Numerical Analysis of Non-Uniformities and Anisotropy in High Temperature Superconducting (HTS) Coils

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***Abstract*—High temperature superconducting (HTS) coils play an important role in a number of large-scale engineering applications, such as electric machines employing HTS coated conductors. Non-uniformities and anisotropy in the properties of the coated conductor along its length and width can have a large impact on the performance of the tape, which influences directly the performance of an HTS electric machine. In this paper, the specific influences of non-uniformity and anisotropy on the DC properties of coils, such as the maximum allowable DC current, and the AC properties, such as AC loss, are analysed using a numerical model based on the *H* formulation. It is found that non-uniformity along the conductor width has a large effect on the AC properties (i.e. AC loss) of a coil, but a relatively small effect on the DC properties (i.e. critical current). Conversely, non-uniformity along the length has a small effect on the AC coil properties, but has a large effect on the DC properties.**

***Index Terms*—AC loss, critical current density (superconductivity), high-temperature superconductors, numerical analysis, superconducting coils, transport ac loss**

I. Introduction

RECENTLY, long lengths of YBCO coated conductor with fairly uniform properties have become available commercially [1]-[3], which have great potential for implementation in large scale HTS applications, such as rotating machines. The material properties of such conductors determine directly the performance of such machines. The performance of HTS coils has been measured and simulated numerically by several groups [4]-[13]. However, few papers investigate the influence of non-uniform material properties on the properties of HTS coils. Gömöry *et al.* [7] recently focused on the non-uniform characteristics along the length and width on a coil consisting of ten turns of YBCO tape. The conductor in an HTS coil can have non-uniformities along the width and length for different reasons, such as an uneven manufacturing process or damage from cutting, handling and winding. Anisotropy in the properties of HTS tape has been reported widely (some examples can be found in [2], [8]-[13]). Non-uniformities and anisotropy can influence directly the electrical properties of the tape: DC properties, such as the maximum allowable DC current in the coil, and AC properties, such as AC loss. Therefore, it is important to consider the combination of anisotropy and non-uniformity in a coated conductor in order to analyse the electrical properties of a coil. Numerical simulation is one of the most effective ways of interpreting experimental results, understanding the physical mechanisms of the material properties, and assisting in optimizing the design of applications utilizing HTS coils.

In this paper, numerical analysis of the effects of anisotropy and non-uniformity on the properties of HTS coils are combined using the ***H*** formulation, implemented in the commercial software package, Comsol Multiphysics 4.3a. An analysis of the DC (i.e. critical current) and AC properties (i.e. AC loss) of coils of various sizes is also performed

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II. HTS Coil Models with Non-uniformities and Anisotropy

A. Finite Element Method (FEM) Modelling Framework

A 2D axisymmetric model of a circular pancake coil [14] is utilized in this section to investigate the electromagnetic properties of the coil, based on the ***H*** formulation [15]-[18]. The model consists of a stack of tapes in the *rz*-plane representing the cross section of the coil. In this model, the inner radius of the coil from the innermost tape is 30 mm, the distance between tapes is 200 µm, the width of the tape is 12 mm and the thickness of the tape is 1 µm. For clarity, only four tapes in the coil are shown in Fig. 1, surrounded by air. Further details of the implementation of the ***H*** formulation in Comsol Multiphysics can be found in [14], [17]-[21].

The critical current density of a superconductor, J_c, varies with the magnitude of magnetic field B and angle θ between magnetic field $\boldsymbol{B}$ and the surface of the tape, which is denoted as $J_c(B,\theta)$ [13]. The measured characteristics of coated conductor can be described by (1), as given in [11]. This behaviour is also called elliptical anisotropy.

$$J_c(B,\theta) = J_0/(1 + B\sqrt{\cos^2\theta + u^2\sin^2\theta}/B_0)^{\beta} \qquad (1)$$

J_0 represents the critical current density in self-field at 77 K. The specific parameters for the elliptical anisotropy used in this paper are $J_0 = 4.602\times10^{10}$ A/m^2, $B_0 = 4.6$ mT, $u = 2.015$ and $\beta = 0.48$, which are consistent with those given in [13].

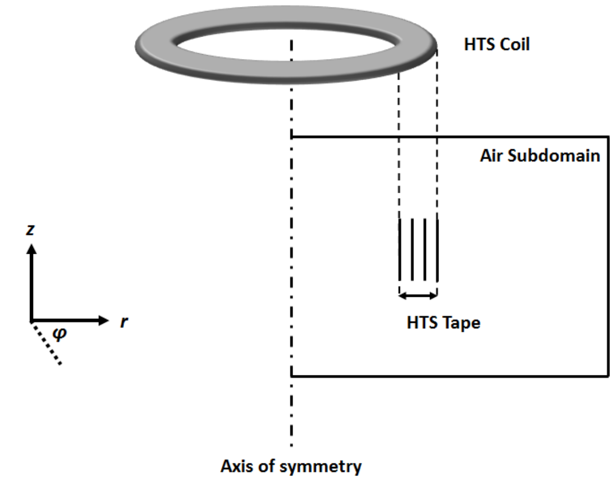

Fig. 1. Schematic configuration of the HTS coil model. For clarity, only four HTS tapes are shown, surrounded by air.

J_0 in (1) is not constant if the effects of non-uniformity along the width of the tape are considered, and decreases gradually along the width towards the edge of the tape based on [7], [22]. Equation (2) is therefore proposed to describe this non-uniformity and to simplify the analysis:

$$J_0(x) = J_0(1-\exp(-((|x|-0.006)/0.001)^2)m) \tag{2}$$

Where $J_0(x)$ represents the critical current density in self-field along the width of tape, parallel to the x-axis. The maximum critical current density of the tape J_0 is the same as that given by (1). The degradation of the critical current density is located along a 2 mm region between ±4 mm and ±6 mm towards both edges of the tape, for a tape of width 12 mm, as shown in Fig. 2. Here $x = 0$ represents the center of the tape. The critical current density decreases to $(1\text{-}m)J_0$ at the edge of the tape, where m is the percentage of degradation at the edge. Here, two cases ($m = 0.4$ and 0.8) are chosen to represent non-uniformity along the width, which is compared to the ideal situation (i.e. uniform properties along the tape width, shown as the solid line in Fig. 2).

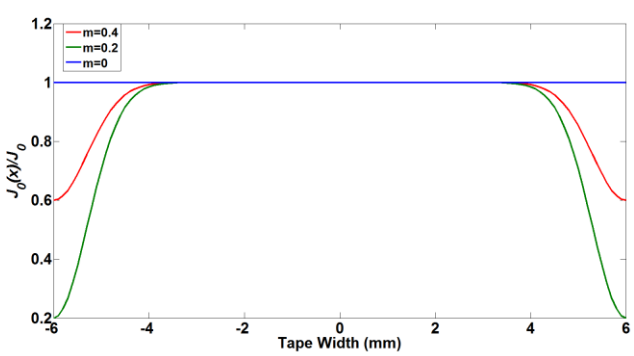

Fig. 2. Current density distribution representing non-uniformity along the width of the tape.

Non-uniformity along the length of the tape simply means there is a decrease in the critical current density in some turns in the HTS coil. For example, for a 10 turn coil, the innermost turn can be numbered as turn 1, and the outermost one as turn 10. Two cases are analysed: where the critical current density decreases for a length of 10 and 20% of the total length of the tape forming the coil. For example, 10 and 20% critical current degradation along the length in a 10 turn coil, corresponds to turn 1 and turns 1 + 2, respectively. In these cases, a 50% reduction in J_0 in these turns is considered for purposes of analysis. The critical current density remains as the original value J_0 for all other turns.

B. Analysis of DC Properties

An analysis of the DC properties (i.e. critical current) was carried out to determine the maximum allowable current, which in an HTS machine, for example, has a large impact on the size and weight of the machine, and hence on power density. In order to draw suitable conclusions, a comparison of DC properties is made for 10, 20 and 50 turn coils. The critical current is assumed here to be the operating current when the voltage across the coil terminals is equivalent to an average electric field (voltage per length) of $E_0 = 1$ μV/cm, consistent with the standard definition of critical current, I_c, for HTS materials. The applied current is ramped up to a maximum current at the same ramp rate of 75 A/s. The average electric field is calculated by the integration of local electric field [7], [14], as given by (3).

$$E_{ave} = \sum_{n=1}^{N} \int 2\pi r_n E \cdot ds \Big/ \sum_{n=1}^{N} 2\pi r_n \cdot s \tag{3}$$

Here N is the total number of turns in the coil, r_n is the radius of the n^{th} turn and E represents the local electric field.

In order to analyse and compare different coils, three situations were considered, each with the different tape characteristics shown in Table I. The modelled DC properties for these characteristics for a 50-turn coil are compared in Fig. 3.

TABLE I
MODELS UNDER ANALYSIS

Model 1	Anisotropy only
Models 2a, 2b	Anisotropy + width non-uniformity (a) m = 0.4, (b) m = 0.8
Models 3a, 3b	Anisotropy + length non-uniformity (a) 20% degraded, (b) 10% degraded

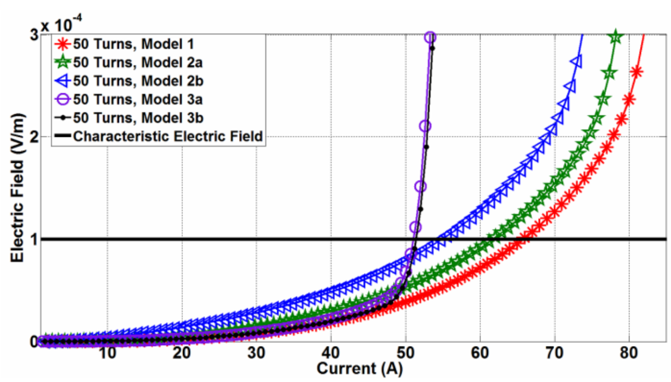

Fig. 3. Comparison of the DC properties of a 50-turn coil for the three example cases.

The critical current of Model 1 is only around 66 A when the effect of the anisotropy on the coil properties is considered, as shown in Fig. 3. The critical currents in Models 2a and 2b (m = 0.4, 0.8) are smaller than that of Model 1. However, the critical current decreases most for Models 3a and 3b (20% and 10% degradation along the length). Similar results are also observed for the cases of 10 and 20 turn coils. Therefore, it can be concluded that non-uniformity along the length has greater effect on the DC properties of the coil than does the non-uniformity along the width. At the same time, it should be noted that Models 3a and 3b have I-V curves of different shapes compared with the other models. The slopes of the I-V curves in Models 3a and 3b are steeper, meaning that the voltage is developed more rapidly.

In order to analyse these findings further, a comparison of particular I-V curves is presented in Figs. 4 and 5, including the results of the 10 and 20 turn coil models. Fig. 4 shows the curves for Models 1, 2a and 2b (comparing uniformity and non-uniformity along the width), and Fig. 5 shows the curves for Models 1, 3a and 3b (comparing uniformity and non-uniformity along the length).

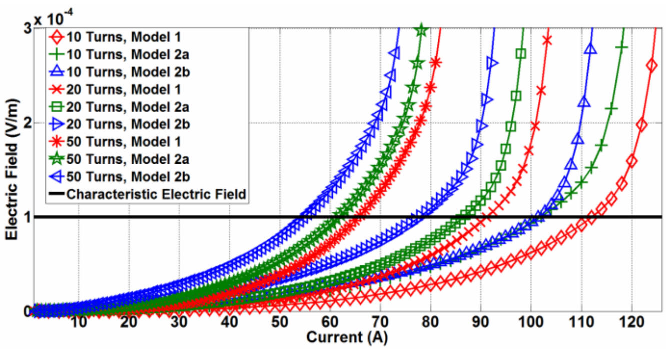

Fig. 4. Comparison of the curves of the DC properties of Models 1, 2a, and 2b.

It can be seen from Fig. 4 that, although Models 2a and 2b almost have the same critical current for the 10 turn coil, greater current degradation along the width (i.e., Models 2b, m = 0.8 for different turns of coil) results in decreasing the critical current of the coil, especially for relative large number of turns in the coil (i.e., 20 and 50 turn coils). There are two reasons for this: increased current degradation along the width lowers J_c more significantly at the edge of the tape, as shown in Fig. 2, but also the larger local magnetic field from increasing numbers of turns in larger coils acts to reduce J_c because of the anisotropic behaviour $J_c(B,\theta)$.

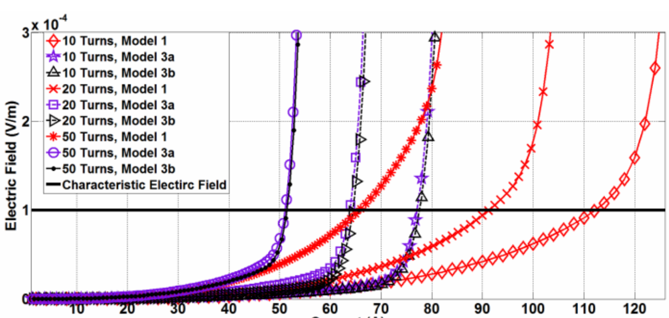

Fig. 5. Comparison of the curves of the DC properties of Models 1, 3a, and 3b.

It can be seen from Fig. 5 that either case of degradation along the length (10 or 20% of the coil) has almost the same effect on the critical current. Therefore, increasing the percentage of the degraded region of the coil almost has no effect on the DC properties of the coil, which are determined by this region of lower J_c. This is explained by the much larger voltage developed in the region with lower J_c, relative to the other regions when the same current is applied. The voltage of the entire coil is equal to the sum of the voltages developed in each turn. Even though the length of the degraded region is very small, the average electric field over the coil is still determined by the large voltage developed in the degraded region. Therefore, if the critical current degradation occurs at some region, the critical current density of the whole coil will decrease by almost the same extent.

It can be seen from Fig. 5 that the critical currents of the 10, 20, 50 turn coils are around 77 A, 64 A, 51 A separately for both Models 3a and 3b. The critical currents of Model 1 are 113 A, 91 A, 66 A for the 10, 20, 50 turn coils. Therefore, the decrease in critical current due to non-uniformity along the length is roughly 32%, 29.6% and 22.5% for 10, 20, 50 turn coils for the two degradation levels assumed. It can be concluded that the increase of number of turns in the coil can reduce the effect on non-uniformity along the length. The main reason may be the fact that critical current has already been reduced due to increased number of turns in the coil. To be precise, the increase in number of turns generates a higher magnetic field for the same applied current, which lowers the critical current of the coil due to the anisotropy. Thus, larger coils are less sensitive to the effects of length non-uniformity.

C. *Analysis of AC Properties*

AC loss can be a significant problem for HTS devices exposed to a time-varying current or magnetic field, which can influence directly the refrigeration load and therefore decrease the device efficiency and increase the complexity of design [6], [15]. The simulation of an HTS coil with anisotropy and non-uniformity is an effective method of analysing the effects on AC loss arising from practical considerations of the material properties. The AC loss in J/Cycle/m can be calculated by integrating the product of the local electric field and current density [18], [23]. The AC loss for the different models is compared for the 50 turn coil in Fig. 6.

It can be seen from Fig. 6 that non-uniformity along the length (Models 3a and 3b) and width (Models 2a and 2b) can both make a contribution to the AC loss. Non-uniformity along the width has greater effect on the AC properties than non-uniformity along the length. These conclusions are also suitable for the cases of 20 and 10 turn coils.

A comparison of AC loss is presented in Figs. 7 and 8 in order to investigate the detailed effects of non-uniformity on AC properties, including the results of the 10 and 20 turn coil models. Fig. 7 shows the curves for Models 1, 2a and 2b (comparing uniformity and non-uniformity along the width), and Fig. 8 shows the curves for Models 1, 3a and 3b (comparing uniformity and non-uniformity along the length).

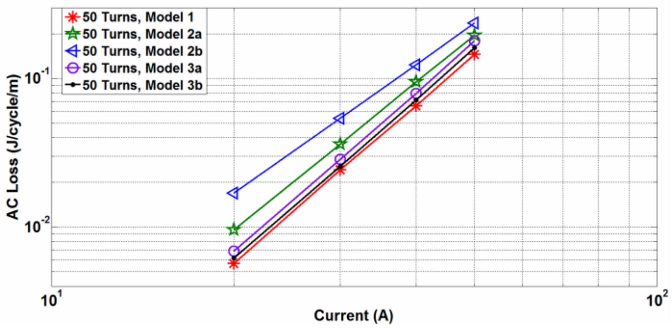

Fig. 6. Comparison of the AC loss for the three models for the 50 turn coil.

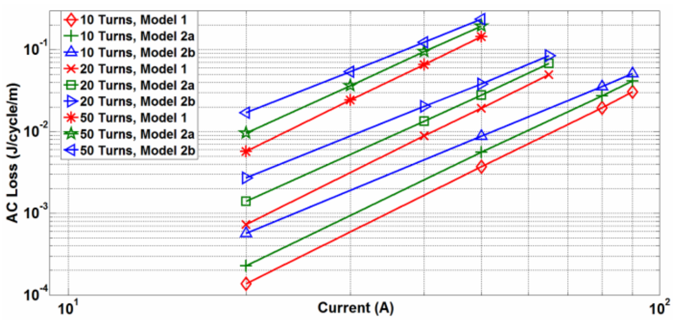

Fig. 7. Comparison of the AC loss for Models 1, 2a, and 2b, comparing uniformity and non-uniformity along the width.

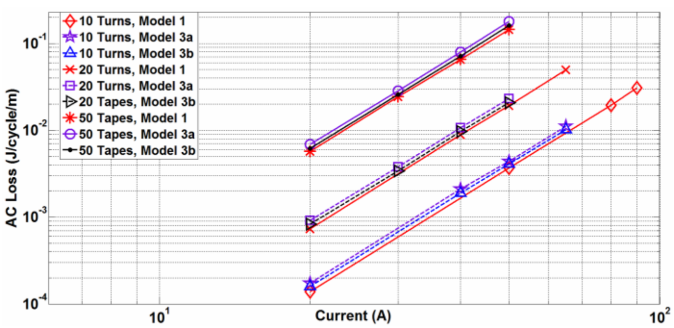

Fig. 8. Comparison of the AC loss for Models 1, 3a, and 3b, comparing uniformity and non-uniformity along the length.

Greater current degradation along the width (i.e., Models 2b, m = 0.8 for different turns of coil) generates higher AC loss compared to the other cases, as shown in Fig. 7. The main reason for this is that current is carried mainly at the edge of tape, particularly for low applied currents, due to the dynamics of the flux penetration for a large aspect ratio coated conductor [19], which corresponds to the degraded region in Models 2a and 2b. A higher AC loss voltage will be generated in the degraded region of lower J_c and therefore the AC loss will be increased for the same applied current.

From Fig. 7, increasing the applied current can decrease the difference of AC loss generated by Models 1, 2a and 2b. The main reason is that more current is carried in the center region of the tape, which is the unaffected region, when the applied current is increased. When more current flows in this unaffected region, there is no obvious difference in AC loss for Models 1, 2a and 2b. Thus, a large applied current, below the critical current, can help reduce the effects on AC loss arising from non-uniformity along the width.

Current degradation along the length (i.e. Models 3a and 3b for different turns of the coil) has a minimal contribution to AC loss, compared to tapes with anisotropy only (i.e. no introduced non-uniformity), as shown in Fig. 8. This may arise from the fact that the region of significant current degradation (e.g. critical current degraded to 0.5 J_c) only occupies 10% and 20% of the whole region of the coil. This conclusion could be very useful for the practical application of HTS tapes in devices. Non-uniformity along the length is a very common phenomenon for HTS tapes, but the percentage of region with serious current degradation is very small along the length of the HTS tape, as in the case of [2]. The effects of this can therefore be ignored for rough estimates of AC loss in HTS devices, which will simplify significantly the design process.

III. Conclusion

Numerical analysis of anisotropy and non-uniformity on the properties of HTS coils have been performed using a 2D finite element model implemented in Comsol Multiphysics 4.3a.

It is found that non-uniformity along the width can decrease the critical current density and increase the AC loss in HTS coils, and that more extensive current degradation along the tape width can increase these effects. Although this has large effects on the AC properties (i.e. AC loss), it has small effects on the DC properties (i.e. critical current) of the coil. The application of a large input current, below the critical current, can help decrease the AC effect arising from the non-uniformity along the tape width.

It can be also seen that non-uniformity along the tape length can decrease the critical current density and increase the AC loss. Increasing the percentage of degraded turns in the coil, for example from 10% to 20%, for a low percentage of turns in the coil with current degradation along the length, has a minor effect on the AC and DC properties of the coil. Most importantly, non-uniformity along the length has a minor effect on the AC properties (i.e. AC loss), but has a relatively large effect on the DC properties (i.e. critical current). This conclusion can help simplify the design process of HTS device based on AC loss considerations. Finally, the increase in number of turns in the coil, for example from 10 to 50 turns in one coil, can decrease the effects of degraded critical current due to non-uniformity along the tape length.

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