MSV solution satisfies

$$\begin{aligned} (I - \beta b - \beta)\, a &= \alpha, && (17) \\ \beta b^2 - b + \delta &= 0, && (18) \\ (I - \beta b)\, c - \beta c\varphi &= \kappa. && (19) \end{aligned}$$

Assume $\left(\bar{a}, \bar{b}, \bar{c}\right)$ is a particular MSV solution. To obtain the conditions for E-stability, we regard equation (16) as the *perceived law of motion* (PLM), and thus the *actual law of motion* (ALM) of y_t can be derived as

$$y_t = (I - \beta b)^{-1} \left[\alpha + \beta a + \delta y_{t-1} + (\kappa + \beta c\varphi)\, \omega_t\right].$$

It implies the mapping from the PLM to the ALM

$$T(a, b, c) = \left((I - \beta b)^{-1} (\alpha + \beta a), (I - \beta b)^{-1} \delta, (I - \beta b)^{-1} (\kappa + \beta c\varphi)\right).$$

Therefore, the E-stability conditions can be stated in terms of following ma-

equilibria and global convergence in a model with a unique equilibrium.

trices

$$\begin{aligned} DT_a(\bar{a},\bar{b}) &= \left(I-\beta\bar{b}\right)^{-1}\beta, \\ DT_b(\bar{b}) &= \left[\left(I-\beta\bar{b}\right)^{-1}\delta\right]'\otimes\left[\left(I-\beta\bar{b}\right)^{-1}\beta\right], \\ DT_c(\bar{b},\bar{c}) &= \varphi'\otimes\left[\left(I-\beta\bar{b}\right)^{-1}\beta\right]. \end{aligned} \tag{20}$$

Evans and Honkapohja (2001) show that the condition for E-stability of the MSV solution $\left(\bar{a},\bar{b},\bar{c}\right)$ is that all eigenvalues of the matrices $DT_a(\bar{a},\bar{b})$, $DT_b(\bar{b})$, and $DT_c(\bar{b},\bar{c})$, given by equation (20), have real parts less than 1. Otherwise, the solution is not E-stable.

Denote $F \equiv \left(I-\beta\bar{b}\right)^{-1}\beta$ and $\Omega \equiv \left(I-\beta\bar{b}\right)^{-1}\delta$. A standard result says that the eigenvalues of a Kronecker product are the products of the eigenvalues of the relevant matrices.[16] Since $|\varphi| < 1$, the condition for E-stability now becomes that all eigenvalues of the matrices F and Ω have real parts less than 1.

We now consider partitioned systems for the E-stability analysis. Rewrite the system (14) as

$$\begin{aligned} \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} &= \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}\begin{pmatrix} E_t y_{1,t+1} \\ E_t y_{2,t+1} \end{pmatrix} + \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}\begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} \\ &\quad + \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}\begin{pmatrix} \omega_{1,t} \\ \omega_{2,t} \end{pmatrix}, \end{aligned} \tag{14'}$$

where $y_t \equiv \begin{pmatrix} y'_{1,t} & y'_{2,t} \end{pmatrix}'$ and $\omega_t \equiv \begin{pmatrix} \omega'_{1,t} & \omega'_{2,t} \end{pmatrix}'$.

If it happens that P_{12}, Q_{12}, and K_{12} are null matrices, the matrices P, Q, K become lower block triangular, which is called as the case (ES1). The system becomes

$$y_{1,t} = P_{11}E_t y_{1,t+1} + Q_{11}y_{1,t-1} + K_{11}\omega_{1,t}, \tag{14-1}$$

$$\begin{aligned} y_{2,t} &= P_{21}E_t y_{1,t+1} + P_{22}E_t y_{2,t+1} + Q_{21}y_{1,t-1} \\ &\quad + Q_{22}y_{2,t-1} + K_{21}\omega_{1,t} + K_{22}\omega_{2,t}, \end{aligned} \tag{14-2}$$

[16] See Magnus and Neudecker (1998).

where the first subsystem (14-1) is independent from the second subsystem (14-2).

It follows that the MSV solutions are in the form

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & \mathbf{0} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} + \begin{pmatrix} c_{11} & \mathbf{0} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} \omega_{1,t} \\ \omega_{2,t} \end{pmatrix}.$$

From (18), matrix $b \equiv \begin{pmatrix} b_{11} & \mathbf{0} \\ b_{21} & b_{22} \end{pmatrix}$ satisfies

$$\begin{cases} P_{11} b_{11} b_{11} - b_{11} + Q_{11} = 0, \\ P_{21} b_{11} b_{11} + P_{22} b_{21} b_{11} + P_{22} b_{22} b_{21} - b_{21} + Q_{21} = 0, \\ P_{22} b_{22} b_{22} - b_{22} + Q_{22} = 0. \end{cases} \tag{21}$$

b_{11}, b_{22}, b_{21} and thus b can be solved from equations (21). Therefore, it follows

$$\begin{aligned} F &\equiv \left(I - P\bar{b}\right)^{-1} P \\ &= \begin{pmatrix} \left(I_{11} - P_{11}\bar{b}_{11}\right)^{-1} P_{11} & \mathbf{0} \\ W P_{11} + \left(I_{22} - P_{22}\bar{b}_{22}\right)^{-1} P_{21} & \left(I_{22} - P_{22}\bar{b}_{22}\right)^{-1} P_{22} \end{pmatrix} \\ &\equiv \begin{pmatrix} F_{11} & \mathbf{0} \\ F_{21} & F_{22} \end{pmatrix}, \end{aligned}$$

where we denote

$$W \equiv \left(I_{22} - P_{22}\bar{b}_{22}\right)^{-1} \left(P_{21}\bar{b}_{11} + P_{22}\bar{b}_{21}\right) \left(I_{11} - P_{11}\bar{b}_{11}\right)^{-1}$$

and

$$\begin{aligned} \Omega &\equiv \left(I - P\bar{b}\right)^{-1} Q \\ &= \begin{pmatrix} \left(I_{11} - P_{11}\bar{b}_{11}\right)^{-1} Q_{11} & \mathbf{0} \\ W Q_{11} + \left(I_{22} - P_{22}\bar{b}_{22}\right)^{-1} Q_{22} & \left(I_{22} - P_{22}\bar{b}_{22}\right)^{-1} Q_{22} \end{pmatrix} \\ &\equiv \begin{pmatrix} \Omega_{11} & \mathbf{0} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}. \end{aligned}$$

Since matrices F and Ω are lower block triangular, the eigenvalues of F are the eigenvalues of F_{11} and F_{22}, and the eigenvalues of Ω are the eigenvalues of Ω_{11} and Ω_{22}. Therefore, to check the learnability of REE for the full system is equivalent to check the eigenvalues of F_{11}, F_{22}, Ω_{11} and Ω_{22}.

If P_{12}, P_{21}, Q_{12}, Q_{21}, K_{12} and $K_{21}$are all null matrices, the matrices P, Q, K become block diagonal, which is called as the case (ES2). The full system (14) can be divided into two independent subsystems

$$\begin{aligned} y_{1,t} &= P_{11}E_t y_{1,t+1} + Q_{11}y_{1,t-1} + K_{11}\omega_{1,t}, \quad &(14\text{-}1)\\ y_{2,t} &= P_{22}E_t y_{2,t+1} + Q_{22}y_{2,t-1} + K_{22}\omega_{2,t}. \quad &(14\text{-}2)\end{aligned}$$

It follows that the MSV solutions for the full system (14) are the same as the MSV solutions for each subsystem, and they satisfy

$$\begin{aligned} y_{1,t} &= a_{11} + b_{11}y_{1,t-1} + c_{11}\omega_{1,t},\\ y_{2,t} &= a_{22} + b_{22}y_{2,t-1} + c_{22}\omega_{2,t}.\end{aligned}$$

Therefore, b can be solved from the first and the third equation of the system (21). In order to make REE of full system (14) learnable, each subsystem of (14-1) and (14-2) should satisfy the condition for learnability. This requirement means that the eigenvalues of F_{11} and F_{22} in subsystem (14-1), and the eigenvalues of Ω_{11} and Ω_{22} in subsystem (14-2) should have real parts less than 1. This result is the same as the discussion for the case of system (ES1).

McCallum (2006) discusses the learnability for a general class of models, based on a class of linear models considered by Evans and Honkapohja (2001), which permits any number of lags, leads, and lags of leads. He proved that if current information is available for individuals, determinacy conditions are always sufficient but not necessary for E-stability and thus least square learnability. However, it is not sufficient if instead only lagged information can be observed in the learning process.[17] This result will be used to obtain the sufficient conditions of learnability for REE under managed regimes.

[17] See Evans and Honkapohja (2001) and McCallum (2006) for examples.

2.6 The parameters

Analytical results have been obtained for most cases. However, the structure of this model, with normally three or more dimensions, is much more complicated than the case in a closed economy with normally two dimensions. Therefore, this paper illustrates the findings using a calibrated case. According to the discussion in Rotemberg and Woodford (1998), the parameters are calibrated by $\eta = 0.47, \rho = 0.16$, and $\sigma = 7.88$, where η is the elasticity of the disutility of producing the differentiated goods, ρ is the inverse of the intertemporal elasticity of substitution in consumption, and σ is the elasticity of substitution across goods produced within a country. The value of the intertemporal discount factor in the consumption preference β is set throughout to 0.99, and the degrees of rigidity is set to vary in $(0, 1)$. Policy rules are calibrated for the parameters ϕ, ψ, μ, μ_1 and μ_2, which appear in the interest rate rules under different exchange rate regimes. The autocorrelations in the Wicksellian rate and natural terms of trade process are set to be 0.4 in each case.

3 Floating exchange rate regime

3.1 The dynamic system

Under a floating exchange rate regime, both countries are assumed to follow simple Taylor-type rules (8) with identical parameters. Therefore, equations (3), (4), (5), (6), and (7), together with the interest rate rules (8), characterize the log-linear equilibrium under the floating exchange rate regime.

From the Open-economy AS Curve (6), (7), the nominal inflation rate differential is

$$\pi_t^F - \pi_t^H = -k_T \left(\hat{T}_t - \tilde{T}_t \right) + \beta E_t \left(\pi_{t+1}^F - \pi_{t+1}^H \right)$$

and the world average inflation rate can be derived as

$$\begin{aligned} n\pi_t^H + (1-n)\,\pi_t^F &= \left(nk_C y_t^W + (1-n)\,k_C y_t^W\right) \\ &\quad +\beta\left(nE_t\pi_{t+1}^H + (1-n)\,E_t\pi_{t+1}^F\right). \end{aligned}$$

Substituting the interest rates with (8), the microfounded "open-economy" IS curve (3) will be

$$\begin{aligned} y_t^W &= E_t y_{t+1}^W - \rho^{-1}\left(\phi\left(n\pi_t^H + (1-n)\,\pi_t^F\right) + \psi\left(y_t^W\right)\right) \\ &\quad +\rho^{-1}\left(nE_t\pi_{t+1}^H + (1-n)\,E_t\pi_{t+1}^F\right) + \rho^{-1}\tilde{R}_t^W. \end{aligned}$$

From the definition of the output gap and aggregate demand functions (2), the output gap differential is derived as

$$\hat{y}_t^H - \hat{y}_t^F = \hat{T}_t - \tilde{T}_t. \tag{22}$$

Therefore, the uncovered interest parity equation (5), combined with interest rate rules (8) means that

$$\begin{aligned} E_t\triangle S_{t+1} &= \phi\left(\pi_t^H - \pi_t^F\right) + \psi\left(\hat{y}_t^H - \hat{y}_t^F\right) \\ &= \phi\left(\pi_t^H - \pi_t^F\right) + \psi\left(\hat{T}_t - \tilde{T}_t\right). \end{aligned}$$

If denote $\pi_t^R = \pi_t^F - \pi_t^H$, and $\pi_t^W = n\pi_t^H + (1-n)\,\pi_t^F$, we can write the full dynamic system as

$$\begin{cases} \pi_t^R = -k_T\left(\hat{T}_t - \tilde{T}_t\right) + \beta E_t\pi_{t+1}^R, \\ \hat{T}_t = \hat{T}_{t-1} + \triangle S_t + \pi_t^R, \\ E_t\triangle S_{t+1} = -\phi\pi_t^R + \psi\left(\hat{T}_t - \tilde{T}_t\right), \\ \pi_t^W = k_C y_t^W + \beta E_t\pi_{t+1}^W, \\ y_t^W = E_t y_{t+1}^W - \rho^{-1}\left(\phi\pi_t^W + \psi\left(y_t^W\right)\right) + \rho^{-1}E_t\pi_{t+1}^W + \rho^{-1}\tilde{R}_t^W. \end{cases}$$

If define $y_{1,t}^{FL} = \begin{pmatrix} \pi_t^R & \hat{T}_t & \triangle S_t \end{pmatrix}'$, $y_{2,t}^{FL} = \begin{pmatrix} \pi_t^W & y_t^W \end{pmatrix}'$, $\omega_{1,t}^{FL} = \left(\tilde{T}_t, 0, 0\right)'$, and $\omega_{2,t}^{FL} = \left(\tilde{R}_t^W, 0\right)'$, the system can be written as the matrix form of

$$\begin{aligned}
\begin{pmatrix} y_{1,t}^{FL} \\ y_{2,t}^{FL} \end{pmatrix} &= \begin{pmatrix} P_{11}^{FL} & \mathbf{0} \\ \mathbf{0} & P_{22}^{FL} \end{pmatrix} \begin{pmatrix} E_t y_{1,t+1}^{FL} \\ E_t y_{2,t+1}^{FL} \end{pmatrix} + \begin{pmatrix} Q_{11}^{FL} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} y_{1,t-1}^{FL} \\ y_{2,t-2}^{FL} \end{pmatrix} \\
&\quad + \begin{pmatrix} K_{11}^{FL} & \mathbf{0} \\ \mathbf{0} & K_{22}^{FL} \end{pmatrix} \begin{pmatrix} \omega_{1,t}^{FL} \\ \omega_{2,t}^{FL} \end{pmatrix},
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
P_{11}^{FL} &= \begin{pmatrix} \frac{\beta\psi}{\psi+\phi k_T} & 0 & \frac{-k_T}{\psi+\phi k_T} \\ \frac{\beta\phi}{\psi+\phi k_T} & 0 & \frac{1}{\psi+\phi k_T} \\ \frac{\beta(\phi-\psi)}{\psi+\phi k_T} & 0 & \frac{(k_T+1)}{\psi+\phi k_T} \end{pmatrix}, P_{22}^{FL} = \begin{pmatrix} \frac{1}{\beta} & -\frac{1}{\beta} k_C \\ \frac{\beta\phi-1}{\beta\rho} & \frac{\psi}{\rho} + \frac{k_C}{\beta\rho} + 1 \end{pmatrix}, \\
Q_{11}^{FL} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, K_{11}^{FL} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, K_{22}^{FL} = \begin{pmatrix} \frac{k_C}{\psi+\rho+\phi k_C} & 0 \\ \frac{1}{\psi+\rho+\phi k_C} & 0 \end{pmatrix}.
\end{aligned}$$

Under the floating exchange rate regime, even though there is no natural separation between two countries, there is complete separation between world variables and relative variables, which implies the full system can be divided into two subsystems, one with world variables, and the other with relative variables.

3.2 Determinacy

Even though the conditions for determinacy under three exchange rate regimes have been discussed in BB (2006b), this paper will re-discuss the conditions for determinacy with alternative proofs of mine.

The full system (23) under the floating regime can be written as two independent systems

$$y_{1,t}^{FL} = P_{11}^{FL} E_t y_{1,t+1}^{FL} + Q_{11}^{FL} y_{1,t-1}^{FL} + K_{11}^{FL} \omega_{1,t}^{FL}, \tag{23-1}$$

$$y_{2,t}^{FL} = P_{22}^{FL} E_t y_{2,t+1}^{FL} + K_{22}^{FL} \omega_{2,t}^{FL}, \tag{23-2}$$

where the subsystem (23-2) is purely forward-looking.

If denote $Y_{1,t}^{FL} = \begin{pmatrix} \pi_t^R & \triangle S_t & \hat{T}_t^l \end{pmatrix}'$, $Y_{2,t}^{FL} = y_{2,t}^{FL} = \begin{pmatrix} \pi_t^W & y_t^W \end{pmatrix}'$, where $\hat{T}_t^l = \hat{T}_{t-1}$, the two subsystems (23-1) and (23-2) can be rewritten in the form of the case (DE2), where

$$
\begin{aligned}
J_{11}^{FL} &= \begin{pmatrix} \frac{k_T+1}{\beta} & \frac{k_T}{\beta} & \frac{k_T}{\beta} \\ -\phi+\psi & \psi & \psi \\ 1 & 1 & 1 \end{pmatrix}, \\
J_{22}^{FL} &= \left(P_{22}^{FL}\right)^{-1}.
\end{aligned}
$$

In order to get the determinate equilibrium for the full system (23), the two subsystems (23-3) and (23-4) should satisfy the condition for determinacy separately. Because in subsystem (23-3) there is one predetermined variable $\hat{T}_t^l$ and two endogenous variables π_t^R and $\triangle S_t$, the determinacy condition for (23-3) is that exactly two of three eigenvalues of matrix J_{11}^{FL} lie outside the unit circle. Since there is no predetermined variable in subsystem (23-4), the determinacy condition for (23-4) is that all of the two eigenvalues of matrix J_{22}^{FL} lie outside the unit circle. These imply following proposition under the floating regime.

Proposition 1 *Under a floating exchange regime defined by the rules of the following form*

$$
\begin{aligned}
\hat{\imath}_t^H &= \phi\pi_t^H + \psi y_t^H, \\
\hat{\imath}_t^F &= \phi\pi_t^F + \psi y_t^F,
\end{aligned}
$$

with ϕ and ψ non negative, if the degrees of rigidity are equal across countries, the necessary and sufficient condition for equilibrium determinacy is

$$
\begin{aligned}
k_T(\phi-1)+\psi(1-\beta) &> 0, && \text{(FL1)} \\
k_C(\phi-1)+\psi(1-\beta) &> 0. && \text{(FL2)}
\end{aligned}
$$

Proof. See Appendix A.1. ■

For simplicity, it has been assumed that the two countries have the same degree of rigidities, and the parameters in policy rules are the same between two countries. The condition (FL2), which is required for the determinacy of world average variables, is the same as that in the closed economy counterpart, while the condition (FL1) required for the determinacy of country relative variables is generally different, depending on the intertemporal elasticity of substitution in consumption ρ^{-1}. When $\rho^{-1} > 1$, i.e. the intertemporal consumption substitutability is larger than the substitutability of goods across countries, we have $k_C < k_T$, and therefore the condition (FL1) implies (FL2), which means the conditions for determinacy of REE is more stringent than in the close economy counterpart. Otherwise, the conditions for determinacy is the same as its closed economy counterpart, when $\rho^{-1} \leq 1$. If $\psi = 0$, the determinacy conditions will be simplified as $\phi > 1$, which is well known as the *Taylor Principle*, and it suggests that the nominal interest rates should be adjusted more than one-for-one with changes in inflation.[18] Finally, it should be noticed that both of the interest rate rules are required to be simultaneously aggressive under the floating regime. Actually, the aggressive policy rule followed by only one country could be no longer sufficient for the determinacy of equilibrium without consideration of the monetary policy rules followed by the other country. The further discussion of this issue will be conducted in the next section.

3.3 Learning stability

Suppose $\tilde{T}_t$ and $\tilde{R}_t^W$ follow $AR(1)$ processes as the following forms

$$\begin{aligned}\tilde{T}_t &= \varphi_1 \tilde{T}_{t-1} + \epsilon_t^1, \\ \tilde{R}_t^W &= \varphi_2^W \tilde{R}_{t-1}^W + \epsilon_t^2,\end{aligned}$$

where $0 < \varphi_1, \varphi_2 < 1$, with ϵ_t^1 and ϵ_t^2 are *iid* stochastic processes. The full system (23) can be divided into two independent subsystems (23-1) and (23-2) as in the case (ES-2), with Q_{22} being null matrix.

[18] See Woodford (1999b, 2000).

The MSV solutions for each subsystem can be respectively written as

$$\begin{aligned} y_{1,t}^{FL} &= a_1^{FL} + b_1^{FL} y_{1,t-1}^{FL} + c_1^{FL} \tilde{T}_t, \\ y_{2,t}^{FL} &= a_2^{FL} + c_2^{FL} \tilde{R}_t^W, \end{aligned}$$

which are also taken as the perceived laws of motion of representative agents. Assume that the time-t information set for each agent is $\left(1, y_{i,t}^{FL\prime}, \omega_{i,t}^{FL\prime}\right)$. The assumptions on PLM and corresponding information set allow the foreign variables are included in the perceived law of motion of a representative agent, and therefore the agents can use information from both countries under learning. Otherwise, if only domestic information is available, it would be insufficient for the agent to learn the REE, as BS (2006) discussed.[19]

Substitute the PLM into (23-1) and (23-2), and it follows that the MSV solutions of subsystem (23-1) satisfy

$$\begin{aligned} (I - P_{11}^{FL} b_1^{FL} - P_{11}^{FL}) a_1^{FL} &= 0, \\ P_{11}^{FL} \left(b_1^{FL}\right)^2 - b_1^{FL} + Q_{11}^{FL} &= 0, \\ (I - P_{11}^{FL} b_1^{FL}) c_2^{FL} - P_{11}^{FL} c_2^{FL} \varphi_1 - K_{11}^{FL} &= 0, \end{aligned} \tag{24}$$

and the MSV solutions of subsystem (23-2) satisfy

$$\begin{aligned} (I - P_{22}^{FL}) a_2^{FL} &= 0, \\ c_2^{FL} - P_{22}^{FL} c_2^{FL} \varphi_2 - K_{22}^{FL} &= 0. \end{aligned} \tag{25}$$

From (24) and (25), the REE is solved for the floating regime as $\left\{\bar{a}_1^{FL}, \bar{b}_1^{FL}, \bar{c}_1^{FL}\right\}$ and $\left\{\bar{a}_2^{FL}, \bar{c}_2^{FL}\right\}$.[20]

The corresponding E-stability conditions are that all the eigenvalues of

[19] The discussion on the perceived law of motion and information set in open economy is controversial. Further investigation is in process.

[20] See Appendix II.

DT_{a_1}, DT_{b_1}, DT_{c_1} and DT_{a_2}, DT_{c_2} have real parts less than 1, where

$$\begin{aligned}
DT_a &= \left(I_{11} - P_{11}^{FL}\overline{b}_1^{FL}\right)^{-1} P_{11}^{FL}, \\
DT_b &= \left[\left(I_{11} - P_{11}^{FL}\overline{b}_1^{FL}\right)^{-1} Q_{11}^{FI}\right]' \otimes \left[\left(I_{11} - P_{11}^{FL}\overline{b}_1^{FL}\right)^{-1} P_{11}^{FL}\right], \\
DT_c &= \left(I_{11} - P_{11}^{FL}\overline{b}_1^{FL}\right)^{-1} P_{11}^{FL}\varphi_1,
\end{aligned}$$

and

$$\begin{aligned}
DT_{a_2} &= P_{22}^{FL}, \\
DT_{c_2} &= P_{22}^{FL}\varphi_2.
\end{aligned}$$

This means that all the eigenvalues of F_{11}^{FL}, Ω_{11}^{FI}, and P_{22}^{FL} have the real parts less than 1, which implies the following proposition.

Proposition 2 *Under a floating exchange rate regime defined by the rules of the following form*

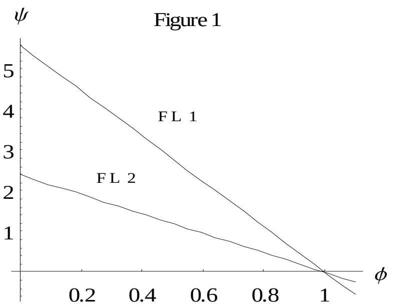
$$\begin{aligned}
\hat{\imath}_t^H &= \phi\pi_t^H + \psi y_t^H, \\
\hat{\imath}_t^F &= \phi\pi_t^F + \psi y_t^F,
\end{aligned}$$

with ϕ and ψ non negative, if the degrees of rigidity are equal across countries, the necessary and sufficient condition for the MSV solution of REE to be stable under adaptive learning is

$$\begin{aligned}
k_T(\phi - 1) + \psi(1 - \beta) &> 0, && \textit{(FL1)} \\
k_C(\phi - 1) + \psi(1 - \beta) &> 0. && \textit{(FL2)}
\end{aligned}$$

Proof. See Appendix A.3. ■

The proposition 1 & 2 implies that the necessary and sufficient conditions for learnability and determinacy coincide when Taylor-type rules are followed by both countries under this floating regime. An important finding is that when $\rho^{-1} > 1$, the condition (FL1) implies (FL2), and therefore the conditions for the determinacy and learnability become more stringent



in this two-country economy than in its close economy counterpart. Only when $\rho^{-1} \leq 1$, the condition (FL2) implies (FL1), and the conditions for determinacy and learnability are the same as in its close economy counterpart.

Given the calibration in Section 2.6, the figure 1 plots the conditions for determinacy and learnability as a function of ϕ and ψ. The two lines are the two conditions of determinacy and learnability. The upper line shows the condition (FL1) with $\rho = 0.16$, and the lower line shows the condition (FL2). The smaller value in ρ, the larger intertemporal elasticity of substitution in consumption, more is the shift of the line (FL2) to northeast.

The Taylor principle, $\phi > 1$, is still sufficient to guarantee the determinacy and learnability no matter what ψ is. Furthermore, for a positive ψ, the region for ϕ to satisfy the condition is allowed to be larger, which means a trade-off between policy parameters ϕ and ψ, i.e. more weight on the reaction towards the output gap in the policy rules, less reaction required towards inflation rate. Line (FL2) describes this trade-off between ϕ and ψ for determinacy and learnability of world average variables; it is the same condition as in its closed economy counterpart. However, when the intertemporal elasticity of substitution in consumption ρ^{-1} is larger than one, (recall that $\rho^{-1} = 0.16^{-1}$ in my calibrated case), k_T becomes larger than k_C, and line (FL1) is above line (FL2). The region for the determinate and learnable interest rate rules is therefore diminished by the condition (FL1), and the trade-off between reactions towards output gap and inflation in policy rules becomes worse.[21] This requirement implies that the interest rate rules need to be relatively more aggressive, when the central bank has open economy consideration and meanwhile the substitutability across goods produced from different countries are less than that of intertemporal consumption. As Figure 1 shows, the two countries have to choose their policy rule coefficients to the northeast of the upper line (FL1), in order to obtain determinacy and learnability for the REE of whole economy. Only when $\rho^{-1} \leq 1$, the line (FL2) shifts above (FL1), and therefore the condition for determinacy and learnability becomes (FL2), which is the same as in closed economy case. This finding supports the result in BS (2006), when they discuss the determinacy and learnability for the similar form of Taylor-type instrument rules followed by both countries. They argue that the degree of openness has quantitative effects on the region for determinacy and learnability, and the conditions for determinacy and learnability are more stringent with the open economy considerations with the non-zero degree of openness.

Intuitively, the change of region for determinate and learnable interest rate rules is due to the terms of trade effects in open economy environments. Since the inflation differential will affect the relative prices of goods across countries, it will improve or worsen the domestic terms of trade, which will shift the goods demand and affect the inflation rate and output gap. For example, when a positive demand shock g_t^H or a negative supply shock a_t^H hits the home economy, the natural rate of terms of trade increases as $\tilde{T}_t = \frac{\eta}{1+\eta}\left(g_t^R - a_t^R\right)$, which will push home inflation rate going up. To reduce the inflation and offset the shock, the home central bank needs to increase the interest rate following the policy rule (8). Without open economy consideration, the policy parameters can be chosen by the home country on the line (FL2), and it will reduce the home inflation and output gap. However, the decrease of the home inflation will worsen the terms of trade or increase $\hat{T}_t \uparrow$ as $\hat{T}_t = \hat{T}_{t-1} + \triangle S_t + \pi_t^F - \pi_t^H$, which implies a cheaper relative price of home goods. The consumption of foreign households, therefore, switches towards home goods, and push the home inflation again. To fully offset the

[21] Empirically, the intertemporal elasticity of substitution in consumption is normally larger than one, for example in Rotemberg and Woodford (1998), and Woodford (2003).

shock and pull the inflation back to the initial level, the home interest rate need to be increased more, for example a policy on line (FL1), which means a more aggressive policy rule for the home country. Only when $\rho^{-1} \leq 1$, the intertemporal elasticity in consumption is larger than substitutability of goods across countries, the consideration of open economy for the home central bank does not affect its decision. Therefore, the inertia of the terms of trade in open economy leads to slower adjustment of prices, and in turn weakens the efficiency of policy rules, compared with its closed economy counterpart.

Finally, the conditions for determinacy and learnability implicitly suggest that both countries should follow aggressive instrument rules simultaneously. Indeed, the failure to satisfy the determinacy and learnability conditions in one country could result in indeterminacy and instability under learning for the whole economy, due to the interdependence across countries. The following section will discuss this issue more explicitly.

3.4 Interdependence across the two countries

The discussion in the above sections has suggested that both countries should simultaneously follow aggressive instrument rules in order to guarantee the determinacy and learnability of REE for whole economy. To see this more explicitly, we relax the assumption of identical parameters in policy rules followed by the two countries, and instead assumes that the home country and foreign country follow the interest rate rules in the forms of

$$\begin{aligned} \hat{\imath}_t^H &= \phi \pi_t^H, \\ \hat{\imath}_t^F &= \phi^* \pi_t^F, \end{aligned} \tag{8*}$$

where the instrument rules only react towards inflation rate with the same or different values of parameters, with $\psi = \psi^* = 0$.

If define $y_{FL,t}^* = \left(y_t^W, \pi_t^H, \pi_t^F, \triangle S_t, \hat{T}_t\right)'$, and $\omega_{FL,t}^* = \left(\tilde{R}_t^W, \tilde{T}_t, \tilde{T}_t, 0, 0\right)'$, the dynamic system becomes

$$A_{FL}^* y_{FL,t}^* = B_{FL}^* E_t y_{FL,t+1}^* + F_{FL}^* y_{FL,t-1}^* + C_{FL}^* \omega_{FL,t}^*,$$

in which the matrices are given in Appendix A.5. It can be written as

$$y^*_{FL,t} = P^*_{FL} E_t y^*_{FL,t+1} + Q^*_{FL} y^*_{FL,t-1} + K^*_{FL} \omega^*_{FL,t}, \tag{23*}$$

where

$$P^*_{FL} = A^{-1*}_{FL} B^*_{FL}, Q^*_{FL} = A^{-1*}_{FL} F^*_{FL}, K^*_{FL} = A^{-1*}_{FL} C^*_{FL}.$$

Then follow the above procedure calibrating the economy, to test the determinacy and learnability of REE for different value of parameters in policy rules.

The only predetermined variable in system (23*) is $\hat{T}_t$. If define $\hat{T}^l_t \equiv \hat{T}_{t-1}$, and $Y^*_{FL,t} = \begin{pmatrix} y^W_t & \pi^H_t & \pi^F_t & \triangle S_t & \hat{T}^l_t \end{pmatrix}$, the system can be written as

$$Y^*_{FL,t+1} = J^* Y^*_{FL,t} + other^*, \tag{DE*}$$

in which J^* is given in Appendix A.5. Therefore, the determinacy condition is that four of all the eigenvalues of matrix J^* lie outside the unit circle, and one of them lies inside the unit circle.

Varying the policy parameters ϕ and ϕ^*, compute the eigenvalues of J to see the determinacy of REE for different pairs of policy rules followed by the two countries.

Suppose the MSV solutions for system (23*) can be written as

$$y^*_{FL,t} = a^*_{FL} + b^*_{FL} y^*_{FL,t-1} + c^*_{FL} \omega^*_{FL,t},$$

which is also PLM of representative agents. Assuming the time-t information set is $\left(1, y^{*'}_{FL,t}, \omega^{*'}_{FL,t}\right)$, substitute the PLM and its expectation into (23*) to solve the REE $\{a^*_{FL}, b^*_{FL}, c^*_{FL}\}$. Then, we can compute the eigenvalues of $F \equiv \left(I - P^{FL}\overline{b}^{FL}\right)^{-1} P^{FL}$ and $\Omega \equiv \left(I - P^{FL}\overline{b}^{FL}\right)^{-1} Q^{FI}$ to test the learnability of REE for different pairs of policy rules. In order to obtain the learnable REE, all the eigenvalues need to have the real parts less than 1. See Appendix A.4 for more details.

Varying the policy parameters ϕ and ϕ^*, we can get the following table to see the learnability of REE for different pairs of policy rules followed by the two countries.[22]

Table-FL1

ϕ	ϕ^*	Det
1.2	1.2	Yes
1.2	0.9	No
1.2	0.6	No
0.9	1.2	No
0.9	0.9	No

Det: Determinacy

Table-FL2

ϕ	ϕ^*	ES
1.2	1.2	Yes
1.2	0.9	No
1.2	0.6	No
0.9	1.2	No
0.9	0.9	No

ES: E-stability

The Proposition 1 and 2 show that given $\psi = 0$, the nominal interest rates should be adjusted more than one-for-one with changes in inflation in order to obtain the determinacy and learnability of the REE. It is well known as the Taylor Principle. The above results in (Table-FL1) and (Table-FL2) show that when the reaction towards inflation in policy rules is large enough, $\phi = \phi^* = 1.2$, the REE is both determinate and learnable. The necessary and sufficient conditions for determinacy and learnability are therefore closely linked to the Taylor's Principle under this floating regime.

However, when the reaction towards inflation in the policy rule followed by one country is smaller than 1, the REE of the economy is neither determinate nor learnable, no matter whether or not the other country follows a rule with reaction towards inflation larger than one. Therefore, the determinacy and learnability of REE for the whole economy require that both countries should follow aggressive interest rate rules simultaneously. This finding is consistent with the result in BS (2006) that the determinacy and learnability conditions must to be met country by country, and otherwise the world economy will be indeterminate and unstable under learning.

[22]This paper does not discuss the learnability of sunspot equilibria.

4 Fixed exchange rate regime

4.1 The dynamic system

There are many ways to specify the instrument rules that can implement a fixed exchange rate regime. BS (2006) introduces one way that one country targets the nominal exchange rate and therefore its optimal interest rate rule is a function of expectations of domestic variables and present values of international variables. Here is another way to obtain a fixed regime, where the nominal interest rate in the foreign country is assumed to react to the nominal interest rate in the home country and the deviations of the nominal exchange rate from a desired target, while the interest rate in the home country just follows a simple Taylor-type rule, as BB (2006b) discussed.

$$\begin{aligned}\hat{\imath}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{\imath}_t^F &= \hat{\imath}_t^H - \mu\hat{S}_t,\end{aligned} \tag{9}$$

where $\mu > 0$, $\hat{S}_t \equiv \ln(S_t/S^*)$, and S^* is the exchange rate target.

Equations (3), (3-1), (3-2), (4), (5), (6), and (7), together with the interest rate rule (9), characterize the log-linear equilibrium under the fixed regime.

Substituting interest rate rules (9) into the uncovered interest parity equation (5), it is obtained that

$$E_t\hat{S}_{t+1} = (1+\mu)\,\hat{S}_t.$$

Therefore, the condition of $\mu > 0$ is sufficient to get a determinate and bounded equilibrium for the nominal exchange rate, which implies the REE of exchange rate satisfies $S_t = S^*$ at all the time t, and $\widehat{S}_t = 0$, and therefore $\hat{\imath}_t^F = \hat{\imath}_t^H$.

Define $\pi_t^R \equiv \pi_t^F - \pi_t^H$, we can rewrite the full dynamic system (3), (3-1), (3-2), (4), (5), (6), (7) and (9) as

$$\begin{cases} \pi_t^R = -k_T\left(\hat{T}_t - \tilde{T}_t\right) + \beta E_t \pi_{t+1}^R, \\ \hat{T}_t = \hat{T}_{t-1} + \pi_t^R, \\ \pi_t^W = k_C y_t^W + \beta E_t \pi_{t+1}^W, \\ \pi_t^H = (1-n)\, k_T\left(\hat{T}_t - \tilde{T}_t\right) + k_C y_t^W + \beta E_t \pi_{t+1}^H, \\ y_t^W = E_t y_{t+1}^W - \rho^{-1}\phi\pi_t^H - \rho^{-1}\psi y_t^H + \rho^{-1} E_t \pi_{t+1}^W + \rho^{-1}\tilde{R}_t^W, \\ y_t^H = E_t y_{t+1}^H - \rho^{-1}\phi\pi_t^H - \rho^{-1}\psi y_t^H + \rho^{-1} E_t \pi_{t+1}^W - (1-n)\left(E_t\hat{T}_{t+1} - \hat{T}_t\right) + rr_t^H. \end{cases}$$

If define $y_{1,t}^{FI} = \begin{pmatrix} \pi_t^R & \hat{T}_t \end{pmatrix}'$, $y_{2,t}^{FI} = \begin{pmatrix} \pi_t^W & \pi_t^H & y_t^W & y_t^H \end{pmatrix}'$, $\omega_{1,t}^{FI} = \begin{pmatrix} \tilde{T}_t & 0 \end{pmatrix}'$, and $\omega_{2,t}^{FI} = \begin{pmatrix} \tilde{R}_t^W & rr_t^H & 0 & 0 \end{pmatrix}'$, the full system can be written as

$$\begin{aligned} \begin{pmatrix} A_{11}^{FI} & \mathbf{0} \\ A_{21}^{FI} & A_{22}^{FI} \end{pmatrix} \begin{pmatrix} y_{1,t}^{FI} \\ y_{2,t}^{FI} \end{pmatrix} &= \begin{pmatrix} B_{11}^{FI} & \mathbf{0} \\ B_{21}^{FI} & B_{22}^{FI} \end{pmatrix} \begin{pmatrix} E_t y_{1,t+1}^{FI} \\ E_t y_{2,t+1}^{FI} \end{pmatrix} \\ &+ \begin{pmatrix} F_{11}^{FI} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} y_{1,t-1}^{FI} \\ y_{2,t-1}^{FI} \end{pmatrix} \\ &+ \begin{pmatrix} C_{11}^{FI} & \mathbf{0} \\ \mathbf{0} & C_{22}^{FI} \end{pmatrix} \begin{pmatrix} \omega_{1,t}^{FI} \\ \omega_{2,t}^{FI} \end{pmatrix}, \end{aligned} \tag{26}$$

in which the matrices are given in Appendix A.5.

Under the fixed exchange rate regime, the complete separation between world variables and relative variables disappears due to the reaction to nominal exchange rate in the policy rules of the foreign country.

4.2 Determinacy

Under the fixed regime, π_t^R, π_t^H, π_t^W, y_t^W, and y_t^H are the nonpredetermined variables, while $\hat{T}_{t-1}$ is the predetermined variable. If denote $\hat{T}_t^l = \hat{T}_{t-1}$, add it to the system, and replace expectations by their true values, we can rewrite the full equilibrium system (26) as

$$\begin{pmatrix} L_{11}^{FI} & \mathbf{0} \\ L_{21}^{FI} & L_{22}^{FI} \end{pmatrix} \begin{pmatrix} Y_{1,t+1}^{FI} \\ Y_{2,t+1}^{FI} \end{pmatrix} = \begin{pmatrix} M_{11}^{FI} & \mathbf{0} \\ M_{21}^{FI} & M_{22}^{FI} \end{pmatrix} \begin{pmatrix} Y_{1,t}^{FI} \\ Y_{2,t}^{FI} \end{pmatrix} + other^{FI}, \tag{27}$$

in which we define $Y_{1,t}^{FI} = \begin{pmatrix} \pi_t^R & \hat{T}_t & \hat{T}_t^l \end{pmatrix}'$ and $Y_{2,t}^{FI} = \begin{pmatrix} \pi_t^W & \pi_t^H & y_t^W & y_t^H \end{pmatrix}'$, and the parameters matrices are given in Appendix A.5.

The matrices L_{21}^{FI} and M_{21}^{FI} are not null under the fixed regime, due to the reaction to the nominal exchange rate in the policy rule followed by the foreign country, where the foreign policymaker concerns international variables.

The reduced form of full system (27) is

$$\begin{pmatrix} Y_{1,t+1}^{FI} \\ Y_{2,t+1}^{FI} \end{pmatrix} = J^{FI} \begin{pmatrix} Y_{1,t}^{FI} \\ Y_{2,t}^{FI} \end{pmatrix} + other^{FI},$$

where the matrix J^{FI} is lower block triangular. According to the results in part 2.5.1, the discussion of condition for equilibrium determinacy for the full system (27) is therefore equivalent to the discussion of determinacy condition for the following two simplified subsystems

$$Y_{1,t+1}^{FI} = J_{11}^{FI} Y_{1,t}^{FI} + other_{1,t}^{FI}, \tag{27-1}$$
$$Y_{2,t+1}^{FI} = J_{22}^{FI} Y_{2,t}^{FI} + other_{2,t}^{FI}, \tag{27-2}$$

where $J_{11}^{FI} = \left(L_{11}^{FI}\right)^{-1} M_{11}^{FI}$, $J_{22}^{FI} = \left(L_{22}^{FI}\right)^{-1} M_{22}^{FI}$. Since there are two nonpredetermined variables and one predetermined variable in subsystem (27-1), and there is no predetermined variable in subsystem (27-2), the determinacy condition for full system (27) is that exactly two of three eigenvalues of J_{11}^{FI} lie outside the unit circle, and all of four eigenvalues of J_{22}^{FI} lie outside the unit circle. This requirement implies the following proposition.

Proposition 3 *Under a fixed exchange regime defined by the rules of the following form*

$$\hat{i}_t^H = \phi \pi_t^H + \psi y_t^H,$$
$$\hat{i}_t^F = \hat{i}_t^H - \mu \hat{S}_t,$$

with ϕ, ψ and μ non negative, if the degrees of rigidity are equal across countries, the necessary and sufficient condition for equilibrium determinacy *is*

$$k_C(\phi - 1) + \psi(1 - \beta) > 0, \tag{FI1}$$

$$\mu > 0. \tag{FI2}$$

Proof. See Appendix A.1. ■

The proposition show that once the nominal exchange rate is determined by the condition of $\mu > 0$, the condition for determinacy is the same as in the closed economy case with Taylor-type rules. Therefore, the determinacy of the worldwide REE only depends on the choice of monetary policy for the home country.

4.3 Learning stability

The full system (26) can be reduced to the following

$$\begin{pmatrix} y_{1,t}^{FI} \\ y_{2,t}^{FI} \end{pmatrix} = P^{FI} \begin{pmatrix} E_t y_{1,t+1}^{FI} \\ E_t y_{2,t+1}^{FI} \end{pmatrix} + Q^{FI} \begin{pmatrix} y_{1,t-1}^{FI} \\ y_{2,t-1}^{FI} \end{pmatrix} + K^{FI} \begin{pmatrix} \omega_{1,t}^{FI} \\ \omega_{2,t}^{FI} \end{pmatrix}, \tag{28}$$

where

$$\begin{aligned} P^{FI} &= \begin{pmatrix} \left(A_{11}^{FI}\right)^{-1} B_{11}^{FI} & \mathbf{0} \\ -\left(A_{22}^{FI}\right)^{-1} A_{21}^{FI} \left(A_{11}^{FI}\right)^{-1} B_{11}^{FI} + \left(A_{22}^{FI}\right)^{-1} B_{21}^{FI} & \left(A_{22}^{FI}\right)^{-1} B_{22}^{FI} \end{pmatrix}, \\ Q^{FI} &= \begin{pmatrix} \left(A_{11}^{FI}\right)^{-1} F_{11}^{FI} & \mathbf{0} \\ -\left(A_{22}^{FI}\right)^{-1} A_{21}^{FI} \left(A_{11}^{FI}\right)^{-1} F_{11}^{FI} & \mathbf{0} \end{pmatrix}, \\ K^{FI} &= \begin{pmatrix} \left(A_{11}^{FI}\right)^{-1} C_{11}^{FI} & \mathbf{0} \\ -\left(A_{22}^{FI}\right)^{-1} A_{21}^{FI} \left(A_{11}^{FI}\right)^{-1} C_{11}^{FI} & \left(A_{22}^{FI}\right)^{-1} C_{22}^{FI} \end{pmatrix}. \end{aligned}$$

Following section 2.5.2, the discussion of learnability for system (28) is equivalent to the discussion of learnability for two simplified subsystems

$$y_{1,t}^{FI} = P_{11}^{FI} E_t y_{1,t+1}^{FI} + Q_{11}^{FI} y_{1,t-1}^{FI} + K_{11}^{FI} \omega_{1,t}^{FI}, \tag{28-1}$$

$$y_{2,t}^{FI} = P_{22}^{FI} E_t y_{2,t+1}^{FI} + K_{22}^{FI} \omega_{2,t}^{FI}, \tag{28-2}$$

where

$$\begin{aligned} P_{11}^{FI} &= \left(A_{11}^{FI}\right)^{-1} B_{11}^{FI}, Q_{11}^{FI} = \left(A_{11}^{FI}\right)^{-1} F_{11}^{FI}, K_{11}^{FI} = \left(A_{11}^{FI}\right)^{-1} C_{11}^{FI}, \\ P_{22}^{FI} &= \left(A_{22}^{FI}\right)^{-1} B_{22}^{FI}, K_{22}^{FI} = \left(A_{22}^{FI}\right)^{-1} C_{22}^{FI}. \end{aligned}$$

Supposing $\omega_{1,t}^{FI}$ and $\omega_{2,t}^{FI}$ follow vector $AR(1)$ processes with serial correlation given by the scalars φ_1 and φ_2, the MSV solutions for the two subsystems are respectively in the form of

$$\begin{aligned} y_{1,t}^{FI} &= a_1^{FI} + b_1^{FI} y_{1,t-1}^{FI} + c_1^{FI} \omega_{1,t}^{FI}, \\ y_{2,t}^{FI} &= a_2^{FI} + c_2^{FI} \omega_{2,t}^{FI}, \end{aligned}$$

Insert them into (28-1) and (28-2), and thus the REE is solved for the fixed regime as $\left\{\overline{a}_1^{FI}, \overline{b}_1^{FI}, \overline{c}_1^{FI}\right\}$ and $\left\{\overline{a}_2^{FI}, \overline{c}_2^{FI}\right\}$.[23]

The corresponding E-stability conditions are that all of the eigenvalues of $F_{11}^{FI} \equiv \left(I_{11} - P_{11}^{FI}\overline{b}_1^{FI}\right)^{-1} P_{11}^{FI}$, $\Omega_{11}^{FI} \equiv \left(I_{11} - P_{11}^{FI}\overline{b}_1^{FI}\right)^{-1} Q_{11}^{FI}$, and P_{22}^{FI} have real parts less than one, which implies the following proposition.

Proposition 4 *Under a fixed exchange rate regime defined by the rules of the following form*

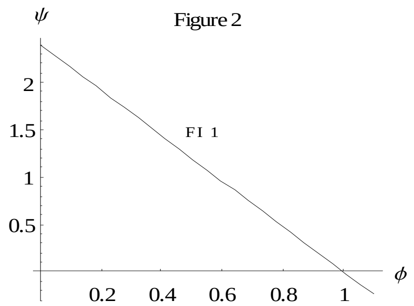
$$\begin{aligned} \hat{\imath}_t^H &= \phi \pi_t^H + \psi y_t^H, \\ \hat{\imath}_t^F &= \hat{\imath}_t^H - \mu \hat{S}_t, \end{aligned}$$

with ϕ, ψ and μ non negative, if the degrees of rigidity are equal across countries, the necessary and sufficient condition for the MSV solution of REE to be stable under adaptive learning is

$$k_C(\phi - 1) + \psi(1 - \beta) > 0,$$
$$\mu > 0.$$

Proof. See Appendix A.3. ■

[23] See Appendix II



Given the calibration in Section 2.6, the figure 2 plots the conditions of determinacy and learnability as a function of ϕ and ψ, which is the same as the lower line (FL2) under the floating regime.

The conditions for determinacy and learnability coincide again as under floating regime. Once the nominal exchange rate is determinate, the conditions of determinacy and learnability for the whole economy depend only on the policy rules in the home country, which is equivalent to the closed economy case. This result shows that the restriction for determinacy of equilibrium under the fixed regime is fairly broad and reasonable, and therefore the fixed regime are not necessarily destabilizing. Actually, under the fixed regime the central bank in the home country can influence the aggregate demand of both domestic and foreign households through the movements of interest rate and nominal exchange rate. Therefore, the central bank in the anchor country is just needed to be inward-looking to obtain the determinacy and learnability for the world economy.

5 Managed exchange rate regimes

Under the managed regimes, it is assumed that the policy rule followed by one country reacts to either the deviations of the level of the nominal exchange rate or the changes in the nominal exchange rate, which implies a 'dirty floating' exchange rate.

5.1 Managed exchange rate regime I

5.1.1 The dynamic system

Under a managed exchange rate regime I, the policy rule followed by the foreign country is assumed to react to the level of the nominal exchange rate, while the home country just follows a simple Taylor-type feedback rule

$$\begin{aligned} \hat{\imath}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{\imath}_t^F &= \phi\pi_t^F + \psi y_t^F - \mu_1 \hat{S}_t, \end{aligned} \tag{10}$$

Equations (3), (4), (5), (6), and (7), together with the interest rate rule (10), characterize the log-linear equilibrium in the managed regime I.

If again denote $\pi_t^R = \pi_t^F - \pi_t^H$, and $\pi_t^W = n\pi_t^H + (1-n)\,\pi_t^F$, following the same procedures as above, the equilibrium conditions are

$$\begin{cases} \pi_t^R = -k_T\left(\hat{T}_t - \tilde{T}_t\right) + \beta E_t \pi_{t+1}^R, \\ \hat{T}_t = \hat{T}_{t-1} + \hat{S}_t - \hat{S}_{t-1} + \pi_t^R, \\ E_t \hat{S}_{t+1} = -\phi\pi_t^R + \psi\left(\widehat{T}_t - \tilde{T}_t\right) + (1+\mu_1)\,\hat{S}_t, \\ \pi_t^W = k_C y_t^W + \beta E_t \pi_{t+1}^W, \\ y_t^W = E_t y_{t+1}^W - \rho^{-1}\phi\pi_t^W - \rho^{-1}\psi y_t^W + \rho^{-1}E_t\pi_{t+1}^W + \rho^{-1}(1-n)\,\mu_1\hat{S}_t + \rho^{-1}\tilde{R}_t^W. \end{cases}$$

If define $y_{1,t}^{MI} = \begin{pmatrix} \pi_t^R & \hat{T}_t & \hat{S}_t \end{pmatrix}'$, $y_{2,t}^{MI} = \begin{pmatrix} \pi_t^W & y_t^W \end{pmatrix}'$, $\omega_{1,t}^{MI} = \begin{pmatrix} \tilde{T}_t & 0 & 0 \end{pmatrix}'$, and $\omega_{2,t}^{MI} = \begin{pmatrix} \tilde{R}_t^W & 0 \end{pmatrix}'$ the full system can be written as

$$\begin{aligned} \begin{pmatrix} A_{11}^{MI} & \mathbf{0} \\ A_{21}^{MI} & A_{22}^{MI} \end{pmatrix} \begin{pmatrix} y_{1,t}^{MI} \\ y_{2,t}^{MI} \end{pmatrix} &= \begin{pmatrix} B_{11}^{MI} & \mathbf{0} \\ \mathbf{0} & B_{22}^{MI} \end{pmatrix} \begin{pmatrix} E_t y_{1,t+1}^{MI} \\ E_t y_{2,t+1}^{MI} \end{pmatrix} \\ &+ \begin{pmatrix} F_{11}^{MI} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} y_{1,t-1}^{MI} \\ y_{2,t-1}^{MI} \end{pmatrix} \\ &+ \begin{pmatrix} C_{11}^{MI} & \mathbf{0} \\ \mathbf{0} & C_{22}^{MI} \end{pmatrix} \begin{pmatrix} \omega_{1,t}^{MI} \\ \omega_{2,t}^{MI} \end{pmatrix}, \end{aligned} \tag{29}$$

in which the matrices are given in Appendix A.5.

Under the managed exchange rate regime I, the complete separation between world variables and relative variables again disappears due to the additional reaction to the international variable, the nominal exchange rate, in the policy rules of the foreign country.

5.1.2 Determinacy

Under this managed regime, π_t^R, $\hat{S}_t$, π_t^W, y_t^W are the nonpredetermined variables, while $\hat{T}_{t-1}$ and $\hat{S}_{t-1}$ are the predetermined variables. If introduce the notation $\hat{T}_t^l = \hat{T}_{t-1}$, $\hat{S}_t^l = \hat{S}_{t-1}$, add them to the system, and then replace expectations by their true values, we can rewrite the full equilibrium system (29) as

$$\begin{pmatrix} L_{11}^{MI} & \mathbf{0} \\ L_{21}^{MI} & L_{22}^{MI} \end{pmatrix} \begin{pmatrix} Y_{1,t+1}^{MI} \\ Y_{2,t+1}^{MI} \end{pmatrix} = \begin{pmatrix} M_{11}^{MI} & \mathbf{0} \\ M_{21}^{MI} & M_{22}^{MI} \end{pmatrix} \begin{pmatrix} Y_{1,t}^{MI} \\ Y_{2,t}^{MI} \end{pmatrix} + other^{MI}, \tag{30}$$

in which we define $Y_{1,t}^{MI} = \begin{pmatrix} \pi_t^R & \hat{S}_t & \hat{T}_t^l & \hat{S}_t^l \end{pmatrix}'$ and $Y_{2,t}^{MI} = \begin{pmatrix} \pi_t^W & y_t^W \end{pmatrix}'$, and the matrices are given in Appendix A.5. The matrix L_{21}^{MI} and M_{21}^{MI} are not null under the managed regime I, due to the reaction to the nominal exchange rate in the policy rule followed by the foreign country, where the foreign policymaker starts to concern international variables.

The reduced form of full system (30) is

$$\begin{matrix} Y_{1,t+1}^{MI} \\ Y_{2,t+1}^{MI} \end{matrix} = J^{MI} \begin{pmatrix} Y_{1,t}^{MI} \\ Y_{2,t}^{MI} \end{pmatrix} + other^{MI},$$

where

$$J^{MI} = \begin{pmatrix} \left(L_{11}^{MI}\right)^{-1} M_{11}^{MI} & \mathbf{0} \\ -\left(L_{22}^{MI}\right)^{-1} L_{21}^{MI} \left(L_{11}^{MI}\right)^{-1} M_{11}^{MI} + \left(L_{22}^{MI}\right)^{-1} M_{21}^{MI} & \left(L_{22}^{MI}\right)^{-1} M_{22}^{MI} \end{pmatrix}$$

and the matrix J^{MI} is lower block triangular. As the discussion in section 2.5.1, to obtain the condition for equilibrium determinacy for the full system (30) is therefore equivalent to calculating the determinacy condition for the following two simplified subsystems

$$
\begin{aligned}
Y_{1,t+1}^{MI} &= J_{11}^{MI} Y_{1,t}^{MI} + other_{1,t}^{MI}, && (30\text{-}1) \\
Y_{2,t+1}^{MI} &= J_{22}^{MI} Y_{2,t}^{MI} + other_{2,t}^{MI}, && (30\text{-}2)
\end{aligned}
$$

where

$$
\begin{aligned}
J_{11}^{MI} &= \left(L_{11}^{MI}\right)^{-1} M_{11}^{MI}, \\
J_{22}^{MI} &= \left(L_{22}^{MI}\right)^{-1} M_{22}^{MI}.
\end{aligned}
$$

Since there are two nonpredetermined variables and two predetermined variables in subsystem (30-1), and there is no predetermined variable in subsystem (30-2), the determinacy condition for full system (30) is that exactly two of four eigenvalues of J_{11}^{MI} lie outside the unit circle, and all of two eigenvalues of J_{22}^{MI} lie outside the unit circle. Notice that the subsystem (30-2) is in the same form as under the floating regime (23-4), which implies the condition (MI2). Therefore, following BB (2006), I get the following proposition.

Proposition 5 *Under the managed exchange regime (I) with following interest rate rules*

$$
\begin{aligned}
\hat{i}_t^H &= \phi \pi_t^H + \psi y_t^H, \\
\hat{i}_t^F &= \phi \pi_t^F + \psi y_t^F - \mu_1 \hat{S}_t,
\end{aligned}
$$

with ϕ, ψ and μ_1 non negative as well as μ_1 non zero, if the degrees of rigidity are equal across countries, the necessary and sufficient condition for equilibrium determinacy is

$$
\begin{aligned}
\mu_1 &> 0, && \textit{(MI1)} \\
k_C(\phi - 1) + \psi(1 - \beta) &> 0. && \textit{(MI2)}
\end{aligned}
$$

Proof. See Appendix A.1. ■

It is easy to see that the Taylor principle is again sufficient for the determinacy of REE under this managed regime. For any positive value of μ_1, the necessary and sufficient condition for determinacy is the same as its closed economy counterpart. Compared with the floating regime case, the region for determinate REE is therefore enlarged under this managed regime, due to the additional reaction towards the nominal exchange rate in the policy rule followed by the foreign country.[24]

5.1.3 Learning stability

The full system (29) can be reduced to the following

$$\begin{pmatrix} y_{1,t}^{MI} \\ y_{2,t}^{MI} \end{pmatrix} = P^{MI} \begin{pmatrix} E_t y_{1,t+1}^{MI} \\ E_t y_{2,t+1}^{MI} \end{pmatrix} + Q^{MI} \begin{pmatrix} y_{1,t-1}^{MI} \\ y_{2,t-1}^{MI} \end{pmatrix} + K^{MI} \begin{pmatrix} \omega_{1,t}^{MI} \\ \omega_{2,t}^{MI} \end{pmatrix}, \quad (31)$$

where

$$\begin{aligned} P^{MI} &= \begin{pmatrix} \left(A_{11}^{MI}\right)^{-1} B_{11}^{MI} & \mathbf{0} \\ -\left(A_{22}^{MI}\right)^{-1} A_{21}^{MI} \left(A_{11}^{MI}\right)^{-1} B_{11}^{MI} & \left(A_{22}^{MI}\right)^{-1} B_{22}^{MI} \end{pmatrix}, \\ Q^{MI} &= \begin{pmatrix} \left(A_{11}^{MI}\right)^{-1} F_{11}^{MI} & \mathbf{0} \\ -\left(A_{22}^{MI}\right)^{-1} A_{21}^{MI} \left(A_{11}^{MI}\right)^{-1} F_{11}^{MI} & \mathbf{0} \end{pmatrix}, \\ K^{MI} &= \begin{pmatrix} \left(A_{11}^{MI}\right)^{-1} C_{11}^{MI} & \mathbf{0} \\ -\left(A_{22}^{MI}\right)^{-1} A_{21}^{MI} \left(A_{11}^{MI}\right)^{-1} C_{11}^{MI} & \left(A_{22}^{MI}\right)^{-1} C_{22}^{MI} \end{pmatrix}. \end{aligned}$$

It is in the same form as system (28) under the fixed regime, and therefore the discussion of learnability for system (31) is equivalent to the discussion of learnability for two subsystems

$$y_{1,t}^{MI} = P_{11}^{MI} E_t y_{1,t+1}^{FI} + Q_{11}^{MI} y_{1,t-1}^{FI} + K_{11}^{MI} \omega_{1,t}^{FI}, \quad (31\text{-}1)$$

$$y_{2,t}^{MI} = P_{22}^{MI} E_t y_{2,t+1}^{MI} + K_{22}^{MI} \omega_{2,t}^{MI}, \quad (31\text{-}2)$$

[24]The condition happens to be the same as under the fixed regime, since we assume the identical parameters in the policy rules with $\phi = \phi^*$, and $\psi = \psi^*$.

where

$$\begin{aligned}
P_{11}^{MI} &= \left(A_{11}^{MI}\right)^{-1} B_{11}^{MI}, Q_{11}^{MI} = \left(A_{11}^{MI}\right)^{-1} F_{11}^{MI}, K_{11}^{MI} = \left(A_{11}^{MI}\right)^{-1} C_{11}^{MI}, \\
P_{22}^{MI} &= \left(A_{22}^{MI}\right)^{-1} B_{22}^{MI}, K_{22}^{MI} = \left(A_{22}^{MI}\right)^{-1} C_{22}^{MI}.
\end{aligned}$$

Supposing $\omega_{1,t}^{MI}$ and $\omega_{2,t}^{MI}$ follow vector $AR(1)$ processes as before, the MSV solutions for the two subsystems are respectively in the form of

$$\begin{aligned}
y_{1,t}^{MI} &= a_1^{MI} + b_1^{MI} y_{1,t-1}^{MI} + c_1^{MI} \omega_{1,t}^{MI}, \\
y_{2,t}^{MI} &= a_2^{MI} + c_2^{MI} \omega_{2,t}^{MI}.
\end{aligned}$$

Insert them into (31-1) and (31-2), and then the REE is solved for the managed regime I as $\left\{\bar{a}_1^{MI}, \bar{b}_1^{MI}, \bar{c}_1^{MI}\right\}$ and $\left\{\bar{a}_2^{MI}, \bar{c}_2^{MI}\right\}$.

The corresponding E-stability conditions are that all of the eigenvalues of $F_{11}^{MI} \equiv \left(I_{11} - P_{11}^{MI}\bar{b}_1^{MI}\right)^{-1} P_{11}^{MI}$, $\Omega_{11}^{MI} \equiv \left(I_{11} - P_{11}^{MI}\bar{b}_1^{MI}\right)^{-1} Q_{11}^{MI}$, and P_{22}^{MI} have real parts less than one. The subsystem (31-2) is again the same as (23-2) under the floating regime, which implies that condition (MI2) is one necessary condition for the learnability of REE in the whole economy. Furthermore, the result of McCallum (2006) means that the condition (MI1), and (MI2) for the equilibrium determinacy is sufficient for the learnability of REE. Therefore, I obtain the following proposition.

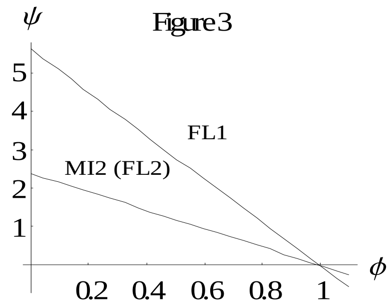
Proposition 6 *Under a managed exchange rate regime I defined by the rules of the following form*

$$\begin{aligned}
\hat{i}_t^H &= \phi \pi_t^H + \psi y_t^H, \\
\hat{i}_t^F &= \phi \pi_t^F + \psi y_t^F - \mu_1 \hat{S}_t,
\end{aligned}$$

with ϕ and ψ non negative as well as μ_1 non zero, if the degrees of rigidity are equal across countries, the necessary and sufficient condition for learnability of REE is

$$\mu_1 > 0, \tag{MI1}$$

$$k_C(\phi - 1) + \psi(1 - \beta) > 0. \tag{MI2}$$



Given the calibration in Section 2.6, the figure 3 plots the conditions of determinacy and learnability as a function of ϕ and ψ. Given any positive value of μ_1, the lower line (MI2) is the condition for determinacy and learnability of REE under this managed regime, which is the same condition as (FL2) for the floating regime. The numerical results suggest that the conditions for determinacy and learnability coincide again under this managed regime.

In particular, the condition (MI2) is the necessary and sufficient condition for determinacy and learnability of REE for the second subsystem of the world average variables (30-2) and (31-2), while additional condition (FL1) is required for the determinate and learnable REE of the first subsystem for relative variables (30-1) and (31-1) when $\mu_1 = 0$. It implies that an additional reaction to the level of nominal exchange rate in the policy rule of the foreign country will generally enlarge the region for determinacy and learnability of REE and therefore improve the trade-off between parameters ϕ and ψ for the central banks, compared with the floating regime. The enlarged region is the part of northeast of line (MI2), which implies the same condition for determinacy and learnability of REE in the closed economy case. Therefore, even though the conditions for determinacy and learnability could become more stringent due to the open economy considerations of the central bank, the restriction for policymakers is not necessarily stricter than the closed economy case when there is additional reaction towards the nominal exchange rate in the policy rules. The enlarged region is due to the terms of trade effects over the output gap. To see this more explicitly, we recall the equation (22), which is derived from the definition of the output gap and aggregate demand functions (2)

$$\hat{y}_t^H - \hat{y}_t^F = \hat{T}_t - \tilde{T}_t.$$

Substitute the terms of trade definition equation (4) into it, we can get

$$\hat{y}_t^H - \hat{y}_t^F = \hat{T}_{t-1} + \hat{S}_t - \hat{S}_{t-1} + \pi_t^F - \pi_t^H - \tilde{T}_t,$$

which implies

$$\hat{S}_t = \left(\hat{y}_t^H + \pi_t^H\right) - \left(\hat{y}_t^F + \pi_t^F\right) - \hat{T}_{t-1} + \hat{S}_{t-1} + \tilde{T}_t.$$

Therefore, the policy rule followed by the foreign country becomes

$$\hat{\imath}_t^F = \phi\pi_t^F + \psi y_t^F + \mu_1\left(\hat{y}_t^F + \pi_t^F\right) - \mu_1\left(\left(\hat{y}_t^H + \pi_t^H\right) - \hat{T}_{t-1} + \hat{S}_{t-1} + \tilde{T}_t\right).$$

This is a more aggressive rule than the Taylor-type rule with the same parameters ϕ and ψ for the foreign country, due to an additional reaction towards domestic inflation rate and output gap by size of μ_1. Equivalently, it implies the condition for determinacy and learnability of the foreign country is less stringent. Intuitively, it is because the home central bank not only influence the domestic demand of households but also the foreign demand through the movements of the terms of trade. The condition for determinacy and learnability of the home country and therefore the world economy is less stringent, which implies explicit or implicit monetary interdependence across countries. Finally, Taylor Principle is again sufficient for determinacy and learnability under this managed regime. In particular, given $\psi = 0$, it is easy to see that $\phi > 1$ is the necessary and sufficient condition to guarantee the determinacy and learnability of REE.

5.2 Managed exchange rate regime II

5.2.1 The dynamic system

Under a managed exchange rate regime II, the rule followed by the foreign country is assumed to react to the changes of the level of the nominal exchange rate from a defined target

$$\begin{aligned} \hat{\imath}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{\imath}_t^F &= \phi\pi_t^F + \psi y_t^F - \mu_2 \triangle S_t. \end{aligned} \tag{11}$$

Equations (3), (4), (5), (6), and (7), together with the interest rate rule (11), characterize the log-linear equilibrium under the managed exchange rate regime II.

Following the same procedures as above, the equilibrium conditions are

$$\begin{cases} \pi_t^R = -k_T\left(\hat{T}_t - \tilde{T}_t\right) + \beta E_t \pi_{t+1}^R, \\ \hat{T}_t = \hat{T}_{t-1} + \triangle S_t + \pi_t^R, \\ E_t \triangle S_{t+1} = -\phi\pi_t^R + \psi\left(\hat{T}_t - \tilde{T}_t\right) + \mu_2 \triangle S_t, \\ \pi_t^W = k_C y_t^W + \beta E_t \pi_{t+1}^W, \\ y_t^W = E_t y_{t+1}^W - \rho^{-1}\phi\pi_t^W - \rho^{-1}\psi y_t^W + \rho^{-1} E_t \pi_{t+1}^W, \\ +\rho^{-1}\tilde{R}_t^W + \rho^{-1}(1-n)\mu_2 \triangle S_t. \end{cases}$$

If define $y_{1,t}^{MII} = \begin{pmatrix} \pi_t^R & \hat{T}_t & \triangle S_t \end{pmatrix}'$, $y_{2,t}^{MII} = \begin{pmatrix} \pi_t^W & y_t^W \end{pmatrix}'$,and $\omega_{1,t}^{MII} = \begin{pmatrix} \tilde{T}_t & 0 & 0 \end{pmatrix}'$, and $\omega_{2,t}^{MII} = \begin{pmatrix} \tilde{R}_t^W & 0 \end{pmatrix}'$, the full system can be written as a matrix form

$$\begin{aligned} \begin{pmatrix} A_{11}^{MII} & \mathbf{0} \\ A_{21}^{MII} & A_{22}^{MII} \end{pmatrix} \begin{pmatrix} y_{1,t}^{MII} \\ y_{2,t}^{MII} \end{pmatrix} &= \begin{pmatrix} B_{11}^{MII} & \mathbf{0} \\ \mathbf{0} & B_{22}^{MII} \end{pmatrix} \begin{pmatrix} E_t y_{1,t+1}^{MII} \\ E_t y_{2,t+1}^{MII} \end{pmatrix} \\ &+ \begin{pmatrix} F_{11}^{MII} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \begin{pmatrix} y_{1,t-1}^{MII} \\ y_{2,t-1}^{MII} \end{pmatrix} \\ &+ \begin{pmatrix} C_{11}^{MII} & \mathbf{0} \\ \mathbf{0} & C_{22}^{MII} \end{pmatrix} \begin{pmatrix} \omega_{1,t}^{MII} \\ \omega_{2,t}^{MII} \end{pmatrix}, \end{aligned} \tag{32}$$

in which the matrices are given in Appendix A.5.

Because of the additional reaction to the change of nominal exchange rate in the policy rules of foreign country, the complete separation between world variables and relative variables disappears under this managed regime II.

5.2.2 Determinacy

Under this managed regime, π_t^R, $\triangle S_t$, π_t^W, y_t^W are the nonpredetermined variables, while $\hat{T}_{t-1}$ is the predetermined variables. Introduce the notation $\hat{T}_t^l = \hat{T}_{t-1}$ and add it to the system, and then replace expectations by their true values. If define $Y_{1,t}^{MII} = \begin{pmatrix} \pi_t^R & \triangle S_t & \hat{T}_t^l \end{pmatrix}'$ and $Y_{2,t}^{MII} = \begin{pmatrix} \pi_t^W & y_t^W \end{pmatrix}'$, the full system (32) can be written as the following matrix form

$$\begin{pmatrix} L_{11}^{MII} & \mathbf{0} \\ \mathbf{0} & L_{22}^{MII} \end{pmatrix} \begin{pmatrix} Y_{1,t+1}^{MII} \\ Y_{2,t+1}^{MII} \end{pmatrix} = \begin{pmatrix} M_{11}^{MII} & \mathbf{0} \\ M_{21}^{MII} & M_{22}^{MII} \end{pmatrix} \begin{pmatrix} Y_{1,t}^{MII} \\ Y_{2,t}^{MII} \end{pmatrix} + other^{MII}, \tag{33}$$

in which the matrices are given in Appendix A.5. Now the matrix M_{21}^{MII} is not null, which is due to the reaction to change of the nominal exchange rate in the policy rule followed by the foreign country.

The reduced form of full system (33) is

$$\begin{matrix} Y_{1,t+1}^{MII} \\ Y_{2,t+1}^{MII} \end{matrix} = J^{MII} \begin{pmatrix} Y_{1,t}^{MII} \\ Y_{2,t}^{MII} \end{pmatrix} + other^{MII},$$

where

$$J^{MII} = \begin{pmatrix} \left(L_{11}^{MII}\right)^{-1} M_{11}^{MII} & \mathbf{0} \\ \left(L_{22}^{MII}\right)^{-1} M_{21}^{MII} & \left(L_{22}^{MII}\right)^{-1} M_{22}^{MII} \end{pmatrix}$$

and the matrix J^{MII} is lower block triangular. As the discussion above, to obtain the condition for equilibrium determinacy for the full system (33) is therefore equivalent to calculating the determinacy condition for the following two simplified subsystems

$$Y_{1,t+1}^{MII} = J_{11}^{MII} Y_{1,t}^{MII} + other_{1,t}^{MII}, \tag{33-1}$$

$$Y_{2,t+1}^{MII} = J_{22}^{MII} Y_{2,t}^{MII} + other_{2,t}^{MII}, \tag{33-2}$$

where

$$\begin{aligned} J_{11}^{MII} &= \left(L_{11}^{MII}\right)^{-1} M_{11}^{MII}, \\ J_{22}^{MII} &= \left(L_{22}^{MII}\right)^{-1} M_{22}^{MII}. \end{aligned}$$

Since there are two nonpredetermined variables and one predetermined variable in subsystem (33-1), one determinacy condition is that exactly two of three eigenvalues of J_{11}^{MII} lie outside the unit circle. The subsystem (33-2) is again in the same form as under the floating regime. Following BB (2006b), I obtain the following proposition.

Proposition 7 *Under the managed exchange regime (II) with ϕ, ψ, and μ_2 non negative, if the degrees of rigidity are equal across countries, the necessary and sufficient condition for equilibrium determinacy is*

$$k_T(\phi + \mu_2 - 1) + \psi(1 - \beta) > 0, \tag{MII1}$$

$$k_C(\phi - 1) + \psi(1 - \beta) > 0, \tag{MII2}$$

Therefore, under managed regime II, additional reaction towards the change of nominal exchange rate does have explicit effects on the conditions for determinacy and learnability, in which the region is generally enlarged given a positive parameter μ_2.

Furthermore, recall the discussion of determinacy for the system (DE2) in Section (2.5). It is suggested that in order to obtain the determinacy for whole economy, not only the full system (15) need to satisfy the condition for determinacy, but also two independent subsystems (15-1) and (15-2) have to satisfy the determinate conditions. For example, when $\phi = 0.7$, and $\psi = 0.1$, with $\mu_2 = 0.3$, the eigenvalues of J_{11}^{-1} for the first subsystem for relative variables are $\{4.02362, 0.993687, 0.825368, 0.\}$, while the eigenvalues of J_{22}^{-1} for the second subsystem for world average variables are $\{1.0491, 0.545589\}$. In this calibrated case, the full system satisfies the condition for determinacy, since there are exactly four eigenvalues of J lying inside unit circle. However, the whole economy is not determinate, because the two subsystems do not satisfy the conditions for determinacy independently.

5.2.3 Learning stability

The full system (32) can be reduced as following

$$\begin{pmatrix} y_{1,t}^{MII} \\ y_{2,t}^{MII} \end{pmatrix} = P^{MII} \begin{pmatrix} E_t y_{1,t+1}^{MII} \\ E_t y_{2,t+1}^{MII} \end{pmatrix} + Q^{MII} \begin{pmatrix} y_{1,t-1}^{MII} \\ y_{2,t-1}^{MII} \end{pmatrix} + K^{MII} \begin{pmatrix} \omega_{1,t}^{MII} \\ \omega_{2,t}^{MII} \end{pmatrix}, \tag{34}$$

where

$$\begin{aligned} P^{MII} &= \begin{pmatrix} \left(A_{11}^{MII}\right)^{-1} B_{11}^{MII} & \mathbf{0} \\ -\left(A_{22}^{MII}\right)^{-1} A_{21}^{MII} \left(A_{11}^{MII}\right)^{-1} B_{11}^{MII} & \left(A_{22}^{MII}\right)^{-1} B_{22}^{MII} \end{pmatrix}, \\ Q^{MII} &= \begin{pmatrix} \left(A_{11}^{MII}\right)^{-1} F_{11}^{MII} & \mathbf{0} \\ -\left(A_{22}^{MII}\right)^{-1} A_{21}^{MII} \left(A_{11}^{MII}\right)^{-1} F_{11}^{MII} & \mathbf{0} \end{pmatrix}, \\ K^{MII} &= \begin{pmatrix} \left(A_{11}^{MII}\right)^{-1} C_{11}^{MII} & \mathbf{0} \\ -\left(A_{22}^{MII}\right)^{-1} A_{21}^{MII} \left(A_{11}^{MII}\right)^{-1} C_{11}^{MII} & \left(A_{22}^{MII}\right)^{-1} C_{22}^{MII} \end{pmatrix}. \end{aligned}$$

P_{12}^{MII}, Q_{12}^{MII}, and K_{12}^{MII} are null, and therefore the discussion of learnability for system (34) is again equivalent to the discussion of learnability for two simplified subsystems

$$y_{1,t}^{MII} = P_{11}^{MII} E_t y_{1,t+1}^{FII} + Q_{11}^{MII} y_{1,t-1}^{FII} + K_{11}^{MII} \omega_{1,t}^{FII}, \tag{34-1}$$

$$y_{2,t}^{MII} = P_{22}^{MII} E_t y_{2,t+1}^{MII} + K_{22}^{MII} \omega_{2,t}^{MII}, \tag{34-2}$$

where

$$\begin{aligned} P_{11}^{MII} &= \left(A_{11}^{MII}\right)^{-1} B_{11}^{MII}, Q_{11}^{MII} = \left(A_{11}^{MII}\right)^{-1} F_{11}^{MII}, K_{11}^{MII} = \left(A_{11}^{MII}\right)^{-1} C_{11}^{MII}, \\ P_{22}^{MII} &= \left(A_{22}^{MII}\right)^{-1} B_{22}^{MII}, K_{22}^{MII} = \left(A_{22}^{MII}\right)^{-1} C_{22}^{MII}. \end{aligned}$$

Supposing $\omega_{1,t}^{MI}$ and $\omega_{2,t}^{MI}$ follow vector $AR(1)$ processes as before, the MSV solutions for the two subsystems are respectively in the form of

$$\begin{aligned} y_{1,t}^{MII} &= a_1^{MII} + b_1^{MII} y_{1,t-1}^{MII} + c_1^{MII} \omega_{1,t}^{MII}, \\ y_{2,t}^{MII} &= a_2^{MII} + c_2^{MII} \omega_{2,t}^{MII}. \end{aligned}$$

Insert them into (34-1) and (34-2), the REE is solved for the managed regime II as $\left\{\overline{a}_1^{MII}, \overline{b}_1^{MII}, \overline{c}_1^{MII}\right\}$ and $\left\{\overline{a}_2^{MII}, \overline{c}_2^{MII}\right\}$.

The corresponding E-stability conditions are that all of the eigenvalues of $F_{11}^{MII} \equiv \left(I_{11} - P_{11}^{MII}\overline{b}_1^{MII}\right)^{-1} P_{11}^{MII}$, $\Omega_{11}^{MII} \equiv \left(I_{11} - P_{11}^{MII}\overline{b}_1^{MII}\right)^{-1} Q_{11}^{MII}$, and P_{22}^{MII} have real parts less than one. The subsystem (34-2) is again the same as (23-2) under the floating regime, which implies one necessary condition for learnability of REE in the whole economy is (MII2). Following the result in McCallum (2006), I get the following proposition.

Proposition 8 *Under a managed exchange regime II defined by the rules of the following form*

$$\begin{aligned}\hat{\imath}_t^H &= \phi\pi_t^H + \psi y_t^H, \\ \hat{\imath}_t^F &= \phi\pi_t^F + \psi y_t^F - \mu_2 \triangle S_t,\end{aligned}$$

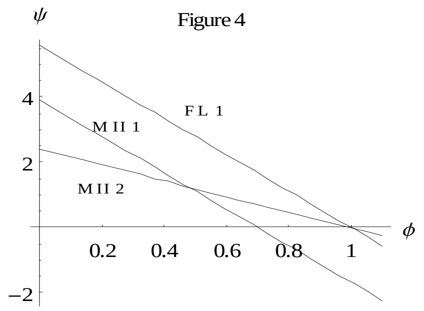
with ϕ, ψ, and μ_2 non negative, if the degrees of rigidity are equal across countries, the necessary and sufficient condition for learnability of REE is

$$k_T(\phi + \mu_2 - 1) + \psi(1 - \beta) > 0, \qquad \text{(MII1)}$$

$$k_C(\phi - 1) + \psi(1 - \beta) > 0. \qquad \text{(MII2)}$$

Given the calibration in Section 2.6, the figure 4 plots the conditions of determinacy and learnability as a function of ϕ and ψ. The middle line and the lower line are respectively the two conditions of determinacy and learnability (MII1) and (MII2), while the upper line describes the condition (FL1) under the floating regime.

Given a positive value for μ_2, the line (FL1) moves parallel to the southwest. When $\mu_2 = 0.3$ in this calibrated case, the line (FL1) moves to the line (MII1). As we discussed before, the condition (MII1) describes the condition for the determinacy and learnability of relative variables in the subsystem (34-1), and the condition (MII2) describes the condition for the determinacy and learnability of world variables in the subsystem (34-2). Therefore, the region for determinate and learnable interest rate rules is the intersection of northwestern parts of both line (MII1) and (MII2). The condition for de-



terminacy and learnability still coincide under this managed regime. Apparently, the region for monetary policy to obtain determinacy and learnability of REE is enlarged under managed regime II than under floating regime, due to the positive parameter of μ_2. When μ_2 is larger, the region is larger, and is at most the northeast part of line (MII2). Therefore, the additional reaction towards the changes of the nominal exchange rate in the policy rule followed by the foreign country will also enlarge the region for determinacy and learnability of REE as under the managed regime I. Recall again the equation (22) and the terms of trade definition equation (4), which imply

$$\triangle S_t = \left(\hat{y}_t^H + \pi_t^H\right) - \left(\hat{y}_t^F + \pi_t^F\right) - \hat{T}_{t-1} + \tilde{T}_t.$$

The policy rule followed by foreign country then becomes

$$\hat{\imath}_t^F = \phi \pi_t^F + \psi y_t^F + \mu_2 \left(\hat{y}_t^F + \pi_t^F\right) - \mu_2 \left(\left(\hat{y}_t^H + \pi_t^H\right) - \hat{T}_{t-1} + \tilde{T}_t\right).$$

The policy rules for the foreign country is more aggressive due to the additional reaction towards domestic inflation rate and output gap by value of μ_2, and thus the condition for ϕ and ψ is less stringent. The enlarged region is also due to the terms of trade effects. Furthermore, Taylor Principle, $\phi > 1$, is still sufficient for determinacy and learnability of REE.

5.3 More on numerical analysis

Above discussion assumes identical parameters before the inflation rate and output gap in policy rules followed by both countries. In this section, we relax this assumption and instead assume that the home country and foreign country follow interest rate rules with different parameters before the inflation rate while there is no reaction towards output gap for simplicity.

Under the managed regime I, the policy rules become

$$\begin{aligned} \widehat{i}_t^H &= \phi\pi_t^H, \\ \widehat{i}_t^F &= \phi^*\pi_t^F - \mu_1\widehat{S}_t. \end{aligned} \tag{10-1}$$

If define $y_{MI,t}^* = \left(y_t^W, \pi_t^H, \pi_t^F, \hat{S}_t, \hat{T}_t\right)'$, and $\omega_{MI,t}^* = \left(\tilde{R}_t^W, \tilde{T}_t, \tilde{T}_t, 0, 0\right)'$, the dynamic system becomes

$$A_{MI}^* y_{FL,t}^* = B_{MI}^* E_t y_{MI,t+1}^* + F_{MI}^* y_{MI,t-1}^* + C_{MI}^* \omega_{MI,t}^*,$$

in which the matrices are given in Appendix A.5.

Varying the parameters ϕ, ϕ^* and μ_1, I check the determinacy and learnability of REE for different pairs of policy rules followed by both countries, which is (Table-MI).

Table-MI

$\mu_1 = 0.000001$

ϕ	ϕ^*	Det	ES
1.2	1.2	Yes	Yes
1.2	0.9	No	Yes
1.2	0.6	No	No
0.9	1.2	No	No
0.9	0.9	No	No

$\mu_1 = 0.1$

ϕ	ϕ^*	Det	ES
1.2	1.2	Yes	Yes
1.2	0.9	Yes	Yes
1.2	0.6	Yes	Yes
0.9	1.2	No	No
0.9	0.9	No	No

The determinacy and learnability conditions coincide again from the numerical result. Proposition 5 and 6 imply that the Taylor Principle is sufficient for the determinacy and learnability of REE under managed regime I; and in particular, given $\psi = 0$, $\phi = \phi^* > 1$ is the necessary and sufficient condition when parameters before inflation rate in policy rules are assumed to be identical. The numerical results in (Table-MI) support the Taylor's intuition. However, it also show that the necessary and sufficient condition for determinacy and learnability is only closely linked to the Taylor's Principle for the home country, while the condition for the foreign country could be quite generous for a not very small reaction towards the nominal exchange rate in foreign policy rule. The larger reaction towards the nominal exchange rate by μ_1, the larger region for policy rules followed by the foreign country. Therefore, the policy rules followed by the two countries are not required to be simultaneously aggressive any more. For example, when $\mu_1 = 0.1$, $\phi^* = 0.1$ can still guarantee the determinacy and learnability for the whole economy.

Under the managed regime II, the policy rules become

$$\begin{aligned} \widehat{i}_t^H &= \phi \pi_t^H, \\ \widehat{i}_t^F &= \phi^* \pi_t^F - \mu_2 \triangle S_t. \end{aligned} \tag{11-1}$$

If define $y^*_{MII,t} = \left(y_t^W, \pi_t^H, \pi_t^F, \triangle S_t, \hat{T}_t\right)'$, and $\omega^*_{MII,t} = \left(\tilde{R}_t^W, \tilde{T}_t, \tilde{T}_t, 0, 0\right)'$, the dynamic system becomes

$$A^*_{MII} y^*_{FL,t} = B^*_{MII} E_t y^*_{MII,t+1} + F^*_{MII} y^*_{MII,t-1} + C^*_{MII} \omega^*_{MII,t},$$

in which the matrices are given in Appendix A.5.

Varying ϕ, ϕ^* and μ_2, check the determinacy and learnability for different pairs of policy rules followed by the two countries, which imply (Table-MII).

Table-MII

$\mu_2 = 0.3, n = 0.4$

ϕ	ϕ^*	Det	ES
1.2	1.2	Yes	Yes
1.2	0.9	No	No
0.9	1.2	No	No
0.9	0.9	No	No

$\mu_2 = 0.7, n = 0.4$

ϕ	ϕ^*	Det	ES
1.2	1.2	Yes	Yes
1.2	0.9	Yes	Yes
0.9	1.2	No	No
0.9	0.9	No	No

$\mu_2 = 0.3, n = 0.6$

ϕ	ϕ^*	Det	ES
1.2	1.2	Yes	Yes
1.2	0.9	Yes	Yes
0.9	1.2	No	No
0.9	0.9	No	No

Again, the Taylor Principle is sufficient for the determinacy and learnability of the REE. However, the above numerical results again show that the necessary and sufficient condition for determinacy and learnability is closely linked to the Taylor's Principle only for the home country, while the policy rule followed by the foreign country can be less aggressive due to additional reaction toward the change of nominal exchange rate, for example, when $\mu_2 = 0.7, n = 0.4$. The enlarged region for the policy rules followed by the foreign country depends on the value of parameter μ_2, and the larger of μ_2 the more enlarged region. Furthermore, the economic size also has effects on the condition for determinacy and learnability of REE. For example, the larger of the home country size n, the less aggressive policy rule is required for the foreign country. Intuitively, the larger economic size of the home country implies the larger monetary influence imported by the foreign country through the terms of trade effects from the home country, and thus the less stringent condition is required for the policy rule followed by the foreign country.

6 Conclusion

This paper discussed determinacy and learnability for monetary policy within a New Keynesian two-country model, based on Benigno and Benigno (2006b). It was found that the open economy consideration by the central bank diminishes the region for determinate and learnable interest rate rules relative to the closed economy counterpart under the floating regime. However, the region is enlarged under other exchange rate regimes, due to the additional reaction towards the change or level of the nominal exchange rate in the policy rules. The terms of trade channel and therefore the monetary interdependence among countries is crucial for the dynamics of the economy in the open economy case, even without the explicit coordination of the policymakers.

The study of learning stability in open economies has provided new insights for the design of monetary policy rules, and there is much further work to be undertaken in the future research. This paper has only discussed the learnability for simple interest rate instrument rules. However, the design of monetary policies in an open economy framework is quite controversial compared with in a closed economy case, since there are more variables toward which monetary policy can react. For example, Ball (1998) suggests a long-run inflation targeting rule and a “monetary conditions index" rule as the policy instruments in an open economy.[25] Zanna (2004) introduces the Purchasing Power Parity (PPP) rules, which is a real exchange rate targeting rule, in emerging economies. Besides, Ghironi (1998), McCallum and Nelson (1998), Monacelli (1998), Svensson (2000) and Weeparana (1998), and others have also analyzed monetary policies in an open-economy framework. Future research should focus on other forms of monetary policies.

Secondly, the baseline model has some strict assumptions. By relaxing some of the assumptions or incorporating more complex models, there will be more interesting findings for the learning problems in an open economy environment. For example, Benigno and Benigno (2006a) introduce an alternative two-country model, which permits variation of the substitutability of goods across countries. It will potentially allow more analysis on shock transmission, monetary interdependence and their implication for policy design. Furthermore, recently there are a large body of literature to discuss the targeting rules. In particular, some recent literature, such as Svensson (2002) and Benigno and Benigno (2006a), has derived inflation targeting rules in open economy environments. A vital topic that requires further research is to analyse whether or not optimal inflation targeting can lead to determinate and learnable REE, especially in open economy settings.

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Appendix A

A.1. Determinacy

Proof of Proposition 1. Under a floating exchange rate regime, in order to get the determinate equilibrium for the full system (23), the two subsystems (23-3) and (23-4) should satisfy the condition for determinacy separately. The determinacy condition for subsystem (23-3) is that exactly two of three eigenvalues of matrix J_{FL}^{11} lie outside the unit circle, and the determinacy condition for subsystem (28-4) is that all of the two eigenvalues of matrix J_{FL}^{22} lie outside the unit circle. The three eigenvalues of J_{FL}^{11} are 0 and

$$\begin{cases} r_1 \equiv \frac{1+k_T+\beta+\beta\psi-\sqrt{(1+k_T+\beta+\beta\psi)^2-4\beta(1+k_T\phi+\psi)}}{2\beta}, \\ r_2 \equiv \frac{1+k_T+\beta+\beta\psi+\sqrt{(1+k_T+\beta+\beta\psi)^2-4\beta(1+k_T\phi+\psi)}}{2\beta}. \end{cases}$$

The determinacy for subsystem (23-3) therefore requires $|r_1| > 1$ and $|r_2| > 1$. Since $r_2 > r_1 > 0$, it is equivalent to prove $r_1 > 1$, which requires

$$\frac{1+k_T+\beta+\beta\psi-\sqrt{\left(1+k_T+\beta+\beta\psi\right)^2-4\beta\left(1+k_T\phi+\psi\right)}}{2\beta} > 1$$

$$\begin{aligned} &\Longrightarrow \quad 1+k_T-\beta+\beta\psi > \sqrt{\left(1+k_T+\beta+\beta\psi\right)^2-4\beta\left(1+k_T\phi+\psi\right)} \\ &\Longrightarrow \quad \left(1+k_T+\beta\psi-\beta\right)^2-\left(1+k_T+\beta\psi+\beta\right)^2+4\beta\left(1+k_T\phi+\psi\right) > 0 \\ &\Longrightarrow \quad \left[k_T\left(\phi-1\right)+\psi\left(1-\beta\right)\right]\beta > 0 \\ &\Longrightarrow \quad k_T\left(\phi-1\right)+\psi\left(1-\beta\right) > 0. \end{aligned}$$

The two eigenvalues of J_{FL}^{22} are:

$$\begin{cases} r_3 \equiv \frac{k_C+\rho+\beta\rho+\beta\psi-\sqrt{-4\beta\rho(\rho+k_C\phi+\psi)+(k_C+\rho+\beta\rho+\beta\psi)^2}}{2\beta\rho}, \\ r_4 \equiv \frac{k_C+\rho+\beta\rho+\beta\psi+\sqrt{-4\beta\rho(\rho+k_C\phi+\psi)+(k_C+\rho+\beta\rho+\beta\psi)^2}}{2\beta\rho}. \end{cases}$$

The determinacy for subsystem (28-4) therefore requires $|r_3| > 1$ and $|r_4| > 1$. Since $r_4 > r_3 > 0$, it is equivalent to prove $r_3 > 1$, which requires

$$\frac{k_C+\rho+\beta\rho+\beta\psi-\sqrt{-4\beta\rho\left(\rho+k_C\phi+\psi\right)+\left(k_C+\rho+\beta\rho+\beta\psi\right)^2}}{2\beta\rho} > 1$$

$$\begin{aligned} &\Longrightarrow \quad k_C+\rho-\beta\rho+\beta\psi > \sqrt{-4\beta\rho\left(\rho+k_C\phi+\psi\right)+\left(k_C+\rho+\beta\rho+\beta\psi\right)^2} \\ &\Longrightarrow \quad \left(k_C+\rho-\beta\rho+\beta\psi\right)^2-\left(k_C+\rho+\beta\rho+\beta\psi\right)^2+4\beta\rho\left(\rho+k_C\phi+\psi\right) > 0 \\ &\Longrightarrow \quad k_C\left(\phi-1\right)+\psi\left(1-\beta\right) > 0. \end{aligned}$$

■

Proof of Proposition 3. Under the fixed exchange rate regime, the subsystem (27-1) can be written as

$$Y_{1,t}^{FI} = \left(J_{11}^{FI}\right)^{-1} Y_{1,t+1}^{FI} + other_{1,t}^{FI},$$

where

$$\left(J_{11}^{FI}\right)^{-1} = \left(M_{11}^{FI}\right)^{-1} L_{11}^{FI} = \begin{pmatrix} \beta & 0 & -k_T \\ 0 & 0 & 1 \\ -\beta & 0 & k_T+1 \end{pmatrix}.$$

Because this subsystem has two predetermined variables, the condition for determinacy is that exactly two of three eigenvalues of J_{11}^{FI} in subsystem (27-1) lie outside the unit circle, which is equivalent to that exactly two eigenvalues of $\left(J_{11}^{FI}\right)^{-1}$ lie inside the unit circle and one outside. The three eigenvalues of $\left(J_{11}^{FI}\right)^{-1}$ are 0 and

$$\begin{cases} \frac{1+k_T+\beta-\sqrt{-4k_T\beta+(1+k_T-\beta)^2}}{2}, \\ \frac{1+k_T+\beta+\sqrt{-4k_T\beta+(1+k_T-\beta)^2}}{2}. \end{cases}$$

Then the determinacy for subsystem (27-1) requires

$$\begin{aligned} \left|\frac{1+k_T+\beta-\sqrt{4k_T\beta+(1+k_T-\beta)^2}}{2}\right| &< 1, \\ \left|\frac{1+k_T+\beta+\sqrt{4k_T\beta+(1+k_T-\beta)^2}}{2}\right| &> 1. \end{aligned}$$

They are always satisfied for any positive values of β and k_T. The proofs are as follows.

From $\left|\frac{1+k_T+\beta-\sqrt{4k_T\beta+(1+k_T-\beta)^2}}{2}\right| < 1$,

$$\begin{aligned} &\Longrightarrow \quad -1+k_T+\beta < \sqrt{4k_T\beta+(1+k_T-\beta)^2} \\ &\Longrightarrow \quad (1-k_T-\beta)^2 - 4k_T\beta - (1+k_T-\beta)^2 < 0 \\ &\Longrightarrow \quad -4k_T < 0. \end{aligned}$$

From $\left|\frac{1+k_T+\beta+\sqrt{4k_T\beta+(1+k_T-\beta)^2}}{2}\right| > 1$,

$$\begin{aligned} &\Longrightarrow \quad \sqrt{4k_T\beta+(1+k_T-\beta)^2} > 1-k_T-\beta \\ &\Longrightarrow \quad 4k_T\beta+(1+k_T-\beta)^2 - (1-k_T-\beta)^2 > 0 \\ &\Longrightarrow \quad 4k_T > 0. \end{aligned}$$

For the subsystem (27-2), because there is no predetermined variable, the condition for determinacy is that exactly all eigenvalues of J_{22}^{FI} lie outside the unit circle. The eigenvalues of J_{22}^{FI} are 1, $\frac{1}{\beta}$, and

$$\left\{ \begin{array}{l} \frac{k_C+\rho+\beta\rho+\beta\psi-\sqrt{-4\beta\rho(\rho+k_C\phi+\psi)+(k_C+\rho+\beta\rho+\beta\psi)^2}}{2\beta\rho}, \\ \frac{k_C+\rho+\beta\rho+\beta\psi+\sqrt{-4\beta\rho(\rho+k_C\phi+\psi)+(k_C+\rho+\beta\rho+\beta\psi)^2}}{2\beta\rho}. \end{array} \right.$$

Then the determinacy requires

$$\left| \begin{array}{l} \frac{k_C+\rho+\beta\rho+\beta\psi-\sqrt{-4\beta\rho(\rho+k_C\phi+\psi)+(k_C+\rho+\beta\rho+\beta\psi)^2}}{2\beta\rho} \\ \frac{k_C+\rho+\beta\rho+\beta\psi+\sqrt{-4\beta\rho(\rho+k_C\phi+\psi)+(k_C+\rho+\beta\rho+\beta\psi)^2}}{2\beta\rho} \end{array} \right| \begin{array}{l} >1, \\ >1, \end{array}$$

which implies the other condition for determinacy

$$k_C(\phi-1)+\psi(1-\beta)>0.$$

The derivation is the same as that in Proof of Proposition 1. ■

Proof of Proposition 5. See the Technical Appendix of BB (2006a). ■

To see the Proof of Proposition 5 more clearly, we assume $\phi=\psi=0$, and then the eigenvalues of J_{11}^{MI} are $1+\lambda$, 0, and

$$\left\{ \begin{array}{l} \frac{1+k_T+\beta-\sqrt{(1+k_T+\beta)^2-4\beta}}{2\beta}, \\ \frac{1+k_T+\beta+\sqrt{(1+k_T+\beta)^2-4\beta}}{2\beta}. \end{array} \right.$$

The eigenvalue $\frac{1+k_T+\beta-\sqrt{(1+k_T+\beta)^2-4\beta}}{2\beta}$ is always inside the unit circle, since

$$0<\frac{1+k_T+\beta-\sqrt{(1+k_T+\beta)^2-4\beta}}{2\beta}<1$$

$$
\begin{aligned}
&\Rightarrow \quad 1+k_T+\beta-\sqrt{(1+k_T+\beta)^2-4\beta}<2\beta \\
&\Rightarrow \quad 1+k_T-\beta<\sqrt{(1+k_T+\beta)^2-4\beta} \\
&\Rightarrow \quad (1+k_T-\beta)^2-(1+k_T+\beta)^2+4\beta<0 \\
&\Rightarrow \quad -4\beta\,(1+k_T)+4\beta<0 \\
&\Rightarrow \quad -4\beta k_T<0,
\end{aligned}
$$

which always holds. The other eigenvalue $\frac{1+k_T+\beta+\sqrt{(1+k_T+\beta)^2-4\beta}}{2\beta}$ is always outside the unit circle, since

$$
\frac{1+k_T+\beta+\sqrt{(1+k_T+\beta)^2-4\beta}}{2\beta}>1
$$

$$
\begin{aligned}
&\Rightarrow \quad 1+k_T+\beta+\sqrt{(1+k_T+\beta)^2-4\beta}>2\beta \\
&\Rightarrow \quad 1+k_T-\beta+\sqrt{(1+k_T+\beta)^2-4\beta}>0,
\end{aligned}
$$

which always holds for $\beta<1$. Actually, there are always two eigenvalues of J_{11}^{MI} inside the unit circle, while the other two are always outside the unit circle, for any positive μ_1.

A.2. REE

Solution 9 (REE under a floating regime) *In the subsystem (23-1)*

$$
y_{1,t}^{FL}=P_{11}^{FL}E_t y_{1,t+1}^{FL}+Q_{11}^{FL}y_{1,t-1}^{FL}+K_{11}^{FL}\omega_{1,t}^{FL},
$$

the solutions for $\overline{b}_1^{FL}$ *is determined by equation*

$$
P_{11}^{FL}\left(b_1^{FL}\right)^2-b_1^{FL}+Q_{11}^{FL}=0.
$$

There are three possible solutions for b_1^{FL}.[26] *Take the only stationary one for b_1^{FL} with all the eigenvalues of b_1^{FI} inside the unit circle. Therefore, under a floating regime, the REE for subsystem (23-1) is solved as*

$$\begin{cases} \overline{a}_1^{FL} = \mathbf{0}, \\ \overline{b}_1^{FL} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \\ \overline{c}_1^{FL} = \begin{pmatrix} \frac{(1-\varphi_1)k_T}{(\phi-\varphi_1)k_T+(1-\varphi_1)(1-\beta\varphi_1)+\psi(1-\beta\varphi_1)} \\ \frac{(\phi-\varphi_1)k_T+\psi(1-\beta\varphi_1)}{(\phi-\varphi_1)k_T+(1-\varphi_1)(1-\beta\varphi_1)+\psi(1-\beta\varphi_1)} \\ \frac{(\phi-1)k_T+\psi(1-\beta\varphi_1)}{(\phi-\varphi_1)k_T+(1-\varphi_1)(1-\beta\varphi_1)+\psi(1-\beta\varphi_1)} \end{pmatrix}, \end{cases},$$

and the REE for subsystem (23-2) is solved as

$$\begin{cases} \overline{a}_2^{FL} = \mathbf{0}, \\ \overline{c}_2^{FL} = \begin{pmatrix} -\frac{k_C}{k_C(\lambda-\phi)-(-1+\beta\lambda)((-1+\lambda)\rho-\psi)} \\ \frac{1-\beta\lambda}{k_C(-\lambda+\phi)+(-1+\beta\lambda)((-1+\lambda)\rho-\psi)} \end{pmatrix}. \end{cases}$$

Solution 10 (REEs under a fixed regime) *In subsystem (28-1)*

$$y_{1,t}^{FI} = P_{11}^{FI} E_t y_{1,t+1}^{FI} + Q_{11}^{FI} y_{1,t-1}^{FI} + K_{11}^{FI} \omega_{1,t}^{FI}, \tag{28-2}$$

the solutions for $\overline{b}_1^{FI}$ are determined by equation

$$P_{11}^{FI} \left(b_1^{FI}\right)^2 - b_1^{FI} + Q_{11}^{FI} = 0.$$

[26] The nonstationary solutions are quite complex, and thus I do not show here.

There are two possible solutions for b_1^{FI}

$$b_{1,1}^{FI} = \begin{pmatrix} 0 & \frac{1+k_T-\beta-\sqrt{4k_T\beta+(1+k_T-\beta)^2}}{2\beta} \\ 0 & \frac{1+k_T+\beta-\sqrt{4k_T\beta+(1+k_T-\beta)^2}}{2\beta} \end{pmatrix},$$

$$b_{1,2}^{FI} = \begin{pmatrix} 0 & \frac{-1+k_T+\beta+\sqrt{4k_T\beta+(1+k_T-\beta)^2}}{2\beta} \\ 0 & \frac{1+k_T+\beta+\sqrt{4k_T\beta+(1+k_T-\beta)^2}}{2\beta} \end{pmatrix}.$$

Take the only stationary one $b_{1,1}^{FI}$ *with all its eigenvalues inside the unit circle. Therefore, under a fixed regime, the REE for subsystem (28-1) is*

$$\overline{a}_1^{FI} = \mathbf{0},$$

$$\overline{b}_1^{FI} = \begin{pmatrix} 0 & \frac{1+k_T-\beta-\sqrt{4k_T\beta+(1+k_T-\beta)^2}}{2\beta} \\ 0 & \frac{1+k_T+\beta-\sqrt{4k_T\beta+(1+k_T-\beta)^2}}{2\beta} \end{pmatrix},$$

$$\overline{c}_1^{FI} = \begin{pmatrix} \frac{2k_T}{1+k_T+\beta+\sqrt{4k_T\beta+(1+k_T-\beta)^2}-2\beta\varphi_1} & 0 \\ \frac{2k_T}{1+k_T+\beta+\sqrt{4k_T\beta+(1+k_T-\beta)^2}-2\beta\varphi_1} & 0 \end{pmatrix}.$$

The REE for subsystem (28-2) is

$$\overline{a}_2^{FI} = \mathbf{0},$$

$$\overline{c}_1^{FI} = \begin{pmatrix} \frac{k_C(\rho(-1+\varphi_2)-\psi)}{\rho Z} & \frac{k_C\psi}{Z} & 0 & 0 \\ \frac{k_C(\rho(-1+\varphi_2)-\psi)}{\rho Z} & \frac{k_C\psi}{Z} & 0 & 0 \\ \frac{-(-1+\beta\varphi_2)(\rho(-1+\varphi_2)-\psi)}{\rho Z} & \frac{\psi-\beta\varphi_2\psi}{Z} & 0 & 0 \\ \frac{k_C(-\varphi_2+\phi)}{\rho Z} & \frac{-\rho(-1+\varphi_2)(-1+\beta\varphi_2)+k_C(\varphi_2-\phi)}{Z} & 0 & 0 \end{pmatrix},$$

where denote

$$Z \equiv (-1+\varphi_2)\left(\rho\left(-1+\varphi_2\right)\left(-1+\beta\varphi_2\right)+k_C\left(\phi-\varphi_2\right)+\psi-\beta\varphi_2\psi\right).$$

A.3. Learnability

Proof of Proposition 2. Under a floating exchange rate regime, the eigenvalues of $\left(I_{11} - P_{11}^{FL}\bar{b}_1^{FL}\right)^{-1} Q_{11}^{FI}$ in subsystem (23-1) are all zeroes, and the eigenvalues of $\left(I_{11} - P_{11}^{FL}\bar{b}_1^{FL}\right)^{-1} P_{11}^{FL}$ in subsystem (23-1) are 0 and

$$\left\{ \begin{array}{l} \frac{1+k_T+\beta+\beta\psi-\sqrt{(1+k_T+\beta+\beta\psi)^2-4\beta(1+k_T\phi+\psi)}}{2(1+k_T\phi+\psi)}, \\ \frac{1+k_T+\beta+\beta\psi+\sqrt{(1+k_T+\beta+\beta\psi)^2-4\beta(1+k_T\phi+\psi)}}{2(1+k_T\phi+\psi)}, \end{array} \right.$$

which means one of the E-stability conditions is

$$k_T\left(\phi - 1\right) + \psi\left(1 - \beta\right) > 0.$$

For the subsystem (23-2), the corresponding eigenvalues of P_{22}^{FL} are

$$\left\{ \begin{array}{l} \frac{k_C+\rho+\beta\rho+\beta\psi-\sqrt{-4\beta\rho(\rho+k_C\phi+\psi)+(k_C+\rho+\beta\rho+\beta\psi)^2}}{2(\rho+k_C\phi+\psi)}, \\ \frac{k_C+\rho+\beta\rho+\beta\psi+\sqrt{-4\beta\rho(\rho+k_C\phi+\psi)+(k_C+\rho+\beta\rho+\beta\psi)^2}}{2(\rho+k_C\phi+\psi)}, \end{array} \right.$$

which means a second condition for E-stability is

$$k_C\left(\phi - 1\right) + \psi\left(1 - \beta\right) > 0.$$

The derivation is the same as that in Proof of Proposition 1. ■

Proof of Proposition 4. Under a fixed exchange rate regime, the eigenvalues of $\left(I_{11} - P_{11}^{FI}\bar{b}_1^{FI}\right)^{-1} P_{11}^{FI}$ and $\left(I_{11} - P_{11}^{FI}\bar{b}_1^{FI}\right)^{-1} Q_{11}^{FI}$ in the subsystem (28-1) are

$$\left\{ \begin{array}{c} 0, \\ \frac{\beta\left(1+k_T+\beta+\sqrt{(1+k_T-\beta)^2-4\beta k_T}\right)}{2(1+k_T)^2}, \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{c} 0, \\ \frac{1+k_T+\beta+\sqrt{(1+k_T-\beta)^2-4\beta k_T}}{2(1+k_T)^2}. \end{array} \right.$$

For the subsystem (28-2), the eigenvalues of P_{22}^{FI} are 1, β and

$$\begin{cases} \frac{k_C+\rho+\beta\rho+\beta\psi-\sqrt{-4\beta\rho(\rho+k_C\phi+\psi)+(k_C+\rho+\beta\rho+\beta\psi)^2}}{2(\rho+k_C\phi+\psi)}, \\ \frac{k_C+\rho+\beta\rho+\beta\psi+\sqrt{-4\beta\rho(\rho+k_C\phi+\psi)+(k_C+\rho+\beta\rho+\beta\psi)^2}}{2(\rho+k_C\phi+\psi)}, \end{cases}$$

which means the E-stability condition is

$$k_C(\phi-1)+\psi(1-\beta)>0.$$

The derivation is the same as that in Proof of Proposition 1. ■

A.4. Interdependence across the countries under learning

Suppose $\tilde{T}_t$ and $\tilde{R}_t^W$ follow $AR(1)$ processes of the following forms

$$\begin{aligned} \tilde{T}_t &= \varphi_1\tilde{T}_{t-1}+\epsilon_t^1, \\ \tilde{R}_t^W &= \varphi_2\tilde{R}_{t-1}^W+\epsilon_t^2, \end{aligned}$$

where $0<\varphi_1,\varphi_2<1$, with ϵ_t^1 and ϵ_t^2 are *iid* stochastic processes. The dynamic system under the floating regime is in the following form

$$y_{FL,t}^* = P_{FL}^* E_t y_{FL,t+1}^* + Q_{FL}^* y_{FL,t-1}^* + K_{FL}^* \omega_{FL,t}^*. \tag{23*}$$

The MSV solutions for system (23*) can be written as

$$y_{FL,t}^* = a_{FL}^* + b_{FL}^* y_{FL,t-1}^* + c_{FL}^* \omega_{FL,t}^*,$$

which is the perceived law of motion of representative agents. Assuming the time-t information set $\left(1, y_{FL,t}^{*'}, \omega_{FL,t}^{*'}\right)$, substitute the PLM into (23*). It follows that the MSV solution of system (23*) satisfies

$$\begin{aligned}(I - P^*_{FL} b^*_{FL} - P^*_{FL}) a^*_{FL} &= 0, \\ P^*_{FL} \left(b^*_{FL}\right)^2 - b^*_{FL} + Q^*_{FL} &= 0, \\ (I - P^*_{FL} b^*_{FL}) c^*_{FL} - P^*_{FL} c^*_{FL} \varphi_1 - K^*_{FL} &= 0.\end{aligned} \tag{24*}$$

From (24*), the REE is solved for the floating regime as $\left\{\overline{a}^*_{FL}, \overline{b}^*_{FL}, \overline{c}^*_{FL}\right\}$.

The corresponding E-stability conditions are that all the eigenvalues of DT_a, DT_b, and DT_c have real parts less than 1, where

$$\begin{aligned}DT_a &= \left(I - P^*_{FL} \overline{b}^*_{FL}\right)^{-1} P^*_{FL}, \\ DT_b &= \left[\left(I - P^*_{FL} \overline{b}^*_{FL}\right)^{-1} Q^*_{FL}\right]' \otimes \left[\left(I - P^*_{FL} \overline{b}^*_{FL}\right)^{-1} P^*_{FL}\right], \\ DT_c &= \left(I - P^*_{FL} \overline{b}^*_{FL}\right)^{-1} P^*_{FL} \varphi_1,\end{aligned}$$

which means that all the eigenvalues of

$$\left(I - P^*_{FL} \overline{b}^*_{FL}\right)^{-1} P^*_{FL} \text{ and } \left(I - P^*_{FL} \overline{b}^*_{FL}\right)^{-1} Q^*_{FL}$$

have the real parts less than 1.

A.5. Parameter Matrices

1. **Section 3.4:**

The matrices are

$$
\begin{aligned}
A_{FL}^{*} &= \begin{pmatrix} 1 & \frac{n\phi}{\rho} & \frac{(1-n)\phi^{*}}{\rho} & 0 & 0 \\ -k_C & 1 & 0 & 0 & -(1-n)\,k_T \\ -k_C & 0 & 1 & 0 & nk_T \\ 0 & \phi & -\phi^{*} & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 \end{pmatrix}, \\
B_{FL}^{*} &= \begin{pmatrix} 1 & \frac{n}{\rho} & \frac{1-n}{\rho} & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\
F_{FL}^{*} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, C_{FL}^{*} = \begin{pmatrix} \rho^{-1} \\ -(1-n)\,k_T \\ nk_T \\ 0 \\ 0 \end{pmatrix}.
\end{aligned}
$$

2. **Section 3.4 (DE^{*})**

The matrix J^{*} is

$$
J^{*} = \begin{pmatrix} 1+\frac{k_C}{\beta\rho} & \frac{n(\beta\phi-1)}{\beta\rho} & \frac{(\beta\phi^{*}-1)(1-n)}{\beta\rho} & 0 & 0 \\ -\frac{k_C}{\beta} & \frac{1+k_T-k_T n}{\beta} & \frac{k_T(n-1)}{\beta} & \frac{k_T(n-1)}{\beta} & \frac{k_T(n-1)}{\beta} \\ -\frac{k_C}{\beta} & -\frac{k_T n}{\beta} & \frac{1+k_T n}{\beta} & \frac{k_T n}{\beta} & \frac{k_T n}{\beta} \\ 0 & \phi & -\phi^{*} & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 \end{pmatrix}.
$$

3. **Section 4.1 (26)**

The matrices are

$$
\begin{aligned}
A_{11}^{FI} &= \begin{pmatrix} 1 & -1 \\ 1 & k_T \end{pmatrix}, A_{21}^{FI} = \begin{pmatrix} 0 & 0 \\ 0 & -(1-n)\,k_T \\ 0 & 0 \\ 0 & -(1-n) \end{pmatrix}, \\
A_{22}^{FI} &= \begin{pmatrix} 1 & 0 & -k_C & 0 \\ 0 & 1 & -k_C & 0 \\ 0 & \rho^{-1}\phi & 1 & \rho^{-1}\psi \\ 0 & \rho^{-1}\phi & 0 & 1+\rho^{-1}\psi \end{pmatrix}, B_{11}^{FI} = \begin{pmatrix} 0 & 0 \\ \beta & 0 \end{pmatrix}, \\
B_{21}^{FI} &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -(1-n) \end{pmatrix}, B_{22}^{FI} = \begin{pmatrix} \beta & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ \rho^{-1} & 0 & 1 & 0 \\ \rho^{-1} & 0 & 0 & 1 \end{pmatrix}, \\
F_{11}^{FI} &= \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}, C_{11}^{FI} = \begin{pmatrix} 0 & 0 \\ k_T & 0 \end{pmatrix}, \\
C_{22}^{FI} &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \rho^{-1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.
\end{aligned}
$$

4. **Section 4.2 (27)**

The matrices are

$$L_{11}^{FI} = \begin{pmatrix} 0 & 0 & 0 \\ \beta & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, L_{21}^{FI} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -(1-n) & 0 \end{pmatrix},$$

$$L_{22}^{FI} = \begin{pmatrix} \beta & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ \rho^{-1} & 0 & 1 & 0 \\ \rho^{-1} & 0 & 0 & 1 \end{pmatrix}, M_{11}^{FI} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & k_T & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$M_{21}^{FI} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -(1-n)\,k_T & 0 \\ 0 & 0 & 0 \\ 0 & -(1-n) & 0 \end{pmatrix}, M_{22}^{FI} = \begin{pmatrix} 0 & -k_C & 0 \\ 1 & -k_C & 0 \\ \rho^{-1}\phi & 1 & \rho^{-1}\psi \\ \rho^{-1}\phi & 0 & 1+\rho^{-1}\psi \end{pmatrix}.$$

5. **Section 5.1.1 (29)**

The matrices are

$$A_{11}^{MI} = \begin{pmatrix} 1 & k_T & 0 \\ -1 & 1 & -1 \\ -\phi & \psi & 1+\mu_1 \end{pmatrix}, A_{21}^{MI} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\rho^{-1}(1-n)\,\mu_1 \end{pmatrix},$$

$$A_{22}^{MI} = \begin{pmatrix} 1 & -k_C \\ \rho^{-1}\phi & 1+\rho^{-1}\psi \end{pmatrix}, B_{11}^{MI} = \begin{pmatrix} \beta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$B_{22}^{MI} = \begin{pmatrix} \beta & 0 \\ \rho^{-1} & 1 \end{pmatrix}, F_{11}^{MI} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix},$$

$$C_{11}^{MI} = \begin{pmatrix} k_T & 0 & 0 \\ 0 & 0 & 0 \\ \psi & 0 & 0 \end{pmatrix}, C_{22}^{MI} = \begin{pmatrix} 0 & 0 \\ \rho^{-1} & 0 \end{pmatrix}.$$

6. **Section 5.1.2 (30)**

The matrices are

$$
\begin{aligned}
L_{11}^{MI} &= \begin{pmatrix} \beta & 0 & -k_T & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & -\psi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, L_{21}^{MI} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\
L_{22}^{MI} &= \begin{pmatrix} \beta & 0 \\ \rho^{-1} & 1 \end{pmatrix}, M_{11}^{MI} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & -1 & -1 & 1 \\ -\phi & 1+\mu_1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \\
M_{21}^{MI} &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho^{-1}(1-n)\mu_1 & 0 & 0 \end{pmatrix}, M_{22}^{MI} = \begin{pmatrix} 1 & -k_C \\ \rho^{-1}\phi & 1+\rho^{-1}\psi \end{pmatrix}.
\end{aligned}
$$

7. **Section 5.2.1 (32)**

The matrices are

$$
\begin{aligned}
A_{11}^{MII} &= \begin{pmatrix} 1 & k_T & 0 \\ -1 & 1 & -1 \\ -\phi & \psi & \mu_2 \end{pmatrix}, A_{21}^{MII} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\rho^{-1}(1-n)\mu_2 \end{pmatrix}, \\
A_{22}^{MII} &= \begin{pmatrix} 1 & -k_C \\ \rho^{-1}\phi & 1+\rho^{-1}\psi \end{pmatrix}, B_{11}^{MII} = \begin{pmatrix} \beta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
B_{22}^{MII} &= \begin{pmatrix} \beta & 0 \\ \rho^{-1} & 1 \end{pmatrix}, F_{11}^{MII} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
C_{11}^{MII} &= \begin{pmatrix} k_T & 0 & 0 \\ 0 & 0 & 0 \\ \psi & 0 & 0 \end{pmatrix}, C_{22}^{MII} = \begin{pmatrix} 0 & 0 \\ \rho^{-1} & 0 \end{pmatrix}.
\end{aligned}
$$

8. **Section 5.2.2 (33)**

The matrices are

$$
\begin{aligned}
L_{11}^{MII} &= \begin{pmatrix} \beta & 0 & -k_T \\ 0 & 0 & -1 \\ 0 & 1 & -\psi \end{pmatrix}, L_{22}^{MII} = \begin{pmatrix} \beta & 0 \\ \rho^{-1} & 1 \end{pmatrix}, \\
M_{11}^{MII} &= \begin{pmatrix} 1 & 0 & 0 \\ -1 & -1 & -1 \\ -\phi & \mu & 0 \end{pmatrix}, M_{21}^{MII} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\rho^{-1}(1-n)\mu & 0 \end{pmatrix}, \\
M_{22}^{MII} &= \begin{pmatrix} 1 & -k_C \\ \rho^{-1}\phi & 1+\rho^{-1}\psi \end{pmatrix}.
\end{aligned}
$$

9. **Section 5.3**

The matrices are

$$
\begin{aligned}
A_{MI}^{*} &= \begin{pmatrix} 1 & \frac{n\phi}{\rho} & \frac{(1-n)\phi^{*}}{\rho} & -\frac{(1-n)\mu_1}{\rho} & 0 \\ -k_C & 1 & 0 & 0 & -(1-n)k_T \\ -k_C & 0 & 1 & 0 & nk_T \\ 0 & \phi & -\phi^{*} & 1+\mu_1 & 0 \\ 0 & 1 & -1 & -1 & 1 \end{pmatrix}, \\
B_{MI}^{*} &= \begin{pmatrix} 1 & \frac{n}{\rho} & \frac{1-n}{\rho} & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\
F_{MI}^{*} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}, C_{MI}^{*} = \begin{pmatrix} \rho^{-1} \\ -(1-n)k_T \\ nk_T \\ 0 \\ 0 \end{pmatrix}.
\end{aligned}
$$

10. **Section 5.3**

The matrices are

$$
\begin{aligned}
A_{MII}^{*} &= \begin{pmatrix} 1 & \frac{n\phi}{\rho} & \frac{(1-n)\phi^{*}}{\rho} & -\frac{(1-n)\mu_2}{\rho} & 0 \\ -k_C & 1 & 0 & 0 & -(1-n)\,k_T \\ -k_C & 0 & 1 & 0 & nk_T \\ 0 & \phi & -\phi^{*} & \mu_2 & 0 \\ 0 & 1 & -1 & -1 & 1 \end{pmatrix}, \\
B_{MII}^{*} &= \begin{pmatrix} 1 & \frac{n}{\rho} & \frac{1-n}{\rho} & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \\
F_{MII}^{*} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, C_{MII}^{*} = \begin{pmatrix} \rho^{-1} \\ -(1-n)\,k_T \\ nk_T \\ 0 \\ 0 \end{pmatrix}.
\end{aligned}
$$