Blast response of laminated glass panels: a critical review of analysis and design methods

Author 1

- Socrates C. Angelides[1*], MEng, MRes
- Department of Engineering, University of Cambridge, Cambridge, UK
- ORCID number: 0000-0002-9498-2134

Author 2

- James P. Talbot[1], MEng, MA, PhD, CEng, FIMechE, MIOA
- Department of Engineering, University of Cambridge, Cambridge, UK
- ORCID number: 0000-0002-0036-9727

Full contact details of corresponding author.

[1]Department of Engineering, University of Cambridge, Trumpington Street, Cambridge, CB2 1PZ, United Kingdom

*sca36@cam.ac.uk

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Abstract

Laminated glass panels are often used to enhance the blast resilience of buildings by replacing the inherently brittle, monolithic glass that has historically been used in building façades. These composite ductile panels offer superior blast resistance and result in reduced glass-related injuries, due to the interlayer's ability to both provide residual resistance, following the fracture of the glass layers, and retain glass fragments. This paper reviews the various analysis methods that have been developed to support the blast design of laminated glass panels and reduce the need for expensive blast testing. The focus is on panels with polyvinyl butyral, as this is the most commonly used interlayer in building façades. The methods identified are categorised into empirical design guidance, analytical models, finite-element analysis and equivalent single-degree-of-freedom methods, thereby enabling a comparison of the modelling principles adopted and the material properties assumed within the different categories. This is informed by first presenting a brief overview of the material properties of laminated glass under blast conditions. The consistency of the underlying structural mechanics principles is discussed by comparing the methodologies across the different categories. Finally, the ease of application is considered, highlighting the methods that are often preferred by practitioners.

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List of notation

PVB	is polyvinyl butyral
FEA	is finite-element analysis
ESDOF	is equivalent single-degree-of-freedom

Introduction

The demand for blast resilient structures has increased in recent years due to the increased risk of external explosions arising from terrorist attacks. Current regulatory requirements are limited to ensuring a robust structural performance that avoids disproportionate collapse in the event of an internal, accidental explosion (HM Government, 2013; EN 1991-1-7, 2014). To enhance the blast resilience of buildings, façades must also be designed to resist external explosions and prevent the blast waves from penetrating the interior, as shown in Figure 1. In general, however, cladding materials, such as monolithic glass, are primarily architectural elements, without the required structural capacity to act as the first barrier of defence in a blast event.

To enhance blast resilience, laminated glass panels are often used to replace monolithic glass in façades. These panels usually consist of two layers of annealed glass with an interlayer that renders the composite panel more ductile than glass alone by providing residual resistance following the fracture of the glass layers. Additionally, glass-related injuries are reduced due to the interlayer's ability to retain glass fragments (Haldimann *et al.*, 2008; O'Regan, 2015).

The behaviour of laminated glass is complex, and its blast performance is typically determined through testing, using either open-field, high-explosives detonations (see Figure 2) or laboratory simulations in shock tubes. Various codes and standards have been developed to evaluate the structural performance (and the so-called hazard rating) of laminated glass from such tests. These include EN 13541, ISO 16933, ISO 16934, EN 13123-1, EN 13123-2, ASTM F1642 and GSA-TS01, as discussed in detail by Johnson (2006), Bedon *et al.* (2014), Hidallana-Gamage *et al.* (2015) and Dellieu *et al.* (2018).

Various analysis methods have been developed to support the design of laminated glass panels and reduce the need for expensive testing. These vary in approach, complexity and ease of application but may be categorised broadly into empirical design guidance, analytical models, finite-element analysis (FEA) and equivalent single-degree-of-freedom methods (ESDOF). This paper presents a review of these by considering both state-of-the-art academic publications and standard industry guidance. A brief overview of the material properties of laminated glass under blast conditions is first presented, which will assist the subsequent review of the various analysis methods. Although many interlayer types are suitable for normal building applications, the UK Centre for the Protection of National Infrastructure recommends using only polyvinyl butyral (PVB) and ionomer interlayers for blast protection (CPNI, 2019). The focus here is on PVB, a thermoplastic polymer, as this is the most common interlayer used in building façades.

2. Material properties

Blast loading is of short duration compared to normal service loading – typically of the order of milliseconds. This results in an enhanced fracture strength of the glass layers of laminated glass panels, as flaws require time to develop into cracks (Angelides *et al.*, 2019). The recommended dynamic fracture strength values for the blast design of monolithic glazing presented by Smith and Cormie (2009) and the UK Glazing Hazard Guide are shown in Table 1. The former were derived by extrapolating the inherent, static strength of annealed glass to the high strain-rates associated with blast loading, using Brown's integral (risk integral) for stress fatigue (also known as sub-critical crack growth), whilst the latter were established from a number of independent blast tests (Morison, 2007). There is broad agreement in the values, although it is worth noting the wide range associated with the various types of toughened glass.

Table 1 Recommended blast design values for glass fracture strength (Smith and Cormie, 2009; Morison, 2007).

Glass type	Fracture strength [MPa]	
	Smith and Cormie (2009)	UK Glazing Hazard Guide (Morison, 2007)
Annealed	80	80
Heat-strengthened	100-120	120
Toughened	180-250	180

The material properties of viscoelastic polymers, including PVB, are strain-rate and temperature dependent. At the higher strain-rates associated with blast loading, the PVB material behaviour observed in uni-axial tension tests resembles an elastic-plastic response with strain hardening, as shown in the stress-strain diagram in Figure 3a (Kott and Vogel, 2003; Bennison *et al.*, 2005; Iwasaki *et al.*, 2007; Morison, 2007; Hooper *et al.*, 2012a; Kuntsche and Schneider, 2014; Zhang *et al.,* 2015b; Chen *et al.,* 2018; Botz *et al.*, 2019). Although the change in slope of the stress-strain diagram is often referred to as yielding, this is simply referring to the bilinear shape of the stress-strain response and not to true plasticity (Angelides *et al.*, 2019). Table 2 summarises the salient enhanced material properties of PVB for strain-rates representative of blast response, which typically range from 7.6 s^{-1} to 30 s^{-1} according to full-scale blast tests (Morison, 2007; Hooper, 2011). These properties are also temperature dependent, with a stiffer / more flexible response observed at low / high temperatures respectively (Bermbach *et al.*, 2016; Chen *et al.*, 2018; Samieian *et al.*, 2018; Kraus, 2019). With many commonly encountered, commercially available PVB products, including Butacite® and Trofisol® (from Kuraray), Saflex® (from Eastman), Lam 51H® (from Everlam) and S-Lec™ (Sekisui), it is clearly important to note that the material properties can vary significantly depending on the type and manufacturer of the PVB (CPNI, 2019).

Table 2: Comparison of PVB material properties from uni-axial tension tests performed at room temperature and at strain-rates ranging from 7.6 s^{-1} to 30 s^{-1}.

Publication	**Specimen tested**	**Strain-rate [s^{-1}]**	**Yield Stress (σ_y) [MPa]**	**Failure stress (σ_f) [MPa]**	**Failure strain (ε_f)**	**Initial Young's Modulus (*E*) [MPa]**
Bennison *et al.* (2005)	PVB (Butacite®)	8	7	30	2	75
Hooper (2011)	Fractured laminated glass with PVB (Saflex®)	10	18	16	1.6	800
		30	34	18	1.3	15,000
Hooper *et al.* (2012a)	PVB (Saflex®)	20	7	34	2.13	228
Zhang *et al.* (2015b)*	PVB	8	3	29	2.05	52
Chen *et al.* (2018)*	PVB (Butacite®)	18	6	75	1.02	55

* Mean values obtained at the same actuator speed.

In addition to temperature and strain-rate, the PVB material properties are also influenced by the presence of the attached glass fragments. Hooper (2011) observed from uni-axial tension tests performed on fractured, laminated glass specimens that the glass fragments result in a stiffer response, compared to the behaviour observed for PVB alone (see Table 2), and the resulting stress-strain diagram resembles an elastic-perfectly-plastic material model, as illustrated in Figure 3b. Although a stiffer response is anticipated at higher strain-rates, the significant jump in the Young's Modulus value from 10 s^{-1} to 30 s^{-1} shown in Table 2 is most likely a consequence of experimental limitations, as Hooper comments that the accuracy of his results diminishes for strain-rates beyond 10 s^{-1}. This composite glass / PVB material response is also dependent on the PVB thickness and the adhesion level between the layers. For thin PVB specimens and high adhesion grades, the material response becomes more brittle (Hooper, 2011; Pelfrene *et al.*, 2016).

3. Empirical design guidance

This section presents a review of the available design guides, for assessing the hazard rating of panels, that include methods developed empirically from blast tests. These are often the first source of guidance used by practitioners, as they enable a rapid assessment of the panel response without performing a detailed analysis. Two guides in particular dominate the literature, and are reviewed here with a focus on the structural capacity of the panels themselves. The design of blast-resilient frames and connections is considered beyond the scope of this review.

3.1 UK Glazing Hazard Guide

The UK Glazing Hazard Guide was published in 1997 by the Security Facilities Executive, an organisation now integrated into the UK Defence Science and Technology Laboratory. Although this guide has restricted access, an overview is provided by Morison (2007). Following blast testing performed in the 1980s, as part of the Evaluation and Development in Counter Terrorism and Sabotage series, so-called damage fragility curves were developed for a variety of panel thicknesses but only two window sizes: 1.25 m x 1.55 m ('large' panel) and 1.25 m x 0.55 m ('small' panel). These empirical charts indicate the level of panel damage based on a specified blast pressure and impulse (or charge size and range). Two limiting contours are included: the lower one, indicating the limit for glass fracturing, and the upper one indicating PVB tearing. These allow practitioners some flexibility in selecting the hazard rating for their design, such as when selecting a panel that may fracture but without PVB tearing under the specified blast load.

3.2 ASTM

ASTM E1300 and ASTM F2248 are two complementary American standards for the blast resistant design of PVB laminated glass panels based on either annealed or heat-strengthened glass layers. Panels with fully-tempered glass layers are not recommended by ASTM F2248, due to the small fragment size produced by this glass and the resulting loss of integrity between a panel and its supporting frame. Using the glass failure prediction model presented by Beason and Morgan (1984) – which is based on Weibull's (1939) failure probability of brittle materials, accounting for the panel surface condition and stress distribution – and Vallabhan's (1983) finite-difference stress and deflection analysis, ASTM E1300 presents the static capacity, or 'load resistance', for a failure probability of 0.008 in a series of design charts for panels with annealed glass layers and a range of geometries and boundary conditions (Haldimann *et al.*, 2008). The higher strength of heat-strengthened glass is accounted for simply by a factor of 2 on the load resistance, termed the 'glass type factor'.

Designing to the ASTM standards requires the load resistance to be greater than the equivalent 3-second duration, static design load, which depends on the standoff distance and TNT equivalent charge weight, according to an empirical chart in ASTM F2248 that converts the blast pressure time-history into an equivalent static load. This chart was originally compiled by comparing the performance of laminated glass panels from blast tests with the load resistance calculated from ASTM E1300. It was first derived for a 60-second equivalent static load by Norville and Conrath (2001), who later updated this to 3 seconds (Norville and Conrath, 2006).

The ASTM charts offer more flexibility compared to the UK Glazing Hazard Guide, with its limitation to two panel sizes. However, the ASTM guide is more restrictive in that it only permits one hazard rating of the panel, known as the 'minimal hazard', as defined in ASTM F2912. This allows the glass layers to fracture but limits the number of detached glass fragments. The full material capacity is therefore not utilised, as opposed to the UK Glazing Hazard Guide which allows design for PVB tearing. This is also discussed in UFC 4-010-01, a guide developed to address the protection of military and U.S. Department of Defence (DoD) facilities from terrorist attacks. This guide permits windows in DoD buildings to be designed to ASTM F2248 but notes that the empirical approach will lead to more conservative designs than those based on some form of blast response analysis.

4. Analytical models

In addition to the established standards and design guides, various analytical models offer an insight into the fundamental structural response that is often obscured in both empirical methods and detailed numerical analyses. Most, if not all of these, are based on two-way spanning plate models, with simplified boundary conditions to enable the derivation of analytical or semi-analytical solutions for the mid-panel displacement time-history. This section reviews the available methods, to introduce the underlying principles behind the blast response of laminated glass.

4.1 Pre-fracture stage

Dharani and Wei (2004) present one of the first analytical models for laminated glass panels during the pre-fracture stage of blast response. A modal analysis was performed to solve the dynamic equation of motion for thin plates, which was derived using Hamilton's principle and assuming small-deflection theory. Both the glass layers and the PVB were assigned a linear-elastic material model. The latter is an approximation based on experimental observations at high strain-rates, as discussed in Section 2, and ignoring the strain-hardening phase, as the interlayer is not expected to yield in the pre-fracture stage due to the dominant stiffness of the glass layers. Shear deflection contributions, which are often significant for sandwich panels with low shear stiffness cores, were assumed to be negligible. The latter is consistent with the enhanced shear modulus of PVB at high strain-rates, which is capable of transferring the horizontal shear forces (Angelides *et al.*, 2019).

Dharani and Wei's original model highlighted the importance of the negative phase of blast loading, which includes the instant of maximum panel displacement as part of the rebound phase of the first vibration cycle – the negative phase is often ignored in the assessment of concrete and steel structures due to the additional mass and stiffness (UFC 3-340-02). The assumed small-deflection theory was also called into question due to predicted deflections exceeding the panel thickness. As a result, Wei and Dharani (2005, 2006) extended their original model to also account for membrane forces. The resulting nonlinear equation of motion, derived from von Karman's large-deflection theory, was solved by explicit integration of the first vibration mode, using Galerkin and Runge-Kutta methods. This solution was finally improved by Del Linz *et al.* (2016, 2018) by adding the contributions from the higher vibration modes.

Both Dharani and Wei and Del Linz *et al.* applied the rules of mixtures to calculate an equivalent Young's Modulus and Poisson's ratio for laminated glass. The derivation of an equivalent homogeneous material is common for fibre-reinforced composites, assuming uniformly distributed fibres. Adopting a similar approach for laminated glass is clearly approximate, as the PVB is positioned between the glass layers. A more accurate approach is to account properly for the through-thickness stress distribution by applying the transformed section approach, which is commonly used in analyses of reinforced concrete and is based on the modular ratio of PVB to glass (Angelides *et al.*, 2019). Laminated glass can also be modelled explicitly as a multi-layered panel. Yuan *et al.* (2017) applied the classical laminated-plate theory presented by Reddy (2004) based on a linear, through-thickness strain distribution (Kirchhoff's plate theory) and separate material properties for each layer.

It is only relatively recently that a damage criterion indicating fracture was introduced (Yuan *et al.*, 2017). This is based on the maximum principal tensile stress in the glass layers reaching the 80 MPa limit for annealed glass (Table 1). The latter may be somewhat conservative: Del Linz *et al.* (2016, 2018) considered a higher limit of 100 MPa, which was determined by comparison with blast tests on laminated glass panels. Given the inherent variability of the tensile strength of glass, it is difficult to conclude which of the two limits is most appropriate.

4.2 Post-fracture stage

The analytical models of Yuan *et al.* (2017) and Del Linz *et al.* (2018) also describe the post-fracture stage of blast response. Both approaches use a plastic yield-line analysis to derive the mid-panel displacement time-history of the fractured panel. The assumed yield-line mechanism was determined from the locations of high crack density observed in blast tests, as shown in Figure 4a. It has since been hypothesised that this consistently observed pattern is attributable to the composite bending action of the fractured panel associated with the interlayer working in tension and the glass fragments working in compression (Angelides *et al.*, 2019 and 2020).

Yuan *et al.* implemented the methodology presented by Yu and Chen (1992), which uses the velocity field of the assumed yield-line mechanism to solve the dynamic equations of motion. In this approach, the yield lines of Figure 4a are allowed to propagate towards the centre of the panel whilst transitioning to the final pattern shown in Figure 4b. Such travelling plastic hinges are a well-known phenomenon in structural dynamics and plasticity (Jones, 2011; Stronge and Yu, 1993). In contrast, Del Linz *et al.* (2018) applied the approximate procedure presented by Jones (1971), which assumes a single, fixed yield-line pattern (Figure 4a). This method equates the rate of external work done to the rate of membrane energy dissipated along the yield lines. Del Linz *et al.* acknowledge that the method could be improved by including the travelling hinges stage (Figure 4b), as the predicted mid-panel displacements underestimate the experimental results for small charge weights (*i.e.* 15 kg TNT equivalent). Nevertheless, overall, both methods demonstrate good agreement with experimental results. The maximum observed error is high in certain cases (∼43% and 23% for Yuan *et al.* and Del Linz *et al.* respectively) but the typical error across the duration of the time-history is significantly less than this and, in the case of Yuan *et al.*, consistently conservative.

The PVB is modelled as a rigid-perfectly-plastic material in both analytical methods but neither include a failure criterion to predict tearing. Del Linz *et al.* (2018) used the material properties presented by Hooper (2011) that account for stiffening effects from the attached glass fragments (Table 2), while Yuan *et al.* (2017) considered a yield strength of 18 MPa obtained from tensile tests performed on PVB alone at a stain-rate of 200 s^{-1}. An explanation for adopting this high strain-rate (which typically range from 7.6 s^{-1} to 30 s^{-1} – see Section 2) is not provided, although the resulting yield strength is indeed close to Hooper's value. Both methods account for the membrane action of the PVB but only Yuan *et al.* include the bending action. It is generally considered a fair assumption to ignore the PVB bending contribution, due to the large span-to-thickness ratio (~1000 for a typical 1.52 mm thick PVB interlayer and 1.5 m panel span).

In conclusion, the yield-line analysis methods reviewed here, which are summarised in Table 3, are challenging to derive and limited to simplified boundary conditions. However, the final expressions are straightforward to implement compared to, for example, more sophisticated/detailed numerical methods (see Section 5). With the inclusion of a failure criterion to predict PVB tearing, these methods would offer a useful tool for practitioners wishing to either predict panel capacity or validate more detailed analyses.

Table 3: Summary comparison of the post-fracture stage analytical models.

Method	Yield line mechanism	PVB material model	Derivation of displacement time-history
Yuan *et al.* (2017)	Travelling yield-line pattern (Figure 4a and 4b).	Rigid-perfectly-plastic (σ_y = 18 MPa).	Solving the dynamic equations of motion, with the velocity field approximated from the assumed yield-line mechanism (methodology introduced by Yu and Chen, 1992).
Del Linz *et al.* (2018)	Single, fixed yield-line pattern (Figure 4a).	Rigid-perfectly-plastic (σ_y depends on strain-rate, using the values presented by Hooper (2011) and shown in Table 2).	Equating the rate of external work done to the rate of membrane energy dissipated along the yield lines (methodology introduced by Jones, 1971).

5. Finite-element analysis

Where panel geometries and/or boundary conditions are neither suitable for analytical methods nor included in the empirical charts of design guides, laminated glass panels can be assessed using finite-element analysis (FEA). This typically involves using commercially available software to perform a transient, non-linear explicit analysis to predict the mid-panel displacement time-history. Additional capabilities include predicting the stress and strain levels in each layer, and simulating the glass fracture pattern. FEA also offers more flexibility than analytical models, which are limited to simplified geometries, boundary conditions and material models. In particular, the supporting frame can be modelled explicitly and effects such as frame out-of-plane displacements can be incorporated into the analysis. More representative material models may also be specified. This section reviews the two different approaches identified in the literature, which generally use either solid or shell elements to represent the panel, as illustrated in Figure 5.

5.1 Solid elements

In the blast analyses identified in the literature, the PVB is assigned either a hyperelastic (Pelfrene *et al.*, 2016) or an elastic-plastic with strain hardening material model (Larcher *et al.*, 2012; Zhang *et al.*, 2013; Zhang and Hao, 2015; Hidallana-Gamage, 2015). PVB is a viscoelastic material and the former model is an approximation that ignores the viscous component; the latter is a simplification based on experimental observations at high strain-rates, as described in Section 2, which requires the material properties (yield strength, Young's Modulus and failure strain) to be defined at a specific strain-rate. Both material models are considered appropriate for the loading phase of the PVB (corresponding to the positive phase of blast loading) but it is clear that further research is required to model properly the unloading phase.

To account properly for the stiffening effect from the attached glass fragments (see Table 2 and Figure 3b), the delamination of the fragments from the PVB also needs to be considered. This was considered explicitly by Zhang *et al.* (2013), who modelled the glass-PVB bond using a tiebreak contact, although the contact failure criterion was based on the tensile and shear adhesive strengths derived by Froli and Lani (2011) from low strain-rate tests. Pelfrene *et al.* (2016) improved the failure criterion by using the fracture energy derived by Franz and Schneider (2014) from high-speed, through-cracked tensile tests, but the final models proved to be unstable and did not reproduce the experimental results. Delamination effects were not included by either Larcher *et al.* (2012) or Hidallana-Gamage (2015), who assumed that the glass fragments remain fully bonded to the PVB.

Although Pelfrene *et al.* used solid elements for the PVB, the glass layers were modelled with linear-elastic shell elements. A tensile stress limit of 81 MPa for annealed glass was considered, referencing DIN 18008-4 (*cf*. 80 MPa in Table 1). To model the fracture pattern, initial modelling attempts deleted the glass elements in the fractured areas based on the Rankine stress criterion. This resulted in an unstable model as a result of excessive element removal compared to experimental observations. To improve this instability in the simulation, von Mises' criterion was later adopted together with a short plastic phase. This modification is described as artificial and unrepresentative of the actual glass fracture process; it also underestimates the capacity of glass layers in compression, as it assumes the same failure limit for both compressive and tensile stresses. Furthermore, the analysis was unable to simulate PVB tearing failure, as wide 'cracks' resulting from the removal of the glass elements prevent large strains from accumulating at the PVB bridges between the remaining attached fragments.

Larcher *et al.* (2012) used solid elements for all layers and considered an improved linear-elastic material law with a failure criterion that allows compressive stresses to develop following the fracture of the glass layers, while the tensile stresses remain zero. A tensile stress limit of 84 MPa was assumed for heat-strengthened glass, although details are not available and a comparison with the data in Table 1 suggests that the high strain-rate enhancement has been ignored. The methodology for removing elements is also not described. However, a failure strain of 0.0012 was assumed, rather than the stress failure criterion adopted by Pelfrene *et al.* (2016), and this resulted in a good prediction of interlayer failure that agrees with experimental results, suggesting that the element erosion technique may be more appropriate when the glass layers are modelled with solid elements.

Zhang *et al.* (2013), Zhang and Hao (2015) and Hidallana-Gamage (2015) also used solid elements for all layers but assigned the alternative Johnson Holmquist Ceramic (JH-2) material model to the glass layers, together with a strain erosion criterion to model the glass fracture pattern. This material model was initially developed to model the ballistic impact response of brittle materials, and it includes the residual compressive capacity of the glass (Johnson and Holmquist, 1994). Modified material parameters, derived by Zhang *et al.* (2015a), were used by Zhang *et al.* (2013) and Zhang and Hao (2015), including an arbitrary limit of 0.03 for the maximum principal strain of annealed glass. Hidallana-Gamage (2015) assumed a strain limit of 0.0024 to improve the accuracy of the model, which is described as an arbitrary, artificial increase to the 0.0012 failure strain cited by Larcher *et al.* (2012). This was an attempt to retain the correct mass of the panel for as long a time as possible, given that deleting elements artificially reduces this as the failure progresses.

The failure criterion introduced by Larcher *et al.* and the Johnson Holmquist Ceramic (JH-2) model both account for the composite bending action of the fractured panel. These models can therefore potentially simulate the yield lines observed from blast tests, thereby potentially resulting in improved response predictions. However, this has yet to be demonstrated and it is evident, from the variable and often arbitrary material properties assumed in different analyses, that the simulation of the post-fracture response with FEA is more challenging than using the empirically derived yield-line mechanisms assumed in analytical models. The situation could be improved with further experimental work to better define the material models for PVB, the glass layers and their bond. However, given the long computation time required for FEA using solid elements (Pelfrene *et al.* (2016) report computation times of up to 50 hours), these methods are unlikely to be considered as the first option of analysis by practitioners.

5.2 Shell elements

Larcher *et al.* (2012) attempted to reduce FEA computation time by modelling laminated glass panels using shell elements. Different modelling options with layered and smeared elements were investigated. The former use the same material properties as the solid elements described in Section 5.1 but now in a single composite element with explicitly defined layers. This resulted in good agreement with experimental results, in particular when predicting the onset of PVB tearing. In contrast, smeared elements were based on two spatially coincident elements that, together, represent the pre-fracture stage, while the response following fracture of one glass layer is described by only one element. Although good agreement was observed with experimental results, this modelling approach is unable to describe the response following the fracture of both glass layers.

Hooper *et al.* (2012b) presented an alternative FEA methodology for use with shell elements that also resulted in improved computation time, achieving 20-60 seconds for a quarter-panel model, compared to 10-20 hours for the same analysis using solid elements. Two separate models were created, with the results of the pre-fracture stage (which was terminated at the 80 MPa tensile limit (Table 1)), used as initial conditions for the post-fracture stage. During the latter, a pure membrane response of the PVB was assumed, with the glass fragments contributing only to the mass of the panel. This results in membrane strains accumulating throughout the panel and not at yield lines, as discussed in Section 4. Hooper *et al.* acknowledge that the omission of the composite bending action within the fractured panel is likely to be a contributing factor to the observed discrepancies between the predicted and measured deflection profiles. The stiffening effect from the attached glass fragments, and their subsequent delamination, which resulted in the modelling challenges discussed in Section 5.1, were accounted for empirically by assigning the Johnson-Cook plasticity model to the PVB. Although this material model was originally derived for metals, it was successfully fitted to match Hooper's (2011) test results, as discussed in Section 2.

The reduced computation time for FEA with shell elements offers a more viable option for practitioners. Additionally, the empirically fitted Johnson-Cook plasticity model for PVB eliminates the need to assign arbitrary material properties, as summarised in Table 4. However, further research is required to assess the implications of the pure membrane response assumed by Hooper *et al.*, which is clearly inconsistent with the yield-line theory adopted in the analytical models described in Section 4.2.

Table 4: Comparison of PVB material models used in FEA.

PVB material model	**Element type**	**Advantages**	**Disadvantages**
Hyperelastic	Solid and shell (layered and smeared)	- Can account for the composite bending action of the fractured panel, when the glass layers are assigned	- Long computation time (for solid elements).

		the failure criterion introduced by Larcher et al. (2012) or the Johnson Holmquist Ceramic (JH-2) model.	- Requires modelling of the stiffening effect from the attached glass fragments, and their subsequent delamination.
Elastic-plastic with strain hardening	Solid and shell (layered and smeared)	- Can account for the composite bending action of the fractured panel, when the glass layers are assigned the failure criterion introduced by Larcher et al. (2012) or the Johnson Holmquist Ceramic (JH-2) model.	- Long computation time (for solid elements). - Requires modelling of the stiffening effect from the attached glass fragments, and their subsequent delamination. - Material properties are defined for a specific strain-rate.
Johnson-Cook plasticity model	Shell	- Improved computation time. - Does not require modelling of the stiffening effect from the attached glass fragments, and their subsequent delamination.	- Assumes pure membrane response. - Only suitable for the post-fracture stage (needs to be combined with a separate FEA for the pre-fracture stage).

6. Equivalent single-degree-of-freedom methods

One way of avoiding the computational effort of FEA is to adopt an equivalent single-degree-of-freedom model (ESDOF). This is a common, generic method of dynamic analysis, for both transient and steady-state loading, when the response of a structure is dominated by a single vibration mode (Biggs, 1964). Figure 6 illustrates how a laminated glass panel subjected to blast loading may be represented using an ESDOF model in order to predict the mid-panel displacement time-history based on the fundamental mode of vibration. Such models are often preferred by practitioners over analytical methods and FEA due to their simplicity and ease of application.

This section reviews the publicly available ESDOF methodologies. Software packages with restricted access, such as HAZL, WINLAC, LamRes Guide, SBEDS-W and WINDAZ, are necessarily excluded.

6.1 Definition of stiffness

A key challenge with ESDOF methods for laminated glass panels is defining the stiffness of the system, which varies significantly with panel deflection. By experimental observation, the latter may be divided into four distinct stages, as shown in Figure 7 (Stage 1 – pre-fracture stage; Stage 2 – single, intact glass layer; Stage 3 – post-fracture stage with PVB elastic; and Stage 4 – post-fracture stage with PVB plastic). However, the stiffness cannot be measured directly from quasi-static bending tests, as the enhanced material properties of both the glass layers and the PVB at high strain-rates are not accounted for (Angelides *et al.*, 2019).

Data points for defining the stiffness, or so-called 'resistance function', of two panel sizes under blast loads are available in tables alongside the fragility curves of the UK Glazing Hazard Guide (Smith, 2001). For the pre-fracture stage (Stage 1 of Figure 7), the data points were produced from a FEA (Morison, 2007) based on Moore's (1980) load-deflection graphs for monolithic glass. The 80 MPa limit for annealed glass (Table 1) is adopted, assuming both layers fracture simultaneously, *i.e.* Stage 2 is ignored (Smith, 2001). Stages 3 and 4 are approximated by a single linear function, which was back-calculated from blast tests to give the same total strain energy as that derived experimentally. Deflection limits of 9.4% (52 mm) and 16% (200 mm) of the short (transverse) span of the panel are assumed, based on experimental observations from the 'small' and 'large' panels described in Section 3.1 (Morison, 2007).

Smith and Cormie (2009) present an alternative semi-empirical methodology to define Stages 3 and 4. Again, these are treated as a single stage but now described by a cubic function due to an assumed pure membrane response of the fractured panel. To define the limiting load of the resistance function, the deflection limit, derived from blast tests, that leads to PVB tearing was first converted into a strain limit (by compatibility). The corresponding stress limit was then calculated using the equivalent Young's Modulus of the fractured panel, which was back-calculated, through trial and error, for a particular panel size that had previously been blast tested. In contrast to the resistance tables of the UK Glazing Hazard Guide, which are limited to two panel sizes, this methodology is applicable to all sizes and thicknesses, as both the Young's Modulus and the strain limit derived semi-empirically are considered to be characteristic material properties, while the limiting load can be adjusted linearly for different PVB thicknesses.

In the ESDOF methodologies of WINGARD, TPS Consult and Morison (2007), the stiffness for Stages 3 and 4 is calculated from first principles, and therefore requires the material properties of the PVB. WINGARD is a commercially available software package, developed by Applied Research Associates, that is recommended in UFC 4-010-01 (see Section 3.2) for the design of windows in DoD buildings by reference to PDC-TR 10-02 (a separate technical report issued by the United States Army Corps of Engineers). It includes a built-in library that defines the PVB as having an assumed yield strength of 20.7 MPa and a strain failure of $\varepsilon_f = 2$ (Morison, 2007). References for these values and the corresponding strain-rate are not provided but a comparison with Table 2 shows that these are close to Hooper's (2011) measurements. The final resistance function is calculated from Timoshenko's (1959) classical solution for square elastic membranes, adjusting the Poisson's ratio for PVB and the aspect ratio for rectangular plates (Morison, 2007).

In the ESDOF approach developed by TPS Consult, a PVB yield strength of 8 MPa is used, which was derived by scaling the value from low strain-rate tensile tests to reproduce the experimental results presented in the UK Glazing Hazard Guide (Morison, 2007). A comparison with Table 2 suggests that this value does not account for stiffening from the attached glass fragments. A deflection limit of 27.8% of the panel span is adopted, which was derived from low strain-rate pressure tests. As the observed failure mechanism was PVB cutting from the attached glass fragments, this deflection limit was considered strain-rate independent. Despite being often cited by practitioners in analyses using both ESDOF methods and finite-elements, this assumption requires further experimental investigation, as the PVB cutting mechanism is not accounted for in the strain failure values reported in Table 2, which are derived purely from uni-axial tension tests. The resistance function in the TPS consult method is ultimately defined by Mansfield's (1989) cubic elastic membrane theory and Timoshenko's and Goodier's (1951) 'soap film' plastic membrane for Stages 3 and 4, respectively (Morison, 2007).

Morison (2007) presents a detailed comparison of the membrane theories adopted by WINGARD and the TPS Consult method, and, in doing so, describes his own detailed modelling to predict the resistance function. In his methodology, the resistance function for Stages 3 and 4 is derived from a FEA using solid elements to model the PVB. An elastic-plastic with strain-hardening material model is considered for the PVB that was derived from tensile tests at a temperature of 23 ˚C and a strain-rate of 40 s^{-1}. This high strain-rate was chosen in an attempt to account for the stiffening effects from the attached glass fragments by applying a strain-rate amplification factor of 4 to account for the anticipated 10 s^{-1} strain-rate of typical blast events (see Section 2). However, comparisons with blast tests required factors varying between 0.6 and 7.1 in order to fit the experimental results. A comparison with Table 2 of the corresponding yield strength for an amplification factor of 4 (7.9 MPa), suggests that a higher amplification should in fact be considered to match Hooper's (2011) measurements.

6.2 Transformation factors

Having described the variable stiffness of a panel, and obtained best-estimates for the describing function, the corresponding stiffness, mass and loading parameters of the ESDOF model are defined by so-called 'transformation factors'. These are applied to the panel parameters, i.e. the stiffness, as derived in Section 6.1, the total mass and the applied load. These are calculated by equating the strain energy, kinetic energy and external work of the two systems assuming a deflected shape function that approximates the fundamental mode of vibration. The UFC 3-340-02 design guide, used for non-military civil engineering structures at risk of high explosive detonations, presents transformation factors for a variety of loading and boundary conditions, using a shape function based on the deflection of the structure under the static application of the actual dynamic load. Cormie and Sukhram (2007), Smith and Cormie (2009) and WINGARD (Morison, 2007) all use these UFC transformation factors for the blast design of laminated glass panels.

The variable nature of the panel stiffness is acknowledged in the TPS Consult method, which considers different transformation factors for each stage. In Stages 1 and 2 of Figure 7, the deflected shape was calculated from Navier's equations presented by Timoshenko (1959) for elastic plates, while the deflected shape for plastic membranes presented by Timoshenko and Goodier (1951) was considered for Stages 3 and 4 (Morison, 2007).

Morison (2007) further improved the accuracy of the transformation factors by using FEA to account for the changes in deflected shape due to the nonlinearity introduced by the membrane forces. This established that the factors varied with deflection, even within the same stage, thereby demonstrating the limitations of assuming constant transformation factors in ESDOF analyses of laminated glass panels.

In conclusion, Morison's methodology is the most sophisticated ESDOF method available for the derivation of both the panel stiffness and the transformation factors. Nevertheless, this method remains inconsistent with the yield-line theory adopted in the analytical models discussed in Section 4.2, as a pure PVB membrane response is assumed when deriving the post-fracture stiffness. Furthermore, previous research on the bending stiffness of a fractured panel under blast loads is not available to enable any comment on the implication of Morison's assumption, nor those of the other ESDOF methodologies. Recent experimental work has indicated a significant enhancement of the post-fracture bending capacity at high strain-rates (Angelides et al., 2020). It is possible that the bending contributions to the mid-panel displacement are accounted for by the empirical factors applied in the various ESDOF methods to fit the experimental results but further research is required to confirm this.

7. Conclusions

This paper has reviewed the various analysis methods that have been developed to support the blast design of laminated glass panels with polyvinyl butyral interlayers. Such methods may be categorised into empirical design guidance, analytical models, finite-element analysis and equivalent single-degree-of-freedom methods.

Two design guides for establishing the hazard rating of panels have been identified, which are often the first sources of guidance used by practitioners. In the case of the UK Glazing Hazard

Guide, this relies on a series of empirically-derived design charts based on full-scale blast tests; in the equivalent American standards, the hazard rating is defined by comparison with the panel resistance calculated from numerical, static analyses using an equivalent static load derived empirically from blast tests. Although both approaches enable rapid assessments, without the need for detailed analyses, the underlying empirical methods are limited to the panel sizes for which blast tests have been performed. Panel sizes not covered by these guides may be designed using analytical models, finite-element analysis or equivalent single-degree-of-freedom methods. These potentially offer improved accuracy and have the benefit of also predicting the panel displacement time-history.

Analytical models offer insights into the fundamental structural response, which is often obscured in the alternative methods. Existing methodologies for simplified boundary conditions, that use a plastic yield-line analysis, offer a potential tool for practitioners to predict panel capacity and validate more detailed analyses performed with finite-elements or equivalent single-degree-of-freedom models. Using the former, the supporting frame can be modelled explicitly and effects such as frame out-of-plane displacements can be incorporated into the analysis, while more representative material models may also be specified. However, inconsistencies observed in the often arbitrary material properties assumed in the various analyses suggest that more detailed material models will not necessarily result in more accurate results unless further research is undertaken to validate these. The associated challenges are attributed to the strain-rate sensitivity of the interlayer, which results in a stiffer response under short-duration blast loads, and the complex interaction of the glass fragments with the interlayer during the post-fracture stage. Further experimental work is clearly required to better define the material models of the glass layers, the interlayer and their bond.

The long computation times required to perform finite-element analyses of laminated glass panels is a significant concern for practitioners, especially when using solid elements rather than shell elements. Equivalent single-degree-of-freedom methods offer a quicker, simplified approach, but further research is required to assess the implications of adopting the simplified stiffness, mass and loading parameters for the equivalent system. Research has shown that more complex, time-varying parameters are required to describe the complete response of laminated glass panels from initial loading to final failure. Furthermore, the assumption of a pure membrane response for the post-fracture stage is inconsistent with observed fracture patterns and the yield-line theory adopted in analytical models. It is possible that bending contributions are accounted for empirically within the equivalent models but further research is required to confirm this.

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Figure captions

Figure 1. Building façades constitute the first barrier of defence in a blast event.

Figure 2. Blast testing of laminated glass panels (Courtesy of D.J. Goode and Associates Ltd).

Figure 3. Schematic stress-strain diagrams typical of uni-axial tension tests performed at high strain-rates: (a) PVB alone and (b) fractured, laminated glass specimens. Not to scale.

Figure 4. Yield-line mechanisms assumed in the post-fracture analytical models of Yuan *et al.* (2017) and Del Linz *et al.* (2018): (a) initial pattern and (b) transition to final pattern.

Figure 5. Different modelling options in FEA for laminated glass panels: (a) solid elements and (b) shell elements.

Figure 6. Conversion of a laminated glass panel into an ESDOF system with an equivalent stiffness (K_{Eq}), mass (M_{Eq}), and external load (P_{Eq}).

Figure 7. Schematic comparison of the mid-panel, static load-deflection response of laminated glass at low and high strain-rates. High strain-rates reproduced from Larcher *et al.* (2011) and Morison and Pullan (2015); low strain-rates reproduced from Morison (2007). Not to scale.