Investigating the modality specific cognitive abilities predictive of arithmetic competence, using a developmental trajectories approach

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Declaration

This dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration except as declared in the Preface and specified in the text.

It is not substantially the same as any that I have submitted, or, is being concurrently submitted for a degree or diploma or other qualification at the University of Cambridge or any other University or similar institution except as declared in the Preface and specified in the text. I further state that no substantial part of my dissertation has already been submitted, or, is being concurrently submitted for any such degree, diploma or other qualification at the University of Cambridge or any other University or similar institution except as declared in the Preface and specified in the text

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Rosemary Penford

Abstract - Investigating the modality specific cognitive abilities predictive of arithmetic competence, using a developmental trajectories approach

Mathematics is complex, with multiple cognitive abilities utilised to solve even relatively simple problems. Research highlights working memory, executive function, intelligence, and numerical acuity as possible predictors of mathematical ability however, findings are inconsistent. While the impact of modality of stimuli presentation has been investigated for working memory and intelligence, it is limited for executive functioning and numerical acuity, with much research focussed on atypical mathematical development, particularly populations with visuospatial deficits. The current study examines which modality specific cognitive abilities are predictive of arithmetic ability in three populations: the general population, girls with Turner syndrome, and children with maths learning disabilities (N = 214; M_{age} = 11.5 years, *SD* = 3.9)

Phase one, a quasi-experimental study, investigates pathways between intelligence, executive functions, number cognition, and arithmetic competence in both auditory and visual modalities, for children (N = 182) across development (4- to 18- years; *M* = 11.6 years, *SD* = 4.1). Structural equation modelling highlighted direct paths between modality specific latent executive functioning and working memory variables, and arithmetic ability, with the visual latent variable showing the strongest associations (auditory: *B* = .40; visual: *B* = .57). Paths between intelligence and age were indirect. Given its complexity, looking to identify a single construct that underpins mathematical outcomes may be erroneous.

Phase two looked to identify differential patterns of development between arithmetic and bimodal cognitive predictors, for each disorder group and a matched typically developing group (Turner syndrome: *n* = 32; typical development matched with Turner syndrome: *n* = 32; maths learning disability; *n* = 40; typical development matched with maths learning disability: *n* = 40), within a developmental trajectories approach. Despite similar difficulties in arithmetic, differential areas of deficit were observed. Deficits were related to both visual and verbal abilities.

Findings highlight the importance of future research overtly considering modality when investigating the cognitive underpinnings of maths ability. Additionally, executive functioning and working memory were found to be a group of abilities with a strong association to arithmetic ability in both typical and atypical development.

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Abbreviations

BERA	British Educational Research Association
BPS	British Psychological Society
CAT-A	Adapted Category Fluency Task
DD	Developmental Disorders
g	general intelligence
IQ	Intelligence
KBIT-II	Kaufman Brief Intelligence Test Second Edition
MLD	Maths learning disability
TD	Typical development
w	Weber fraction
WIAT-II	Wechsler Individual Achievement Test-Second Edition
WJ-R	Woodcock-Johnson Psycho-Educational Battery/Revised
WMTB-C	Working Memory Test Battery for Children

Chapter 1: Introduction

Mathematical skills are vital in the modern world; underpinning technological advances, enabling us to pay for a pint of milk and facilitating the differentiation between Luvo Manyong's 8.37m long jump in the Rio Olympics and Jeff Henderson's 8.38m gold medal winning performance. Increasingly the impact of mathematical literacy is being recognised, as it affects a range of life outcomes as diverse as improving an individual's ability to gain full-time employment (Dowker, 2005) and their chances of staying healthy (Chesney, Bjalkebring, & Peters, 2015).

Governments also acknowledge the importance of a mathematically literate population, given identified links between improved numeracy skills and increased productivity (OECD, 2010) and a reduction in social and economic disadvantage (Every Child a Chance Trust, 2009). In the UK, poor numeracy has been associated with reduced employment opportunities and progress within jobs, indeed the cost attached to poor numeracy skills outweighs those associated with poor literacy skills (Kroesbergen, Van der Van, Kolkman, Van Luit, & Leseman, 2009; Parsons & Bynner, 1997).

As a mathematics teacher, I encountered many children (and parents!) who perceived mathematics as difficult; a subject they were likely to fail. This experience influenced the focus of my MPhil thesis; an investigation into the cognitive and social processes that underpin mathematical competence. The picture that emerged was, perhaps unsurprisingly, complex, with cognitive and social factors all predicting arithmetic competence, although suggestive evidence indicated that general cognitive abilities may be partially mediating all links. Importantly, intervention studies training cognitive abilities associated with mathematical competence have to date failed to impact long term mathematical outcomes, which may be indicative of the cognitive underpinnings of mathematical outcomes not yet being fully understood. It is imperative this is rectified as poor numeracy skills at the start of formal education are maintained throughout the remainder of schooling, above and beyond the influence of intelligence (e.g., Clark, Sheffield, Wiebe, & Espy, 2013; Geary, 2011).

In this thesis, I argue that a more nuanced understanding of these relationships may be gained by examining the impact perceptual abilities have on the predictive power of known cognitive mathematical markers, both general and specific.

Following an analysis of the existing literature, I propose hypotheses designed to investigate key ideas in this area and conduct studies to test them in both typically and atypically developing populations.

The next chapter will consider research investigating some of the proposed cognitive and neuropsychological underpinnings of mathematical understanding in typical development. This will be extended in Chapter 3 which will highlight why some researchers have studied neurodevelopmental disorders to further understanding. Two disorders, maths learning disability and Turner syndrome will be discussed, with a consideration of their aetiology, particularly numerical cognition and perceptual abilities, to determine whether including them within a cross-syndrome design would enhance an investigation into the impact of perceptual and cognitive abilities on numerical cognition.

Chapter 2: Literature Review for the Cognitive Predictors of Mathematical Competence in Typical Development

2.1 Mathematical Competence

Mathematical competence is complex, being the product of interacting cognitive and social factors, which differ across development. Although frequently associated with the ability to work with mathematical symbols (e.g., 8) or words (e.g., nine), particularly the ability to perform numerical computations, mathematical achievement is not limited to symbolic mathematical ability (e.g., Peters & Bjalkebring, 2015). Devlin (2001) suggested a number of cognitive attributes which facilitate mathematical ability (see Table 2.1), the first three of which are necessary for success in arithmetic; the theory of natural numbers, and one of the oldest mathematical disciplines (De Cruz, Neth, & Schlimm, 2010).

For many people arithmetic is synonymous with mathematics, and is typically the first area of mathematics most people learn, with many never progressing beyond it (Devlin, 2001). Arithmetic has many components, including knowledge of arithmetic facts and the ability to carry out arithmetic procedures. Consequently, it is not a unitary construct, even before the onset of formal mathematical instruction (Dowker, 2008).

Arithmetic revolves around numbers, which are abstract entities. Evidence highlights the importance of early arithmetic skills for later mathematical achievement, above and beyond the influence of other cognitive abilities (Geary, 2011). It is therefore important that the cognitive processes underpinning arithmetic ability are understood, a process which begins with an understanding of how the brain represents symbolic numbers.

Table 2.1

Attributes required for Mathematical Ability

Attribute	Description
Number sense	The ability to recognise differences between the number of objects
Numerical ability	The ability to distinguish and compare small numerosities
Algorithmic ability	The ability to learn and follow sequences of operations
Abstractions	The ability to think about abstract entities
A sense of cause and effect	The ability to recognise cause and effect
Construct causal chains	The ability to construct and follow fairly long causal chains
Logical reasoning ability	The ability to construct and follow step-by-step logical arguments
Relational reasoning ability	The ability to recognise the relationships between physical objects, human relationships or abstract objects
Spatial reasoning ability	The ability to reason about space

Note. Adapted from "The maths gene: why everyone has it, but most people don't use it", by K. J. Devlin, 2001, pp. 13-15.

2.2 Human Representation of Symbolic Numbers

Numbers, in all formats (e.g., Arabic, Chinese, Egyptian), are human constructs closely connected to language (De Cruz et al., 2010). Some of the proposed mechanism(s) for how the human brain makes sense of numbers are; utilising a mental number line and/or core number system(s).

2.2.1 The mental number line.

This concept was first forwarded by Moyer & Landauer (1967), following a computerised number comparison experiment where participants indicated, by pressing one of two response keys, the larger of two digits. Response times varied systematically depending on the numbers presented, with increased response times for numbers closer together, hence response times for 5 and 6 were slower than 5 and 18. Additionally, for two pairs of numbers with congruent differences, response times were higher for the numerically larger pair e.g., the reaction time for 15 and 20 was greater than 1 and 6.

In 1982, Henik & Tzelgov asked participants to judge the larger of two digits in either physical or numerical size. Reaction times improved when the irrelevant dimension was congruent with the relevant one e.g., for the number pair 3, 5 participants were faster recognising that 5 was written in the larger font in [3, 5] than the 3 in [3, 5]. It was therefore postulated that numerical distance along the mental number line is automatically computed even when it is not required for the task.

Dehaene, Bossini, & Giraux (1993) investigated the direction of the mental number line by asking participants to indicate, by pressing one of two buttons, whether numbers were odd or even. Reaction times for small numbers were quicker for the left hand and large numbers the right hand, a phenomenon, indicative of the mental number line starting on the left-hand side. Interestingly, a sub-group of Iranian students displayed the opposite pattern, significantly Iranians read from right to left, hence it was postulated that the direction of an individual's mental number line is congruent to the direction in which they read (Devlin, 2001).

General agreement exists for humans possessing a mental number line, similar to the mathematician's number line (Devlin, 2001). However, whilst numbers are spaced evenly along the latter, for the mental number line they become logarithmically compressed (get closer together), making it increasingly difficult to differentiate between numbers further down the line (Dehaene, Piazza, Pinel, & Cohen, 2003); a possible explanation for the reaction time anomalies observed by Moyer and Landauer. Although general consensus supports logarithmic compressions in young children, debate surrounds its retention across development (Feigenson, Dehaene, & Spelke, 2004), or whether age and numerical experience leads to a linear mapping (Siegler & Opfer, 2003).

2.2.2 Number systems.

Behavioural and brain-imaging studies have been used to suggest two core systems (the subitizing and approximate number system) for representing number in human infants and in some non-human animal species (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Mazzocco, Feigenson, & Halberda, 2011). Although, similar evidence has also been forwarded to support a counter theory; the single, overlapping system theory (see Venkatraman, Ansari, & Chee, 2005).

The first or subitizing system, is typically considered to be language and culturally dependent (Ansari & Karmiloff-Smith, 2002). It facilitates the precise tracking of small numbers of individual objects (adults; $x < 6$, children; $x < 3$) via subitizing (Kaufman, Lord, Reese, & Volkmann, 1949); a visual process (Mandler & Shebo, 1982) that enables numerosities, presented for a short duration (to prevent counting), to be rapidly and accurately reported.

The second system, frequently referred to as the approximate number system is an approximate, language independent system thought to facilitate comparison of non-symbolic numerical magnitudes (Dehaene et al., 1999; Mazzocco et al., 2011). Located bilaterally in the intraparietal sulcus (Davis et al., 2009), these discriminations are imprecise, obeying Weber's law (Brannon, 2006), hence the difference between two numerosity needed to detect the change is a constant proportion of the original numerosity. Therefore, if the change in numerosity between 8 and 16 dots is detected, a change will also be detected between 6 and 12 (both a ratio of 2.00) but not necessarily between 6 and 11 (ratio of 1.83).

2.3 Cognitive Markers of Mathematical Competence

Many social and cognitive factors appear to influence mathematical ability, particularly after the onset of formal mathematical education (e.g., Ashcraft, 2002; Cleary & Chen, 2009; Ma & Kishor, 1997). Research looking to establish the cognitive factors underpinning mathematical competence typically divide into separate fields, including those examining the impact of general cognitive abilities, intelligence, specific mathematics related cognitive abilities, and atypical development. Research pertaining to the first three will now be considered, whilst atypical development studies will be reviewed in Chapter 3.

2.3.1 Domain-general abilities.

Domain-general markers of mathematical competence are cognitive abilities that apply not only to mathematics but also to other aspects of cognition such as language, motor planning etc. (Fuchs et al., 2010) and include executive functioning, working memory, sustained attention and reading (e.g., Duncan et al., 2007). A large corpus of literature has attempted to understand the impact of working memory and executive functioning on academic achievement per se, with a subset specifically investigating their impact on mathematical outcomes.

A number of models of working memory (the ability to hold and manipulate information in the mind over short periods of time) have been forwarded, including the prominent Baddeley & Hitch (1974) multi-component model. Here working memory is comprised of two interacting modality and domain-specific subsystems for short-term memory; the phonological loop (verbal information) and visuospatial sketchpad (visual and spatial information), which are co-ordinated by a domain and modality-general central executive system, responsible for controlling resources and monitoring information (Holmes & Adams, 2006).

However, the notion of a unitary central executive is debated with suggestions it is divided into different subsystems or subprocesses (e.g., Baddeley, 1996). One such conceptualisation is for working memory consisting of short-term stores for verbal (phonological loop) and visual (visuospatial sketchpad) information, in addition to a bimodal central executive (e.g., Cragg, Keeble, Richardson, Roome, & Gilmore, 2017). A framework incorporating Baddeley (1996) and the executive function literature is also frequently used, where the central executive remains an important part of the working memory model but is subdivided into three core executive functions: inhibition, switching and updating (Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013; Miyake et al., 2000).

Executive functions are defined as a set of domain-general neurocognitive skills responsible for monitoring tasks that require deliberate, goal-directed behaviour and flexible strategy employment (Cantin, Gnaedinger, Gallaway, Hesson-McInnis, & Hund, 2016; Miyake et al., 2000; Vosniadou et al., 2018). Debate surrounds the core features and developmental trajectory of executive functioning, although it is thought to start as a unitary system, becoming increasingly differentiated across childhood (Lee, Bull, & Ho, 2013). Indeed in adulthood (Miyake et al., 2000), and older children (Lehto, Juujärvi, Kooistra, & Pulkkinen, 2003) executive functioning has been conceptualised as a multicomponent construct comprising several functions, primarily; switching (the ability to switch thought processes fluidly between activities or rules), inhibition (the ability to suppress irrelevant information and inappropriate responses), and the ability to monitor and revise the information that is active in working memory, or updating (Viterbori, Usai, Traverso, & De Franchis, 2015). Despite differences in the conceptualisation of working memory and executive function their importance to mathematical outcomes is acknowledged.

2.3.2 Intelligence.

Intelligence too has been shown to predict academic achievement, and mathematical abilities specifically (e.g., Alloway & Alloway, 2010; Cormier, Bulut, McGrew, & Singh, 2017; Kaufman, Reynolds, Liu, Kaufman, & McGrew, 2012; Kyttälä & Lehto, 2008). Much research investigating the link between intelligence and mathematical outcomes subscribes to the Cattell-Horn-Carroll Theory of Human Cognitive Abilities. This theory evolved when McGrew (1997) amalgamated Cattell and Horn's (Cattell, 1971; Horn, 1968) two-factor model of intelligence; fluid reasoning (the ability for abstract reasoning), verbal comprehension (a command of language and knowledge accumulating with education and age), and Carroll's three-stratum theory of cognitive abilities (Carroll, 1997).

The Cattell-Horn-Carroll model is a hierarchical framework of human cognitive abilities which consists of three strata; general intelligence, or *g* located at the highest level (stratum III), broad cognitive abilities in stratum II, and narrow cognitive abilities at stratum I. The broad cognitive abilities and their definitions can be found in Table 2.2. This model interprets general intelligence and the broad cognitive abilities as operating together within a system of interrelated cognitive abilities (Caemmerer, Maddocks, Keith, & Reynolds, 2018).

A 5-year longitudinal study, conducted in the UK, examined the association between general intelligence, and educational achievement, specifically intelligence at 11-years and educational achievement at 16-years (Deary, Strand, Smith, & Fernandes, 2007). Intelligence was indexed by the Cognitive Abilities Test, second edition (CAT2E; Thorndike, Hagen & France, 1986), and educational achievement by attainment in 25 national examinations (GCSEs). General intelligence (Spearman's *g*) was found to contribute to success in all 25 subjects, with strongest associations being for Mathematics (58.6%) and English (48%), and weakest, Art and Design (18.1%). Overall general intelligence was found to account for between 25-30% of the variance in school achievement (Deary et al., 2007).

Studies examining general intelligence (*g*) solely, highlight a large direct impact on specific test performance, reading, mathematics and writing abilities (e.g., Niileksela, Reynolds, Keith, & McGrew, 2016; Taub, Floyd, Keith, & McGrew, 2008). However, when examined concurrently with broad cognitive abilities the impact of *g* on academic achievement is frequently mediated by broad abilities (Floyd, Meisinger, Gregg, & Keith, 2012; Hajovsky et al., 2018; Niileksela et al., 2016).

Table 2.2

Cattell-Horn-Carroll Broad Abilities and their Definitions

Cluster	Description
Fluid reasoning	The ability to reason, form concepts, and problem solve, often with unfamiliar information or procedures
Comprehension-knowledge	A measure of a person's breadth and depth of vocabulary knowledge
Visual processing	The ability to analyse and synthesise non-linguistic visual stimuli
Auditory processing	The ability to analyses and synthesise auditory linguistic stimuli. Phonological processing
Processing speed	The ability to rapidly perform automatic cognitive tasks, especially when under pressure to maintain focussed concentration
Short-term memory	The ability to temporary store verbal information and then use it within a few seconds
Long-term retrieval	The ability to store information and retrieve it later through association

Note. Adapted from "The Relationship between the WJ-R *GF-GC* Cognitive Clusters and Mathematical Achievement across the Lifespan" by K. McGrew and G. L. Hessler, 1995, *Journal of Psychoeducational Assessment, 13,* p. 25.

2.3.3 Domain-specific abilities.

Mathematical domain-specific markers are cognitive abilities that solely influence mathematical competence and include the ability to make magnitude comparisons (identifying the largest of two sets of dots or numbers), to make dot enumeration (using Arabic numbers to label arrays of dots), and the ability to focus on numerosity in the environment (Feigenson et al., 2004).

The ability to make non-symbolic magnitude comparisons or numerical acuity (typically by indicating the more numerous of two dot arrays) is a widely researched, domain-specific skill (e.g., Feigenson et al., 2004) thought to index the precision or acuity of quantity representations within the approximate number system (see section 2.2.2). This ability relies on basic intuitions and until recently was predominantly considered innate, being found across cultures in some nonhuman animals (Abramson, Hernández-Lloreda, Call, & Colmenares, 2013; Cantlon & Brannon, 2006; Pica, Lemer, Izard, & Dehaene, 2004). This is now disputed with a sense of magnitude (continuous quantities, e.g., density, surface area) suggested as the mechanism behind discriminations (e.g., Gebuis & Reynvoet, 2012; Leibovich, Katzin, Harel, & Henik, 2017), although mathematical modelling suggests the influence of non-numerical factors may decrease with age (Starr, DeWind, & Brannon, 2017).

Numerical acuity improves across childhood; 3-hour old infants display the ability to discriminate between non-symbolic visual arrays in a 3:1 ratio, 6-year olds in a 5:6 ratio, and adults a 10:9 ratio (Halberda & Feigenson, 2008; Izard, Sann, Spelke, & Streri, 2009). However, large individual differences have been observed, even in infancy (Libertus & Brannon, 2010; Mazzocco et al., 2011).

It is generally agreed that numerical acuity is characterised by a ratio or distance effect, hence when numerical differences between two comparison arrays is small or the ratio between them approaches one, performance is slower and less accurate than when the distance is large or ratio small (Halberda & Feigenson, 2008; Libertus & Brannon, 2010; Piazza et al., 2010). The ratio effect can also be described by the Weber fraction (w); the smallest ratio of two numerosities that a person can reliably judge as larger or smaller (Halberda, Mazzocco, & Feigenson, 2008). As children engage in both formal and informal mathematical instruction they increasingly represent magnitudes using symbols, initially via counting words and then Arabic numerals.

While research investigating numerical acuity is typically in the visual domain, some research findings are suggestive of it being multimodal.

2.3.3.1 The impact of modality on numerical acuity.

One of the few studies to investigate temporal numerical acuity was by Lipton & Spelke (2003). They utilised a head turn preference paradigm (where infants tend to orient visually to an attended auditory stimuli), to investigate the sensitivity of 6- and 9-month old infants to numerosity within auditory sequences of naturalistic sounds (e.g., bells, whistles). Presentation of the sounds was through offset speakers occluded behind a curtain, and item length, inter-stimulus interval, sequence length, acoustic energy, and sequence rate were all controlled for.

Younger infants discriminated 8 from 16 sounds (ratio of 2.0) but not 8 from 12 (1.5 ratio), whilst 9-month olds discriminated within a 1.5 but not a 1.25 ratio. The authors concluded that congruent with acuity in visuospatial arrays, 6-month old infants represent large numerosities in auditory-temporal sequences, with acuity of discrimination increasing over development, a finding replicated in subsequent research (VanMarle & Wynn, 2006).

A number of studies, typically involving young children (6- to 8- months), have considered the effect of intermodal stimuli on numerosity acuity, initially within the subitizing range (e.g., Mix, Levine, & Huttenlocher, 1997; Moore, Benenson, Reznick, Peterson, & Kagan, 1987; Starkey, Spelke, & Gelman, 1983). Results were inconsistent possibly due to methodological differences or the lack of an explicit relationship between the visual and auditory displays, thereby making tasks too abstract for young children.

To address this Kobayashi, Hiraki, & Hasegawa (2005) used a violation-of-expectation paradigm, where infants typically look longer at the inconsistent events (e.g., Baillargeon, 1987). This task utilised computer-generated animated movie involving two or three objects (Mickey Mouse). Infants were familiarised to the task by watching the objects impact a surface, whilst accompanied by computer generated auditory tones on impact. There were two conditions, in one the objects motion was visible and in the second the objects' motion was obscured by an opaque screen covering the lower half of the screen. In the test trials a screen gradually rose from the bottom of the screen to cover the display. Auditory notes were then presented (either two or three). The screen then dropped to reveal the visual objects, the number of which were either congruent or incongruent with the number of auditory tones. The authors found that for this ecologically valid intermodal paired-preference procedure, 6-month-old infants were able to match small numerosities in an auditory-visual intermodal matching task, thereby implying modality independence of numerical abilities; a result replicated with 7-month infants (Jordan & Brannon, 2006).

It would therefore appear that homo sapiens are able to make intermodal numerical discriminations within the subitizing range. To determine if this was replicated in the approximate number system, Izard et al. (2009) investigated the ability of new born infants to associate visual-spatial arrays of between four and 18 objects with auditory sequences. A familiarisation paradigm was utilised hence infants were familiarised to a continuous auditory stream consisting of sequences of syllables (chosen from eight syllables of different durations) each repeated a fixed number of times. The infants were subsequently presented with visual arrays of shapes (circle, squares and triangles) whose numerosities were either congruent or incongruent with the auditory stimuli and looking times recorded and coded. Results suggested that infants can associate arrays of large numbers of objects across modalities even with less ecologically valid stimuli; a finding congruent with adult studies (e.g., Arrighi, Togoli, & Burr, 2014; Barth, Kanwisher, & Spelke, 2003).

It is, therefore, typically accepted that the approximate number system is multimodal, with a perceptual mechanism encoding numerical quantity from different senses across space and time. Separate ways of representing numeric information are considered highly connected, feeding into one common representation of number (Arrighi et al., 2014).

Despite the general consensus that working memory, executive functioning, intelligence, and numerical acuity are cognitive abilities which impact mathematical ability, to date there is a paucity of effective interventions to improve mathematical outcomes for children with mathematics learning difficulties. This may be because: an influential marker has yet to been identified, identified markers have not been traced to their core (e.g., constructs are modality specific), or mathematical difficulties arise as a result of the interaction of two or more markers across development.

There follows an examination of research linking working memory, executive functioning, intelligence and numerical acuity to mathematical achievement in typical development.

2.4 Cognitive Markers of Mathematical Competence in Typical Development

The majority of research investigating the cognitive underpinning of typical mathematical competence has examined the impact of general or specific abilities in isolation. It is only more recently that both have been considered concurrently, and in this field, constructs included in studies are heterogeneous, making comparisons of findings difficult. The following sections examine research linking working memory, executive functioning, intelligence and magnitude representation to mathematical competence, before moving to a consideration of research whose aim is to determine whether general or specific abilities, or both, explain individual differences in mathematical ability.

2.4.1 Working memory and executive function.

Intuitively a link between working memory and executive functioning, and mathematics seems salient, as successful completion of mathematical problems requires incoming information to be stored and manipulated (working memory), salient but unhelpful stimuli and responses to be ignored (inhibition), and appropriate strategies selected and maintained, with irrelevant ones disengaged from, or switching (Barrouillet, Fayol, & Lathulière, 1997; Toll, Kroesbergen & Van Luit, 2016; Van Dooren & Inglis, 2015; Yeniad, Malda, Mesman, van IJzendoorn, & Pieper, 2013). Unsurprisingly, a plethora of cross-sectional, and longitudinal studies have reported links, even when intelligence was accounted for (Alloway & Alloway, 2010; Bull, Espy, & Wiebe, 2008; Cantin et al., 2016; Clair-Thompson & Gathercole, 2006; Espy et al., 2004; Van der Ven, Kroesbergen, Boom, & Leseman, 2012). However, no consensus exists for the construct with the strongest association to mathematical achievement, with evidence provided for each component of working memory, and each executive function (e.g., Bull & Scerif, 2001; Cantin et al., 2016; Friso-van den Bos et al., 2013).

Some of the confusion in the literature may be due to differences in the operationalisation of working memory and executive functioning, particularly as the term working memory can refer to the Baddeley & Hitch (1974) model, the subdividing of the central executive into inhibition, switching and updating, or to the central executive component of working memory solely. In this thesis, working memory is conceptualised as the central executive and slave components (visuospatial sketchpad, and phonological loop), with the central executive being subdivided into inhibition, switching and updating, where each construct is considered both independent and inter-related (Frisco-van den Bos et al., 2013; Miyake et al., 2000). To try to bring clarity to this review, where possible the specific component will be highlighted, and as will any subdivision of the central executive component.

A meta-analysis by Peng, Namkung, Barnes, & Sun (2016) consisting of 110 studies, investigated differences in the amount of variance in mathematics explained by working memory. This was pertinent as some studies report R^2 values between 0 and .2 (e.g., Meyer, Salimpoor, Wu, Geary, & Menon, 2010), whilst in others R^2 values are between .5 and .7 (e.g., Passolunghi & Siegel, 2004).

The meta-analysis focussed on three moderators; domains of working memory, types of mathematical skill, and sample type. The domains of working memory specifically considered how the domain of task presentation (visuospatial, visual or numerical) impacted the strength of the relationship between working memory and mathematical ability. Hence determining whether the association between working memory and mathematics is independent of modality (domain-general) or relationships are influenced by domain specificity.

The mathematical skills considered in the meta-analysis were: basic number knowledge, whole number calculations, word-problem solving, fractions, geometry and algebra. The sample type moderator considered whether findings were influenced by the populations included in studies. Three categories were considered: typically developing populations, neurodevelopmental disorders characterised by mathematics specific deficits, and populations with comorbid deficits in addition to mathematics learning difficulties. Correlations between working memory and mathematics for the different domains can be seen in Table 2.3. It should be noted that studies containing working memory tasks that tapped two or more domains were included in the composite working memory group.

No significant differences were found between domains, including when age, type of mathematical skill and sample type were controlled for. However, the type of mathematics skill did impact the relationship between working memory and maths ability, with word problem solving and whole-number calculations showing the strongest correlations, a pattern which remained when age, domains of working memory, and sample type were controlled for. Sample type also impacted the link between working memory and mathematical achievement. Controlling for age, domains of working memory, and types of mathematical skill resulted in individuals with mathematics difficulties associated with other disorders or cognitive deficits, displaying the strongest links.

Table 2.3

Relation Between Working Memory and Mathematics

Variables	*n*	*r*	*p*
Domains			
Verbal working memory	294	.30	< .001
Numerical working memory	268	.34	< .001
Visuospatial working memory	142	.31	< .001
Composite working memory	125	.38	< .001
Types of mathematical skill			
Basic number knowledge	267	.31	< .001
Whole number calculations	326	.35	< .001
Single-digit	85	.33	< .001
Multidigit	55	.27	< .011
Fractions	26	.30	< .001
Word-problem solving	143	.37	< .001
Geometry	40	.23	< .001
Algebra	27	.27	< .05
Sample type			
Typically developing	589	.34	< .001
Mathematics difficulties	50	.25	< .01
Mathematics difficulties with DCD	29	.52	< .001

Note. Adapted from "A meta-analysis of mathematics and working memory: Moderating effects of working memory domain, type of mathematics skill, and sample characteristics," by P. Peng, J. Namkung, M. Barnes, and C. Sun, 2015 in *Journal of Educational Psychology,* p7.
DCD = individuals with mathematics difficulties that are associated with other disorders or cognitive deficits.

Hence as the medium relationship (r = .35) between working memory and mathematics was significantly influenced by the types of mathematics skills and sample type, but not domains of working memory, Peng et al. concluded that working memory was domain-general. However, the working memory domains did not differentiate between tasks that measured the slave components (phonological loop and visuospatial sketchpad) and updating, and the other executive functions (inhibition and switching) were not included in this meta-analysis.

A number of studies have suggested the strength of links between different components of working memory may be age dependent. Geary (2011) conducted a 5-year longitudinal study of children (*N* = 177) between 1st (*M* = 7.0 years) and 5th grade (*M* = 10.7 years). He concluded that the central executive and visuospatial sketchpad were important predictors of arithmetic ability, with verbal updating increasing in importance as complexity of test items increased. However, modality related differences within updating were not investigated.

Another study by Meyer, et al. (2010) found that verbal updating and the phonological loop predicted mathematical reasoning scores for 7-to 8-year olds, whilst performance on both arithmetic and reasoning abilities for 7- to 9-year olds were predicted by the visuospatial sketchpad. Once again measures of inhibition and switching were not included and updating was unimodal.

The weight of evidence highlights bimodal updating, and both slave components of working memory as strong predictors of mathematical competence, however it is still unclear which are the most important and even if age related differences exist.

Evidence for inhibition and switching being robust predictors is even less consistent, with significant correlations found in some studies (Blair & Razza, 2007; Bull & Lee, 2014; Cantin et al., 2016), but not others (Lee et al., 2012; Van der Ven et al., 2012).

A meta-analysis of 18 studies investigated links between switching and maths ability, found substantial and significant links, although intelligence was found to be a stronger predictor, and switching was substantially associated with intelligence (Yeniad et al., 2013). The small number of studies included in the meta-analysis meant it was not possible to determine if switching predicted additional variance beyond the effect of intelligence.

Inhibition has also been found to be an independent predictor of maths ability. For instance, in a sample of 93 between 6- and 8-years (*M* = 7.3 years), a significant association between general maths scores and inhibitory control was observed, even when intelligence and reading ability were accounted for (Bull & Scerif, 2001)

So, whilst both switching and inhibition appear to be independent predictors of mathematical ability, it may be that inhibition and switching contribute unique variance to mathematical competence when studied independently, but not when working memory is included in the model (Bull & Lee, 2014; Cragg et al., 2017). Once again evidence is contradictory.

A study of preschool children (2- to 5- years; *M* = 4.21 years, *SD* = 0.87 years) which studied working memory, inhibitory control and switching found that while inhibitory control and working memory predicted early arithmetic ability, only inhibitory control accounted for unique variance when other executive functions were controlled for (Espy et al., 2004). However, a study of children between 7- and 10-years which utilised path analysis, found switching to be the sole predictor of maths ability, with the effect of working memory and inhibition mediated by reading comprehension (Cantin et al., 2016).

A meta-analysis of 111 studies containing children between 4:00 and 13:11 years by Friso-van den Bos et al. (2013) found medium-sized correlations between each component of working memory, including executive functions, and maths measures (see Table 2.4 for more detail).

Table 2.4

Correlation between Mathematical Performance and Working Memory and Executive Functioning

Construct	*n*	*r*	*p*
Inhibition	29	.27	< .001
Switching	18	.28	< .001
Visuospatial updating	21	.34	< .001
Verbal updating	85	.38	< .001
Visuospatial sketchpad	55	.34	< .001
Phonological loop	65	.31	< .001

Note. Adapted from “Working memory and mathematics in primary school children: A meta-analysis,” by I. Friso-van den Bos, S. H. G. van der Ven, E. H. Kroesbergen, and J. E. H. van Luit, 2013, *Educational Research Review 10,* pp. 36-38.

To highlight the component with the highest correlation to mathematical performance, weighted mean coefficients were compared, which resulted in verbal updating having the strongest correlation followed jointly by the visuospatial sketchpad and visual updating, phonological loop and finally inhibition and switching. At each stage the relationship was significantly stronger than the relationship with other components, *p*s < .001. Hence strongest links to mathematical achievement were with updating, and the strength of associations was impacted by modality. Stronger correlations were also found with general maths measures as opposed to pure arithmetical measures.

Van der Ven et al. (2012) proposed that inconsistency in the findings of executive functioning studies may result from three potential confounds. The first concerns the difficulty of measuring executive function, given the distinct but also interrelated structure of these components (Miyake et al., 2000; Miyake & Friedman, 2012). Hence inconsistent results may occur because alternative executive functions are not controlled for.

A second, possibly more significant explanation is the 'impurity problem'. This refers to issues that arise because executive functions regulate other cognitive functions; consequently, executive functioning tasks also measure non-executive skills, such as verbal speed or visual search efficiency (Hughes, 1998). This makes interpretation of results difficult as non-executive cognitive factors may actually be driving the observed relationship or alternatively masking a relationship.

The third possible confound relates to the development of maths ability. Both mathematical skills and executive functioning show significant development across childhood, with the latter identified as particularly important in all learning processes (Holmes, Gathercole, & Dunning, 2009). It is therefore possible that executive and other cognitive skills influence each other mutually across development (Jones, Gobet, & Pine, 2008).

Many of the above studies suggest that modality may be an important factor in explaining the link between working memory and executive function, and maths, hence predictive power of these abilities may be impacted by the modality of stimuli presentation. Whilst modality can refer to stimuli being presented in a number of formats (e.g., visually, auditorily, through touch), in this field of research typically focusses on the visual and auditory domains. However, the majority of studies in this area have failed to include multimodal measures for all components, possibly because visual tasks are typically easier to administer, and there is a lack of auditory tasks which have been validated and standardised. The majority of studies that have considered modality have focussed on the effect of modality on updating abilities.

2.4.2 Intelligence.

A number of different fields of research have investigated the link between intelligence and mathematical competence including fields studying intelligence per se, those examining general cognitive abilities, and atypical developmental research. This section considers literature relating to intelligence and general cognitive abilities, with atypical development literature reviewed in Chapter 3.

There is heterogeneity in the terms used for two of the most prominent Cattell-Horn-Carroll broad abilities; comprehension knowledge and fluid reasoning. As outside of the pure intelligence literature these constructs are typically referred to as verbal intelligence and non-verbal intelligence, they will be referred to thus forthwith.

2.4.2.1 Intelligence literature.

Research within the intelligence literature typically examines links between the Cattell-Horn-Carroll's measures of broad cognitive abilities (see Table 2.2), and maths ability. A series of studies have examined the relationship using different versions of the Woodcock-Johnson Tests of Cognitive Abilities and Woodcock-Johnson Tests of Achievement.

The first by McGrew & Hessler (1995) investigated the relation between the seven Cattell-Horn-Carroll cognitive abilities included in the Woodcock-Johnson Psycho-educational Battery/Revised (WJ-R; Woodcock & Johnson, 1989), and the WJ-R Basic Mathematics Skills (calculations and basic knowledge) and Mathematics Reasoning (problem solving and applications) subsets. Non-verbal intelligence, verbal intelligence, and processing speed were all related consistently and significantly to both mathematics subsets, with non-verbal and verbal intelligence most consistently related to both aspects of mathematics across the lifespan.

The correlation between processing speed and basic skills was strongest between 5- and 10-years, with a moderate correlation found throughout the life span. Links between processing speed and mathematical reasoning were moderate until approximately 40-years, when they became insignificant. During late adolescence and early adulthood, short-term memory had a negligible correlation with basic skills and a moderate relationship with reasoning.

Floyd, Evans, and McGrew (2003) conducted a similar investigation with 6- to

19-year olds, but utilising the Woodcock-Johnson III (WJ III; Woodcock, McGrew & Mather, 2001) and the Woodcock-Johnson III Tests of Achievement (ACH; Woodcock & Mather, 2001). Strongest associations were between verbal intelligence and maths achievement, and consistent moderate to strong links were observed between non-verbal intelligence and mathematical ability.

This study not only measured short-term memory, but also verbal updating. While both demonstrated moderate relationships with maths achievement, they were stronger and more consistent for updating. The stronger links between processing speed and basic skills was replicated, with only moderate links found with reasoning up to 10-years.

More recently, Cormier et al. (2017) conducted a similar study, this time utilising the Woodcock Johnson Tests of Cognitive Abilities, Fourth Edition (WJ IV COG; Schrank, McGrew & Mather, 2014a), and Woodcock Johnson Tests of Academic Achievement Fourth Edition (WJ IV ACH; Schrank, McGrew & Mather, 2014b). They sought to examine the relationship between the broad abilities and mathematics achievement above and beyond the contribution of general intelligence (*g)*. Similar findings were obtained with significant relationships found between maths calculation skills and non-verbal intelligence, verbal intelligence and processing speed, throughout the school years. Non-verbal and verbal intelligence also demonstrated consistent relationships to math problem solving in the same age span.

These studies suggest that of the Cattell-Horn-Carroll cognitive abilities, verbal and non-verbal intelligence are the strongest predictors of both maths calculation skills and maths problem solving, with processing speed a strong and consistent predictor of calculation skills but not problem solving. The importance of non-verbal intelligence is perhaps unsurprising as it is the ability to reason and solve problems. It is also unsurprising that verbal intelligence impacts both calculations and problem solving, given it is impacted by both age and level of education. Additionally, as problem solving questions tend to be wordy, it is logical to assume that the ability to solve them will be impacted by a person breadth and depth of vocabulary knowledge, or verbal intelligence. Processing speed being more important for calculation skills, than problem solving, also intuitively makes sense as problem solving tends to be a slower process, with more information needing to be read and processed. It also typically involves multiple mathematical techniques, details of which would need to be retrieved from long term memory.

While short-term memory and updating were also highlighted as predictors, associations were weak. This may in part be a result of the tasks used to measure this construct within the Woodcock Johnson materials, as WJ-R short-term memory tasks examined verbal short-term memory solely, and while later versions included some measures for updating, they too were within the auditory domain.

Many studies in this field have utilised the Woodcock-Johnson cognitive and achievement batteries. However, Caemmerer et al. (2018) looked to determine if congruent findings were produced when alternative measures for intelligence and academic achievement were utilised, namely the Wechsler Intelligence Scale for Children, Fifth Edition (WISC-V; Wechsler, 2014), and the Wechsler Individual Achievement Test, Third edition (WIAT-III; Wechsler, 2009). The maths skills included were; maths calculations, problem solving and fluency (simple addition, subtraction and multiplication problems under timed conditions). Tasks used to index working memory were more comprehensive than the Woodcock Johnson measures, including tasks for the phonological loop, visuospatial sketchpad, and verbal updating.

Findings suggest the impact of general intelligence, *g*, was indirect and strong, with a possible isomorphic relationship between *g* and non-verbal intelligence. All mathematics skills were most strongly associated with non-verbal intelligence, and while processing speed also influenced each mathematics skill, associations were stronger for younger students. Links were also found between working memory and fluency and problem solving, but not calculations. Significantly, incongruent with studies utilising the Woodcock Johnson batteries, no significant effects were found between verbal intelligence and either calculations or problem solving.

In the intelligence literature the relationship between intelligence and executive functioning, particularly working memory is acknowledged. Indeed, a possible link between working memory and intelligence has been postulated, with some suggesting they are virtually the same (Kyllonen & Christal, 1990), that the strength of the association between them is modest (Kyttälä & Lehto, 2008), or they operate as distinct constructs (Ackerman, Beier, & Boyle, 2005). The following section examines a body of work investigating the relative strength of associations between intelligence (verbal and non-verbal intelligence), executive functioning and mathematical ability.

2.4.2.2 General cognitive abilities and intelligence.

Studies utilising regression analysis have found that intelligence and working memory jointly contribute to mathematical ability (e.g., Andersson, 2008; Bull & Scerif, 2001). This was supported by Kyttälä & Lehto (2008) who used path analysis to compare the predictive power of the visuospatial sketchpad, visuospatial updating and non-verbal intelligence to maths ability. A link between all three was observed, with the visuospatial sketchpad and non-verbal intelligence predicting general maths ability and mental arithmetic. However, the link between visuospatial updating and maths was mainly mediated by non-verbal intelligence.

Conversely, some studies have found working memory and executive functions to be more important predictors of mathematical competence, or that working memory fully mediates the link between intelligence and maths ability (Kroesbergen, et al., 2009; Passolunghi, Vercelloni, & Schadee, 2007).

To try to determine if differences in the statistical techniques utilised impacted findings, Lee, Lee, Ang, & Stankov (2009) analysed data from three previous studies using regression and path analyses. Entering data into a regression analyses resulted in working memory explaining an additional 5% to 7% of the variance in algebraic proficiency, after controlling for intelligence (both non-verbal and verbal). Conversely, entering the same data into a structural equation model resulted in a direct path between latent intelligence and algebraic proficiency solely, with an indirect path for latent working memory through intelligence.

A recent study Filippetti & Richaud (2017) conducted a comprehensive examination of the relative strength of intelligence (verbal and non-verbal) and executive functioning and working memory (phonological loop, verbal updating, inhibition, and switching), and mathematical skills (number production, mental calculations, and arithmetic problems), for 118, 8- to 12- year olds. They found that working memory (phonological loop and verbal updating), non-verbal intelligence, and age were predictors of both number production and mental calculations, with age and non-verbal intelligence having direct and indirect paths, through working memory. Arithmetic problem-solving was predicted by switching, age, comprehension knowledge, non-verbal intelligence, and gender. Age, and both intelligence measures had direct and indirect paths, this time through switching.

This is suggestive of executive functioning and working memory being stronger predictors of mathematical ability. However, while modality was considered for some constructs (switching was indexed by verbal and visual tasks), this was not the case across all constructs, as working memory tasks were presented verbally and inhibition tasks were visuospatial.

Hence findings from the general cognitive markers of mathematical achievement literature are heterogeneous. Not only is the relative influence of working memory, executive function and intelligence on mathematical ability unclear, but multiple confounds have been suggested including; modality, type of mathematical task, age, gender and statistical techniques.

Next, a body of research investigating links between a specific ability linked to maths competence; numerical acuity will be considered.

2.4.3 Numerical acuity.

A large corpus of literature highlights links between arithmetic competence and an individual's understanding of numerical magnitudes, typically via magnitude comparison tasks, which can be non-symbolic or symbolic (De Smedt, Verschaffel, & Ghesquière, 2009; Halberda et al., 2008; Holloway & Ansari, 2009; Mundy & Gilmore, 2009). Debate surrounds whether non-symbolic (i.e. dots), symbolic (i.e. digits) magnitude comparison tasks, or both are relevant for more advanced mathematical competence (Schneider et al., 2017).

However, probably the greatest debate in this field surrounds the extent to which performance on non-symbolic magnitude comparison tasks predicts mathematical ability. Whilst evidence exists for adults and children, with performance related to prior, concurrent and future mathematical achievement, a sizable corpus of studies has found no associations (e.g., Bonny & Lourenco, 2013; De Smedt, Noël, Gilmore, & Ansari, 2013; Holloway & Ansari, 2009; Inglis, Attridge, Batchelor, & Gilmore, 2011; Libertus, Odic, & Halberda, 2012; Lourenco, Bonny, Fernandez, & Rao, 2012; Rousselle & Noël, 2007).

Possible explanations for inconsistent research findings were addressed in a meta-analysis conducted by Chen and Li (2014). They examined data from 39 studies, with cross-sectional (n = 31) and longitudinal (n = 8) designs. The rationale for the meta-analysis was i) individual studies may lack sufficient power to detect associations, ii) potential moderators may underlie inconsistencies (e.g., age or general cognitive ability), or results may be impacted by; iii) the variability in task formats (addition, paired, sequential or intermixed designs) or, iv) the variable used to represent number acuity (e.g., overall accuracy, Weber fraction and numerical distance effect).

Moderate but significant associations were found between numerical acuity and mathematical outcomes (*r* = .20, 95% CI[.13, .26]), in both cross-sectional and longitudinal studies, which remained when publication bias was corrected for (although the effect size was reduced). Age did not moderate these associations; children (*r* = .25, 95% CI[.18, .31]) and adults (*r* = .22, 95% CI[.12, .33]). This latter finding was incongruent with Fazio, Bailey, Thompson, and Siegler (2014)'s meta-analysis, where age was found to moderate the association between the comparison task and mathematical competence. The largest correlations were seen for children under 6-years (*r* = .40, 95% CI[.33, .47]), in comparison to 6- to 18-year olds (*r* = .17, 95% CI[.12, .21]) and adults (*r* = .21, 95% CI[.19, .23]). However, Chen and Li did not include participants between the ages of 12- and 17-years.

While task format was not found to influence results, the variable used to measure numerical acuity did, with a significant association seen between overall accuracy and the Weber fraction but not the numerical distance effect. However, perhaps the most significant finding was that many studies were underpowered; with only six displaying sufficient power, five of which reported significant associations between numerical acuity and maths outcomes. This finding is an important one, given that in behavioural science low power can lead to a reduction in the replicability of results (Button et al., 2013).

Whilst inhibitory control has been forwarded as a possible cognitive mediator between numerical acuity and mathematical competence (e.g., Fuhs & McNeil, 2013; Gilmore et al., 2013; Purpura & Simms, 2018), Chen & Li (2014) concluded that although controlling for general cognitive abilities did decrease the correlation coefficient (.27 to .16), it remained significant, a result replicated when mathematical modelling was used to parse the impact of numerosity from possible confounds (Starr et al., 2017). Significantly, insufficient studies have compared the predictive power of general cognitive abilities and magnitude comparison tasks in the same study for definitive conclusions to be drawn.

Symbolic magnitude comparison (e.g., choosing the larger Arabic numeral), may be a better predictor of mathematics ability (e.g., Holloway & Ansari, 2009; Xenidou-Dervou, Molenaar, Ansari, van der Schoot, & van Lieshout, 2017). While a link between non-symbolic and symbolic magnitude systems seems likely, to date the relationship between them is unclear. Competing hypotheses suggest it may be; unidirectional, with children learning the meaning of symbolic number words by scaffolding them onto their non-symbolic system, bidirectional, or independent (Gilmore, McCarthy, & Spelke, 2007; Holloway & Ansari, 2009; Matejko & Ansari, 2016; Sasanguie, De Smedt, Defever, & Reynvoet, 2012; Vanbinst, Ceulemans, Peters, Ghesquière, & De Smedt, 2018; Wong, Ho, & Tang, 2016; Xenidou-Dervou et al., 2017).

Performance on symbolic magnitude comparison tasks has been found to improve with age and to be correlated with concurrent and future mathematical ability (e.g., Holloway & Ansari, 2009; Lyons & Beilock, 2011). Performance on these tasks is frequently quantified by calculating the distance effect, which indicates that accuracy increases and reaction time decreases as the numerical distance between two numbers increases.

A recent meta-analysis by Schneider et al. (2017) was the first to consider concurrently the association of non-symbolic and symbolic magnitude comparison and mathematical competence. This meta-analysis consisted of 45 articles, reporting 284 effect sizes, combined via a two-level random-effects regression model, thereby controlling for the lack of independence of some effect sizes, due to multiple reporting of effect size in some studies. Age groups were initially coded as below 6-years, between 6- and 9-years or above 9-years, although when age was investigated as a moderator, they were adjusted to 5-years or below, 6- to 9-years, 10- to 17-years and adults.

Data from all 284 studies (symbolic and non-symbolic) were synthesised, resulting in a significant association with mathematical competence (r = .278, 95% CI [.241, .315]). The authors noted that whilst this only represented an overlap in variance of 8%, given the heterogeneous nature of the tasks, both within and between constructs, this represented a large effect. A statistically significant amount of heterogeneity was found in the data, indicating the association was moderated by a third variable.

One of the identified moderators was task format, with a higher average effect size for symbolic magnitude comparison (r = .302, 95% CI[.243, .361]), than numerical acuity (r = .241, 95% CI[.198, .284]), a small but significant difference. However, non-numerical abilities (e.g., executive functions, working memory and perceptual abilities) were not controlled for, as only a small number of studies included in the meta-analysis controlled for these abilities, and there was heterogeneity in the moderators utilised.

Interestingly, the choice of measures had a greater impact, explaining 14% of the variance of effect size compared to 9% explained by different task formats. The measures displaying substantially stronger effects were accuracy, reaction times and the Weber fraction. This being the case it would seem logical to assume these indices are correlated.

However, this may not be the case. In a review into possible methodological aspects impacting non-symbolic tasks, Dietrich, Huber, & Nuerk (2015) found that only accuracy and Weber fractions were strongly related (r = .89). Additionally, test-re-test reliabilities were strongest for accuracy, although this was smaller in children (r = .47) than adults (r = .65).

Unlike research investigating the impact of working memory, executive function, and intelligence, few if any studies have investigated the impact of modality on the link between numerical acuity and mathematical ability, and to my knowledge no research investigates whether the strength of associations between working memory, executive functions, intelligence and numerical acuity is modality specific. However, some studies have compared the relative strength of a number of these constructs. While each of these studies includes general- and specific- markers of mathematical achievement, typically they are located with one or other of these fields of research, which arguably impacts the research design and focus of interpretation of results.

2.4.4 Comparing the predictive power of general and specific markers.

Congruent with literature looking solely at general or specific markers of mathematical ability, findings from the corpus of literature investigating the concurrent ability of each to predict mathematical ability are heterogeneous. While most studies highlight working memory and intelligence as predictors, links between numerical acuity and mathematical ability in the presence of other cognitive abilities is debated.

One of the first studies to explore the relative importance of general and specific cognitive abilities on mathematical performance did find numerical acuity to be a predictor above and beyond other general abilities (Fuchs et al., 2010). The study investigated the impact of working memory, language, reasoning and attentional measures (all domain-general), and numerical acuity (number lines estimation and precise representation of small numerosities), on the number combination and word problem abilities of 5- to 7-year olds. Both numerical acuity measures were found to predict number combination abilities, with precise representation of small quantities uniquely predicted word problem abilities. However, general abilities also accounted for significant additional variance, hence the authors concluded that different mathematical tasks require different combinations of general and specific cognitive abilities.

More recently Xenidou-Dervou et al. (2018) also highlighted numerical acuity as an important predictor of mathematical ability They utilised latent growth modelling to assess 334 kindergarten children's general and maths-specific cognitive abilities and general maths achievement, longitudinally across four time-points within the first and second grades of primary school (M_{age} = 5.59, *SD* = .35). The cognitive abilities studied were: non-verbal intelligence, phonological loop, visuospatial sketchpad, verbal updating, visuospatial updating, counting abilities, non-symbolic approximate addition, symbolic approximate addition, non-symbolic approximate comparison, symbolic approximate comparison, symbolic exact addition, and general maths achievement.

Latent growth modelling highlighted intelligence, visuospatial sketchpad, phonological loop, verbal updating, counting skills, non-symbolic approximate comparison, symbolic approximate comparison, and symbolic approximate addition as unique predictors of the children's starting point on maths achievement. Symbolic approximate addition was a unique predictor of individual development growth in mathematical achievement from grade 1 (5- to 6-years) to grade 2 (6- to 7-years), however, a large percentage of the variance in individual mathematics developmental growth was unexplained.

In a previous study Xenidou-Dervou, De Smedt, van der Schoot, & van Lieshout (2013) looked at the interrelationship between latent non-symbolic and symbolic approximation skills, working memory (visuospatial sketchpad, phonological loop and updating), and mathematical achievement for 444 kindergarten children (M_{age} = 5:59, *SD* = 0.35 years). Structural equation modelling highlighted direct associations for working memory (β = .74), and symbolic approximation (β = .34), and an indirect association for non-symbolic approximation, through symbolic approximation.

Interestingly paths from working memory to mathematical achievement were also indirect through both numerical acuity latent variables. Hence whilst these studies highlight numerical acuity as an important predictor above and beyond general abilities, there are differences in the actual constructs found to be predictive. However, other studies have found no associations or indirect paths.

In a recent study, developmental changes in the cognitive predictors of mathematical ability were investigated in children between 5- and 7-years (Gimbert, Camos, Gentaz, & Mazens, 2019). A total of 148 children, 73 kindergartners (M_{age} = 5:8 years) and 75 second graders (M_{age} = 7:8 years) completed measures of non-symbolic magnitude comparison, number line estimation, updating, vocabulary, and three mathematics tasks measuring addition, subtraction and verbal problems.

The predictive power of the non-symbolic magnitude comparison task was found to decrease between 5- and 7-years, whilst for updating the opposite was true. Number line estimating abilities were significant predictors of maths ability for both age groups. Additionally, for 5-year olds the link between numerical acuity and mathematical ability was partially mediated by their ability to make number line estimations, whilst this link for 7-year olds was fully mediated by updating abilities.

Findings from these studies are suggestive of both non-symbolic and symbolic numerical acuity being important predictors of maths ability, although links appear strongest with symbolic numerical acuity. However, they also suggest that associations decrease with age, being strongest in early childhood, and the first few years of formal education.

There are, however, a number of studies where no links are observed between numerical acuity and mathematical ability. Passolunghi, Cargnelutti, & Pastore (2014) investigation of the concurrent contribution of general and specific skills to children's early mathematics performance. Children were tested at two timepoints; the beginning ($N = 157$; M_{age} = 6.25 years) and end of first grade (N = 134). Constructs included were verbal and non-verbal intelligence, phonological loop, visuospatial sketchpad, verbal updating, visuospatial updating, and non-symbolic numerical acuity.

At the first timepoint, participants completed the cognitive tasks and an Early Numeracy Test (ENT; van Luit, van der Rijt, & Pennings, 1994), which measures an understanding of numbers and counting abilities. At timepoint two, teachers rated participants' maths abilities on a 5-point Likert-like scale.

Data were analysed via structural equation modelling, with the following latent variables; short-term memory (consisting of measures of the phonological loop and visuospatial sketchpad), working memory (verbal and visuospatial updating measures), intelligence (verbal and non-verbal intelligence), and numerical acuity. Path analysis at timepoint one suggested all variables were significant predictors of mathematics ability, with intelligence (β = .39), working memory (β = .21), and short-term memory (β = .20) having the strongest associations. Intelligence had direct and indirect paths through working memory, short-term memory and numerical acuity.

Splitting the mathematics variable into relational and counting skills resulted in no path between numerical acuity and either skill. For relational skills, intelligence was the strongest predictor (β = .36), followed by short-term memory (β = .17). Associations were similar for counting skills; working memory (β = .19), intelligence (β = .18), and short-term memory (β = .16). In both instances, intelligence had direct and indirect paths (through short-term and working memory). At timepoint two significant paths existed between intelligence (β = .40), and working memory (β = .18), and teacher's ratings.

The authors concluded that intelligence had the greatest impact on mathematical abilities at the start of formal education, followed by working memory and short-term memory. Numerical acuity was not found to make an independent contribution.

This result concurs with the findings of Szucs, Devine, Soltesz, Nobes, and Gabriel (2014) who also failed to find association between numerical acuity (symbolic and non-symbolic) and mathematical ability. Here 98 children between 7- and 10-years were compared on measures of working memory and executive functioning (visuospatial sketchpad, phonological loop, updating, inhibition and switching), in addition to verbal and non-verbal intelligence and numerical acuity. Visuospatial sketchpad and visual updating were found to be robust predictors of mathematical achievement, along with verbal intelligence and general executive functioning. Interestingly non-verbal intelligence was not a predictor.

It has been suggested that the strength of the association between numerical acuity and mathematical ability decreases with age, (e.g., Inglis et al., 2011; Rousselle & Noël, 2007) which may explain why Szucs et al. (2014) failed to find a link. This however is not the case for the Passolunghi et al. (2014) study, which involved participants of a similar age to studies were links have been observed. It may be significant that both Passolunghi et al. (2014), and Szucs et al. (2014) included bimodal measures of intelligence (verbal and non-verbal).

In a slightly different study Skagerlund and Träff (2016) investigated the cognitive abilities predictive of maths ability, however they looked specifically to see whether processing of time, space and number predicts mathematical skill. 133 children aged between 8- and 10-years (M_{age} = 9.7 years, *SD* = 0.90) completed measures of; multidigit calculations, arithmetic equations, arithmetic fact retrieval, general intelligence, processing speed, switching (visuospatial), visuospatial updating, verbal updating, symbolic number comparison, non-symbolic magnitude comparison, spatial transformation, spatial visualisation, and time processing.

Entering all variables into a hierarchical regression analysis resulted in age, switching, and general intelligence emerging as the sole predictors for overall maths ability and each mathematical skill. Mental rotation was a marginal predictor of overall maths ability, and symbolic number comparisons were additional predictors for arithmetic fact retrieval, time discrimination for multidigit calculations, and mental rotation for arithmetic equations. Hence, they concluded that executive function, intelligence and spatial abilities were important predictors, with symbolic and temporal abilities being important for some specific mathematical skills. The importance of spatial abilities to mathematics competence has been suggested elsewhere (Szucs et al., 2014; Verdine, Irwin, Golinkoff, & Hirsh-Pasek, 2014).

Whilst findings from this small body of research are heterogeneous they appear to highlight intelligence and working memory as important predictors of mathematical ability above and beyond the impact of other cognitive abilities. The case for numerical acuity is not as strong, although it may be a stronger predictor early in development. Inconsistent findings may result from numerous methodological differences, including the constructs studied, measures utilised, age of participants, and statistical techniques. Significantly, few studies have included measures of executive function apart from updating and while some considered whether perceptual modality impacted predictive power for some constructs, they did not include bimodal measures of each construct under investigation.

2.5 Research Question One

Findings from the various bodies of research investigating the cognitive markers of mathematical ability highlight the importance of working memory, executive functioning, intelligence and numerical acuity as cognitive predictors of mathematical ability. However, results within each field are inconsistent, and to date no predictors have been definitively confirmed. This is perhaps unsurprising given the complexity of mathematics within and between different areas (e.g., problem solving, arithmetic, shape and space).

Heterogeneity is also seen in findings from the smaller corpus of research considering multiple predictors of mathematical achievement concurrently, although once again they highlight working memory, intelligence, numerical acuity, and executive functioning as possible predictors. It would also seem that at least some of these constructs are modality specific. While inconsistencies may be due to age, to date there is no examination of what is happening across development, as studies typically focus either on the transition to formal education and early school years, or late primary school aged children. To my knowledge no study has overtly considered the impact of modality on a number of the cognitive abilities generally agreed to be predictive of arithmetic ability, particularly switching, inhibition and numerical acuity. This is surprising given evidence from research investigating atypical mathematical development which indicates that difficulties may result from visuospatial deficits. This study will address this by including separate verbal and visuospatial measures for each ability under investigation.

Significantly intervention studies which train specific cognitive abilities (e.g., working memory, numerical acuity) have failed to produce far transfer to mathematical outcomes. This is perhaps unsurprising given the multitude of cognitive abilities needed to solve problems in less complex area of mathematics (e.g., arithmetic). Additionally, most interventions are delivered visually, despite there being no systematic investigation of whether modality of stimuli presentation impacts the links between specific constructs and mathematical ability.

Hence it would appear timely for such an investigation to be conducted, to determine if mathematical ability is determined by a single or multiple construct(s), and whether they are modality-specific. Whilst it would be ideal to include symbolic and non-symbolic numerical acuity in this investigation, including both would make data collection sessions very long particularly for children from vulnerable populations. Hence this study will investigate non-symbolic numerical acuity, and will be designed to ensure there is sufficient power for any associations with arithmetic ability to be detected.

Therefore, my first research question is:

1) Which modality specific cognitive abilities predict arithmetic competence in the general population?

The next chapter examines the literature investigating atypical mathematical development, and considers whether including populations with visuospatial deficits in this investigation can potentially aid our understanding of the cognitive and perceptual underpinnings of mathematical ability.

Chapter 3: Literature Review for Cognitive Predictors of Mathematical Competence in Atypical Development

Research into neurodevelopmental disabilities often seeks to identify differential patterns of development in relation to typically developing control groups (Bruns, Ehl, & Grosche, 2019). Neurodevelopmental disorders are described in the Diagnostic and Statistical Manual of Mental Disorders (APA, 2013) as a group of conditions whose onset is typically early in development and affect neurological structure and function leading to a range of developmental impairments. These disorders are characterised by developmental deficits that produce impairment of personal, social, academic, or occupational functioning, and vary from very specific limitations of learning or control of executive functions to global impairments of social skills or intelligence. Neurodevelopmental disorders frequently co-occur; for example, many individuals with autism spectrum disorder have intellectual developmental disorder. For some disorders, the clinical presentation includes symptoms of excess in addition to deficits and delays in achieving expected milestones (APA, 2013).

Maths learning disability frequently exists in populations with neurodevelopmental disorders, although cognitive deficits underlying the disability are typically syndrome specific (Van Herwegen & Karmiloff-Smith, 2015). This variability facilitates a comparison of how different cognitive and perceptual deficits interact to produce numerical cognition deficits, thereby giving insight into the low-level abilities that may be at the root of mathematical deficits. While the majority of neurodevelopmental disorders displaying maths difficulties are characterised by below average intelligence, this is not the case for some, including maths learning disability and Turner syndrome. It may be advantageous when investigating the cognitive abilities predictive of maths ability to include populations where general cognitive ability in not a confound. Each will therefore be considered in more detail.

3.1 Maths Learning Disability

Poor mathematical achievement in both adults and children is well documented (e.g., Callaway, 2013; Noël, 2015). However, the reasons why such difficulties occur are numerous including problems with education, home environment, and reading ability (Wilkey, Pollack, & Price, 2019). For a subgroup, difficulties are rooted in deficits within the cognitive systems underpinning maths development. Maths learning disability (sometimes referred to as developmental dyscalculia), is thought to affect between 3-6% of the population, and is a learning difficulty specific to maths, particularly the acquisition of knowledge about numbers and arithmetic (Piazza et al., 2010; Szucs, Devine, Soltesz, Nobes, & Gabriel, 2013). The Diagnostic and Statistical Manual of Mental Disorders defines maths learning disability in terms of a discrepancy between performance on maths achievement tests, typically arithmetic, and expected performance based on age, intelligence, and years of education, and for adults it significantly interferes with their daily activities (APA, 2013).

Maths learning disability can be highly selective, affecting individuals with normal intelligence and working memory (Landerl, Bevan, & Butterworth, 2004). Indeed, some individuals with severe maths learning disability have expertise in specific areas of maths such as geometry, whilst others are able to use statistical packages, or complete degrees in computer programming (Butterworth, 2000). Maths learning disability can co-occur with other developmental disorders, such as reading disorders, and attention deficit hyperactivity disorder, and do so more frequently than would be expected by chance (Butterworth, Varma, & Laurillard, 2011; Gross-Tsur, Auerbach, Manor, & Shalev, 1996; Monuteaux, Faraone, Herzig, Navsaria, & Biederman, 2005).

Children with maths learning disability tend to lag behind their peers on a wide range of numerical tasks. These include difficulty in; retrieval of arithmetic facts, using arithmetic procedures, and immature problem-solving strategies, for example using finger counting (Geary, 1993; Jordan, Hanich, & Kaplan, 2003; Shalev & Gross-Tsur, 2001).

There is considerable debate surrounding the origins of maths learning disability, however proposed cognitive impairments include; poor working memory, executive dysfunction, poor phonological processing (including the phonological loop), deficits in attention systems, disorders of visuospatial functioning, poor retrieval of information from memory, or deficits in numerical acuity (Ashkenazi, Rubinsten, & Henik, 2009; Bull, Johnston, & Roy, 1999; De Smedt & Gilmore, 2011; Geary, 1993; Kaufmann, Lochy, Drexler, & Semenza, 2004; Júlio-Costa, Starling-Alves, Lopes-Silva, Wood, & Haase, 2015; Rousselle & Noël, 2007; Shalev, Auerbach, & Gross-Tsur, 1995; Wilkey et al., 2019).

Deficits in numerical acuity or the ability to perceive and manipulate numerical magnitudes have been found in children with maths learning disability (Landerl et al., 2004; Piazza et al., 2010). However, results are inconclusive with other studies failing to find any differences in the numerical acuity of individuals with maths learning difficulties and typically developing controls (De Smedt, et al, 2013). Additionally, amongst researchers who subscribe to the view that numerical acuity deficits underpin maths learning disabilities, the mechanistic nature of these deficits and their causal role in maths learning disabilities is debated (Mazzocco & Räsänen, 2013; Szucs et al., 2013).

Neuroimaging studies have highlighted lower grey matter density in the parietal cortex for individuals with maths learning disability in comparison to neurotypical controls, with some evidence to suggest differential activity for each population in the intraparietal sulcus, and reduced connectivity between parietal and occipito-temporal regions (Isaacs, Edmonds, Lucas, & Gadian, 2001; Rotzer et al., 2008; Rykhlevskaia, Uddin, Kondos, & Menon, 2009). While differences in parietal cortex have been forwarded as evidence for numerical acuity being the source of maths difficulties, there is also evidence highlighting its involvement in a number of cognitive functions, for example working memory (Landerl et al., 2004; Piazza et al., 2010; Szucs, 2016).

3.2 Turner Syndrome

Turner syndrome is a relatively common chromosomal disorder, occurring in between 1 in 1900 and 1 in 2500 live female births (Baker & Reiss, 2016; Mazzocco, 2009). First described in 1805 by Charles Pears, it was Henry Turner who gave his name to the syndrome after studying five women who presented with sexual infantilism, short stature, an abnormality of elbow formation and webbing of the neck (Turner, 1938). Although Turner syndrome typically affects females there are rare cases of its occurrence in males (Rovet, 2004).

Genetically Turner syndrome is caused by the partial or complete loss of one of the two X chromosomes with incidence of the latter condition (approximately 55%) considered the most severe (Murphy, Mazzocco, & McCloskey, 2010; Rovet, 1993). X chromosome loss can be from either parent, although two thirds receive only a maternal X chromosome (Jacobs et al., 1997). The lack of a second X chromosome leads to a failure in ovary development and therefore impairment in oestrogen production (Murphy et al., 2010). Given differences in the proportion of X chromosome lost, it is unsurprising there is heterogeneity in the physical, cognitive and behavioural Turner syndrome phenotype.

Physically, Turner syndrome is characterised by abnormalities in the skeletal, lymphatic and reproductive systems. Skeletal deficits typically result in short stature, an unusual carrying angle of the elbows and arms (cubitus valgus) and a high arched palate, while those of lymphatic system can lead to webbing of the neck and severe edema (Rovet, 2004). Reproductive system deficits, in the absence of medical intervention, result in sexual infantilism and infertility (Rovet, 2004). Additionally, there can be hearing problems, cardiac and renal malformations, and somatic abnormalities (Hutaff-Lee, Bennett, Howell, & Tartaglia, 2019; Simpson, 1975; Turner, 1938).

Neuroimaging studies provide evidence of atypical brain architecture and functioning in Turner syndrome. There are differences in brain organisation; reduced white and grey matter in the bilateral parieto-occipital region, larger volumes in the medial temporal lobes, reduced bilateral caudate, thalamic and hippocampal volumes, and impaired frontal-parietal connections (Haberecht et al., 2001; Murphy et al., 1993; Reiss, Mazzocco, Greenlaw, Freund, & Ross, 1995; Shucard, Shucard, Clopper, & Schachter, 1992).

Unlike the majority of neurodevelopmental syndromes, overall intellectual disability is not a significant feature of the Turner syndrome phenotype (Murphy et al., 2010). Intelligence tends to be low-average to average with a relative strength in verbal as opposed to non-verbal intelligence (Mazzocco, 2001; Murphy et al., 2010). A discrepancy which is marked and significant (Hutaff-Lee et al., 2019; Mazzocco, 2009).

Areas of relative strength are typically found in receptive language skills, music and reading comprehension, although deficits have been observed in reading decoding ability (Mazzocco, 2009; Rovet, 2004; Rovet & Netley, 1982). However, in this population neurocognitive deficits typically increase the risk for learning disabilities. Relative deficits typically include; visuospatial skills, specifically with regard to global processing, poor memory, executive dysfunction, sustained attention, and maths ability (Buchanan, Pavlovic, & Rovet, 1998; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Mazzocco & Hanich, 2010; Rovet, 2004; Williams, Richman, & Yarbrough, 1991).

In the behavioural domain, hyperactivity is frequently noted in children with Turner syndrome, although adults are said to be frequently phlegmatic, with low levels of arousal (Money & Mittenthal, 1970; Sonis et al., 1983). Other behavioural characteristics in children with Turner syndrome include problems with socialisation, emotional maturity, shyness, peer relations, self-esteem and body image (McCauley, Sybert, & Ehrhardt, 1986).

A comparison of the relative strengths and deficits for both maths learning disability and Turner syndrome can be seen in Table 3.1. Whilst both display difficulties in maths ability, importantly overall intelligence within each population is average, with verbal intelligence a strength, and visuospatial skills, which is closely aligned to non-verbal intelligence, an area of deficit. Interestingly, whilst Turner syndrome has a similar cognitive profile to maths learning disability, which frequently co-occurs with Turner syndrome, to my knowledge there has been no investigation to determine whether they display similar or differential patterns of development in the cognitive abilities thought to predict mathematical ability.

Table 3.1

A Comparison of the Relative Strengths and Deficit in Maths Learning Disability and Turner Syndrome.

Domain	Maths learning disability	Turner syndrome
Brain		
Deficit	Right hemispheric dysfunction Reduced grey matter on left intraparietal sulcus Abnormalities in left angular gyrus Reduced white matter in frontal & parietal lobes	Abnormalities in brain organisation Reduced bilateral reduction in white & grey matter in parietal & occipital regions Impaired frontal-parietal connections Reduced bilateral caudate, thalamic & hippocampal volumes
Cognitive		
Strengths	Normal IQ	IQ relatively normal Verbal IQ

Domain	Maths learning disability	Turner syndrome
		Receptive language
		Music
		Reading comprehension
Deficits	Visuospatial deficits	Non-verbal intelligence
	Reading disorder	Visuospatial – global
	Finger agnosia	Slower visually-guided saccades
	Maths	Reading decoding
		Maths
Executive functions		
Strengths	Phonological loop	Planning
	Sustained attention?	Switching?
		Phonological loop?
Deficits	Visuospatial updating	Working memory
	Phonological loop?	Visuospatial working memory
	Visuospatial sketchpad	Inhibition
	Verbal updating?	Switching?
	Inhibition	Inability to search in organised
	Switching	way
Numerical cognition		
Strengths		Reading numbers
		Writing numbers
		Rote counting
		Simple arithmetic
		Quantity
		Mental number line
		Accuracy of fact retrieval
	Subitizing	
Deficits	Numerical acuity?	Numerical acuity
	Naming digits	Subitizing

	Reciting number sequences Procedural arithmetic Retrieving arithmetic facts Learning arithmetic facts Immature problem-solving strategies	Complex arithmetic Procedural/alignment in arithmetic Conceptual/factual maths Performing calculations under pressure Speed of processing Speed of fact retrieval Cognitive estimates Multi-digit calculations

3.3 Numerical Abilities in Maths Learning Disability and Turner Syndrome

Maths learning disability is primarily a severe disability of learning arithmetic, which persists into adulthood (Butterworth, 2000; Butterworth et al., 2011). To date there is no consensus on the cognitive profile of maths learning disability, with a number of theories proposed. Indeed, given the heterogeneity in the profile of individuals displaying maths learning disability, it may be unreasonable to expect a single phenotype, with a number of subtypes proposed (e.g., Bartelet, Ansari, Vaessen, & Blomert, 2014; Geary, 1993). Wilson and Dehaene (2007) suggested four subtypes of maths learning disability based on the putative underlying cognitive deficits in numerical acuity, dysfunctional phonological processing, (including the phonological loop), executive function or working memory deficits, and more tentatively, underlying spatial-attentional deficits.

3.3.1 Number cognition in maths learning disability.

Children with maths learning disability tend to lag behind their peers on a wide range of numerical tasks including; retrieval of arithmetic facts, using arithmetic procedures, and the maturity of problem-solving strategies. Additionally, they show persistent misconceptions of whole numbers, make counting errors linked to poor short-term memory, are less likely to spontaneously focus on quantity as a feature of their environment, make errors reading and writing digits, and have slower and less accurate computational skills (Geary, 1993; Geary et al., 2004; Hannula, Lepola, & Lehtinen, 2010; Jordan et al., 2003; Shalev & Gross-Tsur, 2001).

At a cognitive level, individuals with maths learning disability have been shown to have difficulties in; subitizing, counting, making comparative judgements of symbolic and non-symbolic stimuli, mapping symbolic referents to non-symbolic quantities, automatically processing numbers, and making accurate verbal magnitude comparisons (Ashkenazi, Rosenberg-Lee, Metcalfe, Swigart, & Menon, 2013; Ashkenazi et al., 2009; Cohen Kadosh et al., 2007; Landerl et al., 2004; Mazzocco et al., 2011a; Price, Holloway, Räsänen, Vesterinen, & Ansari, 2007; Schleifer & Landerl, 2011).

To date there is a great deal of debate surrounding the cognitive abilities that explain maths learning disability, with inconsistent findings possibly due to incongruent criteria being utilised to identify participants with maths learning disability, both in terms of intelligence and the cut off used in standardised maths assessments (Geary, 2011; Mazzocco & Räsänen, 2013). Typical selection criteria include overall or verbal intelligence within the average range, and performance on standardised maths achievement tests at or below the 10^{th} percentile, or below the 25^{th} percentile (Geary, 2011). Studies utilising a trichotomous approach compare children with persistent deficits (typically identified as those < 15^{th} or 10^{th} percentile) and moderately low maths ability (< 26^{th} percentile). Whilst such studies found the cognitive profiles of children with persistent and those with moderately low, maths ability are heterogeneous, and not synonymous, boundaries between maths learning disability and other forms of maths difficulty are not clear (Mazzocco & Räsänen, 2013).

Geary, Hoard, Byrd-Craven, Nugent, & Numtee (2007) examined longitudinally, the impact of strict (n = 15; maths achievement scores < 15^{th} percentile) and more lenient (n = 44; maths achievement scores between 23^{rd} and 39^{th} percentile) cut-off criteria. Both groups were compared to a neurotypical control group (n = 46) on a range of maths cognition, working memory (phonological loop, visuospatial sketchpad and auditory updating), and processing speed measures. Children in the strict cut-off criteria group were found to have more pervasive and often severe math cognition deficits, and underlying deficits in working memory and speed of processing. However, the more lenient cut-off identifies children that have more subtle deficits in a few maths domains.

A plethora of research indicates that maths learning disability is related to deficits in numerical acuity, executive functioning or working memory, hence these will be addressed in the following sections, followed by a review of two studies that have looked to compare these constructs concurrently.

3.3.1.1 Numerical acuity in maths learning disability.

A dominant neuroscientific theory of maths learning disability suggests it results from an impaired magnitude representation module, specifically abnormal function bilaterally in the intraparietal sulci (e.g., Butterworth, 2000; Landerl, Fussenegger, Moll, & Willburger, 2009). Two theories have been forwarded to explain this link; the 'defective number module hypothesis' which suggests numerical acuity deficits directly impact number skills, and the 'access deficit hypothesis' which postulates that impairments result from deficits in the links between numerical acuity and numerical symbols rather than numerical acuity deficits per se (e.g., Bartelet et al., 2014; Butterworth, 2000; De Smedt et al., 2013; Piazza et al., 2010; Rousselle & Noël, 2007).

Although numerous studies have examined each hypothesis individually (e.g., Landerl et al., 2004; Piazza et al., 2010), Rousselle & Noël (2007) were the first to contrast the two. They compared the performance of children (M_{age} = 7.4 years) with mathematics difficulties (< 15^{th} percentile on composite maths score) on two tasks; an Arabic number comparison task (symbolic) and a collection comparison task that required non-symbolic processing. The maths learning disability population performed more slowly than controls on the symbolic but not on the non-symbolic magnitude comparison task, providing evidence for the access deficit hypothesis. This finding is congruent with De Smedt & Gilmore (2011), but incongruent with Landerl et al. (2009), who reported reduced performance on both symbolic and non-symbolic comparison tasks in children (*M* = 8.9 years) with maths difficulties. It is possible inconsistencies may result from methodological differences, for example, selection criteria for inclusion in the maths learning disability groups, or the number of trials in the non-symbolic comparison task (see section 2.4.3).

More recently, evidence for the access hypothesis was provided in a longitudinal study which examined children (*N* = 101) over a six-year period (Wong & Chan, 2019). They identified children who showed persistent maths difficulties (consistently below the 25^{th} percentile). When compared to typically developing children, this group scored significantly lower on mapping and symbolic numerical tasks, which remained when verbal updating and non-verbal intelligence were controlled for.

However, it should be noted that other components of working memory, previously highlighted as possible mediators, were not included.

Research in the typically developing population has highlighted the possibility of executive function, particularly inhibition mediating the relationship between non-symbolic numerical acuity and maths outcomes. Hence Wilkey et al. (2019) investigated longitudinally, the relationship between congruent and incongruent trials on a non-symbolic magnitude comparison task, executive function and maths achievement. Participants were children (*n* = 448) aged between 4- and 13-years, who were assigned to one of three groups; maths learning disability (≤ 10th percentile on standardised maths test; *n* = 22), low achievement (10th – 25th percentile; *n* = 12) or the typically developing control group (25th to 95th percentile; *n* = 188).

Accuracy on incongruent but not congruent trials, was found to be significantly lower for the maths learning disability group, when compared to both the low achieving and typically developing groups. This remained when early reading achievement, visuospatial updating, inhibitory control and switching were controlled for. There were however, no significant differences between the low achieving and typically developing groups, suggesting an impairment in the interaction between executive function and non-symbolic numerical acuity is a characteristic of individuals with maths learning disability.

In a second analysis, Wilkey et al. examined the link between non-symbolic numerical acuity and concurrent maths achievement. Accuracy on incongruent trials predicted maths ability even after controlling for early reading achievement, visuospatial updating, inhibitory control and switching, and this remained when the maths learning disability group were removed from the analysis.

Hence while there do appear to be deficits in the numerical acuity abilities of individuals with maths learning disability, the debate surrounding the origins of the deficits is unresolved. Indeed, increasing evidence suggests maths learning disability is not the result of issues with magnitude representation per se, as other cognitive abilities, particularly executive functioning appear to mediate the link between numerical acuity and maths ability.

3.3.1.2 Executive function and working memory in maths learning disability.

A large corpus of studies provide evidence for a link between executive function and working memory deficits and maths learning disability, which are independent of intelligence. However, no consensus exists for the specific components implicated (e.g., Bull et al., 2008; Geary, 2004; Geary et al., 2004; Szucs et al., 2013).

Empirical studies comparing the working memory abilities of children with and without maths learning disability have identified deficits in visuospatial sketchpad and visual updating, which have been collaborated by evidence from neuroimaging studies (Ashkenazi et al., 2013; Bartelet et al., 2014; D’Amico & Guarnera, 2005; Passolunghi & Mammarella, 2010; Szucs et al., 2013). However, other studies identify verbal working memory deficits, both in auditory updating and the phonological loop (Andersson & Lyxell, 2007; Geary et al., 2007).

Passolunghi & Siegel (2004) conducted a longitudinal study, where performance of two groups of students; typically developing (n = 27) and maths learning disability (n = 22) matched on age (*M* = 10.4 years), gender and verbal intelligence, were compared on a range of working and short-term memory tasks. Children with maths learning disability were found to have persistent working memory deficits and concluded that maths learning disability may be the result of generalised and persistent working memory deficits, which may be related to the central executive, specifically inhibitory processes.

It has been suggested that whilst most children with maths learning disability have deficits in all components of executive function and working memory (updating, inhibition, switching, visuospatial sketchpad and phonological loop), executive dysfunction has the greatest impact (Bull et al., 1999; Geary et al., 2007). This may be particularly true for children with a milder form of maths learning difficulty (11^{th} – 25^{th} percentile), as many appear to have normal phonological loop and visuospatial abilities (visuospatial sketchpad and visual updating), but deficits in inhibitory control and switching (Geary et al., 2007; Murphy, Mazzocco, Hanich, & Early, 2007). However, differences have not always been found between the two classifications of maths learning disability.

Mammarella, Caviola, Giofrè, & Szűcs (2018) investigated visual, spatial-sequential and spatial-simultaneous short-term memory performance in children with maths learning disability (n = 24; M_{age} = 9.8 years, SD = 0.6), low maths achievement (n = 24; M_{age} = 9.7 years, SD = 0.5), and a typically developing group (n = 24; M_{age} = 9.8 years, SD = 0.8). Criteria for inclusion into the maths learning disability group was 1) less than 16^{th} percentile on standardised measure of maths performance, 2) a significant discrepancy between verbal intelligence and overall performance on arithmetic academic achievement testing, and 3) average score in reading decoding. Criteria for inclusion in the low maths achievement group were congruent for 2) and 3) but performance on the standardised measure of maths performance was less than the 30^{th} percentile.

Both the maths learning disability and low maths achievement groups displayed low visuospatial working memory function in both spatial-simultaneous and spatial-sequential working memory tasks. Overall although deficits in these domains were greater in the maths learning disability group, differences did not reach significance.

Once again methodological differences may explain inconsistencies. Additionally, it may be significant that studies in this field have typically examined different components of executive function, and working memory. To my knowledge the main three executive functions and all components of working memory have yet to be investigated concurrently in populations with typical and atypical maths development.

3.3.1.3 Comparison studies.

Congruent with studies investigating the cognitive underpinnings of mathematical competence, much research exploring the cognitive roots of maths learning disability has focussed on one domain. However more recently a few studies have investigated theories across domains. Szucs et al. (2013), contrasted the five dominant theories; numerical acuity, working memory, inhibition, attention and spatial processing, in a sample of primary age children (M_{age} = 9.2 years), to determine the relative importance of each.

Children were defined as having a maths learning disability if they were < 16^{th} percentile on the maths tasks, and within the normal range on measures of reading and intelligence. Experimental groups; maths learning disability and typically developing (n = 12 for each), were matched for age, verbal and non-verbal intelligence, socio-economic status and general processing speed.

Multiple measures were utilised (see Appendix 1) and an analysis of accuracy scores indicated that the maths learning disability group performed significantly worse than the control group on measures of the visuospatial sketchpad and visuospatial updating, subitizing, number acuity and inhibition. Indexing the dependent variable on reaction time resulted in the disorder group doing significantly worse on numerical and physical size decision Stroop tasks and spatial processing. When verbal and non-verbal intelligence and processing speed were controlled for significant results remained for visual updating, the visuospatial sketchpad, and inhibition.

The relative predictive power of the three constructs that correlated with mathematical ability (visuospatial memory, inhibition and counting) were subsequently examined via a regression analysis, with a significant association found for visuospatial memory (β = .48, $t(20) = 3.2$, $p = .005$) and a marginally significant result for inhibition (β = .36, $t(20) = 2.1$, $p = .052$).

Although inhibition, specifically, interference suppression, was only a marginal predictor, Szucs et al. suggest that updating and inhibition impairments may be related, which would concur with the proposed 'unity and diversity' structure for executive functions (Miyake & Friedman, 2012). Interestingly while spatial processing was preserved in the maths learning disability group, solution times were slower on the trail making and mental rotation tasks. Hence the authors concluded that spatial skills were preserved but slower in maths learning disability, although it is worth noting that the trail making task is also frequently used as a measure of switching abilities.

The authors concluded that pure maths learning disability could be characterised by a specific impairment of the visuospatial sketchpad and the inhibitory processes crucial to visual updating. They also suggested that the bilateral intraparietal sulci morphology and function differences found for maths learning disability participants, and frequently cited to support the numerical acuity deficits theory, may have an alternative source, as executive functioning and working memory have also been linked to the intraparietal sulcus (e.g., Price, Palmer, Battista, & Ansari, 2012).

Rather than trying to determine whether maths learning disability was rooted in general or specific cognitive deficits, Toll et al. (2016) looked to test the double deficit hypothesis, which had previously been used in developmental dyslexia research, to determine if mathematical weakness results from deficits in both working memory and numerical acuity (Wolf & Bowers, 1999). This longitudinal study built on work by Kroesbergen & van Dijk (2015), who followed children from the end of the first year of kindergarten (M_{age} = 4.96 years) to the end of first grade (M_{age} = 7.02 years). They found numerical acuity and visual working memory had an almost equally important role in the maths development of primary aged children. Additionally, children with deficits in both constructs did significantly worse than children without any deficit or just one deficit.

Measures used by Toll et al. (2016) included two for visual working memory (measures of the visuospatial sketchpad and visual updating) and two for numerical acuity (non-symbolic and symbolic magnitude comparison tasks), in addition to a test of mathematical facts and mathematical problems (arithmetic). Participants (*N* = 670) were divided into four groups; no-deficit, numerical acuity deficit, working memory deficit or double deficit. When data were analysed the numerical acuity deficit group was further divided into three subgroups; symbolic deficits, non-symbolic deficits or deficits in both processes.

Visual working memory and symbolic numerical acuity at the start of kindergarten were found to be related to mathematical performance two years later, with non-symbolic numerical acuity related to problem solving (arithmetic) solely. Importantly, whilst children with a single deficit (working memory or numerical acuity) had lower mathematical abilities than those without weaknesses, the lowest performances were seen for children with both deficits. Splitting the numerical acuity group into its subgroups, indicated that children with weaknesses in non-symbolic representation solely or in combination with visual working memory deficits performed better than participants with symbolic deficits or deficits in both numerical acuity systems, possibly indicating that symbolic numerical acuity is more important for mathematical competence than non-symbolic ones. However, the non-symbolic comparison task contained only 30 trials, which is low for this type of task. This may be significant because studies with low numbers of trials have been found to be underpowered, therefore reliability may have been reduced (Chesney et al., 2015).

Hence, while some research highlights specific subtypes of maths learning disability, others suggest it may be more advantageous to position individuals with maths learning difficulties within a multidimensional parametric space (Szucs, 2016). However, to do so would require a more nuanced understanding of the modality-specific memory subprocesses and supporting executive functions which are relevant for maths learning (Szucs, 2016).

The Turner syndrome cognitive phenotype is aligned with more than one of the proposed subtypes of maths learning disability, although perhaps significantly, there is conflicting evidence as to whether any subtype adequately explains why maths learning disability frequently co-occurs in Turner syndrome (Mazzocco, 2009). There follows an examination of research investigating number cognition on this population.

3.3.2 Number cognition in Turner syndrome.

Number cognition in Turner syndrome has been studied from kindergarten, to middle childhood, through adolescence and into adulthood (e.g., Bruandet, Molko, Cohen, & Dehaene, 2004; Kesler, Menon, & Reiss, 2006; Mazzocco, 2001; Temple, Carney, & Mullarkey, 1996). While there are suggestions that up to 75% of women with Turner syndrome experience some level of maths difficulties, there are inconsistencies (Mazzocco, 2009; Mazzocco, Singh, & Lesniak-Karpiak, 2006), and it is unclear whether difficulties represent a persistent phenotypic characteristic or a short-term developmental delay in this area (Murphy, Mazzocco, Gerner, & Henry, 2006).

Some studies suggest maths difficulties are confined to specific aspects of maths, with simple arithmetic, number comprehension and production, counting, and some aspects of understanding quantity, such as number comparison and estimation, found to be intact among women with Turner syndrome (Bruandet et al., 2004). Similarly, some basic aspects of number sense, including counting, reading and writing numbers and magnitude judgements have been found to be age appropriate among school age girls (Mazzocco, 2001; Temple & Marriott, 1998).

More recently a meta-analysis highlighted meaningful group differences between Turner syndrome and age-matched neurotypical peers across all measures of maths and number aptitude (Baker & Reiss, 2016). Despite this a majority of studies included in the meta-analysis reported non-significant statistical outcomes, hence this area of research may contain high levels of false-negative outcomes (Type II errors), raising the possibility that the severity of maths and number deficits in Turner syndrome has been underestimated.

Given the prevalence of maths learning disability in women and girls with Turner syndrome, it is unsurprising that research looking to understand cognitive deficits that may explain maths difficulties in this population have focused on executive dysfunction and to a lesser extent, numerical acuity, however the following section will begin by considering research into the visuospatial abilities of girls and women with Turner syndrome.

3.3.2.1. Visuospatial abilities in Turner syndrome.

Buchanan et al. (1998) examined the visuospatial deficits seen in girls with Turner syndrome by examining the underlying systems that contribute to this construct, including an investigation of the relative contributions of visual and verbal working memory. Contrary to expectations, girls with Turner syndrome did not display a specific deficit in either the dorsal or ventral streams (neuroanatomical pathways in the visual system that facilitate object location and object identification), instead data suggested a deficit in working memory, specifically visuospatial memory.

This result may be explained by neuroimaging studies which suggest that temporal lobe activation increases with task demand, indicative of verbal strategies being employed opposed to those drawing on executive functioning (Kesler et al., 2006). The latter research also supports the hypothesis that girls with Turner syndrome utilise different or more extensive functional networks when performing executive functioning and maths tasks (e.g., Menon, Rivera, White, Glover, & Reiss, 2000; Tamm, Menon, & Reiss, 2003).

Girls with Turner syndrome have also been found to perform significantly lower on visual-perceptual and visual-motor tasks relative to age matched peers, which may be related to their maths performance (Mazzocco, 1998; Mazzocco, 2001; Rovet & Netley, 1982; Rovet, 1993; Temple & Carney, 1995). However, other studies have reported subtle, widespread visuospatial deficits, in addition to a generalised slowing of responses on tasks involving visuospatial processing (Buchanan et al., 1998; Mazzocco et al., 2006), with suggestions that poor maths performance in Turner syndrome may be independent of visual spatial abilities (Murphy et al., 2006; Rovet, Szekely, & Hockenberry, 1994).

3.3.2.2. Executive functioning and working memory in Turner syndrome.

Executive dysfunction in girls and women with Turner syndrome is a persistent phenotypic feature (Mazzocco & Hanich, 2010), with deficits reported in primary and middle school girls, adolescents, and women (Haberecht et al., 2001; Kesler et al., 2006; Kirk, Mazzocco, & Kover, 2005; Lasker, Mazzocco, & Zee, 2007; Tamm et al., 2003; Temple & Marriott, 1998). Specific areas of executive dysfunction include; attention, working memory, response inhibition, fluency, organisation and planning

(Buchanan et al., 1998; Haberecht et al., 2001; Lepage, Dunkin, Hong, & Reiss, 2011; Murphy & Mazzocco, 2008; Romans, Roeltgen, Kushner, & Ross, 1997).

A study by Temple et al. (1996) investigated the performance of girls with Turner syndrome (diagnosed clinical and genetically) on a range of tasks commonly used to measure executive function. Deficits were found in inhibition, the ability to search in an organised way, and to rapidly access the vocabulary to retrieve specific information, the latter despite good verbal intelligence and relative strengths in planning and switching. However, conversely, inhibition has been found to be an area of relative strength, and deficits found in switching abilities (e.g., Kirk et al., 2005; Romans et al., 1997; Tamm et al., 2003).

Lepage et al., 2011 considered the contribution of executive function to the visuospatial difficulties of girls with Turner syndrome (*n* = 36; Mean age = 9.24 years, *SD* = 1.90). Executive function was found to make a greater contribution to visuospatial performance for Turner syndrome participants than typically developing controls. Additionally, the authors suggested that as a verbal task became more challenging, prefrontal contributions were recruited, resulting in executive dysfunction also becoming apparent in this domain. They concluded that future studies should utilise both verbal and visual tasks.

A recent meta-analysis by Mauger et al (2018) looked at 16 data samples from 13 studies and classified executive functions into; working memory, inhibitory control, switching, and higher order executive functions (e.g., reasoning, problem solving and planning). Whilst significant executive function impairments in Turner syndrome were observed, effect sizes varied from small (inhibitory control), to medium (switching) and large (working memory, higher order executive functions).

Inhibitory control included measures of cognitive inhibition (inhibiting a prepotent response), focussed attention, and response inhibition (acting impulsively). Girls with Turner syndrome were significantly slower than controls on inhibition tasks solely, with focussed attention and response inhibition appearing to be preserved.

Within cognitive switching, results were task specific, with verbal fluency tasks displaying medium to large differences, whilst no significant differences were observed for the Wisconsin card sorting task. The authors suggested differences may be down to each task measuring different aspects of switching (spontaneous versus reactive switching), or because the verbal fluency task was time bound. Additionally, differences may be due to the domain of task presentation.

Working memory differences were present in both visual and auditory domains, with measures of the phonological loop, visuospatial sketchpad, and bimodal updating, included in the analysis. These findings support evidence from a study looking at functional connectivity during working memory, where impairments were observed in both modalities (Bray, Dunkin, Hong, & Reiss, 2011). To date the paucity of studies comparing the impact of modality prohibits definitive conclusions being drawn.

3.3.2.3 Numerical acuity in Turner syndrome.

A study by Bruandet et al. (2004) compared women with Turner syndrome (*n = 12)* to a control group of typically developing adults (*n* = 13) on a range of mathematical concepts including arithmetic and core systems such as subitizing, cognitive estimation (give a sensible estimate for questions whose answer is not known but could be approximately estimated), estimation (estimating the number of squares presented on a screen), digit comparison (identifying the larger of two simultaneously presented Arabic digits) and bisection (identifying the midpoint of two Arabic digits presented side by side). Impairments were found in subitizing and cognitive estimation, and all arithmetic tasks except multiplication. Impairments manifested themselves mostly as increased response time rather than elevated error rates. This study did not, however, include any measure of numerical acuity, hence it was unclear whether deficits existed in processing numerosities above the subitizing range.

Simon et al. (2008) specifically investigated the numerical acuity abilities of 11 girls with Turner syndrome (*M* = 10.4 years, *SD* = 2.3). Participants were asked to indicate the larger of either two blue bars, 2cm wide and varying in height between 1 and 12 cm in length, or two Arabic numbers, again between 1 and 12. The girls with Turner syndrome were found to be impaired on both symbolic and non-symbolic tasks. Additionally, processing speed was not found to be significantly different to typically developing controls.

Given the limited research into the magnitude comparison abilities of women and girls with Turner syndrome it is difficult to draw conclusions, however what research there is suggests there may be deficits in their subitizing and numerical acuity abilities. A recent study took this investigation further by examining symbolic numerical acuity and executive functions concurrently.

3.3.2.4. Comparison studies.

Brankaer, Ghesquière, De Wel, Swillen, & De Smedt (2017) investigated the symbolic numerical abilities (including acuity and executive functioning (phonological loop, visuospatial sketchpad, and auditory updating) for 24 girls with Turner syndrome (*M* = 9.3 years; *SD* = 1.9). Performance on a symbolic numerical acuity task (indicating the numerically larger of two simultaneously presented Arabic digits), was significantly different from typically developing controls, however they became insignificant when the visuospatial sketchpad was accounted for. The authors concluded that associations between performance on symbolic numerical acuity and visuospatial sketchpad tasks were stronger in those with Turner syndrome than those with typical development. They speculated this may be because visuospatial difficulties could disrupt the mental number line, or impact non-symbolic numerical acuity, which may then impact the ability to make symbolic magnitude representations.

3.4 Summary

Research to date appears to be suggestive of maths learning disability being the result of cognitive deficits, and whilst no single deficit has been identified there is evidence to suggest it may be one or more of: executive function, working memory, non-verbal intelligence, verbal intelligence, and numerical acuity. Inconsistencies in this field may result from the majority of studies failing to include a comprehensive range of measures, for example, all components of working memory, and each of the three main executive functions. Additionally, given the proposal that numerical acuity deficits may explain maths learning difficulties in at least one subtype of maths learning disability, it would appear appropriate that this is also included.

Increasingly, it would appear that at least some of the predictors of maths ability are modality specific, and to date typically the modality of task presentation is not made overtly clear in the majority of studies. Additionally, some of the inconsistency in the literature may be due to statements being made about a construct, e.g., updating, when in fact the study utilised either visual or verbal updating tasks solely. To increase clarity, there have recently been calls for both verbal and non-verbal tasks to be utilised, along with an appropriate number of measures (e.g., Szucs, 2016). This will be addressed in the current study, with each construct measured via a visual and an auditory task.

While Turner syndrome has been aligned with a number of the subtypes of maths learning disability, surprisingly there is limited research into the link between deficits in specific cognitive abilities and maths difficulties. Whilst this may be down to maths difficulties being under reported in this population, as deficits have been reported for executive function, working memory, numerical acuity, non-verbal intelligence and visuospatial abilities, they would appear to be an appropriate population to include in an investigation.

3.5 Research Questions

Therefore, in addition to the first research question:

- Which modality specific cognitive abilities predict arithmetic competence in the general population?

this study will address two additional questions regarding the cognitive profile of children with maths learning disability and Turner syndrome:

- Do the cognitive abilities identified as predictors of arithmetic competence in research question one display atypical development for a) children with maths learning disability, and b) children with Turner syndrome?
- Are identified areas of deficits confined to either the visual or auditory domain?

Having identified the research questions the following chapter will consider methodological issues including the research and methodological approaches, ethical considerations and the study design.

Chapter 4: Methodology

The focus of this chapter is the methodological issues pertinent to addressing the research questions:

1) Which modality specific cognitive abilities predict arithmetic competence in the general population?
2) Do the cognitive abilities identified as predictors of arithmetic competence in research question one display atypical development for a) children with maths learning disability, and b) children with Turner syndrome?
3) Are identified areas of deficits confined to either the visual or auditory domain?

4.1 Research Approach

The design of any psychological research is predominantly influenced by the research questions, which in turn are influenced by the researcher's theoretical position, and both impact the approach which grounds the research. Within developmental and cognitive psychology, the nature/nurture debate has impacted theoretical positions, specifically the researcher's beliefs on the extent to which human cognition is domain general or domain specific.

Historically staunch empiricists viewed the brain predominantly as a 'blank slate', where brain development resulted from domain-general processes, via interactions with the physical and social environment (Kirkham, Slemmer, & Johnson, 2002). Conversely, extreme nativists viewed the brain as a series of innate, independent modules which facilitated specific abilities e.g., language (Pinker, 1999) or number (Butterworth, 2000). While nativists acknowledged environmental influences, they considered them predominantly triggers. Taking inspiration from adult neuropsychology and evolutionary psychology (Karmiloff-Smith, 1998), nativists used cognitive profiles observed in the adult brain to make inferences about the cognitive abilities of infants and children. Almost all present-day scientists acknowledge that ontogenetic development results from the interplay of both genes and the environment, however empiricists and nativists disagree on the extent to which development results from predetermined or probabilistic epigenesis.

Towards the end of the 20th century a third approach emerged. Neuroconstructivism neither considers the infant brain as a tabula rasa upon which experience imprints itself, nor a series of specialised modules. Instead biological constraints such as the impact of genes are acknowledged, however gene expression is not considered predetermined, as the influence of the environment is recognised. Hence it subscribes to the concept of probabilistic epigenesis, where development results from cascading interactions across multiple levels of causation (Spencer et al., 2009).

Neuroconstructivism postulates regional differences in the neonate cortex, for example in neuron types, density of neurons, firing thresholds and the balance of neurotransmitters (Carney et al., 2013). These small differences make different parts of the brain more appropriate for certain kinds of processing, hence while brain activity is initially widespread for processing all types of information, competition between regions gradually determines which domain-relevant circuits become domain-specific (Karmiloff-Smith, 1998). Neuroimaging research supports this position with increased interconnectivity of neurons found within the cortical regions of the developing brain (Huttenlocher & de Courten, 1987), and gradual specialisation of the cortex seen across development (Johnson, 2001), as a consequence of synaptogenesis and pruning (Huttenlocher, 1975).

Researchers studying cognitive deficits in a variety of neurodevelopmental disorders have utilised nativist and neuroconstructivist approaches. However, while nativist research advocates the use of double dissociations, usually across syndromes, to provide evidence for innate, specialist cognitive modules, a neuroconstructivist approach typically involves an examination of not only identified deficits but also their source. Congruent with this approach the current study looks to investigate not only the cognitive abilities underpinning arithmetic ability but whether perceptual abilities impact these associations, by comparing typical development with two neurodevelopmental disorders typically associated with visuospatial difficulties.

For research involving neurodevelopmental disorders a number of different methodologies have been utilised, the pros and cons of which will be considered in the following section.

4.2 Methodological Approach

This investigation is cross-syndrome, involving children with maths learning disability, girls with Turner syndrome, and typically developing children. It is quasi-experimental, as random assignment to groups is not possible, and the independent variables being compared are not actively manipulated by the researcher, this is congruent with the majority of neurodevelopmental disorder research (Cohen, Manion, & Morrison, 2011). Within this methodology, research involving developmental disorders has utilised a number of different approaches, the most popular being cross-sectional, matching and more recently a developmental trajectory approach has been suggested.

4.2.1 The cross-sectional approach.

A cross-sectional study produces a snapshot of a population at a particular point in time (Cohen et al., 2011). In neurodevelopmental disorder research this typically involves comparing the abilities of the disorder group in the construct under investigation, with typical development. Comparing data from a number of studies, ideally ranging from infancy to adulthood, enables a determination of whether underlying causes for any given profile were present from infancy (Thomas, Purser, & Van Herwegen, 2012). For example, Halberda & Feigenson (2008) supplemented their investigation into non-symbolic magnitude comparison acuity in 3-, 4-, 5-, and 6-year olds, and adults with data from two studies investigating the same construct in babies aged between 6- and 9- months (Lipton & Spelke, 2003; Xu & Spelke, 2000). Although this method can provide a picture of development across ontogenesis, the validity of conclusions may be influenced by a number of design considerations.

A major limitation of this method is the age-specificity of many tasks used to measure cognitive abilities. Hence, conclusions made about ontogenesis by amalgamating data from different studies, where participants are drawn from incongruent age groups is likely to be flawed when different tasks are utilised. This may result in the constructs investigated being incongruent; a possible criticism of the proposed developmental trajectory of non-symbolic numerical acuity proposed by Halberda & Feigenson (2008). Additionally, even when tasks are congruent, permitted response times may vary, enabling different strategies to be used to generate responses, an issue particularly pertinent in developmental disorder research, where similar or congruent behavioural outcomes can result from very different cognitive processes (Thomas et al., 2012).

A further limitation of this approach is that syndrome specific cognitive profiles can vary across ontogenesis, hence single snapshots may give an incomplete view of the development of specific abilities. For example, language and number abilities in Williams Syndrome look very different in toddlers, where numerosity is a relative strength and language a deficit, and adults who typically display numerosity deficits and relative strengths in language (Paterson, Girelli, Butterworth, & Karmiloff-Smith, 2006; Singer Harris, Bellugi, Bates, Jones, & Rossen, 1997).

4.2.2 The matching approach.

This approach originated as a result of the theoretical debate surrounding intellectual disabilities, particularly two opposing stances; differences and developmental (e.g., Bennett-Gates & Zigler, 1998; Hodapp & Zigler, 1990). The former views learning disability as the result of underlying organic dysfunction, which produce specific deficits in cognitive functioning and qualitatively atypical cognitive development (Thomas et al., 2009).

While advocates of the developmental stance concur with this view for a subset of individuals, they argue that some individuals with learning disabilities fall at the extreme lower end of the distribution of normal individual variability. Hence, while these individuals exhibit impairment when matched with typically developing individuals of the same chronological age, given they can progress through similar developmental milestones, and can have the same overall structure for intelligence, their results may be similar results to individuals matched for mental age (Thomas et al., 2009).

A developmental approach therefore typically matches the disorder group with two control groups; chronological and mental age, as determined by performance on a relevant standardised test (e.g., British Picture Vocabulary Scale). Impaired performance in comparison to the chronological age group solely is indicative of developmental delay, whilst atypical development is implied by impaired performance in comparison to both control groups.

The matching of control and disorder groups is typically carried out in one of two ways. The first matches individuals with the disorder of interest to the study to two other participants, one for chronological and another for mental age. In the second overall mean chronological and mental ages are matched between the disorder and typically developing group. Pros and cons exist for each method, as the former can reduce the generalisability of findings (Mervis & Robinson, 2003), and big differences in the range of ages or abilities of the groups in the latter can introduce spurious differences in behaviour (Paterson, Brown, Gsödl, Johnson, & Karmiloff-Smith, 1999). For example, a mental age control group in an investigation containing pre-school children with neurodevelopmental disorders may well consist of much younger typically developing children, whose maturation level in many systems (e.g., motor control) may well be significantly different to the other groups.

As this approach relies on a standardised test to determine the mental age group, and previous research informs the domain purported to be measured by individual standardised tests, it is a theory dependent approach. Therefore, once the mental age control group has been identified, little flexibility exists to compare the disorder group against alternative measures of mental age. Clearly, care must be taken to ensure the standardised test measures developmental progress in the same domain as the experimental task (Thomas et al., 2010).

Initially, this approach was advocated within neuroconstructivist research and many developmental disorder studies utilise it (e.g., Ansari et al., 2003; Paterson et al., 1999). It is particularly appropriate for studies where the disorder group has a narrow age range or if the experimental measure is sensitive for a specific age range.

4.2.3 The developmental trajectories approach.

The developmental trajectories methodology aims to address the disadvantages of these approaches by assessing how behaviour changes with age, whilst accounting for the multiple constraints associated with studying behaviour over time (Thomas et al., 2009). Some features of neurodevelopmental disorder research are an integral part of this method, particularly cross-syndrome comparisons, where overlapping phenotypic variability is present (e.g., number cognition in Williams and Downs syndromes). As developmental trajectories are constructed for each population on each experimental task (see section 4.5.2), comparisons can provide an insight into how atypical constraints influence the emergence of impaired performance, thereby giving an insight into the constraints that shape typical development (Thomas et al., 2012).

For neurodevelopmental disorder research that aims to understand the development of cognitive processes over time, a number of design issues need to be addressed;

1) the typically developing group must span the youngest mental age for the disorder groups, on all standardised measures, to the oldest chronological age
2) the experimental task(s) must be sensitive across the ability range of all groups, thereby avoiding ceiling and floor effects
3) data should be collected for all constructs deemed relevant to the cognitive process under study.

Whilst a developmental trajectories approach can help to identify the nature of any observed deficits in the cognitive constructs under investigation, it is reliant on the above design considerations being met. Additionally, although there is scope for more than one measure of mental age, those chosen are theory dependent; a possible limitation.

An additional limitation of the developmental trajectory approach is that whilst it looks at developmental pathways, they are cross-sectional, with the term development referring to correlations with indicators for (mental) age (Bruns et al., 20019). Hence this method is exploratory, and findings should be validated via a longitudinal follow-up (Thomas et al., 2009).

All three methodologies are appropriate for studying neurodevelopmental disorders, with advantages and disadvantages for each. While cross-sectional and matching approaches facilitate an examination of the differences between typically developing and disorder groups, at a particular point in time, design constraints frequently make it inappropriate for causal links to be identified. For such links, longitudinal studies are typically advocated (Cohen et al., 2011), as they facilitate the construction of more complex behavioural models (Ruspini, 2002). However, studying the ontogenesis of a construct via a longitudinal study, particularly for research involving neurodevelopmental disorders, is difficult, particularly in terms of cost and problems related to high dropout rates.

For the present study, the lack of similar research makes the cross-sectional approach less appropriate. While the modality specific cognitive abilities underpinning arithmetic ability could be determined for a particular age group, the paucity of similar research means these results would not facilitate a greater understanding of whether modality impacts the predictive power of the cognitive abilities linked to arithmetic ability across development.

Additionally, the scope of the study (investigating the impact of modality on cognitive markers of arithmetic ability) makes the matching approach less applicable, as identifying a single standardised test to produce a mental age control group would be difficult. As a PhD study, this research is time bound, making it infeasible for data to be collected from the same participants across childhood, therefore a longitudinal study is inappropriate. Hence, the greater flexibility offered by the developmental trajectories approach makes it the most appropriate for this investigation. Although it should be noted that for causality to be established, findings from an investigation utilising this approach should be subsequently validated by a longitudinal study (Thomas et al., 2012).

This study will therefore be conducted within a neuroconstructivist approach and a developmental trajectories methodology. Clearly the approach and methodology of a study impact its design, however, first ethical considerations pertinent to an investigation which utilises vulnerable groups will be addressed.

4.3 Ethics

Ethical approval was sought from Cambridge University Faculty of Education ethics committee and I hold a Disclosure and Barring Service Enhanced Certificate, a requirement for anybody working with children. The research adhered to the principles and guidelines of the British Psychological Society (BPS, 2018) and British Educational Research Association (BERA, 2018). Therefore, responsibility towards the participant and the community of educational researchers was taken seriously, particularly regarding respect for the autonomy and dignity of persons, scientific value, social responsibility and maximising benefit and minimising harm, which were particularly pertinent given that the majority of participants were drawn from vulnerable groups.

As an experienced teacher, I am cognizant of the issues of working with children from late primary to adulthood, however this experience has drawbacks, as by their very nature the teacher/student relationship involves unequal power relationships (Kincheloe, 1991). Hence care was taken to ensure appropriate gatekeepers were in place.

4.3.1 Gatekeepers.

Recruitment to the Turner Syndrome group was initially through the Turner Syndrome Support Society, then regional friendship groups, and finally parents, all of whom acted as gatekeepers.

Children in the typically developing and maths learning disability groups were recruited via their school, with a gatekeeper identified in each establishment. Parents of all participants gave active consent to their child's participation (see Appendix 2 and 3).

4.3.2 Informed consent and right to withdraw.

A challenge within this study was to ensure participants were able to give informed consent. With many participants drawn from vulnerable groups, particular care was taken to ensure they were indeed willing to take part in the study, and understood their right to withdraw at any time.

For Turner Syndrome participants, the national organisation, and parents were made aware of the purpose of the study and what was required of participants. Additionally, the researcher's email was provided, so questions could be answered before parent's gave consent.

To ensure participants themselves were able to give informed assent or consent, information sheets for parents and children were produced in addition to a range of consent forms, differentiated by age and cognitive ability were produced (see Appendix 2 to 4), and care was taken to match them to individuals.

At the beginning of a data collection session, participants and if appropriate their parent(s), were reminded of the purpose of the study and what the session entailed. Every effort was made to ensure these details were understood and that participants were willing to take part. Congruent with the BPS Code of Human Research Ethics, throughout the data collection phase, participants from vulnerable groups were regularly monitored (via verbal and non-verbal signs) to ensure their continued willingness to participate.

For the typically developing and maths learning disability populations, headteachers and other relevant staff were informed of the purpose of the study and what was required of the school and individual participants. Having identified potential participants, the school sent to parents an information pack (see Appendix 2), which included details of the mechanism or how to have questions addressed before they gave consent for their child to participate in the study. Parents returned a signed consent form to their child's school to indicate their willingness for their child to be involved in the study. Before commencement of the data collection session(s) participants were reminded of the purpose of the study, what they are required to do and all participants older than 7-years signed a consent form. Children under 7-years completed a sticker chart (see Appendix 4), putting a sticker before commencement of each task to indicate their willingness to attempt it.

The literature sent to parents made clear that potential participants (and their parents) had the right to withdraw from the study. This was reiterated at the beginning of data collection sessions, with particular care taken to ensure that individuals from vulnerable groups understand the mechanism for withdrawal from the study.

4.3.3 Confidentiality and anonymity.

Data collected for this study were anonymised, using individual identity codes. To ensure confidentiality the list of participants and identity codes has been stored separately from the anonymised data. Electronic data were encrypted and stored on a password protected computer, while hard copies are stored in a locked filing cabinet. Access to personally identifying data is restricted to the research team.

Reporting of data will be at a global or group level and participants were made aware on the consent form of how data could be used i.e. PhD report, conferences. This is particularly important as data may be available online if the study is published or part of an open access arrangement.

4.3.4 Assessing risk.

It was not foreseen that this study would be harmful to participants. However, the risk of fatigue was accounted for using breaks and two testing sessions.

4.3.5 Debrief.

At the end of each data collection session a short, informal debrief was conducted to check participants had not experienced any negative feelings as a result of the session. A formal debrief will be provided to the Turner Syndrome Support Society and participating schools on completion of the study. To ensure it is as useful as possible, the format will be negotiated with each organisation separately. Possible formats could include an information letter, assembly or conference talk.

Having addressed ethical considerations, the following sections will discuss design considerations, including the participants and materials involved in the study.

4.4 Design and Procedures

4.4.1 Participants.

The study initially consisted of 216 children, however two did not complete session two (a fifteen-year old girl due to illness and a seventeen year old girl who elected to withdraw from the study), and were removed from the study. Data was therefore considered for 214 children aged between 4- and 19- years (M = 11.68 SD = 4.03), who were drawn from three populations; typical development, maths learning disability, and Turner Syndrome. An a priori power analysis indicated 158 participants would be sufficient to conduct all planned analyses (for calculations see, Appendix 5). Descriptive statistics are detailed in Table 4.1. Congruent with a developmental trajectories approach, participants in the typically developing group spanned the lowest mental age for the disorder groups on all standardised tasks (e.g., WIAT II and KBIT).

Table 4.1

Summary of Participants by Age and Gender

Gender	Status	N	Min	Max	*M*	*SD*
Female	TD	71	4.66	18.47	11.77	4.11
	MLD	18	5.59	19.27	11.59	3.92
	TS	32	4.89	18.70	12.10	3.91
Male	TD	71	4.16	18.69	11.73	4.15
	MLD	22	5.38	18.08	10.64	3.84

Note: TD = typically developing group; MLD = maths learning disability group; TS = Turner syndrome group.

Participants in the typically developing and maths learning disability groups were drawn from nine schools in the East of England, specifically three primary schools (age range: 4- to 11- years), five secondary schools (four schools were 11- to 18- years, one was 11- to 16- years), and one sixth form college (age range: 18+- years). The researcher liaised with schools regarding the profile of participants (e.g., age, gender, mathematical ability), and they subsequently identified potential participants.

In the maths learning difficulty research there are inconsistencies in the criteria used to define individual with maths learning difficulties. Given the time constraints of this study, the maths learning difficulties group was drawn from mainstream schools. Gatekeepers in each school were asked to identify children with a diagnosis of developmental dyscalculia, or maths learning difficulties, in addition to other children who found maths difficult but who were average or above in other subjects. Perhaps unsurprisingly, given the under-diagnosing of maths learning difficulties no children were identified who had a clinical diagnosis. Given the exploratory nature of the second part of this study, a more lenient criteria were utilised to ensure a feasible sample size. Therefore, children whose scores on the arithmetic task were below the 25^{th} percentile, and whose verbal intelligence was in the normal range, were identified, and subsequently formed the maths learning disability group. Given this more lenient criteria findings from the current study capture the profile of children who are struggling with arithmetic, but are not generalisable to children with more profound maths learning difficulties.

Turner Syndrome participants were recruited via the Turner Syndrome Support Society. Initial contact was made at their annual conference, subsequently regional friendship groups were approached to send out information on the study (see Appendices 2 to 4).

The sample reflected the ethnic make-up of the local communities and were predominantly Caucasian. Socioeconomic status information consisted of information on family factors (parent’s years of education, highest academic achievement and occupation), and information on environmental influences (postcode). Summaries by status and gender can be found in Table 4.2.

Table 4.2

Summary of Participants by Socioeconomic Status

Gender	Status		n	Min	Max	*M*	*SD*
F	TD	Family	58	3.00	63.50	46.01	13.53
		Environment	71	5405	30925	20013	5755
	MLD	Family	12	9.00	63.00	38.29	14.79
		Environment	17	8856	31481	19933	6761
	TS	Family	31	9.00	60.50	47.95	11.28
		Environment	32	10012	32723	24224	6644
M	TD	Family	52	9.00	63.50	46.08	14.80
		Environment	76	1792	31481	19592	6210
	MLD	Family	17	12.00	54.00	43.97	14.03
		Environment	15	11470	31535	20100	5726

Note: TD = typically developing group; MLD = maths learning disability group

4.4.2 Materials.

This research aims to consider whether the predictive power of constructs shown to be linked to arithmetic ability is influenced by modality in typical development, and for two populations with known visuospatial deficits; Turner syndrome and maths learning disability. After a consideration of numerical cognition research from a number of different fields, the constructs included for investigation are general and specific markers of numerical cognition, and intelligence. Specifically; visual and auditory measures of working memory, switching, inhibition, numerical acuity and intelligence will be measured alongside arithmetic ability. Processing speed will also be measured as it is a deficit associated with Turner syndrome.

When choosing tasks to measure the constructs of interest a number of factors were considered. Firstly, developmental trajectories approach requires a wide age-range for participants and inclusion of individuals with neurodevelopmental disorders, whose mental age must be considered in addition to chronological age. Materials therefore needed to be sensitive across a wide age range, quick to administer and varied enough to sustain interest. Following a consideration of materials used in similar research (see Appendix 6) the following measures were chosen.

4.4.2.1 Arithmetic competence.

The Wechsler Individual Achievement Test-Second UK Edition (WIAT-II UK; Wechsler, 2005*)* is a comprehensive, individually administered test for assessing the achievement of children and adolescents aged between 4- and 16-years 11 months. It covers the domains of: reading, writing, mathematics and oral language. In this study arithmetic competence is measured using the numerical operations subset.

Normative data for the UK edition of WIAT-II are based on a fully stratified sample (based on UK 2011 Census) of over 700 children and young people. UK norms are available for 4- to 16- years, and US norms up to 85- years. Hence it is sensitive across a large age range, and as raw scores will be utilised, the lack of UK norms for adults did not affect the validity of data. Internal consistency coefficients for the numerical operations subset indicate good to excellent reliability (see Appendix 7), as do test-retest coefficients (range; .85-.98).

The numerical operations subset is a pen and paper task which assesses the ability to identify and write numbers, count using 1:1 correspondence, and solve written calculation problems and simple equations involving the basic operations of addition, subtraction, multiplication, and division. It consists of five items for 5- to 6-years, 16 items for 6- to 7- years and 47 items for individuals above 7- years. Administration time varies depending on the competence of participants but is typically between 10 and 15 minutes.

The starting point is age specific. To establish the basal level the first three items must be correct, otherwise preceding items are administered in reverse order until three consecutive correct answers are achieved. The test is discontinued after six consecutive incorrect responses. Participants receive a point for each correctly answered item, and any questions preceding the basal level. The maximum score is 54, with the analysis in the present study utilising raw scores.

4.4.2.2 General cognitive ability.

As this investigation focusses on the impact of modality on the cognitive predictors of arithmetic ability, and includes two populations typically associated with visuospatial difficulties it was appropriate to include a measure of verbal and non-verbal abilities. The Kaufman Brief Intelligence Test second edition (KBIT-II; Kaufman & Kaufman, 2004) is an individually administered measure of verbal (crystallised) and nonverbal (fluid) cognitive ability, which utilises an easel format. It consists of three subsets; verbal knowledge, nonverbal knowledge and riddles as detailed in Table 4.3. The verbal score is a combination of the verbal knowledge and riddles subsets, which measure verbal, school-related skills by assessing a person's word knowledge, range of general information, verbal concept formation, and reasoning ability. The nonverbal score, measures an individual's ability to solve new problems by assessing their ability to perceive relationships and complete visual analogies, and therefore consists of pictures or abstract designs solely.

Table 4.3

Description of KBIT-II Subtests

Subtest	Number of items	Measures
1	60	Receptive vocabulary Range of general information about the world
2	46	Visual stimuli, both meaningful (people and objects) and abstract (designs and symbols)
3	48	Reasoning and vocabulary knowledge

KBIT-II was selected as it is appropriate across a large age range; 4- to 90- years, and has been used with developmental disorder populations (Libertus, Feigenson, Halberda, & Landau, 2014) for a similar age group (7- to 32- years). Additionally, the internal-consistency reliability statistics indicate good to excellent internal consistency (see Appendix 7), and while the normative sample for KBIT-II is based on 2,120 children and adults from the United States rather than the UK, as raw scores were utilised in the analyses, the validity of the data was not impacted by the lack of UK norms. The manual states that administration time is between 15 minutes (under 9- years) and 25 minutes (16- to 45- years).

The starting point for each subset is age-specific, and contains specific teaching items, where the examiner explains the nature of the task if participants make an error. Subsets have congruent basal and discounting rules; the former requiring participants to answer the first three items correctly. Failure to do so results in the examiner dropping back to the next earlier start point, having first provided appropriate teaching if the failed item is a teaching item. The process of dropping back one start point is repeated until the participant either passes the first three items correctly or drops back to item one. Upon reaching the basal point testing is discontinued following four consecutive incorrect answers.

Correctly answered items elicit a point, hence the verbal score (combined totals from subsets one and three) has a maximum score of 108, and the non-verbal score (subset two), a maximum of 46.

4.4.2.3 Working memory.

In the current study, working memory is conceptualised as consisting of two slave components; visuospatial sketchpad and phonological loop, and two central executive components; auditory and visual updating (see section 2.3.1). These constructs were measured via three subsets and one adapted subset of the Working Memory Test Battery for Children (WMTB-C; Pickering & Gathercole, 2001).

Although normed from 4- to 16- years, WMTB-C was used by Carney et al. (2013) in a study which utilised a developmental trajectories methodology, and whose participants included children and adults with a neurodevelopmental syndrome (age range; 8:2 to 21:10 years) and typically developing children (age range; 4:0 – 9:2 years). The manual reports medium to good reliability coefficients (see Appendix 6) and inter-scorer reliability (.88) for these tasks.

The phonological loop and auditory updating were measured using the forward and backward digit subsets. Both subsets are comprised of seven sections, each containing six trials, a total of 42 items.

In the forward digit recall subset, the researcher speaks sequences of digits at a rate of one per second and participants verbally recall the sequence. The initial section consists of one digit, with subsequent sections increasing by an additional digit. Commencement of the task is preceded by three practice items which consist of one, two and three length sequences. Very young children begin with the first section, whilst older children begin at the section corresponding to the highest sequence correctly replicated in practice trials. However, if they fail to successfully complete four of the six trials in this section, the preceding one is presented and the test continues from there (omitting trials already administered). Congruent with the standard test procedures, if participants respond correctly to four trials within a section, the next section is administered, and credit given for omitted trials. The task terminates when three errors are made within a section.

In the backward digit recall task, participants verbally recall the sequence in the reverse order, with the initial section containing two digits. In the practice trials participants complete two trials of two digits, followed by the two-digit section. If the discontinue rule has not been applied, two three-digit practice trials are administered followed by the remainder of the test. If participants respond correctly to four trials within a section, congruent with standard test procedures, the next section is administered, and credit given for omitted trials. The task terminates when three errors are made.

The visuospatial sketchpad was measured via the block recall subset of WMTB-C. An adapted version, backward block recall was utilised as a measure of visual updating. Both are comprised of nine sections, each containing six trials, thereby producing a raw score between 0 and 54.

In the forward block recall task, the researcher taps a predetermined sequence on a block recall board, at rate of one per second and participants attempt to reproduce it. In the first section a single block is tapped, with the length of the sequence in consecutive sections increasing by an additional tap. Commencement of the task is preceded by three practice items which consist of one, two and three length sequences. Very young children begin test trials at the first section, however older children start at the section congruent to the highest sequence correctly replicated in the practice items. Failure to complete four of the six trials within this section correctly, results in the preceding section being presented, with the test continuing from this point (omitting trials already administered). If participants respond correctly to four trials within a section, the next section is administered, with credit given for omitted trials. The task is discontinued when participants made three errors within a section.

The initial section of the backward block recall task consists of two blocks and participants are required to tap the block in the reverse order. In practice trials participants complete two trials of two blocks, followed by the two-block section. If the discontinue rule is not applied, two three-block practice trials are administered followed by the remainder of the test. If participants respond correctly to four trials within a section, the next section is administered, and credit given for omitted trials. The task is terminated following three consecutive errors.

4.4.2.4 Processing speed, visual inhibition and visual switching.

Processing speed, visual inhibition and visual switching were measured via The

Shape School Extended task (Ellefson, Blagrove, Penford, Espy, 2019, in preparation), which adapts the original Shape School (Espy, 1997), used predominantly with preschool children, to facilitate its use by individuals up to 18-years. The reliability statistics for the original Shape School can be seen in Appendix 7.

The Shape School Extended task has a storybook format, and is divided into four conditions, each containing of 48 trials; control, inhibition, switching and inhibition-switching. The story begins by introducing a school where the children are red or blue, circles or squares; an adapted, simplified version of the original can be seen in Figure 4.1.

The control condition consists of 12 of each of the items (48 in total). In the story the children are lined up to go out to break. Participants are required to name items by colour, as quickly as possible without making any errors. This condition was used as a measure of processing speed.

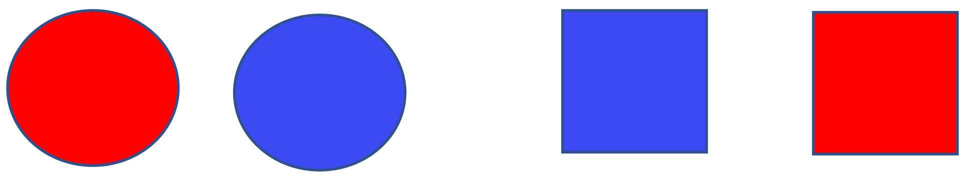


Figure 4.1. Adapted stimuli for Control Condition of Shape School Extended Task.

In the inhibition condition items from Figure 4.1 have either happy or sad faces; six of each, hence 48 in total. Participants are instructed to name the colour of items with happy faces but must ignore those with sad faces.

The switching condition introduces shape as a possible response. Hence if items include a hat, shape is named rather than colour. Again, each item in Figure 4.1 is displayed with and without a hat on 6 occasions, a total of 48 items.

The final inhibition-switching condition, combines the previous two, including sad and happy faces, and hats and without hats. Each item in Figure 4.1 therefore has either a happy or sad face, and a hat or no hat; 3 of each, therefore 48 items in total.

In each condition responses are recorded, then coded, with accuracy and response times noted.

4.4.2.5 Auditory switching.

Auditory switching was measured by a task previously utilised by Menghini, Addona, Costanzo, & Vicari (2010) in a study involving children and adults with Williams syndrome and typically developing children (age range; 6:11-34:9 years), It is an adapted version of the category fluency task (CAT-A; Mäntylä, Carelli, & Forman, 2007) whereby participants generate instances for two separate categories and then alternate between these categories.

Congruent with Mäntylä et al. (2007), in the current study participants were given one minute to generate instances for two separate categories (animals and fruits), followed by two minutes to generate a paired response consisting of one instance from each category. In the separate category conditions one point is awarded for each novel response, and in the paired condition one point is given for each novel pair. Congruent with Mäntylä et al. (2007) a measure was calculated by subtracting the score for paired condition from the mean score from the animal and fruit conditions.

4.4.2.6 Auditory inhibition.

Finding an auditory inhibition task, sensitive across this age range proved difficult. However, an adapted version of the colour association task (Naor-Raz, Tarr, & Kersten, 2003) was included in the pilot (see Chapter 5) to determine its appropriateness. Naor-Raz et al. used a variation of the Stroop paradigm to investigate whether colour is an intrinsic property of object representation. Stroop-like effects were found for pictorial representation of colour-diagnostic objects (defined as objects which tend to have at least one typical colour strongly associated with them, for example a banana and yellow), and priming effects for items that were conceptually related to items shown during colour naming (e.g., banana/monkey) following colour naming of words but not pictures. They concluded that colour is intrinsic to how we learn about, remember, and recognise objects.

For the current study, a task was created that built upon this concept by requiring participants to name the colour they associated with 30 colour-diagnostic objects It consisted of two conditions; control and inhibition. In the control condition, participants heard 30 words with strong colour associations (see Appendix 8), and were asked for the colour they associated with each word. In the inhibition condition the process was repeated however, participants were required to inhibit the colours red and yellow and replace them with “Elmo” and “Big Bird” (see Figure 4.2), characters participants had met in the Panamath task (see section 4.4.2.7).

Figure 4.2. Elmo and Big Bird; characters used in auditory inhibition task.

Responses were recorded, then coded, with one point awarded for each valid association or correct inhibition. However, if younger children were unable to name a colour association in the control condition, to maintain motivation they were told an appropriate colour, and no score was recorded for that item in either condition. The difference between the number of valid associations in each condition was calculated. As this would have resulted in lower scores being indicative of more successful inhibition, to aid the ease of interpretation of data in the analysis, this value was subtracted from the highest difference obtained by a participant.

4.4.2.7 Visual numerical acuity.

Visual numerical acuity was measured via a modified version of the Panamath (2013) software (http://www.panamath.org/), as it is sensitive across a wide age range (3- to 85- years). Whilst concerns have been raised about the Panamath task, particularly as it does not allow for all visual parameters (e.g., convex hull) to be controlled for (Gebuis & Reynvoet, 2012), it has been used in many studies investigating numerical acuity, including studies involving children with neurodevelopmental disorders (e.g., Chesney et al., 2015; Halberda & Feigenson, 2008; Libertus et al., 2014; Purpura & Simms, 2018). Additionally, it is available free of charge, and can easily be downloaded in a form that can be modified.

Presenting stimuli simultaneously rather than sequentially has also been shown to improve reliability and validity (Dietrich et al., 2015). The design was based on Chesney et al. (2015) as the reliability of their adaptation was tested, with large correlations found between commensurate halves for the different size-congruency conditions (.79) and for a random split (.81).

In this version of the Panamath task, participants indicated the numerically larger of two sets of dots, by pressing one of two predetermined keys on a computer keyboard (indicated by red and yellow stickers). Instructions for the task were presented on screen, and given verbally. Following the pilot study (see Chapter 5) the number of trials per session was 84, therefore 168 in total.

Trials were preceded by a centralised white fixation cross, and participants self-initiate trials by pressing the space bar causing the fixation cross to disappear and be replaced by two sets of dots (between 10 and 30), one in each rectangle (see Figure 4.3).

After 750ms the dots were replaced by a yellow and blue snow mask, which lasted for an additional 750ms and covered an area congruent with the rectangles. Once the snow mask disappeared the background grey screen remained until participants responded, thereby causing the fixation cross to reappear. No feedback was given (Dietrich et al., 2015).

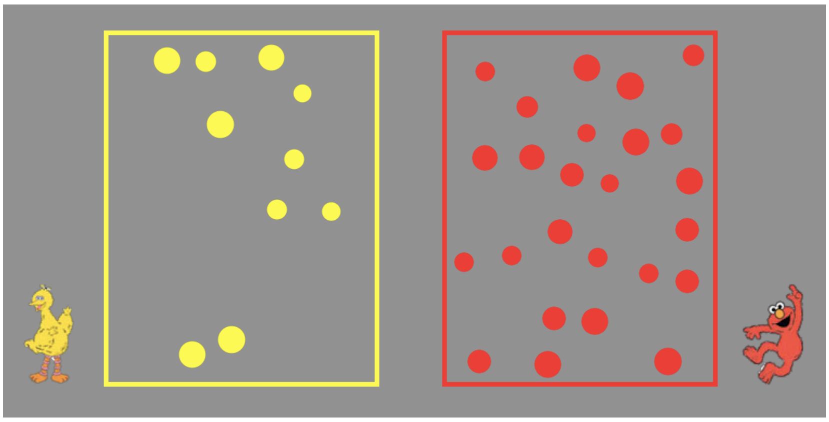


Figure 4.3. Stimuli for Panamath task. Dots appear in each rectangle for 750ms.

The difference in the number of dots contained within the rectangles varied by pre-determined ratios (see Table 4.4), with larger ratios facilitating easier discrimination. Presentation of specific ratios was pseudo-random, following ten practice trials.

Table 4.4

Ratio Bins Utilised in Visual Numerical Acuity Task

Ratio bin	Actual ratios used
1.1	10:11; 20:22
1.2	10:12; 15:18; 20:24; 25:30
1.3	10:13; 20:26
1.4	10:14; 15:21; 20:28
1.6	10:16; 15:24
1.8	10:18; 15:27
2.5	10:25; 12:30

Congruent with other research in this field possible confounds were accounted for in the design of this task (Mazzocco et al., 2011b). Therefore, in half of the trials the side with more dots had a greater total area (size congruent), whilst for the other half of the trials the side with more dots had a smaller total area than the other side (size incongruent). Additionally, six possible “average dot sizes” were used (25, 30, 35, 40, 45 and 50 units) which limited the range of the diameter of the dots in the more numerous sets. The area of individual dots varied randomly by up to 42% of the average dot size (a maximum 19% increase in the dot diameter), with the average being maintained across the set. The 24 trials per ratio therefore consisted of two trials per ratio bin (side with larger number of dots being counterbalanced), per size, per size contingency. All variables were randomly determined by the programme, but as they were based on the same default random seed each participant received the same set.

Research into numerical acuity typically utilises either the Weber fraction (defined as the smallest ratio of two numerosities that a person can reliably judge as larger or smaller), reaction time, or overall accuracy on the trials as the dependent variable. The meta-analysis conducted by Chen & Li (2014) suggests that within their sample of 36 studies, 58% utilised the overall accuracy as their measure of numerical acuity, whereas 47% used w. Initially all three measures will be considered in the exploratory analysis to determine if differences exist.

4.4.2.8 Auditory numerical acuity.

Auditory numerical acuity was measured using an adapted version of the duration comparison task utilised by Rousselle, Dembour, & Noël (2013). This study also included children and adults with a neurodevelopmental disorder (age range; 5:6 – 52:10 years) and typically developing children (age range; 3:8 – 11:8 years). However, harder ratios were included to make the task appropriate for older adolescents e.g., 10/9 and 11/10.

Participants compared the duration of two identical sounds presented sequentially (Range = [375-1500 ms]; audio format: 44100 Hz, 32 bits, Mono), created using NCH Tone Generator v3.22 software, and edited with NCH WavePad Masters software. Ratio bins were congruent to the Panamath task (see Table 4.5). Due to the sequential presentation of the sounds, a silence lasting 700ms was inserted to separate the two durations, with the side of the correct response counterbalanced.

Table 4.5

Ratio Bins Utilised in Auditory Numerical Acuity Task

Ratio bin	Actual ratios used
1.1	650:715; 900:990
1.2	375:450; 750:900
1.3	700:910; 950:1235
1.4	450:630; 850:1190
1.6	500:800; 700:1120
1.8	525:945; 600:1080
2.5	500:1250; 600:1500

Trials were repeated twice (once per session) to ensure parity with the Panamath task, hence ratio bins were presented eight times; a total of 56 trials, double the number of trials in the pilot study (see Chapter 5), where each pairing was presented once. The increased number of trials was to improve task reliability, and to make it congruent to Rousselle et al. (2013).

Whilst the same ratios were presented in both the visual and auditory numerical acuity tasks, because additional visual parameters were controlled for in the visual task, there was a disparity between the number of trials (visual; 168: auditory; 56). Whilst this is not ideal, the pilot study (see Chapter 5) highlighted that this task was challenging for younger participants, hence having more trials may have increased the number of participants failing to complete the task.

4.4.2.9 Socioeconomic status.

In addition to cognitive measures, two metrics for indexing socioeconomic status were included in the study. These were a measure of family influence, via an adapted version of the Hollingshead scale (Hollingshead, Unpublished working paper, 1975), and a measure of environmental influence in the form of postcodes.

The Hollingshead Four Factor Index of Social Status uses parental occupation, years in education, gender and marital status to determine an index of social status (see Appendix 2). Coding for the education and occupation factors can be seen in Table 4.6.

The status score was calculated for each responder, using the following formula:

$$\textit{Status} = (\textit{occupation score} \times 5) + (\textit{education score} \times 3) \qquad (1)$$

If a household contained a single parent their score was used as the index. For two parent households the index was calculated as follows:

1) An average of both parents, if both employed

2) The employed parent's score if only one employed.

Scores range from 3 to 66.

The environmental socioeconomic metric was via participant's postcodes, which enabled deprivation data to be calculated (http://imd-by-postcode.opendatacommunities.org), using 2015 data to postcodes, eliciting an index of multiple deprivation, whereby postal areas of areas of England are ranked from one (most deprived area) to 32,844 (least deprived area). The index of multiple deprivations is a composite score amalgamating seven domain indices; income, employment, education, skills and training, health and disability, crime, barriers to housing and services and living environment.

Table 4.6

Scoring for Hollingshead Scale

Level of school completed	Score	Occupation	Score
Primary school	1	Higher executives, proprietors of large businesses and major professionals	9
No GCSEs / O levels	2	Administrators, lesser professionals, proprietors of medium sized businesses	8
GCSE / AS levels	3	Smaller business owners, farm owners, managers, minor professionals	7
A level	4	Technicians, semi-professionals, small business owners	6
Partial university	5	Clerical and sales workers, small farm owners	5
University graduation	6	Skilled manual workers, craftsmen and tenant farmers	4
Postgraduate degree	7	Machine operators and semiskilled workers	3
		Unskilled workers	2
		Farm laborers, menial service workers	1

A summary of the study measures can be seen in Table 4.7.

4.4.3 Procedures.

The design of the study was between subjects, quasi-experimental, within a developmental trajectory approach, thereby facilitating a comparison of developmental trajectories for each population across a range of different constructs.

It consists of three populations: typically developing, maths learning disability, and Turner syndrome. The typically developing and maths learning disability groups were recruited from schools in the East of England and tested individually in a quiet area of their school. Participants in the Turner syndrome group were recruited via the Turner Syndrome Support Society, and were tested individually in their homes.

Data collection was over two sessions each lasting approximately 45 minutes. Sessions (see Table 4.8 for structure) were conducted between one and six weeks apart (M = 2.0 weeks), with tasks presented in the same order for all participants.

Table 4.7

Summary of Study Measures

Measure	Test type	Range of scores	Interpretation	Use
WIAT-II	Standardised	0 - 54	High = better	RQ All
KBIT-II - verbal	Standardised	0 - 108	High = better	RQ All
KBIT-II - nonverbal	Standardised	0 - 46	High = better	RQ All
Forward digit	Experimental – widely used	0 - 42	High = better	RQ All
Backward digit	Experimental – widely used	0 - 42	High = better	RQ All
Block recall	Experimental – widely used	0 - 42	High = better	RQ All
Backward block recall	Experimental – widely used	0 - 42	High = better	RQ All
Shape School Extended	Experimental - novel	0 – 3.65	High = better	RQ All
CAT-A	Experimental – limited use	0 – 16.5	High = better	RQ All
Colour association	Experimental - novel	0 - 14	High = better	RQ All
Panamath	Experimental – widely used	0 – 100 0 – 2.28 330 - 30020	High = better Lower = better Lower = better	RQ All
Duration comparison	Experimental – limited	0 – 56	High = better	RQ All
Hollingshead Scale	Questionnaire	3 - 66	High = better	RQ 1

Table 4.8

Session Structure

Session	Task	Format	Approximate duration (mins)
1	Panamath	Computer	4
	Duration comparison	Audio recording	4
	Shape School	Picture book	10
	Block recall	Practical	5
	Backward block recall	Practical	5
	Digit recall	Verbal	5
	Backward digit recall	Verbal	5
	Colour association	Verbal	4
2	Panamath	Computer	4
	Duration comparison	Audio recording	4
	KBIT-II	Flip book	15
	Adapted category fluency	Verbal	6
	WIAT-II	Pen & paper	15

4.5 Planned Analyses

To recap, my research questions are:

1) Which modality specific cognitive abilities predict arithmetic competence in the general population?
2) Do the cognitive abilities identified as predictors of arithmetic competence in research question one display atypical development for a) children with maths learning disability, and b) children with Turner syndrome?
3) Are identified areas of deficits confined to either the visual or auditory domain?

As research question 1 deals solely with typical development, and questions 2 and 3 are examining differences between the population, planned analysis will be conducted in two phases.

4.5.1 Phase one.

Phase one will look to identify the modality specific cognitive abilities that predict arithmetic ability in typical development. As the typically developing and maths learning disability populations were recruited from mainstream schools, and none of the participants assigned to the maths learning disability group had a clinical diagnosis, each analysis will be conducted with the typically developing population solely and combined data from typically developing and developmental dyscalculia populations, to elicit whether the addition of the maths learning disability population impacts the typically developing findings. Planned analyse can be found in Table 4.9.

Table 4.9

Summary of Planned Analyses in Phase One

Analysis	Constructs included	Dependent variable	Potential control variables
Descriptive statistics	All	NA	
Zero and partial correlations	All	NA	Age
Independent regression (*z* scores)	Visual and auditory measures of; updating, VSSP, PL, cognitive flexibility, inhibition and numerical acuity	Arithmetic	Age
Hierarchical regressions (*z* scores)	Visual and auditory measures of; updating, VSSP, PL, cognitive flexibility, inhibition and numerical acuity	Arithmetic	Age, SES, processing speed, composite IQ, verbal IQ, nonverbal IQ
Structural equation modelling	Dependent on results of hierarchical regression	Arithmetic	

Note: VSSP = visuospatial sketchpad; PL = phonological loop; SES = socioeconomic status.

4.5.2 Phase two.

Analyses in phase two will address research questions two and three. Firstly, descriptive statistics and zero and age-corrected partial correlation matrices will be produced separately for the maths learning disability and Turner syndrome populations. Subsequent analyses will be conducted within a developmental trajectories methodology. A worksheet which accompanies Thomas et al. (2009) details this approach, and informs the analysis in this phase.

Firstly, developmental trajectories will be constructed for typical development on each measure, thereby facilitating an assessment of the performance of each individual in the neurodevelopmental disorder populations to be made for each measure by determining whether they fit anywhere along the typically developing trajectory. Failure to do so would indicate atypical development within that particular construct (Thomas et al., 2009).

Next developmental trajectories for both developmental disorder groups will be constructed linking their performance on each experimental task with chronological age. As the majority of constructs have two measures, visual and auditory, a mixed-design linear regression, with within-participant factors, will enable several trajectories to be compared simultaneously. Confidence intervals around the regression lines, will be used to facilitate an assessment of whether trajectories converge or diverge and also whether individuals in the disorder group fall outside the range of performance expected for their chronological age. This procedure will be repeated with mental age, to determine whether development is delayed or uneven.

Between-group comparisons for each measure will then be investigated using an adapted analysis of covariance (ANCOVA). A comparison of cross-sectional developmental trajectories for two tasks carried out by the same group will be via a repeated-measures analysis of variance (ANOVA) and an ANCOVA and, a mixed design linear regression will be used to identify if the disorder groups show the same relationship between the development of two abilities as the typically developing group. Data files will be available in advance of my viva.

The next section details the pilot study conducted before commencement of the main study.

Chapter 5: The Pilot Study

The main aim of the pilot study was to ensure materials were appropriate for measuring the constructs under consideration across the proposed age range. Additionally, the design of sessions was reviewed to ensure they were varied enough to sustain interest, of similar length, and the order of task presentation was optimal.

This study consisted of 38 typically developing participants (see Table 5.1) aged between 4- and 64- years (M_{age} = 18.6). Ethical approval was granted by Cambridge University Faculty of Education and adhered to the principles and guidelines of the British Educational Research Association and British Psychological Society. Data were collected in two sessions each lasting approximately 45 minutes.

Table 5.1

Pilot Participants by Age

Age (years)	n	Age (years)	n
4-5	3	14-15	4
6-7	5	16-17	5
8-9	3	18-30	5
10-11	3	>30	5
12-13	2		

After a few minor adjustments, the order of presentation of the tasks was as detailed in Table 5.2. Presenting tasks in this order worked well, particularly having the arithmetic task as the final one, thereby mitigating the concerns of some participants regarding the mathematical nature of the study. Session one was designed to be as varied as possible, containing tasks participants would find interesting and enjoyable. This meant the anxiety displayed by some participants regarding their ability to perform on the tasks (typically because they thought the study would involve predominantly maths-based tasks) was reduced and they were happy and keen to take part in session two.

Typically, tasks were found to be appropriate across the age range, although the youngest children (4- and 5- years) found some conditions in the Shape School Extended task difficult. All failed to complete the final condition, which required participants to both inhibit information and switch between rules. However, as this measure was primarily used to index inhibition and switching abilities independently, the task was retained for the main study.

Table 5.2

Materials tested each Session

Session	Task	Construct measured
1	Panamath	Visual numerical acuity
	Shape School Extended	Visual inhibition and switching
	Block recall	Visuospatial sketchpad
	Backward digit recall	Auditory updating
	Adapted Category Fluency	Auditory switching
	Colour association	Auditory inhibition
2	Panamath	Visual numerical acuity
	Duration comparison	Auditory numerical acuity
	KBIT-II	Verbal and non-verbal cognitive abilities
	WIAT-II	Arithmetic competence

The youngest children also found maintaining focus throughout the 108 trials of the Panamath task difficult, and many were subsequently not motivated to repeat this task in session two. 108 trials are larger than many studies (e.g., De Smedt & Gilmore, 2011; Halberda & Feigenson, 2008; Peng, Yang, & Meng, 2017), and in this pilot the majority of participants did manage to completed 2x108 trials. Having a large number of trials is desirable, as it has been proposed that inconsistent results in non-symbolic magnitude comparison studies may result from the number of studies which utilised small numbers of trials (Chesney et al., 2015). However, as this study includes young children and children with neurodevelopmental disorders it is important they are comfortable with the task. Hence for the actual study the number of trials was reduced to 84 trials per session; 168 in total, still more trials than many studies (e.g., Szucs et al., 2014).

To determine if each task was actually measuring the constructs under investigation, and was appropriate for children between 4- and 18- years, descriptive statistics and an age-corrected partial correlation matrix were produced (see Tables

5.3 & 5.4). The range, standard deviation and variance statistics show a spread of scores for each task indicating they all display sufficient variance. Additionally, as all age-corrected Pearson correlation coefficients were less than .8, multiple tasks do not appear to be measuring the same construct. Additionally, developmental trajectories, constructed using *z* scores for each measure (see Appendix 9) confirm each measure improves across childhood and into early adulthood before stabilising or declining.

Table 5.3

Descriptive Statistics for Measures included in the Pilot Study

Measure	*n*	*M*	*SD*	Range	Min	Max	Variance	*SE*
Age	38	18.62	15.33	59.98	4.51	64.49	234.94	2.49
BDR	36	24.58	6.89	26.00	14.00	40.00	47.51	1.15
Block recall	38	27.61	4.53	17.00	19.00	36.00	20.52	0.73
CAT	36	7.18	2.73	11.50	2.00	13.50	7.45	0.45
SS-CF	37	0.73	0.35	1.71	0.23	1.94	0.12	0.06
Colour association	36	-1.56	1.75	8.00	-6.00	2.00	3.05	0.29
SS-inhibition	37	2.02	0.74	2.70	0.59	3.29	0.55	0.12
Panamath	37	87.43	9.06	34.90	61.40	96.30	82.17	1.49
DC	35	15.31	2.48	14.00	5.00	19.00	6.16	0.42
KBIT-Verbal	34	78.29	18.43	63.00	40.00	103.00	339.67	3.16
KBIT-non-verbal	34	32.50	7.06	30.00	14.00	44.00	49.83	1.21
WIAT-II	35	32.26	13.14	47.00	6.00	53.00	172.79	2.22

Note. BDR = backward digit recall; CAT = Adapted category fluency; SS-CF = Shape School-switching condition; SS-inhibition = Shape School-inhibition condition; DC = duration comparison.

Following the pilot study, a number of adjustments were made to the materials and two additional tasks included. The latter resulted from a recognition that I had not included tasks to measure all the components within my conceptualisation of working memory, which views it as a central executive, and two slave components; the visuospatial sketchpad and phonological loop. Hence the final study included measures of the visuospatial sketchpad (block recall), phonological loop (digit recall), visual updating (backward block recall) and auditory updating (backward digit). While this introduced two additional tasks, they are quick to administer and similar tasks have been used in previous disorder research (e.g., Menghini et al., 2010).

The colour association and category fluency tasks were included in the pilot to determine their suitability for inclusion in the final study, as the former was novel and the latter had been used to measure verbal cognition on only a few occasions. Both tasks displayed sufficient variance (see Table 5.4), and developmental trajectories (see Appendix 9) indicated the lack of ceiling or floor effects, hence given the lack of viable alternatives they were retained.

Table 5.4

Age-Corrected Correlation Matrix for Measures used in the Pilot Study

Measure	1	2	3	4	5	6	7	8	9	10
1. BDR										
2. Block recall	.36									
3. CAT	.24	.10								
4. SS-CF	.35	.33	-.14							
5. Colour association	.30	-.01	.32	.15						
6. SS-inhibition	.44	.29	-.02	.58*	.05					
7. Panamath	.39	.20	.25	-.003	.03	.10				
8. DC	.41	.46	.26	.05	.25	.07	.33			
9. KBIT II- V	.65*	.38	.21	.33	.22	.67*	.36	.37		
10. KBIT II- nonverbal	.77*	.48*	.21	.35	.42	.32	.39	.39	.64*	
11. WIAT-II	.63*	.22	.05	.26	.25	.42	.28	.33	.64*	.60*

Note: BDR = backward digit recall; CAT = category association task; SS-CF = shape school-cognitive flexibility condition; SS-inhibition = shape school-inhibition condition; DC = duration comparison task.

*p < .005 (Bonferroni Corrected threshold for p value)

The following chapter details the preliminary analyses that were conducted on the data.

Chapter 6: Preliminary Analyses

This chapter details preliminary analyses carried out on data before the main analyses for each research questions were conducted. Preliminary analyses are reported both for specific measures (e.g., to identify the most suitable way to index the construct), and general analyses (e.g., descriptive statistics).

6.1 Initial Analysis

Data for each task were scored or coded as appropriate. Next, data for a randomly selected 20 per cent of participants were rescored or coded by independent verifiers to ensure accuracy. In this sample agreement was greater than 99%.

Before embarking on the main analyses, a number of preliminary analyses were conducted to assess:

1) the most appropriate index for two tasks: the Panamath and Shape School Extended
2) whether tasks elicit a spread of responses, and do not produce floor or ceiling effects
3) if multiple tasks measure the same construct.

6.1.1 Task specific analyses.

Two tasks required exploratory analyses to determine the best way to index the constructs under investigation; the Shape School Extended and the Panamath task (measuring visual numerical acuity).

6.1.1.1 Shape School Extended task.

In the Shape School Extended task, older participants tended toward ceiling effects, particularly in the control and inhibition conditions, this is similar to effects observed in other executive functioning tasks (Logan, 1994; Miyake et al., 2000; Rogers & Monsell, 1995). In such cases differentiation is typically measured via reaction times, as age-related improvements for children in accuracy tend to be positively correlated with age-related improvements in reaction time (Ellefson, Ng, Wang, & Hughes, 2017). However, faster reaction time can result from a speed-accuracy trade-off, where faster responses have higher error rates (Bruyer &

Brysbaert, 2011). Hence to account for these problems, and given that participants were instructed to respond as quickly as they could whilst still being accurate, efficiency scores were calculated as follows:

$$\textit{Efficiency} = \frac{\textit{number of correct trials} - \textit{number of incorrect trials}}{\textit{time taken to complete task}} \quad (2)$$

However, efficiency scores can mask response patterns, particularly if responses are random and fast, hence they are most appropriate when the number of errors is less than 10%, and there are high correlations between accuracy and reaction time (Bruyer & Brysbaert, 2011). Additional analyses (see Appendix 10, p. 220) confirmed that all but the switching condition fulfilled these conditions, however as its error rate was only 13%, efficiency was retained as the index for this task.

6.1.1.2 The Panamath task.

To date there is heterogeneity in the measures used to index numerical acuity, although typically it is one or more of the following: accuracy; the percentage of correct trials, Weber fraction (the smallest ratio of two numerosities that a person can reliably judge as larger or smaller) or average reaction time (e.g., Bonny & Lourenco, 2013; DeWind & Brannon, 2012; Holloway & Ansari, 2009; Inglis et al.,, 2011, Mundy & Gilmore, 2009). To determine the most appropriate measure(s) a series of exploratory analyses were conducted.

As both sessions contained the Panamath task, reliability statistics could be calculated (see Table 6.1). Mean scores were also calculated for all trials, which was straightforward when a congruent number of trials were completed in each session, but less so if they were incongruent.

Table 6.1

Reliability Statistics for Panamath task

	Cronbach's α	Average interitem correlation
Accuracy	.85	.75
Reaction time	.39	.26
Weber fraction	.82	.75

In this instance accuracy and reaction time mean scores were calculated by working out the total score for each session, adding them together and dividing the result by the total number of trials. Mean scores for Weber fractions were calculated via a curve fitting spreadsheet provided on the Panamath website (see Appendix 11).

Partial correlations were produced for the resultant mean scores and arithmetic competence (see Table 6.2). They, together with reliability statistics, highlight accuracy and Weber fractions as the most reliable indices of numerical acuity, which also showed the strongest association with arithmetic ability. As this is in line with previous research, reaction time was dropped from future analyses (Schneider et al., 2017).

Table 6.2

Age-Corrected Correlations for Indices of Numerical Acuity and Arithmetic Competence

	Accuracy	Weber	Reaction time
Accuracy			
Weber	$-.68^{***}$		
Reaction time	$-.17^{*}$	$.14^{*}$	
Arithmetic	$.29^{***}$	$-.15^{*}$	-.10

$^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$.

Previous research suggests the association between numerical acuity and arithmetical competence may be driven by inhibitory control (Gilmore et al., 2013), therefore an analysis was conducted to determine if the pattern of results for both accuracy and Weber fractions, were the same for congruent and incongruent trials (see Appendix 12 for results of this analysis). Both accuracy and Weber fractions showed the same pattern of results, whereby congruent and incongruent trials, independently predict arithmetic competence, however when entered into the same model, congruent trials lost their predictive power (for results from accuracy analysis see Table 6.3).

Table 6.3

Regression Analyses for Accuracy Congruent and Incongruent Trials

	t	*p*	β	pr^2
Congruent	0.46	.649	.05	.001
Incongruent	5.38	< .001	.56	.122

Note: Dependent variable = arithmetic

As some participants in this study were very young, and the task consisted of 84 trials per session, there was a possibility they would lose focus and start to answer randomly. Hence a specific mechanism to gauge participant's attention was added via the inclusion of a 2.5 ratio bin (Chesney et al., 2015). All participants should have been able to respond to correctly to the 2.5 ratio bin, as 6-month-old infants have been shown to be able to discriminate numerical ratios of two (Xu & Spelke, 2000). Percentages obtained for this ratio can be seen in Table 6.4.

Table 6.4

Accuracy on 2.5 Ratio for Numerical Acuity Task.

Score achieved	Session 1 (%)	Session 2 (%)	Mean (%)
100	75	77	67
90 ≤ x < 100	12	9	17
80 ≤ x < 90	6	6	7
70 ≤ x < 80	2	3	5
60 ≤ x < 70	2	2	2
50 ≤ x < 60	1	2	2
40 ≤ x < 50	2	1	0.5

Congruent with Chesney et al. (2015) lapse rates were calculated for participants using data from the error rates on this 2.5 trial. As participants should answer these trials correctly if they are paying attention, errors could be attributed to random answering resulting from lapses in attention. Because inattentive participants should choose the correct answer on half of these randomly answered trials, participant's lapse rate was estimated as two times the proportion of incorrect trials (Chesney et al., 2015).

Participants with lapse rates greater than 0.5 on either session, or on the overall mean of both sessions, were identified, as were participants with Weber fractions greater than one (again in either session or overall mean), as values greater than this would indicate abnormal numerical acuity (Chesney et al., 2015). Additionally, participants whose Cook's distances, centred leverage values and/or mahalanobis distances violated assumptions were investigated. The results of these investigations can be found in Appendix 13, however removing visual numerical acuity data for these participants did not significantly impact associations between each measure of numerical acuity (accuracy and Weber fraction) and arithmetic ability, and hence were retained.

Whilst retaining these data improved the validity of the findings of this study, previous research has suggested that Weber fractions can be impacted by low accuracy (Dietrich et al., 2015), hence accuracy scores for incongruent trials will be used to index numerical acuity in subsequent analyses.

Having identified how best to index the Shape School Extended and Panamath tasks, analyses were conducted to address the second and third questions:

2) whether tasks elicit a spread of responses, and do not produce floor or ceiling effects

3) if multiple tasks measure the same construct.

6.1.2 General preliminary analyses

Descriptive statistics were produced for all participants and each population (Tables 6.5, and 6.6).

Table 6.5

Descriptive Statistics for Study Tasks

	n	Missing	*M*	*SD*	Min	Max
Age	214	0	11.68	4.03	4.16	19.27
SES-family	170	44	45.64	13.73	3.00	66.00
SES-environment	213	1	20508	6270	1792	32723
Processing speed	214	0	1.58	0.61	0.25	3.23
KBIT II: Verbal-combined	213	1	62.08	19.96	21.00	97.00
KBIT II: Non-verbal	214	0	27.32	8.57	7.00	44.00
Arithmetic	214	0	25.69	13.05	5.00	53.00
Visual updating	214	0	22.71	7.36	6.00	42.00
Auditory updating	214	0	19.46	6.98	6.00	39.00
Visuospatial sketchpad	214	0	25.08	6.38	7.00	42.00
Phonological loop	214	0	28.61	5.29	17.00	42.00
Visual switching	208	6	0.61	0.29	-.08	1.44
Auditory switching	211	3	6.39	2.97	-0.50	16.5
Visual inhibition	212	2	1.69	0.69	0.31	3.65
Auditory inhibition	213	1	11.24	2.01	0.00	14.00
Visual numerical acuity	212	2	85.12	10.84	45.24	98.81
Auditory numerical acuity	213	1	45.07	6.86	13.00	55.00

Note: SES = socioeconomic status.

Table 6.6

Descriptive Statistics for Study Tasks by Population

	n	Missing	*M*	*SD*	Min	Max
Age						
Typical development	142	0	11.75	4.11	4.16	18.69
Maths learning difficulty	40	0	11.06	3.86	5.38	19.27
Turner syndrome	32	0	12.10	3.91	4.89	18.70
SES-family						
Typical development	110	32	46.04	14.08	3.00	63.50
Maths learning difficulty	29	11	41.62	14.37	9.00	66.00
Turner syndrome	31	1	47.95	11.28	9.00	60.50

SES-environment						
Typical development	142	0	19803	5970	1792	31481
Maths learning difficulty	39	1	20027	6114	8856	31535
Turner syndrome	32	0	24224	6644	10012	32723
Processing speed						
Typical development	142	0	1.66	0.64	0.25	3.23
Maths learning difficulty	40	0	1.46	0.54	0.39	2.66
Turner syndrome	32	0	1.39	0.46	0.27	2.29
KBIT-II: Verbal combined						
Typical development	142	0	64.02	20.85	21.00	97.00
Maths learning difficulty	39	1	56.97	17.97	28.00	91.00
Turner syndrome	32	0	59.66	17.19	30.00	96.00
KBIT II: Non-verbal						
Typical development	142	0	28.93	8.53	7.00	44.00
Maths learning difficulty	40	0	22.85	6.87	12.00	40.00
Turner syndrome	32	0	25.75	8.62	7.00	39.00
Arithmetic						
Typical development	142	0	28.79	13.58	5.00	53.00
Maths learning difficulty	40	0	17.33	7.56	5.00	29.00
Turner syndrome	32	0	22.41	10.69	5.00	43.00
Visual updating						
Typical development	142	0	24.30	7.08	6.00	42.00
Maths learning difficulty	40	0	19.90	7.50	6.00	33.00
Turner syndrome	32	0	19.16	6.25	6.00	30.00
Auditory updating						
Typical development	142	0	20.77	7.38	7.00	39.00
Maths learning difficulty	40	0	16.75	5.09	6.00	32.00
Turner syndrome	32	0	17.03	5.64	6.00	28.00
Visuospatial sketchpad						
Typical development	142	0	26.36	6.09	12.00	42.00
Maths learning difficulty	40	0	23.45	6.57	7.00	38.00
Turner syndrome	32	0	21.47	5.77	7.00	31.00
Phonological loop						

Typical development	142	0	29.54	5.41	17.00	42.00
Maths learning difficulty	40	0	27.43	5.08	19.00	40.00
Turner syndrome	32	0	26.00	3.81	19.00	36.00
Visual switching						
Typical development	138	4	0.64	0.31	-0.08	1.44
Maths learning difficulty	39	1	0.52	0.25	-0.02	1.15
Turner syndrome	31	1	0.58	0.27	0.12	1.03
Auditory switching						
Typical development	141	1	6.51	3.11	-0.50	16.50
Maths learning difficulty	39	1	6.15	2.91	1.00	12.50
Turner syndrome	31	1	6.11	2.40	2.00	12.00
Visual inhibition						
Typical development	140	2	1.79	0.71	0.49	3.65
Maths learning difficulty	40	0	1.53	0.68	0.33	3.14
Turner syndrome	32	0	1.42	0.54	0.31	2.57
Auditory inhibition						
Typical development	141	1	11.41	1.89	1.00	14.00
Maths learning difficulty	40	0	11.13	1.77	6.00	13.00
Turner syndrome	32	0	10.63	2.67	0.00	13.00
Visual numerical acuity						
Typical development	140	2	87.00	9.92	47.74	98.81
Maths learning difficulty	40	0	82.56	11.15	52.78	96.43
Turner syndrome	32	0	80.13	12.31	45.24	96.43
Auditory numerical acuity						
Typical development	141	1	45.76	6.73	21.00	55.00
Maths learning difficulty	40	0	44.53	5.69	25.00	52.00
Turner syndrome	32	0	42.72	8.26	13.00	52.00

Note: SES = socioeconomic status.

Initially two measures of socioeconomic status were obtained (see Section 4.4.2.9). The family measure of socioeconomic status was based around the Hollingshead Four Factor Index of Social Status (Hollingshead, Unpublished working paper, 1975), and the environmental index was based on postcodes and the index of multiple deprivation. Two of the primary schools did not send out to parents the questionnaire utilised to elicit the family socioeconomic data, due to the sensitive nature of this type of information. However, despite missing data, this measure was predictive of arithmetic ability, when placed into a regression analysis, whilst the environmental measure was not. Hence the family socioeconomic status score was used solely in analyses.

Zero and age corrected correlations were produced for all participants and the typically developing population (see Tables 6.7 and 6.8). Correlations for maths learning disability and Turner syndrome participants can be found in Chapter 9.

All tasks appeared to show sufficient variance, and, floor and ceiling effects were not a major issue. Age-corrected correlation statistics for predictor variables were less than .8, suggesting multicollinearity was not an issue (Field, 2013). Interestingly, the zero correlation between visual inhibition and switching was .80, which may indicate these processes are closely aligned across at least part of the age-range of this study. Correlations greater than .8 were found among some of the control variables and arithmetical competence in zero correlations and for partial correlations.

Table 6.7

Zero Order and Age-Corrected Correlation Matrices for All Participants

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. Socioeconomic status		.07	.30*	.25*	.29*	.19	.23	.12	.17	.15	.09	.05	-.07	.16	.07
2. Processing speed	.11		.33*	.41*	.49*	.40*	.44*	.39*	.36*	.51*	.12	.57*	.08	.23	.30*
3. Verbal intelligence	.23	.74*		.57*	.60*	.42*	.43*	.42*	.37*	.40*	.13	.41*	.12	.33*	.25*
4. Non-verbal intelligence	.24*	.67*	.76*		.67*	.53*	.45*	.48*	.32*	.38*	.19	.40*	.13	.43*	.35*
5. Arithmetic	.25*	.77*	.85*	.81*		.56*	.56*	.46*	.40*	.47*	.19	.50*	.09	.35*	.29*
6. Visual updating	.19	.69*	.72*	.72*	.77*		.48*	.63*	.34*	.41*	-.01	.40*	.22	.37*	.28*
87 Auditory updating	.23	.66*	.66*	.64*	.73*	.67*		.45*	.57*	.36*	.14	.40*	.18	.28*	.34*
8. Visuospatial sketchpad	.14	.68*	.72*	.68*	.72*	.79*	.65*		.39*	.30*	.10	.40*	.19	.40*	.35*
9. Phonological loop	.19	.60*	.63*	.55*	.63*	.58*	.70*	.60*		.21	.15	.38*	.10	.22	.23
10. Visual switching	.16	.78*	.78*	.66*	.77*	.70*	.62*	.64*	.53*		.04	.54*	.14	.31*	.29*
11. Auditory switching	.11	.36*	.39*	.38*	.41*	.25*	.32*	32*	.33*	.32*		.13	.02	.21	.09
12.. Visual inhibition	.09	.81*	.78*	.67*	.79*	.69*	.64*	.69*	.62*	.80*	.37*		.06	.34*	.22
13.. Auditory inhibition	-.04	.27*	.31*	.28*	.28*	.35*	.31*	.33*	.24*	.31*	.13	.26*		.15	.15
14. Visual ANS	.17	.56*	.64*	.64*	.63*	.61*	.52*	.63*	.47*	.61*	.39*	.62*	.29*		.38*
15. Auditory ANS	.16	.60*	.61*	.59*	.60*	.56*	.56*	.60*	.48*	.60*	.30*	.56*	.29*	.60*	
16. Age	.08	.73*	.84*	.62*	.76*	.66*	.56*	.65*	.55*	.76*	.39*	.75*	.30*	.59*	.59*

Note: ANS = numerical acuity.

Above diagonal are age corrected partial correlations; below diagonal are zero correlations

*$p < .003$ (Bonferroni corrected for multiple comparison)

Table 6.8

Zero Order and Age-Corrected Correlation Matrices for Typically Developing Participants

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. Socioeconomic status		.08	.30*	.29	.34*	.27	.24	.16	.23	.19	.05	.10	-.14	.25	.12
2. Processing speed	.15		.33*	.37*	.41*	.31*	.38*	.31*	.22	.48*	.08	.54*	-.05	.19	.27
3. Verbal intelligence	.27	.74*		.58*	.59*	.33*	.40*	.35*	.33*	.40*	.12	.34*	.19	.37*	.20
4. Non-verbal intelligence	.30*	.67*	.79*		.63*	.44*	.38*	.46*	.29	.28	.20	.35*	.12	.39*	.33*
5. Arithmetic	.29	.76*	.87*	.81*		.53*	.51*	.38*	.36*	.43*	.16	.48*	.06	.33*	.23
6. Visual updating	.28	.65*	.69*	.69*	.77*		.44*	.57*	.25	.40*	.01	.31*	.11	.34*	.33*
7. Auditory updating	.26	.61*	.63*	.59*	.68*	.63*		.43*	.57*	.32*	.09	.40*	.14	.20	.28
8. Visuospatial sketchpad	.20	.64*	.69*	.69*	.69*	.76*	.62*		.36*	.23	.14	.33*	.12	.27	.22
9. Phonological loop	.26	.51*	.59*	.53*	.60*	.51*	.69*	.57*		.15	.09	.28	.02	.18	.18
10. Visual switching	.21	.76*	.76*	.62*	.77*	.69*	.58*	.59*	.46*		.02	.51*	.01	.25	.25
11. Auditory switching	.10	.37*	.42*	.42*	.44*	.30*	.30*	.38*	.29	.33*		.13	-.03	.30*	.03
12. Visual inhibition	.16	.79*	.75*	.67*	.80*	.66*	.62*	.65*	.54*	.78*	.40*		-.04	.29	.14
13. Auditory inhibition	-.09	.20	.36*	.29	.29	.29	.28	.29	.18	.24	.11	.21		.13	.14
14. Visual ANS	.27	.52*	.64*	.61*	.61*	.58*	.44*	.53*	.42*	.55*	.47*	.58*	.27		.18
15. Auditory ANS	.17	.58*	.58*	.59*	.59*	.59*	.51*	.52*	.43*	.57*	.28	.52*	.29	.45*	
16. Age	.12	.74*	.83*	.66*	.81*	.67*	.53*	.65*	.52*	.73*	.44*	.75*	.31*	.56*	.59*

Note: ANS = numerical acuity.

Above diagonal are age corrected partial correlations; below diagonal are zero correlations

*$p < .003$ (Bonferroni corrected for multiple comparison

With the initial exploration of the data completed, the next chapters will address the research questions:

1) Which modality specific cognitive abilities predict arithmetic competence in the general population?
2) Do the cognitive abilities identified as predictors of arithmetic competence in research question one display atypical development for a) children with maths learning disability, and b) children with Turner syndrome?
3) Are identified areas of deficits confined to either the visual or auditory domain?

As research question one is focussed on typical development, and questions two and three are looking at differences between groups within a developmental trajectories approach, analyses will be conducted in two phases, with the first phase focussed on research question one. Hence the following section will detail the planned analyses for phase one solely.

6.2 Planned Analyses

As none of the participants who fulfilled the criteria for inclusion into the maths learning disability group had a clinical diagnosis of maths learning disability, arguably they can be considered part of the general population (see Section 4.5.1). Hence data for phase one could be participants identified as typically developing solely, or these participants and those identified as having mathematics learning disabilities. Analyses will be conducted with both populations to determine if the pattern of results is similar.

Initial analysis will examine descriptive statistics and zero order and partial correlations. A series of independent regression analyses will then be conducted, with the arithmetic variable as the dependent variable, and each predictor in turn as the independent variable. As age may be a confound, these will be followed by a series of hierarchical regressions, where age is entered into the first step of the model, and the predictor variable in step two, thereby allowing an investigation of the predictive power of each predictor when age is controlled for. This will be followed by a regression analysis where all variables are entered concurrently to facilitate the identification of which predictors explain additional variance when the others are accounted for.

As research question one looks to determine the modality specific cognitive abilities predictive of arithmetic ability, a series of modality specific regression analyses will then be conducted, whereby variables with congruent modality will be entered concurrently into a regression analysis, both with and without age being controlled for.

The results from the regression analyses will inform subsequent analyses which may include mediation or moderation analyses, cluster analysis, path analysis, or structural equation modelling. Until recently fewer studies investigating the cognitive underpinnings of mathematical ability utilised path analysis or structural equation modelling, however, this is starting to change.

It is acknowledged that there is an impurity problem for measures of working memory and executive function as it is difficult if not impossible to produce a task which measure a single executive function (Van der Ven et al., 2012). Therefore, representing executive functions as latent variables may be optimal, as measurement errors in regression analyses can be confounded with true common variance, whilst structural equation modelling uses multiple indicators whose common variance is extracted, and whose measurement errors are modelled explicitly, thereby reducing the confounding effect of tasks which inevitably are not measuring a single construct (Miyake & Friedman, 2012).

Phase one: Results and Discussion

Chapter 7: Analysis for Phase One

Phase one of the data analysis addressed the first research question:

1) Which modality specific cognitive abilities predict arithmetic competence in the general population?

As typically developing and mathematics learning disability populations were drawn from mainstream schools, and no participants in the mathematics learning disability population had an official diagnosis, this group is arguably a subset of the general population. Hence analyses in this phase were conducted with;

a) the typically developing population
b) a combination of this population and the mathematics learning disability group.

Findings from each data set were congruent, hence results for the combined typically developing and mathematics learning disability data are reported, staring with a consideration of the descriptive statistics.

7.1 Descriptive Statistics

Descriptive statistics can be found in Table 7.1, and zero order and partial correlations in Table 7.2. Both suggest data display appropriate variance and constructs appear to be independent. However, there is a possible query over the auditory inhibition and auditory switching tasks, which are not significantly correlated with any other constructs when age is corrected for (Table 7.2), a surprising result given the proposed structure of executive functioning in development (see section 2.3.1).

Table 7.1

Descriptive Statistics for Study Tasks for Typically Developing and Maths Learning Disability Participants

	n	Missing	*M*	*SD*	Min	Max
Age (years)	182	0	11.60	4.06	4.16	19.27
Socioeconomic status	139	43	45.12	14.20	3.00	66.00
Processing speed	182	0	1.61	0.63	0.25	3.23
Verbal intelligence	181	1	62.50	20.43	21.00	97.00
Non-verbal intelligence	182	0	27.59	8.55	7.00	44.00
Arithmetic	182	0	26.27	13.36	5.00	53.00
Auditory updating	182	0	19.89	7.12	6.00	39.00
Visual updating	182	0	23.34	7.38	6.00	42.00
Phonological loop	182	0	29.07	5.40	17.00	42.00
Visuospatial sketchpad	182	0	25.72	6.29	7.00	42.00
Auditory inhibition	181	1	11.35	1.86	1.00	14.00
Visual inhibition	180	2	1.73	0.71	0.33	3.65
Auditory switching	180	2	6.44	3.06	-0.50	16.50
Visual switching	177	5	0.62	0.30	-0.08	1.44
Auditory numerical acuity (%)	181	1	45.49	6.52	21.00	55.00
Visual numerical acuity (%)	180	2	86.01	10.34	47.74	98.81

Table 7.2

Zero Order and Age-Corrected Correlation Matrices for Typically Developing and Maths Learning Disability Participants

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. SES		.12	.33*	.33*	.34*	.25	.26	.16	.21	.09	-.10	.18	.11	.23	.11
2. Processing speed	.18		.35*	.41*	.44*	.36*	.43*	.32*	.25	.48*	-.03	.48*	.07	.20	.27*
3. Verbal intelligence	.29*	.77*		.56*	.60*	.37*	.40*	.37*	.28*	.36*	.17	.39*	.12	.35*	.23
4. Non-verbal intelligence	.34*	.68*	.76*		.69*	.54*	.44*	.46*	.27*	.33*	.10	.31*	.18	.41*	.32*
5. Arithmetic	.32*	.78*	.86*	.83*		.56*	.55*	.39*	.29*	.40*	.05	.43*	.14	.34*	.24
6. Visual updating	.27*	.69*	.72*	.73*	.78*		.43*	.59*	.24	.31*	.11	.38*	-.05	.36*	.28*
7. Auditory updating	.29*	.66*	.65*	.63*	.72*	.64*		.41*	.54*	.36*	.11	.34*	.11	.24	.29*
8. Visuospatial sketchpad	.21	.67*	.72*	.69*	.71*	.78*	.63*		.30*	.31*	.13	.24	.03	.31*	.24
9. Phonological loop	.25*	.58*	.62*	.54*	.61*	.54*	.69*	.58*		.25	.04	.11	.11	.17	.18
10. Visual inhibition	.16	.79*	.79*	.65*	.77*	.67*	.62*	.67*	.59*		-.03	.49*	.09	.33*	.13
11. Auditory inhibition	-.04	.24	.37*	.28*	.29*	.30*	.27*	.32*	.23	.25		.003	-.05	.10	.08
12. Visual switching	.22	.78*	.78*	.63*	.77*	.69*	.61*	.63*	.51*	.80*	.26		-.01	.29*	.21
13. Auditory switching	.16	.36*	.42*	.40*	.41*	.25	.32*	.31*	.33*	.38*	.10	.32*		.18	.03
14. Visual ANS	.26*	.56*	.66*	.64*	.64*	.62*	.49*	.59*	.46*	.63*	.28*	.61*	.39*		.20
15. Auditory ANS	.18	.61*	.62*	.58*	.60*	.58*	.54*	.56*	.48*	.55*	.27*	.58*	.29*	.50*	
16. Age	.14	.77*	.85*	.63*	.79*	.67*	.56*	.68*	.59*	.79*	.34*	.76*	.42*	.60*	.62*

Note: SES = socioeconomic status; ANS = numerical acuity.

Above diagonal are age corrected partial correlations; below diagonal are zero correlations.

*$p < .003$ (Bonferroni corrected for multiple comparisons).

Next, data were explored via a series of regressions analyses. However before embarking upon them, data were investigated to determine if assumptions for regression analyses were met.

7.2 Assumptions for Regression Analyses

A link to the full examination of the assumptions can be found in Appendix 14. In summary, outliers were found in the tasks measuring; processing speed, visual updating, auditory updating, visuospatial sketchpad, auditory switching, visual inhibition, auditory inhibition, visual numerical acuity, and auditory numerical acuity. However, casewise analyses only highlighted two data points in the auditory inhibition task as having a particular influence, and as removing them made no significant difference, they were retained.

All measures but processing speed and visuospatial sketchpad were non-normal, although solution residuals did display normality. To ensure reliability of results, bootstrapping was performed on all regression analyses, using 1000 replications and random-number seed 111. Additionally, given the difference in scoring utilised by the tasks, *z* scores are used in all subsequent analyses.

7.3 Regression Analyses

7.3.1 Investigation of the independent effects of each predictor of arithmetic ability

An initial investigation into the predictive power of each predictor and control variable was conducted via a series of regression analyses (see Table 7.3). Each variable was confirmed as an independent predictor of arithmetic competence. Given the wide age range of participants, these analyses were repeated whilst accounting for age (see Table 7.4).

Adding age to the model resulted in auditory switching becoming a non-significant predictor of arithmetic competence, although it should be noted that this may be due to the task used to index this construct. The next stage of the analysis was to determine if predictive power was retained when variables were entered into the same model.

Table 7.3

Independent Regression Models for each Variable Predicting Arithmetic Ability

	B	95% CI	*SE*	R^2	*z*	*p*
Age	.80	[.71, .89]	.05	.64	17.26	<.001
Socioeconomic status	.32	[.17, .46]	.07	.10	4.36	<.001
Processing speed	.78	[.70, .87]	.04	.61	18.53	<.001
Verbal intelligence	.86	[.77, .94]	.04	.73	19.72	<.001
Non-verbal intelligence	.79	[.68, .89]	.05	.62	14.63	<.001
Auditory updating	.71	[.62, .80]	.05	.51	15.27	<.001
Visual updating	.75	[.65, .86]	.05	.57	14.26	<.001
Phonological loop	.65	[.55, .75]	.05	.42	12.28	<.001
Visuospatial sketchpad	.68	[.59, .77]	.05	.46	14.19	<.001
Auditory inhibition	.32	[.21, .44]	.06	.11	5.63	<.001
Visual inhibition	.78	[.69, .87]	.04	.61	17.35	<.001
Auditory switching	.46	[.35, .57]	.06	.21	8.21	<.001
Visual switching	.77	[.68, .85]	.04	.60	17.57	<.001
Auditory numerical acuity	.58	[.47, .69]	.06	.34	10.53	<.001
Visual numerical acuity	.61	[.48, .73]	.06	.37	9.60	<.001

Note: Each line represents a separate model. Dependent variable = arithmetic.

Table 7.4

Age-Controlled Regression Models per Predictor of Arithmetic Ability

	B	*95% CI*	*SE*	R^2	*z*	*p*
Socioeconomic status	.22	[.11, .32]	.05	.63	4.18	<.001
Processing speed	.39	[.25, .54]	.07	.69	5.45	<.001
Verbal intelligence	.66	[.49, .84]	.09	.74	7.29	<.001
Non-verbal intelligence	.46	[.33, .58]	.06	.75	7.16	<.001
Auditory updating	.36	[.26, .47]	.05	.72	6.71	<.001
Visual updating	.38	[.26, .50]	.06	.71	6.19	<.001
Phonological loop	.24	[.14, .34]	.05	.67	4.52	<.001
Visuospatial sketchpad	.26	[.14, .38]	.06	.67	4.11	<.001
Auditory inhibition	.08	[.01, .16]	.04	.64	2.24	.025
Visual inhibition	.39	[.25, .54]	.07	.69	5.25	<.001
Auditory switching	.09	[-.02, .19]	.05	.64	1.58	.113
Visual switching	.38	[.26, .50]	.06	.71	6.29	<.001
Auditory numerical acuity	.16	[.06, .25]	.05	.65	3.11	.002
Visual numerical acuity	.20	[.10, .30]	.05	.66	3.94	<.001

Note: Each model represents a separate model. Dependent variable = arithmetic.

7.3.2 Comparison of the predictive power of study variables.

Results from entering all variables into an age corrected regression analysis can be seen in Table 7.5. The model was significant, $R^2 = .86$, Wald $\chi^2(15) = 1006.92$, $p < .001$, however only verbal intelligence, non-verbal intelligence, verbal updating and auditory updating retained their predictive power, with coefficients highlighting non-verbal intelligence as the strongest predictor.

Age, socioeconomic status, and processing speed were control variables. As socioeconomic status and processing speed were not significant when other cognitive abilities were accounted for, they were dropped from subsequent analyses. Although age was also non-significant, it was marginally so, and as this study contains a wide age range, it was retained.

Table 7.5

Regression Analysis with all Predictors of Arithmetic Ability Entered Concurrently

	B	*95% CI*	*SE*	*z*	*p*
Constant	-.03	[-.10, .04]	.04	-0.73	.468
Age	.17	[-.02, .35]	.09	1.77	.077
Socioeconomic status	.03	[-.06, -.12]	.04	0.66	.509
Processing speed	.04	[-.11, .18]	.07	0.49	.624
Verbal intelligence	.28	[.07, .49]	.11	2.60	.009
Non-verbal intelligence	.30	[.16, .44]	.07	4.33	<.001
Auditory updating	.17	[.02, .31]	.07	2.27	.023
Visual updating	.17	[.02, .33]	.08	2.21	.027
Phonological loop	-.03	[-.14, .08]	.05	-0.58	.561
Visuospatial sketchpad	-.07	[-.21, .07]	.07	-0.96	.336
Auditory inhibition	-.03	[-.10, .03]	.03	-0.99	.323
Visual inhibition	.05	[-.12, .21]	.08	0.56	.577
Auditory switching	.03	[-.05, .10]	.04	0.69	.490
Visual switching	.07	[-.09, .22]	.08	0.86	.392
Auditory numerical acuity	-.05	[-.17, .07]	.06	-.0.82	.412
Visual numerical acuity	-.02	[-.15, .11]	.06	-0.34	.732

Note: Dependent variable = arithmetic. Model fit statistics; R^2 = .86, Wald $\chi^2(15)$ = 1006.92, *p* > .001.

To determine whether modality was having an impact, the next stage of the analysis investigated the ability of visual and auditory variables to predict arithmetic competence.

7.3.3 The impact of modality.

An initial examination was made of the relative predictive power of concurrently examining visual (see Tables 7.6 and 7.7) and auditory variables (Tables 7.8 and 7.9).

Table 7.6

Visual Predictors of Arithmetic Ability

	B	95% CI	*SE*	*z*	*p*
Constant	-.01	[-.08, .06]	.03	-0.24	.807
Non-verbal intelligence	.39	[.28, .50]	.05.	7.17	<.001
Visual updating	.18	[.04. .31]	.07	2.56	.010
Visuospatial sketchpad	.02	[-.10, .14]	.06	0.37	.710
Visual inhibition	.23	[.10, .36]	.07	3.50	<.001
Visual switching	.22	[.09, .34]	.06	3.37	.001
Visual numerical acuity	-.0002	[-.08, .08]	.04	-0.00	.996

Note: Dependent variable is arithmetic. Model fit statistics; R^2 = .80, Wald χ^2 (6) = 1011.13, p < .001

The model for visual predictors was significant, R^2 = .80, Wald χ^2 (6) = 1011.13, p < .001, with non-verbal intelligence, and visual measures of updating, inhibition and switching, all making a significant contribution to the model.

This pattern of results was similar when age was accounted for (see Table 7.7), R^2 = .82, Wald χ^2 (7) = 971.84, p < .001, although visual inhibition was no longer explain significant additional variance. Interestingly, age was a strong predictor.

Table 7.7

Visual Predictors of Arithmetic Ability Controlling for Age

	B	95% CI	*SE*	*z*	*p*
Constant	-.01	[-.07, .05]	.03	-0.33	.738
Age	.25	[.12, .38]	.07	3.65	<.001
Non-verbal intelligence	.36	[.26, .47]	.05	6.80	<.001
Visual updating	.16	[.03, .28]	.06	2.49	.013
Visuospatial sketchpad	-.01	[-.12, .10]	.06	-0.16	.872
Visual inhibition	.13	[-.02, .27]	.08	1.67	.095
Visual switching	.16	[.03, .30]	.07	2.39	.017
Visual numerical acuity	-.01	[-.09, .08]	.04	-0.21	.831

Note: Dependent variable is arithmetic. Model fit statistic; R^2 = .82, Wald χ^2 (7) = 971.84, p < .001.

Entering auditory predictors concurrently (see Table 7.8) also produced a significant model, R^2 = .76, Wald $\chi^2(6)$ = 660.73 p < .001, although only verbal intelligence and auditory updating made significant contributions, a pattern which remained when age was accounted for, R^2 = .77, Wald $\chi^2(7)$ = 673.45, p < .001 (see Table 7.9). Age was once again found to be a significant predictor.

Table 7.8

Auditory Predictors of Arithmetic Ability

	B	95% CI	*SE*	*z*	*p*
Constant	.004	[-.07, .08]	.04	0.11	.913
Verbal intelligence	.64	[.51, .77]	.07	9.61	<.001
Auditory updating	.24	[.11, .36]	.06	3.81	<.001
Phonological loop	.02	[-.08, .13]	.05	0.48	.631
Auditory inhibition	.03	[-.04, .11]	.04	0.90	.371
Auditory switching	.04	[-.05, .12]	.04	0.86	.387
Auditory numerical acuity	.02	[-.07, .12]	.05	0.49	.623

Note: Dependent variable is arithmetic. IQ = intelligence. Model fit statistics; R^2 = .76, Wald $\chi^2(6)$ = 660.73, p < .001.

Table 7.9

Auditory Predictors of Arithmetic Ability Controlling for Age

	B	95% CI	*SE*	*z*	*p*
Constant	.01	[-.07, .08]	.04	0.15	.879
Age	.19	[.03, .36]	.08	2.31	.021
Verbal intelligence	.49	[.30, .68]	.10	5.00	<.001
Auditory updating	.24	[.12, .37]	.06	3.91	<.001
Phonological loop	.01	[-.10, .11]	.05	0.15	.877
Auditory inhibition	.03	[-.04, .10]	.04	0.82	.414
Auditory switching	.02	[-.06, .11]	.04	0.52	.601
Auditory numerical acuity	.01	[-.09, .10]	.05	0.11	.909

Note: Dependent variable is arithmetic. Model fit statistics; R^2 = .77, Wald $\chi^2(7)$ = 673.45, p < .001.

The final step regression analysis considered auditory and visual variables in the same hierarchical regression, to investigate whether the order of entry of the modality specific variables impacted results. In each case, age was entered into the first step. The final stage of this analysis can be seen in Table 7.10.

Table 7.10

Auditory and Visual Predictors of Arithmetic Ability

	B	95% CI	*SE*	*z*	*p*
Constant	-.01	[-.07, .05]	.03	-0.41	.685
Age	.14	[-.03, .31]	.09	1.60	.110
Verbal intelligence	.28	[.10, .46]	.09	3.01	.003
Auditory updating	.15	[.03, .27]	.06	2.44	.015
Phonological loop	.01	[-.09, .10]	.05	0.16	.875
Auditory inhibition	-.01	[-.06, .05]	.03	-0.22	.823
Auditory switching	.02	[-.05, .09]	.03	0.53	.599
Auditory numerical acuity	-.02	[-.10, .06]	.04	-0.45	.653
Non-verbal intelligence	.25	[.14, .35]	.05	4.49	<.001
Visual updating	.17	[.05, .28]	.06	2.89	.004
Visuospatial sketchpad	-.03	[-.13, .07]	.05	-0.60	.550
Visual inhibition	.04	[-.10, .17]	.07	0.51	.608
Visual switching	.09	[-.04, .22]	.07	1.42	.154
Visual numerical acuity	-.01	[-.10, .09]	.05	-0.12	.901

Note: Dependent variable = arithmetic. Model fit statistics; R^2 = .85, Wald $\chi^2(13)$ = 1102.12, p < .001.

The model was significant, R^2 = .85, Wald $\chi^2(13)$ = 1102.12, p < .001, with the order in which variables were entered into the model having no impact on results. Measures of intelligence and updating in both modalities were found to be the sole significant predictors. In this model, age was no longer a significant predictor.

Results from regression analyses suggest that visual and auditory constructs, specifically intelligence and updating are predictive of arithmetic competence, with both intelligence measures displaying the strongest links. While there were differences for modality specific predictors in each construct, coefficients were very similar. This may be because they are unaffected by modality or that both modalities are equally important predictors of arithmetic ability.

To try to gain a clearer picture of whether modality is having an impact, data were next analysed via structural equation modelling. Given the lack of predictive power for numerical acuity in the presence of other variables they were dropped in subsequent analyses. Hence bimodal measures of intelligence and executive functioning were included in these analyses.

7.4 Structural Equation Modelling

There is a paucity of research utilising structural equation modelling to examine the impact of intelligence and executive functioning on mathematical competence, and most research is centred on updating, hence the theoretical model (see Figure 7.1) draws on the findings of these studies.

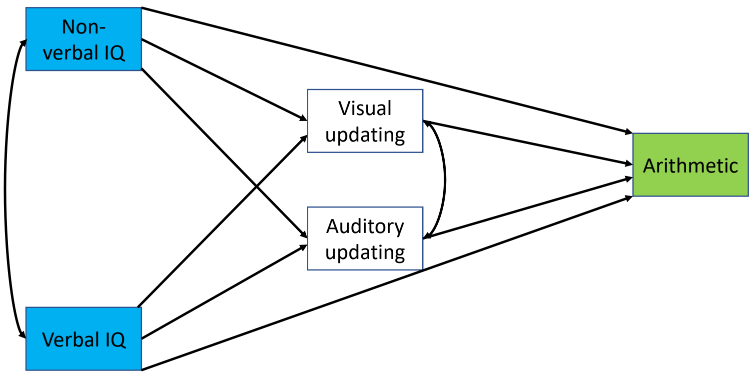


Figure 7.1. Theoretical model for the relationship between bimodal measures of intelligence and working memory.

The initial stage of the modelling was to enter constructs identified as predictors in the regression analyses into a path analysis to investigate how they interacted in more detail. This resulted in a non-specified model, hence an updating latent variable was created. The resultant model (see Figure 7.2) demonstrated a good fit (see Table 7.11 for detail on fit indices); $\chi^2(2) = 1.50$, $p = .682$, RMSEA = .00, 90% CI [00, .10], CFI = 1.00, TLI = 1.00, SRMR = .01, and all paths were significant. Forthwith simplified models with the errors removed will be presented, however full models can be found in Appendix 15.

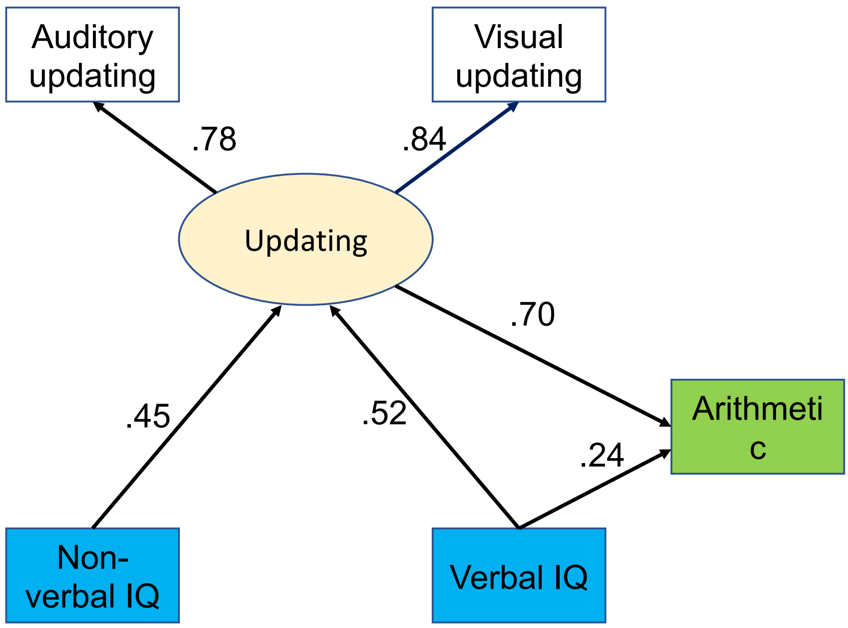


Figure 7.2. A model examining the impact of updating, and intelligence on arithmetic competence. IQ = intelligence.

p < .001 for all paths, except verbal IQ to arithmetic, *p* = .038.

Table 7.11

Fit Indices for Structural Equation Modelling

Statistic	Accepted threshold	Citation
Model chi-square χ^2	p > .05	Hu & Bentler (1992)
χ^2/df	< 5	Wheaton et al. (1977)
	< 2	Tabachnick & Fidell (2007)
RMSEA	< .05 (good)	Hu & Bentler (1992)
	< .08 (reasonable)	
CFI	>.95	Hu & Bentler (1999)
TLI	>.95	Bentler & Hu (1999)
SRMR	< .05 (good)	Byrne (1998)
	< .08 (reasonable)	Hu & Bentler (1999)

Note: df = degrees of freedom; RMSEA = root mean square error of approximation; CFI = comparative fit index; TLI = Tucker-Lewis index; SRMR = standardised root mean square residual.

Adding age to the model depicted in Figure 7.2. also displayed a good fit (see Figure 7.3), $\chi^2(5) = 5.88$, $p = .318$; RMSEA = .03, 90% CI [.00, .11]; CFI = 0.999; TLI = 0.997; SRMR = .01.

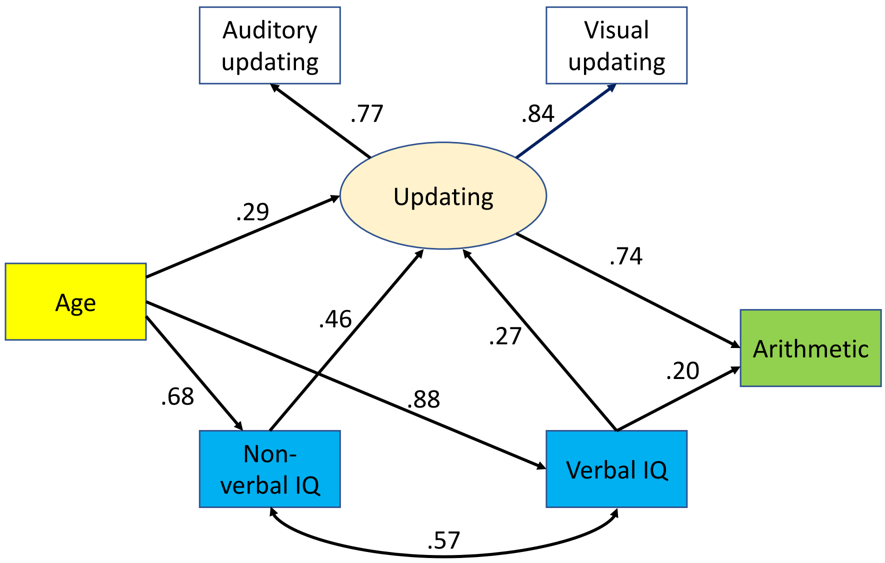


Figure 7.3. A model for bimodal measures of updating, intelligence, and age, as predictors of arithmetic competence. IQ = intelligence.
$p < .001$ for all paths except between verbal IQ and arithmetic, $p = .083$ and verbal IQ and updating, $p = .018$.

Neither model contained a direct path from non-verbal intelligence to arithmetic ability, which was indirect link through updating. This is surprising given the strength of the association in the regression analysis. Links between verbal intelligence and arithmetic competence were direct and indirect, again through updating. However, adding age to the model resulted in the link between verbal intelligence and arithmetic ability becoming marginally significant. The link between age and arithmetic ability was also indirect, through updating, and both intelligence variables, particularly verbal intelligence.

Updating as the strongest predictor of arithmetic ability is in line with a large body of research, as are the indications in regression analyses that visual switching and updating may also be important predictors of arithmetic competence. To investigate this further, the next series analyses looked to determine if the inclusion of the other executive functioning variables gave a clearer picture of how cognitive abilities impact arithmetic competence and the role played by modality.

This began with a consideration of whether executive functioning is best modelled as a single latent variable or split into two modality-specific latent variables. For all models, covariances were only added if required to produce a model with good fit statistics. Once this was achieved no additional covariances were added to the model.

7.4.1 Latent variables.

The first model (see Figure 7.4) contained the executive functioning variables in a single model. Whilst this produced a good fit; $\chi^2(18) = 23.58$, $p = .169$, RMSEA = .04, 90% CI [.00, .09], CFI = 0.993, TLI = 0.989, SRMR = .03. factor loadings suggested that, on the whole, the latent variable was accounting for more variance in the visual variables.

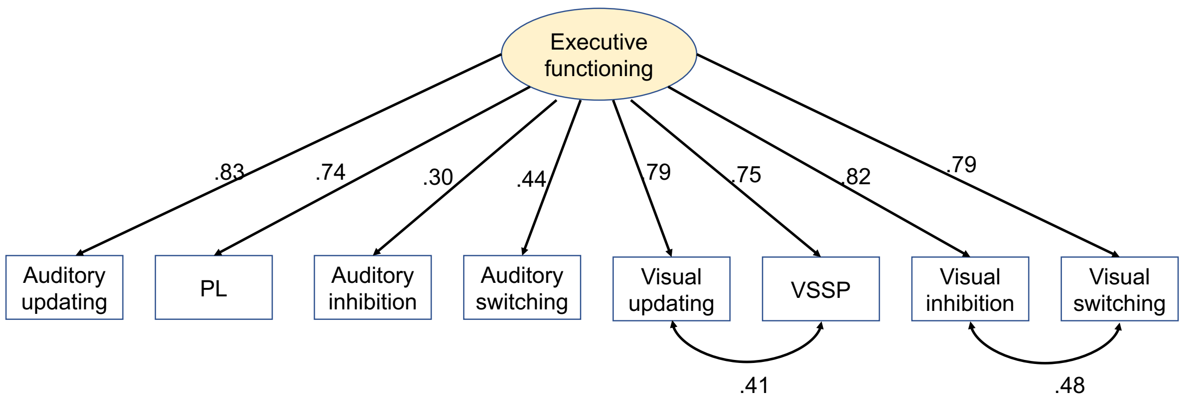


Figure 7.4. Representation of executive functioning as a single latent variable. PL = phonological loop, VSSP = visuospatial sketchpad.
$p < .001$ for all paths.

As the aim of this study is to investigate the impact of modality on cognitive predictors of arithmetic ability, next the impact of splitting the predictors into two latent variables; visual and auditory, was examined. Resultant models can be seen in Figure 7.5. Both displayed a good fit: visual executive function model: $\chi^2(1) = 1.30$, $p = .254$, RMSEA = .04, 90%CI [.00, .21], CFI = 0.999, TLI = 0.996, SRMR = .01, and auditory executive function model: $\chi^2(2) = 0.07$, $p = .967$, RMSEA = .00, 90%CI [.00, .], CFI = 1.00, TLI = 1.04, SRMR = .003.

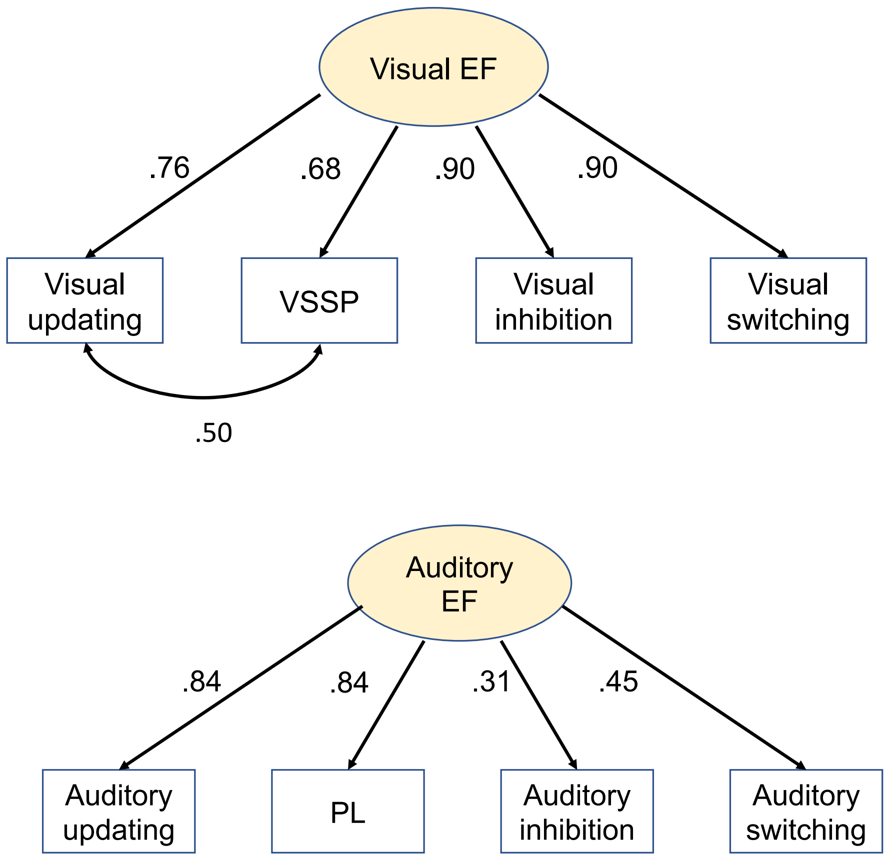
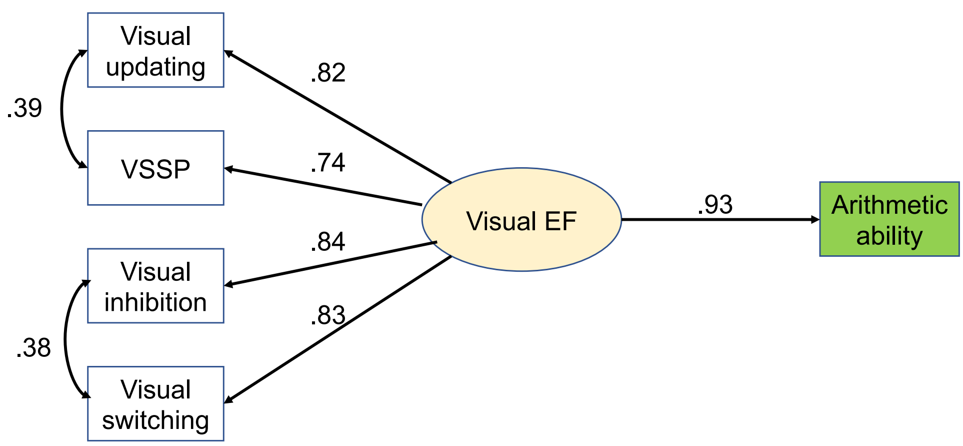


Figure 7.5. Executive functioning variables represented by two modality specific latent variables. EF = executive function, VSSP = visuospatial sketchpad, PL = phonological loop. *p* < .001 for all paths.

The link between the one and two-latent variable models with arithmetic was investigated next (see Figures 7.6 and 7.7).



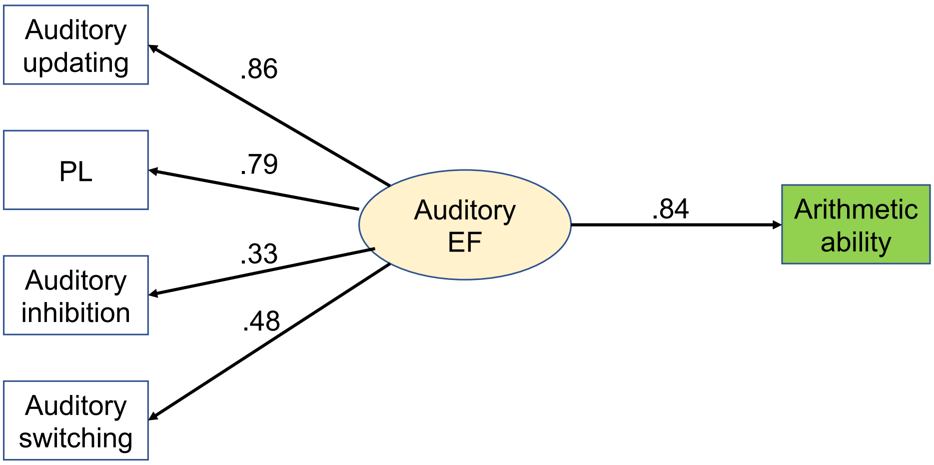


Figure 7.6. Executive functioning variables represented by two modality specific latent variables. EF = executive function, VSSP = visuospatial sketchpad, PL = phonological loop. $p < .001$ for all paths.

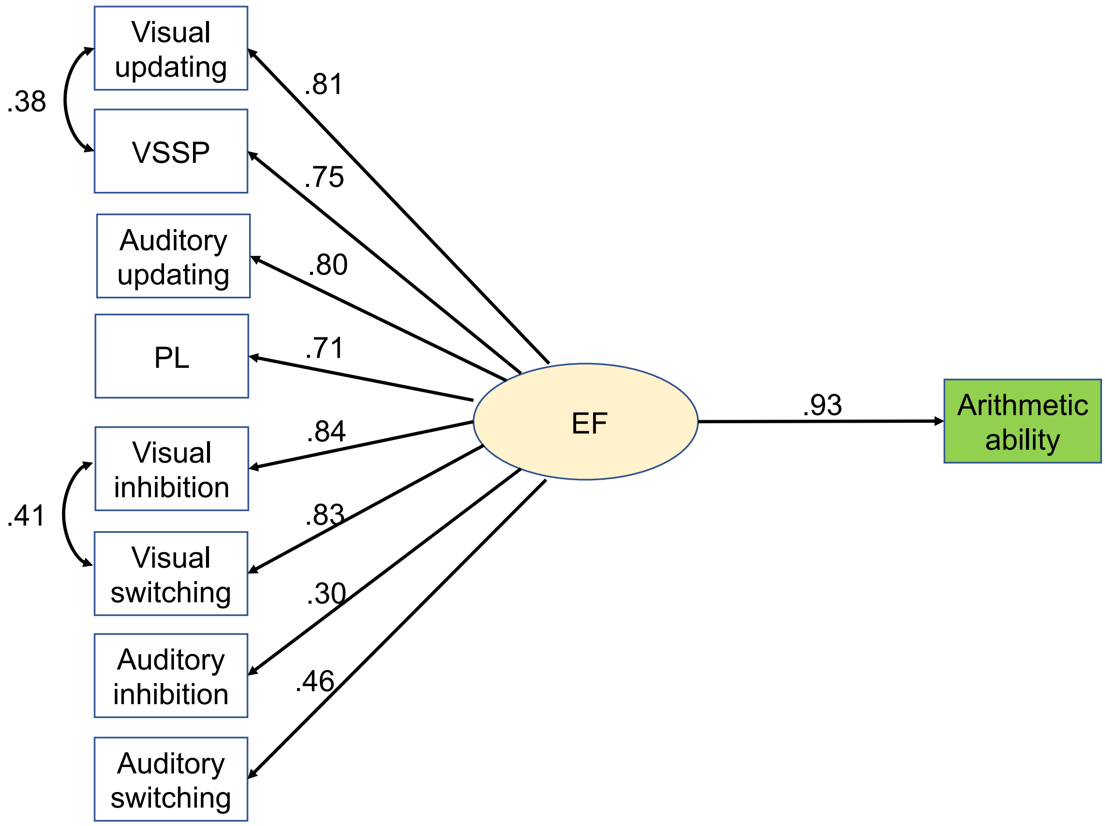


Figure 7.7. Latent executive functioning as a predictor of arithmetic ability. EF = executive function, VSSP = visuospatial sketchpad, PL = phonological loop.
$p < .001$ for all paths.

Once again, all models displayed a good fit (see Table 7.12). It is interesting that the coefficient from the single executive functioning latent variable to arithmetic is the same as the visual latent variable, and the factor loadings for each predictor to the latent variable are very similar. While the predictive power of the auditory latent variable is lower, it is still significant, and the factor loadings for the predictor variables are slightly better than in the single executive function and working memory latent variable.

Table 7.12

Comparison of Fit Indices for Working Memory and Executive Functioning Latent Models as Predictors of Arithmetic Ability

Model	χ^2	*p*	χ^2/df	RMSEA	CFI	TLI	SRMR
Single latent variable	33.68	.115	1.35	.05 [.00, .08]	.99	.99	.03
Visual latent variable	1.39	.708	0.46	.00 [.00, .09]	1.00	1.01	.01
Auditory latent variable	6.87	.233	1.37	.05 [.00, .12]	.99	.99	.02

The impact of other predictors, i.e. verbal and non-verbal intelligence and age were investigated next.

7.4.2 Age and intelligence as predictors of arithmetic competence.

When both modality specific latent variables were placed into a structural equation model along with intelligence (see Figure 7.8), the resultant model displayed a good fit, $\chi^2(36) = 44.53$, $p = .156$, RMSEA = .04, 90% CI [.00, .07], CFI = 0.99, TLI = 0.99, SRMR = .03. Direct paths to arithmetic competence were from both executive functioning and working memory latent variables, with visual executive functioning showing the strongest link. The link between both intelligence variables and arithmetic ability was mediated by visual and auditory executive functioning and working memory.

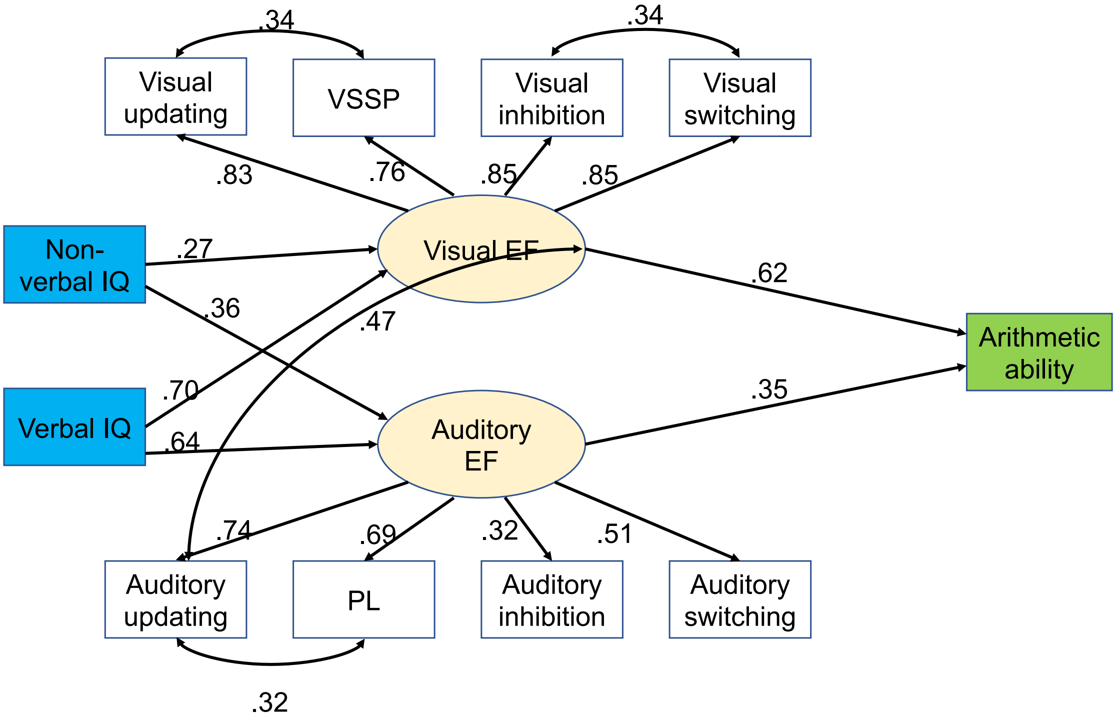


Figure 7.8. Bimodal executive functioning, and intelligence as predictors of arithmetic competence. IQ = intelligence, VSSP = visuospatial sketchpad, EF = executive function, PL = phonological loop.

p < .001 for all paths except auditory EF and arithmetic, *p* = .009.

However, having the single executive function and working memory latent variable with intelligence also produced a model (see Figure 7.9) that displayed a good fit; $\chi^2(40) = 53.14$, $p = .080$, RMSEA = .04 90% CI [.00, .07], CFI = 0.99, TLI = 0.99, SRMR = .03. The fit statistics for each model are very similar, and whilst the one latent variable is more parsimonious, the modality specific latent variable model does highlight the importance of considering modality, with both visual and auditory executive function and working memory predictors playing a role in predicting arithmetic ability.

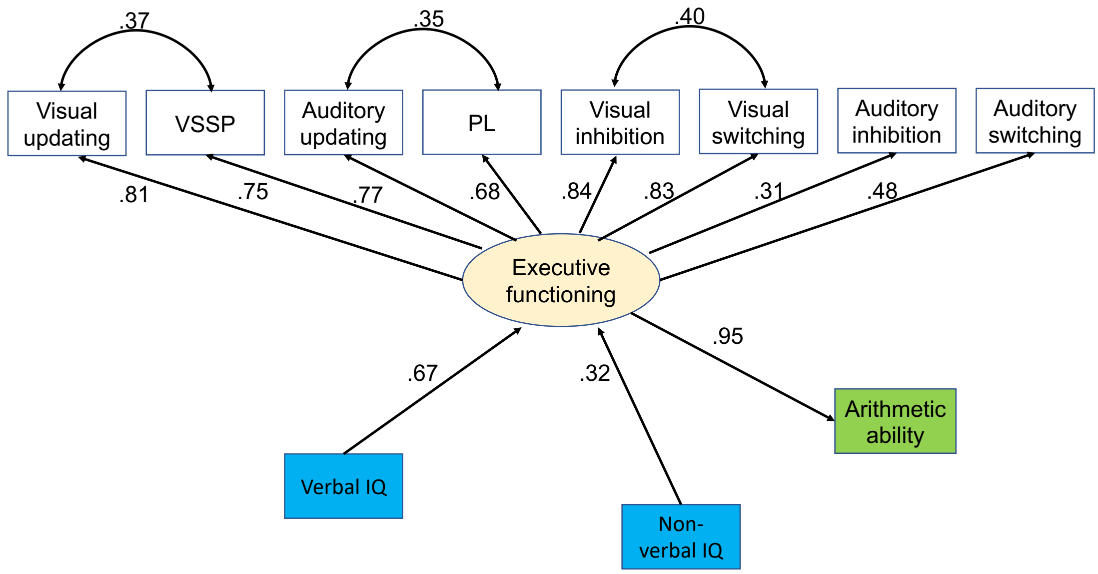


Figure 7.9. Executive functioning, and intelligence as predictors of arithmetic competence. IQ = intelligence, VSSP = visuospatial sketchpad, PL = phonological loop.
p < .001 for all paths.

Adding age to the modality specific two-latent variables model (see Figure 7.10) also produced a good fit, $\chi^2(44) = 54.56$, $p = .132$, RMSEA = .04 90% CI [.00, .07], CFI = 0.99, TLI = 0.99, SRMR = .03, and there were no significantly changes to the relationship between the executive functioning and intelligence predictors and arithmetic competence. The path between age and arithmetic competence was indirect, through verbal intelligence (*B* = .87), non-verbal intelligence (*B* = .66), and visual executive functioning (*B* = .43). It is interesting that whilst age has an impact on visual executive functioning and working memory, that is not the case for the auditory latent variable.

Adding age to the single latent executive functioning and working memory latent variable model (see Figure 7.11) also produced a good fit; $\chi^2(46) = 62.05$, $p = .057$, RMSEA = .05 90% CI [.00, .07], CFI = 0.99, TLI = 0.99, SRMR = .02. Once again, the fit statistics were very similar, and once again there was no direct link between age and arithmetic ability. However, in this model there is a significant path between non-verbal intelligence and arithmetic ability.

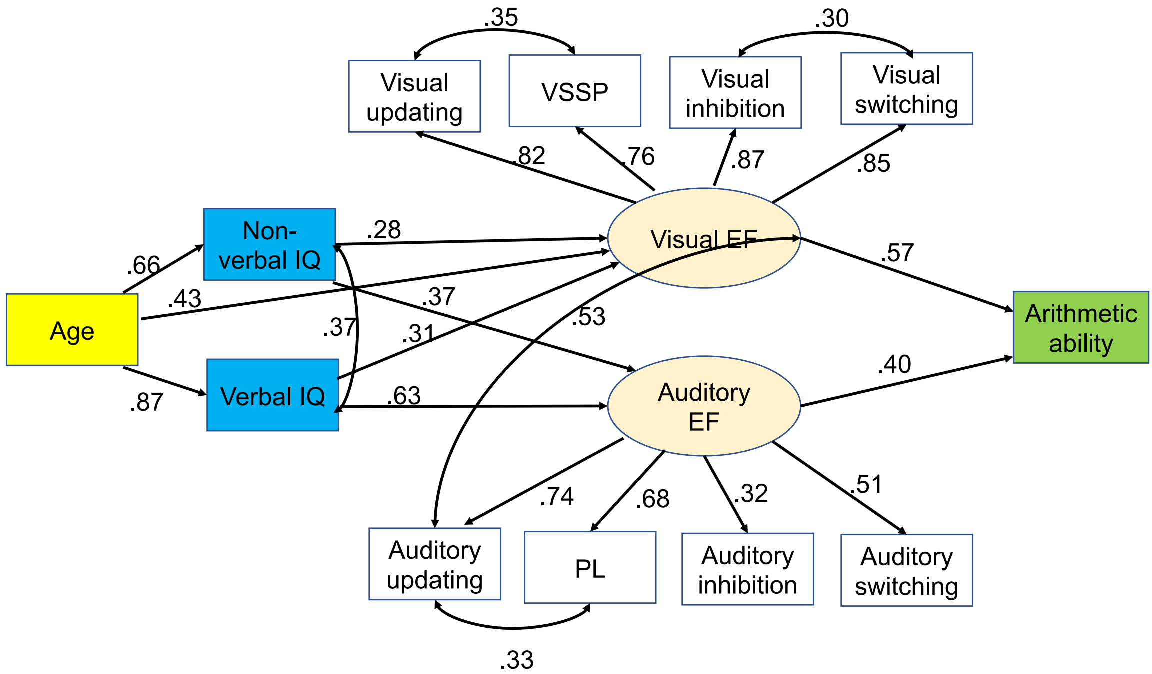


Figure 7.10. Bimodal executive functioning, and intelligence as predictors of arithmetic competence. IQ = intelligence, VSSP = visuospatial sketchpad, EF = executive function, PL = phonological loop.

p < .001 for all paths.

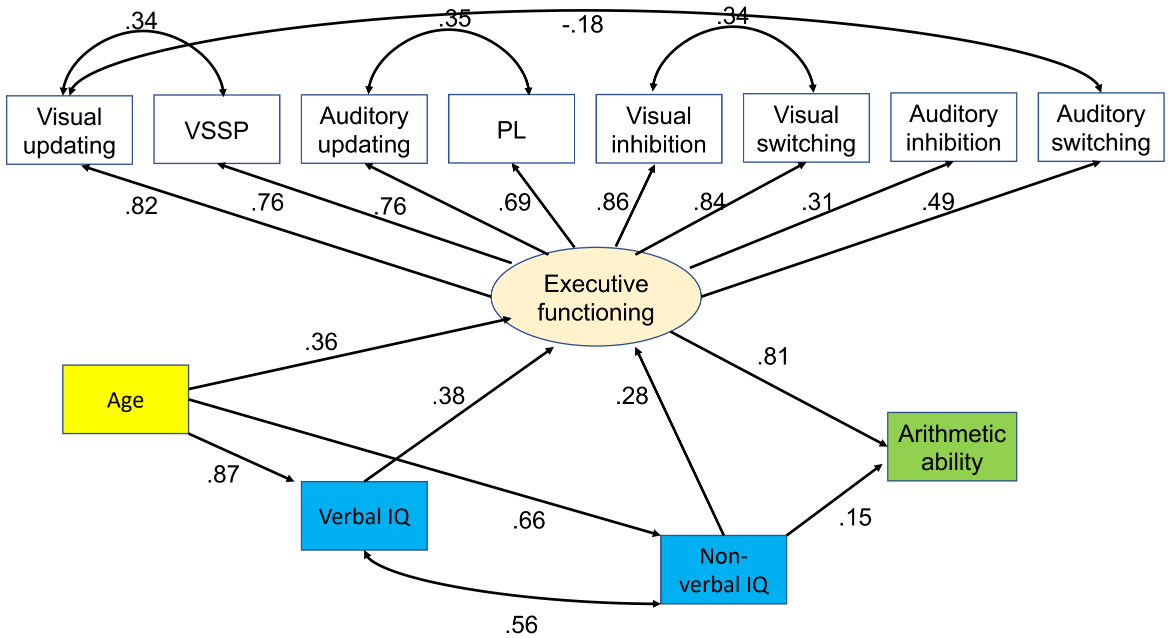


Figure 7.11. Executive functioning, intelligence, and age as predictors of arithmetic competence. IQ = intelligence, VSSP = visuospatial sketchpad, PL = phonological loop.

p < .001 for all paths except visual updating and auditory switching, *p* = .015, and non-verbal intelligence and arithmetic, *p* = .030.

A comparison of fit indices for the final updating and executive functioning models can be seen in Table 7.13. All models represent a good fit, however, although the updating model is more parsimonious, the executive functioning models highlight the importance of other executive function and working memory constructs in predicting arithmetic ability, and given how inter-related these abilities are it may be more appropriate on this occasion not to accept the more parsimonious model. All three models highlight that visual and auditory construct impact the links between cognitive abilities and arithmetic ability.

Table 7.13

Comparison of Fit Indices for Working Memory and Executive Functioning Models

	χ^2	*p*	χ^2/df	RMSEA	CFI	TLI	SRMR
Updating	5.88	.318	1.18	.03 [.00, .11]	1.00	1.00	.01
Single EF	62.05	.057	1.35	.05 [.00, .07]	.99	.99	.02
Two EF	54.56	.132	1.24	.04 [.00, .07]	.99	.99	.03

Note: EF = executive function; df = degrees of freedom; RMSEA = root mean square error of approximation; CFI = comparative fit index; TLI = Tucker-Lewis index; SRMR = standardised root mean square residual.

7.5 Alternative Analysis

Following discussions with experienced researchers, an alternative protocol for analysing the data will now be considered. Following the procedures detailed in Lawson and Farah, 2017, and Wiebe, Espy and Charak, 2008, in this analysis the intelligence and age variables will not be entered into the structural equation model directly, but instead will be entered into a regression analysis and adjusted predictor variables created that control for age and both verbal and non-verbal intelligence.

In the initial step predictor variables (visual updating, auditory updating, visuospatial sketchpad, phonological loop, visual inhibition, auditory inhibition, visual switching and auditory switching) were entered individually into a regression analysis as the dependent variable. The independent variables were age, the standardised verbal score and the standardised non-verbal score; standardised intelligence variables being utilised rather than the raw score as they take account of age, and hence age was not accounted for twice.

Having regressed the predictor variables, postestimation predictions were calculated, to produce adjusted predictor variables, where the impact of age and intelligence had been accounted for. These new variables were then entered in the structural equation models.

7.5.1 A single latent variable.

An updating latent variable containing visual and auditory updating and arithmetic did not converge. Entering all executive and working memory predictors into a single latent variable (see Figure 7.12) resulted in a poor fit with arithmetic, $\chi^2(27) = 24443.57$, $p < .001$, RMSEA = 2.24, 90% CI [.000, .], CFI = 0.17, TLI = -.11, SRMR = .02. Interestingly factor loading were good for all predictors.

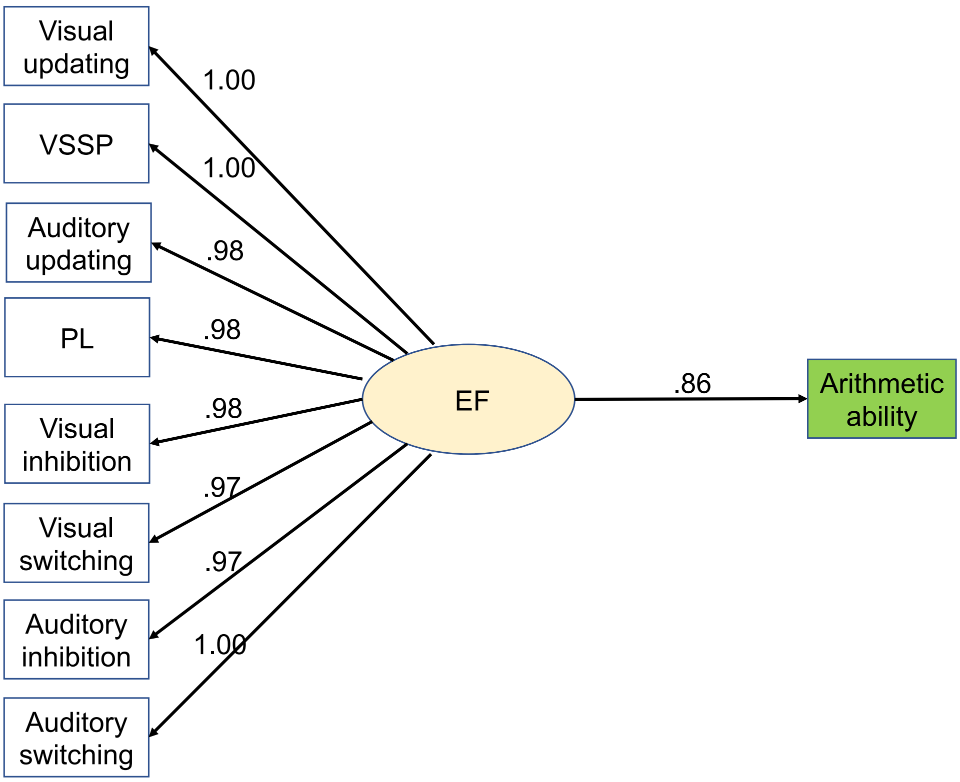


Figure 7.12. Single executive function and working memory latent variable as predictor of arithmetic competence. VSSP = visuospatial sketchpad, PL = phonological loop. *p* < .001 for all paths.

Mirroring previous analyses, the executive functioning and working memory predictors were next separated into two modality specific latent variables and entered into a model with arithmetic.

7.5.2 Visual latent models.

A model consisting of all four visual predictors did not converge, however, each combination of three predictors did (see Figures 7.13, 7.14, 7.15 and 7.16).

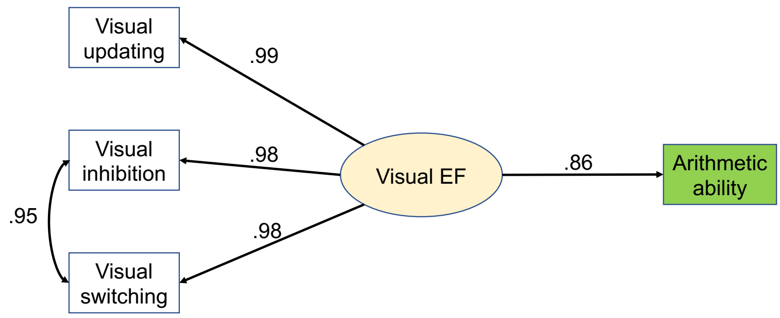


Figure 7.13. Visual executive function latent variable as predictor of arithmetic competence. EF = executive function.

p < .001 for all paths.

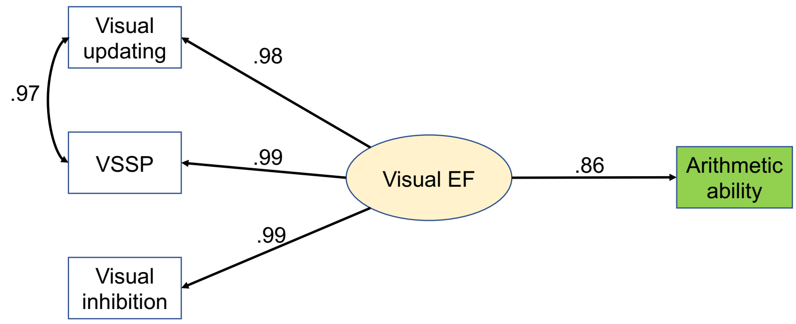


Figure 7.14. Visual executive function latent variable as predictor of arithmetic competence. VSSP = visuospatial sketchpad, EF = executive function.

p < .001 for all paths.

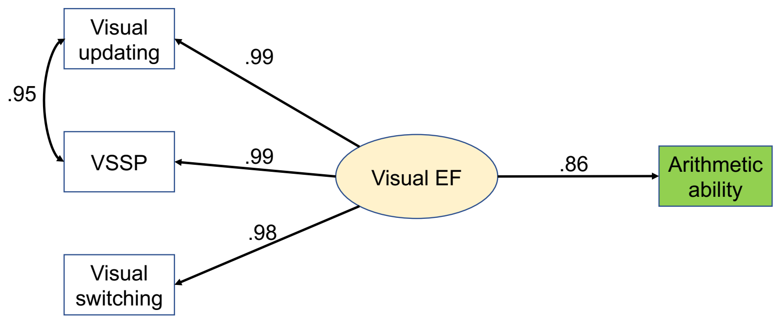


Figure 7.15. Visual executive function latent variable as predictor of arithmetic competence. VSSP = visuospatial sketchpad, EF = executive function.

p < .001 for all paths.

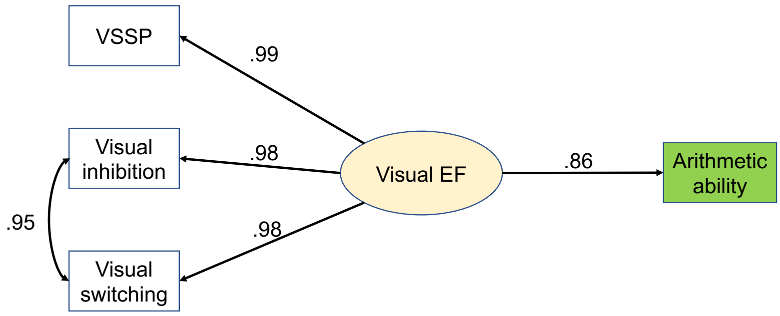


Figure 7.16. Visual executive function latent variable as predictor of arithmetic competence. VSSP = visuospatial sketchpad, EF = executive function.
p < .001 for all paths.

The model fit statistics for each model can be found in Table 7.14. Whilst all but the model consisting of updating, visuospatial sketchpad and inhibition display a good fit, the best is the model which incorporates the executive function variables i.e. updating, inhibition and switching.

Table 7.14
Comparison of Fit Indices for Visual Working Memory and Executive Functioning Models

Figure	χ^2	p	χ^2/df	BIC	RMSEA	CFI	TLI	SRMR
UP, INH, CF	0.29	.591	0.29	1232	.00 [.00, .16]	1.00	1.00	.00
UP, VSSP, INH	5.79	.016	5.79	2135	.16 [.06, .30]	.997	.99	.004
UP, VSSP, CF	1.38	.241	1.38	1867	.05 [.00, .21]	1.00	.999	.001
VSSP, INH, CF	2.45	.134	2.45	1108	.08 [.00, .23]	.999	.996	.002

Note: UP = visual updating; INH = visual inhibition; CF = visual switching; VSSP = visuospatial sketchpad; df = degrees of freedom; RMSEA = root mean square error of approximation; CFI = comparative fit index; TLI = Tucker-Lewis index; SRMR = standardised root mean square residual.

7.5.3 Auditory latent models.

Entering all auditory executive functioning and working memory variables into a single model, also resulted in a non-converging model. However, once again each combination of three variables did converge (see Figures 7.17, 7.18, 7.19 and 7.20.

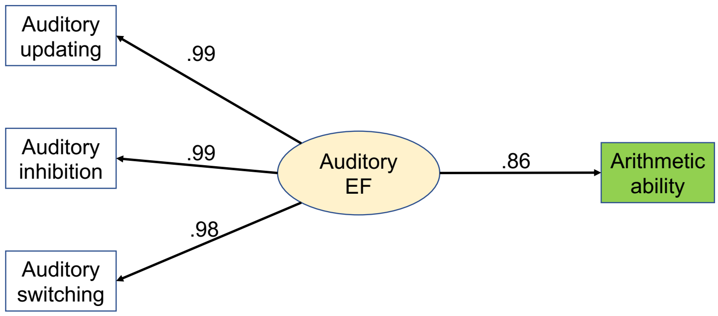


Figure 7.17. Auditory executive function latent variable as predictor of arithmetic competence. EF = executive function.

p < .001 for all paths.

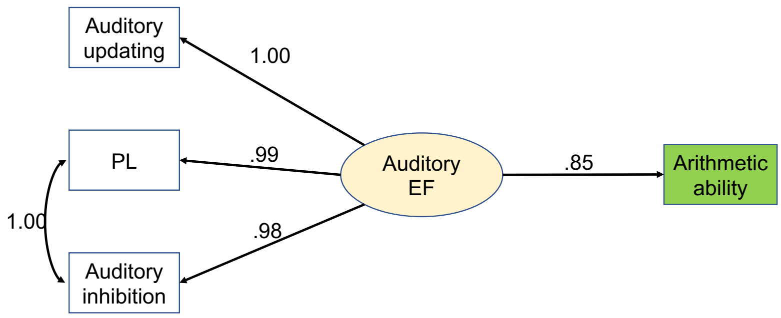


Figure 7.18. Auditory executive function latent variable as predictor of arithmetic competence. EF = executive function; PL = phonological loop.

p < .001 for all paths.

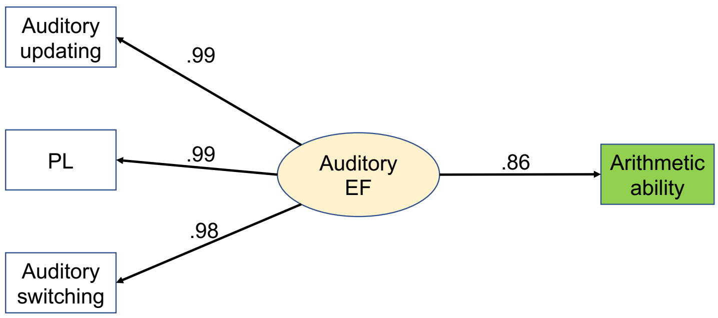


Figure 7.19. Auditory executive function latent variable as predictor of arithmetic competence. EF = executive function; PL = phonological loop.

p < .001 for all paths.

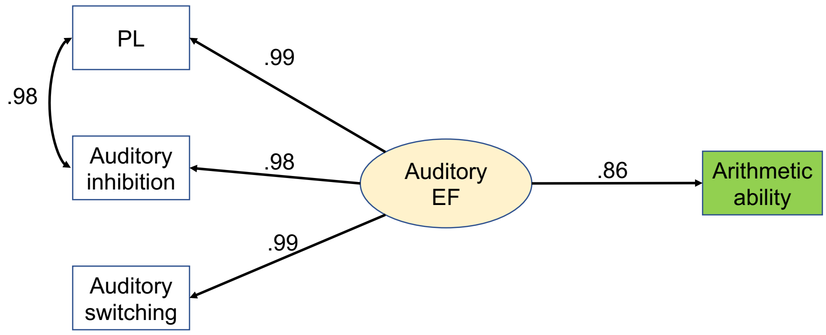


Figure 7.20. Auditory executive function latent variable as predictor of arithmetic competence. EF = executive function; PL = phonological loop.
p < .001 for all paths.

Model fit statistics for each of these models can be found in Table 7.15. Whilst each model indicates a good fit, the one containing the phonological loop, auditory inhibition and auditory switching was arguable to best fit closely followed by auditory updating, auditory inhibition and auditory switching.

Table 7.15
Comparison of Fit Indices for Auditory Working Memory and Executive Functioning Models

Figure	χ^2	*p*	χ^2/df	BIC	RMSEA	CFI	TLI	SRMR
UP, INH, CF	1.38	.501	0.69	2282	.00 [.00, .13]	1.00	1.00	.002
UP, PL, INH	1.96	.161	1.96	1395	.07 [.00, .23]	1.00	.998	.005
UP, PL, CF	2.87	.239	1.43	2819	.05 [.00, .16]	.999	.998	.003
PL, INH, CF	0.02	.898	0.02	1644	.00 [.00, .09]	1.00	1.00	.00

Note: UP = auditory updating; INH = auditory inhibition; CF = auditory switching; PL = phonological loop; df = degrees of freedom; RMSEA = root mean square error of approximation; CFI = comparative fit index; TLI = Tucker-Lewis index; SRMR = standardised root mean square residual.

In this analysis having a single executive function and working memory latent variable did not produce a good fit, however the modality specific latent variables did, with fit statistics for auditory and visual models being similar (see Tables 7.14 and 7.15). However, these models only contained three predictors, and putting an auditory and visual latent variable into the same model did not converge, a finding in keeping with the poor fit for the model where the single executive functioning and working memory latent variable predicts arithmetic ability.

7.5 Summary

The first phase of this research looked to address the following research question:

1) Which modality specific cognitive abilities predict arithmetic competence in the general population?

The cognitive abilities that explained the greatest proportion of the variance in arithmetic ability was impacted by the statistical technique utilised to analyse data. Regression analysis highlighted verbal and non-verbal intelligence, and visual and auditory updating as the constructs which explain additional variance when all other predictors were accounted for. The intelligence predictors showed the strongest associations. Interestingly coefficients for the intelligence predictors were similar, which was also the case for the updating ones, hence it is unclear if modality is an important consideration.

Analysing data via structural equation modelling raised questions about the best model to utilise, with models incorporating updating, a single working memory and executive function latent variable, and two modality specific executive functioning and working memory latent variables all displaying good fit statistics. However, whilst the updating model is the most parsimonious, given both the inter-relatedness of executive function and working memory constructs and the difficulty in finding tasks that measure single executive function or working memory constructs, if may be more appropriate to utilise either the one or two latent executive function and working memory variables. Whichever model is utilised it is clear that the modality of task presentation needs to be considered in future research as both visual and auditory predictors impact arithmetic ability.

In the alternative analysis, the single latent executive function and working memory variable did not display a good fit, and a model containing two modality specific executive function and working memory latent variables in the same model did not converge. However, the models that displayed a good fit again highlighted the importance of a range of modality specific executive function and working memory constructs for arithmetic ability. The results in this alternative analysis may be impacted by a lack of power, and hence future research should address this.

This concludes the analysis for the typically developing population. An interpretation and discussion of findings can be found in the next chapter, Chapter 8.

Chapter 8: Discussion for Research Question One

Phase one of this study looked to address the following research question:

> Which modality specific cognitive abilities predict arithmetic competence in the general population?

To my knowledge this is the first study to investigate concurrently auditory and visual measures of executive functioning (updating, inhibition, switching), intelligence, and numerical acuity, in addition to measure of the visuospatial sketchpad and phonological loop, and the impact they have on arithmetic ability. Representing executive functions and working memory abilities as modality specific latent variables, facilitated an investigation into how the modality in which stimuli are presented impacts links between cognitive abilities and arithmetic competence. Although the creation of latent variables containing updating, switching and inhibition is not new, including the phonological loop and visuospatial sketchpad is an important innovation, as they are not always considered executive functions. However, factor loadings for each were good, which is perhaps unsurprising given the proposed structure of working memory which highlight strong links between each and the central executive (Baddeley, 1996). Hence given these abilities are inter-related and both have previously been identified as predictors of maths ability, it would seem appropriate to include them in the same latent variable.

Both visual and auditory modality specific executive function and working memory latent variables displayed direct paths to arithmetic ability, with the visual latent variable explaining a greater proportion of the variance. Despite intelligence being the strongest predictor in the regression analyses, paths within the structural equation model were indirect, through each latent executive function and working memory variable.

The next section contains a more in-depth discussion of the impact that the choice of statistical technique can have on results. The three structural equation models will then be considered; the first utilises a latent updating variable, the second a single executive function and working memory latent variable, and the third investigated the impact of modality, via two modality specific executive function and working memory variables.

8.1 The Impact of Statistical Techniques

It has been suggested that inconsistencies in research findings surrounding the cognitive underpinnings of mathematical abilities may be influenced by the statistical techniques utilised. This phenomenon could be particularly pertinent when observed variables (i.e., regression analysis) and latent variables (i.e., structural equation modelling) are compared and contrasted (Filippetti & Richaud, 2017; Lee et al., 2009).

This study provides further evidence for this phenomenon as data analysed via regression analyses highlighted intelligence and updating as significant predictors, with intelligence variables (non-verbal and verbal) explaining more of the variance. However, entering data into a structural equation model resulted in latent updating (visual and auditory) emerging as the strongest predictor, with only an indirect path between non-verbal intelligence and arithmetic ability. This is potentially an important finding as the majority of studies in this field utilise correlation or regression techniques. However, given the acknowledged impurity problem for measures of working memory and executive function (Van der Ven et al., 2012), representing executive functions as latent variables may be optimal (Miyake & Friedman, 2012). This is because measurement errors in regression analyses can be confounded with true common variance, whilst structural equation modelling uses multiple indicators whose common variance is extracted, and whose measurement errors are modelled explicitly, thereby reducing the confounding effect of tasks which inevitably are not measuring a single construct. Arguably therefore structural equation modelling facilitates a more precise investigation of the relationships between conceptual constructs (Lee et al., 2009), which is particularly pertinent in studies involving executive functions.

However, this study did not set out to provide evidence for or against the unity and diversity debate, and the range of executive functioning tasks preclude conclusions being drawn.

The statistical technique utilised may explain inconsistencies in the literature investigating the cognitive underpinnings of mathematical ability, and also why intervention studies looking to train a single construct (e.g., working memory, numerical acuity) have to date had limited success.

Most of the intervention studies aimed at improving working memory have focussed on the visual domain, specifically visual updating and the visuospatial sketchpad. While some studies have found gains following adaptive working memory training (e.g., Holmes et al. 2009; Kroesbergen, Noordende, & Kolkman, 2014; St Clair-Thompson, Stevens, Hunt, & Bolder, 2010), others including a meta-analysis and randomised control trial reported near but nor far transfer effects (Dunning, Holmes, & Gathercole, 2013; Melby-Lervåg & Hulme, 2013)

The majority of training programs aimed at developing specific mathematics related abilities involve preschool children and include a range of activities designed to improve early numeracy abilities including counting, recognising and writing numbers, one-to-one correspondence, comparisons of symbolic numerals, change operations and, understanding numbers. Once again stimuli is typically presented visually (e.g., Park & Brannon, 2013; Ramani & Siegler, 2008; Whyte & Bull, 2008; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006). Whilst there is evidence for training improving both symbolic and non-symbolic abilities, the evidence for far transfer to improved mathematical performance is more limited. Indeed it has been suggested that to effect far transfer, interventions should train both specific and general abilities (Passolunghi & Costa; 2016).

This study provides evidence to suggest that rather than focussing solely on visual updating and the visuospatial sketchpad, examining multiple distinct but interrelated general abilities (updating, visuospatial sketchpad, phonological loop, inhibition and switching) affords a better understanding of the cognitive abilities that predict mathematical ability. Hence, forthwith this discussion will focus on findings pertaining to the structural equation modelling. However before doing so it is important to note that whilst this study did not find numerical acuity to be an important predictor of arithmetic ability, this may have been impacted by the task utilised which measured non-symbolic rather than symbolic numerical abilities. Hence future research should examine whether results are congruent when symbolic numerical acuity is investigated concurrently with bimodal executive function and working memory abilities.

8.2 Cognitive Predictors of Arithmetic Ability

Previous research has highlighted executive function, working memory, intelligence and numerical acuity as possible cognitive predictors of mathematical ability. A key finding from this study is the importance of considering multiple, independent but interrelated abilities rather than single constructs, and that modality of stimuli presentation needs to be considered. There follows a discussion of the relative merits of possible models and the impact of modality.

8.2.1 Updating and intelligence.

The initial structural equation model was informed both by the results from the regression analyses and previous research, and utilised a latent updating variable containing visual and auditory updating, in addition to verbal and non-verbal intelligence variables. A large body of evidence supports the importance of updating in predicting mathematical ability, with two recent meta-analyses highlighting both auditory and visual updating (Friso-van den Bos et al., 2013; Peng et al., 2017; Van der Ven et al., 2012), although the former found stronger links for auditory updating.

Previous literature has also highlighted both verbal and non-verbal intelligence as predictors of mathematical ability, however inconsistencies regarding the relative importance of working memory (including updating) and intelligence exist. Regression and path analyses have indicated they jointly contribute, with working memory fully or partially mediating the link between intelligence and mathematical ability, or working memory indirectly influencing mathematical abilities through intellectual skill (e.g., Andersson, 2008; Bull & Scerif, 2001; Kroesbergen et al., 2009; Kyttälä & Lehto, 2008; Lee et al., 2009; Passolunghi et al., 2014). However, the components included in the conceptualisation of working memory varies across studies, and the relative strength of modality specific constructs was not investigated. Additionally, links may be dependent on the mathematical task utilised (Filippetti & Richaud, 2017).

When the structural equation model included a latent updating variable, verbal intelligence, non-verbal intelligence and arithmetic ability, the strongest direct path to arithmetic ability was through updating (B = .66). Of the two intelligence variables, only verbal intelligence displayed a direct path to arithmetic ability, although both had indirect paths through latent updating. Adding age to the model did not majorly change this pattern of results, although the link between verbal intelligence and arithmetic ability was now marginally significant. Age impacted arithmetic ability indirectly through latent updating, and both intelligence variables, with the path between age and arithmetic ability strongest through the intelligence variables, particularly verbal intelligence.

Age influencing verbal intelligence to a greater extent than non-verbal intelligence is perhaps unsurprising given it is defined as a command of language and knowledge accumulating with education and age, whilst non-verbal intelligence is the ability for abstract reasoning. Hence increasing exposure to formal education increases a child's command of language. Arguably, it is far harder to improve non-verbal intelligence overtly through education, as most tasks used to measure it do not contain material participants have been exposed to before, and hence are more reliant on a person's ability to intuitively observe patterns and connections.

However, despite this model displaying a good fit, it is not optimal, as the latent variable consisted of two constructs, and three is considered preferable (Kenny, 2001; http://davidakenny.net/cm/basics.htm#Ident).

8.2.2 Executive function and working memory.

While the regression analyses highlighted updating and intelligence as the most important predictors, inhibition, switching, the visuospatial sketchpad, and phonological loop were all independent predictors. As the structures of working memory and executive function are still debated, and it is difficult to measure these constructs, it would be cogent to consider a model which included all these constructs (updating, inhibition, switching, visuospatial sketchpad and phonological loop) in one latent variable (see Figure 7.4). This model displayed a good fit, although factor loadings were stronger for visual constructs. To address whether modality was impacting the association between cognitive constructs and arithmetic ability, this single latent factor was divided into two modality specific latent variables (see Figure 7.5), both of which displayed a good fit. In all models each construct made a significant contribution to the model.

This is an important finding as evidence for switching and inhibition as important predictors of mathematical ability is mixed, with significant relationships found in some studies (e.g., Blair & Razza, 2007; Cantin et al., 2016; Yeniad et al., 2013), but not others (Cragg et al., 2017; Lee et al., 2012; Van der Ven et al., 2012). Research which predominantly utilises regression analyses has suggested that inhibition and switching explain unique variance when studied independently, which is then accounted for by working memory (updating, phonological loop and visuospatial sketchpad) when it is added to the model (Bull & Lee, 2014; Cragg et al., 2017; Lee & Bull, 2015). Alternatively, inhibition and switching account may account for less variance in mathematical ability than working memory (Cragg et al., 2017; Friso-van den Bos et al., 2013).

In this study inhibition and switching in both modalities were found to be significant components of both a single executive functioning latent variable, and modality specific latent variables. Indeed, factor loadings for visual inhibition and switching were among the highest in both models, and when arithmetic ability was added to each model (see Figures 7.6 and 7.7), they were the highest. Conversely, loadings for auditory inhibition and switching were the weakest, (although still significant), a possible indication they make a lesser contribution to arithmetic achievement. However, before drawing this conclusion, in this study there is the possibility findings may have been impacted by tasks used to index these constructs having low construct validity.

Evidence for the importance of the visuospatial sketchpad and phonological loop as predictors of mathematical ability is inconsistent, with suggestions that links may be age specific, with the phonological loop having more impact early in development, and the visuospatial sketchpad becoming increasingly important as mathematics becomes more complex (e.g., Meyer et al., 2010). However, there is also evidence for the phonological loop being an important predictor in middle childhood (Andersson, 2008).

This study confirmed both the visuospatial sketchpad and phonological loop as independent predictors, which remained when age was accounted for. However, examining them alongside other predictors resulted in both losing predictive power, which was also true when modality specific predictors were investigated. This result is in line with much research (e.g., Geary, 2011), but contrary to others including a meta-analysis (Friso-van den Bos et al., 2013).

The reasons for the loss of predictive power may be understood by looking in more detail at the structural equation modelling (see Figures 7.4 to 7.11), where the visuospatial sketchpad always covaried with visual updating and in more complex models the phonological loop covaried with auditory updating. Covariance between updating and its modality appropriate slave component is not surprising, nor, given the age range of this study, is the finding that the ability to hold information in memory and also manipulate it, explains additional variance.

8.2.3 Possible models for representing executive function and working memory and intelligence.

This study investigated two ways of modelling executive function and working memory constructs. The first included all executive functioning and working memory predictors in a single latent variable (see Figure 7.4), whilst the second split them by modality, hence modelled visual executive function and working memory, and auditory executive function and working memory latent variables (see Figure 7.5). Each model displayed a good fit, which was also the case when arithmetic was added to the model (see Figures 7.6 and 7.7). Importantly each latent variable was a strong predictor of arithmetic ability.

Placing the modality specific latent executive function variables into a model with verbal, and non-verbal intelligence facilitated an investigation into their association with arithmetic ability (see Figure 7.8). In the resultant model direct paths to arithmetic ability were observed from each modality specific executive functioning and working memory variables solely. Paths from verbal and non-verbal intelligence were indirect, through both executive function and working memory latent variables. However, whilst there was a significant path between each modality specific latent variable there was a covariance between auditory updating and the visual latent variable, possibly suggesting that whilst each modality is impacting arithmetic ability, both are important as there are interactions between them.

Representing the executive function and working memory predictors as a single latent variable (Figure 7.9) and placing it in a model with the intelligence predictors, also resulted in a direct path from the latent variable solely, with paths from the intelligence predictors once again being indirect through the latent variable.

When age was included in the two latent variable model (see Figure 7.10), links to arithmetic ability for both intelligence predictors were once again through each executive functioning latent variable. Interestingly, for both verbal and non-verbal intelligence there was a stronger link to the auditory executive function and working memory latent variable. Adding age to the single latent variable model (see Figure 7.11) produced similar relationships, however, there was now a direct path between non-verbal intelligence and arithmetic ability.

In the two latent variable model the link between age and arithmetic ability was indirect through verbal intelligence (B = .87), non-verbal intelligence (B = .66), and visual executive functioning and working memory (B = .43). This finding was replicated in the single latent variable model. Age has been highlighted as a possible confound, with evidence for this phenomenon in general cognitive abilities, and numerical acuity literature (Fazio et al., 2014; Geary, 2011; Meyer et al., 2010), although results are inconsistent (e.g., Chen & Li, 2014). In this study findings appear to converge to age having an indirect influence on arithmetic ability, through its impact on other cognitive abilities. Specifically, age appears to impact measures of intelligence more than executive functioning, with the strength of the impact influenced by modality, as age affected verbal intelligence to a greater extent than non-verbal intelligence, and there was a direct path to the visual executive functioning and working memory latent variable solely.

In the two latent variable model age impacting visual executive functioning but not auditory executive functioning is intriguing. It may be due to auditory processes maturing earlier than visual processes. However, given the paucity of tasks designed to measure auditory inhibition and switching, particularly across a wide age range, results may be due to these tasks displaying lower construct validity. As this study highlights the importance of auditory executive functioning it seems appropriate for reliable measures for auditory switching and inhibition appropriate across development to be developed.

Each of the three models investigated (updating, single executive function and working memory latent variable and two modality specific latent variables) have highlighted the importance of executive functioning and modality in predicting arithmetic ability, above and beyond the impact of other constructs. In deciding which is the preferred model, the updating one is not optimal, given the latent variable is comprised of just two variables. Relationships in the other two are very similar, with the single latent variable being more parsimonious. However, modelling as two modality specific latent variables is valuable, as it highlights the importance of considering modality when investigating the construct that underpin arithmetic ability, as there are significant direct paths from both the visual and auditory latent variables.

The alternative analysis utilised executive function and working memory variables which had been adjusted to account for the effect of age and verbal and non-verbal intelligence. Finding were similar to the main analysis; executive function and working memory predictors in both modalities having a significant impact on arithmetic ability. However, placing all executive function and working memory predictors into a single latent variable with arithmetic produced a model which displayed a poor fit, and placing two modality specific latent variables into a single model with arithmetic did not converge. Modality specific latent variables consisting of combinations of three of the executive function and working memory predictors did produce models which displayed a good fit and significant path to arithmetic ability. It is possible that the poor fit or lack of convergence of the more complex model was due to sample size, however, the models that did display a good fit, also point to executive functioning constructs in both modalities having an impact on arithmetic ability above and beyond the effect of age and verbal and non-verbal intelligence.

8.2.4 Numerical acuity.

Congruent with much research, visual numerical acuity was found to be an independent predictor of arithmetic ability even accounting for age (Fazio et al., 2014; Halberda & Feigenson, 2008; Schneider et al., 2017). However, visual numerical acuity was indexed by incongruent (where numerical and visual properties were incongruent) trials solely, as the link between arithmetic and congruent trials was insignificant.

Entering visual numerical acuity concurrently into a regression analysis with other predictors, resulted in it losing predictive power, hence congruent with previous findings, the link between arithmetic ability and visual numerical acuity appears to be at least partly driven by other cognitive abilities (Gilmore et al., 2013; Price et al., 2012).

Recently it has been suggested that much of the research investigating visual numerical acuity is underpowered (Chen & Li, 2014). This study addressed this by utilising a task which consisted of 168 trials, across two sessions. Reliability across the two sessions was found to be good (Cronbach's $\alpha = .85$).

The auditory numerical acuity task was also included in each session and displayed good reliability (Cronbach's $\alpha = .87$). It was an independent predictor of arithmetic ability, which remained when accounting for age, however, it became insignificant when entered into a regression analysis with other cognitive abilities. As very little research has utilised auditory numerical acuity, there is little to compare this result to, however regression coefficients were similar in magnitude for visual and auditory numerical acuity (see Table 8.1), which may provide additional evidence for this ability being multimodal (Arrighi et al., 2014; Izard et al., 2009).

Although it should be noted that zero and age-corrected correlations between these constructs were less than .8 (see Tables 6.7, 6.8, 9.5 and 9.6).

Table 8.1

Regression Coefficients for Visual and Auditory Numerical Acuity

Regression Model	Auditory numerical acuity	Visual numerical acuity
Independent	.59	.60
Age-corrected independent	.21	.22
All predictors	-.05	-.02

8.3 Summary

Phase one of this study looked to determine the cognitive abilities which are predictive of arithmetic ability in the general population and whether predictive power is impacted by the modality of stimuli presentation. While the way data were analysed produced differing results, the overarching finding was that executive function and working memory constructs had a significant impact on arithmetic ability, even when other predictors; age, verbal intelligence and non-verbal intelligence, were accounted for. A second key finding was the need for future research to overtly consider the modality in which stimuli are presented, as both auditory and visual predictors had an impact on arithmetic ability.

The following chapters will consider data and findings from phase two of this study.

Phase Two: Results and Discussion

Chapter 9: Analysis for Phase Two

Phase two of the data analysis addresses the second and third research questions. Utilising the results from the typically developing analysis the research questions are:

- Do executive function and working memory in both domains, but particularly visual and auditory updating, display atypical development in a) children with maths learning disabilities and b) children with Turner syndrome?
- Are identified areas of deficits confined to either the visual or auditory domain?

9.1 Descriptive Statistics

The populations with developmental disorders (DD) under investigation; Turner syndrome and maths learning disability, each have known visuospatial deficits. Children in each DD population were individually matched by gender and chronological age with an individual from the typically developing population. Hence this analysis consisted of four populations: Turner syndrome, typical development matched with the Turner syndrome group, maths learning disability, and typical development matched with the maths learning disability group. Sample characteristics and descriptive statistics can be found in Table 9.1 (Turner syndrome and their matched typically developing control), Table 9.2 (Maths learning disability and their matched typically developing control). Zero and age-corrected correlations in Tables 9.3 (Turner syndrome) 9.4 (Maths learning disability), 9.5 (Typically development matched with Turner syndrome) and 9.6 (Typically developing matched with maths learning disability).

Table 9.1

Sample Characteristics and Descriptive Statistics for the Turner Syndrome Group and their Matched Typically Developing Group

	TS (*n* = 32)				TDTS (*n* = 32)			
	Min	*Max*	*M*	*SD*	*Min*	*Max*	*M*	*SD*
Sex (male/female)			0/32				0/32	
Age (months)	58.7	224.4	145.2	46.9	55.9	219.9	144.2	47.1
Socioeconomic status	9.0	60.5	48.0	11.3	3.0	63.0	45.3	15.1
Verbal intelligence	30.0	96.0	59.7	17.2	30.0	97.0	67.0	19.7
Non-verbal intelligence	7.0	39.0	25.8	8.6	12.0	42.0	30.0	7.6
Processing speed	0.3	2.3	1.4	0.5	0.7	3.1	1.7	0.5
Arithmetic	5.0	43.0	22.4	10.7	5.0	53.0	29.3	13.1
Visual updating	6.0	30.0	19.2	6.2	12.0	41.0	24.7	5.5
Auditory updating	6.0	28.0	17.0	5.6	11.0	35.0	21.3	6.6
Visuospatial sketchpad	7.0	31.0	21.5	5.8	13.0	42.0	26.5	5.7
Phonological loop	19.0	36.0	26.0	3.8	22.0	42.0	28.4	5.1
Visual inhibition	0.3	2.6	1.4	0.5	0.7	3.1	1.8	0.6
Auditory inhibition	0.0	13.0	10.6	2.7	8.0	13.0	11.3	1.8
Visual switching	0.1	1.0	0.6	0.3	0.1	1.4	0.7	0.3
Auditory switching	2.0	12.0	6.1	2.4	2.0	13.5	6.7	3.3
Visual ANS	45.2	96.4	80.1	12.3	65.5	98.8	88.4	8.2
Auditory ANS	13.0	52.0	42.7	8.3	33.0	53.0	46.0	5.5

Note: TS = Turner Syndrome group; TDTS = typically developing matched with Turner syndrome group participants; ANS = numerical acuity.

Table 9.2

Sample Characteristics and Descriptive Statistics for Maths Learning Disability Group and their Matched Typically Developing Group

	MLD (*n* = 40)				TDMLD (*n* = 40)			
	Min	*Max*	*M*	*SD*	*Min*	*Max*	*M*	*SD*
Sex (male/female)			22/18				22/18	
Age (months)	64.6	231.2	132.8	46.3	62.0	221.7	133.3	46.4
Socioeconomic status	9.0	66.0	41.6	14.4	9.0	62.0	45.4	15.5
Verbal intelligence	28.0	91.0	57.0	18.0	23.0	91.0	62.0	21.0
Non-verbal intelligence	12.0	40.0	22.9	6.9	9.0	41.0	27.8	8.8
Processing speed	0.4	2.7	1.5	0.5	0.2	3.2	1.6	0.7
Arithmetic	5.0	29.0	17.3	7.6	6.0	51.0	26.4	13.4
Visual updating	6.0	33.0	19.9	7.5	6.0	36.0	22.4	7.0
Auditory updating	6.0	32.0	16.8	5.1	7.0	37.0	20.3	7.3
Visuospatial sketchpad	7.0	38.0	23.5	6.6	12.0	39.0	25.0	6.0
Phonological loop	19.0	40.0	27.4	5.1	18.0	40.0	29.6	5.3
Visual inhibition	0.3	3.1	1.5	0.7	0.7	3.1	1.7	0.7
Auditory inhibition	6.0	13.0	11.1	1.8	8.0	14.0	11.5	1.5
Visual switching	-0.02	1.1	0.5	0.3	0.1	1.3	0.6	0.3
Auditory switching	1.0	12.5	6.2	2.9	-0.5	16.5	6.4	3.5
Visual ANS	52.8	96.4	82.6	11.2	47.7	98.8	86.3	11.3
Auditory ANS	25.0	52.0	44.5	5.7	29.0	53.0	46.1	6.4

Note: MLD = maths learning disability group; TDMLD = typically developing matched with maths learning disability group participants; ANS = numerical acuity.

As can be seen from Tables 9.1 and 9.2, each study task displays appropriate variance, with neither floor or ceiling effects observed. Additionally, congruent with a developmental trajectories approach, the age of participants in each matched typically developing group spans the lowest to highest chronological ages of its respective disorder group. This is also the case for the two measures of mental age; verbal and non-verbal intelligence.

Table 9.3

Zero Order and Age-Corrected Correlation Matrices for Turner Syndrome Participants

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. Socioeconomic status		-.15	.33	-.08	.07	.05	.25	.06	.16	-.14	.09	.05	-.14	-.02	.11
2. Processing speed	-.28		-.21	.27	.37	.12	.05	.28	.31	.66*	.22	.54	.09	.07	.23
3. Verbal intelligence	-.03	.47		.54	.37	.37	.29	.30	.46	.32	-.22	.25	-.05	.07	.17
4. Non-verbal intelligence	-.21	.56*	.73*		.55*	.39	.42	.40	.40	.64*	.13	.64*	.09	.41	.38
5 Arithmetic	-.13	.66*	.72*	.73*		.21	.34	.41	.51	.66*	.03	.53	.13	.21	.35
6. Visual updating	-.15	.54*	.73*	.64*	.60*		.38	.50	.34	.41	.40	.38	-.19	.13	.02
7. Auditory updating	.01	.47	.67*	.64*	.64*	.67*		.31	.34	.19	.31	.28	.07	.22	.39
8. Visuospatial sketchpad	-.12	.58*	.65*	.62*	.67*	.72*	.59*		.25	.36	.16	.32	.17	.54	.54*
9. Phonological loop	.12	.52	.62*	.57*	.65*	.55*	.54*	.47		.38	.03	.43	-.03	.09	.15
10. Visual inhibition	-.28	.81*	.70*	.78*	.82*	.70*	.56*	.63*	.57*		.07	.71*	-.04	.12	.30
11. Auditory inhibition	.03	.32	.07	.24	.18	.44	.38	.27	.13	.21		.43	.18	.12	.16
12.. Visual switching	-.17	.77*	.72*	.78*	.77*	.71*	.64*	.64*	.60*	.86*	.45		.14	.20	.43
13.. Auditory switching	-.18	.21	.15	.19	.24	.02	.19	.26	.07	.11	.21	.25		.26	.24
14. Visual ANS	-.17	.45	.53	.62*	.54*	.50	.54*	.71*	.35	.48	.23	.56*	.32		.73*
15. Auditory ANS	-.07	.54*	.57*	.60*	.61*	.43	.63*	.71*	.38	.58*	.26	.67*	.31	.83*	
16. Age	-.26	.67*	.83*	.59*	.69*	.71*	.67*	.63*	.47	.68*	.23	.76*	.21	.61*	.59*

Note: ANS = number acuity.

Above diagonal are age corrected partial correlations; below diagonal are zero correlations

*$p < .003$ (Bonferroni corrected for multiple comparisons).

Table 9.4

Zero Order and Age-Corrected Correlation Matrices for Maths Learning Disability Participants

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. Socioeconomic status		.13	.33	.32	.27	-.02	.24	.04	.07	-.04	.09	.05	.34	.04	.04
2. Processing speed	.15		.23	.39	.49	.33	.55	.18	.33	-.05	.07	.39	-.02	.01	.21
3. Verbal intelligence	.23	.83*		.26	.64*	.27	.08	.30	-.18	.43	.07	.14	.10	.02	.24
4. Non-verbal intelligence	.32	.67*	.62*		.16	.44	.29	.28	.09	.35	.05	.24	.12	.30	.25
5. Arithmetic	.21	.89*	.93*	.58*		.13	.41	.29	-.18	.13	-.05	.24	.04	.12	.32
6. Visual updating	.06	.73*	.72*	.66*	.68*		.02	.52	-.01	.27	.16	.15	-.34	.11	-.01
7. Auditory updating	.24	.79*	.63*	.57*	.74*	.49		.07	.38	-.01	-.12	.24	.22	.15	.27
8. Visuospatial sketchpad	.10	.71*	.77*	.59*	.76*	.78*	.54		-.18	.08	.14	.15	-.43	.27	.18
9. Phonological loop	.12	.80	.68*	.52	.68*	.57*	.71*	.54		-.12	.08	-.28	.30	-.13	.08
10. Visual inhibition	.07	.77*	.89*	.65*	.84*	.72*	.61*	.70*	.70		.003	.32	-.11	.44	-.03
11. Auditory inhibition	.13	.40	.41	.28	.36	.40	.20	.40	.39	.38		-.09	-.16	-.03	-.21
12. Visual switching	.11	.85*	.82*	.61*	.84*	.67*	.68*	.71*	.63*	.86*	.33		-.30	.46	-.06
13. Auditory switching	.36	.29	.35	.29	.33	.02	.38	-.01	.44	.27	.01	.17		-.26	.07
14. Visual ANS	.10	.60*	.63*	.58*	.66*	.55	.55	.65*	.51	.76*	.27	.77*	.07		.13
15. Auditory ANS	.10	.67*	.70*	.55	.72*	.48	.61*	.61*	.59*	.61*	.15	.58*	.28	.54	
16. Age	.10	.87*	.90*	.59*	.90*	.71*	.68*	.75*	.81*	.90*	.42	.88*	.35	.69*	.69*

Note: ANS = number acuity.

Above diagonal are age corrected partial correlations; below diagonal are zero correlations

**p* < .003 (Bonferroni corrected for multiple comparisons)

Table 9.5

Zero Order and Age-Corrected Correlation Matrices for Typically Developing Group Matched with Turner Syndrome Participants

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. Socioeconomic status		.10	.30	.22	.35	.23	.48	.36	.18	.23	-.20	.33	-.12	-.06	.11
2. Processing speed	.05		.05	.15	.19	.40	.38	.57	.39	.48	-.04	.33	.04	-.09	.21
3. Verbal intelligence	.11	.54		.59	.61*	.12	.44	.43	.35	.41	.27	.22	.02	.50	-.01
4. Non-verbal intelligence	.16	.44	.74*		.50	.10	.33	.64*	.41	.35	.16	.14	.29	.41	.17
5 Arithmetic	.14	.60*	.89*	.70*		.59	.54	.36	.39	.67*	.19	.51	-.06	.23	.33
6. Visual updating	.10	.68*	.71*	.50*	.86*		.53	.43	.32	.40	.16	.62*	-.23	.04	.23
7. Auditory updating	.39	.56	.62*	.52	.67*	.68*		.60*	.60*	.57	.16	.37	.17	.23	-.05
8. Visuospatial sketchpad	.26	.72*	.69*	.76*	.66*	.68*	.72*		.66*	.37	.16	.22	.14	.33	.06
9. Phonological loop	.15	.54	.53	.55	.55	.51	.68*	.73*		.49	.11	.11	.24	.26	-.02
10. Visual inhibition	.13	.69*	.74*	.60*	.84*	.72*	.70*	.63*	.62*		.05	.63*	.02	.25	.27
11. Auditory inhibition	-.20	.18	.42	.31	.38	.36	.29	.32	.24	.27		-.11	.20	-.002	.10
12. Visual switching	.20	.61*	.68*	.48	.78*	.82*	.58	.54	.37	.80	.16		-.28	.18	.18
13. Auditory switching	-.13	.27	.35	.44	.31	.18	.33	.34	.37	.29	.31	.09		-.11	.06
14. Visual ANS	-.08	.41	.81*	.65*	.71*	.60	.50	.62*	.47	.63*	.25	.60*	.23		-.02
15. Auditory ANS	.08	.44	.41	.40	.56*	.51	.20	.33	.19	.50	.25	.45	.24	.35	
16. Age	-.05	.61*	.86*	.57	.85*	.79*	.50	.60*	.43	.69*	.34	.69*	.40	.74*	.48

Note: ANS = number acuity.

Above diagonal are age corrected partial correlations; below diagonal are zero correlations

*$p < .003$ (Bonferroni corrected for multiple comparisons).

Table 9.6

Zero Order and Age-Corrected Correlation Matrices for Typically Developing Group Matched with Maths Learning Disability Participants

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1. Socioeconomic status		.07	.32	.37	.37	.48	.13	.08	.07	.15	.003	.05	.15	.52	.22
2. Processing speed	.30		.58*	.55	.66*	.39	.61*	.39	.20	.71*	-.24	.53	.10	.34	.33
3. Verbal intelligence	.45	.84*		.81*	.81*	.53	.57*	.52	.43	.68*	-.15	.53	.13	.42	.47
4. Non-verbal intelligence	.48	.75*	.87*		.76*	.66*	.53	.53	.36	.58*	-.19	.37	.23	.48	.53
5 Arithmetic	.48	.86*	.93*	.86*		.59*	.56*	.52	.17	.72*	-.20	.52	.26	.37	.42
6. Visual updating	.56*	.69*	.76*	.79*	.79*		.41	.51	.03	.42	-.01	.38*	.19	.63*	.63*
7. Auditory updating	.30	.77*	.75*	.70*	.75*	.63*		.38	.52	.55	-.19	.41	.04	.28	.58*
8. Visuospatial sketchpad	.27	.67*	.74*	.71*	.74*	.70*	.61*		.26	.57*	.04	.28	.24	.39	.38
9. Phonological loop	.28	.63*	.76*	.64*	.63*	.47	.71*	.58*		.32	-.34	.26	-.06	-.01	.40
10. Visual inhibition	.35	.88*	.87*	.77*	.88*	.70*	.74*	.76*	.68*		-.24	.46	.29	.34	.20
11. Auditory inhibition	.04	-.07	.01	-.07	-.04	.07	-.08	.10	-.16	-.07		-.15	-.25	.01	-.14
12. Visual switching	.26	.77*	.78*	.64*	.77*	.65*	.65*	.58*	.62*	.74*	-.03		.13	.25	.28
13. Auditory switching	.30	.46	.50	.48	.55*	.46	.34	.49	.34	.56*	-.15	.44		.44	.06
14. Visual ANS	.59*	.64*	.69*	.68*	.66*	.78*	.54	.62*	.43	.64*	.08	.57*	.62*		.47
15. Auditory ANS	.38	.68*	.75*	.73*	.72*	.79*	.75*	.64*	.69*	.61*	-.03	.61*	.39	.69*	
16. Age	.34	.77*	.82*	.62*	.78*	.64*	.59*	.62*	.71*	.75*	.11	.68*	.53	.61*	.68*

Note: ANS = number acuity. Above diagonal are age corrected partial correlations; below diagonal are zero correlations

*$p < .003$ (Bonferroni corrected for multiple comparisons).

The correlation tables indicate that relationships between constructs are as expected, as is the reduction in the degree of covariance when age is controlled for.

As there were two typically developing groups, a series of *t*-tests were conducted to determine if significant differences existed between these groups on any of the variables under investigation. As can be seen from Table 9.7 this was not the case, hence my assumption that they are drawn from one population is a reasonable one.

Table 9.7

T-tests Comparing Predictor Means for each Typically Developing Population

	TDTS/TDMLD	*d*
Age	$t(66.1) = 0.98$, $p = .330$	*.23*
Verbal IQ	$t(68.2) = 1.04$, $p = .302$	*.24*
Non-verbal IQ	$t(69.4) = 1.17$, $p = .248$	*.26*
Processing speed	$t(69.5) = 0.66$, $p = .512$	*.13*
Arithmetic	$t(67.2) = 0.92$, $p = .362$	*.22*
Visual updating	$t(70.0) = 1.52$, $p = .133$	*.05*
Auditory updating	$t(68.8) = 0.67$, $p = .508$	*.15*
Visuospatial sketchpad	$t(67.9) = 1.09$, $p = .278$	*.25*
Phonological loop	$t(67.4) = -0.95$, $p = .344$	*.22*
Visual inhibition	$t(68.55) = 1.02$, $p = .313$	*.23*
Auditory inhibition	$t(61.1) = -0.44$, $p = .659$	*.12*
Visual switching	$t(68.0) = 0.93$, $p = .357$	*.21*
Auditory switching	$t(70.2) = -0.28$, $p = .782$	*.06*
Visual numerical acuity	$t(68.8) = 0.90$, $p = .373$	*.18*
Auditory numerical acuity	$t(71.7) = .04$, $p = .972$	*.01*

Note: TDTS = typically developing group matched with Turner syndrome participants; TDMLD = typically developing group matched with maths learning disability group participants.

9.2 Planned Analyses

Research into neurodevelopmental disabilities often looks to identify differential patterns of development in relation to typically developing control groups (Bruns et al., 2019). While many such studies match a disorder group with two typically developing populations, one matched for chronological age and the other mental age, there are limitations with this approach (see Section 4.2.2 for discussion of these limitations). A developmental trajectories approach looks to overcome these limitations (Thomas et al., 2009), by constructing a function linking performance with age on a specific experimental task, and then determining differences between typically developing and disorder groups.

Analysis for this phase follows the method advocated by Thomas et al. (2009), who showed that traditional group matching is unable to distinguish between differences in onset and rate of development, nor does it allow for different indicators of mental age to be utilised. Thus, a developmental trajectories approach provides a richer taxonomy to describe how developmental pathways can be impaired or different from typical development (Thomas et al., 2009), and has been used in various studies (e.g., Annaz, Karmiloff-Smith, Johnson, & Thomas, 2009; Bruns et al., 2019; Lei et al., 2011). It should be noted that in this study, congruent with others utilising this methodology, the term development refers to correlations with indicators for (mental) age, as data is cross-sectional and not longitudinal (e.g., Bruns et al., 2019).

The analysis of developmental trajectories follows a series of steps of increasing complexity, with results depicted in scatterplots with regression lines and confidence intervals. Regression analyses are carried out to test whether effects are statistically significant (Thomas et al., 2009). In this study the analyses contained three types of predictors: a) three continuous predictors indexing different aspects of development or maturation, which will be referred to as developmental indicators; chronological age, verbal mental age and non-verbal mental age (indexed by verbal and non-verbal raw scores from KBIT-II), b) one between-subjects factor to determine differences between populations; c) ten within-subject predictors which were the scores used to index each cognitive ability under investigation (visual updating, auditory updating, visuospatial sketchpad, phonological loop, visual inhibition, auditory inhibition, visual switching, auditory switching, visual numerical acuity, and auditory numerical acuity).

For between-group comparisons (each disorder group and its typically developing control), age is rescaled to count in months from the youngest age measured in the disorder group when constructing the trajectories. This ensures that group differences are evaluated at the onset of development (the beginning of the trajectory). Effects and interactions of the covariant indicate whether this difference changes with age (Annaz et al., 2009).

When constructing developmental trajectories, the first step is to fit a regression model, in this study using arithmetic ability as the dependent variable, and one of the developmental indicators, as the predictor. This facilitates an investigation into whether any developmental indicator (chronological age, verbal or nonverbal mental age) reliably predicts performance on the arithmetic task. In the second step, the group factor is added to compare groups via their intercept and slopes, which are estimated separately for each group. The main effect of group indicates whether a difference exists in the intercept between groups, whilst the Group x Developmental Indicator interaction highlights whether the rate of development between groups differs. In this study comparisons will be between each disorder group and their matched typically developing group.

In step two, this procedure is repeated for each cognitive predictor of arithmetic ability, to determine which predictors display atypicality, either at onset or in their development. Having identified the cognitive abilities showing atypical development, step three of the analysis utilises a mixed design ANCOVA. In this study, there will be two within-participant factors, Task (arithmetic and an identified atypical cognitive ability), two between-participant factors, Group (the disorder group under investigation and its matched typically developing group) and one covariate, Age (chronological age, verbal mental age or non-verbal mental age).

The output from this analysis will highlight if there is:

a) a main effect of Group, or delayed onset,
b) a main effect of Age,
c) a Task x Group interaction, or whether there are group specific differences in accuracy on each task
d) a Group x Age interaction, or differences in the rate of development,
e) a 3-way interaction between Task x Group x Age, which highlights if groups show the same developmental relationship between the two tasks.

This analysis will allow constructs which display atypicality in each disorder to be identified, and additionally facilitate an understanding of how they manifest (at onset, across development, or differences in the way arithmetic and any given construct develop with time). Deficits found in constructs predictive of arithmetic ability in typical development (executive function and working memory, particularly updating), would suggest the cognitive underpinnings of arithmetic ability are congruent in each population. Predictors displaying atypicality which are predominantly presented in the visual domain, would suggest that visuospatial deficits may be an underlying cause of maths learning difficulties.

9.3 What is the Nature of Atypical Arithmetic Ability in Turner Syndrome and Maths Learning Disability?

The first stage of the analysis looked to identify the precise nature of deficits in arithmetic ability in each population; identifying if it was delayed at onset, across development or both.

Arithmetic ability was assessed using raw scores from the numerical operations subset of WIAT-II, second edition. For the full typically developing group, age accounted for 83% of the variance, $F(1, 139) = 306.20$, $p < .001$. Arithmetic raw scores were significantly lower for the Turner syndrome group than their matched typically developing group, $t(59.7) = -2.29$, $p = .025$, $d = .64$, a pattern repeated for the maths learning disability group; $t(61.6) = -3.73$, $p < .001$, $d = .68$. To understand the nature of these differences, developmental trajectories for the arithmetic ability of each population were produced (see Figure 9.1).

Onset and development for both typically developing groups were similar. Whilst there was no significant intercept effect for either disorder group, $p_{TS} = .905$; $p_{MLD} = .316$, there was a significant Group x Chronological Age effect, Turner Syndrome: $F(1, 60) = 4.77$, $p = .033$, $\eta^2 = .07$, and maths learning disability: $F(1, 76) = 7.89$, $p = .006$, $\eta^2 = .09$. Hence, whilst arithmetic ability for each disorder group was not delayed at onset, their rate of development was slower than typical development, with both developing at 0.64 of the rate of their respective matched typically developing group.

When verbal mental age was the developmental indicator, development for the Turner syndrome group was not significantly different from their matched typically developing group ($p = .243$), a pattern repeated for non-verbal mental age ($p = .874$).

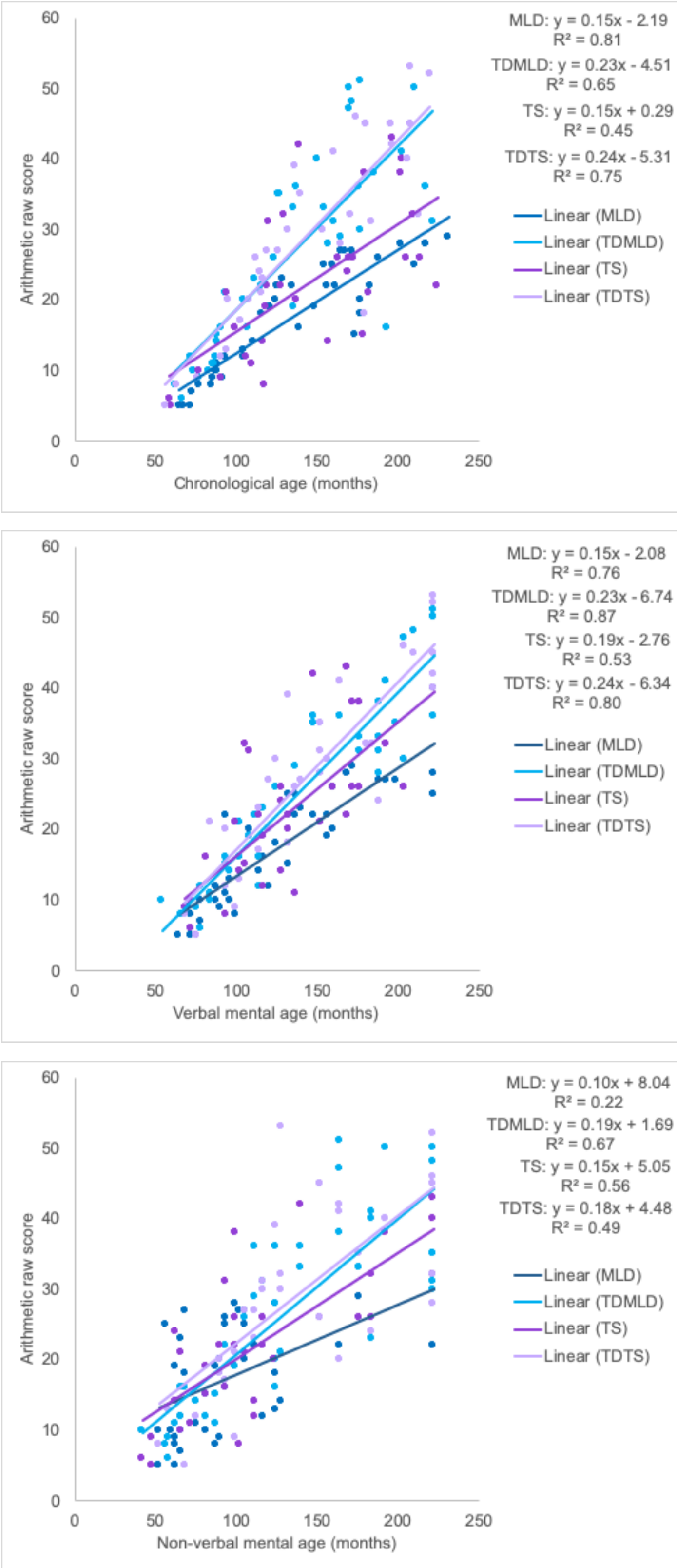


Figure 9.1. Developmental trajectories of arithmetic ability for each population, and each developmental indicator. TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability

For the maths learning disability group, using verbal and non-verbal mental age as the developmental indicator resulted in development continuing to be significantly different; verbal mental age: $F(1, 76) = 13.75$, $p < .001$, $\eta^2 = .15$; non-verbal mental age: $F(1, 76) = 5.77$, $p = .019$, $\eta^2 = .07$.

To determine if differences also existed in the development of the cognitive abilities highlighted as predictors of mathematical competence, developmental trajectories were constructed for each ability under investigation and each developmental indicator (chronological age, verbal and non-verbal mental age).

9.4 Which Cognitive Predictors Display Atypical Development in Turner Syndrome and Maths Learning Disability?

This analysis looked to determine if there were differential developmental patterns for each cognitive predictor (visual updating, auditory updating, visuospatial sketchpad, phonological loop, visual inhibition, auditory inhibition, visual switching, auditory switching, visual numerical acuity, and auditory numerical acuity). This enabled an investigation into whether the development of these abilities in the disorder groups had the same onset as the matched typically developing group, and if they subsequently develop at the same rate. Developmental trajectories were constructed for each developmental indicator, and each disorder group was compared to its matched typically developing group via the intercept (measured as the main group effect) and slope (measured as Group x Developmental Indicator interaction). Results for each cognitive predictor are now presented in turn, starting with visual updating.

9.4.1 Visual updating.

Developmental trajectories for visual updating can be seen in Figure 9.2. When chronological age was the developmental indicator, the Turner Syndrome group displayed a significant group effect; $F(1, 60) = 9.20$, $p = .004$, $\eta^2 = .13$, indicating a significantly lower intercept, however, the interaction of Group x Age was non-significant, $p = .718$. Hence the rate of development between the Turner syndrome group and its matched typically developing control was not significantly different for visual updating. Using verbal mental age as the developmental indicator resulted in the intercept remaining significantly different, $F(1, 60) = 12.36$, $p = .001$, $\eta^2 = .17$, a pattern repeated when non-verbal mental age was the developmental indicator, $F(1, 60) = 7.03$, $p = .010$, $\eta^2 = .11$. However, verbal mental age the rate of development was marginally significant, $F(3, 60) = 3.42$, $p = .069$, $\eta^2 = .05$.

The maths learning disability group showed non-significant effects of group ($p = .165$), and the interaction between Group x Age ($p = .678$), suggesting that onset and development were in line with their matched typical development group.

9.4.2 Auditory updating.

Developmental trajectories for auditory updating by developmental indicator can be seen in Figure 9.3. For each developmental indicator (chronological age, verbal mental age and non-verbal mental age) there were non-significant differences in the intercepts ($p_{\mathrm{TS}} = .098$; $p_{MLD} = .336$), and slopes ($p_{\mathrm{TS}} = .988$; $p_{MLD} = .454$), suggesting that auditory updating ability is similar at onset (in this case at 59 months) in both disorder groups and developing at a similar rate to their matched typically developing controls.

9.4.3 Visuospatial sketchpad.

Developmental trajectories for the visuospatial sketchpad can be seen in Figure 9.4. Onset for the Turner syndrome population was delayed, $F(1, 60) = 6.46$, $p = .014$ $\eta^2 = .10$, but development was in line with their matched typical developing group, $p = .691$. Using verbal mental age as the developmental indictor resulted in onset remaining atypical, $F(1, 60) = 6.39$, $p = .014$, $\eta^2 = .10$, however non-verbal mental age brought it in line with the matched typically developing group, $p = .218$.

For the maths learning disability population both onset, $p = .090$, and development, $p = .267$, were in line with their matched typically developing group.

9.4.4 The phonological loop.

The developmental trajectories for the phonological loop can be found in Figure 9.5. Both the intercept ($p = .540$) and slope ($p = .574$) for the Turner syndrome population were in line with their matched typically developing group. For the maths learning disability group, onset was marginally delayed, $F(1, 76) = 3.23$, $p = .076$, $\eta^2 = .04$, but development was in line with the matched typically developing group, $p = .611$. Using verbal mental age as the developmental indicator, resulted in onset moving in line with the matched typically developing control group, p = .297, a pattern repeated when non-verbal mental age is utilised, p = .688.

9.4.5 Visual inhibition.

Developmental trajectories for visual inhibition can be found in Figure 9.6. For both the Turner syndrome and maths learning disability groups, intercepts; p_{TS} = .165, p_{MLD} = .171, and slopes; p_{TS} = .478, p_{MLD} = .572, were not significantly different from their respective matched typically developing groups.

9.4.6 Auditory inhibition.

Developmental trajectories for auditory inhibition can be found in Figure 9.7. For both the Turner syndrome and maths learning disability groups, intercepts; p_{TS} = .687, p_{MLD} = .141, and slopes; p_{TS} = .837, p_{MLD} = .252, were not significantly different from their matched typically developing groups.

9.4.7 Visual switching

Developmental trajectories for visual switching can be found in Figure 9.8. For both the Turner syndrome and maths learning disability groups, intercepts; p_{TS} = .417, p_{MLD} = .616 and slopes; p_{TS} = .921, p_{MLD} = .639, were not significantly different from their respective matched typically developing groups.

9.4.8 Auditory switching.

Developmental trajectories for auditory switching can be found in Figure 9.9. For both the Turner syndrome and maths learning disability groups, intercepts; p_{TS} = .277, p_{MLD} = .183 were non-significantly different from their respective matched typically developing peers, however slopes; p_{TS} = .087, p_{MLD} = .075, were marginally different.

9.4.9 Visual numerical acuity

Developmental trajectories for visual numerical acuity can be seen in Figure 9.10. In the Turner syndrome population there was a significant difference in the intercepts, $F(1, 59)$ = 10.04, p = .002, η^2 = .15, however, differences in the slopes were non-significant, p = .199. Hence whilst onset was delayed, the Turner syndrome groups were developing at the same rate as their matched typically developing group. A significant difference in the intercept remained both when verbal mental age was the developmental indicator, $F(1, 59) = 7.33$, $p = .009$, $\eta^2 = .01$, and non-verbal mental age, $F(1, 59) = 6.52$, $p = .013$, $\eta^2 = .10$.

For the maths learning disability group, there were non-significant differences in both the intercept, $p = .342$, and slope, $p = .999$.

9.4.10 Auditory numerical acuity.

Developmental trajectories for auditory numerical acuity can be found in Figure 9.11. For the Turner syndrome group and their matched typically developing control, there was a significant difference in the intercepts, $F(1, 60) = 6.39$, $p = .014$, $\eta^2 = .10$, however, the difference between slopes was non-significant, $p = .111$. A significant difference in the intercept remained when verbal mental age was the developmental indicator, $F(1, 60) = 5.91$, $p = .018$, $\eta^2 = .09$, with the slope also becoming atypical, $F(1, 60) = 4.49$, $p = .038$, $\eta^2 = .07$. Using non-verbal mental age as the developmental indicator resulted in differences in the intercepts becoming marginally non-significant, $p = .061$.

For the maths learning disability group, there were non-significant differences in both the intercept, $p = .588$, and slope, $p = .835$.

Hence while the Turner syndrome and maths learning disability groups displayed a similar pattern of atypicality in arithmetic ability (atypical development), when compared to their matched typically developing groups, there were differences in the cognitive predictors which also displayed atypicality. Specifically, the Turner syndrome group displayed atypicality in; visual updating, the visuospatial sketchpad, visual numerical acuity and auditory numerical acuity, whilst marginal atypicality was only observed in the phonological loop and auditory switching for participants with maths learning disability.

Interestingly neither developmental disorder group displayed significant differences in the *rate* of development for the cognitive predictors, however, for the Turner syndrome group there were significant differences from their matched typically developing group in onset for; visual updating, visuospatial sketchpad, visual numerical acuity, and auditory numerical acuity. For the maths learning disability group there were marginal differences from the matched typical group in onset for the phonological loop and auditory switching.

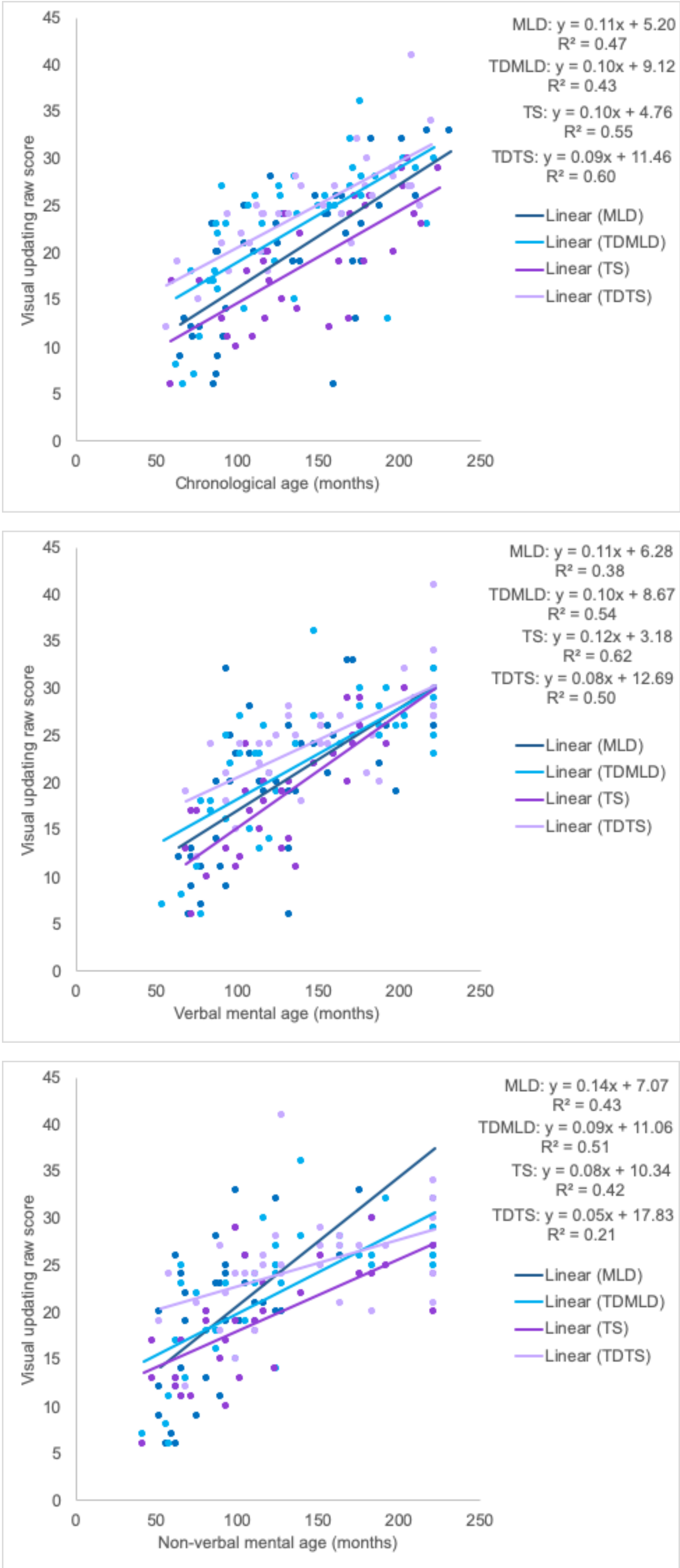


Figure 9.2. Developmental trajectories of visual updating for each population, and each developmental indicator. TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability participants.

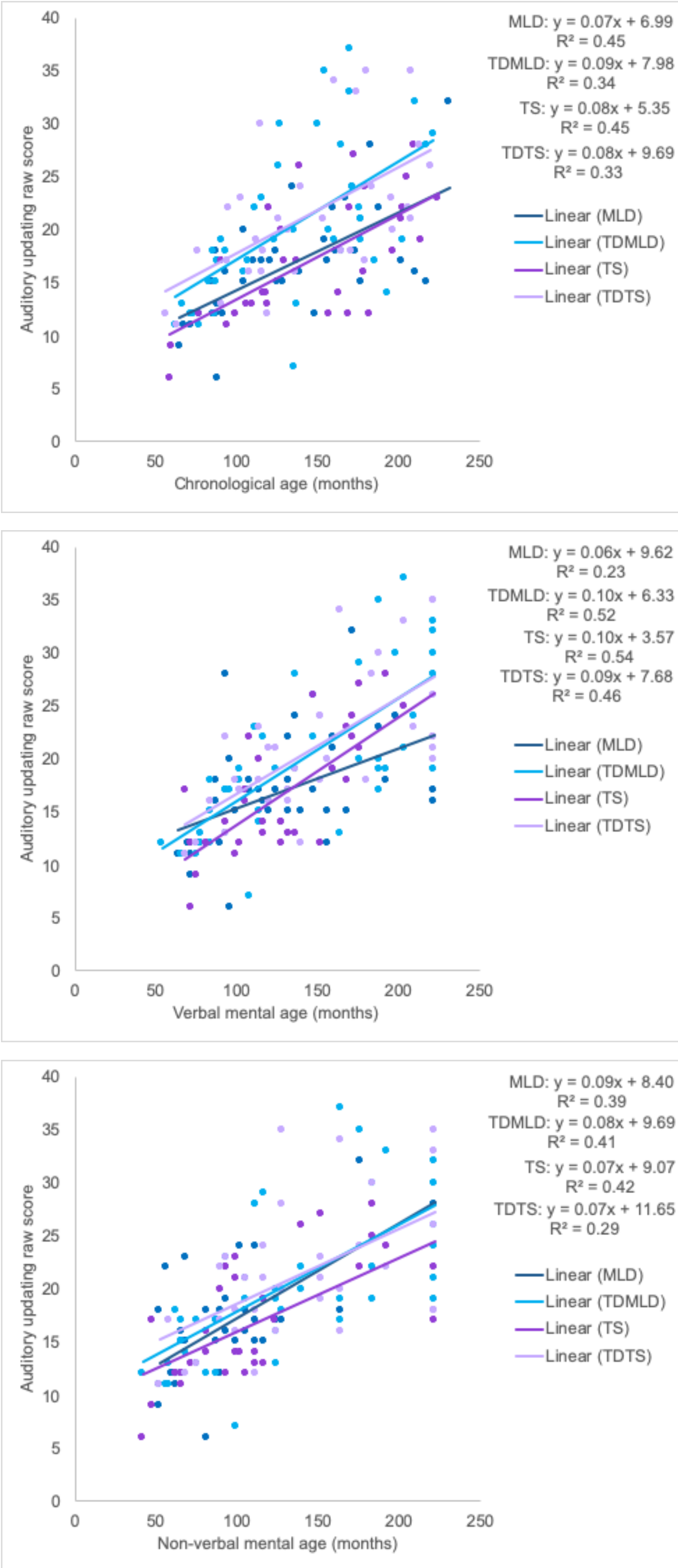


Figure 9.3. Developmental trajectories of auditory updating for each population, and each developmental indicator. TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability participants.

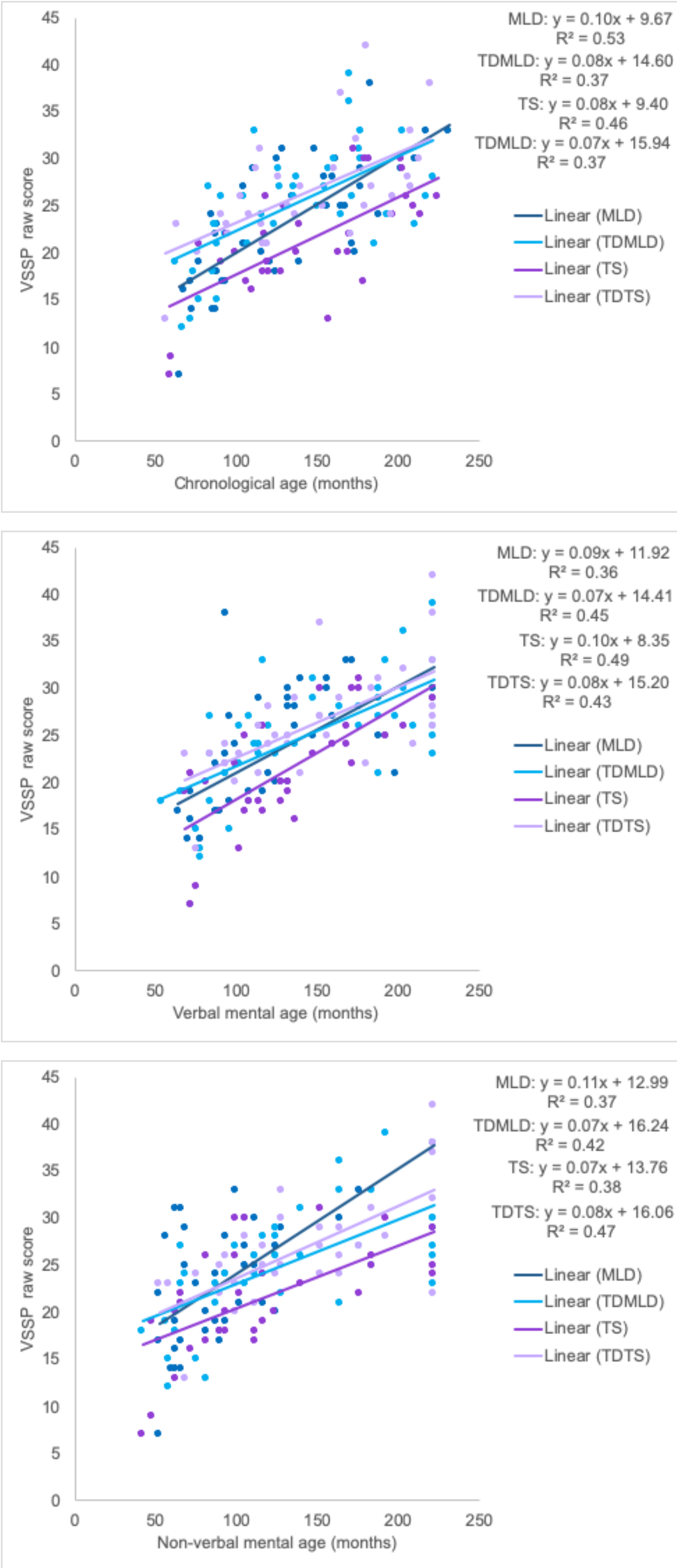


Figure 9.4. Developmental trajectories of visuospatial sketchpad for each population, and each developmental indicator. VSSP = visuospatial sketchpad; TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability participants.

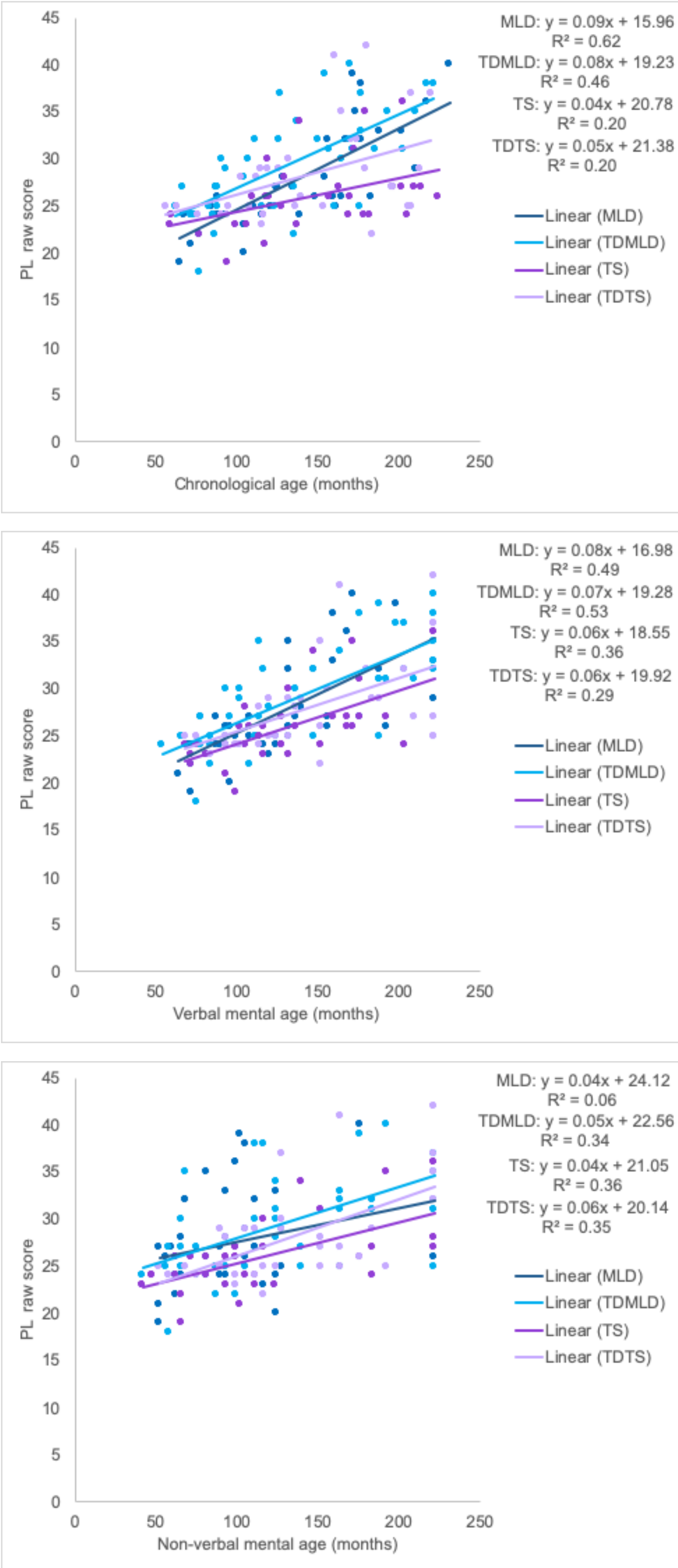


Figure 9.5. Developmental trajectories of the phonological loop for each population, and each developmental indicator. PL = phonological loop; TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability participants.

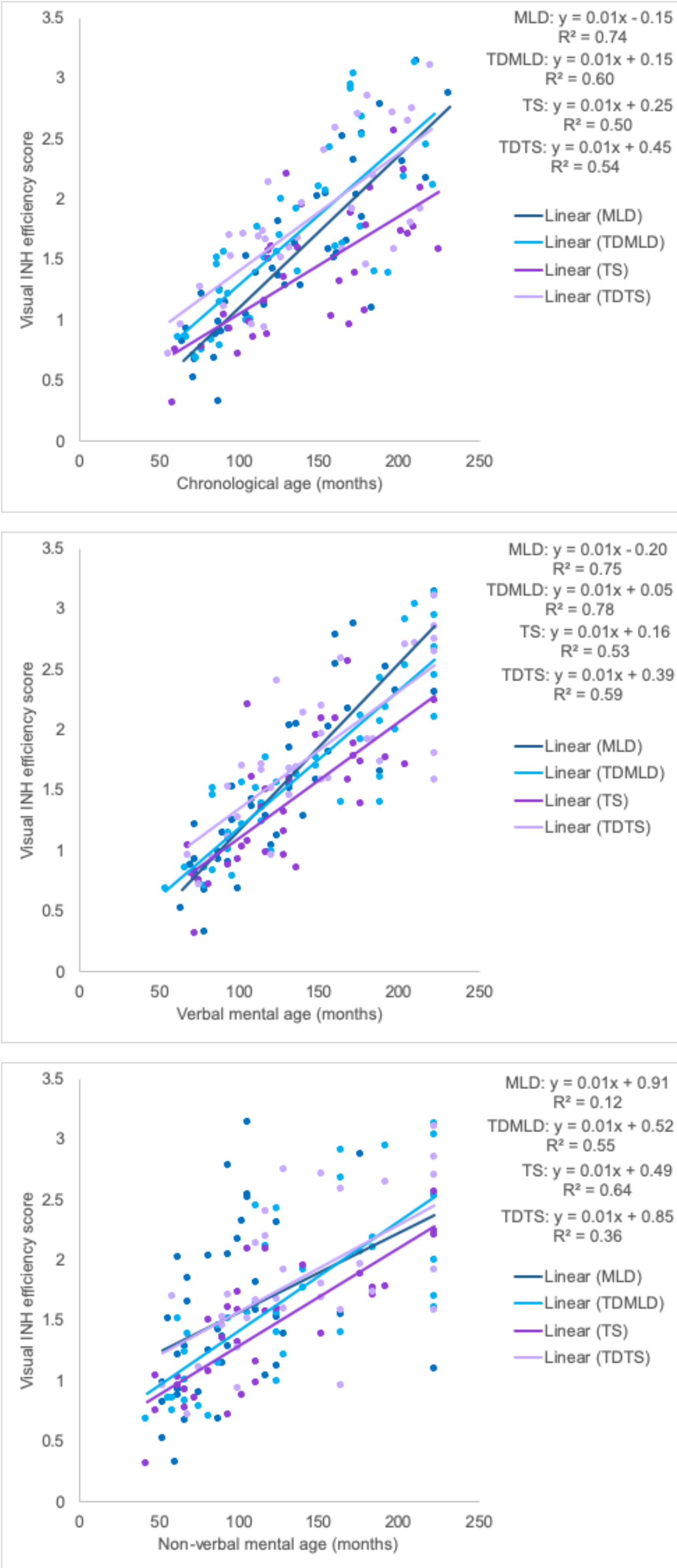


Figure 9.6. Developmental trajectories of visual inhibition for each population, and each developmental indicator. INH = inhibition; TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability participants.

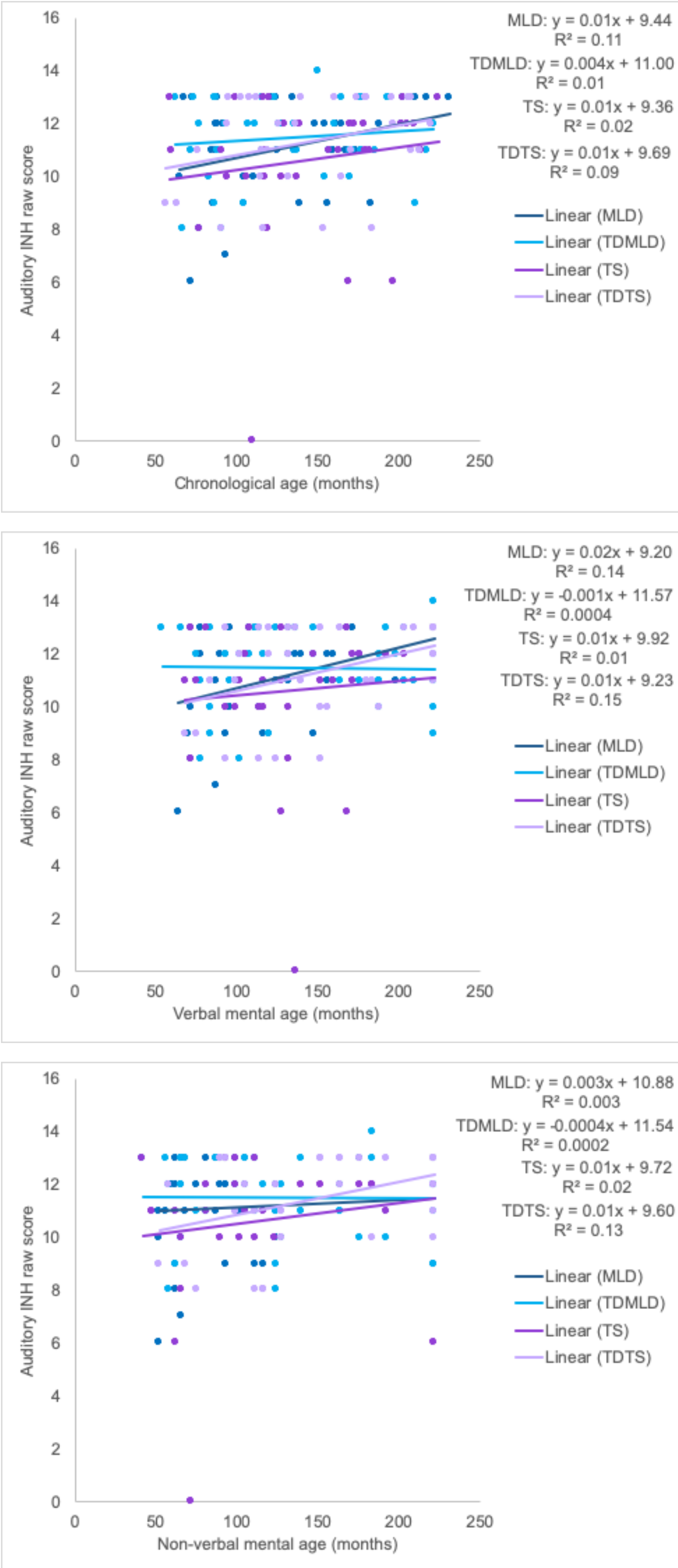


Figure 9.7. Developmental trajectories of auditory inhibition for each population, and each developmental indicator. INH = inhibition; TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability participants.

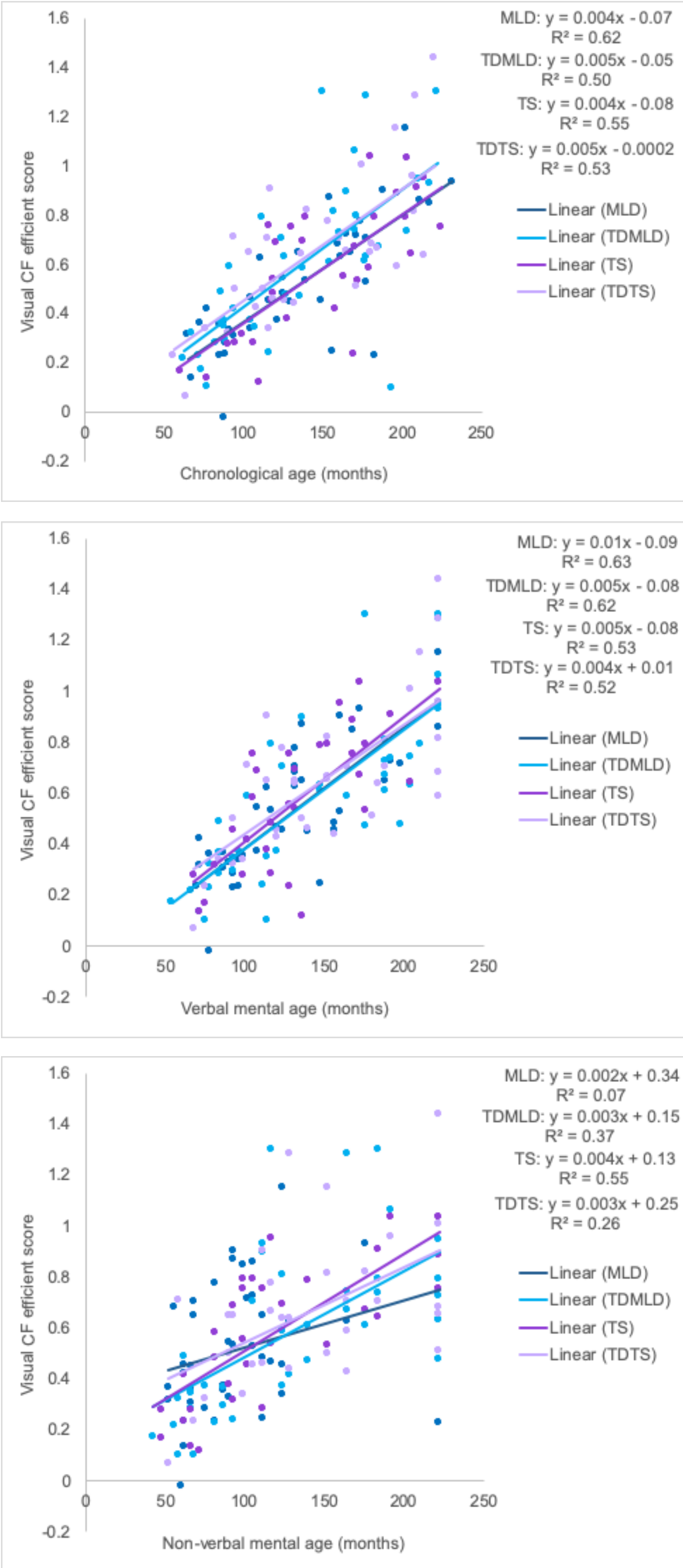


Figure 9.8. Developmental trajectories of visual switching for each population, and each developmental indicator. CF = switching; TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability participants.

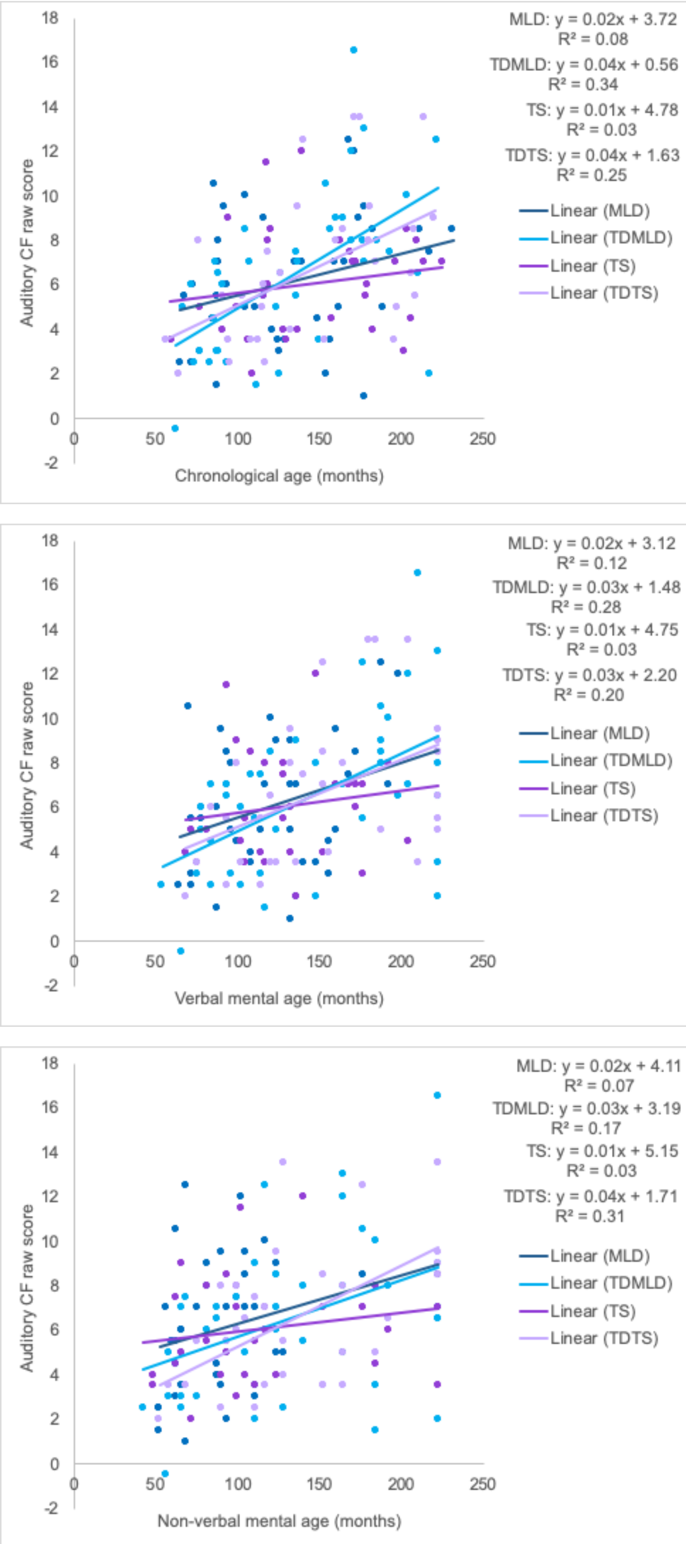


Figure 9.9. Developmental trajectories of auditory switching for each population, and each developmental indicator. CF = switching; TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability participants.

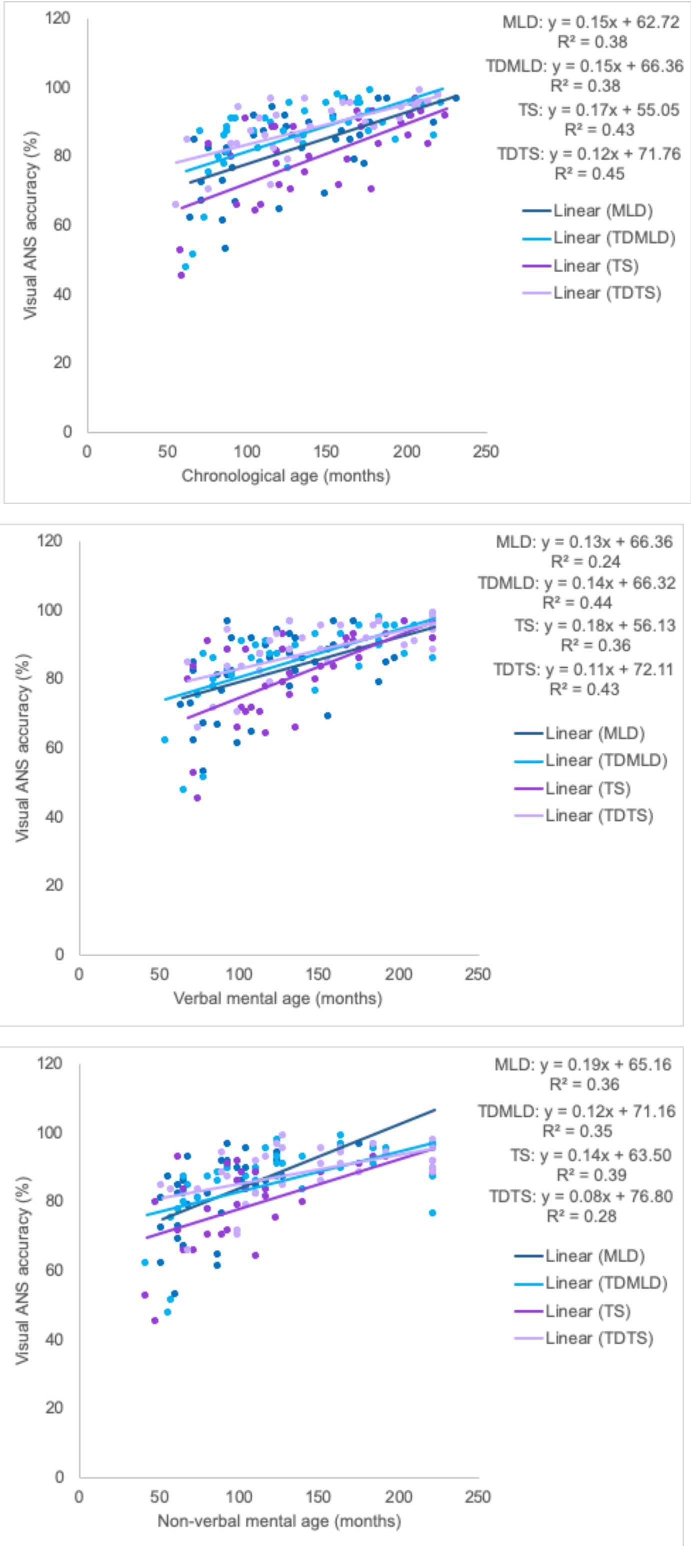


Figure 9.10. Developmental trajectories of visual numerical acuity for each population, and each developmental indicator. ANS = numerical acuity; TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability participants.

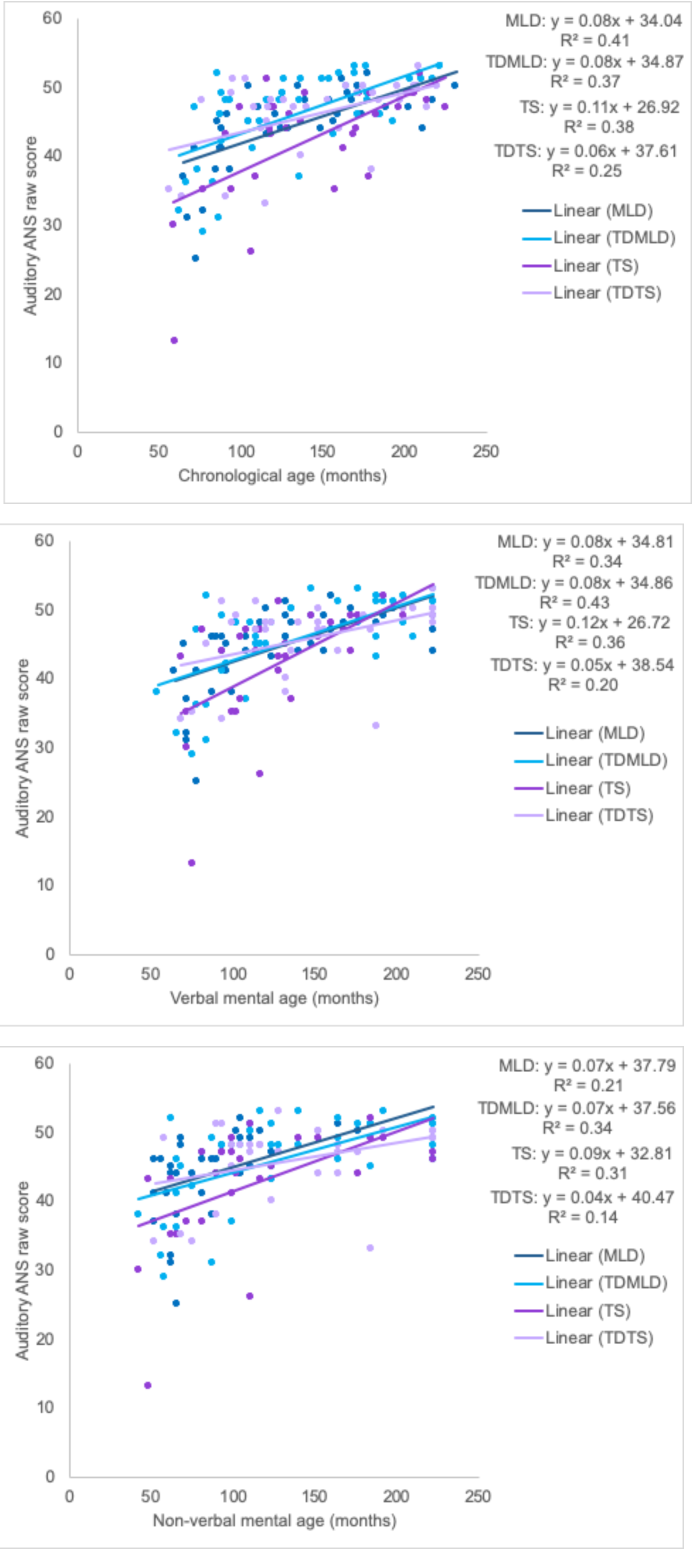


Figure 9.11. Developmental trajectories of auditory numerical acuity for each population, and each developmental indicator. ANS = numerical acuity; TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability participants.

To investigate whether the disorder group and their matched typically developing group show the same developmental relationship between arithmetic ability and the constructs which displayed atypical development, a mixed design ANCOVA was utilised. Group was the between-participants factor, Task the within-participants factor, and Age, the covariant.

9.5 Comparing Developmental Relationships between Arithmetic Ability and Cognitive Predictors

In this stage of the analysis the Turner Syndrome and maths learning disability groups were considered separately.

9.5.1 Turner syndrome.

The cognitive predictors highlighted in previous analyses as showing atypical development were: visual updating, the visuospatial sketchpad, visual numerical acuity, and auditory numerical acuity. In this stage of the analysis each was entered separately into a mixed design ANCOVA, alongside arithmetic, and the Turner syndrome group compared to its matched typically developing group. The analysis was conducted for each developmental indicator (chronological age, verbal mental age, and non-verbal mental age).

9.5.1.1 Arithmetic and visual updating.

The developmental trajectories for the Turner syndrome and their matched typically developing group for arithmetic and visual updating can be found in Figure 9.12.

The first analysis utilised chronological age as the developmental indicator. The main effect of group indicated the Turner syndrome groups exhibited no delay in onset in comparison to the matched typically developing group, $p = .257$, and the main effect of age highlighted chronological age as a strong predictor of performance, $F(1, 60) = 126.43$, $p < .001$, $\eta^2 = .68$. The Group x Age interaction indicated that development for the Turner syndrome groups occurred at the same rate as the matched typically developing group, $p = .129$, whilst the Task x Group interaction suggested the groups showed the same pattern of accuracy on each task, $p = .071$. The 3-way interaction, Task x Group x Age suggested the developmental relationship between arithmetic and visual updating was significantly different between the groups, $F(1, 60) = 6.41$, $p = .014$, $\eta^2 = .10$.

When verbal mental age was the developmental indicator, the developmental relationship between arithmetic and visual updating remained significantly different between the groups, with the pattern of accuracy also becoming significantly different. Non-verbal mental age as a developmental indicator did not result in significant differences in the developmental relationship between these abilities (see Table 9.8).

Table 9.8

Main and Interaction Effects for Arithmetic and Visual Updating using Verbal Mental Age and Non-verbal Mental Age as Developmental Indicators

	Verbal mental age	Non-verbal mental age
Effect of group	$F(1, 60) = 2.04$, $p = .158$, $\eta^2 = .03$	$F(1, 60) = 1.44$, $p = .235$, $\eta^2 = .02$
Effect of mental age	$F(1, 60) = 149.13$, $p < .001$, $\eta^2 = .71$	$F(1, 60) = 60.93$, $p < .001$, $\eta^2 = .50$
Group x Age interaction	$F(1, 60) = 0.01$, $p = .940$, $\eta^2 = .00$	$F(1, 60) = 0.001$, $p = .970$, $\eta^2 = .00$
Task x Group interaction	$F(1, 60) = 5.32$, $p = .025$, $\eta^2 = .08$	$F(1, 60) = 3.00$, $p = .089$, $\eta^2 = .05$
Task x Group X Age interaction	$F(1, 60) = 5.61$, $p = .021$, $\eta^2 = .09$	$F(1, 60) = 2.89$, $p = .094$, $\eta^2 = .05$

9.5.1.2 Arithmetic and the visuospatial sketchpad.

Developmental trajectories for arithmetic and the visuospatial sketchpad can be seen in Figure 9.13.

Utilising chronological age as the developmental indicator resulted in no main effect of group, $p = .296$, indicative of no difference in onset between the groups. The main effect of Age indicated chronological age was a strong predictor of performance, $F(1, 60) = 105.46$, $p < .001$, $\eta^2 = .64$, whilst the insignificant Group x Age interaction was suggestive of no differences in the rate of development, $p = .151$. The Task x Group interaction highlighted non-significant differences in the patterns of accuracy on the two tasks, p = .094, and the Task x Group X Age interaction was indicative of there being a different developmental relationship between arithmetic and the visuospatial sketchpad for the Turner syndrome and their matched typically developing population, $F(1, 60) = 6.25$, $p = .015$, $\eta^2 = .09$.

Results when verbal and non-verbal mental age were the developmental indictors can be seen in Table 9.9. Utilising verbal and non-verbal mental age as developmental indicators, resulted in all effects and interactions coming in line with typical development.

Table 9.9

Main and Interaction Effects for Arithmetic and the Visuospatial Sketchpad using Verbal Mental Age and Non-verbal Mental Age as Developmental Indicators

	Verbal mental age	Non-verbal mental age
Effect of group	$F(1, 60) = 1.22$, $p = .274$, $\eta^2 = .02$	$F(1, 60) = 0.40$, $p = .527$, $\eta^2 = .01$
Effect of mental age	$F(1, 60) = 140.40$, $p < .001$, $\eta^2 = .70$	$F(1, 60) = 81.07$, $p < .001$, $\eta^2 = .58$
Group x Age interaction	$F(1, 60) = 0.18$, $p = .676$, $\eta^2 = .003$	$F(1, 60) = 0.53$, $p = .470$, $\eta^2 = .01$
Task x Group interaction	$F(1, 60) = 3.05$, $p = .086$, $\eta^2 = .05$	$F(1, 60) = 0.27$, $p = .604$, $\eta^2 = .01$
Task x Group X Age interaction	$F(1, 60) = 2.44$, $p = .123$, $\eta^2 = .04$	$F(1, 60) = 0.26$, $p = .609$, $\eta^2 = .004$

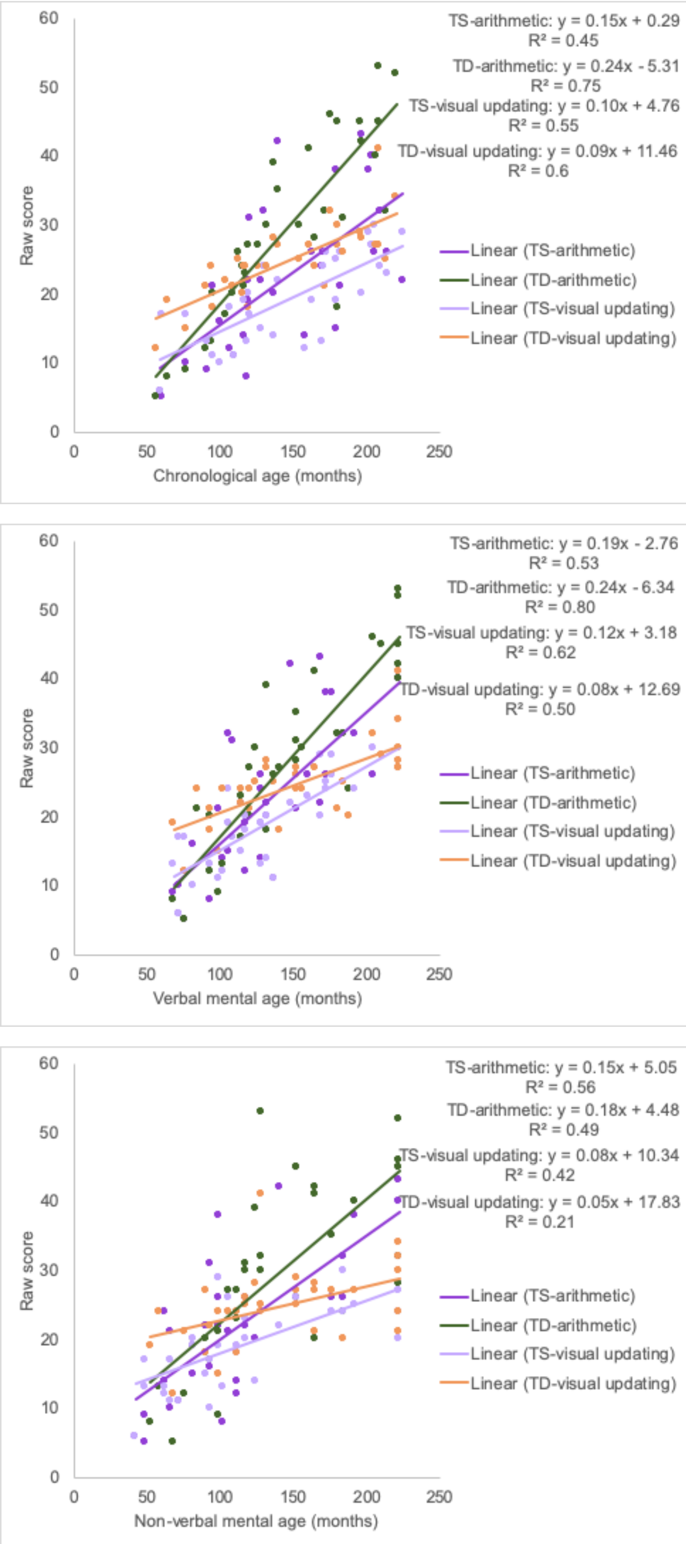


Figure 9.12. Developmental trajectories of arithmetic and visual updating, by developmental indicator. TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants.

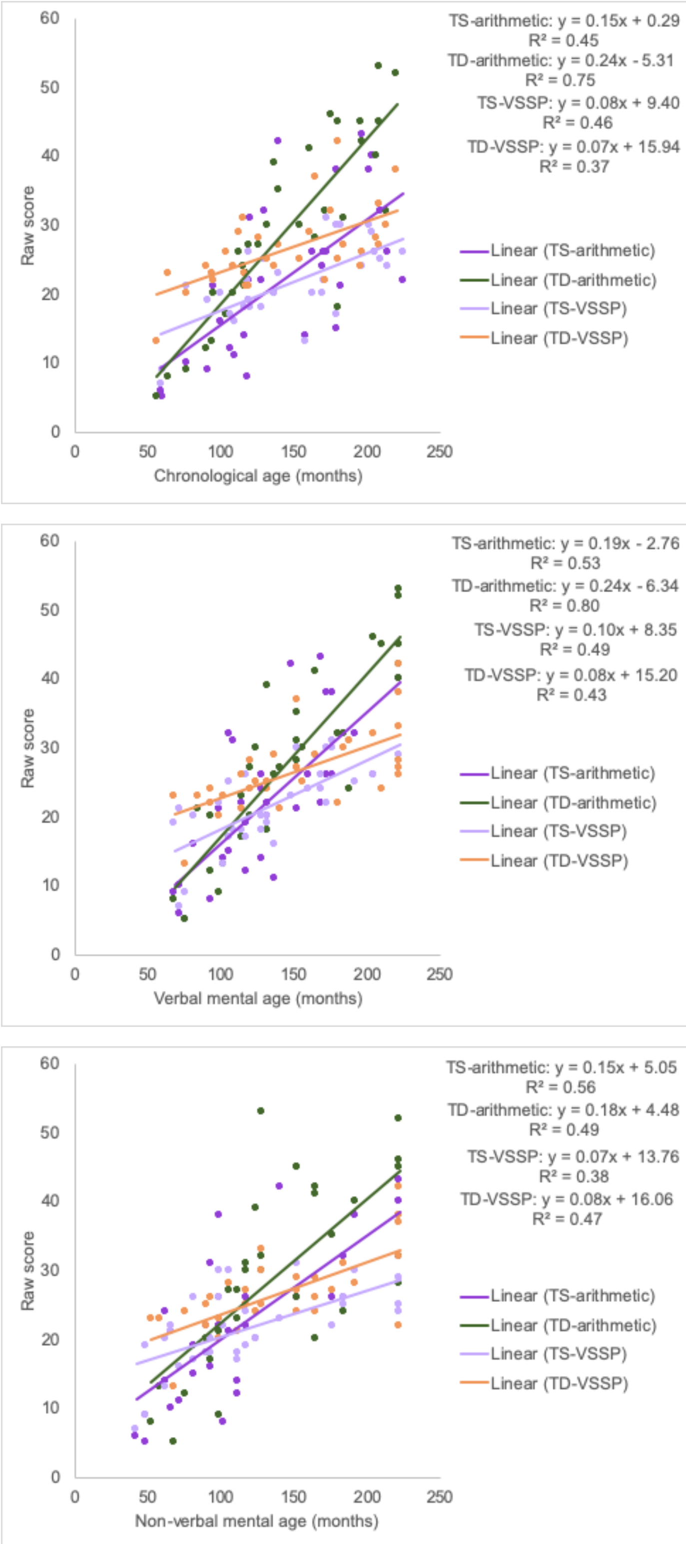


Figure 9.13. Developmental trajectories of arithmetic and the visuospatial sketchpad, by developmental indicator. VSSP = visuospatial sketchpad; TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants.

9.5.1.3 Arithmetic and visual numerical acuity.

Developmental trajectories for arithmetic and visual numerical acuity can be found in Figure 9.14. Utilising chronological age as the developmental indicator resulted in a main effect of group, $F(1, 59) = 4.07$, $p = .048$, $\eta^2 = .07$, hence there were differences in onset between the groups. The main effect of Age indicated chronological age was a strong predictor of performance, $F(1, 59) = 110.52$, $p < .001$, $\eta^2 = .65$, whilst the insignificant Group x Age interaction highlighted no differences in the rate of development, $p = .588$. The Task x Group interaction highlighted different patterns of accuracy on the two tasks, $F(1, 59) = 7.53$, $p = .008$, $\eta^2 = .11$, and the Task x Group X Age interaction was indicative of there being a different developmental relationship between arithmetic and the visual numerical acuity for the Turner syndrome and their matched typically developing population, $F(1, 59) = 8.00$, $p = .006$, $\eta^2 = .12$.

Using verbal mental age as the developmental indicator resulted in the developmental relationship between arithmetic and visual numerical acuity remaining significantly different, however this was not the case when non-verbal mental age was utilised. For both these developmental indicators, the pattern of accuracy on the two tasks remained significantly different (see Table 9.10).

Table 9.10

Main and Interaction Effects for Arithmetic and Visual Numerical Acuity using Verbal Mental Age and Non-Verbal Mental Age as Developmental Indicators

	Verbal mental age	Non-verbal mental age
Effect of group	$F(1, 59) = 3.29$, $p = .075$, $\eta^2 = .05$	$F(1, 59) = 2.69$, $p = .106$, $\eta^2 = .04$
Effect of mental age	$F(1, 59) = 111.22$, $p < .001$, $\eta^2 = .65$	$F(1, 59) = 69.14$, $p < .001$, $\eta^2 = .54$
Group x Age interaction	$F(1, 59) = 0.18$, $p = .677$, $\eta^2 = .003$	$F(1, 59) = 0.20$, $p = .656$, $\eta^2 = .003$
Task x Group interaction	$F(1, 59) = 6.17$, $p = .016$, $\eta^2 = .10$	$F(1, 59) = 4.51$, $p = .038$, $\eta^2 = .07$
Task x Group X Age interaction	$F(1, 59) = 4.77$, $p = .033$, $\eta^2 = .08$	$F(1, 59) = 3.50$, $p = .066$, $\eta^2 = .06$

9.5.1.4 Arithmetic and auditory numerical acuity.

Developmental trajectories for arithmetic and auditory numerical acuity can be seen in Figure 9.15. Utilising chronological age as the developmental indicator highlighted no main effect of group, $p = .209$, hence there were no significant differences in onset between the groups. The main effect of Age indicated chronological age was a strong predictor of performance, $F(1, 60) = 91.38$, $p < .001$, $\eta^2 = .60$. The insignificant Group x Age interaction suggested there were no differences in the rate of development between groups, $p = .533$, whilst the Task x Group interaction highlighted that the groups had different patterns of accuracy in the two tasks, $F(1, 60) = 4.04$, $p = .049$, $\eta^2 = .06$. The Task x Group X Age interaction was indicative of a different developmental relationship between arithmetic and auditory numerical acuity in the Turner syndrome and their matched typically developing population, $F(1, 60) = 10.95$, $p = .002$, $\eta^2 = .15$.

Using verbal mental age as the developmental indicator, resulted in the pattern of accuracy on the two tasks, and the developmental relationship between the two tasks, remaining significantly different. However, when the developmental indicator was non-verbal mental age, there were no significant differences on any of the indicators between arithmetic ability and auditory numerical acuity (see Table 9.11).

Table 9.11

Main and Interaction Effects for Arithmetic and Auditory Numerical Acuity using Verbal Mental Age and Non-verbal Mental Age as Developmental Indicators

	Verbal mental age	Non-verbal mental age
Effect of group	$F(1, 60) = 1.63$, $p = .206$, $\eta^2 = .03$	$F(1, 60) = 1.15$, $p = .288$, $\eta^2 = .02$
Effect of mental age	$F(1, 60) = 99.38$, $p < .001$, $\eta^2 = .62$	$F(1, 60) = 58.50$, $p < .001$, $\eta^2 = .49$
Group x Age interaction	$F(1, 60) = 0.19$, $p = .663$, $\eta^2 = .003$	$F(1, 60) = 0.08$, $p = .780$, $\eta^2 = .001$
Task x Group interaction	$F(1, 60) = 4.61$, $p = .036$, $\eta^2 = .07$	$F(1, 60) = 1.62$, $p = .208$, $\eta^2 = .03$
Task x Group X Age interaction	$F(1, 60) = 8.18$, $p = .006$, $\eta^2 = .12$	$F(1, 60) = 3.52$, $p = .065$, $\eta^2 = .06$

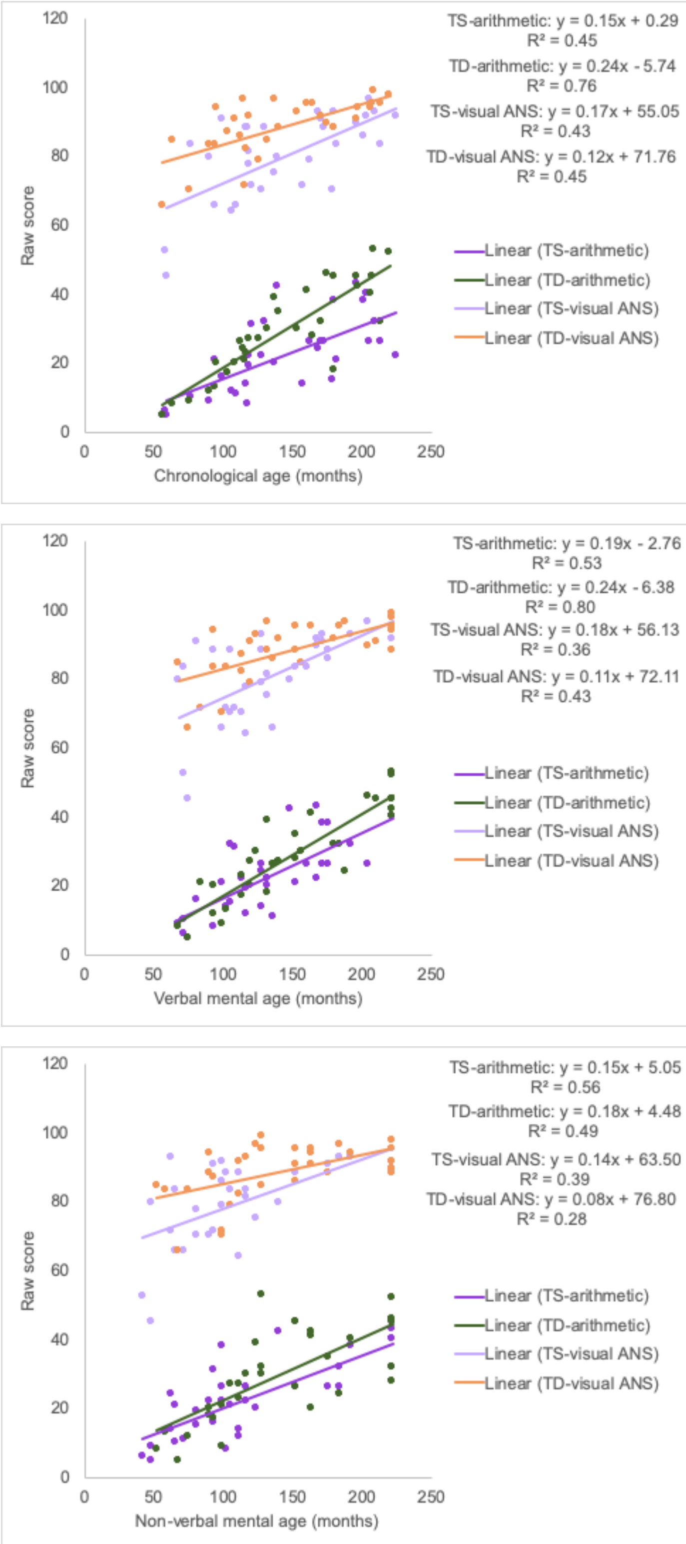


Figure 9.14. Developmental trajectories of arithmetic and visual numerical acuity, by developmental indicator. ANS = numerical acuity; TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants.

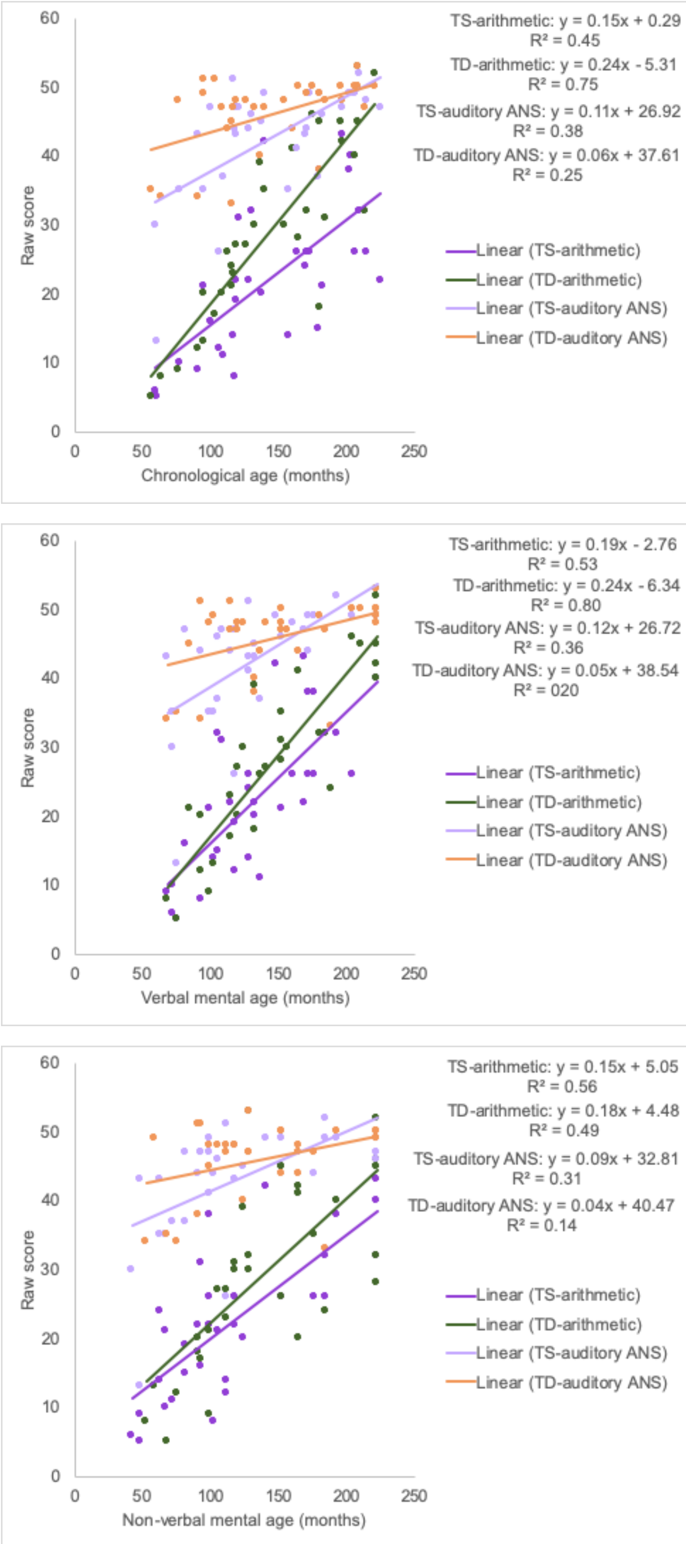


Figure 9.15. Developmental trajectories of arithmetic and auditory numerical acuity, by developmental indicator. ANS = numerical acuity; TS = Turner syndrome group; TDTS = typically developing group matched with Turner syndrome participants.

9.5.2 Maths learning disability.

The cognitive predictors to display atypicality for the maths learning disability group were the phonological loop and auditory switching.

9.5.2.1 Arithmetic and the phonological loop.

Developmental trajectories for arithmetic and the phonological loop can be seen in Figure 9.16. Utilising chronological age as the developmental indicator highlighted no main effect of group, p = .099, hence there were no significant differences in onset between the groups. The main effect of Age indicated chronological age was a strong predictor of performance, $F(1, 76) = 214.96$, $p < .001$, $\eta^2 = .74$. The significant Group x Age interaction suggested there were differences in the rate of development, $F(1, 76) = 4.19$, $p = .044$, $\eta^2 = .05$, and the Task x Group interaction highlighted no significant differences in the patterns of accuracy in the two tasks, p = .975. The Task x Group X Age interaction was indicative of a different developmental relationship between arithmetic and the phonological loop for the maths learning disability and typically developing populations, $F(1, 76) = 8.27$, $p = .005$, $\eta^2 = .10$.

Using verbal mental age as the developmental indicators, resulted in the developmental relationship between the two tasks remaining significantly different, as was the rate of development. This pattern was similar when the developmental indicator was non-verbal mental age (see Table 9.12).

Table 9.12

Main and Interaction Effects for Arithmetic and the Phonological Loop using Verbal Mental Age and Non-Verbal Mental Age as Developmental Indicators

	Verbal mental age	Non-verbal mental age
Effect of group	$F(1, 76) = 0.23$, $p = .636$, $\eta^2 = .003$	$F(1, 76) = 0.58$, $p = .448$, $\eta^2 = .01$
Effect of mental age	$F(1, 76) = 390.49$, $p < .001$, $\eta^2 = .84$	$F(1, 76) = 47.19$, $p < .001$, $\eta^2 = .38$
Group x Age interaction	$F(1, 76) = 5.77$, $p = .019$, $\eta^2 = .07$	$F(1, 76) = 4.07$, $p = .047$, $\eta^2 = .05$
Task x Group interaction	$F(1, 76) = 0.78$, $p = .380$, $\eta^2 = .01$	$F(1, 76) = 0.44$, $p = .509$, $\eta^2 = .01$
Task x Group x Age interaction	$F(1, 76) = 10.70$, $p = .002$, $\eta^2 = .12$	$F(1, 76) = 4.80$, $p = .032$, $\eta^2 = .06$

9.5.2.2 Arithmetic and auditory switching.

Developmental trajectories for arithmetic and auditory switching can be seen in Figure 9.17. Utilising chronological age as the developmental indicator highlighted no main effect of group, $p = .749$, hence there were no significant differences in onset between the groups. The main effect of Age indicated chronological age was a strong predictor of performance, $F(1, 75) = 154.08$, $p < .001$, $\eta^2 = .67$. The significant Group x Age interaction suggested there were differences in the rate of development, $F(1, 75) = 9.72$, $p = .003$, $\eta^2 = .12$, and the Task x Group interaction highlighted no significant differences in the patterns of accuracy in the two tasks, $p = .128$. The Task x Group X Age interaction was indicative of a marginally different developmental relationship between arithmetic and auditory switching for the maths learning disability and typically developing populations, $F(1, 75) = 3.39$, $p = .070$, $\eta^2 = .04$.

Using verbal mental age as the developmental indicators, resulted the developmental relationship between the two tasks, remaining significantly different, as was the rate of development. This pattern was similar when the developmental indicator was non-verbal mental age (see Table 9.13).

Table 9.13

Main and Interaction Effects for Arithmetic and Auditory Switching using Verbal Mental Age and Non-Verbal Mental Age as Developmental Indicators

	Verbal mental age	Non-verbal mental age
Effect of group	$F(1, 75) = 0.25$, $p = .616$, $\eta^2 = .003$	$F(1, 75) = 0.87$, $p = .355$, $\eta^2 = .01$
Effect of mental age	$F(1, 75) = 276.01$, $p < .001$, $\eta^2 = .79$	$F(1, 75) = 51.32$, $p < .001$, $\eta^2 = .41$
Group x Age interaction	$F(1, 75) = 8.74$, $p = .004$, $\eta^2 = .10$	$F(1, 75) = 4.70$, $p = .033$, $\eta^2 = .06$
Task x Group interaction	$F(1, 75) = 0.18$, $p = .671$, $\eta^2 = .002$	$F(1, 75) = 0.38$, $p = .538$, $\eta^2 = .01$
Task x Group x Age interaction	$F(1, 75) = 5.77$, $p = .019$, $\eta^2 = .07$	$F(1, 75) = 6.44$, $p = .013$, $\eta^2 = .08$

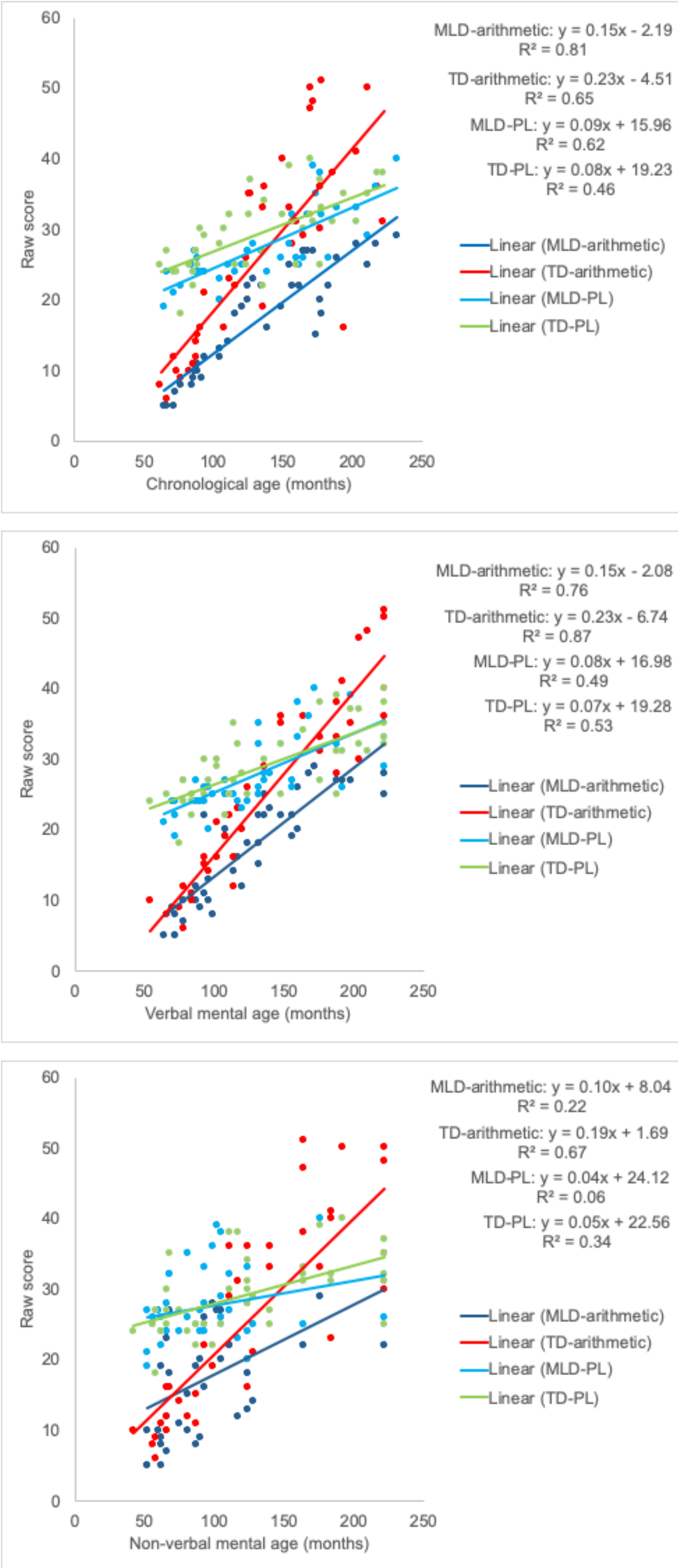


Figure 9.16. Developmental trajectories of arithmetic and the phonological loop, by developmental indicator. PL = phonological loop; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability participants.

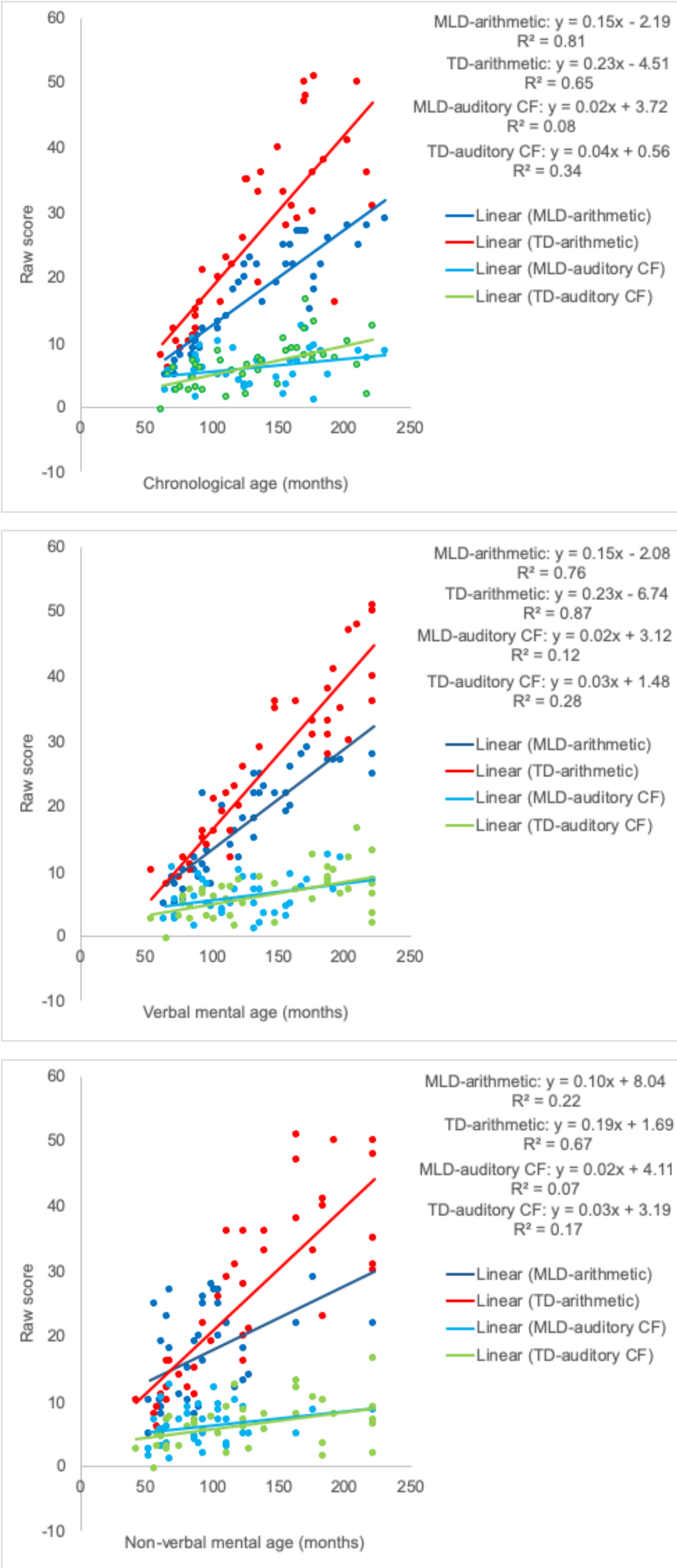


Figure 9.17. Developmental trajectories of arithmetic and auditory switching, by developmental indicator. CF = switching; MLD = maths learning disability group; TDMLD = typically developing group matched with maths learning disability participants.

9.6 Summary

The second phase of the data analysis looked to address research questions two and three:

2) Do executive function and working memory in both domains, but particularly visual and auditory updating, display atypical development in a) children with maths learning disabilities and b) children with Turner syndrome?
3) Are identified areas of deficits confined to either the visual or auditory domain?

Cognitive predictors for arithmetic ability were found to be different for each population. In typical development regression analysis highlighted visual and auditory updating as the predictors which explained the greatest amount of the variance of arithmetic ability. Both the Turner syndrome and maths learning disability groups displayed atypicality in arithmetic ability, specifically in the rate it developed across the school years, however, the cognitive predictors which also showed atypical development were incongruent in their presentation across the different disorder groups.

For the maths learning disability population the phonological loop and auditory switching were the predictors which displayed marginal atypicality, at onset. When the developmental relationship between arithmetic ability and each of these constructs was investigated there were significant differences between the maths learning disability groups and their matched typically developing peers, which remained when verbal and non-verbal mental age were utilised as the developmental indicator. As can be seen in Figure 9.16, this difference is most likely explained by differences in arithmetic development in the two populations rather than differences in development of the phonological loop.

In the Turner syndrome population four predictors displayed atypicality, once again at onset. Of the four, three were visual predictors; updating, the visuospatial sketchpad and numerical acuity. The sole auditory predictor was numerical acuity. It is interesting that numerical acuity in both modalities are highlighted, as in typical development whilst they were independent predictors, they lost predictive power in the presence of other cognitive abilities.

Each of these predictors showed a significantly different developmental relationship in comparison to arithmetic, which remained when verbal mental age was utilised as the developmental indicator. However, when non-verbal mental age was utilised, all came in line with typical development, except the visuospatial sketchpad.

Hence, this study is suggestive of differences in the cognitive abilities which predict arithmetic ability, however there are inter-syndrome differences. Data from the Turner syndrome group is also suggestive of differences in this population may be influenced by visuospatial deficits. The interpretation of results will be discussed for fully in the next chapter.

Chapter 10: Discussion for Phase Two

The second phase of this study looked to determine whether two neurodevelopmental disorders typically associated with maths and visuospatial deficits showed differential development patterns in executive functioning and working memory; the constructs shown to explain a significant proportion of the variance in the typically developing population in phase one. Additionally, congruent with phase one a consideration was made of whether identified areas of deficit were modality specific. Therefore, phase two addressed the following research questions:

- Do executive function and working memory in both domains, but particularly visual and auditory updating, display atypical development in a) children with maths learning disabilities and b) children with Turner syndrome?
- Are identified areas of deficits confined to either the visual or auditory domain?

To facilitate a greater understanding of the nature of identified deficits, a developmental trajectory approach was utilised to analyse data. This approach, unlike traditional matching, enables differences to be distinguished at onset or in the rate of development (Thomas et al., 2009). Additionally, it allows for a number of developmental indicators to be utilised; in this study chronological age, verbal mental age, and non-verbal mental age.

The first part of this discussion will consider the development of arithmetic abilities in each population, before moving on to consider within and between population differences in the development of the cognitive abilities under investigation; executive function, working memory and numerical acuity.

10.1 How does Arithmetic Ability Develop in Turner Syndrome and Maths Learning Disability?

In line with other studies, development of arithmetic ability for both Turner syndrome and maths learning disability was found to be atypical in relation to chronological age (e.g., Baker & Reiss, 2016; Geary, 1993). Specifically, in each population significant differences were found in the rate of development (as determined by the slope of the regression line) rather than at onset (59 months in this study), as indexed by the intercept of the regression line. Further, the girls with Turner syndrome performed above the children with maths learning disability across the school years (see Figure 9.1); a difference that appears to be due to the Turner syndrome group displaying higher arithmetic ability at 59 months, as rates of development were similar.

In comparison to their matched typically developing peers, the arithmetic ability of the Turner syndrome groups was in line or even slightly better at 59 months, when children were in the first year of formal education, however they developed at a significantly slower rate (64% of the rate of the matched typically developing group). Hence differences between two groups increased across the school years. These findings are in line with Rovet (1993), who investigated the psychoeducational characteristics of girls with Turner syndrome. Amongst other results they found significant underachievement in arithmetic for this population. Additionally, assigning participants (both typically developing controls and Turner syndrome) to learning disability subgroups (reading disability, arithmetic disability, or reading and arithmetic disability) if they met specific criteria, resulted in girls with Turner syndrome being placed in the two with arithmetic deficits solely. Of those placed into the arithmetic disability subgroup, 11 of the 13 girls with Turner syndrome were 12-years or above, hence only two were younger than 12-years.

When verbal and non-verbal mental age were the developmental indicators the rate of development for the girls with Turner syndrome was non-significant (verbal mental age; 80%, non-verbal mental age; 84%). Hence it would appear that for this population, arithmetic ability is atypical for their chronological age, but is developing in line with verbal and non-verbal abilities. This is an interesting result as while visuospatial deficits have been implicated in the maths learning difficulties found in Turner syndrome, typically verbal abilities are considered an area of relative strength in this population (Murphy et al., 2010; Rovet, 1993; Temple & Carney, 1995). However, there is evidence to indicate that while general intelligence and receptive language skills for girls and women with Turner syndrome are in the normal range (Rovet & Netley, 1982), there may be selective deficits in verbal abilities, including verbal fluency, planning and switching (Waber, 1979). Importantly, verbal mental age was calculated using a composite score from two tasks whose focus was not solely on receptive language.

The maths learning disability group, lagged behind their matched typically developing peers at 59 months (although this was non-significant), and their rate of development was significantly slower (64%). Significant differences remained in the rate of development when verbal and non-verbal mental age were utilised as developmental indicators (verbal mental age; 67%; non-verbal mental age; 52%). Hence it would appear that arithmetic ability in this population is atypical and not related specifically to either verbal or non-verbal abilities. This finding was unexpected as research in this field has identified particular subgroups of maths learning disability, including weak verbal and visuospatial abilities (e.g., Geary et al., 2004; Mammarella et al., 2018). However, a similar result was reported for children with milder maths learning disability (11^{th} – 25^{th} percentile), and in this study 67.5% of the maths learning disability group matched these criteria (Geary et al., 2007; Murphy et al., 2007). However, while much research in this field has looked to identify particular subgroups of maths learning disability, including weak verbal or visuospatial abilities, this finding appears to support a multidimensional parametric approach rather than the creation of arbitrary subgroups (Szucs, 2016).

Having established that girls with Turner syndrome and children with maths learning disability display atypical arithmetic ability there follows a discussion of the cognitive predictors also found to display atypicality, beginning with the Turner syndrome group.

10.2 The Cognitive Predictors showing Deficits for Turner Syndrome

Four cognitive predictors of arithmetic ability displayed atypicality in the Turner syndrome group, specifically in terms of delay at 59 months (the earliest age of participant's in this study); visual updating, the visuospatial sketchpad, visual numerical acuity, and auditory numerical acuity. In the typically developing population all were independent predictors when placed into a regression analysis, which remained when age was accounted for (see Section 7.3.1). However, only visual updating explained significant additional variance when all the other cognitive abilities were accounted for (i.e., bimodal measures of executive function, working memory, numerical acuity and intelligence). There follows a discussion of the findings for each of these abilities, followed by a consideration of predictors found to be marginally significant.

10.2.1 Visual updating as an area of deficit.

Utilising chronological age as the developmental indicator highlighted atypicality in the visual updating abilities of the girls with Turner syndrome at 59 months, or the first year of schooling. Specifically, this group was 68 months behind their typically developing peers. As the rate of development was similar for each group, the visual updating abilities of the Turner syndrome group did not catch up with their typically developing peers within the age range of this study (maximum age; 18.3-years). While previous research has highlighted atypicality in the visual updating abilities of this population, this study extends this by highlighting performance at onset, rather than the rate of development, as the source of atypicality (Bray et al., 2011; Haberecht et al., 2001).

When verbal mental age was the developmental indicator, atypicality remained at 59 months, with differences between Turner syndrome and their typically developing peers increasing to 84 months. Interestingly, whilst non-significant differences were found in the rate of development for each group, it was marginal ($p = .069$), with the visual updating abilities of girls with Turner syndrome, developing more quickly than their matched typically developing peers (152%). This resulted in similar performance by approximately 18-years.

Utilising non-verbal mental age as the developmental indicator, resulted in the girls with Turner syndrome lagging 128 months behind their matched typically developing peers, a significant difference. Although differences in the rate of development did not reach significance, the visual updating abilities of the Turner syndrome group developed at a faster rate, although they did not catch up with their matched typically developing peers within this age range.

In summary, the visual updating abilities of the girls with Turner syndrome in this study were atypical, specifically at 59 months, and this atypicality was not related to either verbal or non-verbal abilities.

When the developmental relationship between visual updating and arithmetic was considered for each group (the Task x Group x Age interaction), it was significantly different ($p = .014$). Whilst at the start of schooling arithmetic abilities were similar, that was not the case for visual updating abilities. As the updating abilities of both groups were developing at a similar rate across the school years, the girls with Turner syndrome never reached the level of the matched typically developing children. Across the same time span the arithmetic abilities of the girls with Turner syndrome were developing at a much slower rate, and hence they fell further and further behind.

A possible interpretation of this result is that while the visual updating abilities of girls with Turner syndrome develop at a similar rate to their matched typically developing peers, they are always lower. Therefore, as arithmetic complexity increases, visual updating deficits may have an increasing impact on performance. An alternative interpretation is that deficits (some of which may be small) in a number of the cognitive systems underpinning arithmetic competence may interact across development, thereby impacting arithmetic competence to a greater extent than any individual deficit. Hence the impact on arithmetic ability may increase across development. It is also possible that cognitive deficits interact with other factors, such as maths anxiety, to reduce arithmetic prowess over time (e.g., Iuculano, 2016; Szucs, 2016).

However, there were differences in the developmental relationship between visual updating and arithmetic when verbal and non-verbal abilities were utilised as developmental indicators. While non-verbal mental age caused differences in the developmental relationship to become non-significant, this was not the case when verbal mental age was utilised. It would therefore appear that the non-verbal abilities of the Turner syndrome group were impacting these cognitive abilities.

10.2.2 The visuospatial sketchpad as an area of deficit.

When chronological age was the developmental indicator, atypicality was found in the visuospatial sketchpad of the girls with Turner syndrome. Once again this was at the earliest age (59 months) where they were 81 months behind matched typically developing peers. As the rate of development for each group was similar, the Turner syndrome group were still behind their matched typically developing peers at the end of formal education.

Significant differences at 59 months remained when verbal mental age was the developmental indicator (69 months behind). However, the rate of development of the visuospatial sketchpad was faster for the Turner syndrome group (1.32), although this did not reach significance. Conversely, differences at 59 months were non-significant when non-verbal mental age was the developmental indicator (Turner syndrome were 35 months behind), with the rate of development being congruent with verbal mental age. This pattern of results is suggestive of the acuity of the visuospatial sketchpad for the Turner syndrome group being in line with their non-verbal abilities.

The developmental relationship between arithmetic ability and the visuospatial sketchpad for each group was significantly different. However, they became non-significant when either verbal or non-verbal mental age was utilised as the developmental indicator. Therefore, the relationship between arithmetic ability and the visuospatial sketchpad appears to be similar in the Turner syndrome and matched typically developing groups, given their verbal and particularly non-verbal abilities.

Whilst very few studies have examined the visuospatial abilities of girls with Turner syndrome, significantly lower performance has been found in this population when compared to typically developing controls (age range; 6- to 12-years). These significant differences remained when verbal and non-verbal intelligence were accounted for (Brankaer et al., 2017). The current study concurs to some extent with this finding, taking it further by highlighting specific deficits at the start of formal education. However, the atypicality in the visuospatial sketchpad of the girls with Turner syndrome, was in line with their non-verbal abilities, as were differences in the developmental relationship between arithmetic ability and visuospatial sketchpad abilities.

10.2.3 Visual numerical acuity as an area of deficit.

When chronological age was the developmental indicator, visual numerical acuity was significantly delayed at 59 months, being 115 months behind the matched typically developing group. While differences in the rate of development did not meet significance, the Turner syndrome group were developing faster (149%), although there were still differences between this population and their matched typically developing peers at the end of formal education.

Utilising verbal and non-verbal mental age as developmental indicators resulted in visual numerical acuity abilities remaining significantly delayed (verbal: 101 months; non-verbal; 169 months). The girls with Turner syndrome were developing at a faster rate in both (verbal: 168%; non-verbal: 173%), with performance for this population being in line with their matched typically developing peers before the end of the school years. However, while differences in the rate of development were substantial they did not reach significance. Although a power analysis indicates observed power of 0.80 for a sample size of 32, it is possible this result was impacted by the sample size, as it is unusual for such substantial differences in the rate of development to not reach significance (Faul, Erdfelder, Lang, & Buchner, 2007; Faul, Erdfelder, Buchner, & Lang, 2009; http://www.gpower.hhu.de/en.html). A possible explanation for this rapid rate of development could be that the rate of development of the visual numerical acuity abilities of girls with Turner syndrome is comparable to development of younger typical developing children.

The developmental relationship between arithmetic ability and visual numerical acuity was significantly different between the two groups, which remained when verbal mental age was the developmental indicator, but became non-significant when non-verbal mental age was utilised. Hence once again it would appear that the relationship between arithmetic ability and visual numerical acuity was impacted by the non-verbal abilities of the girls with Turner syndrome.

Visual numerical acuity being atypical in Turner syndrome is in line with limited previous research (Simon et al., 2008). However, while this study contained children of a similar age (7- to 14-years), some stimuli were in the subitizing range. Hence the current study gives a clearer picture of the visual numerical acuity abilities of girls with Turner syndrome, highlighting specific deficits at the start of formal schooling. Interestingly, one study which compared multiple constructs reported that significant group differences in symbolic numerical acuity disappeared when visuospatial sketchpad abilities were accounted for (Brankaer et al., 2017).

10.2.4 Auditory numerical acuity as an area of deficit.

The auditory numerical acuity abilities of the Turner syndrome group, when considered for chronological age, were once again significantly delayed at 59 months, this time by 133 months. Although the rate of development did not reach significance, the Turner syndrome group were developing at 187% of their typically developing peers, which meant they caught up with their typically developing peers by the end of formal education.

When verbal mental age was the developmental indicator the Turner syndrome group were 140 months behind their typically developing peers, a significant difference. Differences in the rate of development were also significantly different, with the auditory numerical acuity abilities of the Turner syndrome group developing at 245% of rate of their matched typically developing peers. Hence, this population caught and overtook the matched typically developing peers at about 13-years of age.

For non-verbal mental age, there were marginal differences at 59 months, (143 months behind). While the girls with Turner syndrome were developing at 214% of the rate of their typically developing peers, and overtook them (approximately 16-years), this was a non-significant difference.

The developmental relationship between arithmetic ability and auditory numerical acuity within the two groups were significantly different. This time it was verbal mental age that made this non-significant. Hence verbal abilities appear to impact the relationship between arithmetic ability and auditory numerical acuity.

10.2.5 Summary

Girls with Turner syndrome showed significant atypicality in; visual updating, the visuospatial sketchpad, visual numerical acuity, and auditory numerical acuity, in the early school years (i.e., at onset, 59 months). However, the rate of development in these areas was not significantly different from typically developing peers across the ages spanned by this sample. Of the four abilities displaying atypicality, three were in the visual domain. Visual updating and visual numerical acuity also displayed significant differences in the developmental relationship with arithmetic ability, which were normalised when non-verbal mental age was utilised as the developmental indicator.

Interestingly, despite a number of studies highlighting deficits in the phonological loop in Turner syndrome, this was not the case here, nor were there significant differences in auditory updating abilities (e.g., Lepage et al., 2011; Murphy & Mazzocco, 2008; Rovet et al., 1994). A recent meta-analysis suggested large effect size in studies which examined auditory working memory (Mauger et al., 2018). However, while the meta-analysis combined results for auditory updating and the phonological loop, in the current study they were analysed separately. Additionally, the meta-analysis was relatively small, containing 16 studies, nine of which contained measures of the phonological loop and one, a measure of auditory updating.

Switching abilities have been highlighted as a possible area of deficits in Turner syndrome, specifically verbal switching (Mauger et al., 2018). Findings from this study partially supported previous studies as visual switching was in line with typical development, whilst auditory switching displayed marginal atypicality in the rate of development (p = .087). Mauger et al. (2018) suggested that modality specific differences in the switching abilities of individuals with Turner syndrome, may actually be down to task requirements. Specifically, many verbal tasks measure spontaneous flexibility (as measured by tasks that require generation of diverse answers), whilst visual tasks measure reactive flexibility (where answers need to be adapted to examiners demands or to stimuli). Further research is needed to determine if inconsistent findings are the result of tasks measuring different aspects of switching.

Congruent with phase one of this study, these findings are suggestive of executive functioning and working memory abilities impacting arithmetic ability for girls with Turner syndrome. However, numerical acuity, which was only an independent predictor, also appears to play an important role. In terms of modality, while non-verbal abilities appear to impact the developmental relationship between arithmetic and visual cognitive abilities, it is verbal abilities in the auditory domain. These findings are suggestive of arithmetic ability in this population being related to both non-verbal *and* verbal abilities.

These findings are important, as identifying deficits in these cognitive abilities early in development (preferably before the start of formal schooling), would enable effective interventions to be put in place. As deficits were at onset (59 months), any improvement in the acuity of the defective ability will hopefully mean that the impact on arithmetic competence across formal education will be reduced. Clearly this is an area for future research.

10.3 The Cognitive Abilities showing Atypicality in Maths Learning Disability

Unlike the Turner syndrome group, only two constructs; the phonological loop and auditory switching, displayed atypicality for the maths learning disability group, both of which were marginal.

Differences between the phonological loop in the maths learning disability and their matched typically developing group were marginally significant at onset (p = .076). This atypicality was at 59 months, where the children with maths learning disability were delayed by 28 months. While differences in the rate of development did not reach significance, the maths learning disability group were developing at 74% of the rate of their matched typically developing peers. Onset and the rate of development were similar in both groups when verbal mental age was the developmental indicator, however, for non-verbal mental age the maths learning disability group developed at 72% of their typically developing peers, although this difference did not reach significance.

As can be seen in Figure 9.16, there were differences in the developmental relationship between arithmetic ability and the phonological loop. The phonological loop abilities for both groups were higher than arithmetic competence at 59 months. However, while the phonological loop was developing at a similar rate for both groups, this was not the case for arithmetic ability, where the maths learning disability group was developing at a slower rate. Hence, group differences in the developmental relationship between these tasks appears to be as a result of differences in the rate of arithmetic ability development rather than being linked specifically to the phonological loop. This may point to an as yet unidentified cognitive ability causing the deficit in arithmetic ability, or the phonological loop interacting with other cognitive abilities, or impacting other systems (e.g., language), to cause a reduction in arithmetic competence.

Auditory switching also displayed marginal atypicality in the rate of development (p = .075). At 59 months the auditory switching abilities of this group were above their typically developing peers, however as their rate of development was 42% of the rate of their matched typically developing peers, by about 8-years their auditory switching abilities were worse. Utilising verbal and non-verbal mental ages brought the rate of development closer to that of typical development (verbal: 71%; non-verbal: 87%).

Whilst these findings do not give a clear sense of the cognitive systems responsible for maths learning disability, they highlight the importance of the auditory domain. Although a subtype of maths learning disability associated with weak phonological loop and updating abilities has been forwarded, these findings highlight the importance of executive functioning, specifically auditory switching (e.g., Andersson & Lyxell, 2007; Geary et al., 2007; Murphy et al., 2007; Szucs, 2016).

10.4 Summary

These findings suggest that whilst children with Turner syndrome and maths learning disability possess similar deficits in arithmetic ability, the underlying deficits in cognitive abilities are incongruent. Additionally, deficits in the children with maths learning disability were solely in the auditory domain, whilst the majority of those for the girls with Turner syndrome were in the visual domain. For these abilities; visual updating, the visuospatial sketchpad, and visual numerical acuity, using non-verbal mental age as the developmental indicator brought the developmental relationship between arithmetic ability and each ability in line with typical development. While this is suggestive of non-verbal abilities impacting the development of these abilities it is not the whole story. Verbal mental age brought the developmental relationship in line with typical development for the visuospatial sketchpad and auditory numerical acuity. Therefore, while visuospatial deficits do appear to impact cognitive abilities, verbal deficits are also having an impact.

For the maths learning disability group a clear picture did not emerge, for although there were deficits in arithmetic ability, the only cognitive abilities to also show deficits were marginal. In term of modality, deficits did not appear to be the result of visuospatial deficits, rather they were within the auditory domain, however they did not appear to be impacted by verbal mental age. Results in this population may be down to sample characteristics, as 67.5% of the group could be classed as having moderate maths learning difficulties.

However, it may also be the case that impairment in arithmetic ability is not down to deficits in individual cognitive abilities. Indeed, it has been argued that specific maths weaknesses do not necessarily need specific modular impairment explanations (Szucs et al., 2014; Toll et al., 2016). This explanation would make sense as even a relatively simple area of maths such as arithmetic requires a number of different procedures to be carried out in a particular order and errors cannot be made at any point in the process (Szucs, 2016). It therefore makes sense that to solve even a relatively simple arithmetic problem such as two-digit column addition, a number of different cognitive systems would need to be recruited. Hence small weaknesses in any of these systems (which may not reach significance), may interact, and cascade across development, to produce maths learning difficulties.

Not only would this explanation tie in with the findings from phase one of this study, where two modality specific latent executive function and working memory explained a significant proportion of the variance in arithmetic ability; but the developmental trajectories produced for the cognitive abilities in phase two highlight differences in a number of systems, that did not reach significance.

Chapter 11: Conclusions

This study investigated the modality specific cognitive abilities predictive of arithmetic competence across the school years (i.e., 4- to 18-years). Its aim was to determine the cognitive abilities with the greatest impact on arithmetic ability and whether they were impacted by the modality of stimuli presentation. Hence participants were drawn not only from the general population but from two neurodevelopmental syndromes, with acknowledged visuospatial and maths difficulties. Before conclusions from the study are discussed, strengths and limitations of the study will be considered.

11.1 Strengths and Limitations

To my knowledge, for the first time this study examined concurrently the dominant cognitive predictors of maths ability in two modalities; visual and auditory. This facilitated a greater understanding of the precise links to arithmetic competence, rather than a generic link with working memory, or switching, when the actual link was from auditory updating or visual switching. Additionally, the study looked across the school years, hence differences found across this age range were not the result of different tasks measuring different constructs. Whilst this wide age range was a strength, it also created a potential limitation. However, age was corrected for in regression analyses and it was included in the final structural equation model.

Having so many predictors across such a wide age range, did create some limitations. Whilst all the children were able to access all tasks, finding measures to index auditory switching and auditory inhibition was challenging. For auditory switching an adapted version of the category fluency task was utilised. Whilst it had been used to index auditory switching in another study, there were no significant age-corrected correlations between this construct and any of the others, which is surprising. However, R^2 values in the developmental trajectories were medium to high and marginal deficits were found in this task within the maths learning disability population. It is also interesting that when adjusted values for this task were calculated that accounted for age and both verbal and non-verbal intelligence factor loadings were good in all structural equation models.

Finding a task to index auditory inhibition across this age-range was more difficult, and hence an adaptation of the colour association task was trialled in the pilot. Results indicated sufficient variance, however in the main study variance was limited and R^2 for the developmental trajectories of matched typically developing groups were small. However, once again when this data for this construct was adjusted to account for age and both intelligence predictors, factor loadings for each structural equation model were good. Despite the findings from the alternative analysis an area for future research may be to develop tasks to measure auditory inhibition, and possibly auditory switching, across a wide age range.

There is much debate of the most appropriate way to index numerical acuity, with stronger links between symbolic tasks and maths ability postulated. This study therefore included a large number of trials over two session, thereby enabling test-retest reliability to be measured. The actual task used was chosen as it was freely available, easy to use and adapt. However, a downside was that all suggested confounds, particularly visual parameters (e.g., convex hull) could not be controlled for.

Sample sizes in this study were an area of strength. The 182 participants in phase one enabled data to be analysed via regression and structural equation modelling. The numbers recruited into the disorder groups, particularly the Turner syndrome group, were higher than most research involving these populations. Having 32 participants in the Turner syndrome groups allowed conclusions to be drawn regarding the cognitive abilities displaying deficits in this population. However, as very little research has been conducted in this area, an area for future research would be to see if these results are replicated, ideally with a larger sample and longitudinally. There would then hopefully be a strong body of knowledge to inform the development of interventions to help improve mathematical outcomes in this population. A larger population of girls with Turner syndrome would also facilitate an investigation into whether the nature and extent of X chromosome deletion influences the cognitive abilities that display deficits, including arithmetic ability itself.

Having 40 participants in the maths learning disability group was a strength, however, approximately 67% were between the 15 and 25th percentile on the arithmetic ability measure. As inconsistencies in this field may be the result of classification criteria, another area for future research could be to conduct a study including the same range of measures as the present one, but focussed solely on maths learning disability, so the impact of different cut off criteria can be investigated.

Utilising a developmental trajectories approach to analyse data in phase two enabled nature of deficits to be identified, specifically whether they were at onset (the earliest age in this sample) or in the rate of development. A strength of this approach is the ability to include two measures of mental age, which did prove informative. Obviously, an ideal would be to conduct a longitudinal study, to see if following the same participants across the school years replicated these results.

11.2 Implications for Research and Practice

The finding that there is a strong path to arithmetic ability from auditory executive function and working memory latent variable is intriguing. Much research has highlighted language ability as a predictor of maths ability, hence it is possible that auditory skills may impact verbal ability, and hence linguistic skills (e.g., Toll & Van Luit, 2014; Vukovic & Lesaux, 2013). This area warrants further investigation, however, in order to do so the range of tasks specifically designed to measure auditory executive function and working memory constructs needs to be expanded, including the development of tasks that can be used across a wide age-range.

Findings from both phases of this study highlight executive function and working memory as cognitive abilities which play an important role in determining arithmetic ability. They also highlight the importance of modality. Both these findings have potentially important implications for the development of interventions to help children who struggle with maths. They provide additional evidence that interventions should train multiple abilities, in at least two modalities (e.g., Jordan & Baker, 2011; Jordan, Suanda, & Brannon, 2008).

The current study highlights once again the heterogeneity in the deficits that underpin problems with arithmetical competence. For children with maths learning difficulties there would appear to be differences in the cognitive deficits that underpin more or less severe difficulties (i.e. those whose scores on standardised tests are less than 15th percentile and those who are between the 15th and 25th percentiles). As the makeup of the maths learning difficulties population in this study was predominantly in the less severe category, finding are suggestive that interventions to help this population should contain multiple executive function and working memory constructs. In terms of the modality in which to present stimuli, findings highlight the auditory domain as the predominant area of weakness, hence whilst it would be optimal to present stimuli in both modalities, there should be a concentration on the auditory domain.

Conversely, for the girls with Turner syndrome, cognitive deficits were predominantly in the visual domain, and were not restricted to executive function and working memory constructs, but also included numerical acuity in both the visual and auditory domain. Overall, there was evidence to indicate that deficits in this population were related to both verbal and non-verbal abilities. Hence when devising an intervention for girls with Turner syndrome it would appear cogent to include tasks to train executive function and working memory abilities, particularly updating and the visuospatial sketchpad, and visual numerical acuity, and to ensure stimuli is presented both visually and auditorily.

Whilst the above are suggestions for generic interventions for children in either less severe maths learning difficulty populations and girls with Turner syndrome, it is clear from the current study that there is not just heterogeneity in the deficits that underpin arithmetical deficits in different populations but also within populations. Hence whilst generic interventions can be devised for particular populations, if they are to effect far transfer it may be important to ensure the specific strengths and weaknesses of individual children are investigated and interventions adapted to suit individual needs.

11.3 Conclusions

Congruent with much previous research the current study found within the general population bimodal measures of updating, inhibition, intelligence, and numerical acuity, visual switching, the visuospatial sketchpad, and the phonological loop, are all individual predictors of arithmetic ability, even when age is accounted for.

It also provided additional evidence for recent suggestions that differential results are produced which are dependent on the statistical techniques utilised. Given the inter-relatedness of executive function, working memory, intelligence and even arguably numerical acuity abilities, structural equation modelling may be a more appropriate way to analyse data in this field.

To my knowledge, this study was novel in examining concurrently bimodal executive function, working memory, intelligence and numerical acuity. Findings indicate it may be erroneous to attempt to find a single cognitive construct which predicts arithmetic ability, highlighting executive functioning and working memory as a group of distinct, but interrelated abilities which impact arithmetic ability in the general population. The importance of considering modality is highlighted, hence future research should overtly consider and report the modality of measures utilised.

To date there is limited research investigating the cognitive underpinnings of the maths difficulties observed in girls and women with Turner syndrome. This study is to my knowledge the most comprehensive, highlighting deficits in executive functioning, working memory and numerical acuity. Interestingly, deficits appear to be related to both visual and verbal abilities.

Both the girls with Turner syndrome and children with maths learning disability displayed similar deficits in arithmetic competence, specifically in the rate of development. However, deficits in the cognitive predictors were incongruent, and not related to a single cognitive ability. Hence when considering the source of maths learning difficulties it may be inappropriate to try to identify particular cognitive deficits, rather to consider how multiple deficits interacting across development may result in reduced arithmetic competence.

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Appendix 1: Tasks used by Szucs et al. (2013) to Measure Working Memory, Inhibition, Magnitude Representation and Spatial Processing

Construct	Task	Modality
STM	Digit span	Verbal
	Word recall	Verbal
	Dot matrix	Visuospatial
Updating	Listening span	Verbal
	Odd one out	Visuospatial
Inhibition	Stop-signal	Visuospatial
	Animal Stroop	Visuospatial
Inhibition & magnitude	Numerical magnitude comparison Stoop	Visuospatial
representation	Physical size comparison Stroop	Visuospatial
Magnitude representation	Subitizing	Visuospatial
	Symbolic magnitude comparison	Visuospatial
	Non-symbolic magnitude comparison	Visuospatial
Spatial processing	Trail making	Visuospatial
	Mental rotation	Visuospatial
	Spatial symmetry	Visuospatial
		Visuospatial

Appendix 2: Information sheets and flyers

UNIVERSITY OF CAMBRIDGE

PLAY IN EDUCATION DEVELOPMENT AND LEARNING

Want to get involved in some exciting new research?

The See 1, Hear 1 Study

This research is investigating how sight and hearing affects skills that have been shown to be important for success in maths – general thinking and magnitude comparison skills.

The general thinking skills being investigated are:

- The ability to remember information
- The ability to switch between tasks
- The ability to avoid distractions

The magnitude comparison skill is:

- The ability to know that a box with 18 apples has more apples than one with 10 apples, without actually counting them

This study has received ethical approval from the University of Cambridge, Faculty of Education, and I have undergone the necessary background checks to work with children. I follow the ethical principles for research created by the British Educational Research Association (BERA) and the British Psychological Society (BPS).

What is involved?
You do not have to think you are good at maths to get involved! Children from 4- to 18 have found the tasks fun to do and all but one contain no maths!

If you get involved in this study, you would complete 14 short tasks, all of which are fun to do. Tasks to measure thinking and magnitude comparison skills will require you to look and listen. You will also complete tasks to measure general learning skills and maths ability.

How long will it take?
There are two sessions, each lasting about 45 minutes.

Want to know more?
Contact Rosie Penford email: rcp50@cam.ac.uk

UNIVERSITY OF CAMBRIDGE

PLAY IN EDUCATION DEVELOPMENT AND LEARNING

The See 1, Hear 1 Study

Help us understand why so many women and girls with Turner Syndrome have difficulty with maths!

This research is investigating how sight and hearing affects abilities important for success in maths; general thinking and magnitude comparison skills.

It will involve women and girls with Turner Syndrome aged between 4 and 18 years.

The general thinking skills being investigated are:

- The ability to remember information
- The ability to switch between tasks
- The ability to avoid distractions

The magnitude comparison skill is:

- The ability to know that a box with 18 apples has more apples than one with 10 apples, without actually counting them

What is involved?
The study is made up of 14 short tasks, all of which are fun to do. Some of the thinking skills and magnitude comparison tasks require you to look and others to listen. There are also tasks to measure general learning skills and maths ability.

You do not have to think you are good at maths to get involved! The tasks are fun to do, and all but one contain no maths!

How long will it take?
The tasks are divided into two sessions, each lasting about 45 minutes.

Want to know more?
Contact Rosie Penford email: rcp50@cam.ac.uk

This study has received ethical approval from the University of Cambridge, Faculty of Education, and I have undergone the necessary background checks to work with children. I follow the ethical principles for research created by the British Educational Research Association (BERA) and the British Psychological Society (BPS).

PARENTS INFORMATION LETTER – Turner syndrome

The See 1, Hear 1 Study

What am I researching?

Many women and girls with Turner Syndrome have difficulties in number skills and maths abilities. My research, aims to increase our understanding of how visual and auditory abilities affect brain processes that have been shown to impact mathematical ability. I plan to compare people with Turner Syndrome to other groups of people using tasks known to tap into the underlying thinking skills associated with maths abilities; general thinking skills and magnitude comparison skills. By doing this I hope to learn more about maths development that will help improve understanding and identification of mathematics learning difficulties and inform intervention development and teaching practices for all children, including those with Turner Syndrome.

The general thinking skills being investigated are the ability to remember information, switch between tasks and ignore distractions. The magnitude comparison skills allow us to recognise that a box containing 18 apples has more apples than a box containing 10 apples, without actually counting them.

By doing this research I hope to learn more about maths development that will help improve understanding and identification of mathematics learning difficulties and inform intervention development and teaching practices for all children, including those with Turner Syndrome.

What happens during the study?

If your child gets involved in this study, they would complete 14 activities in total, all of which are fun and easy to do. The tasks involving general thinking skills and magnitude comparison skills, will be presented in two formats – one where they look at the information and the other where they listen to the information. Additional tasks measure general learning skills and arithmetic skills.

The tasks will be completed in two sessions, each lasting about 45 minutes. In my experience, participants find these tasks interesting and fun to complete.

Does your child need to participate?

This study is completely voluntary, which means that it is yours and your child's decision whether or not they participate in this research. You or your child may withdraw from the study at any time, without giving an explanation.

How will I use the data?

The data collected for this study are for research purposes only. It is likely that the results will be reported at professional conferences and/or in academic books or articles. These reports will be written based on group data (for example, the average performance of a certain age group). I will also share my findings in this group format with you, as well as the students, teachers and parents at schools participating in our study. Our research field is becoming more and more open and it is likely that I will have to share the dataset when I publish my findings. In those instances, I do not share identifying information that will allow others to trace those responses back to the person who gave them.

How will the data be protected?

I will store the data using random ID numbers, so that the data cannot be linked back to your child.

Are there any risks to participating?

There are no known risks associated with participating in this study.

Does the study have ethical approval?

This study has received ethical approval from the University of Cambridge Faculty of Education. I follow the ethical principles for research created by the British Educational Research Association (BERA) and the British Psychological Society (BPS).

What if you have questions?

If you have any questions or concerns you would like to discuss, please do not hesitate to email me at rcp50@cam.ac.uk

PARENTS INFORMATION LETTER – Typical development

The See 1, Hear 1 Study

What am I researching?

I am investigating the effect of sight and hearing on some skills that seem to be important for success in mathematics – general thinking skills and magnitude comparison skills.

The general thinking skills being investigated are the ability to remember information, switch between tasks and ignore distractions. The magnitude comparison skills allow us to recognise that a box containing 18 apples has more apples than a box containing 10 apples, without actually counting them.

What happens during the study?

If your child gets involved in this study, they would complete 14 activities in total, all of which are fun and easy to do. The tasks involving general thinking skills and magnitude comparison skills, will be presented in two formats – one where they look at the information and the other where they listen to the information. Additional tasks measure general learning skills and arithmetic skills.

The tasks will be completed in two sessions, each lasting about 45 minutes. In my experience, participants find these tasks interesting and fun to complete.

Does your child need to participate?

This study is completely voluntary, which means that it is yours and your child's decision whether or not they participate in this research. You or your child may withdraw from the study at any time, without giving an explanation.

How will I use the data?

The data collected for this study are for research purposes only. It is likely that the results will be reported at professional conferences and/or in academic books or articles. These reports will be written based on group data (for example, the average performance of a certain age group). I will also share our findings in this group format with you, as well as the students, teachers and parents at schools participating in our study. Our research field is becoming more and more open and it is likely that I will have to share the dataset when I publish my findings. In those instances, I do not share identifying information that will allow others to trace those responses back to the person who gave them.

How will the data be protected?

I will store the data using random ID numbers, so that the data cannot be linked back to your child.

Are there any risks to participating?

There are no known risks associated with participating in this study.

Does the study have ethical approval?

This study has received ethical approval from the University of Cambridge Faculty of Education. I follow the ethical principles for research created by the British Educational Research Association (BERA) and the British Psychological Society (BPS).

What if you have questions?

If you have any questions or concerns you would like to discuss, please do not hesitate to email me at

Appendix 3: Parental consent and background information forms

Parental CONSENT FORM
See 1, Hear 1 Study

If you agree for your child to participate in this study, then please complete this form.

Please circle one:

I agree for my child to take part in this research	***Yes***	***No***
I have read and understood the Participant Information Sheet (with the name and contact details of the researchers)	***Yes***	***No***
I have had the opportunity to ask questions about the study, and had them answered.	***Yes***	***No***
I understand that my child's responses might be audio recorded to ensure accuracy of results. Any audio recordings will be kept confidential and will be kept in a secure location.	***Yes***	***No***
I understand that the data collected for this research project will be kept confidential; all data will be identified by a random code that is not linked back to my child and will be kept in a secured location.	***Yes***	***No***
I understand that my child or I can withdraw from this study at any time without giving a reason.	***Yes***	***No***
I understand that these data may be presented at professional conferences or in academic manuscripts. These results will be written up based on group data.	***Yes***	***No***
I understand that these data might be shared according to open access standards, but no identifying information will allow others to trace my responses back to my child or me.	***Yes***	***No***

Child's name ……………………………….. Your postcode ………………………………..

Signature ……………………………….…………………………………………………………..

Name (please print) …………………………………………… Date ……………………………

Researcher's Signature …………………………………….……………………………...........

The See 1, Hear 1 Study
Family background information sheet

Your child's name: ________________________________

Your relation to your child:
- ❍ Biological mother
- ❍ Step-mother
- ❍ Adoptive mother
- ❍ Foster mother
- ❍ Biological father
- ❍ Step-father
- ❍ Adoptive father
- ❍ Foster father
- ❍ Other

Please specify your relation to your child:
____

What is the highest level of education you have completed?
- ❍ Primary School
- ❍ GCSE / CSE / O-Levels
- ❍ AS Levels
- ❍ A-Levels / GCE / Scottish Highers
- ❍ NVQs / SVQs
- ❍ Some University
- ❍ Bachelors Degree
- ❍ Some post-graduate
- ❍ Masters Degree
- ❍ Some doctoral
- ❍ Doctorate Degree
- ❍ Other

Please specify the highest level of education you have completed:
____

What is your employment status?
- ❍ Full time
- ❍ Part time
- ❍ Homemaker
- ❍ Not currently employed

What is your job title?

____

What is your marital status?

- ❍ Married
- ❍ Have a partner
- ❍ Single (skip the rest of the questions)
- ❍ Widowed (skip the rest of the questions)
- ❍ Divorced (skip the rest of the questions)
- ❍ Separated (skip the rest of the questions)

What is the highest level of education your partner has completed?

- ❍ Primary School
- ❍ GCSE / CSE / O-Levels
- ❍ AS Levels
- ❍ A-Levels / GCE / Scottish Highers
- ❍ NVQs / SVQs
- ❍ Some University
- ❍ Bachelors Degree
- ❍ Some post-graduate
- ❍ Masters Degree
- ❍ Some doctoral
- ❍ Doctorate Degree
- ❍ Other

Please specify the highest level of education you partner has completed:

____

What is your partner's employment status?

- ❍ Full time
- ❍ Part time
- ❍ Home maker
- ❍ Not currently employed

What is your partner's job title?

____

Appendix 4: Participant consent forms

STUDENT CONSENT FORM - 1
The See 1, Hear 1 Study

Please read the following statements and circle either yes or no for each of them, then write your name and sign the bottom of the form. If you have any questions, please ensure you ask.

Please circle one:

I agree to take part in this study	**Yes**	**No**
The reasons for this investigation have been explained to me	**Yes**	**No**
I have read the Participant Information Sheet (which contains the name and contact details of the researcher)	**Yes**	**No**
I have had the chance to ask questions, and any I did ask have been answered.	**Yes**	**No**
I understand that I will complete 14 tasks, which will be split over 2 sessions	**Yes**	**No**
I understand my responses may be audio recorded and that these recordings will be kept confidential and stored in a secure location	**Yes**	**No**
I understand that I can withdraw from this study at any time without giving a reason and I know how to do this	**Yes**	**No**
I understand that the results from this study will be based on group data so no individual can be identified.	**Yes**	**No**
I understand that the results of this research may be presented at conferences or in an academic manuscript.	**Yes**	**No**

Name (please print) ……………………………………………

Signature: ………………………………………………….

Today's Date ………………..

Researcher's Signature ………………………………………………………………............

STUDENT CONSENT FORM - 2
The See 1, Hear 1 Study

Please read the following statements and circle either yes or no for each of them, then write your name and sign the bottom of the form. If you have any questions, please ensure you ask.

Please circle one:

I agree to take part in this study.	**Yes**	**No**
The reasons for this investigation have been explained to me	**Yes**	**No**
I have had the chance to ask questions, and any I did ask have been answered.	**Yes**	**No**
I understand that I will complete 14 tasks, which will be split over 2 sessions	**Yes**	**No**
I understand my answers may be recorded	**Yes**	**No**
I understand that I can withdraw from this study at any time	**Yes**	**No**

Name (please print) ……………………………………………

Signature: ……………………………………………………

Today's Date ……………..

Researcher's Signature ………………………………………………………………….............

CHILDREN CONSENT - 3
The See 1, Hear 1 Study

Game 1	
Game 2	
Game 3	
Game 4	
Game 5	
Game 6	
Game 7	

Name (please print) ……………………………………………

Researcher's Signature …………………………………   Date: ……………………………………

Appendix 5: Power Analyses

Participants will be aged between 4- and 18- years and drawn from three populations; typically developing (142 participants), Maths learning disability (40 participants) and Turner Syndrome (32 participants). This sample size should be sufficient to find large and medium effects in each of the planned analyses (see below).

Research Question 1: Regression – linear multiple regression: Fixed model, R^2 deviation from zero (a priori)
α = .05
power (1 - β) = .80
Predictors = 13
Effect size .15 sample size = 131
. 35 sample size = 64

Research Questions 2 & 3: ANCOVA (a priori)
α = .05
power (1 - β) = .80
Numerator = 2
Number of groups = 3
Number of covariates = 1
Effect size .25 sample size = 158
.40 sample size = 64

Research Questions 2 & 3: ANOVA – repeated measures, between factors (a priori)
α = .05
power (1 - β) = .80
Number of groups = 3
Number of measures = 2
Correlation between measures = .5
Effect size .25 sample size = 120
.40 sample size = 51

Research Questions 2 & 3: ANOVA – repeated measures, within-between interaction (a priori)
α = .05
power (1 - β) = .80
Number of groups = 3
Number of measures = 2
Correlation between measures = .5
Correlation among repeated measures = .5
Nonsphericity correction = 1
Effect size .25 sample size = 42
.40 sample size = 21

All the above found using G* Power (Faul, Erdfelder, Lang, & Buchner, 2007; Faul, Erdfelder, Buchner, & Lang, 2009;

Appendix 6: Materials used in similar studies

https://docs.google.com/spreadsheets/d/1obhMCflTmk0n6U1SEHwxXOugg_X9z5FM2-679lpNvFY/edit#gid=0

https://bit.ly/2FYRwoD

Appendix 7: Reliability Statistics for Materials

Internal-Consistency Reliability Statistics for KBIT-II

Age (years)	Verbal	Nonverbal	IQ composite
4-18[a]	0.90	0.86	0.92
19-90[a]	0.92	0.91	0.95
4-90[a]	0.91	0.88	0.93

Note. [a] Weighted means using Fisher's z transformation.

Adapted from "Kaufman Brief Intelligence Test-Second Edition Manual: by A. S. Kaufman and N. L. Kaufman, p. 52.

Internal-Consistency Reliability Statistics for Number Operations Subtest of WIAT-II

Measure	Average	Range
Age based	.91	.81 - .96
Inter-scorer	.94	.94 - .98

Note. Adapted from "Wechsler Individual Achievement Test-second edition Examiner's Manual" by D. Wechsler, 2005, pp. 85-86

Reliability Statistics for WMTB-C

	Test - retest	
Subtest	Years 1 and 2	Years 3 & 4
Block recall	.63	.43
Digit recall	.81	.82
Backward digit recall	.53	.71

Note. Adapted from "Working Memory Test Battery for Children Manual" by S. Pickering and S. Gathercole, 2001, p.19.

Reliability Statistics for The Original Shape School

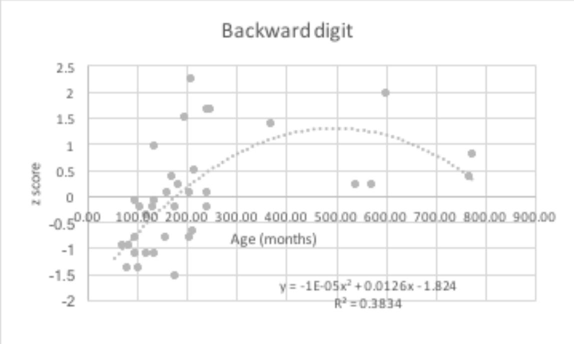
Construct	Cronbach's alpha
Inhibition	.71
Switching	.80
Inhibition-switching	.74

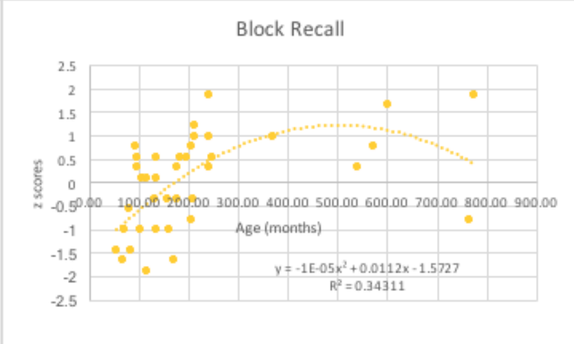
Note. Adapted from "Assessing Executive Functions in Preschoolers Using Shape School Task" by M. Nieto, L. Ros, G. Medina, J. J. Ricarte and J. M. Latorre, 2016, P. 4.

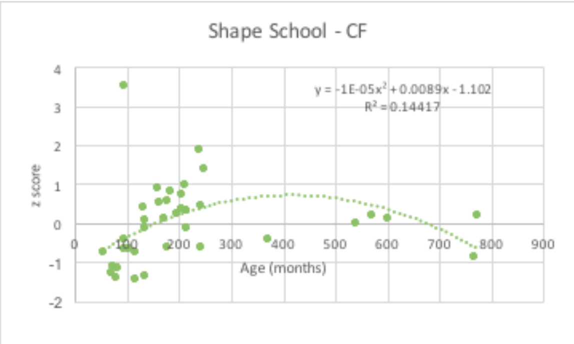
Appendix 8: Stimuli for the Colour Association Task

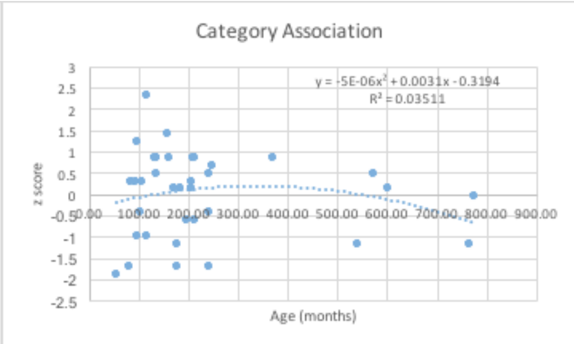
Object	Typical colours	Object	Typical colours
Apple	Red / green	Swan	White
Grass	Green	Egg	Brown / white / yellow
Banana	Yellow	Orange	Orange
Carrot	Orange	Corn	Yellow
Cheese	Orange / yellow	Raspberry	Pink / red
Lemon	Yellow	Pumpkin	Orange
Lime	Green	Pea	Green
Pig	Pink	Sun	Yellow
Strawberry	Red	Sheep	White
Grape	Red / green / purple	Blueberry	Blue
Chocolate	Brown	Rose	Red / yellow / pink
Raisin	Black / purple	Sky	Blue / white
Jam	Red	Milk	White
Ham	Pink	Beans	Green / orange
Cherry	Red	Butter	Yellow

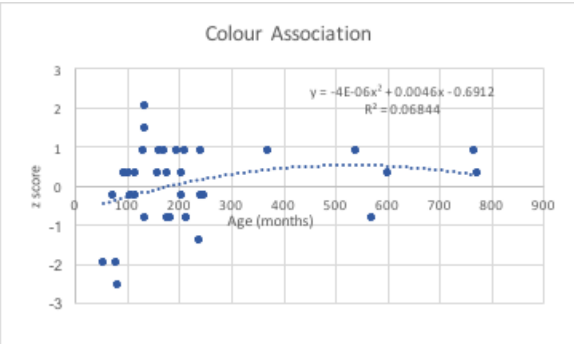
Appendix 9: Developmental Trajectories for Pilot Tasks

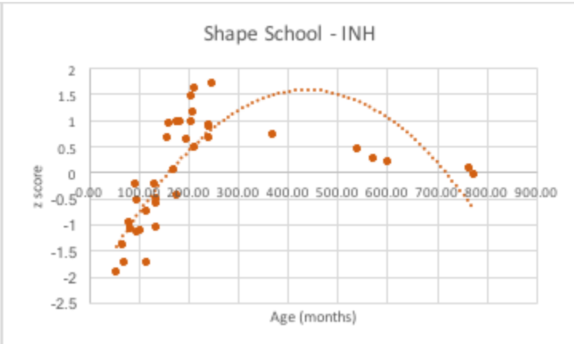


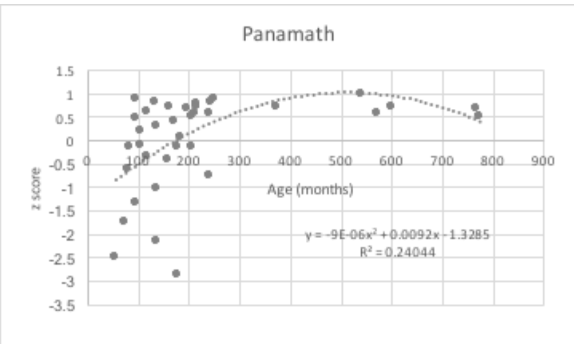


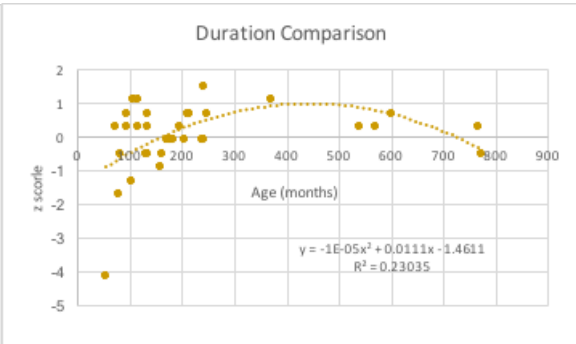


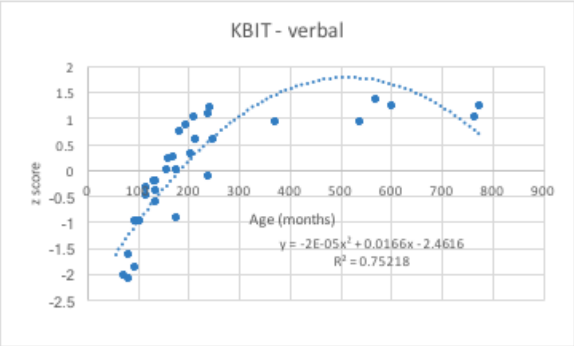


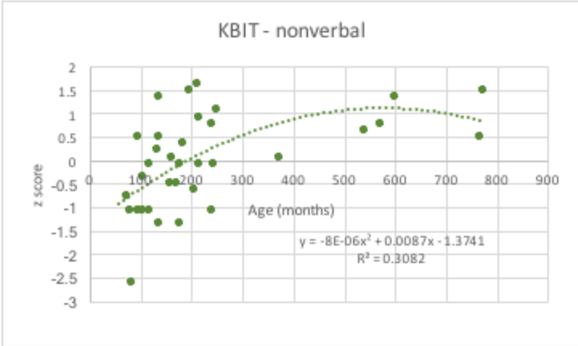


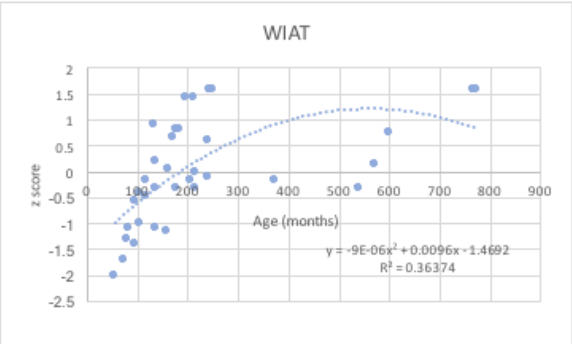







KBIT - verbal
z score
Age (months)
y = -2E-05x² + 0.0166x - 2.4616
R² = 0.75218


KBIT - nonverbal
z score
Age (months)
y = -8E-06x² + 0.0087x - 1.3741
R² = 0.3082


WIAT
z score
Age (months)
y = -9E-06x² + 0.0096x - 1.4692
R² = 0.36374

Appendix 10: Shape School Extended – exploratory analysis

Control – processing speed

	R^2	F	p	t
Efficiency	.61	325.00	< .001	18.03
Accuracy	.13	30.78	< .001	5.55
Response time	.56	266.01	< .001	-16.31

Inhibition

	R^2	F	p	t
Efficiency	.62	342.61	< .001	18.51
Accuracy	.14	35.09	< .001	5.92
Response time	.52	223.24	< .001	-14.94

Switching

	R^2	F	p	t
Efficiency	.60	304.55	< .001	17.45
Accuracy	.17	41.41	< .001	6.44
Response time	.56	257.48	< .001	-16.05

Correlations

Control condition

	M	Response time	Accuracy
Response time (seconds)	32.64		-.44***[a]
Accuracy (%)	96.4	-.26***[b]	

Note: a. zero correlation; b. age corrected correlation

Inhibition condition

	M	Response time	Accuracy
Response time (seconds)	31.12		-.27***[a]
Accuracy (%)	95.4	-.09[b]	

Note: a. zero correlation; b. age corrected correlation

Switching condition

	M	Response time	Accuracy
Response time (seconds)	74.96		-.42***[a]
Accuracy (%)	87.00	-.23**[b]	

Note: a. zero correlation; b. age corrected correlation

Appendix 11: Panamath – Weber Fraction Curve Fitting

https://docs.google.com/spreadsheets/d/1BgGc6UgY5lQBvlr0LrcTh9MUmBwlM4eVEnaUNnqmMSM/edit#gid=1079278864

https://bit.ly/30h8OVG

Appendix 12: Analysis of congruent v incongruent trials from Panamath task

Descriptive Statistics, Zero and Partial Correlations for Accuracy

	N	*M*	*SD*	Congruent	Incongruent	Arithmetic
Congruent	208	88.51	9.36		.79***	.25***
Incongruent	208	85.18	10.91	.85***		.28***
Arithmetic	208	25.79	13.09	.52***	.60***	

Note: Above diagonal are age corrected partial correlations; below diagonal are zero correlations

Paired T-test to determine if congruent and incongruent means are significantly different.

$t(207) = 8.28$, $p < .001$ significantly different means

Age-corrected regression analysis to compare predictive power

$F(3, 204) = 111.87$, $p < .001$, $R^2 = .62$
Congruent: $t(204) = 0.58$, $p = .561$
Incongruent: $t(201) = 2.11$, $p = .036$

Descriptive Statistics, Zero and Partial Correlations for Weber Fraction

	N	*M*	*SD*	Congruent	Incongruent	Arithmetic
Congruent	201	0.20	0.71		.21**	-.17*
Incongruent	201	0.23	0.23	.20***		-.20**
Arithmetic	201	26.25	12.96	-.14	-.50***	

Note: Above diagonal are age corrected partial correlations; below diagonal are zero correlations

Paired T-test to determine if congruent and incongruent means are significantly different.

$t(200) =- 0.53$, $p = .597$ difference between means is not significant

Age-corrected regression analysis to compare predictive power

$F(3, 197) = 95.86$, $p < .001$, $R^2 = .59$
Congruent: $t(197) = -1.92$, $p = .057$
Incongruent: $t(197) = -2.37$, $p = .019$

Independent regression analysis

	R^2	*F*	*p*	*t*
Accuracy - congruent	.27	75.45	< .001	8.69
Accuracy - incongruent	.36	115.56	< .001	10.75
Weber - congruent	.03	5.82	.017	-2.41
Weber - incongruent	.25	66.51	< .001	-8.16
Accuracy	.34	106.97	< .001	10.34
Weber	.15	34.58	< .001	-5.88

Age corrected independent regression analysis

	R^2	ΔR^2	pR^2	F	p	t	β
Accuracy - congruent	.61	.03	.06	162.84	< .001	3.65^{***}	.18
Accuracy - incongruent	.62	.03	.08	168.18	< .001	4.21^{***}	.22
Weber - congruent	.59	.01	.03	145.47	< .001	-2.48^{*}	-.11
Weber - incongruent	.59	.02	.04	140.07	< .001	-2.81^{**}	-.15
Accuracy	.62	.03	.08	167.88	< .001	4.18^{***}	.22
Weber	.59	.01	.02	143.38	< .001	-2.09^{*}	-.10

Appendix 13: Panamath – impact of outliers

1) Accuracy

a) Linear Regression with all participants

$F(1, 203) = 106.6$, $p < .001$, $R^2 = .34$

	b	SE B	β	*t*	*p*
Constant	-42.88	6.49		-6.41	< .001
Accuracy	0.79	0.008	.59	10.33	< .001

b) Looking at Cook's, leverage & Mahalanobis distances suggests ID 47, 81, 84, 89 & 198 are having an influence

ID	Session 1	Session 2	acc_1	acc_2	acc
81	84	45	39.3	48.9	44.1
47	36	36	41.7	50	45.8
198	84	40	51.2	55	53.1
84	73	32	56.2	56.3	56.2
89	84	84	76.2	41.7	58.9
195	84	84	76.2	47.6	61.9

Note: acc_1 = accuracy session 1; acc_2 = accuracy session 2; acc = mean accuracy from both sessions.

c) With all these participants removed:

$F(1, 197) = 114.1$, $p < .001$, $R^2 = .61$

	b	SE B	β	*t*	*p*
Constant	-61.62	8.25		-7.47	< .001
Accuracy	1.00	0.09	.61	10.68	< .001

d) Removing just ID 47, 81, 84, 198 (identified through lapse rates, Weber > 1 and casewise)

$R^2 = .34$, adjusted $R^2 = .34$, $F(1, 199) = 102.44$, $p < .001$, $t(199) = 10.12$

e) Removing IDs 47, 81, 84, 85, 89, 133, 171, 198 (identified through lapse rates and Weber > 1)

$R^2 = .37$, adjusted $R^2 = .37$, $F(1, 195) = 115.78$, $p < .001$, $t(195) = 10.75$

f) Removing IDs 34, 47, 56, 58, 81, 83, 84, 85, 89, 127, 133, 145, 162, 171, 176, 185, 188, 195, 196, 198 (identified through lapse rates > .5)

$R^2 = .32$, adjusted $R^2 = .31$, $F(1, 183) = 85.16$, $p < .001$, $t(183) = 9.23$

2) Weber fraction

a) Linear regression

$F(1, 200) = 34.14$, p < .001, $R^2 = .15$

	b	SE B	β	*t*	*p*
Constant	29.87	1.08		27.69	< .001
Weber	-18.84	3.22	-.38	-5.84	< .001

b) Casewise diagnosis suggested ID84 was having an influence

	Session 1	Session 2
Number of trials	73	32
Weber	2.28	4.17

c) Removing this point results in the following:
$F(1, 199) = 68.07$, $p < .001$, $R^2 = .26$

	b	SE B	β	*t*	*p*
Constant	34.15	1.26		27.13	< .001
Weber	-42.25	5.12	-.51	-8.25	< .001

d) Removing ID 47, 81, 84, 198 (identified through lapse rates, Weber > 1 and casewise)
$R^2 = .26$, adjusted $R^2 = .25$, $F(1, 199) = 68.07$, $p < .001$, $t(199) = -8.25$

e) Removing IDs 47, 81, 84, 85, 89, 133, 171, 198 (identified through lapse rates and Weber > 1)
$R^2 = .28$, adjusted $R^2 = .28$, $F(1, 195) = 76.51$, $p < .001$, $t(195) = -8.75$

f) Removing IDs 34, 47, 56, 58, 81, 83, 84, 85, 89, 127, 133, 145, 162, 171, 176, 185, 188, 195, 196, 198 (identified through lapse rates > .5)
$R^2 = .23$, adjusted $R^2 = .22$, $F(1, 183) = 53.19$, $p < .001$, $t(183) = -7.29$

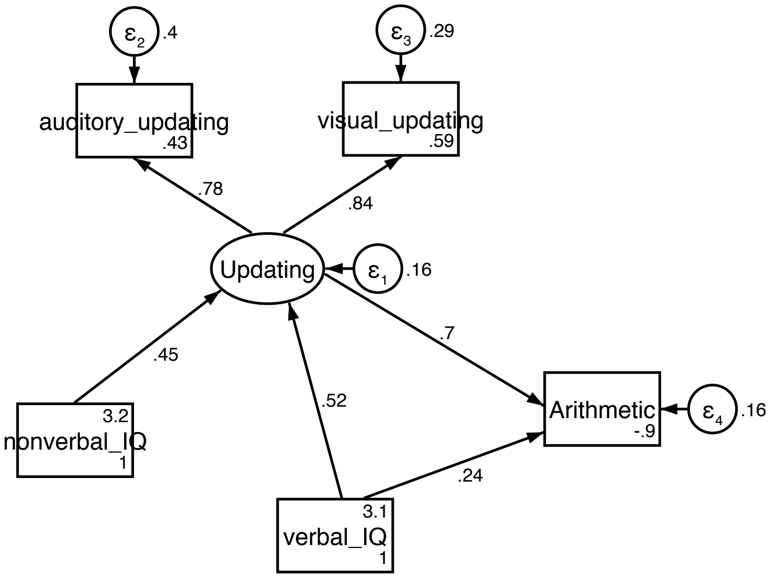
Appendix 14: Regression Assumptions

https://docs.google.com/spreadsheets/d/1TgQLQwLNu3fz2bQX93HXYStmiUtm4ZTPuBupSz1w6KE/edit#gid=1879025717

https://bit.ly/2FX3KxX

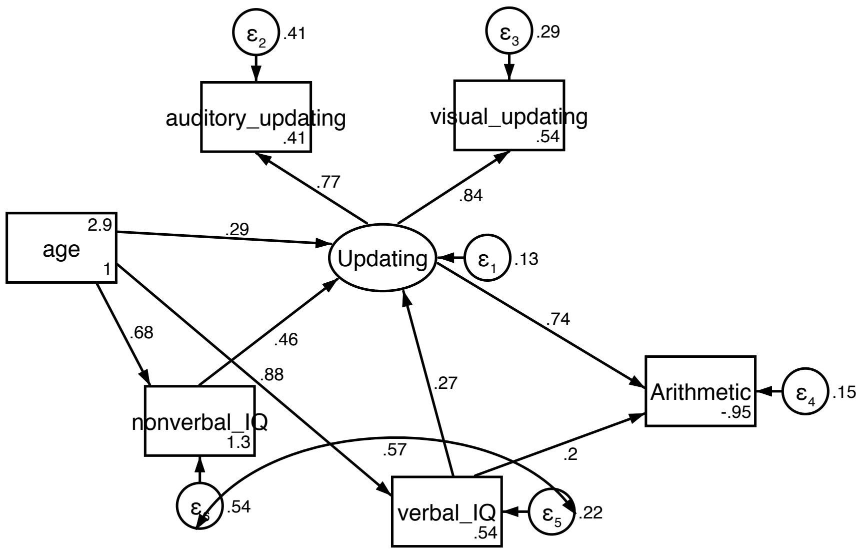
Appendix 15: Structural Equation Models

1. Bimodal Updating and Intelligence Model for Predicting Arithmetic Competence (Figure 7.2).



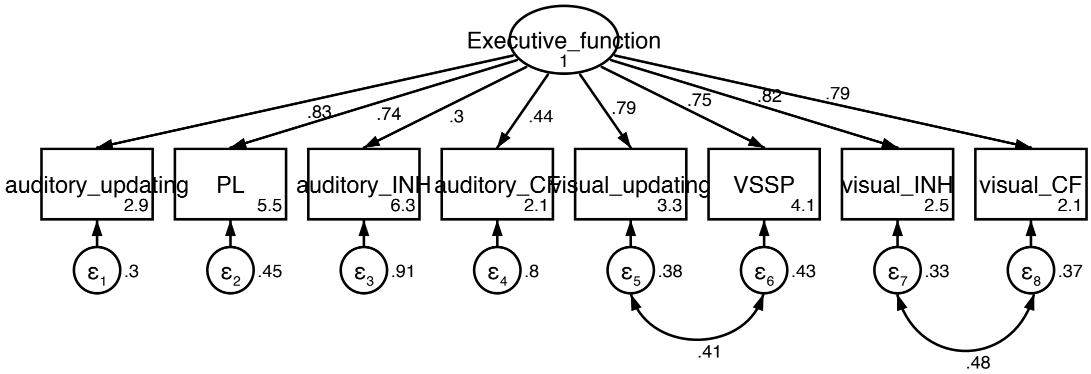
$\chi^2(2) = 1.50$, $p = .682$, RMSEA = .00, 90% CI [00, .10], CFI = 1.00, TLI = 1.00, SRMR = .01

2. Bimodal measures of intelligence and updating, and age as predictors of arithmetic competence (Figure 7.3).



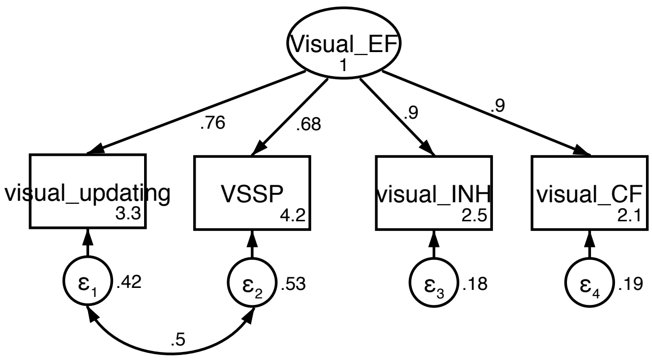
$\chi^2(5) = 5.88$, $p = .318$; RMSEA = .03, 90% CI [.00, .11]; CFI = 0.999; TLI = 0.997; SRMR = .01

3. Single Latent Variable (Figure 7.4).

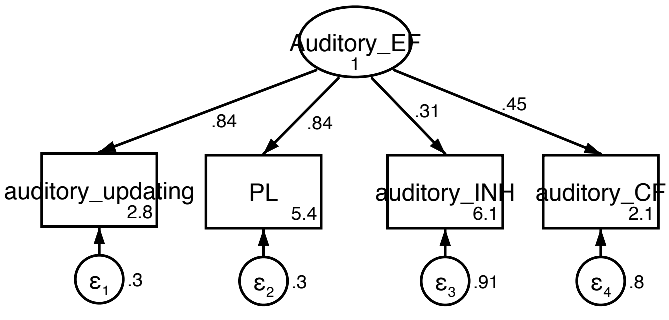


$\chi^2(18) = 23.58$, $p = .169$, RMSEA = .04, 90% CI [.00, .09], CFI = 0.993, TLI = 0.989, SRMR = .03

4. Two Latent Variable Model (Figure 7.5).

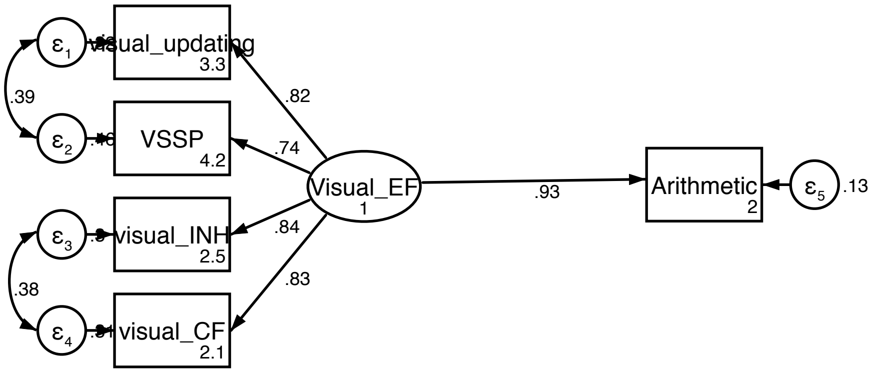


$\chi^2(1) = 1.30$, $p = .254$, RMSEA = .04, 90%CI [.00, .21], CFI = 0.999, TLI = 0.996, SRMR = .01

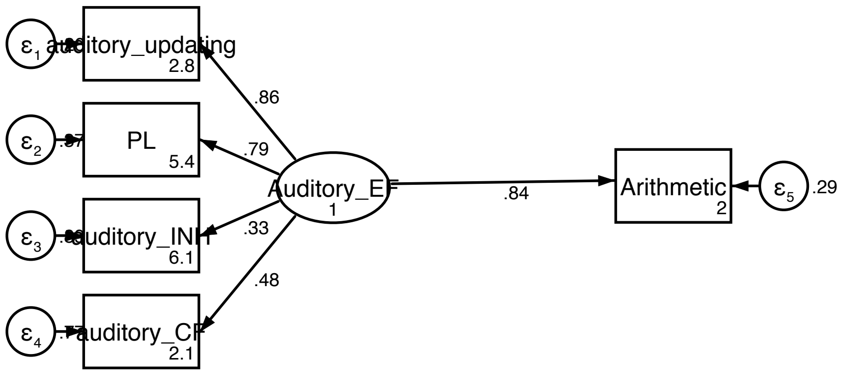


$\chi^2(2) = 0.07$, $p = .967$, RMSEA = .00, 90%CI [.00, .], CFI = 1.00, TLI = 1.04, SRMR = .003

5. Two-latent variable models and arithmetic (Figure 7.6)

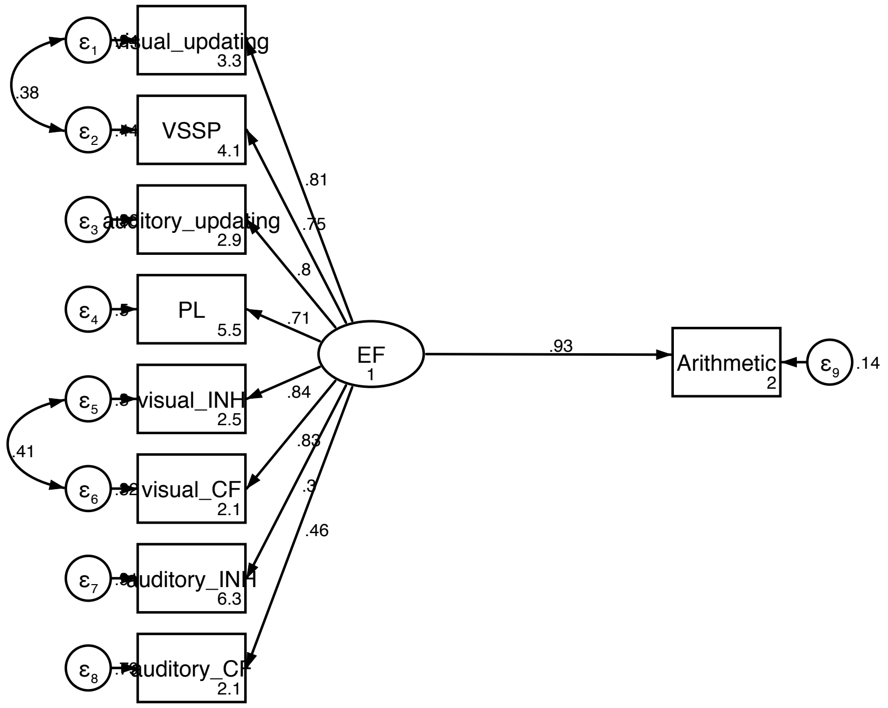


$\chi^2(3) = 1.39$, $p = .708$, RMSEA = .00, 90%CI [.00, .09], CFI = 1.00, TLI = 1.01, SRMR = .01



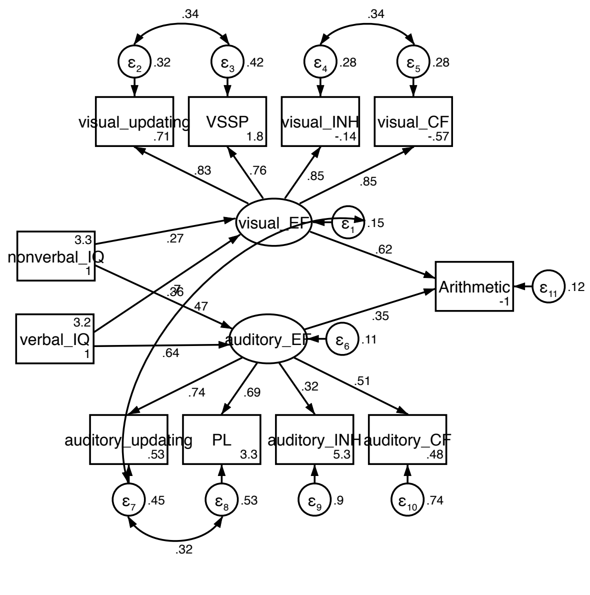
$\chi^2(5) = 6.87$, $p = .233$, RMSEA = .00, 90%CI [.00, .12], CFI = 0.99, TLI = 0.99, SRMR = .02

6. Single latent variable and arithmetic (Figure 7.7)



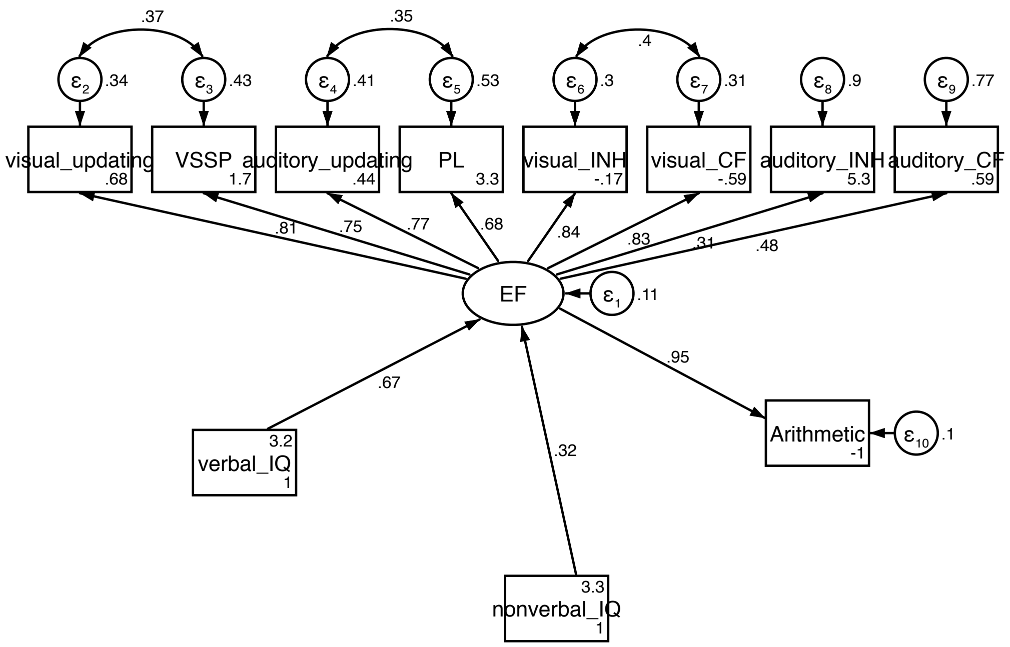
χ^2(25) = 33.68, *p* = .115, RMSEA = .00, 90%CI [.00, .08], CFI = 0.99, TLI = 0.99, SRMR = .03

7. Bimodal measures of intelligence and executive functioning as predictors of arithmetic competence (Figure 7.8).



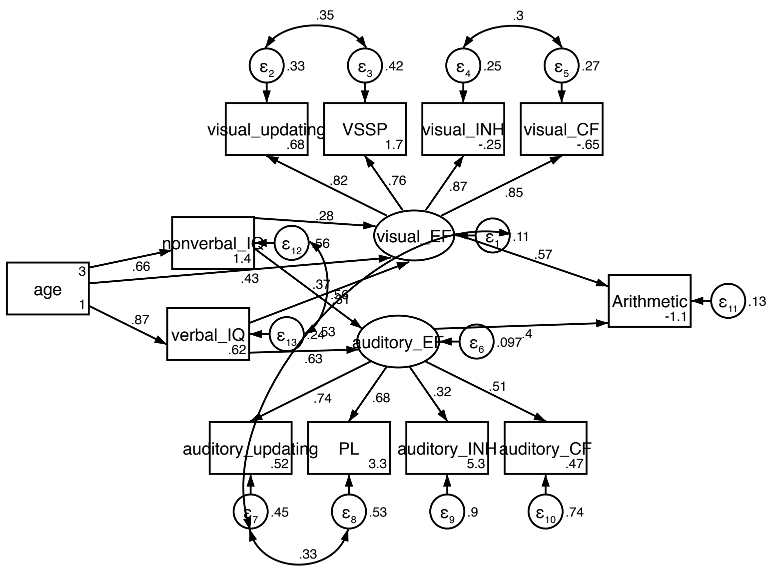
χ^2(36) = 44.53, *p* = .156, RMSEA = .04, 90%CI [.00, .07], CFI = 0.99, TLI = 0.99, SRMR = .03

8. Bimodal measures of intelligence and a single latent executive functioning variable (Figure 7.9)



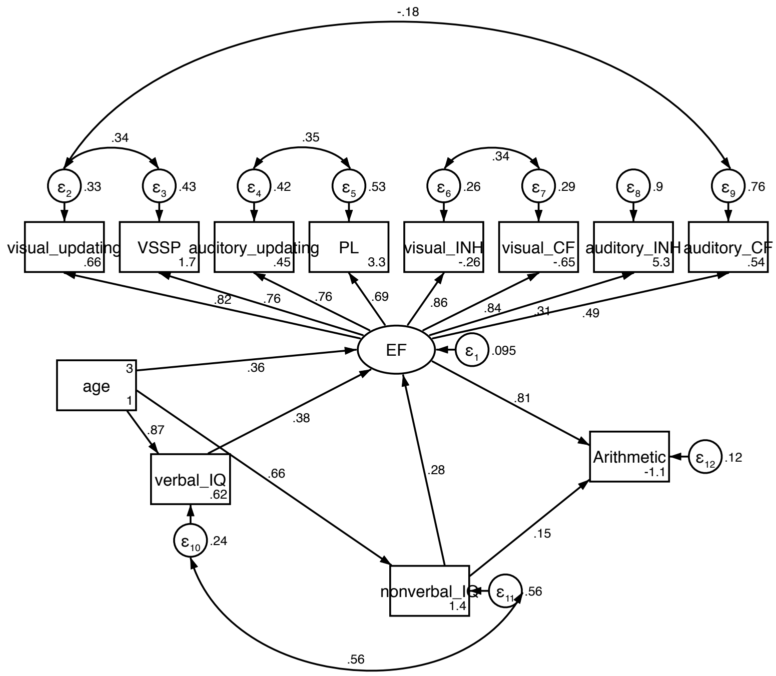
$\chi^2(40) = 53.14$, $p = .080$, RMSEA = .04, 90%CI [.00, .07], CFI = 0.99, TLI = 0.99, SRMR = .03

9. Bimodal measures of intelligence and executive functioning, and age as predictors of arithmetic competence (Figure 7.10).



$\chi^2(44) = 54.56$, $p = .132$, RMSEA = .04, 90%CI [.00, .07], CFI = 0.99, TLI = 0.99, SRMR = .03

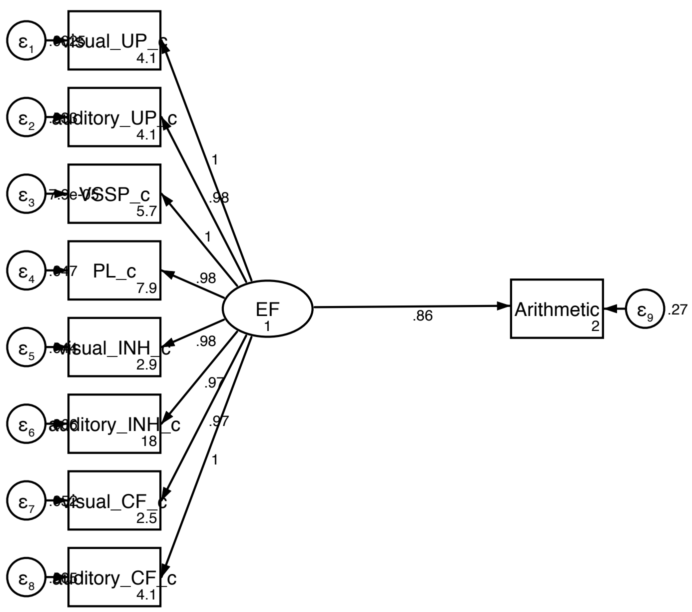
10. Bimodal measures of intelligence and single latent executive functioning, and age as predictors of arithmetic competence (Figure 7.11).



$\chi^2(46) = 62.05$, $p = .057$, RMSEA = .05, 90%CI [.00, .07], CFI = 0.99, TLI = 0.99, SRMR = .02

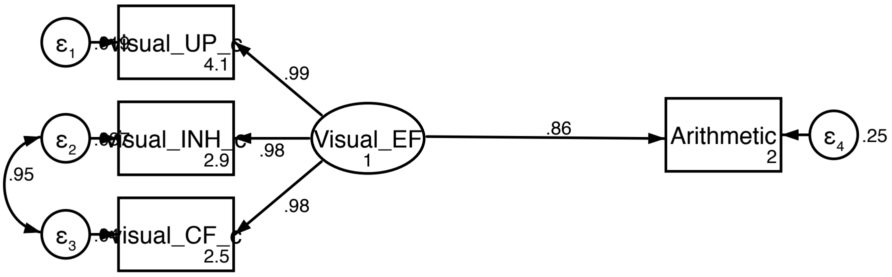
Alternative Analysis

11. Single latent executive function and arithmetic (Figure 7.12)

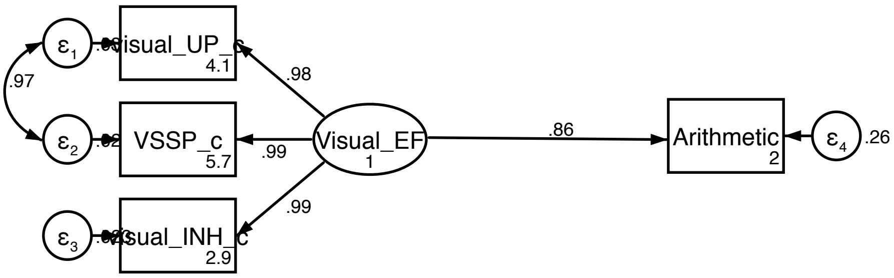


$\chi^2(27) = 24443.57$, $p < .001$, RMSEA = 2.24, 90%CI [.00, .], CFI = 0.17, TLI = -0.11, SRMR = .02

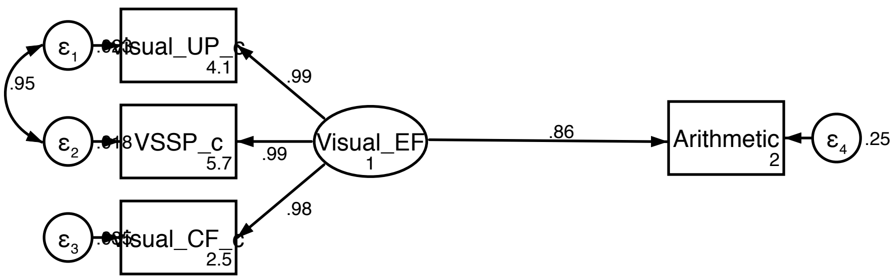
12. Visual latent executive functioning models (Figures 7.13 – 7.16)



$\chi^2(1) = 0.29$, $p = .591$, RMSEA = .00, 90%CI [.00, .16], CFI = 1.00, TLI = 1.00, SRMR = .00

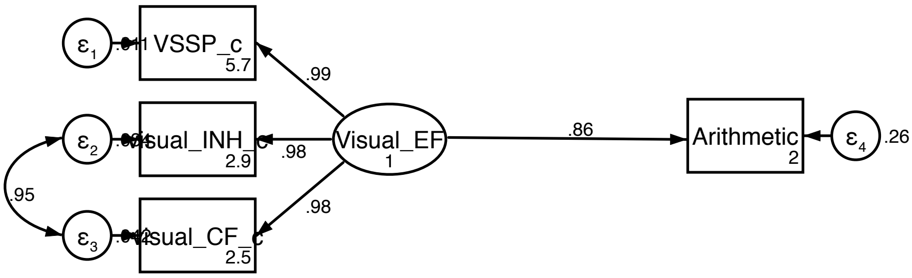


$\chi^2(1) = 5.79$, $p = .02$, RMSEA = .16 90%CI [.06, .30], CFI = 0.997, TLI = .99, SRMR = .004



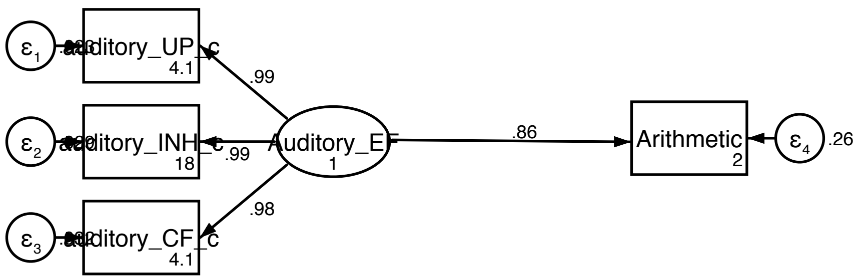
$\chi^2(1) = 1.38$, $p = .24$, RMSEA = .05 90%CI [.00, .21], CFI = 1.00, TLI = .999, SRMR = .001

12. Auditory latent executive functioning models (Figures 7.17 - 7.20)

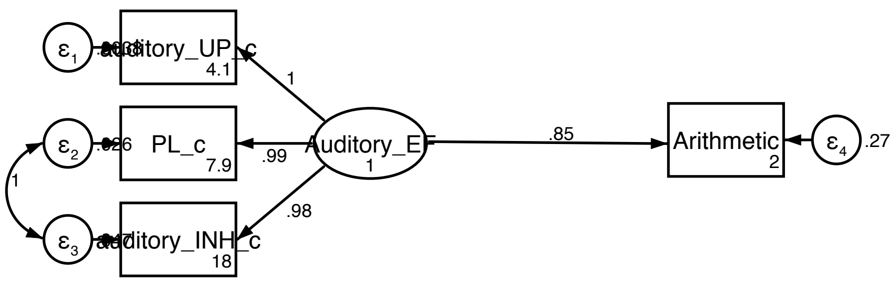


$\chi^2(1)$ = 2.25, *p* = .134, RMSEA = .08 90%CI [.00, .23], CFI = .999, TLI = .996, SRMR = .002

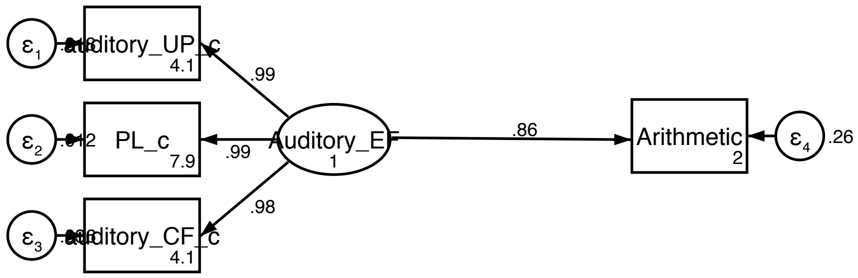
12. Auditory latent executive functioning models (Figures 7.17 - 7.20)



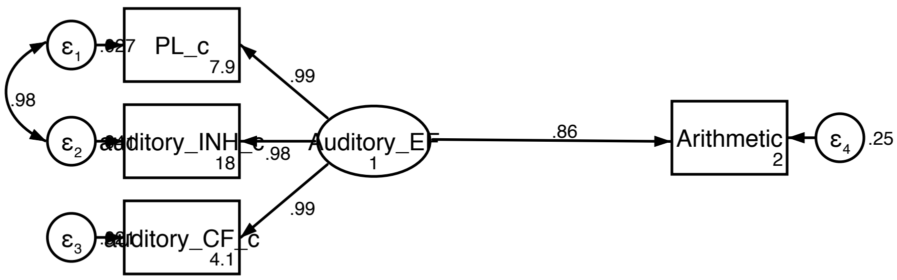
$\chi^2(2)$ = 1.38, *p* = .501, RMSEA = .00 90%CI [.00, .13], CFI = 1.00, TLI = 1.00, SRMR = .002



$\chi^2(1)$ = 1.96, *p* = .161, RMSEA = .07 90%CI [.00, .23], CFI = 1.00, TLI = .998, SRMR = .005



$\chi^2(2) = 2.87$, $p = .239$, RMSEA = .05 90%CI [.00, .16], CFI = .999, TLI = .998, SRMR = .003



$\chi^2(1) = 0.02$, $p = .898$, RMSEA = .00 90%CI [.00, .09], CFI = 1.00, TLI = 1.00, SRMR = .000