

THE LINEAR INSTABILITY OF DILUTE ULTRARELATIVISTIC e^\pm PAIR BEAMS

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Received 2016 August 17; revised 2016 September 27; accepted 2016 September 30; published 2016 December 12

ABSTRACT

The annihilation of TeV photons from extragalactic TeV sources and the extragalactic background light produces ultrarelativistic e^\pm beams, which are subject to powerful plasma instabilities that sap their kinetic energy. Here we study the linear phase of the plasma instabilities that these pair beams drive. To this end, we calculate the linear growth rate of the beam-plasma and oblique instability in the electrostatic approximation in both the reactive and kinetic regimes, assuming a Maxwell–Jüttner distribution for the pair beam. We reproduce the well-known reactive and kinetic growth rates for both the beam-plasma and oblique mode. We demonstrate for the oblique instability that there is a broad spectrum of unstable modes that grow at the maximum rate for a wide range of beam temperatures and wave-vector orientations relative to the beam. We also delineate the conditions for applicability for the reactive and kinetic regimes and find that the beam-plasma mode transitions to the reactive regime at a lower Lorentz factor than the oblique mode due to a combination of their different scalings and the anisotropy of the velocity dispersions. Applying these results to the ultrarelativistic e^\pm beams from TeV blazars, we confirm that these beams are unstable to both the kinetic oblique mode and the reactive beam-plasma mode. These results are important in understanding how powerful plasma instabilities may sap the energy of the ultrarelativistic e^\pm beams as they propagate through intergalactic space.

Key words: BL Lacertae objects: general – gamma rays: general – instabilities – magnetic fields – plasmas

1. INTRODUCTION

The *Fermi* satellite and ground-based imaging atmospheric Cherenkov telescopes such as H.E.S.S., MAGIC, and VERITAS⁹ have demonstrated that the high-energy universe is teeming with energetic very high-energy gamma-ray (VHEGR, $E > 100$ GeV) sources, the extragalactic component of which mainly consists of TeV blazars with a minority population of other sources such as radio and starburst galaxies. These extragalactic VHEGR emitters produce TeV photons that are greatly attenuated via annihilation upon soft photons in the extragalactic background light (EBL) and produce pairs (see, e.g., Gould & Schröder 1967a; Salamon & Stecker 1998; Neronov & Semikoz 2009).

It has been assumed that these ultrarelativistic pairs produced by VHEGR annihilation lose energy exclusively through inverse-Compton (IC) scattering off of the cosmic microwave background, transferring the energy of the original VHEGR to gamma-rays with energies $\lesssim 100$ GeV. The absence of observed secondary IC emission leads a number of authors to argue that this *lack* of emission places lower bounds upon the intergalactic magnetic field (IGMF; see, e.g., Neronov & Vovk 2010; Tavecchio et al. 2010, 2011; Dermer et al. 2011; Dolag et al. 2011; Taylor et al. 2011; Takahashi et al. 2012) with typical numbers ranging from 10^{-19} to 10^{-15} G.

In addition, *Fermi* has also provided the most precise estimate of the unresolved extragalactic gamma-ray background (EGRB) for energies between 200 MeV and 100 GeV. Since inverse-Compton cascades (ICCs) reprocess the VHEGR emission of distant sources into this band, this has been used to constrain the evolution of the luminosity density of VHEGR sources (see, e.g., Narumoto & Totani 2006; Kneiske & Mannheim 2008; Inoue & Totani 2009; Venter 2010). These constraints preclude any dramatic rise in the number of sources by $z \approx 1$ –2 that is seen in the quasar distribution. That is, the comoving number of blazars must have remained essentially fixed, at odds with both the physical picture underlying these systems and with the observed evolution of similarly accreting systems, i.e., quasars and radio galaxies.

These two important conclusions depend on IC cooling dominating the evolution of the ultrarelativistic pairs. However, it was recently found that plasma instabilities driven by ultrarelativistic pair beams likely are the dominant cooling mechanisms (Broderick et al. 2012, hereafter BCP12; Schlickeiser et al. 2012b, 2013), depositing this energy as heat in the intergalactic medium (Chang et al. 2012; Pfrommer et al. 2012). Therefore, the lack of an observed IC halo emission from TeV blazars does not imply the existence of the IGMF as previous groups have argued (BCP12; Schlickeiser et al. 2012b, 2013). We note that the effectiveness of these plasma instabilities is complicated by nonlinear effects, which we briefly discuss below.

The deposition of kinetic energy into the IGM via plasma instabilities produces excess heating, which, over cosmological

⁹ High Energy Stereoscopic System, Major Atmospheric Gamma Imaging Cherenkov Telescope, Very Energetic Radiation Imaging Telescope Array System.

time, may resolve a variety of puzzles, including explaining anomalies in the statistics of the high-redshift Ly α forest (Puchwein et al. 2012; Lamberts et al. 2015) and potentially explaining a number of the X-ray properties of groups and clusters and anomalies in galaxy formation on the scale of dwarfs (Pfrommer et al. 2012; Lu et al. 2013). We have recently shown that *if* the IC halos are ignored, it is possible to quantitatively reproduce the redshift and flux distributions of nearby hard gamma-ray blazars and the extragalactic gamma-ray background spectrum above 3 GeV simultaneously with a unified model of AGN evolution (Broderick et al. 2014a, 2014b). All of these empirical successes provide circumstantial evidence for the presence of virulent plasma beam instabilities.

These potential implications of blazar heating rely on an understanding of the linear and nonlinear physics of these plasma instabilities. Recent work in this area has been inconclusive. For instance, Miniati & Elyiv (2013) argued that these instabilities are physically irrelevant for the cooling of these pair beams because they would saturate at a very low level due to nonlinear Landau damping (NLD). However, Chang et al. (2014) performed a detailed calculation of NLD to show that these plasma processes remain dominant. In addition, Sironi & Giannios (2014) performed particle-in-cell simulations of these plasma processes and argued that these processes saturate at a very low level. It is unclear, however, if the conclusions of their work is applicable to the parameter regime of blazar heating.

Additional nonlinear effects may also be important. For instance, for sufficiently powerful blazars, the modulation instability may operate (Schlickeiser et al. 2012b; Chang et al. 2014; Menzler & Schlickeiser 2015), allowing for a rapid transfer of electrostatic wave energy into thermal energy. For less powerful blazars, the combination of NLD and quasilinear damping, i.e., beam plateauing, will also reduce the rate of damping compared to the linear rate, and alters the resulting IC spectra (Menzler & Schlickeiser 2015). Further study of these effects will help clarify these points.

While a full nonlinear study is required, we focus on the nature of the linear instability in this paper, clarifying its robustness and regimes of applicability. We begin by studying the distribution function of the e^\pm pairs that are produced from VHEGR–EBL photon annihilation. We study the evolution of a distribution function under Lorentz transformations to develop an analytic understanding of how the perpendicular and parallel velocity dispersions transform under boosts. Using this understanding, we then develop a simple description of the distribution function of the beam, which we then use to calculate the unstable modes analytically in both the reactive (hydrodynamic) and kinetic regimes.

Here the reactive instability refers to the instability where the entire beam participates in the instability. In particular, all of the beam particles are resonant with the unstable wave on a timescale longer than the growth time of the instability. The reactive instability is also referred to as the hydrodynamic instability since the instability can be derived from the fluid equations instead of kinetic theory. On the other hand, in the kinetic regime, only a fraction of the beam particles are resonant with the beam over the growth time of the instability, which reduces the growth rate compared to the reactive instability for the same beam density and beam Lorentz factor. We recover the well-known results for the reactive regime for both the beam-plasma and oblique modes. We also derive the

growth rate for these two instabilities in the kinetic regime and delineate the range of applicability for both the reactive and kinetic cases and apply these results to ultrarelativistic e^\pm pair beams.

This paper is organized as follows. In Section 2, we describe the transformation properties of an ultrarelativistic e^\pm beam in terms of its distribution function. We then calculate the various linear instabilities that this beam is subject to in Section 3. In particular, we pay careful attention to both the reactive (or hydrodynamic) and kinetic regimes of the beam-plasma and oblique instabilities and the transition between the two. Applying these results to TeV e^\pm pair beams that arise from TeV photon pair production in Section 4, we demonstrate that despite the extraordinary coldness of the beam we are always in the kinetic regime for the oblique mode, but may be in the reactive regime for the beam-plasma mode. However, for the relevant parameters, the growth rates calculated in either regime are similar. We close with a discussion of the implications of this work and application of these results for nonlinear theory in Section 5.

2. ULTRARELATIVISTIC PAIR BEAMS FROM VHEGRs

As stated in the Introduction, VHEGR photons pair produce on encountering EBL photons as they propagate throughout the universe (Gould & Schröder 1966), and this attenuation of VHEGR flux has been used as a probe of the EBL (Stecker et al. 1992; de Jager et al. 1994; Aharonian et al. 2006). The basic requirement of this process is that the energies of the VHEGR (E_{ph}) and the EBL photon (E_{ebl}) exceed the rest mass energy of the e^\pm pair in the center of momentum (COM) frame, i.e., $2EE_{\text{ebl}}(1 - \cos \theta) \geq 4m_e^2 c^4$, where θ is the relative angle of propagation in the lab frame. As a result, an e^\pm pair can be produced with Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2} \approx E/2m_e c^2$, where v is the velocity of the pairs (Gould & Schröder 1967a). Here, we discuss the distribution function of the pair beam that emerges from this process.

2.1. Distribution Function of the Pair Beam

In the COM frame of the beam, we assume that the distribution function is isotropic, such that $f = f(E)$ is just a function of energy. This equilibrium energy distribution of a relativistic thermal plasma gas is

$$f \propto \exp\left(-\frac{E}{k_B T}\right), \quad (1)$$

where E and T are the dimensionless energy and temperature in terms of a particle's rest mass. In the nonrelativistic case, this reduces to the Maxwell–Boltzmann distribution, while the relativistic version is known as the Maxwell–Jüttner distribution (Jüttner 1911).

The relativistic Maxwellian distribution can be extended to a drifting (or boosted) distribution via an appropriate Lorentz transformation. The relationship between the energies of the lab (boosted) frame and the COM frame is

$$E_{\text{COM}} = \gamma_b (E_L - \beta_b p_{L,\parallel}), \quad (2)$$

where $\gamma_b = \gamma(v_b) = (1 - v_b^2/c^2)^{-1/2}$ is the Lorentz factor of the beam and v_b is the bulk velocity of the pair beam. Inserting this into Equation (1), we find the Maxwell–Jüttner distribution

(Jüttner 1911; Wright & Hadley 1975)

$$f = \frac{n_b m_e c^2}{4\pi\gamma_b k_B T_b K_2(m_e c^2/k_B T_b) m_e^3 c^3} \exp\left(-\frac{\gamma_b(E - \mathbf{v}_b \cdot \mathbf{p})}{k_B T_b}\right), \quad (3)$$

where K_2 is the second-order modified Bessel function and T_b is the comoving temperature of the beam.

The Maxwell–Jüttner distribution leads to an anisotropic velocity spread parallel and perpendicular to the beam’s direction. In Appendix A, we estimate how the parallel and perpendicular velocity spreads scale. The relevant results are

$$\frac{\Delta v_\perp^2}{c^2} \approx \frac{2k_B T_b}{\gamma_b^2 m_e c^2} \quad \text{and} \quad \frac{\Delta v_\parallel^2}{c^2} \approx \frac{k_B T_b}{\gamma_b^4 m_e c^2}, \quad (4)$$

where T is measured in the COM frame of the beam. These simple scalings of the perpendicular velocity dispersion and parallel velocity dispersions can be understood as a result of time dilation between two frames that are boosted relative to each other, giving one factor of γ^{-1} . The coordinates perpendicular to the boost axis remain invariant while the axis along the boost suffers from length contraction and gives an extra scaling of γ^{-1} for the parallel case. In any case, an ultrarelativistic beam has a small velocity spread in both the parallel and perpendicular directions by factors of γ^{-2} and γ^{-1} , respectively. These velocity dispersions will be important in delineating the regime of instability in Section 3.3.

While we have modeled the pair distribution function as a Maxwell–Jüttner distribution, the physical distribution function that is produced by VHEGR photon annihilation is somewhat more complicated (see for instance Schlickeiser et al. 2012a). In particular, the parallel and perpendicular momentum spread will be influenced by the distribution of VHEGR photons and their respective mean free paths. However, a Maxwell–Jüttner distribution is still useful. First, it is also sufficiently simple to allow us to calculate the kinetic instability exactly in the electrostatic approximation. Second, its instability growth rates has been calculated without approximation previously by Bret et al. (2010), allowing a point of comparison for our calculation using the electrostatic approximation (as mentioned below). Third, it possess a continuous (small) distribution of parallel and perpendicular momenta that allow us to elucidate the physics. Finally, the analytic methodology used to calculate the Maxwell–Jüttner distribution may be useful for the full calculation using the physical distribution function.

3. LINEAR THEORY

The Vlasov equation for each species is

$$\frac{\partial f_s}{\partial t} + \mathbf{v}_s \cdot \nabla f_s + q_s \left(\mathbf{E} + \frac{\mathbf{v}_s}{c} \times \mathbf{B} \right) \cdot \nabla_p f_s = 0, \quad (5)$$

where $\mathbf{v}_s = \mathbf{p}_s/\gamma_s m_e$ and $\gamma_s = 1/\sqrt{1 - v_s^2/c^2}$. Here, s is the species label, $+$ for positrons and $-$ for electrons, with $q_\pm = \pm e$. Upon linearizing this in small perturbations about a background distribution, i.e., setting $f_s \rightarrow f_{0s} + \delta f_s$, $\mathbf{B} \rightarrow \delta \mathbf{B}$, and $\mathbf{E} \rightarrow \delta \mathbf{E}$, we obtain

$$\frac{\partial \delta f_s}{\partial t} + \mathbf{v}_s \cdot \nabla \delta f_s + q_s \left(\delta \mathbf{E} + \frac{\mathbf{v}_s}{c} \times \delta \mathbf{B} \right) \cdot \nabla_p f_{0s} = 0. \quad (6)$$

The plasma couples to the field through the Maxwell equations

$$\nabla \times \delta \mathbf{E} = -\frac{1}{c} \frac{\partial \delta \mathbf{B}}{\partial t}, \quad (7)$$

$$\nabla \times \delta \mathbf{B} = \frac{4\pi}{c} \delta \mathbf{j} + \frac{1}{c} \frac{\partial \delta \mathbf{E}}{\partial t}, \quad (8)$$

where $\delta \mathbf{j} = \sum_s q_s \int \mathbf{v} \delta f_s d^3 p$ is the linear current density perturbation.

Here it is useful to work within the electrostatic approximation ($\mathbf{k} \times \delta \mathbf{E} = 0$), where we only need to include Coulomb’s law for the electric field rather than the full Maxwell equations:

$$i\mathbf{k} \cdot \delta \mathbf{E} = 4\pi \delta \rho, \quad (9)$$

where $\delta \rho = \sum_s q_s \int \delta f_s d^3 p$ is the perturbed charge density. By adopting the electrostatic approximation, we have explicitly ignored electromagnetic modes. This would preclude, for example, the Weibel instability. In addition, the electromagnetic terms would introduce corrections to the physics that are not necessarily small in the limit of relativistic particles, i.e., $v/c \rightarrow 1$. However, we make this approximation for two reasons. First, a complete calculation of the unstable modes has already been carried out by Bret et al. (2010), who showed that the oblique mode is mainly electrostatic (modulo the Weibel instability). Hence an electrostatic approximation to the full dispersion relation should recover the essential physics. Second, the electrostatic approximation is much simpler than a full calculation and allows us to analytically calculate the unstable growth rates while permitting a clear exposition of the relevant physics.

We now adopt perturbations of the form $\delta \propto \exp(i\mathbf{k} \cdot \mathbf{r} - i\omega t)$ and without loss of generality assume that $\mathbf{k} = (k_x, 0, k_z)$, where k_z is along the beam direction. Linearizing the Vlasov–Maxwell equations then leads to the dispersion relation:

$$\epsilon = 1 + \sum_s \frac{m_e \omega_{p,s}^2}{k^2} \int \frac{\mathbf{k} \cdot \nabla_p F_s}{\omega - \mathbf{k} \cdot \mathbf{v}} d^3 p = 0, \quad (10)$$

where ϵ is the simplified dielectric function, and for each species $\omega_{p,s}^2 \equiv 4\pi e^2 n_s/m_e$ is the plasma frequency, $n_s \equiv \int f_{0s} d^3 p$ is the number density, and $F_s \equiv f_{0s}/n_s$ is the normalized background distribution function. Upon integrating by parts, Equation (10) becomes

$$\begin{aligned} \epsilon &= 1 - \sum_s \frac{m_e \omega_{p,s}^2}{k^2} \int F_s \mathbf{k} \cdot \nabla_p \frac{1}{\omega - \mathbf{k} \cdot \mathbf{v}} d^3 p \\ &= 1 - \sum_s \frac{\omega_{p,s}^2}{k^2 c^2} \int F_s \frac{k^2 c^2 - (\mathbf{k} \cdot \mathbf{v})^2}{\gamma(\omega - \mathbf{k} \cdot \mathbf{v})^2} d^3 p = 0. \end{aligned} \quad (11)$$

There are two distinct, often qualitatively different, regimes in which we may consider the implications of this dispersion relation. The first is the cold-plasma limit or the hydrodynamic or reactive limit. The hydrodynamic limit is aptly named because the resulting dispersion relation that is found could have also been calculated directly from the continuity equation and the momentum equation. In this limit, the internal distribution of the particles of the background or beam are irrelevant to the physics of the instability and it is only the bulk response that is important. In particular, this means that the

beam particles are resonant with the unstable wave over a timescale much longer than the growth time, i.e., the beam particles do not drift a distance larger than the wavelength of the unstable mode over the growth time of the instability. The second is the kinetic regime, where the internal distribution of beam particles is important to the physics of the instability. Here, only a fraction of beam particles stay within one wavelength of the unstable mode over the growth time of the instability. Moreover, the bulk of the plasma (background or beam) does not respond to the disturbance; instead, only a fraction of particles is relevant for driving (instability) or damping (Landau damping). We discuss below the evaluation of the dispersion relation in these two regimes, which gives two regimes of instability, and the delineation between them.

3.1. Hydrodynamic (Reactive) Instability

Starting with the dispersion relation (11), we first consider the instability of a cold-plasma beam. Taking the limit of Equation (3) as $k_B T_t \rightarrow 0$, for a target plasma $v_0 = 0$ and a beam plasma $v_0 = v_b$ ¹⁰, we find

$$1 - \frac{\omega_{p,t}^2}{\omega^2} - \frac{\omega_{p,b}^2}{\gamma^3(\omega - k_z v_b)^2} \frac{\gamma^2 k_x^2 + k_z^2}{k_x^2 + k_z^2} = 0. \quad (12)$$

For $k_x = 0$, we recover the same beam-plasma instability, which was described in the Appendix of BCP12.

The solution to Equation (12) is given in Appendix B where we show that the associated growth rate (Equation (57)) is

$$\Gamma = \frac{\sqrt{3}}{2^{4/3}} \left(\frac{n_b}{n_t} \right)^{1/3} \left(\frac{\gamma^2 Z_x^2 + 1}{Z_x^2 + 1} \right)^{1/3} \frac{\omega_{p,t}}{\gamma}, \quad (13)$$

where $Z_x = k_x v_b / \omega_{p,t}$ is the dimensionless wave vector perpendicular to the beam direction.

For $k_x = 0 \rightarrow Z_x = 0$, this reduces to the beam-plasma growth rate, which is

$$\Gamma = \Gamma_{TS} \equiv \frac{\sqrt{3}}{2^{4/3}} \left(\frac{n_b}{n_t} \right)^{1/3} \frac{\omega_{p,t}}{\gamma}, \quad (14)$$

which we denote the beam-plasma or “two-stream” growth rate. For the more general case where $Z_x \neq 0$, this becomes the oblique instability studied by Bret et al. (2010). Indeed for $\gamma \gg 1$ and $Z_x \gg 1$, the growth rate approaches the oblique growth rate,

$$\Gamma = \Gamma_{ob} \equiv \frac{\sqrt{3}}{2^{4/3}} \left(\frac{n_b}{n_t} \right)^{1/3} \frac{\omega_{p,t}}{\gamma^{1/3}}, \quad (15)$$

which is much faster than the beam-plasma growth rate, Γ_{TS} .

We should caution in the derivation above that the resonance condition, which is $\omega_{p,t} - k_z v_b$, implies that $k_z \neq 0$. For the case where $k_z \rightarrow 0$, the electrostatic approximation no longer holds and the full dispersion relation must be solved.¹¹ A solution to the full dispersion relation reveals additional modes, including the zero frequency ($k_z = 0$) filamentation or Weibel mode.

Equation (12) can also be solved numerically in terms of k_x and k_z . Here let us specialize to the case of $k_x = 0$, i.e., the

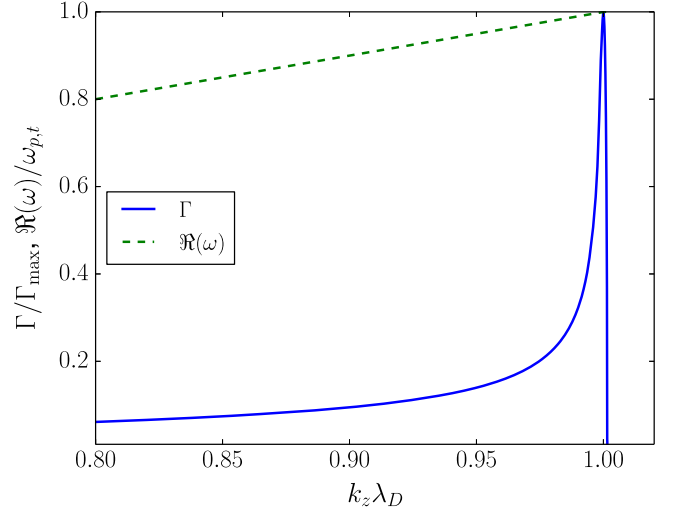


Figure 1. Beam-plasma growth rate (solid line) and unstable wave frequency (dashed line) as a function of k_z for $\gamma_b = 100$ and $n_b/n_t = 10^{-3}$.

beam-plasma case. In this case, we have

$$1 - \frac{\omega_{p,t}^2}{\omega^2} - \frac{\omega_{p,b}^2}{\gamma^3(\omega - k_z v_b)^2} = 0, \quad (16)$$

which we can numerically solve in terms of $\omega/\omega_{p,t}$, $k_z \lambda_D$, v_b/c , and n_b/n_t , where $\lambda_D = c/\omega_{p,t}$ is the skin depth. In Figure 1 we show the real and imaginary parts for $\omega/\omega_{p,t}$ as a function of $k_z \lambda_D$ for the representative case of $v_b/c \approx 1$ and $n_b/n_t = 10^{-3}$ and $\gamma = 100$. For $k_z \lambda_D = 1$, the growth rate reaches its maximum Γ_{\max} and the real part of the frequency is $\Re(\omega) = \omega_{p,t}$, which is the plasma oscillation frequency. This wave would exist in the absence of a tenuous beam. However, as we move away from this frequency toward lower k_z , we still find substantial growth, with $\Gamma \approx 0.1\Gamma_{\max}$ as $k_z \approx 0.9\lambda_D^{-1}$. Interestingly, the real part of the unstable wave has a phase velocity $v_{ph} = \Re(\omega)/k = c$, which is still in resonance with the beam.

In a continuous system, these waves do not matter in comparison to the unstable mode at $k\lambda_D = 1$. However, for discrete numerical systems, which do not sufficiently resolve the most unstable modes, these sub-maximal modes drive the growth of the instability of numerically calculated beam-plasma systems, which may lead to an incorrect nonlinear state in comparison to the physical system.

3.2. Kinetic Instability

The growth rate expressed in Equation (13) is in the reactive (or hydrodynamic) regime as the dispersion relation (Equation (12)) could have been derived from the fluid equations. Here all the particles participate in the instability. However, kinetic theory marks another regime of the instability, where only a fraction of the particle participate in the instability, i.e., the kinetic regime. We now derive the growth rate of the instability in the kinetic regime.

We begin first with the distribution function for the target plasma:

$$F_t = \left(\frac{1}{2\pi m_e k_B T_t} \right)^{3/2} \exp \left(-\frac{p^2}{2m_e k_B T_t} \right), \quad (17)$$

¹⁰ That is, we set $F_s(\mathbf{p}) = \delta^3(\mathbf{p} - \mathbf{p}_{0s})$, where $\mathbf{p}_{0s} \equiv \gamma_0 m_e v_0 \hat{\mathbf{z}}$ is the momentum associated with v_0 .

¹¹ We thank Antoine Bret for helping to clarify this point.

where the target plasma is assumed to be nonrelativistic, $\mathbf{p} = m_e \mathbf{v}$ is the nonrelativistic momentum, and T_t is the temperature of the target background plasma. For the beam plasma, we again adopt the Maxwell–Jüttner distribution (Equation (3)). Inserting these into the dispersion relation (Equation (10)), we find

$$1 - \frac{\omega_{p,t}^2}{k^2 c^2} \int F_t \frac{k^2 c^2 - (\mathbf{k} \cdot \mathbf{v})^2}{\gamma (\omega - \mathbf{k} \cdot \mathbf{v})^2} d^3 p + \frac{m_e \omega_{p,b}^2}{k^2} \int \frac{\mathbf{k} \cdot \nabla_p F_b}{\omega - \mathbf{k} \cdot \mathbf{v}} d^3 p = 0, \quad (18)$$

where we have integrated by parts only the second term, associated with the target plasma.

We discuss the solution to Equation (18) in Appendix C. The associate growth rate for the kinetic oblique instability is

$$\Gamma \approx -\Gamma_0 \frac{\pi \gamma_w^2 \gamma_{ph}^3 (v_{ph} - v_{b,z'})}{4 \gamma_b \mu^2 K_2(\mu) \mathcal{G}^3 c} \times \left[(\mathcal{G}'^2 \mu^2 + 2 \mathcal{G}' \mu + 2) + \frac{\gamma_b^2 v_{b,x'}^2}{2 \mathcal{G}'^2 c^2} (2 \mathcal{G}' \mu + 2) \right] \times \exp(-\mathcal{G}' \mu), \quad (19)$$

where $\mu = m_e c^2 / k_B T_b$, $\gamma_{ph} = (1 - v_{ph}^2 / c^2)^{-1/2}$, $v_{ph} = \omega / k$ is the phase velocity of the wave, $v_{b,z'}$ is the velocity of the beam oriented along the wave vector, $v_{b,x'}$ is the velocity of the beam perpendicular to the wave vector, $\gamma_w = (1 - w^2 / c^2)^{-1/2}$, $w = \gamma_{ph}^{-1} v_{b,x'} / (1 - v_{b,z'} v_{ph} / c^2)$ is the beam velocity transverse to the wave vector in a frame that is comoving with the wave at its phase velocity, and $\mathcal{G}' \equiv \gamma_b \gamma_{ph} (1 - v_{b,z'} v_{ph} / c^2) / \gamma_w$ is the Lorentz factor of a beam particle in a frame that is comoving with the wave at the phase velocity and the transverse (to the wave vector) beam bulk velocity. Finally, we define the typical maximum growth rate, Γ_0 , as

$$\Gamma_0 \equiv \omega_p \gamma_b \frac{n_b}{n_t} \frac{m_e v_b^2}{k_B T_b}. \quad (20)$$

Equation (19) specializes to the beam-plasma growth rate if we take $v_{b,x'} = 0$, which gives

$$\Gamma_{bp} \approx -\Gamma_0 \frac{\pi \gamma_{ph}^3 (v_{ph} - v_{b,z'})}{4 \gamma_b \mu^2 K_2(\mu) \mathcal{G}^3 c} (\mathcal{G}'^2 \mu^2 + 2 \mathcal{G}' \mu + 2) \exp(-\mathcal{G}' \mu), \quad (21)$$

where we have used the fact that $\gamma_w = 1$ for $v_{b,x'} = 0$.

In Figure 2, we plot the growth rate for the oblique instability (Equation (19)) as a function of $\sin \theta$, where $\cos \theta = \hat{\mathbf{k}} \cdot \hat{\mathbf{v}}$, i.e., the angle between the beam and the wave vector, and $k_B T_b / m_e c^2$. Here, it is clear that the growth rate reaches its maximum value at $\sin \theta \gtrsim 1/\gamma_b$, i.e., at an oblique angle. Note that as $\sin \theta \rightarrow 0$, we recover the beam-plasma instability. Moreover, the maximal growth rates, normalized to Γ_0 , vary little and are robust for a broad range of angles between the wave vector and the beam direction. It is clear from this plot that for nearly any combination of wave-vector orientation and beam temperature there exists a broad spectrum of modes that are unstable and grow at nearly the maximum growth rate, Γ_0 , for the parameters of the system, T_b , γ_b , and n_b/n_t , modulo a factor of order unity.

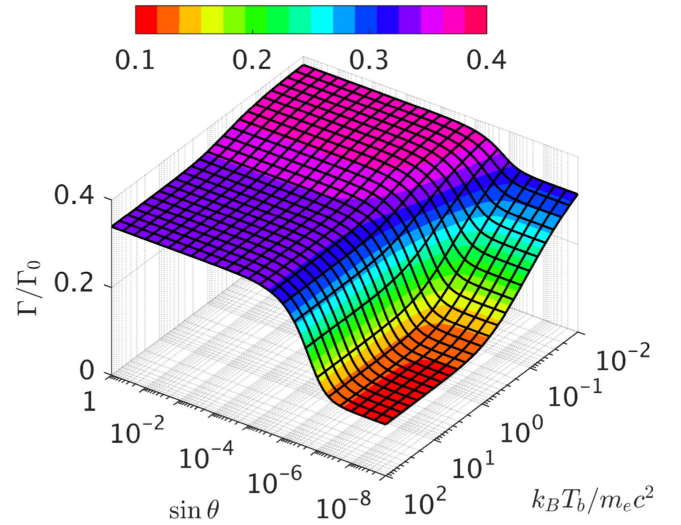


Figure 2. Oblique growth rate maximized over v_{ph} as a function of $\sin \theta$ and $k_B T_b / m_e c^2$, where θ is the angle between the beam direction and wave vector for $\gamma_b = 10^6$. The maximum growth rate occurs when $\sin \theta \gg 1/\gamma_b$. Note that as $\sin \theta \rightarrow 0$, we asymptote to the beam-plasma growth rate.

This does not imply that any individual mode, i.e., a mode with a fixed wave vector, will grow robustly. Any individual mode only grows when the phase velocity of the mode in the direction of the beam are in resonance and this resonant width is narrow. However, the growth is robust since for any combination of wave-vector orientation and beam temperature, there exists some mode that will grow at the maximum rate.

Because there is little variation in the maximal oblique growth rate as a wave-vector orientation, we plot the maximum growth rate as a function of T_b and $\gamma_b - 1$ in Figure 3. Here, for relativistic beams, the maximum growth rate varies little with T_b , varying by less than 10% between hot and cold beams and we find

$$\Gamma_M \approx \begin{cases} 0.38 \Gamma_0 & k_B T_b / m_e c^2 \ll 1 \\ 0.34 \Gamma_0 & k_B T_b / m_e c^2 \gg 1 \end{cases} \quad (22)$$

for $\gamma_b \gtrsim 10$, as seen in Figure 3. Hence, unstable modes exist and robustly grow at roughly $\Gamma \approx 0.4 \Gamma_0$ for nearly any value of $k_B T_b / m_e c^2$, wave-vector orientation, and $\gamma_b \gg 1$.

This can be contrasted with the right panel of Figure 3 where we plot the beam-plasma growth rate (21) as a function of $\gamma_b - 1$ and $k_B T_b / m_e c^2$. Here, we see that the maximum growth rate is somewhat more sensitive to temperature, varying from $\Gamma/\Gamma_0 \approx 0.4$ for $k_B T_b / m_e c^2 \ll 1$ to $\Gamma/\Gamma_0 \approx 0.1$ for $k_B T_b / m_e c^2 \gtrsim 1$. Note, however, that that beam-plasma growth rate remains competitive with the oblique growth rate, i.e., it is not orders of magnitude lower.

Finally, the maximum growth rate that we derived here (Equation (22)) and that of BCP12 (their Equation (16)), which is originally derived from the numerical fit of Bret et al. (2010) are exactly the same. We must note, however, that the our definition of T_b is in the COM frame whereas BCP12 defines T_b in the “lab” frame. As a result, there is a factor of γ_b that is explicit in Equation (22) and in Equation (16) of BCP12.

3.3. The Transition between the Kinetic and Hydrodynamic Instability

The oblique instability exists in two different regimes, raising the important question: how are the two regimes related

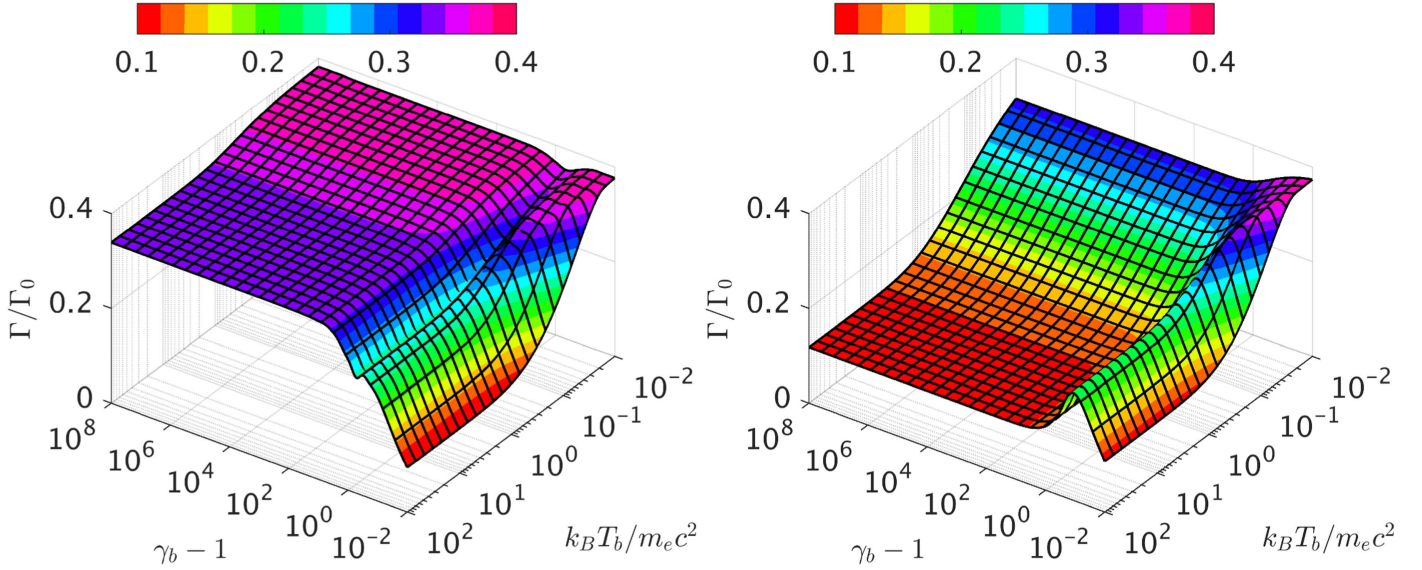


Figure 3. Oblique kinetic growth rate (left) and beam-plasma growth rate (right), and normalized by $\Gamma_0 \equiv \omega_{pt}(n_b/n_t)\gamma_b m v_b^2 / k_B T_b$ as a function of $k_B T_b / m_e c^2$ and $\gamma_b - 1$. The oblique kinetic growth rate has been maximized over θ . Unlike the beam-plasma case on the right, in the case of the oblique kinetic growth rate at high γ_b , the transition between high and low temperature is only marginal, constituting a roughly 10% reduction.

to each other. While this question has been studied by many authors in the context of the beam-plasma or two-stream instability (see for instance Melrose 1986; Boyd & Sanderson 2003), a clear exposition of how these two regimes are related to each other for the oblique instability is lacking.

To begin, let us return to the reactive instability. For the growth rate of the reactive instability in Equation (13) to be valid, the velocity dispersion must be vanishingly small. In particular, over the growth time of the unstable wave, the beam particles may not spread significantly, i.e., their spread is much smaller than one wavelength. Quantitatively, this demands

$$\left| \frac{\mathbf{k} \cdot \Delta \mathbf{v}}{\Gamma} \right| \ll 1. \quad (23)$$

For $k_{\perp} \approx \omega_p / v_b$, this gives

$$\frac{\Delta v_{\perp}}{v_b} \ll \left(\frac{n_b}{\gamma_b n_t} \right)^{1/3} \quad (24)$$

where we have dropped constant factors of order unity and assumed that $Z_x \propto O(1)$ and that the velocity dispersion is dominated by the perpendicular (to the beam) component. Hence, this defines the upper limit on the velocity dispersion of the plasma for the cold-plasma approximation to hold and, hence, the range of validity for the reactive oblique growth rate (Equation (13)). For $Z_x \ll \gamma^{-2}$, we recover the condition for the relativistic, reactive beam-plasma instability:

$$\frac{\Delta v_{\parallel}}{v_b} \ll \gamma_b^{-1} \left(\frac{n_b}{n_t} \right)^{1/3}. \quad (25)$$

Applying the scaling of the perpendicular and parallel velocity dispersions (Equations (4)) to these results and assuming $v_b \approx c$, we find

$$\frac{\Delta v_{\perp}}{v_b} \approx \gamma_b^{-1} \sqrt{\frac{k_B T_b}{m_e c^2}} \quad \text{and} \quad \frac{\Delta v_{\parallel}}{v_b} \approx \gamma_b^{-2} \sqrt{\frac{k_B T_b}{m_e c^2}}. \quad (26)$$

Hence, the conditions for the reactive regime for the oblique mode (Equation (24)) and beam-plasma mode (Equation (25)) can be reduced to

$$1 \ll \begin{cases} (k_B T_b / m_e c^2)^{-1/2} \gamma_b^{2/3} (n_b / n_t)^{1/3} & \text{oblique} \\ (k_B T_b / m_e c^2)^{-1/2} \gamma_b (n_b / n_t)^{1/3} & \text{beam plasma} \end{cases}. \quad (27)$$

We now proceed to study the range of validity for the kinetic growth rate for the beam-plasma mode (Equation (21)) and oblique mode (Equation (19)). Following the argument of Boyd & Sanderson (2003), the growth occurs over a range where the distribution function is positive or $v_b - \Delta v < \omega/k < v_b$. Hence the bandwidth over which the distribution powers grows is $\Delta \omega \approx k \Delta v$. For the kinetic growth rate to be valid, the bandwidth, $\Delta \omega$, must be large compared to the growth rate; otherwise, the entire beam contributes to the growth and, hence, the reactive regime applies. For the beam-plasma case, the growth rate is roughly

$$\Gamma \approx \frac{n_b}{\gamma^3 n_t} \left(\frac{c}{\Delta v_{\parallel}} \right)^2. \quad (28)$$

The bandwidth, $\Delta \omega$, is then greater than the growth rate if

$$\frac{\Delta v_{\parallel}}{v_b} \gtrsim \gamma^{-1} \left(\frac{n_b}{n_t} \right)^{1/3}, \quad (29)$$

which connects with the condition on the reactive beam-plasma instability from Equation (25). Similarly for the oblique mode, the bandwidth, $\Delta \omega$, is then greater than the growth rate if

$$\frac{\Delta v_{\perp}}{v_b} \gtrsim \left(\frac{n_b}{\gamma n_t} \right)^{1/3}, \quad (30)$$

which can similarly be compared to the condition on the reactive oblique mode from Equation (24).

Combining these two kinetic condition and our result again from Section 2, we find

$$1 \geq \begin{cases} (k_B T_b / m_e c^2)^{-1/2} \gamma_b^{2/3} (n_b / n_t)^{1/3} & \text{oblique} \\ (k_B T_b / m_e c^2)^{-1/2} \gamma_b (n_b / n_t)^{1/3} & \text{beam plasma} \end{cases}, \quad (31)$$

which in combination with Equation (27) denotes the transition between the reactive and kinetic regimes.

4. APPLICATION TO ULTRARELATIVISTIC e^\pm BEAMS

As discussed in the Introduction, the annihilation of VHEGRs and EBL photons produce ultrarelativistic e^\pm beams that are unstable to the beam-plasma and oblique modes discussed above. To apply the above results to the ultrarelativistic e^\pm beams, we now calculate their initial conditions.

4.1. Average COM Energy of the e^\pm Beam

To find the effective velocity dispersion of the ultrarelativistic e^\pm beam, we must first estimate the average COM energy of the beam. To do so, we consider the process of photon-photon annihilation. For a mono-energetic population of VHEGR photons with energy E_{ph} , the angle-averaged production rate of e^\pm on EBL photons is

$$\begin{aligned} \Gamma_\pm(E_{\text{ph}}) &= \frac{1}{4\pi} \int \sigma c n_{\text{EBL}} d\Omega \\ &= \frac{1}{2} \int \sigma(E_{\text{ph}}, E_{\text{EBL}}, \theta) c \frac{dn_{\text{EBL}}}{dE_{\text{EBL}}} dE_{\text{EBL}} d\cos\theta, \end{aligned} \quad (32)$$

where Γ_\pm is the rate of pair production, σ is the pair-production cross section, n_{EBL} is the number density of EBL photons, E_{EBL} is the energy of the EBL photons, and θ is the angle between the momentum of the VHEGR photon and the EBL photon. There are two important components to this calculation—the cross section, σ , and the spectrum of the EBL.

For σ , we use the results from Nikishov (1962) and Gould & Schröder (1967b), who considered a high-energy photon with energy E_{ph} moving along the x -axis annihilating on an EBL photon with energy E_{EBL} moving at an angle θ with respect to the x -axis. The total cross section for this process is (Nikishov 1962; Gould & Schröder 1967b)

$$\begin{aligned} \sigma &= \frac{1}{2} \pi r_e^2 \left(1 - \frac{v_e^2}{c^2} \right) \left[\left(3 - (v_e/c)^4 \right) \ln \frac{1 + v_e/c}{1 - v_e/c} \right. \\ &\quad \left. - 2 \frac{v_e}{c} \left(2 - \frac{v_e^2}{c^2} \right) \right], \end{aligned} \quad (33)$$

where $r_e = e^2 / m_e c^2$ is the classical electron radius and v_e is the electron velocity in the COM frame of the generated pair.¹² To find v_e , we use the energy of the electron in the COM frame, $E_{e,\text{COM}}$, which is

$$E_{e,\text{COM}} = \frac{m_e c^2}{\sqrt{1 - v_e^2/c^2}} = \sqrt{\frac{1}{2} E_{\text{EBL}} E_{\text{ph}} (1 - \cos\theta)}. \quad (34)$$

Pair production occurs when $E_{e,\text{COM}} / m_e c^2 \geq 1$.

The second ingredient is the spectrum of the EBL, which is not well constrained. Here we use the constraints from Aharonian et al.

¹² In this section, the COM frame and subscript “COM” refer to the center of momentum frame of the pair that is produced by a single $\gamma - \gamma$ annihilation.

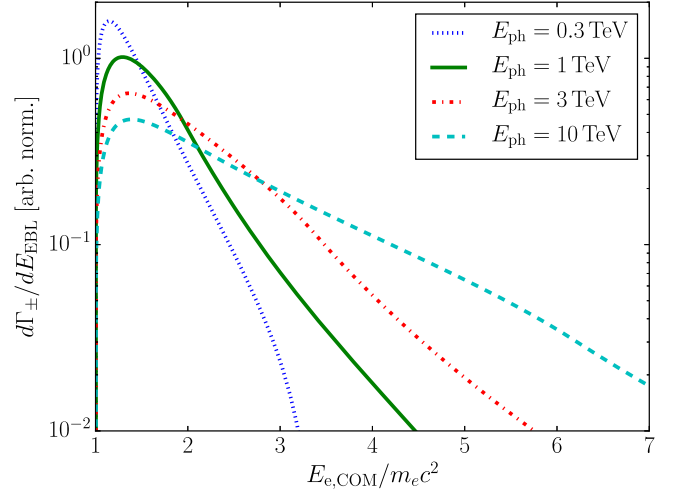


Figure 4. Differential rate of pair production as a function of the COM energy of the electron (and positron) for photon energies of $E_{\text{ph}} = 0.3$ (dotted line), 1 (solid line), 3 (dashed-dotted line), and 10 TeV (dashed line). The effect of the EBL spectrum can be seen in the different features in this plot. The cutoff in $d\Gamma_\pm/dE_{\text{EBL}}$ above $E_{e,\text{COM}}/m_e c^2 \approx 3$ for $E_{\text{ph}} = 1$ TeV is due to the cutoff in the EBL spectrum above 10 eV. The break in $d\Gamma_\pm/dE_{\text{EBL}}$ at the same position for $E_{\text{ph}} = 10$ TeV is due to the change in the EBL spectrum at 1 eV. The average COM energies of the produced electrons are $\bar{E}_{e,\text{COM}}/m_e c^2 \approx 1.5, 1.7, 2.2$, and 2.8 for $E_{\text{ph}} = 0.3, 1, 3$, and 10 TeV, respectively.

(2006), who demonstrated that VHEGR emission from H2356-309 and 1ES 1101-232 places an upper limit on the EBL that is close to the lower limit of the integrated light from galaxies (Madau & Pozzetti 2000). Looking at Figure 1 of Aharonian et al. (2006), we note that the EBL has a flat spectrum, i.e., constant $dn_{\text{EBL}}/dE_{\text{EBL}}$ below 1 eV and a falling spectrum $dn_{\text{EBL}}/dE_{\text{EBL}} \propto E_{\text{EBL}}^{-1.5}$ with a spectral index of ≈ 1.5 above 1 eV with a rapid cutoff above 10 eV. Thus, we adopt a simplified model:

$$\frac{dn_{\text{EBL}}}{dE_{\text{EBL}}} \propto \begin{cases} E_{\text{EBL}}^0 & E_{\text{EBL}} \leq 1 \text{ eV} \\ E_{\text{EBL}}^{-1.5} & 1 \text{ eV} < E_{\text{EBL}} \leq 10 \text{ eV} \\ 0 & E_{\text{EBL}} > 10 \text{ eV}. \end{cases} \quad (35)$$

In Figure 4, we plot the differential rate of pair production as a function of the COM energy of the electron (and positron), $E_{e,\text{COM}}$, for photon energies $E_{\text{ph}} = 0.3$ (dotted line), 1 (solid line), 3 (dashed-dotted line), and 10 TeV (dashed line). Note that the distribution of the COM energy for the electrons (and positrons) depends on the initial photon energy. This is because different energy photons probe different regimes of the EBL spectrum. Due to the rapid cutoff in the EBL above 10 eV, lower energy VHEGRs produce colder beams. This is seen in the average COM energies of the produced electrons, which are respectively, $\bar{E}_{e,\text{COM}}/m_e c^2 \approx 1.5, 1.7, 2.2$ and 2.8 for $E_{\text{ph}} = 0.3, 1, 3$ and 10 TeV. Hence we expect that these pairs are in the sub-relativistic to mildly relativistic regimes in their COM frame.

4.2. Regime of Instability

Given that the range of $k_B T_b / m_e c^2 = E_{e,\text{COM}} / m_e c^2 - 1$ falls between 0.5 and 2 for $E_{\text{ph}} = 1\text{--}10$ TeV, we now determine whether or not the reactive or kinetic instabilities apply to these beams. First, it is necessary to determine n_b/n_t . Here the target is the background IGM, so $n_t = n_{\text{IGM}}$, where $n_{\text{IGM}} \approx 2 \times 10^{-7} (1 + \delta)(1 + z)^3 \text{ cm}^{-3}$ is the mean density of the IGM, z is the redshift, and δ is the overdensity. The number density of the TeV beam is more complicated as the production rate of pairs must be

balanced against their loss due to plasma instabilities or ICC. This is discussed extensively in BCP12 and will not be repeated here. However, we note that the important issue here is the loss rate due to plasma instabilities, which is a nonlinear process. In BCP12, we assumed that the nonlinear loss rate was the same as the linear growth rate. This remains to be shown and is the focus of ongoing work, for which this paper lays the initial foundation.

Still some progress can be made if we use the IC rate as a lower limit to the beam cooling rate. This allows us to get an upper limit on the beam density. The ratio of the beam-plasma density to the IGM, n_b/n_{IGM} , is then (BCP12)

$$\begin{aligned} \frac{n_b}{n_{\text{IGM}}} &\approx \frac{L_E}{2\pi D_{\text{pp}}^3 \Gamma_{\text{IC}} n_{\text{IGM}}} \\ &\approx 2.3 \times 10^{-16} \left(\frac{1+z}{2} \right)^{3\zeta-7} \\ &\quad \times \left(\frac{EL_E}{10^{45} \text{ erg s}^{-1}} \right) \left(\frac{E}{\text{TeV}} \right) \text{cm}^{-3} \end{aligned} \quad (36)$$

at the mean density of the IGM, where L_E is the isotropic luminosity per unit energy of the VHEGR source, E is the energy of the VHEGR photon, Γ_{IC} is the inverse-Compton cooling rate, and the mean free path of a VHEGR

$$D_{\text{pp}}(E, z) = 35 \left(\frac{1+z}{2} \right)^{-\zeta} \left(\frac{E}{1 \text{ TeV}} \right)^{-1} \text{Mpc}, \quad (37)$$

where $\zeta = 4.5$ for $z < 1$ and $\zeta = 0$ for $z \geq 1$ (Kneiske et al. 2004; Neronov & Semikoz 2009).

In Section 3.3, we derived the controlling parameter that delineates the reactive (Equation (27)) and kinetic regimes (Equation (31)) by comparing the frequency spread of resonant waves, $\Delta\omega \approx k\Delta v$, with the growth rate, Γ . Applying these conditions (Equations (27) and (31)) to the ultrarelativistic e^\pm pair beams of interest, we find, for the controlling parameter,

$$\begin{aligned} \frac{\gamma_b^{2/3}}{\sqrt{k_B T_b/m_e c^2}} \left(\frac{n_b}{n_{\text{IGM}}} \right)^{1/3} &= 3.2 \times 10^{-2} \frac{\gamma_6^{2/3}}{\sqrt{k_B T_b/m_e c^2}} \\ &\times \left(\frac{1+z}{2} \right)^{\zeta-7/3} \left(\frac{EL_E}{10^{45} \text{ erg s}^{-1}} \right)^{1/3} \\ &\times \left(\frac{E}{\text{TeV}} \right)^{1/3}, \end{aligned} \quad (38)$$

and

$$\begin{aligned} \frac{\gamma_b}{\sqrt{k_B T_b/m_e c^2}} \left(\frac{n_b}{n_{\text{IGM}}} \right)^{1/3} &= 3.2 \frac{\gamma_6}{\sqrt{k_B T_b/m_e c^2}} \\ &\times \left(\frac{1+z}{2} \right)^{\zeta-7/3} \left(\frac{EL_E}{10^{45} \text{ erg s}^{-1}} \right)^{1/3} \\ &\times \left(\frac{E}{\text{TeV}} \right)^{1/3}, \end{aligned} \quad (39)$$

where $\gamma_6 = \gamma/10^6$. We see from our reactive (Equation (27)) and kinetic (Equation (31)) conditions that the oblique instability always exists in the kinetic regime, but the beam-plasma instability is in the reactive regime for $z \gtrsim 1$, for sufficiently

cold beams $k_B T_b/m_e c^2 \lesssim 0.5$, which occurs for $E_{\text{ph}} \lesssim 0.3 \text{ TeV}$, or for large γ , which occurs for $E_{\text{ph}} \approx 10 \text{ TeV}$ at $z = 0$.

In BCP12, we compared the cold-plasma growth rates of the oblique and beam-plasma instabilities and noted that the oblique cold growth rate is larger. While we also noted that the oblique instability was in the kinetic regime in BCP12, which we confirmed above, we made no effort to study the regime of instability of the beam-plasma case. Here we have shown that the oblique growth rate is kinetic and the beam-plasma rate is marginally reactive. This implies that the growth rate of the beam-plasma instability is similar to that of the oblique instability. In any case, we do not expect that the beam-plasma mode will have a major effect on our earlier results. First, plasma instabilities losses on the TeV pairs could easily push the beam-plasma mode into the kinetic regime by reducing n_b , but this requires a proper estimate of the effect of the nonlinear instability. This is a part of ongoing work and will be presented in a future publication. Second, while it seems that the beam-plasma mode may be in the reactive regime, it is not too far from the kinetic regime, i.e., the controlling parameter, $(\gamma_b/\sqrt{k_B T_b/m_e c^2})(n_b/n_{\text{IGM}})^{1/3}$, is of order unity. Thus, both the reactive and the kinetic growth rates are similar and it likely makes little difference for the beam-plasma mode which regime is assumed (in terms of growth rate). Therefore, the use of the kinetic growth rate for the oblique mode (and beam-plasma mode) in BCP12 is valid, and the results of this paper buttresses the results of Broderick et al. (2012), Chang et al. (2012), Pfrommer et al. (2012), Puchwein et al. (2012), and Lamberts et al. (2015).

5. SUMMARY AND CONCLUSION

The ultrarelativistic e^\pm beams that result from VHEGR–EBL annihilation are subject to powerful plasma beam instabilities including the beam-plasma and oblique instability. In this work, we examined these linear instabilities as they would apply to the ultrarelativistic pair beams. Our main findings are as follows.

1. We analytically calculated the growth rate of the beam-plasma and oblique instabilities in both the reactive and kinetic regimes. We have recovered the reactive scalings for the beam-plasma mode $\Gamma \approx \gamma^{-1}(n_b/n_t)^{1/3}$ and the oblique mode $\Gamma \approx (n_b/\gamma n_t)^{1/3}$. In the kinetic regime, we have shown that the growth rate for both modes have the same scaling and similar normalization. Finally, we have shown that the growth rate of the kinetic oblique instability has broad support. Namely, there exists unstable modes that grow at $\approx 0.4\Gamma_0$ for any value of beam temperature and wave-vector orientation for relativistic beams.
2. We also delineated the regime of applicability of the kinetic and reactive calculation and found, while the kinetic growth rates are similar for both the beam-plasma and oblique mode, the condition for transition between the kinetic and reactive regimes are different. In particular, the beam-plasma mode transitions at a lower value of γ in comparison to the oblique mode. This is due to a difference of $\gamma^{1/3}$ scaling between the two modes.
3. We calculate the average COM energy of the ultrarelativistic pair beam using a simplified model of the spectrum of the EBL. We found that the average energy of these beams range from $E_{e,\text{COM}}/m_e c^2 = 1.5\text{--}2.8$ for $E_{\text{ph}} = 0.3\text{--}10 \text{ TeV}$, with colder beams at lower energies. The average COM energies of the generated pairs imply that the oblique instability is in the kinetic regime, validating our results from BCP12.

We thank A. Bret for sharing his notes of the oblique instability and for extensive and enlightening discussions. P.C. is supported by the UWM Research Growth Initiative, the NASA ATP program through NASA grant NNX13AH43G, and NSF grant AST-1255469. A.E.B. and M.S. receive financial support from the Perimeter Institute for Theoretical Physics and the Natural Sciences and Engineering Research Council of Canada through a Discovery Grant. Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research and Innovation. C.P. gratefully acknowledges support by the European Research Council under ERC-CoG grant CRAGS-MAN-646955 and by the Klaus Tschira Foundation. E.P. gratefully acknowledges support by the Kavli Foundation. Support for A.L. was provided by an Alfred P. Sloan Research Fellowship, NASA ATP Grant NNX14AH35G, and NSF Collaborative Research Grant #1411920 and CAREER grant #1455342. G.V. acknowledges support from the Australian Research Council, project number DE140101960.

APPENDIX A LORENTZ FACTOR DEPENDENCE OF THE DISTRIBUTION FUNCTION AND VELOCITY DISPERSION

Here we explicitly derive the scaling of the parallel and perpendicular velocity dispersions with the Lorentz factor upon boosting the distribution function to the lab frame. Let us begin with a distribution function that is isotropic in the COM frame and depends only on energy. Therefore in the COM frame, which we denote with the subscript “COM,” the distribution function is $f_{\text{COM}}(E_{\text{COM}}(\mathbf{x}_{\text{COM}}, \mathbf{p}_{\text{COM}}))$. When we move to the lab (denoted with subscript “L”) frame, the integral of the distribution function remains invariant, i.e., the total number or

$$N \equiv \int f_L d^3p_L d^3x_L = \int f_{\text{COM}} d^3p_{\text{COM}} d^3x_{\text{COM}}. \quad (40)$$

It is well known that under Lorentz transformations (Landau & Lifshitz 1975),

$$d^3p_L d^3x_L = d^3p_{\text{COM}} d^3x_{\text{COM}}, \quad (41)$$

so therefore

$$f_L[\mathbf{x}_L(\mathbf{x}_{\text{COM}}, \mathbf{p}_{\text{COM}}), \mathbf{p}_L(\mathbf{x}_{\text{COM}}, \mathbf{p}_{\text{COM}})] = f_{\text{COM}}(E_{\text{COM}}). \quad (42)$$

Now let us consider moments of the distribution function. For clarity, it is helpful to consider moments of the distribution function first in the COM frame. The velocity moment is

$$\bar{\beta}_{\text{COM}} = N^{-1} \int \frac{\mathbf{p}_{\text{COM}}}{\gamma_{\text{COM}} m_e c} f_{\text{COM}} d^3p_{\text{COM}} d^3x_{\text{COM}} = 0. \quad (43)$$

We consider the lab frame to be boosted along the x -axis by β_b . More precisely, the initial inertial frame is the COM frame and the lab frame is moving with velocity $\beta_b = -|\beta_b|$ with respect to the COM frame. This gives

$$\begin{aligned} \bar{\beta}_L &= N^{-1} \int \frac{\mathbf{p}_L}{\gamma_L m_e c} f_L d^3p_L d^3x_L \\ &= N^{-1} \int \frac{\mathbf{p}_L(\mathbf{p}_{\text{COM}}, \mathbf{x}_{\text{COM}})}{\gamma_L(\mathbf{p}_{\text{COM}}, \mathbf{x}_{\text{COM}}) m_e c} f_{\text{COM}} d^3p_{\text{COM}} d^3x_{\text{COM}}. \end{aligned} \quad (44)$$

Breaking the components of $\bar{\beta}_L$ into components parallel and perpendicular to the boost, we find

$$\bar{\beta}_{L,\parallel} = N^{-1} \int \frac{\beta_{\text{COM},\parallel} + \beta_b}{1 + \beta_b \beta_{\text{COM},\parallel}} f_{\text{COM}} d^3p_{\text{COM}} d^3x_{\text{COM}} \quad (45)$$

$$\bar{\beta}_{L,\perp} = N^{-1} \int \frac{\beta_{\text{COM},\perp}}{\gamma_b (1 + \beta_b \beta_{\text{COM},\parallel})} f_{\text{COM}} d^3p_{\text{COM}} d^3x_{\text{COM}}, \quad (46)$$

where $\gamma_b = (1 - \beta_b^2)^{-1/2}$ is the Lorentz factor of the boost between the lab and COM frame. For $|\beta_{\text{COM}}|$, $\beta_b \ll 1$, we recover the Galilean invariant result, $\bar{\beta}_L \approx \bar{\beta}_{\text{COM}} + \beta_b \hat{\mathbf{x}}$. However, this Galilean result no longer holds for relativistic motion.

Now we consider the dispersion around $\bar{\beta}$. In components, the COM frame is

$$\overline{\Delta\beta^2}_{\text{COM},i} = N^{-1} \int \beta_i^2 f_{\text{COM}} d^3p_{\text{COM}} d^3x_{\text{COM}}. \quad (47)$$

In the lab frame, it is again useful to break it into components—the parallel component becomes

$$\begin{aligned} \overline{\Delta\beta^2}_{L,\parallel} &= N^{-1} \int (\beta_{L,\parallel} - \bar{\beta}_{L,\parallel})^2 f_L d^3p_L d^3x_L \\ &= N^{-1} \int \beta_{L,\parallel}^2 f_L d^3p_L d^3x_L - \bar{\beta}_{L,\parallel}^2 \\ &= N^{-1} \int \left(\frac{\beta_{\text{COM},\parallel} + \beta_b}{1 + \beta_{\text{COM},\parallel} \beta_b} \right)^2 f_{\text{COM}} d^3p_{\text{COM}} d^3x_{\text{COM}} - \bar{\beta}_{L,\parallel}^2, \end{aligned} \quad (48)$$

while the perpendicular component becomes

$$\begin{aligned} \overline{\Delta\beta^2}_{L,\perp} &= N^{-1} \int \beta_{L,\perp}^2 f_L d^3p_L d^3x_L \\ &= N^{-1} \gamma_b^{-2} \int \frac{\beta_{\text{COM},\perp}^2}{(1 + \beta_{\text{COM},\parallel} \beta_b)^2} f_{\text{COM}} d^3p_{\text{COM}} d^3x_{\text{COM}}. \end{aligned} \quad (49)$$

It is easier to first look at the perpendicular component and study how velocity dispersions scale between the center of mass frame and the lab frame for the nonrelativistic center of mass velocity dispersion. Hence, for $|\beta_{\text{COM}}| \ll 1$, Equation (49) becomes, to lowest order in β_{COM} ,

$$\overline{\Delta\beta^2}_{L,\perp} \approx \frac{\overline{\Delta\beta^2}_{\text{COM},\perp}}{\gamma_b^2} \approx \frac{2k_B T_b}{\gamma_b^2 m_e c^2}. \quad (50)$$

This simple scaling of the perpendicular velocity dispersion can be understood as a scaling with time between two frames boosted relative to each other, where the coordinates perpendicular to the boost axis remain invariant. This result is also in line with the transformation of temperature as $T \rightarrow T/\gamma$ under a boost, i.e., $mv^2 \approx kT$ —two factors of $1/\gamma$ from the perpendicular velocity dispersion is countered by one factor of γ from the mass. Let us now consider the parallel component (Equation (48)) again to lowest order in β_{COM} :

$$\overline{\Delta\beta^2}_{L,\parallel} \approx \frac{\overline{\Delta\beta^2}_{\text{COM},\parallel}}{\gamma_b^4} \approx \frac{k_B T_b}{\gamma_b^4 m_e c^2}, \quad (51)$$

Here, the scaling of the parallel velocity dispersion can be understood as a double scaling of both time and coordinate (along the boost axis) between the same two frames boosted

relative to each other, giving an extra scaling of γ^{-2} . This scaling of the parallel component of the velocity dispersion has important consequences that we explore in the main part of the paper.

APPENDIX B SOLUTION FOR THE REACTIVE REGIME

We begin with the dispersion relation (Equation (11)), which is

$$\begin{aligned}\epsilon &= 1 - \sum_s \frac{m_e \omega_{p,s}^2}{k^2} \int F_s \mathbf{k} \cdot \nabla_p \frac{1}{\omega - \mathbf{k} \cdot \mathbf{v}} d^3p \\ &= 1 - \sum_s \frac{\omega_{p,s}^2}{k^2 c^2} \int F_s \frac{k^2 c^2 - (\mathbf{k} \cdot \mathbf{v})^2}{\gamma(\omega - \mathbf{k} \cdot \mathbf{v})^2} d^3p = 0.\end{aligned}\quad (52)$$

We then take the limit of Equation (3) as $k_B T_i \rightarrow 0$, which yields a δ function. For the target plasma, we set $v_0 = 0$, and for a beam plasma $v_0 = v_b$.¹³ This leads to Equation (12), which we reproduce below:

$$1 - \frac{\omega_{p,t}^2}{\omega^2} - \frac{\omega_{p,b}^2}{\gamma^3(\omega - k_z v_b)^2} \frac{\gamma^2 k_x^2 + k_z^2}{k_x^2 + k_z^2} = 0. \quad (53)$$

Equation (53) can be rewritten as

$$\begin{aligned}(\omega^2 - \omega_{p,t}^2) \left[(\omega - k_z v_b)^2 - \frac{\omega_{p,b}^2}{\gamma^3} \frac{\gamma^2 k_x^2 + k_z^2}{k_x^2 + k_z^2} \right] \\ = \frac{\omega_{p,t}^2 \omega_{p,b}^2}{\gamma^3} \frac{\gamma^2 k_x^2 + k_z^2}{k_x^2 + k_z^2},\end{aligned}\quad (54)$$

where we have added a factor of $\gamma^{-3} \omega_{p,t}^2 \omega_{p,b}^2 / (\gamma^2 k_x^2 + k_z^2) / (k_x^2 + k_z^2)$ to both sides. To solve the dispersion relation (54), we take $\omega = \omega_{p,t} + \Delta\omega$ and expand to lowest order in $\Delta\omega$ and $\omega_{p,b}$. This gives

$$2\Delta\omega\omega_{p,t}(\Delta\omega + \omega_{p,t} - k_z v_b)^2 = \frac{\omega_{p,t}^2 \omega_{p,b}^2}{\gamma^3} \frac{\gamma^2 k_x^2 + k_z^2}{k_x^2 + k_z^2}. \quad (55)$$

For $\Delta\omega \ll \omega_{p,t} - k_z v_b$, $\Delta\omega$ is real and there is no instability. However, if $k_z = \omega_{p,t}/v_b$, we then have

$$\Delta\omega^3 = \omega_{p,t}^3 \frac{\omega_{p,b}^2}{2\gamma^3 \omega_{p,t}^2} \frac{\gamma^2 Z_x^2 + 1}{Z_x^2 + 1}, \quad (56)$$

where we have multiplied the fraction on the right-hand side by $(v_b/\omega_{p,t})^2/(v_b/\omega_{p,t})^2$ and $Z_x = k_x v_b/\omega_{p,t}$ is the dimensionless wave vector perpendicular to the beam direction. Equation (56) gives three solutions for $\Delta\omega$: one real and two imaginary (one growing and one damping). The maximum growth rate is then

$$\Gamma = \frac{\sqrt{3}}{2^{4/3}} \left(\frac{n_b}{n_t} \right)^{1/3} \left(\frac{\gamma^2 Z_x^2 + 1}{Z_x^2 + 1} \right)^{1/3} \frac{\omega_{p,t}}{\gamma} \quad (57)$$

¹³ That is, we set $F_s(\mathbf{p}) = \delta^3(\mathbf{p} - \mathbf{p}_{0s})$, where $\mathbf{p}_{0s} \equiv \gamma_0 m_e v_0 \hat{\mathbf{z}}$ is the momentum associated with v_0 .

APPENDIX C SOLUTION FOR THE KINETIC REGIME

To find the growth rate for the kinetic regime, we begin first with the distribution function for the target plasma:

$$F_t = \left(\frac{1}{2\pi m_e k_B T_t} \right)^{3/2} \exp\left(-\frac{p^2}{2m_e k_B T_t} \right). \quad (58)$$

We assume that the target plasma is nonrelativistic with a momentum $\mathbf{p} = m_e \mathbf{v}$, and T_t is the temperature of the target background plasma. For the beam plasma, we adopt the Maxwell–Jüttner distribution (Equation (3)). Inserting these into the dispersion relation (Equation (10)), we find

$$\begin{aligned}1 - \frac{\omega_{p,t}^2}{k^2 c^2} \int F_t \frac{k^2 c^2 - (\mathbf{k} \cdot \mathbf{v})^2}{\gamma(\omega - \mathbf{k} \cdot \mathbf{v})^2} d^3p \\ + \frac{m_e \omega_{p,b}^2}{k^2} \int \frac{\mathbf{k} \cdot \nabla_p F_b}{\omega - \mathbf{k} \cdot \mathbf{v}} d^3p = 0,\end{aligned}\quad (59)$$

where we have integrated by parts only the second term, associated with the target plasma.

As the target plasma is nonrelativistic, we can take $v \ll c$ and $\gamma \rightarrow 1$. Expanding the denominator in powers of v , we find¹⁴

$$\begin{aligned}\int F_t \frac{k^2 c^2 - (\mathbf{k} \cdot \mathbf{v})^2}{\gamma(\omega - \mathbf{k} \cdot \mathbf{v})^2} d^3p \\ \approx k^2 c^2 \int F_t \left(\frac{1}{\omega^2} + \frac{2\mathbf{k} \cdot \mathbf{v}}{\omega^3} + \frac{3(\mathbf{k} \cdot \mathbf{v})^2}{\omega^4} \right) d^3p \\ \approx \frac{k^2 c^2}{\omega^2} (1 + 3k^2 \lambda_D^2),\end{aligned}\quad (60)$$

where the second term is zero because it is odd, $\lambda_D^2 = k_B T_t/m_e \omega_p^2$ is the Debye length, and we have assumed that $k^2 \lambda_D^2 \ll 1$ and $\omega \approx \omega_p$ in the last term on the rhs.¹⁵ If we ignore the third term in the kinetic dispersion relation (18), this yields two plasma modes: an undamped plasma oscillation mode with $\omega = \omega_{p,t}$ and a longitudinal electron plasma wave, i.e., a Langmuir wave, with

$$\omega \approx \omega_{p,t} \left(1 + \frac{3}{2} k^2 \lambda_{D,t}^2 \right). \quad (61)$$

To compute the contribution from the beam term, we will reorient our coordinate system and define the z' -axis along the wave vector, \mathbf{k} . In this case we have the beam taking on a non- z' component, $\mathbf{v}_b = v_{bz'} \hat{\mathbf{z}}' + v_{bx'} \hat{\mathbf{x}}'$. This frame moves with a velocity $\mathbf{v}_{ph} = \omega \mathbf{k}/k^2$. With an eye toward computing the residue that will appear in Equation (18), we define $p_{z'} = \gamma_{z'} v_{z'} E_{\perp}$,

¹⁴ An alert reader will note that the Lorentz factor, γ , and the second term in the numerator both contribute to the expansion in powers of v at second order. These contributions are the result of the minor deviations from the Lorentz factor of the nonrelativistic electrons and the subtle difference between the momentum and velocity at order v^2/c^2 . These corrections correct the plasma frequency, ω_p , at order v^2/c^2 , but do not change the physics of the oscillations, i.e., they are independent of the wave vector. Hence, we ignore these effects while keeping the $O(v^2/c^2)$ correction that determine the Langmuir wave because these corrections depend on the wave vector.

¹⁵ A direct solution to Equation (60) without approximating $\omega \approx \omega_p$ will reveal waves with nontrivial growth or damping rates. These waves are not legitimate and result from the Taylor expansion of the denominator of Equation (60). A correct treatment of Equation (60) with the appropriate Landau contours will give the correct growing or damping behaviors for waves with phase speeds approximately that of the electron phase speeds.

$E = \gamma_z E_\perp$, and $E_\perp = \sqrt{m^2 + p_\perp^2}$ is the perpendicular energy. In this case, we can rewrite the beam distribution function as

$$F_B = \frac{m_e c^2}{4\pi\gamma_b k_B T_b K_2(m_e c^2/k_B T_b) m_e^3 c^3} \times \exp\left(-\frac{\gamma_b(E - v_{b,z'} p_{z'} - v_{b,x'} p_{x'})}{k_B T_b}\right) = \frac{\mu m_e^{-3} c^{-3}}{4\pi\gamma_b K_2(\mu)} \exp\left(-\frac{\gamma_b \gamma_z'(c^2 - v_{b,z'} v_{z'}) E_\perp}{k_B T_b c^2}\right) \times \exp\left(\frac{\gamma_b v_{b,x'} p_{x'}}{k_B T_b}\right), \quad (62)$$

where we define $\mu = m_e c^2/k_B T_b$. Inserting this equation into Equation (59) and using the results of Equation (60), we find

$$1 - \frac{\omega_{p,t}^2}{\omega^2} (1 + 3k^2 \lambda_D^2) + i \frac{\pi n_b}{n_t} m_e v_B^2 R = 0, \quad (63)$$

which involves the integral of

$$R \equiv \int \frac{k \partial F_b / \partial p_{z'}}{\omega - k_{z'} v_{b,z'}} d^3 p, \quad (64)$$

where R is the residue for $p_{z'}$ such that $v_{z'} = v_{ph}$.

We can assume that $k\lambda_D \ll 1$ since the thermal velocity of the background plasma is much smaller than the speed of the ultrarelativistic beam. We then take $\omega = \omega_r + i\Gamma$, where $\omega_r = \Re(\omega)$ is the real part of ω and the growth rate $\Gamma \ll \omega_r$, to find

$$\Gamma \approx -\omega_p \frac{\pi n_b}{2n_t} m_e v_B^2 R. \quad (65)$$

Here two elements contribute to the pole:

$$\left. \frac{\partial}{\partial p_{z'}} (\omega - k v_{b,z'}) \right|_{\text{pole}} = -k \left(\frac{c^2}{E} - \frac{p_{z'}^2 c^4}{E^3} \right) \bigg|_{\text{pole}} = -\frac{k c^2}{\gamma_{ph}^3 E_\perp}, \quad (66)$$

and

$$\begin{aligned} k \frac{\partial F_b}{\partial p_{z'}} \bigg|_{\text{pole}} &= -\frac{k \gamma_b}{k_B T_b} \left(\frac{p_{z'}^2 c^2}{E} - v_{b,z'} \right) F_b \bigg|_{\text{pole}} \\ &= -\frac{k (v_{ph} - v_{b,z'}) \mu^2}{4\pi m_e^4 c^5 K_2(\mu)} \\ &\quad \times \exp\left(-\frac{\gamma_b \gamma_{ph} (c^2 - v_{b,z'} v_{ph}) E_\perp}{k_B T_b c^2}\right) \\ &\quad \times \exp\left(\frac{\gamma_b v_{b,x'} p_{x'}}{k_B T_b}\right). \end{aligned} \quad (67)$$

Putting this all together, the residue is

$$R = \frac{\gamma_{ph}^3 (v_{ph} - v_{b,z'}) \mu^2}{4\pi K_2(\mu)} \frac{\mathcal{I}}{m_e^4 c^7} \quad \text{where} \quad \mathcal{I} \equiv \int d^2 p_\perp E_\perp \exp\left(-\frac{\mathcal{G}(E_\perp - w p_{x'})}{k_B T_b}\right), \quad (68)$$

$\mathcal{G} \equiv \gamma_b \gamma_{ph} (1 - v_{b,z'} v_{ph}/c^2)$, and $w \equiv \gamma_b v_{b,x'}/\mathcal{G} \leq 1$. This latter inequality is guaranteed as

$$\mathcal{G} E_\perp - \gamma_b v_{b,x'} p_{x'} = \gamma_b (E - v_{b,z'} p_{z'} - v_{b,x'} p_{x'})|_{v_{b,z'}=v_{ph}} > 0 \quad (69)$$

is the energy in beam frame and is therefore positive definite. Noting that the $\exp(-w p_{x'})$ term appears as a boosted distribution, we boost by w along the x' -axis, removing the anisotropic term from the exponential.

Thus, we define $p_{x'}' = \gamma_w (p_{x'} - w E_\perp/c^2)$ and $p_{y'}' = p_{y'}$ and find

$$E_\perp = \gamma_w (E_\perp' + w p_{x'}') \quad \text{and} \quad dp_x dp_y = \frac{E_\perp}{E_\perp'} dp_x' dp_y'. \quad (70)$$

Inserting this into Equation (68), we find

$$\begin{aligned} \mathcal{I} &= \int d^2 p_\perp' \frac{E_\perp'}{E_\perp} \exp\left(-\frac{\mathcal{G}' E_\perp'}{k_B T_b}\right) \\ &= \pi \gamma_w^2 \int_0^\infty dp_\perp'^2 \left(E_\perp' + \frac{w^2 p_\perp'^2}{2 E_\perp'}\right) \exp\left(-\frac{\mathcal{G}' E_\perp'}{k_B T_b}\right), \end{aligned} \quad (71)$$

where $\mathcal{G}' \equiv \mathcal{G}/\gamma_w$. Note in the second line that we have used isotropy in \mathbf{p}_\perp' to eliminate terms linear in \mathbf{p}_\perp' . Using the following integrals:

$$\begin{aligned} \int_0^\infty dx \sqrt{1+x} e^{-a\sqrt{1+x}} &= \frac{2e^{-a}}{a^3} (a^2 + 2a + 2) \\ \text{and} \quad \int_0^\infty dx \frac{x}{\sqrt{1+x}} e^{-a\sqrt{1+x}} &= \frac{4(a+1)}{a^3} e^{-a}, \end{aligned} \quad (72)$$

we find

$$\begin{aligned} \mathcal{I} &= \frac{2\pi \gamma_w^2 m_e^3 c^4}{\mathcal{G}'^3 \mu^3} [(\mathcal{G}'^2 \mu^2 + 2\mathcal{G}' \mu + 2) \\ &\quad + \frac{w^2}{2c^2} (2\mathcal{G}' \mu + 2)] \exp(-\mathcal{G}' \mu). \end{aligned} \quad (73)$$

Inserting this into (68) yields

$$\begin{aligned} R &= \frac{\gamma_{ph}^3 \gamma_w^2 (v_{ph} - v_{b,z'})}{2\mu \mathcal{G}'^3 K_2(\mu) c} [(\mathcal{G}'^2 \mu^2 + 2\mathcal{G}' \mu + 2) \\ &\quad + \frac{\gamma_b^2 v_{b,x'}^2}{2\mathcal{G}'^2 c^2} (2\mathcal{G}' \mu + 2)] \exp(-\mathcal{G}' \mu) \end{aligned} \quad (74)$$

and therefore,

$$\begin{aligned} \Gamma &\approx -\Gamma_0 \frac{\pi \gamma_w^2 \gamma_{ph}^3 (v_{ph} - v_{b,z'})}{4\gamma_b \mu^2 K_2(\mu) \mathcal{G}'^3 c} [(\mathcal{G}'^2 \mu^2 + 2\mathcal{G}' \mu + 2) \\ &\quad + \frac{\gamma_b^2 v_{b,x'}^2}{2\mathcal{G}'^2 c^2} (2\mathcal{G}' \mu + 2)] \exp(-\mathcal{G}' \mu), \end{aligned} \quad (75)$$

where $\Gamma_0 \equiv \omega_p \gamma_b (n_b/n_t) (m_e v_B^2/k_B T_b)$ is the typical maximum growth rate.

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