Diversification Potential in Real Estate Portfolios ⋆

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Abstract

In this paper, we study the international and sectoral diversification potential in real estate portfolios. Building on a unique dataset of direct real estate markets covering 16 OECD countries over the period 1999-2018, we introduce a statistical test to compare country-level and sector-level diversification potential. This new diversification test provides investors and analysts with a valuable tool as it delivers both estimates and robust significance levels. The empirical findings for real estate investments broadly reveal that international diversification dominates sectoral diversification.

Keywords: *Portfolio diversification, Real estate markets.*

JEL Classification: C23, F21, G11, R33

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1. Introduction

Commercial real estate attracts considerable investments in both direct and indirect markets due to its purported features. Chief among these are a perception of low or inverse correlation with stock and bond markets, potential inflation hedging benefits and outperformance vis-a-vis most other asset classes. Global real estate investments have enjoyed significant growth rates in recent years from USD 8.9 trillion in 2018 to USD 9.6 trillion in 2019 (Teuben and Neshat, 2020). A common strategy of institutional investors is to employ a top-down asset allocation, deciding first on a set of specific countries and property types and then moving into more fine-grained analysis and allocation at the individual asset level. The a priori selection of countries and property types can either be driven by macroeconomic and real estate sector analysis or by practical considerations, for example the expertise of investors or managers in particular markets, geographical proximity and ease of access to the assets under management. By definition, the top-down approach excludes investment opportunities that fall outside of the pre-defined areas and sectors, some of which may potentially enhance the risk-adjusted returns of a portfolio but cannot be pursued. By contrast, a bottom-up strategy does not set stringent exclusionary criteria but is prone to lacking a distinct strategic vision and coherent asset allocation rules. In practice, very few investors follow a pure top-down or bottom-up approach but typically use a combination of the two approaches (Worzala and Newell, 1997).

Despite a well established and increasingly sophisticated body of literature on the diversification benefits of real estate, it is still not well understood if diversification is generally best achieved at a global scale by diversifying portfolios across (1) property types such office, retail, industrial, hotels etc or (2) geographic markets in different countries. The present study investigates this question in the context of the diversification loss for a large number of countries and sectors for a dataset of real estate returns covering 16 countries in the $1999 - 2018$ period. The traditional measure of the diversification loss in a portfolio based on correlation has been widely criticized by Calvet et al. (2007). They introduce a relative

Sharpe ratio loss ($RSRL$) approach and we extend their work proposing a modified version of this measure taken into account the modified Value-at-Risk and which is robust to non-normality ($mRSRL$). Following Ledoit and Wolf (2008) and Ardia and Boudt (2015; 2018), we use a studentized circle block-bootstrap procedure to build robust confidence intervals for both measures. It turns out that a diversification strategy along geographic markets generally outperforms property type diversification.[1] Such a finding is of major importance for asset managers specialized in the real estate sector.

The remainder of this paper is organised as follows. After reviewing the state of research on measuring portfolio diversification, both in real estate and other asset classes, we outline our research strategy and methodology, notably the Relative Sharpe Ratio Loss ($RSRL$) approach, and propose (i) a modified version of the $RSRL$ that is robust to non-normality, i.e., the modified RSRL ($mRSRL$), and (ii) robust confidence intervals associated with both $RSRL$ and $mRSRL$ estimates. To summarize, our contribution is to propose a diversification test that is robust to finite samples of non-normal returns and non-iid residuals. We then proceed with the empirical analysis using the dataset mentioned above and present results on the diversification potential of the two principal strategies. Finally, we draw conclusions and review the implications for global real estate investors.

2. State of Research

Since Markowitz's seminal work in 1952, measuring portfolio diversification has become of major interest for both scholars and practitioners. In line with the modern portfolio theory (MPT), correlation and cluster analysis have long been used to measure diversification po-

[1] Strictly speaking, our empirical analysis deals with the estimation of the diversification potential. As we empirically validate our statistical test from efficient portfolios (that we compute using the input parameters derived from our dataset), we estimate portfolio dominance rather than portfolio diversification. For brevity purposes, we use the following terms "diversification potential" and "portfolio dominance" indifferently to refer to the diversification potential concept.

tential. Other methods have been used as proxies for diversification, such as the Herfindahl - Hirschman Index. However, focusing only on the weighting scheme of portfolios enables accounting for asset risks, but not for overall risk at a portfolio level. Inversely, focusing only on asset returns correlation is quite limited because it does not provide information about asset reward-risk ratios. As argued by Statman and Scheid (2008), both weighted asset returns and their correlations are needed to compute overall portfolio risks and returns, and hence to measure diversification.

2.1. Measuring diversification

Recent works have attempted to build diversification indices that reconcile these two different approaches. Rudin and Morgan (2006) proposed the Portfolio Diversification Index (PDI) based on a principal component analysis (PCA). Describing the most diversified portfolio, Choueifaty and Cognard (2008) introduced the diversification ratio (DR). The DR is defined as the ratio of the portfolio's weighted average variance to its overall variance (see also Choueifaty et al. (2013)). While this diversification measure has been popular among practitioners, the DR is based on correlations, which means that it only accounts for the first two moments of the return distribution. To overcome this drawback, Tasche (2006) introduced a diversification index based on the Value-at-Risk. Meucci (2009) proposed the effective number of bets (ENB), a diversification index based on the relationship between information entropy and portfolio diversification. Indeed, the Shannon entropy refers to the uncertainty related to an entire statistical distribution; i.e., higher diversification induces lower uncertainty and entropy. Thus, the common advantage of both Tasche's (2006) and Meucci's (2009) respective indices is to take into account idiosyncratic risks of individual assets using more than the first two moments of the return distribution. Vermorken et al. (2012) introduced a new measure of diversification: the diversification delta (DD). The objective of this new measure is to capture benefits from an entropy-based diversification proxy and to be interpreted as a classic correlation coefficient. Highlighting several drawbacks of the

DD, Flores et al. (2017) proposed a revised measure of this index: the diversification delta star (DD^*). Their purpose was to overcome the drawbacks of the original DD while retaining its attractive properties, such as its ease of computation and interpretation. Specifically, they revised the original DD to satisfy the risk measure criteria of Artzner et al. (1999): homogeneity and subadditivity. Last, they also built the DD^* to be bounded between 0 and 1.

The recent literature offers a set of several competing diversification measures. Easy to compute and to interpret, they nevertheless suffer from several common drawbacks. First, these measures are built to quantify the diversification level of a given portfolio, but not to compare diversification levels of several portfolios. Additionally, most of these recent measures are constructed from practitioners' specifications, being weakly connected to economic background. Last but not least, none of these diversification measures considers building confidence intervals, and neither conduct hypothesis testing. Considering these weaknesses, we follow another strand of the literature that offers interesting perspectives about diversification measurement. While investigating the ability of households to invest in efficient portfolios, Calvet et al. (2007) introduced the relative Sharpe ratio loss ($RSRL$) and the return loss (RL). Their aim is to quantify the household investment mistakes by comparing households' portfolios and a benchmark portfolio. In a classic mean-variance framework, the authors derive the $RSRL$ and RL from $CAPM$; these indicators measure the extent of financial loss from holding a subefficient portfolio. Broadly speaking, the $RSRL$ and RL proxy the distance between a given portfolio and the nearest portfolio on the efficient frontier. This measure of underdiversification is also used in Calvet et al. (2009) and Tang et al. (2010). These latest authors computed the empirical distribution of the $RSRL$ and, from confidence intervals, infer the $RSRL$ significance. More recently, Roche et al. (2013), Von Gaudecker (2015) and Dimmock et al. (2018) used the $RSRL$.

2.2. Diversification potential in real estate portfolios

The diversification benefits that can be achieved by extending the geographical coverage of an investment portfolio to multiple countries have been recognised in previous studies following Markowitz' paradigm (Markowitz 1952, Solnik 1974). At the same time, many of these studies warn that sufficient diversification in a direct real estate portfolio is only achievable for very large institutional portfolios (Byrne and Lee, 2003). A possible solution to this problem are indirect real estate investments, notably Real Estate Investment Trusts (REITs) whose return profiles offer significant but not perfect correlation with direct real estate returns (Hoesli and Oikarinen, 2012). A common caveat is that REITs are generally found to be positively correlated with equity markets (Peterson and Hsieh, 1997; Hoesli and Serrano, 2010; Lizieri, 2013). Hence, it may be argued that REITs only offer a partial exposure to the real estate market whilst being a less than ideal diversifier away from stock markets.

Notwithstanding these limitations, REITs and other real estate investment vehicles ought to be able to benefit from their underlying geographic and property type asset base. However, the proposition that diversification strategies yield a net benefit for real estate companies is not universally accepted. Hartzell et al. (2014) report that geographically diversified REITs were valued lower than their more geographically focused counterparts but further analysis showed that this discount can be mitigated by the involvement of large institutional owners in the REIT who actively monitor the REIT investment strategy. Feng et al. (2019) add another criterion for diversification benefits to materialise. In their study of equity REITs from 2010-16, the authors find diversified REITs with a high degree of transparency exhibit higher values whereas more opaque REITs appear to fare better when they are less diversified.

Apart from diversification, investors also seek to achieve an inflation hedge through their direct and indirect real estate investments. Hoesli et al. (2008) empirically test the long-term relationship between real estate yields and inflation by studying the private and public real estate markets in the United Kingdom and the United States. They find that this negative relationship is stronger for the private market than for the public market. While overall real estate market returns are on average uncorrelated with inflation, there is considerable variation in this relationship across direct and indirect real estate investment vehicles. In an earlier study, Eichholtz et al. (1995) analyse data from the USA and UK to compare efficient frontier portfolios of different property types and regions. The authors find that diversification by property type is not always necessary to achieve full diversification benefits. For example, they show that a US-retail portfolio is nearly as diversified as a fully diversified portfolio. However, they find that the UK shows less regional variation in retail returns than the USA and therefore diversification by property type appears to be the superior strategy in that market.

The hybrid nature of indirect real estate investments means that direct brick-and-mortar investments remain an attractive proposition to many diversification-seeking investors despite the identified problems with minimum lot size, indivisibility, illiquidity, high transaction costs, high idiosyncratic risk, opaque markets and high management costs. How successful the diversification strategy of a direct real estate portfolio will be, depends crucially on the investment horizon. Heaney and Sriananthakumar (2012), Lizieri (2013) and Sing and Tan (2013) show that the relationship between equity market returns and those of the direct real estate market is not stable over time. However, the authors emphasize that this relation remains weak. Regression coefficients are reversed during economic crises and/or remain insignificant for certain periods. Regarding the public real estate market, the empirical literature offers more empirical evidence. Many studies describe a change in the correlation pattern between real estate market returns and other asset classes during a market downturn. Goldstein and Nelling (1999), Chatrath et al. (2000), Clayton and MacKinnon (2001), and more recently Hoesli and Reka (2013; 2015) show that there is an asymmetry in the correlation between REIT and equity market returns, suggesting considerable co-movement between real estate and equity markets during market downturns and financial crises. However, Chiang et al. (2004) and Simon and Ng (2009) take the opposite view and argue that the observed changes in co-movement during different phases of the market cycle are due to underlying risk factors and that extreme dependencies are lower for REITs than they are for the equity market.

Despite the long-term nature of many real estate investments, relatively few studies investigate the long-term relationship between the real estate market and other asset classes. MacKinnon and Al Zaman (2009) find that the real estate proportion in an optimal multi-asset portfolio increases with the investment horizon and note that REITs are redundant if direct real estate can be incorporated into the portfolio. In a similar vein, Rehring (2012) highlights the crucial role of return predictability for the role of real estate in a multi-asset portfolio. More recently, Leone and Ravishankar (2018) investigated if sector-region diversification is still a viable strategy for improving portfolio-specific efficiency. Using Stochastic Frontier Analysis in a UK portfolio context, they report that empirically there is indeed scope for improving portfolio efficiency and lowering the variability of portfolio efficiency levels over time.

The consensus emerging from the extant literature appears to be that diversification is generally achievable by adding real estate to a portfolio both in the short and long run, although these benefits may be weaker in indirect investments and during periods of economic and financial turmoil.

3. Testing diversification loss

3.1. Estimating diversification loss: from the RSRL to mRSRL

Following Calvet et al. (2007), diversification losses can be proxied by comparing the Sharpe ratio of a given portfolio to the Sharpe ratio of a benchmark portfolio. Formally, we focus on the *ex post* Sharpe ratio, defined as follows:

Definition 1 (Sharpe, 1994)**.** *Let μ_i and σ_i be the mean return and volatility of the portfolio i. Let μ_j and σ_j be the mean return and volatility of the benchmark portfolio j. It is assumed that the benchmark portfolio is the risk-free rate, so $\mu_j = r_f$ and $\sigma_j = 0$. It is assumed* *that these two moments are perfectly known and perfectly summarize the distribution of the portfolio return i. We note the Sharpe ratio S_i as follows:*

$$S_i = \frac{\mu_i - r_f}{\sigma_i}. \tag{1}$$

For any portfolio h, we denote by μ_h and σ_h the mean and standard deviation of the excess return on the risky portfolio and by S_h the corresponding Sharpe ratio. Similarly, S_b defines the Sharpe ratio of the benchmark portfolio. The loss from imperfect diversification with respect to the benchmark can be quantified by the relative Sharpe ratio loss.

Definition 2 (Calvet et al., 2007)**.** *Let S_h and S_b be the Sharpe ratios of the portfolio h and the benchmark portfolio b, respectively. The relative Sharpe ratio loss is as follows:*

$$RSRL_h = 1 - \frac{S_h}{S_b}. \tag{2}$$

As illustrated in Figure 1, the $RSRL$ relates the normalized difference of angles $\alpha_h - \alpha_b$. The $RSRL$ is defined on $[0, 1]$; by construction, it equals zero for identical portfolios and one if the overall risk of portfolio h is idiosyncratic only.

Figure 1: The $RSRL$ as a measure of underdiversification

Scheme_1.PNG

Notes: This figure illustrates the $RSRL$ roots in a mean-variance analysis. From this figure, trigonometry helps to understand the $RSRL$ as a measure of underdiversification.

Based on Sharpe ratios, this (under)diversification measure implicitly assumes that returns are fully characterized by their two first moments, i.e., the hypothesis of returns' normality. In case of a non-normal distribution of asset returns ($student - t$, for example), the $RSRL$ depends on higher moments of the distribution (skewness, kurtosis, etc.). Therefore, the

$RSRL$ no longer constitutes an indicator of diversification. In such a case, Ardia and Boudt (2015) recommended substituting the standard deviation with the modified Value-at-Risk and to build a modified Sharpe ratio (mS) robust to higher moments. Several modified Sharpe ratios coexist in the literature. These ratios are based on different modified Value-at-Risk measures: Favre and Galeano (2002) and Gregoriou and Gueyie (2003) used a Cornish-Fisher (1937) expansion to calculate a modified VaR analytically. More recently, Bali et al. (2013) simply used the $VaR^{99\%}$ to replace the standard deviation. Both approaches enable building a modified Sharpe ratio which is robust to non-normal returns. We choose Gregoriou and Gueyie's modified Value-at-Risk (2003), as suggested in Ardia and Boudt (2015).

Definition 3 (Gregoriou and Gueyie, 2003)**.** *Let* μ_i *and* $mVaR_i^{\alpha\%}$ *be the mean return and the* $1-\alpha$ *% Cornish-Fisher's approximation of the Value-at-Risk of the portfolio* i *and* r_f *the risk-free rate. We note the modified Sharpe ratio* $g_m(\mu_i, mVaR_i^{\alpha\%}) = mS_i$*:*

$$mS_i = \frac{\mu_i - r_f}{mVaR_i^{\alpha\%}}. \tag{3}$$

The literature shows that the Cornish-Fisher approximation provides a more accurate estimation of the Value-at-Risk. This has been confirmed in a recent study; Amédée-Manesme et al. (2015) have also shown that, given the properties of commercial real estate indices, Value-at-Risk should be adjusted by a Cornish-Fisher approximation associated with a rearrangement procedure as in Chernozhukov, Fernández-Val and Galichon (2010) (essentially a simple reordering).

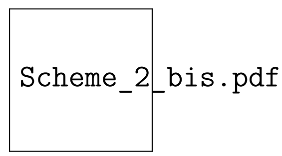
It is then possible to extend the $RSRL$, introducing the modified relative Sharpe ratio loss ($mRSRL$) robust to non-normally distributed returns:

Definition 4. *Let* mS_h *and* mS_b *be the modified Sharpe ratios of the portfolio* h *and the* *benchmark portfolio b, respectively. The modified relative Sharpe ratio loss is as follows:*

$$mRSRL_h = 1 - \frac{mS_h}{mS_b}. \tag{4}$$

Similarly to the $RSRL$, the $mRSRL$ is defined on $[0;1]$ by construction[2] and is a monotonic and decreasing function with respect to diversification.

Figure 2: The $RSRL$ and $mRSRL$ as measures of underdiversification



Notes: This figure illustrates the $RSRL$ (resp. $mRSRL$) rooted in a mean-variance (resp. mean-$mVaR$) analysis. From this figure, trigonometry helps to understand the $RSRL$ (resp. $mRSRL$) as a measure of underdiversification.

3.2. Empirical counterparts for the RSRL and mRSRL

In the previous section, it was assumed that the moments were exactly known and perfectly defined the distribution of the portfolio returns. We now consider the empirical counterpart (i.e., when the moments of the distribution are estimated).[3] Under the assumption of normality of the portfolio returns' distribution, estimating the mean and variance is sufficient. Without loss of generality, the empirical counterpart of the $RSRL$ can then be expressed as:

$$RS\hat{R}L_h = 1 - \frac{\hat{S}_h}{\hat{S}_b}, \tag{5}$$

where

$$\hat{S}_i = \frac{\hat{\mu}_i - r_f}{\hat{\sigma}_i}, \quad i = (h, b), \tag{6}$$

[2]It equals zero if the composition of portfolio h (P_h) is identical to that of portfolio b (P_b) but is lower than 1, as we assume that the benchmark portfolio is the most diversified one.

[3]Estimated moments are indicated with a hat.

$\hat{\mu_i}$ and $\hat{\sigma_i}$ being unbiased estimates of the mean and the variance of the portfolio returns. Similarly, $mR\hat{S}RL_h = 1 - \frac{m\hat{S_h}}{m\hat{S_b}}$, which requires an unbiased estimate of $mVaR_i^{\alpha\%}$.

To better understand the tests to be implemented in order to test for the diversification ability from the $RSRL$ and $mRSRL$, let us consider the two portfolios (b and h), with the following features:

$$\mu = \begin{pmatrix} \mu_h \\ \mu_b \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} \sigma_h^2 & \sigma_{hb} \\ \sigma_{hb} & \sigma_b^2 \end{pmatrix}, \text{ where } \mu \text{ is the return and } \sigma \text{ the risk.}$$

A proper test to compare the diversification possibility of portfolio h with respect to the benchmark b would consist of testing for the nullity of the $RSRL$, i.e., $RSRL_h = 1 - \frac{S_h}{S_b} = 0$. It is straightforward to notice that this null hypothesis is strictly equivalent to $\Delta = \mathrm{S}_h - \mathrm{S}_b = \frac{\mu_h}{\sigma_h} - \frac{\mu_b}{\sigma_b} = 0$. It is thus statistically strictly equivalent to test for the nullity of the $RSRL$ or the nullity of the Sharpe ratio difference Δ. Hence, whether the null $\Delta = 0$ is not rejected, portfolio h does not offer diversification advantages, whereas it does under the alternative $\Delta \neq 0$.

Under normal iid returns, testing for diversification, and thus loss, boils down to:

$$H_0 : RSRL \equiv S_b - S_h = 0. \tag{7}$$

Similarly, when higher moments are characterizing the returns' distribution, testing for diversification loss should be conducted at a particular risk level (α) using the modified Sharpe ratios at α, denoted as $mRSRL(\alpha)$ (see Ardia and Boudt, 2015). We then focus on testing the significance of $mRSRL$:

$$H_0 : mRSRL \equiv mS_B(\alpha) - mS_h(\alpha) = 0, \tag{8}$$

Asymptotic distribution of both tests is relatively easy under these strong assumptions (see Memmel, 2003). However, considering "real world" financial data features as such strong assumptions (i.e., skewed distribution, autocorrelation, and heteroscedasticity to name but a few), such assumptions would likely be violated. Just as a practical example, the finite sample framework (< 30 observations per series) often encountered does not advocate in favor of asymptotic properties. In such a case, and following Ledoit and Wolf (2008) or Ardia and Boudt (2015; 2018), a bootstrap version of the test appears as the solution. Both studies propose a studentized circular block version of the traditional bootstrap approach. Circular block-bootstrap has been proposed by Politis and Romano (1992). Contrary to conventional block-bootstrap methods, such as the moving block-bootstrap or the non-overlapping block-bootstrap, the circular block-bootstrap uses elements from the periodically extended series. Studentization consists in considering as the bootstraped test statistic $mRSRL^b/s(mRSRL^b)$, where ^{b} indicates the b-th bootstrap draw and $s(.)$ its standard error instead of $mRSRL^b$. The studentized test statistic is asymptotically distributed as the absolute value of a standard normal random variable.

4. Empirical analysis

4.1. Data

Revisiting the diversification potential in real estate portfolios, we aim at investigating the relative size of both international and sectoral diversification benefits. Based on REITs data, most studies focus on either economic or geographical regions within several countries. Here, the objective is to compare diversification by country with diversification by property type. Our empirical contributions to the literature are to extend the empirical investigation to a panel of 16 OECD countries and 4 sectors (residential, retail, office and industrial) and to simultaneously consider the sectoral as well as the geographical dimensions of diversification.

Indeed, an international analysis of the diversification potential in the real estate market requires data returns at a disaggregated level for both the geographical and sectoral dimensions. Among the few data providers of international real estate returns, we therefore chose the MSCI (ex IPD) database.[4] This database is the only one which fits our specifications, as it provides returns per country and per property type. However, because of missing values in the dataset, arbitrage between the number of countries and the number of observations (and data frequency) restricts the initial dataset. The constraints imposed by the needs of our empirical study led us to choose a panel of 16 OECD member countries covering the period 1999-2018. Annual returns are time-weighted (i.e., annual returns are calculated by compounding 12-monthly returns) and available at both country- and sector-level. Descriptive statistics of the resulting database are presented in Table 1.

Table 1: Real estate data returns - Descriptive statistics

	AUS	CAN	DEN	FIN	FRA	GER	IRL	NLD
Mean	8.758	10.080	8.200	6.893	9.283	5.741	7.666	7.436
Median	9.575	9.850	8.200	6.250	8.900	5.325	9.250	8.100
Std Error	4.109	4.435	4.880	1.999	5.712	2.843	14.513	4.623
Kurtosis	2.157	0.885	2.266	1.587	0.723	-0.324	0.768	-1.101
Skewness	-1.323	-0.511	1.429	1.415	0.150	0.577	-0.974	-0.298
Country level	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector level	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	NZL	NOR	POR	SAF	SPA	SWE	UK	USA
Mean	10.009	10.078	5.788	13.842	6.947	9.705	9.497	8.912
Median	9.875	10.425	6.575	10.825	8.850	9.900	10.425	11.775
Std Error	5.258	5.809	3.165	6.477	7.662	5.222	8.612	8.496
Kurtosis	-0.156	2.493	-0.456	0.056	-0.372	0.814	7.086	6.250
Skewness	-0.383	-0.262	-0.410	1.140	-0.481	-0.026	-2.234	-2.377
Country level	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sector level	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: This table reports descriptive statistics of real estate data returns. The initial dataset, covering 16 countries and 4 sectors over a period from 1999 to 2018, is extracted from the MSCI Property Indexes (ex IPD) database.

4.2. Diversification potential in real estate portfolios

In this section, we aim at investigating the short-term diversification potential of direct real estate markets. Using the MSCI Property Indexes (ex IPD) database, we first build a set

[4] Documentation about IPD Property Fund Indexes can be found here: `https://www.msci.com/real-estate-indexes`.

of three real estate portfolios from country-level, property-level and global indices, respectively. Specifically, we use two optimization frameworks: the Markowitz and Black-Litterman models. As a first step in our analysis, we compute efficient frontiers in both cases. Mean-variance graphs are plotted in Figure 2.

Figure 3: Comparing diversification potential - OECD - World

Efficient_Frontiers.JPG

Notes: This figure illustrates Markowitz and Black-Litterman $mean - variance$ analysis. This figure enables the comparison of the diversification potential from the measured differences between the benchmark portfolio and the two subportfolios with respect to investing at country- and sector-level, respectively.

Results illustrated in Figure 1 indicate that investing in country- or sector-levels indices only reduces diversification possibilities. Indeed, with the *Overall* efficient frontier being the benchmark portfolio, the distance between this efficient frontier and the two others is related to diversification loss. Furthermore, we observe that the *Country* efficient frontiers lie closer to the *Overall* efficient frontier than to the *Sectors* efficient frontier. Thus, investing in country-level indices only appears to be more efficient than investing in sector-level indices alone. In both the Markowitz and Black-Litterman frameworks, mean-variance analysis exhibits similar features. Country-level diversification dominates sector-level diversification in both cases. Furthermore, in the Black-Litterman framework, the results appear to be amplified. Compared with the *Overall* efficient frontier, we observe that investing in country-level indices only induces a slight diversification loss, whereas investing in sector-level indices induces a greater one.

In a second step, we analyze the diversification loss induced by restricting the set of opportunity. Extending the $RSRL$ proposed by Calvet et al. (2007) and the $mRSRL$, its modified version (in its studentized and not studentized version), we can test for the significance of the diversification loss. Using analytical results from Section 3, we compute estimates and associated significance levels in both the Markowitz and Black-Litterman frameworks. Studentized and percentile circular block-bootstrap exhibit the same results about significance levels. The results are reported in Table 2.

Table 2: Estimating diversification loss - OECD - World

	Framework	Countries	Sectors
Test	Markowitz	0.45**	0.74***
$RSRL$	Black-Litterman	0.39	0.19***
Test	Markowitz	0.14*	0.28**
$mRSRL$	Black-Litterman	0.05	0.09**

Notes: This table enables the comparison of the diversification potential from the $RSRL$ (resp. $mRSRL$) differences between the benchmark portfolio and the two subportfolios with respect to investing at country- and sector-level, respectively. Significance levels of the $RSRL$ (resp. $mRSRL$) coefficient are computed via a studentized circular block-bootstrap as in Ardia and Boudt (2015). Results are computed using R 3.6.0 (R Core Team, 2020) and *DiversificationR* (*v0.1.0*; Hasse, 2021) package. The full reproducible code is available on CRAN. Labels $***$, $**$ and $*$ indicate significance at 99%, 95% and 90% levels, respectively.

It first turns out that the studentized and the non-studentized circular block-bootstrap diversification test lead to the same outcomes. In both the Markowitz and Black-Litterman frameworks, the results indicate that the diversification loss is stronger for sector-level diversification than for country-level diversification. Indeed, the $RSRL$ (resp. $mRSRL$) estimates for sector-level portfolios are twice greater than the one for country-level portfolios. However, significance levels differ in the Markowitz and Black-Litterman cases. The diversification loss is highly significant in every case, except at the country-level for the Black-Litterman framework. These results are in line with our findings illustrated in Figure 1. In the two risk-return graphs, the distance between *Overall* and *Country* efficient frontiers is smaller than that between *Overall* and *Sectors*. Moreover, this feature is the most pronounced in the Black-Litterman case, in which the distance loss induced by investing at country-level without diversifying by property type turns out to be negligible. Estimating both coefficient and significance level of the $RSRL$ and $mRSRL$ statistics constitutes an intuitive and relevant way to measure diversification of a given portfolio relative to a benchmark portfolio.

In a third step, we empirically validate the relevance of our extensions of the $RSRL$. To do so, we compare the empirical results with those obtained considering the other diversification measures. Specifically, we focus on the four most recent diversification measures found in the literature. These measures are listed in Table 3.

Table 3: Diversification potential - Diversification measures - OECD - World

Name	ID	References
Portfolio Diversification Index	PDI	Rudin and Morgan (2006)
Diversification Ratio	DR	Choueifaty and Coignard (2008)
Diversification Delta	DD	Vermorken et al. (2012)
Diversification Delta Star	DD^*	Flores et al. (2017)
Relative Sharpe Ratio Loss	$RSRL$	Calvet et al. (2007)

Notes: This table reports a set of the five main diversification measures from the recent literature.

Contrary to early diversification measures (e.g., Herfindahl - Hirschman Index and Shannon entropy), recent measures described in Table 3 take both weights and covariance into account. Our set of diversification measures aims at achieving a fair comparison between those, as well as the new ones ($RSRL/mRSRL$). The outcomes of the formal diversification tests are reported in Tables 4 (Markowitz framework) and 5 (Black-Litterman framework).

Table 4: Diversification potential - Markowitz framework - OECD - World

	Countries & Sectors	Countries	Sectors
Portfolio Diversification Index (PDI)	2.669	1.762	1.790
Diversification Ratio (DR)	2.729	1.504	1.017
Diversification Delta (DD)	0.585	0.291	0.014
Diversification Delta Star (DD^*)	0.634	0.335	0.017
Relative Sharpe Ratio Loss ($RSRL$)		0.45**	0.74***
Modified Relative Sharpe Ratio Loss ($mRSRL$)		0.14*	0.28**

Notes: This table reports the estimation results of six different diversification measures on Markowitz $mean - variance$ optimal portfolios. This table enables the comparison of the diversification potential from the measured differences between the benchmark portfolio and the two subportfolios with respect to investing at country- and sector-level, respectively. Significance levels of the $RSRL$ (resp. $mRSRL$) coefficient are computed via a studentized circular block-bootstrap as in Ardia and Boudt (2015). Results are computed using R 3.6.0 (R Core Team, 2020) and the *DiversificationR* (*v0.1.0*; Hasse, 2021) package. The full reproducible code is available on CRAN. Labels $***$, $**$ and $*$ indicate significance at 99%, 95% and 90% levels, respectively.

The results indicate that diversification measures are statistically higher at a country-level than at a sector-level. The only exception is the PDI scores, for which the country-level and sectoral-level diversification are roughly similar. Such a finding derives its justification from the definition of the PDI scores. They are built from the eigenvalues of the covariance matrix of the portfolio via a principal component analysis. However, if these diversification measures are relevant to quantify the level of diversification of a given portfolio, they only constitute

Table 5: Diversification potential - Black-Litterman framework - OECD - World

	Countries & Sectors	Countries	Sectors
Portfolio Diversification Index (PDI)	2.669	1.762	1.790
Diversification Ratio (DR)	1.558	1.155	1.036
Diversification Delta (DD)	0.246	0.075	0.032
Diversification Delta Star (DD^*)	0.358	0.134	0.035
Relative Sharpe Ratio Loss ($RSRL$)		0.39	0.19***
Modified Relative Sharpe Ratio Loss ($mRSRL$)		0.05	0.09**

Notes: This table reports the estimation results of six different diversification measures on Black-Litterman $mean - variance$ optimal portfolios. This table enables the comparison of the diversification potential from the measured differences between the benchmark portfolio and the two subportfolios with respect to investing at country- and sector-level, respectively. Significance levels of the $RSRL$ (resp. $mRSRL$) coefficient are computed via a studentized circular block-bootstrap as in Ardia and Boudt (2015). Results are computed using R 3.6.0 (R Core Team, 2020) and the *DiversificationR* (*v0.1.0*; Hasse, 2021) package. The full reproducible code is available on CRAN. Labels $* * *$, $**$ and $*$ indicate significance at 99%, 95% and 90% levels, respectively.

some absolute measures of diversification. In other words, these diversification measures are designed to provide an absolute score from given portfolio weights and covariance. Thus, comparing absolute and relative diversification scores computed from several efficient frontiers (i.e., comparing diversification benefits across subsets of investment opportunities) leads to interpretation biases.

To summarize, we empirically validate the relevance of our diversification test based on the $RSRL$ (resp. $mRSRL$) estimates and confidence intervals. Indeed, the $RSRL$ is an intuitive statistical measure offering several advantages compared to other diversification measures. First, the $RSRL$ (resp. $mRSRL$) is a relative measure of diversification loss, based on the efficiency loss between a given portfolio and its benchmark portfolio. This efficiency loss is intuitively related to the distance between the optimal portfolios located on the tangent of the two efficient frontiers. Thus, the $RSRL$ is designed as a statistical measure allowing comparison of diversification abilities of several portfolios. Last but not least, finite and asymptotic inference for the $RSRL$ (resp. $mRSRL$) is precisely known, which is not the case for the other diversification measures. It is therefore possible to set up a test for diversification loss from these diversification measures.

4.3. Robustness checks

In this subsection, we replicate our empirical analysis focusing exclusively on European countries. Testing for diversification within this sub-sample leads to similar results, i.e., higher opportunities for country- than sector-level diversification. Figure 4 illustrates the comparison of diversification potential across all sectors and across European countries. Specifically, similarly to our previous results, Figure 4 indicates that investing in country- or sector-level indices only reduces diversification potential. Indeed, the visual configuration of efficient frontiers for the sub-sample is similar to the one we observe in Figure 3. In both the Markowitz and Black-Litterman frameworks, it appears that investing in country-level indices only appears to be more efficient than investing in sector-level indices alone.

Figure 4: Comparing diversification potential - OECD - Europe

Revision/Efficient_Frontiers_Europe_Corrected.jpeg

Notes: This figure illustrates Markowitz and Black-Litterman $mean - variance$ analysis. This figure enables the comparison of the diversification potential from the measured differences between the benchmark portfolio and the two subportfolios with respect to investing at country- and sector-level, respectively.

Tables 5 and 6 report the outcomes of the different diversification measures and tests in both Markowitz and Black-Litterman frameworks. The results are similar to those obtained in the previous analysis from the global sample. Without exception, the results indicate that diversification measures are higher for country-level than for sector-level diversification. Diversification tests exhibit similar results compared to those obtained from the global sample (i.e., OECD - World). The empirical findings clearly indicate that international diversification potential dominates that of property type diversification. To summarize, our empirical findings over the sub-sample (i.e., OECD - Europe) are very similar to those of the full sample (i.e., OECD - World). Efficient frontiers exhibit similar features, and the different diversification measures and tests are robust to the sub-sampling process. This first robustness check further validates the relevance of our diversification test.

In the previous analysis, the empirical analysis was related to full diversification across all property types and countries. To check the robustness of our results, we now focus on diversification within each property type but across all countries. Figure 5 illustrates the

Table 6: Diversification potential - Markowitz framework - OECD - Europe

	Countries & Sectors	Countries	Sectors
Portfolio Diversification Index (PDI)	2.274	1.515	1.395
Diversification Ratio (DR)	2.443	1.354	1.000
Diversification Delta (DD)	0.546	0.249	0.000
Diversification Delta Star (DD^*)	0.591	0.261	0.000
Relative Sharpe Ratio Loss ($RSRL$)		0.43**	0.73***
Modified Relative Sharpe Ratio Loss ($mRSRL$)		0.14**	0.28**

Notes: This table reports the estimation results of six different diversification measures on Markowitz $mean - variance$ optimal portfolios. This table enables the comparison of the diversification potential from the measured differences between the benchmark portfolio and the two subportfolios with respect to investing at country- and sector-level, respectively. Significance levels of the $RSRL$ (resp. $mRSRL$) coefficient are computed via a studentized circular block-bootstrap as in Ardia and Boudt (2015). Results are computed using R 3.6.0 (R Core Team, 2020) and the *DiversificationR* (*v0.1.0*; Hasse, 2021) package. The full reproducible code is available on CRAN. Labels $***$, $**$ and $*$ indicate significance at 99%, 95% and 90% levels, respectively.

Table 7: Diversification potential - Black-Litterman framework - OECD - Europe

	Countries & Sectors	Countries	Sectors
Portfolio Diversification Index (PDI)	2.274	1.515	1.395
Diversification Ratio (DR)	1.534	1.265	1.030
Diversification Delta (DD)	0.183	0.123	0.026
Diversification Delta Star (DD^*)	0.348	0.210	0.030
Relative Sharpe Ratio Loss ($RSRL$)		0.18	0.09**
Modified Relative Sharpe Ratio Loss ($mRSRL$)		0.03	0.04**

Notes: This table reports the estimation results of six different diversification measures on Black-Litterman $mean - variance$ optimal portfolios. This table enables the comparison of the diversification potential from the measured differences between the benchmark portfolio and the two subportfolios with respect to investing at country- and sector-level, respectively. Significance levels of the $RSRL$ (resp. $mRSRL$) coefficient are computed via a studentized circular block-bootstrap as in Ardia and Boudt (2015). Results are computed using R 3.6.0 (R Core Team, 2020) and the *DiversificationR* (*v0.1.0*; Hasse, 2021) package. The full reproducible code is available on CRAN. Labels $***$, $**$ and $*$ indicate significance at 99%, 95% and 90% levels, respectively.

four efficient frontiers for diversification within each property type (residential, retail, office and industrial) but across all countries. The graphical analysis of efficient frontiers indicates features similar to previous results illustrated in Figures 3 and 4; i.e., diversification across all sectors and countries outperforms diversification within a given sector but across all countries. Consequently, our diversification analysis is robust with respect to the hypothesis of whether investment is carried out in a sequential stage. To go further, we replicate previous diversification measurements and tests across each strategy and present empirical evidence that our results are not sensitive to top-down or bottom-up approach. Tables 8 and 9 report the comparison of the diversification potential between each strategy. The results are similar to those obtained in previous analyses. Diversification measures are higher for full diversification across all property types and countries than for diversification within each property type. Last, the results of diversification tests broadly indicate that full diversification across all property types and countries is the most efficient strategy. Finally, this second robustness check verifies the latter finding, yielding another line of evidence for the relevance of our diversification test.

Figure 5: Comparing diversification potential per property type - OECD - World

Revision/efficient_frontiers_sectors.jpeg

Notes: This figure illustrates Markowitz and Black-Litterman $mean - variance$ analysis. This figure enables the comparison of the diversification potential from the measured differences between the benchmark portfolio and the four sub-portfolios with respect to investing at residential-, retail-, office- or industrial-level, respectively.

5. Conclusions

The diversification potential of real estate is well acknowledged among scholars and practitioners. However, whether diversification can be best achieved on a global scale by diversifying portfolios across property types or geographic markets in different countries is still not well understood. Our study addresses this issue, revisiting the diversification potential in real estate portfolios and developing a new diversification test.

Table 8: Diversification potential per property type - Markowitz framework - OECD - World

	All Sectors	Residential	Retail	Office	Industrial
Portfolio Diversification Index (PDI)	2.669	2.132	1.811	2.026	2.039
Diversification Ratio (DR)	2.729	2.135	1.891	2.256	1.232
Diversification Delta (DD)	0.585	0.457	0.429	0.524	0.109
Diversification Delta Star (DD^*)	0.634	0.532	0.471	0.557	0.188
Relative Sharpe Ratio Loss ($RSRL$)		0.14	0.42*	0.49***	0.54***
Modified Relative Sharpe Ratio Loss ($mRSRL$)		0.03	0.12	0.15***	0.18**

Notes: This table reports the estimation results of six different diversification measures on Black-Litterman $mean - variance$ optimal portfolios. This table enables the comparison of the diversification potential from the measured differences between the benchmark portfolio and the two subportfolios with respect to investing at country- and sector-level, respectively. Significance levels of the $RSRL$ (resp. $mRSRL$) coefficient are computed via a studentized circular block-bootstrap as in Ardia and Boudt (2015). Results are computed using R 3.6.0 (R Core Team, 2020) and the *DiversificationR* (*v0.1.0*; Hasse, 2021) package. The full reproducible code is available on CRAN. Labels $***$, $**$ and $*$ indicate significance at 99%, 95% and 90% levels, respectively.

Table 9: Diversification potential per property type - Black-Litterman framework - OECD - World

	All Sectors	Residential	Retail	Office	Industrial
Portfolio Diversification Index (PDI)	2.669	2.132	1.811	2.026	2.039
Diversification Ratio (DR)	1.558	1.505	1.260	1.319	1.281
Diversification Delta (DD)	0.246	0.199	0.091	0.155	0.160
Diversification Delta Star (DD^*)	0.358	0.277	0.206	0.242	0.220
Relative Sharpe Ratio Loss ($RSRL$)		0.12*	0.16***	0.29***	0.24***
Modified Relative Sharpe Ratio Loss ($mRSRL$)		0.06*	0.10**	0.15***	0.09***

Notes: This table reports the estimation results of six different diversification measures on Black-Litterman $mean - variance$ optimal portfolios. This table enables the comparison of the diversification potential from the measured differences between the benchmark portfolio and the two subportfolios with respect to investing at country- and sector-level, respectively. Significance levels of the $RSRL$ (resp. $mRSRL$) coefficient are computed via a studentized circular block-bootstrap as in Ardia and Boudt (2015). Results are computed using R 3.6.0 (R Core Team, 2020) and the *DiversificationR* (*v0.1.0*; Hasse, 2021) package. The full reproducible code is available on CRAN. Labels $***$, $**$ and $*$ indicate significance at 99%, 95% and 90% levels, respectively.

Several diversification measures coexist in the literature. Most of these are easy to compute but are only descriptive, weakly connected to the economic background and lack information on confidence intervals. Considering these drawbacks, we follow Calvet et al. (2007)'s approach based on the measure of financial loss from holding a subefficient portfolio. Specifically, we extend their diversification measure by building a diversification test that allows for inference asymptotically but also in finite non-iid residuals. Based on Ardia and Boudt (2015)'s test for equality of modified Sharpe ratios, we first compute robust confidence intervals for the estimator $R\hat{S}RL$ via a studentized circular block bootstrap procedure. Then, we introduce a modified version of Calvet et al. (2007)'s $RSRL$, the $mRSRL$, to have an estimator that is robust to non-normality.

Focusing on direct real estate, we use the MSCI (ex IPD) database to build a unique dataset of real estate market returns covering 16 OECD countries over the period $1999-2018$. The resulting balanced panel dataset includes the returns for each country but also considers different property types (residential, retail, office and industrial), enabling a precise evaluation of the diversification potential of real estate portfolios. Comparing the efficient frontiers of each strategy in both the Markowitz and Black-Litterman frameworks, the preliminary results indicate that the diversification loss is stronger at the sectoral level than it is at the country level. Diversification loss is significant in every case, except at the country level for the Black-Litterman framework. Then, using four different diversification measures from the literature, we investigate the diversification potential of real estate portfolios. Our empirical findings broadly reveal that international diversification strategies outperform sectoral diversification strategies in terms of real estate assets. Our results have several implications for practitioners. First, allowing for a robust estimation of the $RSRL$ and $mRSRL$ statistics constitutes an intuitive and relevant way to measure the diversification of a given portfolio relative to a benchmark portfolio. Second, in line with Heston and Rouwenhorst's (1994) early results, our empirical findings reveal that international diversification strategies outperform sectoral diversification strategies in a real estate context. Last, despite of substantial data and methodology differences, our findings corroborates Eichholtz et al. (1995)'s main results. Our empirical results are also consistent with the recent findings of Ciochetti et al. (2015) and Yang et al. (2018).[5]

Finally, the econometric approach proposed in this paper may be applied in future research to other asset classes suffering from similar data and statistical constraints, such as private equity, cryptocurrency or hedge funds. Our diversification test can also be used by practitioners, investors and financial advisors to investigate the diversification level of mutual funds by comparing them to their benchmark.

[5]See also Al-Abduljader (2018) about international diversification in frontier real estate markets.

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Appendix A - Portfolios Composition

This appendix reports overall, country-level and sector-level optimal portfolio weights computed in Section 4.2. Our empirical analysis is based on two popular portfolio optimization models, namely, the Markowitz and Black-Litterman approaches. Those two frameworks complement each other, which enables a robust empirical analysis of the diversification potential of real estate portfolios. On the one hand, the classic Markowitz mean-variance model is the most appropriate framework to test for diversification; the concept of diversification finds its theoretical roots in this model. However, this approach suffers from several drawbacks, which leads to highly concentrated portfolios and high input sensitivity. On the other hand, Black-Litterman is a more flexible approach that enables more stable estimations and avoids results with unstable weights, as well as counter-intuitive or corner solutions. Thus, in our empirical analysis described in Section 4, we use both portfolio optimization models. In line with the literature, we find that Markowitz-optimal portfolios are more concentrated and exhibit more corner solutions than Black-Litterman-optimal portfolios. Specifically, Tables A1, A2 and A3 report portfolio weights at the sector-level, country-level and overall, respectively.

Table A1: Sectors - Optimal portfolio weights - OECD - World

Sectors	Markowitz	Black-Litterman
Retail	00.00%	25.05%
Office	00.00%	24.99%
Industrial	29.82%	25.04%
Residential	70.18%	24.92%

Table A2: Countries - Optimal portfolio weights - OECD - World

Countries	Markowitz	Black-Litterman
Australia	00.00%	03.15%
Canada	00.00%	04.78%
Denmark	00.00%	01.06%
Finland	59.24%	00.76%
France	00.00%	08.71%
Germany	33.26%	09.76%
Ireland	00.00%	01.50%
Netherlands	00.00%	02.52%
New-Zealand	00.00%	00.35%
Norway	00.00%	01.28%
Portugal	00.00%	00.63%
South Africa	07.50%	00.62%
Spain	00.00%	04.39%
Sweden	00.00%	01.52%
UK	00.00%	10.21%
USA	00.00%	48.75%

Table A3: Overall - Optimal portfolio weights - OECD - World

Overall	Markowitz	Black-Litterman	Overall / Optim	Markowitz	Black-Litterman
Australia Retail	00.00%	00.70%	New Zealand Retail	00.00%	00.82%
Australia Office	00.00%	00.00%	New Zealand Office	00.00%	01.51%
Australia Industrial	00.00%	00.89%	New Zealand Industrial	00.00%	04.83%
Australia Residential	05.38%	00.00%	New Zealand Residential	00.00%	02.73%
Canada Retail	00.00%	01.93%	Norway Retail	00.00%	00.00%
Canada Office	00.00%	02.03%	Norway Office	00.00%	00.00%
Canada Industrial	00.00%	02.24%	Norway Industrial	00.00%	00.00%
Canada Residential	00.00%	01.31%	Norway Residential	11.06%	00.08%
Denmark Retail	02.83%	04.49%	Portugal Retail	00.00%	00.00%
Denmark Office	00.00%	03.18%	Portugal Office	00.00%	03.27%
Denmark Industrial	00.00%	00.02%	Portugal Industrial	00.00%	01.82%
Denmark Residential	00.00%	02.15%	Portugal Residential	00.00%	01.54%
Finland Retail	00.00%	01.12%	Sweden Retail	00.00%	00.69%
Finland Office	00.00%	01.99%	Sweden Office	00.29%	02.30%
Finland Industrial	00.00%	03.35%	Sweden Industrial	00.00%	03.41%
Finland Residential	55.00%	06.11%	Sweden Residential	00.00%	02.41%
France Retail	00.00%	00.00%	South Africa Retail	06.74%	02.27%
France Office	00.00%	00.00%	South Africa Office	00.00%	00.72%
France Industrial	00.00%	00.00%	South Africa Industrial	00.00%	00.95%
France Residential	00.00%	00.25%	South Africa Residential	00.30%	01.40%
Germany Retail	00.00%	05.62%	Spain Retail	00.00%	01.33%
Germany Office	17.63%	06.13%	Spain Office	00.00%	00.08%
Germany Industrial	00.00%	01.35%	Spain Industrial	00.00%	01.58%
Germany Residential	00.00%	01.39%	Spain Residential	00.00%	00.00%
Ireland Retail	00.00%	00.00%	UK Retail	00.78%	01.07%
Ireland Office	00.00%	02.72%	UK Office	00.00%	01.02%
Ireland Industrial	00.00%	00.22%	UK Industrial	00.00%	01.81%
Ireland Residential	00.00%	02.33%	UK Residential	00.00%	00.00%
Netherlands Retail	00.00%	00.61%	USA Retail	00.00%	01.11%
Netherlands Office	00.00%	01.15%	USA Office	00.00%	00.00%
Netherlands Industrial	00.00%	02.08%	USA Industrial	00.00%	02.53%
Netherlands Residential	00.00%	02.99%	USA Residential	00.00%	00.38%