Evaluation of a minimum flexural reinforcement ratio using fracture-based modelling

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Abstract

Minimum reinforcement ratios are specified for reinforced concrete structures to provide enough ductility. The aims are to control cracking in the serviceability limit state and to prevent sudden failure by ensuring sufficient ductility after the loss of tensile stress in concrete due to cracking. This can provide a warning before collapse and time to take preventive or remedial measures. A review of past research reveals that there are large variations, and sometimes contradictions, in proposed minimum reinforcement requirements for flexural members. In this paper, a fracture mechanics-based model is used to study different local phenomena such as tensile and compressive concrete softening to more precisely describe the behaviour of reinforced concrete beams. The findings show a decrease in the minimum reinforcement ratio with increasing beam size. This contradicts the provisions of prevailing codes and standards which suggest no change in the minimum reinforcement ratio with size. Therefore, there is a need to review the minimum reinforcement provisions.

Keywords: Flexural reinforcement; minimum reinforcement; reinforced concrete, fracture, flexural crack.

1 Introduction

A minimum flexural reinforcement requirement is specified in most concrete codes and standards. These provisions are generally intended to control cracking in the serviceability limit state and to prevent sudden failure of structures that have a very small amount of tensile reinforcement by providing a reasonable ductility after the loss of tensile stresses in the concrete. This can provide warning before collapse and time to take preventive or remedial measures. A review of previous research on minimum reinforcement percentages reveals that there are significant variations, and some contradictions, in the results obtained. This may be attributed to two reasons: (1) the minimum reinforcement requirement can be a function of many parameters i.e. the concrete properties, the reinforcement properties, the interaction between the concrete and the reinforcement, the beam shape and size effects; and (2) the derivations of most minimum reinforcement formulae are based on empirical approaches. This has led to a certain controversy over the years. In this paper, a fracture mechanics-based model is used to study the ductility of reinforced concrete and, hence, to inform code provisions relating to minimum flexural reinforcement requirements.

2 Previous models

Existing codes and studies have defined the minimum flexural reinforcement using different approaches. In the following sections, some of the prevailing codes provisions and theoretical studies will be presented and the underlying analysis approaches will be highlighted to clarify some of the differences between the various models.

2.1 Codes provisions

According to ACI318M-11 [1], to prevent sudden failures, the computed moment resistance of a reinforced concrete section using a cracked section analysis should not be less than that of the corresponding unreinforced concrete section computed from its modulus of rupture. The minimum reinforcement ratio is determined by equating the moment resistance with the cracking moment. The proposed formula for the minimum area of reinforcement $A_{s,min}$ is

$$A_{s,min} = \frac{0.25\sqrt{f_c'}\ b\ d}{f_y} \quad (1)$$

and not less than

$$\frac{1.4\ b\ d}{f_y} \quad (2)$$

where f_c' is the concrete cylinder compressive strength, b is the width of the concrete beam, d is the depth of the beam and f_y is the reinforcement yield strength. Equation (2) does not include the concrete strength as a variable and was proposed in earlier editions of the ACI code. It will control when the concrete strength is less than 31 MPa. An empirical formula for the modulus of rupture f_r was used in the analysis where $f_r = 0.62\sqrt{f_c'}$ (for normal weight concrete). According to equation (1) the minimum reinforcement ratio is a function of the concrete compressive strength and reinforcement yield strength. The effect of size is not considered though the existence of a size effect in the structural behaviour has been proved by many researchers and approaches have been proposed to model its influence [2–5]. Another concern relates to the assumption that once the concrete cracks, it cannot sustain any tensile stress.

A similar logic was applied in the European code (EC2) [6] to determine the minimum reinforcement requirement, however, the cracking moment was calculated using the mean flexural tensile strength of reinforced concrete members which depends on the tensile strength of concrete f_{ctm}.

$$A_{s,min} = \frac{0.26\ f_{ctm}\ b\ d}{f_y} \quad (3)$$

and not less than

$$0.0013\ b\ d \quad (4)$$

Different codes such as the Indian Standards (IS:456-2000) [7] and the fib Model code (2010) [8] used the same approach.

2.2 Analytical studies

The criteria of equating the cracking moment with the bending moment was also applied by Ozbolt and Bruckner [9] but with some modifications. They studied the problem numerically by allowing strain-softening within a finite element framework. In contrast to code provisions which assume independence between the minimum reinforcement ratio ρ_{min} and beam size, they showed an increase in the required minimum reinforcement with an increase in beam height. The use of a finite element framework makes it difficult for this model to be considered as a simple analytical model.

Gerstle et al. [10] simplified some assumptions related to the fictitious crack model (FCM) developed by Hillerborg [11] to develop an analytical solution for flexural cracks in reinforced concrete beams. Using the balance of forces and considering the deformation in concrete, a dimensionless moment was obtained as a function of the crack length for given concrete and steel properties. The minimum reinforcement was defined as the point at which there was no further unstable crack propagation and this was associated with a continuous positive slope in the curve of moment capacity versus crack length. A minimum reinforcement formula was proposed as

$$\rho_{min} = \frac{E_c}{E_s}\sqrt{\sqrt{(0.0081 + 0.0148\beta} - 0.09} \quad (5)$$

where β is a dimensionless material-scale parameter (that increases with increasing size). According to Equation (5), the minimum reinforcement ratio increases with increasing beam depth. Rao et al. [12] used the same model as that of Gerstle et al. [10] to further investigate the minimum flexural reinforcement ratio. They came to the same conclusion that the minimum reinforcement increases as the depth of the beam increases and also as the concrete strength increases. It is worth mentioning that although the model includes rational assumptions related to the behaviour of concrete in tension, it assumes perfect bond between concrete and steel which can be a conservative assumption when studying cracked concrete.

Ruiz et al. [13] use the cohesive model to develop a numerical model for crack propagation and consider the bond-slip behaviour between concrete and steel. Many numerical parameters were included in the model and this presents difficulties for incorporation in practical codes and standards. A non-linear fracture mechanics model was recently developed by Carpinteri et al. and used to evaluate the minimum reinforcement ratio [8]. It was found that the reinforcement ratio decreases as the beam depth increases. The interaction between the steel and concrete along the reinforcement bar was not modelled. A summary of some of the previous models is presented in Table (1).

Table 1. Minimum reinforcement according to various models

Reference	**Trend (ρ_{min} with beam size)**	**Approach/ Commentary**
ACI318 M-11 [1]	Independent	- Employs the criteria $M_{cr} = M_y$ (equating the moment capacity associated with steel yielding and the cracking moment) - Uses empirical relationships in the derivation - Does not consider size effects - Assumes that the concrete tensile strength after cracking is zero (Conservative assumption)
European code (EC2) [6]	Independent	- Uses the same underlying approach as that of ACI318-11
Ozbolt and Bruckner [9]	Increase *after reaching a critical beam size*	- Employs the criteria $M_{cr} = M_y$ but allows strain-softening within a finite element framework - Uses a finite element framework that makes it difficult to be considered among the simple analytical models
Gerstle et al. [10]	Increase	- Is based on Non-Linear Fracture Mechanics (NLFM) - Considers the equilibrium of the tensile and compressive forces with concrete tensile softening - Defines the minimum reinforcement as that at which the crack propagation process becomes stable - Does not consider the interaction between concrete and steel or the softening in the concrete

Reference	Trend (ρ_{min} with beam size)	Approach/ Commentary
Rao et al. [12]	Increase	- Uses the same approach as Gerstle et al. [10]
Bosco and Carpinteri [14]	Decrease	- Is a linear fracture mechanics approach - Proposes a brittleness number which is a function of the reinforcement ratio and the beam size - Does not consider the fracture process zone in concrete (strain softening) - Does not consider the interaction between concrete and steel
Riuz et al. [13]	decrease	- Is a non-linear fracture mechanics (cohesive model) approach. - Employs numerical modelling - Uses an effective slip- model for reinforcement and concrete interaction - Is difficult to apply as it includes many numerical parameters
Carpinteri et al. [8]	Decrease	- Uses a numerical approach based on non-linear fracture mechanics - Does not model the interaction between steel and concrete along the reinforcement bar

3 Fracture-based modelling

As mentioned previously, many of the prevailing codes provisions are based on empirical formulae that were derived from tests conducted on small specimens and do not consider the size effect. The size effect can be studied using fracture mechanics and the study of the crack growth process. Figure (1) shows the relationship between a 'nominal' strength and 'characteristic' size. According to the graph, a strength criteria analysis gives results that do not depend on size whereas a linear elastic fracture mechanics (LEFM) analysis predicts a decrease of the nominal strength with characteristic size. It was shown in [15] that the results of many tests show a transition between the two trends. This is believed to be due to the considerable size of the fracture process zone (FPZ) ahead of the crack tip which can be explained by non-linear fracture mechanics (NLFM).

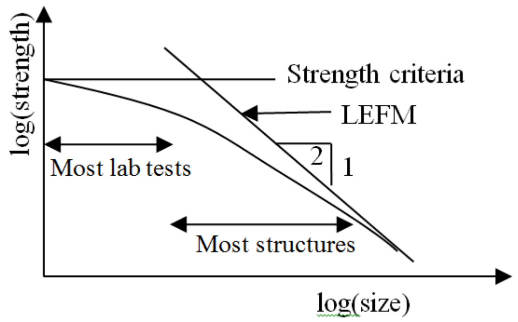

Figure 1. Strength versus size

Concrete is a very complicated material and it contains materials of different scales. So concrete cracking requires the consideration of local phenomena to understand the overall behaviour of structural elements.

Fracture mechanics provides the basis for a rational approach to study the fracture of reinforced concrete (RC). Information about the ductility of RC beams, which is the main criterion in determining the minimum reinforcement requirements to avoid brittle failure, can also be obtained.

3.1 Integrated fracture-based model description

An integrated fracture-based model was developed to predict the behaviour of lightly reinforced concrete beams [16]. The proposed model incorporates post-cracking tensile stresses in the concrete, the bond-slip behaviour between the reinforcement and concrete, and compression softening in the concrete compressive zone. The stages of the analysis include: the development of a crack, crack propagation with tension softening, concrete compressive softening and rotation. The bond between the steel and concrete was modelled using a bond-slip model which gives a more accurate representation than the perfect bond assumption and helps to reveal the main parameters affecting the behaviour. Assumptions related to the material behaviour were used to minimize the variables in the model and develop a closed-form solution for the stresses associated with concrete cracking. For example, the tension softening relationship was assumed to be linear. The developed model includes many variables that were found to affect the cracking process. These variables include the beam depth d, beam width b, concrete compressive strength f_c', reinforcement ratio ρ, critical crack mouth opening C_r, maximum shear stress between the reinforcement and concrete τ_{max}, elastic modulus of concrete E_c and elastic modulus of steel E_s. The reader is referred to [16] for more explanation of these parameters and for a full description of the model. The model is capable of describing the cracking process in concrete including the initiation and propagation of flexural cracks and was validated against a sample of experimental results [16].

3.2 Ductility definition according to the model

Ductility can be defined as the ability of a material to provide sufficient deformation, and hence warning prior to failure. However, there is debate about the specific meaning of ductility because deformation can be translated into different measures such as curvature, rotation and displacement [17]. These terms can then be connected with other phenomena which are in turn the subject of controversy.

The developed model assumes a gradual loss of tensile strength with crack propagation and that the concrete loses its tensile strength completely when the crack opening equals a critical crack opening. Those stresses are then transferred to the reinforcement. If the reinforcement is not able to compensate for the loss of the concrete tensile stresses, the resistance decreases causing unstable crack propagation. Stable behaviour is associated with the development of the crack with increasing load whereas unstable behaviour is associated with the propagation of the crack under decreasing load. Under unstable crack propagation, if the applied load increases, steel fracture is expected and, therefore, at that point a sudden failure will occur. Therefore, a minimum area of reinforcement is required to avoid this unstable cracking process.

3.2.1 The effect of concrete strength

Using the developed model, the behaviour of three RC beams is shown in Figure (2). The figure shows predictions of relative crack length versus dimensionless moment for three beams which have the same size, reinforcement ratio and steel properties but the concrete compressive, and hence tensile, strengths differ. There are some kinks in the graphs which indicate a change between the different stages of behaviour. It is of note that the beam with a tensile strength of 1.9 MPa (f_c= 20MPa) exhibits more stable crack growth than the equivalent beam with a strength of 2.6 MPa (f_c= 30MPa). In this case, due to the low value of tensile strength of concrete, the reinforcement is better able to compensate for the loss of the tensile stresses and accordingly a large strength reduction does not occur when the concrete cracks. This means that for a low concrete tensile strength, smaller values of minimum reinforcement are required to prevent unstable crack growth.

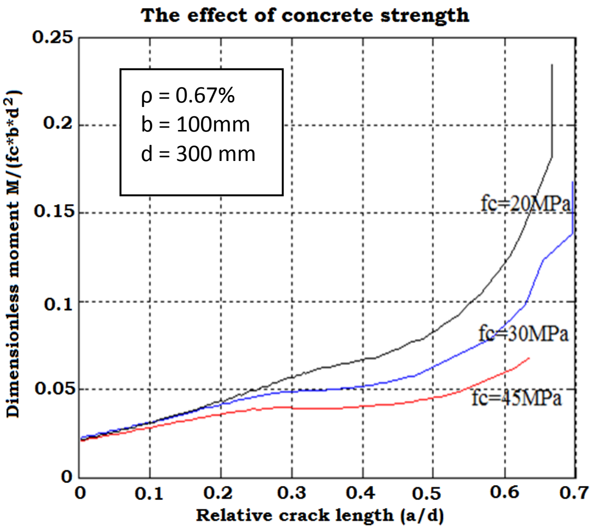

Figure 2. The effect of concrete strength

3.2.2 The effect of beam size

The effect of the beam size is presented in Figure (3) which shows the predictions for three beams with similar properties but different depths. The beam with a depth of 100mm exhibits a large decrease in moment capacity after a relative crack length of around *0.43*. In the beams with higher depths the reductions in moment are relatively small. It is of note that the three beams have the same reinforcement ratio. This means that a small beam needs more reinforcement to compensate for the loss of the concrete tensile strength. At the same time, the same reinforcement is enough for larger beams to have a stable crack growth. This means that the ductility of the beam “avoiding brittle failure” is a function of the reinforcement ratio and the beam size. Once the size increases, a lower reinforcement ratio is required to obtain stable crack propagation.

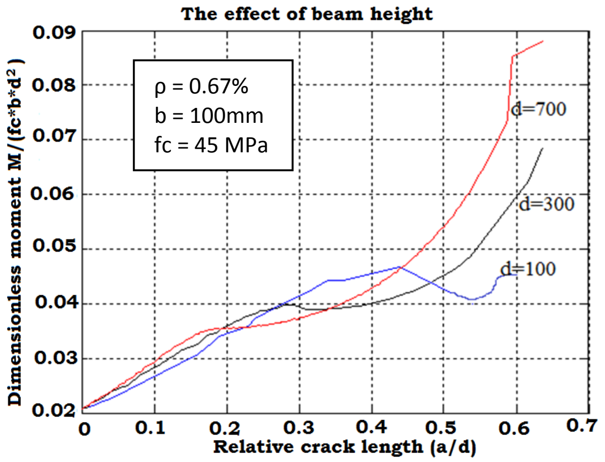

Figure 3. The effect of beam height

4 Minimum reinforcement requirements

The minimum reinforcement ratios required to avoid unstable crack growth were found from the model predictions for different beam heights. The minimum value was defined by the ratio where the moment capacity followed a continuous increasing trend with increasing crack length. The results are shown in Figure (4) where the minimum reinforcement ratio is found to be a decreasing function of the beam size. This contradicts formulae which suggest a constant ratio or an increase in the minimum reinforcement ratio with size. Figure (5) presents a comparison between the minimum reinforcement requirements according to the developed model, ACI 318 and EC2.

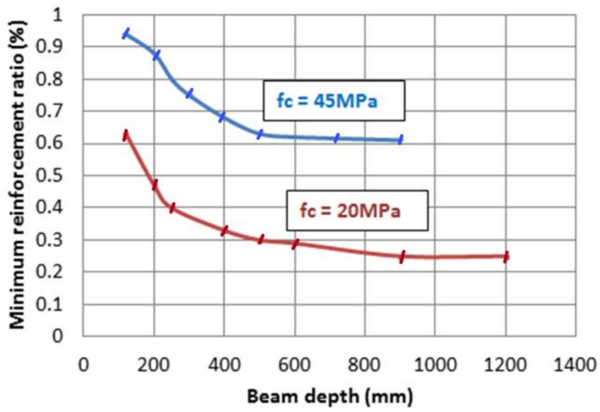

Figure 4. Minimum reinforcement requirements

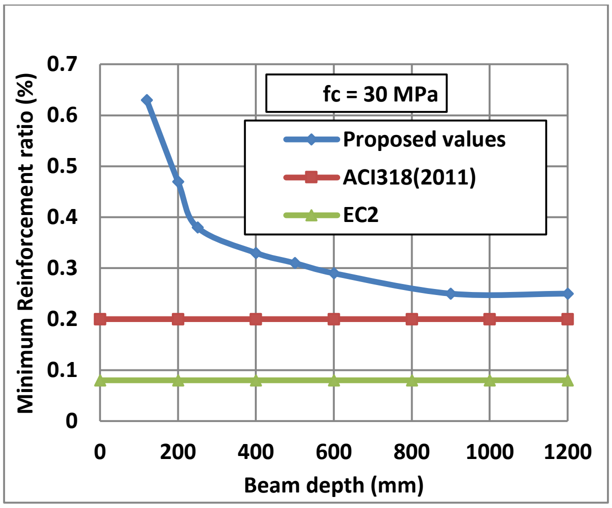

Figure 5. Comparisons with codes provisions

According to the developed model, the minimum reinforcement ratio decreases with increasing beam size and the codes underestimate the necessary minimum reinforcement for beams.

5 Conclusions

A minimum amount of flexural reinforcement is essential to ensure ductile behaviour and avoid sudden failure. Many models in the literature are based on empirical derivations and there is a need for a rational approach that considers the local phenomena in concrete to gain a better understanding of RC behaviour. A fracture-based model is developed to evaluate the required minimum reinforcement. Local phenomena such as tensile and compressive concrete softening have been integrated into the developed model to more precisely describe the behaviour of reinforced concrete beams. The findings show that the ductility of a beam is a function of its size. Therefore, when the beam size increases, a smaller minimum reinforcement is required. This contradicts the provisions of codes and standards that suggest no change in the minimum reinforcement ratio with beam size. This paper focuses on the effect of the beam size and concrete compressive strength because they have been found to have a noticeable effect on ductility; however, there are many other parameters such as ρ, C_r, τ_{max}, E_c and E_s that affect the ductility and need to be reflected in general formulae to describe minimum reinforcement ratios.

6 Acknowledgment

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