Queueing Systems Vol. 100
Special Issue: 100 views on queues

Resource pooling in electricity grids: wind, storage and transmission

Frank Kelly

February 14, 2022

1 Introduction

The decarbonisation of electricity networks gives rise to many interesting problems for mathematical scientists, reviewed in [1,2]. Some of the most challenging involve simultaneously game theory, randomness and network constraints. Our aim in this note is to illustrate this using a very simple model of storage across a transmission grid.

As motivation, Figure 1 illustrates the transmission constraints that were active at various times during a recent week in Great Britain. Wind generation in the north and west was often more than could be transferred to demand elsewhere. If the grid had *no* transmission constraints, then the generators whose auction bids are lowest would be used first: this ranking is called the *merit order*. The constraints illustrated in Figure 1 forced departures from the merit order which cost the grid about £70 million that week [3]. These costs are expected to rise to several billion pounds annually in the next few years [4].[1]

The analysis of the auctions producing the merit order is in itself an important problem (reviewed in [5]), and a not surprising feature is that a reduction in competition between generators can lead to perverse incentives that substantially reduce overall benefit.[2] Note that the constraints in Figure 1 dynamically fragment the market for generation: when an area cannot get more energy from outside, there are fewer competitors to moderate the bidding strategies of generators located within the area.

Centre for Mathematical Sciences, University of Cambridge, Cambridge CB3 0WB, United Kingdom. E-mail: fpk1@cam.ac.uk. The author is indebted to Chris Dent, Paul Plumtre, Stan Zachary and Bert Zwart for helpful discussions.

[1] The transmission constraints are also called *thermal* constraints. In an AC network there are also *voltage* and *stability* constraints which impose relatively low costs in the GB transmission network [3,4], motivating our later use of the *DC approximation*.

[2] To indicate the potential magnitude of this effect, suppose electricity demand is $D(p) = p^{-1/\alpha}$ as a function of price, where $-1/\alpha$ is the price elasticity of demand, and total generating capacity is $\sum C_n$. Even if generator m has *no* short-run production costs, generator m will increase its profit by *lowering* the capacity C_m it offers iff $C_m > \sum C_n/\alpha$. A short-run elasticity of -0.1 [6], corresponding to $\alpha = 10$, would require a generator to contribute no more than 10% of total capacity to avoid this perverse incentive.

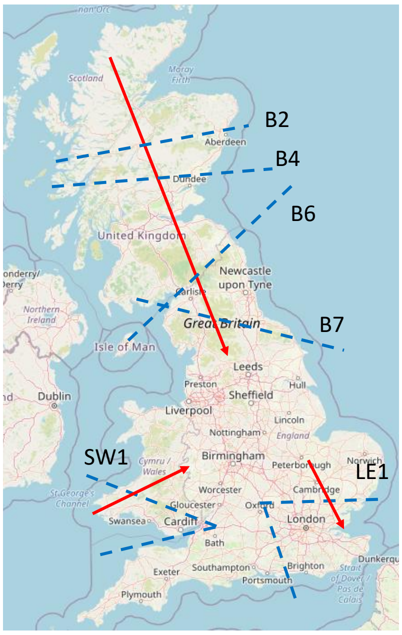

Fig. 1 The transmission constraints that were tight in Great Britain at times in the week ending 19 December 2021 are indicated by interrupted lines and the direction of flow by solid arrows [3].

2 Problem statement

A graph has nodes labelled $i \in \mathscr{I}$. At time $t \in \mathscr{T}$ node i generates an amount $w_i(t)$ of wind energy, and has a demand for energy of d_i. Energy can be transferred from node i to j, subject to transmission constraints, and energy can be efficiently stored at node i, subject to storage constraints. If wind energy cannot be used, stored or transferred, it is lost. If demand for energy cannot be met from wind energy, it is met from other (local, carbon generating) sources, and the aim is to meet energy demand from wind rather than these other sources.

Consider, then, the following finite horizon optimization problem. Maximize

$$\sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{I}} w_i(t)$$

subject to the constraints

$$s_i(t+1) = s_i(t) + w_i(t) - d_i - \sum_{j \neq i} x_{ij}(t) \tag{1}$$

where $0 \leq s_i(t) \leq S_i, 0 \leq w_i(t) \leq W_i(t)$ for $i \in \mathscr{I}$ and for $t \in \mathscr{T}$. Here S_i is the storage capacity at node i and $W_i(t)$ is the wind energy available at node i at time t. Note that the wind energy generated $w_i(t)$ may be less than $W_i(t)$ if it is not possible to use, store or transfer all the wind energy available. The energy transferred from node i to node j at time t is $x_{ij}(t) = -x_{ji}(t)$, where $|x_{ij}(t)| \leq X_{ij}$ for $i, j \in \mathscr{I}$ captures the finite capacity of the transmission grid.

As expressed, this problem is a classical network flow problem, where the max-flow min-cut theorem holds. The minimum cut, however, is defined on the graph with nodes $(i,t) \in \mathscr{I} \times \mathscr{T}$ where each node has a directed edge $(i,t) \to (i,t+1)$ of capacity S_i as well as its edges with (j,t) for $j \in \mathscr{I}$. Thus a cut in this graph could, for example, correspond to constraint B7 on Monday and constraint B4 on Tuesday with storage in the intermediate area going from full to empty in the period between.

3 Discussion

But of course the available wind energies $\{W_i(t)\}$ are highly variable and not easy to predict. Is there a modelling approach to the resulting stochastic problem which keeps the simplicity of the notion of a cut? Such a notion would be helpful for insight into architectural issues, such as whether storage should be considered: as just another competitor in the auction market for generation; or as part of the regulated monopoly transmission network, in view of its formal comparability with transmission links and its role in pooling resources and reducing perverse incentives for generators.

As a simple illustration, suppose that $S_i = S$ for each $i \in \mathscr{I}$ and consider the effect of increasing S. For S small enough, storage makes no difference to the cut; there is no linkage between different time periods, each potentially having a geographic cut. And for S large enough, wind energy is pooled across time periods and the transmission network has only to cope with the mean geographic imbalances between wind and demand. The intermediate regime is where the interplay between wind, storage and transmission produces cuts involving storage as well as transmission.

If there is only one node the recursion becomes the familiar queueing recursion

$$s_1(t+1) = [s_1(t) + W_1(t) - d_1]_0^{S_1}$$

for a buffer of size S_1. In queueing networks, and in the Internet, the idea of a pooled resource has been influential. A comment from [7] is that “Resource pooling is such a powerful tool that designers at every part of the network will attempt to build their own load-shifting mechanisms. A network architecture is effective overall, only if these mechanisms do not conflict with each other.”

References

1. National Academies of Sciences, Engineering, and Medicine. Analytic research foundations for the next-generation electric grid. National Academies Press, 2016.
2. Mancarella, P., Moriarty, J., Philpott, A., Veraart, A., Zachary, S., Zwart, B. (eds.): The mathematics of energy systems. Phil. Trans. Roy. Soc. **379** (2021)
3. National Grid Electricity System Operator. Operational Transparency Forum, 22 December 2021.
4. National Grid Electricity System Operator. Modelling constraint costs, Network Options Assessment 2020/21.
5. Acemoglu, D., Kakhbod, A., Ozdaglar, A.: Competition in electricity markets with renewable energy sources. Energy Journal **38** KAPSARC Special Issue (2017)
6. Csereklyei, Z.: Price and income elasticities of residential and industrial electricity demand in the European Union. Energy Policy **137**: 111079 (2020)
7. Wischik, D., Handley, M., Braun, M. B.: The resource pooling principle. ACM SIGCOMM Computer Communication Review, **38**(5): 47-52 (2008)