DISTORTION IN WELDED STEEL PLATES

Dissertation submitted to the University of Cambridge for the Degree of Doctor of Philosophy

by

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SUMMARY

An experimental and theoretical investigation of transverse shrinkage and angular distortions caused by welding of steel plates is described. A review of previous research reveals few measurements or formulae that are directly comparable; even when comparisons can be made the results are widely scattered.

The experimental work reported in this thesis include shrinkage measurements at over 160 weld beads. The distortion at submerged arc bead-on-plate welds on large unrestrained plates of Grade 43 weldable structural steel are taken as the basic case with which all other results are compared. The parameters investigated include weld conditions, weld process, plate dimensions and restraint. The effects of multipass welding, root gap, edge preparation, and tacking of butt welds have also been explored.

A computer model of transverse distortion at single pass welds is described. The program follows the accumulation of distortion as the temperature distribution associated with the weld pool moves along the plate centre-line. Temperature calculation is based on previously published analytical solutions to the heat flow equation, but uses empirically adjusted values of the thermal properties which give improved temperature prediction. The high temperature zone near the path of the weld pool is split longitudinally and through the thickness into layertype elements which have non-linear, elasto-plastic, load history and temperature dependent material properties. The remainder of the plate is split into two dimensional elastic elements which model in-plane and out-of-plane distortions and stresses.

A comparison was made of computed and experimental distortions at bead-on-plate welds. The theoretical results agreed with the principal conclusions of the experimental investigations. There were some areas of quantitative disagreement: these highlighted the critical importance of the shape of the fusion zone and the accompanying temperature distribution.

The investigation has yielded new insights into the underlying mechanisms of weld distortion. These have led to simple formulae for transverse shrinkage and angular distortion at single and multipass welds, and for the reduction in angular distortion due to rotational restraint.

DECLARATIONS

I hereby declare that my thesis entitled "Distortion in Welded Steel Plates" is not substantially the same as any that I have submitted for a degree or diploma or other qualification at any other University.

I further state that no part of my thesis has already been or is being concurrently submitted for any such degree, diploma, or other qualification.

This thesis is the result of my own work and includes nothing which is the outcome of work done in collaboration. Its total length is 194 pages.

R.H. Leggatt.

June, 1980.

R. H. Leggatt

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NOMENCLATURE

Symbols which appear only once are defined in context and are excluded from this list. Vector and matrix quantities are underlined.

A	vector array of element x dimensions
A _w	area of weld
B	vector array of element y dimensions
D	flexural rigidity = $Et^3/12(1 - v^2)$
E, E _o	Young's modulus, room temperature value
E', E _m	work hardening modulus
F, <u>F</u>	force, vector of element forces
<u>F</u> o	vector of temperature equivalent forces
I	current
K	stiffness matrix
Ko	Bessel function of the second kind of zero order.
K _e , K _i , K _j	external, internal, and joint stiffness (see 8.4)
K w	strain direction indicator (see 6.5.3)
L	number of layers in deformation zone
М	number of elements in elastic zone in x direction
^M _x , ^M _y	moments about x and y.
Ν	number of elements in elastic zone in y direction
N	number of passes
PS	approximate proof stress.
<u>s</u>	stiffness matrix
Т	temperature
т _b	temperature above which material strength is negligible
Tm	melting temperature
т _р	peak temperature
T _{pb}	peak temperature at back of plate.
v	voltage

a	throat of fillet weld.
a _i	x dimension of i th row of elements.
b	plate width.
b'	free plate width between restraints.
^b i	y dimension of i th row of elements.
b _m	average width of fusion zone.
b _w	width of bead.
d	separation of plate centre and joint centre.
g	gap
k, k _o	conductivity, room temperature value.
k e	effective conductivity (see 2.3).
l	plate length.
m	thermal property gradient (see 2.2.1).
p	penetration (i.e. depth) of weld bead.
q	heat input per second (= IV).
S	rise time to peak temperature.
s'	time above 80 % of peak temperature.
t	plate thickness.
tc	joint thickness.
v	welding velocity.
х	direction parallel to weld.
У	direction transverse to weld.
z	direction through plate thickness.

bead width or deformation zone width

*

W

*

α	coefficient of expansion.
β	angular distortion about weld line, per pass.
β _o	angular distortion in unrestrained plate, per pass.
βι	preset angular distortion before welding.
β _p	predicted angular distortion.
β*	angular distortion scaled by specified factor to correct for
	small variations in q/vt^2 .
δ	transverse shrinkage, per pass.
δ _o	theoretical maximum transverse shrinkage.
δ _b	transverse shrinkage at back surface of plate.
δ _c	transverse shrinkage at joint centre.
δ _f	transverse shrinkage at front surface of plate.
^δ x, ^δ y	node displacements in x and y directions.
δ*	transverse shrinkage scaled by specified factor to allow for
	small variations in q/vt.
δε _p	change in plastic strain.
ε	strain, total strain.
εc	mechanical strain (i.e. total - thermal strain).
εe	elastic strain.
εp	plastic strain.
$\varepsilon_{\rm T}$	thermal strain.
ε _w	strain origin (i.e. plastic strain at most recent strain reversal)
εy	yield strain.
η	process efficiency (i.e. proportion of electrical energy
	entering plate as heat).
θ _x , θ _y	rotation about x axis, y axis.
ν, ν _e , ν _p	Poisson's ratio, elastic and plastic.
ξ	distance forward from heat source.
ρ c , ρc _ο	volumetric specific heat, room temperature value.
ρc _e	effective volumetric specific heat (see 2.3)

*

λ

*

σ		stress
σ,	σ0.2	yield stress, 0.2 $\%$ proof stress.
ω		displacement in z direction.

ABBREVIATIONS

EZ	Elastic Zone
FEWM	Finite Element Weld Model
DZ	Deformation Zone
GTA	Gas Tungsten Arc
MMA	Manual Metal Arc
Sub Arc	Submerged Arc.
2D, 3D	Two dimensional, three dimensional.

CONVENTIONS

Shrinkage (δ) is taken as a positive quantity.

Angular distortion (β) is positive if shrinkage is greatest on the side being welded.

All distortions are incremental (i.e. changes per pass) unless otherwise stated.

1. INTRODUCTION

1.1 WELD SHRINKAGE

The distortion of welded structures is one aspect of the more general phenomenon of weld shrinkage, which may be described as a set of permanent plastic strains arising from the laying of one or more weld beads. These strains are predominantly contractile, and are present in both the weld metal and the surrounding parent plate. They are caused by compressive yield of the metal heated by the energy of the weld, as it simultaneously tries to expand and suffers a loss of strength. The subsequent thermal contraction during cooling gives rise to permanent overall shrinkage and, if restraint is present, to tensile residual stresses. Restraint may arise either internally, from strain differentials within the parts being welded, or from external reaction forces in the surrounding structure.

Residual stresses and distortions may affect the fitness-forpurpose of a welded structure in a number of areas. The fabricator must make due allowance for weld distortion if he is to achieve design tolerances. Tensile residual stresses contribute to the initiation and propagation of cracks. The buckling strength of structural elements is affected by out-of-flatness caused by weld distortion, and by initial compressive stresses in the structure balancing the tensile residual stresses in the weld. This project in fact arose from the continuing programme of research into plate buckling which has been conducted at Cambridge University Engineering Department under the leadership of J. B. Dwight for a number of years. It is however concerned specifically with the cause - weld distortion - and not with the resulting loss of structural strength.

The overall state of distortion and reaction stresses in a welded structure are functions of the three weld shrinkage parameters illustrated on Figure 1. The TRANSVERSE SHRINKAGE, δ , (Figure 1b) is the inwards displacement at mid-thickness of the abutting plate edges.

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The ANGULAR DISTORTION, or "WRAP-UP", β (Figure 1(c)), is the relative rotation of the abutting plate edges. The TENDON FORCE, F, (Figure 1(d)) is the force in an imaginary tendon running along the weld which would cause the observed compressive stresses in the plates. All three parameters are, in general, functions of the welding procedure, the plate and joint geometry, and of the degree of restraint which is applied. They may vary along the length of the weld.

Transverse shrinkage at the weld causes overall transverse contraction of free plates, or tensile transverse stresses in restrained plates. Angular distortion causes either whole body rotation or local cusping and corresponding bending stresses in the welded plates. The tendon force, by definition, gives rise to compressive stresses away from the weld. If its line of action is displaced from the plate centroid, it will cause in-plane or out-of-plane bowing, and there will be stress gradients in the plates. Stress variations may also be caused by the variation of transverse shrinkage along the weld line, by the locking-in by the weld of applied distortions, and by the decay of the tendon force at the ends of the weld.

All the stresses mentioned above may be classified as reaction stresses - the elastic response of the structure to the shrinkage parameters δ , β , and F. It should be noted however that these parameters give no information about the distribution or magnitude of local non-uniform stresses within the region heated by the weld.

This thesis is concerned with the two parameters which describe the transverse distortion at the weld line, transverse shrinkage and angular distortion. The third parameter, tendon force, has been investigated by White in a parallel project (52, 54). The overall objective of the present project has been to gain greater understanding of the mechanisms by which transverse weld distortion is originated, and hence to obtain reliable methods for predicting distortions in bead-on-plate or butt welded plates.

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1.2 RESEARCH PROGRAMME

The temperature distribution accompanying the moving weld pool is the starting point for research into weld distortion, since it not only generates the thermal strains which cause distortion, but also affects the mechanical properties of the heated metal. Chapter 2 commences with a review of previous research into heat flow during welding, and goes on to propose an empirical method for allowing for the effect of varying thermal properties on temperatures calculated using analytical solutions to the heat flow equation. This method is shown to give improved temperature prediction in the range 300 to 1000^oC.

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Previous research into weld distortion is reviewed in Chapter 3, of which one section is devoted to a quantitative comparison of existing weld distortion formulae.

Experimental measurements of shrinkage and angular distortion are described in Chapters 4 and 5. The laying of a single weld bead along the centreline of a large unrestrained Grade 43 steel plate using the Submerged Arc welding process is considered as the BASIC CASE, with which other sets of welding conditions are compared. The distortions are characterized by the steady-state values which are achieved over the central portion of long welds. Chapter 4 is concerned with bead-on-plate welds. The first and most important variables to be considered are the heat input per unit length and the plate thickness. Thicknesses from 6 to 20 mm were used in the experimental programme. Secondary parameters which are investigated include the welding parameters (current, voltage and velocity), the welding process, second pass welds, variation of distortion along the weld length, plate width, rotational restraint, and material yield strength. In Chapter 5, this parametric survey is extended to butt welds. The variables introduced are edge-preparation, fit-up gap, tacking, and multipass welding.

A mathematical model of the development of transverse distortions during welding is described in Chapter 6. The review of previous research indicated that existing models were essentially two dimensional, and considered stresses and distortions either in the plane of the plate, or in a section transverse to the weld. These two types of analysis are combined in the present model, which considers the transient transverse stresses and strains, both through the thickness and along the length of weld, and the balancing in-plane and out-of-plane stresses in the abutting plates.

Experimental and computed distortions are compared in Chapter 7, in which the description of the effects of the various relevant parameters follows the order in which the original experimental results were presented in Chapter 4.

Qualitative descriptions of the mechanisms which control the development of distortion during welding are postulated in Chapter 8. These descriptions lead on to quantitative formulae for transverse shrinkage and angular distortion in single and multipass welds, and for the reduction of angular distortion due to external restraint. The formulae are compared with experimental results.

The principal conclusions are summarized in Chapter 9.

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2. THE TEMPERATURE DISTRIBUTION AROUND A WELD POOL.

2.1 INTRODUCTION.

Since both the loading (i.e. thermal strains) and material properties in a welded joint are functions of temperature, accurate knowledge of the temperature distribution is the first component of any analysis of weld distortion.

An expression for the temperature distribution during welding was obtained by Rosenthal (ref. 36), who derived an analytical solution to the equation of heat flow from a moving point source of heat. This solution requires the thermal properties of the material to be constant with temperature, and ignores the presence of the molten region. Consequently, it becomes increasingly inaccurate with proximity to the weld pool boundary.

Previous attempts to obtain improved temperature prediction fall into two categories: modifications to Rosenthal's solution, and rigorous solutions of the heat flow equation using numerical techniques.

Since the temperature distribution is to be used in a stress analysis program which will itself use considerable computer resources, it is desirable to avoid numerical techniques if possible. The previously published modifications to Rosenthal's formula are found to be not applicable to the present situation, so a new approach is adopted.

The essence of this approach is an empirical method of determining "EFFECTIVE VALUES" of the relevant material properties which, when substituted into Rosenthal's expression, give the correct temperatures. The hypothesis is made that these effective values are unique functions of the temperature to be determined. This hypothesis is tested by deriving expressions which relate the effective values of volumetric specific heat and conductivity to key parameters of the temperature-time history. The effective values are then evaluated experimentally, as functions of temperature, over a wide range of welding conditions.

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2.2 REVIEW OF PREVIOUS RESEARCH

2.2.1 Analytical solutions to the heat flow equation

Rosenthal (ref. 36) simplified the equation for the heat flow during welding by making the following idealizations:

- The heat source is considered to be a point moving across a solid medium.
- (2) Heat losses from the specimen are neglected.
- (3) Heat sources or sinks caused by phase changes are ignored.
- (4) Material properties are taken as constant.

The heat flow equation under these conditions becomes

$$2\lambda \frac{\partial T}{\partial t} = \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial x^2} \right)$$
(2.1)

where

$\lambda = \frac{\rho_c}{2k} = \frac{\text{volumetric specific heat}}{2. \text{ conductivity}}$

Assuming that the heat source is moving at a constant velocity over a distance sufficient for quasi-stationary conditions (i.e. stationary with respect to axes moving with the heat source), Rosenthal obtained expressions for the temperature distribution under different plate boundary conditions which are summarized by Myers et al. (ref. 30).

The basic solution is for the 3D case - a heat source moving at constant velocity over the surface of a semi-infinite body. For plates of finite thickness, the effect of the bottom surface of the plate is allowed for by the method of images, and the so-called 2.5D solution is obtained. At points distant by more than twice the thickness from the weld centre line, variation of temperature through the thickness is negligible, and a 2D distribution is used. Expressions for the 2D and 2.5D distributions are given in the Appendices as equations Al.1 and A2.1.

Carslaw and Jaeger (ref. 5) give an expression for the temperature distribution around a moving heat source of finite rectangular area, but this solution has not been applied to the welding problem.

Grosh, Trabant and Hawkins (ref. 14) give solutions to the heat flow equation for the case where conductivity (k) and volumetric specific heat (ρ c) both vary in the same manner with temperature according to the laws

(2.2)

 $\rho c = \rho c_{o} (1 + mT)$ and $k = k_{o} (1 + mT)$ where m, ρc_{o} and K_o are constants.

2.2.2 Temperature prediction using analytical solutions

Papers which compare experimental measurements of temperature in plates during welding with theoretical results obtained using Rosenthal's solution to the heat flow equation have the following features in common: they allow the thermal properties to vary with temperature (even though the solution is only valid if the properties are constant): and they find that the theoretical results become increasingly inaccurate with proximity to the weld pool. Where they differ is the manner in which values are ascribed to the thermal properties.

The relevant properties are the conductivity, k, the volumetric specific heat, ρc , and a derived quantity which may be expressed as λ (= $\rho c/2k$) or as thermal diffusivity (= $k/\rho c$).

Rosenthal and Schmerber (ref. 37) assume properties as shown in Figure 2.1. k is assumed to be constant, while λ increases with temperature. Temperature prediction was good in five of the seven edge weld, thin-plate specimens, at distances of more than 8 mm from the heat source.

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Christensen et al. (ref. 6) carried out a much more extensive programme, using thick plates for which the 3D distribution was appropriate. The thermal properties used were a constant value of diffusivity and actual values of heat content (= $T\rho c$), which implies a varying ρc . Calculated peak temperatures were accurate "at locations sufficiently remote from the centre-line".

Masubuchi et al. (ref. 27) use actual thermal properties appropriate to the temperature being determined. This method was found to be sufficiently accurate in a 1D longitudinal stress analysis program. An example of its use will be given in Section 2.5.

Temperature distributions in materials whose properties approximately obey equation 2.2 have been verified experimentally by Grosh and Trabant (ref. 14) and Kellock (ref. 25). Good agreement was obtained. Unfortunately, common structural steels have properties which cannot be expressed in the required format: in fact, ρc increases with temperature whilst k decreases.

Jhaveri, Moffat and Adams (ref. 20) differentiated Rosenthal's temperature equation with respect to time, to give expressions for cooling rates which were found to be independent of weld velocity for a given rate of heat input (energy per unit length). This led to the concept of the instantaneous (or infinite velocity) weld, which reduced the dimension of the problem by one, and allowed simplified expressions for cooling rate to be derived. Those expressions were analysed numerically, to produce non-dimensionalized charts which gave cooling rates in terms of weld power and temperature. It was noted that zero cooling rate corresponds to peak temperature, giving rise to the following expression for peak temperature at the back of the plate

$$T_{pb} = \frac{0.47}{\rho c} \cdot \frac{q}{vt^2}$$
(2.3)

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In a later paper, Barry, Paley and Adams (ref. 3) found that this formula consistently predicted peak temperatures which were higher than those measured experimentally. A revised formula was presented that apparently allowed for the variation in material properties, and the finite width and temperature of the heat source. However, there are inconsistencies in the derivation of this formula and the present author has found it to be incompatible with his own experimental evidence.

2.2.3 Weld pool dimensions

One test of a theoretical temperature distribution is its ability to predict the dimensions of the weld pool, which should correspond to the melting temperature isotherm. Christensen et al. (ref. 6) have conducted an extensive comparison of experimental and theoretical weld pool-dimensions.

Although the predictions of the cross sectional area of the pool were reasonable, prediction of the width was poor and of the penetration, worse. Indeed, according to Savage et al. (ref. 40), "any attempt to predict the puddle-configuration on the basis of heat input is a futile task". This is because the shape of the weld pool section is not, as in the theory, a semi-circle, but is sensitive to the individual weld parameters: in particular, its depth is largely controlled by the current, and the bead width is principally a function of welding velocity.

Savage and co-workers related the dimensions of the weld pool to the welding conditions by means of a power formula:

dimension =
$$KI^{\alpha}v^{\beta}v^{\gamma}$$
 (1.1t)⁶ (2.4)

where K, α , β , γ , and δ are constants which were determined by statistical methods from experiments in which measurements were made of the width and depth of weld pools produced by the gas tungsten arc (GTA) process.

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Jackson and Shrubshall (ref. 19) report similar work by Gunnert, who investigated the penetration of the submerged arc and manual covered electrode processes. The two formulae for penetration are markedly different:

Savage* :
$$p = 0.58I^{39}V^{04}v^{-.45}$$
 (2.5)

Gunnert :
$$p = KI^{1.33}V^{-.6}v^{-.33}$$
 (2.6)

Jackson and Shrubshall suggest that K lies between 0.019 and 0.036, depending on the process and electrode composition. Penetration data from several sources (Hex, Renwick, Jones (refs. 16, 35, 21) and the author) are plotted on Figure 2.2 against the Gunnert parametric group (expressed as $(I^4/V^2v)^{\frac{1}{3}}$). 90 % of the data falls within the limits suggested by Jackson and Shrubshall, and this formula would appear to be the best available for predicting weld penetration. Savage's formula, having a low power of current, fails to predict the high penetrations which are known to result from high currents.

Friedman (ref. 10) has published a description of a finite element model of a stationary gas tungsten arc weld pool, which allows for arc pressure, gravity, and surface tension, but does not include magnetic or fluid flow effects. He used his program for quantitative assessment of the importance of the modelled variables in determining the shape of the weld pool boundary.

In another paper (ref. 9), Friedman assumes that the distribution of heat transfer from the arc to the workpiece has a normal distribution characterized by a "heat input distribution parameter". He then found values of this parameter, as functions of current and weld velocity, which, when input to a heat flow model, gave values of bead width and penetration which correlated to experimental results.

* Both formulae have been converted to mm and mm/sec units. In Savage's formula, the power δ of thickness is small, and this term has been included in the constant, assuming t = 19 mm.

2.2.4 Solution of the heat flow equation using numerical methods

Several workers have produced specific solutions to the equation of heat flow during welding using numerical methods. These methods enable provision to be made for the variability of thermal properties, for the effects of phase changes, and, if empirical data is available, for irregularities in the shape of the weld pool boundary.

Pavelic et al. (ref. 34) considered tungsten inert gas (TIG) welding in thin plates. The shape of a weld pool in two dimensions was defined by empirical constants, and was used as one boundary of the mesh. Peak temperature distributions were within 10 % of experimental values, which was considered a significant improvement over previous methods. The computing resources required were considerable.

Masubuchi and Tsai (ref. 28) describe a 3D program which models the heat flow in the region close to, but outside, the weld pool boundary, whose shape is given by an empirically modified Rosenthal type isothermal contour. The heat flow across the pool boundary is given by an energy balance within the weld pool. Underwater welding was investigated: allowance was made for heat loss by boiling.

Friedman (ref. 9) points out that computation time may be much reduced if, as in Rosenthal's analytical solution, the problem is effectively reduced to two dimensions by calculating the heat flow relative to a set of axes moving with the heat source and ignoring heat flow parallel to the welding direction. At low speeds, where parallel heat flow is significant, it may be modelled as an equivalent internal heat generation term. Allowance must also be made for convective heat transfer within the weld pool by assuming a very high value of conductivity.

Other programs, which do not allow for irregularities in the weld pool boundary, cannot be expected to be accurate in its immediate vicinity. These are described in papers by Hibbit and Marcal (ref. 17), who use the finite element method and Ueda et al. (ref. 47) and Krutz and Segerlind (ref. 26), who use finite differences. The first two papers are concerned mainly with residual stress analysis, the third with metallurgical effects.

2.3 THE REQUIREMENT FOR A NEW METHOD OF TEMPERATURE PREDICTION

The requirement in the present project is for a temperature distribution which

- (a) Is accurate up to 750°C, the temperature above which steel has no significant strength.
- (b) Uses as small an amount of computing resources as possible, since it will be used in conjunction with a mechanical model whose resource requirements will inevitably be considerable.

In order to meet the second requirement it was decided to use an analytical model, based on Rosenthal's solution. Consequently , it was necessary firstly to devise some rational method of allowing for the variation in material properties, and secondly to recognize that the model predicts a weld pool of semi-circular cross-section, and will not be valid if the weld conditions are such as to produce a radically different shape. This is not such a serious limitation as might at first appear: good welding practice demands that the welding parameters should be selected in such a way that gross irregularities in the weld pool shape are avoided. If the weld pool boundary (i.e. melting temperature isotherm) is *reasonably* regular, the 750°C isotherm will be more so: any weld section etching shows that low temperature isotherms (e.g. the recrystallization boundary, at 910° C) are smoother than the fusion boundary.

Existing methods of allowing for the variation of material properties (see 2.2.2) appear to be rather arbitrary. Therefore, a systematic empirical method for determining the most suitable values of the material properties k and ρc for use in Rosenthal's equation was devised. The procedure was as follows

(1) Expressions for the peak temperature (T_p) and rise time (s) were derived from Rosenthal's general formulae for the temperature at any point (see Appendices A1 and A2). These allowed the apparent or "effective" values of k and ρc to be expressed as functions of measured values of peak temperature, rise time, welding conditions, plate thickness, and position relative to the heat source:

2.5D case (y << t)

$$\rho c_{e} = \frac{4}{e\pi} \cdot \frac{\eta q}{v(t^{2} + y^{2})} \cdot \frac{1}{T_{p}}$$
(2.7)

$$k_{e} = \frac{(t^{2} + y^{2})}{4} \cdot \frac{\rho c_{e}}{s}$$
 (2.8)

2D case (y > 2t)

$$\rho c_{e} = \frac{1}{\sqrt{2e\pi}} \cdot \frac{\eta q}{v t y} \cdot \frac{1}{T_{p}}$$
(2.9)

$$k_{e} = \frac{y^2}{2} \cdot \frac{\rho c_{e}}{s}$$
 (2.10)

(2) Measured values of T_p and s from fifty experimental thermal cycles were substituted into equations 2.7 to 2.10, enabling the effective values of ρc and k corresponding to each peak temperature to be evaluated.

(3) The effective values were then plotted as functions of temperature, and empirical curves fitted to the data.

(4) The empirical curves were then used for the calculation not only of peak temperatures (from which they were derived), but also of complete thermal cycles. Since the effective value at any point is itself a function of the temperature being calculated, an iterative solution procedure was required. In order to be of use in temperature prediction, the effective values must be unique functions of temperature, for a given material. This unique relationship cannot be proven theoretically, but seems reasonable in view of the fact that for all locations experiencing a particular temperature, T, the heat flow paths from the source are similar (namely, they pass first through a region of molten metal, and then through a region where temperature decreases from melting point to T). The relationship must, however, be demonstrated experimentally.

2.4 EXPERIMENTAL STUDY OF TEMPERATURES DURING WELDING

2.4.1 Scope

The aim of the investigation was to record temperature histories in steel plates during welding, for a wide range of welding conditions.

2.4.2 Specimens

The specimens were all Grade 43 steel. Their lengths and thicknesses are given in Tables 2.1 and 2.2. The widths were all greater than 300 mm. Each specimen was equipped with run-on and run-off pieces, tacked on as shown in Figure 2.3, so that the weld cross section was constant over the length of the specimen.

For a steady-state temperature distribution, the specimens should theoretically be infinitely wide and long. In practice the heat flow is substantially one dimensional except in the immediate vicinity of the weld pool, so that the short but wide specimens used were quite adequate. However, specimens WC4b to WC11, whose prime experimental purpose was not temperature measurement, were very short, and their temperature distributions may have been disturbed at the interface between the specimen and the run-on and run-off pieces. Results from those specimens are therefore regarded as less reliable than the others.

2.4.3 Welding processes

Submerged Arc or Metal Inert Gas processes were used as indicated in Tables 2.1 and 2.2. The welding heads were mounted on an MPE 15C side beam machine. Satinarc BX10 flux and Bostrand 3.2 mm diameter wire were used with the submerged arc process. The MIG welds were shielded with 95 % Argon, 5 % CO₂ and used a 1.2 mm diameter wire.

2.4.4 Welding conditions

Weld voltage and current were recorded continuously. Weld velocity was indicated by the machine controls, and checked by direct measurement. The range of weld passes used represents the extreme range of power practical for each given thickness: it is limited at the lower end by arc stability and at the higher end by burn-through.

2.4.5 Temperature measurement

40 SWG Chromel-Alumel thermocouples were flash welded to the back of each specimen. The second junction was kept at room temperature. The calibration was 24.0 °C/mV. The thermocouple voltage was recorded continuously on a chart recorder. The chart velocity was checked by direct measurement. The transverse displacement from the weld centre line (y) of each thermocouple was measured after welding.

2.4.6 Experimental errors

Significant sources of error in the equations for effective conductivity and volumetric specific heat (equations 2.7 to 2.10) were assessed as:

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	Term	Description	Maximum Error	%
	I	Current	300 ± 10 A	3.3
	v	Voltage	30 ± 0.5 V	1.7
	η	Arc Efficiency	80 ± 3 %	3.8
($t^2 + y^2$)	Radius	64 ± 1 mm	1.7
giving	^{ρc} e	Effective Volumetric Spe	cific Heat	10.5
	S	Rise Time	10 ± 0.5 secs	5.0
and	k e	Effective Conductivity		15.5

2.4.7 Experimental results: effective thermal properties

The experimental results are summarized in Tables 2.1 and 2.2. Values of ρc_e determined from the fifty measured thermal cycles using equations 2.7 and 2.9 are plotted on Figure 2.4. It can be seen that within the limits of experimental error (estimated at ±10 %) the evidence supports the hypothesis that ρc_e is a unique function of temperature, (though some of the low temperature data, produced by undergraduate projects, is slightly less consistent). The arc efficiency was assumed to be 80 % at all times.

Actual values of volumetric specific heat (taken from Woolman and Mottram (ref. 55)) are also plotted on Figure 2.4. It can be seen that the effective value is approximately equal to the actual value at room temperature, but exceeds it by an increasing margin at higher temperatures. This is presumably because the value relevant to a particular isothermal contour is the average value of the whole high temperature region (including the weld pool) within that contour.

For the purposes of computation, the effective values of volumetric specific heat are characterized as:

$$pc_{e} = 0.00032 \,\sqrt{T}$$
 (2.11)

This expression covers the range 200 to 1,000°C,

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For the limiting case of zero temperature rise, the actual room temperature value must be appropriate. Therefore, below 200⁰, the curve is blended into the room temperature values using a cubic function.

Equations 2.8 and 2.10 give expressions for the effective conductivity in terms of the rise time, s, and other parameters. However, in the case of thermal cycles measured directly under the weld bead (y = 0 approx.), the rise time was short and could not be determined very accurately. Instead, the time interval between 80 % T_p (rising) and 80 % T_p (falling) was measured. This quantity is referred to as the 80/80 time, s'.

Then:

if y << t and 200 < T < $1000^{\circ}C$

$$k_{e} = \frac{\rho c_{e} (t^{2} + y^{2})}{1.91s'}$$
(2.12)

The derivation of s' given in Appendix A.3 utilizes the empirical relationship for ρc_{ρ} (equation 2.11).

Effective and actual values of conductivity are compared on Figure 2.5. Effective conductivity is approximately constant at temperatures above 400°C at a value equal to the high temperature value of actual conductivity. This may be interpreted by regarding the high-temperature region as a low-conductivity barrier which regulates the temperature at more distant regions. The higher values of effective conductivity at temperatures below 400°C may reflect heat loss from the plate, which is not allowed for in the heat flow equations:

For the purposes of computation, the effective conductivity is characterized as:

$$k_{e} = 0.023$$
 (2.13)

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Below 400[°]C, there is an increase in effective conductivity which is modelled by a cubic function.

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2.5 COMPARISON OF COMPUTED AND EXPERIMENTAL THERMAL CYCLES

Two Fortran sub-routines have been written. The first, given co-ordinates, welding conditions, and assumed effective values of material properties, calculates the temperature at any point in a plate during welding according to Rosenthal's equations. The second organizes the iterative procedure which ensures that the effective values correspond to the temperature calculated, as defined by equations 2.11 and 2.13.

Figures 2.6 to 2.9 show computed and experimental thermal cycles for a selection of weld conditions. Figure 2.6 shows temperature cycles measured at the back of the plate directly under the weld (y = 0 approx.). The close agreement (within 5 %) was typical of the majority of the measured cycles. The exceptions (shown on Figure 2.7) were:

> WC16: delayed rise from 0 to 6 secs; chart speed error suspected. WC18: computed values up to 10 % high: reason unknown. WC22: peak temperature exceeded 1000[°]C: insufficient data. WG5b (not shown): reason unknown.

Figure 2.8 shows thermal cycles measured at locations away from the weld centre line, with peak temperatures up to $200^{\circ}C$.

Three different methods of calculating thermal cycles are compared on Figure 2.9. Line 1 shows temperatures calculated using room temperature values of the material properties: the peak is too high and occurs too early. Line 2 uses the method recommended by Masubuchi (ref. 27) whereby actual, temperature dependent values of material properties are used. Although the peak temperature is lowered slightly, the agreement is still poor. Line 3 utilizes the currently proposed "effective values" of material properties. The most striking improvement occurs in the early stages, when the temperatures are in excess of 300 °C. At low temperatures, all methods are reasonably accurate. This is reflected in the fact that the effective values are close to the actual values at the low end of the temperature range. An expression for the length of the weld pool is derived from Rosenthal's solution to the heat flow equation in Appendix A4. The expression is

$$\xi' = \eta q / 2\pi k T_m \qquad (2.14)$$

No measurements of weld pool length were made during the present investigation, but Christensen et al. (ref. 5) have published the data reproduced on Figure 2.10. The heavy line on the figure is a good fit through the points corresponding to measurements made in mild steel: it has the equation

$$\xi' = 1.9 \, \eta q / 2 \pi k T_{m}$$
 (2.15)

There is a clear discrepancy between the empirical and theoretical formulae - the factor 1.9. This may be largely removed by using the effective, rather than room temperature value of conductivity, assuming that the constant value of effective conductivity found below 1000 ^OC may be extrapolated up to melting temperature.

Then
$$k_{e1000}/k_{RT} = 0.023/0.046 = \frac{1}{2}$$

 $\vdots \xi' = 2.\eta q/2\pi kT_m$
(2.16)

2.7 CONCLUSIONS

It has been demonstrated that the use of "effective values" of volumetric specific heat and conductivity give improved prediction of temperatures near welds in steel in the range 300 to 1000°C, and of weld pool length.

The effective values are determined empirically as described in 2.3. The values obtained for weldable structural steel are shown on Figures 2.4 and 2.5, as functions of temperature up to 1000° C. The temperature at any point is calculated using these effective values in conjunction with Rosenthal's equations Al.l and A2.l. An iterative solution procedure is required.

3. WELD DISTORTION: REVIEW OF PREVIOUS RESEARCH

3.1 INTRODUCTION

This chapter is concerned mainly with transverse distortion at welded joints, although research into the resultant distortion of welded structures, and the effect on their performance, is reviewed briefly in Section 3.7.

Papers on weld distortion are categorised in this chapter according to the approach adopted. In Sections 3.2 and 3.3, the approach may be characterized as postulating a relationship between the distortion parameters and the welding parameters, and then determining this relationship experimentally. Sections 3.4 and 3.5 describe research where authors have attempted to model the state of stress and strain in and around the weld bead, and thus determine the distortion analytically or numerically.

Formulae resulting from the different approaches are compared in Section 3.6.

3.2 TRANSVERSE SHRINKAGE

In a series of papers, Spraragen et al. (refs. 43, 44, 45) reviewed papers on research into weld shrinkage published prior to 1944. They found that practically no two items of data were directly comparable, and that where comparison was possible, agreement was often poor, possibly as a result of the simplifying assumptions that had to be made.

In his last paper (ref. 45) Spraragen gave an overall formula for transverse shrinkage at a butt weld, relating it to the area of the fusion zone, the plate thickness, and the pre-set root gap. This and all other formulae mentioned in this section are listed in Appendix A5. Spraragen

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states that the values given by his formula should be reduced by up to twenty percent if the plates being welded are thinner than 25 mm, narrower than 75 mm, or shorter than ten times their thickness. The value may also be affected by the degree of external restraint, the welding position, or by the welding process selected. A second formula relates the transverse shrinkage at a fillet weld to the ratio leg length to plate thickness (equations A5.1, A5.2).

Hansen (ref. 15) presents a graph in which the shrinkage at a butt weld is plotted as a function of the mean width of the fusion zone. Another graph relates the shrinkage at butt welds or full penetration T-joints to plate thickness for given angles of preparation of the plate edges. For fillet welds at T-joints, Hansen relates the shrinkage to the ratio throat thickness to plate thickness (equations A5.3 to A5.5).

Early theoretical formulae for transverse shrinkage are based on the assumption that the welded joint is restrained against expansion but is free to contract. The total shrinkage is given by the sum of the solidification contraction of the weld pool and the thermal contraction of solid metal. The shrinkage can thus be related to the size of the weld pool.

Malisius (see ref. 43) relates the shrinkage to the cross-sectional area and width of the weld pool, whilst Wortmann and Mohr (see ref. 43) relate it to the mass of metal deposited. Both these formulae contain arbitrary factors representing the net temperature drop in the weld pool. The formula by Gilde (ref. 12) relates the shrinkage directly to the heat input to the weld and the thermal properties of the parent metal. The derivation of this formula is not stated (equations A5.6 to A5.8).

The remaining formulae in Appendix A5 are derived from more complex analyses which will be described in subsequent sections.

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3.3 ANGULAR DISTORTION

where

Early investigations into angular distortion, reviewed by Spraragen and Cordovi (ref. 44) generally considered one particular weld geometry and measured the effect of varying the number of passes in which the weld was laid. The general conclusion was that angular distortion was proportional to the number of passes. Values of between 0.7 and 1.3 degrees per pass were obtained for double fillet welds.

The variability of these empirical results is due to the fact that the actual shape and size of the weld is not taken into account. Hansen (ref. 15) gives a graph relating angular distortion to the throatto-plate-thickness ratio for given numbers of passes up to three. For butt welds with plate edge preparation between fifty and sixty degrees, Hansen quotes a rotation of 0.75 degrees per pass (equations A6.1, A6.2).

Reference 18 gives three methods for calculating the angular distortion at fillet welds, none of which take into account the number of passes. These and other formulae are listed in Appendix A6. The first, by Blodgett, relates the distortion to leg length and plate thickness. Hirai and Nakamura's results are presented in graphical form: angular distortion is related to weight of metal deposited and plate thickness.

If the joint is restrained as shown in Figure 3.1, then the angular distortion is reduced by the factor

1/(1 + 2D/lc)	(3.1)
$D = Et^3/12(1 - v^2)$ (units not stated)	
l = span between stiffeners	
$c = t^{4} / (1 + w/5)$	
t = plate thickness, mm	
w = mass of metal deposited, gm/cm	

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The third method is by Okerblom (ref. 32). He considers both the contraction across the face of the fillets, which tends to bend the through-plate over the end of the outstand, and non-uniform contraction in the plane of the through-plate, which adds to its angular distortion. The latter factor will be discussed in the next section. Okerblom distinguishes between one side welding, two side sequential deposition, and two side simultaneous deposition (equations A6.3 to A6.8).

3.4 TWO DIMENSIONAL ANALYSIS IN

PLANE PERPENDICULAR TO WELD LINE

The first attempt to analyse the state of stress and strain in and around the weld bead was by Okerblom (ref. 32). He considers a weld bead laid across the short dimension of a narrow strip of metal. The bead might be one pass (not the root) of a multipass weld, or it might be a fillet attaching an outstand to the strip. He cites experimental evidence in support of his assumption that all deformation takes place within a width of plate equal to the maximum width of the weld bead. All the metal in the weld bead is assumed to have a certain thermal contraction. Given the depth, width, and shape of the fusion zone, the distortion is then determined by satisfying conditions of equilibrium and compatibility in the deformation zone, whilst conforming with the material properties. Plasticity of the weld bead is permitted, which leads to solutions which are expressed graphically.

Okerblom then assumes that the fusion zone is parabolic in section, and that its area may be established as a function of the proportion of weld energy used for fusing the base metal and the electrode. The distortions may then be related to the heat input. In the case of a fillet weld, the heat input to each incoming plate at a joint is assumed to be in proportion to its thickness. His graphs for the transverse shrinkage and angular distortion at a short weld as functions of heat input and bead shape are reproduced as Figure 3.2. It can be seen that both graphs have a linear upper envelope, passing through the origin. In other words, the maximum distortion is proportional to the heat input.

Okerblom recognizes that weld distortion increases as the length of the weld increases, up to a limiting value. For angular distortion, he states (without analysis) that the maximum distortion is three times the short-weld value. However, for the transverse shrinkage in a long weld, he abandons the above analysis. Instead, by making the assumption that the integrals across the plate of the transverse and longitudinal plastic strains are equal, he relates it to the heat input to the plate (equation A5.9).

Vinokurov (ref. 49) presents a graph, attributed to Kuzminov, which is reproduced as Figure 3.3, in which angular distortion is related to heat input, weld velocity, and the total thickness of accumulated weld deposit. The basis upon which this graph was constructed is not stated.

Watanabe and Satoh (ref. 50) describe an analysis which is superficially similar to Okerbloms. However, their formulae contain a large number of empirical factors whose values must be determined from the results of distortion experiments for any given set of welding conditions: it is therefore impossible to establish whether the formulae have any general validity (equations A5.10, A5.11, A6.9).

Watanabe and Satoh also consider the effect on weld distortion of external restraint. They introduce the concept of restraint factors, which are defined as follows:

> (i) For transverse shrinkage, the average value of transverse stress along the weld line necessary to produce average shrinkage of 1 mm along the weld line.

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(ii) For angular distortion, the average value of maximum bending stress along the weld line necessary to produce average angular change of 1/t radians, where t is plate thickness in mm.

The restraint factor for a given structural geometry is determined by elastic analysis. The authors give experimental evidence, for a small number of structural geometries, which supports the hypothesis that the reduction in weld distortion caused by external restraint is a unique function of the appropriate restraint factor.

Papers by Sekine and Takashi (ref. 42) and Ueda et. al., (ref. 47) also analyse stresses in the plane perpendicular to the weld line, but their primary aim is to establish the micro-scale distribution of stresses rather than the weld distortion. Sekine and Takashi's paper describes a purely mathematical method of modelling the distribution of stresses by calculating the elastic response to a hypothetical "angle of disclination" - that is, the disappearance of a wedge shaped piece of metal at the weld. Calculated results are said to be "appropriate" to experimental measurements.

On the other hand, Ueda and co-workers have produced a computer program which includes both a finite difference analysis of the heat flow around the weld pool and strains in the deformation zone. The program allows for variation with temperature of the thermal and mechanical properties of the parent, heat-affected, and weld metal. It is capable of analysing either multi-pass welding or thermal stress relief of the joint, and of calculating the residual stress distribution on the basis of surface or sub-surface measurements of strain changes. The model assumes very thick walled structures (up to 300 mm) in which the deformation zone is effectively fully restrained against any net distortion. The computing resources required for this program are such that its use could only be considered for structures whose safety must be guaranteed at all costs - in other words, nuclear pressure vessels. The main limitation to its accuracy is said to be the current state of knowledge of high temperature material properties.

A much simpler finite element program is described by Rybicki et. al., (ref. 30). This does not follow the thermal history of the joint. Instead, the transverse section of the weldment during each successive pass of a multipass weld is divided into zones representing the current weld bead, the heat affected zone, and the remainder. The size and shape of the zones is determined from experimental data. The deformation at each pass is determined by assigning a contractile thermal strain and a value of Young's modulus to each zone, and then finding the equilibrium position assuming elastic behaviour. Only two cases were studied - an 11 pass weld in 25 mm plate and a 6 pass weld in 19 mm plate. The correlation between predicted and measured angular distortions was good.

3.5 TWO DIMENSIONAL ANALYSIS IN PLANE OF PLATE

In the analyses reviewed in the previous section, the interaction between the deformation zone and the remainder of plates being welded was not taken into account. In this section, the stresses and distortions in the plate during welding are considered, but only average in-plane stresses: through-thickness variation of stress, and hence angular distortion, is ignored.

Vinokurov's (ref. 49) approach is analytical. He starts with an expression by Rykalin (ref. 39) for the temperature distribution in an infinite plate due to an elementary instantaneous heat source. The stress distribution due to the temperature strains are determined by elastic analysis. The plate is cut along the projected weld line. The metal movement necessary to relieve the transverse stress along the cut edges is

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then determined. By summing the metal movements due to the elementary heat sources for the whole welding period, the free metal movement is found as a function of the distance from the current position of the electrode. This function has the form shown in Figure 3.4.

In the weld pool there is no resistance to metal movement. At some point behind the electrode the bead solidifies, and the final shrinkage is equal to the metal movement at this critical point. Curiously, Vinokurov does not attempt to establish the value of the final shrinkage in terms of the length of the weld pool; instead, he gives it as an empirical percentage of the maximum possible metal movement, δ_0 , as given in Equation A5.12. Vinokurov's work is particularly related to electroslag welding.

Muraki et. al., (ref. 29) and Kamtekar (ref. 24) have tackled the same problem using elasto-plastic numerical computer models. Muraki used the finite-element method whilst Kamtekar used finite differences. Apart from this, the analyses were very similar. Both used Rosenthal's two-dimensional temperature distribution (ref. 36) and both used Von Mises yield criterion in conjunction with the normality rule for determining changes in plastic strain.

Both found good agreement between computed and experimental values of the distribution of longitudinal stresses. Kamtekar was specifically interested in longitudinal stresses, and made no measurements or calculations of transverse shrinkage. Muraki's experimental evidence is rather sparse: he found only one case where the agreement between predicted and computed transverse shrinkage was good.

A paper by Kamichika et. al., (ref. 22) is concerned with the problem of heat affected zone cracking due to excessive rate of change of plastic strain. The critical region (a strip transverse to the weld line and near to the end of the plate) is modelled as a series of parallel,

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homogeneous elements transmitting transverse stresses only, whilst the rest of the plate is modelled using a hybrid analytic and numerical technique.

3.6 COMPARISON OF WELD DISTORTION FORMULAE

Of the twelve formulae for transverse shrinkage at butt welds listed in Appendix A5, no two relate to the same parametric group. However, by substituting the original author's suggested values for material and empirical constants, and by equating the weld area to the product of weight per unit length and density or average width and thickness, as appropriate, four of the formulae (A5.1, 3, 6, 7) may be expressed in terms of weld area divided by thickness, and three (A5.8, 9, 12) in terms of heat input per unit length per unit thickness. If we then assume that heat input is related to weld area as

$$A_{w} = 0.012 \text{ q/v},$$
 (3.2)

then the seven formulae may be directly compared, as shown on Figure 3.5. There would appear to be a consensus that transverse shrinkage is proportional to heat input, but there is a scatter in the suggested constants of proportionality from 0.0003 to .00336 mm^3/J - a factor of eleven.

Four ways of calculating angular distortion at a double fillet weld are plotted on Figure 3.6.

Again, the range of predicted values of wrap-up is large, though Hansen's two empirical curves, for one and two pass welds, span most of the other results.

^{*} This formula is based on empirical values of weld area and heat input per unit length measured by Kamtekar (ref. 23) for manual, submerged arc, and fusarc processes. The constant has the dimensions mm³/J.

3.7 STRUCTURAL DISTORTIONS AND LOSS OF STRENGTH DUE TO WELDING

Okerblom (ref. 32) offers the most comprehensive and detailed discussion of distortions in structures due to weld shrinkage. He gives many worked examples, including:

Curvature in made-up beams due to longitudinal welds displaced from the centroid.

Curvature in beams due to transverse shrinkage at fillets attaching diaphragms and ribs.

Curvature of tubes due to nozzle attachments.

Additional curvature in made-up curved beams due to welding. Bending of plating due to angular distortion at stiffeners. Control of distortion by clamping during welding.

Buckling of flanges and webs due to longitudinal stresses. With particular reference to curvature in beams caused by transverse welds, Okerblom considers two cases.

- (a) Transverse shrinkage at a butt joint in the flange plate, where the flange is unattached near the joint (Figure 3.7(a)). The rotation is given by $\delta Az/I$ where δ is the shrinkage, Az is the first moment of area of the flange and I is the second moment of area of the beam.
- (b) Transverse shrinkage at a short, eccentric fillet. There is a large restraint against shrinkage, resulting in transverse yield at the weld. Okerblom assumes that all metal heated up to or above twice the "yield temperature" ($\sigma_0/\alpha E$) will be at tensile yield: hence, the curvature can be calculated.

Other examples of weld-induced structural distortion considered by previous workers are:

Twisting of made-up box sections due to shear at corner joints (Vinokurov, ref. 49).

Stress fields and buckling due to circular welds in flat plates. (Vinokurov).

Distortions and stresses in stiffened cylinders due to tendon force and angular distortion at circumferential welds and transverse shrinkage at longitudinal welds (White, ref. 53, 54).

The Welding Institute (ref. 51) publish a practical guide dealing with the principles of distortion prevention, control and correction, which includes an extensive bibliography.

Stability problems in welded structures have been extensively investigated. The International Ship's Structures Congress review paper (ref. 18) gives results of statistical analysis of "unfairness" (i.e. maximum lateral distortion) of plating, and gives the following formula for the average distortion:

$$\frac{\overline{x}}{t} = 0.1 \left(\frac{s}{t}\right)^2 \cdot \frac{\sigma}{E} \cdot \frac{t}{t}$$

where s = frame spacing $t_w = web thickness.$

The buckling of distorted plating under lateral loading is considered, and allowable levels of imperfection are suggested.

Fujita and Nomoto (ref. 11) have analysed buckling of stiffened plating due to angular distortion and shrinkage forces: their analysis predicts the critical heat input for square plate elements of side 500 or 1000 mm. Ueda and Yao (ref. 48) and Bradfield and Moxham (ref. 4) have considered the problem of buckling due to end loading of plate elements containing residual stresses and distortions, using finite element and energy methods respectively.

Faulkner (ref. 8) believes that stiffened cylinders are more sensitive to fabrication imperfections than are flat plated structures, but that strength is more likely to be degraded by residual stresses than by shape imperfections.

3.8 CONCLUSIONS

The detailed comparison of available empirical and theoretical formulae for transverse distortion at welds given in 3.6 serves to underline the difficulty found by an earlier reviewer (ref. 43) in finding any consistency between data from different sources. Whether this should be attributed to systematic differences (different processes, materials, weld geometries, and welding conditions) or to random errors in the measurement of these parameters is not apparent. What is clear, however, is that distortion data is worthless without a full description of the conditions under which it was obtained, and that no general conclusions can be reached unless a very wide range of welding condition is considered.

The advent of the computer has added a new dimension to the possibilities for the theoretical study of weld shrinkage, and its application to the problem is now widespread, especially in America and Japan. The published computer models are remarkable for the variety of methods and assumptions employed: this is a reflection of the fact that, in spite of the computing power now at our disposal, the mechanics of the welding process are so complex as to make a single, complete and detailed model of all aspects of the problem almost inconceivable. Different workers, with different interests - distortion, residual stresses, stability, fracture mechanics, metallurgical properties - must therefore produce models which concentrate attention on those aspects which they believe to be important, and neglect others.

In view of the incompleteness of such models, experimental validation over a wide range of welding conditions is essential. This has not been a noticeable feature of previously published models.

4. EXPERIMENTAL DETERMINATION OF DISTORTION

AT BEAD-ON-PLATE WELDS.

4.1 INTRODUCTION

The overall aim of the experimental programme described in this and the following chapter is to establish the dependence of transverse weld distortion on those factors which may affect it - welding conditions, plate geometry, and material properties.

As with other aspects of welding technology, the main problems in conducting systematic research are the large number of parameters involved and the ever-present scatter in experimental results. To examine every combination of parameters would be a near-infinite task. The approach in this programme of research was to concentrate first on an idealized basic case, and to regard all other sets of welding parameters as variations from this standard.

The <u>basic case</u> is the laying of a bead along the centre-line of a single, large, unrestrained Grade 43 steel plate using the submerged arc process. The use of a bead-on-plate, rather than a butt weld gives more consistent results, and eliminates fit up gap, angular preset, edge preparation and tacking from the list of input variables.

A "large" plate is taken to be one whose width and length are great enough for the distortion to be independent of these dimensions.

The submerged arc process is particularly suitable for experimental welds because of the wide range of currents and velocities which can be used and because of the consistency of these quantities over the weld length. This chapter is entirely devoted to bead-on-plate welds: both to *basic case* welds, where the effect on transverse shrinkage and angular distortion of the primary welding parameter - heat input per unit length on plates of various thickness is investigated; and to other bead-on-plate tests, where secondary parameters such as welding procedure, plate geometry, and material properties are considered.

Chapter 5 will extend this experimental parametric survey to butt welds.

4.2 EXPERIMENTAL METHOD

4.2.1 Welding processes and method

The processes used were submerged arc (sub arc), and metal inert gas (MIG). These are fully automatic processes - i.e. the wire feed and welding speed are machine controlled to preset values - and give much more consistent welds than the manual metal arc process (MMA).

The welds were laid using an MPE15C side beam welding machine (see Figure 4.0). Table 4.1 gives details of welding conditions general to each process. The carriage control panel carried dials indicating current, voltage, and welding speed. Current and voltage were continuously recorded on a Chessel pen recorder, and the welding velocity and wire feed were measured with stopwatch and ruler.

4.2.2 Distortion measurement

The principle device used was a 50 mm gauge length electrical demountable extensometer of the type described by Denston and White (Reference 7) and shown in Figure 4.1. The ball feet of the extensometer locate in indentations in the surface of the specimen: the relative displacement of the feet causes resistance changes in strain gauges which are attached to the waisted section at the top of the cantilever arm. The output was recorded using a Peekel digital strain gauge recorder. The instrument was calibrated in conjunction with the recorder on a micrometer controlled calibration table, to give a direct relationship between displacement and output. The calibration factor was checked regularly, and found to be constant (Figure 4.2). The range of the readings did vary from day to day, and a reading from an "Invar" 50 mm standard bar was included in every set of measurements. Every reading was taken three times: the median value was used.

The indentations in the plate were made by lightly tapping 1 mm diameter ball bearings into its surface (Figure 4.3). The actual contact between the ball feet and the specimen was around the rim of the indentations (Figure 4.4).

At each location where distortion was to be measured, two pairs of indentations were made across the weld line, one on each side of the specimen.

Measurements were made of the separation of the gauge points before and after each weld pass. The distortions are given by

$$\delta = (\delta_{f} + \delta_{b})/2$$

$$\beta_{o} = (\delta_{f} - \delta_{b}) \times 180 / \pi t'$$

where

δ = shrinkage at mid thickness over 50 mm $δ_f = shrinkage at front of plate$ $δ_b = shrinkage at back of plate$ $β_o = angular distortion over 50 mm (degrees)$ t' = thickness of plate plus allowance for offset of ball centres from plate surface = (t + 1.3) mm.

The values of distortion given in the results section are the average of all measurements made in the middle half of the plate length. All measurements were made with the plates free from any restraint. The after-welding measurements were not taken until the plates had cooled to within 3° C of their pre-welding temperature, (as indicated by a limpet thermometer)

eliminating the necessity for temperature correction. The repeatability of the extensometer readings was generally better than $\pm 2\frac{1}{2}$ units, indicating an overall accuracy of $\pm .005$ mm on shrinkage or $\pm .075^{\circ}$ in the angular distortion in an 8 mm plate.

Angular distortion was also measured with two inclinometers, placed simultaneously on the plate on either side of the weld. The inclinometers measure absolute rotations, and were used to check the changes derived from the extensometer readings.

4.2.3 Specimen Preparation

The stages of preparation of the specimens were cutting, grinding, and denting. A typical specimen is shown in Figure 4.5. They were cut from 1220 x 2440 mm sheets of weldable structural steel, using a tractordriven oxyacetylene torch. Except where otherwise stated, the welding direction was parallel to the rolling direction.

Surface scale in the region of the measurement points was removed using a hand held grinder. The extensometer dents were then made as described in the previous section. The location and depth of the dents were controlled by a template consisting of two sheets of 22 gauge tinplate with holes in the appropriate places. The two sheets were drilled in tandem, and clamped together using a spacer bar of thickness equal to the plate thickness. The whole assembly was clamped to the plate during denting, ensuring accurate location of the pips on the back and front of the plate. The balls were held in the templates by masking tape, and were used up to six times each.

The specimens were not stress relieved at any stage, on the assumption that transverse distortion is the product of permanent plastic strains in the immediate vicinity of the weld, where any preexisting stresses will be relieved by the heat of the weld, and cannot effect the final distortion.

4.3 EXPERIMENTAL RESULTS AND DISCUSSION

4.3.1 Basic case - effect of heat input and plate thickness

As described in the introduction, the laying of a weld bead along the centre-line of a single large unrestrained Grade 43 steel plate using sub arc welding is regarded as the basic case with which all other distortion data will be compared. This section describes the results of 25 basic case welds. The plates were all at least 300 mm wide and 1200 mm long, with thicknesses varying from 6.1 to 20.0 mm. Heat inputs varied from 400 to 3600 J/mm. The plate dimensions, welding conditions and distortions are given in Table 4.2. The range of heat inputs used was as wide as possible: it was limited at the bottom end by arc stability and at the top by burnthrough, except in the case of the 20 mm plate where the maximum current was limited by the capacity of the power source.

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The distortions quoted in this section are the average of all values measured in the central half of the plate length, and are derived from measurements on the back and front of the plate as described in 4.2.2.

The transverse shrinkages are plotted against the parameter q/vt (heat input per unit thickness per unit length) in Figure 4.6. Also plotted (dotted line) is the equation

 $\delta = 0.0035 \text{ q/vt} \qquad (4.1)$ which is a purely theoretical formula for the maximum possible shrinkage at a weld, derived in Chapter 8.

The data appears to be grouped about a line parallel to, but below equation 4.1. The following is suggested as a simple summary formula

 $\delta = 0.0035 (q/vt - 50)$ (4.2)

Only the 20 mm thickness data show a consistent deviation from this line: the results are all low by about 0.1 mm. For reasons which will be given in the next section, it is believed that this is caused by the high voltage used, and is not a function of plate thickness. The angular distortions are plotted against the parameter q/vt^2 on Figure 4.7, together with the line

 $\beta_0 = 0.22 \text{ q/vt}^2$ (4.3)

At each thickness, the data points follow the line as heat input increases until some critical value, beyond which the distortion decreases. This occurs because angular distortion is a function of the temperature differential through the plate thickness. When the heat input is large, the temperature is high right through the plate. The most obvious index of the penetration of heat into the plate is the penetration of the weld bead. The bead penetrations were not measured, but have been calculated according to the empirical formula given as equation 2.6, using an average value for the constant of K = 0.027. The magnitudes of the penetration to thickness ratio are given in Table 4.2. The values of this ratio at maximum angular distortion for the four thicknesses tested were 0.53, 0.51, 0.42 and 0.47. All higher power welds had higher penetration ratios. It seems reasonable to conclude that the angular distortion will obey equation 4.3, provided that the bead penetration is less than half the plate thickness.

Attempts to relate post-maximal values of angular distortion to the welding conditions have not been successful. Where the bottom of the fused zone is within a few millimetres of the back of the plate, the penetration is rather unstable: a small fluctuation in current can cause a large variation in penetration, which in turn affects the angular distortion.

In conclusion, equations 4.2 and 4.3 are proposed as basic formulae for transverse weld shrinkage and angular distortion: the latter applying only where the weld penetration is less than half the plate thickness.

4.3.2 Current, voltage, and velocity

In the preceding section, heat input per unit length was treated as a single variable. It does in fact have three components: current, voltage and velocity:

$$v = IV/v$$

Although these three quantities are controlled independently by the operator, many factors will restrict the welding engineer in his choice of parameters. The voltage is largely a function of the operating characteristics of the chosen welding process. At a prescribed heat input and voltage, the current/velocity ratio is also fixed. A high current and high velocity will give a deep narrow bead, whilst a low current and velocity bead will be wide and shallow.

In this section, the effect of this change in weld pool shape is examined by considering the effect of the two different velocities used for welding the 6.1 mm plates (WF series) described in 4.3.1.

The transverse shrinkages and angular distortions are plotted on Figures 4.8 and 4.9. The angular distortion of -5.5° at q/vt^2 of 32.3 J/mm^3 by plate WF4 is not shown: the phenomenon of negative angular distortion will be discussed in Section 5.3.1. The most significant difference between the two velocities is that peak angular distortion occurs at 18 J/mm³ for the high speed weld and at 26 J/mm³ for the low speed weld. This is a direct result of the change in weld pool shape. The high velocity welds are also high current, and hence high penetration welds: thus the peak angular distortion occurs at a lower heat input.

A secondary effect is that in the region where the two sets of points overlap, the shrinkage is about 10 % higher for the high velocity, high current welds. This is another effect of the increased penetration: there is a reduced thickness of unmelted metal under the weld bead, and hence less resistance to transverse shrinkage. The effect on distortion of varying voltage has not been systematically investigated. However, in sections 4.3.1 and 4.3.3, low points on the transverse shrinkage graphs (Figures 4.6 and 4.10) are all associated with high voltages. This too is presumably caused by low penetration: at a given heat input, a high voltage implies a low current.

It is tempting at this point to suggest that weld distortion may be reduced by the use of high voltages and low velocities. This is not however a practical proposition. The object of any structural weld is to achieve a molten zone of a specified depth. The use of a high voltage or low velocity will not reduce the required current: it will merely widen the molten zone, and increase the cost of the operation. The conclusion of Section 4.3.1 could be modified to say that equations 4.2 and 4.3 are the basic formulae for transverse shrinkage and angular distortion at efficient welds: inefficient welds may produce less distortion.

4.3.3 MIG welds

Results from seventeen bead-on-plate welds using the Metal Inert Gas process are summarized in Table 4.3. The MIG process operates in two distinct regimes. At low current and voltage, the electrode dips into the weld pool, where it melts. At high current and voltage, the electrode vaporizes above the weld pool, and is transferred into it as a spray. The two regimes, known as dip and spray, are differentiated on the shrinkage and angular distortion graphs (Figures 4.10 and 4.11). Looking first at transverse shrinkage, all the dip and several of the spray points lie within the range already established for sub arc welds. As before, a few points lie well below the line. The cluster of empty squares (t = 8 mm, spray transfer) were all welded using an uncooled

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welding torch which was not capable of operating at more than 220 amps: when this was attempted, (plates WB10 and WB11) the welding was interrupted by the overheating (and eventual melting) of the contact tip. The higherpower, spray transfer welds were therefore achieved by increasing the voltage without increasing the current; the result was lower than normal shrinkage.

On the angular distortion graph (Figure 4.11), the points lie within 10 % of the line already established for sub arc welds, for heat inputs less than 15 J/mm^3 . At higher heat inputs, the angular distortions are relatively low, either because of excess voltages (as above) or because of excess penetration (as in 4.3.1). In general, equations 4.2 and 4.3 apply to both sub arc and MIG welds.

4.3.4 Second pass welds

When access allows, thin plates are often welded together using two welding passes - one on either side of the plate - in the hope that the angular distortion of the second pass will cancel out that of the first. In this section, distortion changes due to second pass welds are compared with the first pass results described in 4.3.1.

Second passes were laid on the backs of the WE and WG series plates: the welding conditions are given in Table 4.4 and the distortions per pass are plotted on Figures 4.12 and 4.13. The transverse shrinkage is plotted against the usual heat input parameter (q/vt), and there appears to be no significant difference between first and second pass results. The same is also true for angular distortion, but only if the second pass results are plotted against q/vt_c^2 , where t_c is the "current" thickness of the joint, that is, the plate thickness plus the height of the first pass reinforcement (see inset sketch, figure 4.13). These results suggest that the presence of the first bead reduces the angular distortion, but not the transverse shrinkage. This can be explained, qualitatively, if we assume that two effects are at work: the extra thickness of the joint, which would act against either type of distortion; and transverse tensile stresses in the first pass bead, which would increase shrinkage but reduce angular distortion.

The reduced angular distortion at the second pass is reflected in established welding procedures, which normally specify that the second pass should be larger than the first.

4.3.5 Weld length

The definition of a basic case weld (4.1) included the requirements that the transverse distortion should be independent of the plate length. Figures 4.14 and 4.15 show the variation of distortions along the whole length of some of the WB series plates (for welding conditions, see Tables 4.3 and 4.5). Ignoring odd kinks in the curves (which are thought to be bad readings caused by damage to the extensometer location dents) all the graphs show essentially the same features; reduced distortion at the ends, with a level plateau along the central section of the plate. The reduced shrinkage occurs over a length approximately equal to the plate width (300 mm), and hence it is concluded that steady state distortion is achieved over the middle half of a plate whose length is four times its width. All plates welded after the WB and WH series were therefore gauged only over the central portion. The variation of measured shrinkages and rotations on a selection of *basic case* specimens is shown on Figure 4.16 and 4.17. The graphs are reasonably flat, although the WG results were rather unsatisfactory, and would have been more convincing had more readings been taken.

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4.3.6 Plate width (unrestrained plates).

Two series of bead-on-plate welds on unrestrained plates of varying width were performed. The welding parameters are summarized in Table 4.5, and the distortions (scaled to allow for small variations in heat input) are plotted in Figures 4.18 and 4.19. All the graphs have the same features: reduced distortion at low width, rising steeply at first and then levelling off at greater widths. The reduction in shrinkage in narrow plates lends support to the description of the shrinkage mechanism to be put forward in Chapter 8: namely, that the transverse shrinkage is produced as a result of the plate as a whole resisting the outwards expansion of the heated zone, and causing it to yield in compression. When the plate is narrow, it has a small in-plane stiffness, and hence causes less yield and a reduced final shrinkage. This effect is especially strong in the WB series of plates, which, as was noted in 4.3.3, were welded at an unrealistically low current: the penetration was low and the heated zone was therefore particularly resistant to transverse shrinkage.

The reduction of angular distortion at low width gives another interesting insight into weld distortion. As a weld proceeds, a twist is set up between the rotated section behind the electrode and the flat plate ahead (Figure 4.20). If angular distortion was merely the result of weld bead contraction, as is often supposed, the increased torsional stiffness of a wide plate would tend to reduce the angular distortion. In fact angular distortion increases with plate width. The extra in-plane stiffness causes increased compressive stresses at the weld line, which, since the back of the plate is cooler and therefore stronger than the front, causes increased angular distortion.

It will be recalled that the definition of *basic case* welds includes the requirement that the distortion shall be independent of the plate dimensions. We now see that this requirement is not strictly fulfilled:

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at 300 mm, both types of distortion are measurably lower than for wider plates. The extent of this reduction can be quantified by plotting the data on reciprocal axes (Figure 4.21), where the relationship appears to be approximately linear and can be extrapolated to give an estimate of the distortion at infinite plate width. Hence, the apparent reductions in distortion in plates of width 300 mm were:

			WI	4	WE	3	
Reduction	in	8	9	%	8	%	
Reduction	in	ß	5	%	21	%	

The largest reduction is in the angular distortion of the WB plates, which have already been branded as unrepresentative owing to the low currents used (see 4.3.3). The reductions in the WH plates, which fall into the *basic case* category, are less than 10 %.

4.3.7 Plates restrained against angular distortion

The objective of this part of the experimental programme was to investigate the effect on angular distortion of out-of-plane restraint. The plates were held during welding by the restraint rig shown in Figure 4.22. The bed of the rig consisted of four 12" x 4" channels, bolted together; their combined moment of inertia was more than one hundred times that of the thickest specimens. Two 8" x $3\frac{1}{2}$ " channels were bolted to the bed, and effectively encastered the plate edges. Packing pieces held the plate off the table, such that its centre line was free to move downwards during welding. The clamping channels could be moved to accommodate different plate widths. In order to prevent in-plane restraint affecting the final distortions, the edge restraint was released soon after each weld was finished. This did not affect the angular distortion, which, as indicated on Figure 4.20, achieves a steady value a few seconds after the electrode has passed. Overall in-plane shrinkage, on the other hand, occurs later, as the plate loses heat to the surroundings. The plates may therefore be considered to be restrained out-of-plane, but free in-plane.

There were six sets of experiments: three being series of restrained plates of varying width, and three consisting of pairs of free and restrained plates. For each set, the plate thickness and welding conditions were held approximately constant. The welding conditions and measured distortions are summarized in Table 4.6. Where unplanned variations in welding power occurred (series WH, WM, and WN7-9) the angular distortions were scaled in the ratio q/v^* : q/v where q/v^* was taken as 500, 1000, 1200 J/mm respectively.

The angular distortions are plotted as functions of the width between restraints on Figure 4.23. As expected, the distortion was considerably reduced in narrow plates. This reduction may best be quantified in terms of the theoretical formula which will be derived in section 8.4. Several possible versions of the new formula are postulated: the one which is found to give the best overall representation of the present data is equation 8.12c, which may be rewritten as

$$\frac{\beta_{o}}{\beta} = 1 + K \frac{w}{b'} \qquad (4.4)$$

where β_0 / β = the ratio of free to restrained angular distortion

K = material dependent constant

w = joint width, taken as equal to bead width

b' = width between restraints (taken as infinity for unrestrained plates) The ratio β_0/β is plotted against w/b on Figure 4.24. With the

glaring exception of plate WN6, the agreement with the form of equation 4.4 is remarkably good, and is represented by

$$\frac{\beta_{0}}{\beta} = 1 + \frac{25w}{b'}$$
 (4.5)

The low angular distortion of WN6, and its unrestrained partner WN5, are indicative of their being in the post-maximal range. The penetration was high; the back of the plate (under the weld) yielded in compression; and the heated region, though capable of producing a

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significant rotation in an unrestrained plate, produced a negligible amount under external restraint.

4.3.8 Yield strength and rolling direction

The majority of tests used grade 43 plate, with a nominal yield stress of 250 N/mm², but having measured values between 269 and 396 N/mm² (see Table 4.7). The WR series plates were cut from Grade 50 steel, and were included as a direct comparison with the WE series, which had the same dimensions. Results from both series are plotted on Figure 4.25. Also plotted are results from the two WRX specimens, which, unlike all the other plates, were welded at right angles to the rolling direction. The three sets of results are indistinguishable, which suggests that neither yield stress nor rolling direction have any significant effect on weld distortion.

4.4 CONCLUSIONS

The transverse shrinkage and angular distortion occurring when weld beads were laid on unrestrained structural steel plates are summarized in equations 4.2 and 4.3. The distortions were found to be independent of rolling direction, mechanical properties, welding process (sub arc or MIG), welding conditions, and plate length, with the following restrictions:

- (a) The plate length must be sufficient for the distortion to reach a steady state value.
- (b) The weld must be "efficient": the weld velocity and voltage must be appropriate to the current used, which is largely dependent on the penetration required.
- (c) When the penetration is high (greater than half the plate thickness) the angular distortion is reduced and equation 4.3 cannot be applied.

(d) When the bead is a second pass, the angular
distortion in equation 4.3 is related to t_c
(= plate thickness + height of first pass bead).

Distortion was found to be sensitive to the width of the plate. In unrestrained plates the sensitivity was low: the estimated reduction in distortion of a 300 mm plate (as used in most of the tests) compared with a very wide plate, was less than 10 %. Where the plates are restrained against out-of-plane bending, the reduction in angular distortion is given by equation 4.5.

5. EXPERIMENTAL DETERMINATION OF DISTORTION AT BUTT WELDS

5.1 INTRODUCTION

The empirical formulae for transverse distortion at bead-on-plate welds presented in the previous chapter will now be used as a basis for assessing the distortions arising in the more realistic case of butt-welded joints. The presence of the joint introduces several new variables to the problem - edge preparation, fit-up gap, tacking, and multipass welding. The aim of this chapter is to establish if, and under what conditions, the formulae derived for bead-on-plate distortion are also applicable to butt welds.

5.2 EXPERIMENTAL METHOD

5.2.1 Welding Processes and Equipment

The welding processes used were submerged arc and MIG, using the equipment described in 4.2.1 and the general welding conditions given in Table 4.1. The root runs of two of the thick plate multipass specimens were welded using the manual metal arc (MMA) process.

5.2.2 Distortion Measurement

The distortion measurement techniques were identical to those described in 4.2.2.

However, in the case of the thick plate multipass specimens (WL series), extensometer readings were taken on only one side of the plate after each pass: these readings were used in conjunction with the inclinometer readings to calculate the shrinkage at the middle of the plate. A final extensometer reading on the back of the plate confirmed overall agreement between the two methods.

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5.2.3 Specimen Preparation

The stages of preparation for the butt welded specimens were cutting, edge preparation, tacking, grinding, and denting. Of these, edge preparation and tacking were not required on the bead-on-plate specimens and will now be described.

The edge preparations were produced by a variety of methods. The square edges on the WA and WP series plates were left as cut by oxyacetylene, with light hand grinding to remove any slag or other protuberances that would have prevented a close butt. The square edges on the WJ series and the single vee on the WL series plates were cut on a planing machine. The double vee preparations on the WS and WT series plates were produced by a shearing machine. The actual dimension of the vee preparations will be given in the results section.

The two parts of each specimen were held together by MMA tacks. The plates were clamped to a flat table during tacking. The tacks were 50 mm long and spaced at 300 mm, with a 100 mm long tack at the end of the weld run (Figure 5.1). The tacks were made on the opposite side of the joint to that on which the first pass was to be laid.

5.3 EXPERIMENTAL RESULTS AND DISCUSSION

5.3.1 Two pass butt welds - unrestrained

The WS series consisted of fourteen pairs of 8.1 mm thick plates, each with a close-butted double vee edge preparation (Figure 5.2). The welding process was MIG. The welding and distortion parameters are given in Table 5.1. The changes in shrinkage for the first and second passes are plotted against q/vt on Figure 5.3, and are compared with equation 4.2, the summary formula for transverse shrinkage at bead-on-plate welds. The second pass points lie close to the line, which is not surprising since a completed two pass bead-on-plate weld is physically identical to a genuine two pass butt welded joint. On the other hand, the first pass shrinkages were higher than the corresponding bead-on-plate result, and only tended towards equation 4.2 at higher heat inputs. If the shrinkages are plotted against q/vt_c (not shown) where t_c is current joint thickness (Figure 5.2) then the points lie on the line at low heat inputs, but below it at higher heats. Good agreement could be obtained, by adopting a variable effective thickness, whose value ranged from t_c at low heat to t at high heat. In practice, the lower heat inputs are probably unrealistic for two pass welds, as they may lead to lack of penetration at the root, or to the bead failing to fill the preparation. Hence, the use of q/vt for predicting transverse shrinkage using equation 4.2 will generally be satisfactory for two pass butt welds.

In section 4.3.4, it was established that for weld penetrations of up to half the plate thickness, the change in angular distortion at a second pass bead-on-plate weld was given by

$$\beta_0 = 0.22 \text{ q/vt}_0^2$$
 (5.1)

The butt weld results are compared with equation 5.1 on Figure 5.4. As before, the points fall into two categories: those on or near the line, and those well below it. This latter category seem to follow two distinct branches, one of which produces negative angular distortion at high heat inputs. Any plate which had a positive angular distortion before welding (i.e. concave on the welding side) followed the upper branch: convex plates followed the lower branch: approximately flat plates could go either way. The result was that it was virtually impossible to produce a flat plate using high heat input welds: whichever way the plate was distorted before welding, each pass increased the angle. This fact may give a clue to the cause of the bistable effect: it could be caused by the in-plane compressive stresses in the heated zone around the weld causing the plate to "pop-out" in the direction of preset. This effect, though interesting, was not thought important enough to warrant further investigation. In a practical situation, this eccentric behaviour could easily be supressed by the application of external restraint.

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5.3.2 Two pass butt welds - restrained.

Seven pairs of 7.9 mm (WA series) and four pairs of 6.4 mm thick plates (WJ series) were welded using edge restraint as shown in Figure 5.5. This clamping arrangement relies for its stiffness on the stiffness of the trolley structure, and for its alignment on the flatness across the width of the 12" x 4" channels; in neither respect was it felt to be wholly satisfactory, though the degree of misalignment and flexibility was hard to quantify and will be ignored in the preliminary interpretation of the results.

The transverse shrinkages for passes one and two are plotted against q/vt on Figures 5.6 and 5.7, and are in close agreement with the bead-on-plate results as represented by equation 4.2.

The angular distortion for passes one and two are plotted on Figures 5.8 and 5.9. The results show more than the usual amount of scatter, and except for some of the WA series second pass welds, are low compared with the unrestrained bead-on-plate summary formula (equation 4.3). In order to account for these discrepancies, allowance must be made not only for the effect of out-of-plane restraint (which was investigated in 4.3.7) but also for the possible effects of preset angular distortion, and reinforcement. Under these circumstances, the expected reduction in angular distortion is given by

$$\beta = (\beta_{0} - K_{c} w\beta_{1}/b') / (1 + K_{c}w/b') \begin{cases} 5.2\\ 8.12f \end{cases}$$

where

 β = predicted angular distortion

 $\beta_{o} = 0.22q/vt_{c}^{2} = free angular distortion$ $\beta_{1} = angle of preset$ $K_{c} = Kt^{3}/t_{c}^{3}$

K is a material dependent constant, taken as 25 (see 4.3.7)

K and K are dimensionless

t = plate thickness

 t_c = plate thickness plus reinforcement of previous passes. w = bead width

b' = width between restraints.

Actual values of β are plotted against predicted values in Figure 5.10. Although the scatter is reduced, agreement is not as good as was hoped. For the second pass welds, some of the measured values were higher than predicted; this could be due to the inadequacies of the restraint rig already noted. The first pass distortions tended to be below predicted values: flexibility in the rig would produce the opposite effect, so that it may be that angular distortion at the first pass of a butt weld is actually lower than that at a corresponding bead-on-plate.

5.3.3 One pass butt welds - the effects of gap, tacking, and plate length.

A series of four butt welds on 12.8 mm thick plates was used to investigate the effect on transverse shrinkage of varying the initial gap between the square edges of the plates. The gaps varied from zero to 4.5 mm. The plates were restrained during welding as shown in Figure 5.11. A backing bar was used to prevent burnthrough. It also supported the plates along the weld centreline, such that the restraint against positive angular distortion was large (the centre support increased the rotational stiffness fourfold).

In order to allow the shrinkage to reach a maximum value, the plates were tacked at the ends only (Figure 5.12). This had the effect of producing continuously varying shrinkage over the centre section of the plates where measurements were made (Figure 5.13). Only WP1 (zero gap) shows signs of achieving a stable value: at 0.77 mm it compares closely with equation 4.2, which gives 0.78 mm for $q/vt = 273 \text{ J/mm}^2$. In the other plates, the presence of a gap allowed a greater shrinkage to occur.

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The change in shrinkage seems to be independent of the size of the gap: plates WP2 and WP3 had almost identical values: plate WP4 was lower, but had a lower heat input (see table 5.3). The shrinkages can be seen to fall steadily along the length of the plate (Figure 5.13): extrapolation of the graphs to the end of the plate would give shrinkages no greater than the zero gap value. It seems likely that the increased shrinkage was caused by the freedom of the plates to move inwards between the end tacks, and that a well tacked joint <u>with</u> a gap would suffer the same shrinkage as a bead-on-plate or zero-gap butt weld.

To investigate this possibility, a fifth pair of plates (WP5 see Figure 5.14) were welded. This specimen had tacks at every 300 mm, and 28 measurement points over its 2510 mm length. The gap was 3.8 mm. Over the first half of the plate, the backing bar became permanently attached, making shrinkage readings on side 2 impossible. However, as with all the plates in this series, the combination of high restraint and high penetration restricted angular distortion to a negligible value, and the side one shrinkages plotted on Figure 5.15 are close to the mid-plate values. The figure shows that the shrinkage oscillates along the plate length, with minima at the tacks. The shrinkage variations (measured over a 50 mm gauge length) must cause local stress distributions which will balance out at greater distances from the weld. The mean of the oscillations is high at the start of the weld, but levels down to about 0.73 mm over the final 1500 mm of the plate length. Equation 4.2 predicts a shrinkage of 0.77 mm. This close comparison supports the suggestion made above that for a well tacked weld, the presence of a gap causes no increase in transverse shrinkage, provided that the heat input remains constant.

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5.3.4 Multipass Welding

Four specimens were used to study the accumulation of distortion during multipass welding. WLl and WL2 were 20 mm thick and had a 60° single vee edge preparation. The welding procedures were based on recommendations from the Welding Handbook (ref. 2) and consisted of one MIG sealing pass on the back (subsequently ground flush) followed by 3 or 6 submerged arc passes on the front. WT1 and WT2 were 25 mm thick and had a double vee edge preparation. They were welded using procedures approved for North Sea oil rig construction, which consisted of one MMA sealing run and two submerged arc runs on the back, followed by arc-air gouging (for WT1 only) and ten or eleven more submerged arc passes on the front. Apart from the sealing runs on WL1 and WL2, the plates were unrestrained. The welding conditions and measured distortions are summarized in Table 5.4. The weld geometry and the accumulation of angular and shrinkage distortions are shown in Figures 5.16 to 5.18. It can be seen that although the total heat inputs for each pair of specimens (WL or WT) were similar, the final distortions varied considerably. WL2, with three large passes, showed more shrinkage but less wrap up than WL1. This can be explained with reference to the bead-on-plate results. At high power, shrinkage is greater, prorata, than at low power, whilst angular distortion falls off when the heat input is very high.

In Figure 5.17, both WT1 and WT2 show negative shrinkage (i.e. expansion) during the first three passes. This was a consequence of the welding sequence and the edge preparation. The angular distortion of the backing beads caused the large vee preparation at the front of the plates to open, with the abutting noses of the prepared plate edges acting as a pivot. Although the bead itself contracted, there was a net expansion at the mid-section of the plate.

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During the finishing passes, WT1 showed much greater distortions than WT2 (Figures 5.17, 5.18). This also can be attributed to the weld geometry, which, in the case of WT1, was modified substantially by the arc-air gouging carried out after the backing passes were complete. The shrinkage actually caused by the arc-air gouging is not shown. The section was very thin after gouging, so that the shrinkages and angular distortions of the post-gouging passes were very large, and led to overall distortions, of 1.88 mm and 10.3° , compared with 0.28 mm and -1.3° in WT2.

The above comments have concentrated on a qualitative interpretation of the behaviour of multipass welds in terms of the cumulative effects of each pass. In Section 8.5, this process is taken a step farther: equations 8.13 to 8.15 make quantitative predictions of the distortions of multipass welds. These formulae incorporate the empirical expressions for single pass shrinkage and rotation which were presented in Chapter 4. They are plotted on Figures 5.16 to 5.18 along with the experimental results, and are seen to agree with them to within about 10 % for the WT specimens or about 20 % for WL1 and WL2.

5.4 CONCLUSIONS

It was found that the summary formula for transverse shrinkage at bead-on-plate welds (equation 4.2) applied also to the transverse shrinkage at each pass of a one or two pass butt weld, provided that the plates were well tacked before welding. The only exception occurred at low heat input, first pass welds at joints with double-vee edge preparations, at which the shrinkage could be predicted by using the current joint thickness t_c (see Figure 5.2) instead of t in equation 4.2. The same modified formula also applied to multipass welds, at which the heat input of each pass is necessarily small relative to the plate thickness. However,

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the formula gives the shrinkage at the current centre of the joint, which may be displaced from the centre of the plate. If so, a correction must be made for angular distortion using equation 8.15. The angular distortion at multipass welds was given by equation 8.14, which is a modified version of the bead-on-plate formula (equation 4.3) with t replaced by t_c .

The angular distortion at two pass butt welds showed a disappointing degree of scatter. Unrestrained double vee butt joints exhibited a bistable effect, the sign of the rotation apparently being controlled by the sign of any pre-existing angular distortion. Measurements of rotation at restrained square-edge butt joints were inconclusive: it was felt that the stiffness and alignment of the restraint rig were unsatisfactory.

6. COMPUTER PROGRAM

FOR THE ANALYSIS OF TRANSVERSE DISTORTION DURING WELDING

6.1 INTRODUCTION

This chapter describes a numerical model for the development of transverse shrinkage and angular distortion during welding. In common with previous analyses of weld shrinkage and related phenomena reviewed in Chapter 3, the present model represents only a partial solution of the overall problem. Some simplification is essential, as the computing resources required for a full three dimensional transient elasto-plastic analysis using conventional techniques would be prohibitive, in view of the large numbers of elements and loading increments which would be necessary to model the extreme thermal gradients in the vicinity of the moving weld pool.

The most comprehensive of previous models of weld shrinkage have been two dimensional, and have considered stresses and strains either in a plane perpendicular to the weld (section 3.4) or in the plane of the plate (3.5). In each case, the researcher used his engineering judgement to choose which set of simplifying assumptions were most applicable to the particular aspect of weld shrinkage under investigation. It is interesting to note that both in-plane and transverse-section analyses have been applied to the problem of transverse shrinkage, notably by Vinokurov (in-plane, ref. 49) who drew attention to the importance of internal in-plane restraint in determining transverse shrinkage, and by Okerblom (transverse section, ref. 32) whose analysis of weld bead contraction highlighted the interaction between transverse shrinkage and angular distortion in a partial penetration weld.

The present program could be regarded as an amalgamation and extension of these two types of two-dimensional analysis, as it not only models the transverse stresses in a cross-section of the weld, but simultaneously models the stresses in parallel sections along the whole length of the weld line, and balances the resulting longitudinal distribution of transverse shrinkage and angular distortion in the weld against in-plane and bending distortions in the plate.

6.2 MESH LAYOUT

In designing a mesh to represent the structural behaviour of a welded plate, the objective has been to model those features of the behaviour which influence the development of transverse distortion. In choosing the mesh, therefore, the following basic assumptions about the mechanics of weld distortion have been taken into account:

- (i) Permanent plastic strains occur only in the very hot region near to the path of the weld pool.
- (ii) Transverse distortion is controlled by the interaction between the deformations of the heated region and the elastic response of the remainder of the plate.

The mesh used in the program is shown in Fig. 6.1. The plate is assumed to have two regions, the elastic zone (EZ) and deformation zone (DZ). In-plane and bending stresses in the elastic zone are modelled by the superposition of two separate sets of four node finite elements, which share the same rectangular mesh. The corner nodes have two degrees of freedom modelling in-plane displacements (δ_x , δ_y , see Fig. 6.2) and three degrees of freedom modelling bending effects (ω , θ_x , θ_y , see Fig. 6.3). The deformation zone is split into a single row of rectangular elements. Each element is split into a number of layers of equal thickness. Each layer transmits only transverse stresses (σ_y): longitudinal, through thickness and shear stresses are not modelled in this zone. The transverse stresses are transmitted through inflexible through-thickness node lines to those elastic zone nodes which lie on the DZ/EZ interface. There is no other structural connection between deformation zone elements. The transverse displacement and rotation (δ_y, θ_x) of the interface node-lines determine the strains in the deformation zone layers and are the independent variables upon which the whole solution procedure is based.

6.3 PROGRAM ORGANIZATION

The activity list of the FORTRAN programs which implement the mathematical model of weld distortion described in this chapter is as follows:

PROGRAM FEWM1 (Eliminates elastic zone)

1. Read plate parameters. 2. Create overall out-of-plane stiffness matrix. 3. Reduce to interface stiffness matrix. 4. Create overall in-plane stiffness matrix. 5. Reduce to interface stiffness matrix. 6. Store interface stiffness matrices. PROGRAM FEWM2 (Models the weld distortion) 1. Read elastic zone data including interface stiffness matrices. 2. Read material properties. 3. Read weld parameters. 4. Calculate temperatures relative to electrode position. 5. Calculate thermal strains and temperature dependent material properties, and store. 6. Set electrode position, XE = 0. 7. Read deformation zone material properties for current electrode position from store. 8. Guess distortions. Calculate interface forces. 9. 10. Adjust distortions to achieve equilibrium. 11. Increment load history parameters. 12. Print output for current electrode position. 13. Advance the electrode. 14. If weld not finished, go to 7.

15. Stop.

The assumption of elasticity outside the deformation zone allows the displacements of nodes other than the DZ/EZ interface nodes to be eliminated from the calculation. Program FEWM1 (Finite Element Weld Model 1) assembles overall stiffness matrices of the elastic zone, and then reduces them to

"interface stiffness matrices", which relate the forces and moments at the interface nodes to their displacements and rotations. These reduced matrices are then stored on disc or magnetic tape for use by FEWM2.

FEWM2 models the development of transverse shrinkage and angular distortion as the temperature distribution associated with the weld pool moves along the plate. Since the three dimensional temperature distribution is stationary with respect to the heat source, it is possible to calculate temperatures at all required locations relative to the electrode before commencing the solution procedure. Corresponding values of the temperature dependent parameters - Young's modulus, proof stress, and thermal strain are calculated at the same time, and stored in a master array, from which the particular values for each element at each electrode position are read off. A Newtonian routine then adjusts the displacements and rotations of the interface nodes until equilibrium of the elastic zone and deformation zone forces and moments is achieved. The details of the different parts of programs FEWM1 and FEWM2 are given in the following sections.

6.4 FEWM1

FEWM1 is the program in which the overall stiffness matrices describing the in-plane and out-of-plane behaviour of the elastic zone of the plate (see Fig. 6.1) are first assembled and then reduced to *interface* stiffness matrices, which relate the forces and moments at the interface node lines to their displacements and rotations.

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6.4.1 Input parameters

The parameters input to FEWM1 are:

M, N	Numbers of nodes in x and y directions, respectively
ν	Poisson's ratio.
Е	Young's modulus.
t	Plate thickness.
<u>A, B</u>	Vector arrays containing element dimensions.

6.4.2 Out-of-plane elements

The displacements, rotations, forces and moments on a typical rectangular out-of-plane plate element are shown on Fig. 6.3. Each node has three degrees of freedom, and each element has four nodes:

Nodal displacement =
$$\underbrace{\delta}_{n} = \begin{bmatrix} \omega \\ \theta_{x} \\ \theta_{y} \end{bmatrix}_{n}$$
 Nodal force = $\underbrace{F}_{n} = \begin{bmatrix} F_{z} \\ M_{x} \\ M_{y} \end{bmatrix}_{n}$ $\underbrace{M_{y}}_{n}$

Element displacement =
$$\underline{\delta}_{e} = \begin{bmatrix} \underline{\delta}_{1} \\ \underline{\delta}_{2} \\ \underline{\delta}_{3} \\ \underline{\delta}_{4} \end{bmatrix}_{e}$$
 Element force = $\underline{F}_{e} = \begin{bmatrix} \underline{F}_{1} \\ \underline{F}_{2} \\ \underline{F}_{3} \\ \underline{F}_{4} \end{bmatrix}_{e} = \dots (6.2)$

The displacements and forces of the nodes of element e are related by

$$\underline{\mathbf{F}}_{\mathbf{e}} = \underline{\mathbf{K}}_{\mathbf{e}} \cdot \underline{\boldsymbol{\delta}}_{\mathbf{e}} \tag{6.3}$$

The out-of-plane element stiffness matrix $\frac{K}{-e}$ is taken from reference 56. It assumes a shape function of the form

$$w = \alpha_{1} + \alpha_{2} x + \alpha_{3} y + \alpha_{4} x^{2} + \alpha_{5} xy + \alpha_{6} y^{2} + \alpha_{7} x^{3} + \alpha_{8} x^{2} y + \alpha_{9} xy^{2} + \alpha_{10} y^{3} + \alpha_{11} x^{3} y + \alpha_{12} xy^{3}, \qquad (6.4)$$

where all alphas are constants.

6.4.3 In-plane elements

The displacements and forces on a typical rectangular in-plane element are shown on Figure 6.2. Each node has two degrees of freedom, and each element has four nodes:

Nodal displacement =
$$\underline{\delta}_{n} = \begin{bmatrix} \delta_{x} \\ \delta_{y} \end{bmatrix}$$
 Nodal Force = $\underline{F}_{n} = \begin{bmatrix} F_{x} \\ F_{y} \end{bmatrix}_{n}$... (6.5)
Element displacement = $\underline{\delta}_{e} = \begin{bmatrix} \underline{\delta}_{1} \\ \underline{\delta}_{2} \\ \underline{\delta}_{3} \\ \underline{\delta}_{4} \end{bmatrix}_{e}$ Element Force = $\underline{F}_{e} = \begin{bmatrix} \underline{F}_{1} \\ \underline{F}_{2} \\ \underline{F}_{3} \\ \underline{F}_{4} \end{bmatrix}_{e}$... (6.6)

The element displacements and forces are related by

$$\underline{\mathbf{F}}_{\mathbf{e}} = \underline{\mathbf{K}}_{\mathbf{e}} \cdot \underline{\boldsymbol{\delta}}_{\mathbf{e}}$$

The element stiffness matrix $\frac{K}{e}$ is taken from reference 41. As this reference is not widely available, $\frac{K}{e}$ is shown on Fig. 6.4. It is based on a shape function of the form

$$\mathbf{u} = \alpha_1 + \alpha_2 \mathbf{x} + \alpha_3 \mathbf{y} + \alpha_4 \mathbf{xy}$$

$$\mathbf{v} = \alpha_5 + \alpha_6 \mathbf{x} + \alpha_7 \mathbf{y} + \alpha_8 \mathbf{xy}$$

$$(6.7)$$

where all alphas are constants.

6.4.4 Assembly of overall stiffness matrices

The element and node numbering system is shown on Fig. 6.1. As the individual components of the element stiffness matrices are calculated, they are added in to the overall stiffness matrices which relate the displacements of and forces at all the nodes in the elastic zone. These matrices are symmetrical and banded - only the upper diagonal bands need to be stored. For the mesh of MxN nodes shown in Figure 6.1 the maximum bandwidth is (2N + 4) for the in-plane matrix and (3N + 6) for the out-of-plane matrix. Both matrices are stored in the same memory locations, as they are each immediately reduced to the corresponding interface stiffness matrix, and do not need to be stored permanently.

6.4.5 <u>Reduction to Interface Stiffness Matrices</u> *

At the nodes which lie on the EZ/DZ interface, the transverse forces F_y and axial moments M_x in the elastic zone are balanced by the stresses in the layers of the deformation zone elements. At all other nodes in the elastic zone, either the forces are in internal equilibrium, or else are subject to a known displacement. It is therefore possible to reduce the overall stiffness matrices to "interface stiffness matrices" which relate the interface displacements δ_y and θ_x to the corresponding interface forces and moments F_y and M_x .

The procedure is the same for both in-plane and out-of-plane matrices. First, to prevent rigid body displacement, δ_x and ω for the first node and θ_y for the last node are set equal to zero, and the rows and columns of the overall stiffness matrix corresponding to these displacements are deleted. Next, any restraints at edges other than the interface are applied. This again involves deleting rows and columns corresponding to those degrees of freedom which have been removed. The following boundary conditions are allowed:

(i) Edge parallel to interface:

(a) free

or (b) clamped against rotation and vertical displacement

- $(\theta_x, \theta_y, \omega)$
- or (c) clamped against transverse displacement
- or (d) fully clamped.

*Footnote: The reduction of a multidimensional finite element model to a boundary stiffness matrix is called substructuring, and is described in ref. 56.

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(ii) Edges perpendicular to interface: free.

We may then write, for the whole plate,

$$\underline{\mathbf{F}} = \underline{\mathbf{K}} \cdot \underline{\boldsymbol{\delta}} + \underline{\mathbf{F}} \tag{6.8}$$

where \underline{F}_{0} is the vector of initial nodal forces corresponding to the current temperature loading in the plate (see 6.5.2). If subscript 'i' is used to denote interface forces and deflections, and subscript 'j' for other forces and deflections, equation 6.8 may be rearranged and partitioned as

$$\begin{bmatrix} \underline{F}_{i} \\ \underline{F}_{j} \end{bmatrix} = \begin{bmatrix} \underline{K}_{ii} & \underline{K}_{ij} \\ \underline{K}_{ji} & \underline{K}_{jj} \end{bmatrix} \begin{bmatrix} \underline{\delta}_{i} \\ \underline{\delta}_{j} \end{bmatrix} + \begin{bmatrix} \underline{F}_{io} \\ \underline{F}_{jo} \end{bmatrix}$$
(6.9)

Setting $\underline{F}_j = 0$ for internal equilibrium allows $\underline{\delta}_j$ to be eliminated. Hence

$$\underline{F}_{i} = \underline{S}_{ii} \cdot \underline{\delta}_{i} + \underline{S}_{ij} \cdot \underline{F}_{jo} + \underline{F}_{io}$$
(6.10)

where

$$\underline{S}_{ii} = \underline{K}_{ij} - \underline{K}_{jj} \cdot \underline{K}_{jj}^{-1} \cdot \underline{K}_{ji}$$

$$\underline{S}_{ij} = -\underline{K}_{ij} \cdot \underline{K}_{jj}^{-1}$$
(6.11)
(6.12)

and

In addition to the in-plane and out-of-plane interface stiffness matrices,

$$\underline{S}_{ii}$$
, the in-plane version of matrix \underline{S}_{ij} is also stored for transfer to
program FEWM2, as it allows for the effect at the interface of temperature
loading in the elastic zone.

The following precautions are taken to reduce the storage requirements in FEWM1.

- (1) Only the upper diagonal bands of \underline{K}_{ii} and \underline{K}_{jj} are stored.
- (2) $\frac{K}{ij}$ is not square or symmetrical but is stored in banded form.

(3) $\frac{K}{jj}$ ⁻¹ is very large, and is not banded. Only one column is stored at any one time, and immediately used to calculate the corresponding column of \underline{S}_{ij} . (4) Except for \underline{S}_{ii} , the same storage space is used for in-plane and out-of-plane matrices. \underline{S}_{ij} (out-of-plane) may be overwritten by \underline{S}_{ij} (in-plane) since \underline{F}_{jo} (out-of-plane) is zero. Both use the storage locations previously allocated to the original unpartitioned K matrix.

6.4.6 Validation tests

A comparison was made of the load/distortion behaviour of edge loaded rectangular plates calculated using

- (i) The interface stiffness matrices generated by FEWM1.
- (ii) Simple structural theory.

The procedure adopted was to postulate a self-balancing set of loads, and to calculate the corresponding distortions using the appropriate simple theory. These distortions were then multiplied by the interface stiffness matrix, and the resulting forces compared with the original applied loads.

The mesh used is shown in Figure 6.5(a), and the specified forces for the in-plane case are shown in Fig. 6.5(b). A smooth distribution of discrete forces was chosen, in order to avoid load discontinuities which could be modeled by the program but not by the simple theory. The distribution was sinusoidal, with the end loads halved to obtain overall equilibrium. The corresponding distortions were found by using simple beam and shear theory. For a load of F_x at x, the displacement at \bar{X} is

$$S_{X,x} = \left(\frac{x_1}{YL}\right)^3 \left(\frac{3x_2}{x_1} - 1\right) \frac{2}{Et} \cdot F_x - \frac{x_1}{YL} \cdot \frac{2(1+\nu)}{Et} \cdot F_x \quad (6.13)$$
(bending) (shear)

where x_1 is the smaller of x and X and x_2 is the greater, and all distances are measured from the centre-line, at which the displacement and rotation are zero. The total displacement at any node is found by summing the effects of all the applied loads. The resulting vector of nodal displacements was multiplied by the in-plane stiffness matrix to obtain the corresponding "computed" forces, which are compared with the specified forces on Figure 6.5(b). The agreement is within 6 %. The fact that, for the same distortion pattern, the computed forces are always marginally lower than those predicted by simple theory is due to the extra degrees of freedom (e.g. plane sections are not constrained to remain plane) that are allowed in the program.

The out-of-plane routine was checked by a similar procedure. The specified and computed moments at the interface nodes are shown in Figure 6.5(c). For a moment of M_x at x, the rotation at X of a rectangular bar is given by Timoshenko (ref. 57) as

$$\theta_{x} = \frac{x_{1}}{YL} \cdot \frac{6(1+v)}{E+} \cdot M_{x}$$
(6.14)

where the meaning of x_1 and the boundary conditions are as above, and YL is very much greater than t. The agreement between specified and computed moments was better than 2 % at all nodes except the end one, where the discrepancy was 50 %. This occurred because the simple theory assumed that the moment was applied to the end of the plate as a whole, whilst in the program it was applied at the plate edge, and therefore caused local bending in addition to the overall twist.

The overall level of agreement between the computed and specified forces indicates the ability of the program to calculate the forces arising from distortions of the interface with an acceptable degree of accuracy, and demonstrates that the stiffness matrix assembly and reduction procedures were correctly implemented in the program.

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6.5 PROGRAM FEWM2

FEWM2 is the program which actually models the development of transverse distortions in a welded plate. The description of it given in the following sub-sections follows the order of the activity list in Section 6.3.

6.5.1 Input parameters

The parameters input to FEWM2 are as follows:

Elastic zone data (from FEWM1)

XL, YL, t	Elastic zone dimensions			
Eo	Young's modulus			
ν	Poisson's ratio			
M, N	Numbers of nodes in x and y directions,			
	respectively.			
<u>A</u> , <u>B</u>	Vector arrays containing element dimensions			
<u>S</u> ii	Interface stiffness matrices			
Sij	Elastic zone thermal load factor matrix.			

Deformation zone material properties data

Eo	Room temperature Young's Modulus
σο	Room temperature proof stress.
Em	Work hardening modulus.
αo	Room temperature coefficient of expansion.
T _a , T _b	constants defining shape of material
f_{a}, f_{b}	property versus temperature curves.
Weld data	
V, I	Voltage, current
ν, η	velocity, arc efficiency.

w deformation zone width. L number of layers.

6.5.2 Temperature and thermal strain

The number of positions at which temperatures and dependent variables have to be calculated is limited by ensuring that the movements of the electrode, and the mesh spacings in the x direction, are multiples of a common basic length. Then, values of the temperature dependent variables at all required stations relative to the electrode position can be calculated before the solution procedure begins; the value for a given element and a given electrode position is simply read off from the master matrix.

The temperature at any point relative to the electrode position is calculated using the methods outlined in Chapter 2, in which Rosenthal's analytical solution to the flow of heat from a moving point source is used in conjunction with the "effective values" of conductivity and volumetric specific heat defined in 2.4.7. Within the deformation zone, the temperature distribution is computed using the method of images to allow for the finite plate thickness (equation A2.1). In the elastic zone, a two dimensional temperature distribution is assumed (equation A1.1).

The average temperature in any elastic zone element or in any layer of the deformation zone elements is found by numerically averaging the calculated temperatures along its y direction centre line. The temperature strains in the elastic zone elements are converted to equivalent nodal forces:

$$F_{01} = -F_{04} = \begin{bmatrix} F_{ox} \\ F_{oy} \end{bmatrix}$$
$$F_{02} = -F_{03} = \begin{bmatrix} F_{ox} \\ -F_{oy} \end{bmatrix}$$

(6.15)

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where
$$F_{ox} = \frac{TE \alpha t b_i}{2(1 - \nu)}$$

and $F_{oy} = \frac{TE \alpha t a_i}{2(1 - \nu)}$ (6.16)

The element node numbering system is as shown in Figure 6.2. The temperature loads are those forces necessary to prevent the thermal expansion corresponding to the average temperature rise T, and are added into the overall temperature loading vector \underline{F}_{0} (see equation 6.8).

The free thermal expansion in the deformation zone elements is αT . However, the internal restraint of the structure is such that there is heavy restraint against expansion or contraction in the longitudinal direction. The interaction between longitudinal and transverse effects is allowed for in the model on the basis of two assumptions.

(i) that the total strain in the longitudinal direction is zero(ii) that the temperature changes are sufficient to cause yield.

Then, the longitudinal thermal expansion (αT) gives rise to a counteracting longitudinal plastic strain (- αT), which in turn, by the conservation of volume, causes plastic strains of $+\frac{1}{2}\alpha T$ in the y and z directions. Thus, the transverse strain due to the thermal loading is 1.5 αT in the deformation zone elements.

6.5.3 Stress-strain relationships

uniaxial

The algorithm relating stress and strain in the deformation zone element layers is capable of allowing for the following effects:

- (i) Non-linear elasto-plastic stress-strain curves.
- (ii) Temperature dependent mechanical properties.
- (iii) Load reversals.

Footnote: The factor multiplying the free thermal strain is called "the shrinkage ratio". It is the only allowance made for biaxial interaction in the deformation zone. The effects of this simplification are discussed on page 114. The generalized monotonic stress-strain curve shown in Figure 6.6 has been adopted. It is defined in terms of three mechanical properties, E (Young's modulus), E_m (work-hardening modulus) and PS, which is related to experimental values of the 0.2 % proof stress (though it is not actually a proof stress). The values of the mechanical properties used, and their assumed variation with temperature, will be described in 7.2.3.

For a given set of mechanical properties, the stress in a layer of the deformation zone is calculated from its applied mechanical strain, ε_{c} (i.e. total minus thermal strain) as illustrated on Figure 6.7. The load history is defined by three parameters, ε_{w} , the plastic strain at the last strain reversal (zero if no reversals yet); ε_{p} , the plastic strain at the last load increment, and K_{w} , an indicator which takes the value +1 if the load was tensile at the last load stage (as illustrated) or -1 if it was compressive. The bold line defines the relationship between stress and strain for the current loadstage. It has three parts. If the mechanical strain is within the elastic range, AC, the stress is given by

$$\sigma = E \left(\varepsilon_{c} - \varepsilon_{p} \right) \tag{6.17}$$

If the strain is outside the elastic zone and increasing, the load point lies (as it did for the previous load stage) on the monotonic stressstrain curve whose origin is at $(\varepsilon_w, 0)$. If the strain is outside the elastic region and decreasing, the load point lies on the stress-strain curve whose origin is at $(\varepsilon_p, 0)$ and whose orientation is rotated through 180° , but whose shape is the same as the original curve.

The response to load reversals defined by this algorithm lies between the responses predicted by the well-known "kinematic" and "isotropic" models (ref. 58). Using isotropic hardening, which is widely favoured because of its simplicity, the elastic range would be A'C (see strain axis of Figure 6.7) where A'B = BC. Using kinematic hardening, which allows more realistically for the Bauschinger effect, the elastic range would be reduced to A"C, where A"C = $2\varepsilon_y$, and ε_y = PS/E, the yield strain. The present algorithm, for which the author proposes the name "metastatic hardening", is alone amongst the three in having no shift in stress space of the origin of the stress-strain curves. This is essential in the presence of drastically changing mechanical properties, as a shift in the stress origin from a previous load stage might otherwise place the load point outside the permitted stress range (± UTS) of the new properties.

6.5.4 Forces and moments at the interface

The forces and moments at the node lines where the elastic and deformation zone elements meet are functions of the interface distortions. The forces and moments acting on the elastic zone elements are given by equation 6.10, which was

$$\underline{F}_{i} = \underline{S}_{ii} \cdot \underline{\delta}_{i} + \underline{S}_{ij} \cdot \underline{F}_{j0} + \underline{F}_{i0}$$
(6.13)
(6.10)

 \underline{F}_{i} and $\underline{\delta}_{i}$ may represent either the in-plane forces and displacements of the nodes (\underline{F}_{y} and $\underline{\delta}_{y}$) or the out-of-plane moments and rotations (\underline{M}_{x} and $\underline{\theta}_{x}$). For the out-of-plane case, the temperature loading vectors \underline{F}_{jo} and \underline{F}_{io} are zero.

The calculation of the forces and moments acting on the deformation zone is more complex. The force and moment at the ith node line (F_{yi} and M_{yi}) are found by summing the stresses in the layers of the DZ elements

$$\mathbf{F}_{\mathbf{v}\mathbf{i}} = \Sigma \quad \sigma_{\mathbf{i}\mathbf{k}} \cdot \mathbf{b}_{\mathbf{i}} \cdot \mathbf{d}\mathbf{z}_{\mathbf{k}}$$
(6.19)

$$M_{xi} = \Sigma \sigma_{ik} \cdot b_i \cdot (z_k - \frac{t}{2}) \cdot dz_k$$
(6.20)

where σ_{ik} is the stress in the kth layer of the ith element, b_i is its length, and z_k and dz_k are the z coordinate and depth of the kth layer. The stress in any layer is calculated from its mechanical strain (i.e. total minus thermal strain) using the stress-strain relationship described

*Footnote: Goodier and Hodge (ref. 59) describe several hardening models, including one which they call "fixed hardening", whose treatment of load reversals is identical to metastatic hardening.

in 6.5.3. The total strain in layer k of element i is calculated from the nodeline displacements $\frac{\delta}{vi}$ and $\frac{\theta}{xi}$

$$\varepsilon_{ik} = 2\left[\delta_{yi} + \theta_{xi}\left(z_k - \frac{t}{2}\right)\right] / w \qquad (6.21)$$

where w/2 is the width of the deformation zone elements.

6.5.5 Solution Procedure

The objective of the solution procedure is to adjust the guesses of the interface nodeline distortions $\frac{\delta}{y}$ and $\frac{\theta}{x}$ until equilibrium at the interface is achieved. The equilibrium condition is simply that the sum of the forces or moments acting on the abutting elastic and deformation zone elements at each nodeline should be zero. This is achieved using a library routine (ref. 31) which uses a combination of the Newton and steepest descent methods in such a way as to give a steady progress and and a fast rate of ultimate convergence.

The routine is supplied with initial guesses for the distortions, a suggested steplength for the initial determination of the Jacobian matrix relating the rates of change of the out-of-balance forces to the distortions, and a solution acceptance limit, which is expressed as the sum of the squares of the remaining out-of-balance forces. It must have access to a subroutine which relates the forces to the distortions, as described in 6.5.4.

The initial guesses for the distortions are based on the assumption that the distribution of distortions along the weld line is stationary with respect to the electrode, such that if the electrode has advanced $\delta \mathbf{x}$ since the last load stage, the distortion at x is found by interpolation at $(\mathbf{x} - \delta \mathbf{x})$ of the previous distribution of distortions. Once a solution is obtained, the load history parameters for each layer of each deformation zone element, as defined in 6.5.3, are updated to their new values. They are not updated during the solution procedure, as the guess of distortion from which they are calculated may be incorrect. The new plastic strain is given by

$$\varepsilon_{\rm p} = \varepsilon_{\rm c} - \sigma/E \tag{6.22}$$

The strain origin and load direction indicator are unchanged unless there has been a plastic strain reversal, in which case ε_w is set equal to the old value of ε_p and K changes sign.

6.5.6 Output and electrode movement

The following data are available at every load stage (i.e. every electrode position).

For each node line:

Shrinkage	δ
Wrap-up	β
Force	F y
Moment	Mx
Temperature component	
of EZ force	F

For each layer of each deformation zone element:

Femperature		
Proof stress	PS	
Young's modulus	Ε	
Fotal strain	ε	
Thermal strain	$\epsilon_{\rm T}$	
Elastic strain	ε e	
Plastic strain	ε _p	
Strain origin	εw	
Strain indicator	ĸ	
Stress	σ	

A full output of the above data for a typical weld would require about 7k lines of print, so output is restricted to values relating to a mid-plate node line for all but the final loadstage.

The shrinkage and angular distortion (or "wrap-up") are deduced from the node line displacements

$$\delta_{i} = 2 \delta_{yi} \tag{6.23}$$

$$\beta_{i} = 2.180 \ \theta_{xi} / \pi \ (degrees) \tag{6.24}$$

As discussed in 6.5.2, the electrode position advanced in steps whose length was a multiple of a fixed basic length. A typical value was 15 mm, with electrode movements of one basic length per load stage along the entire length of the plate. Additional load stages were considered in which the quasi-stationary temperature distribution accompanying the electrode moved off the end of the plate, again in steps of one basic length. This allowed the equalization of temperatures after welding to be modelled, and was continued until no further plastic straining occurred, at which point all temperatures were set to zero and a final solution obtained.

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7. COMPARISON OF COMPUTED AND EXPERIMENTAL WELD DISTORTIONS

7.1 INTRODUCTION

The computer programs FEWM1 and FEWM2 have been used in a parametric survey of transverse distortions at bead-on-plate welds. The range of parameters considered is based on that of the experimental results described in Chapter 4. A separate section (7.3) is devoted to the effects of parameters whose values are arbitrary, in as much as they define the subdivision of the plate into finite elements, or of the welding process into finite load stages, but do not have any physical significance.

7.2 VALUES OF MATERIAL PROPERTIES USED IN FINITE ELEMENT WELD MODEL

The material property values given in this section are the values used for all runs of the program unless otherwise stated.

7.2.1 Thermal properties for temperature analysis

The values of volumetric specific heat (ρc_e) and conductivity (k_e) used were the "effective" values defined by equations 2.11 and 2.12 and illustrated on Figures 2.4 and 2.5.

7.2.2 Plate properties - FEWM1

The properties of the plate elements were assumed to be constant within the narrow range of temperatures experienced by areas outside the weld zone. The values used were:

Young's modulus, E	210,000	MPa
Coefficient of expansion, $\boldsymbol{\alpha}$	12×10^{-6}	per deg C.
Poisson's ratio, v	0.3	

Apart from v, these are the room temperature values of the temperaturedependent functions to be described in the next section.

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7.2.3 Deformation zone properties - FEWM2

The temperature-dependent properties were represented by linear sections chosen to model the published data as closely as possible. Adopted and published values of yield stress, Young's modulus and coefficient of expansion are shown on Figures 7.1 to 7.3. The published elevated temperature mechanical properties data is both sparse and scattered, and there is certainly scope for further research on this aspect of material behaviour.

The room temperature value of 0.2 % proof stress was set at 307 MPa a typical value for the Grade 43 plates used in this investigation (see Table 4.7). The work hardening modulus, E_m , was set equal to a constant value of 3900 MPa. The value ascribed to E_m is rather arbitrary, as it is based on very little data, and is sensitive to the strain range considered and the condition and composition of the material. In retrospect, the value chosen appears to be rather low. A brief discussion of values for the work-hardening modulus extracted from published data, and their effect on weld distortion, is given in 7.4.4. The stresses were not allowed to rise indefinitely (as implied by Fig. 6.6) but were limited to an arbitrary ultimate value of twice the proof stress. 7.3 SENSITIVITY OF COMPUTED RESULTS TO ARBITRARY PARAMETERS

The subdivision of the structure into finite elements, or of the welding operation into finite timesteps is arbitrary in as much as the chosen intervals have no physical significance. It is therefore necessary to demonstrate that the discretization is sufficiently fine that a further reduction will have a negligible effect on the computed results.

The standard mesh spacing for the *basic case* results (see next section) is illustrated on Figure 7.4. The elastic zone was split into a 3 x 16 mesh of 60 x 60 mm square elements, and the deformation zone elements were each split into ten layers. The first position of the electrode for which a solution was obtained was x = 0. Subsequently, the

electrode was moved along the plate centreline in steps of one quarter of the mesh spacing, i.e. 15 mm. At each solution point, the criterion for a successful solution was that the squares of the out-of-balance forces and moments, summed over all the interface nodes, should be less than 10^6 (N, mm units). This corresponded to a maximum error in terms of the direct or bending stress at any one node of 3 MP $_{\odot}$

The width of the deformation zone does have some physical significance, as it defines the region within which all yield is assumed to occur. However, the assumption that the interface between the elastic and plastic regions should be an xz plane, and that the width of the deformation zone should be equal to the width of the weld bead, is arbitrary. Assumed values for the width of the DZ for the various plate thicknesses were:

t(mm)	6	8	13	20
w(mm)	12	15	18	18

The greatest change in computed shrinkage or angular distortion (output) caused by gross changes in the arbitrary parameters (input) for a range of heat inputs in 8 mm thick plate are listed below:

Parameter	Units	Standard Value	Trial Value	Percentage Input	change Output
Layers	-	10	8	-20	+1
Electrode step	mm	15	30	+100	+4
Accuracy	MPa	3	1.5	-50	0
DZ width	mm	15	12	-20	-4
DZ width	mm	15	18	+20	-3
Mesh size	mm	60	45	-25	+7

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It is considered that in view of the scatter of \pm 15 % or more which is quite normal in experimental determinations of weld distortion, changes of 5 % or less in the computed results were acceptable. By this criterion, it would have been desirable to adopt a smaller standard mesh size (although the 7 % discrepancy was an isolated value, the other changes being 5 % or less). However, the use of a finer mesh in the x-direction increases the number of independent variables, and hence the overall solution time. In order to avoid further restrictions on the number of cases that could be run, the larger mesh size was used.

It is interesting to note that the distortion was relatively insensitive to the deformation zone width. This is consistent with the description of the mechanism of transverse shrinkage given in Section 8.1, in which permanent plastic deformation at the weld centreline is related to the thermal strain integrated across both the plastic *and* the elastic regions: the exact position of the boundary is not significant.

7.4 COMPARISON OF COMPUTED AND EXPERIMENTAL WELD DISTORTIONS 7.4.1 Basic case - the effect of heat input and plate thickness

Program FEWM2 has been used to calculate mid plate transverse distortions corresponding to the experimental *basic case* welds described in Section 4.3.1, which demonstrated the effect of varying heat input in bead-on-plate welds on material of thicknesses from 6 to 20 mm. The computed values of transverse shrinkage and angular distortion are plotted against the usual heat input parameters on Figures 7.5 to 7.12, and are compared with the relevant experimental results, which consist of not only the *basic case* data, but also all data from sets of welding conditions which gave rise to distortions lying within the *basic case* scatter band, and include MIG welds, butt welds with square preparations, the second passes of V butt welds, and plates of Grade 50 material.

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Examination of the figures shows that the computed values follow the general trend of the experimental results and that the agreement is closest at mid-range: the program tends to overestimate both types of distortion at low heat inputs, and to underestimate the shrinkage at high heat inputs. The experimental plots of angular distortion versus q/vt^2 show a maximum which varies from 2.5° in 13 mm plate to 4.5° in 6 mm plates, whilst the equivalent computed maxima vary only from 2.9° to 3.4°.

It is suspected that the areas of disagreement stem from inaccuracies in the thermal model. It will be recalled from Chapter 2 that the temperature calculating routine adopted herein was based on Rosenthal's analytical solution to the heat flow equation, but used empirically determined "effective" values of the thermal properties, and that this procedure gave satisfactory predictions of thermal cycles at the back of the plate. However, no solution based on the assumption of a point source of heat can accurately model the weld pool boundary, whose shape is sensitive to welding parameters, especially current. The empirical formula for weld bead penetration given in 2.3.3 suggests that it is proportional to current to the power $\frac{4}{3}$, compared with the power of $\frac{1}{2}$ implicit in the analytically based models. Clearly, the calculated penetration will be too high at low current, low heat input welds, and too low at the other end of the scale. The satisfactory calculation of temperatures at stations displaced from the weld line, as reported in Chapter 2, suggests that the effect of this discrepancy dies away with distance from the weld pool boundary. Nevertheless, there will be an intermediate region in which the accuracy of the calculated values is affected.

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The difference between the computed and empirical distortions can be attributed directly to errors in the calculated penetrations. Excessive calculated penetrations at low heat input welds would cause the observed overestimation of shrinkage and angular distortion. It can be shown that the analytical model predicts that the penetration to thickness ratio (p/t) is equal to a half at a fixed value of q/vt^2 of about 15 J/mm³. As in the experiments (see 4.3.1), maximum angular distortion occurs when $p/t = \frac{1}{2}$: but if the penetration is incorrect the point of maximum distortion will also be wrong.

It is therefore apparent that better agreement between computed and experimental results could only be achieved by employing a numerical temperature model which used the actual shape of the weld pool as a boundary condition, such as those described in 2.2.4. Of these, only Friedman's model (Ref. 9) has been applied to a distortion problem: no experimental confirmation of the predicted transverse shrinkage of a toroidal seal weld was offered.

In conclusion, transverse distortions calculated by the present model are most representative of experimental values when the welding conditions are such that the calculated and empirical weld bead penetrations coincide.

7.4.2 Welding Velocity

As explained in the previous section, the point-source temperature distribution used in FEWM2 cannot model the variations in weld pool section which are caused by changing current, voltage or velocity at constant heat input, and will therefore not produce the significant change in peak angular distortion that was observed experimentally when the welding velocity was changed (see Figure 4.9). However, the use of a higher velocity also produces an elongation of the isotherms, which is correctly modelled.

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Hence, the rate of change of distortions with respect to distance from the heat source is increased, and the balance of forces and moments between the joint and the plate is affected. This may produce a small change in the final distortion, as seen on Figures 7.5 and 7.6, where the doubling of the weld velocity has produced a change of up to 10 % in the resultant shrinkage or angular distortion.

7.4.3 Variation of distortion along the weld length

Computed and experimental values of transverse shrinkage and angular distortion for specimen WB14 are plotted on Figures 7.13 and 7.14 as functions of the distance along weld. The computed transverse shrinkage follows the experimental value very closely, with a reduction in shrinkage at both ends, a maximum at x = 200 mm, and a plateau in the central region The heat input to WB14 lay in a region in which the program tended to overestimate the angular distortion: it has succeeded however in modelling the way in which β builds up at the start of the weld and then reaches a steady state value. A strange anomaly occurs at the end of the weld: the experimental angular distortion falls off, whilst the computed values show a small increase. This occurred because, unlike sections at mid-length, the end of the joint yielded to a small positive angle of distortion before the electrode reached it. Why this does not happen in real plates is not known.

The reader may have noticed that the standard plate length used in the runs of the program was 960 mm, compared with actual plate lengths of between 1200 and 1400 mm. This shortening (which produced a proportional decrease in computing time) is justified by the existence of the central plateaux in both distortion graphs. Once steady state conditions have been reached, further computation of distortions at stations remote from the ends of the plate serves no useful purpose.

7.4.4 Rotational restraint

The program has been used to investigate the theoretical formula for the effect of rotational restraint on angular distortion which will be put forward in Chapter 8:

$$\frac{\beta_{o}}{\beta} = 1 + \frac{\frac{1}{b'}}{\frac{1}{K_{1}\lambda vt^{2}} + \frac{E'}{Ew}}$$
(8.12a)

The relationship was investigated by considering a fixed set of welding conditions (q/v = 1000 J/mm, t = 8 mm, b' = 360 mm) and then finding the effect on β_0/β of variations in λ^* , v, E', and w. The results are plotted on Figure 7.15. They show that the ratio of free to restrained angular distortion, β_0/β , was insensitive to changes in the term Ew/E'b', which describes the relative stiffness of the joint, but varied in a near-linear manner with respect to $\lambda vt^2/b'$, which reflects the ratio between the permanent moments caused by the action of the external restraint, and the transient moments caused by distortion differentials during welding. In other words, the second term on the bottom line of equation 8.11 was found to be negligible, and the relationship was given by

$$\frac{\beta_{0}}{\beta} = 1 + \frac{0.72 \,\lambda v t^{2}}{b'}$$
(7.1)

The reason why the joint stiffness was negligible was apparent from the computer output: at each section of the weld, the joint was at some stage loaded to its ultimate moment, even in a free plate. In other words, the joint stiffness (at the crucial time) was zero.

Footnote

 λ is defined as volumetric specific heat divided by twice the conductivity, $\rho c/2k$. The values of these quantities used in the program are not constant (see 7.2 and Figures 2.4, 2.5). For the purposes of this section, λ is characterized by the constant values of ρc_e and k_e that are assumed to occur at temperatures of over 1000°C.

Please note that E_m and E' are synonymous.

Experimental and computed values of β_0/β for a range of free spans are plotted on Figure 7.16. The solid line was obtained using the standard values of mechanical and thermal properties described in 7.2. Clearly, it failed to model the reduction in angular distortion that was observed at increasing levels of restraint. For the reasons outlined above, no improvement could be obtained by adjustments to the mechanical properties of the joint. Of the two thermal properties which define λ , the volumetric specific heat cannot be altered without proportionately altering the peak temperature reached, and hence the magnitude of the free as well as the restrained distortion. On the other hand the conductivity does not affect the magnitude of temperatures, but does affect the time and distance over which the distribution is spread, and therefore has a direct effect on the "internal stiffness" of the plate in resisting distortion differentials. In order to match the experimental results, it was necessary to reduce the constant value of k (which occurs at temperatures in excess of $400^{\circ}C$, see Figure 2.5) from 0.023 to 0.008 J/mm sec deg C. As the former value of ${\bf k}$ was obtained from measured rise-times at the back of the joint, this adjustment cannot be justified by independent experimental evidence. Nor can it be argued that the use of effective rather than actual values has caused the discrepancy, since all the empirical changes have increased λ and thus lessened the problem. The inference is that the effective values of k in the middle of the plate are even lower than the observed values at the back surface.

The effect of changes in conductivity on the distortions in free plates is identical to that of proportional changes in the welding velocity, which was discussed in 7.4.2.

7.4.5 Mechanical properties

The effect of a 33 % increase at all temperatures in the proof stress of the joint material is illustrated on Figures 7.9 and 7.10. Although the increased strength of the joint does make it more resistant to transverse shrinkage, and able to develop a slightly larger maximum angular distortion, the changes are marginal, as observed in practice (see 4.3.8).

The effect of changes in elevated temperature properties only has also been investigated. For the standard runs, the proof stress was assumed to vary from 20 % of the room temperature proof stress at $750^{\circ}C$ to zero at $1100^{\circ}C$ (see Figure 7.3). Distortions calculated with the $750^{\circ}C$ strength factor reduced to 10 % are plotted on Figures 7.7 and 7.8. The change in properties does have a significant effect: transverse shrinkage at high powers is increased, and the maximum angular distortion is reduced by 20 %. It is therefore possible that changes in elevated temperature material properties are partly responsible for the scatter in the experimental results.

7.5 CONCLUSIONS

The computer programs FEWM1 and FEWM2 have been used to model the development of transverse distortions for a range of welding conditions, plate geometry, and material properties, and compared with experimental measurements.

The computed results agreed qualitatively with the principal conclusions of the experimental results reported in Chapter 4, namely

- (i) The transverse shrinkage is primarily a function of the heat input parameter q/vt.
- (ii) The angular distortion in an unrestrained plate is primarily a function of the heat input parameter q/vt^2 .
- (iii) External rotational restraint reduces the angular distortion in such a way that there is a linear relationship between its reciprocal and the bending stiffness of the free span of the plate.

- (iv) Transverse distortions are insensitive to changes in theroom temperature proof stress of the plate material.
- (v) Both types of transverse distortion achieve steady state values over the central part of a weld whose length is equal to or greater than about three times the width of the plate.

The quantitative agreement between computed and experimental values was best at the middle of the range of heat inputs, when the shape of the weld fusion zone was most likely to coincide with the shape predicted by the thermal model. The program tended to overestimate both types of distortion at low heat inputs and to underestimate the transverse shrinkage at high heat inputs. The maximum angular distortion did not display the sensitivity to welding current which had been observed in practice. Additionally, the program failed to model the full extent the reduction of angular distortion caused by the application of rotational restraint. This latter disagreement could be removed by substantially reducing the input value of thermal conductivity, but this adjustment could not be supported by independent experimental evidence. All the quantitative discrepancies mentioned above can be related to presumed inaccuracies in the temperature model, and highlight the critical importance of the temperature distortion.

The program is, as far as the author is aware, the only one to have been published which simultaneously models transient in-plane and out-ofplane distortions over the length of a plate during welding. It has provided an invaluable test bed for the ideas which will be presented in Chapter 8.

8. THE MECHANICS OF WELD DISTORTION

8.1 TRANSVERSE SHRINKAGE - A THEORETICAL MAXIMUM

The term "weld shrinkage" conjures up a mental image of a molten bead solidifying and cooling, and pulling in the adjacent plate edges as it contracts. Deeper consideration reveals that this is not the case. In a cross section through a plate during welding (Figure 8.1(a)) the weld bead is the hottest, and therefore the weakest part of the section. The adjacent plate edges are heated to a lesser temperature, and suffer a net thermal expansion. The heated region is small at first, and is entirely surrounded by a jaw-shaped region of cold metal which tends to prevent any outwards expansion. The expansion therefore acts inwards, causing compressive yield at the weakest point - the weld bead. Transverse shrinkage is controlled not by the contraction of the enfeebled bead, but by the inwards movement of the abutting plate edges.

The reduced shrinkage of narrow plates observed in Chapter 4 can be accounted for in terms of the above description of the mechanism of transverse shrinkage. In a narrow plate, the inplane stiffness is low, and some outwards expansion of the heated region can occur. Accordingly, the yield at the centre line, and hence the final shrinkage, is reduced.

At the other end of the scale, if there is an infinite resistance to outwards expansion, and no resistance to inwards expansion, the final transverse shrinkage will have a maximum value which is quantifiable in terms of the heat input to the weld, the plate thickness, and the material properties.

Figure 8.1(b) shows a typical pattern of isothermal contours in a large plate during the laying of a full penetration weld, as predicted by Rosenthal's solution for the two dimensional heat flow from a moving line heat source (reference 36). We consider a particular transverse section through the plate, where the peak temperature (i.e. the centre-line temperature) is equal to T_{b} , the temperature above which the weld metal has no measurable strength (Figure 8.2). We assume that, since the isothermal contours are long and thin, there is very little heat flow parallel to the welding direction, so that the total heat per unit length in any section is equal to the heat input per unit length from the torch to the plate. Hence, the free thermal expansion of the heated region is given by

$$\delta = \frac{\alpha}{\rho c} \frac{\eta q}{v t} \quad (mm) \tag{8.1}$$

where

v = weld speed (mm/sec)

 $q = I \times V = weld power (J/sec)$

 η = proportion of weld power entering the plate

- t = plate thickness (mm)
- ρc = volumetric specific heat (J/mm³ °C)

 α = coefficient of thermal expansion (/^oC)

Since the heated region has zero strength at its centre-line, any transverse expansion will cause a corresponding amount of centre-line yield. However, there is restraint against thermal expansion in the <u>longitudinal</u> direction. In an infinite plate, all longitudinal expansion will be prevented, and this will cause additional expansion in the transverse direction. If the longitudinal thermal expansion is accommodated elastically, the elastic strain in an element heated to $T^{O}C$ will be $-\alpha T$, and the total strain in the transverse direction will be $(1 + \nu) \alpha T$, compared with a free expansion of αT . The inwards movement will be increased by a similar factor, giving a final shrinkage of

$$\delta_{o} = \frac{(1+v)\alpha}{\rho c} \cdot \frac{\eta q}{vt}$$
(8.2)

In practice, the temperature changes will be large enough to cause yield in the longitudinal direction. This will cause yield in the transverse direction, which will not only affect the metal movement at the centre-line, but also cause permanent dimensional change in the heated region. A full elasto-plastic analysis is given in Appendix A7. The analysis shows that the net effect on the shrinkage of the longitudinal restraint in an element heated to a certain temperature is a function of that temperature and of the material properties. For common weldable structural steels, the average value of the factor increasing shrinkage is 1.36, compared with 1.30 (i.e. $(1+\nu)$) for the purely elastic case. Since the difference is small, it is convenient, for the sake of generality, to ignore the effect of plasticity and to regard the transverse shrinkage, as given by equation 8.2, as a theoretical maximum with which experimental values can be compared.

The above derivation may be compared with an analysis by Vinokurov (reference 49) which was described briefly in Section 3.5. Both derivations start with the same premise, that the shrinkage is controlled by metal movement at the abutting edges: but Vinokurov, by a rather circuitous route which involves numerical analysis of the two dimensional temperature and elastic stress fields during welding, arrives at a version of equation 8.2 in which the factor (1 + v) is replaced by a factor of 2. However, he later applies an empirical correction factor of between 0.5 and 0.7, which brings the overall factor down to 1.0 to 1.4; the two formulae then become compatible.

8.2 TRANSVERSE SHRINKAGE IN WELDABLE STRUCTURAL STEEL

For a given welding process, the constants in equation 8.2 can be evaluated to give a direct relationship between maximum shrinkage and heat input per unit length per unit thickness. Published values of linear expansion and volumetric specific heat for a weldable structural steel are plotted against temperature on Figure 8.3. The temperature at which the steels have negligible strength is approximately 750° C. Although the individual material properties vary considerably in the range 0 to 750° C, the ratio $\alpha/\rho c$ is approximately constant, and has an average value of

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 $0.0034 \text{ mm}^3/\text{J}.$

The value to be ascribed to the process efficiency η is debatable. Christensen (reference 6) quotes 0.66 to 0.69 for MIG welds in steel, and 0.91 to 0.99 for submerged arc welds. However, in White's experimental and theoretical study of longitudinal shrinkage in welded steel plates (reference 54), it was found that there was no discernible difference between the two processes and that the efficiency appeared to lie in the range 0.7 to 0.9. A value of 0.8 will be used here.

Hence, with Poisson's ratio equal to 0.3, the maximum theoretical transverse shrinkage is given by

$$\delta_{0} = 0.0035 \frac{q}{vt}$$
 (8.3)

Experimental measurements of transverse shrinkage at bead-on-plate and butt welds were compared with the shrinkage predicted by equation 8.3 in chapters 4 and 5. The evidence showed that on a graph of δ versus q/vt, the results lay on a line which was parallel to, but below the predicted line, and could be summarized as

$$\delta = 0.0035 (q/vt - 50) (mm)$$
 (8.4)

It will be recalled that the original formula (equation 8.3) gave the maximum possible shrinkage at a weld. It seems that a fixed amount of the expansion of the heated zone is absorbed in the plate, rather than pushing inwards and causing yield at the weld centre-line.

Equation 8.4 is compared with the summary graph of published weld shrinkage formulae on Figure 3.5. Previous workers found that transverse shrinkage was proportional to either q/vt or a related parameter, but offered widely varying constants of proportionality. The present formula suggests an interpretation by which the apparent spread of previous results may be explained: it is compatible with each of them *over a limited range*. The range of applicability of the previous results was not closely defined, but there was a general trend from the low heat input bead-on-plate welds considered by Okerblom to the massive Electroslag welds of Vinokurov.

8.3 FREE ANGULAR DISTORTION

The description of the mechanism of transverse shrinkage given in 8.1 considered the case of a full penetration weld, where the temperature distribution is uniform through the plate thickness. In a partial penetration weld, however, there is a large temperature differential between the molten metal at the top of the joint and the cool metal below it. There is a significant time lag before this temperature differential is dissipated by heat flow through the thickness, so that at stations behind the electrode, the top of the joint is hotter than the bottom. The top therefore has a greater free thermal expansion and a lower strength, so that, under the action of the adjacent heated plate edges pushing in towards the weld line, the joint yields non-uniformly through the thickness, and a permanent plastic rotation is locked into it.

On purely geometrical grounds, a maximum value for this rotation would be obtained if the maximum shrinkage δ_0 given by equation 8.3 was not uniform through the thickness, but varied linearly from $2\delta_0$ at the top of the plate to zero at the back. Then,

$$\beta_{\max} = \frac{2\delta_0}{t} \ge \frac{180}{\pi} = 0.40 \text{ q/vt}^2$$
 (8.5)

The experimental evidence in Chapters 4 and 5 confirms that angular distortion is proportional to q/vt^2 , but the *empirical* constant of proportionality was 0.22. Hence, the angular distortion in an unrestrained plate is given by

 $\beta_0 = 0.22 \, \mathrm{q/vt}^2$ (8.6)

where β_0 is in degrees and q/vt^2 has the dimensions J/mm^3 . This formula applies only when the penetration of the weld bead is less than half the plate thickness. At higher penetrations, the back of the joint is softened by the heat of the weld, and the angular distortion is reduced.

The occurrence of a maximum in the graph of angular distortion against heat input has been demonstrated theoretically by Okerblom (ref. 32 - see section 3.4), in his analysis of the transverse distortions due to the contraction of the weld bead. However, this analysis ignores the effect of the heat outside the weld bead and leads to predictions of distortion which are much lower than those observed in practice. Nevertheless, Okerblom's results support the view that angular distortion in the postmaximal range is critically dependent on the penetration-to-thickness ratio. Since the penetration is unstable when the bottom of the bead is close to the back of the plate, it is the present author's opinion that it is not possible to make reliable predictions of the angular distortions of welds whose penetration to thickness ratio is high.

8.4 RESTRAINED ANGULAR DISTORTION

Figures 8.4, (c) and (d) show the variation along the weld (as computed by the program described in Chapter 6) of the moments per unit length and angular distortion for two plates with identical welding conditions, but of which one was unrestrained and the other was subjected to rotational restraint. In both, the angular distortion builds up to 95 % of its final value over a length of about 120 mm. The difference in rotation between sections ahead of and behind the electrode represents a twist in the plate which is balanced by a moment on the joint. The joint moment has a maximum value at 120 mm from the electrode, and then, in the free plate, dies away to zero as the distortion stabilizes. In the restrained plate, the moments on the joint must also carry the overall bending moments caused by the external restraint, so that the moments build up to a higher maximum and then die back to a finite value.

The ratio between the maximum moments in the free plate and the final angular distortion may be used to characterise the "internal stiffness" of the plate for the given set of welding conditions:

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$$K_{i} = M_{max} / \beta_{o}$$
 (8.7)

The internal stiffness is a function of the flexural rigidity of the plate, D, and the wave length of the disturbance (in this case 120 mm) which, by analogy with the rise time formula given in equation A2.7, is proportional to λvt^2

i.e.
$$K_{i} = D/K_{1}\lambda vt^{2}$$
 (8.8)

where D = Et³/12(1 - v^2) and K₁ is a non-dimensional constant.

The external stiffness of the restrained plate is the ratio of its steady state moments to its steady state rotation:

$$K_{e} = \frac{M_{e}}{\beta} = \frac{D}{b}$$
(8.9)

where b is the free span between infinitely stiff rotational restraints.

The combined internal and external moments $(M_e + M_{max})$ act on the joint, which at this stage is still hot and is likely to be in a work hardening regime, in which case it has a stiffness which can be characterised by

$$K_{j} = \frac{E't^{3}}{(1 - v^{2})w} = \frac{D}{w} \cdot \frac{E'}{E}$$
 (8.10a)

where w is the bead width and E' is the average work hardening modulus. The above definitions of the external, internal and joint stiffnesses allow a simple formula for the reduction in angular distortion due to restraint to be derived. We consider the section at which the moments are at a maximum, and compare the maximum moments in a restrained plate with those in the free plate. Then, the additional moments necessary to prevent the joint from contracting to β_0 , its free value, are equal to the external moments less the reduction in internal moments due to the reduced distortion, i.e.

$$K_{j} (\beta_{o} - \beta) = K_{e} \beta - K_{i} (\beta_{o} - \beta)$$

$$\frac{\beta_{o}}{\beta} = 1 + \frac{K_{e}}{K_{i} + K_{j}}$$
(8.11a)

or

Substitution from equations 8.8 to 8.10 gives

$$\frac{\beta_{o}}{\beta} = 1 + \frac{\frac{1}{b'}}{\frac{1}{K_{1}\lambda vt^{2}} + \frac{1}{wE/E'}}$$
(8.12a)

Since the values of K_1 and E' are not well defined, this equation is not seen as a precise method of predicting restrained angular distortion, but as an indication of the form of the relationship. For a given set of welding conditions, all the quantities on the bottom line are constant, and the equation reduces to

$$\frac{\beta_{o}}{\beta} = 1 + \frac{\text{constant}}{b'}$$
(8.12b)

which is its most generally applicable form. If the heat input parameter q/vt^2 is low, and the weld velocity v is high, then the internal plate stiffness may be negligible compared with the joint stiffness, so that

$$\frac{\beta_0}{\beta} = 1 + \frac{wE}{b'E'}$$
(8.12c)

If q/vt^2 is high, then the joint stiffness tends to zero, and

$$\frac{\beta_{o}}{\beta} = 1 + \frac{K_{1}\lambda vt^{2}}{b'}$$
(8.12d)

For a range of values of plate thickness t, the joint width w will generally increase with t, whilst the welding velocity is usually decreased to obtain bigger welds. Thus, both the terms on the bottom line of 8.12(a) are very roughly proportional to 1/t, and the equation may be written as

$$\frac{\beta_0}{\beta} = 1 + \frac{K_2 t}{b'}$$
 (8.12e)

where ${\tt K}_2$ is an empirically determined constant.

Of the various versions of equation 8.12 listed above, the one which correlated best with the experimental data presented in Section 4.3.7 was 8.12c, with the constant E/E' set equal to 25. For E = 210000 MPa, this suggests a value of E' = 8400 MPa. There is little published data on work hardening behaviour at elevated temperatures. However, stress-strain curves published by Owen (ref. 33) in the temperature range 223 to $526^{\circ}C$ and for strains between 0.2 and 0.5 % have an average gradient of 6500 MPa. Similarly, proof stresses given by Woolman and Mottram (55) suggest an average value of E' of 10,700 MPa. Thus, the empirical value is of the correct order.

In spite of the success of equation 8.12c in bringing together the results of experimental restraint tests, it must be emphasized that the quantity of data is insufficient to establish its general validity, and that any new data should be assessed in the first instance on the basis of equation 8.12b.

In the case of a second pass weld, account must be taken of two additional factors. The welded joint is thickened by the first pass reinforcement to a total thickness of t_c , and there will be an initial rotation of β_1 resulting from the first pass weld. If β is the change in angle due to the second pass (β_1 is taken with the same sense as β , and will therefore normally be negative), then the moment equation 8.11a becomes

$$K_{j} (\beta_{o} - \beta) = K_{e} (\beta_{1} + \beta) - K_{i} (\beta_{o} - \beta)$$

$$\frac{\beta_{o} - K_{e}\beta_{1}/(K_{i} + K_{j})}{\beta} = 1 + \frac{K_{e}}{K_{i} + K_{j}}$$
(8.11b)

or

and the joint stiffness Kj is now given by

$$K_{j} = \frac{E't}{(2(1-v^{2}))} = \frac{D}{w} \cdot \frac{E'}{E} \cdot (\frac{t}{c})$$
(8.10b)

The corresponding version of equation 8.12c, in which $\mbox{K}_{\mbox{i}}$ is assumed to be negligible, is

$$\frac{\beta_{o} - K_{c} w \beta_{1} / b'}{\beta} = 1 + K_{c} w / b'$$
(8.12f)
where $K_{c} = \frac{E}{E}$, $\left(\frac{t}{t_{c}}\right)^{3}$

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In Section 5.3, it was found that this formula failed to give a good prediction of the angular distortion at restrained two pass butt welds. The evidence, however, was inconclusive, as there was doubt about the efficacy of the restraint arrangements.

8.5 MULTIPASS WELDING

If equations 8.4 and 8.6 are rewritten in terms of the current thickness of the joint, t_c (see Figure 8.5), then they can be used to calculate the distortion at each pass of a multipass weld. The equations become

$$\delta_{c} = 0.0035 (q/vt_{c} - 50) \qquad (8.13)$$

$$\beta = 0.22 q/vt_{c}^{2} \qquad (8.14)$$

The use of t_c in equation 8.14 is consistent with the procedure adopted in chapters 4 and 5 for comparing angular distortions at the first and second pass of two pass welds, and implies that the angular distortion is dependent on the temperature differential through the joint, and hence on the joint thickness. However, the previous version of equation 8.13 used t rather than t_c , and the choice between the two requires some clarification. In Section 5.3.1, it was found that the parameter which gave the best prediction of transverse shrinkage at the first pass of two-pass double-V butt welds was q/vt_c for low power welds, but was q/vt for higher powers. In other words, for small beads, the heat is concentrated in the joint region, whose thickness is t_c , whilst the heat of a larger weld is spread over a wider region whose average thickness tends to t. Thus, for one or two-pass welds, for which the heat input relative to the plate thickness must be large if a full penetration joint is to be made, the shrinkage is related to q/vt; but for each small pass of a multipass joint, equation 8.13 applies. Equation 8.9 gives δ_c , the shrinkage at the centre of the joint. Since the centre of the joint will in general be displaced from the centre of the plate (Figure 8.5), the presence of angular distortion causes an additional shrinkage component at the plate centre:

$$\delta = \delta_{c} + \beta d\pi / 180 \qquad (8.15)$$

where d is the separation of the plate and joint centres.

The development of transverse shrinkage and angular distortion during the laying of a multipass-weld can be found by applying equations 8.13 to 8.15, and summing the results. Where q/vt_c is less than 50 J/mm², δ_c is taken as zero. Where the plate is clamped during the root run, β is taken as zero.

This procedure was used to calculate the distortions in the four multipass specimens described in 5.3.4, and the agreement between the overall shape and final magnitude of the measured and calculated values was remarkably good: within 10 % for two of the specimens, and within 20 % for the other two.

8.6 CONCLUSIONS

In this chapter, an attempt has been made to describe the underlying mechanisms which control the development of transverse shrinkage and angular distortion during welding. These descriptions of weld shrinkage mechanisms are based on the increased understanding which has been gained through the experimental and theoretical investigations described in the previous four chapters. They have led to new or improved formulae for predicting weld distortion under a variety of conditions, which have been shown to correlate well with experimental results.

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9. CONCLUSIONS

The distortions that arise whenever two pieces of metal are joined together by welding must be allowed for at some stage of the design and fabrication process. Angular distortion may be avoided by the use of balanced welding procedures, or prevented by the application of restraint, or compensated for by presetting, or corrected afterwards by mechanical or thermal means. Components may be made oversize to allow for transverse shrinkage, and fabrication sequences may be specified which allow transverse shrinkage to take place freely without giving rise to transverse reaction stresses. Any methods of distortion control that are used are likely to be based on practical experience, rule-of-thumb allowances, or shop-floor experimentation, rather than on published data or theoretical calculation. There are good reasons why it is difficult to make a reliable prediction of the distortion that may arise in a given welding situation - the large number of variables involved, the scatter inherent in even the most closely controlled experimental results, the variability, as seen in Chapter 3, of supposed "constants" of proportionality, and the anomalies that can occur, such as negative angular distortion at joints with preset, or transverse expansion during the early passes of multipass welds.

In view of the practical difficulties in applying the results of research into weld distortion, what then is its role? The objective of the present project has been to gain an increased understanding of the mechanisms by which transverse distortions are created, and thus better to be able to anticipate and overcome the associated problems. This objective has been realized by the assessment and development of the results of previous research, by experimentation, and by the writing of a computer program which allows the assumed distortion mechanisms to be quantified and tested. The principal conclusions from this research project will now be summarized.

9.1 TRANSVERSE SHRINKAGE

Transverse shrinkage is primarily a function of the heat input per unit plate thickness per unit length of the weld and the coefficient of expansion and volumetric specific heat of the weldments. A theoretical maximum value for the shrinkage at a single pass weld can be obtained by assuming that the distribution of temperatures around the weld is such that all thermal expansion is prevented, but shrinkage takes place freely. Then, the maximum shrinkage is given by

$$\delta_{\mathbf{o}} = (1 + \nu) \frac{\alpha}{\rho \mathbf{c}} \frac{\eta \mathbf{q}}{\mathbf{v} \mathbf{t}}$$
(8.2)

The experimental measurements of transverse shrinkage in Grades 43 and 50 weldable structural steel were best summarized by

$$\delta = 0.0035 \left(\frac{q}{vt} - 50\right)$$
(4.2)

where δ is the shrinkage in mm, 0.0035 represents an average value of the parametric group $(1 + \nu)\eta\alpha/\rho c$ over the temperature range 0 to $750^{\circ}C$, q/vt has the units J/mm^2 , and the constant -50 allows for the thermal expansion that occurs despite the compressive stresses acting in the vicinity of the weld. The ability of the plate to resist the expansion of the heated zone is reduced at the ends of the weld, or in a narrow plate: the transverse shrinkage is correspondingly decreased. Room temperature yield stress, rolling direction, and welding process (submerged arc or MIG) were found to have no significant effect on the measured shrinkages.

Equation 4.2 was derived with reference to single pass bead-on-plate welds, but applies also to each pass of a one or two pass butt weld, even in the presence of a root gap or prepared edges, provided that the passes are large enough to make a full penetration weld, and that the component parts are tacked and restrained so as to prevent gross metal movement ahead of the electrode.

During multipass welding, the size of the fusion zone of each pass is small relative to the plate thickness, and its heat is concentrated in the joint region. The shrinkage due to each additional pass is given by

$$\delta = 0.0035 \left(\frac{q}{vt_c} - 50\right) + \beta d \frac{\pi}{180}$$
 (8.15)

where t_c is the current joint thickness and the final term makes allowance for the displacement of the joint centre from the mid thickness of the plate (see Figure 8.5). The overall shrinkage is found by summing the contributions of the individual passes.

9.2 ANGULAR DISTORTION

The present investigation corroborates the well known fact that a graph of angular distortion versus heat input at single pass welds exhibits a maximum, beyond which the angular distortion falls as the heat input rises. The maximum occurs when the weld penetration (i.e. the depth of the fusion zone) is approximately equal to half the plate thickness. At lower heat inputs the angular distortion at single pass welds, or at each pass of a multipass weld is given by

$$\beta_0 = 0.22 \text{ q/vt}_c^2$$
 (4.3)

where β_0 is in degrees, q/vt_c^2 has the units J/mm^3 , and t_c is the current thickness of the joint (see Figures 4.13 and 8.5). When the penetration exceeds half the plate thickness, the angular distortion becomes sensitive to the elevated temperature mechanical properties of the joint material and the depth of the unmolten metal below the fusion zone, and it may be that small variations in these quantities are responsible for the large scatter in measured angular distortions in this region.

If a plate is restrained at its edges by rigid rotational restraints, then the angular distortion is reduced in accordance with

$$\frac{\beta_{o}}{\beta} = 1 + \frac{\text{constant}}{b'}$$
(8.12b)

where β_0/β is the ratio of unrestrained to restrained angular distortion under identical welding conditions, and b' is the free span between the restraints. Other types of rotational restraint, such as the inherent rotational stiffness at a girth weld in a cylinder, can be expressed as equivalent free spans. Experimental measurements confirm the linearity of the relationship between the reciprocals of the free span and the resulting angular distortion, and suggest that the constant in equation 8.12b is a function of the width of the weld bead. A speculative consideration of the balance of moments at the joint lead to a more general relationship (equation 8.12a) but the experimental evidence was insufficient to establish its validity.

9.3 COMPUTER PROGRAM FOR THE ANALYSIS OF TRANSVERSE DISTORTIONS DURING WELDING

A computer model of the transverse stresses and distortions developed during the laying of a single pass weld has been written. The high temperature zone near to the path of the weld pool is split longitudinally and through the thickness into layer type elements which have non-linear, elastoplastic, load history and temperature dependent material properties. The remainder of the plate is assumed to remain elastic, and is split into a two dimensional mesh of rectangular elements, which model the in-plane and out-of-plane reactions in the plate caused by shrinkage and rotation at the weld line. The structure of the model reflects the concepts of the mechanisms controlling weld distortion which are put forward in Chapter 8. Hence, its success in confirming the principal qualitative conclusions of the experimental investigations supports the validity of these concepts. Quantitative agreement between observed and computed distortions was closest for medium heat input welds whose current and velocity were such that the shape of the fusion zone coincided most closely with that predicted by the program. The calculation of the temperature distribution caused by the weld was based on previously published analytical solutions to the flow of heat from a moving point source, but used empirically modified values of the thermal properties. Despite the demonstrable success of this method in improving the prediction of temperatures at the back of the plate, it could not model the effect of the welding conditions on the width to depth ratio of the weld pool. Inaccuracies in the calculated temperature distribution were believed to be the main cause of quantitative differences between observed and computed distortions. A more accurate representation of the temperature distribution would require a numerical model of the flow of heat from the weld pool, which could take into account the actual shape of the weld pool, the release and reabsorption of latent heat, surface cooling, and the variability of the thermal properties. The incorporation of such a model into the present program is seen as the most promising way forward for future research.

APPENDIX A1

DERIVATION OF FORMULA RELATING PEAK TEMPERATURE AND RISE TIME

TO MATERIAL PROPERTIES AND WELDING CONDITIONS - 2D CASE

Rosenthal's (refs. 30, 36) formula for the temperature rise at (ξ, y) due to a moving point heat source in a plate of small thickness t:

$$T = \frac{\eta q}{2\pi k t} e^{-\lambda v \xi} K_{0}(\lambda v r) \qquad (A1.1)$$
If $\lambda v r > 5$, $K_{0}(\lambda v r) \simeq e^{-\lambda v r} \sqrt{(\pi/2)} v r$

$$T = \frac{\eta q}{2k t \sqrt{(2\pi \lambda v r)}} e^{-\lambda v (\xi + r)} \qquad (A1.2)$$

$$\frac{\partial T}{\partial \xi} = \frac{\eta q}{2k t \sqrt{(2\pi \lambda v)}} \{ r^{-\frac{1}{2}} e^{-\lambda v (\xi + r)} \cdot \frac{\partial}{\partial \xi} (-v(\xi + r)) + e^{-\lambda v (\xi + r)} \cdot \frac{\partial}{\partial \xi} (-v(\xi + r)) + e^{-\lambda v (\xi + r)} \cdot \frac{1}{2} r^{-\frac{3}{2}} \cdot \frac{\partial r}{\partial \xi} \}$$

but
$$\mathbf{r} = \sqrt{(\xi^2 + y^2)}$$

 $\therefore \frac{\partial}{\partial \xi} (\xi + \mathbf{r}) = \frac{\xi + \mathbf{r}}{\mathbf{r}}$
 $\therefore \frac{\partial \mathbf{T}}{\partial \xi} = \frac{q e^{-\lambda \mathbf{v}(\xi + \mathbf{r})}}{2kt \sqrt{(2\pi \ \lambda \mathbf{v}\mathbf{r})}} \left\{ \frac{-\lambda \mathbf{v}(\xi + \mathbf{r})}{\mathbf{r}} - \frac{1}{2r} \cdot \frac{\xi}{\mathbf{r}} \right\}$

For a maximum

 $\lambda \mathbf{v} \ (\boldsymbol{\xi} + \mathbf{r}) = -\frac{1}{2} \frac{\boldsymbol{\xi}}{\mathbf{r}}$

Since this maximum occurs when ξ is large and negative, ξ \simeq - r

 $\lambda v (\xi + r) = \frac{1}{2}$ (A1.3)

. . $\lambda v \sqrt{(r^2 - y^2)} + \lambda vr = \frac{1}{2}$

 $\therefore - (\lambda vy)^2 = \frac{1}{4} - \lambda vr$

 $. \lambda vr \simeq (\lambda vy)^2$ (the constant term is negligible) (A1.4)

Substituting Al.4 and Al.3 into Al.2 gives

$$T_{p} = \frac{\eta q}{2kt \sqrt{(2\pi\lambda^{2}v^{2}y^{2})}} e^{-\frac{1}{2}}$$

But $2k\lambda = \rho c$ by definition

$$T_{p} = \frac{1}{\sqrt{(2e\pi)}} \cdot \frac{\eta q}{v t y} \cdot \frac{1}{\rho c}$$
 A1.5

The rise time from zero to max temp is given by $\ensuremath{\texttt{Al.4}}$

time =
$$\xi/v \simeq r/v = \lambda/y^2$$

... s = $\rho cy^2/2k$

A1.6

APPENDIX A2

DERIVATION OF FORMULAE RELATING PEAK TEMPERATURE AND RISE TIME TO MATERIAL PROPERTIES AND WELDING CONDITIONS - 2.5D CASE

Rosenthal's equation (refs. 30, 36) for the temperature rise at (ξ, y, z) due to a moving point heat source on a plate of finite thickness:

$$T = \frac{\eta q}{2\pi k} \sum_{-\infty}^{+\infty} \frac{e}{r_n}$$
(A2.1)

where $r_n = \sqrt{(\xi^2 + y^2 + (2nt + z)^2)}$

On the centreline of the back of the plate (y = 0, z = t) only the terms n = 0 and n = -1 are significant.

:
$$r_0 = r_1 = r = \sqrt{(\xi^2 + t^2)}$$

Hence $T_b = \frac{\eta q}{\pi k r} e^{-\lambda v (\xi + r)}$ (A2.2)

Define a non dimensional quantity L:

 $L = \lambda v(r + \xi)$ (A2.3)

. L = $\lambda vr + \lambda v \sqrt{(r^2 - t^2)}$

$$L^2 - 2L \lambda vr = - (\lambda vt)^2$$

In the region - $\xi >> t$, $r \simeq -\xi$ and L^2 is negligible.

 $\therefore \mathbf{r} = \frac{\lambda \mathbf{v} \mathbf{t}^2}{2\mathbf{L}} \tag{A2.4}$

substituting A2.3 and A2.4 into A2.2 and putting $k\lambda$ = $\rho c/2$ (by definition) gives

$$\Gamma_{b} = \frac{4}{\pi} \cdot \frac{\eta q}{vt^{2}} \cdot \frac{Le^{-L}}{\rho c}$$
(A2.5)

Noting that $\frac{dL}{d\xi} = \frac{L}{r}$

$$\frac{dT_{b}}{d\xi} = \frac{4}{\pi\rho c} \cdot \frac{\eta q}{vt^{2}} \cdot (-Le^{-L} + e^{-L}) \cdot \frac{L}{r}$$

$$\therefore L = \infty \text{ or } \underline{L} = 1$$
So $T_{pb} = \frac{4}{e\pi} \cdot \frac{\eta q}{vt^2} \cdot \frac{1}{\rho c}$
(A2.6)
And rise time = $\frac{\xi_p}{v} \approx \frac{r_p}{v} = \frac{\lambda t^2}{2}$ from A2.4
$$\therefore \text{ Rise time } = s = \frac{\lambda t^2}{2} = \frac{\rho c t^2}{4k}$$
(A2.7)

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APPENDIX A3

80/80 RISE TIME - 2.5D CASE

Using equations A2.5 and A2.6 and the empirical relationship given by equation 2.11:

$$\rho c = m \sqrt{T_p}$$
(A3.1)

At peak temperature

$$(T_{pb})^{3/2} = \frac{4}{m\pi} \cdot \frac{\eta q}{vt^2} \cdot e^{-1}$$
 (A3.2)

At 80 % peak temperature

$$(T_b)^{3/2} = \frac{4}{m\pi} \cdot \frac{\eta q}{vt^2} \cdot Le^{-L}$$
 (A3.3)

Dividing A3.3 by A3.2

$$(0.8)^{\frac{3}{2}} = Le^{-L} / e^{-1}$$

L = 2.06 or 0.388

From A2.4

$$r/v = \lambda t^2/2L$$

 $\therefore s' = \frac{r}{v} - \frac{r}{2} = \frac{\lambda t^2}{2} (\frac{1}{.388} - \frac{1}{2.06}) = \frac{\lambda t^2}{.956}$

But k = $\rho c/2\lambda$

.'. 80/80 Rise Time, s' = $\rho ct^2/1.91 \ k$

APPENDIX A4

THE LENGTH OF THE WELD POOL

Rosenthal's expression for the temperature distribution in a thick plate (3D case)

$$T = \frac{\eta q}{2\pi kr} e^{-\lambda v(\xi + r)}$$

At the rear end of the weld pool,

 $\xi = -\xi'$ (weld pool length) y = 0 z = 0 $T = T_m$ (melting temperature) $\therefore r = \xi'$

$$\therefore T_{m} = \frac{\eta q}{2\pi k\xi'} e^{0}$$

 $\therefore \xi' = \eta q/2\pi k T_m$

APPENDIX A5

PUBLISHED TRANSVERSE SHRINKAGE FORMULAE

1.	Spraragen and Ettinger (ref. 45) - Butt welds.	
	$\delta = 0.2 A_w/t + 0.05 g$ (any units)	A5.1
2.	Spraragen and Ettinger - Fillet welds.	
	$\delta = d/t$ (mm units)	A5.2
	where d = leg length.	
3.	Hansen (ref. 15) - Butt welds.	
	$\delta = 0.17 \text{ b}_{\text{m}}$	A5.3
	or $\delta = 0.8 + Kt$ (mm units)	A5.4
	where $K = 0.093$ for 60° V preparation,	
	K = 0.077 for 50 ⁰ V preparation,	
	$K = 0.048$ for 60° X preparation,	
	$b_m = mean width of fusion zone.$	
	× .	
4.	Hansen - Fillet welds.	
	$\delta = K a/t$ (mm units)	A5.5
	where $K = 0.57$ for two pass double fillets,	
	K = 0.85 for single pass double fillets,	
	a = throat dimension of fillet.	
5.	Malisius (see ref. 43) - Butt welds.	
	$\delta = 1.3 (\alpha \Delta T' K A_W/t + \alpha \Delta Tb_m) (mm units)$	A5.6
	where $\alpha \Delta T = 0.0093$, assumed thermal strain.	
	$\alpha \Delta T' = 0.0044$	
	K = 26 for bare wire arc welding	
	K = 27 to 33 for covered wire arc welding.	

Wortmann and Mohr (see ref. 43) - Butt welds.

$$\delta = \alpha Kw/\rho ct + \alpha \Delta Tb_{w} \qquad (cgs units) \qquad A5.7$$

where $\alpha \Delta T = 0.0103$

Kw = specific heat content of molten weld metal.

 $\alpha K/\rho c = 0.0141$ for a.c. welding.

 $\alpha K/\rho c = 0.0220$ for d.c. welding.

7. Gilde (ref. 12) - Butt welds.

 $\delta = \eta \alpha Bq/vt \ K \qquad (cgs units) \qquad A5.8$ where $\alpha B/K = 16.5 \times 10^{-6}$, $0.42 \le \eta \le 0.75$

8. Okerblom (ref. 32) - Butt welds.

$$\begin{split} \delta &= \ \mbox{K} \mbox{η} \mbox{η} \mbox{χ} \m$$

9. Watanabe and Satoh (ref. 50). Bead-on-plate.

 $\delta = KI^2/t^2 v \qquad (cgs units) \qquad A5.10$ where K = 0.153 for 4 mm dia. Illumenite type rod.

10. Watanabe and Satoh. Butt welds.

 $\delta = K A_w/t^2 \cdot \log_e(w/w_o) + K_2\sqrt{A_w}/t \text{ (cgs units) A5.11}$ where w_o = mass of deposited metal per pass $K_1 = 0.102$

 K_2 = 0.0584 $\,\}$ for above rod at I = 150 Amps and v = 0.3 cm/sec.

11. Vinokurov (ref. 49). Butt welds.

 $\delta = 2 \text{ K} \alpha q/\rho c \text{ vt}$ (any units) A5.12 where K = 0.3 (approx.) for Electroslag welding

K = 0.5 to 0.7 for arc welding.

 $\alpha/\rho c = 0.0024 \text{ J/mm}^2$

APPENDIX A6

PUBLISHED ANGULAR DISTORTION FORMULAE

1.	Hansen (ref. 15) - Double fillet welds.	
	$\beta = K a/t degrees$	A6.1
	where $K = 6$ for 1 pass welds (β max. = 3°)	
	K = 10 for 2 pass welds	
	K = 12 for 3 pass welds.	
2.	Hansen - Butt welds	
	β = .75 N degrees approx.	A6 _° 2
	for $N < 20$	
3.	Blodgett (see ref. 18) - Double fillet welds.	
	$\beta = 0.77 d^{1.3/t^2}$ radians (mm units)	A6.3
4.	Hirai and Nakamura (see ref. 18) - Double fillet we	lds.
	$\beta = \log_{10} \text{ w/lOt} - 0.01 \text{ radians approximately}$	
	(cgs units)	A6.4
	for $1 \leq t \leq 2.5$ cm	
5.	Okerblom (ref. 32) - Bead-on-Plate - short welds.	
	$\beta_0 = 10.5 \times 10^{-6} \text{ ng/vt}^2 \text{ radians approx.}$	
	(cgs units)	A6.5
	for $p/t < 0.6$	
	and $b_w/t < 1.5$	
	where $\eta = 0.3$	
6.	Okerblom - Bead-on-Plate - Long welds.	
	$\beta_1 = 3\beta_0$	A6,6
	where β_0 is given by above formula.	

7。	Okerblom - Double fillet welds.	
	$\beta = 2\beta_2 + (0.024 - \beta_2)/(1 + 4/K_0)$ for sequential fillets.	A6.7
	$\beta = 2\beta_2 + 2(0.024 + \beta_2)/(1 + 2/K_0)$ for simultaneous	
	fillets.	A6.8
	where $\beta_2 = \beta_1 \cdot 2t/(2t + t_0)$	
	$K = 0.7d_{\circ}^2 (3t_0 + 2d)/t^3$	
	$t_0^{=}$ outstand thickness.	
	d = leg length.	
8.	Watanabe and Satoh (ref. 50) - Bead-on-plate	

 $\beta = K_1 (I/t\sqrt{vt})^2 \cdot 5 \quad \exp(-K_2 I/t\sqrt{vt}) \text{ radians (cgs units)} \qquad A6.9$ where $K_1 = 0.131 \times 10^{-6}$

 $K_2 = 10.0 \times 10^{-3}$ for 4 mm dia. Illumenite rod.

APPENDIX A7

ELASTO-PLASTIC ANALYSIS OF TRANSVERSE SHRINKAGE

CAUSED BY HEATING AN ELEMENT TO TEMPERATURE T AND COOLING BACK TO ZERO

Assumptions (See 8.1 for background. Transverse stress, σ_v is zero.)

- (1) Total longitudinal strain = $\varepsilon_x = 0$.
- (2) Total transverse strain after rise to temperature T is ϵ_{yl} . This strain acts inwards, causing net shrinkage at the centreline.
- (3) Total transverse strain after cooling is $\epsilon_{y^2},$ which represents a net expansion.
- (4) Plastic Poisson's ratio equals 0.5. Hence incremental plastic strain $\delta \varepsilon$ in longitudinal direction causes -0.5 $\delta \varepsilon$ in transverse px and through-thickness directions.
- (5) The material is linear elastic and perfectly plastic, with Young's modulus E and yield stress σ_0 .
- (6) Compatibility of strains: total strain equals the sum of elastic, thermal, plastic, and incremental plastic strains.

i.e. $\varepsilon = \sigma/E + \alpha T + \varepsilon_{p} + \delta \varepsilon_{p}$

(7) The "shrinkage ratio" is defined as the ratio of the gross shrinkage due to the heating/cooling cycle to the free thermal expansion.

i.e. shrinkage ratio = $(\epsilon_{y1} - \epsilon_{y2})/\alpha T$

Derivation

- (i) <u>Purely elastic</u> ($\alpha T < \sigma_0 / E$)
 - (a) Heating to T

$$\varepsilon_{x} = 0 = \sigma_{x}/E + \alpha T$$

$$\cdot \cdot \sigma_{x}/E = -\alpha T$$

$$\varepsilon_{y1} = -\nu\sigma_{x}/E + \alpha T$$

$$\cdot \cdot \varepsilon_{y2} = (1 + \nu) \alpha T$$

(b) Cooling to zero

$$\varepsilon_{x} = 0 = \sigma_{x}/E$$
$$\therefore \sigma_{x}/E = 0$$
$$\therefore \varepsilon_{y2} = -\nu\sigma_{x}/E = 0$$

(c) Shrinkage ratio

$$(\varepsilon_{y1} - \varepsilon_{y2}) / \alpha T = (1 + v)$$

(ii) <u>Heating plastic/cooling elastic</u> $(\sigma_0/E \le \alpha T \le 2\sigma_0/E)$

(a) Heating to T $\varepsilon_{x} = 0 = -\sigma_{o}/E + \alpha T + \delta \varepsilon_{px}$ $\therefore \delta \varepsilon_{px} = \sigma_{o}/E - \alpha T$ $\therefore \delta \varepsilon_{py} = (\alpha T - \sigma_{o}/E)/2$ $\varepsilon_{y1} = + \nu \sigma_{o}/E + \alpha T + \delta \varepsilon_{py}$ $\therefore \varepsilon_{y1} = 1.5 \alpha T + (\nu - 0.5)\sigma_{o}/E$

(b) Cooling to zero

$$\varepsilon_{x} = 0 = \sigma_{x}/E + (\sigma_{o}/E - \alpha T)$$
(where $(\sigma_{o}/E - T)$ = previous plastic strain)
 $\therefore \sigma_{x}/E = \alpha T - \sigma_{o}/E$
 $\varepsilon_{y2} = -\nu \sigma_{x}/E + (\alpha T - \sigma_{o}/E)/2$
 $\therefore \varepsilon_{y2} = (0.5 - \nu)\alpha T + (\nu - 0.5)\sigma_{o}/E$

(c) Shrinkage ratio

$$(\varepsilon_{y_1} - \varepsilon_{y_2}) / \alpha T = (1 + v)$$

(iii) <u>All plastic</u> $(2\sigma)/E \leq \alpha T$)

(a) Heating to T : same as (ii)(a).

(b) Cooling to zero

$$\varepsilon_x = 0 = \sigma_0 / E + (\sigma_0 / E - \alpha T) + \delta \varepsilon_{px}$$

 $\therefore \delta \varepsilon_{px} = \alpha T - 2\sigma_{o}/E$

 $\therefore \delta \varepsilon_{pv} = \sigma_o / E - 0.5 \alpha T$

$$\epsilon_{\rm w2} = -v\sigma_0/E + (\alpha T - \sigma_0/E)/2 + (\sigma_0/E - 0.5\alpha T)$$

(c) Shrinkage ratio

$$(\varepsilon_{v_1} - \varepsilon_{v_2})/\alpha T = 1.5 - (1 - 2\nu)\sigma_0/\alpha TE$$

<u>The shrinkage ratio</u> has a constant value of (1 + v) up to $T = \sigma_0 / \alpha E$, and then increases to 1.5 at $T = \infty$. For the range of temperatures at which weldable structural steels have significant strength (0 to 750°C) it has an average value of 1.36 (see Figure 8.6).

<u>Note</u>: The shrinkage ratio allows the effect of longitudinal restraint to be included in the theoretical formula for transverse shrinkage, equation 8.1. It also casts light on the errors introduced by the arbitrary division of the plate into elastic and plastic zones in the finite element model described in Chapter 6. In a real plate, temperatures outside the "deformation zone" in excess of $2\sigma_0/\alpha E$ (approx. 245°C) can cause a shrinkage ratio in excess of the elastic value, 1.3, to which the elastic model is limited. Inside the deformation zone, a shrinkage ratio of 1.5 is assumed in the model, though this can in practice only be achieved at infinite temperature. However, since the range of the shrinkage ratio is small (1.3 to 1.5) and the errors due to the elastic and deformation zone assumptions are opposite in sign, it is reasonable to assume that their combined effect on the transverse strains is very small.

Specimen	L mm	t mm	y mm	I A	v v	v mm/s	$q/v(t^2+y^2)$ J/mm	T _{pb} oC	ρc _e J/mm ^{3o} C	k _e J/mm ^o Cs	s' secs
WC1 WC4 b	240 16	7.6		370 570	31.5 [/] 30.0	15.8	12.8 17.0	614 780	.00779	.0192 .0221	12.2
WC6 WC7 WC9	16 16 16	7.6	-	425 370	30.0 31.5 32.5	17.4 17.4 17.4	14.9 13.3 12.0	628 667 580	.00891 .00748 .00773	.0194 .0224 .0253	13.9 10.1 9.2
WC10 WC11 WC12	16 16	7.6	-	506 580 334	31.5 32.0 33.5	17.4 17.4	15.9 18.5 4 4	732 751 294	.00812 .00924	.0227	10.8 10.3
WC14 WC16	100 100 100	12.8	-	342 550	33.0 31.0	5.3	13.0 19.3	596 820	.00817	.0220	31.8 38.0
WC18 WC19 WC20	180 180 180	7.6 7.6 7.6		300 290 240	32.0 32.5 32.5	15.2 18.8 18.8	10.9 8.7 7.2	504 450 424	.00813 .00725 .00637	.0227 .0261 .0226	10.8 8.4 8.5
WC21 WC22 WC23	180 180	7.6	-	560 585	31.0 31.0	14.1	21.3 28.5 25.3	876 1116 949	.00912 .00957	.0230 .0234	12.0 12.4
WGla WG2a	960 960	7.9 7.9	1.0 1.0	280 400	32.0 31.0	21.5 21.5	6.6 9.1	374 463	.00658	.0244 .0271 .0238	7.9 10.2
WG3a WG1 b WG2 b	960 960 960	7.9 8.8 8.9	2.5 0 1.0	410 290 400	30.0 30.5 31.0	16.7 21.7 21.7	10.7 5.3 7.1	546 361 440	.00736 .00546 .00606	.0195 .0270 .0220	12.3 8.1 11.4
WG3b WG4b WG5b	960 960 960	8.9 9.2 9.4	0 1.0 0	400 410 510	31.0 30.0 28.0	17.0 13.2 13.2	9.2 10.9 12.2	523 561 708	.00661 .00726 .00648	.0207 .0167 .0167	13.3 19.2 18.0
				0.00							

TAB.	LE	2.	1

TEMPERATURE MEASUREMENT SPECIMENS (y = 0 approx.)

Note (1) Temperatures are relative to room temperature.

(2) "b" specimens were second pass welds: t includes reinforcement.

1	1					1	· · · · · · · · · · · · · · · · · · ·	1		r	r
Specimen	Process	l	t	I	V	v	У	Tp	s	pce 30	ke
		mm	mm	A	V	mm/s	mm	C	secs	J/mm ^{3°} C	J/mm ⁻ Cs
SW1	MIG	300	8.01	191	26.5	5.95	21 5	202	28	00475	0359
				/		0.00	31 5	149	50	00440	0437
							41 5	119	73	.00416	0471
SW2	MIG	300	7 92	195	26 5	5 34	18.5	245	25	.00410	.0471
2.0-		000		100	20.0	0.01	29.0	166	15	.00330	.0339
							53.5	100	40	.00493	.0441
SW3	MIG	300	8 01	189	26.9	5 78	20.0	217		.00430	.0344
2		000	0.01	100	20.0	0.10	30.5	149		.00493	.0390
							50.0	96	95	.00434	.0480
BNI	SA	300	8 04	420	31 0	11 79	20.0	308	55	.00485	.0515
	, SII	000	0.01	120	51.0	11.15	30.0	223		.0043	.0234
							40.0	170		.0040	.0328
BN2	SA	300	7 95	420	32 0	14 18	21.0	241		.0039	.0361
2112	Dir	000		120	52.0	14.10	31 0	160		.0043	.0340
							41 0	136		.0044	.0393
BN3	SA	300	81	420	30.0	9 98	21.0	305		.0041	.0477
2110	, Dir	000	0.1	420	50.0	5.30	30.5	228		.0049	.0272
				V.			41 0	182		.0045	.0356
WGla	SA	960	76	280	32.0	21 5	21.5	117	22.8	.0042	.0395
WG2a	SA	960	7.6	400	31.0	21 5	20.0	167	20.4	00421	0412
WG3a	SA	960	7.6	410	30.0	16 7	22.5	187	28.2	00428	0384
WG1b	SA	960	7.6	290	30.5	21 7	18.0	151	18.9	00367	0314
WG2b	SA	960	7.6	400	31 0	21.7	15.5	174	21 3	00518	0292
WG3b	SA	960	7.6	400	31.0	17.0	15.0	230	19.8	.00519	.0295
WG4b	SA	960	7.6	410	30.0	13.2	14.0	299	22.2	.00545	.0235
WG5b	G A	960	7.6	510	28.0	13.2	15.0	337	27.6	.00524	0213
WG5D	DA	500	1.0	510	20.0	13.2	15.0	551	21.0	.00524	.0215
						1					

TEMPERATURE MEASUREMENT SPECIMENS (y > t)

TABLE 2.2

Note: (1) SW and BN specimens were an undergraduate project performed by Messrs. Stobart, Woods, Bryant and Norton.

(2) Temperatures are relative to room temperature.

General Welding Conditions

Specimen Series	WA, WE, WF WG, WH, WJ WM, WT	WL, WK, WP	WB, WS WL (root)	WN
Process	SA	SA	MIG	MIG
Rectifier	FRCA 8000	FRCA 800	SMR 400	FRCA 8000
Gas or Flux	Murex BS10	Murex BX10	A/CO ₂ 95/5	A/CO ₂ 95/5
Gas Flow (ft ³ /h)	·	-	30	30
Contact tip height (mm)	18	18	20	20
Nozzle height (mm)	13	13	15	15
Torch angle (⁰)	0 ⁰	0	10	о
Filler type	Bostrand Sl	Bostrand S1	Bostrand LW1	Bostrand LW1
Wire diameter (mm)	3.2	4.0	1.2	1.6
Water cooling?	N.A.	N.A.	No	Yes

SPECIMEN	t mm	b mm	I A	v v	v mm/s	q/v J/mm	q/vt J/mm ²	q/vt ² J/mm ³	δ mm	β°	p/t
WF1	6.1	310	250	30.5	11.8	641	105	17.3	197	3.8	31
WF2			300	32.0	11 8	813	133	21 8	258	4.4	30
WF3			370	30.5	11 8	956	156	25.6	326	5.2	.55
WF4			460	31 0	11 8	1208	107	20.0	.520	5.5	.00
WF5			250	31 5	19.4	1200	197	11 5	. 508	-0.0	. /1
WEG			240	20.0	10.4	420	70	11.5	.096	2.0	.26
WE 7			340	30.0	18.4	554	91	14.9	.147	3.6	.41
N IN			400	31.0	18.4	674	110	18.0	.230	4.4	.50
W£8			470	30.5	18.4	779	127	20.8	.288	3.6	.63
WG1	7.9	380	280	32.0	21.5	417	52	6.6	.050	1.6	.22
WG2			400	31.0	21.5	568	72	9.2	.104	2.4	.37
WG3			410	30.0	16.7	736	93	11.8	.150	2.8	.42
WG6	7.9	420	510	31.0	15.0	1054	132	16.8	.337	3.0	.57
WG7			540	33.0	12.0	1485	188	23.8	. 523	2.4	.64
WG8	8.0	300	480	32.0	15.4	997	125	15.6	.271	3.34	.51
WG9	~		590	32.5	15.4	1245	156	19.5	.394	3.31	.66
WG10			680	31.0	15.4	1369	171	21.4	.478	2.13	.82
WE1	12.4	400	440	30.5	11.3	1188	95	7.7	.109	1.61	.33
WE2			580	30.5	8.0	2211	179	14.4	.436	0.95	.54
WE3			530	30.0	8.0	1987	160	12.9	. 338	1.63	.48
WE4			510	31.0	9.8	1613	130	10.5	.202	2.23	.42
WE5		i i	420	30.5	10.9	1175	94	7.6	.135	1.71	. 32
WE6			320	32.0	14.9	687	56	4.5	.047	0.85	.19
WKl	20.0	415	460	34.0	7.6	2057	104	5.2	.092	1.04	.23
WK2			640	34.5	7.6	2905	146	7.3	.196	1.78	. 36
WK3			790	35.0	7.6	3638	182	9.1	.353	1,99	.47

Basic Case - 1st Pass bead-on-plate sub arc welds

Note: A blank space implies ditto.

MIG bead-on-plate welds

												1	
Plate	L mm	b mm	t mm	I A	v	v mm/s	q/v J/mm	q/vt J/mm ²	q/vt ² J/mm ³	p/t mm	δ mm	β ⁰	Regime
WB3	1380	422	8.0	217	28.8	6.11	1023	128	16.0	.26	.229	2.89	Spray
WB4	1375	310	8.0	213	28.2	6.12	983	123	15.4	.26	.201	2.70	Spray
WB5	1375	543	8.0	214	29	6.16	1009	126	15.8	.26	.229	2.98	Spray
WB6	1375	349	8.0	218	29	6.11	1036	130	16.2	.26	.209	2.56	Spray
WB7	1380	330	8.0	208	28.8	6.14	975	122	15.2	.26	.202	2.54	Spray
WB8	1375	310	8.0	222	18.0	4.12	971	121	15.2	.42	.244	2.16	Dip
WB9	1375	310	8.0	274	25.5	7.06	989	124	15.5	.36	.218	2.32	Spray
WB10	1375	310	8.0	263	34	7.35	1218	152	19.0	.28	.226	2.82	Spray
WB11	1375	310	8.0	212	18.0	6.15	622	78	9.7	.34	.107	1.99	Dip
WB12	1375	310	8.0	179	35.9	6.14	1047	131	16.4	.17	.175	2.56	Spray
WB13	1375	312	8.0	210	21	6.14	720	90	11.3	. 30	.138	2.27	Dip
WB14	1375	325	7.9	235	20.5	6.14	784	99	12.6	. 36	.156	2.51	Dip
WB15	1375	310	13.0	210	28	6.05	972	75	5.8	.15	.097	1.18	Spray
WNT	1280	280	9.1	390	20	15 3	730	01	11.2	41	149	2 54	Sprov
1012	1200	200	0.1	330	20	15.5	755	51	11.5	.41	.142	2.04	spray
WN3	1280	280	6.4	390	29	15.3	739	115	18.0	. 52	.249	2.91	Spray
WN5	1280	280	8.1	390	29	9.0	1256	155	19.1	.48	.335	2.12	Spray
WN7	1280	400	12.2	390	33	10.0	1287	105	8.6	.29	.132	1.92	Spray
6.00			ļ										

Specimen	t	tc	I	v	v	q/vt	q/vt _c ²	δ	β _o
	mm	mm	A	v	m/s	J/mm ²	J/mm ³	mm	0
WG1	7.9	8.8	290	30.5	21.7	52	5.3	.023	0.76
WG2		8.9	400	31.0	21.7	72	7.2	.049	1.21
WG3		8.9	400	31.0	17.0	92	9.2	.084	1.70
WG4		9.2	410	30.0	13.2	118	11.0	.174	2.29
WG5		9.4	510	28.0	13.2	137	12.2	.273	2.49
WE2	12.4	15.1	630	31.5	7.5	213	11.6	.545	2.20
WE 3		14.9	520	31.0	7.8	167	9.3	.313	2.49
WE4		14.5	500	31.0	10.0	125	7.4	.170	1.94
WE5		14.1	440	31.0	11.3	97	6.1	.090	1.07
WE6		13.7	320	31.0	15.2	53	3.5	.027	0.49
· · · ·	1.1.1.2.20	1.12	$F_{-1}=\pm\infty,$						

2nd Pass Bead-on-Plate Sub Arc Welds

	N 720									
5	t	b	I	v	v	q/v	δ	β _o	*۵	β*
Specimen	mm	mm	A	v	mm/s	J/mm	mm	0	mm	ο
ā ^v							14			
WH2	6.4	711	270	31.0	15.4	543	.113	2.92	.104	2.69
WH4	6.4	305	270	31.0	15.4	543	.109	2.84	.100	2.62
WH9	6.4	125	270	30.5	15.4	534	.092	2.38	.086	2.43
WB1	8.0	111	214	29.0	6.2	1009	.177	1.90	.175	1.88
WB2	8.0	210	212	28.2	6.1	984	.194	2.22	.197	2.26
WB3	8.0	422	218	28.8	6.1	1027	.229	2.89	.223	2.81
WB4	8.0	310	213	28.2	6.1	983	.201	2.70	.204	2.75
WB5	8.0	543	214	29.0	6.2	1009	.229	2.98	.227	2.95
WB6	8.0	349	218	29.0	6.1	1036	. 209	2.66	.202	2.56
WB 7	8.0	310	208	28.8	6.1	975	.202	2.54	.207	2.60
	5 × 1	X 3 9 6 6	4 T B	-8 I - 8 R	$(k_1,k_2) = (k_1,k_2)$					

Unrestrained plates of varying width

Note (1) WH series (Sub. arc) δ^{*} and β^{*} are scaled by 500/q/v

(2) WB series (MIG) δ^{*} and β^{*} are scaled by 1000/q/v

													-
Specimen	t mm	w mm	b mm	b' mm	I A	v v	v mm/s	q/v J/mm	q/vt J/mm ²	q/vt ² J/mm ³	ර mm	ФО	β* 0
WH1	6.4	9.0	813	711	270	31	18.5	453	71	11.1	.087	2.01	2.22
WH2			711	∞	270	31	15.4	543	85	13.3	.113	2.92	2.69
WH 3			406	305	270	31	15.4	543	85	13.3	.077	1.79	1.65
WH8			508	406	370	29	18.5	583	91	14.2	.133	2.15	1.84
WH10			305	203	270	30.5	15.4	534	83	13.0	.091	1.35	1.26
WMl	8.3	13	320	8	410	31	11.8	1073	129	15.6	.274	2.68	2.49
WM2			625	∞	395	32		1067	129	15.5	.284	2.70	2.53
WM3			280	203	390	31.5		1077	130	15.6	.224	.96	.89
WM4			385	305	400	31		1050	127	15.2	.248	1.24	1.18
WM5			690	605	390	31		1024	123	14.9	.256	1.77	1.73
WNl	8.1	9	280	00	390	29	15.3	739	91	11.2	.142	2.54	-
WN2		E.	280	203							.123	1.31	
WN3	6.4	9	280	80	390	29	15.3	739	115	18.0	.249	2.91	-
WN4	~		280	203							.211	1.22	-
WN5	8.1	13	280	œ	390	29	9.0	1256	155	19.1	.335	2.12	-
WN6			280	203							. 332	0.43	-
WN7	12.2	13	400	∞	390	33	10.0	1287	105	8.6	.132	1.92	1.79
WN8		1	475	405	360	33.5	10.0	1206	99	8.1	.109	.94	.94
WN9			880	810	360	34	10.0	1224	100	8.2	.119	1.35	1.32

Plates with varying degrees of rotational restraint

Notes: (1) b' = width between rotational restraints.

b = total plate width.

(2) Blank spaces imply "ditto".

		1											-
Specimen	t mm	w mm	b mm	b' mm	I A	v v	v mm/s	q/v J/mm	q/vt J/mm ²	q/vt ² J/mm ³	δ mm	β <mark>ο</mark>	β* 0
WH1	6.4	9.0	813	711	270	31	18.5	453	71	11.1	.087	2.01	2.22
WH2			711	∞	270	31	15.4	543	85	13.3	.113	2.92	2.69
WH3			406	305	270	31	15.4	543	85	13.3	.077	1.79	1.65
WH8			508	406	370	29	18.5	583	91	14.2	.133	2.15	1.84
WH1O			305	203	270	30.5	15.4	534	83	13.0	.091	1.35	1.26
WMl	8.3	13	320	8	410	31	11.8	1073	129	15.6	.274	2.68	2.49
WM2		-	625	∞	395	32		1067	129	15.5	.284	2.70	2.53
WM3			280	203	390	31.5		1077	130	15.6	.224	.96	.89
WM4			385	305	400	31		1050	127	15.2	.248	1.24	1.18
WM5			690	605	390	31		1024	123	14.9	.256	1.77	1.73
WN1	8.1	9	280	80	390	29	15.3	739	91	11.2	.142	2.54	-
WN2		,	280	203							.123	1.31	-
WN3	6.4	9	280	œ	390	29	15.3	739	115	18.0	.249	2.91	-
WN4			280	203							.211	1.22	-
WN5	8.1	13	280	∞	390	29	9.0	1256	155	19.1	. 335	2.12	-
WN6			280	203							. 332	0.43	-
WN7	12.2	13	400	œ	390	33	10.0	1287	105	8.6	.132	1.92	1.79
WN8			475	405	360	33.5	10.0	1206	99	8.1	.109	.94	.94
WN9			880	810	360	34	10.0	1224	100	8.2	.119	1.35	1.32

Plates with varying degrees of rotational restraint

Notes: (1) b' = width between rotational restraints.

b = total plate width.

(2) Blank spaces imply "ditto".

Tensile Test Data

Plate Series	Specimen	Measured Yield Stress (N/mm ²)	Average Yield Stress (N/mm ²)	Grade
WA	WA1 WA2	292 301	297	43
WB	WV2 WQ2 WS2 WV1	340 349 350 346	347	43
WE	WE WEl WE2	320 318 316	318	43
WF	WF2	307	307	43
WG(1-7)	WG3	321	321	43
WG(8-10), WM WN(1, 2,5,6)	WG1O WM	400 409	405	50
WH, WJ	WJ3	396	396	43
WK, WL	WK	269	269	43
WR	WR	385	385	50
WRX	WRX	416	416	50

Miscellaneous

SPECIMEN	t mm	b mm	q/v J/mm	q/vt J/mm ²	q/vt ² J/mm ³	δ mm	βo	SPECIALITY
WR1X WR2X	12.9	400	1056 1860	82 144	6.3 11.2	.074 .268	1.26 2.12	Welded across rolling direction
WR3	12.9	400	1264	98	7,6	121	1.60	High yield
WR4			1650	128	9.9	.185	1.96	strength
WR5			2201	171	13.2	.453	0.94	$\int \sigma_0 = 405 \text{ N/mm}^2$

Miscellaneous

SPECIMEN	t mm	b mm	q/v J/mm	q/vt J/mm ²	q/vt ² J/mm ³	δ mm	βo	SPECIALITY
WR1X WR2X	12.9	400	1056 1860	82 144	6.3 11.2	.074 .268	1.26 2.12	Welded across rolling direction
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WR4			1650	128	9.9	.185	1.96	strength
WR5			2201	171	13.2	.453	0.94	$\int \sigma_0 = 405 \text{ N/mm}^2$

2 Pass Double Vee Butt Welds

Plate	Details		Weld	ing Parame	Distortion			
Plate	Pass	tc	q/v	q/vt	q/vt ²	δ	β _o	βı
		mm	J/mm	J/mm^2	J/mm ³	mm	ο	O
WS1	1	5.1	793	98	30.5	.245	63	73
	2	8.1	783	97	11.9	.156	2.83	1.00
WS5	1	5.1	528 1480	65 183	20.3	.124	.42	-2.91 2.49
	2	0.1	1400	100	22.0	FIF.	0.00	-3 87
WS7	2	5.1 8.1	975	124 120	38.8	.312	-1.80 2.80	5.67
ws9	1	5.1	1080	133	41.5	292	-2.05	-1.88
	2	8.1	914	113	13.9	.203	2.85	3.93
WS11	1	5.1	724	89	27.8	.221	86	-1.46
	2	8.1	1248	154	19.0	.291	3.11	2.32
WS15	1	5.1	1200	148	46.1	.377	-1.59	-1.56
	2	8.1	801	99	12.2	.154	2.31	3.15
WS17	1	5.1	332	41	12.8	.073	2.04	-2.78
	2	8.1	1638	202	25.0	.525	2.17	.74
WS19	1	5.1	422	52	16.2	.083	1.26	50
	2	8.1	947	117	14.4	.262	2.27	70
WS21	1	5.1	855	106	32.9	.230	32	78
	2	0.1	1095	155	10.7	.201	5.45	1.10
WS23	1	5.1	861	106	33.1	.210 .262	-1.55	69 2.24
WCOZ	1	5.1	1010	100	10.0	,202	1 00	2.21
W521	2	8.1	426 726	53 90	10.4	008	2.18	99
WS29	1	5.1	420	52	16.1	.147	2.10	- 31
	2	8.1	1643	203	25.0	.540	-1.16	-1.79
WS33	1	5.1	970	120	37.3	.217	2.37	3.00
	2	8.1	970	120	14.8	.216	1.41	-5.37
WS35	1	5.1	700	86	26.9	.181	1.79	3.03
	2	8.1	1338	165	20.4	.342	42	-4.82

N.B. Plate thickness, t = 8.1 mm. Plate width, b = 320 mm. Plate length, & = 1400 mm.

 β_0 = change in angle during each pass (positive

if shrinkage is greatest on side being welded).

 β_1 = angle before each pass (taken in same sense as following pass).

Restrained 2 Pass Butt Welds

Plat	e Det	ails	Welding Parameters								Measurements			
Ref	b'	t _c	Pass	I	v	V mm/s	q/v	q/vt	q/vt_c^2	w	δ	β ⁰ 1	β ^ο	β ^o p
WA2	305 305	7.9 8.8	1 2	310 640	29.5 30	16.9 16.9	540 1140	68 144	8.6 14.7	10.4 12.8	.117	.72	.15 .91	.69 -
WA3	305 305	7.9 8.9	1 2	380 515	29 29.5	16.9 16.9	650 900	82 114	10.4 11.4	11.5 11.2	.124 .230	35 92	$1.27 \\ 2.70$	1.35 1.89
WA4	305 305	7.9 8.9	1 2	410 400	29 30	16.9 16.9	704 710	89 90	11.3 9.0	11.6 11.4	.130 .109	95 54	1.49 1.87	$1.75 \\ 1.41$
WA5	305 305	7.9 8.9	1 2	390 310	29.5 30	16.9 16.9	680 550	86 70	10.9 6.9	11.1 10.7	.112 .043	65 46	1.11 1.08	1.56 1.12
WA6	305 305	7.9 9.4	1 2	530 400	29 29.5	16.9 16.9	910 700	115 89	14.6 7.9	11.2 11.6	.260 .090	94 99	1.93 1.53	$2.12 \\ 1.47$
WA8	610 610	7.9 8.9	1 2	418 530	30.4 30.1	16.9 16.9	752 944	95 119	12.0 11.9	11.0 11.2	.160 .262	-1.37 11	1.48 2.89	2.24 2.01
WA9	102 102	7.9 8.9	1 2	415 520	30.0 30.0	16.9 16.9	737 941	93 119	11.8 11.9	11.0 11.2	.114 .182	63 50	1.13 1.19	1.16 1.22
WJ1	305 305	6.4 7.9	1 2	480 480	30 31	15.4 15.4	935 966	146 151	22.8 15.5	11 11	.347 .327	.11 67	.56 2.40	_ 2.52
WJ2	305 305	6.4 7.6	1 2	430 480	28 31	$15.4 \\ 15.4$	781 966	122 151	19.1 16.7	11 11	.266 .323	27 -1.53	1.80 2.66	$\begin{array}{c} 2.34\\ 2.92 \end{array}$
WJ3	305 305	6.4 7.2	1 2	240 375	30.5 30	15.4 15.4	475 730	74 114	11.6 14.1	11 11	.058 .210	53 67	1.20 2.65	1.59 2.16
WJ4	305 305	6.4 7.4	1 2	340 540	29 32	15.4 15.4	640 1122	100 175	15.6 20.5	11 11	.105 .473	.09 -1.07	.98 1.67	1.76 -

Notes (1) Process - Sub Arc

(2) Edge preparation - square

(3) $\beta_p = \beta$ predicted, see equation 5.1

(4) Plate length, $\ell = 1170$ mm.

(5) β_1 = angle before each pass.

(6) β = change in angle during each pass.

WP series (Gap varies)

			mm
Length	WP	1-4	1250
Length	WP	5	2510
Width			400
Width be	etween	restraint	305

Pla	te						
Plate	t	I	v	v	q/v	q/vt	gap
	mm	A		mm/s	J/mm	J/mm J/mm ²	
WP1	12.8	760	34.5	7.5	3496	273	0
WP2	12.8	720	36	7.5	3456	270	1.3
WP3	12.8	740	34.5	7.5	3404	266	2.9
WP4	12.8	700	34.5	7.5	3220	252	4.5
WP5	12.2	690	36	7.5	3312	271	3.8
					L.		

Multipass Welds

	Plate			Weldi	1	Distortions			
Plate	Pass	t	Process	I	V	v	q/v	δ	β
		mm		A	v	mm/s	J/mm	mm	0
WL1	1 2 3 4 5 6 7	7 13 14 16 18 19 20	MIG SA	370 480 510 510 515 520	- 35.5 30.5 30.0 29.0 29.5 30.0	- 11.9 11.8 11.8 11.8 11.9 12.0	- 1104 1230 1297 1253 1275 1300	- .183 .167 .186 .161 .032 .060	0 1.31 1.53 1.41 .68 .49 .12
WL2	1 2 3 4	7 8 15 20	MIG SA	216 660 690 700	28 32 32 32 32	8.8 7.4 7.46 7.45	689 2854 2959 3006	.218 .599 .405 .273	0 .80 1.87 2.07
WT1	1 2 3 4 5 6 7 8 9 10 11	8.5 11.5 12 6 9 11 12.5 15 15 18 22 23	MMA SA	- 350 350	- 26 26	- 8.3 3.03	500 1100 3000	.028 027 046 .303 .335 .336 .269 .241 .133 - .332	1.51 1.33 1.05 .18 1.22 1.77 1.37 1.27 .91 2 - 1.86
WT2	1 2 3 4 5 6 7 8 9 10	8 11 12 17 18 20 22 23 25 25	MMA · SA	- 350	26	- 8.3	500 1100	.054 015 019 .131 .060 .037 .045 .015 .014 .007	1.66 1.30 1.02 .76 .51 .23 .30 .15 .01 01

NOTES: (1) Blank space implies "ditto".

(2) " - " implies not measured.

(3) WL dimensions 20 x 410 x 1250 mm

(4) WT dimensions 25 x 380 x 1550 mm

(5) Distortions are per pass.



Fig.1. Weld shrinkage parameters.



Fig.2.1. (After Ref.37) Variation of thermal properties of mild steel used by Rosenthal and Schmerber.




Fig.2.1. (After Ref.37) Variation of thermal properties of mild steel used by Rosenthal and Schmerber.



Fig.2.2. Measured penetrations v. empirical penetration parameter.





Fig.2.5. Effective conductivity v. temperature.



Fig.2.6. Comparison of computed and experimental thermal cycles close to weld.



Fig.2.7. Comparison of computed and experimental thermal cycles: cases with poor agreement.



Fig.2.8. Comparison of computed and experimental thermal cycles remote from the weld.



Fig.2.9. Comparison of several methods of computing a thermal cycle.



Fig.2.10. After Christensen et al (Ref.5). Non-dimensional plot of weld pool length v. welding conditions.







Fig.3.2. (From Okerblom, ref. 32) Variation of shrinkage and angular distortion with heat input per unit thickness squared, and bead shape.



Fig.3.3. (From Vinokurov, ref. 49) Nomogram for determining angular distortion at butt and T welds in relation to welding conditions and calculated thickness, δ_{p} .



Fig.3.4. (After Vinokurov, ref. 49) Build up of transverse shrinkage due to movement of plate edges.







Fig.3.6. Comparison of angular distortion at double fillet welds derived from published formulae or graphs.



(a)



(Ь)

Fig.3.7. (After Okerblom, ref. 32) Transverse shrinkage leading to curvature of beam sections: a) Butt joint in flange; b) eccentric fillet.



Fig.4.0. Submerged arc welding on side beam machine.



Fig.4.0. Submerged arc welding on side beam machine.



Fig.4.1. Electrical extensometer.



Fig.4.2. Calibration of 50mm extensometer with 'Peekel' data logger.



Fig.4.1. Electrical extensometer.



Fig.4.2. Calibration of 50mm extensometer with 'Peekel' data logger.



Fig.4.3. Making indentations in specimen surface.



Fig.4.4. Location of extensometer foot on rim of dent.



Fig.4.5. Typical bead-on-plate specimen showing layout of extensometer dents.



Fig.4.7. Basic case – variation of angular distortion with heat input parameter q/vt².











Fig.4.11. Angular distortion at MIG welds.



Fig.4.12. Incremental transverse shrinkage at first and second pass.



Fig.4.13. Incremental angular distortion at first and second pass welds. (Sign convention - angular distortion is positive if shrinkage is greatest on side being welded.)



Fig.4.14. Variation of transverse shrinkage along plate length.



Fig.4.15. Variation of angular distortion along the plate length.



Fig.4.16. Variation of shrinkage over central section - basic case welds.



Fig.4.17. Variation of angular distortion over central section - basic case welds.



Fig.4.18. Variation of shrinkage with plate width (unrestrained plates).



Fig.4.19. Variation of angular distortion with plate width (unrestrained plates).



Fig.4.20. Buildup of angular distortion during welding.







End elevation



Fig.4.22. Rig for restraining specimens against angular distortion.





Fig.4.23. Restrained plates. Angular distortion v. width between restraints.



Fig.4.24. Restrained plates. Non-dimensional, reciprocal plot of angular distortion v. plate width.



Fig.4.25. The effect of yield stress and rolling direction on weld distortion.



Fig.5.1. Typical butt weld specimen.



Fig.5.2. WS series – edge preparation and definition of current joint thickness, tc.



Fig.5.3. Transverse shrinkage at unrestrained double V butt welds.



Fig.5.4. Angular distortion of unrestrained double V butt specimens (WS).



Fig.5.5. Welding trolley used for restraining butt weld specimens.



Fig.5.6. Transverse shrinkage at first-pass of butt weld.



Fig.5.7. Transverse shrinkage at second-pass of butt weld.



Fig.5.8. Angular distortion at first-pass of butt weld.



Fig.5.9. Angular distortion at second-pass of butt weld.



Fig.5.10. Measured angular distortions v. prediction of eq.5.2.



Fig.5.11. Restraint of WP series plates.



Fig.5.12. Layout of WP series plates.



Fig.5.13. Variation of shrinkage along length of butt weld with gap.



Fig.5.14. Plate WP5.

Fig.5.15. Transverse shrinkage (WP5).



Fig.5.16. Accumulation of transverse shrinkage and angular distortion during multipass welding of WL specimens.



Fig.5.17. Buildup of transverse shrinkage during multipass welding of 25mm plate (sealing pass and arc air gouging excluded).



Fig.5.18. Buildup of angular distortion during multipass welding of 25mm plate.



End view

Plan view

Fig.6.1. Mesh layout, dimensions, and node numbering.


Fig.6.2. In-plane elements.



Fig.6.3. Out-of-plane elements.

$$Et/12ab(1 - v^2)$$
 ×

1	4(βa ² + b ²)							1
5	2(βa ² - 2b ²)	$4(\beta a^2+b^2)$						
7	- 2(βa ² + b ²)	2(b² - 2βa²)	$4(\beta a^2 + b^2)$		Symn	netric		
3	2(b ² - 2βa ²)	$-2(\beta a^2 + b^2)$	2(βa ² - 2b ²)	$4(\beta a^2 + b^2)$			<i>*</i>	
2	$\operatorname{Sab}\gamma$	3ab(β - ν)	-3ab γ	3ab(ν - β)	$4(a^2+\beta b^2)$			
6	3ab(ν - β)	-3ab γ	3ab(β - ν)	3ab γ	2(a ² - 2βb ²)	$4(a^2+\beta b^2)$		
8	- 3ab γ	3ab(ν - β)	$\operatorname{Sab}\gamma$	3ab(β - ν)	$-2(a^2+\beta b^2)$	2(βb ² - 2a ²)	$4(a^2 + \beta b^2)$	
4	3ab(β - ν)	$\operatorname{Sab}\gamma$	3ab(ν - β)	-3ab γ	2(βb ² - 2a ²)	$-2(a^2 + \beta b^2)$	2(a ² – 2βb ²)	$4(a^2 + \beta b^2)$
		5	7	3	2	6	8	4]
	where $\beta = \frac{1}{2}(1 - \nu)$							
$\gamma = \gamma_2(1 \pm \nu)$								

Fig.6.4. In-plane element stiffness matrix.

Note. The matrix is taken from reference 41. The numbering of the rows and columns indicates the reordering necessary to conform with the ordering of the degrees of freedom specified in Figure 6.2 and equations 6.2 and 6.1.



Fig.6.5. Validation of FEWMI: a) Test case. Plate 960 x 180 x 6.1mm. 16 x 3 mesh of 60 x 60mm square elements; b) in-plane loading; c) out-of-plane loading.

(a)

(b)

(c)



Fig.6.6. Generalised monotonic stress-strain curve.



Fig.6.7. Algorithm for stress calculation.



Fig.7.1. Variation of coefficient of expansion with temperature.



Fig.7.2. Variation of Young's modulus with temperature.



Fig.7.3. Variation of non-dimensionalised proof stress with temperature.



Side view

Fig.7.4. Basic case - mesh layout and electrode movement.



Fig.7.5. Transverse shrinkage in 6mm thick plates.



Fig.7.6. Angular distortion in 6mm thick plates.



Fig.7.7. Transverse shrinkage in 8mm thick plates.



Fig.7.8. Angular distortion in 8mm thick plates.



Fig.7.9. Transverse shrinkage in 13mm thick plates.



Fig.7.10. Angular distortion in 13mm thick plates.



Fig.7.11. Transverse shrinkage in 20mm thick plates.



Fig.7.12. Angular distortion in 20mm thick plates.



Fig.7.13. Variation of transverse shrinkage along plate.



Fig.7.14. Variation of angular distortion along plate.



Fig.7.15. Effect of internal and joint stiffness on restrained angular distortion.



Fig.7.16. Variation of angular distortion with free span.





Fig.8.1. Temperature distribution in plate during welding: a) Cross-section on XX; b) plan, showing isothermal contours.



Fig.8.2. Variation of non-dimensionalised proof stress of low carbon steel with temperature.



Fig.8.3. Variation of thermal properties of weldable structural steel with temperature (Ref.55).



Fig.8.4. Angular distortion in restrained and unrestrained plates: a) Unrestrained. Rotation = β_0 ; b) restrained. Rotation = β .



Fig.8.4. continued. (c) Variation of moments during welding; (d) variation of angular distortion during welding.



Fig.8.5. Current joint thickness, t_c, and joint centre displacement, d, in a multipass joint.



Fig.8.6. Variation of shrinkage ratio with non-dimensional temperature parameter (see Appendix A7).

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