Volatility, Growth and Labour Elasticity^{*}

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Abstract

We study the relationship between growth and variability in a DSGE model with nominal rigidities and growth driven by learning-by-doing. We show that this relationship may be positive or negative depending on the impulse source of fluctuations. A key role is also played by the Frisch elasticity of labour supply and by institutional features of the labour market. Our general findings are that monetary shocks volatility will generally have a negative effect on growth, while the opposite tends to be true for fiscal and productivity shocks. These findings are consistent with the existing empirical evidence: data show, in fact, a somewhat ambiguous relationship between output growth and real variability, but a generally negative relationship between output growth and nominal variability.

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1 Introduction

In macroeconomics growth and business cycles have been considered as two separate areas of research. In this paper we attempt to bridge these two areas by proposing a model of endogenous growth that gives a central position to uncertainty.

Until the 1980's macroeconomists regarded short-term economic fluctuations (or business cycles) as deviations around a smooth and stable trend growth path of GDP. Nelson and Plosser (1982) started a debate on the by now accepted fact that output does not show a strong tendency to return to trend after a shock: this fact questions the separation between growth and business cycles analysis. Indeed, the evidence on the persistence of the output process was interpreted by real business cycle theorists as a sign of the nature of the disturbances that caused business cycles, i.e. technological shocks. An alternative explanation to the high persistence of fluctuations comes from models where growth is endogenous: in fact a generally neglected key implication of these models, which are often deterministic, is that any temporary disturbance that has an effect on the amount of growth-enhancing activities can produce permanent effects on the level of output.

Further evidence of a link between growth and cycles is provided by a number of empirical studies which report statistically significant correlations between output growth and output volatility using various cross-section and time series data. Following the seminal Ramey and Ramey (1995) paper, cross-country studies have consistently found that volatility exerts a significant negative impact on long-run (trend) growth, which is is however stronger in poorer countries (see Martin and Rogers 2000, Kose et al. 2005, Hnatkovska and Loayza 2005). As to time series methods, using a univariate GARCH model on US data, Caporale and McKiernan (1998) find a positive effect, while Grier and Perry (2000) find no effect in a symmetric bivariate GARCH model of inflation and output growth, and Dawson and Stephenson (1997) reach the same conclusion from an examination of state level data. Inflation and/or money average growth and volatility are generally found to negatively affect output growth using cross-section (e.g. Barro 1997 and 2001), panel data (e.g. Andrés and Hernando 1997, Judson and Orphanides 1999), and time series methods (see Grier and Perry 2000 and Elder 2004). A careful examination of the problems of the different approaches is offered in the overview by Temple (2000). Finally, using multivariate GARCH models, Grier et al. (2004), Fountas et al. (2006) and Andreou et al. (2008) find a generally negative effect of the volatility of money shocks on output growth in G7 countries and a positive effect of growth volatility on output growth.

The existence of a relationship between growth and volatility has important policy implications as it suggests the possibility that policies designed to stabilize short-run fluctuations might also affect the long-run performance of the economy. Depending on whether this relationship is negative or positive, there is the presumption that successful stabilization would also entail either an improvement or deterioration in growth prospects. The potential significance of this is obvious, especially considering that it takes only small changes in the growth rate to produce substantial cumulative gains or losses in output.

It is therefore unsurprising that the relationship between growth and cycles is also receiving an increasing attention in the theoretical literature.¹ In the so called 'schumpeterian' approach recessions have a positive impact on growth by reducing the opportunity cost of technological improvements. Aghion and Banerjee (2005) note that in this kind of models the relationship between volatility and growth is likely to be positive. The relationship will become negative if credit constraints are pervasive, so that R&D has to be financed by current profits, a condition more relevant for developing countries. However this view is challenged by the empirical evidence on the procyclicality of R&D expenses (see Walde and Woitek 2004 and Barlevy 2007) in developed countries. In 'arrovian' models where growth takes the form of learning-by-doing, revived by Romer (1986), recessions have a negative effect on growth (e.g. Blackburn

 $^{^1\}mathrm{A}$ comprehensive overviews $% ^{-1}\mathrm{A}$ are in Gaggl and Steindl (2007) and in Aizenman and Pinto (2005) .

1999, Pelloni 1997 and Stadler 1990). Martin and Rogers (1997) and (2000), Blackburn and Galindev (2003), show that when the knowledge (embodied or disembodied) accumulation externality works only through labour, volatility will be detrimental to growth. However, De Hek (1999), going back to Romer's (1986) specification of learning by doing shows that volatility will have a positive effect on growth if the elasticity of the marginal utility of consumption is higher than one. Canton (2002) finds a positive relationship in a model where growth is driven by human capital accumulation and Jones et al. (2005) show that in a large class of convex models of endogenous growth, the relationship between growth and volatility is positive, even when preferences have less curvature than in the logarithmic case, with the magnitude of the effect being U-shaped with respect to the intertemporal elasticity of substitution.

These insights have been established within the context of purely real models of the economy with real shocks and real propagation mechanisms. Dotsey and Sarte (2000) and Varvarigos (2008) explore the role of monetary factors. The former study a simple AK model with a cash in advance constraint and show that an increase in variance of monetary growth (or inflation) will cause an increase in the mean of output growth again through precautionary savings. The latter proposes a model of human capital accumulation and a transactions facilitating role for money, and again finds a positive effect of money volatility on growth.

Finally, Blackburn and Pelloni (2004) and (2005) propose a model with technology \dot{a} la Romer (1986) where money enters the utility function and there is nominal wage-setting by unions. They derive a negative relationship between growth and the volatility of nominal shocks and a positive relationship between growth and the volatility of shocks to the rate of subjective time discount.

In this paper we confirm their insight that the sign of the link between growth and volatility may depend on the nature of the shocks behind it. However, we use less restrictive assumptions and find many new results. First, we show that an important parameter in determining the effects of increased uncertainty is the Frisch (compensated) elasticity of labour supply (FELS).² We consider both the case of a competitive labour market and of nominal wage setting, and include fiscal and technology shocks as well as monetary shocks. In all cases we consider how the persistence of the shocks is affecting the results.

We find that with nominal rigidity monetary shocks volatility will generally have a negative effect on growth, which however becomes positive when the FELS is high. Uncertainty due to technology shocks leads to higher long-run growth than in a deterministic environment, however the effect is lower the lower is the FELS. Moreover for given variance of the shocks, a higher autocorrelation coefficient in the process governing the shocks will, above a certain threshold, induce a lower average and a higher variance of growth: this means that even if uncertainty increases growth, a negative relationship between growth and its volatility is likely to be observed in the data. This holds under both labour market organizations.

The effect of the volatility of the fiscal shocks on growth is positive with a high elasticity of labour supply, but becomes negative when the elasticity is lower (but still consistent with empirical evidence), or when, for a given level in the variance of the shocks, the serial correlation in the shocks is high enough. Moreover, as is the case with technology shocks, the persistence in the fiscal shocks is a crucial factor for determining the relationship between growth and volatility.

Finally, we show that the institutional features of the labour market are also important. Not only money shocks, and their variance, have real effects only with nominal wage setting, but the effect of technological variability is higher under nominal wage setting, while the volatility of the fiscal has larger effects on growth in a competitive labour market.

Coming to our solution techniques, it is common practice in macroeconomics to

 $^{^{2}}$ A parameter that measures the elasticity of total effort with respect to its return is found to influence the relationship between growth and volatility when the utility of leisure is a linear function of human capital by Blackburn and Varvarigos (2008).

solve nonlinear dynamic stochastic systems using linear methods. However, these methods are not suitable to study the effects of the volatility of the exogenous shocks in complex dynamics environments, since by adopting a linear method all the second order effects will be wiped off. Until recently, this has constrained the literature jointly analysing business cycles and growth to using only models that could be solved analytically. However some methodological contributions have appeared recently that allow researchers to circumvent this limitation. To evaluate the effects of the volatility of shocks on the endogenous variables of our model we use the perturbation method proposed by Schmitt-Grohé and Uribe (2004), which amounts to a second-order Taylor approximation around a deterministic steady state.³ To better understand the interplay between model specification, volatility, and growth, after presenting the model we consider an analytically solvable special case, which is helpful in interpreting the results. We also conduct two types of sensitivity analysis: we vary some preference and technological parameters and consider alternative decompositions of the shock volatility between innovation variance and autocorrelation.

The paper is organised as follows: section two describes the basic stochastic growth model incorporating exogenous monetary disturbances in the process governing money growth, technology and fiscal shocks; section three summarizes the general equilibrium conditions of the model, section four describes some preliminary analytical results, section five applies the perturbation method of Schmitt-Grohé and Uribe (2004) to evaluate the effects of volatility on growth. Section six concludes.

2 The Model

We consider an artificial economy in which there are constant populations (normalised to one) of identical, immortal households and identical, competitive firms. Time is

³Schmitt-Grohé and Uribe (2004) also provide Matlab codes to compute second-order approximations for any rational expectation model, whose equilibrium conditions can be written in a given form they describe. We are able to use these codes as the model we propose has the required form.

discrete and indexed by $t = 0, 1...\infty$.

2.1 Firms

The representative firm combines N_t units of labour with K_t units of capital to produce Y_t units of output according to

$$Y_t = b_t (\overline{K}_t N_t)^{\alpha} K_t^{1-\alpha}, \quad \alpha \in (0,1)$$
(1)

$$b_t = C_b + \rho_b b_{t-1} + \varepsilon_{b,t},\tag{2}$$

where C_b and ρ_b are constants. The shock $\varepsilon_{b,t}$ is assumed to be identically and independently distributed with mean zero, variance equal to $\sigma_{\varepsilon_b}^2$ and bounded support. The term \overline{K}_t represents an index of knowledge which is freely available to all firms and which is acquired through serendipitous learning-by-doing, as in the classic Romer (1986) paper. There is a vast empirical literature that documents the pervasive presence of learning-by-doing effects in the economy. Some recent evidence and references to other studies can be found in Thornton and Thompson (2001), Cooper and Johri (2002) and Jovanovic and Rousseau (2002).

Labour and capital are hired from households at the real wage rate $\frac{W_t}{P_t}$ and real rental rate R_t , respectively, where W_t is the nominal wage and P_t is the price of output. Profit maximisation implies that factors are paid at their marginal productivity:

$$\frac{W_t}{P_t} = \alpha b_t \overline{K}_t^{\alpha} N_t^{\alpha-1} K_t^{1-\alpha} = \alpha b_t N_t^{\alpha-1} K_t, \tag{3}$$

$$R_t = b_t (1 - \alpha) \overline{K}_t^{\alpha} N_t^{\alpha} \overline{K}_t^{-\alpha} - \delta = b_t (1 - \alpha) N_t^{\alpha} - \delta,$$
(4)

where δ is the depreciation rate and the second equalities in the expressions above are the result of a symmetry assumption for firms.

2.2 Households

The representative household derives lifetime utility, U, according to

$$U = \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \mu \log \left(\frac{M_t}{P_t} \right) - \lambda L_t^{\eta} \right], \quad \beta \in (0, 1), \quad \lambda, \mu > 0, \ \eta > 1, \tag{5}$$

where C_t denotes consumption, $\frac{M_t}{P_t}$ denotes real money balances and L_t denotes labour.

To generate a demand function for money, we adopt the familiar short-cut device of introducing money directly into the utility function, rather than specifying explicitly a separate transactions technology. The quantity M_{t-1} is understood to denote beginning-of-period t (i.e., end-of-period t-1) nominal cash balances which are augmented by a proportional monetary transfer, a_t .⁴ We assume that the disturbance is governed by the following process:

$$a_t = C_a + \rho_a a_{t-1} + \varepsilon_{a,t}.$$
(6)

The shock $\varepsilon_{a,t}$ is assumed to be identically and independently distributed with mean zero, variance equal to $\sigma_{\varepsilon_a}^2$ and bounded support. C_a is a constant, the scalar ρ_a is assumed to be less than one. Defining Z_t as real assets, the budget constraint for the household is given by

$$C_t + \frac{M_t}{P_t} + Z_{t+1} = \frac{W_t}{P_t} L_t + \frac{M_{t-1}a_t}{P_t} + (1+R_t)Z_t - S_t,$$
(7)

where Z_t represents wealth and S_t is lump-sum taxation.

Each household confronts the problem of maximising the expected value of intertemporal utility in (5), subject to the sequence of budget constraints in (7) and

⁴In some models, it is end-of-period (rather than beginning-of-period) money holdings that serve as the reference point. To the extent that money yields utility by facilitating transactions, it seems more reasonable to adopt the present formulation. The assumption that monetary transfers are proportional (rather than lump-sum) is made largely for analytical convenience, as in other investigations (e.g., Bénassy 1995).

initial conditions for Z_0 and M_0 . The information set conditioning expectations consists of the values of all parameters, the current and past values of all variables and the probability distributions of all shocks. The problem is solved, in part, by choosing plans for consumption, money balances and asset holdings that satisfy the following conditions:

$$C_t^{-1} = \beta E_t C_{t+1}^{-1} (1 + R_{t+1}), \tag{8}$$

and

$$\frac{\mu}{M_t} = \frac{\gamma}{C_t P_t} - E_t \beta \frac{\gamma a_{t+1}}{C_{t+1} P_{t+1}}.$$
(9)

Using the transversality condition $\lim_{\tau\to\infty} \beta^{\tau} E_t \left(\frac{M_{t+\tau-1}a_{t+\tau}}{P_{t+\tau}C_{t+\tau}}\right) = 0$, equation (9) can be solved forward to give:

$$\frac{M_t \gamma}{C_t P_t} = \mu (1 - \beta)^{-1}.$$
 (10)

We consider two alternative scenarios for the labour market. In the first scenario, the labour market is characterised by monopolistic unions. Bewley's (1999) detailed study of firms' wage policies based on interviews with managers finds ample evidence of downward nominal wage rigidities. More recently, the multi-country study of Dickens et al. (2007) uncovers evidence of significant downward nominal and real wage rigidities in most of the countries in their sample. We assume that wage setting takes place prior to the realisations of shocks, on the basis of one-period contracts. In this case, therefore, the economy displays nominal rigidities, as in the early contracting models of Gray (1976) and Fischer (1977), as well as those of a more recent vintage (e.g., Bénassy 1995). The nominal wage is fixed at the level that maximizes the expected utility of the households, taking into account the constraint given by labour demand. The optimality condition for the nominal wage is found to be:

$$\eta \lambda E_{t-1} L_t^{\eta} = \alpha W_t E_{t-1} L_t (P_t C_t)^{-1}.$$
(11)

This just says that in equilibrium the marginal expected benefit of working is equal to the expected cost. In the second scenario, the labour market is perfectly competitive. Labour supply turns out to be:

$$\frac{W_t}{P_t}C_t^{-1} = \eta \lambda L_t^{\eta-1},\tag{12}$$

From (12) we can see that the Frish elasticity (FELS) is equal to $1/(\eta - 1)$. The equilibrium of the household is now characterised completely by the first-order conditions in (8) and (10), the optimal condition for the nominal wage (11) or for labour supply (12), the budget constraint in (7), the initial conditions for money holdings and financial wealth and the transversality conditions:

$$\lim_{\tau \to \infty} \beta^{\tau} E_t \left(\frac{M_{t+\tau-1} a_{t+\tau}}{P_{t+\tau} C_{t+\tau}} \right) = \lim_{\tau \to \infty} \beta^{\tau} E_t \left(\frac{Z_{t+\tau+1}}{C_{t+\tau}} \right) = 0.$$
(13)

3 General Equilibrium

The solution of the model is computed by combining the equilibrium conditions for households listed above, with the profit maximization condition for firms (3) and (4), the market clearing conditions for capital, $K_t = Z_t$, for labour, $N_t = L_t$, for goods, $C_t + K_{t+1} + G_t = Y_t + (1 - \delta)K_t$, where G_t denotes government spending in period t, and for the money market. Money supply, H_t , moves in conformance with:

$$H_t = a_t H_{t-1}.\tag{14}$$

The equilibrium condition for the money market is then $M_t = H_t$.

We assume that the government runs a continuously balanced budget, so that $G_t = S_t$. Government expenditure is assumed to evolve according to:

$$G_t = u_t C_t, \tag{15}$$

$$u_t = C_u + \rho_u u_{t-1} + \varepsilon_{u,t}.$$
(16)

The shock $\varepsilon_{u,t}$ is assumed to be identically and independently distributed with mean zero, variance equal to $\sigma_{\varepsilon_u}^2$ and bounded support, C_u is a constant and the scalar ρ_u is assumed to be less than one.

The labour market equilibrium condition with nominal wage setting can be rewritten as (derivation in the Appendix):

$$L_{t} = \left(\frac{\alpha^{2} a_{t} b_{t} K_{t} E_{t-1} L_{t} a_{t}^{-1}}{\eta \lambda C_{t} E_{t-1} L_{t}^{\eta}}\right)^{1/(1-\alpha)}.$$
(17)

When the labour market is competitive, its equilibrium condition, obtained just by equating labor demand (3) and labour supply (12), is:

$$L_t = \left(\frac{K_t \alpha b_t}{C_t \eta \lambda}\right)^{1/(\eta - \alpha)}.$$
(18)

4 Some Analytical Results

To pin down some of the mechanisms relating growth and uncertainty, in this section we consider a set of very restrictive assumptions under which the model admits a closed form solution. The findings in this section will be useful in interpreting the results we get by simulation in the extended model.

When $\delta = 1$ and the fiscal shock u_t is i.i.d., the following relation, derived in the appendix, holds:

$$\frac{K_{t+1}}{Y_t} = \frac{\varkappa (1+u)}{(1-\varkappa)(1+u_t) + \varkappa (1+u)}.$$
(19)

where $\varkappa \equiv (1 - \alpha)\beta$, and $u \equiv E_t u_{t+i}$ for $i = 1, 2..\infty$. This implies that the rate of investment is a decreasing and convex function of the fiscal shocks and therefore will increase on average when the variance of the shocks increases, by Jensen's inequality. We'll label this the 'precautionary saving effect'.

4.1 Money Wage Setting

Consider the nominal wage setting scenario. Assume that $\eta = 1$, and that all shocks are i.i.d and uncorrelated. We can then derive (see the Appendix):

$$L_t = \frac{\alpha^2 a_t \left[(1 - \varkappa)(1 + u_t) + \varkappa (1 + u) \right]}{(1 - \varkappa)\lambda a}.$$
(20)

where $a \equiv Ea_{t+i}$ for $i = 1, 2...\infty$. Labour is an increasing linear function of the fiscal and the money shock. In fact both these shocks cause an increase in aggregate demand and therefore in labour demand.

We can now state the following:

Proposition 1 Assume $\delta = 100\%$, all shocks are *i.i.d.* and uncorrelated with each other and η is equal to one. Then the following expression for the rate of growth of output in the presence of money wage setting obtains:

$$\frac{Y_{t+1}}{Y_t} - 1 = b_{t+1} \frac{\left(\frac{\alpha^2 a_{t+1}[(1+u_{t+1})(1-\varkappa)+\varkappa(1+u)]}{(1-\varkappa)\lambda a}\right)^{\alpha} \varkappa(1+u)}{(1+u_t)(1-\varkappa)+\varkappa(1+u)} - 1.$$
(21)

Proof. See Appendix. \blacksquare

As we have seen the TFP shock has no effect, given the assumed specifications of preferences and technology, on the marginal choice between leisure and consumption or on the time path of consumption. Growth is then a linear function of the technology shock b_{t+1} , through a direct effect on the production function. This means the variance of this shock will not affect average growth. Growth is instead a strictly concave function of the monetary shock, so its variance will have a negative effect on average growth. The term inside the large parenthesis in (21) is just L_{t+1} . Even if expected labour is not affected by σ_a^2 , as from (20) labour is a linear function of the money shock, however, through the diminishing marginal productivity of labour, i.e. since $\alpha < 1$, the expected rate of growth will be affected by σ_a^2 . We could label this the 'diminishing returns to labour' effect. The effects of a mean-preserving spread in the distribution of the fiscal shocks are less immediately readable from (21): the rate of growth is in fact a concave function of the current realizations of the shocks - again through the 'diminishing returns to labour' effect- but a convex function of the lagged realizations of the shocks - through the rate of investment, as can be seen from (19). Calculating a second-order approximation of the rate of output growth as in (21) and taking expectations we get:

$$E\frac{Y_{t+1}}{Y_t} - 1 \simeq A - 1 + A_a \sigma_a^2 + A_u \sigma_u^2,$$
(22)

with $A \equiv b\varkappa \left(\frac{\alpha^2(1+u)}{(1-\varkappa)\lambda}\right)^{\alpha}$, $A_a \equiv \frac{A\alpha(\alpha-1)a^{-2}}{2} < 0$, $A_u \equiv \frac{A}{2} \left(\frac{1-\varkappa}{1+u}\right)^2 \left[2 + \alpha(\alpha-1)\right] > 0$.

We conclude that an increase in σ_u^2 causes an increase in precautionary savings which more than offsets the 'diminishing returns to labour effect': the net effect on expected growth is positive.

4.2 Competitive Labour Market

When the labour market is competitive equilibrium employment is given by (the derivation is in the Appendix):

$$L_t = \left(\frac{\alpha \left[(1-\varkappa)(1+u_t) + \varkappa(1+u)\right]}{(1-\varkappa)\eta\lambda}\right)^{1/\eta}.$$
(23)

By comparing (20) and (23) we see that employment will be higher than in the presence of unions, i.e. when labour is sold monopolistically. Labour is an increasing and concave function of u_t This means that a mean-preserving spread in the distributions of the fiscal shock will induce employment to decrease on average. The effects of volatility on expected labour are referred in the rest of the paper as 'employment effects'. We are now ready for:

Proposition 2 Assume $\delta = 100\%$ and all shocks are *i.i.d.* and uncorrelated. Then the following expression for the rate of growth of output holds if the labour market is competitive:

$$\frac{Y_{t+1}}{Y_t} - 1 = \frac{b_{t+1}\varkappa \left(\frac{\alpha[(1+u_{t+1})(1-\varkappa)+\varkappa(1+u)]}{(1-\varkappa)\eta\lambda}\right)^{\alpha/\eta} (1+u)}{(1-\varkappa)(1+u_t) + \varkappa(1+u)}.$$
(24)

Proof. See Appendix. \blacksquare

As in the previous case, growth is a linear function of the technology shock b_{t+1} , so the variance of this shock will not affect unconditional average growth. The fiscal shocks enter (24) in a fashion analogous to that in which they enter (21), so we expect the effects of an increase in σ_u^2 to be similar to those we have seen in the economy with unions. In fact we have the following approximation:

$$E\frac{Y_{t+1}}{Y_t} - 1 \simeq B - 1 + B_u \sigma_u^2, \tag{25}$$

with $B \equiv b\varkappa \left(\frac{\alpha(1+u)}{(1-\varkappa)\eta\lambda}\right)^{\alpha/\eta}$, $B_u \equiv \frac{B}{2} \left(\frac{1-\varkappa}{1+u}\right)^2 \left[2 + \frac{\alpha}{\eta} \left(\frac{\alpha}{\eta} - 1\right)\right] > 0.$

Intuitively, as in the previous case, an increase in σ_u^2 causes an increase in precautionary savings which more than offsets the negative 'employment' and 'diminishing returns to labour' effects. We can also notice that expected growth will be higher than when labour is sold monopolistically (by comparing A and B with $\eta = 1$) and that the positive effect of volatility on growth will be higher as well (by comparing A_u and B_u with $\eta = 1$).

5 Simulating the Model

In order to study the effects of volatility of the exogenous shocks on growth in the general case the model is solved following the numerical method based on accurate second-order approximation to the policy functions representing the optimum paths for the control and the endogenous state variables devised by Schmitt-Grohé and Uribe (2004).⁵

5.1 Inducing Stationarity

In this economy a number of variables, such as output, consumption etc. will not be stationary along the balanced-growth path. We therefore perform a change of variables, so as to obtain a set of equilibrium conditions that involve only stationary variables. We note that non stationary variables at time t are cointegrated with K_t , while the same variables at time t+1 are cointegrated with K_{t+1} . We divide variables by the appropriate cointegrating factor and denote the corresponding stationary variables with lowercase letters.

Using (15), the economy wide resource constraint can be written as:

$$1 + g_{t+1} = \frac{c_{t+1}}{c_t} \left[b_t L_t^{\alpha} - c_t (1 + u_t) + 1 - \delta \right].$$
(26)

where $c_t \equiv \frac{C_t}{K_t}$ and $g_{t+1} = \frac{C_{t+1}}{C_t} - 1$ i.e. g_t indicates consumption growth.

Using (4) and (9), the Euler equation becomes:

$$1 = E_t \frac{\beta \left[(1 - \alpha) b_{t+1} L_{t+1}^{\alpha} + (1 - \delta) \right]}{1 + g_{t+1}}.$$
(27)

Equation (10) can be written as:

$$m_{p,t} = \frac{\mu c_t}{1 - \beta}.$$
(28)

 $^{{}^{5}}$ Kim et al. (2003) propose an alternative algorithm for calculating second order approximations to the solutions to nonlinear stochastic rational expectation models based on the "state free" approach described in Sims (2001). More recently Lombardo and Sutherland (2007) have proposed a methodology for computing second-order accurate solutions of non-linear rational expectation models using a two-step algorithm devised for the solution of linear expectation models. Their algorithm generates identical results to those reported by Schmitt-Grohé and Uribe (2004) in their example on a stochastic growth model.

where $m_{pt} \equiv \frac{M_t}{P_t K_t}$.

Equation (10) also tells us that inflation $\pi_{t+1} = \frac{P_{t+1}}{P_t} - 1$ evolves according to:

$$(1+\pi_t)(1+g_t) = a_t.$$
 (29)

Coming to the labour market equilibrium, (17) in terms of stationary variables becomes:

$$L_t = \left(\frac{\alpha^2 a_t b_t E_{t-1} L_t a_t^{-1}}{\eta \lambda c_t E_{t-1} L_t^{\eta}}\right)^{1/(1-\alpha)},\tag{30}$$

and (18) becomes:

$$L_t = \left(\frac{\alpha b_t}{\eta \lambda c_t}\right)^{1/(\eta - \alpha)}.$$
(31)

A stationary competitive equilibrium is a set of stationary processes $\{c_t, g_t, m_{pt}, \pi_t, L_t\}$ satisfying (26), (27), (28), (29), and (30) (in case of monopolistic wage setting) or (31) (if the labour market is competitive), given the exogenous stochastic processes $\{a_t, b_t, u_t\}$ and the transversality conditions:

$$\lim_{\tau \to \infty} \beta^{\tau} E_t \left(\frac{m_{t+\tau}}{c_{t+\tau}} \right) = \lim_{\tau \to \infty} \beta^{\tau} E_t \left(\frac{z_{t+\tau+1}}{c_{t+\tau}} \right) \left(b_t L_{t+\tau}^{\alpha} - c_{t+\tau} (1+u_{t+\tau}) + 1 - \delta \right) = 0.$$

5.2 Deterministic Balanced Growth Path

We have now to pin down the deterministic balanced growth path equilibrium, which will be the centre of our approximation. To indicate steady-state variables we drop the time subscript, i.e. x is the steady state value of the generic variable x_t . We have the following steady-state relationships:

$$1 + g = bL^{\alpha} - c(1 + u) + 1 - \delta, \tag{32}$$

$$m_p = \frac{\mu c}{(1-\beta)},\tag{33}$$

$$1 + g = \beta \left[(1 - \alpha)bL^{\alpha} + 1 - \delta \right], \tag{34}$$

$$(1+\pi)(1+g) = a, (35)$$

$$L = \left(\frac{b\alpha^2}{\eta\lambda c}\right)^{1/(\eta-\alpha)}.$$
(36)

If the labour market is competitive instead of (36) we have

$$L = \left(\frac{b\alpha}{\eta\lambda c}\right)^{1/(\eta-\alpha)}.$$
(37)

Finally, from (2), (6) and (16):

$$C_b = (1 - \rho_b)b,\tag{38}$$

$$C_a = (1 - \rho_a)a,\tag{39}$$

$$C_u = (1 - \rho_u)u. \tag{40}$$

We will study the implications of higher uncertainty on growth considering first the case of monopolistic nominal wage setting, and then the case of a competitive labour market.

5.3 Calibration

To implement the simulation method we have to choose values for the parameters appearing in the equations. The time period in the model is assumed to be one year. For some of these parameters, estimates are available in the empirical literature, others are chosen in order to make the steady-state values of the variables consistent with the data of the US economy. For each parameter we choose a benchmark value. To check for the robustness of our results we then consider a range of other possible values for some of the parameters, fixing the other parameters at their benchmark level. The baseline calibration of the model is reported in Table 1, where most of benchmark parameter values are set along the lines of the existing literature. Consistently with Schmitt-Grohé and Uribe (2007) the subjective discount factor β equals 0.96, the annual rate of depreciation of capital δ equals 0.1 and the cost share of labour α is set to 0.7. We set η at 1.5, while μ and λ are implied values.

The persistence of the money supply shock and the annual standard deviation of the innovation have been estimated over the period 1980-2007 using FRED data for seasonally adjusted M2. As in Schmitt-Grohé and Uribe (2007) the autoregressive parameters in the driving forces b_t and u_t are set equal to 0.85 and 0.87, respectively, while the standard deviations of the innovations are $\sigma_{\varepsilon_b} = 0.0212$ and $\sigma_{\varepsilon_u} = 0.0102$.

As in Galí et al. (2007) we set the share of government purchases in value added to be 20 percent in steady state, which is in line with the observed U.S. postwar average. In the U.S. M2 was on average about 52 percent of annual GDP over the period 1980 to 2007. King and Rebelo (1999) suggest that the average GDP-capital ratio in the U.S. is about 50% on annual basis. This gives steady-state values for m_p and s equal to 0.26 and 0.1, respectively. The steady-state inflation rate is assumed to be 4 percent per year. This value is consistent with the average U.S. consumer price index change over the period 1980-2007. The steady-state value for L is 0.2. Following Jones et al. (2005) the value for the non-stochastic growth rate of consumption is set to 2%.

5.4 Volatility and Growth under Monopolistic Wage Setting

We first consider the effects of nominal and real volatility on growth under the assumption that the nominal wage is set by a monopolistic union prior to the realisations of shocks. In this case monetary shocks and their variance have real effects.

Tables 2-4 report the effects on mean consumption growth, E(g), of the volatility of the exogenous shocks. We also report the standard deviations of the growth rate of output, σ_g , and of each relevant shock, σ_a , σ_b , σ_u in turn. The first row of each table reports the results obtained by using benchmark values for the parameters of Table 1. To check for the robustness of our findings we study the sensitivity of our results using alternative values for the standard deviation of the innovation, holding all other parameters fixed. Then we vary the persistence of each shock, holding all other parameters at their benchmark values. We also check how expected growth changes when shocks' serial correlation changes, while their variance is fixed. Finally, we vary the labour supply coefficient η , the subjective discount factor β , the rate of capital depreciation δ , and the steady-state employment level L.

5.4.1 Monetary Shocks Volatility

Table 2 shows that increased monetary policy variability results in lower growth and a lower level of equilibrium employment. First we notice that, given equation (27), abstracting from the real shocks, to a first order the rate of growth goes up if labour goes up as well. So we first focus on this variable to interpret this result. In particular, the employment rate will be lower the higher is η . Our intuitive explanation is built around the optimal condition for the nominal wage (11). We can see that for given price level and consumption, the wage is the ratio of the expected value of a convex function of employment and of the expected value of a linear function of employment. So for a given level of employment, if the variance of employment increases, then the numerator will increase more than the denominator, pushing up the target real wage, or in other terms, through labour demand, pushing down equilibrium employment. So we have 'target real wage effect', involving a negative 'employment effect'. Notice however that, by (3), labour demand is a convex function of the real wage, so that an increase in the variability of the real wage tends, through this mechanism, to increase employment. This is the 'labour demand effect'.

The sign of the relationship between nominal volatility and consumption growth tends to be positive for low values of the coefficient η . Intuitively, the lower is η the weaker the 'target real wage effect' and the implied 'employment effect', as Table 2 shows. The sum of the positive 'precautionary saving effect' and 'labour demand effect' then prevails. Looking again at (27) from a different angle, we have to consider that the expression on the right hand side is concave in labour and convex in growth, so an increase in the variance of labour moves the expression down and an increase in the variance of growth moves it up, making it possible for labour and growth to move in opposite directions with increased uncertainty (in Table 2 this is seen to happen for $\eta = 1.2$).

Conversely, the higher η , the larger in absolute value the negative effects produced by monetary volatility on the unconditional mean of consumption growth. In fact, the lower the FELS, the higher the positive response of the nominal wage to the variance of the monetary shock i.e. the 'target real wage effect', and, as a consequence, the stronger the negative 'employment' effect, which dominates the positive 'precautionary saving effect' and 'labour demand effect'.

Finally, we notice that the persistence of the monetary shock has no effect on the relationships derived, which is explained by the fact that in the model only money surprises have real effects. In fact this also explains the fact that an increase in the variance of the monetary shock has negligible effects on the variance of growth. As we will see, for all shocks when the coefficient of autocorrelation is set to zero, the effect of their variance on growth volatility is negligible. If the model had featured for instance staggered wage contracts, monetary shocks would have had more persistent effects and money shocks volatility would have had bigger effects on growth volatility: including such mechanisms in the model is a direction for future research.

5.4.2 Technological Shocks Volatility

Table 3 shows that in the presence of technological variability uncertainty increases average consumption growth and average employment. This is partly due to the convexity of labour demand with respect not only to the real wage, as described above, but also with respect to b_t , as easily readable from (3). These two 'labour demand effects' prevail over the 'target real wage effect', so employment on average goes up. The largest impact of uncertainty upon growth is observed for low levels of the parameter η : when η is high the 'target real wage effect' tends to dampen the 'labour demand effects' and the 'precautionary saving' effect.

We observe that changes in the variability of the innovation, σ_{ε_b} , have a larger impact on the unconditional mean of growth rates than changes in the persistence of the shock, ρ_b . Nevertheless, both sources of volatility have a monotonically increasing effect on the average growth rate. Table 3 also shows how expected growth changes when the autocorrelation coefficient in the exogenous state variable changes, while keeping its variance fixed. This means that even if technological uncertainty in itself increases growth, however a negative correlation between growth and its variance may be detected in the data. This shows that it could be important in empirical analysis to decompose total variability into a pure 'risk' element, due to the innovation variance, and the increase in persistence, which is a predictable element, a point raised in Wolf (2005).

5.4.3 Fiscal Shocks Volatility

The interplay between the various effects of uncertainty, i.e the positive ones through precautionary savings and the convexity of labor demand with respect to the real wage and the negative one through the higher target real wage, is particularly difficult to disentangle in the case of fiscal shocks. Table 4 shows that consumption growth increases as fiscal variability rises if the FELS if is sufficiently high, and if the coefficient of autocorrelation of fiscal shocks is lower than 0.95.

We notice however that varying the serial correlation of the fiscal shock has a non-linear effect on average growth rate: we observe first a positive, then a negative relationship. Similarly, even for a high FELS ($\eta = 1.5$), fixing the variance of the shock at its benchmark value, if the autocorrelation of the shocks is above 0.7 expected growth will be decreasing in ρ_u , while its standard deviation will be increasing so, as in the case of the technological shock, a negative correlation between growth and its variance will appear empirically even for configurations of parameters conductive to higher growth under uncertainty. This shows the importance in applied work to separate expected volatility from unexpected volatility.

5.4.4 The Effect of the Frisch Elasticity of Labour

From the above results it clearly emerges that the size of the Frisch (compensated) elasticity of labour supply is a crucial parameter in determining the effects of real and nominal volatility. We recall that available estimates from micro data vary widely, due to measurement errors and sample selection bias problems, but most surveys report a "consensus" estimate for labour supply elasticity of -0.1 for males, with the income effect being the double in absolute value of the substitution effect. Blundell and Macurdy (1999) and Borjas (2008) offer recent overviews of the literature. This would correspond to a value for η equal to 11. The elasticity of labour supply for females is estimated to be higher (see for instance Blau and Kahn, 2006). Aggregate models often assume elastic labor supply, despite the low estimates from empirical studies based on individual data. The explanation offered is that fluctuations of hours are mainly accounted for by participation rates, so that the important margin is the extensive rather than the intensive (see, for instance, Rogerson and Wallenius 2007 and Chang and Kim 2006). Given the ongoing debate on the issue, we offer results on a wide range of values of the FELS. Figures 1-3 plot the relationship between the unconditional mean of consumption growth, E(q), and the standard deviation of each shock, for four different levels of the parameter η . In particular, in each plot we vary the standard deviation of the innovation, holding the serial correlation and all other parameters fixed at their benchmark levels.

For high values of the parameter η we observe that uncertainty has a negative effect on growth in the case of money and fiscal shocks, while the positive effect is lowered by a high value of η in the case of the technology shock. We also notice that the relationship between mean growth rates and volatility is monotonic for each shock.

5.5 Volatility and Growth in a Competitive Labour Market

Consider now the effects of uncertainty on average growth under competitive labour markets with flexible wages. Tables 5-6 present the effects on the unconditional mean of consumption growth E(g), and on mean employment, E(L), of the volatility of the real shocks.

Close inspection of the results reveals that in general the sign of the relationship between the volatility of the shocks and average consumption growth is not affected by the labour market wage setting mechanism, however the magnitude of the observed effects may change considerably.

Table 5 shows that the effects of uncertainty of the technology shock are still positive with a competitive labour market, but the size of the effects is systematically lower than in a monopolistic market with nominal rigidities.

When we vary the persistence of the technology shock, ρ_b , holding σ_b and all other parameters fixed, again we observe a non-linearity, with expected growth and its volatility being negatively related for $\rho_b > 0.7$.

Larger positive effects are instead observed under a competitive labour market when the source of economic fluctuation is given by fiscal policy (see Table 6): this confirm the result we found in the analytic setting. As seen in the case of nominal wage rigidities expected growth is lower than in a deterministic case for a high value of the shock autocorrelation. Again we study how expected growth changes when the autocorrelation coefficient in the fiscal shock changes, while keeping its variance fixed. Note that again increasing the autocorrelation coefficient of the exogenous fiscal process has a non-linear effect on the average growth rate E(g).

Finally we observe that, as in the case of monopolistic wage setting, the effect of uncertainty on consumption growth varies with the parameter η affecting the FELS. The lower the FELS, the lower the positive effect of a given amount of uncertainty upon growth. In particular, Figures 4-5 plot consumption growth rates as a function of the serial correlations for each shock when a higher value of η is considered, keeping all other parameters constant. We observe that when volatility is due to the fiscal shock, for a lower FELS there is a wider gap between the effects on growth rates produced by uncertainty under monopolistic nominal wage setting and those observed in a competitive labour market. Unsurprisingly the role played by labour market institutions in affecting the relationship between growth and volatility tends to be higher, the lower the FELS.

6 Concluding Remarks

In this paper we explore the links between short-run (cyclical) phenomena and long-run technological trend of output. The study of the issue has important policy implications as it opens the possibility that stabilization policies affect the long-run performance of the economy, with cumulative effects.

The impact of volatility on economic growth has been the subject of considerable investigation in the empirical literature. However, the evidence provided on the variability-growth relationship is mixed, withe cross-section studies finding a negative correlation and time-series studies finding a positive correlation, especially when nominal volatility is jointly considered. Our results help to explain this evidence: in fact in the model presented the relationship between growth and volatility depends critically on the source of the stochastic fluctuations in the economy.

A key role is also played by the Frisch elasticity of labour supply and by labour

market institutions. We observe that in general consumption growth is adversely affected by the volatility of the money supply shock, unless the Frisch elasticity of labour supply is low (but still consistent with available estimates). Conversely, the volatility of the technological shock has a positive impact on growth: however this will not always give rise to an observed positive correlation between growth and its standard deviation, unless the innovation variance is isolated from the predictable component of volatility. Finally, the volatility of government spending will have a negative effect on growth for a low enough Frisch elasticity and/or for a high enough degree of serial correlation in the process governing the shocks.

Coming to the effect of the organization of the labour market, we show that money volatility will have a negative effect on growth in the presence of nominal wage setting, while the effect of technology variability will be stronger and those of fiscal volatility weaker than with a competitive labour market.

Appendix

Derivation of (17): When $L_t = N_t$ using (10) we can rewrite (11) as: $\frac{(1-\beta)M_{t-1}\eta\lambda E_{t-1}L_t^{\eta}}{\alpha\mu E_{t-1}L_t a_t^{-1}} = W_t$, while, combining (3) and (10) we get $W_t = \alpha b_t L_t^{\alpha-1} K_t \frac{(1-\beta)\gamma M_t}{\mu C_t}$. Eliminating W_t from these two equations we get: $\frac{(1-\beta)M_{t-1}\eta\lambda E_{t-1}L_t^{\eta}}{\alpha\mu E_{t-1}L_t a_t^{-1}} = \alpha b_t L_t^{\alpha-1} K_t \frac{(1-\beta)\gamma M_t}{\mu C_t}$ which gives us (17) considering: $M_t = a_t M_{t-1}$.

Derivation of (19): When $\delta = 100\%$ we have: $K_{t+2} + C_{t+1} + G_{t+1} = Y_{t+1} = K_{t+1}(1+R_{t+1})(1-\alpha)^{-1}$ where the second equality comes from (4). We can then write (8) as:

$$C_t^{-1}K_{t+1} = \beta E_t C_{t+1}^{-1} (K_{t+2} + C_{t+1} + G_{t+1}) (1 - \alpha).$$
(41)

When u_t is i.i.d. (and uncorrelated with each other) we can write $E_t u_{t+i} \equiv u$ for all i = 1, 2..n. Iterating and considering the transversality condition we then have:

$$C_t = \frac{1 - \varkappa}{\varkappa (1+u)} K_{t+1},\tag{42}$$

where $\varkappa \equiv \alpha(1-\beta)$. Using: $C_t(1+u_t) + K_{t+1} = Y_t$ we can easily derive (19) and:

$$C_t = \frac{1 - \varkappa}{(1 + u_t)(1 - \varkappa) + \varkappa(1 + u)} Y_t.$$
(43)

Derivation of (20): Combining (10) and (43) we have:

$$M_t(1-\beta)\mu^{-1} = \frac{(1-\varkappa)}{\gamma \left[(1+u_t)(1-\varkappa) + \varkappa(1+u) \right]} Y_t P_t.$$

Using (3) and rearranging we have:

$$L_t = \frac{\alpha \gamma M_t (1 - \beta) \left[(1 + u_t) (1 - \varkappa) + \varkappa (1 + u) \right]}{\mu (1 - \varkappa) W_t}.$$
(44)

We can then compute:

$$\eta \lambda L_t^{\eta} = \eta \lambda \left(\frac{\gamma \varkappa M_t \left[(1+u_t)(1-\varkappa) + \varkappa (1+u) \right]}{\mu (1-\varkappa) W_t} \right)^{\eta},$$

and

$$\eta \lambda E_{t-1} L_t^{\eta} = \eta \lambda E_{t-1} \left(\frac{M_t \varkappa \left[(1+u_t)(1-\varkappa)\gamma_t + \varkappa \gamma(1+u) \right]}{\mu(1-\varkappa)W_t} \right)^{\eta}.$$
 (45)

Using (43), (11) can be rewritten as:

$$\eta \lambda E_{t-1} L_t^{\eta} = \alpha E_{t-1} W_t L_t (Y_t P_t)^{-1} (1 - \varkappa)^{-1} \gamma \left[(1 + u_t (1 - \varkappa) + \varkappa (1 + u)) \right]$$

= $\frac{\alpha^2}{1 - a} \gamma E_{t-1} \left[(1 + u_t (1 - \varkappa) \gamma + \varkappa (1 + u)) \right].$

The second equality is obtained using (3). Finally, using the assumed properties

of the distribution of the shock u_t we have $\eta \lambda E_{t-1} L_t^{\eta} = \frac{\alpha^2 \gamma(1+u)}{1-\varkappa}$, using which in (45) we arrive to:

$$W_t^{\eta} \frac{\alpha^2 \gamma(1+u)}{1-\varkappa} = \eta \lambda E_{t-1} \left(\frac{\gamma M_t \varkappa \left[(1+u_t)(1-\varkappa) + \varkappa (1+u) \right]}{\mu(1-\varkappa)} \right)^{\eta}$$

To proceed further we assume $\eta = 1.^6$ Assuming $\rho_a = 0$ and $\rho_{\lambda} = 0$, the expression above becomes:

$$W_t = \frac{M_{t-1}\varkappa\lambda a}{\alpha^2\mu}.$$

This, using (44), can be rewritten as (20).

Proof of Proposition 1: Using the production function to write $Y_{t+1} = K_{t+1}b_{t+1}L_{t+1}^{\alpha}$ and the expression for Y_t in terms of K_{t+1} given by (19), we can write the rate of growth of output as:

$$\frac{Y_{t+1}}{Y_t} = \frac{b_{t+1}L_{t+1}^{\alpha}\varkappa(1+u)}{(1+u_t)(1-\varkappa)+\varkappa(1+u)},\tag{46}$$

Substituting (20) in (46) we find (21).

Derivation of (23): If we combine (43) and (12), while considering that labour income is α times total income, we get (23).

Proof of Proposition 2: Substituting (23) in (46) we have (24).

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 $^{^{6}\}mathrm{Another}$ possibility, pursued in Blackburn and Pelloni (2005) is to treat u_{t} as constant through time.

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Table 1: Baseline Calibration								
Parameters								
β	0.96	Subjective discount factor						
δ	0.1	Depreciation rate						
α	0.7	Cost share of labour						
γ	1	Preference parameter						
η	1.5	Preference parameter						
ρ_a	0.69	Autoregressive parameter in the monetary shock						
$ ho_b$	0.85	Autoregressive parameter in the technology shock						
ρ_u	0.87	Autoregressive parameter in the fiscal shock						
σ_{ε_a}	0.0190	Standard deviation of the innovation ε_a						
σ_{ε_b}	0.0212	Standard deviation of the innovation ε_b						
σ_{ε_u}	0.0102	Standard deviation of the innovation ε_u						
Rati	os over (Capital and Steady-State Values						
m_p	0.26	Real money balances						
s	0.1	Government spending						
π	1.04	Inflation						
L	0.2	Labour supply						
q	2%	Consumption growth rate						

Table 1: Baseline Calibration

Benchmark	σ_{ε_a}	ρ_a	σ_a	E(q)	σ_a	E(L)	σ_L
	0.019	0.69	0.0263	1.9995	0.0000	0.1999	0.0000
The effect of σ_{ε_a} on growth rates	σ_{ε_a}	ρ_a	σ_a	E(g)	σ_q	E(L)	σ_L
	0.01	0.69	0.0138	1.9999	0.0000	0.2000	0.0000
	0.03	0.69	0.0415	1.9988	0.0000	0.1998	0.0000
	0.05	0.69	0.0691	1.9966	0.0000	0.1994	0.0000
	0.08	0.69	0.1105	1.9913	0.0000	0.1985	0.0000
	0.10	0.69	0.1382	1.9864	0.0000	0.1976	0.0000
	0.15	0.69	0.2072	1.9694	0.0000	0.1946	0.0000
The effect of ρ_a on growth rates	$\sigma_{arepsilon_a}$	ρ_a	σ_a	E(g)	σ_{g}	E(L)	σ_L
	0.019	0	0.0190	1.9995	0.0000	0.1999	0.0000
	0.019	0.50	0.0219	1.9995	0.0000	0.1999	0.0000
	0.019	0.90	0.0436	1.9995	0.0000	0.1999	0.0000
The effect of ρ_a on growth rates	$\sigma_{arepsilon_a}$	ρ_a	σ_a	E(g)	σ_g	E(L)	σ_L
for benchmark variance	0.0263	0	0.0263	1.9995	0.0000	0.1999	0.0000
	0.0227	0.50	0.0263	1.9995	0.0000	0.1999	0.0000
	0.0114	0.90	0.0263	1.9995	0.0000	0.1999	0.0000
The effect of η on the relationship				E(g)	σ_{g}	E(L)	σ_L
between volatility and growth	$\eta = 1.2$			2.0012	0.0000	0.1999	0.0000
	$\eta = 3$			1.9912	0.0000	0.1998	0.0000
	$\eta = 5$			1.9802	0.0000	0.1996	0.0000
Sensitivity				E(g)	σ_g	E(L)	σ_L
	$\beta = 0.9$	5		1.9995	0.0000	0.1999	0.0000
	$\beta = 0.9$	7		1.9995	0.0000	0.1999	0.0000
	$\delta = 0.0$	75		1.9993	0.0000	0.1999	0.0000
	$\delta = 0.12$	25		1.9997	0.0000	0.1999	0.0000
	L = 0.1	7		1.9995	0.0000	0.1699	0.0000
	L = 0.3			1.9995	0.0000	0.2999	0.0000

Table 2: Volatility of the Monetary Shock under Monopolistic Nominal Wage Setting

Ponchmark	σ	0	σ	$\frac{1}{E(a)}$	σ	$\overline{E(L)}$	<u> </u>
Denchmark	$0 \varepsilon_b$ 0.0212	$\frac{\rho_b}{0.85}$	0.0402	2.0112	0.0037	0.2001	0.0009
The effect of σ on growth rates	σ	0.00	σι	E(a)	σ	E(L)	σ.
The effect of $\mathcal{O}_{\mathcal{E}_b}$ on growth rates	$0 0 \varepsilon_b$	$\frac{\rho_b}{0.85}$	0 0190	2.0025	0.0017	0.2000	0.0004
	0.01	0.85	0.0150 0.0759	2.0020	0.0011	0.2000	0.0001 0.0017
	0.04	0.85	0.0100	2.0000 2 0895	0.0010	0.2002	0.0017
	0.00	0.85	0.1100	2.0000 2 1592	0.0100	0.2009	0.0020 0.0035
	0.00	0.85	0.1013	2.1002 2.2487	0.0110 0.0174	0.2000 0.2014	0.0000
	$0.10 \\ 0.15$	0.85	0.1000 0.2847	2.5596	0.0262	0.2011	0.0015
The effect of ρ_L on growth rates	σ_{ϵ}	ρ _ι	σ_{h}	E(q)	σ_{a}	E(L)	σ_L
	0.021	0	0.0212	2.0054	0.0000	0.2001	0.0000
	0.021	0.10	0.0213	2.0057	0.0003	0.2001	0.0001
	0.021	0.30	0.0222	2.0064	0.0008	0.2001	0.0004
	0.021	0.50	0.0245	2.0075	0.0015	0.2001	0.0006
	0.021	0.70	0.0297	2.0092	0.0024	0.2001	0.0008
	0.021	0.95	0.0679	2.0131	0.0065	0.2000	0.0009
	0.021	0.99	0.1503	2.0142	0.0146	0.2000	0.0013
The effect of $ ho_b$ on growth rates	σ_{ε_b}	ρ_b	σ_b	E(g)	σ_{g}	E(L)	σ_L
for benchmark variance	0.0402	0	0.0402	2.0195	0.0000	0.2003	0.0000
	0.0400	0.10	0.0402	2.0203	0.0005	0.2003	0.0003
	0.0384	0.30	0.0402	2.0211	0.0015	0.2002	0.0008
	0.0369	0.40	0.0402	2.0210	0.0020	0.2002	0.0009
	0.0349	0.50	0.0402	2.0203	0.0025	0.2002	0.0011
	0.0287	0.70	0.0402	2.0169	0.0033	0.2001	0.0011
	0.0175	0.90	0.0402	2.0083	0.0038	0.2000	0.0008
	0.0057	0.99	0.0402	2.0010	0.0039	0.2000	0.0003
The effect of η on the relationship				E(g)	σ_g	E(L)	σ_L
between volatility and growth	$\eta = 1.2$			2.0117	0.0038	0.2001	0.0012
	$\eta = 3$			2.0101	0.0034	0.2001	0.0004
	$\eta = 5$			2.0094	0.0033	0.2001	0.0002
Sensitivity				E(g)	σ_g	E(L)	σ_L
	$\beta = 0.9$	5		2.0095	0.0036	0.2001	0.0008
	$\beta = 0.9$	7		2.0126	0.0037	0.2001	0.0010
	$\delta = 0.0$	75		2.0152	0.0038	0.2001	0.0013
	$\delta = 0.12$	25		2.0085	0.0036	0.2000	0.0007
	L = 0.1	7		2.0089	0.0033	0.1700	0.0007
	L = 0.3			2.0197	0.0049	0.3002	0.0018

Table 3: Volatility of the Technology Shock under Monopolistic Nominal Wage Setting

Developments	<u> </u>	0	σ	$\frac{E(a)}{E(a)}$	σ	$\frac{E(I)}{F(I)}$	σ-
Benchmark	0_{ε_u} 0.0102	$\begin{array}{c} \rho_u \\ 0.87 \end{array}$	0.0206	2.0003	0.0008	0.2000	0.0016
The effect of σ on growth rotes	σ	0.01	σ	$\frac{E(a)}{E(a)}$		$\frac{E(L)}{E(L)}$	
The effect of O_{ε_u} on growth rates	$0 \varepsilon_u$	ρ_u	0 u	$\frac{D(y)}{2.0001}$	0_g	D(D) = D(D)	0 L
	0.000	0.87	0.0101	2.0001 2.0028	0.0004 0.0025	0.2000	0.0008
	0.05	0.87	0.0000	2.0028	0.0020	0.1999	0.0040
	0.05 0.07	0.07	0.1014 0.1490	2.0070 2.0150	0.0042	0.1998	0.0077
	0.07	0.07	0.1420	2.0100 2.0206	0.0059	0.1990	0.0108
	0.10 0.125	0.87	0.2028	2.0300 2.0477	0.0084	0.1992 0.1097	0.0100
	0.123	0.87	0.2000	2.0477	0.0105	0.1987	0.0194
The effect of ρ_u on growth rates	σ_{ε_u}	ρ_u	σ_u	E(g)	σ_g	E(L)	σ_L
	0.010	0	0.0102	2.0001	0.0000	0.2000	0.0000
	0.010	0.10	0.0102	2.0001	0.0000	0.2000	0.0000
	0.010	0.30	0.0107	2.0002	0.0000	0.2000	0.0002
	0.010	0.50	0.0117	2.0003	0.0002	0.2000	0.0003
	0.010	0.70	0.0142	2.0003	0.0004	0.2000	0.0007
	0.010	0.95	0.0326	1.9998	0.0016	0.1980	0.003
	0.010	0.99	0.0721	1.9951	0.0040	0.1930	0.0073
The effect of ρ_u on growth rates	$\sigma_{arepsilon_u}$	$ ho_u$	σ_u	E(g)	σ_{g}	E(L)	σ_L
for benchmark variance	0.0206	0	0.0206	2.0005	0.0000	0.2000	0.0000
	0.0205	0.10	0.0206	2.0006	0.0000	0.2000	0.0001
	0.0197	0.30	0.0206	2.0007	0.0002	0.2000	0.0003
	0.0179	0.50	0.0206	2.0008	0.0003	0.2000	0.0006
	0.0165	0.60	0.0206	2.0008	0.0004	0.2000	0.0008
	0.0147	0.70	0.0206	2.0007	0.0006	0.2000	0.001
	0.0124	0.80	0.0206	2.0005	0.001	0.2000	0.0013
	0.0090	0.90	0.0206	2.0002	0.001	0.2000	0.0017
	0.0029	0.99	0.0206	1.9996	0.0011	0.2000	0.0021
The effect of η on the relationship				E(g)	σ_q	E(L)	σ_L
between volatility and growth	$\eta = 1.2$			2.0003	0.0011	0.2000	0.002
	$\eta = 3$			1.9998	0.0004	0.19998	0.0007
	$\eta = 5$			1.9988	0.0002	0.19996	0.0004
Sensitivity				E(g)	σ_q	E(L)	σ_L
	$\beta = 0.9$	5		2.0003	0.0001	0.2000	0.0017
	$\beta = 0.9$	7		2.0003	0.0008	0.2000	0.0015
	$\delta = 0.0'$	75		2.0004	0.0007	0.2000	0.0015
	$\delta = 0.12$	25		2.0003	0.0010	0.2000	0.0017
	L = 0.1	7		2.0003	0.0008	0.1700	0.0013
	L = 0.3			2.0003	0.0008	0.3000	0.0023

Table 4: Volatility of the Fiscal Shock under Monopolistic Nominal Wage Setting

Benchmark	σ_{c}	01	σ_{h}	E(q)	σ_{a}	E(L)	σ_I
	0.021	0.85	0.0402	2.0112	0.0037	0.20006	0.0009
The effect of σ_{ε_h} on growth rates	σ_{ε_h}	ρ_b	σ_b	E(g)	σ_q	E(L)	σ_L
-0	0.01	0.85	0.0190	2.0022	0.0017	0.2000	0.0004
	0.04	0.85	0.0759	2.0353	0.0070	0.2002	0.0017
	0.06	0.85	0.1139	2.0794	0.0105	0.2005	0.0026
	0.08	0.85	0.1519	2.1411	0.0140	0.2008	0.0035
	0.10	0.85	0.1898	2.2205	0.0174	0.2012	0.0043
	0.15	0.85	0.2847	2.4962	0.0262	0.2029	0.0065
The effect of $ ho_b$ on growth rates	$\sigma_{arepsilon_b}$	$ ho_b$	σ_b	E(g)	σ_{g}	E(L)	σ_L
	0.021	0	0.0212	2.0035	0.0000	0.20007	0.0000
	0.021	0.10	0.0213	2.0038	0.0003	0.20007	0.0001
	0.021	0.30	0.0222	2.0045	0.0008	0.20007	0.0004
	0.021	0.50	0.0245	2.0057	0.0015	0.20007	0.0006
	0.021	0.70	0.0297	2.0075	0.0024	0.20006	0.0008
	0.021	0.95	0.0679	2.0123	0.0065	0.20004	0.0009
	0.021	0.99	0.1503	2.0137	0.0146	0.19997	0.0012
The effect of ρ_b on growth rates	$\sigma_{arepsilon_b}$	$ ho_b$	σ_b	E(g)	σ_g	E(L)	σ_L
for benchmark variance	0.0402	0	0.0402	2.0127	0.0000	0.20026	0.0000
	0.0400	0.1	0.0402	2.0135	0.0005	0.20025	0.0003
	0.0384	0.3	0.0402	2.0148	0.0015	0.20023	0.0007
	0.0349	0.5	0.0402	2.0152	0.0020	0.20018	0.0011
	0.0287	0.7	0.0402	2.0153	0.0025	0.20012	0.0011
	0.0266	0.75	0.0402	2.0130	0.0034	0.2001	0.0011
	0.0175	0.9	0.0402	2.0075	0.0038	0.2000	0.0008
	0.0057	0.99	0.0402	2.001	0.0039	0.2000	0.0003
The effect of η on the relationship				E(g)	σ_{g}	E(L)	σ_L
between volatility and growth	$\eta = 1.2$			2.0110	0.0038	0.20006	0.0012
	$\eta = 3$			2.0081	0.0034	0.20006	0.0004
	$\eta = 5$			2.0075	0.0033	0.20006	0.0002
Sensitivity				E(g)	σ_g	E(L)	σ_L
	$\beta = 0.9$	5		2.0133	0.0036	0.20005	0.0008
	$\beta = 0.9$	7		2.0076	0.0037	0.20007	0.001
	$\delta = 0.0$	75		2.0133	0.0038	0.20008	0.0013
	$\delta = 0.12$	25		2.0076	0.0036	0.2000	0.0007
	L = 0.1	7		2.0079	0.0033	0.1700	0.0007
	L = 0.3			2.0175	0.0049	0.3001	0.0018

Table 5: Volatility of the Technology Shock under Competitive Labour Market

Benchmark	σ_{ε}	ρ_{a}	σ_u	E(q)	σ_a	E(L)	σ_L
	0.010	0.87	0.0206	2.0011	0.0008	0.2^{-1}	0.0016
The effect of σ_{ε_u} on growth rates	σ_{ε_u}	ρ_u	σ_u	E(g)	σ_q	E(L)	σ_L
_	0.005	0.87	0.0101	2.003	0.0004	0.2000	0.0008
	0.03	0.87	0.0608	2.0091	0.0023	0.1999	0.0046
	0.05	0.87	0.1014	2.0253	0.0038	0.1999	0.0077
	0.07	0.87	0.1420	2.0495	0.0053	0.1998	0.0108
	0.10	0.87	0.2028	2.1095	0.0084	0.1996	0.0155
	0.125	0.87	0.2535	2.1711	0.0105	0.1993	0.0194
The effect of ρ_u on growth rates	$\sigma_{arepsilon_u}$	ρ_u	σ_u	E(g)	σ_g	E(L)	σ_L
	0.010	0	0.0102	2.0003	0.0000	0.2000	0.0000
	0.010	0.10	0.0102	2.0003	0.0000	0.2000	0.0000
	0.010	0.30	0.0107	2.0004	0.0000	0.2000	0.0002
	0.010	0.50	0.0117	2.0006	0.0002	0.2000	0.0003
	0.010	0.70	0.0142	2.0008	0.0003	0.2000	0.0007
	0.010	0.95	0.0326	2.0008	0.0015	0.2000	0.003
	0.010	0.97	0.0419	2.0002	0.0020	0.2000	0.0040
	0.010	0.98	0.0025	1.9993	0.0511	0.2000	0.0051
	0.010	0.99	0.0721	1.9967	0.0037	0.2000	0.0074
The effect of $ ho_u$ on growth rates	$\sigma_{arepsilon_u}$	ρ_u	σ_u	E(g)	σ_{g}	E(L)	σ_L
for benchmark variance	0.0206	0.10	0.0206	2.0012	0.0000	0.2000	0.0000
	0.0197	0.30	0.0206	2.0015	0.0002	0.2000	0.0003
	0.0179	0.50	0.0206	2.0017	0.0003	0.2000	0.0006
	0.0165	0.60	0.0206	2.0017	0.0004	0.2000	0.0008
	0.0147	0.70	0.0206	2.0017	0.0005	0.2000	0.0010
	0.0090	0.90	0.0206	2.0008	0.0008	0.2000	0.0016
	0.0029	0.99	0.0206	1.9997	0.0011	0.2000	0.0021
The effect of η on the relationship				E(g)	σ_g	E(L)	σ_L
between volatility and growth	$\eta = 1.2$			2.0007	0.0011	0.2000	0.0020
	$\eta = 3$			2.0022	0.0004	0.2000	0.0007
	$\eta = 5$			2.0027	0.0002	0.2000	0.0004
Sensitivity				E(g)	σ_{g}	E(L)	σ_L
	$\beta = 0.9$	5		2.0012	0.0001	0.2000	0.0017
	$\beta = 0.9$	7		2.0010	0.0008	0.2000	0.0015
	$\delta = 0.0$	75		2.0010	0.0007	0.2000	0.0015
	$\delta = 0.12$	25		2.0012	0.0010	0.2000	0.0017
	L = 0.1	7		2.0011	0.0008	0.1700	0.0013
	L = 0.3	8		2.0011	0.0008	0.3000	0.0024

Table 6: Volatility of the Fiscal Shock under Competitive Labour Market





Figure 2: Volatility of the Technology Shock under Monopolistic Nominal Wage Setting, $\rho_b=0.85$







Figure 4: Persistence of the Technology Shock, $\sigma_{\varepsilon_b}=0.021,\,\eta=5$



Figure 5: Persistence of the Fiscal Shock, $\sigma_{\varepsilon_u}=0.010,\,\eta=5$

