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Energy Consumption and Jet Multiplicity from the Leading Log BFKL Evolution

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Abstract: We study the associated jet multiplicity arising from t–channel BFKL gluon evolution in forward dijet production at hadron colliders. Previous results have shown that the effect of conserving overall energy and momentum is to introduce a pdf suppression that completely compensates the predicted exponential BFKL rise with rapidity difference between the leading dijets. However, we show that there is still expected to be a significant amount of BFKL radiation, especially in the central region, and we give predictions for the multiplicity of the resulting mini–jets at the LHC.

Contents

1. Introduction

The leading log BFKL formalism [1, 2, 3] resums large logarithms in QCD processes with two large but different energy scales. In dijet production at hadron colliders the resummation is relevant in the forward region [4], when $\hat{s} \gg |\hat{t}|$ with $\hat{s}$ the parton centre of mass energy and $\hat{t}$ the square of the momentum transfer. This process has been promoted as the cleanest case at hadron colliders for studying the BFKL evolution of the dominating t–channel gluon exchange, since other processes where the BFKL formalism is applicable, for example the DIS structure functions at low x, are plagued by an interplay between perturbative and non–perturbative effects.

One of the most striking parton–level prediction of the leading log BFKL formalism is an exponential rise in the dijet cross section with increasing rapidity span of the dijets. However, as has been pointed out [5, 6, 7], this BFKL enhancement may not be visible in the hadronic cross section. This is because the exponential rise of the cross section relies on the emission of gluons from the BFKL evolution of the t–channel gluon exchange, but the suppression from the parton distribution functions (pdfs) for increasing partonic centre of mass energy $\hat{s}$, at either the Tevatron or LHC, more than compensates for the exponential rise. While this is certainly true for dijet production, which is driven by the steeply falling (in x) gluon pdf, recent results [8] suggest that for processes depending instead on the much flatter (in the relevant region) *valence quark* distributions, the BFKL evolution might indeed lead to an increase in the hadron–level cross section over the LO result.

Since the exponential rise in the cross section as a function of the rapidity span of the dijets cannot therefore be considered a precision BFKL prediction, other signatures of the BFKL evolution have been proposed. In particular, the weakening of the angular correlation of the leading dijets with increasing rapidity span, as predicted by BFKL at the partonic level, is still present after the convolution with the pdfs [5, 9]. This angular decorrelation is a result of the multiple emission of gluons, strong ordered in rapidity but unordered in transverse momentum, resulting from the BFKL evolution. It is also interesting to study the *jet multiplicity* resulting from the BFKL evolution, since this could be another experimentally verifiable signature of the theory. Such studies have been undertaken previously [10, 11, 12, 13], but these did not take into account the impact of the BFKL gluon energies on the parton luminosities. In this paper we present a study which does include this effect and thereby conserves overall energy and momentum.

We will report on a study of the jet multiplicity from the BFKL evolution of the t–channel gluon exchange in dijet production at hadron colliders. We will also examine the energy of the multi–jet events, in order to understand better why the exponential rise of the partonic dijet cross section evidently comes at too big a price in total energy to survive the convolution with the pdfs. Both of these studies are performed using the BFKL Monte Carlo approach of Ref. [5]. The paper is organised as follows: we first briefly review BFKL applied to dijet production and the technique of the BFKL MC in Section 2, before reporting on the results obtained for the total energy of the partonic BFKL events (Section 3). In Section 4 we calculate the jet multiplicity characteristic of BFKL evolution in dijet production, both in the partonic and the hadronic case, and finally we present our conclusions in Section 5.

2. Dijet Production at Hadron Colliders

In the high energy limit of $\hat{s} \gg |\hat{t}| \gg 0$, the gluon–gluon scattering cross section to leading order in $\ln \hat{s}/|\hat{t}|$ but summed to all orders in α_s is given by [4]

$$\frac{\mathrm{d}\hat{\sigma}_{gg}(\Delta y)}{\mathrm{d}^2\mathbf{p}_{a\perp}\mathrm{d}^2\mathbf{p}_{b\perp}} = \left(\frac{C_A\alpha_s}{p_{a\perp}^2}\right) f(\mathbf{p}_{a\perp}, -\mathbf{p}_{b\perp}, \Delta y) \left(\frac{C_A\alpha_s}{p_{b\perp}^2}\right), \tag{2.1}$$

where $\mathbf{p}_{a\perp}$ ($\mathbf{p}_{b\perp}$) is the transverse momentum of the most forward (backward) jet, and Δy is the rapidity difference between them. The function $f(\mathbf{p}_{a\perp}, -\mathbf{p}_{b\perp}, \Delta y)$ resums the logarithms in $\hat{s}/|\hat{t}|$ arising from both virtual corrections to, and rapidity ordered emission from, the t–channel gluon exchange, and so solves the BFKL equation. Although an analytic form for f can be obtained, and the result for the gluon–gluon scattering including the resummation of the BFKL logarithms in Eq. (2.1) thereby also solved analytically, such an approach will potentially pose a problem when it comes to calculating the hadronic cross section. This is because in order to obtain an analytic solution, the gluons emitted from the BFKL evolution cannot be counted in the contribution to the parton momentum fractions, and therefore the BFKL gluons that lead to the exponential rise in the partonic cross section are emitted at no cost in energy. Although these energy–conserving contributions to the centre of mass energy are formally subleading compared to the contribution from the leading dijets, they can have a huge impact on the parton distribution functions and therefore on the normalisation of the hadronic cross section[14], since the pdfs (and in particular the gluon pdfs) are decreasing very rapidly in the relevant region. The reformulation of the solution to the BFKL equation in terms of an explicit sum and integration over the emitted gluons and their rapidity ordered phase space was devised to solve this problem[5, 15]. In particular one finds for the solution to the leading log BFKL equation[5]

$$f(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y) \;=\; \sum_{n=0}^{\infty} f^{(n)}(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y)\;. \tag{2.2}$$

where we have set $\mathbf{q}_{a\perp} = \mathbf{p}_{a\perp}, \mathbf{q}_{b\perp} = -\mathbf{p}_{b\perp}$ and

$$\begin{aligned} f^{(0)}(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y) &= \left[\frac{\mu^2}{q_{a\perp}^2}\right]^{\bar{\alpha}_s \Delta y} \frac{1}{2}\,\delta^{(2)}(\mathbf{q}_{a\perp} - \mathbf{q}_{b\perp})\,, \\ f^{(n\geq 1)}(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y) &= \left[\frac{\mu^2}{q_{a\perp}^2}\right]^{\bar{\alpha}_s \Delta y} \left\{\prod_{i=1}^{n} \int \mathrm{d}^2\mathbf{k}_{i\perp}\, \mathrm{d}y_i\, \mathcal{F}_i\right\} \frac{1}{2}\,\delta^{(2)}(\mathbf{q}_{a\perp} - \mathbf{q}_{b\perp} - \sum_{i=1}^{n} \mathbf{k}_{i\perp})\,, \\ \mathcal{F}_i &= \frac{\bar{\alpha}_s}{\pi k_{i\perp}^2}\,\theta(k_{i\perp}^2 - \mu^2)\,\theta(y_{i-1} - y_i)\, \left[\frac{(\mathbf{q}_{a\perp} + \sum_{j=1}^{i-1} \mathbf{k}_{j\perp})^2}{(\mathbf{q}_{a\perp} + \sum_{j=1}^{i} \mathbf{k}_{j\perp})^2}\right]^{\bar{\alpha}_s y_i}\,, \end{aligned} \tag{2.3}$$

with $\bar{\alpha}_s = C_A \alpha_s/\pi$[1] and μ the resolution scale of the Monte Carlo. For small μ the sum in Eq. (2.2) is only weakly dependent on μ. This Monte Carlo formulation has been applied to studies of the dijet production rate and angular decorrelation at large rapidity separation at hadron colliders. One finds that the decrease in parton flux as a result of the increased centre of mass energy when the BFKL radiation is taken into account more than compensates for the BFKL exponential rise in the partonic cross section. The details of this effect obviously depends on the specific shape of the pdfs. In this paper we will therefore first study directly the impact of the BFKL radiation on the centre of mass energy for gluon–gluon scattering.

3. Energy Consumption of the BFKL evolution

Using the solution of the BFKL equation in the form of Eqs. (2.2,2.3) together with the Monte Carlo implementation of the integrations, we can answer the question of how much energy goes into creating the LL BFKL radiation. When energy and momentum conservation is applied, the parton momentum fractions are given by

$$\begin{aligned} x_a =& \frac{p_{a\perp}}{\sqrt{s}} e^{y_a} + \sum_{i=1}^{n} \frac{k_{i\perp}}{\sqrt{s}} e^{y_i} + \frac{p_{b\perp}}{\sqrt{s}} e^{y_b} \\ x_b =& \frac{p_{a\perp}}{\sqrt{s}} e^{-y_a} + \sum_{i=1}^{n} \frac{k_{i\perp}}{\sqrt{s}} e^{-y_i} + \frac{p_{b\perp}}{\sqrt{s}} e^{-y_b}, \end{aligned} \tag{3.1}$$

with the overall centre of mass energy squared given by $\hat{s} = x_a x_b s$ where $\sqrt{s}$ is the energy of the hadron collider.

[1] This is the fixed α_s result. The solution for running α_s is only slightly more complicated and is given for example in Ref. [5]. In the numerical results that follow we use a fixed coupling with a value of $\alpha_s(20\text{GeV}) = 0.1635$.

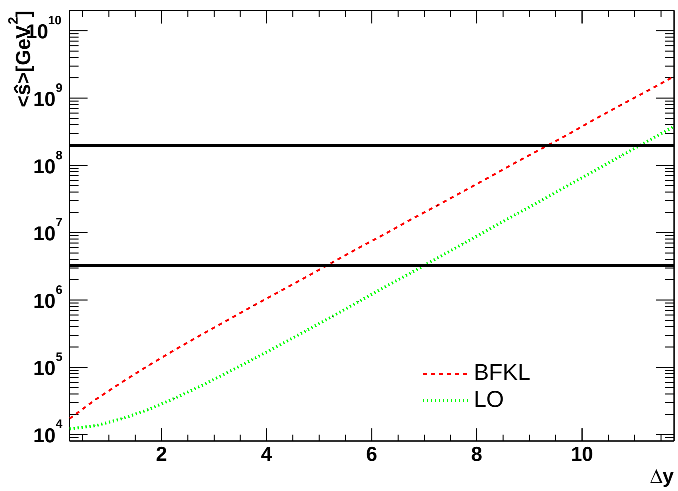

Figure 1: The average centre of mass energy in $gg \to gg$ scattering with (red/dashed) and without (green/dotted) BFKL evolution of the t channel gluon, with $p_{\perp\min} = 20$ GeV for the dijets and $\alpha_s = 0.1635$. Also plotted is the hadronic centre of mass energy squared for the Tevatron ($(1.8\text{TeV})^2$) and the LHC($(14\text{TeV})^2$).

In Fig. 1 we show the average centre of mass energy squared for dijet production in fixed leading order QCD (green/dotted) and for dijet production in the high–energy limit with BFKL evolution of the t–channel gluon exchange (red/dashed). The prediction for pure dijet production is indistinguishable in this plot from the 'standard' prediction for BFKL evolution, when the BFKL equation is solved analytically and the contribution from the BFKL gluons to the centre of mass energy is neglected.

The contribution from the BFKL radiation to the centre of mass energy is formally subleading compared to the contribution from the leading dijets. Indeed we see in Fig. 1 that asymptotically (which evidently is reached quickly), the two curves have the same slope, even though they are offset by about 1.5 units of rapidity. This means that the kinematic limit of dijet production with BFKL evolution at hadron colliders is reached about 1.5 units of rapidity before an estimate based on the energy of the leading dijets only. We have also indicated the hadronic centre of mass energy squared for the Tevatron ($(1.8\text{TeV})^2$) and the LHC ($(14\text{TeV})^2$). The rapid decrease of the pdfs as the kinematic limit is approached means that the horizontal lines of interest for a given hadron collider lie considerably lower than the lines indicated on the figure. Therefore the effect of including the contribution from the BFKL radiation in the overall energy and momentum conservation is larger than may appear at first glance. When considering the implications on the cross section, it must also be remembered that the BFKL evolution predicts an exponential rise with the rapidity span Δy, i.e. $\hat{\sigma} \sim \exp(\lambda \Delta y)$. Therefore effectively reducing the available rapidity span has a sizeable impact on the prediction for the cross section.

We can see explicitly why the two curves in Fig. 1 have the same asymptotic slope. As we will see later, the radiation from the BFKL chain is distributed evenly in rapidity along the chain. We can approximate the Bjorken x's given by Eq. (3.1) by assuming that all the $k_{\perp i}$ are equal to k and that the n BFKL gluons are distributed evenly over the rapidity span Δy, separated by δy such that $\Delta y = (n+1)\delta y$. The Bjorken x's for the $2 \to 2+n$ scattering then become (with $z = e^{-\delta y}$ and assuming that $y_0 = -\Delta y/2,\ y_{n+1} = +\Delta y/2$ — the centre of mass energy is independent of this assumption)

$$x_a = x_b = \frac{k}{\sqrt{s}} e^{\Delta y/2}(1 + z + z^2 + \cdots + z^n) = \frac{k}{\sqrt{s}} e^{\Delta y/2} \frac{1 - z^{n+1}}{1 - z}. \tag{3.2}$$

In the large Δy limit, with $n \to \infty$ for evenly spaced radiation, we find

$$\hat{s} \propto k^2 e^{\Delta y} \frac{1}{(1 - e^{-\delta y})^2}, \tag{3.3}$$

to be compared with the pure dijet prediction $\hat{s} \propto k^2 e^{\Delta y}$, which has the same dependence on Δy. It is radiation from the region of the chain close to the endpoints that contributes most to $\hat{s}$, since the middle part of the chain will give exponentially suppressed contributions to the energy (this is just a refinement of the asymptotic argument for dropping the contribution from the chain altogether). This explains why asymptotically there is only a difference in the normalisation and not the shape of the two curves in Fig. 1. From Eq. (3.3) we see that the smaller the δy, the bigger the difference in normalisation. Note that a smaller δy can be achieved by increasing α_s, thereby increasing the amount of BFKL radiation everywhere, and specifically also in the region close to the endpoints of the BFKL chain.

The observation that there is insufficient energy available at present–day colliders for all the BFKL radiation resummed in the analytic approach to be emitted without penalty motivated the introduction in Ref. [6] of a 'reduced effective rapidity separation', to be used when making phenomenological predictions of BFKL signatures for comparison with data. The idea behind this is to emulate the reduction of phase space for BFKL radiation, dictated by energy and momentum conservation, by reducing the rapidity span Δy that is used in the solution to the BFKL equation.

4. Jet Multiplicity of the BFKL Evolution

In this section we will study the jet multiplicity from the leading log BFKL evolution, first for partonic gluon–gluon scattering and then for the full hadronic cross section when proper account is taken of the energy and momentum carried by the BFKL gluons. The parton multiplicity, as predicted from leading log BFKL evolution, is equivalent (up to sub–leading terms) to the result obtained in the CCFM approach, in which colour coherence effects are taken into account [11, 16]. This supports the correspondence between gluons emitted from the BFKL chain and jets appearing in the detector.

4.1 Mini–jet Parton Multiplicities

In this section we study the mini–jet multiplicity from the BFKL chain. A mini–jet is here defined as a gluon with transverse momentum $q_{i\perp} < \mu_m$, where μ_m is the maximum allowed transverse momentum of a mini–jet, and the scales of the problem are ordered according to

$$\mu < q_{i\perp} < \mu_m \ll q_{a,b\perp}. \tag{4.1}$$

Recall that μ is the resolution scale of the BFKL MC solution in Eq. (2.3). Defined in this way, the sum over n–mini–jet rates with a resolution scale μ and upper scale μ_m should equal the 0–jet rate with a resolution scale of μ_m (up to terms of order μ^2/μ_m^2). Introducing a cut–off in the transverse momentum integral in the definition of $f^{(1)}(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y)$ in Eq. (2.3) and denoting the function with constrained integration $f_c^{(1)}(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y)$, we find

$$f_c^{(1)}(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y) = \left[\frac{\mu^2}{q_{a\perp}^2}\right]^{\bar{\alpha}_s \Delta y} \int \mathrm{d}^2\mathbf{k}_{1\perp} \int_0^{\Delta y} \mathrm{d}y_1 \frac{1}{2} \delta^{(2)}(\mathbf{q}_{a\perp} - \mathbf{q}_{b\perp} - \mathbf{k}_{1\perp}) \tag{4.2}$$

$$\theta(k_{1\perp}^2 - \mu^2)\,\theta(\mu_m^2 - k_{1\perp}^2) \frac{\bar{\alpha}_s}{\pi k_{1\perp}^2} \left[\frac{(\mathbf{q}_{a\perp})^2}{(\mathbf{q}_{a\perp} + \mathbf{k}_{1\perp})^2}\right]^{\bar{\alpha}_s y_1}. \tag{4.3}$$

If we approximate the term in the square brackets by unity (by virtue of the applied ordering of momenta in (4.1)) we find that $f_c^{(1)}(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y)$ can be approximated by

$$f_c^{(1)}(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y) \approx \left[\frac{\mu^2}{q_{a\perp}^2}\right]^{\bar{\alpha}_s \Delta y} \frac{1}{2} \delta^{(2)}(\mathbf{q}_{a\perp} - \mathbf{q}_{b\perp})\, \Delta y\, \bar{\alpha}_s \ln \frac{\mu_m^2}{\mu^2}, \tag{4.4}$$

where we have set $\delta^{(2)}(\mathbf{q}_{a\perp} - \mathbf{q}_{b\perp} - \mathbf{k}_{1\perp}) \approx \delta^{(2)}(\mathbf{q}_{a\perp} - \mathbf{q}_{b\perp})$. Similarly, one finds

$$f_c^{(n)}(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y) \approx \left[\frac{\mu^2}{q_{a\perp}^2}\right]^{\bar{\alpha}_s \Delta y} \frac{1}{2} \delta^{(2)}(\mathbf{q}_{a\perp} - \mathbf{q}_{b\perp}) \frac{\Delta y^n}{n!} \left(\bar{\alpha}_s \ln \frac{\mu_m^2}{\mu^2}\right)^n. \tag{4.5}$$

Therefore the contribution to the sum $f(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y)$ in Eq. (2.2) with no gluon emission with transverse momentum squared greater than μ_m^2 calculated this way is

$$\begin{aligned}
\sum_n f_c^{(n)}(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y) &\approx \left[\frac{\mu^2}{q_{a\perp}^2}\right]^{\bar{\alpha}_s \Delta y} \frac{1}{2} \delta^{(2)}(\mathbf{q}_{a\perp} - \mathbf{q}_{b\perp}) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\Delta y\, \bar{\alpha}_s \ln \frac{\mu_m^2}{\mu^2}\right)^n \\
&= \left[\frac{\mu^2}{q_{a\perp}^2}\right]^{\bar{\alpha}_s \Delta y} \frac{1}{2} \delta^{(2)}(\mathbf{q}_{a\perp} - \mathbf{q}_{b\perp}) \exp\left(\Delta y\, \bar{\alpha}_s \ln \frac{\mu_m^2}{\mu^2}\right) \\
&= \left[\frac{\mu_m^2}{q_{a\perp}^2}\right]^{\bar{\alpha}_s \Delta y} \frac{1}{2} \delta^{(2)}(\mathbf{q}_{a\perp} - \mathbf{q}_{b\perp}),
\end{aligned} \tag{4.6}$$

which we recognise as $f^{(0)}(\mathbf{q}_{a\perp}, \mathbf{q}_{b\perp}, \Delta y)$ evaluated with a resolution scale μ_m. This serves as a check of the consistency of the physical picture emerging from the Monte Carlo solution to the BFKL equation.

4.2 Partonic Jet Multiplicities

It proves much harder to obtain analytic predictions for multi–jet multiplicities or predictions for the rates of harder mini–jets from the BFKL chain, i.e. with the strong ordering

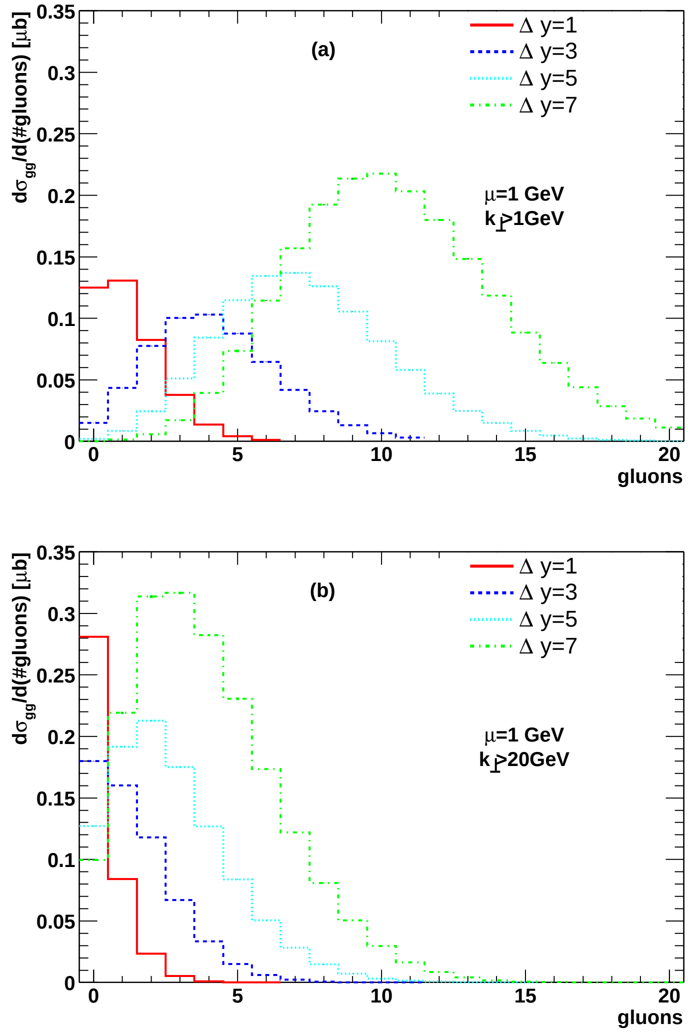

Figure 2: The contribution to the partonic cross section for choices of the rapidity separation from different numbers of resolved gluons with (a) $k_{i\perp} > 1$ GeV and (b) $k_{i\perp} > 20$ GeV. The leading dijets have $p_{\perp \min} = 20$ GeV in both cases.

$\mu_m \ll q_{a,b\perp}$ constraint relaxed. The results which follow are therefore obtained using the Monte Carlo approach. In all of these calculations we have chosen a resolution scale for the Monte Carlo implementation of Eqs. (2.2,2.3) of $\mu = 1$ GeV and a fixed value for the coupling $\alpha_s = 0.1635$. In Fig. 2(a) we plot the contribution to the partonic cross section from different numbers of resolved (i.e. $k_{i\perp} > \mu = 1$ GeV) gluons from the BFKL chain for a selection of rapidity spans of the chain. In calculating the partonic cross section (Eq. (2.1)) we have chosen a cut–off on the minimum transverse momentum of the jets a, b of 20 GeV. Such a simple cut–off, in which the threshold is the same for jet a and jet b, is known to cause incomplete cancellations of virtual and real IR divergences in a full NLO QCD calculation [17], but as was shown in Ref. [7] this effect is not present in the case of BFKL evolution. This is essentially because that at NLO the transverse momentum of the third gluon is by momentum conservation determined by the two others, and so the collinear (and soft) phase space of the third gluon can be restricted by cuts in the allowed phase space for the two harder jets. However, when more gluons are emitted (e.g. by BFKL evolution) this is no longer true, and the extra gluons can populate the IR regions irrespective of the cuts on the harder jets. We can therefore safely choose just a simple cut–off for our calculation.

The cross section for a given value of the rapidity difference between the leading dijets is given by the integral of the curves in Fig. 2(a). If one were to plot the average number $\langle n \rangle$ of emitted BFKL gluons as a function of the rapidity span of the BFKL chain, one would find that $\langle n \rangle$ increases linearly with the rapidity span. This feature will be evident in later plots.

Fig. 2(b) contains the same curves as in (a), but now for harder BFKL gluons with $k_{i\perp} > 20$ GeV. These latter curves are obtained by running the Monte Carlo with a resolution scale $\mu_r = 1$ GeV and then counting how many gluons with $k_{i\perp} > 20$ GeV the event contains. These plots are for the partonic cross section, and so these harder gluons from the BFKL chain contribute significantly to the increase in the average centre of mass energy seen in Fig. 1 for BFKL in comparison with the LO prediction.

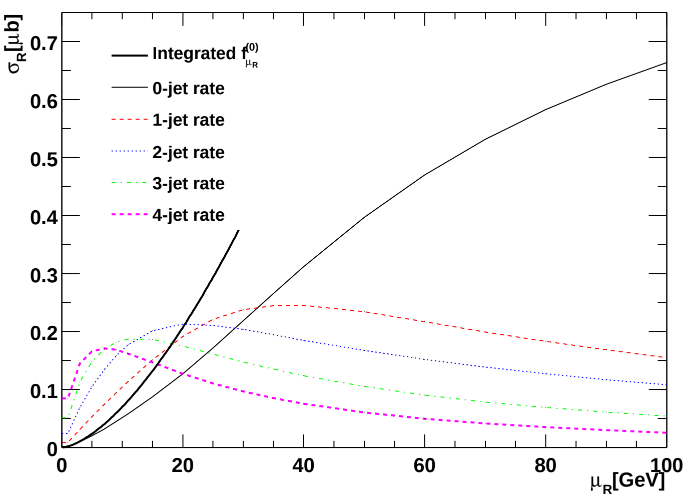

Figure 3: The 0–, 1, 2–, 3– and 4–jet parton–level cross sections as a function of μ_R, for a rapidity span of $\Delta y = 5$ and $p_{\perp\min} = 20$ GeV for the leading dijets. Also shown is the analytic 0–jet prediction valid for small μ_R.

Fig. 2 also shows that the jet multiplicity varies with the minimum $k_{i\perp}$ in the expected way. This is also seen in Fig. 3, where the 0–, 1–, 2–, 3– and 4–jet cross sections are shown as a function of μ_R for a rapidity span of $\Delta y = 5$ and $p_{\perp\min} = 20$ GeV for the leading dijets. Note that for $\mu_R \approx p_{\perp\min}$ the multijet cross sections are all of similar magnitude (for the relatively small number of jets considered here). Also shown in Fig. 3 is the analytic prediction for the zero jet rate, valid for $\mu_R \ll p_{\perp\min}$, obtained by integrating Eq. (4.7) over $q_{a,b\perp}$:

$$\sigma_R = \frac{\alpha_s^2 C_A^2 \pi}{2p_{\perp\min}^2} \frac{1}{1+\bar{\alpha}_s \Delta y} \left(\frac{\mu_R^2}{p_{\perp\min}^2}\right)^{\bar{\alpha}_s \Delta y} . \tag{4.7}$$

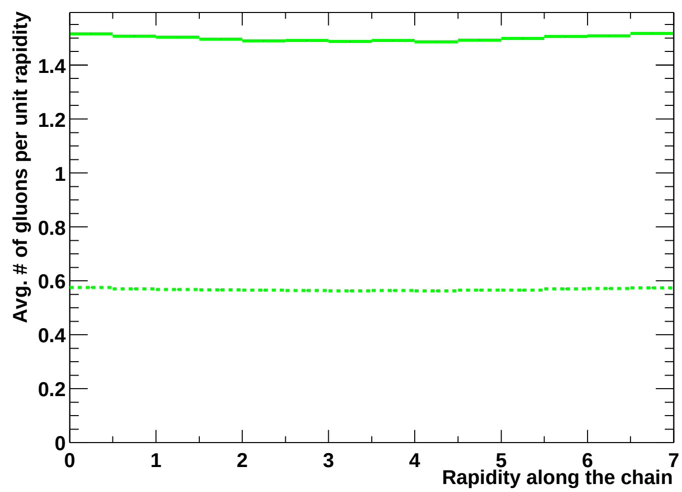

Figure 4: The average density of emitted gluons along the BFKL chain. Please see text for further details.

In Fig. 4 we show the average density in rapidity of the gluons emitted from the BFKL chain (not counting the two leading jets) in the case of gluon–gluon scattering for a gluon chain spanning 7 units of rapidity. The two lines on the plot correspond to the average density of resolved BFKL gluons ($k_{i\perp} > 1$ GeV) and harder gluons ($k_{i\perp} >$ 20 GeV). The density of gluon emission along the chain is the observable best suited for illustrating the effects of taking into account the energy of this radiation when calculating the parton momentum fractions for the hadronic cross section. As expected from the analytic approach, the density in rapidity of emitted gluons is (relatively) constant along the chain.

4.3 Hadronic Jet Multiplicities

If the contributions from the BFKL gluon radiation to the parton momentum fractions in Eq. (3.1) are neglected, the parton–level result of an exponential growth (over the LO result) in cross section when BFKL evolution is taken into account obviously carries through to the hadronic cross section. As has been discussed before (see for example Refs. [5, 6]), this will no longer be the case when energy and momentum conservation is imposed by taking into account the BFKL gluon radiation in evaluating the parton momentum fractions. This is because the dijet cross section is driven by the gluon pdf, which falls off very sharply at medium and high x. So despite the fact that the contribution of the BFKL gluons to the parton momentum fractions is formally subleading, the numerical impact for dijet production is large and gets magnified by the sharply decreasing gluon pdf (see for example

Ref. [14] for arguments on the error in the normalisation of the result when neglecting the contribution of the BFKL gluons to the parton momentum fraction). The impact on the pdfs counteracts the expected exponential rise in cross section with growing rapidity difference between the leading dijets [5], and results in an almost no–change situation for the cross section. The fine details obviously depend on the exact shape of the pdfs, but the conclusions do not. In the following we will use the fits to the pdfs of Ref.[18].

The observation that the shape of the differential cross section as a function of rapidity is not expected to change dramatically when including BFKL evolution of the t–channel gluon has led to the study of other observables such as the azimuthal correlation of the leading dijets[14], where the dependence on the pdfs is expected to be less pronounced.

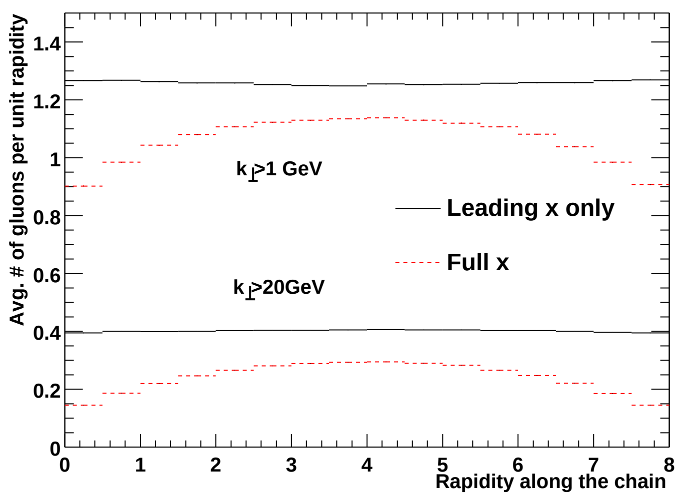

Figure 5: The average density of emitted gluons along the BFKL chain for the constant coupling formalism in the case hadronic dijet production. Please see text for further details.

Here we will study how energy and momentum conservation influences the associated gluon (i.e. jet) multiplicity for hadronic dijet production at the LHC. If the effect of the energy and momentum conserving constraint was to suppress the emissions of BFKL gluons completely, then BFKL would be irrelevant at such collider energies and fixed–order calculations would be adequate: there simply would not be enough phase space available for the gluon emission resummed through the BFKL equation. In Fig. 5 we plot the average density (in rapidity) of gluons emitted from a BFKL chain with ends fixed at rapidities -3.5 and 3.5 in the case of the hadronic cross section, i.e. with pdfs included. Just as for Fig. 4, we plot the density of both resolved ($k_{i\perp} > 1$ GeV) and harder ($k_{i\perp} > 20$ GeV) gluons, and we have chosen to plot the results obtained by two choices for evaluating the parton momentum fractions, namely the full version of Eq. (3.1) and the version where the BFKL gluon contribution is ignored.

Let us start by discussing the two upper curves of Fig. 5, corresponding to the density of resolved gluons. The upper line is obtained by ignoring the BFKL gluon contribution to the parton momentum fractions, and so corresponds to the case where one obtains an exponential increase over the LO cross section. The result for the gluon density is to be compared with the upper line in Fig. 4. We see that, just as expected, the density of gluons is not modified significantly in this case[2]. This is needed in order to maintain the exponential rise in cross section over the LO result. However, when we include the contribution of the BFKL gluons to the parton momentum fraction, the result is changed to the dashed (red) curve, and we see that the density of radiated gluons is reduced, especially at the ends of the chain where the effect on the energy consumption is exponentially enhanced compared to the effect of radiation in the middle of the chain (in this symmetric configuration).

The corresponding curves for harder gluons with $k_{i\perp} > 20$ GeV are also plotted in Fig. 5, and we see that in this case the relative effect of conserving energy and momentum is larger, especially at the end points. This is of course because the effect of energy and momentum conservation is most severe for hard radiation near the end points of the chain.

But the most important conclusion to be drawn from Fig. 5 is that the suppression of the BFKL evolution is not as dramatic as suggested by the cancellation of the BFKL rise in the dijet cross section by the pdf suppression. Conservation of energy and momentum still leaves sufficient phase space (at the LHC) to allow for a significant number of BFKL gluons to be emitted, and therefore for the BFKL evolution to be relevant. Therefore one cannot conclude from the predicted lack of rise in the hadronic dijet cross section that BFKL evolution is irrelevant. It is the abundance of BFKL radiation even when applying the full parton momentum fractions that results in sizeable angular decorrelations predicted between dijets at the LHC[5].

Fig. 5 also supports the idea of introducing an effective reduced rapidity span of the BFKL chain in analytic calculations. The BFKL evolution is obviously most important in a reduced rapidity span[6], where the BFKL emission (serving as a measure of the importance of BFKL evolution in the presence of energy and momentum conservation) is only slightly reduced compared to the partonic prediction.

It is not clear *a priori* in which region of rapidities an analytic prediction (ignoring energy and momentum conservation) will be valid, since first “asymptotic” values of the rapidity span have to be reached for the formalism to be valid, but these “asymptotic values” cannot be too large in order for the total energy available at the collider not to restrict too much the phase space available for BFKL gluon emission. In essence, examining this problem is what the construction of the BFKL Monte Carlo approach is all about.

5. Conclusions

We have examined the jet multiplicity predicted from BFKL evolution of a t–channel gluon exchange in dijet production, at both the partonic and hadronic levels, at the LHC. Previous results have shown that the effect of conserving overall energy and momentum at

[2]The small difference compared to Fig. 4 is caused by the fact that the transverse momentum spectrum of the leading dijets is softened after convolution with the pdfs. The softening of the leading dijets results also in a softening of the BFKL radiation.

present and future hadron colliders is to introduce a pdf suppression that completely compensates the predicted exponential BFKL rise with rapidity difference between the leading dijets. However, in the present analysis we have shown that there is still predicted to be a significant amount of BFKL radiation, and therefore that BFKL evolution *will* be relevant for QCD observables less dependent on the parton distribution functions. These include the angular decorrelation of the leading dijets. Furthermore, this multi–jet emission will of course provide a background to multi–jet observables in processes beyond the Standard Model at hadron colliders.

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