Growth and Human Capital: A Network Approach *

Short Title: Growth, Human Capital and Networks

Tiago V. V. Cavalcanti   Chryssi Giannitsarou

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Abstract

We study the interactions of human capital, growth and inequality by embedding networks into an endogenous growth model with overlapping generations. Human capital depends on investment in education and the average human capital of a household's neighborhood. High network cohesion leads to long run equality, while for low network cohesion inequality is high and persists more often. During transition, high overall growth is achieved when the network has high degree centralization, and high individual growth is achieved when the household has low human capital relative to its neighborhood and is located in a neighborhood with high average human capital.

Keywords: Human capital, growth, inequality, local externality, networks

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Corresponding author: Chryssi Giannitsarou, Faculty of Economics, University of Cambridge, Austin Robinson Building, Sidgwick Avenue, Cambridge CB3 9DD, UK. *E-mail*: cg349@cam.ac.uk.

This paper revisits an old question with a new approach: what is the role of local externalities for the accumulation of human capital, and therefore for economic growth and inequality? We assume that local externalities represent peer effects via a network structure and investigate how societies that are populated by agents with different initial human capital levels, but are otherwise identical in terms of economic primitives such as preferences, technology and endowment, can have different equilibrium dynamics and long run behavior, depending on whether and how economic agents are linked.

Our model is a stylized economy with overlapping generations, in which altruistic parents allocate their income between consumption and investment in the education of their offspring. A key ingredient of our analysis is the evolution of human capital: we assume that future human capital depends on two factors, namely investment in education and a local externality. Our novelty is in the way that the local externality is modeled. We describe the social structure by a connected network and assume that the local externality is the average human capital of the household's network neighbors.[1] There are four main advantages of modelling local externalities using networks, which also differentiate our analysis from previous contributions. First, by using networks, we can disentangle differences in the dynamics and possible long run outcomes that would not be detectable in a framework with a reduced form local externality. Understanding such differences is important for designing policies that promote high growth and high human capital levels in the long run. Second, a local externality modeled by a network can incorporate as special cases a variety of other known, analyzed frameworks, by exploiting the fact that network neighborhoods can be overlapping and do not need to be restricted to isolated communities. Therefore, our general framework can go beyond such stylized benchmark frameworks and generate more informed analysis. Third, modelling households as discrete nodes on a network, allows us to make concrete statements about the evolution of human capital and income of individuals and address topics beyond the behavior of macroeconomic aggregates, such as social mobility. Fourth, our analysis provides an operational framework for modelling local externalities, which is useful for quantitative purposes, as we illustrate in Section 5.

Our results can be collected into two groups of statements, one regarding long run outcomes and another on transition dynamics of the whole economy and individual households. First, heterogeneity in initial

[1] In this respect, our paper borrows the idea of Coleman (1988) that social capital or networks might be an important input in the formation of human capital.

human capital may imply long run outcomes in which inequality persists. We show analytically the non-trivial but very intuitive result, that if the network that describes the economy has enough *cohesion*, then human capital heterogeneity and thus inequality is eliminated in the long run, but at the cost of lower economic growth.[2] The threshold for which this happens depends on the incentives households have to privately invest in the education of their children. If a large amount of education is provided as a public good and households have less urgency to individually invest in education, then the network externality effect for human capital accumulation becomes relatively more important; for a connected network, less cohesion is required in order to smooth out differences across households and reach equality in the long run. This result is very powerful because it links a simple, one-dimensional network statistic (network cohesion) to long run outcomes with income equality. When the condition linking network cohesion and investment in education is not satisfied, we show numerically that heterogeneity and inequality persist in the long run. Moreover, the lower the network cohesion is, the higher the long run growth and inequality are.

In the long run, along any balanced growth path, the human capital for all households grows at the same growth rate. However, during transition, the human capital of individual households may grow at different rates, depending on the initial condition and the location of the households on the network. During transition, the human capital of a household will grow faster when it is relatively low compared to the average human capital of the household's network neighbors, and the household's neighborhood has high human capital relative to the average of the economy, i.e. whenever the household is a 'small fish in a big pond'. This decomposition for individual growth provides an implicit prediction regarding the chances of a household for upward mobility. By being located in a relatively affluent neighborhood, a low income household can grow faster than when it is located in low income neighborhoods. Therefore younger generations have more chances of climbing upwards in the distribution ranking. Recent work by Chetty *et. al.* (2014), Chetty and Hendren (2015) and Chetty *et. al.* (2015) provides strong evidence that supports exactly this view, i.e. that when young children from low income households relocate with their families to high income areas, they are very likely to have substantially higher income when they become adults than their parents, therefore increasing their chances of upward social mobility.[3]

[2] *Network cohesion* is a statistic introduced and analyzed extensively in a companion paper by Cavalcanti *et al.* (2015). We provide the formal definition in Appendix D and some discussion in Section 3. Network cohesion can be interpreted as a measure of how uniform or fragmented a network is, or as a measure of the tendency of a network to synchronize, and is related to spectral homophily (see Golub and Jackson, 2012).

[3] In fact, the work by Chetty and Hendren (2015) and Chetty *et al.* (2015) suggests causal relationships between education

Turning to the growth rate of average human capital during transition, we do extensive numerical experiments that resemble impulse response analysis. Specifically, we assume that before period $t = 0$, the economy has converged to a balanced growth path with equality and then in some period, an arbitrary household gets a positive exogenous shock which increases its human capital relative to the rest of the economy. We then trace the effect of this shock to the whole economy and show that network characteristics are very important for the transition. We find that the growth rate of average human capital during transition is highest whenever the network has both of the following two characteristics: (i) there are many nodes linked to the shocked one, so that many nodes benefit from the local externality (local positive shock) and (ii) the non-shocked nodes have small degrees, so that this positive effect is not mitigated by the inactivity of other non-shocked nodes. Networks that satisfy these properties have high degree centralization; the star network has the highest degree centralization and also generates the highest growth rate during transition when the centre is shocked. Finally, we show that network cohesion also matters for the speed of convergence to the balanced growth path and for the levels of inequality during transition. Convergence is faster, and consequently inequality will generally be lower during transition, for networks with cohesion that is far from the threshold that determines the long run equality outcome.

Our work belongs to the vast literature that studies the relationship and interactions of inequality and growth. This literature is reviewed in Barro (2000), Helpman (2004) and Garcia-Penalosa and Turnovsky (2006). Lucas (1988) emphasizes the role of external effects on human capital accumulation and productivity. Following this, there is a large number of articles that also assume spillover effects on human capital formation. For instance, De la Croix and Doepke (2003) investigate how inequality affects economic growth in a model with fertility differentials. There is also an important strand of literature that examines the importance of capital market imperfections for human capital formation and the dynamics of inequality and growth (e.g. Galor and Zeira, 1993; Banerjee and Newman, 1993). This literature generally does not model network structures explicitly. Bénabou (1993) shows that local externalities affect the incentives of agents to segregate and examines the effects of this on productivity. His model, however, is static and focuses on location choice. Durlauf (1996) also demonstrates how endogenous community formation can generate segregated societies resulting in inequality persistence and poverty traps. Kempf and Moizeau (2009) examine

quality and peer effects, and individual development and upward mobility. This provides extra support for our assumptions on the technology for accumulation of human capital.

the dynamics of inequality and growth by assuming that economic agents belong to neighborhoods (clubs), the membership of which changes endogenously, depending on the performance of agents in terms of human capital levels.

More closely related to our paper is the work of Bénabou (1996), who studies a similar question to ours and investigates how social community structure affects growth and inequality. In his model, the acquisition of human capital also reflects the influence of family, local and economy wide factors.[4] Last, in a more recent article, Mookherjee, *et. al.* (2010) emphasize that geographical location affects parents' aspirations. In their model, which features an exogenous location, a household will have higher aspirations if it lives in a neighborhood with a large fraction of educated neighbors.

An important difference of our paper to this literature is that we explicitly model networks and focus on how outcomes depend on the different networks, without the need to only explore extreme scenarios of complete segregation versus full integration. Our framework is more tractable and flexible for analyzing a variety of networks representing how different communities are interlinked (see also De la Croix and Doepke, 2004; Fernandez and Rogerson, 1996). Due to the tractability of our framework, we are able to map simple summary measures such as network cohesion and degree centralization onto the dynamics of the economy. Last, a major difference of our model to the existing literature is the fact that we treat households as discrete, individual agents instead of a continuum of different types. This means that we can trace the performance of individual households during transition, and make statements about topics that other papers cannot address, e.g. social (upward) mobility. To our knowledge, we are the first to explicitly incorporate networks into a standard model of overlapping generations and endogenous economic growth.[5]

There is also a growing literature relating networks to a variety of other economic questions. Goyal (2007) and Jackson (2008) provide overviews of the recent research and models on networks in economics and techniques for analyzing different economic issues. Our article is closer to papers that investigate the effects of social networks on equilibrium allocations. For example, Calvó-Armengol and Jackson (2004) embed networks in a model of the labor market and study their implications on employment status and unemployment duration. Chantarat and Barrett (2012) build a two period model with two technologies

[4] The similarities and differences of our paper with the framework of Bénabou (1996) are discussed in more detail in later sections, once we have presented our model and laid out our main assumptions.

[5] In a recent article, Fogli and Veldkamp (2012) also explore how stylized types of regular networks affect economic growth. The focus of their paper is on technology diffusion, while we investigate externalities on human capital dynamics. Also, we do not restrict attention to specific network structures.

(low and high) to investigate the effects of networks on poverty traps and Van der Leij and Buhai (2010) study how social interactions map into path dependence on occupational segregation; as in Bénabou (1993), they show that segregation is an equilibrium outcome and can be welfare enhancing. Ghiglino and Goyal (2010)„ show how social networks shape equilibrium allocations and prices in a standard exchange economy in which the utility of an individual is negatively affected by the consumption of their neighbors. Last, Carvalho (2010) and Acemoglu *et. al.* (2012) use networks to depict intersectoral trade relationships and show how these can aggravate macroeconomic fluctuations via aggregation of idiosyncratic sectoral shocks.

The paper proceeds as follows. Section 1 presents the environment in terms of economic primitives: preferences, endowments and technologies. We also explain how we embed networks to the model and define the competitive equilibrium. In Section 2 we derive analytical results, including the main proposition which shows how convergence to a long run equilibrium with equality depends on the network cohesion measure. In Section 4 we present an extensive numerical investigation of long run outcomes, as well as transition for all non-labeled connected networks with $n = 4, 5, 6$ and 7 nodes. Sections 5 and 5 close the paper with some further comments on related literature, possible extensions of the current framework, as well as a short discussion of an empirical application of our framework using Swiss school data.

1. The Model

1.1. *Environment*

The economy consists of n households indexed by $i = 1,\ 2, ..., n$. Throughout the paper, we use the terms *household* and *node* interchangeably.[6] Each household lives for two periods, childhood and adulthood and consists of one adult and one child, such that the household population is constant. The initial endowment of human capital is heterogeneous across households. The adult in node i at time t has human capital h_{it} and cares about her own consumption c_{it} and the future human capital of her offspring, h_{it+1}. She works and her productivity is proportional to her skill. Production takes place in a centralized market. The decisions of household i at time t are made by the parent, who chooses consumption and investment in the education of her offspring, e_{it}. At the end of period t, the parent of household i dies, and the offspring becomes the adult/parent in the next period $t + 1$, who then has a new child, and so on.

[6] We use the term *node* to refer to the elements of a network that here are the households. We will be specific about the definition of the network in a later section.

There is a continuum of measure one of identical firms, with one production input, human capital. The production technology is linear in the efficiency unit of labor, H_t:

$$Y_t = H_t. \tag{1.1}$$

Since the labor productivity of workers is proportional to their human capital, the income for household i at time t is equal to the human capital of the parent, h_{it}. Note that the introduction of physical capital does not change any of the qualitative implications of the model in a significant way. Therefore, in order to focus on the effects of different network structures on growth and inequality, we abstract from physical capital accumulation.

The utility function for the household i is given by

$$\ln c_{it} + \psi \ln h_{it+1},$$

where $\psi > 0$ is the altruism factor.

The human capital h_{it+1} for the child in node i may generally depend on various factors. To gain intuition, here we focus on only two factors namely *investment in education*, captured by e_{it}, and *peer effects* (via the local externality), captured by $\bar{h}_{it}$. We then express the evolution of human capital as

$$h_{it+1} = (\theta + e_{it})\bar{h}_{it}, \tag{1.2}$$

where $\theta > 0$. The assumption $\theta > 0$ allows for the possibility that the parent may opt not to educate her child, since it implies that $h_{it+1} > 0$ even in the case of $e_{it} = 0$. From a technical perspective, it also ensures that for any non-zero initial human capital, h_{it+1} can never be (optimally) zero, and therefore the utility function of the household is well-defined. The parameter θ has the natural interpretation of the basic public education provision, which is available to everyone and ensures that all agents have some minimum level of human capital.[7]

[7] In the most general version of the model we assume that it depends multiplicatively on four factors. First, it depends on education e_{it}; second, on the inherited human capital of her parent h_{it}; third, on a local externality $\bar{h}_{it}$ defined as the average human capital of node i's neighborhood, which we will define precisely in Section 1.3; fourth on possibly a global externality, summarized by the average human capital $\bar{h}_t$ for the whole economy, which can be thought of as the human capital of the

The budget constraint for household i is given by

$$c_{it} + e_{it}p_t = h_{it}, \tag{1.3}$$

where $e_{it}p_t$ is the cost of education at period t. The problem of this household is then summarized as follows:

$$\max_{c_{it}, e_{it}} (\ln c_{it} + \psi \ln h_{it+1}) \tag{1.4}$$

$$s.t. \;\; c_{it} + e_{it}p_t = h_{it}, \tag{1.5}$$

$$h_{it+1} = (\theta + e_{it})\bar{h}_{it}, \tag{1.6}$$

$$e_{it}, c_{it} \geq 0, \tag{1.7}$$

$$h_{i0} > 0, \text{ given} \tag{1.8}$$

The solution to the household's problem is:

$$e_{it} = \begin{cases} 0, & \text{if } \frac{h_{it}}{p_t} \leq \frac{\theta}{\psi} \\ \frac{\psi}{1+\psi}\left(\frac{h_{it}}{p_t} - \frac{\theta}{\psi}\right), & \text{if } \frac{h_{it}}{p_t} > \frac{\theta}{\psi} \end{cases} \quad \text{and} \quad c_{it} = \begin{cases} h_{it}, & \text{if } \frac{h_{it}}{p_t} \leq \frac{\theta}{\psi} \\ \frac{p_t}{1+\psi}\left(\frac{h_{it}}{p_t} + \theta\right), & \text{if } \frac{h_{it}}{p_t} > \frac{\theta}{\psi} \end{cases}. \tag{1.9}$$

This means that if income relative to the price of education is sufficiently low for a given household, the parent will optimally choose not educate her children and she will therefore consume all her income. The prevalence of the corner solution depends on the importance of investment in education, captured by θ. As θ increases, i.e. as the public good provision of education increases, the need of a household to invest in further education of their offspring is less pressing, and therefore, more households optimally choose not to spend private resources on the education of their children but benefit instead from the local externality.

1.2. *Education*

We assume that children are educated by teachers under a common school curriculum. There is a notional continuum of teachers, whose average human capital is equal to the average human capital in the economy

teachers or the general level of the national school curriculum. We can then assume that

$$h_{it+1} = (\theta + e_{it})^{\eta} h_{it}^{1-\beta_1-\beta_2} \bar{h}_{it}^{\beta_2} \bar{h}_t^{\beta_1}$$

where $0 < \eta \leq 1$ and $0 < \beta_1 + \beta_2 \leq 1$, and the results shown in the paper carry through in a natural way.

$\bar{h}_t$. We assume that teachers could also work in the production sector, i.e. that the time used for education by teachers is time not allocated to production of consumption goods, which implies that the total cost of education for node i is $e_{it}\bar{h}_t$. As in De la Croix and Doepke (2003), not only do all parents face the same price of education, but as the average human capital grows, the price of education relative to the price of the consumption good increases. These assumptions are consistent with the empirical evidence provided by Theil and Chen (1995), who show that there is a positive correlation between income and the relative price of education in a cross section of countries. Also, the Commonfund Institute in the United States calculates the Higher Education Price Index (HEPI), which is an index of the cost of higher education in the United States. According to the HEPI (2009) report, from 1983 to 2009 the HEPI rose (it went from 1 to 2.79) by roughly 28% more than the Consumer Price Index (which increased from 1 to 2.18). A similar index constructed by *Universities UK* grew at a faster rate than the RPI index in the period 1996-2009.

There are different approaches in the literature to determine p_t in equilibrium. We could have assumed that investment in education is in terms of parent's time (home schooling), such that the budget constraint could be written as $c_{it} = h_{it}(1 - e_{it})$. This is a standard approach when agents are homogenous (e.g. Galor and Weil, 2000), since in this case it does not matter who provides education to the children (i.e. parents or teachers). However, when agents are heterogenous, this would imply that parents would face different prices for investment in education and prices would be lower for relatively poor households. As a result, in equilibrium parents would invest the same fraction of their income in education regardless of their income level.[8] We could also assume that the relative price of education in terms of the consumption good is constant over time, for example that $p_t = 1$, such as in Galor and Moav (2004). This would imply that education is relatively more expensive for poor than for rich parents, and consequently the latter will invest more in education than the former; however this does not allow for time variation in the relative price of education.

Since we observe changes in the relative price of education over time, we prefer the specification for which education is provided by teachers whose human capital level is equal to the average level of human

[8] Glomm and Ravikumar (1992) and Bénabou (1996) use this assumption, but they also consider another variable in the offspring's human capital formation, namely education quality, and parents face the same relative price for the quality of education. Although parents' time devoted to help their offspring to learn would be the same, investment in the quality of education varies with income.

capital. For $p_t = \bar{h}_t$, we have that households' decisions depend only on their relative human capital

$$x_{it} = \frac{h_{it}}{\bar{h}_t}. \tag{1.10}$$

Relatively rich households will always invest more in education than relatively poor parents.

In order to ensure that investment in education is strictly positive when all households are identical, we must assume that

$$\psi > \theta. \tag{1.11}$$

This is generally true when the society is highly altruistic (i.e. ψ is large) or for example when the threshold θ for having non-zero human capital is relatively small; in fact the parameter θ can be taken arbitrarily close to zero, to have a well defined utility function, but it can never be exactly zero. Finally, we assume that

$$\psi\theta \geq 1, \tag{1.12}$$

in order to ensure that the economy always has a positive growth rate.

1.3. *Local externalities and network structures*

The economy consists of the n nodes or households that are indexed by $i = 1,\ 2, ..., n$. Nodes may be separated or linked. The generic interpretation of a link is that it represents the local, peer effects that households may have on each other by interacting. The local effect may materialize in many different ways. For example, in Section 5, where we present an illustrative example, two nodes i and j are linked, whenever there is a school that children from both these two nodes attend.

We consider a static network in which the number of nodes n and links between them remain unchanged over time. A household born in node i always stays in this node. Let $g_{ij} \in \{0, 1\}$ be a relationship between two nodes i and j. It is assumed that $g_{ij} = g_{ji}$ (i.e. that the network is undirected) and that $g_{ij} = 1$ if there is a link between nodes i and j and $g_{ij} = 0$ otherwise. We also assume that $g_{ii} = 1$. This notation allows us to represent the network with an *augmented adjacency matrix* $\mathbf{G}$.[9] Given that we consider an undirected network, this is a symmetric matrix of zeros and ones, of which the $ij-$th entry is g_{ij}. We now formally

[9] In the terminology of the network literature, we include 'loops' apart from the usual links. In this case, the matrix that describes the network has ones in the main diagonal, which is why we call it *augmented*.

define the local externality variable $\bar{h}_{it}$ as the average human capital of node i's neighborhood, that is

$$\bar{h}_{it} = \frac{\sum_{j=1}^{n} g_{ij} h_{jt}}{\sum_{j=1}^{n} g_{ij}}. \tag{1.13}$$

Last, when node i is not linked to any other node, then $\bar{h}_{it} = h_{it}$.

Next, we highlight some specific, important assumptions behind the local externality formulation we have defined. First, this is only one of various possible ways of defining the local externality. Other formulations for $\bar{h}_{it}$ include an aggregate measure of the human capital of all neighborhood nodes, or a measure of human capital that calculates the average human capital of a node's neighbors plus the human capital of the nodes' neighbors, etc. We choose this average-based formulation as a natural starting point for analysis, that has a straightforward interpretation, as discussed in Golub and Jackson (2010) and Acemoglu and Ozdaglar (2011).

Second, the assumption that $g_{ii} = 1$ can be interpreted in two different ways. It can be thought of as the fact that a household i is part of its own neighborhood, and therefore it is sensible to include its human capital to the average of the neighborhood. Alternatively, one can think of it as the relative contribution of intergenerational transmission of human capital from parent to offspring. This formulation allows for well-defined non-zero human capital levels even when a household has no links with anyone else. It also allows us to consider extreme cases of the empty or disconnected networks and compare them to the structures that we are more interested in, namely connected networks.

Third, and related to the second point, we focus our analysis on cases of *connected* networks only.[10] If the underlying network were disconnected, the local (network) externality would lose its role as a potential vehicle for long run convergence. Consider for example a network with two components. By construction, our economy will behave as two separate worlds with no spill-overs from one to the other, that only 'communicate' to the extent that we average out the evolution of their human capital for comparison purposes. In the extreme case of an empty network, defined by $g_{ij} = 0$ for all $i \neq j$ (every man is an island), and in the absence of any other mechanism that generates feedback from the individual households to the aggregate economy, the dynamics for all households diverge.[11] On the other hand, when the network is connected,

[10] A network is *connected* if there is no isolated node, i.e. there is no node that has no links with any other node.

[11] As will become clear later, adding a global externality summarized by the average human capital of the whole economy $\bar{h}_t$ in the human capital accumulation function is enough to restore convergence to a balanced growth path, even if its contribution is small.

local effects from overlapping neighborhoods of the network that may partially interact with each other, will have a long run impact on the whole economy and will lead, as we will see, to balanced growth paths. The extreme case of a complete network (i.e. $g_{ij} = 1$ for all i, j) implies that $\bar{h}_{it} = \bar{h}_t$ and effectively turns the externality into *global* rather than local: everyone's human capital depends on the average human capital in the economy. It will generally be useful to have the complete network (global externality) as a comparison benchmark for the analysis of human capital dynamics.

We also explain here how our framework is related and differs from that of Bénabou (1996). In his paper, Bénabou (1996) focuses on two polar cases, that of *perfect segregation*, such that all households are sorted into completely homogeneous communities, and *perfect integration*, where all households benefit from the same levels and composition of local and global externalities. The former scenario corresponds to a world in our framework where the network is disconnected, but each component is itself complete, i.e. all nodes within the component are linked: this ensures that all households within a community (component) are identical, but communities do not interact with each other. Assuming the same degree of complementarity of individual human capitals at the local and global levels (i.e. in the notation of Bénabou (1996), assuming that $\sigma = \varepsilon$), allows us to map the latter scenario to a world where the network is complete and households can generally be heterogeneous. As in Bénabou (1996), we will show later that in this case, the economy always converges to a balanced growth path with equality. In addition to the analysis of Bénabou (1996), our novelty is to examine a whole range of economies between these two extreme cases, parametrized by the network structure. This allows us to quantify precisely under what conditions an *imperfectly integrated* society (i.e. a connected but not complete network) converges to long run equality. It will also allow us to analyze what determines long run outcomes where inequality persists, even if the economy does not exhibit complete segregation, when such conditions are not satisfied.

We close this subsection with a comment on our assumption of one household per network node. Generally, our framework allows us to give several interpretations to the network nodes. The first one, which we use throughout the paper, is that each node is simply a unique household. However, we can also interpret each node as a continuum of identical households, each with the same measure (e.g. measure one for simplicity) and interpret the links between them accordingly. Perhaps, more interestingly, we can also think of each node as a finite set of households that are all linked to each other (a community). This 'super-node', or region, would then be linked to another such super-node, if all its households are linked to the households of the other node. In other words, we can treat the network as an aggregation of a larger, finer network of many households with augmented adjacency matrix that contains block sub-matrices with ones in all entries. An example of such aggregation can be found in Cavalcanti *et. al.* (2015). If all regions have the same number of households, then the resulting aggregate network is non-weighted. If regions have different numbers of households, then the aggregate network will be described by a weighted adjacency matrix. In Cavalcanti *et. al.* (2015) we show that networks aggregated in this way share important properties, and therefore the properties of the local dynamics of the economy are similar under either underlying network structure. With this discussion in mind, we also make an implicit assumption that although the initial human capital may differ across nodes, it is identical within a node i, i.e. that all households in a region or node start with the same human capital.

This assumption can also be relaxed if one assumes that the number of households within each node is large enough. Under some mild and reasonable assumptions, and in a framework that is related to ours, Golub and Jackson (2012) show that initial heterogeneity within a node can be eliminated in one period, and that it is therefore possible to work directly with a representative household from each node. Proposition 3 from Golub and Jackson (2012), can be directly applied to our setting for all connected networks that satisfy the requirements of their Definition 3, i.e. sufficient density, no vanishing groups, interior homophily and comparable densities. Alternatively, we can think of the households of each node as a complete subnetwork, where each individual may have different initial human capital. As will become clear in Section 2, heterogeneity within a completely linked community can be eliminated quite fast and will always lead to an average, representative household (see Proposition 2 and its Corollary). We can then heuristically justify our assumption as if enough time has passed for internal (within community/node) heterogeneity to have been eliminated as we start the analysis of the dynamics.

1.4. *Competitive equilibrium*

We are now ready to define the competitive equilibrium in this economy. Given a network structure described by the matrix $\mathbf{G}$, and initial human capital of all households $\{h_{i0}\}_{i=1}^{n}$, with $h_{i0} > 0$ for all $i = 1, ..., n$, a competitive equilibrium at time $t = 1, 2, ...$ is a collection of households' allocations $\{c_{it}, e_{it}\}_{i=1}^{n}$, human capital $\{h_{it}\}_{i=1}^{n}$, firm output H_t, and price $p_t = \bar{h}_t$, such that (i) given education price $p_t = \bar{h}_t$, the parent of household i chooses $\{c_{it}, e_{it}\}$ to solve problem (1.4), subject to constraints (1.5), (1.6) and (1.7); (ii) output,

H_t, maximizes the representative firm's profits and (iii) the goods market clears, i.e.

$$\sum_{i=1}^{n} c_{it} + \sum_{i=1}^{n} e_{it} p_t = \sum_{i=1}^{n} h_{it} = H_t. \tag{1.14}$$

We note that optimal consumption and investment in human capital given by (1.9) do not depend on the local externality. This externality only plays an indirect role, via the expression for the evolution of human capital (1.2). Therefore, from the perspective of a household i, the equilibrium in a given period does not depend on neighbors' decisions. This is true because of the way we have defined the human capital formation technology and due to the assumption of a 'warm glow' utility function.[12] Consequently, in any given period t, the decision of household i is the same irrespective of the strategies of all other households, since this is based on *state* variables only and not on concurrent decisions by other households. This implies that there is a unique competitive equilibrium given the state variables in each period.

2. Human Capital Dynamics

We first provide some definitions that we will use in the remainder of the paper. Let the growth rate of the average human capital and the growth of human capital for household i be

$$\gamma_t = \frac{\bar{h}_{t+1}}{\bar{h}_t}, \text{ and } \gamma_{it} = \frac{h_{it+1}}{h_{it}}, \tag{2.1}$$

respectively. We also define the following

$$\gamma^* = \frac{\psi(1+\theta)}{1+\psi}. \tag{2.2}$$

[12] The 'warm glow' utility function, whereby the offspring's human capital enters directly in the utility function, has been widely used in economic growth models. See, for instance, Galor and Weil (2000) and De la Croix and Doepke (2003). In addition, it appears to be more consistent with empirical evidence (e.g. Andreoni, 1989).

In equilibrium, the dynamic system is described by the following system of difference equations (see Appendix A for a derivation):

$$x_{it+1} = \frac{\gamma_{it}}{\gamma_t} x_{it} = \frac{x_{it}\gamma_{it}}{\frac{1}{n}\sum_{k=1}^{n} x_{kt}\gamma_{kt}}, \text{ for } i = 1, ..., n, \tag{2.3}$$

$$\gamma_{it} = \left(\theta + \max\left\{0, \frac{\psi x_{it} - \theta}{1+\psi}\right\}\right)\left(\frac{\sum_{j=1}^{n} g_{ij} x_{it}}{\sum_{j=1}^{n} g_{ij} x_{jt}}\right)^{-1}, \text{ for } i = 1, ..., n, \tag{2.4}$$

$$x_{i0} > 0, \text{ for all } i = 1, ..., n, \tag{2.5}$$

$$\frac{1}{n}\sum_{i=1}^{n} x_{i0} = 1. \tag{2.6}$$

The above expressions define the evolution of relative human capital for all nodes, given a set of initial conditions that satisfy (2.5) and (2.6). The first condition (2.5) is satisfied since we assume that all households start off with some strictly positive human capital (see (1.8)), and the second condition (2.6) holds by the definition of relative human capital. We also note that the dynamics of the levels of human capital for all households, h_{it}, and the growth rate of the average human capital γ_t can be entirely and uniquely recovered by the dynamic system (2.3)-(2.6) via the definitions (1.10), (2.1) and the initial conditions $h_{i0} > 0$. We can then track average growth in this economy by γ_t.[13]

The dynamic system can be interpreted in an intuitive way that demonstrates the interactions between the network structure, the externalities and the evolution of human capital for each household. Expression (2.4) contains the term

$$\frac{\sum_{j=1}^{n} g_{ij} x_{it}}{\sum_{j=1}^{n} g_{ij} x_{jt}} = \frac{h_{it}}{\bar{h}_{it}} \equiv \bar{x}_{it} > 0 \text{ for all } i = 1, ..., n, \tag{2.7}$$

where $\bar{x}_{it}$ is interpreted as the human capital of household i relative to the average human capital of the household's neighborhood (*local relative human capital*). We can then see that the growth rate of a household increases in its relative human capital x_{it}, but decreases in its local relative human capital $\bar{x}_{it}$. The growth rate for household i can also be written as (see Appendix A):

$$\gamma_{it} = \begin{cases} \gamma^* \left(\frac{1+\psi}{\psi}\frac{\theta}{1+\theta}\frac{\bar{h}_{it}}{h_{it}}\right) & \text{, if } x_{it} \le \theta/\psi \\ \gamma^* \left(\frac{\theta}{1+\theta}\frac{\bar{h}_{it}}{h_{it}} + \frac{1}{1+\theta}\frac{\bar{h}_{it}}{h_t}\right) & \text{, if } x_{it} > \theta/\psi \end{cases}. \tag{2.8}$$

[13] In fact, in Appendix A we effectively present the reverse process of deriving (2.3)-(2.6) from the equilibrium dynamics of h_{it} and the initial conditions h_{i0}, $i = 1, ..., n$.

This expression has some powerful interpretation in relation to the individual growth, human capital and ranking of a household in the income distribution. In both cases, the growth rate of household i depends negatively on the household's human capital relative to that of its neighborhood: the lower the local relative human capital for household i, the faster i grows. As long as $x_{it} > \psi/\theta$, the deviation of the growth rate for household i from γ^* is therefore a weighted average of $\bar{h}_{it}/h_{it}$ and $\bar{h}_{it}/\bar{h}_t$. A household will grow fast, i.e. will have large γ_{it}, whenever it is located in a network neighborhood that is doing better on average than the whole economy (large $\bar{h}_{it}/\bar{h}_t$) and it is also doing worse relative to its own neighborhood (low local relative human capital $h_{it}/\bar{h}_t$). However, if a household is doing relatively well in its neighborhood but is located in a neighborhood where his peers are not doing so well relative to the whole economy, its growth slows down. This decomposition also provides an implicit prediction regarding the chances of a household for upward mobility: when located in a relatively high income neighborhood, a low income household can grow faster, than when it is located in low income neighborhoods; therefore younger generations have more chances of climbing upwards in the distribution ranking, thus increasing their chances of upward social mobility. The importance of each of these channels for the growth of a household depends on θ. As θ increases, that is when households care less about individually investing in education because for example there is more provision of free, state funded training, the weight on low local relative human capital is larger, since the specifics of the particular neighborhood are less important for individual growth.

We now return to the dynamic system for the evolution of relative human capital, and translate the system into the more compact form

$$\mathbf{x}_{t+1} = \mathbf{W}(\mathbf{x}_t), \tag{2.9}$$

$$x_{i0} > 0, \text{ for all } i = 1, ..., n, \tag{2.10}$$

$$\frac{1}{n}\sum_{i=1}^{n} x_{i0} = 1. \tag{2.11}$$

The following result establishes existence and uniqueness of a competitive equilibrium path, given the initial state of human capital across nodes, and follows directly from the fact that $\mathbf{W}$ is a single-valued function and the recursivity of the dynamic system. See also Theorem B.16 in Acemoglu (2009).

PROPOSITION 1. ***Existence and uniqueness of equilibrium path:*** *For a given set of initial conditions $\mathbf{x}_0$, the dynamic system of difference equations (2.3)-(2.6) has a unique trajectory (solution).*

Generally, a non-linear system of first order difference equations may be characterized by existence, or non-existence of steady-state (stationary) equilibria, and the existence of chaotic behavior. Asymptotically, the system may diverge to plus/minus infinity, converge to a steady state equilibrium, converge to a period orbit or behave chaotically. We focus our analysis on existence and characterization of steady-state equilibria, and in particular on only those that are consistent with a balanced growth path. A *steady state equilibrium* of the non-linear system (2.3)-(2.6) is a vector $\mathbf{x}$ such that $\mathbf{x} = \mathbf{W}(\mathbf{x})$. This highly non-linear system of n equations and n unknowns, has possibly many steady state equilibria. A *balanced growth path* is defined by imposing the same growth rate for all nodes, i.e. $\gamma_{it} = \gamma_t = \gamma$. Along the balanced growth path, the human capital of all nodes grows at the same rate γ and the relative human capitals denoted by x_i remain constant. In other words, we focus on steady state equilibria that satisfy the following, for some constant γ:

$$\frac{1}{n}\sum_i x_i = 1 \text{ and } \left(\theta + \max\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\}\right)\left(\frac{\sum_{j=1}^n g_{ij}x_j}{\sum_{j=1}^n g_{ij}x_i}\right) = \gamma \text{ for all } i = 1, ..., n. \tag{2.12}$$

We can then establish the following:

PROPOSITION 2. ***Existence of balanced growth paths:*** *All the steady-state equilibria of the system (2.3)-(2.6) are consistent with balanced growth.*

Proof. See Appendix B. ■

The importance of showing this statement is twofold. First, it excludes mathematical solutions to the system of equations $\mathbf{x} = \mathbf{W}(\mathbf{x})$ of the form $x_i = 0$ for some i (this hinges on the fact that the initial conditions of human capital is strictly positive); if they existed, such solutions would imply unbalanced growth in the long run. Second, having established that such solutions cannot exist and that $x_i \neq 0$ for all i, it is then shown that these steady state equilibria always imply balanced growth in the long run.

Such steady-state equilibria represent the different possible long run outcomes in our economy. The special case where $x_{it} = x^* = 1$, for all i as $t \to \infty$, corresponds to an economy with long run *equality.* Supposing that initially different nodes/households may have different levels of human capital, this long-run outcome prevails whenever the local network externality encourages more human capital accumulation by poorer, less educated households, relative to their rich, more educated peers, so that eventually the human capital levels of all households converge. In that case, the growth rate along the balanced growth path is

$$\gamma^* = \frac{\psi(1+\theta)}{1+\psi}. \tag{2.13}$$

Combining the above propositions with the fact that the vector $\mathbf{x} = (1, ..., 1)'$ is a fixed point of the system (2.9), formally ensures existence of a balanced growth path with equality.

Any other steady state equilibrium that allows for generally $x_i \neq x_j$ in the long run, represents outcomes where the local externality is not sufficient to make households converge to the same levels of human capital, and thus the economy is characterized by persistent long run *inequality*. In outcomes of long run inequality, the growth rate γ along the balanced growth path is given by

$$\gamma = \left(\gamma^* - \frac{1}{n}\sum_{i=1}^{n} \min\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\}\right)\left(\frac{1}{n}\sum_{i=1}^{n} \bar{x}_i\right)^{-1} \geq \gamma^*. \tag{2.14}$$

A few important observations are in place.[14] First, the growth rate along the balanced growth path generally depends on the network structure, through the adjacency matrix, whenever there is long run inequality. Second, the long run growth rate is larger, as the long run relative human capitals x_i take more extreme values, i.e. as they are more disperse. Third, generally the growth rate on the balanced growth path is always *larger* under long run inequality than under long run equality, that is $\gamma > \gamma^*$. This is because in this economy, each of the two factors that determine human capital serves a different purpose: investment in education is the engine of growth (without it, there would be no endogenous increase of human capital), while the local network externality facilitates redistribution, and thus more equality among agents. When inequality increases, households with high human capital tend to invest more in education, since it is relatively cheaper, and thus accelerating growth more than if there was equality.

A steady state equilibrium x_i, $i = 1, ..., n$ cannot be globally stable, since there can be multiple ones that can potentially be locally asymptotically stable. This assertion can be verified easily numerically, because by varying the networks and the key parameters of the model, we can find parameter configurations such that two sets of distinct initial conditions may lead to two different steady state equilibria.

For a given set of parameter values and a network, the steady state equilibria can be split into two types:

[14] The derivation of γ and proofs for the statements that follow can be found in Appendix C.

(a) those that are such that $x_i > \theta/\psi$ for all $i = 1, ..., n$, and (b) those for which there exist some i, such that $x_i \leq \theta/\psi$. Steady state equilibria from the latter group imply higher inequality in the long run: since the long run relative human capital for some nodes is very low and below the threshold that justifies investment in education, it must be that some other nodes have relatively high human capital, i.e. the x_is are more disperse than in the former case. Also, as argued earlier, the growth rate along such balanced growth paths is generally higher than for equilibria from the former group.

The distinction between these two groups is important for analyzing the local asymptotic properties of the steady state equilibria. Generally, there are theorems that provide sufficient (and under some assumptions necessary) conditions for local asymptotic stability of a specific steady state equilibrium, via the evaluation of the Jacobian of $\mathbf{W}$ at such an equilibrium, but these hinge on the local differentiability of the function $\mathbf{W}$. These theorems cannot be applied to the steady state equilibria of high inequality, from category (b). However, for steady state equilibria such that $x_i > \theta/\psi$ for all $i = 1, ..., n$, a sufficient condition for local asymptotic stability is that the eigenvalues of the Jacobian of $\mathbf{W}$ evaluated at the steady state equilibrium are strictly within the unit circle. The following proposition provides a general expression for the Jacobian in such cases:

PROPOSITION 3. Consider the system (2.3)-(2.6), and a steady state equilibrium such that $x_i > \theta/\psi$ for all $i = 1, ..., n$. Then the Jacobian of the system evaluated at $\mathbf{x}$ is

$$\mathbf{J}(\mathbf{x}) = \left(\mathbf{I}_n - \frac{1}{n} diag(\mathbf{x}) \mathbf{e}_n \mathbf{e}_n^T\right) (diag(\mathbf{q}) + \mathbf{R}(\mathbf{x})), \tag{2.15}$$

where $\mathbf{q}$ is a $n \times 1$ vector with entries

$$q_i = \frac{x_i}{\theta + x_i}, \tag{2.16}$$

$\mathbf{R}$ is a $n \times n$ matrix with entries

$$r_{ij} = \frac{g_{ij} x_j}{\sum_k g_{ik} x_k}, \tag{2.17}$$

and $\mathbf{e}_n$ is an $n \times 1$ vector with entries equal to 1.

Proof. See Appendix D. ■

As we can see from the expression for the Jacobian, conditions for local asymptotic stability of a balanced growth path require that we know the exact set of long run relative human capitals x_i. It turns out that for the special case of equality ($x_i = 1$ for all $i = 1, ..., n$), the Jacobian for the system (2.3)-(2.6) evaluated at $x_i = x^* = 1$ is

$$\mathbf{J}^* = \left(\mathbf{I}_n - \frac{1}{n}\mathbf{e}_n\mathbf{e}_n^T\right)\left(\frac{1}{1+\theta}\mathbf{I}_n + \mathbf{R}^*\right). \tag{2.18}$$

We can derive a simple, interpretable sufficient condition for local asymptotic stability for this case. The condition makes use of *network cohesion* κ, a statistic introduced and analyzed extensively in Cavalcanti *et. al.* (2015), and briefly defined in (D.16), in Appendix D. Network cohesion can be interpreted as a measure of how uniform or fragmented a network is, i.e. as a measure of connectedness. It varies between zero and one: the complete network has the largest possible network cohesion, equal to 1, while the empty network (or any disconnected network) has the lowest cohesion, equal to 0. The star network has intermediate cohesion equal to 1/2.

PROPOSITION 4. *If*

$$\kappa > \frac{1}{1+\theta}, \tag{2.19}$$

where κ is the network cohesion defined in (D.16), then the balanced growth path with $x_i = x^* = 1$ (equality) is locally asymptotically stable.

Proof. See Appendix D. ∎

Perhaps not surprisingly, the long run growth rate with equality γ^* does not depend on the network structure; this is because when everyone has identical levels of human capital, externalities do not matter for the growth rate of human capital. However the local dynamics of this balanced growth path do depend on the particular network that generates the local externality, via the summary statistic κ. The stability condition can be nicely interpreted as an expression that gives a lower bound of the importance of the network externality relative to the importance of a household's own investment in education for achieving equality in the long run. In accumulating human capital, there is a tension between investment in education and the network externality: as θ increases, and households have less urgency to invest in education, the lower bound of network cohesion κ that is required for long run equality becomes smaller, i.e. the stability condition is satisfied for more networks. However, in the absence of network cohesion ($\kappa = 0$ for disconnected or empty networks), the condition is never satisfied, since $\theta > 0$, confirming our earlier intuition.

An immediate consequence of Proposition 2 is the following:

CORROLARY 1. *Higher network cohesion κ implies faster convergence to the balanced growth path with long run equality.*

Proof. See Appendix D. ■

This result is interesting and intuitive. Cohesion κ captures a notion of connectivity of the network; the higher it is, the easier it is for the network to synchronize. It is in this sense that the higher κ is, the quicker it is for all the nodes in the economy to synchronize (converge) to the common balanced growth path. In the language of Bénabou (1996), "*A [more] integrated society is better at reducing heterogeneity. It converges faster to a homogeneous outcome.*"

3. Numerical Simulations

Whenever the externality parameters are such that the inequality (2.19) is not satisfied, it is not possible to find closed form solutions to the system or characterize the dynamics analytically. Therefore, in order to have an informed overview of possible long run outcomes in such cases, we perform an extensive series or numerical simulations of the system by varying the networks, the number of nodes n, the initial conditions, and the parameters of the model. Simulations are also employed for analyzing the transition dynamics of the economy.

We describe in detail the configurations of the experiments we performed. We consider four cases for the number of nodes, namely $n = 4$, 5, 6 and 7, and generate all non-labeled connected networks, as listed in Read and Wilson (1998); a small number of nodes allows us to generate easily all possible connected networks. There are 6, 21, 112 and 853 such networks for $n = 4$, 5, 6 and 7 nodes respectively, i.e. we are working with 992 networks in total. We have two parameters to choose, ψ and θ. They also need to satisfy the conditions $\psi > \theta$ and $\psi\theta \geq 1$. We set $\psi = 3$ and $\theta = 1$ so that $\gamma^* = 1.5$, and the stability condition for local asymptotic convergence to equality is $\kappa > 0.5$.[15] The vectors of initial human capitals are drawn from a generalized Pareto distribution with shape parameter and scale parameter 0.7654 and threshold parameter equal to 1. This distribution and parameters replicate well income distributions of developed countries, such as the US, with an approximate Gini coefficient for income of 60-65% (see Antunes *et. al.*, 2012). With

[15] With this set of parameters we are not aiming at calibrating specific characteristics of an economy. The parameters are chosen to give convenient numerical values of γ^* and the stability condition relating to network cohesion. Nevertheless, our parameter values are entirely plausible: for a time period of 25 years (i.e. one generation), our $\gamma^* = 1.5$ is consistent with a long run growth rate of 1.64% per year.

# long run outcomes	$n = 7$	$n = 6$	$n = 5$	$n = 4$
1	157	36	11	4
2	515	67	9	2
3	172	9	1	0
4	9	0	0	0
# networks	853	112	21	6

Table 3.1: Number of long run outcomes.

this in place, and for all the groups of unlabeled connected networks with $n = 4, 5, 6, 7$ nodes, we run 1,000 simulations with different realizations of initial conditions for human capital. Last, for any simulation, the recursion is repeated for a number of periods T until convergence to a balanced growth path. The criterion for convergence is that both the change in the x_is is of an order of magnitude of 10^{-6} (convergence) and that the difference between any γ_is is of an order of magnitude of 10^{-6} (balanced growth). The time period at which these requirements are satisfied also serves as a measure for the speed of convergence of the system to the balanced growth path. The results from the simulation experiments are presented in the next few subsections, supported by corresponding figures and tables.

3.1. *Number of long run outcomes*

Table 3.1 reports the number of networks that generate a specific number of long run outcomes. We see that for the majority of networks, there are two long run stable outcomes, while for many networks there is a unique outcome, or three outcomes. As the number of nodes increases, there will occasionally be four distinct long run outcomes. For high network cohesion, above 0.6, there is a unique long run outcome, for network cohesion between 0.5 and 0.6 there are at most two long run outcomes, while for cohesion below 0.5 there can be from one up to four long run outcomes. Networks with cohesion less than 0.2 always have two or more long run outcomes.

3.2. *Prevalence of long run equality*

The results from the following numerical experiment provide additional support to the statement of Proposition 2. Figure 3.1 shows how often in the simulations the economy convergences to long run equality, plotted against network cohesion. We see that when cohesion is well above the stability condition threshold ($\kappa = 0.5$), the unique long run outcome is that of long run equality, suggesting global asymptotic stability

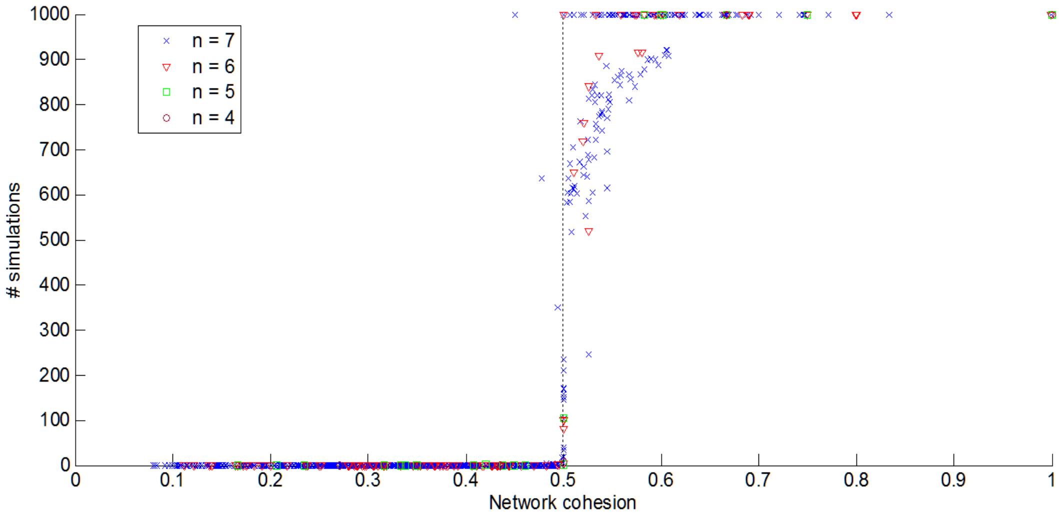

Figure 3.1: *Prevalence of Long Run Equality.* Each point in the figure shows how often in the simulations the economy convergences to long run equality, plotted against network cohesion. Based on 1,000 simulations for each network group of $n = 4, 5, 6, 7$ nodes.

for these cases. In contrast, for network cohesion below the threshold, long run equality is reached very rarely. For network cohesion that satisfies the stability condition but is not very high (generally between 0.5 and 0.6) the set of initial conditions that lead to long run equality is smaller, and long run equality is attained less frequently for some networks, as their cohesion gets closer to 0.5.

3.3. *Ranking and frequency of long run outcomes*

Next, we discuss the ranking of possible different long run outcomes and how often these prevail in the simulations. Figure 3.2 shows the minimum and the maximum long run growth rates plotted against network cohesion, for all non-labeled connected networks with $n = 7$ from 1,000 simulations. For networks with high cohesion, above 0.6, there is a unique outcome, that of long run equality with $\gamma = \gamma^*$, as already established. Networks with cohesion in the range 0.5 to 0.6 have either one unique outcome (equality) or at most one additional long run outcome. In the latter case, the growth rate of the second possible long run outcome is in the range 1.7 to 1.9 and becomes higher as cohesion decreases. It also prevails more frequently in the simulations as network cohesion gets closer to 0.5. When the network cohesion is below 0.5, most networks will have more than one long run outcome. Network cohesion forces a decreasing lower bound for the long run growth rate: the lower the cohesion of a connected network is, the less likely it is that it will

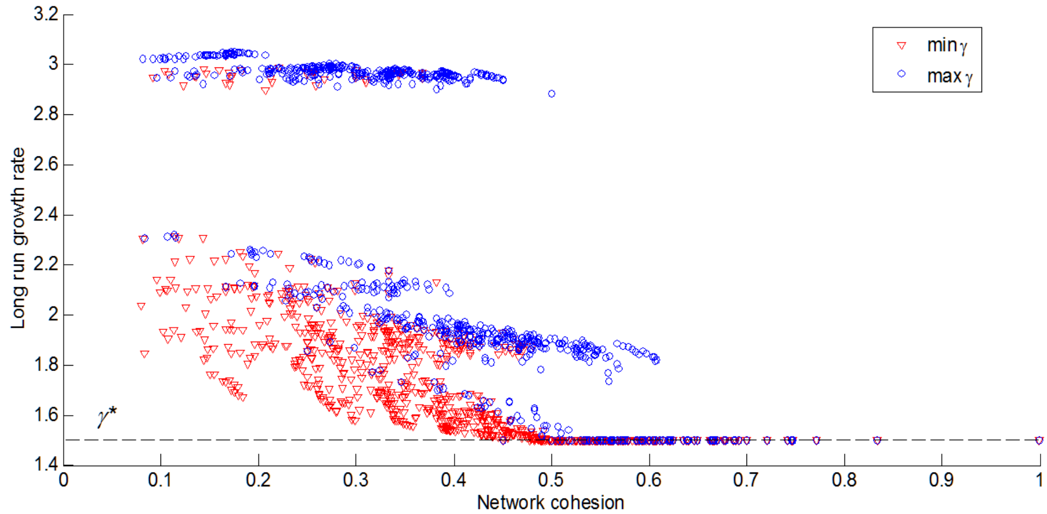

Figure 3.2: *Ranking of Long Run Growth Rates.* For any given network, a red triangle marks the minimum γ and a blue circle marks the maximum γ, plotted against network cohesion. Based on 1,000 simulations of all non-labeled connected networks with $n = 7$.

lead to a low growth rate in the long run.

As indicated in Figure 3.2, most networks generate long run outcomes with growth rates that may be in a broad range of relatively low values, but many networks lead to outcomes with high long run growth rates, in the range of 2.9 to 3. These are the cases for which $\min\{0, (\psi x_i - \theta)/(1+\psi)\}$ is often strictly negative and $\left(\sum_{i=1}^{n} \bar{x}_i\right)/n$ is a small number, and thus from (2.14), γ must be high. The simulations do not indicate a clear frequency pattern for the different possible outcomes. Long run outcomes with very high growth rate, above 2.8, occur in 24% of all the cases.

3.4. *Inequality in the long run*

The measure of inequality we employ in our simulations is the Gini coefficient. When we make statements about the relationship between growth and inequality we refer to the growth rate of the average human capital γ and the Gini coefficient of the relative human capital of all households. It is easy to confirm for all our simulations that, for *a given network*, the relationship between growth and inequality is unambiguously positive. In other words, high long run growth rates are associated with high inequality. Therefore the lower the network cohesion is, the less likely it is to have low inequality in the long run. Likewise, when network cohesion is high, inequality is always eliminated.

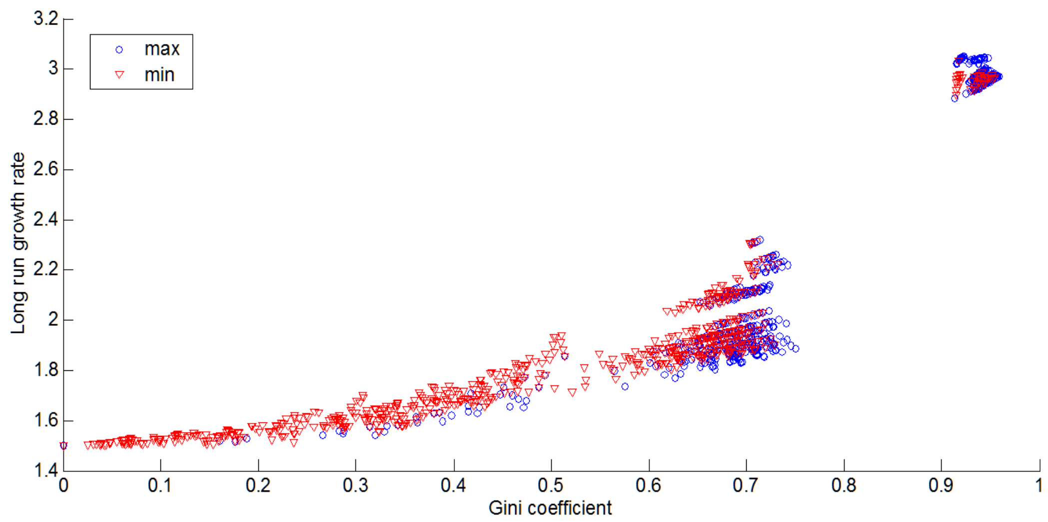

Figure 3.3: *Growth and Inequality.* For any given network, a red triangle marks the minimum γ and a blue circle marks the maximum γ, plotted against the corresponding Gini coefficient in the long run. Based on 1,000 simulations of all non-labeled connected networks with $n = 7$.

However, the relationship between long run growth and inequality *across* networks is not as clear-cut. Figure 3.3 demonstrates this.[16] For any given network, a red triangle marks the minimum γ and a blue circle marks the maximum γ, plotted against the corresponding Gini coefficient in the long run, from 1,000 simulations of all non-labeled connected networks with $n = 7$. The pattern that emerges is indeed that growth and inequality are positively correlated. However, it is possible to have distinct network structures for which the same long run growth rate is associated with a wide range of possible long run Gini coefficients. For example there are several network structures that lead to long run outcomes with growth rate approximately equal to 1.8, with Gini coefficients ranging from 43% to 67%, as indicated in Figure 3.3.

3.5. *Speed of convergence*

Next, we examine the relationship between network cohesion and speed of convergence to a balanced growth path. Corollary 2 suggests that whenever equality is a locally asymptotically stable outcome (i.e. whenever $\kappa > 0.5$), higher cohesion should imply faster convergence. Figure 3.4 plots the number of periods to convergence to a balanced growth path, against network cohesion for all non-labeled connected networks

[16] In Figure 3, we have done the following: for each of the 853 networks with seven nodes we have run 1000 simulations, from which we pick the maximum and minimum γ and their corresponding Gini coefficients. We then put all these in a scatter plot that have γ and the Gini on the two axes. We note that this does *not* necessarily imply that for a given network the blue circle (max) and the red triangle (min) should be on the same vertical line.

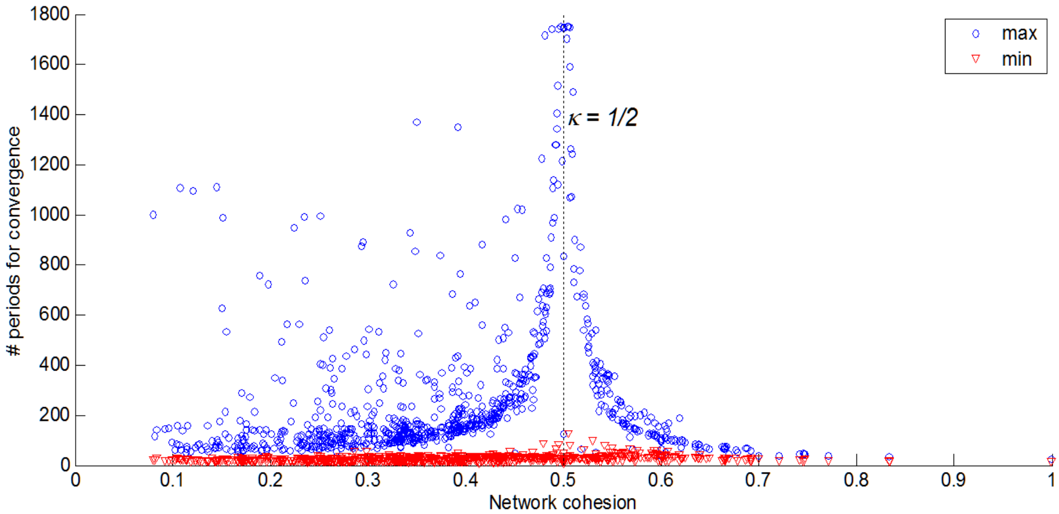

Figure 3.4: *Speed of Convergence.* Number of periods to convergence to a balanced growth path, plotted against network cohession for all non-labeled connected networks of $n = 7$ nodes. The blue circles mark the maximum number of periods and the red triangles mark the minimum number of periods from 1,000 simulations for each network.

of $n = 7$ nodes. The blue circles mark the maximum number of periods and the red triangles mark the minimum number of periods from 1,000 simulations for each network. The minimum numbers of periods to convergence are low and they generally correspond to cases for which the initial conditions are 'close' enough to the corresponding balanced growth path. Looking at the maximum number of periods to convergence, the numerical results confirm Corollary 2. As network cohesion approaches the threshold of the stability condition (2.19), convergence becomes slower. Whenever cohesion is well below the threshold, convergence (to high growth outcomes) is most often faster as network cohesion decreases, but there are several networks for which convergence can be quite slow. These are the networks that may lead to more than two long run outcomes.

3.6. *Varying parameters*

Next we use our numerical experiments to establish how our results change when we alter the number of nodes n and the parameter values ψ and θ. We start with the number of nodes of the network, n. Our numerical simulations indicate that the maximum possible growth rate in balanced growth paths with inequality increases with the number of nodes. Indeed, our numerical simulations for all networks with $n = 4, 5, 6, 7$ generate maximum long run growth rates 1.9032, 2.2914, 2.6730 and 3.050 respectively. The reason why the maximum growth rates increase with n, is that a larger number of nodes generally allows for the possibility of more inequality, i.e. more disperse relative human capitals x_i. More inequality generally implies that both $\min\{0,(\psi x_i-\theta)/(1+\psi)\}$ is non-zero more often and that $\sum_{i=1}^{n}\bar{x}_i$ is smaller, thus allowing $\left(\gamma^*-\frac{1}{n}\sum_{i=1}^{n}\min\{0,(\psi x_i-\theta)/(1+\psi)\}\right)\left(\frac{1}{n}\sum_{i=1}^{n}\bar{x}_i\right)^{-1}$ to be larger. Importantly, we also record from our simulations that as n increases from 4 to 7, such high growth outcomes occur less frequently (the frequencies of occurrence are 38%, 37%, 32% and 24 % respectively).

Next, we consider the sensitivity of our results to changes in the altruism parameter ψ. Changes in ψ affect growth rates directly: γ^* is increasing in ψ and for all our numerical simulations, an increase in ψ makes all long run growth rates shift up; we also observe that such increases are relatively larger for the outcomes with high inequality and high growth rates γ, for similar patterns of inequality. Other things equal, stronger altruism implies more investment in education for households with high human capital, therefore more polarization and inequality whenever the local externality is not strong enough. Therefore more altruism, i.e. larger ψ, implies higher growth rates and more inequality in the long run.

Last, we examine the sensitivity of the results to changes in the parameter θ, which is a key parameter of the model. Recall that the interpretation of θ is the minimum education available to all households (e.g. free public schooling) that ensures that no one ever has zero human capital. As θ increases, households have less incentives to invest in further educating their children and the network cohesion required for the stability condition (2.19) to hold is lower. As θ increases, the growth rate γ^* clearly increases. The simulations suggest that growth rates of long run outcomes with inequality also increase, but in this case such increases are relatively larger for the outcomes with mid-range growth rates γ and smaller for the high level growth rates. In other words, for larger θ the range of possible growth rates in the long run is less wide. Long run outcomes with equality prevail more often, since the condition for local asymptotic stability of such outcomes is satisfied for more networks. Also, although growth rates are generally higher, inequality is always lower as θ increases.

3.7. *Transition to balanced growth path*

We do the following transition experiments. Suppose that before time zero, the economy has converged to a balanced growth path with equality and that $h_{i,0}=1$, so that $x_{i,0}=1$ for all i and $\gamma_0=\gamma^*$. We then assume that in period 1, an arbitrary node gets an exogenous positive shock, i.e. a shock which *increases* its human capital relative to the rest of the economy. Our task is then to trace the transition of the dynamic system to a balanced growth path. Recall that the growth rate of the average human capital is an average of the individual growth rates of all the nodes, weighted by the corresponding relative human capitals. The average growth rate can therefore be decomposed into the weighted growth rate of the shocked node plus the sum of the weighted growth rates of all the non-shocked nodes. As the shock occurs, the relative human capital of the shocked node increases, while the relative human capitals of the non-shocked nodes are identical and drop, since there is one node that has higher human capital than all the rest. We use the interpretation of (2.8) to argue that the sum of the weighted growth rates of all the non-shocked nodes is relatively high, if many of the non-shocked nodes are linked to the shocked node. That is, when many nodes do worse relative to their neighborhood (which includes the shocked node). This effect is stronger if such neighborhoods include fewer other nodes, so that any given non-shocked node does a lot worse relative to the shocked node. In other words, we anticipate the average growth rate following the shock to be high whenever (i) many nodes are linked to the shocked node, so that many nodes benefit from the local externality (local positive shock) and (ii) the non-shocked nodes have low number of links, so that this positive effect is not mitigated by the inactivity of other non-shocked nodes.

This discussion suggests that networks that seem to give high growth during transition would be networks with high *degree centralization*, when we shock the node with the highest degree. Centralization of a network is a general measure of how central its most central node is in relation to how central all the other nodes are (see Freeman, 1979). It can be defined for every measure of centrality, including degree (which is relevant in our case). It is the ratio of the sum in differences in centrality between the most central node in a network and all other nodes over the theoretically largest such sum of differences in any network of the same degree. By construction, centralization varies between 0 and 1 and the star network has the highest centralization, equal to 1.

First, we generate scatter plots that plot the growth rate of average human capital one period after shocking the node with the highest degree in the network (i.e. γ_2) against the centralization of networks. Figure 3.5 shows such a scatter plot of γ_2 against degree centralization, when the shocked node has the highest degree in the network, for all 853 unlabeled connected networks with $n = 7$ nodes. The shock is set to $\varepsilon = 0.05n$.[17] The scatter plot confirms our insights, i.e. that the growth rates after the impact are

[17] We set the size of the shock as a function of the number of nodes n, so that the impact effect on the average growth rate is identical for all n. Clearly, if a network has many nodes, then a relatively large shock ε is required to generate a sizeable change for the average growth rate compared to a network with few nodes. Such normalization ensures that all transition experiments

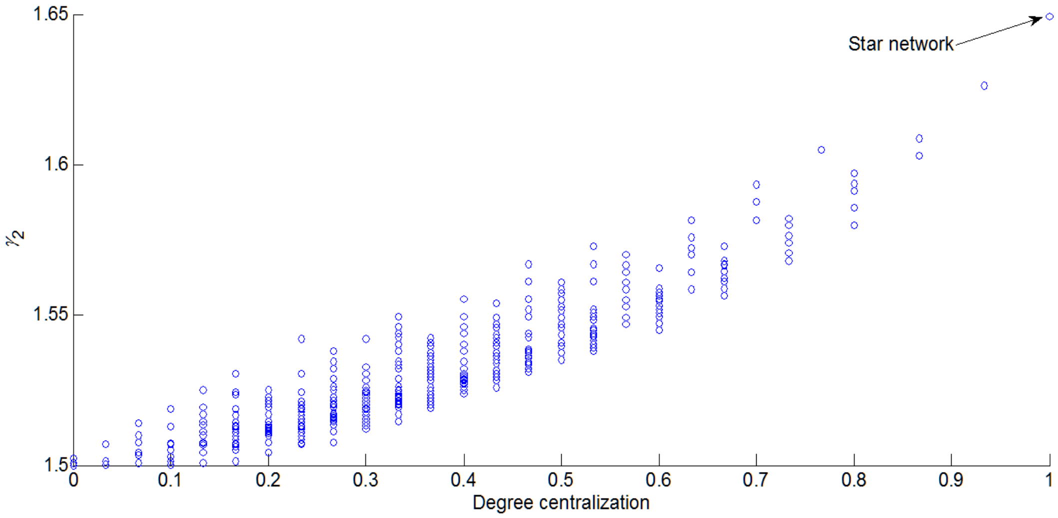

Figure 3.5: *Growth in Transition*: Scatterplot of γ_2 against degree centralization when the shocked node has the highest degree in the network, for all unlabeled connected networks with $n = 7$ nodes.

broadly increasing in the network centralization. The star network has the highest centralization and the highest possible growth rate a period after the shock.

Second, we observe that during transition networks with high cohesion have generally lower inequality. Figure 3.6 shows a scatter plot of the Gini coefficient four periods after the shock has occurred, plotted against network cohesion, using the same experiment data as in Figure 3.5. The higher variation of the Gini coefficient for lower cohesions is due to the fact after four periods, fewer networks will have converged, but there can be many differences among different networks because of their structure. Here, κ provides an upper bound for how much the Gini index can be four periods after the positive shock.

3.8. *Summary of results from simulations*

The general picture that emerges from this simulation analysis is that, in identifying possible long run outcomes for our economy, there is a combination of *history* and *location* dependence. History dependence is a well known and understood feature of many models of endogenous growth and a characteristic of dynamic systems with multiple steady state solutions that may be locally asymptotically stable: different

are easily comparable, irrespective of the size of the network.

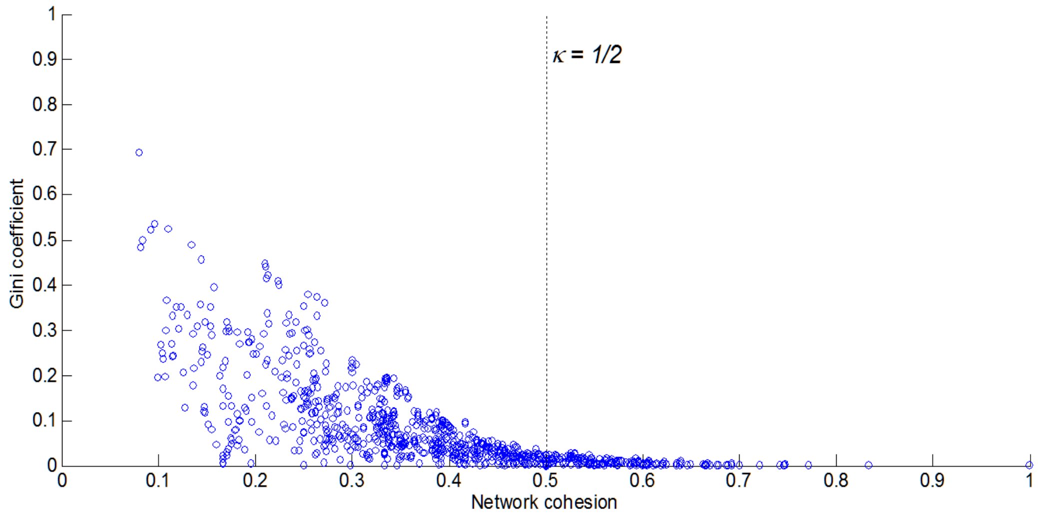

Figure 3.6: *Inequality During Transition.* Network cohesion κ against Gini coefficient, four periods after the sock occurs, when the shocked node has the highest degree in the network, for all non-labeled connected networks with $n = 7$.

levels of initial human capital may lead to quite different long run levels of relative human capitals.[18] Location dependence is an additional complication layer of our analysis: for a specified vector of initial human capital, two distinct network structures may lead to very different long run outcomes.

4. An Empirical Illustration

We next discuss, via a simple example, how one can map our model economy to a true economy. In order to do this, we first present a simple extension of the model that is more suitable to for calibration. Future human capital h_{it+1} may now depend on two additional factors, namely intergenerational human capital transmission (inherited capital), captured by h_{it}, and a global externality (a common education system or school curriculum), captured by the average human capital in the economy, $\bar{h}_t$. The human capital accumulation equation then becomes

$$h_{it+1} = (\theta + e_{it})^{\eta} h_{it}^{1-\beta_1-\beta_2} \bar{h}_{it}^{\beta_2} \bar{h}_t^{\beta_1} \tag{4.1}$$

[18] History dependence here does not mean that the system exhibits chaotic behavior, because small perturbations in the initial conditions do not change the long run outcomes.

where $0 < \beta_1 + \beta_2 \leq 1$. Much of the analysis we have presented here can be extended to this more general formulation. The growth rate on the balanced growth path with equality is then

$$\gamma^* = \left[\frac{\psi\eta(1+\theta)}{1+\psi\eta}\right]^\eta \tag{4.2}$$

and the sufficient condition for local asymptotic stability of the steady state equilibrium with equality is

$$\beta_1 + \kappa\beta_2 > \frac{\eta}{1+\theta}. \tag{4.3}$$

The interpretation of this condition remains broadly the same: i.e. it is a condition that gives a lower bound of the importance of the two externalities relative to the importance of a household's own investment in education, summarized by $\eta/(1+\theta)$, in order to achieve equality in the long run. It shows that the global externality (captured by β_1) and the local externality (captured by $\kappa\beta_2$) have to be stronger than own investment in education in order to eliminate inequality in the long run.

One important advantage of this extension is that it allows us to also consider networks that are disconnected and have cohesion $\kappa = 0$. The global externality ensures that such environments can converge to steady state equilibria that are consistent with balanced growth. Long run growth rates and inequality are generally decreasing in β_1 and β_2, due to the concavity of the human capital accumulation function (4.1).

Using this more general version of the model, we now discuss how to calibrate the model from data. We use school data from Switzerland, for which there is good availability; also the federal structure of Switzerland allows us to work with a variety of networks, which are nevertheless comparable since they belong to the same country. The analysis serves as an illustration only and we do not aim at providing a formal econometric analysis and identification of spillover channels of growth from one Swiss region to another. We are aware of reverse causality issues here since the network structure might be a consequence of inequality and income levels, and not the other way around.

The details of how the model parameters are calibrated or estimated, as well as all the detailed information on the data, are available in a companion online Appendix. In what follows, we present a brief description of the procedure. In order to set the parameters of the model, we used the following strategy.

Canton	n	Correlations of commune growth rates, models with data 1980-2007 Model	Compl	Global	Gini index for models and data 1980 Data	2007 Data	2007 Model	2007 Compl	2007 Global
Geneva (0.52)	45	0.1554	0.0794	0.0813	0.157	0.221	0.110	0.094	0.107
Lucerne (0.46)	95	0.6375*	0.5762*	0.5758*	0.145	0.112	0.093	0.088	0.099
Obwalden (0.83)	7	-0.1027	-0.1236	-0.1203	0.064	0.080	0.042	0.040	0.045
Schwyz (0.12)	30	0.0057	-0.1032	-0.1045	0.167	0.279	0.121	0.104	0.117
Schwyz† (0.12)	27	0.6514*	0.6430*	0.6447*	0.157	0.136	0.108	0.097	0.109
Uri (0.58)	19	0.7796*	0.7566*	0.7554*	0.153	0.102	0.093	0.095	0.107
Zurich (0.36)	171	0.4847*	0.4377*	0.4388*	0.151	0.140	0.105	0.091	0.103
All Cantons	368	0.3019*	0.2299*	0.2305*	0.2260	0.198	0.199	0.193	0.199
All Cantons†	**365**	**0.4433***	**0.3866***	**0.3872***	**0.227**	**0.188**	**0.200**	**0.194**	**0.200**

*Denotes correlation that is statistically different from zero at 95 percent confidence level.

†Excludes three communes from Schwyz for which the real gross growth rate between 1980 and 2007 is larger than 3.

Table 4.1: Model vs Data: Growth Correlations and Iinequality. The number in parentheses are cohesions for the corresponding networks.

First, we calibrate ψ, η and θ using the Swiss data such that along a balanced growth path the model economy matches the following moments of the data: (i) the long run annual growth rate of output per capita is 1.97%, which is consistent with data from the Penn World Table for the period 1950-2007; (ii) returns to investment in education is 9.5% (from OECD data); and (iii) the ratio of investment in education to final consumption is roughly 9.4% (from OECD data). Using these facts, we find that

$$\theta = 3.9049, \eta = 0.379125 \text{ and } \psi = 11.5157. \tag{4.4}$$

In this case $\delta = \frac{\eta}{1+\theta} = 0.0773$ and, using an annual growth rate 1.97%, the implied growth rate at the balanced growth path for the 27 year period is $\gamma^* = 1.69$.

Next, we 'calibrate' the networks. We work with six Swiss Cantons for which we have data available, namely Geneva, Lucerne, Obwalden, Schwyz, Uri and Zurich. Each Canton as a separate network and consists of administrative regions known as *communes*, which represent the nodes of each network. Swiss communes are the smallest government division in Switzerland (municipalities) and are a proxy for the nodes of our model. We say that two communes within a Canton are *linked*, if there is at least one school that children from both communes attend. Using school data from academic year 2007-08, we can then generate the exact networks and adjacency matrices that describe the above relationships. We set the income per capita in 1980 in each commune as the initial conditions for the human capital. Feeding these initial conditions into the model and given that the model period is 27 years, we get the model predicted level of income per capita for each commune in 2007.

Last, using these network structures and income data (at the Canton and Commune level) we estimate the externality parameters β_1 and β_2. Taking logs of the law of motion of human capital we get

$$\log(h_{it+1}) = \alpha_0 + \alpha_1 \log(x_{it} + \theta) + \alpha_2 \log(h_{it}) + \alpha_3 \log(\bar{h}_{it}) + \alpha_4 \log(\bar{h}_t), \tag{4.5}$$

where $\alpha_0 = \eta \log(\frac{\eta\psi}{1+\eta\psi})$, $\alpha_1 = \eta$, $\alpha_2 = 1 - \beta_1 - \beta_2$, $\alpha_3 = \beta_2$, and $\alpha_4 = \beta_1$. Then, using the calibrated parameters for ψ, η and θ, we restrict the value for parameters α_0 and α_1 and estimate the following restricted non-linear least squares regression:

$$\log(h_{it+1}) = \alpha_0 + \alpha_1 \log(x_{it} + \theta) + \alpha_2 \log(h_{it}) + \alpha_3 \log(\bar{h}_{it}) + \alpha_4 \log(\bar{h}_t) + \epsilon_{it}. \tag{4.6}$$

We find point estimates for the externality parameters that are $\beta_1 = 0.01$, and $\beta_2 = 0.44$. Therefore, we have that $1 - \beta_1 - \beta_2 = 0.55$. While β_1 is not statistically different from zero at 95 percent confidence level, the point estimate for β_2 is.

With the calibrated and estimated parameters in place, we now examine the performance of our model. Table 4.1 presents the correlations between the growth rates of income per capita in each commune γ_i in the data and in the model for each Canton (columns titled *Model*). We also present the correlation between growth rates in the data and in two alternative cases: the first one (columns titled *Compl*) is when we use the same parameters as estimated previously, but assume that the underlying network is complete (in which case, the local externality becomes global and the parameter for the global externality becomes $\beta_1 + \beta_2$); the second one (columns titled *Global*) also considers a model that only has a global externality, but for which we now re-estimate the externality parameter under this assumption. Therefore, we impose $\beta_2 = 0$ and re-estimate the value of parameter β_1 using the data (see online Appendix for more details). The correlation in growth rates between the data and the model is positive and statistically different from zero for the following three Cantons: Lucerne, Uri and Zurich. It is also positive for Schwyz when we remove the growth rates for three specific communes (Feusisberg, Freienbach, Wollerau) that had a real gross growth rate above 3 in that period. The correlation in growth rates for all communes is about 0.30 when we use all data and 0.44 when we abstract from the communes with high growth rates in Schwyz.[19] The model, when the local externality is at work and the network structure is explicitly taken into account, provides a better fit to the data (always higher correlations of model and data growth rates) than the two models in which we abstract from the network structure. Table 4.1 also reports inequality in income per capita among communes for each Canton in the data and in the model. The initial measure of inequality is the same in the data and in the model, since the initial conditions of the model are taken from the data. The inequality for the whole sample decreases from 0.226 in 1980 to 0.198 in 2007. The model with the local externality and network structure yields a similar Gini index in 2007 when we consider all communes. For most Cantons, the model predicts a higher decrease in inequality than what is observed in the data. The Gini index in 2007 for both the data and the model is roughly decreasing in network cohesion (only Geneva does not fit this monotonic ranking), which is qualitatively consistent with our analytical results.

We note here that our model can be employed to evaluate different policy interventions. For example, one important counterfactual exercise is the design of schools catchment areas: consider two different neighbourhoods in which children from each community attend two separate schools. Then suppose that these two schools are merged into one.[20] This could have important effects on individual and community outcomes, since these depend on local externalities. Our model can provide a framework for analysing the effects of such a policy change on the growth and inequality of these neighbourhood communities. Another type of policy which can be analysed within our model is a voucher school program in which families could choose which school to enroll their children to. See Angrist *et. al.* (2002) for an empirical evaluation of a voucher program in Colombia on students performance.

[19] However, the correlations are not statistically different from each other at usual confidence levels. The p-value for the difference in the correlations produced by the model with the network structure and the other two models (*Compl* and *Global*) are 0.29 and 0.36, respectively.

[20] In a real life example, recently in Cambridge, UK, a relatively well ranked state school (Parkside) was merged with a school (Coleridge) which was under special measures to improve. Now these two schools form the Parkside Federation Academies, which run two secondary schools and an international Sixth Form in Cambridge.

5. Closing comments

Identifying the effects of social structures on the process of economic development is an important challenge in the social sciences. Here, we developed a theory for understanding how network externalities affect human capital dynamics, economic growth and inequality. We use tools from network theory to characterize the dynamics of an economy in a standard overlapping generations model with an explicitly modeled network. We show that the nature and properties of long run outcomes depend on the network structure of a particular economy, via a measure of network cohesion. Higher network cohesion makes it is more likely that the economy will converge to a path of long run equality, while lower network cohesion leads to long run outcomes with persistent inequality.

Our findings are complementary to, yet distinct from the results of related papers such as Bénabou (1996) and Galor and Tsiddon (1997). The paper by Galor and Tsiddon (1997) focuses on the complementarity and interactions between inherited human capital and investment in education. Using our notation, human capital in their paper accumulates in equilbrium according to

$$h_{it+1} = \phi\left(h_{it}, e_{it}\right) = \phi\left(h_{it}, \xi\left(h_{it}, \lambda\left(\bar{h}_t\right)\right)\right). \tag{5.1}$$

The curvature of these functions determines the existence and dynamic properties of steady state equilibria. For them, the '*local*' effect is captured by ϕ_1, i.e. the importance of inherited human capital (or home environment). The '*global*' effect feeds through the impact that each household has to the average human capital $\bar{h}_t$ and thus the decision to invest. They show that depending on the properties of the functions ϕ, ξ and λ, their economy may be characterised by *convergence* (long run equality) or *polarization* (long run inequality). In our setting, the starting point is polarization: this would correspond to the case of an empty network, with the local effect being restricted to 'home environment' h_{it} and our functions are such that with an empty network, there is always polarization. Here we argue that the local effect may include more broadly the *neighborhood* environment $\bar{h}_{it}$ rather than only the home environment and polarization can be avoided, as long as the whole network economy has enough network cohesion. As already discussed in section 2.3, our framework is more general and encompasses the insights of Bénabou (1996), as well as Galor and Tsiddon (1997), i.e. that attempts to mix and integrate communities early in the development process of an economy may have an adverse effect on long run performance and growth. Additionally, our framework allows us to broadly rank long run aggregate outcomes with persistent inequality based on network cohesion. It also provides a natural plattform for analyzing long run income distribution and rankings for individual households, based on the features of their network neighbrohood.

We close the paper with a comment on what we believe to be the most interesting extension of the present work, that of developing a framework where links between households are formed endogenously.[21] In such a world, the whole network then forms endogenously and evolves dynamically in order to accumulate human capital. While the exogenous static network of our current framework is a useful starting point for understanding how network structures affect growth dynamics, we do recognize that endogenous network formation for the determination of local externalities is an attractive assumption. We should highlight, however, that our exogenous network approach considers several network structures that might represent some stable network equilibria, which may prevail under endogenous network formation. We defer this kind of analysis to future work.

[21] In a similar framework to ours, Kempf and Moizeau (2009) examine the dynamics of inequality and growth by assuming that economic agents belong to neighborhoods (clubs), the membership of which changes endogenously, depending on the performance of agents in terms of human capital levels. They conclude that segregation and inequality persists in the long run. In our terminology, their framework corresponds to a *disconnected* network with a number of distinct components. The component size changes over time, but irrespective of that, the underlying assumed network structure always has zero cohesion, and thus leads to persistent inequality.

APPENDIX

A. Dynamic System and Balanced Growth Path

We have that

$$h_{it+1} = (\theta + e_{it})\bar{h}_{it} = \left(\theta + \max\left\{0, \frac{\psi x_{it} - \theta}{1+\psi}\right\}\right)\bar{h}_{it}. \tag{A.1}$$

Therefore

$$x_{it+1} = \frac{h_{it+1}}{\bar{h}_{t+1}} = \frac{1}{\gamma_t}\left(\theta + \max\left\{0, \frac{\psi x_{it} - \theta}{1+\psi}\right\}\right)\left(\frac{\sum_j g_{ij} x_{jt}}{\sum_j g_{ij}}\right). \tag{A.2}$$

Also,

$$\begin{aligned} \gamma_t &= \frac{\bar{h}_{t+1}}{\bar{h}_t} = \frac{1}{n}\sum_k \left[\left(\theta + \max\left\{0, \frac{\psi x_{kt} - \theta}{1+\psi}\right\}\right)\frac{\sum_j g_{kj} x_{jt}}{\sum_j g_{kj} x_{kt}}\right] x_{kt} \\ &= \frac{1}{n}\sum_k \gamma_{kt} x_{kt}. \end{aligned} \tag{A.3}$$

In other words the system is described by

$$x_{it+1} = \frac{\gamma_{it}}{\gamma_t} x_{it} = \frac{x_{it}\gamma_{it}}{\frac{1}{n}\sum_{k=1}^{n} x_{kt}\gamma_{kt}}, \tag{A.4}$$

$$\gamma_{it} = \left(\theta + \max\left\{0, \frac{\psi x_{it} - \theta}{1+\psi}\right\}\right)\left(\frac{\sum_j g_{ij} x_{it}}{\sum_j g_{ij} x_{jt}}\right)^{-1}, \tag{A.5}$$

i.e. the growth rate of the average human capital is the weighted average of the growth rates of all households, weighted by their relative human capital.

At a given point in time t, and for a given x_{it}, the growth rate for household i can be either

$$\gamma_{it} = \frac{\theta}{\bar{x}_{it}} = \theta\frac{\bar{h}_{it}}{h_{it}}, \text{ if } x_{it} \leq \theta/\psi, \tag{A.6}$$

or

$$
\begin{aligned}
\gamma_{it} &= \frac{\psi}{1+\psi}\frac{(\theta+x_{it})}{\bar{x}_{it}} = \frac{\psi\theta}{1+\psi}\frac{1}{\bar{x}_{it}} + \frac{\psi}{1+\psi}\frac{x_{it}}{\bar{x}_{it}} \\
&= (\gamma^*-\phi)\frac{1}{\bar{x}_{it}} + \phi\frac{x_{it}}{\bar{x}_{it}} = \frac{\gamma^*}{\bar{x}_{it}} + \phi\left(\frac{x_{it}}{\bar{x}_{it}} - \frac{1}{\bar{x}_{it}}\right) = \gamma^*\left(\frac{1}{\bar{x}_{it}} + \frac{\phi}{\gamma^*}\left(\frac{x_{it}}{\bar{x}_{it}} - \frac{1}{\bar{x}_{it}}\right)\right) \\
&= \gamma^*\left(\frac{1}{\bar{x}_{it}} + \frac{1}{1+\theta}\left(\frac{x_{it}}{\bar{x}_{it}} - \frac{1}{\bar{x}_{it}}\right)\right) = \gamma^*\left(\frac{\theta}{1+\theta}\frac{1}{\bar{x}_{it}} + \frac{1}{1+\theta}\frac{x_{it}}{\bar{x}_{it}}\right) \\
&= \gamma^*\left(\frac{\theta}{1+\theta}\frac{\bar{h}_{it}}{h_{it}} + \frac{1}{1+\theta}\frac{\bar{h}_{it}}{\bar{h}_t}\right), \text{ if } x_{it} > \theta/\psi.
\end{aligned}
\tag{A.7}
$$

With these expressions in place, we get that

$$
x_{it}\gamma_{it} = \begin{cases} \theta\frac{\bar{h}_{it}}{\bar{h}_t}, & \text{if } x_{it} \leq \theta/\psi \\ \gamma^*\left(\frac{\theta}{1+\theta}\frac{\bar{h}_{it}}{h_{it}} + \frac{1}{1+\theta}\frac{\bar{h}_{it}}{\bar{h}_t}\right), & \text{if } x_{it} > \theta/\psi \end{cases}, \tag{A.8}
$$

and

$$
\gamma_t = \frac{1}{n}\sum_{k=1}^{n} x_{kt}\gamma_{kt}. \tag{A.9}
$$

B. Proof of Proposition 2

We first show the following Lemma:

LEMMA 1. *For initial conditions $x_{i0} > 0$, for all $i = 1, ..., n$, relative human capital satisfies $0 < x_{it} < n$ for all $i = 1, ..., n$ and $t = 1, 2, ...$.*

Proof. We show the result by induction. We start by $x_{i0} > 0$. Then, for all $i = 1, ..., n$,

$$
\gamma_{i0} = \left(\theta + \max\left\{0, \frac{\psi x_{i0} - \theta}{1+\psi}\right\}\right)\frac{\sum_j g_{ij}x_{j0}}{\sum_j g_{ij}x_{i0}} > 0, \tag{B.1}
$$

because $x_{i0} > 0$, $\theta > 0$ and the matrix G is never zero (at least $g_{ii} = 1 \neq 0$). Therefore,

$$
\gamma_0 = \frac{1}{n}\sum_{i=1}^{n} x_{i0}\gamma_{i0} > 0. \tag{B.2}
$$

For period $t = 1$, we then have that for all $i = 1, ..., n$

$$x_{i1} = \frac{\gamma_{i0}}{\gamma_0} x_{i0} > 0. \tag{B.3}$$

Now assume that at period τ, we have that $x_{i\tau} > 0$ for all $i = 1, ..., n$. Then

$$\gamma_{i\tau} = \left(\theta + \max\left\{0, \frac{\psi x_{i\tau} - \theta}{1+\psi}\right\}\right) \frac{\sum_j g_{ij} x_{j\tau}}{\sum_j g_{ij} x_{i\tau}} > 0, \tag{B.4}$$

and thus

$$\gamma_\tau = \frac{1}{n} \sum_{i=1}^{n} x_{i\tau} \gamma_{i\tau} > 0, \tag{B.5}$$

which implies that at period $\tau + 1$, we will have that for all $i = 1, ..., n$

$$x_{i\tau+1} = \frac{\gamma_{i\tau}}{\gamma_\tau} x_{i\tau} > 0. \tag{B.6}$$

The second part of the inequality follows directly from the fact that $\Sigma_{i=1}^{n} x_{it} = n$ for all t. ■

The proof of Proposition 2 is now straightforward: A steady-state solution of the system (2.3)-(2.6) needs to satisfy $x_{it+1} = x_{it} = x_i$ for all i, as $t \to \infty$. From the above Lemma, $x_{it} \neq 0$, for all i and t. Therefore,

$$x_{it+1} = x_{it} = \frac{\gamma_{it}}{\gamma_t} x_{it} \tag{B.7}$$

implies that, as $t \to \infty$

$$\gamma_{it} = \gamma_t = \gamma \tag{B.8}$$

for all $i = 1, ..., n$, which the requirement for a balanced growth path.

C. Growth Rate Along a Balanced Growth Path

A balanced growth path is defined by having the same growth rate for all nodes, i.e. $\gamma_{it} = \gamma_t = \gamma$ so that the relative human capitals denoted by x_i remain constant along the balanced growth path. We are essentially looking for constants x_i and γ such that

$$\sum_i x_i = n \text{ and} \tag{C.1}$$

$$\left(\theta + \max\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\}\right) \frac{\sum_{j=1}^n g_{ij} x_j}{\sum_{j=1}^n g_{ij} x_i} = \gamma \text{ for all } i = 1, ..., n. \tag{C.2}$$

This implies

$$\gamma \left(\frac{\sum_{j=1}^n g_{ij} x_j}{\sum_{j=1}^n g_{ij} x_i}\right)^{-1} = \theta + \max\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\}. \tag{C.3}$$

Summing across all nodes and using the fact that $\sum_i x_i = n$ we get

$$\gamma \sum_{i=1}^n \left(\frac{\sum_{j=1}^n g_{ij} x_i}{\sum_{j=1}^n g_{ij} x_j}\right) = \theta n + \sum_{i=1}^n \max\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\} \tag{C.4}$$

$$\gamma = \left(\theta n + \sum_{i=1}^n \max\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\}\right) \left(\sum_{i=1}^n \left(\frac{\sum_{j=1}^n g_{ij} x_i}{\sum_{j=1}^n g_{ij} x_j}\right)\right)^{-1}. \tag{C.5}$$

Now we note that

$$\frac{\psi n - \theta n}{1+\psi} = \sum_{i=1}^n \frac{\psi x_i - \theta}{1+\psi} = \sum_{i=1}^n \min\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\} + \sum_{i=1}^n \max\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\}, \tag{C.6}$$

where the first term of the sum of the right hand side expression contains only negative terms and the second term contains only positive terms. Then

$$\sum_{i=1}^n \max\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\} = \frac{\psi n - \theta n}{1+\psi} - \sum_{i=1}^n \min\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\}. \tag{C.7}$$

Therefore

$$\begin{aligned}
\gamma &= \left(\theta n + \frac{\psi n - \theta n}{1+\psi} - \sum_{i=1}^n \min\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\}\right) \left(\sum_{i=1}^n \left(\frac{\sum_{j=1}^n g_{ij} x_i}{\sum_{j=1}^n g_{ij} x_j}\right)\right)^{-1} \\
&= \left(n\gamma^* - \sum_{i=1}^n \min\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\}\right) \left(\sum_{i=1}^n \left(\frac{\sum_{j=1}^n g_{ij} x_i}{\sum_{j=1}^n g_{ij} x_j}\right)\right)^{-1} \\
&= \left(n\gamma^* - \sum_{i=1}^n \min\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\}\right) \left(\sum_{i=1}^n \bar{x}_i\right)^{-1}.
\end{aligned} \tag{C.8}$$

Since $\sum_{i=1}^{n} \min\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\}$ contains only negative terms, a lower bound for γ is given by

$$\gamma \geq \gamma^* \left(\frac{1}{n}\sum_{i=1}^{n} \bar{x}_i\right)^{-1}. \tag{C.9}$$

This lower bound corresponds cases where $\psi x_i - \theta > 0$ for all i. But in such cases, all x_i are relatively large (compared with θ/ψ), and because they all sum to a fixed number, n, they cannot be very dispersed. If there were some very large x_i s there should be some very small ones too that would generate negative terms in $\sum_{i=1}^{n} \min\left\{0, \frac{\psi x_i - \theta}{1+\psi}\right\}$. This suggests that more inequality, i.e. more varied long run x_i s, is associated with higher growth rates along the balanced growth path. Generally $\gamma \geq \gamma^*$. This is equivalent to showing that

$$\sum_{i=1}^{n} \bar{x}_i \leq n, \tag{C.10}$$

which follows from the fact that for any i, $\bar{x}_i$ is always between x_i and 1 (this can be confirmed numerically).

D. Proofs for Long Run Dynamics

D.1. *Derivation of general Jacobian*

The Jacobian is derived for steady state equilibria that satisfy $x_i > \frac{\theta}{\psi}$, for all $i = 1, ..., n$. Let $\phi = \frac{\psi}{1+\psi}$, and define $z_{it} = \theta + x_{it}$. We have that

$$x_{it}\gamma_{it} = \phi z_{it}\left(\frac{\sum_{j=1}^{n} g_{ij}x_{jt}}{\sum_{j=1}^{n} g_{ij}}\right) = \phi\left(\theta + x_{it}\right)x_{it}\left(\frac{\sum_{j=1}^{n} g_{ij}x_{jt}}{\sum_{j=1}^{n} g_{ij}x_{it}}\right). \tag{D.1}$$

First

$$\frac{\partial z_{it}}{\partial x_{it}} = \left(\frac{1}{\theta + x_{it}}\right) z_{it} \tag{D.2}$$

and

$$\frac{\partial z_{kt}}{\partial x_{it}} = 0. \tag{D.3}$$

Also,

$$\frac{\partial}{\partial x_{it}}\left(\frac{\sum_{j=1}^{n} g_{ij}x_{jt}}{\sum_{j=1}^{n} g_{ij}}\right) = \beta\frac{r_{ii,t}}{x_{it}}\left(\sum_{l=1}^{n}\frac{r_{il,t}}{x_{lt}}\right)^{-1} \tag{D.4}$$

and

$$\frac{\partial}{\partial x_{it}}\left(\frac{\sum_{j=1}^{n} g_{kj}x_{jt}}{\sum_{j=1}^{n} g_{kj}}\right)=\beta\frac{r_{ki,t}}{x_{kt}}\left(\sum_{l=1}^{n}\frac{r_{kl,t}}{x_{lt}}\right)^{-1}. \tag{D.5}$$

Then

$$\frac{\partial\left(x_{it}\gamma_{it}\right)}{\partial x_{it}}=\left(\frac{1}{\theta+x_{it}}+\beta\frac{r_{ii,t}}{x_{it}}\right)x_{it}\gamma_{it} \tag{D.6}$$

and if $k\neq i$

$$\frac{\partial x_{kt}\gamma_{kt}}{\partial x_{it}}=\beta r_{ki,t}\gamma_{kt}. \tag{D.7}$$

Last,

$$\frac{\partial\gamma_{t}}{\partial x_{it}}=\frac{1}{n}\frac{x_{it}\gamma_{it}}{\theta+x_{it}}+\frac{1}{n}\sum_{k=1}^{n}r_{ki,t}\gamma_{kt}. \tag{D.8}$$

Using the above, the diagonal elements of the Jacobian are given by

$$J_{ii}\left(\mathbf{x}_{t}\right)=\frac{1}{\gamma_{t}}\left[\left(1-\frac{x_{it+1}}{n}\right)\left(\frac{1}{\theta+x_{it}}\right)x_{it}\gamma_{it}+\left(r_{ii,t}\gamma_{it}-\frac{x_{it+1}}{n}\sum_{k=1}^{n}r_{ki,t}\gamma_{kt}\right)\right], \tag{D.9}$$

and the off-diagonal elements are given by

$$J_{il}\left(\mathbf{x}_{t}\right)=\frac{1}{\gamma_{t}}\left[-\frac{x_{it+1}}{n}\left(\frac{1}{\theta+x_{lt}}\right)x_{lt}\gamma_{lt}+\left(r_{il,t}\gamma_{it}-\frac{x_{it+1}}{n}\sum_{k=1}^{n}r_{kl,t}\gamma_{kt}\right)\right]. \tag{D.10}$$

In a steady-state equilibrium, along the balanced growth path with $\gamma_i=\gamma$ and $x_{it}=x_i$ which satisfies $\sum_i x_i=n$ the elements of the Jacobian become

$$J_{ii}\left(\mathbf{x}\right)=\left(\frac{x_i}{\theta+x_i}\right)\left(1-\frac{x_i}{n}\right)+\left(r_{ii}-\frac{x_i}{n}\sum_{k=1}^{n}r_{ki}\right), \tag{D.11}$$

$$J_{il}\left(\mathbf{x}\right)=\left(\frac{x_l}{\theta+x_l}\right)\left(-\frac{x_i}{n}\right)+\left(r_{il}-\frac{x_i}{n}\sum_{k=1}^{n}r_{kl}\right), \tag{D.12}$$

which in matrix form can be written as

$$\mathbf{J}\left(\mathbf{x}\right)=\left(\mathbf{I}_n-\frac{1}{n}diag\left(\mathbf{x}\right)\mathbf{1}_{n\times n}\right)\left(diag\left(\mathbf{q}\right)+\mathbf{R}\left(\mathbf{x}\right)\right). \tag{D.13}$$

D.2. *Sufficient condition for balanced growth path with equality*

We now consider the special case of equality along the balanced growth path, i.e. $x_i = 1$ for all i, and $\gamma = \gamma^*$. The vector of the steady state equilibrium is $\mathbf{e}_n = \begin{pmatrix} 1 & \dots & 1 \end{pmatrix}^T$. For a network defined by the augmented adjacency matrix $\mathbf{G}$, the elements of the matrix $\mathbf{R}$ are now given by

$$r_{il} = \frac{g_{il}}{\sum_j g_{ij}} \tag{D.14}$$

and

$$f_{il} = r_{il} - \bar{r}_l, \tag{D.15}$$

where $\bar{r}_l = \left(\sum_i r_{il}\right)/n$. Network cohesion κ is then defined as

$$\kappa = 1 - \rho, \tag{D.16}$$

where ρ the largest modulus eigenvalue of matrix $\mathbf{F} = \{f_{il}\}$. The Jacobian matrix reduces to

$$\mathbf{J}(\mathbf{e}_n) = \left(\mathbf{I}_n - \frac{1}{n}\mathbf{e}_n\mathbf{e}_n^T\right)\left(\frac{1}{1+\theta} + \mathbf{R}\right). \tag{D.17}$$

Letting

$$\mathbf{C}_n = \mathbf{I}_n - \frac{1}{n}\mathbf{e}_n\mathbf{e}_n^T \tag{D.18}$$

be the centering matrix of order n, and noting that $\mathbf{F} = \mathbf{C}_n\mathbf{R}$, the Jacobian can now be written as

$$\mathbf{J}(\mathbf{e}_n) = \mathbf{F} + \frac{1}{1+\theta}\mathbf{C}_n. \tag{D.19}$$

Next, we derive the condition that ensures stability of the steady state equilibrium with equality. First, we show that the set of eigenvalues of $\mathbf{J}(\mathbf{x}^*)$ are $\left\{\lambda_i + \frac{1}{1+\theta}, 0\right\}$, where λ_i are eigenvalues of matrix $\mathbf{F}$. To see this, first note that $\mathbf{e}_n\mathbf{e}_n^T\mathbf{F} = 0$, and $\mathbf{F}\mathbf{e}_n\mathbf{e}_n^T = 0$, and therefore

$$\mathbf{F}\mathbf{C}_n = \mathbf{F} - \mathbf{F}\left(\frac{1}{n}\mathbf{e}_n\mathbf{e}_n^T\right) = \mathbf{F} - \left(\frac{1}{n}\mathbf{e}_n\mathbf{e}_n^T\right)\mathbf{F} = \mathbf{C}_n\mathbf{F}, \tag{D.20}$$

i.e. the matrices $\mathbf{F}$ and $\mathbf{C}_n$ commute. From Theorem 2.4.9 in Horn and Johnson (1990), the set of eigenvalues of the sum of commuting matrices consists of combinations of sums of the eigenvalues of these matrices. The eigenvalues of $\mathbf{C}_n$ are 0 with multiplicity 1 and 1 with multiplicity $n-1$. Let the eigenvalues of $\mathbf{F}$ be $\lambda_1, ..., \lambda_n$. One of these eigenvalues is 0, with corresponding eigenvector e_n, since $\mathbf{F}e_n = 0$. For both $\mathbf{C}_n$ and $\mathbf{F}$ the eigenvector corresponding to eigenvalue 0 is $\mathbf{e}_n$. This implies that 0 is also an eigenvalue of $\mathbf{J}(\mathbf{e}_n)$ with eigenvector $\mathbf{e}_n$. From Horn and Johnson (1990), ex. 8 in 1.4 (matrix deflation), we have that the remaining eigenvalues of $\mathbf{J}(\mathbf{e}_n) = \mathbf{F} + \frac{1}{1+\theta}\mathbf{C}_n$ are those of the matrix in the lower right block of the matrix:

$$\begin{pmatrix} 0 & * \\ 0 & \mathbf{F}_1 + \frac{1}{1+\theta}\mathbf{I}_{n-1} \end{pmatrix}. \tag{D.21}$$

Then, the remaining eigenvalues of $\mathbf{J}(\mathbf{e}_n)$ are the eigenvalues of $\mathbf{F}_1 + \frac{1}{1+\theta}\mathbf{I}_{n-1}$, i.e. $\lambda_i + \frac{1}{1+\theta}$ where λ_i are eigenvalues of $\mathbf{F}$ other than the zero eigenvalue.

For the balanced growth path to be stable, we require that

$$\left|\lambda_i + \frac{1}{1+\theta}\right| < 1, \tag{D.22}$$

or equivalently

$$\left|1 - (1-\lambda_i) + \frac{1}{1+\theta}\right| < 1, \tag{D.23}$$

for all i. First we note that the eigenvalues of $\mathbf{F}$ are real. This is because (as shown in Cavalcanti *et. al.*, 2015) they coincide with the eigenvalues of the row stochastic matrix $\mathbf{R}$ apart from eigenvalue $\lambda_0 = 1$ and $\mathbf{R}$ is similar to the symmetric matrix $\mathbf{D}^{-1/2}\mathbf{G}\mathbf{D}^{-1/2}$, where $\mathbf{G}$ is the augmented adjacency matrix and $\mathbf{D}$ is a diagonal matrix with typical diagonal element $\sum_j g_{ij}$. From Theorem 3.2 in Cavalcanti *et. al.*, (2015), it also follows that $0 \leq \lambda_i \leq 1$. Therefore

$$1 - (1-\lambda_i) + \frac{1}{1+\theta} \geq 0. \tag{D.24}$$

It follows that, requiring condition (D.23) for all i is equivalent to requiring that for all i

$$1 - (1-\lambda_i) + \frac{1}{1+\theta} < 1, \tag{D.25}$$

or that $(1-\lambda_i) > \frac{1}{1+\theta}$, for all i. But this is equivalent to requiring

$$\kappa > \frac{1}{1+\theta}, \tag{D.26}$$

where κ is the network cohesion, i.e. one minus the maximum eigenvalue of $\mathbf{F}$ in absolute value.

D.3. *Rate of convergence for balanced growth path with equality*

We have shown that the dynamic system that describes the economy is

$$\mathbf{x}_{t+1}= \mathbf{W}(\mathbf{x}_t), \tag{D.27}$$

where W is a (non-homogeneous) and highly non-linear function of $\mathbf{x}_t$. The first order approximation of this around the balanced growth path with long run equality is given by

$$\mathbf{x}_{t+1} = \mathbf{e}_n + \mathbf{J}\left(\mathbf{e}_n\right)\left(\mathbf{x}_t - \mathbf{e}_n\right), \tag{D.28}$$

where $\mathbf{J}\left(\mathbf{e}_n\right)$ is the Jacobian matrix evaluated at $\mathbf{e}_n = \begin{pmatrix} 1 & \dots & 1 \end{pmatrix}'$. It has already been shown that the eigenvalues of $\mathbf{J}\left(\mathbf{e}_n\right)$ are $\left|1-(1-\lambda_i)+\frac{1}{1+\theta}\right| \leq 1-\kappa+\frac{1}{1+\theta}$. As cohesion κ increases, these become smaller and convergence of the system is faster.

Affiliations:

Tiago V. V. Cavalcanti: *Faculty of Economics, University of Cambridge and Sao Paulo School of Economics, FGV*

Chryssi Giannitsarou: *Faculty of Economics, University of Cambridge and CEPR*

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