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Market Power and Innovation in the Intangible Economy

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Market Power and Innovation in the Intangible Economy[*]

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1. Introduction

The decline of productivity growth has played a prominent role in recent academic and policy debates. Average productivity growth in the United States was less than 0.5% between 2005 and 2018, well below the long-term average of 1.3% (Figure 1a). A similar slowdown occurred across most of Europe, causing productivity in countries such as France and the United Kingdom to flatline (Adler et al. 2017). The slowdown followed after a decade of above-average growth, fueled by rapid improvements in information technologies (Fernald 2015). Moreover, the slowdown occurred despite an increase in productivity-enhancing investments: indeed, U.S. investments in corporate research and development have increased by 65% as a fraction of GDP over the last 30 years (Figure 1b). Rather than seeming to be driven by a lack of effort to become more productive, the slowdown therefore comes from a decline in the effect of innovative investments on productivity growth (e.g. Bloom et al. 2020).

The initial surge and subsequent decline in productivity growth coincided with two other trends: the slowdown of business dynamism and the rise of markups. Signs that dynamism is weakening include the decline in the rate at which workers reallocate to different firms (e.g. Decker et al. 2014), the decline in skewness of the firm-growth distribution (e.g. Decker et al. 2016) and the decline of entry rates (e.g. Pugsley and Şahin 2018). The rise of markups has recently attracted attention and has been linked to the decline of the labor share (e.g. De Loecker et al. 2020). Despite the growing body of evidence detailing these trends, there is thus far no consensus on what has caused them.

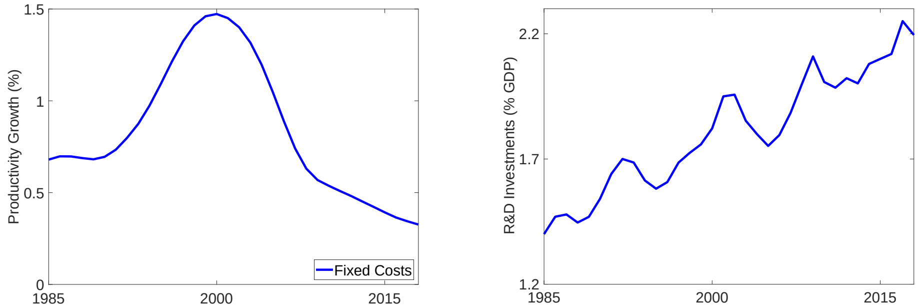
This paper claims that the trends in productivity growth, business dynamism and markups are reflections of a structural change in the way firms produce. Specifically, I show that an increase in the use of intangible inputs can drive these patterns. Intangible inputs are inputs that are used in production, but that are not physically embodied. Information technology and software are prominent examples. The rise of intangible inputs has been dramatic over the last 30 years: software alone is now responsible for 18% of U.S. corporate investments, up from 3% in 1980 (BEA).

Intangible inputs alter the relationship between profitability, firm-level innovation and aggregate growth because intangibles have two features: they are scalable, and firms differ in the efficiency with which they deploy them. Intangibles are scalable in the sense that they can be duplicated at close-to-zero marginal cost (e.g. Haskel and Westlake 2017, Hsieh and Rossi-Hansberg 2021). This causes the cost structure to change when firms use intangible inputs in production. Firms invest in the development and maintenance of intangible inputs but face minimal additional costs when production is scaled up. Firms that sell products that include software (e.g. the operating system of a phone, a car's drive-by-wire-system), for example, face minimal costs of reproducing software in additional units. The rise of intangibles therefore shifts costs away from the marginal towards the fixed component. As a consequence, firms take these costs as given when making decisions about pricing and production.

Firms differ in the extent to which they adopt intangible inputs to reduce marginal costs. A 2018 European Investment Bank survey finds that over 40% of American and European manufacturing firms do not use state-of-the-art digital technologies, while less than 15% organize their entire operation around digital technologies (Veugelers et al. 2020).[1] A likely driver is the fact that firms, even within

[1]A full literature review on firm-level determinants of IT adoption is provided in Haller and Siedschlag (2011).

Figure 1. Trends in Productivity Growth and Research & Development



(a) Productivity Growth (b) R&D Investments

Notes: Figure 1a plots annual productivity growth from the Fernald series (FRBSF). The plot is smoothed using an HP filter with an annual smoothing parameter of 100. Figure 1b plots private R&D as a percentage of GDP. Data is from the BEA NIPA tables.

narrowly defined industries, differ in the efficiency with which they are able to use intangibles. A rich literature provides evidence on this. Bloom et al. (2012), for example, show that American-owned European establishments achieve greater productivity improvements from information technology (IT).[2] They find that IT productivity is a firm characteristic, especially because the IT productivity of establishments increases when they are *acquired* by an American firm. Schivardi and Schmitz (2019) furthermore show that inefficient management practices can explain not only the low IT adoption by Italian firms but also why the productivity gains that these firms obtain from using IT are limited.[3]

I show that a rise in the use of intangibles that is unequal across firms alters both the rate and the efficiency with which firms engage in research and development, causing the trends depicted in Figure 1. I derive these results in an endogenous growth model – in the spirit of Klette and Kortum (2004)– that is tractable yet sufficiently rich to quantitatively analyze the effect of intangibles on growth, dynamism and market power. Firms produce one or multiple goods and invest in research and development (R&D) to create higher-quality versions of goods produced by other firms. Successful innovation causes the innovator to become the new producer, while the incumbent ceases to produce the good. Step-wise improvements to goods through this process of creative destruction drive aggregate growth.

Firms in the model are able to reduce marginal costs by committing to the purchase of fixed-cost intangibles. As firms are heterogeneous in the efficiency with which they deploy intangibles, some firms reduce marginal costs by a greater fraction than others do. This introduces a tradeoff between *quality* and *price* to the Klette and Kortum (2004)-framework. In the standard model, firms that develop a higher-quality version of a good become its sole producer. Other firms have the same costs but are unable to produce the same quality and hence cannot compete. Intangibles change this result, as

[2] Software and IT are used throughout this paper as examples of inputs that are both scalable and deployed at heterogeneous efficiency, as investments in these inputs have increased rapidly over the last 30 years. Other inputs that satisfy both requirements could be used to explain the trends in productivity growth, business dynamism and markups in the framework.

[3] Bloom et al. (2014) also find that structured management practices are closely related to IT adoption in American firms. Evidence also suggests that workplace organization and organizational capital affect a firm's IT productivity (e.g. Crespi et al. 2007, Bartel et al. 2007). Changes to organization design come at the price of high adjustment costs, which makes IT productivity a persistent firm characteristic (e.g. Bresnahan et al. 2002).

high-intangible firms are able to produce at lower costs than others can – enabling them to sell at lower prices. When a firm with lower intangible-adoption develops a higher quality version of a good sold by one of these firms, the incumbent could undercut the innovator on price. Only if the quality difference is sufficiently large to offset the gap in marginal costs would the innovator become the new producer. The presence of firms with a high take-up of intangibles therefore deters other firms from entering new markets. Firms with high-intangible productivity can therefore negatively affect productivity growth.

It follows that the effect of the rise of intangibles on growth critically depends on how inclusive that rise is. A broad-based shift towards intangibles can raise growth because their fixed-cost nature enhances profitability and therefore incentivizes innovation – thereby stimulating growth and raising welfare. An unequal rise of intangibles, however, incentivizes innovation only for high-intangible firms, while making it harder for other firms to enter new markets. This reduces growth and welfare.

To understand the degree to which intangibles explain the macroeconomic trends, I introduce high-intangible entrants to an economy where firms initially use similar levels of intangibles. Over the transition path, high-intangible firms initially cause a boom in productivity growth. As these firms have a greater incentive to invest in R&D, they serve to "disrupt" sectors, and economic activity concentrates disproportionately around these firms. Their entry raises productivity because high-intangible firms produce at a lower cost. The increase in aggregate productivity is not matched by wages, however, because high-intangible firms set proportionally higher markups. As the economy transitions to the new balanced growth path, there is a decline in entry, as most start-ups are unable to compete with high-intangible incumbents. Low-intangible incumbents similarly have weaker incentives to innovate. This causes a gradual decline in productivity growth, which falls below the initial steady-state within 20 years after high-intangible firms first enter the market. Although overall R&D increases, it concentrates around a smaller group of incumbents. Because returns are concave, the concentration of R&D lowers its effectiveness. Combined with the fact that a fraction of innovations fail because high-intangible incumbents undercut innovators on price, this explains why growth falls while R&D increases.

The model offers three main theoretical insights. First, it shows that the relationship between aggregate R&D and aggregate growth depends on how R&D is distributed across firms. As firm-level innovation is concave in spending, concentration of R&D can negatively affect growth. As R&D concentration increases when profitability diverges across firms, heterogeneous profits come at a dynamic cost.[4] Policies such as R&D subsidies should therefore be designed with heterogeneity in firm-level incentives in mind. Second, the model introduces a tradeoff between quality and price to Schumpeterian growth models. High-intangible firms are able to sell at lower prices, which can compensate for lower quality and therefore be used to undercut innovators. Because long-term growth arises from improvements in quality, differences in intangible productivity across firms reduce the effect of R&D on growth. An increase in the variance of intangible productivity can therefore negatively affect welfare even if the variance rises because of the entry of firms with above-average productivity. Third, the model shows that it is important to distinguish between sources of cross-sectional productivity differences that arise from productivity upon which other firms can build (denoted as quality in the model) and those that

[4] The dynamic costs form an additional inefficiency from profit heterogeneity, on top of the static costs of heterogeneous profits and markups that arise through low production rates by high-profit firms (e.g. Edmond et al. 2022, Peters 2020).

arise from rival technologies in the form of intangibles. Improvements in quality have higher public returns because future innovators stand on the shoulders of today's R&D.[5] Statically, both productivities act as substitutes. Dynamically, substitution towards intangibles can carry significant welfare costs.

I quantify the model using two structural estimations, one for the U.S. and one for France. The French estimation relies on administrative data for the universe of firms while its U.S. counterpart relies on data for listed firms. While evidence on the macroeconomic trends is stronger for the U.S., I show that the trends are largely visible for France as well. Using a new measure of fixed costs, I show that the share of fixed over total costs gradually rose from 13 to 23% in the U.S. between 1979 and 2015, and from 9 to 14% in France between 1994 and 2016. I also use the micro data to confirm model-implied conditional correlations between intangibles, fixed costs, markups, and innovation.

The quantitative model explains a significant part of the slowdown of productivity growth, the decline in business dynamism and the rise of markups. The model predicts a slowdown in steady-state growth of 0.3 percentage points in the U.S., after an initial boom in growth of eight years. For France, the model's predictions are more modest, with a 0.1 percentage point slowdown. Markups increase by 14.7 and 6.4 percentage points in the respective calibrations. Entry rates fall by 4.5 and 1 percentage points, respectively. If markups are held constant, the model predicts a greater decline in productivity growth and business dynamism. The rise of markups stimulates innovative investments by high-intangible firms, and therefore mitigates the decline in productivity growth in the baseline model.[6]

Related literature This paper contributes a new mechanism to the literature that links trends in productivity growth, business dynamism and market power. Closest to this paper is Aghion et al. (2022), who also point to technology as the driver of the trends. In a model with creative destruction, they analyse the effect of an increase in span of control, which enables firms with higher (ex-ante) production efficiency to expand. Their short-term predictions overlap with mine: the efficient firms expand and cause a burst in growth, while aggregate markups rise because economic activity reallocates to profitable firms. In the long run, Aghion et al. predict that efficient firms face increasing competition from other efficient firms. This diminishes their incentives to innovate, so that growth and investments in R&D decline.[7] In my model, high-intangible firms invest persistently more in R&D, as they remain more profitable than their competitors. The additional investments have low returns because innovation is concave in a firm's R&D. As high-intangible firms can undercut others, they lower both the level and the effectiveness of R&D by other firms. Intangible inputs therefore help us to understand why the slowdown of productivity growth occurred despite a continued rise in corporate R&D.

Alternative explanations of the macroeconomic trends focus on the long-term decline in productivity growth, rather than the initial burst and subsequent slowdown. Akcigit and Ates (2022, 2021) hy-

[5] This highlights an important difference between R&D, which is often referred to as intangible capital (e.g. Corrado et al. 2009, McGrattan and Prescott 2014, McGrattan 2020), and intangible inputs, such as software.

[6] This paper's analysis is therefore robust to concerns about the firm-level measurement of markups (Traina 2018, Bond et al. 2021) as well as recent evidence that the labor share is stable outside of the U.S. (Gutierrez and Piton 2020).

[7] Profitability and the incentives to innovate fall over time because a greater fraction of producers (firms with the highest quality patent for a product) face second-best competitors that also have a high production efficiency. Under Bertrand competition, this reduces the limit-price markup for the producing firm, so that the incentive to innovate falls.

pothesize that imitation rates between leaders and followers have declined, and provide evidence that intellectual property rights are increasingly used anti-competitively. Olmstead-Rumsey (2022) documents that the innovation efficiency of laggard firms has fallen over time. This causes a rise in markups and a decline in aggregate R&D in a quantitative growth model. I contribute to their findings by offering an endogenous driver of the decline in the innovation efficiency of laggards: as high-intangible firms become dominant, low-intangible (laggard) firms are more likely to be undercut when they try to enter a new market, thereby lowering the returns to their research activities.

I also view my paper as complementary to the growing body of work that explains the long-term decline in productivity growth through demographics. Peters and Walsh (2021) relate the decline of entry to the fall in labor force growth, which is consistent with evidence in Hopenhayn et al. (2022) and Karahan et al. (2019). The lack of entry stimulates expansion by incumbents, which raises firm concentration and markups, and slows down productivity growth.[8] Liu et al. (2022) relate productivity and business dynamism to low interest rates, which may itself be caused by aging (e.g. Eggertsson et al. 2019). Low interest rates increase investment most strongly for the market leader which, through strategic interaction, dissuades followers from investing in R&D. This in turn reduces optimal R&D by leaders, and therefore slows productivity growth.[9] I offer a complementary mechanism that combines these paper's predictions of a long-term slowdown in growth with an increase in aggregate R&D.[10]

This paper also provides a theory of the trends in productivity growth, business dynamism and market power that is consistent with their micro properties. As emphasized by Van Reenen (2018), there has been a substantial divergence in profitability and productivity across firms over time. Andrews et al. (2016) show that productivity growth of the most productive firms has not declined. Decker et al. (2020) find an increase in productivity dispersion within the U.S.[11] The rise in markups in De Loecker et al. (2020) is also strongest in the highest deciles.[12] I propose that intangible inputs are at the root of the growing differences across firms, and show that the negative externality that high-intangible firms impose on others can drive aggregate trends in productivity growth, business dynamism and markups.

This paper is additionally consistent with Baqaee and Farhi (2020)'s finding that aggregate markups have increased because output has reallocated towards high-markup firms. Their finding, in turn, aligns with the finding of Autor et al. (2020) and Kehrig and Vincent (2021) that the decline in the labor share is driven by reallocation. Most other theories of rising profits and falling growth predict a

[8] Peters and Walsh (2021) note that, with an alternative calibration, their model can predict both a rise in R&D and a decline in growth after a decline in population growth. Salgado (2020) notes that there has also been a decline in the fraction of college graduates that become entrepreneurs. Bornstein (2018) relates the rise of markups to aging and price sensitivity.

[9] Liu et al. (2022) predict that a shock to interest rates initially causes a burst of productivity growth, but a slowdown follows immediately and productivity growth falls below the initial steady-state level within a quarter.

[10] Cavenaile et al. (2020) extend the Schumpeterian growth model with oligopolistic competition. They estimate their framework on data before and after the slowdown of productivity growth, and find that the cost parameters for innovation are higher in the latter period. My model yields an endogenous rise in the cost of innovation. Brynjolfsson et al. (2021) claim that adopting artificial intelligence requires unmeasured investments, causing productivity to initially decline but eventually rise. Rachel (2021) notes that leisure-enhancing technologies can exacerbate the understatement of utility in observed TFP.

[11] Kehrig and Vincent (2019) note that an increase in productivity dispersion at the establishment level may reflect an improvement in factor allocation and a reduction of internal credit market frictions.

[12] This result has been confirmed for several countries (Díez et al. 2021, Calligaris et al. 2018). Recent summaries of the debate on markup estimation are found in Syverson (2019), Basu (2019) and Bond et al. (2021). Altomonte et al. (2021) remark that credit frictions can cause dispersion in intangibles and markups across firms.

within-firm rise of markups instead.[13] I predict that high-intangible, high-markup firms endogenously expand through R&D. Even though firm-level markups are constant, aggregate markups rise over time.

Other papers also relate the rise of intangible inputs to firm concentration. Hsieh and Rossi-Hansberg (2021) suggest that intangibles explain the rise of concentration in services across geographic markets, as software can be deployed across markets after paying a fixed cost.[14] Korinek and Ng (2019) and Weiss (2020) relate intangibles that raise fixed costs and reduce marginal costs to the rise of firm concentration and markups. Lashkari et al. (2019) share these predictions and provide evidence on the non-homothetic role of information technology in production.[15] Mariscal (2018) adds that information technology complements the efficiency of managers, and may explain wage polarization. I view my paper as complementary, as I show that fixed-cost intangibles can also explain the trends in productivity growth: its initial burst and subsequent slowdown, as well as the simultaneous rise in R&D.

My theoretical predictions are in line with empirical work that relates productivity growth, business dynamism and market power to intangibles. Crouzet and Eberly (2019) show that intangibles cause an increase in market power and productivity for leading U.S. public firms. McKinsey (2018) and Ayyagari et al. (2018) show that firms with high profitability and growth invest more in software and R&D. Bessen and Righi (2019) find that productivity of U.S. firms increases persistently after an increase in the stock of their IT staff. Farhi and Gourio (2018) show that unmeasured intangibles can explain the rising wedge between the measured marginal product of capital and risk-free rates. Bajgar et al. (2019) find that sectors with high intangible investments experienced a greater increase in concentration. Bessen (2017) finds a positive sector-level relationship between concentration and the use of IT systems, and stresses that the scalability of intangibles is advantageous to firms that are already large. Calligaris et al. (2018) find a positive correlation between the use of digital technologies and the rise of markups. Bijnens and Konings (2018), documenting a decline in Belgian business dynamism, remark that the decline is strongest in industries with a high IT intensity.[16]

The theoretical framework builds on Schumpeterian growth models of creative destruction in the tradition of Aghion and Howitt (1992) and Grossman and Helpman (1991). It is part of the strand of models where firms produce multiple products (Klette and Kortum 2004). The Klette and Kortum framework is attractive because it is analytically tractable, yet able to replicate many empirical features of firm dynamics (Lentz and Mortensen 2008). The framework was recently used to study the reallocation of innovative activity (Acemoglu et al. 2018), to discern the effect of innovation policy (Atkeson and Burstein 2019) and to assess sources of innovation (Akcigit and Kerr 2018, Garcia-Macia et al. 2019). It has also been used to analyze misallocation with endogenously heterogeneous markups (Peters 2020).

Outline The paper proceeds as follows. Section 2 presents the growth model and discusses the mechanism. Section 3 verifies the model's main predictions empirically. I structurally estimate the model in Section 4, and discuss results in Section 5. Section 6 presents extensions, and Section 7 concludes.

[13] Aghion et al. (2022) is a notable exception. Theories of rising profits through reallocation outside of the endogenous growth literature include Helpman and Niswonger (2021), Hubmer and Restrepo (2021) and De Loecker et al. (2021).

[14] Babina et al. (2020) find evidence that artificial intelligence has a similar effect on concentration.

[15] Martinez (2019) and Martinez (2021) relate automation to the labor share.

[16] The paper also relates to the literature on rising profits (e.g. Gutiérrez and Philippon 2019, Barkai 2020, Karabarbounis and Neiman 2019) and how measures of profits may relate to intangibles (e.g. Koh et al. 2020).

2. Intangibles, Firm Dynamics and Growth

This section describes the endogenous growth framework with creative destruction and intangible inputs that I use to explain the trends in productivity growth, business dynamism and market power.

2.1. Preferences and Market Structure

There is a continuum of identical households with unit mass that choose the path of consumption to maximize the following utility function:

$$U = \int_0^\infty \exp(-\rho t) \ln C_t \, dt, \tag{1}$$

where C_t is consumption and ρ is the discount factor.[17] Time is continuous and indexed by t, which is suppressed when convenient. The household is endowed with a single unit of labor, which it supplies inelastically. The consumption good is composed of a continuum of intermediate goods, indexed by j. Each good can be produced by the set of firms I_j that own the production technology, a patent, to produce good j at a level of quality q_{ij}. Quality determines the value that each unit of a good produced by a firm $i \in I_j$ contributes to aggregate consumption. The intermediate goods are competitively aggregated with the following Cobb-Douglas technology:

$$Y = \exp \int_0^1 \ln \left(\sum_{i \in I_j} q_{ij} y_{ij} \right) dj,$$

where Y denotes aggregate output, and y_{ij} is the amount of good j that is produced by firm i. As all output is consumed, $Y = C$. Firms that own the patent to produce good j compete à la Bertrand.[18] While multiple firms own such patents, the profit-maximizing aggregator therefore only demands j from the firm that offers the highest combination of output and quality at a given expenditure. In other words, goods are produced by the firm that offers the lowest quality-adjusted price p_{ij}/q_{ij}.

2.2. Firms and Intangibles

There is a continuum of firms, indexed by i. In the spirit of Klette and Kortum (2004), firms potentially produce more than one good, as they can produce any good for which they own a patent.

Firms produce each of their goods using two inputs, a tangible and an intangible input. The intangible input is an input that allows firms to reduce a good's marginal cost by some desired fraction.[19]

[17] It is straightforward to generalize the setup to feature a CRRA utility function. This would change the Euler equation — and the relationship between discount factor ρ and interest rate r — and hence require a different calibration of ρ.

[18] The Cobb-Douglas aggregator implies that the demand function has a unit elasticity; hence, prices of producers are bound by the minimum price over quality that the producer's competitors can offer. A generalization to CES would imply a similar bound on prices, up to the point that the limit-pricing markup exceeds the monopolist markup (see, e.g., Lentz and Mortensen 2008). This gives rise to a kink in the profit function and puts a ceiling on the model's markups. Given the absence of such a ceiling on markups in the data, and to preserve tractability, I instead rely on the Cobb-Douglas technology.

[19] This definition applies to a subset of the total of possible intangible assets and inputs that firms may deploy. It might not apply, for example, to research and development expenses, which are treated separately in the model. Throughout the text, the term 'intangible inputs' refers to inputs for which the definition applies.

To preserve tractability, the only tangible input is production labor l_{ij}, so that intangibles allow firms to cut the amount of production labor required to produce an additional unit of output.[20] Denoting by $s_{ij} \in (0,1]$ the fraction of marginal costs that a firm incurs, the production function reads

$$y_{ij} = l_{ij}/s_{ij}, \tag{2}$$

so that the marginal cost for firm i of producing j equals $mc_{ij} = s_{ij}w$, where w is the wage rate.

To lower s_{ij} in order to reduce marginal costs, firms must raise spending on intangible inputs. Intangibles differ from production labor in two ways. First, firms differ in the efficiency with which they deploy intangibles. A firm-specific cost parameter, ϕ_i, determines by how much a firm's marginal costs fall for a given expenditure on intangibles. This causes some firms to produce more efficiently than others. Firms draw their ϕ_i from a known discrete distribution $G(\phi)$ at birth and benefit from their intangible efficiency on each good that they produce. Second, firms commit to their spending on intangibles before they observe the marginal costs of competitors and before they set prices. Their intangibles are sunk when firms make pricing and production decisions; firms incur these costs regardless of how much they produce, which is why intangibles represent a fixed cost. Note that firms pay the fixed costs at each t. This keeps the model's tradeoffs extremely tractable, and limits the difference between tangible and intangible inputs to the minimum needed for the results.[21] Additionally, recurrent costs are in line with the high depreciation rate of software, which Li and Hall (2020) estimate at 30 to 40%. Firms must therefore constantly invest in order to maintain constant levels of software.[22]

The function $f(s_{ij}, \phi_i)$, which relates marginal costs to the firm's spending on intangibles, reads

$$f(s_{ij}, \phi_i) = w\phi_i \left(s_{ij}{}^{-\theta} - 1 \right), \tag{3}$$

where $\theta > 0$. The function declines convexly in $s_{ij} \in (0,1]$, rises in $\phi_i > 0$, and satisfies $f(1, \phi_i) = 0$ and $\lim_{s_{ij} \to 0} f(s_{ij}, \phi_i) = \infty$. The latter ensures that all firms have positive marginal costs in equilibrium, while the former yields that firms do not pay for intangibles if they do not reduce marginal costs.

The timing of production decisions is as follows. Firms first observe the intangible cost parameters and quality levels of their competitors for the products that they can produce. For each of their products, they then separately choose whether to produce at the baseline marginal costs w, or to retain – by paying a vanishingly small sunk cost – the option to reduce marginal costs by spending on intangibles.[23] Firms then choose their marginal costs and pay the associated fixed costs on intangibles. Finally, firms observe the marginal costs of their competitors, set prices, and produce the quantity of each product that is demanded from them.

[20] I generalize the production function with multiple tangible inputs when mapping the model to the data in Section 3.

[21] Note that firms in the model also accumulate intangible capital in the spirit of Corrado et al. (2009): they invest in research and development to expand the set of goods that they produce, which persistently affects both firm size and GDP.

[22] There has furthermore been an increase in the share of enterprise software that is sold *as a service* (SaaS), where firms pay periodic fees instead of a one-off fee for perpetual use. For example, 35% of Microsoft's enterprise sales in Q2 of 2019 came from SaaS, at an annual growth rate of 48%.

[23] The small cost $\epsilon \to 0$ that enables firms to choose positive intangibles represents, e.g., the cost of setting up an IT department. A similar assumption simplifies Nash equilibrium pricing in Akcigit and Kerr (2018) and Acemoglu et al. (2018).

2.3. Static Equilibrium

Before proceeding to how firms obtain new patents to produce additional goods, consider the static equilibrium where the set of firms with a patent to produce each good is taken as given. The equilibrium's main novelty is the identity of a good's producer, which will be determined not only by the quality of the firms' patents, but also by the efficiency with which they use intangibles inputs.

In the baseline Klette and Kortum (2004) model, the firm with the highest quality patent is the sole producer. This is because firms have equal marginal costs, so that the highest quality producer can offer the lowest price per quality unit. In the present model, marginal costs differ across firms through their choice of intangibles, the fixed costs of which are determined by their intangible cost parameter ϕ_i. This means that firms can sell at different prices, which can compensate for lower levels of quality. As I show formally below, the equilibrium producer is the firm which – after adjusting for quality differences – has the lowest *choke price*. This choke price is the price at which a firm breaks even, were it to optimize intangibles as the sole producer selling at that price. Firms with a relatively low ϕ_i have a lower choke price, which means that they can reduce marginal costs by a greater fraction than other firms can before producing goods at a loss. A firm with a lower quality patent may thus still be able to offer the best combination of prices and quality, and therefore to produce in equilibrium.

To derive this subgame-perfect Nash equilibrium, note that an equilibrium satisfies two necessary conditions. First, there is only one firm $i \in I_j$ that actively produces good j in any subgame perfect equilibrium. If there were more than one firm, ruinous Bertrand competition would imply that at least one firm ends up setting prices equal to marginal costs in the pricing stage. As intangible inputs are sunk costs at that stage, that firm would produce at a loss. Second, the unique active firm in I_j is the firm that can propose the lowest profitable yet entry-proof quality-adjusted price. Formally, define firm i's choke price as the lowest price at which it does not incur a loss if it is the good's sole producer:

$$p_i^c = \inf\left\{p > 0 : \max_{s_{ij}\in(0,1]} (p - w s_{ij}) Y p^{-1} - w\phi_i \left(s_{ij}^{-\theta} - 1\right) \geq 0\right\},$$

which uses that demand for good j for the producer is $y_{ij} = Y p_{ij}^{-1}$. Because demand does not depend on quality, a firm's choke price is independent of product characteristics and solely determined by the firm's intangible cost parameter – in which the choke price monotonically increases.[24] We can therefore write $p_i^c = p^c(\phi_i)$. By construction, if for two competing firms i and $\tilde{i}$, we have that i has a lower quality-adjusted choke price, then firm i can price $\tilde{i}$ out of the market at any price at which $\tilde{i}$ is willing to be active. Hence firm $\tilde{i}$ will avoid incurring sunk costs on intangibles, thus never setting $s_{\tilde{i}j} < 1$. It follows that the single active firm, henceforth firm i, satisfies

$$\frac{p^c(\phi_i)}{q_{ij}} = \min_{h\in I_j} \frac{p^c(\phi_h)}{q_{hj}}. \tag{4}$$

It remains for us to characterize the equilibrium price and marginal costs of the producer. Because quality units by the potential producers of a good are perfect substitutes, the producer's price is con-

[24] Appendix A.1 derives this formally.

strained by the marginal costs of its nearest competitor. Anticipating that they will be undercut, the producer's competitors do not invest in intangibles. This means that, in line with the standard model, the equilibrium price is equal to the wage – adjusted for the quality of the closest firm's patent.

Formally, the producer charges the following price in the final stage of the interaction:

$$p_{ij}^{*} = w \times \frac{q_{ij}}{\max_{\tilde{i} \in \tilde{I}_j} q_{\tilde{i}j}}, \tag{5}$$

where $\tilde{I}_j = I_j \setminus \{i\}$ are firm i's competitors in the production of good j. Anticipating that firms in $\tilde{I}_j$ do not purchase fixed-cost intangibles, the minimum price at which they are willing to sell is w. The firm in the most advantageous position to compete with firm i is therefore the firm with the highest level of quality, irrespective of that firm's choke price. The price (5) therefore deters any competitor from undercutting i – which means that the producer engages in limit pricing.

As demand for the producer's output depends on the price it sets, the firm internalizes (5) when it backwardly induces optimal fixed costs in the intangibles-setting stage. Minimizing the sum of variable costs $ws_{ij}y_{ij}$ and fixed costs along (3) gives the following product-specific first-order condition:

$$s_{ij}^{*} = \min\left[\left(p_{ij}^{*} Y^{-1} \theta \phi_i\right)^{\frac{1}{\theta+1}}, 1\right]. \tag{6}$$

Firms with lower intangible cost parameters ϕ_i thus incur a smaller fraction of their marginal costs although their exact marginal costs differ per product, as prices p_{ij}^{*} depend on the firm's relative quality.

Finally, the initial stage of the strategic interaction – in which firms pay a small sunk cost to enable the use of intangibles – ensures that the equilibrium is unique. While it is necessary for a Nash equilibrium to satisfy that (4) identifies the producer, which sets prices and intangibles along (5) and (6), the initial stage ensures that other firms do not use intangibles. In the absence of the small sunk cost, competitors $\tilde{i} \in \tilde{I}_j$ may find it profitable to set $s_{\tilde{i}j} < 1$ in response to (6). The initial stage rules out this indeterminacy, as competitors anticipate that they will not produce and therefore avoid sunk costs.

2.4. Innovation

2.4.1. Research and Development

Firms expand their portfolio of patents by investing in research and development (R&D). When investing, firms choose the Poisson flow rate $x_i \geq 0$ with which a new patent is added to their portfolio. In exchange for achieving x_i, firms employ rd^x researchers along

$$rd^x(x_i, n_i) = \eta^x x_i^{\psi^x} n_i^{-\sigma}, \tag{7}$$

where $\psi^x > 1$ and $0 \leq \sigma \leq \psi^x - 1$. The number of researchers that the firm employs is convex in the rate of innovation and declines in the number of goods that the firm produces, n_i. The former implies that the marginal return to R&D is diminishing within each time t. The latter is an assumption from Klette and Kortum (2004), and reflects the idea that large firms have more in-house knowledge or organizational capital than small firms do. Practically, the presence of $n_i^{-\sigma}$ governs the relationship between firm size and firm growth. For $\sigma = \psi^x - 1$, the model satisfies Gibrat's law of constant firm growth in size, while for $\sigma = 0$ firm growth declines rapidly in size. Following Akcigit and Kerr (2018), I allow for an intermediate case $\sigma \in [0, \psi^x - 1]$ so that the relationship between size and growth matches the data.

A firm that innovates successfully becomes the owner of a state-of-the-art patent for a random good j. Innovation is not directed, in the sense that firms are equally likely to innovate on all products. As in Aghion and Howitt (1992), the state-of-the-art patent allows firm i to produce its new good at a quality level that is a multiple $\lambda_{ij} > 1$ of the level of the current producer of the good:

$$q_{ij} = q_{\tilde{i}j}\lambda_{ij}, \tag{8}$$

where $\tilde{i}$ denotes the incumbent of good j while λ_{ij} denotes the realized innovation step size, which is a continuous random variable with counter-cumulative distribution function $H(\lambda)$.

Innovation in the model is different from the usual Klette and Kortum (2004) setup because the innovator of a certain good will not necessarily become its new producer. Section 2.3 shows that, here, the innovator only becomes the new producer of the good if its quality improvement λ_{ij} is sufficiently large for the innovator to become the firm with the lowest quality-adjusted choke price. It follows that when the innovator is firm i and the incumbent is $\tilde{i}$, the innovation is successful if

$$\lambda_{ij} > \frac{p^c(\phi_i)}{p^c(\phi_{\tilde{i}})}. \tag{9}$$

When this condition is satisfied, the innovator receives its patent and becomes the firm with the lowest quality-adjusted choke price. This always happens if the innovator has an equal or lower choke price than the incumbent does, as it can offer the good at the same price but at a superior quality. If the incumbent has a lower choke price (implying that it uses intangibles more efficiently) the innovator must have a sufficiently large innovation step to take over. In case the innovator's quality improvement is also insufficiently large, the innovation fails; product j is not added to the innovator's portfolio J_i and the innovation is lost. As the innovator never produces good j, future innovators are unable to learn from the lost innovation and only improve j's quality over the level at which the incumbent produces.

2.4.2. Innovation and Intangibles

It is useful to highlight the difference between quality and price in the model. In most models of growth through creative destruction, the two are isomorphic. Prices reflect the ability of firms to produce at low marginal costs (that is; with high productivity). It may seem that this is equivalent to quality, in the sense that a firm can increase its effective output $q_{ij}y_{ij}$ using the same quantity of tangible inputs by either selling at higher quality or by using a greater amount of intangibles.

The difference between the two lies in their contribution to long-term growth. Innovation raises the quality with which good j can be produced. If an innovating firm successfully takes over production, this offers a positive externality: all future innovations on j are improvements over q_{ij}, the innovation by firm i allows good j to be produced at a permanently higher level of quality. This makes quality improvements the source of long-term economic growth. Intangibles do not come with a similar externality. They improve production efficiency only for the current producer. Intuitively, the fact that the incumbent is efficient at using software applications to reduce marginal costs does not benefit an innovating firm when it takes over production.

2.4.3. Innovators' Equilibrium Intangibles, Markups and Profits

It follows from (9) that a good's equilibrium producer is the firm that innovated most recently. As innovators that fail to take over production lose their patent, the difference between the quality of the producing firm and the firm that is closest on the quality ladder is λ_{ij}. This means that the limit price along (5) is $p^*_{ij} = w\lambda_{ij}$, while the producer's cost-minimizing use of intangibles along (6) is such that

$$s^*_{ij} = \min\left[\left(\lambda_{ij}\frac{w}{Y}\theta\phi_i\right)\right)^{\frac{1}{\theta+1}}, 1\right]. \tag{10}$$

The markup μ_{ij} is found by dividing the price p^*_{ij} by firm i's optimal marginal cost ws^*_{ij}:

$$\mu_{ij} = \frac{\lambda_{ij}}{s^*_{ij}}, \tag{11}$$

which yields that markups increase in the difference in quality between the current producer and the previous producer, λ_{ij}, as well as the producer's use of intangibles. Note that profits π_{ij} do not rise proportionally to markups when firms use more intangibles, as firms also face higher fixed costs:

$$\pi_{ij} = \left(1 - \mu_{ij}^{-1}\right)Y - f(s^*_{ij}, \phi_i), \tag{12}$$

where the first term represents variable operating profits, while the second term represents fixed costs. As the firm's s^*_{ij} is pinned down by the combination of ϕ_i and λ_{ij}, profits can be written as $\pi(\phi, \lambda_{ij})$.

2.5. Entry and Exit

There is a unit mass of entrepreneurs that invest in R&D to obtain patents to produce goods that are currently owned by incumbents. The R&D cost function is analogous to that of incumbents:

$$rd^e(e) = \eta^e e^{\psi^e}, \tag{13}$$

where $rd^e(e)$ denotes the number of researchers employed by potential entrants to achieve start-up rate e, and where $\eta^e > 0$, $\psi^e > 1$. Entrepreneurs that draw an innovation improve the quality of a random good that is currently produced by an incumbent. In similar spirit to models where firms draw idiosyncratic productivities at birth (e.g. Hopenhayn 1992, Melitz 2003), entrants then draw their intangible cost parameter $\phi_e \in \Phi$ from the known probability distribution $G(\phi)$, and learn about their incumbent's intangible costs. The entrant becomes the new producer if it has drawn a sufficiently large step-size to overcome any difference in choke prices along condition (9).

A firm exits the economy if it does not produce any good in its patent portfolio. This happens when entrants or other incumbents develop higher-quality versions of the sole good that a firm produces.

2.6. Creative Destruction

Firms cease to produce a good if a different incumbent or an entrant successfully innovates on that product. The rate at which this happens is the rate of creative destruction, $\tau(\phi_i)$. The rate of creative destruction is endogenous, as it is determined by the respective efforts that incumbents and entrants put into innovation. It is a function of the firm's intangible cost parameter ϕ_i, because a firm with relatively low intangible costs is more likely to be able to undercut an innovative challenger on price. The rate of creative destruction for a firm with intangible cost ϕ_i is given by

$$\tau(\phi_i) = \sum_{\phi_h \in \Phi} \text{Prob}\left(\lambda_{hj} \geq \frac{p^c(\phi_h)}{p^c(\phi_i)}\right)\left[\sum_{n=1}^{\infty} M_n(\phi_h)x_n(\phi_h) + eG(\phi_h)\right], \tag{14}$$

where $x(\phi_h, n)$ and $M(\phi_h, n)$ respectively denote optimal innovation rates and the measure of firms with intangible cost ϕ_h that produce n products. The outer-summation reflects that an incumbent with intangible cost ϕ_i faces innovative competitors from each intangible-cost level $\phi_h \in \Phi$. Within the summation there are two terms: the probability that an innovation by a firm with cost ϕ_h is successful, multiplied by innovation efforts by firms with that level of intangible costs. Under cumulative density function $1 - H(\lambda)$, the probability that (9) is satisfied when i is the incumbent and h innovates equals

$$\text{Prob}\left(\lambda_{hj} \geq \frac{p^c(\phi_h)}{p^c(\phi_i)}\right) = H\left(\frac{p^c(\phi_h)}{p^c(\phi_i)}\right). \tag{15}$$

This probability is strictly lower when the incumbent is a low-ϕ firm, as these have a lower choke price. The term for innovation effort in (14) contains two parts. The first captures innovation by incumbents of type ϕ_h. As is shown below, a firm's innovation effort is a function of its intangible cost parameter and product count, which explains the inclusion of the summation over n. The Poisson rate is multiplied by measure $M(\phi_h, n)$ to obtain the innovation rate. The second term measures innovation by entrants. It is the product of the entry rate e and the probability $G(\phi_h)$ that the entrant is of type ϕ_h.

2.7. Equilibrium

I now characterize the full stationary equilibrium where productivity, output and wages grow at rate g.

2.7.1. Optimal Innovation Decisions

Firms choose the level of spending on research and development that maximizes firm value. The associated value function, where notation is borrowed from Akcigit and Kerr (2018), reads as

$$rV_t(\phi_i, J_i) - \dot{V}_t(\phi_i, J_i) = \max_{x_i} \left\{ \begin{array}{c} \sum_{j \in J_i} \left(\pi_t(\phi_i, \lambda_{ij}) + \tau(\phi_i)\left[V_t(\phi_i, J_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i, J_i)\right]\right) \\ +x_i \mathscr{P}(\phi_i)\mathbb{E}_{\phi_i}\left[V_t(\phi_i, J_i \cup_+ \lambda_{ih}) - V_t(\phi_i, J_i)\right] \\ -w_t \eta_x (x_i)^{\psi^x} n_i^{-\sigma} - F(\phi_i, n_i) \end{array} \right\}, \tag{16}$$

where r is the interest rate and where $\dot{V}_t$ denotes the change in V_t with time. The top right-hand line contains the sum of all good-specific items. It is the sum of contemporaneous profits along (12) and the change in firm value if the firm ceases production of good j due to creative destruction. $V_t(\phi_i, J_i \setminus \{\lambda_{ij}\})$ denotes the value of producing the set of goods J_i except good j with innovation realization λ_{ij}. The bottom two lines are not specific to goods. The first of these gives the expected increase in firm value from innovation. $\mathbb{E}_{\phi_i} V(\phi_i, J_i \cup_+ \lambda_{ih})$ denotes the firm's value if it successfully takes product h, taking conditional expectations over λ_{ih} for firm type ϕ_i. The change in value is multiplied by the innovation rate and the probability $\mathscr{P}(\phi_i)$ that the firm is able to offer a sufficiently low quality-adjusted price. The final line gives the costs of R&D and a fixed term $F(\phi_i, n_i)$. Firms must pay the latter in order to operate, and it is assumed to equal the option value of R&D. This ad-hoc restriction, borrowed from Akcigit and Kerr (2018), ensures that the value function is linear in the number of goods that firms produce, so that the model admits an analytical first-order condition. In Section 6 I remove this assumption and show that, though significantly reducing tractability, the results are qualitatively and quantitatively robust.

Proposition 1. *The value function of a firm with intangible cost ϕ_i that produces a portfolio of goods J_i with cardinality n_i grows at rate g along the balanced growth path and is given by*

$$V(\phi_i, J_i) = \textstyle\sum_{j \in J_i} \pi(\phi_i, \lambda_{ij})(r - g + \tau(\phi_i))^{-1},$$

which is decreasing in ϕ_i. The optimal rate of innovation reads as

$$x_{n_i}(\phi_i) = \left(\mathscr{P}(\phi_i)\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)}\right](\eta^x \psi^x w)^{-1}\right)^{\frac{1}{\psi^x - 1}} n_i^{\frac{\sigma}{\psi^x - 1}}. \tag{17}$$

The optimal entry rate is given by

$$e = \left(\textstyle\sum_{\phi_e \in \Phi} G(\phi_e)\mathscr{P}(\phi_e)\mathbb{E}_{\phi_h}\left[\frac{\pi(\phi_e, \lambda_{ej})}{r - g + \tau(\phi_e)}\right](\eta^e \psi^e w)^{-1}\right)^{\frac{1}{\psi^e - 1}}. \tag{18}$$

Proof: Appendix A.

First-order condition (17) is intuitive. Firms engage in more innovation when the expected increase in value is higher, and invest less when the innovation cost-parameters are high. Innovation increases in the firm-size n_i — although if $\sigma < \psi^x - 1$, the firm's expected growth rate will decline with size. Firms with a lower intangible cost parameter ϕ_i choose a higher innovation rate because their ability to reduce marginal costs increases profitability. They furthermore face a lower rate of creative destruction, which decreases the effective discount rate. Firms with a lower ϕ_i also have a higher probability of successfully becoming the new producer of products upon which they innovate. Jointly, these effects cause a negative relationship between ϕ_i and the rate of innovation.

Innovation by entrants (18) is such that the marginal cost of increasing the entry rate e is equal to the expected value of producing a single good, adjusted for the probability that the entrant is able to take over production from the incumbent by offering a sufficiently low quality-adjusted price. Because entrants only learn about their type after they have drawn an innovation, the expectation of the value of producing a good is taken over the distribution of firm types at entry $G(\phi)$.

2.7.2. Dynamic Optimization by Households

Maximizing life-time utility with respect to consumption and savings gives the usual Euler equation,

$$\frac{\dot{C}}{C} = r - \rho, \tag{19}$$

combined with the transversality condition. Along the balanced growth path, consumption grows at the same rate as output and productivity, so that $r - g = \rho$.

2.7.3. Firm Measure and Size Distribution

The optimal innovation rate in (17) is a function of a firm's intangible input costs ϕ_i and the number of goods n_i it produces. The rate of creative destruction (and hence the growth rate of output and productivity) therefore depends on the equilibrium distribution of n and ϕ across firms. Along the balanced growth path, these distributions are stationary. To find the stationary distributions, consider the law of motion for the measure of firms that produce more than one product:

$$\begin{aligned} \dot{M}_n(\phi_i) \quad &= \big(M_{n-1}(\phi_i)x(\phi_i, n-1) - M_n(\phi_i)x_n(\phi_i)\big)\mathscr{P}(\phi_i) \\ &+ \big(M_{n+1}(\phi_i)[n+1] - M_n(\phi_i)n\big)\tau(\phi_i), \end{aligned} \tag{20}$$

where the first term captures entry into and exit out of measure $M_n(\phi_i)$ through innovation by firms of type ϕ_i with $n-1$ products and n products, respectively. The second term captures entry and exit of firms with $n+1$ and n products that ceased producing one of their products through creative destruction. For the measure of single-product firms, the law of motion reads as

$$\dot{M}_1(\phi_i) = \big(eG(\phi_i) - x_1(\phi_i)M_1(\phi_i)\big)\mathscr{P}(\phi_i) + \big(2M_2(\phi_i) - M_1(\phi_i)\big)\tau(\phi_i). \tag{21}$$

The stationary firm-size distribution follows from setting both equations to zero for each n.

The fraction of goods that is produced by firms with intangible cost ϕ_i is given by

$$K(\phi_i) = \frac{\sum_{n=1}^{\infty} nM_n(\phi_i)}{\sum_{\phi_h\in\Phi}\sum_{n=1}^{\infty} nM_n(\phi_h)}. \tag{22}$$

2.7.4. Labor Market Equilibrium

The solutions to the static and dynamic optimization problems of firms allow the labor market equilibrium conditions to be defined. Labor is supplied inelastically by households at a measure standardized to 1. Equilibrium on the labor market requires that

$$1 = L^p + L^f + L^{rd} + L^e,$$

where L^p is the labor used to produce intermediate goods. Inserting the unit-elastic demand function, markup (11) and intangible first-order condition (10) into $L^p = \int_0^1 \mathbf{1}_{j\in J_i} l_{ij} di\ dj$ yields

$$L^p = \int_0^1 \int \mathbf{1}_{j\in J_i} \frac{Y}{w} \left[1 - \left(\lambda_{ij} \frac{w}{Y} \theta \phi_i \right)^{\frac{1}{\theta+1}} \right] \lambda_{ij}^{-1} \, di \, dj,$$

where $\mathbf{1}_{j\in J_i}$ is the indicator function that equals one when firm i produces good j. L^f is the labor used to fulfill the intangible fixed costs:

$$L^f = \int_0^1 \int \mathbf{1}_{j\in J_i} \left[\left(\lambda_{ij} \frac{w}{Y} \theta \phi_i \right)^{-\frac{\theta}{\theta+1}} - 1 \right] \phi_i \, di \, dj.$$

L^{rd} is the labor involved with research and development carried out by existing firms:

$$L^{rd} = \sum_{\phi_i \in \Phi} \sum_{n=1}^{\infty} \left[M_n(\phi_i) \eta^x \, x_n(\phi_i)^{\psi^x} n^{-\sigma} \right],$$

while L^e is the labor involved with research and development carried out by entrants $L^e = \eta^e e^{\psi^e}$, where innovation rates $x_n(\phi_i)$ and e are dynamically optimized along (17) and (18).

2.7.5. Aggregate Variables

I can now characterize the economy's aggregate variables. The equilibrium wage is given by

$$w = \exp\left(\int_0^1 \int \mathbf{1}_{j\in J_i} \ln \left[\frac{q_{ij}}{s_{ij}} \right] di \, dj \right) \exp\left(\int_0^1 \int \mathbf{1}_{j\in J_i} \ln \left[\frac{s_{ij}}{\lambda_{ij}} \right] di \, dj \right). \tag{23}$$

The first term of (23) is the standard CES productivity term. The second term is the inverse of the expected markup. Note that a rise in the intangibles has no effect on the level of the wage because s_{ij} cancels out. While a firm that deploys more intangibles becomes productive, it is able to proportionally raise its markups. These have offsetting effects on the level of the wage. Aggregate output is given by

$$Y = L^p \exp\left(\int_0^1 \int \mathbf{1}_{j\in J_i} \ln \left[\frac{q_{ij}}{s_{ij}} \right] di \, dj \right) \frac{\exp \int_0^1 \int \mathbf{1}_{j\in J_i} \ln \mu_{ij}^{-1} \, di \, dj}{\int_0^1 \int \mathbf{1}_{j\in J_i} \mu_{ij}^{-1} \, di \, dj}. \tag{24}$$

Derivations are provided in Appendix A.3. As in the model with heterogeneous markups and misallocation by Peters (2020), the last term captures the loss of efficiency due to the dispersion of markups. If all markups are equalized, the term is equal to 1, while it declines as the variance of markups increases. Total factor productivity is the product of the second- and the last term in (24).

Equation (24) reveals that a rise in intangibles has two counteractive effects on the level of output. The spread of markups increases when the average s_{ij} falls along (11), because a lower s_{ij} amplifies the heterogeneity in markups caused by the heterogeneous innovation steps (the second term in (24)). On the other hand, the decrease in s_{ij} has a direct positive effect on productivity because it increases the CES productivity index (the first term in (24)). As will be clear below, the second effect dominates in feasible calibrations. That means that a rise in intangibles initially has a positive effect on the level of output and on productivity. The next section shows, however, that this may not be the case for growth.

2.7.6. Growth

The growth rate of total factor productivity and output is a function of creative destruction.

Proposition 2. *The constant growth rate of productivity, consumption C, output Y and wages w is*

$$g = \sum_{\phi_i \in \Phi} K(\phi_i)\tau(\phi_i)\mathbb{E}_{-\phi_i}(\lambda_{hj} - 1), \tag{25}$$

where $\mathbb{E}_{-\phi_i}(\lambda_{hj})$ *is the expected realization of* λ_{hj} *when a firm with* ϕ_i *is the incumbent of a good before a different firm h becomes the new producer due to successful innovation.* **Proof:** Appendix A.

The proposition shows that growth is the expected increase in quality multiplied by the rate of creative destruction, weighted by the fraction of product lines that firms of each intangible cost own.

Equation (25) shows the counteracting effects of an increase in ϕ_i at a subset of firms. On the one hand, firms with a lower ϕ_i have a greater incentive to invest in research and development, which raises the rate of creative destruction. On the other hand, even at a constant innovation rate, the presence of low-ϕ_i firms has a negative effect on the rate of creative destruction because high-ϕ_i firms have a lower probability of becoming the new producer. This has not only a direct effect on growth at given innovation rates, but also an indirect effect, as these firms reduce their expenditure on R&D.

2.7.7. Equilibrium Definition

Definition 1. *The economy is in a balanced growth path equilibrium if for every t and for every intangible productivity* $\phi_i \in \Phi$*, the variables* $\{r, e, L^p, g\}$ *and functions* $\left\{x_{n_i}(\phi_i), K(\phi_i), M_{n_i}(\phi_i), s^*_{ij}, \tau(\phi_i)\right\}$ *are constant,* $\{Y, C, w,\}$ *grow at a constant rate g that satisfies (25), interest rates follow from (19), Y satisfies (24), innovation rates* $x(n_i, \phi_i)$ *satisfy (17), the entry rate e satisfies (18), firm distribution* K_{ϕ_i} *and measure* M_{ϕ_i} *are constant and satisfy (A.25) and (21), markups* μ_{ij} *satisfy (11), the fraction of marginal costs reduced through intangibles* s^*_{ij} *satisfies (10) for all* λ_{ij}*, the rate of creative destruction* $\tau(\phi_i)$ *satisfies (14), and both goods and labor markets are in equilibrium so that* $Y = C$ *and* $L^p = 1 - L^s - L^{rd} - L^e$*.*

3. Model Meets Data

This section empirically validates the main mechanisms of the model. Section 3.1 introduces firm-level micro-data for two countries, the United States and France. Section 3.2 derives predictions that are tested with this data, using a new fixed costs measure detailed in Section 3.3. Evidence on the empirical relationship between fixed costs, intangibles, innovation and markups is presented in Section 3.4.

3.1. Data

To perform the empirical analysis for the United States I use micro data from financial statements on listed firms, while for France I use administrative tax data on the universe of firms. Appendix D, replicating the macroeconomic trends that motivate this paper for France, confirms that the country has incurred a decline in productivity growth and business dynamism, and a modest rise in markups.

Table 1: Descriptive Statistics

	Mean	Std. Dev.	Median	10th Pct.	90th Pct.	Obs.
U.S. Compustat Firms (1979-2015)						
Sales (revenue)	1,733,510	5,154,239	1,94,318.5	13,329.7	3,603,252	127,682
Operating expenses	1,466,022	4,326,827	169,248.4	12,957.5	3,074,281	127,682
Cost of goods sold	1,111,563	3,315,834	116,819.2	7,133.9	2,351005	127,682
Selling, General, and Adm. expenses	335,829.9	1,046,907	39,864.5	3,918.3	6,501,041	127,682
Capital stock	1,109,350	3,714,094	79,352.3	4,910.0	2,131,897	127,682
French Firms in FICUS-FARE (1994-2016)						
Sales (revenue)	4,684	103,285	617	149	4,996	9,913,058
Employment (headcount)	19	356	5	1	28	9,913,058
Wage bill	622	10,753	144	38	831	9,913,058
Capital stock	1,738	131,183	92	12	895	9,913,058
Intermediate inputs and raw materials	2,234	58,699	136	0	1,923	9,913,058
Other operating expenses	1,211	35,656	124	33	1168	9,913,058

Notes: Nominal figures in thousands of deflated dollars (U.S.) and euros (France). Sales, operating costs and materials are deflated with KLEMS sector deflators; the wage bill and capital are deflated with the GDP deflator.

Data for U.S. firms comes from S&P's Compustat. Compustat contains balance sheet and income statement data for all listed firms in the U.S. I restrict the sample to firms outside of finance, insurance and real estate between 1979 and 2015, and drop firms with missing or negative sales, (fixed) assets and operating expenses. The sample covers 11,750 firms across 788 6-digit NAICS industries.

The French data comes from two administrative datasets on the universe of firms outside of finance and agriculture (FICUS, from 1994 to 2007, and FARE, from 2008 to 2016), both based on data from the tax office DGFiP. FICUS and FARE share an identifier (the *siren code*) that consistently tracks firms over time. The data contains the full balance sheet and income statement, with detailed breakdowns of revenues and costs. The sample covers 1,087,726 firms across 651 NACE industries.[25]

Details on variable definitions and data construction are provided in Appendix B. Summary statistics are provided in Table 1. All variables are deflated and are winsorized at their 1% tails.

3.2. Testable Predictions

These micro datasets enable a test of the model's main predictions. Before deriving the predictions, I generalize production function (2) to facilitate a mapping to the data. Assumptions on the remainder of the framework follow Section 2. I generalize (2) by assuming that firms produce along a first-degree homogeneous production function $z(z_{ijt,1}, z_{ijt,2}, .., z_{ijt,k})$ with k tangible production factors or intermediate inputs. Intangibles allow firms to reduce marginal costs, in exchange for higher fixed costs. Denoting by s_{ijt} the share of marginal costs that a firm keeps, the production function reads

$$y_{ijt} = s_{ijt}^{-1} z(z_{ijt,1}, z_{ijt,2}, .., z_{ijt,k}). \tag{26}$$

The first-degree homogeneity of $z(\cdot)$ implies that firm i's marginal cost is $mc_{ijt} = s_{ijt}\boldsymbol{c}(w_{1t}, w_{2t}, .., w_{kt})$, where w_{kt} denotes the factor price of tangible production factor k at time t.

[25]The FICUS-FARE panel was created for Burstein et al. (2019), who kindly allowed me to use the data in this paper.

The generalized model yields a number of empirical predictions. First, one of the core assumptions of the model is that intangible inputs are fixed costs. It follows directly that if one observes both (a subset of) a firm's intangibles and its fixed costs, these should correlate positively. Second, the model yields equilibrium relationships between intangibles, research and development expenditures and markups, as summarized by the following proposition:

Proposition 3. *For equilibria where firms choose a positive level of spending on intangibles, the model implies the following firm-level relationships between intangibles, markups and innovation:*

a. The cost-minimizing fixed- over total cost ratio decreases in a firm's intangible cost parameter ϕ_i.

b. Firms with lower intangible cost parameters ϕ_i have higher spending on research and development, and have higher markups across their products than other firms do.

Proof: Appendix A.

Combined, parts (a) and (b) of the proposition invite regressions that relate firms' ratios of fixed costs over total costs to research and development and markups, with a positive predicted correlation.

3.3. Measuring Fixed Costs

Testing these predictions involves two main challenges. First, firms do not identify whether costs are variable or fixed in financial statements. Second, the administrative datasets provide data at the firm level, while production function (26) operates at the product level. I address both issues below.

3.3.1. Measure

I derive a new time-varying measure of fixed costs from the difference between marginal cost markups and the profit rate, which equals operating profits over revenue. Past work typically uses the sensitivity of a firm's operating costs or profits to sales shocks to measure fixed costs, under the assumption that all variable costs are set freely.[26] This is problematic when firms face adjustment costs for some variable inputs (e.g. when adjusting their labor force), and it does not yield a time-varying measure of fixed costs at the firm level. As posts on the income statement are broad, they too cannot be classified as fixed costs, even in the French data. The new measure instead takes advantage of marginal costs being constant within firm-product-years, implying that the accounting definition of the profit rate is

$$\frac{\pi_{ijt}}{p_{ijt}y_{ijt}} = \frac{\left(p_{ijt} - mc_{ijt}\right)y_{ijt}}{p_{ijt}y_{ijt}} - \frac{f(s_{ijt},\phi_i)}{p_{ijt}y_{ijt}},$$

which follows from dividing profits (12) by revenue. Isolating fixed costs on the left, and defining the markup μ_{ijt} for the ratio of prices to marginal costs, yields a model-consistent measure of fixed costs:

$$\frac{f(s_{ijt},\phi_i)}{p_{ijt}y_{ijt}} = \left(1 - \frac{1}{\mu_{ijt}}\right) - \frac{\pi_{ijt}}{p_{ijt}y_{ijt}}. \tag{27}$$

[26]Appendix C shows that trends in alternative measures of fixed costs are similar to the trend in the new measure.

The equation shows that profits differ from markups because the latter measure *marginal* profitability, while the profit rate measures *average* profitability. A firm with positive fixed costs should have a profit rate below the markup. This implies that rising markups do not necessarily reflect rising profitability.

It is straightforward to aggregate the product-level (27) to a model-consistent measure of fixed costs at the firm level. A firm-level measure is needed because the micro datasets contain firm-level revenues $py_{it} = \sum_{j\in J_{it}} p_{ijt}y_{ijt}$ and operating profits $\pi_{it} = py_{it} - tc_{it}$, where tc_{it} denotes firms' total costs and where J_{it} is a firm's product portfolio. Because firms may face fixed costs that are not related to intangible spending on individual products (such as overhead), I write total costs tc_{it} as follows:

$$tc_{it} = \tilde{f}_{it} + \sum_{j\in J_{it}} \left(y_{ijt}s_{ijt}\boldsymbol{c}(w_{1t}, w_{2t}, .., w_{kt}) + f(s_{ijt}, \phi_i)\right),$$

where $\tilde{f}_{it}$ are the additional firm-level fixed costs that are unrelated to intangibles and that apply at the firm level. The following proposition applies:

Proposition 4. *A firm's total fixed costs $f_{it} = \tilde{f}_{it} + \sum_{j\in J_{it}} f(s_{ijt}, \phi_i)$ are identified by*

$$\frac{f_{it}}{py_{it}} = \left(1 - \frac{1}{\mu_{it}}\right) - \frac{\pi_{it}}{py_{it}}, \tag{28}$$

where μ_{it} is the harmonic average of the firm's product-level markups μ_{ijt}. **Proof:** Appendix A.

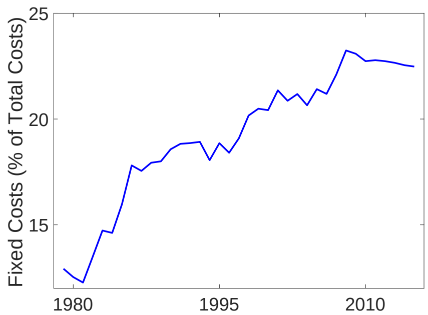
It follows that firm-level fixed costs can be identified from firm-level data on operating profits and revenue (both observable on financial statements), as well as markups. Markups are not observable because financial statements lack data on marginal costs and prices. I therefore estimate markups using the method proposed by Hall (1988). He shows that under cost minimization, a firm's markup is given by the product of the output elasticity of a variable input multiplied by the ratio of a firm's sales to its expenditure on that input. Appendix A.8 derives that, if a firm's products have equal output elasticities, Hall's firm-level markup estimates are the harmonic average of product-level markups.[27]

The advantage of the Hall (1988) methodology to estimate markups is that it does not assume a particular market structure or competition, and that it is consistent with the framework in Section 2. The measure requires firm-level data on revenue and spending on some variable input, which is assumed to be set flexibly and without adjustment costs. Other inputs may be fixed, variable or a combination of both: as long as one freely-set variable input is observed, markups can be estimated consistently. For the French data I use intermediate inputs as the variable input, while for U.S. data I use markups from De Loecker et al. (2020), which use cost of goods sold. Revenue and expenditure on the input are observed on the income statement, while I obtain the output elasticity by estimating a translog production function using the procedure proposed by De Loecker and Warzynski (2012).[28]

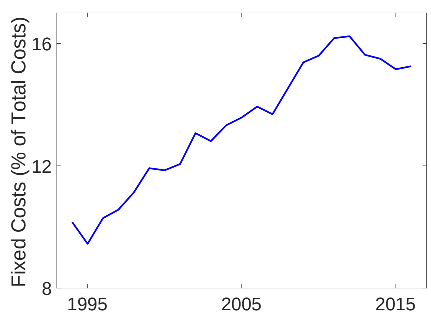
[27] The quantified model in Section 4 shows that most firms produce a single product, in line with the data. Hall's markups directly correspond to product-level markups for these firms. For multi-product firms with heterogeneous output elasticities across their products, (28) measures fixed costs with error. The error is firm-specific; fixed costs may be over- or underestimated, so that there is no clear bias in aggregates. The error would, however, cause attenuation bias in regressions.

[28] Details are provided in Appendix C.

Figure 2. Weighted-Average Ratio of Fixed Costs to Total Costs



(a) United States



(b) France

Notes: Sales-weighted average of fixed costs as a percentage of total costs, U.S. listed firms (left) and universe of French firms (right). Fixed costs are derived from the difference between markups and profit rates at the firm level along (28).

A final data issue is the mapping of Proposition 3 from the product to the firm level in the presence of firm overhead costs $\tilde{f}_{it}$. When firms produce additional goods, the fixed-cost ratio declines mechanically because firm-level $\tilde{f}_{it}$ is now spread across more products. As firm concentration has increased over time in the data, an analysis of aggregate trends in fixed costs underestimates the rise in fixed costs that is driven by fixed-cost intangibles. In the regressions I address this by controlling for (log) revenue, as revenue is proportional to firm size ($py_{it} \propto n_{it}$) in the model. Revenue, therefore, controls for the mechanical decline in the ratio of $\tilde{f}_{it}$ over total costs in firm size.

3.3.2. Fixed Cost Estimates

I use the data on operating expenses, revenue and markups to measure firm-level fixed costs for France and the United States along (28). On average, fixed costs comprise 23.7% of operating costs in the U.S. data and 19.4% in the French data, with respective standard deviations of 19.6% and 22.6%.

Figure 2 plots the revenue-weighted average ratio of fixed to total costs. I weigh by revenue, because revenue is proportional to the firm's weight in aggregate fixed costs in the model. The measure shows a persistent increase in both countries. Fixed costs made up less than 13% (9.5%) of costs for U.S. (French) firms at the start of the sample, and close to 23% (14%) at the end. Over the full episode, the increase is greater for U.S. firms but this seems to be due to the difference in time samples. Between 1995 and 2015, the increase in the fixed-costs share is around 5 percentage points in both datasets.[29]

Appendix C provides various robustness checks for the trend in fixed costs. It shows that the results are robust to alternative estimates of markups, as well as to various approaches to the adjustment of profits for capital costs.[30] The appendix also confirms that alternative measures of fixed costs from the literature yield similar levels and trends as those in Figure 2. Finally, the appendix also contains an

[29]The level of the fixed-cost measure depends on the estimated output elasticity of the variable input. Some estimations of these elasticities are consistently lower than the level used for fixed costs in Figure A3, and therefore imply a lower level of fixed costs. The trend was similar across estimations, however. Appendix C contains a full robustness check.

[30]The definition of operating profits for Figure A3 accounts for depreciation costs, but it does not adjust profits for the rental costs of capital. The appendix derives multiple series of required rental costs r_t^K from the estimates of risk-free interest rates and capital risk premia in Caballero et al. (2017), and shows that the trends in Figure 2 are robust to the adjustment.

illustration of the sectoral composition of fixed costs (Figure A4). It shows that fixed costs are especially high in the information sector, while variable costs are relatively important in retail and wholesale. Nearly all broad sectors have seen an increase in their share of fixed costs in total costs, and a formal between-within decomposition in Table A3 confirms that the increase occurs largely within sectors.

A key assumption in the model is that intangible inputs are fixed costs. While there is no exhaustive information on firm-level spending on intangible inputs, external data on software investments is available. As software spending is a subset of total intangible spending, we should see it positively correlate with fixed costs. To assess whether this is the case in the data, I estimate

$$\ln\left(\frac{f_{it}}{py_{it}}\right) = \beta \ln\left(\frac{f_{it}^{s}}{py_{it}}\right) + \gamma' x_{it} + \eta_{it}, \tag{29}$$

where software spending is denoted by f_{it}^{s}, so that $\beta > 0$ if f_{it}^{s} is indeed part of the firm's fixed costs $\sum_{j \in J_{it}} f(s_{ijt}, \phi_i)$. Both the dependent and the explanatory variable are standardized by revenue because revenue is proportional to a firm's product count in the model. This controls for the mechanical correlation between fixed costs and software spending that arises when firms produce more goods.[31]

To measure software for U.S. firms, I use estimates of annual software budgets in the Ci Technology Database (CiTDB), produced by the marketing company Harte-Hanks (e.g., Bloom et al. 2012). The CiTDB collects site-level IT data through phone surveys, which is sold for the purpose of commercial acquisition. It contains consistently defined estimates of software budgets for 2010 and for 2012-2015.[32] Appendix B details the data construction. The dataset contains 6,585 observations.

For France, data on software and IT comes from the *Enquête Annuelle d'Entreprises* (EAE), which is an annual survey that (post-weighting) representatively samples around 12,000 firms between 1994 and 2007. The survey provides a comprehensive panel of firms with more than 20 employees, and samples smaller firms in most sectors. I use this survey for data on software investments, developed in-house or purchased externally.[33] The matched data contains 106,865 firm-years across 31,005 firms.

Results are presented in Table 2. Column I presents univariate regressions, which show a significantly positive relationship between software spending and fixed costs in both countries. Column II adds log revenue to control for firm size n_{it}. Adding 2-digit industry- and year fixed effects in columns III and IV has limited effects on the estimated correlations, which are comparable in size for France and the U.S. The coefficients shrink substantially when adding firm fixed effects. This is consistent with the model, as most of the variation in intangibles comes from ϕ_i, which is fixed within the firm.

Results in Table 2 are robust to alternative specifications, such as controlling for size through costs instead of revenue as a control, or to a broader measure of information technology budgets to measure software spending in the U.S. data. Table A5 shows that there is also a positive correlation between

[31] Alternatively, the regression could be performed with *controls* for the log of revenue. This yields similar results.

[32] I thank Nick Bloom and John Van Reenen for graciously providing an extract from the CiTDB to perform this analysis. I additionally observe firm-level adoption of personal computers. While personal computers are not directly a measure of intangible inputs, they are a common proxy for IT intensity at the firm level (see, e.g., Bloom et al. 2012), and this data is available for a longer sample from 1997 to 2015. The dataset also contains detailed data on the kind of IT systems that firms have installed, but it is not possible to distinguish between missing entries and entries where a system was not installed.

[33] This survey was also used to measure software by Lashkari et al. (2019). Details are provided in Appendix B.

Table 2: Relationship between Software Spending and Fixed-Cost Share

Fixed-Cost Share (log)	I	II	III	IV	V	VI
United States (CiTDB, 2010 to 2015)						
Software budget over revenue (log)	0.093	0.067	0.054	0.055	0.022	0.019
	(0.008)	(0.010)	(0.010)	(0.010)	(0.007)	(0.007)
R^2	0.054	0.063	0.170	0.172	0.015	0.038
France (EAE, 1994 to 2007)						
Software investments over revenue (log)	0.130	0.117	0.0719	0.0678	0.0456	0.0324
	(0.002)	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)
R^2	0.039	0.056	0.192	0.197	0.032	0.129
Year fixed effects	No	No	No	Yes	No	Yes
Firm fixed effects	No	No	No	No	Yes	Yes
Industry fixed effects	No	No	Yes	Yes	No	No
Revenue (product-count) control	No	Yes	Yes	Yes	Yes	Yes

Notes: Dependent variable is fixed costs over revenue (log). Explanatory variable is (log) software investments over revenue. Revenue is deflated with the sector-specific gross output deflator, software with the input deflator from EU-KLEMS. Variables are winsorized at 1% tails. Firm-clustered standard errors are given in parentheses. Revenue in logs, as it is proportional to a firm's product count in the model. Sector fixed effects are at the 2-digit level. U.S. regressions have 6,635 observations; French regressions have 106,865 observations.

fixed costs and personal computer intensity. I also find a significantly positive relationship when f_{it}^s is proxied with dummies for the adoption of specific technologies.[34] I conclude that there is a robustly positive relationship between observed intangible inputs and fixed costs, in line with the model.

3.4. Correlations between Fixed Costs, Research and Development, and Markups

To further explore the empirical validity of the model's mechanisms, I now assess the firm-level conditional correlations implied by the model's equilibrium (Proposition 3) in the micro data.

3.4.1. Research and Development

I first assess the relationship between fixed costs and innovative investments, measured through R&D intensity. R&D intensity is measured as the ratio of R&D over revenue, as is standard in the literature (e.g. Hall et al. 2010). In equation (A.8) of Appendix A I show that the firm's R&D intensity is a function of three terms. The first term captures the value of becoming the producer of an additional good, which is higher for firms with low intangible costs ϕ_i. Along the balanced growth path, the term is entirely captured by a firm fixed effect. The second term captures that innovation intensity falls in firm size. The final term is a time fixed effect. The firm's intangible costs ϕ_i are unobservable, but Proposition 3 shows that high-intangible firms have higher ratios of fixed costs over total costs. I therefore estimate

$$\ln\left(xrd_{it}/py_{it}\right) = \beta \ln\left(f_{it}/tc_{it}\right) + \gamma \ln\left(py_{it}\right) + \xi_t + \varepsilon_{it}, \tag{30}$$

where ξ_t are time fixed effects and ε_{it} are residuals, while revenue is included to control for firm size.

[34]Data on technology adoption is available for France. I use the *Enquête sur les Technologies de l'Information de la Communication* (TIC), which details a firm's installed IT systems from 2008 to 2016. It samples around 10,000 firms annually. Appendix F's Table A4 regresses (log) fixed costs over revenue on IT system adoption and finds a strong relationship.

Table 3: Relationship between Fixed-Cost Share and Research & Development

R&D intensity (log)	I	II	III	IV	V	VI
United States						
Fixed costs over total costs (log)	0.693	0.623	0.566	0.536	0.179	0.163
	(0.018)	(0.018)	(0.018)	(0.018)	(0.012)	(0.012)
R^2	0.175	0.231	0.306	0.320	0.028	0.043
France						
Fixed costs over total costs (log)	0.412	0.336	0.123	0.128	0.044	0.053
	(.017)	(0.017)	(0.015)	(0.015)	(0.024)	(0.025)
R^2	0.045	0.103	0.275	0.278	0.015	0.026
Year fixed effects	No	No	No	Yes	No	Yes
Firm fixed effects	No	No	No	No	Yes	Yes
Industry fixed effects	No	No	Yes	Yes	No	No
Revenue (product-count) control	No	Yes	Yes	Yes	Yes	Yes

Notes: Dependent variable is R&D intensity (log). Explanatory variable is fixed costs over total costs (log). Variables are winsorized at 1% tails. Firm-clustered standard errors are given in parentheses. Revenue in logs, as it is proportional to a firm's product count in the model. Sector fixed effects are at the 2-digit level. U.S. regressions have 58,246 observations; French regressions have 20,666 observations.

I measure xrd_{it} as R&D on the income statement in Compustat, which is feasible because R&D is expensed under U.S. accounting rules.[35] For France I obtain R&D from the *Enquête Communautaire sur L'Innovation* (CIS). The CIS is an innovation survey that was conducted in 1996 and 2000, and biannually since 2004. The variable for xrd_{it} that I take from this dataset is expenditures on innovation activities, including externally purchased R&D and expenditures on external knowledge or innovation-related capital expenditures. The dataset contains 20,666 firm-years across 12,879 firms.

Table 3 presents the results. Estimates in column (I) come from a univariate regression, which shows that firms with relatively high fixed costs indeed have higher R&D intensities than other firms. Controlling for firm size through revenue slightly lowers the coefficient, in line with the prediction of the model that both R&D intensity and fixed costs over total costs (through overhead $\tilde{f}_{it}$) decline in firm size. The estimates imply that a one percent increase in fixed over total costs raises R&D intensity by 0.3 to 0.6 percent, on average. This result is robust to industry- and year fixed effects.

The model predicts that, in the steady state, fixed costs are orthogonal to innovation rates after conditioning on firm effects. Columns V and VI show that this is not the case in the data, although the coefficients are significantly smaller than in the columns with fewer controls. As Table 3 uses the full sample, it is unlikely that the economy is in the steady state, explaining the positive estimates.[36]

3.4.2. Markups

I next relate fixed costs to markups. As shown in Proposition 3, fixed costs should correlate positively with markups. In a regression, two counteracting forces govern this relationship. To see this, note that

[35] Because R&D is expensed, it negatively affects profits. When calculating fixed costs, I do not include R&D in the definition of costs, thereby preventing a mechanically positive relationship between fixed costs and R&D.

[36] Results in Table 3 are robust to various alternative specifications, including the use of fixed costs over revenue as the explanatory variable. For the U.S. it is also feasible to use software investments from the CiTDB (Appendix Table A6). For France this is not feasible, because the R&D and software data comes from different surveys with insufficient overlap.

Table 4: Relationship between Fixed-Cost Share and Markups

	OLS		2SLS		Reduced Form	
Markup (log)	I	II	III	IV	V	VI
United States (CiTDB, 2010 to 2015)						
Fixed costs over total costs (log)	0.271	0.061	0.439	-2.423	0.043	0.002
	(0.009)	(0.007)	(0.026)	(9.585)	(0.003)	(0.002)
R^2 (First-stage F.)	0.504	0.073	(380.70)	(9.37)	0.074	0.002
France (EAE, 1994 to 2007)						
Fixed costs over total costs (log)	0.173	0.071	0.306	0.254	0.040	0.003
	(.001)	(0.001)	(0.007)	(0.036)	(0.001)	(0.000)
R^2 (First-stage F.)	0.457	0.449	(2391.6)	(106.35)	0.032	0.051
Year fixed effects	No	Yes	No	Yes	No	Yes
Firm fixed effects	No	Yes	No	Yes	No	Yes

Notes: Dependent variable is the markup (log). Explanatory variable is fixed costs over revenue (log). Variables are winsorized at 1% tails. Firm-clustered standard errors are given in parentheses. The instrument for the 2SLS regressions is the ratio of software spending over sales. U.S. regressions have 6,635 observations; French regressions have 106,795 observations.

product-level markups in the model are $\ln \mu_{ijt} = \ln \lambda_{ij} - \ln s_{ijt}$, where λ_{ij} denotes the innovation step size and where time subscripts were added to s_{ijt}. Given Cobb-Douglas demand, this equals

$$\ln \mu_{ijt} = -\ln s_{ijt} + \ln Y_t - \ln y_{ijt} - \ln w_t.$$

The first counteracting force is captured by the final term, as a higher s_{ijt} directly translates to lower fixed costs. This creates the positive simple correlation between markups and fixed costs. The second force operates through y_{ijt} and λ_{ij}: a greater step λ_{ij} raises markups and lowers fixed costs, because higher markups reduce demand. This creates a negative correlation between markups and fixed costs. Indeed, solving for λ_{ij} in terms of s_{ijt} using first-order condition (10), we can denote markups as

$$\ln \mu_{ijt} = \theta \ln s_{ijt} - \ln \phi_i + \ln Y_t - \ln w_t - \ln \theta.$$

It follows that the relation between fixed costs and markups should be positive in a regression without fixed effects. Conversely, given that fixed costs are higher when s_{ijt} is low, the relationship could be negative when controlling for firm fixed effects.[37]

To test these predictions, I run a firm-level regression of the (log) markup on the (log) ratio of fixed costs over total costs. Compared to the other regressions there is an added complication: fixed costs are derived from markups, so that measurement error in markups causes a positive correlation. I address this by instrumenting total fixed costs with the observable subset of fixed costs: software spending.[38]

Table 4 presents the results. As expected, OLS results in columns I and II show a strong positive correlation between markups and fixed costs, regardless of controls. When addressing bias through correlated measurement error in 2SLS results (columns III and IV), we see that the correlation is strongest

[37] Firm fixed effects orthogonalize fixed costs from ϕ_i up to the first order. In the model, the relationship between ϕ_i and s_{ijt} is non-linear, so that a linear regression of markups on fixed costs with firm effects may still yield a positive coefficient.

[38] The instrument also controls for measurement error in f_{it} caused by the firm-level overhead $\tilde{f}_{it}$. Controlling for $\tilde{f}_{it}$ by adding the log of revenue as an additional control, as in Table 3, does not affect the qualitative results in Table 4.

in the univariate regressions. Those estimates are economically significant: a one-percent increase in fixed costs raises markups by 0.3 to 0.4 percent. Consistent with the model, fixed effects reduce the relationship (France) or even reverse its direction (U.S.). The reduced-form regressions confirm that software-intensive firms have higher markups, but not when account is taken of fixed effects.

In summary, the model implies positive correlations between fixed costs and intangibles, markups and innovation, especially when no account is taken of firm fixed effects. The correlations in the micro data are broadly in line with these patterns. I next use the data to quantify the model.

4. Quantification

This section outlines how the model is quantified. I first discuss the calibration and structural estimation strategy, and then discuss the extent to which the model is able to replicate a set of targeted and untargeted moments along the original balanced growth path.

4.1. Calibration

In the baseline calibration all firms have equal intangible costs ϕ, which leaves nine parameters to be calibrated. Five parameters are structurally estimated, while four others are taken from the literature. I conduct the estimation separately for the U.S. and France, using the micro data from Section 3.

4.1.1. Externally Calibrated Parameters

The model is calibrated at an annual frequency. I calibrate the curvature of R&D for entrants (ψ^e) and incumbents (ψ^x) to 2. This is a key parameter because it determines the concavity of the return to R&D. If innovative activities concentrate among fewer firms, the fact that $\psi^x > 1$ implies that the average effect of these investments on growth is lower. The literature that studies the elasticity of R&D with respect to the user costs ($\epsilon_{x,w}$) of such activities finds elasticities around -1.0 for tax credit changes (see, e.g., Bloom et al. 2002 for a review).[39] The parameter ψ^x is related to $\epsilon_{x,w}$ along

$$\psi^x = -\frac{\epsilon_{x,w} - 1}{\epsilon_{x,w}},$$

and is therefore set to 2. The same value is used in Akcigit and Kerr (2018) and Acemoglu et al. (2018).

I calibrate the curvature parameter θ of fixed cost function (3) to match empirical estimates of the pass-through of marginal costs to markups. To see how these relate, note that the quality-step equation (8) and first-order conditions for markups (11) and intangibles (10) imply an equilibrium log markup

$$\ln \mu_{ijt} = \ln \lambda_{ij} - \tfrac{1}{\theta+1} \ln \left(\lambda_{ij} \tfrac{w_t}{Y_t} \theta \phi_i \right). \tag{31}$$

The elasticity of marginal costs with respect to wages is $(\theta+1)/(\theta+2)$, so that the elasticity of markups with respect to marginal costs at a given level of Y is $-(\theta+2)^{-1}$. I set θ to 2, which achieves a pass-

[39] A recent large-scale analysis from cross-country micro data by Appelt et al. (2020) finds an average cost elasticity of -0.66. The corresponding ψ^x is 2.53, which yields a greater reduction in productivity growth.

through of -25%. Empirical estimates of this elasticity vary. Amiti et al. (2019) find a pass-through of -35% in their main results. In robustness checks on the full sample they find values between -39% and -25%. For firms with fewer than 100 employees they find coefficients of -3%. Table A13 in Appendix F shows that the results are robust to $\theta = 0.86$, yielding a -35% pass-through.

As a further robustness check, I calibrate θ from the relationship between variable costs $vc_{ijt} = w_t s_{ijt} y_{ijt}$ and fixed costs. From the first-order condition for intangible inputs, it follows that

$$vc^*_{ijt} = (s^*_{ijt}, \phi_i) +_t \phi_i.$$

I run a firm-level regression of variable costs on fixed costs in Table A12 in Appendix F. Estimates of θ range from 0.86 to 1.34, similar to the θs derived from markup pass-through. Appendix Table A13 shows that the main results are robust to a calibration of θ based on the relationship between fixed and variable costs.

The distribution of the innovation step size λ_{ij} is Pareto, so that

$$H(\lambda) = \lambda^{-\frac{\overline{\lambda}}{\overline{\lambda}-1}},$$

where the shape parameter is a function of the average innovation step size $\overline{\lambda}$, which I structurally estimate along with the other parameters in the next section. The Pareto distribution ensures that the quality of innovations follows a power law, in line with empirical evidence (e.g. Harhoff et al. 2003, Kogan et al. 2017). It also delivers an exponential distribution for log markups, as in Peters (2020).

The discount rate ρ is set to 0.01, which gives rise to a 2.3% risk-free rate.

4.1.2. Structurally Estimated Parameters

The remaining five parameters are estimated using indirect inference by matching moments from either the U.S. Compustat data on listed firms or the French administrative data. The U.S. calibration targets moments around 1980, which is the first year that firm variables from Compustat can be complemented by administrative data on business dynamism. The French calibration targets moments in the first year of the data (1994), or the first available year for surveys.

The structural estimation chooses the set of parameters that satisfies the objective function:

$$\min \sum_{k=1}^{5} \frac{|\, \text{model}_k - \text{data}_k \,|}{(|\, \text{model}_k \,| + |\, \text{data}_k \,|)\, 0.5} \Omega_k, \tag{32}$$

where model_i and data_i respectively refer to the simulation and data for moment i with weight Ω_i. I solve the model as a fixed point along the algorithm described in Appendix E. Using the equilibrium values for innovation and entry rates, the firm-size distribution, rates of creative destruction and aggregate quantities such as the efficiency wedge, wages and output, I simulate the economy for 32,000 firms until the distribution of s_{ij} has converged, and simulate ten more years to collect moments.

The following moments are used for the U.S. calibration. I calibrate the intangible efficiency parameter ϕ to match the 1979 ratio of fixed to variable costs of 12.9% in Section 3. Appendix A.9 derives

Table 5: Overview of Parameters

Parameter	Description	Method	Value (U.S.)	Value (France)
ρ	Discount rate	External	.010	.010
θ	Intangibles cost elasticity	External	2.00	2.00
ψ^x	Cost elasticity of innovation (incumbents)	External	2.00	2.00
ψ^e	Cost elasticity of innovation (entrants)	External	2.00	2.00
η^x	Cost scalar of innovation (incumbents)	Indirect inference	3.41	1.73
η^e	Cost scalar of innovation (entrants)	Indirect inference	2.47	2.87
$\overline{\lambda}$	Average innovation step size	Indirect inference	.060	.064
σ	Relationship firm size and firm growth	Indirect inference	.519	.636
ϕ	Intangible costs	Indirect inference	.215	.279

that this ratio falls in ϕ. The cost scalar of R&D by entrants (η^e) is estimated by targeting the entry rate of 13.8% for 1980 in the Business Dynamics Statistics. The cost scalar of innovation by existing firms (η^x) is estimated by targeting the average ratio of R&D over sales for firms with positive expenditures in 1980, at 2.5%.[40] Following Akcigit and Kerr (2018), I calibrate the parameter that governs the extent to which R&D scales with size (σ) by targeting an OLS regression of size on growth along

$$\Delta_i\left(py\right) = \alpha_s + \beta \ln\left(p_i y_i\right) + \varepsilon_i, \tag{33}$$

where the left-hand side is the growth rate of sales using the measure of growth in Davis et al. (2006), while α_s is a sector fixed effect. Akcigit and Kerr (2018) estimate (33) on Census data and find β=-0.035, which implies that a firm with 1% greater sales is expected to grow 0.035% less. I estimate average innovation step size $\overline{\lambda}$ by targeting productivity growth along the balanced growth path of 1.3%, which equals average growth of total factor productivity between 1969 and 1980 in the Fernald series.[41]

The calibration for France relies on French counterparts of the U.S. moments. Intangible cost parameter ϕ is calibrated to match the 1994 ratio of fixed to variable costs of 9.5% in Section 3. The cost scalar of research and development by entrants (η^e) is estimated by targeting an entry rate of 10%. This is the fraction of firms that enter the FARE-FICUS dataset for the first time in 1995, the second year for which data is available and therefore the first year that entry is observed. The cost scalar of innovation by existing firms (η^x) is estimated by targeting the average ratio of R&D over sales in the CIS for 1996, which is 3.1%. I calibrate σ to match the β in (33) using the French micro data for 1994-1995. The estimated β is -0.035, coincidentally the same as for the U.S. I target a productivity growth rate of 1.3%, which is the average TFP growth rate between 1969 and 1994 in the Penn World Tables.

Table 5 presents an overview of the calibrated and estimated parameters. The lower R&D intensity of U.S. firms gives rise to a higher innovation-cost scalar η^x, while the higher ratio of fixed- to variable costs of U.S. firms causes their baseline estimated intangible costs ϕ to be lower than that of the French firms. The estimated innovation-step size is similar for both countries.

[40] This value is in line with the literature and similar to the value in Akcigit and Kerr (2018) and Cavenaile et al. (2020).

[41] I measure productivity growth with Fernald's utilization-adjusted series. This series is closest the model because it adjusts for temporary fluctuations in the utilization of labor and capital, as such fluctuations do not occur in the model.

Table 6: Comparison of Theory and Data for Targeted Moments

Parameter	Moment	Weight Ω	United States Model	United States Target	France Model	France Target
$\bar{\lambda}$	Long-term growth rate of productivity	1	1.3%	1.3%	1.3%	1.3%
ϕ	Fixed costs as a fraction of total costs	2	12.9%	12.9%	9.5%	9.5%
σ	Relation between firm growth and size	1	-.035	-.035	-.035	-.035
η^e	Entry rate (fraction of firms age 1 or less)	1	13.1%	13.8%	8.5%	10%
η^x	Ratio of research and development to sales	1	2.5%	2.5%	2.7%	3.2%

4.2. Model Properties

A comparison of theoretical and empirical targeted moments is provided in Table 6. The first column lists the parameter that corresponds most closely to the moment, the second column describes the moment, and the third column summarizes the moment's weight in the structural estimation. All moments receive the same weight except the share of fixed costs, which is assigned a weight of two. The model is able to match moments on growth, fixed costs, and the relationship between firm growth and firm size precisely for both countries. R&D intensities are also well-matched, while the estimated model underestimates French entry by 1.5 percentage points and U.S. entry by 0.7 percentage points.

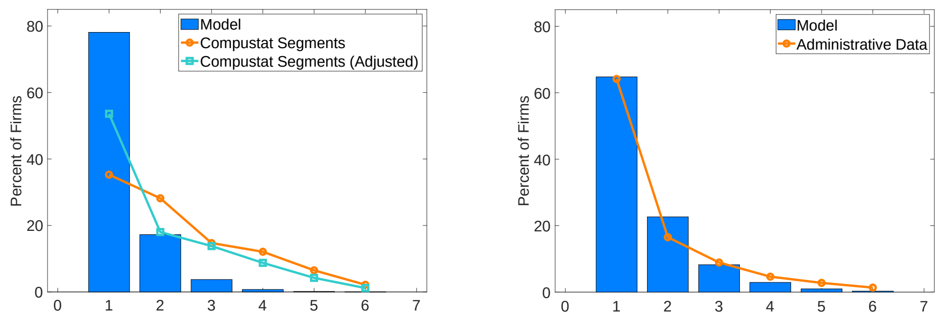
The firm-size distribution is untargeted. The Cobb-Douglas aggregator implies that a firm's revenue is determined by the number of goods that it produces, which is plotted against data in Figure 3. I rely on the Compustat Segments data for the U.S. to count the number of NAICS industries in which firms operate (Figure 3a).[42] This is the orange-circled line. Results show that U.S. listed firms operate in more sectors than the model predicts. Note that the Compustat segments are an imperfect measure of the number of products that firms produce because firms apply heterogeneous reporting standards on what a segment is. Further, 29.5% of firms do not report their segments at all. The green-squared line plots an alternative distribution of the product count, setting the number of products to one for non-reporting firms. This brings the distribution closer to what is predicted. The difference between the fraction of firms with 2 and 3 (and 3 and 4) products is also accurately predicted. Figure 3b plots the same for France. Data comes from the *Enquête Annuelle de Production dans L'Industrie* (EAP). This dataset is available only for firms in manufacturing and contains identifiers for each product that the firm sells.[43] The figure shows that the distribution of the number of product count is closely matched.

Markups are also untargeted. The model predicts average markups of 1.41 for the U.S. calibration and 1.29 for the French calibration. This overstates the actual markups at the beginning of the sample, which average 1.27 and 1.17 in the U.S. and French data, respectively.[44]

Table A7 in Appendix F presents a set of additional untargeted moments. The first panel analyzes the relationship between size and age. Size is measured as sector-deflated sales, while age is measured as years since creation in France, and as years since entry into Compustat for the U.S. Both are transformed to within-year quartiles indexed from 1 to 4.[45] For the U.S., the model accurately predicts that

[42] The first year with NAICS segment codes is 1990, which is plotted here. Details are provided in Data Appendix B.
[43] The first year of the survey is 2009, which is plotted here. Details are provided in Data Appendix B.
[44] Appendix G presents an alternative calibration that targets markups, and shows that results are qualitatively robust.
[45] E.g. the first entry implies that firms in age quartile 1 have a 1.16 average score on a 1-4 scale of the size quartiles.

Figure 3. Number of Products by Firm: Theory and Data



(a) United States (b) France

Notes: U.S. data are taken from the Compustat Segments file and count the number of primary NAICS codes that firms report to operate in during 1990. Adjusted segments data assign a segment count of 1 for firms that are not included in the segments file. French data are taken from the *Enquête Annuelle de Production dans L'Industrie* (manufacturing only, 2009).

small firms are more likely to exit and less likely to stop producing a product, but cannot explain the relationship between exit and age. This could be because U.S. exits are calculated in Compustat, which may be for other reasons than firm closure. The model correctly predicts that young firms are smaller than older firms, as they have had less time to accumulate patents. The model also matches for France that young and small firms are more likely to exit and less likely to stop producing a product.

Tables A8 to A11 additionally compare regressions of simulated data between the model and the data. Table A8 estimates the elasticity of fixed-over-total costs with respect to fixed costs; Table A9 estimates the relationship between fixed costs and firm size. Both are well-matched. Table A10 and Table A11 present estimations that relate fixed costs to markups and R&D as in Section 3. These are not well-matched because all firms have equal intangible efficiencies, but improve in the next section.

5. Quantitative Analysis

This section contains the main exercise: a quantitative analysis of the effect of a rise of intangibles on productivity growth, business dynamism and markups. Section 5.1 first outlines how high-intangible firms are introduced, while Section 5.2 analyzes how they change the balanced growth path. The transition path from the old to the new steady state is discussed in Sections 5.3 to 5.5.

5.1. Introducing Heterogeneous Intangible Efficiency

To model the rise of intangibles, I introduce a group of firms with lower intangible costs than the homogeneous intangible cost in the initial calibration of Section 4.[46] Two parameters characterize the introduction of these firms: their cost parameter, $\overline{\phi}$, and the fraction of entrants that receive it, $G(\overline{\phi})$. To calibrate $G(\overline{\phi})$, I target the decline in entry. Entry depends on the share of firms with a higher intangible

[46]The model is able to analyse the effect of any finite combination of intangible efficiencies. Computational complexity is exponential in the number of different levels of ϕ, however, which is why this calibration sticks to two types.

efficiency because the latter determines what fraction of entrants benefit from the rise of intangibles. For low levels of $G(\bar{\phi})$ there is little chance that an entrant is highly efficient at intangibles. Because high-intangible firms expand strongly, however, entrants are likely to face a high-intangible incumbent when they attempt to enter. This raises effective entry costs and lowers the incentive to enter.[47] I calibrate $\overline{\phi}$ by targeting the rise in the average ratio of fixed costs over total costs using the estimates from Section 3.4. For the initial calibration, it is straightforward to show that fixed costs over total costs rise when the homogeneous ϕ is calibrated to a lower level. Showing that the relationship between fixed costs and $\overline{\phi}$ is similarly monotonic is not feasible analytically, as $\overline{\phi}$ affects the new equilibrium's entry rates, research labor and markup dispersion. A simple comparative static in Appendix Figure A10, which plots $\overline{\phi}$ against average fixed costs over total costs, confirms the existence of a monotonic relationship between fixed costs and $\overline{\phi}$ in the model's calibration.[48]

In the U.S. calibration, 8.4% of all new firms benefit from a 26% reduction in intangible costs. In the French calibration, I assign the low intangible costs to 5.4% of all new entrants, and set $\overline{\phi}$ 17% lower than that of other firms.[49]

I analyse two experiments on the introduction of these high-intangible firms. In the main experiment, I start with an economy where the share of incumbents with $\overline{\phi}$ is zero. That is, the rise of high-intangible firms is entirely driven by firms that were not initially operative. This experiment aligns with the observation that the rise of IT-intensity in the 1990s was concentrated in young firms and that the decline of dynamism occurred later for these firms (Haltiwanger et al. 2014). In the alternative experiment, I smooth the introduction of high-intangible firms by gradually raising $G(\overline{\phi})$ from its initial value to its final value over the first 15 years of the transition. I further allow a fraction $G(\overline{\phi})$ of incumbents in the initial balanced growth path to see the gradual improvement in their intangible costs from the original, homogeneous, level of efficiency to the lower cost $\overline{\phi}$. The latter assumes that salient differences in intangible costs across firms always existed, but that changes in the availability of technology have made these differences relevant. This experiment aligns with the finding that older firms contributed to the speedup and slowdown in productivity growth since the 1990s in Klenow and Li (2020). Besides their difference in narrative, the main difference between both experiments is that the path of entry is more consistent with the data if high-intangible entrants and incumbents are gradually introduced.

5.2. Balanced Growth Path

The effect of introducing high-intangible firms is summarized in Table 7, which presents the variables of interest in differences from the original balanced growth path. Two changes are targeted: the increase in fixed costs as a percentage of total costs, and the entry rate. The rise of fixed costs is well

[47] For the U.S., I target the decline in entry in the Business Dynamics Statistics between 1980 and 2016. For France, I impute the decline in entry from the decline in the employment share by entrants in FICUS-FARE, from 1994 to 2016.

[48] To structurally estimate the new parameters, I minimize a loss function similar to (32) with the rise of fixed costs and the decline in entry as moments. To assure that the model does not overshoot the rise in fixed costs, an additional penalty is applied to calibrations where the predicted rise of fixed costs exceeds the empirical rise in fixed costs.

[49] The tables with additional untargeted moments that relate fixed costs to R&D (A10) and markups (A11) show a better match for the final steady state because there is heterogeneity in θ. Fixed effects regressions are still not well-matched, which may be because simulations rely on steady state data. Tables A8 and Table A9 are well-matched across specifications.

Table 7: Balanced Growth Path Change due to Reduction in Intangible Costs of Top Firms

		United States		France	
	Targeted	**Δ Model**	**Δ Data**	**Δ Model**	**Δ Data**
Cost Structure					
Average Fixed-Cost Share	Yes	10.6 pp	10.5 pp	4.5 pp	4.5 pp
Slowdown of Productivity Growth					
Productivity Growth Rate	No	-0.3 pp	-0.9 pp	-0.1 pp	-1.3 pp
Aggregate R&D over Value Added	No	34.8%	64.5%	22.1%	5.6%
Decline of Business Dynamism					
Entry rate	Yes	-4.5 pp	-5.8 pp	-1.0 pp	-3.8 pp
Reallocation Rate	No	-35.9%	-23%	-17.0%	-23%
Rise of Market Power					
Average Markup	No	14.7 pt	27 pt	6.4pt	11 pt
Model Wedges					
Labor Wedge	No	6.7 pt	N.A.	3.6 pt	N.A.
Efficiency Wedge	No	.04 pt	N.A.	.04 pt	N.A.

Notes: Data columns present the empirical moments, while model columns present the theoretical moments. The change in productivity growth is the difference between growth from 1969-1979 (U.S.) or 1969-1994 (France) to growth post-2005. Other U.S. moments equal the difference between 1980 and 2016. Other French moments equal the difference between 1994 and 2016.

matched, while the decline in entry is underestimated in both calibrations. The remainder of Table 7 presents results for untargeted objects. These include the slowdown of productivity growth, the decline in business dynamism and the rise of markups. In the U.S. calibration, the model is able to explain about one-half of the rise of markups and one-third of the slowdown of productivity growth. In the French calibration, the model is able to explain most of the decline in the reallocation rate and just over one-half of the rise of markups. The model predicts a 0.1 percentage-point decline in productivity growth. It therefore seems that intangibles are not responsible for most of the slowdown of growth in France. The model overestimates the decline in the reallocation rate in the United States, because all growth in the model occurs through creative destruction.[50]

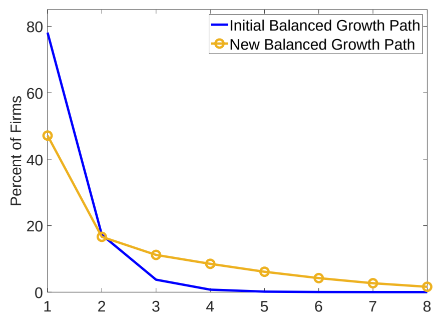
The bottom of Table 7 presents the changes in the labor and efficiency wedges. The labor wedge measures the difference between wages and marginal product, and grew by 6.7 and 3.6 in the U.S. and French calibrations, respectively, due to the rise of markups. The efficiency wedge measures efficiency loss from markup heterogeneity, which increases modestly because low-ϕ firms have higher markups.

The model predicts a decline in productivity growth despite an *increase* in aggregate research and development, in line with the data in France and the United States.[51] In a model with homogeneous firms this would be paradoxical, because there is a direct relationship between aggregate R&D and growth. Higher investments and lower growth co-exist in this model because innovation activity is con-

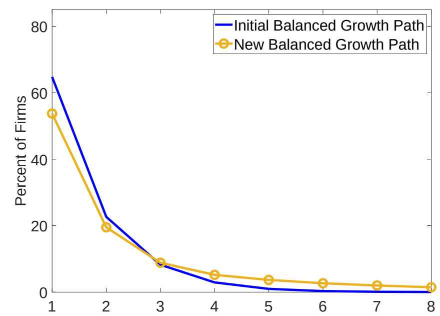
[50]An empirically relevant additional source of innovation is the improvement of goods that firms already produce (e.g. Garcia-Macia et al. 2019, Akcigit and Kerr 2018). In the context of the model, internal innovation would be affected similarly by the rise of intangibles, as high-intangible firms have lower discount rates. In a model like Peters (2020), however, internal innovation primarily raises a firm's market power, hence exacerbating the rise of markups and the decline in wages.

[51]The French increase in Table 7 is measured over 1994-2016, while the U.S. increase is over 1980-2016. France experienced a 49.4% increase in R&D over national income between 1980-2016, which is closer to what the model predicts.

Figure 4. Number of Products before and after a Reduction in Intangible Costs for Top Firms



(a) United States



(b) France

Notes: Lines plot the fraction of firms that produce the number of products on the horizontal axis. Solid lines are from the original calibration. Squared lines present the counterpart for the balanced growth path after the introduction of high-intangible firms.

centrated in a smaller group of high-intangible firms, and because some innovations by low-intangible entrants and incumbents fail to enter the market.

The increase in firm concentration is illustrated in Figure 4, which plots the distribution of firms over the number of products that they produce. This is the most direct measure of concentration in the model. The original balanced growth path is characterized by a lower concentration, featuring more firms that produce one or two goods than is the case in the new balanced growth path. Conversely, the right tail of the firm-size distribution is fatter, indicating that there are more large firms. Note that the increase in concentration is endogenous: high-intangible firms have higher markups and therefore have more incentives to invest in research and development. This causes them to produce a disproportionate fraction of all goods and to grow larger than other firms.

5.3. Transition Path

The analysis thus far has studied the effect of a rise in intangibles along the balanced growth path. This section shows that short-term dynamics are substantially different. To quantify the transition path, I numerically solve for the path of productivity, markups and wages.[52] This section presents the results from the experiment in which none of the incumbents are assigned a lower intangible cost parameter. The alternative experiment, together with a comparison with the data, are provided in Section 5.5.

The path of productivity growth is presented in Figure 5. Figure 5a presents results for the U.S. calibration, Figure 5b for the French calibration.[53] The solid blue line plots the path of growth in total factor productivity as defined in (24). The yellow dash-dotted line plots the increase in productivity due to the step-wise improvement of quality, which is the source of long-term growth.

When low-ϕ firms start entering the economy in year 0, there is initially a jump in productivity growth compared to the original steady state (the black upper-dashed line). This is partly because of

[52] The computational algorithm is described in Appendix E.

[53] Figures in the remainder of this section only plot results for the U.S. calibration because the results are qualitatively similar in both calibrations. Full French results are provided in Appendix F, Figure A11.

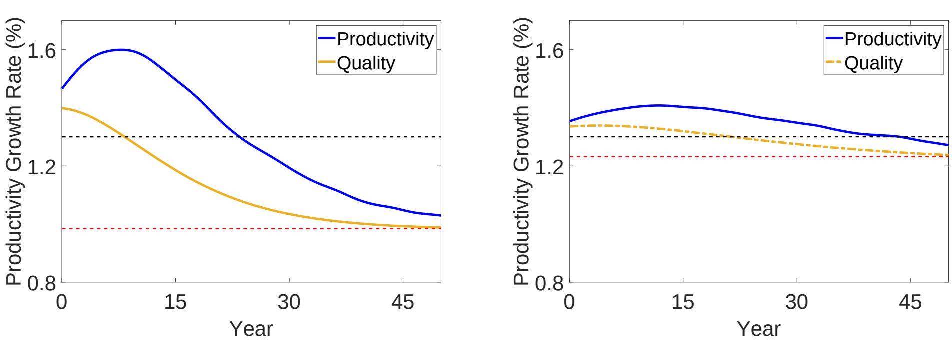
a rise in entry, driven by the fact that new firms now have a positive probability of being the profitable low-ϕ type, while the high-ϕ entrants do not face low-ϕ incumbents yet (Figure 6a). Entry jumps above its initial steady state value as the figures plot the effect of an immediate, permanent shock to $G(\phi)$.[54]

As the low-cost firms enter the economy there is a further increase in productivity because they reduce the marginal costs of any good that they produce through the use of intangibles. This causes productivity growth to exceed the growth rate of quality. At peak growth, eight years after the introduction of low-ϕ entrants, this boosts growth to a level of up to 1.6%. The transitional boom evolves more slowly in France, because a smaller fraction of start-ups benefit from the lower intangible costs (5.4% for France versus 8.4% for the U.S.). The extraordinary growth is predominantly driven by cost reductions from intangibles, consistent with the finding that above-average productivity growth from the mid-1990s to the mid-2000s was primarily caused by IT (Fernald 2015).

A slowdown occurs from year 8 onwards in the U.S. calibration. Entry declines because low-ϕ incumbents produce an increasingly large share of all products. The probability that an entrant benefits from drawing a low ϕ therefore falls below the probability that it faces a low-ϕ incumbent, which increases the likelihood of a failed innovation.

The decline in productivity growth is mirrored by an *increase* in the average ratio of R&D over sales, also known as R&D intensity (Figure 6b). The increase is large: average R&D intensity increases from 2.5 to 7.1%. This is quantitatively very similar to the data. Among U.S. public firms with positive R&D, the average R&D intensity increased from 2.5 (the calibration target) to 8.7%.[55] It aligns with the result that 'ideas are getting harder to find' in Bloom et al. (2020), who argue that the effect of innovative investments on growth has diminished. The model offers a potential explanation for their result. As high-intangible firms have higher markups, they have a greater incentive to innovate. Because the returns to R&D are concave, these additional investments have limited effects on growth but increase

Figure 5. Transition: Growth Rate of Total Factor Productivity



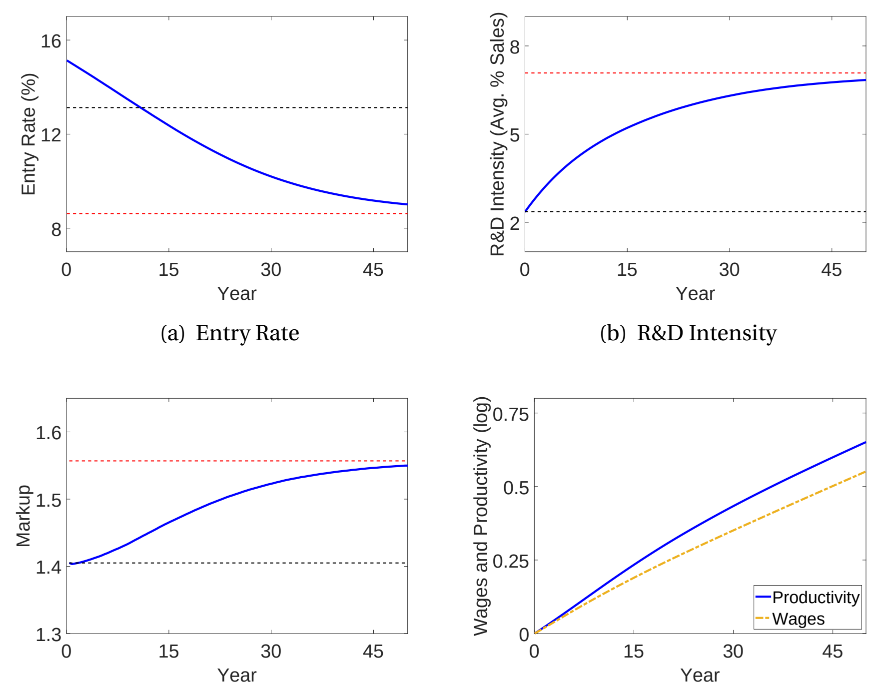
(a) United States (b) France

Notes: Black- and red dashed lines (respectively) indicate the original and the new steady state.

[54]This is an abstraction, as the true change in the composition of intangible costs was likely more gradual. It may also be the case that some incumbents have seen a reduction in their intangible efficiency over time. In Section 5.5, I show that entry is in line with the data when entrants and incumbents with low intangible cost parameters are gradually introduced.

[55]R&D intensity among all public firms increased from 2.0 to 6.7%, again similar to the increase in the model. French R&D expenditure over sales increased from 3.1% among positive spenders (the calibration target) to 4.0%.

Figure 6. Transition Path for Entry, R&D, Markups, Wages



(a) Entry Rate (b) R&D Intensity

(c) Markups (d) Wages and Productivity

Notes: Black- and red dashed lines (respectively) in (a) to (c) indicate the original and the new steady state. Figure (a) presents the entry rate, (b) the average ratio of R&D to sales, (c) the average markup, (d) the path of wages (dashed yellow) and productivity (solid blue).

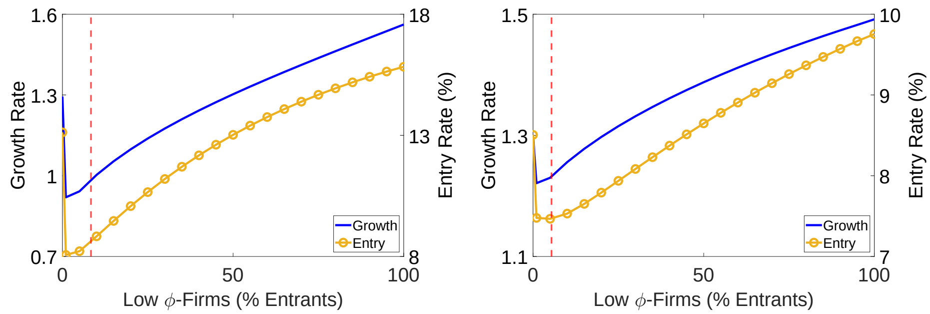
average R&D intensity considerably, causing the decline in research effectiveness. The presence of low-ϕ incumbents further means that a fraction of the innovations fail to be introduced to the market, again diminishing the effect of research on growth. Section 5.6 elaborates on these inefficiencies.

The model also sheds light on why wages have not kept up with productivity growth in the past 20 years, which has caused a decline in the labor share (Kehrig and Vincent 2021). While the reallocation of economic activity to lower-ϕ firms leads to a reduction of marginal costs and an increase in productivity, there is no increase in wages because productivity is offset by higher markups (Figure 6c). Note that markups increase because activity reallocates towards high-markup firms, in line with empirical evidence (e.g. Baqaee and Farhi 2020, Autor et al. 2020). This leads to a decoupling of wages and productivity (Figure 6d). Wages continue to grow at the rate of quality improvements, but do not benefit from the transitory increase in productivity growth from intangible adoption.

5.4. Results: Welfare

The welfare effect of the rise of intangibles is given by the change in the discounted sum of log consumption. Two counteracting effects are at play. The initial boom in growth raises the level of productivity, which is positive for welfare. The subsequent slowdown of productivity growth lowers output, which reduces welfare. The permanent rise of R&D worsens this negative effect, because a smaller fraction of the labor force is dedicated to the production of consumption goods.

Figure 7. Balanced Growth Path Effects of an Increase in Intangible Efficiency for Top Firms



(a) United States (b) France

Notes: Balanced growth path growth- and entry rates for various levels of $G(\overline{\phi})$. Figure 7a plots the U.S. calibration, in which $\overline{\phi}$ is below the ϕ of other firms by 33%. Figure 7b plots results for the French calibration, in which $\overline{\phi}$ is 28% lower than the ϕ of other firms. Yellow-squared lines present $G(\overline{\phi})$ in the calibration of Table 7. The lowest $G(\overline{\phi}) > 0$ plotted is 1%.

The model predicts that utility falls by 10.4% in the U.S. calibration and by 1.06% in the French calibration. The decline is modest because consumers place greater weight on current consumption, which is boosted by the initial spike in productivity growth. The welfare effect is determined by the fraction of firms that have access to the lower intangible costs, $G(\overline{\phi})$. The effect of $G(\overline{\phi})$ on growth and entry is illustrated in Figure 7. At $G(\overline{\phi}) = 0$, the economy is in the original steady state. As the share of entrants with low-intangible costs becomes positive there is a substantial decline in growth and entry. This is because the smaller $G(\overline{\phi}) > 0$, the greater the increase in variance and the smaller the reduction of the expected intangible costs. If all firms see a decrease in ϕ, then average markups would increase, as would the incentive to innovate. A sufficiently homogeneous decrease in intangible costs therefore raises entry and growth above the old steady-state level.[56] Conversely, a mean-preserving spread of ϕ has a negative effect on growth because it reduces incentives to enter. Any change in technology that improves the diffusion of intangibles therefore positively affects entry, growth and welfare.

Changes in welfare under alternative calibrations for $G(\overline{\phi})$ are summarized in Table 8. In the main exercise, 8% (5%) of U.S. (French) firms receive the low cost parameter. The greater the fraction of entrants that receive a lower intangible cost parameter, the higher the welfare. The potential gain is substantial: if $G(\overline{\phi})$ is increased to 50%, both calibrations display an *increase* in welfare of over 15%.

Table 8: Welfare Change at Various Levels of Intangible Adoption

	United States				France			
Fraction of low-intangible cost firms ($G(\overline{\phi})$):	0.08	0.10	0.25	0.50	0.05	0.10	0.25	0.50
Δ Welfare	-10.4%	-9.13%	1.93%	16.1%	-1.06%	0.61%	6.96%	15.6%

Notes: Percentage change from original balanced growth path. $G(\overline{\phi}) = 0.08$ for the U.S. and $G(\overline{\phi}) = 0.05$ for France in the main analysis.

[56] In Figure 7 this happens when around 45% of entrants receive the lower costs in both calibrations. Note that this is an exaggeration because the figure does not correct for the fact that the increase in the model's steady-state intangible share would exceed the empirical increase when a larger fraction of entrants receive $\overline{\phi}$.

5.5. Alternative Experiment: Smooth Introduction of Low Intangible Costs

The previous section plotted a transition where the fraction of entrants that receive the low intangible cost parameter $\overline{\phi}$ jumps overnight, and where all incumbents retain their original intangible costs. I now analyse the transition where, instead, the rise in $G(\overline{\phi})$ is gradually introduced over 15 years, while the same fraction of incumbents receive the lower $\overline{\phi}$. This experiment represents, for example, the case in which heterogeneous intangible costs are a salient feature of firms, which becomes relevant when gradual technological advancement enables the use of intangible inputs to reduce marginal costs.

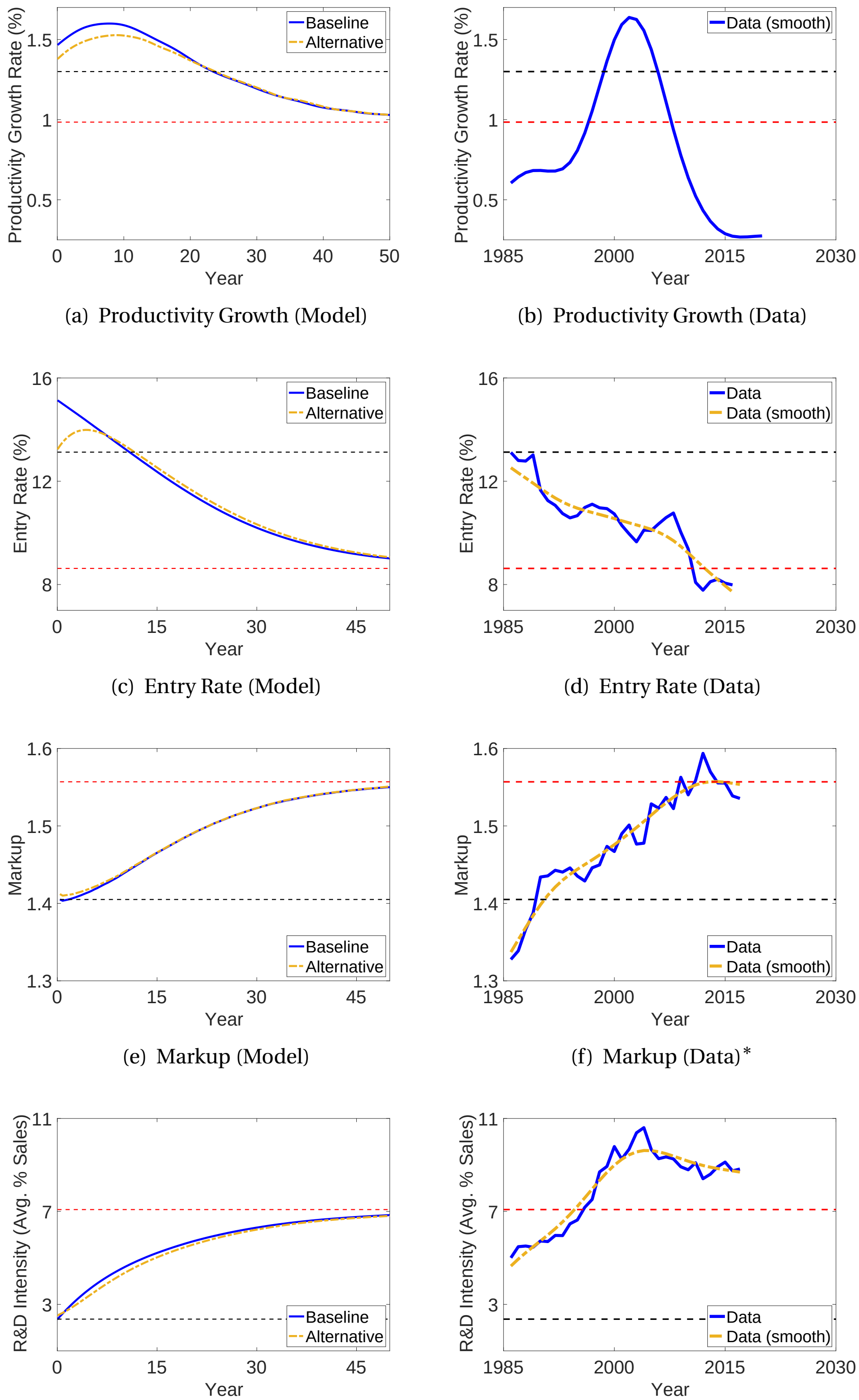
Results for the U.S. calibration are presented in the left-hand plots of Figure 8, which combine plots from the previous section (solid blue lines) and the alternative experiment (dashed yellow lines). Figure 8a plots productivity growth. When firms with low intangible costs are gradually introduced among entrants and incumbents, the initial increase in growth is slightly smaller. This is primarily due to the behavior of the entry rate, which no longer jumps in the first year (Figure 8c). Instead, entry increases by a modest 0.7 percentage points over the first three years of the transition, consistent with occasional increases in entry in the data (Figure 8d). After ten years, the behavior of entry and productivity in both transitions is similar. The rise of markups and R&D intensity are nearly identical in both transitions. Overall, the transition is qualitatively similar with the gradual introduction of low-intangible cost firms. Because the initial boom in productivity is smaller, the alternative transition does raise the fall in welfare: welfare drops by 11.4%, compared to 10.4% in the baseline transition.

The figures on the right-hand side of Figure 8 plot empirical counterparts to the transition path. The horizontal axes start in 1985 when business dynamism starts trending, and span 45 years to match the theoretical plots. The model is largely able to explain the qualitative features of the data. The model predicts that it takes approximately 45 years for entry rates and markups to converge to the new steady state, while convergence in the data takes 30 years. By that time, the model series have approached levels that are close to their new steady states. The transition duration therefore matches the data.

Productivity growth in the data contains the initial boom and subsequent decline, although its timing differs from the model. The boom occurs between 1995 and 2002, while the model predicts a boom right after high-intangible firms are introduced. The paths of variables in the model also appear more muted than the data, in line with the steady state results. Note, however, that no part of the transition path is targeted. The magnitude of the boom, with growth spiking at 1.6% in the model and 1.7% in the smoothed data, is similar. The boom also lasts for eight years in both series. The model is therefore capable of replicating most qualitative features of the path of productivity growth. Its predictions for entry are closer to the data when a high-intangible firms are gradually introduced, though the paths of R&D and markups are in line with the data in either transition.

Figure A12 in Appendix F presents the French transition path, which is qualitatively similar. An empirical comparison is complicated by the fact that data on entry and business dynamism is available only from 1994, so that the effects of intangibles are likely to predate the figures. The ability of the model to fit the time path of growth is worse than for the U.S., furthermore, because productivity growth in France was negative for most years after 2005. The model replicates the duration of the transitions of entry and markups well, though it only explains a modest fraction of the trends for France.

Figure 8. Transition Path: Model Predictions versus Data (Untargeted)



(a) Productivity Growth (Model)
(b) Productivity Growth (Data)
(c) Entry Rate (Model)
(d) Entry Rate (Data)
(e) Markup (Model)
(f) Markup (Data)*
(g) R&D Intensity (Model)
(h) R&D Intensity (Data)

Notes: Black- and red dashed lines (respectively) indicate the original and the new steady state. U.S. calibration. Productivity growth in Figure 8b only includes the smoothed series, as the raw series is highly volatile. HP-filter smoothing parameter is 100. Data sources: productivity growth from Fernald (FRBSF), R&D from Compustat, entry from the BDS, markups from Compustat.

5.6. Innovation Inefficiencies and Policy

I now take a closer look at the drivers behind the slowdown of productivity growth. The model simultaneously predicts lower productivity growth and higher aggregate R&D expenditures in the final steady state. It is able to do so because the efficiency of R&D declines endogenously.[57] This section describes two new sources of inefficiency that arise when firms have heterogeneous intangible costs, and discusses implications for innovation policy that seeks to remedy them.[58] I first outline these sources and then measure their relative importance.

5.6.1. Sources of Inefficiency

The first inefficiency is that a fraction of innovations are not implemented, because innovators may be undercut by a high-intangible incumbent. I refer to this as the lost innovation channel. The second is that firms of different intangible efficiencies innovate at different rates. Because $\psi^x > 1$, the model features diminishing returns to innovation at the firm level. Differences in innovation rates therefore reduce the average output of researchers. High-intangible firms have higher profits, lower discount rates and lower choke prices than low-intangible firms, and therefore invest more in R&D. As the heterogeneous innovation rates are unrelated to a firm's innovation efficiency, low-intangible firms innovate inefficiently little compared to high-intangible firms. I refer to this as research misallocation.

5.6.2. Measuring Inefficiency

I quantify both inefficiency channels by calculating the maximum amount of growth that the number of researchers in the final steady state would generate if the inefficiencies were removed. I abstract from the link between a firm's growth and its own innovation, and instead assume that the collective innovation raises the quality of random goods, causing aggregate productivity to grow along (24).

I conduct five experiments. Experiment 1 quantifies the lost innovation channel. It assumes that each innovation successfully raises the quality of a good, but holds innovation rates across entrants and incumbents (of all sizes) constant. The resulting growth rate is the product of the average innovation step size $\overline{\lambda}$ and the sum of innovation rates from incumbents and entrants.

Experiment 2 additionally reallocates researchers in a way that maximizes their innovation output for a given total number of researchers $\overline{L}^{rd}$ that are employed by incumbents. This quantifies the part of the research misallocation channel that is driven by incumbents. I first calculate $\overline{L}^{rd}$, and then allocate them efficiently across firms. The efficient allocation is so that the marginal research output is equal across researchers of differently-sized firms. Note that a firm's intangible efficiency does not affect the number of researchers that it should employ, as research productivity does not depend on a firm's ϕ_i. The experiment takes the firm-size distribution as given. Experiment 3 extends experi-

[57] Lehr (2021) provides empirical evidence of an increase in misallocation of researchers across firms in the United States.

[58] The model also features the common sources of innovation inefficiency of models with creative destruction: firms discount the value of their innovations at higher rates than the social planner, causing underinvestment in research. The new channels capture inefficiency in the performance of research, rather than in the aggregate level of research spending.

Table 9: Allocative Efficiency of Researchers: Productivity Growth by Allocation

	United States			France		
	Max.	Actual	Improve (%)	Max.	Actual	Improve (%)
Initial steady state						
1. Keep unsuccessful innovations	1.29	1.29	0.0	1.30	1.30	0.0
2. Reallocate incumbent R&D	1.29	1.29	0.0	1.30	1.30	0.0
3. Reallocate incumbent and entrant R&D	1.29	1.29	0.0	1.30	1.30	0.0
4. Real. incumbent R&D, optimal firm size	1.33	1.29	3.1	1.37	1.30	5.4
5. Real. incumbent and entrant R&D, opt. firm size	1.33	1.29	3.1	1.38	1.30	6.2
Final steady state						
1. Keep unsuccessful innovations	1.17	0.98	19.4	1.29	1.23	4.9
2. Reallocate incumbent R&D	1.23	0.98	25.5	1.35	1.23	9.8
3. Reallocate incumbent and entrant R&D	1.43	0.98	45.9	1.38	1.23	12.2
4. Real. incumbent R&D, optimal firm size	1.44	0.98	46.9	1.52	1.23	23.6
5. Real. incumbent and entrant R&D, opt. firm size	1.57	0.98	60.2	1.53	1.23	24.4

Notes: Columns headed 'Max.' present maximum productivity growth from reallocating R&D as described in the row. 'Actual' presents actual steady-state growth. 'Improve (%)' presents maximum growth as a percentage improvement of actual growth, which measures the efficiency of the actual steady-state allocation of R&D. All rows keep unsuccessful innovation; they assume that the probability that an innovation is successful is 1. Rows 'Reallocate incumbent R&D' calculate growth when researchers are reallocated across incumbents, holding the firm-size distribution constant. Rows 'Reallocate incumbent and entrant R&D' also reallocate researchers dedicated to entry. Rows with 'optimal firm size' assume that all firms produce a single product.

ment 2 by additionally reallocating the researchers $\overline{L}^e$ that are employed by potential entrants. This experiment measures the full extent of the research misallocation channel.

In experiments 4 and 5 I repeat experiments 2 and 3, respectively, but alleviate the constraint that the measure of firms M_n is equal to its steady-state value. Instead, I assume that all researchers are employed by firms with a single product, and that the measure of these firms is one. Because research productivity does not fully scale with firm size, this quantifies the reduction in innovations driven by diminishing returns to scale in research. These experiments offer a benchmark of maximum innovation capacity in an economy where a fraction $\overline{L}^{rd}$ (or $\overline{L}^{rd} + \overline{L}^e$) of workers are researchers.

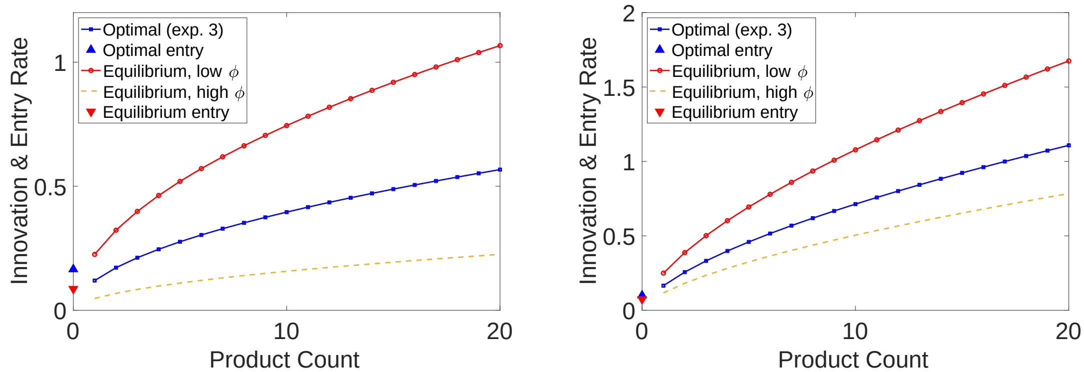
The optimal allocations of researchers and the resulting growth rates are derived in Appendix H.

5.6.3. Results

The growth rate of productivity under each of the five experiments is presented in Table 9. The top panel presents growth rates for the initial calibration in which intangible cost parameters are homogeneous; the bottom panel reports results for the final calibration in which some entrants have higher intangible efficiency. Columns 'Max' contain the growth rate $\tilde{g}$ belonging to each of the experiments. Columns 'Actual' contain the equilibrium growth rate in that calibration. Columns 'Improve (%)' give the percentage by which productivity growth would improve under the experiment's allocation.

The upper panel confirms that the allocation of innovation resources is efficient under homogeneous intangible costs. Firms have equal choke prices, so that all innovations are successful. The marginal research product is also equalized across entrants and incumbents of all sizes, because firms have equal expected profits and creative destruction rates. There is a modest loss from the diminishing

Figure 9. Entry Rates and Innovation Rates by Firm Size: Actual versus Optimal



(a) United States (b) France

Notes: Figure plots innovation rates $x_n(\phi_i)$. Blue-squared lines present the optimal innovation rate according to experiment 3. Red-circled lines present equilibrium innovation rates in the final steady state for low-ϕ firms, while the yellow-dashed lines present steady-state rates for high-ϕ firms. The blue triangles are optimal entry according to experiment 3; red triangles are entry in the final steady state.

returns to innovation: growth rates would be 3.1% (5.4%) percent higher if all research was conducted in single-product firms in the United States (French) calibration.

In contrast, the bottom panel shows that innovation is inefficient when firms have heterogeneous intangible costs. In particular, experiment 3 in the bottom panel shows that when researchers are efficiently allocated across firms and no innovation is wasted, maximum growth would be 1.49% (1.45%) in the United States' (French) calibration, which is well above growth in the initial steady state. In the actual equilibrium, growth falls from 1.3% to 1% (1.2%) in the United States' (French) calibration, despite a significant increase in $\overline{L}^{rd}$. From comparing the maximum growth in experiments 1 and 3, we learn that both the lost innovation and the research misallocation channel contribute significantly to the overall reduction in innovation efficiency. Experiment 1 shows that if researcher allocations are held constant but the lost innovation is removed, growth would have been 1.17% (1.29%) in the respective calibrations. This means that the misallocation channel comprises 61% (60%) of the total research inefficiency, with the lost-innovation channel accounting for the remainder.

Figure 9 offers a closer inspection of the misallocation drivers. It plots innovation rates against the number of products that firms produce. The figure shows that, compared to the efficient allocation in experiment 3, low-ϕ (high-intangible) firms innovate excessively. Their high innovation rates are driven by their profitability and low choke price, rather than by a talent for innovation. The figure furthermore shows that the fraction of researchers that is dedicated to entry is insufficient, particularly in the U.S.: the efficient entry rate in experiment 3 is more than twice as high as the actual entry rate in the final steady state. The inefficient entry rate accounts for a large fraction of the misallocation costs to innovation. From comparing experiments 2 and 3 in Table 9, it follows that allowing incumbents' researchers to contribute to entrant innovation yields a much larger improvement in the maximum growth rate than the improvement had from redistributing R&D efficiently across incumbents.

5.6.4. Discussion and Policy Implications

The analysis in this section shows that inefficiency arising from both the lost innovation channel and the misallocation channel causes a significant loss of potential productivity growth. Resolving the inefficiencies is challenging, especially for lost innovation. The firm that produces a good in equilibrium is the firm that, statically, is able to do so at the most favorable terms to consumers. This means that traditional antitrust policy, for example, is unlikely to raise welfare in this setting. The main way to overcome lost innovation is to stimulate the diffusion of intangible technologies. This involves broadening the fraction of entrants that has access to the lower ϕ_i, or creating an efficient market for the innovations that do not make it to production. This, however, falls outside standard policy tools.

Addressing inefficiency arising from the misallocation of researchers may, however, be more straightforward. The misallocation occurs because there is heterogeneity in the private returns to innovation that does not relate to the efficiency with which researchers at the firm produce patents. Policy that alters the private incentives for research can therefore promote a more efficient allocation.[59] It turns out, however, that the subsidy or tax that removes the misallocation inefficiency at a given level of aggregate research effort would have to be sizable. Appendix H analytically derives the rate at which profits would have to be multiplied at entrants and high-ϕ firms to equate their private research incentives. The differences between the required subsidies are substantial. For entrants, the policy would have to multiply the equilibrium private value of innovation by a factor 3.61 (2.01) in the U.S. (French) calibration to bring them in line with the low-ϕ firms. The moderate inequality in intangible efficiency therefore significantly alters the private benefits of research.

6. Extensions

This section explores three extensions. I first show that the model's predictions for productivity growth and business dynamism also hold if markups are constant. I then show that the results in the previous section are robust when firms internalize the diminishing option value of innovation. Finally, I show that the results also hold if intangibles raise fixed costs at the firm level, rather than at the product level.

6.1. Constant Markups

The analysis thus far has explained the decline in productivity growth and business dynamism jointly with the rise of markups. Recent evidence shows that the labor share in Europe is constant outside of the residential housing sector (Gutierrez and Piton 2020), while markups may be hard to measure accurately in the absence of data on prices (e.g. Bond et al. 2021).[60]

This section shows that the model predicts a *larger* decline in productivity growth if markups are constant. To make that point, I impose that all firms charge a constant markup $\overline{\mu}$ over their marginal

[59] I refrain from a full optimal policy exercise: the model features additional inefficiencies that are common across Schumpeterian growth models that optimal policy would have to address. Because labor is supplied inelastically, there are furthermore no meaningful tradeoffs that such an exercise would bring to light.

[60] Appendix C discusses the model's robustness to measurement issues in markups in the absence of price data.

costs. The markup is calibrated to match the average endogenous markup of 1.29 in the French- and 1.41 in the U.S. calibration of the model. The remainder of the model is left unchanged. In particular, I do not alter the demand system to endogenously arrive at a fixed markup. This facilitates a direct comparison with the main results.[61] The first-order condition for s_{ij} reads $s_{ij} = \left(wY^{-1}\theta\phi_i\bar{\mu}\right)^{\frac{1}{\theta}}$, which follows from inserting the new pricing rule into first-order condition (10). Because markups are homogeneous, the expressions for output and wages respectively simplify to

$$Y = \exp\left(\int_0^1 \int \mathbf{1}_{j\in J_i} \ln\left[\frac{q_{ij}}{s_{ij}}\right] \, di dj\right) L^p, \text{ and } w = \exp\left(\int_0^1 \int \mathbf{1}_{j\in J_i} \ln\left[\frac{q_{ij}}{s_{ij}}\right] \, di dj\right) \bar{\mu}^{-1}.$$

Appendix Table A15 compares the change in the steady-state values in the model with variable markups (columns headed Var. μ) and constant markups (columns headed Fixed $\overline{\mu}$). The rise of high-intangible firms causes productivity growth to fall significantly more when markups are constant: growth now falls by 0.5 and 0.12 percentage points in the U.S. and French calibrations, respectively. When markups are endogenous, high-intangible firms are profitable and invest strongly in R&D. This offsets a part of the decline in growth induced by the fact that high-intangible firms undercut other firms on price. When markups are exogenous there is no motive for R&D by high-intangible firms, worsening the decline in growth. Reallocation rates mirror the additional decline in productivity growth when markups are constant, and now fall well in excess of their empirical decline. In contrast to the data, the table displays a decline in research and development. This is intuitive: constant markups mean that a shift from marginal costs to fixed costs, induced by intangibles, reduces the profit rate. This limits the incentive to expand by high-intangible firms. Combined with the lack of R&D incentives for low-intangible firms due to the low success rate, this explains the R&D decline.

6.2. Value Function Specification

The preceding analysis relied on a simplified dynamic optimization problem where firms did not internalize the change in their innovation capacity when they added a new product to their portfolio. This assumption significantly improves tractability, as it allows for a closed-form expression of the first-order conditions for innovation. This section shows that the results are qualitatively and quantitatively robust to removing this assumption. The new value function is characterized by

$$rV_t(\phi_i, J_i) - \dot{V}_t(\phi_i, J_i) = \max_{x_i} \left\{ \begin{array}{c} \sum_{j\in J_i} \pi_t(\phi_i, \lambda_{ij}) + \tau(\phi_i)\left[V_t(\phi_i, J_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i, J_i)\right] \\ +x_i \mathscr{P}(\phi_i)\mathbb{E}_{\phi_i}\left[V_t(\phi_i, J_i \cup_+ \lambda_{ih}) - V_t(\phi_i, J_i)\right] - w_t\eta_x(x_i)^{\psi^x} n_i^{-\sigma}). \end{array} \right\}.$$

The solution of this function is considerably less tractable than the solution in Section 2 because the function no longer scales linearly in firm size. As firms get larger, the option value of investing in R&D increases, causing them to choose a higher innovation rate. R&D does not fully scale with size, however, because the parameter σ is estimated so that the model matches the negative empirical relationship between firm size and growth. Proposition A.1 in Appendix A gives the new value function's solution.

[61] The introduction of CES utility, for example, would require functional-form changes in order to maintain a balanced growth path. In particular, the fixed-cost function (3) would have to be multiplied by a product's relative quality.

I perform the same experiment as in Section 5.1. To ease the comparison with the main analysis, I retain most of the previous calibration. I re-estimate σ so that the model matches the empirical relationship between firm size and growth. Under an unchanged calibration, the model would predict a strongly negative relationship between firm-growth and firm-size. This is because firms now internalize that the additional option value from producing a good diminishes in n_i. Appendix Table A14 details the new model's calibration and main moments. Compared to the original calibration, there is an increase in the value of σ for both France and the U.S. The higher parameter value ensures that the empirical deviation from Gibrat's Law is still matched by the model.

Appendix Table A16 compares the effect of introducing a calibrated group of high-intangible firms in the model with the new value function specification to the effect in the main analysis. The alternative specification of the model yields a slightly larger decline in productivity growth of 0.4 percentage points in the U.S.. The predicted declines in entry are similar, as are the changes in the reallocation rate. The rise of markups and the changes in the labor and efficiency wedge are also nearly identical to the main results. These findings are intuitive - conditional on the recalibration of σ, the model displays a similar relationship between firm size and firm growth. Because the value function specification in this section differs from the value function in the main analysis only in this regard, the results are both qualitatively and quantitatively robust to the use of the full value function.

6.3. Firm-Level Intangibles

In a final extension, I explore a model where firms can use their intangibles across each product they produce. Instead of facing a tradeoff between fixed- and marginal costs for each product in their portfolio, firms can reduce their marginal costs an equal fraction across all of their products, in exchange for higher fixed costs at the level of the firm. This assumption creates strong increasing returns to scale at the firm-level, which are amplified when the efficiency with which firms use intangibles rises.[62]

The model is presented in Appendix I. I show that the rise of high-intangible firms causes a qualitatively similar slowdown of productivity growth, fall in business dynamism and rise in markups as the main model does. The effects are quantitatively larger than the results in the main analysis. The model with firm-level intangibles predicts a strong decline of the rate of creative destruction in firm size, so that the model is only solvable if innovation declines rapidly in firm size. I therefore conclude that, while robust to firm-level intangibles, the model in the main text is preferred.

7. Conclusion

This paper proposes a unified explanation for the decline of productivity growth, the fall in business dynamism and the rise of markups. I hypothesize that the rise of intangible inputs — in particular, information technology and software — can explain these trends.

[62] Complementarities across products that create positive returns to scale on a firm's demand side are explored in the Klette and Kortum (2004)-framework in a recent paper by Feijoo-Moreira (2021).

Central to the theory is that intangible inputs shift costs from variable to fixed costs, and that firms differ in the efficiency with which they deploy these inputs. I embed intangibles in an endogenous growth model with heterogeneous multi-product firms, variable markups and realistic entry and exit dynamics. The model suggests that when a subset of new firms becomes more efficient at using intangible inputs, the aggregate rise of intangibles is accompanied by a decline in entry and long-term growth. I structurally estimate the model to match micro data on U.S. listed firms and the universe of French firms, and find that intangibles cause a decline of long-term productivity growth of 0.3 percentage points in the U.S. calibration and 0.1 percentage points in the French calibration. Despite the decline of growth, there is an increase in R&D expenditures, in line with empirical evidence. R&D becomes less effective because it is concentrated among a small number of firms and because a fraction of innovators are unable to beat high-intangible incumbents.

While the rise of intangibles negatively affects growth in the long run, its short-run effect is positive. By numerically solving the transition path between the original and the new balanced growth path, I show that growth initially increases for eight years. This is because high-intangible firms initially disrupt sectors by producing goods at lower costs. The overall effect on consumption is negative, although technologies that raise the diffusion of intangibles across firms yield significant welfare gains.

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Appendix A. Proofs and Derivations

A.1. Derivation of the Choke Price

The choke price p_i^c of firm i is the minimum price at which a cost-minimizing producer breaks even:

$$p_i^c = \inf\left\{ p > 0 : \max_{s\in(0,1]} \left(p - ws\right) Y p^{-1} - w\phi_i \left(s^{-\theta} - 1\right) \geq 0 \right\},$$

where the maximized function measures the producing firm's profits, net of fixed-cost intangibles, at price p. Note that none of the terms on the right-hand side are specific to the good that a firm produces. This means that the firm's choke price, p_i^c, is homogeneous across the goods that a firm produces. Setting profits to zero yields an expression for the choke price conditional on the fraction of marginal costs that a cost-minimizing firm would incur if selling at its choke price, s_i^c:

$$p_i^c = w s_i^c (Y - w\phi_i([s_i^c]^{-\theta} - 1))^{-1} Y.$$

The expression is not closed form because optimal intangibles depend on prices. Under cost minimization, first-order condition (6) at the choke price can be written as :

$$s_i^c = \min\left[\left(p_i^c Y^{-1} \theta \phi_i\right)^{\frac{1}{\theta+1}}, 1 \right]. \tag{A.1}$$

For a given wage rate w and aggregate output Y, the choke price is thus entirely determined by a firm's intangible cost parameter ϕ_i, so that $p_i^c = p^c(\phi_i)$. In closed form, the choke price is given by:

$$p^c(\phi_i) = \begin{cases} w\left(\theta^{1/(1+\theta)} + \theta^{-\theta/(1+\theta)}\right)^{(1+\theta)/\theta} \left(\left[\left(\frac{Y}{w\phi_i}\right)^{\theta/(1+\theta)} \right] \left[\frac{Y}{w\phi_i} + 1 \right]^{-1} \right)^{(1+\theta)/\theta} & \text{if } \phi_i < Y/(\theta w) \\ w & \text{if } \phi_i \geq Y/(\theta w), \end{cases}$$

where $Y/(\theta w)$ is the lowest value of the intangible cost parameter such that $s_i^c = 1$.

Two properties of the choke price are worth pointing out. First, the choke price is homogeneous of degree one in *(w,Y)*. This is clear from the choke-price equation above, and means that the relative choke prices across firms are constant along a balanced growth path where w and Y are growing at the same constant rate. Second, for $s_i^c < 1$, the choke price strictly increases in a firm's intangible cost parameter ϕ_i. To see this, note that the choke price strictly increases in the term

$$\left(\left[\left(\frac{Y}{w\phi_i}\right)^{\theta/(1+\theta)} \right] \left[\frac{Y}{w\phi_i} + 1 \right]^{-1} \right)^{(1+\theta)/\theta},$$

and that this term, in turn, increases in ϕ_i for $0 < \phi_i < Y/(\theta w)$. Hence $\partial p^c(\phi_i)/\partial\phi_i > 0$ for $\phi_i < Y/(\theta w)$.

A.2. Proof of Proposition 1

The value function is given by the following Bellman equation:

$$rV_t(\phi_i, J_i) - \dot{V}_t(\phi_i, J_i) = \max_{x_i} \left\{ \begin{array}{c} \sum_{j\in J_i} \left[\begin{array}{c} \pi_t(\phi_i, \lambda_{ij}) + \\ \tau(\phi_i) \left[V_t(\phi_i, J_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i, J_i) \right] \end{array} \right] \\ + x_i \mathscr{P}(\phi_i) \mathbb{E}_{\phi_i} \left[V_t(\phi_i, J_i \cup_+ \lambda_{ih}) - V_t(\phi_i, J_i) \right] \\ - w_t \eta_x (x_i)^{\psi^x} n_i^\sigma - F(\phi_i, n_i) \end{array} \right\}$$

Guess that the solution takes the following form:

$$V_t(\phi_i, J_i) = \sum_{j\in J_i} v_t(\phi_i, \lambda_{ij})$$

where $v_t(\cdot)$ (and hence V_t) grows at a constant rate g in the balanced growth equilibrium. Then $v_t(\phi_i, \lambda_{ij})$ can be written as:

$$\left[r - g + \tau(\phi_i) \right] v_t(\phi_i, \lambda_{ij}) = \pi_t(\phi_i, \lambda_{ij}) + \Gamma$$

where Γ is the option value of innovation adjusted for the fixed term $F(\phi_i, n_i)$:

$$\Gamma = \max_{x_i} \left[\frac{x_i}{n_i} \mathscr{P}(\phi_i) \mathbb{E}_{\phi_i} \left[v_t(\phi_i, \lambda_{ih}) \right] - w_t \eta_x (x_i)^{\psi^x} n_i^{\sigma-1} \right] - \frac{F(\phi_i, n_i)}{n_i} \tag{A.2}$$

which is a function Γ. In order for the value function to scale with size along the guess (a simplification that is removed in Section 6), Γ must not change with the number of goods that the firm produces. I achieve that by choosing $F(\phi_i, n_i)$ so that $\Gamma = 0$. To find the $F(\phi_i, n_i)$ that achieves this, use that the first-order condition satisfies:

$$\mathscr{P}(\phi_i) \mathbb{E}_{\phi_i} \left[v_t(\phi_i, \lambda_{ih}) \right] = \psi^x w_t \eta_x (x_i)^{\psi^x - 1} n_i^\sigma$$

so that if $\Gamma = 0$, the fixed term satisfies:

$$F(\phi_i, n_i) = (\psi^x - 1) w_t \eta_x \left[x(\phi_i, n_i) \right]^{\psi^x} n_i^\sigma$$

With this constraint, optimal research and development expenditures satisfy the equation in Proposition 1:

$$x(\phi_i, n_i) = \left(\mathscr{P}(\phi_i) \frac{\mathbb{E}_{\phi_i} \left[\frac{\pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)} \right]}{\eta^x \psi^x w_t} \right)^{\frac{1}{\psi^x - 1}} n_i^{\frac{\sigma}{\psi^x - 1}}$$

It follows that

$$V_t(\phi_i, J_i) = \frac{\sum_{j\in J_i} \pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)}$$

where operating profits satisfy:

$$\pi_t(\phi_i, \lambda_{ij}) = \left[1 - \frac{\left(\lambda_{ij}\frac{w_t}{Y_t}\phi_i\right)^{\frac{1}{\theta+1}}}{\lambda_{ij}}\right] Y_t - w_t\phi_i\left(\left[\lambda_{ij}\frac{w_t}{Y_t}\phi_i\right]^{\frac{-\theta}{\theta+1}} - 1\right)$$

which increases at rate g along the balanced growth path, confirming the initial guess.

A.3. Derivation of Aggregate Quantities and Proof of Proposition 2

The equilibrium wage is derived as follows. Start with the definition of aggregate output when each sector is in a betrand equilibrium:

$$\ln Y = \int_0^1 \int \mathbf{1}_{j\in J_i} \ln\left(q_{ij} y_{ij}\right) di\, dj$$

Inserting the firm's production function $y_{ij} = l_{ij}/(s_{ij})$ and demand function $y_{ij} = Y/p_{ij}$ yields:

$$\ln Y = \ln Y + \int_0^1 \int \mathbf{1}_{j\in J_i} \ln\left(q_{ij}(w[s_{ij}])^{-1}\mu_{ij}^{-1}\right) di\, dj$$

Isolating wage on the left hand side gives:

$$\ln w = \int_0^1 \int \mathbf{1}_{j\in J_i} \ln\left[\frac{q_{ij}}{s_{ij}}\right] di\, dj + \int_0^1 \int \mathbf{1}_{j\in J_i} \ln\left[\frac{s_{ij}}{\lambda_{ij}}\right] di\, dj$$

The derivation of GDP is as follows. Labor market equilibrium requires:

$$L^p = \int_0^1 \int \mathbf{1}_{j\in J_i} l_{ij} di\, dj$$

Inserting the firm's production function $y_{ij} = l_{ij}/s_{ij}$ and demand function $y_{ij} = Y/p_{ij}$ yields:

$$L^p = \int_0^1 \int \mathbf{1}_{j\in J_i} Y p_{ij}^{-1} s_{ij} di\, dj$$

Isolate Y on the left hand side, insert the first-order condition for pricing, and insert the equilibrium wage to obtain:

$$Y = L^p \exp\left(\int_0^1 \int \mathbf{1}_{j\in J_i} \ln\left[\frac{q_{ij}}{s_{ij}}\right] di\, dj\right) \frac{\exp \int_0^1 \int \mathbf{1}_{j\in J_i} \ln \mu_{ij}^{-1} di\, dj}{\int_0^1 \int \mathbf{1}_{j\in J_i} \mu_{ij}^{-1} di\, dj} \tag{A.3}$$

Define total factor productivity Q_t as the terms to the right of L^p in expression (A.3). A balanced growth path equilibrium is characterized by constant type-shares $K(\phi_i)$. Given that markups equation λ_{ij}/s_{ij} where s_{ij} is given by equation (10), the law of large numbers assures that the third term in (A.3) is constant. Hence $g \equiv \partial \ln Q/\partial t$ is given by:

$$g = \int_0^1 \int \mathbf{1}_{j\in J_i} \frac{\partial \ln q_{ij}}{\partial t}\, di\, dj = \sum_{\phi_i \in \Phi} K(\phi_i)\tau(\phi_i)\mathbb{E}_{-\phi_i}(\lambda_{hj} - 1)$$

which uses that $K(\phi_i)\tau(\phi_i)$ is the fraction of goods that changes producer each instance and where initially produced by ϕ_i-type firms.

A.4. Proposition on Shape of Alternative Value Function

Proposition A.1. *The value function of a firm with intangible cost parameter ϕ_i that produces a portfolio of goods J_i with cardinality n_i grows at rate g along the balanced growth path and is given by*

$$V_t(\phi_i, J_i) = \sum_{j\in J_i} \Upsilon_t^1(\phi_i, \lambda_{ij}) + \Upsilon_{t,n_i}^2(\phi_i),$$

where Υ_1 is the present value of the profit flow from producing good j. Matching coefficients gives

$$\Upsilon_t^1(\phi_i, \lambda_{ij}) = \pi_t(\phi_i, \lambda_{ij})(r - g + \tau(\phi_i))^{-1},$$

while Υ_{2n_i} is the option value of research and development which evolves along this sequence:

$$\begin{aligned} \Upsilon_{t,n_i+1}^2(\phi_i) \quad = \quad & \left[\left((r-g)\Upsilon_{t,n_i}^2(\phi_i) + n_i\tau(\phi_i)\left[\Upsilon_{t,n_i}^2(\phi_i) - \Upsilon_{t,n_i-1}^2(\phi_i)\right]\psi^x - 1\right)(\psi^x - 1)^{-1}\right]^{\frac{\psi^x-1}{\psi^x}} \\ & \left[\mathscr{P}(\phi_i)\right]^{-1}\psi^x\left(\eta w_t\right)^{\psi^{x-1}} n_i^{-\frac{\sigma}{\psi^x}} + \Upsilon_{t,n_i}^2(\phi_i) - \Upsilon_t^1(\phi_i, \lambda_{ij}), \end{aligned}$$

so that the first-order conditions for optimal research and development and entry read

$$x(\phi_i, n_i) = \left(\mathscr{P}(\phi_i)\frac{\mathbb{E}_{\phi_i}\left[\Upsilon_t^1(\phi_i, \lambda_{ij}) + \Upsilon_{t,n_i+1}^2(\phi_i) - \Upsilon_{t,n_i}^2(\phi_i)\right]}{\eta^x\psi^x w_t}\right)^{\frac{1}{\psi^x-1}} n_i^{\frac{\sigma}{\psi^x-1}},$$

$$e = \left(\sum_{\phi_h\in\Phi} G(\phi_h) H\left(\frac{p^c(\phi_h)}{p^c(\phi_{-i})}\right)\frac{\mathbb{E}_{\phi_h}\left[\Upsilon_t^1(\phi_h, \lambda_{hj}) + \Upsilon_{t,1}^2(\phi_h)\right]}{\eta^e\psi^e w_t}\right)^{\frac{1}{\psi^e-1}}. \tag{A.4}$$

Proof:

The value function is given by the following Bellman equation:

$$rV_t(\phi_i, J_i) - \dot{V}_t(\phi_i, J_i) = \max_{x_i}\left\{\begin{array}{c} \sum_{j\in J_i}\left[\begin{array}{c}\pi_t(\phi_i, \lambda_{ij}) + \\ \tau(\phi_i)\left[V_t(\phi_i, J_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i, J_i)\right]\end{array}\right] \\ +x_i\mathscr{P}(\phi_i) \\ \mathbb{E}_{\phi_i}\left[V_t(\phi_i, J_i \cup_+ \lambda_{ij}) - V_t(\phi_i, J_i)\right] - w_t\eta_x(x_i)^{\psi^x} n_i^{-\sigma}) \end{array}\right\}$$

Guess that the solution takes the following form:

$$V_t(\phi_i, J_i) = \sum_{j\in J_i} \Upsilon_t^1(\phi_i, \lambda_{ij}) + \Upsilon_{t,n_i}^2(\phi_i)$$

where firm i produces a portfolio of goods J_i with cardinality n_i, and where $\Upsilon^1_t()$ and $\Upsilon^2_{t,n_i}()$ (and hence V_t) grow at a constant rate g in the balanced growth equilibrium. Grouping terms yields:

$$(r-g+\tau(\phi_i))\Upsilon^1_t(\phi_i,\lambda_{ij}) = \pi_t(\phi_i,\lambda_{ij}) \Rightarrow \Upsilon^1_t(\phi_i,\lambda_{ij}) = \frac{\pi_t(\phi_i,\lambda_{ij})}{r-g+\tau(\phi_i)}$$

The proof of proposition 1 showed that profits grow at rate g, confirming the guess. Furthermore:

$$\left(r-g\right)\Upsilon^2_{t,n_i}(\phi_i) = \max_{x_i}\left\{\begin{array}{c} n_i\tau(\phi_i)\left[\Upsilon^2_{t,n_i-1}(\phi_i)-\Upsilon^2_{t,n_i}(\phi_i)\right]+x_i\mathscr{P}(\phi_i) \\ \mathbb{E}_{\phi_i}\left[\Upsilon^2_{t,n_i+1}(\phi_i)-\Upsilon^2_{t,n_i}(\phi_i)+\Upsilon^1_t(\phi_i,\lambda_{ij})\right]-w_t\eta_x(x_i)^{\psi^x}n_i^{-\sigma}) \end{array}\right\}$$

The first-order condition of the maximization reads:

$$\mathscr{P}(\phi_i)\mathbb{E}_{\phi_i}\left[\Upsilon^2_{t,n_i+1}(\phi_i)-\Upsilon^2_{t,n_i}(\phi_i)+\Upsilon^1_t(\phi_i,\lambda_{ij})\right] = w_t\psi^x\eta_x(x_i)^{\psi^x-1}n_i^{-\sigma}$$

Inserting the first-order condition and isolating $\Upsilon^2_{t,n_i+1}(\phi_i)$ and $\Upsilon^1_t(\phi_i,\lambda_{ij})$ on the left hand side gives the sequence for Υ^2_{t,n_i+1} along:

$$\begin{aligned}\Upsilon^2_{t,n_i+1}(\phi_i)+\Upsilon^1_t(\phi_i,\lambda_{ij}) \quad &= \quad \left[\frac{(r-g)\Upsilon^2_{t,n_i}(\phi_i)+n_i\tau(\phi_i)\left[\Upsilon^2_{t,n_i}(\phi_i)-\Upsilon^2_{t,n_i-1}(\phi_i)\right]}{\psi^x-1}\right]^{\frac{\psi^x-1}{\psi^x}} \\ &\quad \mathscr{P}(\phi_i)^{-1}\psi^x\left(\eta w_t\right)^{\psi^{x-1}}n_i^{-\frac{\sigma}{\psi^x}}+\Upsilon^2_{t,n_i}(\phi_i).\end{aligned}$$

A.5. Proof of Proposition 3

Part (a) I first show that, holding all else constant, a decline in ϕ_i raises a firm's cost-minimizing ratio of fixed over variable costs. To see this, consider the s^*_{ijt} that minimizes costs

$$tc_{ij} = y_{ij}s_{ij}\boldsymbol{c}(w_1,w_2,..,w_k)+f(s_{ij},\phi_i).$$

where time subscripts are omitted for ease of exposition. Given Cobb-Douglas demand, s^*_{ij} is implicitly given through the first-order condition

$$y_{ij}\boldsymbol{c}(w_{1t},w_{2t},..,w_{kt}) = -\frac{\partial f(s^*_{ij},\phi_i)}{\partial s_{ij}}. \tag{A.5}$$

for an interior solution where $s^*_{ij}<1$. The ratio of fixed costs over total costs whenever $f(s^*_{ij},\phi_i)>0$ is therefore given by:

$$\frac{f(s^*_{ij},\phi_i)}{tc_{ij}} = \frac{f(s^*_{ij},\phi_i)}{f(s^*_{ij},\phi_i)-s^*_{ij}\frac{\partial f(s^*_{ij},\phi_i)}{\partial s^*_{ij}}}.$$

Dividing both the numerator and denominator by cost-minimizing fixed costs, this simplifies to:

$$\frac{f(s_{ij}^*,\phi_i)}{tc_{ij}} = \frac{1}{1-\varepsilon_{f(s_{ij}^*,\phi_i),s_{ij}^*}}. \tag{A.6}$$

where $\varepsilon_{f(s_{ij},\phi_i),s_{ij}} < 0$ is the elasticity of fixed costs with respect to s_{ij}. It follows that a reduction in intangible cost parameter ϕ_i leads to an increase to the cost-minimizing firm's fixed costs ratio if the elasticity declines in ϕ_i. This condition is satisfied in the model:

$$\begin{aligned} \frac{\partial \varepsilon_{f(s_{ij}^*,\phi_i),s_{ij}^*}}{\partial \phi_i} &= -\theta\left(\frac{\partial {s_{ij}^*}^{-\theta}/({s_{ij}^*}^{-\theta}-1)}{\partial \phi_i}\right), \\ &= -\theta\left(\frac{\theta}{\theta+1}\frac{1}{\phi_i}\frac{{s_{ij}^*}^{-\theta}}{({s_{ij}^*}^{-\theta}-1)^2}\right) < 0. \end{aligned} \tag{A.7}$$

Part (b) I next show that firms with lower intangible efficiencies ϕ_i innovate at higher rates and, on average, charger higher markups. A firm's rate of innovation is given by first-order condition (17), hence

$$\frac{\partial \ln x_{n_i}(\phi_i)}{\partial \phi_i} = \frac{1}{\psi^x-1}\left(\frac{\partial}{\partial \phi_i}\ln \mathscr{P}(\phi_i) + \frac{\partial}{\partial \phi_i}\ln \mathbb{E}_{\phi_i}\left[\frac{\pi(\phi_i,\lambda_{ij})}{r-g+\tau(\phi_i)}\right]\right).$$

Regardless of whether firms have equal or unequal intangible efficiencies, a lower ϕ_i raises innovation through profitability. Under the Cobb Douglas aggregator, profits are given by $Y - tc_{ij}$. Hence, if a lower ϕ_i reduces total costs, profits fall in ϕ_i. It is straightforward to show that this is the case. With slight abuse of notation, I denote tc_{ij}^* as a firm's total costs under cost minimization,

$$tc_{ij}^* = f(s_{ij}^*,\phi_i) - s_{ij}^*\frac{\partial f(s_{ij}^*,\phi_i)}{\partial s_{ij}^*}.$$

These costs rise in a firm's intangible cost parameter, as is clear from the following total derivative:

$$\frac{dtc_{ij}^*}{d\phi_i} = \frac{\partial f(s_{ij}^*,\phi_i)}{\partial \phi_i} + \frac{\partial f(s_{ij}^*,\phi_i)}{\partial s_{ij}^*}\frac{\partial s_{ij}^*}{\partial \phi_i} - \frac{\partial s_{ij}^*}{\partial \phi_i}\frac{\partial f(s_{ij}^*,\phi_i)}{\partial s_{ij}^*} > 0,$$

which uses that the derivative term in total costs is a constant through first-order condition (A.5). Hence low-ϕ_i firms, ceteris paribus, are more profitable. This incentivises them to spend more on R&D regardless of intangible cost parameters of other firms. If firms have heterogeneous intangible efficiencies ($|\Phi| > 1$), $x_{n_{it}}$ is higher for firms with a lower ϕ_i through two additional channels. First, the probability of success in innovation is given by

$$\mathscr{P}(\phi_i) = \textstyle\sum_{\phi_{-i}\in\Phi} K(\phi_{-i}) H\left(\frac{\phi_i}{\phi_{-i}}\right),$$

we have that $\partial \mathscr{P}(\phi_i)/\partial \phi_i > 0$ as long as $|\Phi| > 1$ and $f(s_{ij}^*,\phi_i) > 0$. Second, the rate of creative destruction $\tau(\phi_i)$ along (14) obeys $\partial \tau(\phi_i)/\partial \phi_i < 0$ as long as $|\Phi| > 1$ and $f(s_{ij}^*,\phi_i) > 0$. Hence $\partial \ln x_{n_i}/\partial \phi_i > 0$. The positive relationship between markups and fixed costs follows from the first-order condition:

$$\ln \mu_{ij} = \lambda_{ij} - \ln mc_{ij}$$

where $\tilde{\lambda}_{ij}$ is the quality gap between the current and previous producer of j. Given that $mc_{ij} = s_{ij}\boldsymbol{c}(w_1, w_2, .., w_k)$, we have $\partial \ln \mu_{ij}/\partial \ln s_{ij} < 0$. Combined with (A.11), this confirms the proposition.

A.6. Derivation of estimation equation (30)

To derive the estimation equation, I start from the first-order equation for R&D in the model from Section 2. Combining (17) with (7) and defining R&D spending $xrd_{it} = w_t rd_{it}$, I write

$$\ln\left(\frac{xrd_{it}}{py_{it}}\right) = \frac{\psi^x}{\psi^x - 1}\ln\left(\mathscr{P}(\phi_i)\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)}\right]\right) + \left(\frac{\sigma}{\psi^x - 1} - 1\right)\ln n_{it} + \ln\left(\frac{w_t \eta^x}{Y_t}(\eta^x \psi^x w_t)^{\frac{\psi^x}{1-\psi^x}}\right), \tag{A.8}$$

The right-hand side of the equation contains three terms described in the main text. The first term captures the value of becoming the producer of an additional good, which is higher for firms with low intangible costs ϕ_i. Along the balanced growth path, the term is entirely captured by a firm fixed effect. The second term captures that innovation intensity falls in firm size. The final term is the time fixed effect. This motivates the reduced-form estimation equation (30).

A.7. Proof of Proposition 4

The proposition claims that firm-level fixed costs are identified by measured fixed costs $\widehat{f}_{it}$ along

$$\frac{\widehat{f}_{it}}{p_{it} y_{it}} = \left(1 - \frac{1}{\widehat{\mu}_{it}}\right) - \frac{\pi_{it}}{p_{it} y_{it}},$$

where notation follows the main text, as long as $\widehat{\mu}_{it}$ is the harmonic average of product-level markups $\mu_{ijt} = p_{ijt}/mc_{ijt}$. Given observed profits $\pi_i = \sum_{j \in J_{it}} \left(p_{ijt} y_{ijt} - y_{ijt} mc_{ijt} - f_{ijt}\right) - \tilde{f}_{it}$, it follows that measured fixed costs $\widehat{f}_{it}$ equal the firm's true fixed costs $\tilde{f}_{it} + \sum_{j \in J_{it}} f_{ijt}$ as long as:

$$\widehat{\mu}_{it} \quad = \quad \frac{\sum_{j \in J_{it}} p_{ij} y_{ij}}{\sum_{j \in J_i} y_{ijt} mc_{ijt}} = \frac{\sum_{j \in J_{it}} p_{ij} y_{ij}}{\sum_{j \in J_i} y_{ijt} p_{ijt} \mu_{ijt}^{-1}}.$$

Proposition 4 is verified by inserting the Cobb Douglas demand function $p_{ijt} y_{ijt} = Y_t$:

$$\widehat{\mu}_{it} = \left(n_i^{-1} \sum_{j \in J_i} \mu_{ijt}^{-1}\right)^{-1},$$

which is the harmonic average of the true product-level markups.

A.8. Derivation: Product and Firm-level Markups with the Hall (1988) equation

To derive the conditions under which measured firm-level markups from the Hall (1988) equation equal the harmonic average of true product-level markups, I first show that the Hall equation is valid in the model. Firms solve the following cost minimization problem for tangible inputs $z_{ijt,h}$:

$$\min_{z_{ijt,h} \forall j,t,h} \sum_{h=1}^{k} z_{ijt,h} w_{ht} \text{ s.t. } y_{ijt} = \frac{1}{s_{ijt}} z(z_{ijt,1}, z_{ijt,2}, .., z_{ijt,k})$$

The first-order condition for a particular tangible input $z_{ijt,h}$ is given by:

$$w_{ht} = \lambda_{ijt} \frac{1}{s_{ijt}} \frac{\partial z(z_{ijt,1}, z_{ijt,2}, .., z_{ijt,k})}{\partial z_{ijt,h}},$$

where λ_{ijt} is the Lagrange multiplier of the cost-minimization problem, which measures marginal costs. Dividing both sides by output y_{ijt} and prices p_{ijt}, and multiplying both sides by $z_{ijt,h}$ gives:

$$\frac{z_{ijt,h} w_{ht}}{y_{ijt} p_{ijt}} = \frac{\lambda_{ijt}}{p_{ijt}} \frac{1}{s_{ijt}} \frac{\partial z(z_{ijt,1}, z_{ijt,2}, .., z_{ijt,k})}{\partial z_{ijt,h}} \frac{1}{y_{ijt}},$$

where the first term on the right-hand side is the inverse of the product's true markup. Rewriting gives:

$$\mu_{ijt} = \alpha_{ijt} \left(\frac{y_{ijt} p_{ijt}}{z_{ijt,h} w_{ht}} \right), \tag{A.9}$$

where α_{ijt} is the elasticity of y_{ijt} with respect to $z_{ijt,h}$. This is the Hall (1988) equation.

In practice, with lack of data on product-level input and revenue data, markups are calculated at the firm level. In the empirical analysis, I measure firm-level markups along

$$\widehat{\mu}_{it} = \widehat{\alpha}_{it} \left(\frac{py_{it}}{w_{ht} z_{it,h}} \right),$$

where $\widehat{\alpha}_{it}$ is an estimate of the elasticity of output with respect to tangible input $z_{it,h} = \sum_{j \in J_{it}} z_{ijt,h}$. Expressed in terms of product-level revenue and input spending, firm-level markups are measured as:

$$\widehat{\mu}_{it} = \widehat{\alpha}_{it} \left(\frac{\sum_{j \in J_{it}} p_{ijt} y_{ijt}}{\sum_{j \in J_{it}} w_{ht} z_{ijt,h}} \right),$$

From (A.9), product-level spending on materials can be substituted with product-level markups along:

$$\widehat{\mu}_{it} = \widehat{\alpha}_{it} \left(\frac{\sum_{j \in J_{it}} p_{ijt} y_{ijt}}{\sum_{j \in J_{it}} \alpha_{ijt} p_{ijt} y_{ijt} \mu_{ijt}^{-1}} \right) = \left(\frac{1}{n_i} \sum_{j \in J_{it}} \frac{\alpha_{ijt}}{\widehat{\alpha}_{it}} \mu_{ijt}^{-1} \right)^{-1}, \tag{A.10}$$

where the final step uses the Cobb-Douglas demand function.

It follows that firm-level markups from the Hall (1988) equation measure the harmonic average of the product-level markups, as required for Proposition 4, in two cases. For multi-product firms, (A.10) shows that measured firm markups indeed equal the harmonic average of true product levels as long as $\alpha_{ijt} = \widehat{\alpha}_{it}$, so that:

$$\widehat{\mu}_{it} = \left(\frac{1}{n_i} \sum_{j \in J_{it}} \mu_{ijt}^{-1} \right)^{-1},$$

as required. In other words, as long as product-level output elasticities are the same across a firm's products and the elasticity estimate is correct, the firm-level markups can be used to identify the firm's total fixed costs. This is the case, for example, for the production function in Section 2, where $\alpha_{ijt} = 1$ and where the sole input that satisfies the assumptions on $z_{ijt,h}$ is production labor l_{ijt}.

For single-product firms, the markup is correctly measured as long as the output elasticity of input $z_{ijt,h}$ is correctly estimated. To assure this, I build on the production function estimation literature, as detailed in Appendix C. Both in the data and in the calibrated model, the majority of firms produces a single product, and estimates of fixed costs for these firms are consistent.

A.9. Derivation of Average Fixed Costs as a Function of homogeneous ϕ

I next show that the average ratio of fixed costs over total costs across firms in a calibration where firms have a homogeneous $\phi_i = \phi$ declines in ϕ. A homogeneous rise in ϕ is different from an idiosyncratic decline in ϕ_i because the homogeneous rise will affect the labor share, which enters in the first-order condition for intangibles. The average ratio of fixed-costs over total costs is:

$$\int_0^1 \frac{f(s_j^*, \phi_i)}{tc_j} dj = \int_0^1 \frac{1}{1 - \varepsilon_{f(s_j^*, \phi), s_j^*}} dj.$$

where firm subscripts are omitted as all firms have equal intangible cost parameters. To understand how the average ratio of fixed costs over total costs changes in ϕ, consider the following total derivative:

$$\frac{d \int_0^1 \frac{f(s_j^*, \phi_i)}{tc_j} dj}{d\phi} = \int_0^1 (1 - \varepsilon_{f(s_j^*, \phi), s_j^*})^{-2} \frac{d\varepsilon_{f(s_j^*, \phi), s_j^*}}{d\phi}$$

Hence it suffices to show that the final term on the right-hand side is negative to show that the ratio of fixed costs over total costs declines in ϕ. This is the case if:

$$\begin{aligned} \int_0^1 \frac{d\varepsilon_{f(s_{ij}^*, \phi_i), s_{ij}^*}}{d\phi} dj &= -\theta \int_0^1 \left(\frac{d {s_{ij}^*}^{-\theta} / ({s_{ij}^*}^{-\theta} - 1)}{d\phi_i} \right) dj, \\ &= -\theta \int_0^1 \left(\theta \frac{{s_{ij}^*}^{-\theta-1}}{({s_{ij}^*}^{-\theta} - 1)^2} \frac{ds_{ij}^*}{d\phi} \right) dj, \\ &< 0. \end{aligned} \tag{A.11}$$

It follows that the right-hand side must be positive for average fixed costs to rise in ϕ. To see that this is the case, define $\tilde{s} \equiv \left[\frac{w}{Y}\theta\phi\right]^{\frac{1}{\theta+1}}$ such that $s_j = \tilde{s}\lambda_j^{\frac{1}{\theta+1}}$, where firm subscripts have been omitted due to the homogeneity in ϕ. This yields the following expression for the labor share:

$$\frac{w}{Y} = \frac{\tilde{s}}{L^p}\int \lambda_j^{-\frac{\theta}{\theta+1}} dj.$$

which is the ratio of (23) and (24). Inserting this into the first-order condition for intangibles (10) gives

$$\tilde{s}\lambda_j^{\frac{1}{\theta+1}} = \left(\left(1+\phi-\phi\tilde{s}_j^{-\theta}\int_0^1 \lambda_h^{-\frac{\theta}{\theta+1}} dh - L^e - L^r\right)^{-1} \tilde{s}\int \lambda_h^{-\frac{\theta}{\theta+1}} dh\theta\phi\right)^{\frac{1}{\theta+1}} \lambda_j^{\frac{1}{\theta+1}}$$

Isolating $\tilde{s}$ yields the following closed-form expression:

$$\tilde{s} = \left[\left(\frac{1-L^e-L^r}{\phi}+1\right)\frac{1}{1+\theta\Lambda}\right]^{-\frac{1}{\theta}}.$$

where $\Lambda \equiv \int \lambda_h^{-\frac{\theta}{\theta+1}} dh$ is a constant that does not depend on ϕ. It then follows from $s_j = \tilde{s}\lambda_j^{\frac{1}{\theta+1}}$ that for a given L^e and L^r, we have $\frac{ds_{ij}^*}{d\phi} > 0$, as required.[63]

[63]The fraction of employment that is dedicated to research, L^r, is constant as the estimation targets R&D spending. While lower values of ϕ may raise R&D spending, the calibration of the R&D cost parameter offsets any effect on L^r by raising the R&D cost scalar. Employment to create new firms, L^e, falls in ϕ. The estimation targets the entry rate, so that lower incentives to enter when ϕ rises are offset by a higher cost scalar η^e. As employment of entrants is the product of wages (which do not depend on ϕ, the entry rate, and η^e, this strengthens the negative relationship between the average ratio of fixed costs over total costs and ϕ.

Appendix B. Data

B.1. Construction of the Datasets

B.1.1. Compustat Data

Data on the income statement and balance sheet for U.S. listed firms is obtained from S&P's Compustat. The panel to estimate markups comes Burstein et al. (2019). It is cleaned by dropping firms with sales, costs of good sold, operating costs and physical assets that are missing, negative, of less than 1000 dollars in value. Following Baqaee and Farhi (2020), I also drop firms with ratios of sales to the cost of goods sold or of sales to selling, general, and administrative expenses outside of the 2.5-97.5 percentile range. I restrict the sample to firms outside of finance, insurance and real estate and start the sample in 1979 to match the start of the Business Dynamics Statistics.

I merge the Compustat data with IT data from the CiTDB for a subset of years. For 2003 to 2009, the data contains ticker symbols for most listed firms that can be matched to Compustat. I use these codes to obtain an initial match for the 1997 to 2009 years. For 2010 to 2015 I match the datasets based on the name of the parent company of a site. I first standardize the names by removing spaces and capitalization, as well as an extensive list of common words in firm names, such as 'Inc', 'Company' or 'Ltd'. The code to perform the standardization was kindly provided by Hazell et al. (2021). I then perform a precise merge on firm names, which is successful for 64% of firms in the Compustat data. I also use the name matching to complement the ticker-based matching for the 1997-2009 years.

B.1.2. French Administrative Data

Balance Sheet and Income Statement The main firm-level datasets are FICUS from 1994 to 2007 and FARE from 2008 to 2016. I obtained access to the merged FICUS and FARE panel from Burstein et al. (2019). They developed the merge of FICUS and FARE, with code that was partly provided by Isabelle Mejean. I thank them for their help in obtaining data access and for permission to use the data for this project. They append FICUS with FARE using a firm identifier (the *siren code*) that consistently tracks firms over time. I keep all firms in legal category 5, which means all non-profit firms and private contractors are excluded from the sample. I also drop firms with operating subsidies in excess of 10% of revenues. Firms in financial industries and firms with missing or negative sales, assets, materials or employment are also excluded. From 2004, INSEE starts to group firms that are owned by the same company in single *siren* codes. This treatment has been gradually extended over time, which means that data on groups in later years of the data contain more consolidated firms. The unit of observation is a legal entity (*unité légale*), although subsidiaries of the largest companies are grouped as a single entity. From 2009 onwards, data is provided separately for the underlying firms (legal entities) and for the group. To have a consistent panel (and prevent an artificial increase in firm concentration), Burstein et al. (2019) group firms along the pre-2009 definitions and extend that treatment backwards and forwards.

Software and IT Data on software comes from the Annual Enterprise Survey (*Enquête Annuelle d'Entreprises*, EAE), which is an annual survey of around 12,000 firms between 1994 and 2007. There are separate surveys for major industries (agriculture, construction, manufacturing, services, transportation) which differ in variables and coverage. The survey is comprehensive for firms with at least 20 employees, and smaller firms are sampled for all sectors except manufacturing. The survey is merged to FARE-FICUS using the SIREN firm identifier. The level of observation is the legal unit, for firms that are aggregated prior to 2009 by INSEE as discussed in the main text. From 2008 onwards I use data from the E-Commerce Survey (*Enquête sur les Technologies de l'Information de la Communication* - TIC). This survey contains questions on the use of IT systems annually from 2008 to 2016. This dataset contains dummies on the adoption of specific IT systems such as Enterprise Resource Planning and Customer Resource Management.

Research and Development Data on R&D comes from the Community Innovation Survey (*Enquête Communautaire sur L'Innovation* - CIS). The CIS is carried out by national statistical offices throughout the European Union, and is coordinated by Eurostat. The survey is voluntary, but sample weights are adjusted for non-response to create nationally representative data. The French survey is carried out by INSEE, and contains consistent variables on research and development expenditures in 1996, 2000, 2004, 2006, 2008, 2010, 2012, 2014 and 2016.

Product Count The number of products by firm comes from the Annual Production Survey (*Enquête Annuelle de Production*, EAP). This survey is used for annual data on industrial production for the EU's PRODCOM statistics. The survey is available for manufacturing only, from 2009 to 2016. I count the number of unique products each year by firm, excluding products on which the firm acts as outsourcer, or was only involved in product design (M1 and M5).

B.2. Variable Definitions

B.2.1. Compustat Data

Revenue is total sales. The Compustat Fundamentals variable is SALE.

Cost of goods sold involves all direct costs involved with producing a good. This includes the cost of materials and other intermediate inputs, as well as the labor directly used to produce a good. It is observed on the income statement. The Compustat variable is COGS.

Selling, general and administrative expense are all direct and indirect selling, general and administrative expenses. They include overhead costs and costs such as advertisement or packaging and distribution. It is observed on the income statement. The Compustat variable is XSGA.

Operating expenses are the sum of cost of goods sold and selling, general, and administrative expenses. The Compustat variable is XOPR.

Capital stock The firm's production capital is defined as the contemporaneous balance sheet value of gross property, plants and equipment (tangible fixed assets). The Compustat variable is PPEGT.

Operating profits are measured as income before extraordinary items. I add expenditures on research and development because these are expensed in the American data yet not in the French data. This furthermore prevents a spuriously positive correlation between the fixed cost measure (which declines in profits) and research and development. The Compustat variable is IB.

Research and development expenditures include all the costs incurred for the development of new products and services. They also include R&D activities undertaken by others for which the firm paid. They are observed on the income statement. The Compustat variable is XRD.

Product count is obtained from the Compustat Historical Segments File. I count the number of products that firms produce as the number of unique primary 6-digit NAICS codes of business segments that firms report. In the adjusted count I assign a count of 1 for firms absent in the segments file.

IT usage is obtained from the CiTDB. I derive two variables from the dataset. The first is the firm's number of personal computers and laptops per employee. Although it does not directly speak to software, this variable is the most common variable derived from the database in prior work. It is the only variable that the dataset consistently collects over time, and is available for 1997 to 2015, except for 2011. This measure is frequently used as a measure IT intensity, and examples of prior papers that rely on this measure include Bloom et al. (2012), Bloom et al. (2016) and Hershbein and Kahn (2018). I calculate the firm-level value of this variable by taking the sum of PCs and employees across all sites that are linked to a firm, and then take the ratio of both. The second variable is the firm's software budget. I take budgets from the data on the firm's headquarter, because coverage of sites varies over time. This variable has been used by, e.g., He et al. (2021). The variable speaks directly to spending on intangibles in the data, but it has two shortcomings. First, the variable is only available for 2010, 2012, 2013, 2014 and 2015. Second, the budget is estimated based on a combination of a survey and a site's characteristics, such as the type of information technologies that the site has installed. The data provider validates the data extensively, but details of the exercise are not provided. I therefore test robustness of results based on software spending with results from IT intensity.

B.2.2. French Administrative Data

Revenue is total sales, including exports. In FICUS years this is CATOTAL, in FARE years this is REDI_R310. In regressions, firm-size is controlled for by a third degree polynomial of log revenue.

Employment is the full-time equivalent of the number of directly employed workers by the firm averaged over each accounting quarter. In FICUS, the data is based on tax records for small firms, and on a combination of survey and tax data for large firms (variable name: EFFSALM). In FARE the variable is REDI_E200, which is based on the administrative DADS dataset.

Wage bill is defined as the sum of wage payments (SALTRAI in FICUS, REDI_R216 in FARE) and social security contributions (CHARSOC in FICUS, REDI_R217 in FARE).

Direct production inputs are calculated as the sum of merchandise purchases (goods intended for resale) and the purchase of raw materials, corrected for fluctuations in inventory. In FICUS, the respective variables are ACHAMAR, ACHAMPR, VARSTMA, and VARSTMP. The corresponding variables in FARE are REDI_R210, REDI_R212,REDI_R211, and REDI_213.

Other purchases are defined as purchases of services form other firms. This includes outsourcing costs, lease payments, rental charges for equipment and furniture, maintenance expenses, insurance premiums, and costs for external market research, advertising, transportation, and external consultants (AUTACHA in FICUS, REDI_R214 in FARE).

Operating profits is defined as revenue minus the wage bill, expenditure on direct production inputs, other purchases, import duties and similar taxes (IMPOTAX in FICUS, REDI_R215 in FARE) capital depreciation (DOTAMOR in FICUS), provisions (DOTPROV in FICUS), and other charges (AUTCHEX in FICUS). The sum of the wage bill, material input expenses, capital depreciation, provisions, and other charges is REDI_R201 in FARE.

Capital stock is measured as fixed tangible assets. This includes land, buildings, machinery, and other installations. The associated variable is IMMOCOR in FICUS, and IMMO_CORP in FARE. Capital is not calculated using the perpetual inventory method because investment is missing for 2008.

Industry codes are converted to NACE Rev. 2 codes using official nomenclatures. Firms that are observed before and after changes to industry classifications are assigned their NACE Rev. 2 code for all years, while other firms are assigned a code from official nomenclatures. Industries without a 1-to-1 match in nomenclatures are assigned the NACE Rev. 2 that is observed most frequently for firms with their industry codes. Firms that switch codes are assigned their modal code for all years.

Research and Development are measured as all innovative expenditures by firms as reported in the CIS. Subcategories of expenditures fluctuate with each version of the survey, but total expenditures seems consistently defined.

Software Investments The variable for software investments closely follows the definition in Lashkari et al. (2019). The underlying variables are observed from 1994 to 2007 in the EAE.[64] The main variable for software is I460. This variable contains all software investments and is available for all sectors. Because missing observations are coded as 0, I drop these firm-years when analysing software. An additional sub-division into externally purchased and internally developed software is available for a subset of firms (I461, I462, I463, I464, I465). Where available, I use this to clean cases where I460 is smaller than I461-I465, and verify that summary statistics match Lashkari et al. (2019).

[64] As coverage is consistent from 1995 onwards, all analysis with software investments starts in that year.

Appendix C. Markup and Fixed Costs Estimation

This appendix summarizes the iterative GMM approach by De Loecker and Warzynski (2012) that is used to estimate the output elasticity of a variable input, in order to calculate fixed costs along (27). I first outline the production function estimation procedure for both France and the U.S., and subsequent discuss the robustness of the resulting series for fixed costs. I also discuss the implication of recent criticisms on the method that I use to calculate markups from the production function.

C.1. Estimation Procedure

I follow the literature that estimates production functions to measure markups along De Loecker and Warzynski (2012). The model in Section 3 assumes that output is a function of tangible inputs (through $z(z_{ijt,1}, z_{ijt,2}, .., z_{ijt,k})$) and intangible inputs, which collapse to s_{ijt}. Tangible inputs are assumed to be flexible, while intangible inputs are assumed to be fixed within periods. In practice datasets contain limited information on individual inputs at the firm-level.

Besides the firm-level aggregation problem discussed in A, this yields two complications. First, the production function can only be estimated with the broad categories as inputs. To maximize the flexibility of the estimation, I approximate this general production function by estimating a translog function that contains the (squared) log of all observed inputs. Second, the broad categories usually are as broad as labor, capital and intermediate inputs. Most of these categories contain a combination of tangible and intangible inputs in the context of the model. In order for estimated markups to obey equation (A.9), the input h may only consist of a flexibly set tangible input. Both in Compustat and in the French data I use h that is most commonly assumed to be a flexible input in the literature.

De Loecker and Warzynski (2012)'s estimation then identifies the translog production function. The procedure is designed to deal with two empirical complications that are not in the model. First, output may be observed with error, causing attenuation bias in the estimates. Second, if firms have different unobserved idiosyncratic total factor productivities, inputs and outputs may correlate through productivity, again causing bias. Both problems are addressed in a separate stage in the estimation. In the first step, output is non-parametrically regressed on all observed inputs in the production function. The fitted value of this regression is output cleaned of measurement error, which serves as the dependent variable in the remainder of the analysis. In the second stage, the production function is identified under the assumption that the idiosyncratic productivity follows an AR(1) process. The production function is identified under the assumption that flexible inputs and lagged fixed inputs (set before the shock occurs), are respectively orthogonal to the lagged and current productivity shock.[65]

Below I detail the exact estimation of the output elasticity of h for both datasets.

[65]An algorithm then iterates over the parameters of the production function. For each iteration, productivity is calculated as the difference between cleaned output and the product of inputs and the assumed production function parameters. These estimates are then auto-regressed to obtain AR(1) productivity shocks, which are then correlated with the inputs. The iteration continues until the correlation between the productivity shock and the current fixed variables and lagged flexible variables is zero.

C.1.1. Implementation: France

The production function estimation for France come from Burstein et al. (2019) who analyse the cyclical properties of French markups, and I thank the authors for permission to use their estimates for this project. In line with their work, I use markups based on the estimated elasticity of output with respect to materials m, which are least likely to contain intangbile inputs in the context of the model. The main estimates of the production function use data on capital k_{it}, labor l_{it} and materials m_{it} and estimate the production function for each 2-digit industry with at least 12 firms in the data, along:

$$y_{it} = \beta^l l_{it} + \beta^{ll} l_{it}^2 + \beta^k k_{it} + \beta^{kk} k_{it}^2 + \beta^m m_{it} + \beta^{mm} m_{it}^2 + \omega_{it} + \epsilon_t \tag{A.12}$$

where cross-terms are omitted to prevent measurement error in one of the inputs to directly affect the estimated elasticity of other inputs.[66] Capital is measured through fixed tangible assets, labor is the number of employees and materials equal firm purchases. In contrast to (i.e.) U.S. Census data, data on materials is available annually for firms in all industries. I instrument k_{it} with its current value, I assume that firms cannot increase capital in response to a contemporaneous productivity shock. By instrumenting l_{it} and m_{it} by their lagged value I assume that they may depend on contemporaneous productivity shocks, but require autocorrelation in factor prices.[67]

The three-factor production function is commonly used in the literature and is therefore the basis of estimates in the main text. To assess the robustness of these estimates, I also estimate a more extensive production function with four production factors. The FARE-FICUS dataset allows materials to be divided into direct production inputs v_{it} (intermediate goods for resale and expenses on primary commodities) and other purchases o_{it}, which include the purchase of external services like advertising. An output elasticity can only be used to estimate markups when the factor is freely set each period, which seems most likely to hold for v_{it}. Direct production inputs v_{it} are most likely to only be tangible, as they only include expenses on intermediate goods for resale or expenses on primary commodities. That is why I use the elasticity of output with respect to v_{it} to measure markups from the four-factor production function. The production function reads:

$$y_{it} = \beta^l l_{it} + \beta^{ll} l_{it}^2 + \beta^k k_{it} + \beta^{kk} k_{it}^2 + \beta^v v_{it} + \beta^{vv} v_{it}^2 + \beta^o o_{it} + \beta^{oo} o_{it}^2 + \omega_{it} + \epsilon_t \tag{A.13}$$

Because of the large number of firms in the data, I estimate this more extensive production function separately for each 4-digit industry.

Gross output in the production function is measured through sales, which has been criticized in a number of recent papers. While a review of the debate goes beyond the scope of this paper, a particularly relevant critique is presented in Bond et al. (2021). They show that when markups are measured by multiplying the inverse of a factor's share in revenue with the revenue function elasticity rather than the production function elasticity, the resulting markup is biased to an average value of 1.

[66] This follows De Loecker et al. (2020) in their treatment of capital.

[67] For France it is reasonable to assume that labor is, in fact, not set freely and could therefore be instrumented by contemporaneously. This turns out to have no significant effect on the estimated production function.

In practice, markups estimated with the De Loecker and Warzynski (2012) methodology do not measure the revenue elasticity as revenue is purged from factors unrelated to inputs in the first stage and output is deflated with sector deflators (see, e.g., De Loecker 2021). The French data furthermore allows for a comparison of markups obtained from data on revenue versus data on quantities, because the French product-level data on manufacturing (the *EAP*) contains price data from 2009. Using this data, Burstein et al. (2019) show that markups based on quantity data have a 0.83 correlation coefficient with markups based on revenue data. Note, furthermore, that the model only relies on fixed costs in order to calibrate the initial level of intangible efficiency. Bias in markup estimates therefore only affect the initial calibration of ϕ_i. Appendix C.4 furthermore shows that the average fixed costs derived from the De Loecker and Warzynski 2012 markups align with averages from alternative measures.

C.1.2. Implementation: United States

Markups and production function elasticities for U.S. publicly listed firms come from De Loecker et al. (2020). I estimate the elasticities using code created for Burstein et al. (2019) that replicates De Loecker et al. (2020), and merge the elasticities with my data to calculate markups and fixed costs. A constraint of the analysis of markups for these firms is that data on materials and the wage bill is not available from the income statement. Instead, De Loecker et al. (2020) use a broad category of operating expenses (cost of goods sold) that captures all expenditures that are directly related to the cost of production. Results in the main text use these markup estimates.

One critique on using a production function estimation with capital and cost of goods sold is that it does not account for selling, general, and administrative expenses (SG&A), which have become more important over time. Adding SG&A to cost of goods sold to form a single input in a production function is evenly problematic because 1) a large part of SG&A are fixed overhead costs as well as expenditures on intangible inputs,[68] and 2) it assumes all operating expenses are perfect substitutes. Instead, I test the robustness the main results by adding SG&A as an input in a production function along (A.12).

C.2. Robustness of Fixed Cost Trends

C.2.1. France

The results in the main text are robust to using the more extensive four-factor production function. After estimating the industry-level production function coefficients, I calculate the firm-level markup as the product of the input elasticity and the inverse of the input's revenue share. I then calculate the fixed cost share along (27). Markups at the firm-level are summarized in Table A1. The table shows that the extensive production function estimates a very similar average markup to the markup from the standard three-factor production function. The variance of markups, however, is significantly greater when using the four-factor production function. This is likely due to the additional parameters that need to be estimated at the 4-digit level, or because firms have some flexibility in what costs fall under direct production inputs v versus other purchases o.

[68] Heterogeneity in fixed costs across firms will then cause an underestimation of the input elasticities and markups.

Table A1: Summary Statistics on Estimated Markups

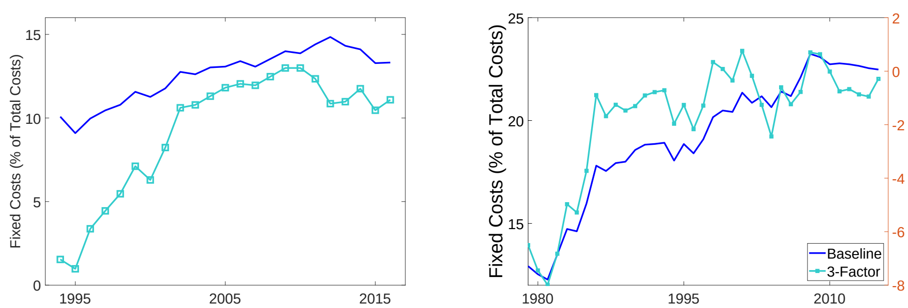
	Mean	Std. Dev.	Median	10th Pct.	90th Pct.	Observations
France						
Basic production function	1.38	0.46	1.26	0.96	1.91	9,913,058
Extended production function	1.47	5.17	1.01	0.53	2.59	8,477,467
United States						
COGS production function	1.50	.58	1.33	1.01	2.25	127,682
COGS and SG&A production function	1.30	.51	1.15	0.87	1.94	127,682

The trends of aggregate fixed costs are plotted in Figure A1. The solid-blue line is replicated from the main text and is for the three-factor standard production function, while the squared-green line uses the four-factor extensive production function. Both figures show that the sales-weighted average fixed cost share has increased strongly over the 1994 to 2016 sample, with the largest increase occurring between 1994 and 2010, after which the increase moderates.

C.2.2. United States

Markups from the two-factor and three-factor production functions are highly correlated. The bottom panel of TableA1 presents summary statistics for both and shows that they mainly differ in terms of their their level. When adding SG&A, over 30% of all firms have markups below 1 and the median markup is 1.15. Though the 2-factor admits markups around 15 percentage points above that at most percentiles, both series co-move strongly. The firm-level correlation is 0.92. While the correlation of the markup series is close, the difference in levels between the series have a large effect on the predicted level of fixed costs. The right plot in Figure A1 shows that the 3-factor production function predicts *negative* average fixed costs as a percentage of total costs between 1980 and 2004. This is likely to be driven by an underestimation of the markup; of the firms with a 3-factor markup below unity, 63% report positive profits. The predicted increase in fixed costs over the sample is 13 percentage points, which is similar to the predicted increase in the main text. Figure A1 does raise concerns about the correct calibration target for the initial level of fixed costs. Appendix C.4 shows that fixed costs from alternative measures imply a similar average level to the level in the main text.

Figure A1. Robustness of Trends in Aggregate Fixed Cost Share



(a) France (b) United States

C.3. Fixed Costs and Capital Costs

Fixed costs in the main text are calculated using data on estimated markups and measured operating profits from the income statement. Operating profits account for capital costs in the form of depreciation, but they do not account for other rental costs of capital that the firm owns. An exact estimate of these costs is difficult because these rental costs must account for the fair risk premium of purchasing capital, which is not directly observed.

In this appendix I show that the trends and cross-sectional distribution of fixed costs is robust to various controls for rental costs. To resolve the lack of directly observable measures of capital rental costs, I rely on various estimates of risk-free interest rates and capital risk premia in Caballero et al. (2017), each with slight differences in underlying assumptions.[69] I use four alternative approaches to calculate r_t^K. Caballero et al. (2017) calculate rental rates of capital either under the assumption that there has not been capital-biased technological change over the sample, or that markups have remained flat. Under the first assumption, they calculate capital required returns for three cases, corresponding to different elasticities of substitution between capital and other factors (1.25, 1.00 and 0.80 in cases 1, 2, and 3, respectively). I focus on these series, given the role of markups in the paper.[70] While neither is perfectly in line with the model, they can be used to assess whether accounting for capital costs with common methods affects the empirical results.

I calculate profits along:

$$\pi_{it} = \tilde{\pi}_{it} - r_t^K K_{it}$$

where $\tilde{\pi}_{it}$ is the original series of operating profits excluding depreciation and K_{it} is the firm's stock of property plants and equipment. The same series is used for France and the US. I then calculate fixed costs with the alternative profit rates and compare results.

Results in the paper are robust to the alternative definition of operating profits. Figure A2 compares trends in the weighted average of various fixed cost series. The original series is blue-solid, other

Table A2: Correlations Between Alternative Fixed Cost Series

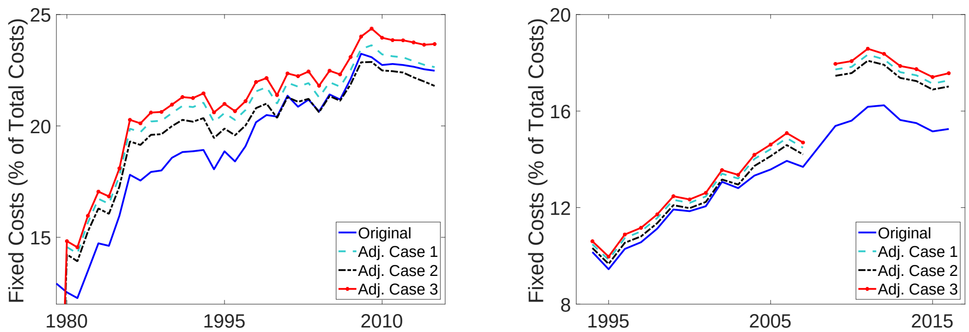
	Original	Adj. Case 1	Adj. Case 2	Adj. Case 3
United States				
Original	1.00	0.98	0.98	0.98
Adjusted Case 1	0.98	1.00	>0.99	>0.99
Adjusted Case 2	0.98	>0.99	1.00	>0.99
Adjusted Case 3	0.98	>0.99	>0.99	1.00
France				
Original	1.00	0.99	0.99	0.99
Adjusted Case 1	0.99	1.00	>0.99	>0.99
Adjusted Case 2	0.99	>0.99	1.00	>0.99
Adjusted Case 3	0.99	>0.99	>0.99	1.00

Notes: Correlations between the main fixed cost series (Original; accounting for capital costs through depreciation) and the rental-cost adjusted series using estimates from Caballero et al. (2017). See text for differences between adjusted series.

[69] Caballero et al. (2017) provide these estimates for years at the beginning, middle and end of my sample. I interpolate the variables linearly to obtain annual estimates for the risk-free interest rates and capital risk premia.

[70] Results from the series under assumption (b) are very similar with firm-level correlations of fixed costs exceeding 0.99.

Figure A2. Comparison of Fixed Cost Series Accounting for Capital Costs



(a) United States (b) France

Notes: Sales-weighted average of fixed costs as a percentage of total costs, U.S. listed firms (left) and universe of French firms (right). Fixed costs are inferred from the difference between profits as a percentage of sales and the marginal cost markup. A discontinuity in the French investment and depreciation data prevents the calculation of capital-adjusted fixed costs for 2008.

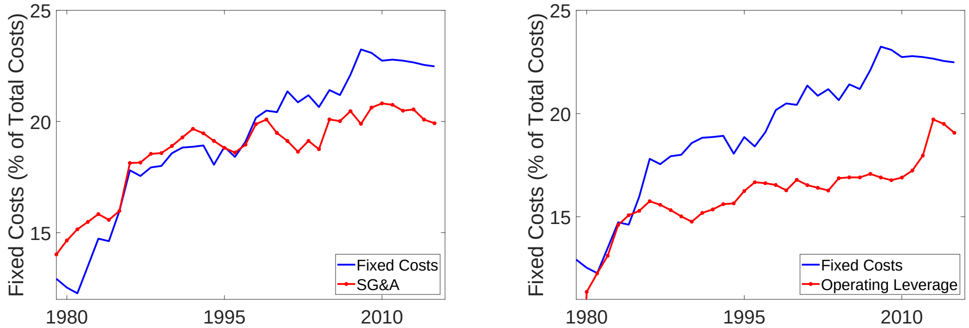
series denote the various cases described above. The series generally show that, accounting for the required rate of return, fixed costs are higher. This is expected because accounting for capital costs lowers profits.[71] The series become slightly closer over the sample because of the decline in the risk-free rate, although the decline is moderated by a rise in capital risk premia. The French data has a discontinuity around 2008, when investment and depreciation data change definitions. At the firm-level, Table A2 presents correlations of fixed costs across the specifications. The original fixed costs estimates have a firm-level correlation with the alternative estimates of at least 0.98 for all of the specifications.

C.4. Alternative Approaches to Fixed Cost Calculation

There are two alternative common approaches to the calculation of fixed costs in the literature. The first is to assign particular costs on the profits and loss (P&L) statement to either fixed costs or variable costs. This approach is used for U.S. firms in De Loecker et al. (2020), who assume that SG&A represents fixed costs. The advantage of this approach is that the fixed cost estimates are firm-specific and time-varying, and that they can be obtained without uncertainty as long as one believes the classification. Figure A3a in this letter plots the trend in fixed costs as measured as the ratio of SG&A over total costs. It shows that the average SG&A-ratio was slightly higher than the measure of fixed costs in this paper (14.6%), and that the increase over time was smaller (5.7 percentage points). This could be explained by the presence of some variable costs in SG&A, which are predicted to fall in my paper's model. Broadly, however, fixed costs according to the SG&A-ratio are in line with the main measure of fixed costs in the paper. The firm-level correlation between the measures is 0.66. A second practice is to measure fixed costs from the responsiveness of costs to changes in sales. This approach originates from the empirical corporate finance literature, where the ratio of fixed costs over total costs is

[71] The original (blue-solid) series does account for depreciation costs as reported by the firm, whereas the adjusted cases assume a fixed depreciation rate of 7.3% in line with Caballero et al. (2017). This explains why there are some years in which the adjusted case 2 (black-dashed) is lower than the original series.

Figure A3. Alternative Measures for the Ratio of Fixed Costs to Total Costs



(a) Fixed Costs versus SG&A   (b) Fixed Costs versus Operating Leverage

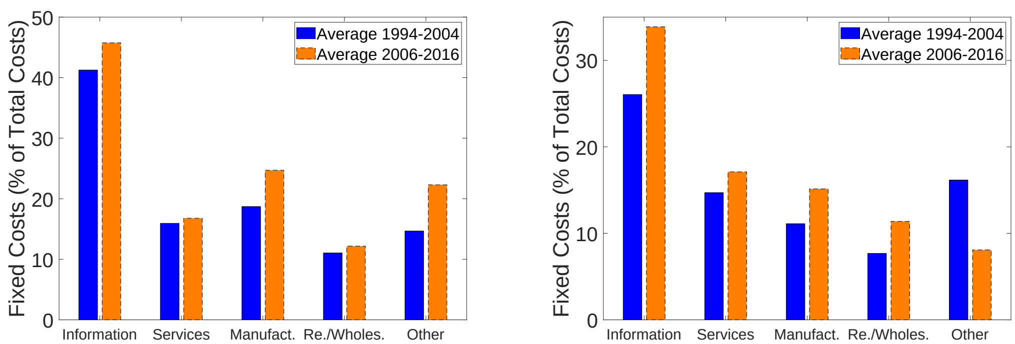
Notes: Sales-weighted average of fixed costs as a percentage of total costs for U.S. listed firms, versus SG&A over total costs (left) and average operating leverage based on the sensitivity of firm-costs to firm-sales from Saibene (2017) (right). Fixed costs in the solid-blue lines are the main series from the paper.

also known as *operating leverage*. García-Feijóo and Jorgensen (2010) summarize this literature, and Saibene (2017) provides estimates for Compustat firms. He estimates operating leverage from a firm-specific regression of costs on sales and plots the annual average of this across Compustat firms using all firms operating in that year. A limitation of this approach is that the elasticity cannot be estimated by firm-year, which complicates a panel analysis like in Section 2. The series are reproduced here as the red-marked line in Figure A3b.

D4. Within versus Between Sector Changes in Rise of Fixed Costs

Figure A4 illustrates the sectoral composition of fixed costs. It shows that fixed costs as a fraction of total costs are especially high in the information sector (NAICS industry 51 for the U.S. and NACE

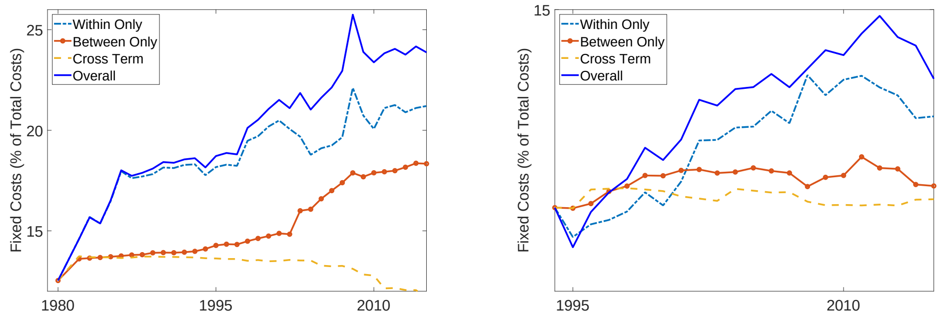
Figure A4. Weighted-Average Ratio of Fixed Costs to Total Costs across Sectors



(a) United States   (b) France

Notes: Sales-weighted average of fixed costs fraction by sector for U.S. listed firms (left) and the universe of French firms (right). Sectors are ordered by the average fixed-cost share in the last ten years of the French sample. Industry definitions for the United States (NAICS): 51 for information, 64 and above for services, 31, 32 for manufacturing, and 42, 44, 45 for wholesale and retail; for France (NACE/ISIC): JB, JC for information, I, M, N for services, B, C, D, E for manufacturing, and G for wholesale and retail.

Figure A5. Within-Between Decomposition of the Rise of Fixed Costs



(a) United States (b) France

Notes: Within-between decomposition of the rise of fixed costs for U.S. listed firms (left) and the universe of French firms (right).

industry JB and JC for France). The distribution of fixed costs across sectors is similar for the U.S. and France and the majority of sectors have seen an increase in their average ratio of fixed- to variable costs. The latter suggests that fixed costs have increased at the aggregate level because of an increase in the importance of fixed costs within sectors and not because high-fixed costs sectors have become larger over time. To formally show that the aggregate rise of fixed costs is driven by within-sector reallocation, I perform the following within-between decomposition:

$$\Delta\frac{\tilde{F}_t}{TC_t} = \sum_{j\in J} s_{jt-1}\Delta\frac{\tilde{F}_{jt}}{TC_{jt}} + \sum_{j\in J}\Delta s_{jt}\frac{\tilde{F}_{jt-1}}{TC_{jt-1}} + \sum_{j\in J}\Delta s_{jt-1}\Delta\frac{\tilde{F}_{jt}}{TC_{jt}}$$

where $\tilde{F}_t/TC_t$ is the aggregate fixed cost share, $\tilde{F}_{jt}/TC_{jt}$ the sector-level counterpart, and s_j the fraction of sales by sector j. The first term captures changes due to increases in fixed costs within sectors. The second term captures the 'between' share: changes because of changes in the relative size of sectors. The last term is the interaction of both. I perform the decomposition annually and regress each term on the change in the aggregate fixed cost share.

The coefficients are presented in Table A3. Figure A5 illustrates the contribution of within and between shares over time, by plotting the development of fixed costs holding other contributors constant. The results show that within-sector reallocation was largely responsible for the rise of fixed costs.

Table A3: Decomposition of Changes in Aggregate Fixed Cost Share

	Within Sectors	Between Sectors	Cross Term	Total
United States	0.88	0.11	0.01	1
	(0.06)	(0.05)	(0.03)	
France	0.80	0.17	0.03	1
	(0.004)	(0.003)	(0.003)	

Notes: Standard errors in brackets.

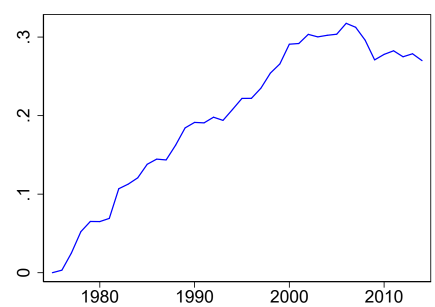
Appendix D. Macroeconomic Trends in France

The introduction summarizes three recent trends: the slowdown of productivity growth, the fall in business dynamism and the rise of corporate profits. This appendix gives an overview of the macroeconomic trends for France.[72]

The slowdown of productivity growth is depicted in Figure A6. It plots an index of the log of TFP at constant prices, standardized to 0 in 1975. The figure shows that TFP was growing at a steady rate for most years between 1975 and 2000. There was a significant slowdown in the early 2000s, and productivity growth over the 2005-2020 era has been slightly negative.

The decline in business dynamism is summarized with three statistics, following the literature. The first is the reallocation rate in Figure A7a, which is the sum of job destruction and creation rates. I calculate the reallocation rate across French firms using the FARE-FICUS dataset for 1994-2016. Because this sample coincides with the Great Recession, which brought a strong transitory increase in reallocation due to job destruction, I plot the HP trend. The second fact is the decline of entry of new firms. Figure A7b captures this trend by plotting the fraction of employees that work for a firm that enters the FARE-FICUS dataset in a given year. Note that this may include firms that have undergone significant organizational changes that have caused their firm identifier to change. The figure shows that employment by entrants has declined by almost half within the 1994-2016 sample. The third fact is the decline of skewness of the firm growth distribution. As discussed by Decker et al. (2017), small (young) high-growth firms have historically been an important contributor to productivity growth. They infer the decline in skewness of the growth distribution from the decline between the 90th and 10th, and between the 90th and 50th percentile of the growth distribution. Figure A8 shows that both have declined by around 40% between 1994-2016. The difference between the 50th and 10th percentile has remained flat, in line with U.S. evidence.

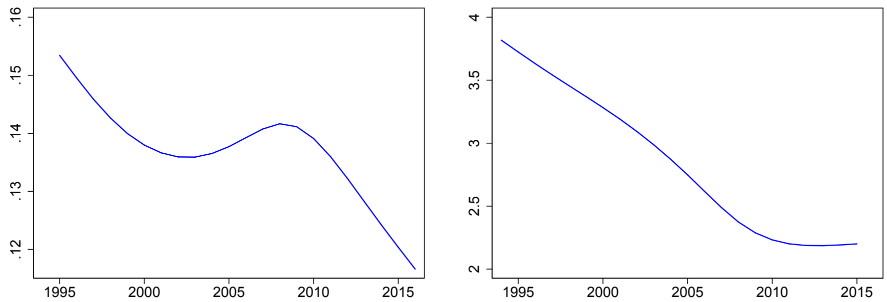
Figure A6. Total Factor Productivity in France



Notes: Log TFP at constant prices, 1975=0. Data: Penn World Tables.

[72] Whether market power is increasing across advanced economies remains a subject of debate. The slowdown of productivity growth and the decline of start-ups have been widely documented (e.g. Adler et al. 2017 and Calvino et al. 2016), while the rise of market power and firm concentration seems to be larger in the U.S. Döttling et al. (2017) and Cavalleri et al. (2019) find no increase in industry concentration in Europe between 2000 and 2013, using Orbis data. Bajgar et al. (2019) document a rise in concentration in most of Europe when accounting for ownership structures and the coverage of small firms in Orbis. Aquilante et al. (2019) also find an increase in U.K. industry concentration between 1998 and 2016.

Figure A7. Business Dynamism in France

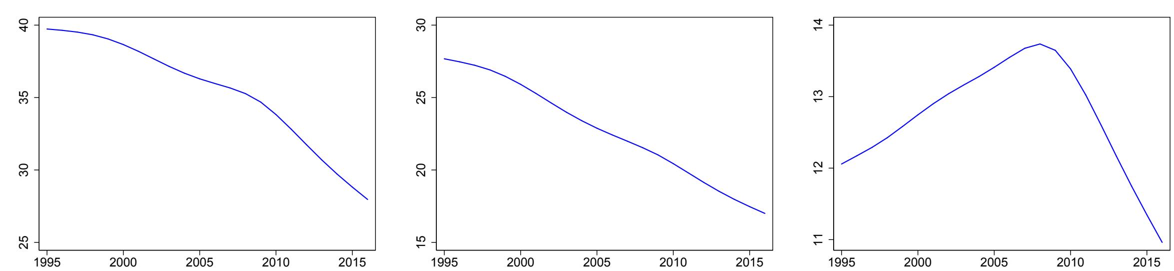


(a) Reallocation Rate (b) Entrants

Notes: Both figures plot HP trends. Left figure: sum of job creation and job destruction rates across companies. Right figure: Percentage of employment by new firms (≤ 1yr) in private sector employment. HP trend.

The rise of corporate profits is measured through the marginal cost markup. This is a measure of marginal rather than average profits, a distinction that is key in Section 3. Figure A9a plots the average sales-weighted markups for French firms between 1994 and 2016. The markups has increased modestly, in line with previous evidence (e.g. IMF 2019). Though not directly measuring market power, concentration also displays a modestly positive trend over the sample. This is shown in Figure A9b, which depicts the average Herfindahl Index across 5-digit industries. The rise of concentration has been linked to the decline in the labor share by Autor et al. (2020) through the reallocation of activity to firms with low labor shares. This result has been replicated for France for 1994-2007 by Lashkari et al. (2019). Note that the increase in concentration depends on measurement. The graph below presents an average of the Herfindahl across sectors. Weighing sectors by value added gives an increase in the Herfindahl index from 2008 from 0.087 in 1994 up to 0.122 in 2008, but a modest decline to 0.117 afterwards.

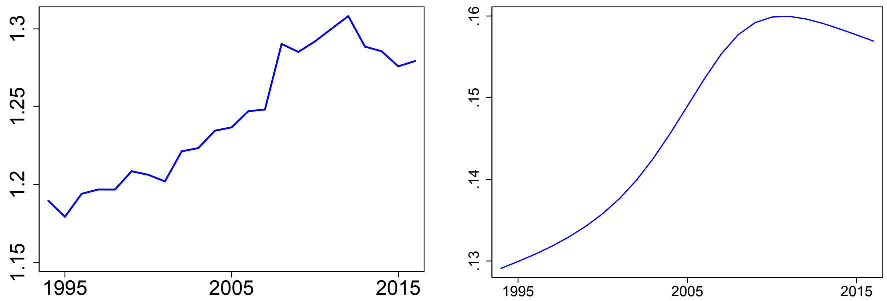
Figure A8. Skewness of the Employment-Growth Distribution



(a) 90-10 Difference (b) 90-50 Difference (c) 50-10 Difference

Notes: Difference (perc. point) in growth between percentiles of the employment-growth distribution. HP trend.

Figure A9. Markups and Firm Concentration in France



(a) Markups (b) Concentration

Notes: Left figure: sales-weighted marginal cost markups using the Hall (1988) equation with production function elasticities estimated with iterative GMM as in De Loecker and Warzynski (2012). Details in Appendix C . Right figure: average Herfindahl index across 5-digit NACE industries. HP trend.

Appendix E. Computational Algorithm

The balanced growth path equilibrium along definition 1 is found by solving the system of detrended equilibrium equations as a fixed point. The algorithm works as follows:

1. Solve the fixed point:

   (a) Guess a level of Y/Q, w/Q, $\tau(\phi)$, and $K(\phi)$.

   (b) Collect choke prices by solving:

   $$\left(p^c(\phi_i) - w[1 - s^*(\phi_i)]\right) Y - w\phi_i \left([1 - s(\phi_i)]^{-\theta} - 1\right) = 0 \text{ where } \phi_i \in \Phi$$

   (c) Given the vector of choke prices and the guess for $K(\phi)$, calculate the following objects:

   - a $|\Phi| \times |\Phi|$ matrix $\mathbf{P}$ with probabilities that a firm of type $\phi_i \in \Phi$ successfully innovates when facing $\phi_{-i} \in \Phi$ along (15) and a vector with the weighted average over this probability $\sum_{\phi_{-i} \in \Phi} K(\phi_{-i})\mathbf{P}(\phi_i, \phi_{-i})$ with the probabilities that a type's innovation is successful in general.
   - the set of distributions of $\lambda_{ij} \sim H(\lambda)$ for each combination of $\phi_i \in \Phi$ and $\phi_{-i} \in \Phi$ truncated at $p^c(\phi_i)/p^c(\phi_{-i})$.
   - the expectation of markups along (11) given the truncated distributions and the guess for $K(\phi)$.
   - the optimal innovation efforts by incumbents and entrants given markups, $\mathbf{P}$, Y, w, $\tau(\phi)$, and $K(\phi)$.

   (d) Calculate Y along (24) and w along (23). Use the innovation effort by incumbents and entrants to calculate $\tau(\phi)$ along (14) and (A.25), (21) and (22) to find $K(\phi)$.

   (e) Repeat from step (b) until the model has converged.

2. Perform the firm simulation, building computationally on Akcigit and Kerr (2018) and Acemoglu et al. (2018):

   (a) Collect the equilibrium Y, w, $\tau(\phi)$, $K(\phi)$, $x(\phi, n)$, e for all n and all $\phi_i \in \Phi$.

   (b) Discretize time by introducing a sufficiently large number of instances per year so that $x(\phi, n) < 1$ and $e < 1$.

   (c) Initialize the firm-size distribution along (A.25) and (21).

   (d) Simulate firms until the markup distribution has converged, then collect moments.

3. In the structural estimation, the resulting moments are then compared to the targets using the penalty function described in the main text. The parameters are updated along either a genetic algorithm or particle swarm algorithm – to optimize fit – until the penalty function is minimized.

The transitional dynamics are numerically solved using the following algorithm:

1. Create a fine grid with a T-year horizon, allowing each year to consist of $\tilde{T}$ instances.

2. Guess an initial value function of innovation activities $V()$ equal to the new steady-state level for each type in $\phi_i \in \Phi$ at each point of the grid. Similarly guess the paths of wages w/Q and output Y/Q at their new steady-state level.

3. Initialize the firm-size and type distribution $K(\phi)$ and $M(\phi, n)$ to their original steady state.

4. Iterate over the path of the value function as follows:

   (a) Solve the static optimization problem and the dynamic innovation decisions for incumbents and entrants for each point on the grid using the initial guess for $V()$.

   (b) Given the innovation and static decisions, simulate the development for a large (N) number of products and track the innovation step-sizes λ in $N \times (T\tilde{T})$ matrix Λ and similarly a matrix of ownership types using a forward loop over the grid.[73]

   (c) Update the value function using the new sequences for Y, w, the firm-type and -size distribution, and distributions for markups and λs implied by Λ. This involves calculating:

      i. the expectation of profits $\pi_{kt}(\phi_i, \lambda_{ij})$ at each instance t on the grid $t = 1, .., T$ separately for each cohort of patents k.

      ii. the value of obtaining the patent to produce an additional product for incumbents of type ϕ at time k as follows:

$$V^k(\phi_i) = \mathbb{E}^k_{\phi_i}\left[\sum_{t=k+1}^{\varepsilon T} \prod_{h=k+1}^{t} \left(\frac{1-\tau_h(\phi_i)}{1+\rho}\right) \pi_{kt}(\phi_i, \lambda_{ij})\right]$$

[73] This simulation is needed because the changing composition of firm types means the distribution of realized λs has no analytical representation. I then use the resulting distribution of markups to calculate the efficiency wedge along (24), as well as a path for Y and w. These serve as the basis for the algorithm's next iteration.

which is a discretization of the original value function, where ϵ is set so that the present value of profits in instances exceeding ϵT approaches zero.[74]

(d) Use the resulting value for each type on each point of the grid as the guess for $V()$ in step (a) in the next iteration. Continue until the path of the value function converges.

(e) Smooth the transition paths for productivity growth to remove noise induced by simulating software growth for a finite number of firm-products.

Appendix F. Additional Figures and Tables

Table A4: Relationship between Technology Adoption and Fixed-Cost Share (France)

	Software Adoption					
Fixed-Cost Share (log)	ERP	CRM	SCM	CAD	RFID	Spec.
Adoption Dummy	0.060	0.030	0.022	0.080	0.141	0.230
	(0.013)	(0.014)	(0.019)	(0.030)	(0.032)	(0.020)
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Industry fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Revenue (product-count) control	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.252	0.252	0.257	0.248	0.286	0.265
Observations	54,709	59,190	39,033	17,139	14,656	41,619

Notes: Explanatory variable is a dummy for the adoption of the technology specified in the column header. Industry-fixed effects at the 5-digit NACE level in lieu of firm fixed effects, as firms are randomly sampled. Observations are weighted by sample weights. Firm-clustered standard errors in parentheses. Observation counts differ, as not every measure was included in each survey year.

Table A5: Relationship between PC Intensity and Fixed-Cost Share (United States)

Fixed-Cost Share (log)	I	II	III	IV	V	VI
PC Intensity (log)	0.163	0.135	0.107	0.097	0.017	0.003
	(0.017)	(0.016)	(0.015)	(0.017)	(0.007)	(0.007)
R^2	0.023	0.047	0.137	0.139	0.005	0.027
Observations	16,806	16,806	16,806	16,806	16,806	16,806
Year fixed effects	No	No	No	Yes	No	Yes
Firm fixed effects	No	No	No	No	Yes	Yes
Industry fixed effects	No	No	Yes	Yes	No	No
Revenue (product-count) control	No	Yes	Yes	Yes	Yes	Yes

Notes: Dependent variable is fixed costs over total costs (log). Explanatory variable is IT intensity, measured through the number of personal computers per employee. Revenue is deflated with the sector-specific gross output deflator from EU-KLEMS. Variables are winsorized at 1% tails. Firm-clustered standard errors are given in parentheses. Revenue is in logs, as it is proportional to product count in the model. Sector fixed effects are at the 2-digit level.

[74] I set $T = 3000$ (corresponding to 60 years), $\epsilon = 11$ (a profit horizon of 600 years), and set $N = 10000$.

Table A6: Relationship between Software Budget and Research & Development (United States)

R&D intensity (log)	I	II	III	IV	V	VI
Software budget over total costs (log)	0.266	0.243	0.215	0.219	0.019	0.006
	(0.016)	(0.019)	(0.019)	(0.019)	(0.008)	(0.008)
R^2	0.163	0.166	0.285	0.286	0.035	0.064
Observations	3,787	3,787	3,787	3,787	3,787	3,787
Year fixed effects	No	No	No	Yes	No	Yes
Firm fixed effects	No	No	No	No	Yes	Yes
Industry fixed effects	No	No	Yes	Yes	No	No
Revenue (product-count) control	No	Yes	Yes	Yes	Yes	Yes

Notes: Dependent variable is R&D intensity (log). Explanatory variable is software budget as a percentage of total costs (log). Revenue is deflated with the sector-specific gross output deflator, software with the input deflator from EU-KLEMS. Variables are winsorized at 1% tails. Firm-clustered standard errors are given in parentheses. Revenue is in logs, as it is proportional to product count in the model. Sector fixed effects are at the 2-digit level.

Table A7: Comparison of Theory and Data for Untargeted Moments

		United States			France		
	Quartile	**Model**	**Data**	**St. Dev.**	**Model**	**Data**	**St. Dev.**
Size and Age	1st (Age)	1.20	2.04	(1.04)	1.26	1.98	(1.01)
	2nd (Age)	1.20	2.33	(1.05)	1.26	2.39	(1.06)
	3rd (Age)	1.20	2.52	(1.09)	2.08	2.67	(1.08)
	4th (Age)	2.04	2.88	(1.08)	2.27	3.02	(1.04)
Exit Rate and Age	1st (Age)	.151	.110	(.318)	.136	.087	(.282)
	2nd (Age)	.151	.113	(.317)	.136	.056	(.231)
	3rd (Age)	.151	.105	(.306)	.090	.041	(.197)
	4th (Age)	.113	.060	(.265)	.081	.039	(.194)
Exit Rate and Size	1st (Size)	.162	.121	(.333)	.151	.122	(.327)
	2nd (Size)	.162	.101	(.312)	.151	.055	(.229)
	3rd (Size)	.162	.083	(.287)	.025	.038	(.191)
	4th (Size)	.024	.063	(.251)	.003	.030	(.171)
Product Loss Probability and Age	1st (Age)	.172	.038	(.190)	.170	.058	(.234)
	2nd (Age)	.172	.054	(.226)	.170	.077	(.266)
	3rd (Age)	.172	.057	(.232)	.234	.096	(.295)
	4th (Age)	.228	.063	(.244)	.254	.120	(.325)

Notes: U.S. data is from Compustat data (1980 to 2016). French data is from the full FICUS-FARE dataset (1994-2016). Size is measured as sector-deflated sales, age as the number of years since creation or Compustat entry. Exit is a dummy equal to 1 if a firm no longer appears in Compustat/FICUS-FARE in subsequent years. Product loss is a dummy equal to 1 if a firm produces fewer goods the subsequent year in the segment/EAP data. Items under 'model' and 'data' are the mean of the variable within the quartile considered.

Table A8: Data vs Model: Elasticity of Fixed-over-Total Costs Ratio with respect to Fixed Costs

	I			II		
Fixed costs over total costs (log)	Initial S.S.	Final S.S.	Data	Initial S.S.	Final S.S.	Data
United States						
Fixed costs (log)	0.49	0.74	0.95	0.49	0.46	0.88
	(.001)	(.002)	(.002)	(.001)	(.001)	(.003)
R^2	0.99+	0.89	0.96	0.99+	0.98	0.94
France						
	0.69	0.77	0.92	0.67	0.69	0.93
Fixed costs (log)	(.039)	(.015)	(.001)	(.018)	(.028)	(.000)
R^2	0.97	0.95	0.95	0.98	0.96	0.97
Firm Fixed Effects	No	No	No	Yes	Yes	Yes

Notes: Dependent variable is the log of the ratio of fixed costs over total costs. Explanatory variable is the log of fixed costs. Firm-clustered standard errors in parentheses. Columns headed 'Initial S.S.' are from simulated data for the initial steady state where ϕ_i is homogeneous across firms, while columns headed 'Final S.S.' are for the final simulation. The U.S. regression uses 115,564 observations from Compustat (1980-2015), the French regression uses 7,648,443 observations (1994-2015). Data regressions control for size through the log of revenue and time fixed effects, while regressions on the simulated data directly control for the log of product count.

Table A9: Data vs Model: Relationship between Firm Size and Fixed Cost Intangibles

	I			II		
Fixed costs (log)	Initial S.S.	Final S.S.	Data	Initial S.S.	Final S.S.	Data
United States						
Product count / Revenue (log)	1.00	1.07	0.90	1.00	1.00	0.90
	(.001)	(.001)	(.004)	(.001)	(.000)	(.007)
R^2	0.94	0.98	0.82	0.95	0.97	0.63
France						
Product count / Revenue (log)	1.01	1.09	0.80	1.01	1.00	0.52
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
R^2	0.91	0.96	0.54	0.92	0.92	0.16
Firm Fixed Effects	No	No	No	Yes	Yes	Yes

Notes: Dependent variable is the log of fixed costs. Explanatory variable is firm-size, measured through the log of product count in model columns and the log of revenue in data columns. Firm-clustered standard errors in parentheses. Columns headed 'Initial S.S.' are from simulated data for the initial steady state where ϕ_i is homogeneous across firms, while columns headed 'Final S.S.' are for the final simulation. The U.S. regression uses 115,564 observations from Compustat (1980-2015), the French regression uses 7,648,443 observations (1994-2015). Data regressions control for time fixed effects.

Table A10: Data vs Model: Relationship between Innovation and Fixed Cost Intangibles

	I			II		
R&D Intensity (log)	Initial S.S.	Final S.S.	Data	Initial S.S.	Final S.S.	Data
United States						
Fixed costs over total costs (log)	0.00	12.9	0.58	0.00	0.00	0.16
	(.000)	(.062)	(.019)	(.000)	(.001)	(.012)
R^2	1.00	0.81	0.25	1.00	1.00	0.04
France						
Fixed costs over total costs (log)	0.00	2.97	0.34	0.00	0.00	0.05
	(.000)	(.377)	(.017)	(.001)	(.001)	(.003)
R^2	1.00	0.52	0.11	1.00	1.00	0.03
Firm Fixed Effects	No	No	No	Yes	Yes	Yes

Notes: Dependent variable is the log of ratio of R&D spending over sales. Explanatory variable is the log of fixed costs over total costs. Firm-clustered standard errors in parentheses. Columns headed 'Initial S.S.' are from simulated data for the initial steady state where ϕ_i is homogeneous across firms, while columns headed 'Final S.S.' are for the final simulation. The U.S. regression uses 58,246 observations from Compustat (1980-2015), the French regression uses 20,666 observations (1994-2015). Data regressions control for firm size through log of revenue, model regressions through the log of product count.

Table A11: Data vs Model: Relationship between Markups and Fixed Cost Intangibles

	I			II		
Markup (log)	Initial S.S.	Final S.S.	Data	Initial S.S.	Final S.S.	Data
United States						
Fixed costs over total costs (log)	-0.84	0.11	0.44	-0.83	-0.91	-2.42
	(.003)	(.003)	(.036)	(.003)	(.008)	(9.59)
R^2	0.99	0.22	N.A.	0.99	0.93	N.A.
France						
Fixed costs over total costs (log)	-0.90	0.09	0.31	-0.36	-0.30	0.25
	(.073)	(.005)	(.007)	(.041)	(.049)	(.04)
R^2	0.69	0.19	N.A.	0.80	0.66	N.A.
Firm Fixed Effects	No	No	No	Yes	Yes	Yes

Notes: Dependent variable is the log of the firm's markup. Explanatory variable is the log of fixed costs. Firm-clustered standard errors in parentheses. Columns headed 'Initial S.S.' are from simulated data for the initial steady state where ϕ_i is homogeneous across firms, while columns headed 'Final S.S.' are for the final simulation. Data regressions are based on Table 4 columns III and IV.

Table A12: Relationship between Fixed Costs and Variable Costs (Estimation of θ)

	United States		France	
Variable Costs over Purged Sales	I	II	III	IV
Fixed Costs over Purged Sales (θ)	1.07	0.86	1.34	1.12
	(0.015)	(0.029)	(0.002)	(0.002)
Value in main calibration [robustness]	2.00 [0.86]	2.00 [0.86]	2.00 [0.86]	2.00 [0.86]
R^2	0.94	0.92	0.91	0.91
Observations	115,673	115,673	7,648,443	7,648,443
Year Fixed Effects & ϕ control	Yes	Yes	Yes	Yes
Firm fixed effect	No	Yes	No	Yes

Notes: Dependent variable is the ratio of variable costs over sales, purged for time effects. Explanatory variable is fixed costs over purged sales. Firm-clustered standard errors in parentheses. All regressions include a third-degree polynomial of the ratio of fixed costs over total costs, interacted with time fixed effects, to control for ϕ_i and w_t. Columns II and IV additionally include firm fixed effects.

Table A13: Balanced Growth Path Comparison - Robustness Checks for θ

	United States				France					
	Δ Model	**Δ Model**	**Δ Model**	**Δ Data**	**Δ Model**	**Δ Model**	**Δ Model**	**Δ Model**	**Δ Model**	**Δ Data**
	($\theta = 2$)	($\theta = 0.86$)	($\theta = 0.86$)		($\theta = 2$)	($\theta = 0.86$)	($\theta = 0.86$)	($\theta = 1.12$)	($\theta = 1.12$)	
	Shock A	Shock B	Shock A		Shock A	Shock B	Shock A	Shock B	Shock A	
Cost Structure										
Fixed cost (%)	10.6 pp	10.7 pp	16.1 pp	10.5 pp	4.5 pp	4.5 pp	12.2 pp	4.5 pp	6.5 pp	4.5 pp
Productivity										
Prod. Growth	-0.32 pp	-0.14 pp	- 0.47 pp	-0.9 pp	-0.07 pp	-0.04 pp	-0.32 pp	-0.03 pp	-0.08 pp	-1.3 pp
R&D/v.a.	34.8%	25.4%	48.5%	64.5%	22.1%	8.5%	27.4%	20.9%	30.0%	5.6%
Business Dyn.										
Entry rate	-4.5 pp	-1.6 pp	-6.6%	-5.8 pp	-1.0 pp	-0.5 pp	-3.4 pp	-0.7 pp	-1.3 pp	-3.8 pp
Realloc. Rate	-35.9%	-24.2 %	-42.5%	-23%	-17.0%	-13.2 %	-36.3 %	-13.2%	-19.3 %	-23%
Model Objects										
Labor Wedge	6.7 pt	5.5	10.7	N.A.	3.6 pt	1.8 pt	6.4 pt	3.6 pt	5.2 pt	N.A.
Efficiency Wedge	0.04 pt	0.04 pt	0.03 pt	N.A.	0.036 pt	0.023 pt	0.051 pt	0.056 pt	0.08 pt	N.A.

Notes: This table contains a robustness check for the balanced growth path results in Table 7. Rather than estimating the model with $\theta = 2$, the model is estimated with $\theta = 0.86$. This achieves a pass-through of marginal cost shocks to markups of -35% rather than -25%, in line with the main results in Amiti et al. (2019). For France there is an additional column with $\theta = 1.12$, in line with the relationship between fixed costs and variable costs in Table A12. Data columns present the empirical moments, while model columns present the theoretical moments. Columns headed Shock A provide the change in the steady state variables when the same shock is applied to firms as in the main calibration. Shock B presents results where the shock is re-estimated for each new calibration. They are included separately, as the re-calibrated shock assigns the higher intangible efficiency to a larger fraction of entrants than in the baseline calibration, such that the model predicts significantly smaller declines in entry than the baseline calibration and the data. For shock B the respective calibrations (in order of columns) for $[\overline{\phi}/\phi, G(\overline{\phi})]$ is [0.85, 0.251], [0.92, 0.121], and [0.86, 0.083]. Shock A is [0.74,0.084] for the U.S., [0.83,0.054] for France.

Table A14: Structural Estimation - Alternative Value Function Specification

		United States				France			
Par.	Moment	Par. Value	Data	Model	Model	Par. Value	Data	Model	Model
		(Old/New)	(Main)	(Full)	Target	(Old/New)	(Main)	(Full)	Target
$\overline{\lambda}$	Productivity Gr.	0.06/0.06	1.4%	1.3%	1.3%	0.064/0.064	1.4%	1.3%	1.3%
ϕ	Fixed Costs (%)	0.215/0.215	11.9%	12.9%	12.9%	0.279/0.279	9.3%	9.5%	9.5%
σ	Gibrat's Law	0.519/0.55	-0.036	-0.035	-0.035	0.636/0.690	-0.035	-0.035	-0.035
η^e	Entry Rate	2.47/2.47	13.9%	13.1%	13.8%	1.73/1.73	9.6%	8.5%	10.0%
η^x	R&D Intensity	3.41/3.41/	2.7%	2.5%	2.5%	2.87/2.87	3.1%	2.7%	3.2%

Table A15: Comparison of Steady States - Constant Markup

	United States			France		
	Δ Model (Var. μ)	**Δ Model** (Fixed $\overline{\mu}$)	**Δ Data**	**Δ Model** (Var. μ)	**Δ Model** (Fixed $\overline{\mu}$)	**Δ Data**
Cost Structure						
Average Fixed-Cost Share	10.6 pp	12.1 pp	10.5 pp	4.5 pp	2.7 pp	4.5 pp
Slowdown of Productivity Growth						
Productivity Growth Rate	-0.3 pp	-0.5 pp	-0.9 pp	-0.1 pp	-0.12 pp	-1.3 pp
Aggregate R&D over Value Added	34.8%	-29.6%	64.5%	22.1%	-10.7%	5.6%
Decline of Business Dynamism						
Entry rate	-4.5 pp	-4.7 pp	-5.8 pp	-1.0 pp	-0.4 pp	-3.8 pp
Reallocation Rate	-35.9%	-51.0%	-23%	-17.0%	-15.9%	-23%

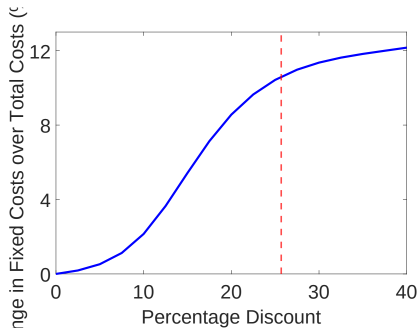
Notes: Data columns present the empirical moments, while model columns present the theoretical moments. Model - $\bar{\mu}$ columns present theoretical moments where markups are exogenous and homogeneous across firms in both steady states. The change in productivity growth is the difference between growth from 1969-1994 (France) or 1969-1979 (U.S.) to growth post-2005. Other French moments equal the difference between values in 1994 and in 2016. Other U.S. moments equal the difference between 1980 and 2016.

Table A16: Comparison of Steady States - Alternative Value Function Specification

	United States			France		
	Δ Model (Main)	**Δ Model** (Full Val.)	**Δ Data**	**Δ Model** (Main)	**Δ Model** (Full Val.)	**Δ Data**
Cost Structure						
Average Fixed-Cost Share	10.6 pp	10.8 pp	10.5 pp	4.5pp	5.0 pp	4.5 pp
Slowdown of Productivity Growth						
Productivity Growth Rate	-0.3 pp	-0.4 pp	-0.9 pp	-0.1 pp	-0.1 pp	-1.3 pp
Aggregate R&D over Value Added	34.8%	46.4%	64.5%	22.1%	34.1%	5.6%
Decline of Business Dynamism						
Entry rate	-4.5 pp	-5.1 pp	-5.8 pp	-1.0 pp	-1.3 pp	-3.8 pp
Reallocation Rate	-35.9%	-37.9%	-23%	-17.0%	-17.4%	-23%
Rise of Market Power						
Average Markup	14.7 pt	13.9pt	30 pt	6.4pt	6.1 pt	11 pt
Model Objects						
Labor Wedge	6.7 pt	6.4 pt	N.A.	3.6 pt	3.5 pt	N.A.
Efficiency Wedge	.04 pt	.04 pt	N.A.	.04 pt	.04 pt	N.A.

Notes: Data columns present the empirical moments, while Model - Main columns present the theoretical moments from the model in the main analysis. Model - Full Val. columns present moments where the value function includes the R&D option value. The change in productivity growth is the difference between growth from 1969-1994 (France) or 1969-1979 (U.S.) to growth post 2005. Other French moments equal the difference between values in 1994 and in 2016. Other U.S. moments equal the difference between 1980 and 2016.

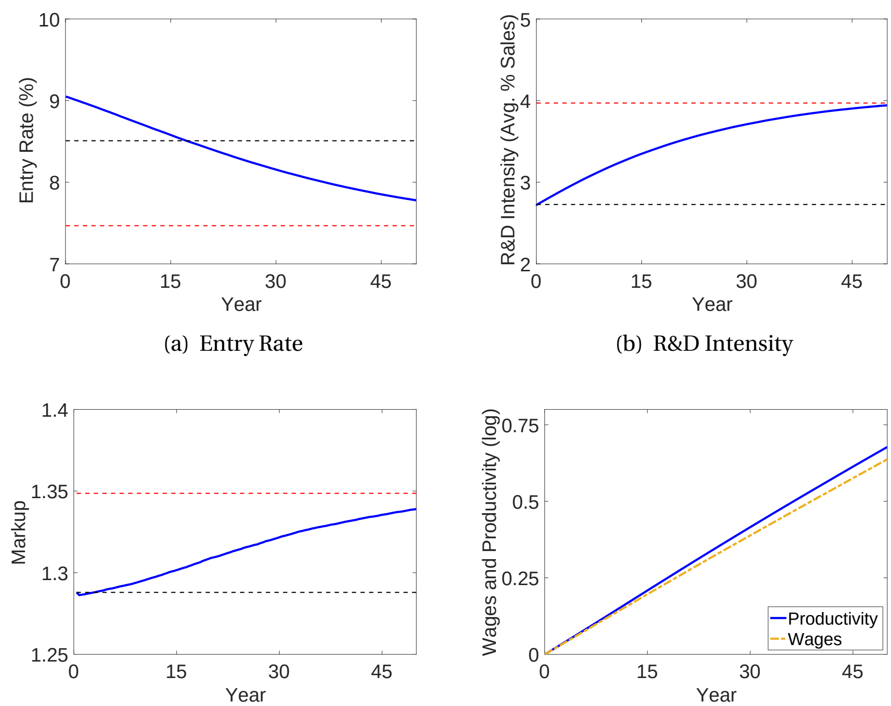
Figure A10. Relationship between Average Fixed Costs and Intangible Costs $\overline{\phi}$



(a) United States (b) France

Notes: The figure presents a comparative static, plotting the effect of the intangibles cost parameter $\overline{\phi}$ on the average ratio of fixed costs over total costs along the balanced growth path. The horizontal axis plots $(1-\overline{\phi}/\phi)\times 100\%$, that is, it plots the percentage discount on intangible costs that the low-cost firms receive. Vertical-red lines plot the percentage discount in the calibration of the final steady state. The vertical axis measures average fixed costs across products, which is the model-consistent counterpart of the revenue-weighted firm-level fixed costs (plotted in Figure A3) as revenue is proportional to a firm's product count in the model.

Figure A11. Transition Path for Various Variables (France)

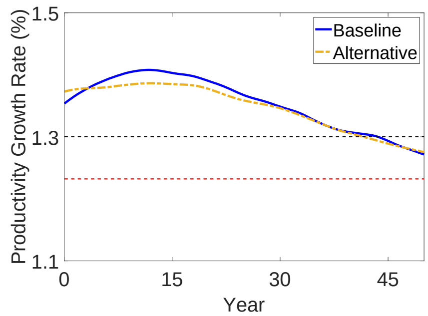


(a) Entry Rate (b) R&D Intensity

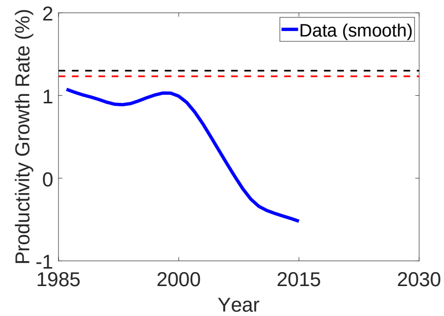
(c) Markups (d) Wages and Productivity

Notes: Black and red dashed lines (respectively) indicate the original and the new steady state. Figure (a) presents the entry rate, (b) presents R&D intensity (the average ratio of R&D over sales), (c) presents the average markup, (d) presents the path of wages (which tracks quality) and productivity (which tracks quality and intangibles).

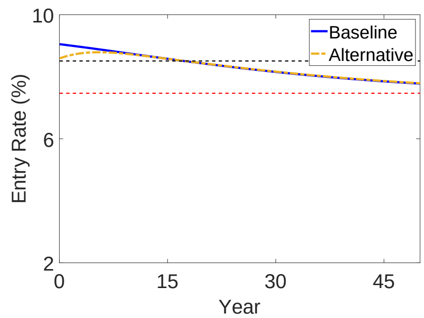
Figure A12. Transition Path: Model Predictions versus Data (France)



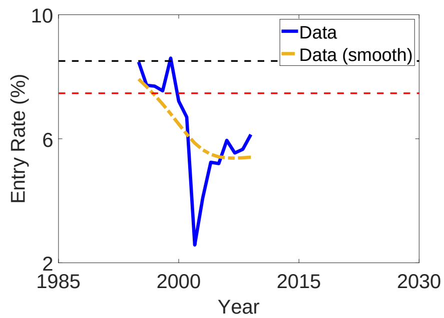
(a) Productivity Growth* (Model)



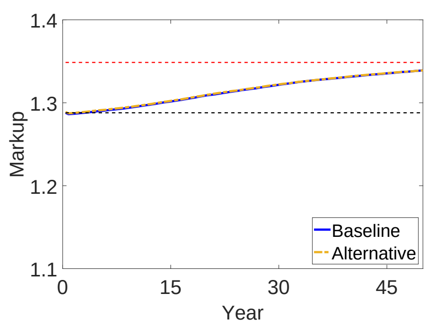
(b) Productivity Growth (Data)



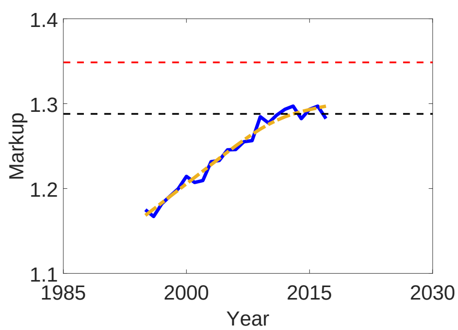
(c) Entry Rate (Model)



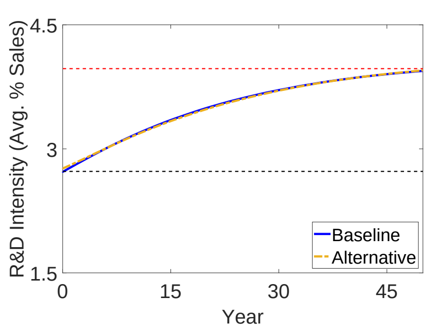
(d) Entry Rate (Data)



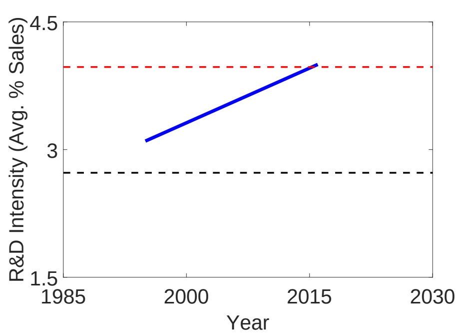
(e) Markup (Model)



(f) Markup (Data)



(g) R&D Intensity (Model)



(h) R&D Intensity (Data)*

Notes: Black and red dashed lines (respectively) indicate the original and the new steady state. Calibration is for France. Productivity growth in Figure 8b only includes smoothed data as productivity growth is highly volatile. HP-filter smoothing parameter is 100. Data sources: productivity growth from Penn World Tables, R&D from CIS (1996, 2016), entry (imputed using trend in entrant employment weights) and markups from FICUS-FARE. Note that productivity growth is not plotted on the same scale between model and data.

Appendix G. Alternative Calibration: Markup Target

The model in the main analysis predicts average markups of 1.45 for the U.S. calibration and 1.32 for the French calibration. This overstates the actual markups at the beginning of the sample. Markups are overestimated because the ratio of fixed costs over total costs is used to calibrate initial intangible cost ϕ_i. As markups are given by the ratio of innovation steps λ_{ij} and the fraction of marginal costs retained s_{ij}, lower intangible costs yield higher markups. No separate parameter disciplines markups.

In this robustness check I recalibrate the model to match average markups. Markups replace fixed-over-total costs as the moment related to ϕ_i. I target initial markups of 1.27 and 1.17 in the U.S. and French calibration, respectively. Table A17 presents the estimation results. The new calibrations have higher intangible cost parameters, so that firms chose a higher s_{ij} and therefore have lower markups than in the main calibration. Because the lower markups reduce the incentive to innovate, the new calibrations have lower innovation cost scalars to preserve innovation and growth rates. Table A18 presents the model's performance on the main targets. The model matches aggregate growth and the relationship between firm size and firm growth perfectly, while it understates U.S.entry and French R&D intensity. Average markups are matched with precision in both calibrations. In exchange for the well-matched markups, the model now underpredicts fixed costs as a percentage of total costs. One interpretation is that fixed costs in the data comprise of intangibles-induced fixed costs, which raise markups, and other fixed costs. Other fixed costs are not present in the model and therefore create a tradeoff in simultaneously matching markups and fixed costs.

The main experiment, in which a group of high-intangible firms is introduced in the economy, is rerun in Table A19. The experiment now targets changes in entry rates and in average markups. The table presents changes in the balanced growth path values of the variables of interest. Columns headed 'Main' are reproductions of Table A16 in the main text while columns headed 'Markup Calib.' contain results for the new calibration. Overall, results for the new calibration are in line with the original results. Compared to the main calibration, the model performs better at matching the (targeted) rise of markups, as well as the (untargeted) decline of the reallocation rate. The model is still able to explain around one third of the slowdown of productivity growth, although the predicted increase in R&D now vastly exceeds the increase in the data. The rise of fixed costs is well-matched for the U.S., although it is overstated in the French calibration. Overall, the rise of intangibles has a similar qualitative effect in the model: it reduces productivity growth and dynamism, and raises market power and R&D.

Table A17: Overview of Parameters - Markup Target

Parameter	**Description**	**Main** Calib. (U.S.)	**Markup** Calib. (U.S.)	**Main** Calib. (Fr.)	**Markup** Calib. (Fr.)
η^x	Cost scalar innov. (incumbents)	3.41	2.05	1.73	1.56
η^e	Cost scalar innov. (entrants)	2.47	2.58	2.87	1.78
$\overline{\lambda}$	Average innovation step size	.060	.067	.064	.067
σ	OLS reg. firm-size and growth	.519	.574	.636	.552
ϕ	Intangible costs	.215	.292	.279	.369

Notes: This table presents values for the structurally estimated parameters. Columns headed 'Main' contain parameters in the main analysis and reproduce the entries in Table A17. Columns headed Markup Calib. contain the calibration that uses the markups target.

Table A18: Comparison of Theory and Data for Targeted Moments: Calibration with Markup Target

			United States		France	
Parameter	**Moment**	**Weight Ω**	**Model**	**Target**	**Model**	**Target**
$\overline{\lambda}$	Long-term growth rate of productivity	2	1.3%	1.3%	1.3%	1.3%
ϕ	Average markup	2	1.27	1.27	1.17	1.17
ϕ	Fixed costs as a fraction of total costs	0	9.2%	12.9%	5.4%	9.5%
σ	Relation between firm growth and size	1	-.035	-.035	-.035	-.035
η^e	Entry rate (fraction of firms age 1 or less)	1	9.4%	13.8%	9.9%	10.0%
η^x	Ratio of research and development to sales	1	2.3%	2.5%	1.6%	3.2%

Table A19: Comparison of Steady States - Calibration with Markup Target

	United States			France		
	Δ Model (Main)	**Δ Model** (Markup Calib.)	**Δ Data**	**Δ Model** (Main)	**Δ Model** (Markup Calib.)	**Δ Data**
Cost Structure						
Average Fixed-Cost Share	10.6 pp	10.4 pp	10.5 pp	4.5 pp	6.8 pp	4.5 pp
Slowdown of Productivity Growth						
Productivity Growth Rate	-0.3 pp	-0.3 pp	-0.9 pp	-0.07 pp	-0.04 pp	-1.3 pp
Aggregate R&D over Value Added	34.8%	91.7%	64.5%	22.1%	47.1%	5.6%
Decline of Business Dynamism						
Entry rate	-4.5pp	-5.8 pp	-5.8 pp	-2.4 pp	-1.0 pp	-1.2 pp
Reallocation Rate	-35.9%	-27.8%	-23%	-17.0%	-17.2%	-23%
Rise of Market Power						
Average Markup	14.7 pt	27.8 pt	30 pt	6.4 pt	11 pt	11 pt
Model Objects						
Labor Wedge	6.7 pt	14.2 pt	N.A.	3.6 pt	7.3 pt	N.A.
Efficiency Wedge	0.04 pt	0.02 pt	N.A.	0.04 pt	0.12 pt	N.A.

Notes: Data columns present the empirical moments, while Model - Main columns present the theoretical moments from the model in the main analysis. Model - Markup Calib. columns present moments when the calibration uses the markup moment for intangible use. The change in productivity growth is the difference between growth from 1969-1994 (France) or 1969-1979 (U.S.) to growth post 2005. Other French moments equal the difference between values in 1994 and in 2016. Other U.S. moments: difference between 1980 and 2016.

Appendix H. Optimal Allocation of Researchers

Growth in experiment (1)

The growth rate of total factor productivity is the product of the measure of patents that researchers generate and the average improvements to quality that these patents embody:

$$\tilde{g} = \overline{\lambda}\left(e + \sum_{\phi_i \in \Phi} \sum_{n=1}^{\infty} M_n(\phi_i) x_n(\phi_i)\right),$$

where the innovation by entrants and incumbents is multiplied by the average innovation step size $\overline{\lambda}$ because all innovation is successful.

Growth in experiment (2)

To calculate the efficient growth rate, I first calculate the number of researchers $\overline{L}^{rd}$ that incumbents employ in the final steady state:

$$\overline{L}^{rd} = \sum_{\phi_i \in \Phi} \sum_{n=1}^{\infty} M_{\phi_i,n} \eta^x x(\phi_i, n)^{\psi^x} n^{-\sigma},$$

I then allocate them across firms of different sizes so that the marginal research productivity of researchers is equalized:

$$\widetilde{rd}_n^x = \overline{L}^{rd} \left(n^{\frac{\sigma}{\psi^x - 1}} \left[\sum_{n=1}^{\infty} M_n n^{\frac{\sigma}{\psi^x - 1}} \right]^{-1} \right), \tag{A.14}$$

where $M_n = \sum_{\phi \in \Phi} M_n(\phi)$ is the measure of size n firms in the final steady state and where $\widetilde{rd}_n^x$ is the optimal measure of researchers assigned to firms of size n. The growth rate of productivity is:

$$\tilde{g} = \overline{\lambda} \left(e + \sum_{n=1}^{\infty} M_n \left(\widetilde{rd}_n^x n^{\sigma} / \eta^x \right)^{1/\psi^x} \right). \tag{A.15}$$

Growth in experiment (3)

The total number of researchers available is $\overline{L}^{rd} + \overline{L}^e$, where the second term measures the number of researchers employed by entrants, $\overline{L}^e = \eta^e e^{\psi^e}$. Given $\psi^x = \psi^e$, the optimal allocation to incumbents is:

$$\widetilde{rd}_n^x = \left(\overline{L}^{rd} + \overline{L}^e \right) \left((\eta^x / \eta^e)^{\frac{1}{\psi^x - 1}} + \sum_{n=1}^{\infty} M_n n^{\frac{\sigma}{\psi^x - 1}} \right)^{-1} n^{\frac{\sigma}{\theta - 1}}, \tag{A.16}$$

while the optimal allocation to entrants is:

$$\widetilde{rd}^e = \left(\overline{L}^{rd} + \overline{L}^e \right) \left((\eta^x / \eta^e)^{\frac{1}{\psi^x - 1}} + \sum_{n=1}^{\infty} M_n n^{\frac{\sigma}{\psi^x - 1}} \right)^{-1} (\eta^x / \eta^e)^{\frac{1}{\psi^x - 1}}. \tag{A.17}$$

Hence the growth rate of productivity is:

$$\tilde{g} = \overline{\lambda} \left(\left(\widetilde{rd}^e / \eta^e \right)^{1/\psi^e} + \sum_{n=1}^{\infty} M_n \left(\widetilde{rd}_n^x n^{\sigma} / \eta^x \right)^{1/\psi^x} \right). \tag{A.18}$$

Growth in experiments (4) and (5): set $M_1 = 1$ and $M_n = 0$ for $n > 1$ in growth equations (A.15) and (A.18) and in R&D equations (A.14), (A.16), and (A.17).

Research subsidy to resolve misallocation

In the remainder of this appendix, I quantify the wedge between the private returns to innovation across entrants and incumbents with different intangible efficiencies. To do so, I calculate the factor by which a targeted research subsidy, Ξ_i^1, would have to multiply the private value of research at high-ϕ and entrants, relative to the subsidy (or tax) imposed on low-ϕ firms. I also introduce a homogeneous subsidy (or tax) on R&D, Ξ^2, which controls the aggregate fraction of resources that is devoted to R&D.

My focus is on the Ξ_i^1 that equates the marginal research productivity across researchers, to quantify the difference in private rates of return to R&D across firms. I abstract from optimizing the homogeneous Ξ^2, as this is the tool through which the social planner would address the usual Schumpeterian innovation distortions, which the model has in common with the rest of the literature.[75] Taxes enter an incumbent's first-order condition as follows:

$$\left(x_i/n_i^{\frac{\sigma}{\psi^x-1}}\right)=\left(\mathscr{P}(\phi_i)\Xi_i^1\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i,\lambda_{ij})}{r-g+\tau(\phi_i)}\right](\eta^x\psi^x w\Xi^2)^{-1}\right)^{\frac{1}{\psi^x-1}}, \tag{A.19}$$

where I standardize the equation by product-count because the wedge between the firm's private return from research and its research productivity does not depend on n_i. The Ξ_i^1 that implements the statically optimal allocation for experiment 2 (A.14) and experiment 3 (A.16) are given by:

$$\Xi_i^1=\overline{mpr}\left(\mathscr{P}(\phi_i)\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i,\lambda_{ij})}{r-g+\tau(\phi_i)}\right]\right)^{-1}$$

in both cases, where $\overline{mpr}$ denotes the (common) marginal research product. In experiment 2, this is

$$\overline{mpr}=\left[\overline{L}^{rd}\left(\left[\sum_{n=1}^{\infty}M_n n^{\frac{\sigma}{\psi^x-1}}\right]^{-1}\right)\right]^{\frac{1-\psi^x}{\psi^x}}(\eta^x)^{\frac{1}{\psi^x}}(\psi^x)^{-1},$$

where $\overline{L}^{rd}$ is pinned down by Ξ^2. Conversely, when reallocating all researchers (experiment 3), $\overline{mpr}$ is

$$\overline{mpr}=\left[\left(\overline{L}^{rd}+\overline{L}^{e}\right)\left((\eta^x/\eta^e)^{\frac{1}{\psi^x-1}}+\sum_{n=1}^{\infty}M_n n^{\frac{\sigma}{\psi^x-1}}\right)^{-1}\right]^{\frac{1-\psi^x}{\psi^x}}(\eta^x)^{\frac{1}{\psi^x}}(\psi^x)^{-1}, \tag{A.20}$$

The first-order condition for entry in the presence of the research subsidies (or taxes) is given by

$$e=\left(\sum_{\phi_e\in\Phi}G(\phi_e)\mathscr{P}(\phi_e)\Xi_e^1\mathbb{E}_{\phi_h}\left[\frac{\pi(\phi_e,\lambda_{ej})}{r-g+\tau(\phi_e)}\right](\eta^e\psi^e w\Xi^2)^{-1}\right)^{\frac{1}{\psi^e-1}}. \tag{A.21}$$

The marginal research product equals that of incumbents along (A.20). Given that $\psi^e=\psi^x$, the subsidy that equates the marginal research product of entrants to that of incumbents is given by

$$\Xi_e^1=\overline{mpr}\left(\sum_{\phi_e\in\Phi}G(\phi_e)\mathscr{P}(\phi_i)\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i,\lambda_{ij})}{r-g+\tau(\phi_i)}\right]\right)^{-1}\eta^e/\eta^x.$$

The results are presented in Table A20. It expresses the optimal Ξ_i^1 for entrants and high-ϕ incumbents relative to Ξ_i^1 for low-ϕ incumbents. By normalizing the Ξ_i^1 this way, the table quantifies the wedge between the private value of research across firms with different intangible efficiencies for a given Ξ^2.

Table A20: Required Relative R&D Subsidy to Offset Researcher Misallocation

	United States		France	
	Experiment 2 (reallocate inc. R&D)	Experiment 3 (reallocate all R&D)	Experiment 2 (reallocate inc. R&D)	Experiment 3 (reallocate all R&D)
Incumbents, high ϕ	4.74	4.74	2.14	2.14
Entrants	N.A.	3.61	N.A.	2.01

Notes: The table gives the R&D subsidy Ξ_i relative to Ξ_i for low-ϕ firms that equate marginal research products across researchers.

[75] The model also features a single inelastically supplied input, labor, so that the tradeoffs of a usual optimal taxation and subsidy exercise are not present. I therefore focus on Ξ_i^1, which addresses the new distortions in the model.

Appendix I. Alternative Production Function: Firm-Level Intangibles

I.1. Framework

I.1.1. Production and Intangibles

The framework follows the setup in Section 2, except where discussed in this appendix. There are two main changes. The first is that firms reduce the marginal costs across all their products in exchange for a fixed cost levied at the firm level. Total costs for firm i that produces portfolio J_i are therefore:

$$tc_i = \sum_{j \in J_i} w s_i y_{ij} + \phi_i w \left(s_i^{-\theta} - 1 \right),$$

where the first term denotes the firm's total variable costs and the second term denotes the firm's total fixed costs. The firm-level s_i identifies the share of marginal costs that the firm keeps. The optimal s_i now depends on both firm size n_i and the firm's intangible costs ϕ_i. As a second change, I assume that a firm's optimal markup is $\mu_i = \overline{\lambda}/s_i$, where $\overline{\lambda}$ is (say) the average innovation step size. The simplified pricing rule is needed, as the markup would depend on the use of intangibles by the firm's competitors in the rigorous Bertrand-Nash equilibrium.[76] The cost-minimizing firm sets intangibles so that

$$s_i = \min \left[\left(\left[\sum_{j \in J_i}^{n_i} y_{ij} \right]^{-1} \theta \phi_i \right)^{\frac{1}{\theta+1}}, 1 \right]. \tag{A.22}$$

The marginal cost of producing any of the firm's goods is $w s_i$, so that output under the optimal markup is given by $y_{ij} = Y/(w\overline{\lambda})$. Inserting this into the first-order condition above implies that s_i can be written in terms of a firm's number of products n_i and its intangibles cost parameter ϕ_i:

$$s_{n_i}(\phi_i) = \min \left[\left(n_i^{-1} w Y^{-1} \overline{\lambda} \theta \phi_i \right)^{\frac{1}{\theta+1}}, 1 \right],$$

where n_i is the cardinality of J_i, which implies that firms with more products choose higher fixed costs, as they benefit from lower marginal costs across a greater number of products.

I.1.2. Choke Prices and Creative Destruction

The rate of creative destruction declines in the number of products that a firm produces. High-n_i firms are able to produce at lower average costs, and are therefore able to sell at lower prices. Unless an innovator draws a sufficiently large innovation step, a higher-n_i incumbent can therefore keep producing the good. Recall that the rate of creative destruction is the product of the arrival rate of innovations by other firms on goods that the firm currently produces, and the probability that these firms have a sufficiently low choke price. If incumbent i faces an innovator of type ϕ_h that currently produces n_h goods

[76] Competing firms no longer set intangibles to zero in the alternative model, because they use intangibles across their products. The competitors' own use of intangibles, furthermore, changes over time if they start or cease to produce goods, and depends on the use of intangibles by the firms with which they, in turn, compete. Because the model is characterized by a continuum of products and firms, it is unfeasible to track the resulting infinite-dimensional object.

and that develops a higher-quality version of good j, the probability that firm h successfully takes over production is given by:

$$\text{Prob}\left(\lambda_{hj} \geq \frac{p^c_{n_h+1}(\phi_h)}{p^c_{n_i}(\phi_i)}\right) = \min\left[\left(\frac{p^c_{n_i}(\phi_i)}{p^c_{n_h+1}(\phi_h)}\right)^{\frac{\overline{\lambda}}{\overline{\lambda}-1}}, 1\right], \tag{A.23}$$

which uses the Pareto distribution of innovation steps with mean step size $\overline{\lambda}$. The choke price $p^c_{n_i}(\phi_i)$ is the price at which a type-ϕ_i firm is indifferent between producing n_i goods, including some good j, or $n_i - 1$ goods. The rate of creative destruction follows from taking the product of the probability (A.23) with the flow of innovative patents from firms of each size n_h, and entrants, summed across firm types. All firms face equal flows of innovations to each of their products, but the size-dependent probability of success yields that creative destruction rates strictly decline in n_i. Using notation from the main text, we have:

$$\tau_{n_i}(\phi_i) = \sum_{\phi_k \in \Phi}\left[\sum_{n_h=1}^{n_i} M_{n_h}(\phi_k)x_{n_h}(\phi_k)\left(\frac{P^c_{n_i}(\phi_i)}{P^c_{n_h+1}(\phi_k)}\right)^{\frac{\lambda}{\overline{\lambda}-1}} + \sum_{n_h=n_i+1}^{\infty} M_{n_h}(\phi_k)x_{n_h}(\phi_k) + G(\phi_k)e\left(\frac{P^c_{n_i}(\phi_i)}{P^c_1(\phi_k)}\right)^{\frac{\lambda}{\overline{\lambda}-1}}\right].$$

The creative destruction rate therefore hinges on the relative choke price across firms. To find the choke price, I compare profits for a type-ϕ_i firm that produces $n_i - 1$ goods under cost minimization and optimal markups

$$\pi_{n_i-1}(\phi_i) = (n_i - 1)(1 - s_{n_i-1}(\phi_i)\lambda^{-1})Y - \phi_i w(s_{n_i-1}(\phi_i)^{-\theta} - 1),$$

to profits for a firm i that produces some additional good j and sells it at price p_{ij},

$$\tilde{\pi}_i = \Big([n_i - 1][1 - \tilde{s}_i\lambda^{-1}]Y + (1 - p_{ij}^{-1} w\tilde{s}_i)Y\Big) - \phi_i w(\tilde{s}_i^{-\theta} - 1),$$

where optimal intangibles depend on p_{ij} because output of good j affects the firm's overall output, in line with first-order condition (A.22). The choke price for j sets $\tilde{\pi}_i = \tilde{\pi}_{n-1}$, and therefore solves

$$\tilde{s}_i^{\theta} s_{n_i-1}(\phi_i)^{\theta} - (n_i - 1)\lambda^{-1}\tilde{s}_i^{\theta} s_{n_i-1}(\phi_i)^{\theta}\left(\tilde{s}_i - s_{n_i-1}(\phi_i)\right) - \phi_i\frac{w}{Y}(s_{n_i-1}(\phi_i)^{\theta} - \tilde{s}_i^{\theta}) - w\tilde{s}_i^{\theta+1} s_{n_i-1}(\phi_i)^{\theta} p_{ij}^{-1} = 0,$$

which yields that for a given parameterization, wage and aggregate output, the choke price is only a function of n_i and ϕ_i. There is no analytical solution for the choke price as it appears with various powers through the intangibles first-order condition, although it is straightforward to see that $\lim_{n_i\to\infty} p^c_{n_i}(\phi_i) = 0$; as n_i becomes large, $s_{n_i-1}(\phi_i)$ converges to zero and optimal marginal costs eventually approach zero. The choke price therefore converges to zero. From the diminishing choke price in n_i it follows that $\lim_{n_i\to\infty} \tau_{n_i}(\phi_i) = 0$, that is, firms become unbeatable as their size increases.

I.1.3. Innovation

Firms maximize their value by choosing the flow rate x_i at which they receive a patent to produce goods that they do not currently produce. As in the main model, firms hire rd_i^x researchers for research and development as a function of x_i along

$$rd_{n_i}^x(x_i) = \eta^x x_i^{\psi^x} n_i^{-\sigma}. \tag{A.24}$$

The associated value function, with notation from the main text, reads as

$$rV_{tn_i}(\phi_i) - \dot{V}_{tn_i}(\phi_i) = \max_{x_i} \left\{ \begin{array}{c} \sum_{j \in J_i} \pi_{tn_i}(\phi_i) + \tau_{n_i}(\phi_i)\left[V_{tn_i-1}(\phi_i) - V_{tn_i}(\phi_i)\right] \\ +x_i \text{Prob}\left(\lambda_{ij} \geq \frac{p^c_{n_i+1}(\phi_i)}{p^c_{n_{-i}}(\phi_i)}\right)\left[V_{tn_i+1}(\phi_i) - V_{tn_i}(\phi_i)\right] - w_t \eta_x (x_i)^{\psi^x} n_i^{-\sigma}) \end{array} \right\},$$

where the main difference with the main text is that the value function, profits per product and the rate of creative destruction are now also a function of n_i. The solution to the value function is similar to the solution to the extended value function in Section 6.2, as the following proposition makes clear.

Proposition I.1. *The value function of a firm that produces a portfolio of goods J_i with cardinality n_i grows at rate g along the balanced growth path and is given by*

$$V_{n_i}(\phi_i) = n_i \Upsilon^1_{n_i}(\phi_i) + \Upsilon^2_{n_i}(\phi_i),$$

where $\Upsilon^1_{n_i}$ is the present value of the per-product, size-dependent profit stream for a firm that produces n_i goods and where time-subscripts are omitted for readability:

$$\Upsilon^1_{n_i}(\phi_i) = \frac{\pi_{n_i}(\phi_i)}{r - g + \tau_{n_i}(\phi_i)}.$$

while $\Upsilon^2_{n_i}$ is the option value of research and development, which evolves along the sequence

$$\begin{aligned} \Upsilon^2_{n_i+1}(\phi_i) \quad = \quad & \frac{(r-g)\Upsilon^2_{n_i}(\phi_i) - n_i \tau_{n_i}(\phi_i)\left(\Upsilon^2_{n_i-1}(\phi_i) + \Upsilon^2_{n_i-1}(\phi_i)\cdot(n_i - 1) - \Upsilon^2_{n_i}(\phi_i)\right)}{x_{n_i}(\phi_i)\cdot(1-\psi^x)\cdot \text{Prob}\left(\lambda_{ij} \geq \frac{p^c_{n_i+1}(\phi_i)}{p^c_{n_{-i}}(\phi_{-i})}\right)} \\ & + \Upsilon^2_{n_i}(\phi_i) + n_i \Upsilon^1_{n_i}(\phi_i) - \Upsilon^1_{n_i+1}(\phi_i)\cdot(n_i+1), \end{aligned}$$

where $x_{n_i}(\phi_i)$ is the value-maximizing rate of innovation. The dynamic first-order conditions are

$$x_{n_i}(\phi_i) = \left(\text{Prob}\left(\lambda_{ij} \geq \frac{p^c_{n_i+1}(\phi_i)}{p^c_{n_{-i}}(\phi_i)}\right) \frac{\left[(n_i+1)\Upsilon^1_{n_i+1}(\phi_i) - n_i \Upsilon^1_{n_i}(\phi_i) + \Upsilon^2_{n_i+1}(\phi_i) - \Upsilon^2_{n_i}(\phi_i)\right]}{\eta^x \psi^x w_t}\right)^{\frac{1}{\psi^x - 1}} n_i^{\frac{\sigma}{\psi^x - 1}}.$$

$$e = \left(\sum_{\phi_e \in \Phi} \text{Prob}\left(\lambda_{ej} \geq \frac{p^c_1(\phi_e)}{p^c_{n_{-i}}(\phi_{-i})}\right) \frac{\left[\Upsilon^1_1(\phi_e) + \Upsilon^2_1(\phi_e)\right]}{\eta^e \psi^e w}\right)^{\frac{1}{\psi^e - 1}}.$$

Proof: Closely follows proof for Proposition A.1 in Appendix A.

It follows that innovation rate $x_{n_i}(\phi_i)$ depends on firm-size through three channels. First, profits increase in n_i because large firms have lower average costs and higher markups because they deploy more intangibles. The rate at which profits from acquiring a product are discounted is also lower for high-n_i firms because of their lower rates of creative destruction. The third channel is through innovation costs: the inclusion of $n_i^{-\sigma}$ in (A.24) implies that the number of researchers that firms hire for a given innovation rate depends on n_i. In the main model, $\sigma > 0$ so that large firms have lower innovation costs. This in line with Akcigit and Kerr (2018), and assures that the model matches empirical evidence on the firm-size, firm-growth relationship. As will be clear from the next section, the model with firm-level intangibles requires $\sigma < 0$ in order to have a solution.[77]

I.1.4. Firm-size distribution

Finally, consider the implication of firm-level intangibles for the firm-size distribution. The firm-size distribution is stationary along the balanced growth path, if the model admits one. To find the stationary distributions, consider the law of motion for the measure of firms with more than one product:

$$\begin{aligned}
\dot{M}_{n_i}(\phi_i) \quad = \quad & M_{n_i-1}(\phi_i)x_{n_i-1}(\phi_i)\text{Prob}\left(\lambda_{ij} \geq \frac{p^c_{n_i}(\phi_i)}{p^c_{n_{-i}}(\phi_{-i})}\right) - M_{n_i}(\phi_i)x_{n_i}(\phi_i)\text{Prob}\left(\lambda_{ij} \geq \frac{p^c_{n_i+1}(\phi_i)}{p^c_{n_{-i}}(\phi_{-i})}\right) \\
& + \left(M_{n_i+1}(\phi_i)\tau_{n_i+1}(\phi_i)\cdot(n_i+1) - n_i M_{n_i}(\phi_i)\tau_{n_i}(\phi_i)\right).
\end{aligned}$$

The first term captures entry and exit out of the measure of firms with n_i products through innovation by firms with $n_i - 1$ and n_i products, respectively. The second term captures entry and exit of firms with $n_i + 1$ and n_i products that ceased producing one of their products through creative destruction. For the measure of single-product firms, the law of motion reads as

$$\dot{M}_1(\phi_i) = \left(\text{Prob}\left(\lambda_{ij} \geq \frac{p^c_1(\phi_i)}{p^c_{n_{-i}}(\phi_i)}\right)e - x_1(\phi_i)M_1(\phi_i)\text{Prob}\left(\lambda_{ij} \geq \frac{p^c_2(\phi_i)}{p^c_{n_{-i}}(\phi_i)}\right)\right) + \left(2M_2(\phi_i)\tau_2(\phi_i) - M_1(\phi_i)\tau_1(\phi_i)\right),$$

where e is the entry rate. The stationary firm-size distribution follows from setting both equations to zero for each n_i. The model features a stationary firm-size distribution as long as expected firm growth remains negative as $n_i \to \infty$. The expected growth rate $\text{E}(g_i)$ of a firm that produces n_i products and has intangible costs ϕ_i is given by

$$\text{E}(g_i) = \frac{x_{n_i}(\phi_i)}{n_i} - \tau_{n_i}(\phi_i),$$

where the value-maximizing per-product innovation rate is given by

$$\frac{x_{n_i}(\phi_i)}{n_i} = \left(\text{Prob}\left(\lambda_{ij} \geq \frac{p^c_{n_i+1}(\phi_i)}{p^c_{n_{-i}}(\phi_i)}\right)\frac{\left[(n_i+1)\Upsilon^1_{n_i+1}(\phi_i) - n_i\Upsilon^1_{n_i}(\phi_i) + \Upsilon^2_{n_i+1}(\phi_i) - \Upsilon^2_{n_i}(\phi_i)\right]}{\eta^x\psi^x w_t}\right)^{\frac{1}{\psi^x-1}} n_i^{\frac{\sigma}{\psi^x-1}-1}.$$

[77] A fourth channel is that the change in the innovation option value depends on n_i, but the direction of this effect depends on the relative magnitude of the other three channels.

Table A21: Overview of Estimated Parameters: Model with Firm-Level Intangibles

Parameter	Description	Method	Value (U.S.)	Value (France)
$\overline{\lambda}$	Average innovation step size	Indirect inference	.138	.129
ϕ	Intangible costs	Indirect inference	.579	.834
σ	Relationship firm-size and firm-growth	Indirect inference	-7.82	-4.54
η^e	Cost scalar of innovation (entrants)	Indirect inference	2.69	8.82
η^x	Cost scalar of innovation (incumbents)	Indirect inference	6.91	4.42

Given that profits and the success probability increase in firm-size, it follows that a sufficiently low σ is needed to guarantee that expected firm-growth does not rise above zero when firm-size increases, to offset the decline of the creative destruction rate.

I.2. Quantification

Because firm-level intangibles change the incentive to reduce marginal costs and the relationship between firm-size and firm-growth, I perform a new structural estimation for the initial steady state. As in Section 6.1 and 6.2, I leave the percentage reduction in intangible costs and the fraction of entrants that receive the lower costs is the same as in the main calibration. This facilitates a direct comparison of the effect of introducing a high-intangible group of firms in the models with product- and firm-level intangibles. The moments and the algorithm in the structural estimation are also unchanged.

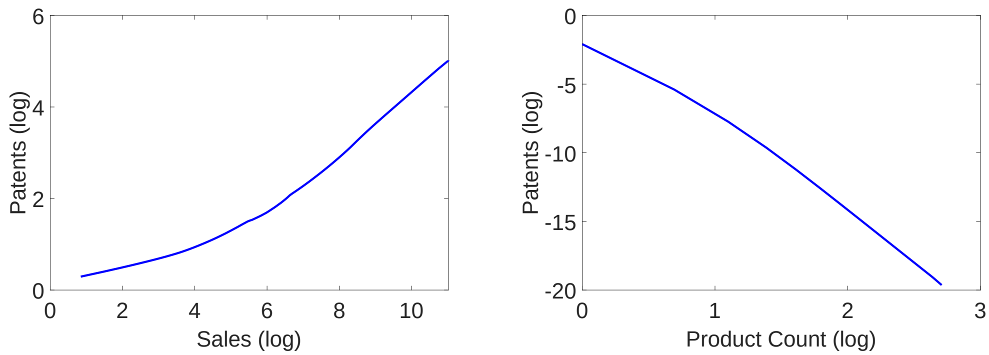
The parameter estimates are summarized in Table A21. The calibration has changed in two main ways from the main model in Table 5. First, there is a substantial increase in the initial (homogeneous) intangible costs ϕ, which offsets the additional incentive to reduce marginal costs because intangibles now apply across goods that firms produce. Second, the parameter σ, which governs the relationship between firm-size and innovation costs, is now large and negative. At -4.50 in the U.S. calibration, the parameter yields that the same number of researchers in a firm that produces $n_i = 2$ goods will on average create 82% fewer patents than the same number of researchers in a firm that produces $n_i = 1$ goods.[78] This contrasts with other Klette and Kortum (2004) models, which assume that the same number of researchers produce more patents in larger firms. The model therefore predicts a steep decline in innovation productivity with size.

The negative effect of size on research productivity creates strong counterfactual predictions on the relationship between size and innovative activity. The model predicts, for instance, that the flow rate of patents to incumbents, $x_{n_i}(\phi_i)$, falls rapidly with size. It is straightforward to contrast this with empirical evidence from U.S. patents.[79] I obtain patent data from Stoffman et al. (2019), who extend the dataset of Kogan et al. (2017) until 2015, and merge the patents to the Compustat sample using CRSP identifiers. Figure A13a plots this relationship with (log) sales on the horizontal axis and the (log) number of patents that firms produce, both winsorized at 1% sales. The solid-blue line presents estimates from a lowess regression between both, which shows a strong and near-linear positive relationship for all values of sales. An OLS regression yields a coefficient of 0.46 with a standard error

[78] This follows from the fact that the expected number of patents created by $\overline{rd}$ researchers according to the innovation-cost function is $\overline{x}_{n_i} = (\eta^x)^{-1/\psi^x} n^{\sigma/\theta} (\overline{rd})^{1/\psi^x}$, where $\psi^x = 2$ and $\sigma = -4.5$.

[79] At the time of writing, there exists no publicly available link of French patents to firms in the FARE-FICUS dataset.

Figure A13. Relationship between Firm Size and Innovation: Data versus Model



(a) Data (b) Model

Notes: Left-hand figure plots the relationship between log of patents expenditures (vertical axis) and log sales from a lowess regression. Data is from the Compustat sample from the manuscript. Right-hand figure plots log optimal patents ($x_{n_i}(\phi_i)$) in the model, against the log number of products that firms produce.

of 0.01. Figure A13b plots the model's counterpart in the form of the relationship between the log-optimal innovation rate $x_{n_i}(\phi_i)$ and the log-number of goods that the firm produces.[80] The figure confirms that, with the negative σ required to have a non-degenerate firm-size distribution, the model features a clear counterfactual prediction for the relationship between innovation and size.

The model's ability to match its targeted moments is displayed in Table A22. The table shows that the model with firm-level intangibles is well able to predict firms' average fixed costs, the steady-state growth rate of total factor productivity along the balanced growth path, as well as the relationship between firm-growth and firm-size. Compared to the main model, the model with firm-level intangibles struggles to match the empirical entry rate and the average ratio of research and development to sales in both the U.S. and the French structural estimation. Overall, the model's ability to match empirical moments is slightly worse, which is likely due to the interrelationship between parameters: because profitability and creative destruction rates now strongly depend on firm size, the parameters governing innovation costs, firm growth, and aggregate growth are now closely connected.

Table A22: Comparison of Theory and Data for Targeted Moments

			United States		France	
Parameter	**Moment**	**Weight Ω**	**Model**	**Target**	**Model**	**Target**
$\bar{\lambda}$	Long-term growth rate of productivity	2	2.1%	1.3%	1.3%	1.3%
ϕ	Fixed costs as a fraction of total costs	2	13.0%	12.9%	9.2%	9.5%
σ	Relation between firm growth and size	1	-.036	-.035	-.037	-.035
η^e	Entry rate (fraction of firms age 1 or less)	1	10.7%	13.8%	5.9%	10%
η^x	Ratio of research and development to sales	1	5.4%	2.5%	4.0%	3.2%

[80] Sales is the appropriate measure of size, as it is proportional to the number of products that firms sell in the model.

Table A23: Comparison of Steady States, Firm-Level Intangibles

	United States			France		
	Δ Model (Main)	**Δ Model** (Firm-Intan)	**Δ Data**	**Δ Model** (Main)	**Δ Model** (Firm-Intan)	**Δ Data**
Cost Structure						
Average Fixed-Cost Share	10.6 pp	11.9 pp	10.5 pp	4.5 pp	10.4 pp	4.5 pp
Slowdown of Productivity Growth						
Productivity Growth Rate	-0.3 pp	-0.8 pp	-0.9 pp	-0.1 pp	-0.4 pp	-1.3 pp
Aggregate R&D over Value Added	34.8%	75.9%	64.5%	22.1%	37.5%	5.6%
Decline of Business Dynamism						
Entry rate	-4.5 pp	-2.7pp	-5.8 pp	-1.0 pp	-0.5 pp	-3.8 pp
Reallocation Rate	-35.9%	-54.4%	-23%	-17.0%	-39.1%	-23.0%
Rise of Market Power						
Average Markup	14.7pt	12.9 pt	29.7 pt	6.4 pt	8.9 pt	11 pt

Notes: Data columns present the empirical moments, while Model - Main columns present the theoretical moments from the model in the main analysis. Firm-Intan. columns present moments for the model with firm-level intangibles. The change in productivity growth is the difference between growth from 1969-1994 (France) or 1969-1979 (U.S.) to growth post 2005. Other French moments equal the difference between values in 1994 and in 2016. Other U.S. moments are the difference between 1980 and 2016.

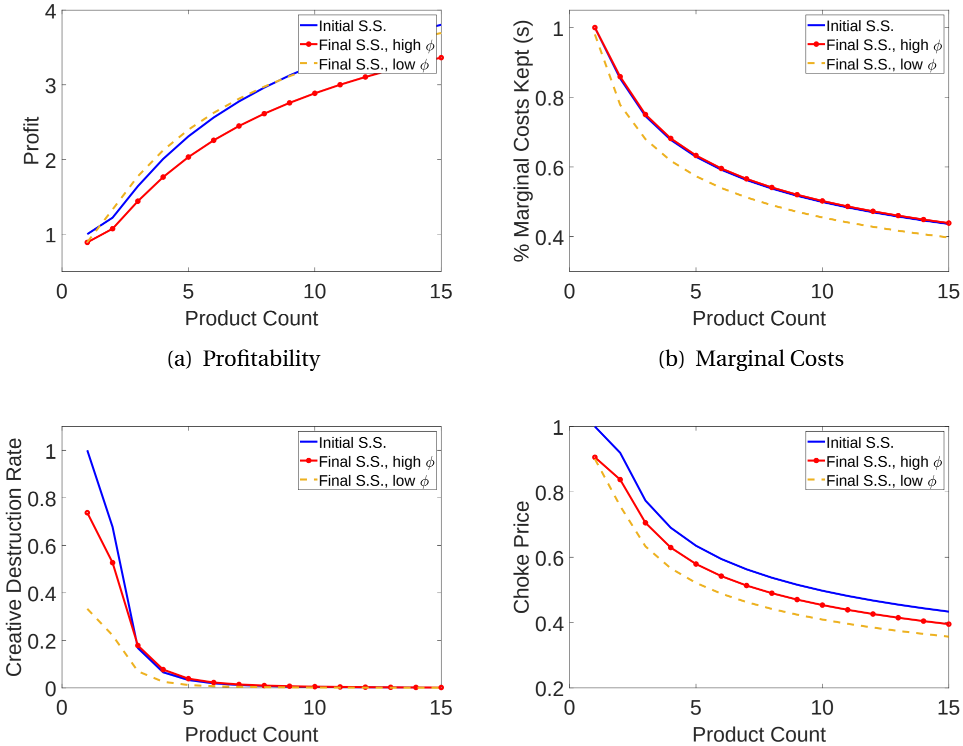
I.3. Rise in Intangibles

This section presents the effect of a rise in intangibles, and shows that the mechanisms and effects are approximately in line with the product-level intangibles model. I start with the experiment in the main text where a fraction of entrants are awarded a lower intangibles cost parameter ϕ_i. In line with the main calibration, low-cost firms have a 33% discount on their intangible costs and comprise 12% of potential entrants in the U.S. calibration. In the French calibration, low-cost firms have a 28% discount and form 6% of entrants.

The effect of the introduction of high-intangible firms is presented in Table A23. Results from the original model are presented under columns headed 'Main', the new results are presented under 'Firm-Intan'. The table shows that the model with firm-level intangibles predicts the same qualitative effects of the rise of a group of high-intangible firms. The model predicts a decline in productivity growth despite an increase in aggregate research and development, a fall in entry and reallocation rates, and a rise in average markups. As expected when significantly altering the structure of production in the model, the quantitative results do differ from the main model. For the same introduction of high-intangible firms, the new model predicts overall increases in fixed costs and intangible shares well above the actual increases in the data. The additional increase is driven by a denominator effect: while the effect of a reduction in ϕ_i on s_i is not amplified in the new model, the same increase in fixed costs now reduces marginal costs across a greater number of products, reducing overall variable costs and raising the fixed-to-total cost ratio further. An illustration is provided in Figure A14 for the U.S. calibration.[81] The figure plots the relationship between size and either profits, marginal costs, creative destruction in the original (solid-blue) and final steady state (red-circled and yellow-dashed). Sub-figure (b) plots the relationship between n_i and s_i. It shows that the low-ϕ_i firms indeed choose lower

[81] The plot for the French calibration is qualitatively similar and available upon request.

Figure A14. Relationship between Size and Relevant Variables (U.S. Calibration)



(a) Profitability

(b) Marginal Costs

(c) Creative Destruction

(d) Choke Price

Notes: Plots depict the relationship between n_i (horizontal axis) and the respective variable. All plots are standardized with the value of a single-product firm in the initial steady state. Solid-blue lines plots are for the initial steady state where all firms draw the high ϕ_i. Dashed-yellow (circled-red) lines are for the low-ϕ_i (high-ϕ_i) firms in the final steady state.

marginal costs, but that the effect does not depend strongly on the firms' size. Table A23 also shows that the new model features a greater decline in productivity growth than the initial model. This is because the R&D investments by low-ϕ_i now have an additional long-term effect: the investments cause a rise in firm concentration and average firm-size that further reduces the ability of other firms to innovate on their products. This lowers the rate of creative destruction for firms of all types, as shown in sub-figure (c). The new model predicts a smaller absolute decline in entry: the rate falls by 1.6 percentage points instead of 5.8 percentage points, but this is largely driven by the lower initial entry rate.[82]

The results above are conditional on a sufficiently small fraction of firms receiving the lower ϕ_i. As in the main model, a reduction in ϕ_i across all firms increases the profitability of all firms (Figure A14a) and would therefore raise the incentive for all firms to invest in R&D. I conclude that the model with firm-level intangibles features broadly the same mechanisms as the main model.

[82]The initial entry rate is 13.7% in the main model, so that the fall in entry yields a 42% decline. The new model has an initial entry rate of 6%, so that the decline remains economically significant at 27%.